18.06 Professor Strang/Ingerman Quiz 2 October 25, 2002

**Your name is:** ______________________________

**1 (40 pts.)** (a) Find the projection matrix $P_C$ onto the column space of $A$ (after looking closely at the matrix!)

$$A = \begin{bmatrix} 3 & 3 & 6 \\ 1 & 1 & 2 \end{bmatrix}$$

(b) Find the 3 by 3 projection matrix $P_R$ onto the row space of $A$. What is the closest vector in the row space to the vector $\boldsymbol{b} = (1, 0, 0)$?

(c) Multiply $P_C A$ and then $P_C A P_R$. Your answers should be a little surprising—can you explain?

(d) Find a basis for the subspace of all vectors orthogonal to the row space of $A$.

**2 (30 pts.)** (a) Choose $c$ and the last column of $Q$ so that you have an orthogonal matrix:

$$Q = c\begin{bmatrix} 1 & -1 & -1 & \mathrm{x} \\ -1 & 1 & -1 & \mathrm{x} \\ -1 & -1 & -1 & \mathrm{x} \\ -1 & -1 & 1 & \mathrm{x} \end{bmatrix}$$

(b) Project $\boldsymbol{b} = (1,1,1,1)$ onto the first column of $Q$. Then project $\boldsymbol{b}$ onto the plane spanned by the first two columns.

(c) Suppose the last column of the 4 by 4 matrix (where the x's are) was changed to $(1,1,1,1)$. Call this new matrix $A$. If Gram-Schmidt is applied to the 4 columns of $A$, what would be the 4 outputs $\boldsymbol{q}_1, \boldsymbol{q}_2, \boldsymbol{q}_3, \boldsymbol{q}_4$? (Don't do a lot of calculations... please.)

**3 (30 pts.)** (a) If you multiply all $n!$ permutations together into a single $P$, is the product odd or even? (Answer might depend on $n$.)

(b) If you know that $\det A = 6$, what is the determinant of $B$?

$$\det A = \begin{vmatrix} \text{row } 1 \\ \text{row } 2 \\ \text{row } 3 \end{vmatrix} = 6 \qquad \det B = \begin{vmatrix} \text{row } 3 + \text{row } 2 + \text{row } 1 \\ \text{row } 2 + \text{row } 1 \\ \text{row } 1 \end{vmatrix} = ?$$

(c) Prove $\det A = 0$ for the 5 by 5 *all-ones matrix* (all $a_{ij} = 1$) in ***two ways***:

(1) Using Properties 1–10 of determinants

(2) Using the "big formula" = sum of 120 terms.