18.06 Professor Strang Exam 3 May 6, 1998

Your name is: ______________________________

Grading 1
2
3
4

1 Find the eigenvalues and eigenvectors of these matrices:

(a) **(10)** Projection $P = \dfrac{aa^T}{a^Ta}$ with $a = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

(b) **(10)** Rotation $Q = \begin{bmatrix} .6 & -.8 \\ .8 & .6 \end{bmatrix}$

(c) **(8)** Reflection $R = 2P - I$

2 (a) **(10)** Find the eigenvalues λ_1, λ_2, λ_3 (NOT the eigenvectors x_1, x_2, x_3) of this Markov matrix:

$$A = \begin{bmatrix} .6 & .6 & 0 \\ .2 & .2 & .2 \\ .2 & .2 & .8 \end{bmatrix}$$

(b) **(10)** Suppose u_0 is the sum $x_1 + x_2 + x_3$ of the three eigenvectors that you didn't compute. What is $A^n u_0$?

(c) **(4)** As $n \to \infty$ what is the limit of $A^n u_0$?

3 (a) **(2 each)** Suppose M is any invertible matrix. Circle all the properties of a matrix A that remain the same for $M^{-1}AM$:

same rank

same nullspace

same determinant

real eigenvalues

orthonormal eigenvectors

symmetric positive definiteness

(b) **(2 each)** This is a similar question but now Q is an orthonormal matrix. Circle the properties of A that remain the same for $Q^{-1}AQ$:

same column space

A^k approaches zero as k increases

orthonormal eigenvectors

symmetric positive definiteness

projection matrix

4 (a) **(3 each)** Suppose the 5 by 4 matrix A has independent columns. What is the most information you can give about

the eigenvalues of A^TA: ______________________

the eigenvectors of A^TA: ______________________

the determinant of A^TA: ______________________

(b) **(9)** Find the singular value decomposition (SVD) for this matrix:

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 4 & 0 \end{bmatrix}.$$

(c) **(8)** When the input basis is $v_1, \ldots, v_n$ and the output basis is $w_1, \ldots, w_n$ and the matrix of the linear transformation T using these bases is the identity matrix, what is $T(v_1 + v_2)$?