18.06 Professor Strang Quiz 3 December 6, 2000

Your name is: ______________________________

1 (30 pts.) (a) Find the diagonalization $A = S\Lambda S^{-1}$ of

$$A = \begin{bmatrix} 0.5 & 3 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

(b) What is the limit of A^k as $k \to \infty$?

(c) Suppose B^k approaches I (the 2 by 2 identity) as $k \to \infty$. How do you know that $B = I$? Explain using eigenvalues and Jordan forms like

$$J = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

2 (40 pts.) (a) Suppose the diagonalization $A = S\Lambda S^{-1}$ is exactly the same as the singular value decomposition $A = U\Sigma V^{\mathrm{T}}$ (so $S = U = V$ and $\Lambda = \Sigma$). What information does this give about A? Can it be singular?

(b) What are the eigenvalues of a 3 by 3 Markov projection matrix that has trace 2? Create one matrix that has these properties.

(c) Here is a matrix with orthogonal columns. Find its SVD $A = U\Sigma V^{\mathrm{T}}$.

$$A = \begin{bmatrix} 3 & 0 \\ 4 & 0 \\ 0 & 7 \end{bmatrix}$$

(d) Suppose A is similar to a 3 by 3 matrix B that has eigenvalues 1, 1, 2. What can you say about

1. the eigenvalues of A
2. diagonalizability of A
3. symmetry of A
4. positive definiteness of A

In each of (2) (3) and (4) decide if A can't have or might have or must have this property.

3 (30 pts.) (a) Find the eigenvalues of the matrix (and fill in the blanks)

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}.$$

These eigenvalues are all __________ because this matrix A is __________

(b) If the eigenvectors are x_1, x_2, x_3 (not required to compute them) describe the general solution to the differential equation $\frac{du}{dt} = Au$.

(c) At what time T is the solution $u(T)$ guaranteed to equal its initial value $u(0)$?