18.06 Fall 2003 Quiz 1 October 1, 2003

Your name is:

Grading:

Question	Points	Maximum
Name + rec		5
1		25
2		15
3		5
4		35
5		15
Extra credit:		(10)
Total:		100

Remarks:
Do all your work on these pages.
No calculators or notes.
Putting your name and recitation name correctly is worth 5 points.
The exam is worth a total of 100 points.

1. a) (15 points) Find an LU-decomposition of the 3×3 matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix}.$$

b) (10 points) Solve $Ax = b$ where

$$b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

2. (15 points) Let A be an unknown 3×3 matrix, and let

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Consider the augmented matrix $B = [A \mid P\,]$. After performing row operations on B we get the following matrix

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 2 & -3 & -4 \\ 0 & 1 & 0 & -1 & 2 & 2 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array}\right].$$

What is A^{-1}?

3. (5 points) Find a matrix A such that

$$A \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} x - y \\ x + y + 2w \end{bmatrix}.$$

4. All of the questions below refer to the following matrix A

$$A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}.$$

a) (5 points) What is the rank of A?

b) (5 points) Do all pairs of columns span the column space, $C(A)$, of A? If yes, explain. If no, give a pair of columns that do not span the column space.

c) (10 points) Find a basis for the nullspace $N(A)$ of A.

d) (5 points) Does there exist a vector $b \in R^2$ such that $Ax = b$ has no solution?

e) (10 points) Find all solutions of

$$Ax = \begin{bmatrix} 0 \\ 2 \end{bmatrix}.$$

Express your solution in the form

$$x = x_{particular} + c_1 x_1 + c_2 x_2$$

where x_1, x_2 are special solutions.

5. a) (6 points) How many 3×3 permutation matrices are there (including I)?

b) (9 points) Is there a 3×3 permutation matrix P, besides $P = I$, such that $P^3 = I$? If yes, give one such P. If no, explain why.

6. **Extra Credit (10 points)** The matrix in question 1 is a Pascal matrix. Find an LU-decomposition of the 6×6 Pascal matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 3 & 6 & 10 & 15 & 21 \\ 1 & 4 & 10 & 20 & 35 & 56 \\ 1 & 5 & 15 & 35 & 70 & 126 \\ 1 & 6 & 21 & 56 & 126 & 252 \end{bmatrix}$$

Note: you don't need to write the entire matrix again, just explain how to get the LU-decomposition.