18.06 Exam 2 April 12, 2000 Closed Book

Your name is: ____________________

Note: Make sure your exam has 4 problems.

Problem		**Points possible**
1	______	30
2	______	16
3	______	30
4	______	24
Total	______	100

Note: Some problems are worth more than others.

1 (30 pts) Let

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ -2 & 4 \end{bmatrix}.$$

(a) Find orthonormal vectors q_1, q_2, and q_3 so that q_1 and q_2 form a basis for the column space of A.

(b) Which of the four fundamental subspaces contains q_3?

(c) Find the projection matrix P projecting onto the left nullspace (not the column space!) of A.

(d) Find the least squares solution to $Ax = (1, 2, 7)$.

Note: You must show your work to receive credit for this problem.

2 (16 pts) Compute the determinant of

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}.$$

Note: You must show your work to receive credit for this problem.

3 (30 pts) Consider this sequence: $G_0 = 0$, $G_1 = 1$ and $G_{k+2} = (G_k + G_{k+1})/2$. (So G_{k+2} is the average of the previous two numbers G_k and G_{k+1}.) This problem will find the limit of G_k as $k \to \infty$.

(a) Find a matrix A which satisfies

$$\begin{bmatrix} G_{k+2} \\ G_{k+1} \end{bmatrix} = A \begin{bmatrix} G_{k+1} \\ G_k \end{bmatrix}.$$

(b) Find the eigenvalues and eigenvectors of A.

(c) Write $A^k = S\Lambda^k S^{-1}$, where Λ is a diagonal matrix. You do **not** need to multiply this out to get a single matrix.

(d) Find the limit as $k \to \infty$ of the numbers G_k.

Note: You must show your work to receive credit for this problem.

4 (24 pts) Suppose A is a 3×3 matrix with eigenvalues 0, 1, and 2. Find the following:

(a) the rank of A.

(b) the determinant of $A^{\mathrm{T}}A$.

(c) the determinant of $A + I$.

(d) the eigenvalues of $(A + I)^{-1}$.

Note: You must show your work to receive credit for this problem.