18.06 Professor Strang Quiz 3 May 5, 2004

Grading

Your name is: ____________________

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Please circle your recitation:

1 (40 pts.) This question deals with the following symmetric matrix A:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

One eigenvalue is $\lambda = 1$ with the line of eigenvectors $x = (c, c, 0)$.

(a) That line is the nullspace of what matrix constructed from A?

(b) Find (in any way) the other two eigenvalues of A and two corresponding eigenvectors.

(c) The diagonalization $A = S\Lambda S^{-1}$ has a specially nice form because $A = A^{\mathrm{T}}$. Write all entries in the three matrices in the nice symmetric diagonalization of A.

(d) Give a reason why e^A *is or is not* a symmetric positive definite matrix.

2 (30 pts.) (a) Find the *eigenvalues* and *eigenvectors* (depending on c) of

$$A = \begin{bmatrix} .3 & c \\ .7 & 1-c \end{bmatrix}.$$

For which value of c is the matrix A *not diagonalizable* (so $A = S\Lambda S^{-1}$ is impossible)?

(b) What is the *largest range of values of* c (real number) so that A^n approaches a limiting matrix A^∞ as $n \to \infty$?

(c) What is that limit of A^n (still depending on c)? You could work from $A = S\Lambda S^{-1}$ to find A^n.

3 (30 pts.) Suppose A (3 by 4) has the Singular Value Decomposition (with real orthogonal matrices U and V)

$$A = U\Sigma V^{\mathrm{T}} = \begin{bmatrix} & & \\ u_1 & u_2 & u_3 \\ & & \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} & & & \\ v_1 & v_2 & v_3 & v_4 \\ & & & \end{bmatrix}^{\mathrm{T}}.$$

(a) Find the *rank* of A and a *basis* for its column space $C(A)$.

(b) What are the eigenvalues and eigenvectors of $A^{\mathrm{T}}A$? (You could first multiply A^{T} times A.)

(c) What is Av_1? You could start with $V^{\mathrm{T}}v_1$ and then multiply by Σ and U to get $U\Sigma V^{\mathrm{T}}v_1$.