1 8.06	Professor Strang	Exam 3	May 6, 1998
Your 1	name is:		_ Grading 1 2 3 4

 ${\bf 1}$ $% ({\bf 1})$ Find the eigenvalues and eigenvectors of these matrices:

(a) (10) Projection
$$P = \frac{aa^T}{a^T a}$$
 with $a = \begin{bmatrix} 3\\ 4 \end{bmatrix}$

(b) (10) Rotation
$$Q = \begin{bmatrix} .6 & -.8 \\ .8 & .6 \end{bmatrix}$$

(c) (8) Reflection
$$R = 2P - I$$

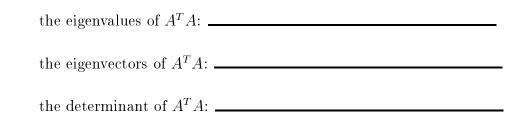
2 (a) (10) Find the eigenvalues λ_1 , λ_2 , λ_3 (NOT the eigenvectors x_1 , x_2 , x_3) of this Markov matrix:

$$A = \begin{bmatrix} .6 & .6 & 0 \\ .2 & .2 & .2 \\ .2 & .2 & .8 \end{bmatrix}$$

- (b) (10) Suppose u_0 is the sum $x_1 + x_2 + x_3$ of the three eigenvectors that you didn't compute. What is $A^n u_0$?
- (c) (4) As $n \to \infty$ what is the limit of $A^n u_0$?

- 3 (a) (2 each) Suppose M is any invertible matrix. Circle all the properties of a matrix A that remain the same for $M^{-1}AM$:
 - same rank same nullspace same determinant real eigenvalues orthonormal eigenvectors symmetric positive definiteness
 - (b) (2 each) This is a similar question but now Q is an orthonormal matrix. Circle the properties of A that remain the same for $Q^{-1}AQ$:

same column space A^k approaches zero as k increases orthonormal eigenvectors symmetric positive definiteness projection matrix 4 (a) (3 each) Suppose the 5 by 4 matrix A has independent columns. What is the most information you can give about



(b) (9) Find the singular value decomposition (SVD) for this matrix:

$$A = \left[\begin{array}{rrr} 0 & 0 & 0 \\ 0 & 4 & 0 \end{array} \right] \,.$$

(c) (8) When the input basis is v_1, \ldots, v_n and the output basis is w_1, \ldots, w_n and the matrix of the linear transformation T using these bases is the identity matrix, what is $T(v_1 + v_2)$?