18.06 Professor Strang Exam 2 April 8, 1998

Your name is: ______________________________

Grading	1
	2
	3
	4

1 (a) **(15)** Find an orthonormal basis for the subspace S of $\mathbf{R}^4$ spanned by these three vectors:

$$a_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad a_2 = \begin{bmatrix} 3 \\ 1 \\ 1 \\ 3 \end{bmatrix} \qquad a_3 = a_1 + a_2$$

(b) **(15)** Find the closest vector p in that subspace S to the vector

$$b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

2 (a) **(15)** Start with the same subspace S. Find a basis (not necessarily orthonormal) for its orthogonal complement $S^\perp$ (the space of all vectors perpendicular to S).

(b) **(10)** Find the closest vector q in $S^\perp$ to the same vector b.

3 (a) **(10)** Find the determinant of this matrix A_4:

$$A_4 = \begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

(b) **(10)** How many terms are *nonzero* out of the 24 terms in the big formula

$$\det A = \Sigma(\pm) a_{1\alpha} a_{2\beta} a_{3\gamma} a_{4\omega}$$

and what are those nonzero terms?

Suppose the matrices A_n all follow the same pattern as A_4, with 2's on the main diagonal and 1's on the *second* diagonals above and below. Thus

$$A_1 = \begin{bmatrix} 2 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \qquad A_3 = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$

(c) **(10)** Use cofactors along row 1 of A_n to *find the relation between* $\det A_n$, $\det A_{n-1}$ and $\det M_{n-2}$.

That mysterious matrix M_{n-2} is not the same as A_{n-2}. Start with $n = 4$, and use cofactor to find M_{n-2} when this submatrix is 2 by 2. Decribe M_{n-2} for larger n.

4 (a) **(10)** Give the formula for the projection matrix P onto the column space of a matrix A. Where does the formula assume that A has independent columns?

(b) **(5)** The two properties of all these projection matrices are $P^2 = P$ and $P^T = P$. Suppose v^T is the first row of P and v_1 is the first entry in that row. Prove that $v^T v = v_1$.