18.06 Exam 1 March 1, 2000 Closed Book

Your name is: ______________________

Note: Make sure your exam has 5 problems.

Problem		**Points possible**
1	______	20
2	______	20
3	______	20
4	______	20
5	______	20
Total	______	100

1 (20 pts) Suppose the 3×3 matrix A has row $1 +$ row $2 =$ row 3.

(a) Explain why $Ax = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ cannot have a solution.

(b) Which right hand side vector $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ might allow a solution $Ax = b$?

(Give the best answer you can, based on the information provided.)

(c) Why is A not invertible?

2 (20 pts) Let

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 5 & 0 \\ 0 & 3 & 2 \end{bmatrix}.$$

(a) Factor A as $A = LU$, where L is lower triangular and U is upper triangular.

(b) Find a basis for the column space of A.

(c) What is the rank of A?

3 (20 pts) Suppose

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 7 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 4 & 5 \\ 0 & 1 & 2 & 2 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

(a) What is the rank of A?

(b) Find a basis for the nullspace of A.

(c) Find the complete solution to

$$Ax = \begin{bmatrix} 10 \\ 15 \\ 85 \end{bmatrix}.$$

4 (20 pts) Let M be the vector space of all 2×2 matrices and let

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}.$$

(a) Give a basis for M.

(b) Describe a subspace of M which contains A and does not contain B.

(c) True (give a reason) or False (give a counterexample): If a subspace of M contains A and B, it must contain the identity matrix I.

(d) Describe a subspace of M which contains no diagonal matrices except for the zero matrix.

5 (20 pts) If $A^2 = 0$, the zero matrix, explain why A is not invertible.