18.06 Quiz 2 November 10, 1999 Closed Book

Your name is: ______________________________ **Grading** 1
2
3
4
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1 (25 pts.) (a) Find equations (**do not solve**) for the coefficients C, D, E in $b = C + Dt + Et^2$, the parabola which best fits the four points $(t, b) = (0,0), (1,1), (1,3)$ and $(2,2)$.

(b) In solving this problem you are projecting the vector $b =$ __________ onto the subspace spanned by __________. The projection in terms of C, D, E is $p =$ __________.

2 (28 pts.) Let

$$A = \begin{bmatrix} 3 & 4 & 6 \\ 0 & 1 & 0 \\ -1 & -2 & -2 \end{bmatrix}.$$

(a) Find the eigenvalues of the singular matrix A.

(b) Find a basis of $\mathbb{R}^3$ consisting of eigenvectors of A.

(c) By expressing $(1, 1, 1)$ as a combination of eigenvectors or by diagonalizing $A = S\Lambda S^{-1}$, compute

$$A^{99} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

3 (25 pts.) Start with two vectors (the columns of A):

$$a_1 = \begin{bmatrix} \cos\theta \\ 0 \\ \sin\theta \end{bmatrix} \qquad \text{and} \qquad a_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

(a) With $q_1 = a_1$ find an orthonormal basis q_1, q_2 for the space spanned by a_1 and a_2 (column space of A).

(b) What shape is the matrix R in $A = QR$ and why is $R = Q^T A$? Here Q has columns q_1 and q_2. Compute the matrix R.

(c) Find the projection matrices P_A and P_Q onto the column spaces of A and Q.

4 (22 pts.) (a) If Q is an orthogonal matrix (square with orthonormal columns), show that $\det Q = 1$ or -1.

(b) How many of the 24 terms in $\det A$ are nonzero, and what is $\det A$?

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$