

18.06 Professor Strang/Ingerman Quiz 3 December 6, 2002

Your name is: _____

- 1 (36 pts.) (a) What are the eigenvalues of the 5 by 5 matrix $A = \mathbf{ones}(5)$ with all entries $a_{ij} = 1$? Please look at A , not at $\det(A - \lambda I)$.

- (b) Solve this differential equation to find $\mathbf{u}(t)$:

$$\frac{d\mathbf{u}}{dt} = A\mathbf{u} \quad \text{starting from } \mathbf{u}(0) = (0, 1, 1, 1, 2).$$

First split $\mathbf{u}(0)$ into two eigenvectors of A .

- (c) Using part (a), what are the *eigenvalues* and *trace* and *determinant* of the matrix $B =$ same as A except zeros on the diagonal.

- 2 (20 pts.)** (a) If A is similar to B show that e^A is similar to e^B . First define “similar” and e^A !!
- (b) If A has 3 eigenvalues $\lambda = 0, 2, 4$, find the eigenvalues of e^A .
- Using part (a) explain this connection with determinants:

$$\text{determinant of } e^A = e^{\text{trace of } A}$$

3 (22 pts.) Suppose the SVD $A = U\Sigma V^T$ is

$$A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} 9 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

- (a) For which angles θ and α (0 to $\frac{\pi}{2}$) is A a positive definite symmetric matrix? No computing needed.
- (b) What are the eigenvalues and eigenvectors of $A^T A$? No computing!

- 4 (22 pts.) Multinational companies in the US, Asia, and Europe have assets of \$ 12 trillion. At the start, \$ 6 trillion are in the US, \$ 6 trillion in Europe. Each year half the US money stays home, $\frac{1}{4}$ each goes to Asia and Europe. For Asia and Europe, half stays home and half is sent to the US.

$$\begin{bmatrix} \text{US} \\ \text{Asia} \\ \text{Europe} \end{bmatrix}_{\text{year } k+1} = \begin{bmatrix} .5 & .5 & .5 \\ .25 & .5 & 0 \\ .25 & 0 & .5 \end{bmatrix} \begin{bmatrix} \text{US} \\ \text{Asia} \\ \text{Europe} \end{bmatrix}_{\text{year } k}$$

- (a) The eigenvalues and eigenvectors of this *singular* matrix A are

- (b) The limiting distribution of the \$ 12 trillion as the world ends is

$$\begin{aligned} \text{US} &= \\ \text{Asia} &= \\ \text{Europe} &= \end{aligned}$$