18.06 Professor Strang Quiz 2 April 2, 2004

Your name is: ______________________________

Please circle your recitation:

Grading

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1 (20 pts.) We are given two vectors a and b in $\mathbb{R}^4$:

$$a = \begin{bmatrix} 2 \\ 5 \\ 2 \\ 4 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

(a) Find the projection p of the vector b onto the line through a. **Check** (!) that the error $e = b - p$ is perpendicular to (**<u>what</u>**?)

(b) The subspace S of all vectors in $\mathbb{R}^4$ that are *perpendicular to this* a is 3-dimensional. Find the projection q of b onto this perpendicular subspace S. The numerical answer (it doesn't need a big computation!) is $q =$ ______.

2 (30 pts.) Suppose q_1, q_2, q_3 are 3 orthonormal vectors in $\mathbb{R}^n$. They go in the columns of an n by 3 matrix Q.

(a) What inequality do you know for n?

Is there any condition on n for $Q^{\mathrm{T}}Q = I$ (3 by 3)?

Is there any condition on n for $QQ^{\mathrm{T}} = I$ (n by n)?

(b) Give a nice matrix formula involving b and Q, for the projection p of a vector b onto the column space of Q.

Complete this sentence: p **is the closest vector** ____________ ...

(c) Suppose the projection of b onto that column space is $p = c_1 q_1 + c_2 q_2 + c_3 q_3$. Find a formula for c_1 that only involves b and q_1. (You could take dot products with q_1.)

3 (20 pts.) Suppose the 4 by 4 matrix M has four equal rows all containing a, b, c, d. We know that $\det(M) = 0$. The problem is to find by any method

$$\det(I+M) = \begin{vmatrix} 1+a & b & c & d \\ a & 1+b & c & d \\ a & b & 1+c & d \\ a & b & c & 1+d \end{vmatrix}.$$

Note If you can't find $\det(I+M)$ in general, partial credit for the determinant when $a = b = c = d = 1$.

4 (30 pts.) We are looking for the parabola $y = C + Dt + Et^2$ that gives the least squares fit to these four measurements:

$$y_1 = 1 \text{ at } t_1 = -2,\ y_2 = 1 \text{ at } t_2 = -1,\ y_3 = 1 \text{ at } t_3 = 1,\ y_4 = 0 \text{ at } t_4 = 2\,.$$

(a) Write down the four equations (not solvable!) for the parabola $C + Dt + Et^2$ to go through those four points. This is the system $Ax = b$ to solve by least squares:

$$A \begin{bmatrix} C \\ D \\ E \end{bmatrix} = b\,.$$

What equations would you solve to find the best C, D, E?

(b) Compute $A^{\mathrm{T}}A$. Compute its determinant. Compute its inverse. NOT ASKING FOR C, D, E.

(c) The first two columns of A are already orthogonal. From column 3, subtract its projection onto the plane of the first two columns. That produces what third orthogonal vector v? Normalize v to find the third orthonormal vector q_3 from Gram-Schmidt.