18.06 Professor Strang Quiz 3 May 9, 1997

1.(a) (20) Compute $u_{20} = A^{20}u_0$ starting from

$$A = \begin{bmatrix} .7 & .5 \\ .3 & .5 \end{bmatrix} \quad \text{and} \quad u_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

(b) (10) For the same A, which real numbers c have the property that $(A - cI)^n\, u_0$ approaches zero as $n \to \infty$?

(c) (10) Find the eigenvalues and eigenvectors of $A^{-1} + A^{20}$ without computing A^{-1} (for the same A as above).

2.(a) (10) If you transpose $S^{-1}AS = \Lambda$ you learn that

The eigenvalues of A^T are ________________________

The eigenvectors of A^T are ________________________

(b) (10) Complete the last row so that B is a singular matrix, with real eigenvalues and orthogonal eigenvectors:

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ - & - & - \end{bmatrix}.$$

(c) (10) C is a 3×3 matrix. I add row 1 to row 2 to get K:

$$K = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} C\,.$$

This probably changes the eigenvalues. What should I do to the *columns* of K (answer in words) to get back to the original eigenvalues of C?

3.(a) (10) For which numbers c is this matrix positive definite?

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & c & -1 \\ 0 & -1 & 1 \end{bmatrix}.$$

What function $F(x_1, x_2, x_3)$ has A as its matrix of second derivatives?

(b) (12) What is the 2×2 matrix P that projects every vector onto the "θ-line" containing all multiples of $\mathbf{a} = (\cos\theta, \sin\theta)$? What are the *eigenvalues* of P?

(c) (8) That projection is a linear transformation. Suppose we choose the basis vectors $v_1 = (\cos\theta, \sin\theta)$ along the θ-line and $v_2 = (-\sin\theta, \cos\theta)$ perpendicular to the θ-line. What matrix represents P with respect to this basis?