MODIFICATION OF THE ATMOSPHERIC SEMI-DIURNAL LUNAR TIDE BY OCEANIC AND SOLID EARTH TIDES

by

ANTHONY HOLLINGSWORTH

B.Sc., National University of Ireland
(University College Cork)
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Signature of Author..
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Certified by..
Thesis Supervisor

Accepted by..
Chairman, Departmental Committee
on Graduate Students

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ABSTRACT

The effects of the earth tide and the ocean tide on the semi-diurnal lunar tide in the atmosphere have been ignored in nearly all studies of this air tide. Elementary arguments show that these boundary effects are not trivial.

Using linear theory we calculated the combined effect of the lunar potential, the earth tide and the ocean tide on a realistic model atmosphere. Love's theory was used to represent the earth tide. Numerical calculation by Bogdanov and Magarik (1967) and by Pekeris and Accad (1969) were used to represent the ocean tide. Our results indicate that the ocean tide has a significant and probably a dominant effect on the lunar air tide. The ocean tide of Pekeris and Accad yielded results that agreed better with the observations.

We calculated the effect of a tide in a "small" or "point" ocean on the atmosphere and found that its effects were global. Hence differences between the observations and our calculations of the lunar air tide cannot easily be reduced by simple manipulation of the forcing function, the ocean tide, in the immediate vicinity of the places where discrepancies occur.

The forcing functions of the problem were represented as Fourier-Hough series, involving 232 Hough functions. The expansions of these Hough functions in terms of Associated Legendre Polynomials are presented in the

Appendix.

Computations of the semi-diurnal lunar tidal winds at 98 km are presented and compared with observations.

Thesis Supervisor: Norman A. Phillips
Title: Professor of Meteorology

DEDICATION

Dom' Tuismuitheóirí

agus

Dom' Bhean Chéile

Bríd

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Chapter I

1.1 Introduction

The study of the tides of the atmosphere is at least as old as Laplace and has been pursued with varying intensity since his time. There are several works available that give comprehensive reviews of the whole field among which one might mention Wilkes' book (1949) and the valuable review article by Siebert (1961), and most recently the book by Chapman and Lindzen (1970). We will be concerned here with purely gravitational tides.

The present study was prompted by the results of a study by Geller (1969, 1970). Geller was mainly concerned with the effect of the seasonal variations of the vertical temperature profile of the atmosphere on the phase of the lunar tide. He considered only the direct forcing by the lunar tidal potential and ignored vertical motions of the earth-atmosphere interface. The amplitudes he found at the equator were typically of the order of $30\mu b$ ($1\mu b = 10^{-6}$ bar = 1 dyne/cm^2). Moreover, because of the fact that he considered only one mode of oscillation, these amplitudes were independent of longitude. A perusal of Fig. 1, taken from Haurwitz and Cowley (1970), shows that the amplitudes near the equator are somewhat greater than $30\mu b$ and that the amplitudes do vary with longitude. Our purpose in this investigation is to study the

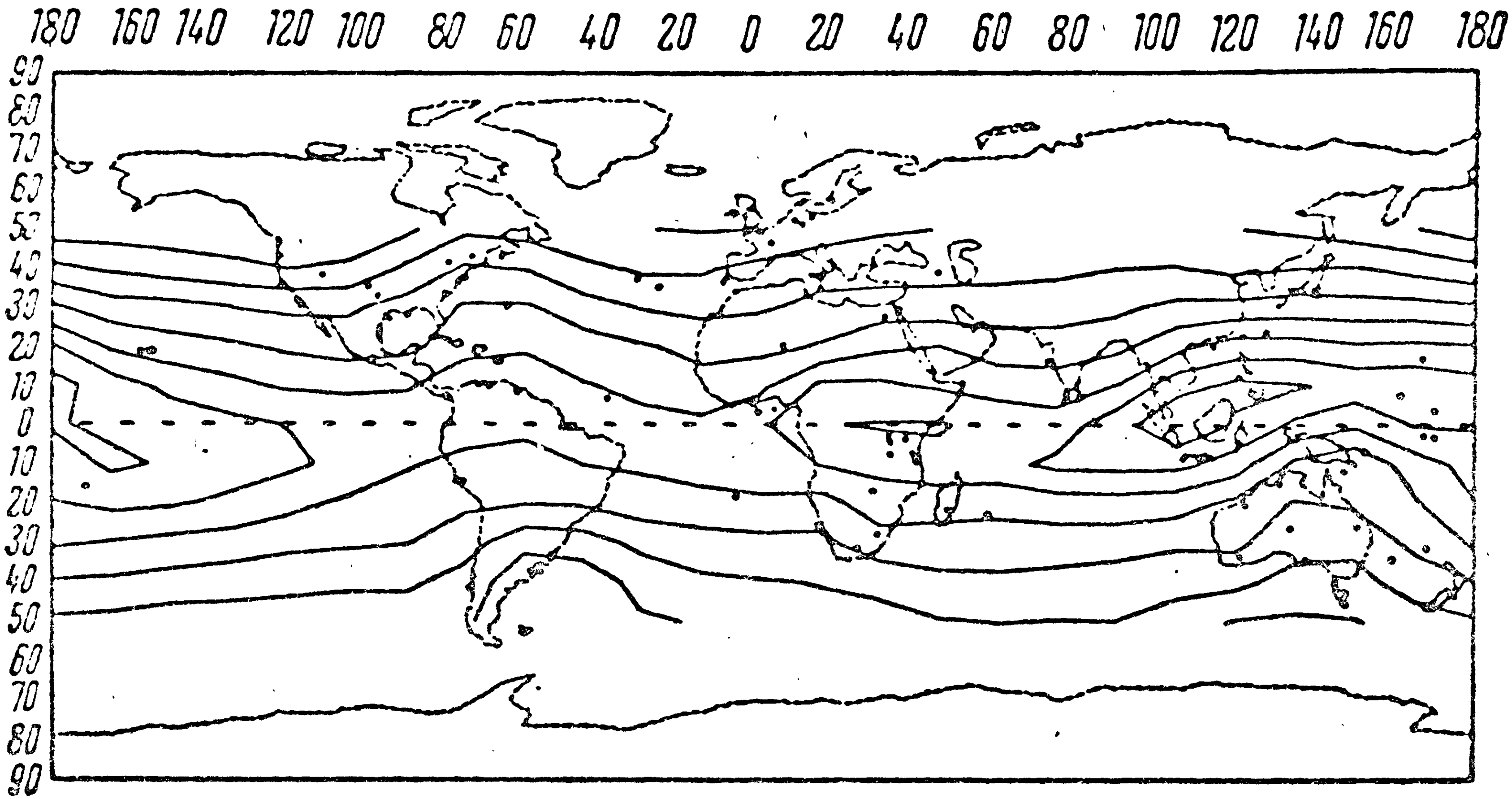


Fig. 1 Distribution of the amplitude of the semi-diurnal lunar air tide (in μb , contour interval 10 μb). The amplitude maximum over Indonesia is 80 μb . From Haurwitz and Cowley (1970).

reasons for the discrepancy. Assuming, as we do, that Geller's model is a reasonable representation of the atmosphere the discrepancy suggests that there is some other mechanism exciting the atmosphere at the lunar semi-diurnal frequency. Since there is negligible heating at this frequency, Geller, and other writers, have suggested that the lunar semi-diurnal ocean tides and earth tides provide an energy input to the atmosphere. If only the earth tide is taken into account, it can be shown that the amplitudes Geller calculated should be multiplied by about 0.7, thus increasing the discrepancy and making a study of the effect of the ocean tide even more interesting.

1.2 Interpretation of the Data

In the following discussion potentials are defined so that $\underline{F} = -\nabla\phi$ where $\underline{F}$ is the force due to the potential. The net potential at a point due to the moon is given by

$$\bar{\Phi} = -\frac{3}{4} g \frac{M}{E}\left(\frac{a}{D}\right)^3 \frac{(a+z)^2}{a}\Big[3\cos^2\left(\Delta-\tfrac{1}{3}\right)\left(\cos^2\vartheta-\tfrac{1}{3}\right) + \sin 2\Delta . \sin 2\vartheta . \cos(\alpha+\lambda) + \sin^2\Delta . \sin^2\vartheta . \cos 2(\alpha+\lambda)$$

where

g = acceleration of gravity

M = mass of the moon

E = mass of the earth

a = mean radius of the earth

D = distance between the centers of the earth and the moon

z = height above the surface of the earth

Δ, α = north polar distance and the hour angle of the moon

ϑ, λ = colatitude and longitude

D, Δ and α have complicated time variations; Doodson (1922) has performed the most extensive harmonic development of the lunar tidal potential. According to his computations the largest component Φ of the lunar potential is given by

$$\Phi = -0.90812 \left(\frac{a+z}{a}\right)^2 G \sin^2\vartheta \cos 2(\tau+\lambda)$$

$$= -0.90812 \left(\frac{a+z}{a}\right)^2 G \frac{\sqrt{15}}{4} P_2^2(\mu) \cos 2(\tau+\lambda) \quad (1.2.1)$$

where

$$G = 26{,}206 \text{ cm}^2\text{sec}^{-2}$$

$$\mu = \cos\vartheta$$

and $P_2^2(\mu)$ is the fully normalized Associated Legendre Polynomial. τ in this expression increases by 2π in 1 mean lunar day and hence this potential is periodic of period half a mean lunar day.

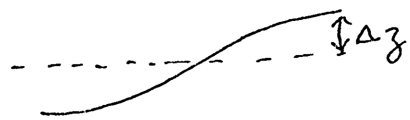
Throughout this thesis we shall mean by semi-diurnal lunar frequency that frequency whose period is half a mean lunar day. For the oceanic tide, this is called the "M_2"

tide. [Geller (1969) ignored the numerical factor .90812 in this last expression.]

It is the response of the atmosphere to periodic forces at this frequency that we shall be discussing and this response is usually spoken of as the semi-diurnal lunar tide. Observations of the tide (often spoken of as determinations) have been made at 104 stations over the globe. The tide as seen in surface pressure is a phenomenon of very small amplitude and it is masked by much larger non-periodic events. Hence, it is no easy task to separate the tide from the noise. The method used to make most of the determinations now available is due to Chapman and Miller (1940). Chapman (Chapman and Lindzen (1970)) points out that this method is "truly harmonic." By this he means that it follows the practice of Doodson (1922) who perfected the method of harmonic analysis for sea-tide analysis.

Thus, the determinations of the semi-diurnal lunar tide in surface pressure are determinations of the regular variation of pressure with period half a lunar day. These determinations are not affected by events arising from such factors as variations in the moon's declination or distance from the earth.

It is worthwhile to consider the physical interpretation of the determinations of the lunar tide. Consider the pressure as observed by a barometer fixed to the ground and moving vertically with it. Let $p(t, z)$ be the pressure at the barometer, $\left.\frac{\mathcal{D}p}{\mathcal{D}t}\right|_{bar}$ be the time derivative following the barometer and w_{bar} be the vertical velocity at the boundary.



Then

$$\left.\frac{\mathcal{D}p}{\mathcal{D}t}\right|_{bar} = \lim_{\Delta t \to 0} \frac{p(t+\Delta t, z+\Delta z) - p(t,z)}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{p(t+\Delta t, z+\Delta z) - p(t, z+\Delta z)}{\Delta t} + \frac{p(t, z+\Delta z) - p(t,z)}{\Delta t}$$

$$= \left(\frac{\partial p}{\partial t}\right)_z + \lim_{\Delta t \to 0} \left(\frac{\partial p}{\partial z}\right)_t \frac{\Delta z}{\Delta t}$$

$$= \left(\frac{\partial p}{\partial t}\right)_z + \left(\frac{\partial p}{\partial z}\right)_t w_{bar}$$

In our model we will assume that the pressure field is composed of a mean field $p_{oo}(z)$ with a small perturbation p' superimposed. Hence, to first order in the perturbations

$$\left.\frac{\mathcal{D}p}{\mathcal{D}t}\right|_{bar} = \frac{\partial p'}{\partial t} + w_{bar} \frac{\partial p_{oo}}{\partial z}$$

Again

$$\frac{dp}{dt} = \omega = \frac{\partial p}{\partial t} + \frac{u}{a\cos\theta}\frac{\partial p}{\partial \lambda} + \frac{v}{a}\frac{\partial p}{\partial \theta} + w\frac{\partial p}{\partial z}$$

and to first order

$$\omega = \frac{\partial p'}{\partial t} + w \frac{\partial p_{oo}}{\partial z}.$$

But at the ground, i.e. at the lower boundary of the atmosphere

$$w = w_{bar}$$

this being the kinematic boundary condition.
Thus, to first order in the perturbations

$$\left.\frac{\partial p}{\partial t}\right|_{bar} = \omega \Big|_{\text{lower boundary}}$$

and we see that the time derivative of the pressure as observed by a barometer fixed to the earth-atmosphere interface is, to first order, the same as the substantial derivative.

Before concluding this section we consider briefly the effect of tidal variations in gravity on barometric readings. Let ρ_{Hg} denote the density of mercury, H_b the height of the column of mercury in a barometer, $\bar{g}$ the standard value of gravity at a given place, g the actual value of gravity at the place, p_{obs} the pressure reported at a station, p_{true} the true pressure at the station, δp the variation of p due to the tide. p_{obs} is computed according to the formula

$$p_{obs} = \rho_{Hg}\, \bar{g}\, H_b$$

while p_{true} is given by

$$p_{true} = \rho_{Hg}\, g\, H_b$$

Let

$$g' = g - \bar{g}$$

Then

$$p_{true} = \frac{g}{\bar{g}}\, p_{obs} = \left(1 + \frac{g'}{\bar{g}}\right) p_{obs}$$

and, to first order,

$$\delta p_{true} = \delta p_{obs} + \frac{g'}{\bar{g}}\, p_{oo}$$

Now

$$\delta p_{obs} \sim 50\ \mu b$$
$$p_{oo} \sim 10^6\ \mu b$$
$$g' \sim \Phi/a \sim 4.10^{-5}\ cm\,sec^{-2}.$$

Thus

$$\frac{g'}{\bar{g}}\, p_{oo} \sim 0.04\ \mu b \ll \delta p_{obs}$$

Hence, tidal variations in the force of gravity have negligible effect on determinations of the tide in surface pressure.

1.3 An Earlier Study

Some writers (Siebert 1961, Chapman, Pramanik and Topping 1931) have referred very briefly to the fact that motion of the earth and ocean could be a source of tidal energy in the atmosphere. More recently Sawada (1965) made an approximate calculation of the effect. He calculated the effect of the semi-diurnal lunar tidal potential on an atmosphere with a realistic temperature structure above an ocean-covered globe. The ocean was of uniform depth and the ocean and atmosphere were coupled by the kinematic and dynamic boundary conditions at their interface. Problems such as this are most usefully discussed in terms of Hough functions. An ocean covering the entire globe has free oscillations of any given zonal wave number at the semi-diurnal lunar frequency provided the depth of the ocean takes on one of a discrete set of values. The latitudinal structure of such a free mode is given in terms of the appropriate Hough function. Only the latitudinally symmetric modes are relevant in Sawada's problem. There are latitudinally symmetric free oscillations at this frequency and of zonal wave number two provided the depth of the ocean takes one of the values 7077m, 1849m, --- etc. The Hough function appropriate to these oscillations would be denoted by H_2^2, H_4^2 --- etc.

The semi-diurnal lunar potential can be written as a sum of these (symmetric) Hough functions multiplied by certain factors. Sawada considered separately the effect of the first two terms of this sum on his model ocean-atmosphere system. For the first term, that involving H_2^2, he found that if the depth of the ocean were near 7077m the response to this part of the forcing became infinite, as one would expect. For depths away from this value the presence of the ocean had little effect on the phase of the atmospheric pressure oscillation but its presence did amplify the oscillation. The effect of the second term of the forcing function, that involving H_4^2, was, in the presence of the ocean, somewhat different. For depths away from the critical depth of 1849m the ocean had little effect on the amplitude of the oscillation but it's presence changed the phase of the oscillation markedly.

These results were interesting but "the effect of limited oceans closely resembling those actually occurring on the earth remains to be discussed." Sawada did not examine the resonance behavior of the model atmosphere he used. This behavior depends on the value of the separation constant β (cf.§ 4.1), $\frac{4\Omega^2 a^2}{g\ell}$ in Sawada's notation, and so we cannot say whether the atmosphere he used was resonant near the value of ℓ for which his model ocean was resonant.

Chapter II

2.1 Mathematical Theory

We will regard the lunar tidal motions as small perturbations about a mean state (Dickinson 1969). The mean state we choose is one in which the mean velocities are zero and we assume the perturbations to be small enough that linear theory is valid. The temperature in the mean state is assumed to vary only with height, and electro-magnetic and viscous effects are ignored. We will also assume that the perturbations are hydrostatic, that the atmosphere is of uniform composition, that "standard gravity" is a constant, and that the ellipticity of the earth is kinematically negligible (Lamb, 1932 § 214).

This model atmosphere differs from the mean state of the real atmosphere in a number of ways, particularly in the neglect of meridional temperature and velocity gradients. The effect of meridional temperature gradients has been examined by Chiu (1953) and Siebert (1957), while Sawada (1966) studied the effects of zonal winds with vertical shear. Their approximate results would indicate that the effects were generally small. (The full linear problem under these conditions is rather intractable as the equations are non-separable.)

The linearised equations of motion are (cf. Chapman and Lindzen, 1970)

$$\frac{\partial u}{\partial t} - 2\Omega \sin\theta\, v + \frac{1}{a\cos\theta}\frac{\partial \phi'}{\partial \lambda} = 0 \qquad \text{a}$$

$$\frac{\partial v}{\partial t} + 2\Omega \sin\theta\, u + \frac{1}{a}\frac{\partial \phi'}{\partial \theta} = 0 \qquad \text{b}$$

$$\frac{1}{a\cos\theta}\left[\frac{\partial u}{\partial \lambda} + \frac{\partial}{\partial \theta} v\cos\theta\right] + \frac{\partial W}{\partial Z} - W = 0 \qquad \text{c} \qquad (2.1.1)$$

$$\frac{\partial T}{\partial t} + W\left[\frac{d\bar{T}}{dZ} + \varkappa \bar{T}\right] = 0 \qquad \text{d}$$

$$\frac{\partial \phi'}{\partial Z} - RT = 0 \qquad \text{e}$$

where $u = a\cos\theta\dot{\lambda}$, $v = a\dot{\theta}$, λ = longitude, θ = latitude, a = mean radius of the earth, p = pressure, p_{oo} = 1013.25mb, mean sea level pressure, $Z = -\ln p/p_{oo}$, $W = \frac{dZ}{dt}$, $\phi = gz$ = = standard geopotential, Φ = lunar potential, $\phi' = \phi + \Phi$, $\bar{T} = \bar{T}(z)$, the temperature of the undisturbed atmosphere, T = perturbation temperature, $\varkappa = R/C_p$, R = gas constant for air, C_p = specific heat of air at constant pressure. We have ignored all heating effects. Since $\frac{\partial \Phi}{\partial z} \ll g$ we have also made the approximation

$$\frac{\partial \Phi}{\partial Z} = 0$$

as indeed has already been assumed in the standard development leading to (1.2.1). The lower boundary condition is

$$W\Big|_{Z=0} = -\frac{1}{R\bar{T}}\left[\frac{\partial}{\partial t}(\phi' - \Phi) - g\,w\right]\Big|_{Z=0} \qquad (2.1.1f)$$

where w is the vertical velocity of the lower boundary.

If we assume solutions of the form

$$\{f(\theta,z,\lambda,t)\} = R_l\, e^{i\sigma t} \sum_{l=-\infty}^{\infty} \{f(\theta,z)\}_l\, e^{il\lambda} \tag{2.1.2}$$

and if we write $\mu = \sin\theta$ the equations become

$$i\sigma U_l - 2\Omega\mu V_l + \frac{il}{a(1-\mu^2)^{\frac{1}{2}}}\phi'_l = 0 \qquad \text{a}$$

$$i\sigma V_l + 2\Omega\mu U_l + \frac{(1-\mu^2)^{\frac{1}{2}}}{a}\frac{\partial\phi'_l}{\partial\mu} = 0 \qquad \text{b}$$

$$\frac{il}{a(1-\mu^2)^{\frac{1}{2}}}U_l + \frac{1}{a}\frac{\partial}{\partial\mu}V_l(1-\mu^2)^{\frac{1}{2}} + \frac{\partial W_l}{\partial z} - W_l = 0 \qquad \text{c}$$

(2.1.3)

$$i\sigma T_l + W_l\left(\frac{d\bar{T}}{dz} + \kappa\bar{T}\right) = 0 \qquad \text{d}$$

$$\frac{\partial\phi'_l}{\partial z} = RT_l \qquad \text{e}$$

$$W_l\big|_{z=0} = -\frac{1}{R\bar{T}}\left\{i\sigma(\phi'_l - \Phi_l) - g\,w_l\right\}\Big|_{z=0} \qquad \text{f}$$

Solving the horizontal momentum equations for U_l , V_l and substituting these expressions in the continuity equation we get

$$U_l = \frac{1}{\sigma^2 - 4\Omega^2\mu^2}\left\{\frac{-l\sigma}{a(1-\mu^2)^{\frac{1}{2}}}\phi'_l + \frac{2\Omega\mu(1-\mu^2)^{\frac{1}{2}}}{a}\frac{\partial\phi'_l}{\partial\mu}\right\} \qquad \text{a}$$

(2.1.4)

$$V_\ell = \frac{1}{\sigma^2 - 4\Omega^2\mu^2}\left\{ \frac{-2\Omega\mu i \ell}{a(1-\mu^2)^{\frac{1}{2}}}\phi'_\ell + \frac{i\sigma(1-\mu^2)^{\frac{1}{2}}}{a}\frac{\partial \phi'_\ell}{\partial \mu}\right\} \qquad \text{b}$$

and

$$i\sigma F(\phi'_\ell) + (2\Omega a)^2\left\{\frac{\partial W_\ell}{\partial Z} - W_\ell\right\} = 0 \qquad (2.1.5)$$

where F is the so-called Hough operator:

$$F = \frac{d}{d\mu}\frac{1-\mu^2}{f^2-\mu^2}\frac{d}{d\mu} - \frac{1}{f^2-\mu^2}\left\{\frac{\ell}{f}\frac{f^2+\mu^2}{f^2-\mu^2} + \frac{\ell^2}{1-\mu^2}\right\} \qquad (2.1.6)$$

where

$$f = \frac{\sigma}{2\Omega}$$

is the non-dimensional frequency.

Eliminating T_ℓ from the hydrostatic and thermodynamic relations we get a second relation between ϕ'_ℓ and W_ℓ viz.

$$R^{-1} i\sigma \frac{\partial \phi'_\ell}{\partial Z} + W_\ell\left(\frac{d\bar{T}}{dZ} + \kappa\bar{T}\right) = 0 \qquad (2.1.7)$$

As discussed in §2.3 the operator F has eigenfunctions, known as Hough functions, denoted by $H_n^\ell(\mu)$. Thus

$$F(H_n^\ell(\mu)) = -\beta_n^\ell H_n^\ell(\mu) \qquad (2.1.8)$$

where β_n^ℓ is the eigenvalue associated with $H_n^\ell(\mu)$. The

$H_n^\ell(\mu)$ are orthogonal and are presumed complete. Let us now develop W_ℓ, ϕ'_ℓ, Φ_ℓ and w_ℓ as series of Hough functions:

$$\phi'_\ell(\mu, z) = \sum_{n=|\ell|}^{\infty} \phi'^\ell_n(z)\, H_n^\ell(\mu) \qquad \text{a}$$

$$W_\ell(\mu, z) = \sum_{n=|\ell|}^{\infty} W_n^\ell(z)\, H_n^\ell(\mu) \qquad \text{b}$$

(2.1.9)

$$\Phi_\ell(\mu, z) = \sum_{n=|\ell|}^{\infty} \Phi_n^\ell(z)\, H_n^\ell(\mu) \qquad \text{c}$$

$$w_\ell(\mu, z) = \sum_{n=|\ell|}^{\infty} w_n^\ell(z)\, H_n^\ell(\mu) \qquad \text{d}$$

and let us substitute these series in our equations. From (2.1.5) we have

$$\phi'^\ell_n = \frac{1}{i\sigma} \frac{(2\Omega a)^2}{\beta_n^\ell}\left(\frac{\partial W_n^\ell}{\partial z} - W_n^\ell\right), \qquad (2.1.10)$$

and this, together with (2.1.7) implies

$$\frac{\partial^2}{\partial z^2} W_n^\ell - \frac{\partial}{\partial z} W_n^\ell + \frac{\beta_n^\ell R}{(2\Omega a)^2}\left(\frac{d\bar{T}}{dz} + \varkappa \bar{T}\right) W_n^\ell = 0. \qquad (2.1.11)$$

The lower boundary condition takes the form

$$W_n^\ell \Big|_{z=0} = -\frac{1}{R\bar{T}}\left[i\sigma \left\{ \frac{(2\Omega a)^2}{i\sigma \beta_n^\ell}\left(\frac{\partial W_n^\ell}{\partial z} - W_n^\ell\right) - \Phi_n^\ell \right\} - g\, w_n^\ell \right]\Big|_{z=0} \qquad (2.1.12)$$

If we make the substitution

$$W_n^\ell(z) = e^{z/2}\, Y_n^\ell(z) \qquad (2.1.13)$$

then (2.1.11) reduces to

$$\frac{d^2}{dZ^2} Y_n^\ell + \left[-\frac{1}{4} + \beta_n^\ell S \right] Y_n^\ell = 0 . \qquad (2.1.14)$$

Here $S = S(z)$ represents the static stability of the atmosphere:

$$S = \frac{g}{(2\Omega a)^2} \left[\frac{d\bar{H}}{dz} + \kappa \bar{H} \right] = \frac{\bar{H}^2 N^2}{4\Omega^2 a^2}$$

where N is the Brunt-Väisälä frequency and $\bar{H} = R\bar{T}/g$.
The lower boundary condition becomes

$$\frac{d}{dZ} Y_n^\ell + \left(\frac{\bar{H}}{h_n^\ell} - \frac{1}{2} \right) Y_n^\ell = \left. \frac{i\sigma \Phi_n^\ell + g w_n^\ell}{g h_n^\ell} \right|_{Z=0} , \qquad (2.1.15)$$

where h_n^ℓ is the "equivalent depth" (Taylor, 1936):

$$h_n^\ell = \frac{(2\Omega a)^2}{g \beta_n^\ell} .$$

Within this theoretical framework the steps involved in calculating the effect of a forcing potential or of a known vertical oscillation of the bottom boundary are as follows: One first must calculate the appropriate Hough functions and their eigenvalues in order to make the expansions (2.1.9). Knowing S one then solves the vertical equation (2.1.14) for each mode subject to the lower boundary condition (2.1.15) and an upper boundary condition to be discussed shortly (§ 2.2).

2.2 The Stability Profile

The profile for $S(z)$, the stability function, (see Fig. 2) used herein is the "mean annual" profile prepared by Geller (1969). Geller constructed this profile as follows. The profile between 30 km and 100 km was calculated from the temperature profile of the 1965 CIRA mean atmosphere. The lower part of the profile was calculated by averaging temperatures collected in a five-year study by the Planetary Circulations Project under Professor V. Starr at M.I.T. These temperatures were first time-averaged for the five years and then averaged with respect to area over the Northern Hemisphere. Above 100 km the CIRA atmosphere was approximated by a straight line

$$S = aZ + b$$

$$a = 0.029825$$

$$b = -0.41222$$

The upper boundary condition is that at high levels in the atmosphere the solutions correspond to waves whose energy is propagating away from the center of the earth, the so-called "radiation condition". Following Geller (1970) we

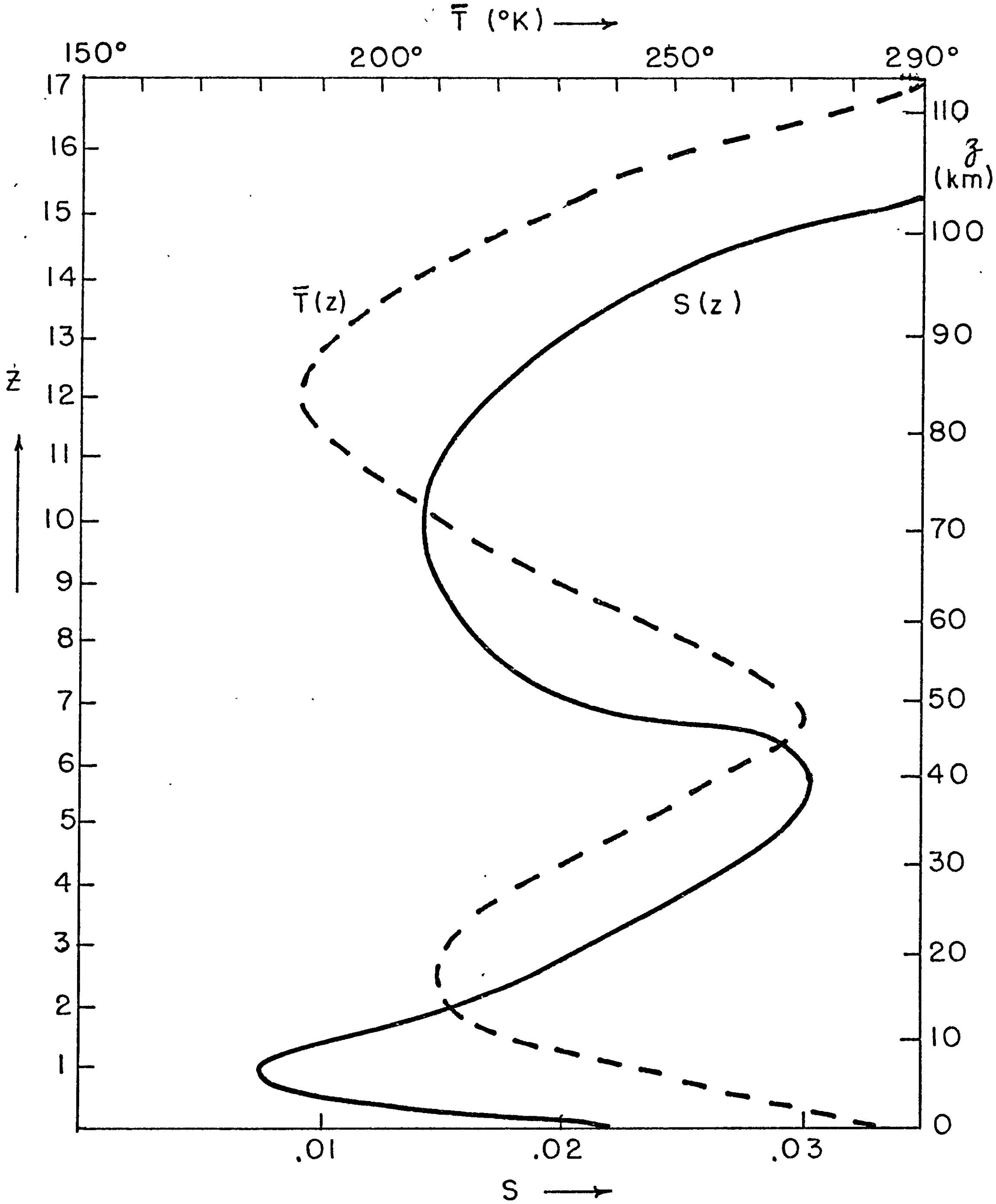


FIG. 2. Mean Temperature, T̄ (- - -) and Mean Stability, S (——) Profiles

note that with

$$S = aZ + b$$

and

$$x = -\frac{1}{(\beta a)^{2/3}}\left[\beta(aZ+b) - \frac{1}{4}\right]$$

The solutions of

$$\frac{d^2y}{dZ^2} + \left[\beta(aZ+b) - \frac{1}{4}\right]Y = 0$$

are the Airy functions $Ai(x)$, and $Bi(x)$. For $X < 0$ and $|x| >> 0$ these behave as

$$Ai(x) \sim \frac{1}{\pi^{\frac{1}{2}}|x|^{\frac{1}{4}}} \sin(\zeta + \tfrac{\pi}{4})$$

$$Bi(x) \sim \frac{1}{\pi^{\frac{1}{2}}|x|^{1/4}} \cos(\zeta + \tfrac{\pi}{4})$$

where $\zeta = \frac{2}{3}|x|^{3/2}$.

At large negative X (large positive Z), then,

$$Bi(x) + i\, Ai(x) \sim \frac{1}{\pi^{\frac{1}{2}}|x|^{\frac{1}{4}}} \exp i\left\{\tfrac{2}{3}|x|^{3/2} + \tfrac{\pi}{4}\right\}. \qquad (2.2.2)$$

The time dependence of these solutions is $e^{i\sigma t}$ and hence in this region the solutions are proportional to

$$\exp i\left\{\tfrac{2}{3}|x|^{3/2} + \sigma t\right\}.$$

The phase ψ is

$$\psi = \tfrac{2}{3}|x|^{3/2} + \sigma t .$$

To keep ψ constant as t increases $|x|$ must decrease. Hence the phase velocity is in the direction of increasing

x or, equivalently, decreasing z . Thus the phase velocity of the waves (2.2.2) is downwards and by standard theorems the group velocity of the wave is directed upwards.

Alternatively, we may use Wilkes' (1949) conclusion that the radiation condition is equivalent to

$$\mathrm{Im}\left(y^* \frac{dy}{dz}\right) > 0 \qquad (2.2.3)$$

In terms of our x this is

$$\mathrm{Im}\left(y^* \frac{dy}{dx}\right) < 0$$

For $y = K[\mathrm{Bi}(x) + i\,\mathrm{Ai}(x)]$, K real, we find

$$\mathrm{Im}\left(y^* \frac{dy}{dx}\right) = -K^2 W(\mathrm{Ai}(x), \mathrm{Bi}(x)) = -\frac{K^2}{\pi}$$

so that the combination (2.2.2) is indeed the proper solution in the thermosphere. Our model thermosphere begins at

$$z = z_{top} = 14 \cdot 8$$

and at this level we require the solution in the lower region and its first derivative to be continuous to a solution of the form

$$K[\mathrm{Bi}(x) + i\,\mathrm{Ai}(x)]$$

and its first derivative where K is a constant.

Following Geller still the two conditions on the function and its derivative may be rewritten, so as to eliminate K , in the form

$$\frac{d}{dz}\mathrm{Re}\, y_n^\ell = a_n^\ell\, \mathrm{Re}\, y_n^\ell + b_n^\ell\, \mathrm{Im}\, y_n^\ell \Big|_{z = z_{top}} \qquad \text{a} \quad (2.2.4)$$

$$\frac{d}{dz} \mathrm{Im}\, Y_n^\ell = - b_n^\ell \,\mathrm{Re}\, Y_n^\ell + a_n^\ell \,\mathrm{Im}\, Y_n^\ell \Big|_{z=z_{top}} \qquad \text{b}$$

where

$$a_n^\ell = -(a\beta_n^\ell)^{\frac{1}{3}} \left[\frac{Bi(x_n^\ell)Bi'(x_n^\ell) + Ai(x_n^\ell)Ai'(x_n^\ell)}{Ai^2(x_n^\ell) + Bi^2(x_n^\ell)} \right]$$

$$b_n^\ell = -(a\beta_n^\ell)^{\frac{1}{3}} \left[\frac{Ai(x_n^\ell)Bi'(x_n^\ell) - Bi(x_n^\ell)Ai'(x_n^\ell)}{Ai^2(x_n^\ell) + Bi^2(x_n^\ell)} \right]$$

with

$$x_n^\ell = -\frac{1}{(a\beta_n^\ell)^{\frac{2}{3}}} \left[\beta_n^\ell (a z_{top} + b) - \frac{1}{4} \right].$$

Given a knowledge of S, a_n^ℓ, b_n^ℓ the equation (2.1.14) can be solved, subject to the boundary conditions (2.1.15), (2.2.4), using exactly the same method used by Geller. The equations were written out in their real and imaginary parts and cast into finite difference form, giving a set of simultaneous linear equations. These were solved using an IBM-supplied routine GELB which is suitable for solving band-structured matrices. The interval Δz was 0.1, corresponding roughly to $\Delta z = 0.8$ km. The reader is referred to Geller (1969) for further details.

2.3 Hough Functions

Hough functions are defined as solutions of the eigenvalue problem

$$\frac{d}{d\mu}\left(\frac{1-\mu^2}{f^2-\mu^2}\frac{d\Psi}{d\mu}\right)-\frac{1}{f^2-\mu^2}\left(\frac{\ell^2}{1-\mu^2}+\frac{\ell}{f}\frac{f^2+\mu^2}{f^2-\mu^2}\right)\Psi=-\beta\Psi \qquad (2.3.1)$$

for $\mu \in [-1, 1]$ where ℓ is an integer and the boundary conditions are that Ψ be finite at ± 1.

This equation was first treated by Laplace (1775, 1776) and it is known as Laplace's tidal equation. Margules (1893) and Hough (1897, 1898) presented solutions of the equation and Hough (1898) presented asymptotic solutions valid for β small and positive. Longuet-Higgins (1968) has extended the analysis to all values of β while Flattery (1967), using Hough's methods, presents some computations of the Hough functions for different values of f and ℓ.

We are interested in solutions for the semi-diurnal lunar frequency i.e. for

$$f = 0.963499$$

and for ℓ a positive or negative integer. Longuet-Higgins showed that for fixed ℓ and $|f| < 1$ the problem has two families of eigenfunctions; for one family the eigenvalues have an accumulation point at $+\infty$ and for the other family the accumulation point is at $-\infty$. For the second family he derived relatively simple asymptotic expressions in the limit $|f| \to 1$. In this limit the eigenfunctions assumed a boundary layer character, being of appreciable size only near one of the poles and decaying exponentially towards the equator beyond a certain turning point co-latitude, ϑ_t. This turning co-latitude in the Northern Hemisphere is given by (cf. his equation 11.6)

$$-\frac{1}{4} + \frac{k}{x'} - \frac{4m^2-1}{4x'^2} = 0 \qquad (2.3.1)$$

where $x' = (-\beta)^{\frac{1}{2}} \vartheta^2$

ϑ = co-latitude in the Northern Hemisphere

$k = \frac{1}{2}\{ |\ell-1| + (2\nu+1)\}$

$2m = |\ell-1|$

ℓ = zonal wave number

ν = any positive integer

β = eigenvalue in question

The asymptotic relationship between the frequency and the eigenvalue is then

$$f = 1 - \frac{|\ell-1| + (2\nu+1)}{(-\beta)^{\frac{1}{2}}} + O\left(\frac{1}{\beta}\right) \qquad (2.3.2)$$

Taking the value for f appropriate to the lunar semi-diurnal tide it is easy to show that no matter which mode (i.e. value of ν) is chosen

$$\vartheta_t \leq 17.1^{\circ}$$

Beyond this latitude the eigensolutions decay as

$$e^{-\frac{1}{2}(-\beta)^{\frac{1}{2}}\vartheta^2}$$

Since $1-\varsigma$ is small it is clear from (2.3.2) that $(-\beta)^{\frac{1}{2}}$ is large (≥ 80). By latitude 60N these functions have decayed to insignificance. Exactly similar arguments apply to the Southern Hemisphere.

Since the calculations of the ocean tide that we used did not extend much beyond 60N or 60S it was decided on the grounds of the arguments just presented, to ignore the modes having negative eigenvalues. These modes are insignificant in this problem because the non-dimensional frequency is so close to unity. For lower frequencies, however, such as the diurnal tide, they become very important (cf. Lindzen 1967).

Sets of Hough functions were calculated for twenty nine wave numbers ℓ

$$-14 \leq \ell \leq 14$$

For each zonal wave number eight Hough functions were calculated, the four gravest symmetric and the four gravest anti-symmetric modes. The only exception was the case of $\ell = 0$. In this case there is a certain degeneracy in that the gravest symmetric mode is a constant and has eigenvalue $\beta = 0$. This mode was therefore excluded and the next four lowest symmetric modes were computed. The numerical method used was that of Flattery (1967). It is a slight modification of Hough's original method, in which the eigenvalues are found as the zeros of certain continued fractions. The Hough functions are expressed as sums of Associated Legendre

Polynomials and a knowledge of the eigenvalue enables one to compute the coefficients in these sums. The eigenvalues and coefficients are presented in the Appendix.

Chapter III

3.1 Motivation

As mentioned in the introduction this study was prompted by Geller's (1970) work where he found that the maximum surface response to the lunar tidal forcing to be about 30μb while the observed tide has a maximum amplitude of 90μb. It seemed clear that some additional forcing must be acting. There is negligible heating at the lunar semi-diurnal frequency so the next matter to be investigated is the tidal oscillation of the lower boundary of the atmosphere. If we look at the lower boundary condition (2.1.15) and put $w_n^l = i\sigma \zeta_n^l$ where ζ_n^l is the amplitude of the vertical excursion then we see that if $\zeta_n^l \sim \frac{\Phi_n^l}{g} \sim$ $\sim$20 cm the effect of the vertical oscillation of the lower surface is just as important as the effect of the forcing potential. The oscillation of the lower boundary of the atmosphere has two components, one due to the earth tide over land and the other due to the combined effect of the earth and ocean tides over the ocean.

3.2 Equilibrium Tide

A concept much used in discussions of tides is the notion of an equilibrium tide.

Suppose a thin ocean completely covered the globe, and suppose a gravitating body like the moon or sun remained in the same position relative to the earth and exerted a potential on the earth. Then the surface of the water would adopt a position where it coincided with an equipotential of the combined potential fields of the earth and the heavenly body. Elementary theory then shows that if ζ is the deviation of the free surface from its position in the absence of any perturbation then

$$\zeta = -\Phi/g \tag{3.2.3}$$

where Φ is the potential of the disturbing body.

Suppose the lunar ocean tide were an equilibrium tide in the sense that the surface deviation is given by (3.2.3). From our lower boundary condition (2.1.15) we then have

$$w_n^\ell = i\sigma\zeta_n^\ell = -i\sigma\Phi_n^\ell/g$$

and thus

$$i\sigma\Phi_n^\ell + g w_n^\ell = 0$$

The forcing term in our problem now vanishes and the only solution for $Y_n^\ell(z)$ is the trivial one

$$Y_n^\ell(z) \equiv 0$$

At $z = 0$ we have

$$Y_n^\ell(z) = \frac{\omega_n^\ell(0)}{p_{oo}}$$

and so if the deviation of surface height were given by (3.2.3) then a barometer moving up and down with the ocean surface would observe no tidal pressure change. The same argument would apply to the earth's surface in the absence of an ocean if the earth tide were an equilibrium tide. We therefore conclude that (within the framework of the standard linear analysis) the observed atmospheric lunar tidal oscillation depends on the extent to which the air-ocean and air-land interfaces deviate from an equilibrium tide.

3.3 Earth Tides

Until recently relatively little was known about the world wide distribution of the earth tide. Data have been accumulated rapidly during the last twenty years or so and now a fair deal is known about the phenomenon.

The free modes of oscillation of the earth have frequencies of the order of about an hour at most (Press, 1962). Hence, one would expect that static theory would be a good approximation in the study of the phenomenon. Love (1911) developed a theory based on this approximation. According to this theory, if Φ is the disturbing potential then the potential Φ_1 at the surface of the earth due to the deformation caused by Φ is given by $\Phi_1 = k\Phi$ where k is a constant; and the elevation ζ of the surface of the earth caused by these two potentials acting together is given by

$$\zeta = -\frac{h\Phi}{g}$$

where h is another constant.

Various observable effects of the earth tide, such as the variation of the local vertical with respect to the earth's axis, can be described within this theory by simple combination of k, h and two other numbers; these numbers are known collectively as Love's numbers. Melchior (1966) gives an up to date discussion of the field.

It appears from observations that Love's theory gives a good approximation to the phenomenon. The observed phase lags of the earth tides are never more than 10 minutes; their theoretical phase lag should be zero. The distribution of the amplitude is less well behaved but nonetheless is such that Love's theory still provides a very useful framework in which to organize the results. The Love numbers can also be calculated from observations of the Chandler wobble and of changes in the rate of the earth's rotation. Considering these various sources of information Melchior gives the following values for k and h as the most satisfactory to date

$$k = 0.290$$
$$h = 0.584$$

Let us now suppose that there are no oceans on the earth and that the earth tide responds according to Love's theory. Then we see that our lower boundary condition assumes the form

$$\frac{d Y_n^\ell}{dz} + \left(\frac{\bar{H}}{h_n^\ell} - \frac{1}{2}\right) Y_n^\ell = \left. \frac{i\sigma(1+k)\Phi_n^\ell - i\sigma g h \Phi_n^\ell / g}{g h_n^\ell} \right|_{z=0}$$

$$= \frac{i\sigma(1+k-h)\,\Phi_n^\ell}{g h_n^\ell}\Bigg|_{z=0}$$

Thus we see that our forcing term is reduced to about 70% of what it was when the earth was assumed rigid. We carried out this computation, using four Hough functions to represent the forcing potential (see Appendix for the expansion of P_2^2 as a sum of Hough functions); (Geller used only one term of this expansion for his forcing; and also ignored the numerical coefficient .90812 in equation (1.2.1) for the tidal potential.) The amplitude of the response was of course independent of longitude. Fig. 3 shows the latitudinal variation of the amplitude. (The minor "wiggles" at the equator may be due to the truncated representation of P_2^2 by only 4 Hough functions, and the details at very high latitudes may be changed slightly if the functions with negative β had been included.) The amplitude near the equator is $\sim 28\mu b$. Had we considered the earth rigid this amplitude would be $\sim 40\mu b$. These amplitudes are the amplitudes that a barometer fixed to the moving surface of the earth would see. This reduction of the amplitude when one takes the earth tides into account makes it even more interesting to take the ocean tides into consideration.

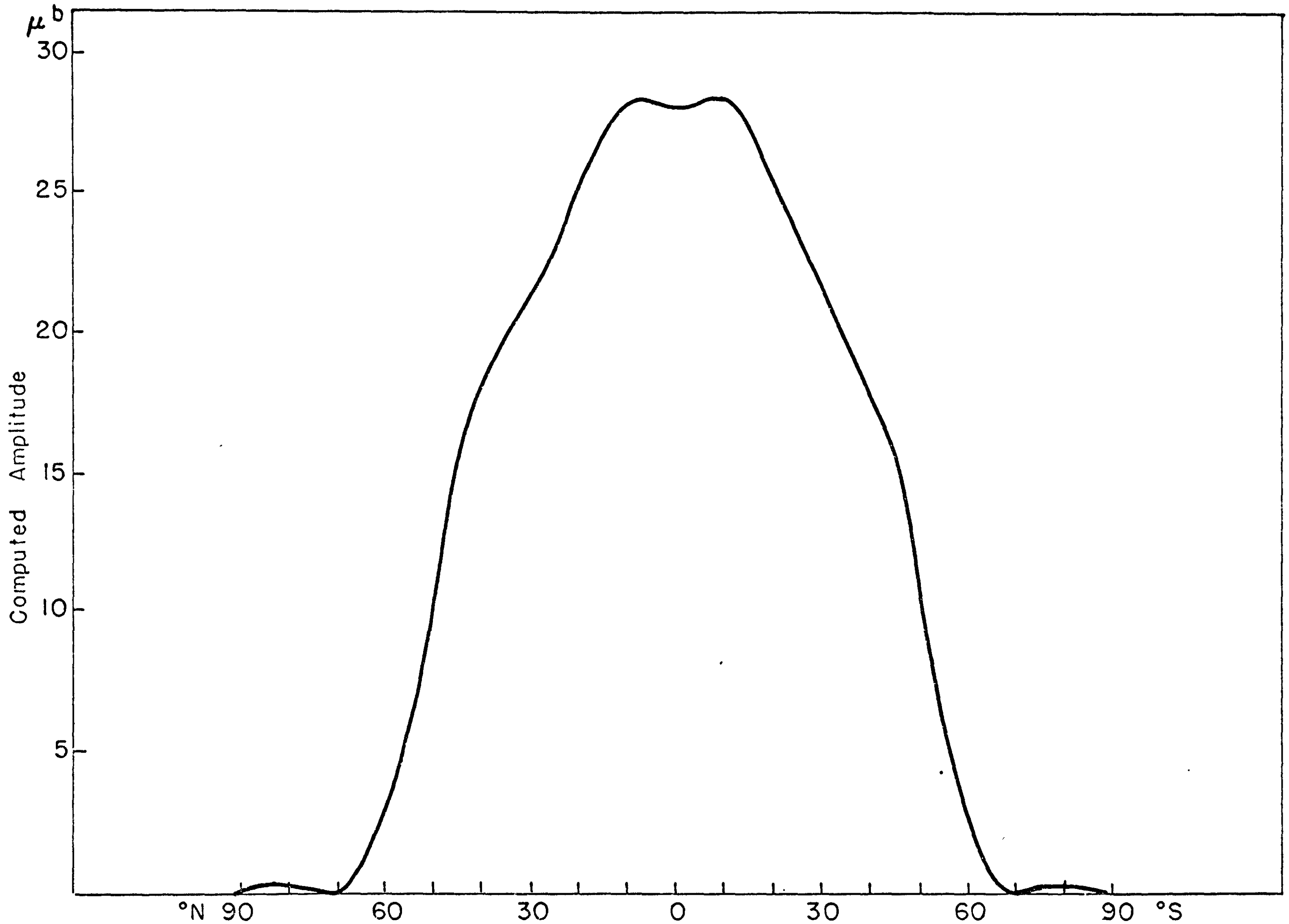


Fig. 3. Amplitude of response as a function of latitude when the atmosphere is excited by the lunar potential and the earth tide.

3.4 Ocean Tides

The phenomena of ocean tides and earth tides are closely inter-related (Kuo et al., 1970, Hendershott and Munk 1970) and ultimately they must be treated as two aspects of the same problem. This task has not been undertaken as yet. All published calculations of tides in the open ocean have regarded the ocean bottom as fixed relative to the center of the earth.

Even with the simplification of ignoring the effect of the earth tides the problem of calculating the tides in the open ocean is arduous. The first attempts to deduce the character of the tides in the open oceans were based on deductions from coastal tide-gauge data. Until quite recently, with the development of sensitive pressure sensors which can be placed on the ocean floor, no observations had been made of the tides in the deep ocean. Tide guage data is not a very reliable guide to the behavior of the tide in adjacent regions of the ocean because a multiplicity of local effects complicate the water motions near coasts. Despite this difficulty some authors have prepared co-tidal charts for various ocean basins, based on coastal data and the theory of flow in various idealised ocean basins. The classic paper is that of Dietrich (1944) who presented co-tidal charts for two diurnal tides as well as for the lunar and solar semi-diurnal tides. However, no attempt was made to deduce the amplitudes of these tides on a global basis.

The accumulation of observations of the tides in the deep ocean will be slow and expensive. Meanwhile research on the problem is advancing rapidly on a third front, namely solving the tidal equations for realistic geometries using electronic computers. A good survey of the work done in this field is presented by Hendershott and Munk (1970). At the time our work was undertaken, two separate calculations of the semi-diurnal lunar tide in the world ocean were available to us, one due to Pekeris and Accad (1969) (denoted by P & A), Fig. 4, and the other due to Bogdanov and Magarik (1967) (denoted by B & M), Fig. 5.

Boganov and Magarik solved Laplace's tidal equations in a model of the world-ocean basin with the boundary condition that the amplitude at coasts and islands take on specified values; these values being taken from tide-guage data. Pekeris and Accad solved the tidal equations in a more detailed representation of the world-ocean basin than B & M, using the boundary condition of zero normal velocity at coasts.

Neither of these solutions are altogether satisfactory. The treatment of dissipative processes is very simplified in P & A and there is no dissipation in the model of B & M.

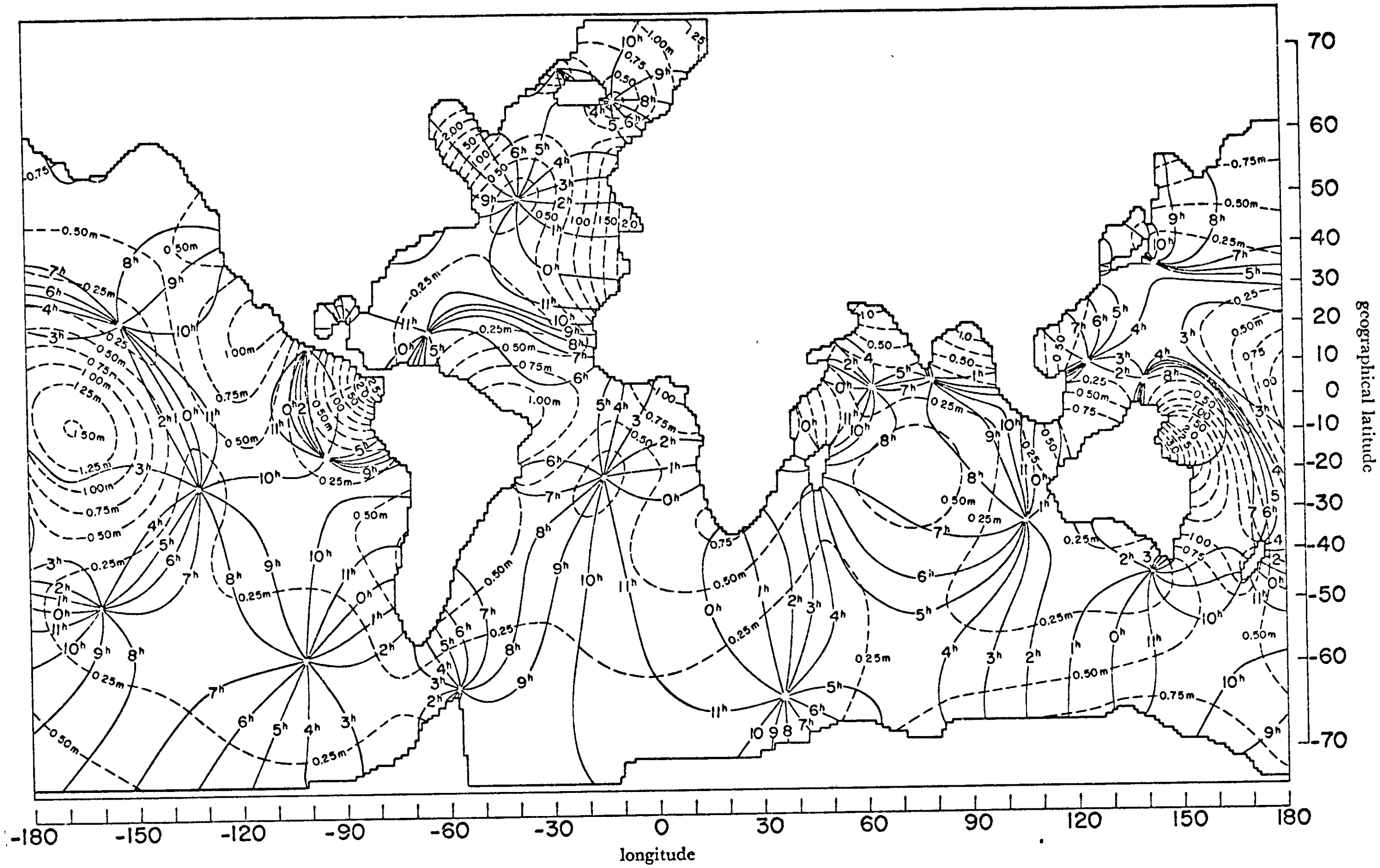


Fig. 4. Computation of M_2 ocean tide by Pekeris and Accad (1969). Isophase lines denoted by (——). Isoamplitude lines denoted by (- - -). Amplitudes given in meters, phases in mean lunar hours. Convention for the phase: $\ell_2 \cos(\sigma t - \epsilon)$

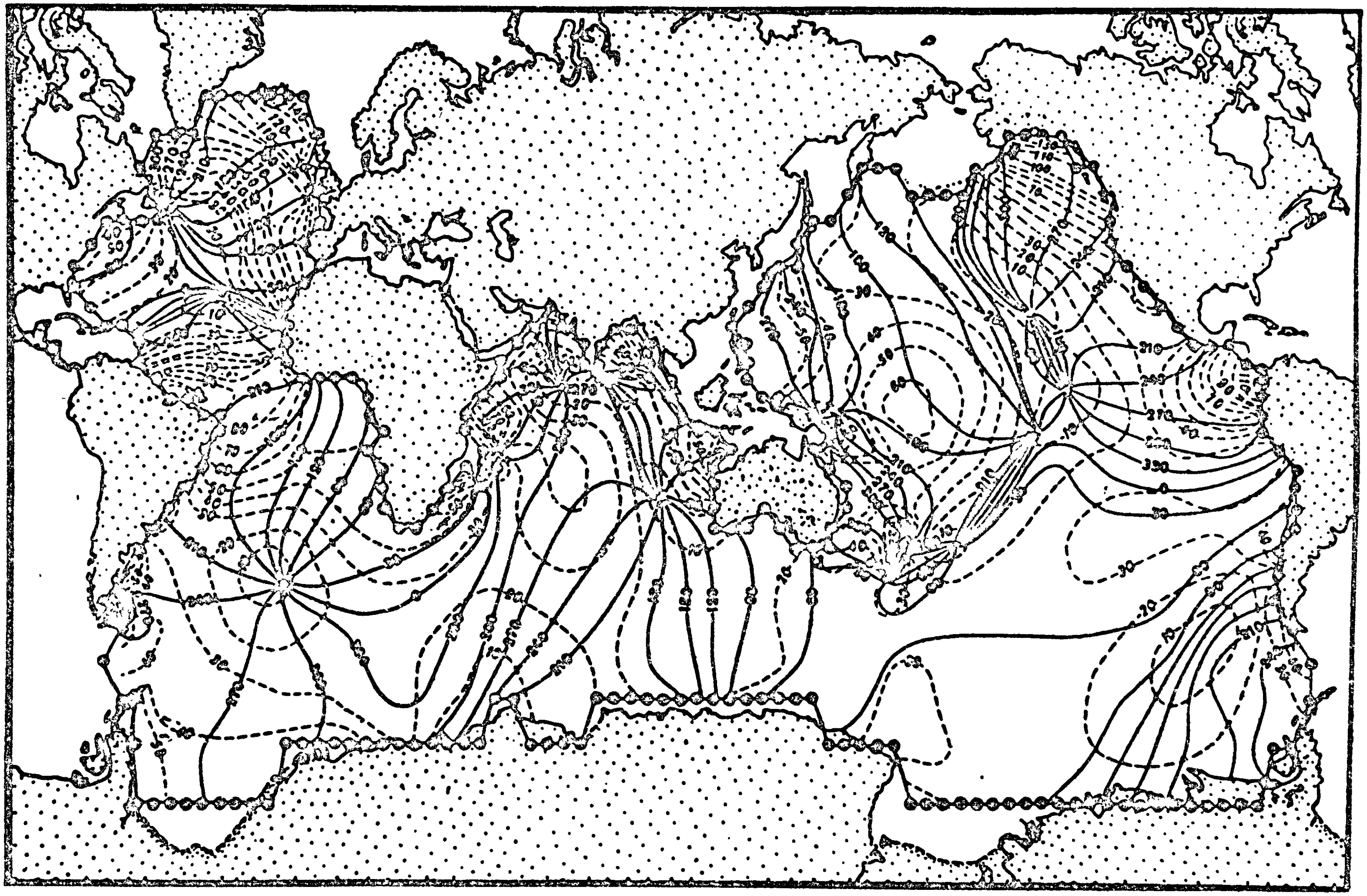


Fig. 5. Computation of M_2 ocean tide by Bogdanov and Magarik (1967). Isophase lines denoted by (———) and Isoamplitude lines denoted by (- - -) . Amplitudes given in cm, phases in degrees. Convention for the phase ϵ is $l_2 \cos(\sigma t - \epsilon)$.

In P & A the results are very sensitive to apparently small changes in the shape of the boundaries. We may quote Hendershott and Munk (1970): "The results suggest that one or more normal modes of the world ocean lie close to the driving frequency. This means that numerical models of the global tides are much more sensitive to details of discretization---than one would have expected."

It was felt that if one were to take account of the ocean tides in this study one should also take the earth tide into account. It is a very difficult problem to correctly take both effects into account so the following crude approximation was adopted. Over land it was assumed that the vertical displacement ζ at the bottom of the atmosphere was given by the Love theory earth tide, ζ_E :

$$\zeta_{land} = \zeta_E$$

Over the ocean ζ was given by

$$\zeta_{ocean} = \zeta_E + \zeta_o$$

where ζ_o represents the ocean tide solutions by B & M or P & A.

Finally, for completeness, the effect of the additional potential due to the ocean tide itself was taken into account. Suppose ζ is the surface elevation of the ocean. Let

$$\zeta = Rl \sum_{l=-\infty}^{\infty} \sum_{r=|l|}^{\infty} C_r^l \, P_r^l(\mu) e^{il\lambda} e^{i\sigma t} \tag{3.4.1}$$

Then the potential Φ_ζ due to this deformation is given by

$$\Phi_\zeta = \rho \ell \sum_{\ell=-\infty}^{\infty} \sum_{r=|\ell|}^{\infty} C_r^\ell \frac{3\rho g}{\rho_0 (2r+1)} P_r^\ell(\mu) e^{i\ell\lambda} e^{i\sigma t} \tag{3.4.2}$$

where ρ is the density of the water and ρ_0 is essentially the mean density of the earth (Hough 1898).

3.5 Treatment of the Data

The expansions of the Hough functions in terms of Associated Legendre Polynomials that are presented in the Appendix were prepared using the method of Hough (1897, 1898) as presented by Flattery (1967). Standard methods of evaluating the Associated Legendre Polynomials were used in conjunction with these expansions to evaluate the Hough functions (and their derivatives) at intervals of ten degrees in latitude. We now had to calculate the expansions (2.1.9), the Fourier-Hough expansions, for the ocean tidal elevation calculated by P & A and B & M.

B & M and P & A calculated the sea surface height in the form

$$\zeta = a(\theta,\lambda)\cos(\sigma t - \epsilon(\theta,\lambda))$$

and they presented their results as maps of a and ϵ. From these maps we read values of a and ϵ at intervals of ten degrees in latitude and longitude. We denote such a point value of ζ (or a) by

$$\zeta_{mn} = \zeta(\theta_m, \lambda_n)$$

where

$$\theta_m = \frac{\pi}{2} - \frac{\pi}{18} m \qquad 0 \leq m \leq 18$$

$$\lambda_n = \frac{n\pi}{18} \qquad 0 \leq n < 36$$

ξ_{mn} was written in the form

$$\xi_{mn} = Rl\, A_{mn} e^{i\sigma t}$$

where

$$A_{mn} = a_{mn} e^{-i\epsilon_{mn}}$$

For each latitude circle (i.e. for each m) a complex Fourier analysis of A_{mn} was performed, the Fourier coefficients being denoted by κ_{lm}. A second quantity

$$\xi^F_{mn}$$

was calculated for each point by summing the Fourier series. Thus

$$\xi^F_{mn} = Rl\, e^{i\sigma t} \sum_{l=-s}^{s} \kappa_{lm} e^{il\lambda_n} \qquad s \leq 18$$

These sums were calculated for three values of s, $s = 6, 9, 14$

A correlation coefficient r between ξ and ξ^F was calculated as follows

$$r = \frac{\langle \sum_m \sum_n \xi_{mn} \xi^F_{mn} \cos\theta_m \rangle}{\left[\langle \sum_m \sum_n \xi^2_{mn} \cos\theta_m \rangle \langle \sum_m \sum_n (\xi^F_{mn})^2 \cos\theta_m \rangle \right]^{\frac{1}{2}}}$$

where $\langle \quad \rangle$ indicates an average over one period. The values of r are shown in Table 3.5.1.

σ	P & A	B & M
6	.8968	.9074
9	.9513	.9516
14	.9844	.9835

Table 3.5.1 Correlation coefficients between ξ and ξ^F for three values of σ.

The latitudinal behavior of the coefficients $c_{\ell m}$ (ℓ fixed) now had to be expressed by expansions in sums of Hough functions. Since the Hough functions are real, the real and imaginary parts of $c_{\ell m}$ could be handled separately. For each ℓ we had a set of nineteen numbers, $\mathrm{Re}\, c_{\ell m}$ or $\mathrm{Im}\, c_{\ell m}$ to be represented in a least squares sense by a sum of eight Hough functions. This number, eight, was chosen as a compromise between the desire for accuracy and the cost of calculating the expansions in the Appendix. This problem in the method of least squares is well known, (Hildebrand 1956). To solve it one must solve the so-called normal equations. Two IBM supplied routines APFS and APLL (IBM 1968) were used to set up and solve these equations, using the values of the Hough functions already calculated as data. The solutions yielded the coefficients b_r^ℓ in the expression

$$c_{\ell m} \sim \sum_{r=|\ell|}^{|\ell|+7} b_r^\ell H_r^\ell(\mu_m)$$

The Fourier-Hough expansions were then summed for each point to yield a new quantity ζ_{mn}^{FH} given by*

$$\zeta_{mn}^{FH} = R\ell \, e^{i\sigma t} \sum_{\ell=-s}^{s} e^{i\ell\lambda_n} \sum_{r=|\ell|}^{|\ell|+7} b_r^{\ell} \, H_r^{\ell}(\mu_m) \qquad (3.5.1)$$

A correlation coefficient between ζ and ζ^{FH} was calculated according to a formula similar to that given above. The results are shown in Table 3.5.2

s	P & A	B & M
6	.8503	.8538
9	.9011	.8908
14	.9301	.9146

Table 3.5.2 Correlation coefficients between ζ and ζ^{FH} for three values of s .

It was felt that taking 14 wavenumbers was sufficient since with such a representation one accounted for over 80% of the variance of the functions being fitted. Expressions

* $H_0^0(\mu)$ is a constant $(= 2^{-\frac{1}{2}})$ and the term in $H_0^0(\mu)$ represents a purely radial oscillation of the lower boundary of the atmosphere. Such an oscillation is physically impossible and so this term must be excluded. In order to use eight Hough functions for each sum on r the limits on this sum in the case $\ell = 0$ were $|\ell|+1$ to $|\ell|+8$.

of the form (3.5.1) with $S = 14$ were used in all further calculations involving the ocean tidal elevation.

The amplitude and phase of ζ^{FH} for P & A and B & M are plotted in Figs. 6 and 7 respectively. (Note that in these figures the solid lines are lines of equal amplitude and the dashed lines are lines of equal phase.) The coefficients w_r^l in the expansion of the vertical velocity at the lower boundary are then given by

$$w_r^l = i\sigma b_r^l$$

The expansion of the lunar tidal potential as a Fourier-Hough series was straightforward since only wave number two was involved and an expression for P_2^2 as a sum of Hough functions had already been calculated.

The gravitational potential due to the displacement of the ocean waters was relatively small, being in magnitude about 10% of the lunar potential. This "ocean tide" potential was computed in straightforward fashion. The Associated Legendre Polynomials were evaluated at intervals of ten degrees in latitude and a truncated Fourier-Legendre series of the form

$$Rl \sum_{l=-14}^{14} \sum_{r=|l|}^{|l|+7} c_r^l P_r^l(\mu) e^{il\lambda} e^{i\sigma t}$$

for the ocean tide was generated in exactly the same way as the Fourier-Hough series already discussed. The calculation

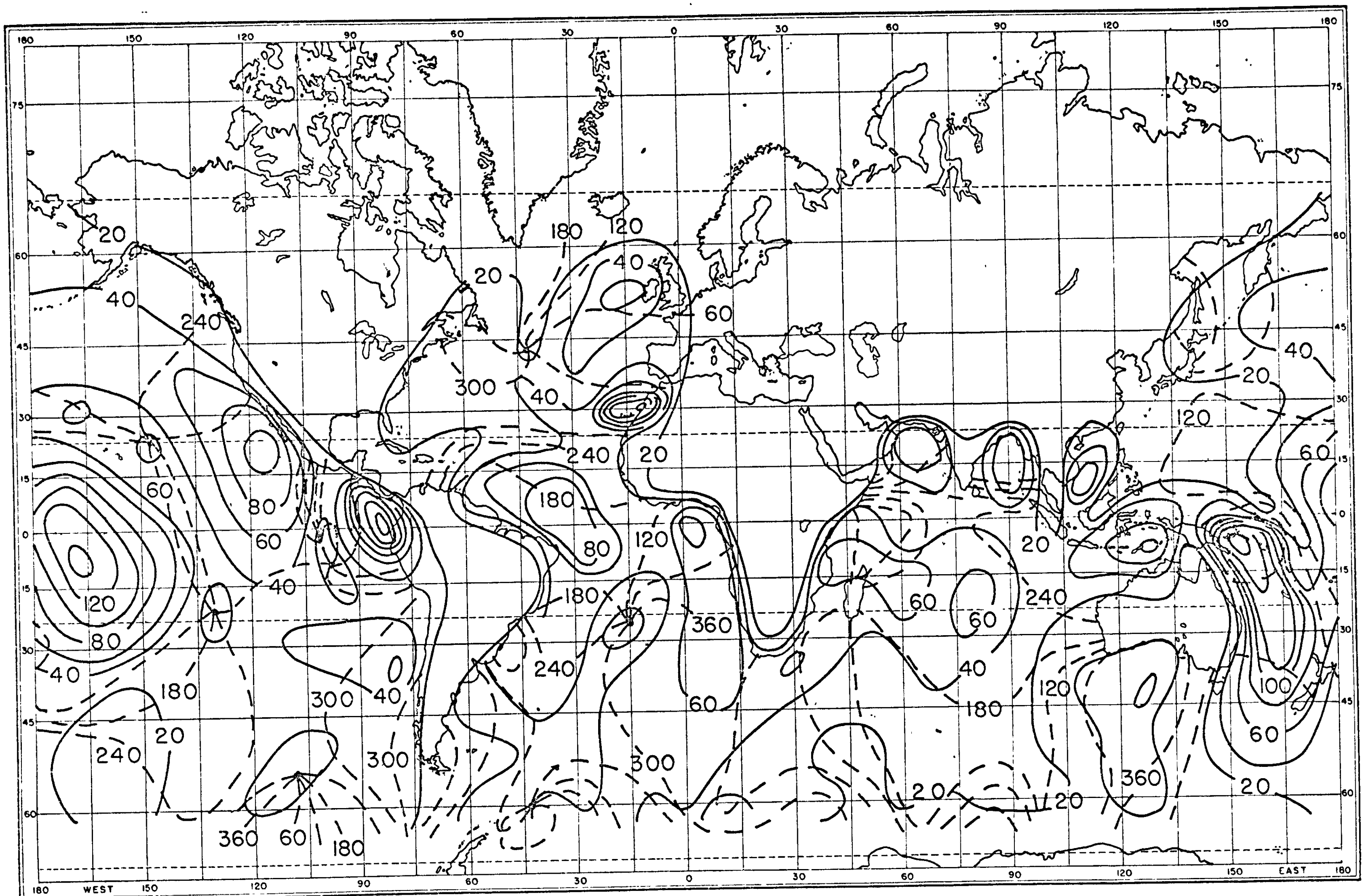


Fig. 6. Plot of ζ^{FH} for P & A, with $s = 14$. Isoamplitude lines denoted by (—). Isophase lines denoted by (- - -). Amplitude lines plotted in intervals of 20 cm. Phase lines plotted in intervals of 60°. The convention for the phase ϵ is $\ell_2 \cos(\sigma t - \epsilon)$

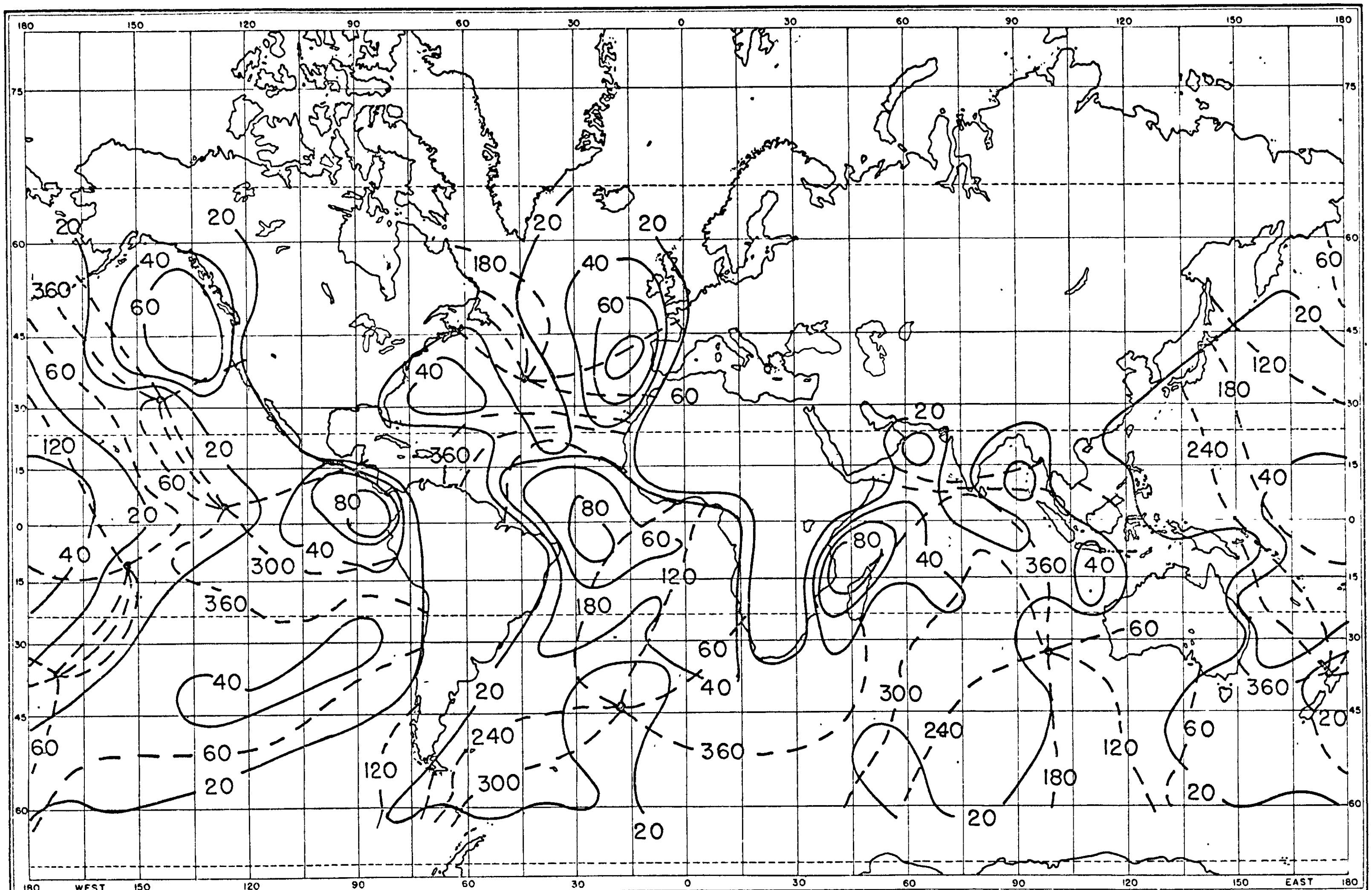


Fig. 7. Plot of ζ^{FH} for B & M, with $s = 14$. Isoamplitude lines denoted by (—). Isophase lines denoted by (- - - -). Amplitude lines plotted in intervals of 20 cm. Phase lines plotted in intervals of 60°. The convention for the phase ϵ is $\ell_2 \cos(\sigma t - \epsilon)$

of the potential in the form*

$$\mathrm{Re} \sum_{l=-14}^{14} \sum_{r=|l|}^{|l|+7} C_r^l \frac{3\rho g}{\rho_0(2r+1)} P_r^l(\mu) e^{il\lambda} e^{i\sigma t} \tag{3.5.2}$$

analogous to (3.4.2) was then a simple matter. The expansions for the Associated Legendre Polynomials in terms of Hough functions (cf. Appendix) were then used to transform the Fourier-Legendre series into a Fourier-Hough series. For a given l the largest values of C_r^l were those for which $|l| \leq r \leq |l|+2$. The expressions for the Associated Legendre Polynomials are fairly accurate for $r < |l|+4$. For $r > |l|+4$ the expressions underestimate the variance of the Legendre functions by a not insignificant amount. However, this was not thought a serious error since these coefficients were relatively small to begin with and the factor $\frac{1}{2r+1}$ in the expression (3.5.2) further reduces them relative to the coefficients for $|l| \leq r \leq |l|+2$.

When the expressions for the ocean tidal height and the potential due to this displacement were calculated we were then in a position to solve the vertical equation (2.1.14) with the boundary conditions (2.1.15) and (2.2.4)

* $P_0^0(\mu)$ is a constant $(=2^{-\frac{1}{2}})$ and the term in $P_0^0(\mu)$ in this series represents a purely radial oscillation of the lower boundary of the atmosphere. Such an oscillation is physically impossible; it is precluded by the effective incompressibility of the earth and the oceans, and so this term must be excluded (cf. footnote to (3.5.1)).

for each mode of interest. With $-14 \leq \ell \leq 14$ and 8 coefficients for each value of ℓ this meant that the vertical equation had to be solved 232 times.

Once Y_n^ℓ was known as a function of Z it was a simple matter to compute the value of W (and hence ω), U, V at any point, using the expressions for these quantities presented in Chapter II. We discuss the results of these computations in the next chapter.

Chapter IV

4.1 The Resonance Curve of the Model Atmosphere

Before considering the response of our model atmosphere to the forcing functions described in previous chapters it is important to investigate its resonance behavior. Once the forcing function $A = i\sigma \Phi_n^l + g w_n^l$ is fixed the solution of the vertical equation depends only on the atmospheric stability function S and the separation constant β. The atmosphere is said to have a free mode if there is a value of β for which there is a non-zero solution of the vertical equation when the forcing term is zero or equivalently if there is a value of β for which the solution to the vertical equation is infinite when the forcing term is finite. For such a value of β the atmosphere is said to be resonant.

For an arbitrary value of the forcing function A the ratio

$$M = \frac{Y'_{z=0}}{A}$$

was computed, at intervals of 0.1 in β for $0 \leq \beta \leq 35$. The amplitude and phase of M are shown in Fig. 8. For $\beta < 8 \cdot 8$ the value of the phase was between -0.001° and 0° but this cannot be shown on a log-graph. This curve is called

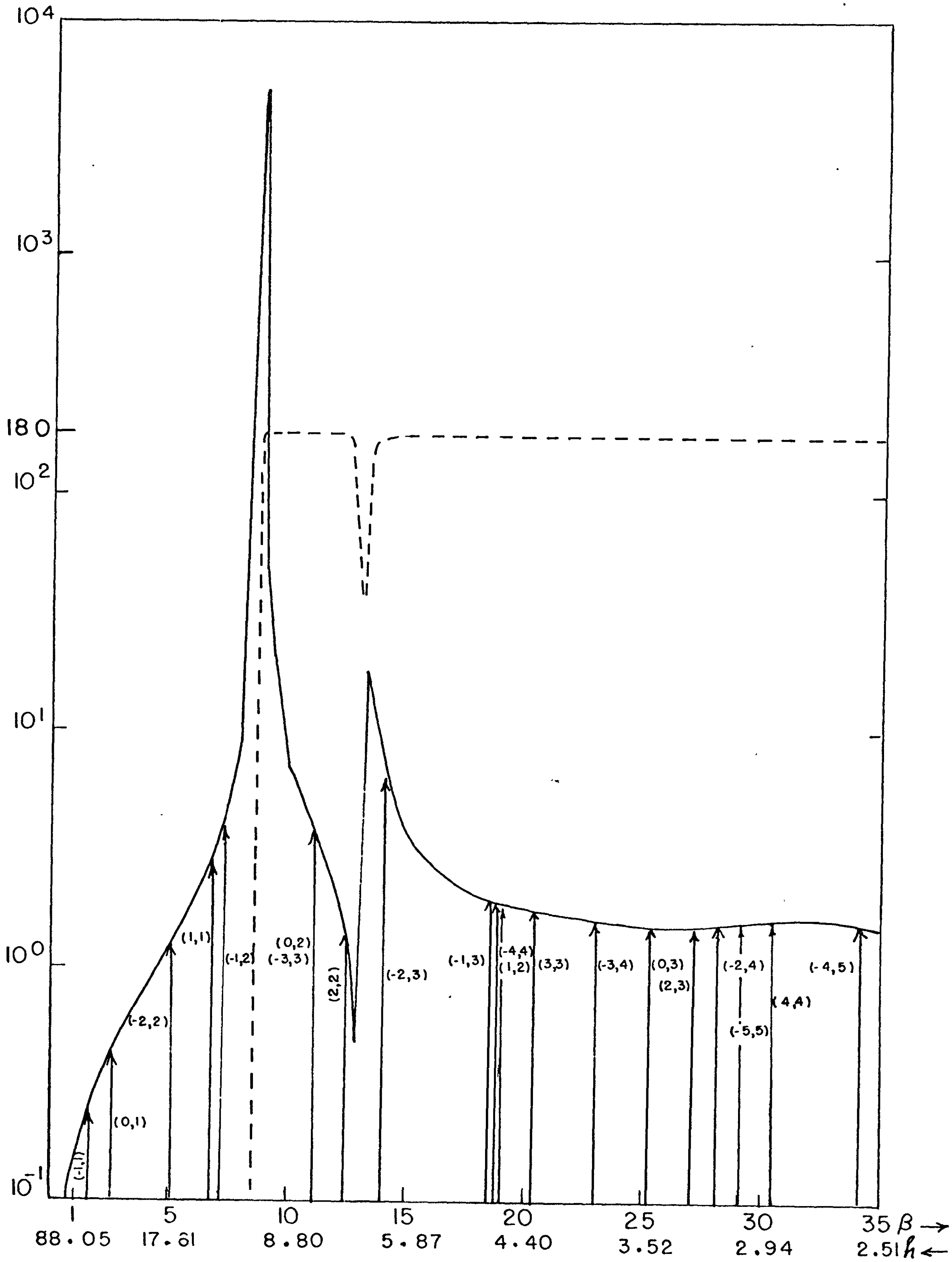


Fig. 8. Resonance curve for the model atmosphere showing the amplitude (——) and phase (– – –) of M as a function of β or h. The vertical arrows indicate the values of M and β_n^ℓ for the mode numbers (ℓ,n) indicated ($h = \frac{4\Omega^2 a^2}{g\beta}$).

a resonance curve.

There are two peaks in the resonance curve. The major peak occurs at $\beta = 8.8$ corresponding to an equivalent depth of 10.006 km. This may be compared with Taylors (1929) calculation based on the waves generated by the Krakatoa explosion, that the atmosphere had a free mode with an equivalent depth of 10.3 km. We note too that at this value of β a 180° shift occurs in the value of the phase of M. There is a second maximum in the resonance curve at $\beta = 13.42$ corresponding to an equivalent depth of 6.56 km*.

This second maximum is not a true resonance but it is associated with some changes in the phase. This feature of the resonance curve may be compared with a similar feature in a resonance curve presented by Jacchia and Kopal (1952). They were seeking to construct an atmospheric temperature profile which met the requirements of the resonance theory of the solar tides (Pekeris 1937) and was consistent with what was then known about the vertical temperature structure of the atmosphere. The profile they settled on had two maxima in its resonance curve. The larger maximum occurred at $\beta = 8.47$ (equivalent depth 10.388 km) and appeared to be a true resonance. The smaller maximum occurred at $\beta = 11.05$ (equivalent depth 7.93 km). At this maximum the value of $|M|$

* M was computed at intervals of 0.001 in β for $12 \leq \beta \leq 14$

was 81. We have no information on the phase of their M and so we cannot say if this was a true resonance. The profile which Jacchia and Kopal were led to had an unrealistically high temperature of about 350° near 50 km, and they found that the location and magnitude of the lesser maximum on the resonance curve were very sensitive to small changes in temperature near 50 km. It seems reasonable to suppose then that the peak which Jacchia and Kopal found near $\beta = 11.05$ is simply an unrealistic modification of the secondary peak on Fig. 8.

For $\beta > 35$ * the factor M decreases slowly, as β increases, to a value of ~ 1.2 at $\beta \sim 1000$. The phase does not change significantly in this region.

Also shown in Fig. 8 are the seventeen values of β in the range $0 \leq \beta \leq 35$ and the corresponding mode numbers for which the equation was solved in our computations of the response to oceanic forcing. For all but four of these we have $|M| < 2$; the largest value of $|M|$ is about 6. It is clear therefore that we were not exciting any resonant modes of the atmosphere.

* M was computed at intervals of 2.0 in β for $35 \leq \beta \leq 1000$

4.2 Influence of a Small Ocean

As a further exploration of the behavior of the model atmosphere the following experiment was performed.

It was supposed that the entire surface of the globe was immovable land with the exception of a small square ocean ten degrees of latitude and longitude on a side (i.e. one point in the latitude-longitude grid mesh of section 3.5). The surface of the ocean was supposed to execute a vertical oscillation of amplitude one meter at the lunar semi-diurnal frequency. This may be thought of then as an approximation to a point source of ϖ. We calculated the amplitude and phase of the resulting oscillation as it would be seen by a barometer fixed to the ground. For the "oceanic" region we applied the correction discussed at the beginning of the next section. All effects of the lunar potential and the earth tide were ignored. The result of the calculation is shown in Fig. 9, for an ocean centered on 40^{o}N and 0^{o}E whose "tide" had zero phase lag relative to local lunar time.

The effect of this localized forcing is felt globally. In the calculations presented below there are differences between the calculated and the observed pressure oscillations. The conclusion to be drawn from Fig. 9 is that these differences cannot easily be reduced by simple

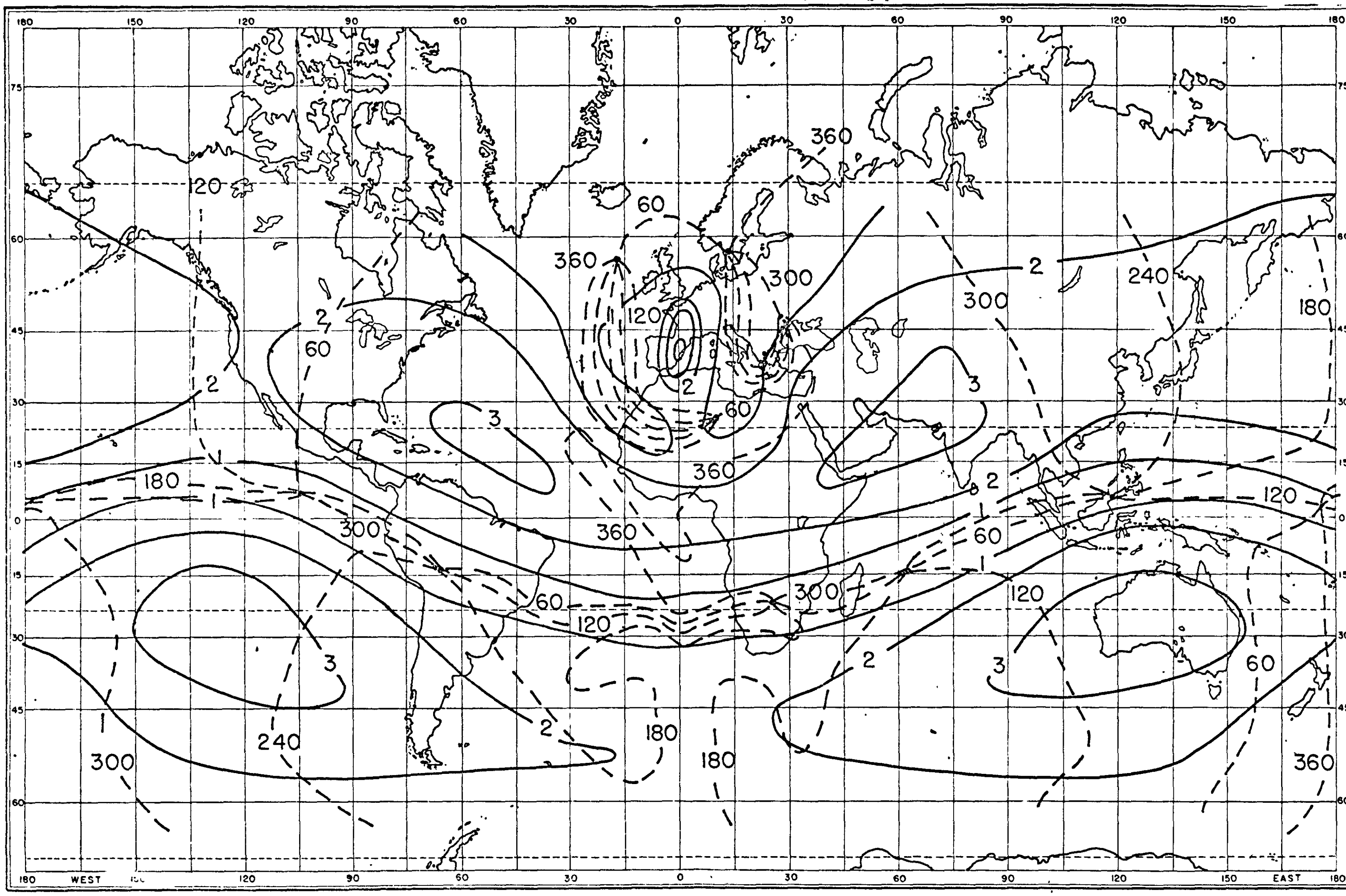


Fig. 9 Response in surface pressure to a tide in a "point" ocean at 40°N, 0°E (cf. § 4.2). Amplitudes (——) in μb, phases (- - -) in degrees. The convention for the phase ϵ is $\ell_2 \cos(\sigma t + \epsilon)$

manipulation of the forcing function, (the imperfectly known ocean tide) in the immediate vicinity of the places where discrepancies occur.

4.3 Calculated response when the ocean tide is taken into account.

The results of the calculations described at the end of § 3.5 are shown in Figs. 10 and 11 for the ocean tides of B & M and P & A respectively. The result of solving (2.1.14) was to give an expression for $\omega = \frac{dp}{dt}$. As discussed in section 1.2, this would be the pressure as seen by a barometer fixed to the moving ocean or land surface. Now the "oceanic" barometers for the lunar tidal observations are fixed to islands, which we shall assume to move vertically with the Love earth tide. In order to compare the theoretical results with observations in an oceanic area, we must therefore compute the pressure change as seen by a barometer which moves up and down with the earth tide. This requires a correction over the ocean. Now

$$\omega\Big|_{z=0} = \frac{\partial p}{\partial t} - g\rho(w_E + w_0)$$

where w_E is the vertical velocity due to the earth tide and w_0 is the vertical velocity due to the ocean tide. The quantity we wish to compute,

$$\frac{\partial p}{\partial t} - g\rho w_E$$

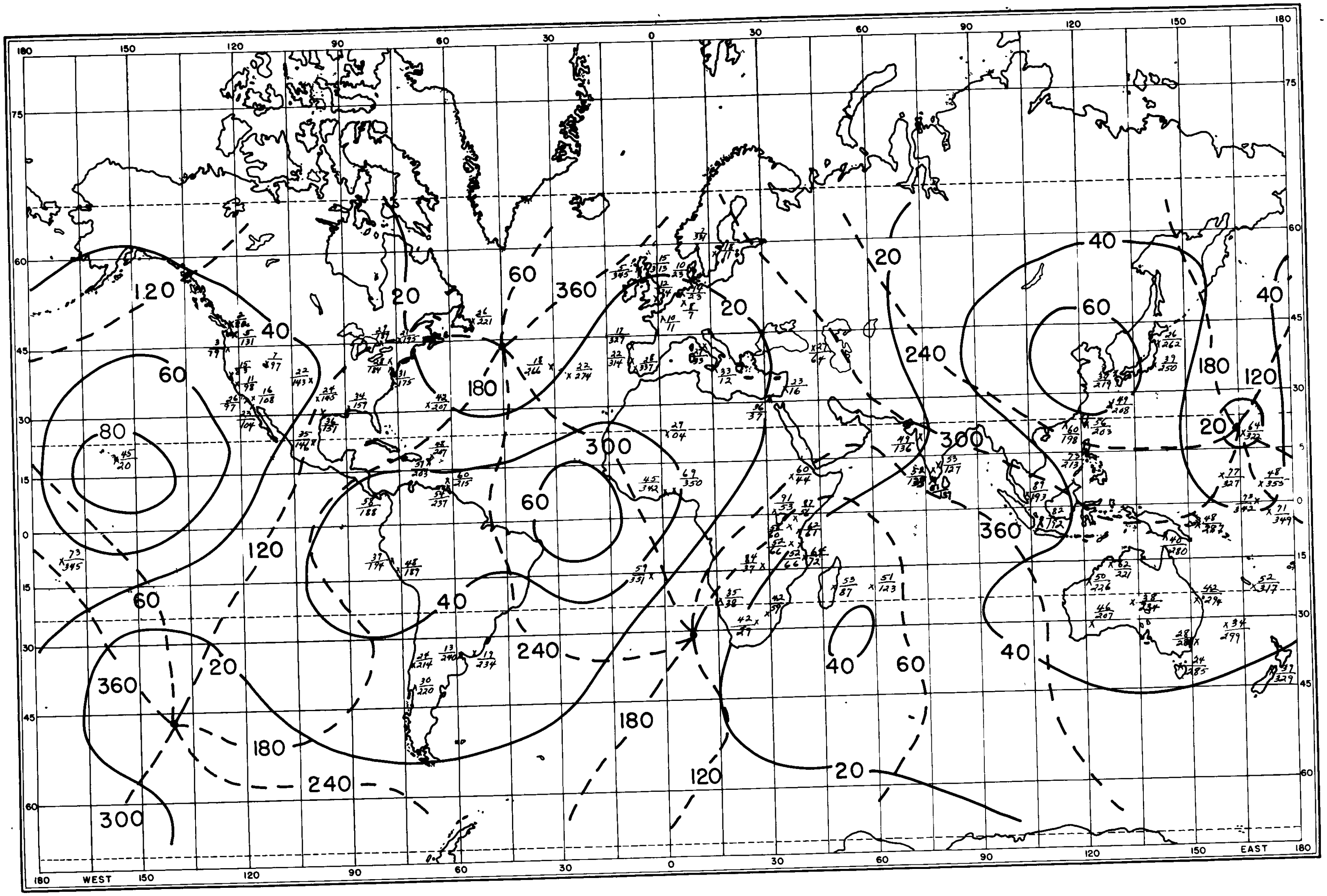


Fig. 10. Tidal response in surface pressure to forcing by lunar potential, earth tide and ocean tide due to B & M. Amplitudes (——) in μb. Phases (- - -) in degrees. The convention for the phase ϵ is $l_2 \cos(\sigma t+\epsilon)$. Script letters l/ϵ are observed amplitudes and phases.

is given by

$$\frac{\partial p}{\partial t} - g\rho w_E = \omega\Big|_{z=0} + g\rho w_0$$

This quantity $\frac{\partial p}{\partial t} - g\rho w_E$ is plotted in Figs. 10 and 11 for the two cases in which w_0 corresponds respectively to the B & M and P & A data. As discussed earlier, in both cases the forces perturbing the atmosphere were

1) The force due to the lunar tidal potential
2) The force due to the potential caused by the earth tide
3) The effect of the vertical oscillation of the lower boundary of the atmosphere due to the earth tide
4) The effect of the vertical oscillation of the lower boundary of the atmosphere due to the ocean tide
5) The effect of the potential due to the ocean tide

It turned out that the introduction of the forcing due to the ocean tide had quite marked effects on the atmosphere. For convenience the data on which Fig. 1 is based are entered on Figs. 10 and 11 in the form ℓ/ϵ where ℓ gives the amplitude and ϵ the phase, when the oscillation is written in the form

$$\ell \cos(\sigma t + \epsilon)$$

ϵ being in degrees. When we speak of the tidal crest moving in a given direction we mean that the phase ϵ decreases in that direction. This convention is opposite to the oceanic convention used on Figs. 4, 5, 6, and 7. The data was prepared from the tabulations of Haurwitz and Cowley (1970).

We consider first the response when B & M was used. The comparison is best made on a regional basis.

Melanesia and Micronesia The computations yield an amphidrome in this region. The computed amplitudes are small by a factor of two. The computed phases appear to be of the right size and the direction of motion of the computed tidal crest agrees with the observations in the region 10S - 15N.

Eastern Asia and Indonesia The observed amplitude maximum over Indonesia is not found in the computations; rather a maximum occurs over Korea with a local minimum over Indonesia. The computed tidal crest moves north-eastward, not westward, so that while the computed and observed phases agree near southern Japan, they disagree over the rest of the region.

Australia The computed amplitudes are of the right order of magnitude but here too the computed tidal crest moves towards the northeast rather than towards the west.

India The computed amplitudes are too small again by a factor of two, roughly. The computed phases differ from the observed phases by 180^{o}. The computed tidal crest moves towards the northeast; the motion of the observed tidal crest has an easterly component also.

Africa East of 15E The calculated amplitudes disagree very much with the observed amplitudes, being too small by a factor of 3 or more. The computed phases typically lead the observed phases by 30°; the computed tidal crest moves northwest while the observed tidal crest moves westward.

West Africa The calculated amplitudes appear to be of the right order of magnitude. The calculated phases lag behind the observed phases by about 60°. The computed and observed tidal crests both move towards the southwest.

Europe The calculated amplitudes are too high, in some cases by a factor of two. However, the computed amplitudes do decrease towards the north. The computed phase was very nearly uniform at $\sim 345^{\circ}$ over most of Europe west of 30E. Thus the computed tidal crest moves towards the southeast over northwestern Europe, towards the southwest over southern Europe and North Africa and towards the northeast over Russia. The observed tidal crest moves towards the northwest over northern Europe and towards the southwest over southern Europe.

North America The computed amplitudes disagree markedly with the observations in western North America. The observations show a noticeable increase in amplitude from west to east across the continent while the computations show the amplitude increasing from east to west. The observed tidal crest moves westward while the computed crest moves north-westward over the United States.

Latin America The agreement between the computed and observed amplitudes is good except in the neighborhood of the River Plate; this region will be discussed further shortly. The computed phases agree fairly well with the observed phases and the computed tidal crest moves more or less due west.

Isolated Island Stations:

Bermuda The computed amplitude is too small by a factor of two. The computed phase lags the observed phase by about 60^{o}.

Azores The computed amplitudes are about right. The computed phases lead the observed phase by about 50^{o}.

St. Helena The computed amplitude is small by a factor of two. The computed phase lags the observed phase by 60°.

Honolulu The computed amplitude is too large by a factor of two. The computed phase leads the observed phase by 70°.

We turn now to consider the response when P & A is used as the ocean tide forcing.

Melanesia and Micronesia The computed amplitudes are too small by a factor of two. The computed phases agree fairly well with the observed phases and the direction of motion of the computed crest is westward.

Eastern Asia and Indonesia The computed amplitudes agree quite well with the observed amplitudes in this region. The computations show a maximum in amplitude, of about 80 μb in the China sea and the computed amplitude declines towards the north at about the same rate as does the observed amplitude. In northern Japan the computed phases lead the observed phases by about 60° but over the rest of the region the agreement between the two is much better and consequently the directions of motion of the computed and observed tidal crests agree well.

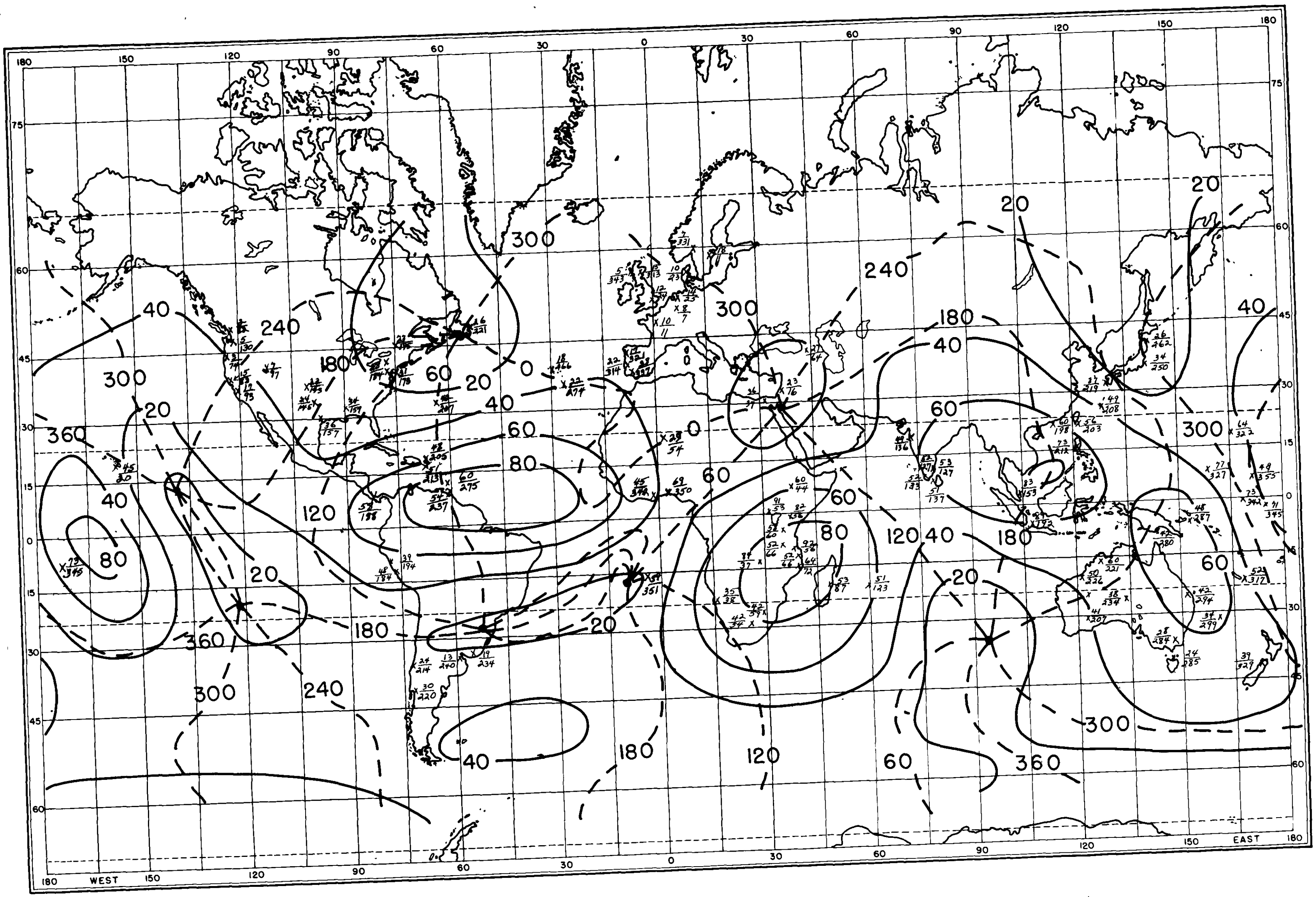


Fig. 11. Tidal response in surface pressure to forcing by lunar potential, earth tide and ocean tide due to P & A. Amplitudes (—) in μb . Phases (- - -) in degrees. The convention for the phase ϵ is $l_2 \cos(\sigma t+\epsilon)$. Script letters l/ϵ are observed amplitudes and phases.

Australia The computed amplitudes do not agree well with the observations, particularly near Tasmania. The computed phases seldom differ from the observed phases by more than $\sim 30^{\circ}$ and the computed motion of the tidal crest has a large westerly component.

India The computed amplitudes agree fairly well with the observations as do the computed phases. The computed motion of the tidal crest is towards the southwest while the observed crest appears to have an eastward motion.

Africa East of 15E There is controversy about some of the determinations of the lunar tide in this region. Haurwitz and Cowley (1967) point out that some of the amplitudes determined in earlier studies turned out to be erroneously large. It may be that the true amplitudes in the region of Tanzania are nearer 60 μb than 90 μb. In any event our computations yielded a maximum amplitude of 90 μb at 40°E 10°S. The computed amplitudes were too high by a factor of two in southern Africa. In the region where observations are available the computed phases were typically 100° thus leading the observed phases by $\sim 40^{\circ}$.

The Middle East The existence of the amphidrome indicated by the computations does not agree with the available observations.

West Africa The calculated amplitudes agree well with two of the three available observations but do not agree well with the large amplitude reported for Lagos. The computed phases do not agree well with the observations, for the computed tidal crest moves north-westward while the observed tidal crest waves moves south-westward.

Europe The computed amplitudes are too high by a factor of two or three. The computed phases do not agree with the observed phases because the computed tidal crest moves northwards while the observed tidal crest moves mainly westward.

North America As with the computations using B & M the present computations yield a pattern in which the amplitude declines from west to east across the continent. The computed tidal crest moves towards the southeast rather than towards the west.

Latin America The computed amplitudes in the northern part of the continent agree fairly well with the observations. However, the computed tidal crest moves eastward while the observed tidal crest moves westward. The calculations yield an amphidrome near the east coast of the continent in the vicinity of the River Plate. Haurwitz and Cowley (1967) point out that the determinations for Montevideo and Buenos Aires yield amplitudes that are considerably lower than those for other stations at the same latitude or further south. The computed amplitudes agree quite well with the observations in the southern part of Latin America. The calculated phases agree fairly well with the observations also.

Isolated Island Stations:

The Azores The comments on the computations for Europe also apply here.

Bermuda The computed amplitude is a little low in comparison with the observed amplitude; the computed phase lags the observed phase by 140°.

St. Helena The existence of the amphidrome predicted by the computations in the vicinity of St. Helena is contradicted by the large observed amplitude at St. Helena.

Honolulu The computed amplitude is a little low but there is fair agreement between the computed and observed phases.

We might summarise these results as follows. Forcing of our model atmosphere with the lunar potential and the solid earth tides produces a response which is uniform in longitude and has a maximum amplitude of ~28 μb at the equator. The lines of equal phase run almost due north-south, the only amphidromes being at the poles. The introduction of an additional forcing due to ocean tides changes the picture considerably. The longitudinal uniformity of the amplitude of the response is destroyed and the pattern of equal-phase lines becomes more complex. Certain features of the calculated response seem to be directly related to similar features in the forcing. For example, over the central and southern Atlantic there is a distinct resemblence between the patterns of ocean tidal amplitude and atmospheric air tide amplitude when P & A was used. On the other hand, over land there is no such simple explanation for the results of the computation.

The results using P & A gave fairly good agreement with the observations over Asia, East Africa, and South America and poor agreement over Europe and North America. The agreement between the results using B & M and the observations was not as good.

In order to further illustrate the difference that the introduction of the ocean tide makes to these calculations two scatter diagrams, Figs. 12 and 13, were prepared. For each station listed by Haurwitz and Cowley (1970) we plotted the pressure amplitude determined from observations against the pressure amplitude for that station predicted by one of two computations. Each figure has the observed amplitude as abscissa. Fig. 12 has as ordinate the predicted amplitude when only the lunar potential and the earth tides are taken into account (i.e. the amplitudes predicted in Fig. 3). Fig. 13 has as its ordinate the amplitude predicted when in addition to these effects the ocean tide due to P & A is taken into account (i.e. the amplitudes predicted in Fig. 11). Points falling on the diagonal are perfect predictions. It is clear from a comparison of these two figues that the ocean tide is an important and possibly the most important influence on the lunar air tide.

4.4 Tidal Winds

The subject of lunar tides in the ionosphere is a subject of some interest (Matsushita 1967). We computed the tidal winds at $Z = 14.8$, i.e. at a height of $\sim$ 98 km, in our model when the forcing consisted of the lunar potential, the earth tide (potential and surface movement) and the ocean tide and its potential according to P & A. The computations were made at ten-degree intervals in latitude and longitude. The results are presented in the form

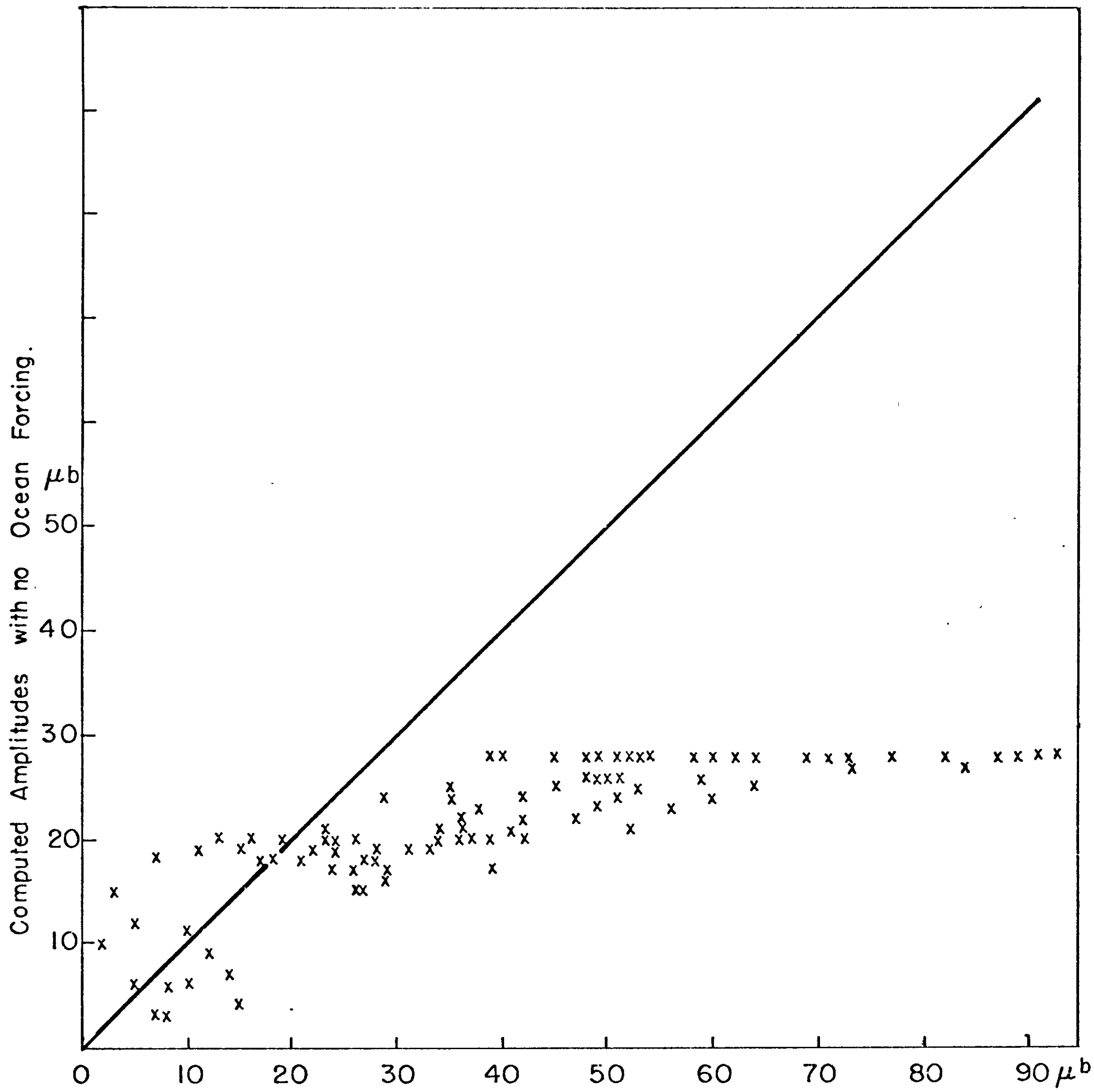


Fig. 12. Scatter diagram of observed value of tidal pressure amplitude at a station versus the amplitude predicted for that station using no ocean forcing.

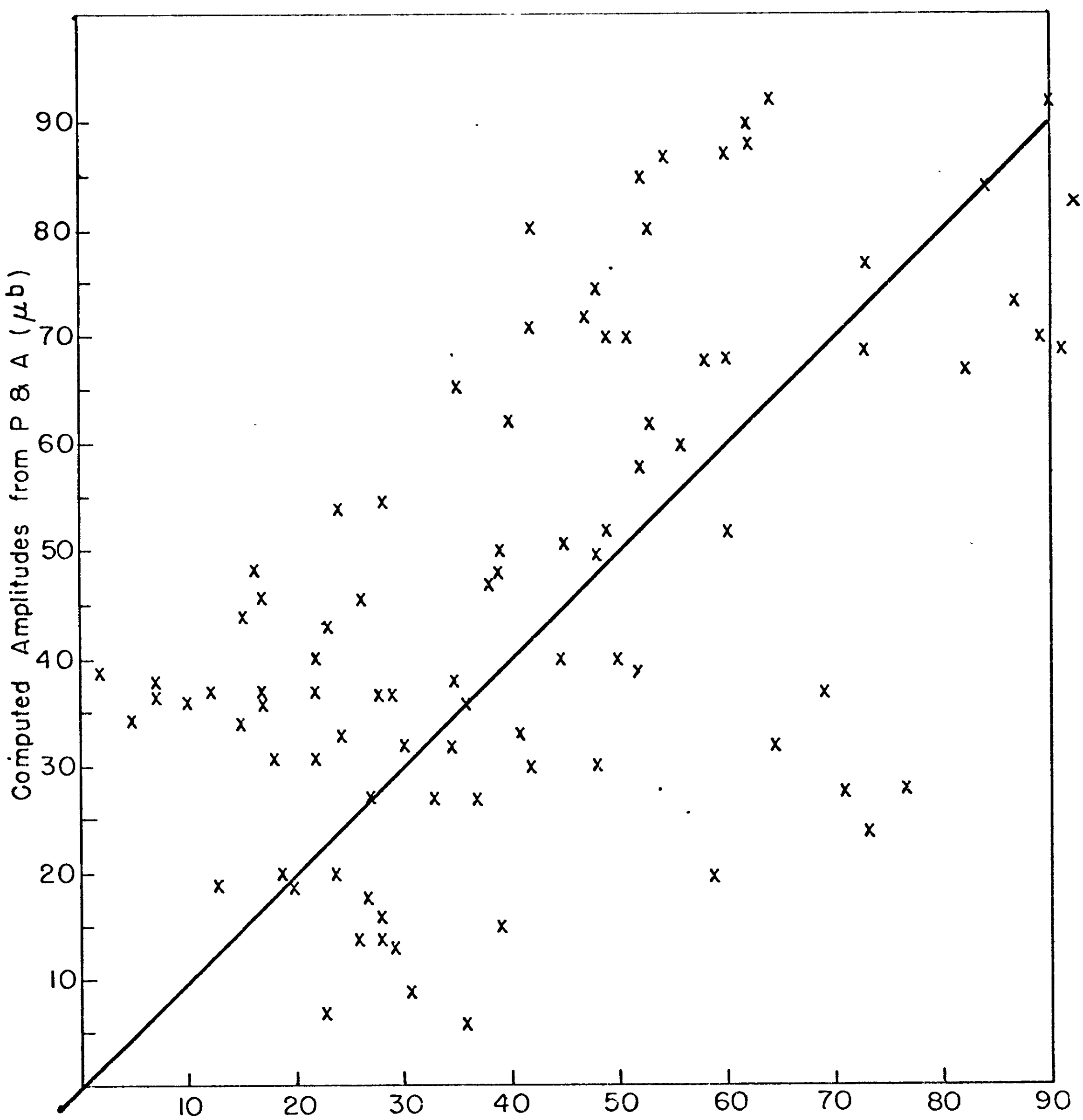


Fig. 13. Scatter diagram of observed value of tidal pressure amplitude at a station versus the amplitude predicted for that station using P & A's tide as the ocean tide forcing.

of tidal ellipses in Fig. 14. The line that is flagged at each of these points indicates (1) the sense of rotation of the wind vector (the wind rotates in the sense in which the tip of the flag points) and (2) the wind direction when the mean moon is in upper or lower transit at Greenwich i.e. the phase of the wind oscillation. For example, consider the wind at 40N, 30W near the Azores. At upper or lower transit of the mean moon at Greenwich the wind at this location is towards 30.24° west of south. Some 1.6 mean lunar hours before this transit at Greenwich the wind had attained its maximum speed of 5 m/sec while blowing towards 12.5° south of west. Some 4.4 mean lunar hours after this transit at Greenwich the wind at 40N, 30W will again attain its maximum speed while blowing towards 12.5° north of east. The tidal ellipses were computed by first computing

$$\underline{u} = u\,\underline{i} + v\,\underline{j}$$

at the point in the form

$$\underline{u} = A \cos(\sigma t + \alpha)\,\underline{i} + B \cos(\sigma t + \beta)\,\underline{j}$$

We then calculated A', B', δ, θ, so that the velocity could be written

$$\underline{u} = A' \cos(\sigma t + \delta)\,\underline{i}' + B' \sin(\sigma t + \delta)\,\underline{j}'$$

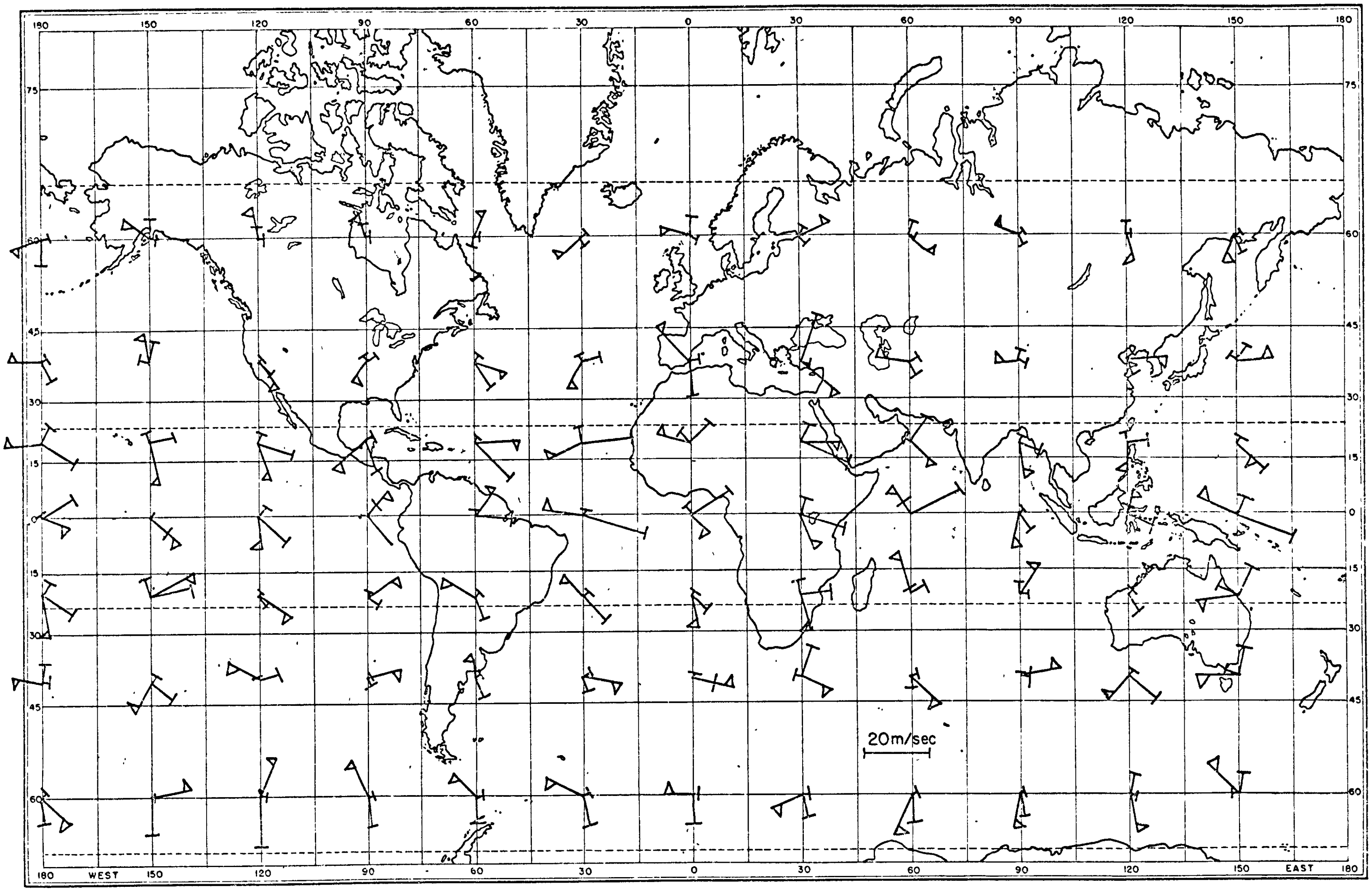


Fig. 14. Tidal wind velocities, represented as tidal ellipses, at Z = 14.8 (~98 km), when the atmosphere is forced by the lunar potential, the earth tide and the ocean tide due to P & A. Flagged line at each point indicates the sense of rotation of the wind vector and the direction towards which the wind is blowing at lunar transit at Greenwich.

where

$$\underline{i}' \cdot \underline{i} = \cos\theta$$

$$\underline{i}' \cdot \underline{j} = \sin\theta$$

The appropriate formulae for the transformation are

$$\theta = \frac{1}{2}\tan^{-1}\frac{2AB\cos(\alpha-\beta)}{A^2 - B^2}$$

$$\delta = \tan^{-1}\frac{2A\cos\alpha\sin^2\theta - B\cos\beta\sin 2\theta}{B\sin\beta\sin 2\theta - 2A\sin\alpha\sin^2\theta}$$

$$A' = A\frac{\cos\alpha + \sin(\alpha-\delta)\sin\delta}{\cos\theta\cos\delta}$$

$$B' = A\frac{\sin(\alpha-\delta)}{\sin\theta}$$

Most observations of motions in the region 80-110 km fall into two classes. The first class consists of radar observations of meteor trails. It is generally accepted that the velocities computed by this method represent motions of the neutral atmosphere. The second class consists of "drift" observations by the radio fading technique. It is not known whether the velocities derived from these latter observations pertain to the neutral atmosphere or to the charged species (Rawer, 1968). Even if these velocities do pertain to the neutral atmosphere there is some doubt as to the height at which the velocities are being measured.

Greenhow and Neufeld (1961) from meteor wind observations at Jodrell Bank derived an upper bound of 2 m/sec on the semi-diurnal lunar tidal velocity. Müller (1966) made an extensive study of tides in meteor wind data from Sheffield, England but did not report any determinations at the semi-diurnal lunar period. Müller (1968, 1970) reports an experiment he carried out to compare velocities measured by the two methods. He found evidence that the velocities measured by the radio fading method do represent motions of the neutral atmosphere at levels somewhat higher than the levels where meteor trails are measured. In his experiment he found that meteor-wind observations gave winds that pertained to an average height of 95 km while the average E-region reflection height was 103 km. Our calculations were made for an intermediate level, 98 km.

Using drift observations Phillips (see Briggs and Spencer, 1954) determined lunar tidal winds at Cambridge, England, Chapman (1953) did a similar calculation for Montreal, and Ramana and Rao (1962) did one for Waltair, India. Table 4.4.1 shows a comparison between these determinations and our computations for the points indicated. t' is local lunar time.

	Observed	Computed	At: Co-ordinates
Cambridge	$u = 16 \sin(2t' - 81°)$	$u = 0.4 \sin(2t' - 160°)$	50N,0E
52.2N,0.1E	$v = 14 \sin(2t' + 3°)$	$v = 10.3 \sin(2t' - 208°)$	
Montreal	$u = 21 \sin(2t' - 106°)$	$u = 3.0 \sin(2t' - 32°)$	50N,70W
45.4N,73.8W	$v = 24 \sin(2t' + 3°)$	$v = 1.0 \sin(2t' - 43°)$	
Waltair	$u = 12 \sin(2t' + 21°)$	$u = 4.8 \sin(2t' + 250°)$	20N,80E
17.7N,83.3E	$v = 5 \sin(2t' + 90°)$	$v = 4.0 \sin(2t' + 288°)$	

Table 4.4.1 Tidal velocities determined from E-region drifts and computed tidal velocities at 98 km. Unit is 1 m/sec.

The drift velocities tend to be somewhat larger than the computed velocities, which is what one would expect, given the height difference between the levels to which the velocities pertain.

Some determinations of tidal winds at the surface have been made (Chapman and Lindzen, 1970). A comparison of the observed winds and the computed winds is shown in Table 4.4.2. Velocities at $z=0$ were computed using P & A as the ocean tidal forcing. The determinations are presented in the form

$$l_2 \sin(2t' + \lambda_2)$$

where t' is the local lunar time.

	Observed U ℓ_2	Observed U λ_2	Observed V ℓ_2	Observed V λ_2	Computed U ℓ_2	Computed U λ_2	Computed V ℓ_2	Computed V λ_2	At co-ordinates	
Upsala	0.75	179°	0.6	24°	0.1	333°	0.4	269°	60N	20E
Greensboro, NC	1.8	80°	1.3	11°	0.6	30°	0.6	249°	40N	280E
Hong Kong	2.2	69°	1.0	278°	0.3	253°	0.6	6°	20N	110E
San Juan, P.R.	0.6	253°	1.4	87°	0.8	329°	0.5	204°	20N	290E
Aguadilla	1.5	100°	1.5	65°	0.8	329°	0.5	204°	20N	290E
Balboa, Panama	0.6	195°	1.2	29°	1.2	41°	0.6	162°	10N	280E
Mauritius	1.0	220°	1.2	356°	1.2	355°	0.8	288°	20S	60E

Table 4.4.2 Comparison of observed surface tidal winds and winds computed at the coordinates shown. Unit is 1cm/sec.

There is no agreement between the computed and observed phases. The computed wind speeds generally underestimate observed wind speed.

4.5 Summary and Conclusions

The results of the previous chapters show that the lunar air tide is significantly influenced by both oceanic tides and solid earth tides. Our computations exhibited some measure of agreement with the observations, but the agreement was by no means perfect.

However, given the uncertainties of our knowledge of the ocean tide it is encouraging that the agreement was as good as it was.

It would appear that a good deal of work remains to be done before one could say that the semi-diurnal lunar air tide is fully understood. On the observational side there have been a total of 104 determinations of the tide in surface pressure. Many large gaps in our knowledge of the distribution of tidal parameters remain. For instance, there are no determinations available over most of Asia. As regards the ocean tide hardly any observations of the tide in the deep ocean have been made.

It was disappointing that over North America, where the air tide is well known, agreement between theory and observation was so poor. It was interesting nonetheless that P & A and B & M produced similar distributions of amplitude in this region. This raises the question of the effect of topography on the lunar tide. Wallace and Hartranft (1969) show that topography markedly affects the solar diurnal tide in the troposphere and lower stratosphere. However, it is not clear to what extent the effect is mechanical rather than thermal.

The model we used neglected dissipation. Geller (1969) studied the effect of infra-red cooling to space and found that our knowledge of the physical processes involved, particularly those involving ozone, was poor and uncertain. Because of this uncertainty it was thought best, at this time, not to pursue the matter in the present study.

In summary, it is felt that this study has achieved its primary purpose of showing that solid earth tides and ocean tides particularly must have a considerable effect on the semi-diurnal lunar air tide.

Appendix

Hough functions were computed following the method of Hough as presented by Flattery (1967). This method yields expressions for the Hough functions as sums of (completely normalized) Associated Legendre Polynomials, thus

$$H_n^l(\mu) = \sum_{r=|l|} C_{n,r}^l \, P_r^l(\mu) \qquad \text{A.1}$$

The $H_n^l(\mu)$ are normalized so that

$$\int_{-1}^{1} H_n^{l\,2}(\mu)\, d\mu = 1 \qquad \text{A.2}$$

or equivalently

$$\sum_{r=|l|} C_{n,r}^{l\,2} = 1 \qquad \text{A.3}$$

It follows that

$$P_r^l(\mu) = \sum_{n=|l|}^{\infty} C_{n,r}^l \, H_n^l(\mu) \qquad \text{A.4}$$

and

$$\sum_{n=|l|}^{\infty} C_{n,r}^{l\,2} = 1 \qquad \text{A.5}$$

For any given Hough function coefficients were calculated successively until the stage was reached where the addition of the square of the last coefficient when added to the sum of squares of the previous coefficients changed this sum by less than one part of 10^{10}. No further coefficients were determined and the coefficients were normalized so that the sum of their squares was 1.

To illustrate the use of the tables, suppose we want to find $C_{17,21}^{-14}$, the coefficient of $P_{21}^{-14}(\mu)$ in the expansion of $H_{17}^{-14}(\mu)$. In the table of expansion coefficients for anti-symmetric Hough functions of zonal wave number -14 we find the column headed $H(-14,17)$. This column lists the $C_{17,r}^{-14}$. In that row of the column labeled $P(-14,21)$ we find the value of $C_{17,21}^{-14}$ to be 0.101577.

The degree to which the Associated Legendre Polynomials are represented by the resulting sums of the form A.4 is shown by the sum of the squares of the relevant coefficients in the rightmost column. For a perfect representation this sum would be 1.

Shown above the expansion coefficients of a Hough function is the value of the eigenvalue pertaining to it. These are further tabulated in Table A.1.

TABLE OF EIGENVALUES β_n^ℓ

ZONAL WAVE NO. L	N: |L|	|L|+1	|L|+2	|L|+3	|L|+4	|L|+5	|L|+6	|L|+7
-14	217.3	263.4	314.6	371.1	433.0	500.2	572.9	651.0
-13	187.8	230.7	278.9	332.3	391.1	455.3	524.9	600.0
-12	160.4	200.3	245.3	295.7	351.4	412.5	479.0	551.1
-11	135.2	172.0	213.9	261.2	313.8	371.8	435.4	504.4
-10	112.1	145.8	184.7	228.8	278.4	333.4	393.9	460.0
-9	91.2	121.8	157.6	198.6	245.1	297.1	354.6	417.7
-8	72.5	99.9	132.6	170.6	214.0	263.0	317.5	377.6
-7	55.9	80.2	109.8	144.8	185.1	231.1	282.6	339.8
-6	41.5	62.7	89.2	121.1	158.4	201.4	249.9	304.2
-5	29.2	47.3	70.7	99.5	133.9	173.8	219.5	270.9
-4	19.0	34.1	54.4	80.2	111.6	148.6	191.4	239.9
-3	11.1	23.0	40.3	63.0	91.5	125.6	165.6	211.4
-2	5.2	14.1	28.3	48.1	73.7	105.0	142.2	185.4
-1	1.6	7.3	18.5	35.5	58.3	87.0	121.6	162.3
0	0.0	2.7	11.1	25.4	45.6	71.9	104.3	142.8*
1	6.7	18.8	37.1	61.7	92.6	129.9	173.6	223.6
2	12.5	27.1	47.7	74.2	106.7	145.4	190.2	241.2
3	20.5	38.0	61.3	90.4	125.4	166.3	213.3	266.3
4	30.6	51.2	77.3	109.2	146.9	190.4	239.9	295.3
5	43.0	66.5	95.6	130.4	170.9	217.2	269.3	327.3
6	57.5	84.1	116.2	153.9	197.3	246.4	301.3	362.0
7	74.1	103.8	138.9	179.6	225.9	277.9	335.7	399.2
8	93.0	125.7	163.9	207.6	256.8	311.7	372.4	438.7
9	113.9	149.7	191.0	237.7	289.9	347.8	411.3	480.6
10	137.1	176.0	220.2	270.0	325.3	386.1	452.6	524.7
11	162.3	204.3	251.7	304.5	362.8	426.6	496.0	571.1
12	189.8	234.9	285.3	341.1	402.4	469.3	541.7	619.7
13	219.4	267.6	321.1	380.0	444.3	514.2	589.6	670.6
14	251.1	302.4	359.0	420.9	488.3	561.2	639.7	723.7

* $\beta_8^0 =$ 187.4

TABLE A.1

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L=-14

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	217.281	314.629	432.990	572.884	
	H(-14,14)	H(-14,16)	H(-14,18)	H(-14,20)	SUM OF SQUARES
P(-14,14)	0.993105	0.115039	0.021765	0.005368	0.999993
P(-14,16)	-0.116399	0.945198	0.289659	0.089696	0.998895
P(-14,18)	0.013866	-0.298831	0.820911	0.436794	0.954176
P(-14,20)	-0.001464	0.062921	-0.467060	0.614712	0.599977
P(-14,22)	0.000135	-0.010321	0.149446	-0.587030	0.367044
P(-14,24)	-0.000011	0.001391	-0.034757	0.266966	0.072481
P(-14,26)	0.000001	-0.000158	0.006407	-0.083491	0.007012
P(-14,28)	-0.000000	0.000016	-0.000977	0.020121	0.000406
P(-14,30)	0.000000	-0.000001	0.000127	-0.003947	0.000016
P(-14,32)	-0.000000	0.000000	-0.000014	0.000651	0.000000
P(-14,34)	0.000000	-0.000000	0.000001	-0.000092	0.000000
P(-14,36)	0.0	0.000000	-0.000000	0.000011	0.000000
P(-14,38)	0.0	-0.000000	0.000000	-0.000001	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	263.368	371.147	500.219	651.031	
	H(-14,15)	H(-14,17)	H(-14,19)	H(-14,21)	SUM OF SQUARES
P(-14,15)	0.977580	0.203751	0.050647	0.014934	0.999966
P(-14,17)	-0.207778	0.893449	0.368963	0.137607	0.996491
P(-14,19)	0.033834	-0.386661	0.727511	0.488227	0.918290
P(-14,21)	-0.004520	0.101577	-0.535391	0.485598	0.532788
P(-14,23)	0.000509	-0.020009	0.205294	-0.617777	0.424195
P(-14,25)	-0.000049	0.003176	-0.055642	0.331407	0.112937
P(-14,27)	0.000004	-0.000421	0.011781	-0.118716	0.014232
P(-14,29)	-0.000000	0.000048	-0.002046	0.032329	0.001049
P(-14,31)	0.000000	-0.000005	0.000300	-0.007111	0.000051
P(-14,33)	-0.000000	0.000000	-0.000038	0.001309	0.000002
P(-14,35)	0.000000	-0.000000	0.000004	-0.000206	0.000000
P(-14,37)	0.0	0.000000	-0.000000	0.000028	0.000000
P(-14,39)	0.0	-0.000000	0.000000	-0.000003	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L=-13

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(-13,13)	H(-13,15)	H(-13,17)	H(-13,19)	SUM OF SQUARES
EIGEN VALUES	187.762	278.905	391.094	524.872	
P(-13,13)	0.993198	0.114213	0.021830	0.005506	0.999994
P(-13,15)	-0.115643	0.945335	0.289131	0.090523	0.998823
P(-13,17)	0.013504	-0.298860	0.820042	0.436757	0.952725
P(-13,19)	-0.001382	0.062273	-0.468932	0.610929	0.597011
P(-13,21)	0.000122	-0.010019	0.149465	-0.590070	0.370623
P(-13,23)	-0.000009	0.001314	-0.034371	0.268705	0.073385
P(-13,25)	0.000001	-0.000145	0.006224	-0.083589	0.007026
P(-13,27)	-0.000000	0.000014	-0.000927	0.019926	0.000398
P(-13,29)	0.000000	-0.000001	0.000117	-0.003848	0.000015
P(-13,31)	-0.000000	0.000000	-0.000013	0.000622	0.000000
P(-13,33)	0.0	-0.000000	0.000001	-0.000086	0.000000
P(-13,35)	0.0	0.000000	-0.000000	0.000010	0.000000
P(-13,37)	0.0	-0.000000	0.000000	-0.000001	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(-13,14)	H(-13,16)	H(-13,18)	H(-13,20)	SUM OF SQUARES
EIGEN VALUES	230.744	332.330	455.256	599.988	
P(-13,14)	0.977727	0.202929	0.050991	0.015319	0.999965
P(-13,16)	-0.207187	0.893298	0.368783	0.139070	0.996248
P(-13,18)	0.033245	-0.387581	0.725420	0.487895	0.915601
P(-13,20)	-0.004333	0.101118	-0.538045	0.479786	0.529931
P(-13,22)	0.000472	-0.019622	0.206070	-0.620614	0.428012
P(-13,24)	-0.000044	0.003046	-0.055401	0.334193	0.114764
P(-13,26)	0.000003	-0.000393	0.011566	-0.119376	0.014384
P(-13,28)	-0.000000	0.000043	-0.001971	0.032244	0.001044
P(-13,30)	0.000000	-0.000004	0.000282	-0.007003	0.000049
P(-13,32)	-0.000000	0.000000	-0.000035	0.001268	0.000002
P(-13,34)	0.000000	-0.000000	0.000004	-0.000196	0.000000
P(-13,36)	0.0	0.000000	-0.000000	0.000026	0.000000
P(-13,38)	0.0	-0.000000	0.000000	-0.000003	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L=-12

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(-12,12)	H(-12,14)	H(-12,16)	H(-12,18)	SUM OF SQUARES
EIGEN VALUES	160.398	245.335	351.357	479.033	
P(-12,12)	0.993305	0.113267	0.021886	0.005623	0.999995
P(-12,14)	-0.114770	0.945491	0.288529	0.091434	0.998734
P(-12,16)	0.013097	-0.298889	0.819052	0.436705	0.951065
P(-12,18)	-0.001293	0.061546	-0.471048	0.606648	0.593698
P(-12,20)	0.000109	-0.009687	0.149497	-0.593463	0.374642
P(-12,22)	-0.000008	0.001232	-0.033951	0.270673	0.074418
P(-12,24)	0.000000	-0.000131	0.006029	-0.083717	0.007045
P(-12,26)	-0.000000	0.000012	-0.000875	0.019721	0.000390
P(-12,28)	0.000000	-0.000001	0.000107	-0.003743	0.000014
P(-12,30)	-0.000000	0.000000	-0.000011	0.000592	0.000000
P(-12,32)	0.0	-0.000000	0.000001	-0.000080	0.000000
P(-12,34)	0.0	0.000000	-0.000000	0.000009	0.000000
P(-12,36)	0.0	-0.000000	0.000000	-0.000001	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(-12,13)	H(-12,15)	H(-12,17)	H(-12,19)	SUM OF SQUARES
EIGEN VALUES	200.274	295.670	412.459	551.126	
P(-12,13)	0.977894	0.201983	0.051354	0.015743	0.999960
P(-12,15)	-0.206504	0.893126	0.368576	0.140692	0.995961
P(-12,17)	0.032583	-0.388624	0.723048	0.487493	0.912537
P(-12,19)	-0.004128	0.100607	-0.541029	0.473235	0.526803
P(-12,21)	0.000433	-0.019198	0.206955	-0.623739	0.432250
P(-12,23)	-0.000038	0.002908	-0.055146	0.337327	0.116839
P(-12,25)	0.000003	-0.000363	0.011336	-0.120137	0.014562
P(-12,27)	-0.000000	0.000038	-0.001891	0.032167	0.001038
P(-12,29)	0.000000	-0.000003	0.000264	-0.006891	0.000048
P(-12,31)	-0.000000	0.000000	-0.000031	0.001225	0.000002
P(-12,33)	0.000000	-0.000000	0.000003	-0.000185	0.000000
P(-12,35)	0.0	0.000000	-0.000000	0.000024	0.000000
P(-12,37)	0.0	-0.000000	0.000000	-0.000003	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L=-11

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(-11,11)	H(-11,13)	H(-11,15)	H(-11,17)	SUM OF SQUARES
EIGEN VALUES	135.187	213.920	313.781	435.369	
P(-11,11)	0.993428	0.112157	0.021950	0.005774	0.999994
P(-11,13)	-0.113750	0.945671	0.287841	0.092451	0.998633
P(-11,15)	0.012636	-0.298916	0.817914	0.436637	0.949145
P(-11,17)	-0.001197	0.060723	-0.473458	0.601762	0.589969
P(-11,19)	0.000096	-0.009323	0.149547	-0.597273	0.379186
P(-11,21)	-0.000006	0.001146	-0.033492	0.272919	0.075608
P(-11,23)	0.000000	-0.000116	0.005819	-0.083885	0.007071
P(-11,25)	-0.000000	0.000010	-0.000821	0.019507	0.000381
P(-11,27)	0.000000	-0.000001	0.000097	-0.003634	0.000013
P(-11,29)	-0.000000	0.000000	-0.000010	0.000561	0.000000
P(-11,31)	0.0	-0.000000	0.000001	-0.000073	0.000000
P(-11,33)	0.0	0.000000	-0.000000	0.000008	0.000000
P(-11,35)	0.0	-0.000000	0.000000	-0.000001	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(-11,12)	H(-11,14)	H(-11,16)	H(-11,18)	SUM OF SQUARES
EIGEN VALUES	171.959	261.167	371.830	504.449	
P(-11,12)	0.978088	0.200882	0.051756	0.016241	0.999952
P(-11,14)	-0.205707	0.892930	0.368343	0.142508	0.995625
P(-11,16)	0.031832	-0.389816	0.720327	0.486999	0.909009
P(-11,18)	-0.003905	0.100035	-0.544409	0.465792	0.523366
P(-11,20)	0.000392	-0.018732	0.207976	-0.627195	0.436979
P(-11,22)	-0.000033	0.002760	-0.054877	0.340880	0.119218
P(-11,24)	0.000002	-0.000333	0.011091	-0.121025	0.014770
P(-11,26)	-0.000000	0.000034	-0.001808	0.032100	0.001034
P(-11,28)	0.000000	-0.000003	0.000245	-0.006775	0.000046
P(-11,30)	-0.000000	0.000000	-0.000028	0.001181	0.000001
P(-11,32)	0.000000	-0.000000	0.000003	-0.000174	0.000000
P(-11,34)	0.0	0.000000	-0.000000	0.000022	0.000000
P(-11,36)	0.0	-0.000000	0.000000	-0.000002	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L=-10

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	112.131	184.661	278.370	393.886	
	H(-10,10)	H(-10,12)	H(-10,14)	H(-10,16)	SUM OF SQUARES
P(-10,10)	0.993573	0.110849	0.022019	0.005947	0.999994
P(-10,12)	-0.112541	0.945881	0.287047	0.093591	0.998512
P(-10,14)	0.012110	-0.298942	0.816589	0.436545	0.946902
P(-10,16)	-0.001092	0.059787	-0.476229	0.596132	0.585743
P(-10,18)	0.000082	-0.008921	0.149621	-0.601580	0.384364
P(-10,20)	-0.000005	0.001054	-0.032989	0.275506	0.076993
P(-10,22)	0.000000	-0.000102	0.005594	-0.084108	0.007106
P(-10,24)	-0.000000	0.000008	-0.000765	0.019284	0.000372
P(-10,26)	0.000000	-0.000001	0.000087	-0.003520	0.000012
P(-10,28)	-0.000000	0.000000	-0.000008	0.000530	0.000000
P(-10,30)	0.0	-0.000000	0.000001	-0.000067	0.000000
P(-10,32)	0.0	0.000000	-0.000000	0.000007	0.000000
P(-10,34)	0.0	0.0	0.000000	-0.000001	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	145.799	228.823	333.371	459.965	
	H(-10,11)	H(-10,13)	H(-10,15)	H(-10,17)	SUM OF SQUARES
P(-10,11)	0.978314	0.199610	0.052197	0.016801	0.999949
P(-10,13)	-0.204767	0.892697	0.368075	0.144548	0.995210
P(-10,15)	0.030975	-0.391191	0.717178	0.486387	0.904907
P(-10,17)	-0.003660	0.099391	-0.548270	0.457263	0.519581
P(-10,19)	0.000350	-0.018218	0.209167	-0.631035	0.442288
P(-10,21)	-0.000028	0.002603	-0.054597	0.344940	0.121971
P(-10,23)	0.000002	-0.000302	0.010829	-0.122072	0.015019
P(-10,25)	-0.000000	0.000029	-0.001720	0.032049	0.001030
P(-10,27)	0.000000	-0.000002	0.000226	-0.006657	0.000044
P(-10,29)	-0.000000	0.000000	-0.000025	0.001136	0.000001
P(-10,31)	0.000000	-0.000000	0.000002	-0.000163	0.000000
P(-10,33)	0.0	0.000000	-0.000000	0.000020	0.000000
P(-10,35)	0.0	-0.000000	0.000000	-0.000002	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= -9

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(-9, 9)	H(-9,11)	H(-9,13)	H(-9,15)	SUM OF SQUARES
EIGEN VALUES	91.230	157.557	245.123	354.590	
P(-9, 9)	0.993744	0.109283	0.022065	0.006164	0.999994
P(-9,11)	-0.111088	0.946128	0.286115	0.094879	0.998363
P(-9,13)	0.011504	-0.298963	0.815032	0.436423	0.944252
P(-9,15)	-0.000978	0.058711	-0.479451	0.589572	0.580917
P(-9,17)	0.000068	-0.008476	0.149733	-0.606488	0.390320
P(-9,19)	-0.000004	0.000957	-0.032438	0.278519	0.078626
P(-9,21)	0.000000	-0.000088	0.005353	-0.084405	0.007153
P(-9,23)	-0.000000	0.000007	-0.000706	0.019054	0.000364
P(-9,25)	0.000000	-0.000000	0.000077	-0.003401	0.000012
P(-9,27)	-0.000000	0.000000	-0.000007	0.000497	0.000000
P(-9,29)	0.0	-0.000000	0.000001	-0.000061	0.000000
P(-9,31)	0.0	0.000000	-0.000000	0.000006	0.000000
P(-9,33)	0.0	0.0	0.000000	-0.000001	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(-9,10)	H(-9,12)	H(-9,14)	H(-9,16)	SUM OF SQUARES
EIGEN VALUES	121.792	198.638	297.087	417.682	
P(-9,10)	0.978581	0.198094	0.052683	0.017435	0.999941
P(-9,12)	-0.203640	0.892422	0.367766	0.146855	0.994705
P(-9,14)	0.029987	-0.392797	0.713489	0.485615	0.900077
P(-9,16)	-0.003391	0.098662	-0.552721	0.447393	0.515407
P(-9,18)	0.000306	-0.017648	0.210575	-0.635321	0.448286
P(-9,20)	-0.000023	0.002434	-0.054307	0.349625	0.125193
P(-9,22)	0.000001	-0.000270	0.010550	-0.123320	0.015319
P(-9,24)	-0.000000	0.000025	-0.001629	0.032022	0.001028
P(-9,26)	0.000000	-0.000002	0.000207	-0.006536	0.000043
P(-9,28)	-0.000000	0.000000	-0.000022	0.001090	0.000001
P(-9,30)	0.000000	-0.000000	0.000002	-0.000152	0.000000
P(-9,32)	0.0	0.000000	-0.000000	0.000018	0.000000
P(-9,34)	0.0	-0.000000	0.000000	-0.000002	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= -8

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	72.482	132.610	214.045	317.488	
	H(-8, 8)	H(-8,10)	H(-8,12)	H(-8,14)	SUM OF SQUARES
P(-8, 8)	0.993949	0.107380	0.022081	0.006372	0.999993
P(-8,10)	-0.109308	0.946423	0.285011	0.096335	0.998176
P(-8,12)	0.010801	-0.298977	0.813171	0.436255	0.941069
P(-8,14)	-0.000856	0.057463	-0.483244	0.581835	0.575360
P(-8,16)	0.000054	-0.007982	0.149898	-0.612133	0.397240
P(-8,18)	-0.000003	0.000855	-0.031832	0.282073	0.080579
P(-8,20)	0.000000	-0.000073	0.005093	-0.084802	0.007217
P(-8,22)	-0.000000	0.000005	-0.000646	0.018821	0.000355
P(-8,24)	0.000000	-0.000000	0.000067	-0.003278	0.000011
P(-8,26)	-0.000000	0.000000	-0.000006	0.000465	0.000000
P(-8,28)	0.0	-0.000000	0.000000	-0.000055	0.000000
P(-8,30)	0.0	0.000000	-0.000000	0.000006	0.000000
P(-8,32)	0.0	0.0	0.000000	-0.000000	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	99.941	170.613	262.983	377.612	
	H(-8, 9)	H(-8,11)	H(-8,13)	H(-8,15)	SUM OF SQUARES
P(-8, 9)	0.978901	0.196270	0.053197	0.018160	0.999929
P(-8,11)	-0.202266	0.892091	0.367402	0.149487	0.994068
P(-8,13)	0.028837	-0.394699	0.709111	0.484627	0.894321
P(-8,15)	-0.003095	0.097830	-0.557913	0.435835	0.510800
P(-8,17)	0.000260	-0.017014	0.212262	-0.640130	0.455111
P(-8,19)	-0.000018	0.002254	-0.054013	0.355093	0.129013
P(-8,21)	0.000001	-0.000238	0.010252	-0.124830	0.015688
P(-8,23)	-0.000000	0.000021	-0.001534	0.032032	0.001028
P(-8,25)	0.000000	-0.000001	0.000187	-0.006416	0.000041
P(-8,27)	-0.000000	0.000000	-0.000019	0.001044	0.000001
P(-8,29)	0.0	-0.000000	0.000002	-0.000142	0.000000
P(-8,31)	0.0	0.000000	-0.000000	0.000016	0.000000
P(-8,33)	0.0	-0.000000	0.000000	-0.000002	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= -7

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(-7, 7)	H(-7, 9)	H(-7,11)	H(-7,13)	SUM OF SQUARES
EIGEN VALUES	55.889	109.819	185.141	282.593	
P(-7, 7)	0.994200	0.105014	0.022054	0.006593	0.999991
P(-7, 9)	-0.107081	0.946779	0.283682	0.097996	0.997936
P(-7,11)	0.009976	-0.298980	0.810908	0.436024	0.937178
P(-7,13)	-0.000724	0.056002	-0.487778	0.572569	0.568899
P(-7,15)	0.000041	-0.007431	0.150145	-0.618691	0.405377
P(-7,17)	-0.000002	0.000749	-0.031167	0.286329	0.082956
P(-7,19)	0.000000	-0.000060	0.004815	-0.085341	0.007306
P(-7,21)	-0.000000	0.000004	-0.000583	0.018589	0.000346
P(-7,23)	0.000000	-0.000000	0.000057	-0.003151	0.000010
P(-7,25)	0.0	0.000000	-0.000005	0.000432	0.000000
P(-7,27)	0.0	-0.000000	0.000000	-0.000049	0.000000
P(-7,29)	0.0	0.000000	-0.000000	0.000005	0.000000
P(-7,31)	0.0	0.0	0.000000	-0.000000	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(-7, 8)	H(-7,10)	H(-7,12)	H(-7,14)	SUM OF SQUARES
EIGEN VALUES	80.243	144.751	231.067	339.771	
P(-7, 8)	0.979292	0.194035	0.053761	0.019010	0.999914
P(-7,10)	-0.200558	0.891682	0.366978	0.152521	0.993255
P(-7,12)	0.027484	-0.396988	0.703822	0.483340	0.887338
P(-7,14)	-0.002769	0.096877	-0.564047	0.422114	0.505721
P(-7,16)	0.000215	-0.016306	0.214324	-0.645548	0.462933
P(-7,18)	-0.000013	0.002062	-0.053727	0.361560	0.133616
P(-7,20)	0.000001	-0.000205	0.009936	-0.126688	0.016149
P(-7,22)	-0.000000	0.000017	-0.001436	0.032096	0.001032
P(-7,24)	0.000000	-0.000001	0.000168	-0.006298	0.000040
P(-7,26)	-0.000000	0.000000	-0.000016	0.000997	0.000001
P(-7,28)	0.0	-0.000000	0.000001	-0.000131	0.000000
P(-7,30)	0.0	0.000000	-0.000000	0.000015	0.000000
P(-7,32)	0.0	-0.000000	0.000000	-0.000001	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= -6

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	41.451	89.186	158.416	249.922	
	H(-6, 6)	H(-6, 8)	H(-6,10)	H(-6,12)	SUM OF SQUARES
P(-6, 6)	0.994514	0.101997	0.021948	0.006875	0.999990
P(-6, 8)	-0.104216	0.947219	0.282058	0.099916	0.997624
P(-6,10)	0.009001	-0.298967	0.808095	0.435706	0.932319
P(-6,12)	-0.000585	0.054268	-0.493296	0.561260	0.561299
P(-6,14)	0.0	-0.006816	0.150521	-0.626399	0.415078
P(-6,16)	0.0	0.000638	-0.030437	0.291520	0.085911
P(-6,18)	0.0	0.0	0.004518	-0.086088	0.007432
P(-6,20)	0.0	0.0	-0.000520	0.018369	0.000338
P(-6,22)	0.0	0.0	0.0	-0.003022	0.000009
P(-6,24)	0.0	0.0	0.0	0.000399	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	62.701	121.056	201.351	304.182	
	H(-6, 7)	H(-6, 9)	H(-6,11)	H(-6,13)	SUM OF SQUARES
P(-6, 7)	0.979781	0.191233	0.054344	0.019992	0.999893
P(-6, 9)	-0.198380	0.891163	0.366473	0.156052	0.992181
P(-6,11)	0.025873	-0.399801	0.697310	0.481622	0.878711
P(-6,13)	-0.002411	0.095779	-0.571407	0.405557	0.500162
P(-6,15)	0.000169	-0.015513	0.216896	-0.651678	0.471969
P(-6,17)	0.0	0.001858	-0.053468	0.369328	0.139266
P(-6,19)	0.0	-0.000173	0.009603	-0.129022	0.016739
P(-6,21)	0.0	0.0	-0.001334	0.032244	0.001041
P(-6,23)	0.0	0.0	0.000149	-0.006188	0.000038
P(-6,25)	0.0	0.0	0.0	0.000951	0.000001
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000001	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= -5

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	29.166	70.712	133.882	219.498	
	H(-5, 5)	H(-5, 7)	H(-5, 9)	H(-5,11)	SUM OF SQUARES
P(-5, 5)	0.994916	0.098022	0.021698	0.007133	0.999988
P(-5, 7)	-0.100402	0.947772	0.280030	0.102138	0.997201
P(-5, 9)	0.007835	-0.298931	0.804498	0.435249	0.926081
P(-5,11)	-0.000440	0.052184	-0.500166	0.547153	0.552266
P(-5,13)	0.0	-0.006128	0.151103	-0.635583	0.426835
P(-5,15)	0.0	0.000525	-0.029641	0.297994	0.089679
P(-5,17)	0.0	0.0	0.004201	-0.087142	0.007611
P(-5,19)	0.0	0.0	-0.000456	0.018178	0.000331
P(-5,21)	0.0	0.0	0.0	-0.002894	0.000008
P(-5,23)	0.0	0.0	0.0	0.000366	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	47.312	99.531	173.850	270.878	
	H(-5, 6)	H(-5, 8)	H(-5,10)	H(-5,12)	SUM OF SQUARES
P(-5, 6)	0.980407	0.187622	0.054922	0.021129	0.999862
P(-5, 8)	-0.195515	0.890482	0.365872	0.160213	0.990714
P(-5,10)	0.023926	-0.403350	0.689085	0.479260	0.867792
P(-5,12)	-0.002021	0.094511	-0.580407	0.385179	0.494172
P(-5,14)	0.000125	-0.014623	0.220196	-0.658626	0.482488
P(-5,16)	0.0	0.001642	-0.053270	0.378839	0.146360
P(-5,18)	0.0	-0.000141	0.009258	-0.132026	0.017517
P(-5,20)	0.0	0.0	-0.001230	0.032520	0.001059
P(-5,22)	0.0	0.0	0.000130	-0.006093	0.000037
P(-5,24)	0.0	0.0	0.0	0.000908	0.000001
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000001	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= -4

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	19.035	54.401	111.555	191.360	
	H(-4, 4)	H(-4, 6)	H(-4, 8)	H(-4,10)	SUM OF SQUARES
P(-4, 4)	0.995448	0.092552	0.021178	0.007358	0.999985
P(-4, 6)	-0.095091	0.948483	0.277438	0.104736	0.996605
P(-4, 8)	0.006437	-0.298865	0.799732	0.434576	0.917790
P(-4,10)	-0.000297	0.049644	-0.508968	0.529046	0.541402
P(-4,12)	0.0	-0.005360	0.152039	-0.646695	0.441359
P(-4,14)	0.0	0.000412	-0.028790	0.306299	0.094648
P(-4,16)	0.0	0.0	0.003867	-0.088676	0.007878
P(-4,18)	0.0	0.0	-0.000392	0.018047	0.000326
P(-4,20)	0.0	0.0	0.0	-0.002771	0.000008
P(-4,22)	0.0	0.0	0.0	0.000335	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	34.079	80.186	148.591	239.910	
	H(-4, 5)	H(-4, 7)	H(-4, 9)	H(-4,11)	SUM OF SQUARES
P(-4, 5)	0.981237	0.182797	0.055422	0.022454	0.999817
P(-4, 7)	-0.191592	0.889544	0.365154	0.165201	0.988625
P(-4, 9)	0.021540	-0.407983	0.678361	0.475894	0.853563
P(-4,11)	-0.001601	0.093053	-0.591669	0.359464	0.487947
P(-4,13)	0.000085	-0.013624	0.224574	-0.666479	0.494814
P(-4,15)	0.0	0.001418	-0.053202	0.390763	0.155528
P(-4,17)	0.0	-0.000111	0.008910	-0.136012	0.018579
P(-4,19)	0.0	0.0	-0.001127	0.033003	0.001090
P(-4,21)	0.0	0.0	0.000112	-0.006027	0.000036
P(-4,23)	0.0	0.0	0.0	0.000868	0.000001
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000001	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= -3

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	11.058	40.256	91.463	165.571	
	H(-3, 3)	H(-3, 5)	H(-3, 7)	H(-3, 9)	SUM OF SQUARES
P(-3, 3)	0.996176	0.084576	0.020113	0.007450	0.999981
P(-3, 5)	-0.087233	0.949419	0.274032	0.107799	0.995721
P(-3, 7)	0.004768	-0.298776	0.793102	0.433535	0.906253
P(-3, 9)	-0.000165	0.046500	-0.520676	0.504924	0.528214
P(-3,11)	0.0	-0.004509	0.153611	-0.660379	0.459717
P(-3,13)	0.0	0.000302	-0.027912	0.317349	0.101490
P(-3,15)	0.0	0.0	0.003523	-0.090996	0.008293
P(-3,17)	0.0	0.0	-0.000330	0.018036	0.000325
P(-3,19)	0.0	0.0	0.0	-0.002662	0.000007
P(-3,21)	0.0	0.0	0.0	0.000307	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	23.000	63.036	125.621	211.362	
	H(-3, 4)	H(-3, 6)	H(-3, 8)	H(-3,10)	SUM OF SQUARES
P(-3, 4)	0.982386	0.176039	0.055676	0.023938	0.999746
P(-3, 6)	-0.185932	0.888165	0.364315	0.171289	0.985473
P(-3, 8)	0.018571	-0.414319	0.663773	0.470857	0.834306
P(-3,10)	-0.001160	0.091408	-0.606180	0.325970	0.482068
P(-3,12)	0.000050	-0.012511	0.230652	-0.675235	0.509300
P(-3,14)	0.0	0.001187	-0.053397	0.406170	0.167826
P(-3,16)	0.0	-0.000083	0.008579	-0.141515	0.020100
P(-3,18)	0.0	0.0	-0.001026	0.033830	0.001146
P(-3,20)	0.0	0.0	0.000095	-0.006015	0.000036
P(-3,22)	0.0	0.0	0.0	0.000835	0.000001
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000001	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= -2

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	5.233	28.291	73.661	142.245	
	H(-2, 2)	H(-2, 4)	H(-2, 6)	H(-2, 8)	SUM OF SQUARES
P(-2, 2)	0.997212	0.071942	0.017875	0.007129	0.999977
P(-2, 4)	-0.074568	0.950658	0.269415	0.111437	0.994314
P(-2, 6)	0.002848	-0.298745	0.783215	0.431799	0.889132
P(-2, 8)	-0.000062	0.042565	-0.537079	0.471109	0.512210
P(-2,10)	0.0	-0.003581	0.156436	-0.677547	0.483555
P(-2,12)	0.0	0.000201	-0.027101	0.332802	0.111492
P(-2,14)	0.0	0.0	0.003182	-0.094707	0.008980
P(-2,16)	0.0	0.0	-0.000272	0.018268	0.000334
P(-2,18)	0.0	0.0	0.0	-0.002586	0.000007
P(-2,20)	0.0	0.0	0.0	0.000282	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	14.077	48.111	105.020	185.381	
	H(-2, 3)	H(-2, 5)	H(-2, 7)	H(-2, 9)	SUM OF SQUARES
P(-2, 3)	0.984072	0.165933	0.055261	0.025453	0.999634
P(-2, 5)	-0.177148	0.885927	0.363396	0.178922	0.980318
P(-2, 7)	0.014837	-0.423591	0.642715	0.462791	0.806908
P(-2, 9)	-0.000724	0.089681	-0.625610	0.280433	0.478073
P(-2,11)	0.0	-0.011294	0.239623	-0.684569	0.526181
P(-2,13)	0.0	0.000958	-0.054142	0.426900	0.185176
P(-2,15)	0.0	0.0	0.008309	-0.149537	0.022430
P(-2,17)	0.0	0.0	-0.000934	0.035276	0.001245
P(-2,19)	0.0	0.0	0.0	-0.006106	0.000037
P(-2,21)	0.0	0.0	0.0	0.000816	0.000001
SUM OF SQUARES	1.000001	1.000001	1.000001	1.000001	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= -1

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(-1, 1)	H(-1, 3)	H(-1, 5)	H(-1, 7)	SUM OF SQUARES
EIGEN VALUES	1.556	18.535	58.270	121.614	
P(-1, 1)	0.998679	0.049264	0.012806	0.005559	0.999982
P(-1, 3)	-0.051378	0.952150	0.263017	0.115812	0.991820
P(-1, 5)	0.000938	-0.299262	0.766798	0.428497	0.861147
P(-1, 7)	0.0	0.037664	-0.561875	0.420007	0.493528
P(-1, 9)	0.0	-0.002614	0.162046	-0.699403	0.515430
P(-1,11)	0.0	0.000116	-0.026624	0.356053	0.127482
P(-1,13)	0.0	0.0	0.002880	-0.101164	0.010242
P(-1,15)	0.0	0.0	-0.000222	0.019036	0.000362
P(-1,17)	0.0	0.0	0.0	-0.002586	0.000007
P(-1,19)	0.0	0.0	0.0	0.000267	0.000000
SUM OF SQUARES	1.000001	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(-1, 2)	H(-1, 4)	H(-1, 6)	H(-1, 8)	SUM OF SQUARES
EIGEN VALUES	7.310	35.485	86.962	162.267	
P(-1, 2)	0.986742	0.149281	0.053028	0.026463	0.999457
P(-1, 4)	-0.161979	0.881689	0.362637	0.188863	0.970787
P(-1, 6)	0.010160	-0.438673	0.609477	0.448472	0.765127
P(-1, 8)	-0.000337	0.088350	-0.653013	0.214582	0.480278
P(-1,10)	0.0	-0.010043	0.254114	-0.693032	0.544969
P(-1,12)	0.0	0.000743	-0.056138	0.456475	0.211521
P(-1,14)	0.0	0.0	0.008213	-0.162213	0.026381
P(-1,16)	0.0	0.0	-0.000864	0.037956	0.001441
P(-1,18)	0.0	0.0	0.0	-0.006415	0.000041
P(-1,20)	0.0	0.0	0.0	0.000828	0.000001
SUM OF SQUARES	1.000000	1.000001	1.000001	1.000001	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= 0

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(0, 2)	H(0, 4)	H(0, 6)	H(0, 8)	SUM OF SQUARES
EIGEN VALUES	11.101	45.617	104.274	187.445	
P(0, 0)	0.0	0.0	0.0	0.0	0.0
P(0, 2)	0.952265	0.254643	0.121459	0.073174	0.991758
P(0, 4)	-0.303573	0.733842	0.420358	0.267869	0.879134
P(0, 6)	0.032122	-0.604289	0.332460	0.358185	0.605023
P(0, 8)	-0.001716	0.175243	-0.726672	-0.096282	0.568036
P(0,10)	0.000055	-0.027412	0.395561	-0.600398	0.517698
P(0,12)	0.0	0.002728	-0.114186	0.587402	0.358087
P(0,14)	0.0	-0.000189	0.021211	-0.270813	0.073790
P(0,16)	0.0	0.0	-0.002789	0.078727	0.006206
P(0,18)	0.0	0.0	0.000275	-0.016185	0.000262
P(0,20)	0.0	0.0	0.0	0.002509	0.000006
P(0,22)	0.0	0.0	0.0	-0.000306	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(0, 1)	H(0, 3)	H(0, 5)	H(0, 7)	SUM OF SQUARES
EIGEN VALUES	2.703	25.373	71.900	142.778	
P(0, 1)	0.991408	0.117078	0.045234	0.024754	0.999256
P(0, 3)	-0.130722	0.871187	0.363024	0.202577	0.948878
P(0, 5)	0.004661	-0.468204	0.548326	0.418120	0.694722
P(0, 7)	-0.000077	0.089609	-0.694633	0.109364	0.502505
P(0, 9)	0.0	-0.009060	0.281306	-0.694556	0.561622
P(0,11)	0.0	0.000572	-0.061520	0.502887	0.256681
P(0,13)	0.0	0.0	0.008643	-0.185214	0.034379
P(0,15)	0.0	0.0	-0.000855	0.043637	0.001905
P(0,17)	0.0	0.0	0.0	-0.007288	0.000053
P(0,19)	0.0	0.0	0.0	0.000917	0.000001
SUM OF SQUARES	1.000000	1.000000	1.000001	1.000001	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= 1

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(1, 1)	H(1, 3)	H(1, 5)	H(1, 7)	SUM OF SQUARES
EIGEN VALUES	6.734	37.095	92.604	173.567	
P(1, 1)	0.937637	0.257775	0.135686	0.088029	0.971771
P(1, 3)	-0.346085	0.632025	0.383567	0.259137	0.733507
P(1, 5)	0.032555	-0.696041	0.137349	0.208198	0.547744
P(1, 7)	-0.001384	0.220006	-0.747753	-0.277007	0.684272
P(1, 9)	0.000034	-0.034773	0.482477	-0.491967	0.476024
P(1,11)	-0.000001	0.003351	-0.151042	0.656672	0.454043
P(1,13)	0.000000	-0.000219	0.029200	-0.345514	0.120233
P(1,15)	-0.000000	0.000010	-0.003896	0.108729	0.011837
P(1,17)	0.0	-0.000000	0.000383	-0.023534	0.000554
P(1,19)	0.0	0.000000	-0.000029	0.003772	0.000014
P(1,21)	0.0	-0.000000	0.000002	-0.000469	0.000000
P(1,23)	0.0	0.000000	-0.000000	0.000047	0.000000
P(1,25)	0.0	0.0	0.000000	-0.000004	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(1, 2)	H(1, 4)	H(1, 6)	H(1, 8)	SUM OF SQUARES
EIGEN VALUES	18.808	61.679	129.896	223.620	
P(1, 2)	0.825998	0.366408	0.220813	0.152498	0.888541
P(1, 4)	-0.553053	0.390970	0.323962	0.246856	0.624614
P(1, 6)	0.108415	-0.763298	-0.095008	0.062183	0.607270
P(1, 8)	-0.010310	0.351536	-0.652583	-0.388353	0.700366
P(1,10)	0.000583	-0.080954	0.590795	-0.289251	0.439259
P(1,12)	-0.000022	0.011502	-0.242018	0.666004	0.502267
P(1,14)	0.000001	-0.001121	0.060625	-0.448789	0.205088
P(1,16)	0.0	0.000080	-0.010490	0.174033	0.030398
P(1,18)	0.0	-0.000004	0.001343	-0.045965	0.002115
P(1,20)	0.0	0.0	-0.000133	0.008971	0.000080
P(1,22)	0.0	0.0	0.000011	-0.001360	0.000002
P(1,24)	0.0	0.0	-0.000001	0.000166	0.000000
P(1,26)	0.0	0.0	0.0	-0.000017	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= 2

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	12.462	47.660	106.743	190.214	
	H(2, 2)	H(2, 4)	H(2, 6)	H(2, 8)	SUM OF SQUARES
P(2, 2)	0.962708	0.228431	0.105893	0.062925	0.994161
P(2, 4)	-0.268877	0.750658	0.421970	0.266311	0.884763
P(2, 6)	0.029910	-0.594196	0.342066	0.364523	0.603850
P(2, 8)	-0.001739	0.174547	-0.723269	-0.092896	0.562217
P(2,10)	0.000062	-0.028058	0.395859	-0.597958	0.515045
P(2,12)	-0.000001	0.002888	-0.115799	0.587225	0.358251
P(2,14)	0.000000	-0.000208	0.021874	-0.272972	0.074992
P(2,16)	-0.000000	0.000011	-0.002931	0.080172	0.006436
P(2,18)	0.000000	-0.000000	0.000295	-0.016672	0.000278
P(2,20)	0.0	0.000000	-0.000023	0.002616	0.000007
P(2,22)	0.0	-0.000000	0.000001	-0.000323	0.000000
P(2,24)	0.0	0.000000	-0.000000	0.000032	0.000000
P(2,26)	0.0	0.0	0.000000	-0.000003	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	27.121	74.177	145.409	241.189	
	H(2, 3)	H(2, 5)	H(2, 7)	H(2, 9)	SUM OF SQUARES
P(2, 3)	0.888712	0.355676	0.195323	0.126179	0.970386
P(2, 5)	-0.449822	0.561821	0.423556	0.304346	0.790009
P(2, 7)	0.088118	-0.689011	0.115459	0.256909	0.561834
P(2, 9)	-0.009316	0.281261	-0.692005	-0.260292	0.625818
P(2,11)	0.000620	-0.062649	0.503138	-0.451779	0.461178
P(2,13)	-0.000029	0.009006	-0.187203	0.633965	0.437038
P(2,15)	0.000001	-0.000914	0.044675	-0.365669	0.135710
P(2,17)	-0.000000	0.000069	-0.007570	0.129912	0.016934
P(2,19)	0.000000	-0.000004	0.000967	-0.032457	0.001054
P(2,21)	-0.000000	0.000000	-0.000097	0.006111	0.000037
P(2,23)	0.0	-0.000000	0.000008	-0.000906	0.000001
P(2,25)	0.0	0.000000	-0.000001	0.000109	0.000000
P(2,27)	0.0	-0.000000	0.000000	-0.000011	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= 3

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(3, 3)	H(3, 5)	H(3, 7)	H(3, 9)	SUM OF SQUARES
EIGEN VALUES	20.450	61.276	125.379	213.292	
P(3, 3)	0.972315	0.207817	0.085685	0.046175	0.998058
P(3, 5)	-0.231893	0.803626	0.421437	0.246234	0.937829
P(3, 7)	0.028728	-0.535644	0.450442	0.429225	0.674872
P(3, 9)	-0.002079	0.153039	-0.692498	0.030425	0.503905
P(3,11)	0.000098	-0.025572	0.349571	-0.633159	0.523744
P(3,13)	-0.000003	0.002847	-0.100123	0.538898	0.300444
P(3,15)	0.0	-0.000228	0.019134	-0.236562	0.056328
P(3,17)	0.0	0.000014	-0.002651	0.067900	0.004617
P(3,19)	0.0	-0.000001	0.000280	-0.014086	0.000198
P(3,21)	0.0	0.0	-0.000023	0.002237	0.000005
P(3,23)	0.0	0.0	0.000002	-0.000282	0.000000
P(3,25)	0.0	0.0	0.0	0.000029	0.000000
P(3,27)	0.0	0.0	0.0	-0.000003	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(3, 4)	H(3, 6)	H(3, 8)	H(3,10)	SUM OF SQUARES
EIGEN VALUES	38.001	90.377	166.336	266.285	
P(3, 4)	0.914728	0.336790	0.168334	0.100186	0.988528
P(3, 6)	-0.396181	0.644654	0.453122	0.304480	0.870566
P(3, 8)	0.078918	-0.638191	0.238805	0.354340	0.596101
P(3,10)	-0.009208	0.246194	-0.691316	-0.153864	0.562288
P(3,12)	0.000710	-0.055014	0.451499	-0.523044	0.480454
P(3,14)	-0.000039	0.008217	-0.161376	0.599037	0.364956
P(3,16)	0.000002	-0.000887	0.038216	-0.320588	0.104238
P(3,18)	0.0	0.000073	-0.006560	0.109856	0.012111
P(3,20)	0.0	-0.000005	0.000862	-0.027043	0.000732
P(3,22)	0.0	0.0	-0.000090	0.005089	0.000026
P(3,24)	0.0	0.0	0.000008	-0.000762	0.000001
P(3,26)	0.0	0.0	0.0	0.000093	0.000000
P(3,28)	0.0	0.0	0.0	-0.000010	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= 4

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(4, 4)	H(4, 6)	H(4, 8)	H(4,10)	SUM OF SQUARES
EIGEN VALUES	30.624	77.317	146.895	239.899	
P(4, 4)	0.977301	0.193726	0.072442	0.035731	0.999171
P(4, 6)	-0.210008	0.834156	0.413655	0.225647	0.961947
P(4, 8)	0.027830	-0.496544	0.519861	0.461379	0.730456
P(4,10)	-0.002346	0.139656	-0.665695	0.120177	0.477102
P(4,12)	0.000136	-0.024202	0.318918	-0.646908	0.520785
P(4,14)	0.0	0.002890	-0.090608	0.502749	0.260975
P(4,16)	0.0	-0.000254	0.017641	-0.213090	0.045718
P(4,18)	0.0	0.0	-0.002537	0.060616	0.003681
P(4,20)	0.0	0.0	0.000282	-0.012675	0.000161
P(4,22)	0.0	0.0	0.0	0.002054	0.000004
P(4,24)	0.0	0.0	0.0	-0.000267	0.000000
P(4,26)	0.0	0.0	0.0	0.0	0.0
P(4,28)	0.0	0.0	0.0	0.0	0.0
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(4, 5)	H(4, 7)	H(4, 9)	H(4,11)	SUM OF SQUARES
EIGEN VALUES	51.157	109.205	190.444	295.298	
P(4, 5)	0.929039	0.320838	0.147865	0.081563	0.994567
P(4, 7)	-0.362547	0.694930	0.462367	0.292462	0.913686
P(4, 9)	0.073219	-0.601194	0.322574	0.411085	0.639840
P(4,11)	-0.009234	0.223799	-0.682100	-0.068822	0.520168
P(4,13)	0.000801	-0.050528	0.415591	-0.561994	0.491107
P(4,15)	0.0	0.007851	-0.145242	0.569023	0.344944
P(4,17)	0.0	-0.000900	0.034505	-0.290409	0.085529
P(4,19)	0.0	0.0	-0.006047	0.097615	0.009565
P(4,21)	0.0	0.0	0.000821	-0.023973	0.000575
P(4,23)	0.0	0.0	0.0	0.004555	0.000021
P(4,25)	0.0	0.0	0.0	-0.000695	0.000000
P(4,27)	0.0	0.0	0.0	0.0	0.0
P(4,29)	0.0	0.0	0.0	0.0	0.0
SUM OF SQUARES	1.000001	1.000001	1.000001	1.000000	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= 5

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(5, 5)	H(5, 7)	H(5, 9)	H(5,11)	SUM OF SQUARES
EIGEN VALUES	42.965	95.639	170.911	269.323	
P(5, 5)	0.980312	0.183652	0.063317	0.028881	0.999584
P(5, 7)	-0.195571	0.853986	0.404731	0.207979	0.974602
P(5, 9)	0.027081	-0.468446	0.568401	0.478485	0.772202
P(5,11)	-0.002548	0.130334	-0.643204	0.188673	0.466302
P(5,13)	0.000172	-0.023300	0.296719	-0.651542	0.513092
P(5,15)	0.0	0.002953	-0.084054	0.474436	0.232163
P(5,17)	0.0	-0.000281	0.016688	-0.196284	0.038806
P(5,19)	0.0	0.0	-0.002487	0.055670	0.003105
P(5,21)	0.0	0.0	0.000291	-0.011775	0.000139
P(5,23)	0.0	0.0	0.0	0.001951	0.000004
P(5,25)	0.0	0.0	0.0	-0.000262	0.000000
P(5,27)	0.0	0.0	0.0	0.0	0.0
P(5,29)	0.0	0.0	0.0	0.0	0.0
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(5, 6)	H(5, 8)	H(5,10)	H(5,12)	SUM OF SQUARES
EIGEN VALUES	66.523	130.411	217.200	327.320	
P(5, 6)	0.938037	0.308094	0.132514	0.068285	0.997058
P(5, 8)	-0.339422	0.728778	0.463823	0.278180	0.938841
P(5,10)	0.069222	-0.573059	0.383587	0.446685	0.679854
P(5,12)	-0.009277	0.207966	-0.670867	-0.000004	0.493398
P(5,14)	0.000885	-0.047491	0.388729	-0.585206	0.495832
P(5,16)	-0.000063	0.007651	-0.133926	0.543647	0.313546
P(5,18)	0.000004	-0.000926	0.032041	-0.268257	0.072989
P(5,20)	-0.000000	0.000088	-0.005741	0.089145	0.007980
P(5,22)	0.000000	-0.000007	0.000807	-0.021955	0.000483
P(5,24)	-0.000000	0.000000	-0.000092	0.004228	0.000018
P(5,26)	0.000000	-0.000000	0.000009	-0.000659	0.000000
P(5,28)	-0.000000	0.000000	-0.000001	0.000085	0.000000
P(5,30)	0.000000	-0.000000	0.000000	-0.000009	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= 6

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(6, 6)	H(6, 8)	H(6,10)	H(6,12)	SUM OF SQUARES
EIGEN VALUES	57.469	116.186	197.281	301.286	
P(6, 6)	0.982313	0.176141	0.056727	0.024148	0.999765
P(6, 8)	-0.185352	0.867859	0.396258	0.193316	0.981927
P(6,10)	0.026444	-0.447262	0.604253	0.487747	0.803762
P(6,12)	-0.002700	0.123406	-0.624358	0.242647	0.463937
P(6,14)	0.000204	-0.022643	0.279785	-0.651783	0.503614
P(6,16)	0.0	0.003018	-0.079207	0.451588	0.210215
P(6,18)	0.0	-0.000308	0.016021	-0.183525	0.033938
P(6,20)	0.0	0.0	-0.002468	0.052050	0.002715
P(6,22)	0.0	0.0	0.000302	-0.011149	0.000124
P(6,24)	0.0	0.0	0.0	0.001869	0.000004
P(6,26)	0.0	0.0	0.0	-0.000261	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(6, 7)	H(6, 9)	H(6,11)	H(6,13)	SUM OF SQUARES
EIGEN VALUES	84.074	153.898	246.395	361.996	
P(6, 7)	0.944188	0.297889	0.120776	0.058568	0.998245
P(6, 9)	-0.322549	0.753088	0.462000	0.264418	0.954541
P(6,11)	0.066222	-0.550965	0.430041	0.470059	0.713839
P(6,13)	-0.009311	0.196092	-0.659627	0.056538	0.476843
P(6,15)	0.000959	-0.045264	0.367749	-0.599617	0.496829
P(6,17)	0.0	0.007530	-0.125459	0.522103	0.288388
P(6,19)	0.0	-0.000956	0.030266	-0.251131	0.063984
P(6,21)	0.0	0.0	-0.005541	0.082861	0.006897
P(6,23)	0.0	0.0	0.000804	-0.020514	0.000421
P(6,25)	0.0	0.0	0.0	0.004008	0.000016
P(6,27)	0.0	0.0	0.0	-0.000639	0.000000
SUM OF SQUARES	1.000001	1.000001	1.000001	1.000000	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= 7

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(7, 7)	H(7, 9)	H(7,11)	H(7,13)	SUM OF SQUARES
EIGEN VALUES	74.131	138.932	225.933	335.656	
P(7, 7)	0.983731	0.170343	0.051782	0.020747	0.999855
P(7, 9)	-0.177750	0.878083	0.388636	0.181184	0.986491
P(7,11)	0.025899	-0.430722	0.631788	0.492627	0.828029
P(7,13)	-0.002817	0.118032	-0.608454	0.286217	0.466076
P(7,15)	0.000234	-0.022132	0.266400	-0.649799	0.493698
P(7,17)	-0.000015	0.003078	-0.075451	0.432745	0.192970
P(7,19)	0.000001	-0.000333	0.015526	-0.173458	0.030329
P(7,21)	-0.000000	0.000029	-0.002465	0.049268	0.002433
P(7,23)	0.000000	-0.000002	0.000313	-0.010688	0.000114
P(7,25)	-0.000000	0.000000	-0.000033	0.001851	0.000003
P(7,27)	0.0	-0.000000	0.000003	-0.000263	0.000000
P(7,29)	0.0	0.000000	-0.000000	0.000032	0.000000
P(7,31)	0.0	-0.000000	0.000000	-0.000003	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(7, 8)	H(7,10)	H(7,12)	H(7,14)	SUM OF SQUARES
EIGEN VALUES	103.799	179.620	277.929	399.154	
P(7, 8)	0.948640	0.289608	0.111589	0.051265	0.998871
P(7,10)	-0.309701	0.771360	0.458791	0.251958	0.964883
P(7,12)	0.063871	-0.533173	0.466565	0.485888	0.742123
P(7,14)	-0.009332	0.186825	-0.649059	0.103648	0.467011
P(7,16)	0.001024	-0.043544	0.350866	-0.608712	0.495535
P(7,18)	-0.000088	0.007450	-0.118850	0.503666	0.267861
P(7,20)	0.000006	-0.000987	0.028917	-0.237431	0.057211
P(7,22)	-0.000000	0.000105	-0.005403	0.077987	0.006111
P(7,24)	0.000000	-0.000009	0.000808	-0.019430	0.000378
P(7,26)	-0.000000	0.000001	-0.000099	0.003852	0.000015
P(7,28)	0.000000	-0.000000	0.000010	-0.000627	0.000000
P(7,30)	0.0	0.000000	-0.000001	0.000086	0.000000
P(7,32)	0.0	-0.000000	0.000000	-0.000010	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= 8

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	92.950	163.862	256.827	372.354	
	H(8, 8)	H(8,10)	H(8,12)	H(8,14)	SUM OF SQUARES
P(8, 8)	0.984785	0.165744	0.047944	0.018197	0.999903
P(8,10)	-0.171879	0.885915	0.381896	0.171073	0.989498
P(8,12)	0.025430	-0.417454	0.653574	0.494949	0.847048
P(8,14)	-0.002906	0.113730	-0.594907	0.322086	0.470597
P(8,16)	0.000260	-0.021718	0.255539	-0.646693	0.483983
P(8,18)	-0.000019	0.003133	-0.072443	0.416933	0.179091
P(8,20)	0.000001	-0.000356	0.015140	-0.165289	0.027550
P(8,22)	-0.000000	0.000033	-0.002469	0.047056	0.002220
P(8,24)	0.000000	-0.000003	0.000326	-0.010334	0.000107
P(8,26)	-0.000000	0.000000	-0.000036	0.001826	0.000003
P(8,28)	0.0	-0.000000	0.000003	-0.000267	0.000000
P(8,30)	0.0	0.000000	-0.000000	0.000033	0.000000
P(8,32)	0.0	-0.000000	0.000000	-0.000003	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	125.689	207.551	311.746	438.694	
	H(8, 9)	H(8,11)	H(8,13)	H(8,15)	SUM OF SQUARES
P(8, 9)	0.952005	0.282785	0.104251	0.045624	0.999231
P(8,11)	-0.299595	0.785575	0.455064	0.240913	0.972007
P(8,13)	0.061974	-0.518550	0.496001	0.496845	0.765608
P(8,15)	-0.009342	0.179378	-0.639365	0.143410	0.461618
P(8,17)	0.001080	-0.042168	0.336972	-0.614445	0.492873
P(8,19)	-0.000099	0.007394	-0.113530	0.487753	0.250846
P(8,21)	0.000007	-0.001017	0.027853	-0.226196	0.051941
P(8,23)	-0.000000	0.000113	-0.005304	0.074081	0.005516
P(8,25)	0.000000	-0.000010	0.000815	-0.018581	0.000346
P(8,27)	-0.000000	0.000001	-0.000104	0.003737	0.000014
P(8,29)	0.000000	-0.000000	0.000011	-0.000621	0.000000
P(8,31)	0.0	0.000000	-0.000001	0.000087	0.000000
P(8,33)	0.0	-0.000000	0.000000	-0.000011	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= 9

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(9, 9)	H(9,11)	H(9,13)	H(9,15)	SUM OF SQUARES
EIGEN VALUES	113.926	190.967	289.939	411.335	
P(9, 9)	0.985599	0.162010	0.044902	0.016250	0.999933
P(9,11)	-0.167212	0.892096	0.375967	0.162573	0.991577
P(9,13)	0.025023	-0.406577	0.671219	0.495734	0.862218
P(9,15)	-0.002976	0.110204	-0.583259	0.352090	0.476313
P(9,17)	0.000283	-0.021371	0.246542	-0.643050	0.474753
P(9,19)	-0.000022	0.003181	-0.069974	0.403476	0.167700
P(9,21)	0.000001	-0.000378	0.014831	-0.158519	0.025348
P(9,23)	-0.000000	0.000037	-0.002478	0.045250	0.002054
P(9,25)	0.000000	-0.000003	0.000338	-0.010054	0.000101
P(9,27)	-0.000000	0.000000	-0.000038	0.001810	0.000003
P(9,29)	0.000000	-0.000000	0.000004	-0.000271	0.000000
P(9,31)	0.0	0.000000	-0.000000	0.000035	0.000000
P(9,33)	0.0	-0.000000	0.000000	-0.000004	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(9,10)	H(9,12)	H(9,14)	H(9,16)	SUM OF SQUARES
EIGEN VALUES	149.743	237.676	347.809	480.557	
P(9,10)	0.954633	0.277083	0.098265	0.041175	0.999450
P(9,12)	-0.291443	0.796933	0.451225	0.231195	0.977097
P(9,14)	0.060407	-0.506326	0.520213	0.504544	0.785201
P(9,16)	-0.009343	0.173256	-0.630562	0.177350	0.459167
P(9,18)	0.001128	-0.041035	0.325331	-0.617974	0.489418
P(9,20)	-0.000110	0.007353	-0.109147	0.473902	0.236550
P(9,22)	0.000009	-0.001045	0.026989	-0.216802	0.047733
P(9,24)	-0.000001	0.000122	-0.005229	0.070875	0.005051
P(9,26)	0.000000	-0.000012	0.000824	-0.017899	0.000321
P(9,28)	-0.000000	0.000001	-0.000108	0.003650	0.000013
P(9,30)	0.000000	-0.000000	0.000012	-0.000619	0.000000
P(9,32)	0.0	0.000000	-0.000001	0.000089	0.000000
P(9,34)	0.0	-0.000000	0.000000	-0.000011	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= 10

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	137.057	220.242	325.252	452.568	
	H(10,10)	H(10,12)	H(10,14)	H(10,16)	SUM OF SQUARES
P(10,10)	0.986245	0.158921	0.042439	0.014703	0.999951
P(10,12)	-0.163413	0.897094	0.370746	0.155349	0.993068
P(10,14)	0.024669	-0.397501	0.685790	0.495579	0.874523
P(10,16)	-0.003031	0.107258	-0.573153	0.377539	0.482554
P(10,18)	0.000303	-0.021075	0.238965	-0.639199	0.466124
P(10,20)	-0.000025	0.003223	-0.067905	0.391887	0.158197
P(10,22)	0.000002	-0.000398	0.014575	-0.152810	0.023563
P(10,24)	-0.000000	0.000041	-0.002489	0.043744	0.001920
P(10,26)	0.000000	-0.000004	0.000349	-0.009826	0.000097
P(10,28)	-0.000000	0.000000	-0.000041	0.001801	0.000003
P(10,30)	0.000000	-0.000000	0.000004	-0.000276	0.000000
P(10,32)	0.0	0.000000	-0.000000	0.000036	0.000000
P(10,34)	0.0	-0.000000	0.000000	-0.000004	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	175.958	269.985	386.098	524.703	
	H(10,11)	H(10,13)	H(10,15)	H(10,17)	SUM OF SQUARES
P(10,11)	0.956739	0.272250	0.093315	0.037593	0.999590
P(10,13)	-0.284730	0.806210	0.447477	0.222645	0.980853
P(10,15)	0.059089	-0.495959	0.540454	0.510004	0.801663
P(10,17)	-0.009338	0.168131	-0.622595	0.206619	0.458672
P(10,19)	0.001171	-0.040084	0.315435	-0.620027	0.485540
P(10,21)	-0.000120	0.007321	-0.105469	0.461752	0.224392
P(10,23)	0.000010	-0.001071	0.026272	-0.208826	0.044299
P(10,25)	-0.000001	0.000129	-0.005172	0.068192	0.004677
P(10,27)	0.000000	-0.000013	0.000834	-0.017337	0.000301
P(10,29)	-0.000000	0.000001	-0.000113	0.003582	0.000013
P(10,31)	0.000000	-0.000000	0.000013	-0.000618	0.000000
P(10,33)	-0.000000	0.000000	-0.000001	0.000091	0.000000
P(10,35)	0.0	-0.000000	0.000000	-0.000012	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= 11

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(11,11)	H(11,13)	H(11,15)	H(11,17)	SUM OF SQUARES
EIGEN VALUES	162.344	251.682	362.757	496.031	
P(11,11)	0.986769	0.156324	0.040390	0.013468	0.999963
P(11,13)	-0.160264	0.901215	0.366127	0.149155	0.994170
P(11,15)	0.024358	-0.389814	0.698019	0.494850	0.884657
P(11,17)	-0.003075	0.104758	-0.564312	0.399377	0.488934
P(11,19)	0.000321	-0.020818	0.232492	-0.635323	0.458122
P(11,21)	-0.000028	0.003259	-0.066144	0.381805	0.150160
P(11,23)	0.000002	-0.000416	0.014360	-0.147928	0.022089
P(11,25)	-0.000000	0.000044	-0.002502	0.042469	0.001810
P(11,27)	0.000000	-0.000004	0.000361	-0.009637	0.000093
P(11,29)	-0.000000	0.000000	-0.000044	0.001795	0.000003
P(11,31)	0.000000	-0.000000	0.000005	-0.000282	0.000000
P(11,33)	0.0	0.000000	-0.000000	0.000038	0.000000
P(11,35)	0.0	-0.000000	0.000000	-0.000004	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(11,12)	H(11,14)	H(11,16)	H(11,18)	SUM OF SQUARES
EIGEN VALUES	204.331	304.470	426.595	571.103	
P(11,12)	0.958463	0.268114	0.089149	0.034659	0.999685
P(11,14)	-0.279107	0.813921	0.443907	0.215104	0.983691
P(11,16)	0.057964	-0.487061	0.557621	0.513891	0.815613
P(11,18)	-0.009329	0.163777	-0.615388	0.232091	0.459479
P(11,20)	0.001208	-0.039272	0.306916	-0.621072	0.481471
P(11,22)	-0.000129	0.007295	-0.102335	0.451018	0.213943
P(11,24)	0.000012	-0.001096	0.025665	-0.201965	0.041450
P(11,26)	-0.000001	0.000137	-0.005127	0.065912	0.004371
P(11,28)	0.000000	-0.000014	0.000844	-0.016866	0.000285
P(11,30)	-0.000000	0.000001	-0.000117	0.003527	0.000012
P(11,32)	0.000000	-0.000000	0.000014	-0.000620	0.000000
P(11,34)	-0.000000	0.000000	-0.000001	0.000093	0.000000
P(11,36)	0.0	-0.000000	0.000000	-0.000012	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= 12

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(12,12)	H(12,14)	H(12,16)	H(12,18)	SUM OF SQUARES
EIGEN VALUES	189.786	285.287	402.445	541.708	
P(12,12)	0.987203	0.154106	0.038680	0.012453	0.999969
P(12,14)	-0.157610	0.904670	0.362027	0.143793	0.995010
P(12,16)	0.024082	-0.383222	0.708420	0.493772	0.893109
P(12,18)	-0.003111	0.102609	-0.556519	0.418310	0.495234
P(12,20)	0.000337	-0.020591	0.226900	-0.631526	0.450733
P(12,22)	-0.000031	0.003292	-0.064626	0.372953	0.143282
P(12,24)	0.000002	-0.000433	0.014175	-0.143703	0.020852
P(12,26)	-0.000000	0.000048	-0.002514	0.041372	0.001718
P(12,28)	0.000000	-0.000005	0.000371	-0.009479	0.000090
P(12,30)	-0.000000	0.000000	-0.000047	0.001793	0.000003
P(12,32)	0.000000	-0.000000	0.000005	-0.000287	0.000000
P(12,34)	0.0	0.000000	-0.000000	0.000040	0.000000
P(12,36)	0.0	-0.000000	0.000000	-0.000005	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

	H(12,13)	H(12,15)	H(12,17)	H(12,19)	SUM OF SQUARES
EIGEN VALUES	234.863	341.127	469.290	619.738	
P(12,13)	0.959899	0.264530	0.085611	0.032228	0.999751
P(12,15)	-0.274330	0.820430	0.440555	0.208429	0.985893
P(12,17)	0.056993	-0.479341	0.572351	0.516651	0.827530
P(12,19)	-0.009316	0.160029	-0.608857	0.254439	0.461142
P(12,21)	0.001240	-0.038569	0.299507	-0.621420	0.477357
P(12,23)	-0.000138	0.007272	-0.099632	0.441474	0.204878
P(12,25)	0.000013	-0.001118	0.025145	-0.196000	0.039050
P(12,27)	-0.000001	0.000144	-0.005090	0.063948	0.004115
P(12,29)	0.000000	-0.000016	0.000855	-0.016465	0.000272
P(12,31)	-0.000000	0.000001	-0.000122	0.003484	0.000012
P(12,33)	0.000000	-0.000000	0.000015	-0.000622	0.000000
P(12,35)	-0.000000	0.000000	-0.000002	0.000095	0.000000
P(12,37)	0.0	-0.000000	0.000000	-0.000013	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= 13

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	219.383	321.052	444.310	589.588	
	H(13,13)	H(13,15)	H(13,17)	H(13,19)	SUM OF SQUARES
P(13,13)	0.987568	0.152198	0.037225	0.011595	0.999975
P(13,15)	-0.155343	0.907606	0.358370	0.139112	0.995661
P(13,17)	0.023838	-0.377507	0.717371	0.492489	0.900246
P(13,19)	-0.003140	0.100742	-0.549601	0.434873	0.501335
P(13,21)	0.000352	-0.020390	0.222019	-0.627863	0.443920
P(13,23)	-0.000034	0.003320	-0.063303	0.365122	0.137333
P(13,25)	0.000003	-0.000449	0.014015	-0.140010	0.019799
P(13,27)	-0.000000	0.000051	-0.002526	0.040419	0.001640
P(13,29)	0.000000	-0.000005	0.000382	-0.009343	0.000087
P(13,31)	-0.000000	0.000000	-0.000049	0.001792	0.000003
P(13,33)	0.000000	-0.000000	0.000006	-0.000292	0.000000
P(13,35)	0.0	0.000000	-0.000001	0.000041	0.000000
P(13,37)	0.0	-0.000000	0.000000	-0.000005	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	267.552	379.953	514.174	670.592	
	H(13,14)	H(13,16)	H(13,18)	H(13,20)	SUM OF SQUARES
P(13,14)	0.961114	0.261402	0.082562	0.030173	0.999799
P(13,16)	-0.270220	0.825993	0.437427	0.202489	0.987628
P(13,18)	0.056145	-0.472581	0.585126	0.518593	0.837797
P(13,20)	-0.009302	0.156770	-0.602925	0.274192	0.463363
P(13,22)	0.001269	-0.037954	0.293004	-0.621288	0.473291
P(13,24)	-0.000146	0.007253	-0.097274	0.432935	0.196948
P(13,26)	0.000014	-0.001139	0.024693	-0.190765	0.037002
P(13,28)	-0.000001	0.000151	-0.005061	0.062239	0.003899
P(13,30)	0.000000	-0.000017	0.000865	-0.016119	0.000261
P(13,32)	-0.000000	0.000002	-0.000126	0.003448	0.000012
P(13,34)	0.000000	-0.000000	0.000016	-0.000625	0.000000
P(13,36)	-0.000000	0.000000	-0.000002	0.000098	0.000000
P(13,38)	0.0	-0.000000	0.000000	-0.000013	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

EXPANSION COEFFICIENTS FOR
HOUGH FUNCTIONS OF ZONAL WAVE NUMBER L= 14

SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	251.135	358.979	488.348	639.663	
	H(14,14)	H(14,16)	H(14,18)	H(14,20)	SUM OF SQUARES
P(14,14)	0.987879	0.150537	0.035965	0.010882	0.999978
P(14,16)	-0.153386	0.910130	0.355088	0.134997	0.996176
P(14,18)	0.023619	-0.372505	0.725154	0.491094	0.906339
P(14,20)	-0.003164	0.099104	-0.543423	0.449478	0.507170
P(14,22)	0.000364	-0.020209	0.217721	-0.624363	0.437640
P(14,24)	-0.000036	0.003345	-0.062138	0.358146	0.132141
P(14,26)	0.000003	-0.000463	0.013874	-0.136753	0.018894
P(14,28)	-0.000000	0.000055	-0.002538	0.039583	0.001573
P(14,30)	0.000000	-0.000006	0.000391	-0.009225	0.000085
P(14,32)	-0.000000	0.000000	-0.000052	0.001793	0.000003
P(14,34)	0.000000	-0.000000	0.000006	-0.000297	0.000000
P(14,36)	0.0	0.000000	-0.000001	0.000043	0.000000
P(14,38)	0.0	-0.000000	0.000000	-0.000005	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

ANTI-SYMMETRIC EIGENFUNCTIONS AND EIGENVALUES

EIGEN VALUES	302.397	420.944	561.240	723.655	
	H(14,15)	H(14,17)	H(14,19)	H(14,21)	SUM OF SQUARES
P(14,15)	0.962154	0.258650	0.079919	0.028424	0.999836
P(14,17)	-0.266649	0.830800	0.434523	0.197182	0.989022
P(14,19)	0.055399	-0.466615	0.596304	0.519932	0.846707
P(14,21)	-0.009286	0.153910	-0.597522	0.291766	0.465935
P(14,23)	0.001294	-0.037411	0.287249	-0.620819	0.469329
P(14,25)	-0.000153	0.007236	-0.095199	0.425255	0.189957
P(14,27)	0.000016	-0.001158	0.024296	-0.186133	0.035237
P(14,29)	-0.000001	0.000157	-0.005036	0.060736	0.003714
P(14,31)	0.000000	-0.000018	0.000875	-0.015818	0.000251
P(14,33)	-0.000000	0.000002	-0.000130	0.003418	0.000012
P(14,35)	0.000000	-0.000000	0.000017	-0.000628	0.000000
P(14,37)	-0.000000	0.000000	-0.000002	0.000100	0.000000
P(14,39)	0.0	-0.000000	0.000000	-0.000014	0.000000
SUM OF SQUARES	1.000000	1.000000	1.000000	1.000000	

BIBLIOGRAPHY

Bogdanov, K. T. and Magarik, V. A., 1967: Numerical solution of the distribution problem for the semi-diurnal tidal waves (M_2 and S_2) in the world ocean, Dokl. Akad. Nauk SSSR, 172, 1315-1317.

Briggs, B. H. and Spencer, M., 1954: Horizontal movements in the ionosphere, Rep. Progr. Phys., 17, 245-280.

Chapman, J. H., 1953: A study of winds in the ionosphere by radio methods, Canad. J. Phys., 31, 120-131.

Chapman, S. and Lindzen, R. S., 1970: Atmospheric Tides, Thermal and Gravitational, D. Reidel Publ. Co. Dordrecht Holland, 200 pp.

Chapman, S., Pramanik, S. K. and Topping, J., 1931: The world wide oscillations of the atmosphere, Gerland's Beitr. z. Geophys., 33, 246-260.

Chiu, W. C., 1953: On the oscillations of the atmosphere, Arch. Meteorol. Geophys. Biokl. A, 5, 280-303.

CIRA, 1965: (COSPAR Intern. Reference Atmosphere), North-Holland Publ. Co. 313 pp.

Dickinson, R. E., 1969: On the formulation of a non-linear atmospheric tidal theory from the meteorological primitive equations, Pure and Applied Geophysics, 72, 1969/I 198-203.

Dietrich, G., 1944: Die Gezeiten des weltmeeres als geographische erscheinung, Zeitschrift der Gesellschaft für Erdkunde zu Berlin, No. 3/4, 69-85.

Doodson, A. T., 1922: The harmonic development of the tide generating potential, Proc. Roy. Soc. A 100, 305-329.

Flattery, T. W., 1967: Hough Functions, Tech. Rept. 21, Dept. Geophys. Sci., Univ. of Chicago.

Geller, M. A., 1969: The Lunar Tide in the Atmosphere. Ph.D. Thesis, Massachusetts Institute of Technology.

Geller, M. A., 1970: An investigation of the semi-diurnal lunar tide in the atmosphere, J. Atmos. Sci. 27, 202-218.

Greenhow, J. S. and Neufeld, E. L., 1961: Winds in the upper atmosphere, Quart. J. Roy. Meteorol. Soc., 87 472-489.

Haurwitz, B. and Cowley, A. D., 1967: New determinations of the lunar barometric tide, Beitr. Phys. Atmos., 40, 243-261.

Haurwitz, B. and Cowley, A. D., 1970: The lunar barometric tide, its global distribution and annual variation, Pure and Applied Geophysics, 75, 1-29.

Hendershott, M. and Munk, W., 1970: Tides, in Annual Review of Fluid Mechanics, Vol. 2, Annual Reviews, Inc., Palo Alto, California, 205-224.

Hildebrand, F. B., 1956: Introduction to Numerical Analysis, McGraw-Hill, New York, 511 pp.

Hough, S. S., 1897: On the application of harmonic analysis to the dynamical theory of tides, Part I. On Laplace's 'Oscillations of the first species', and on the dynamics of ocean currents, Phil. Trans. Roy. Soc. A, 189, 201-257.

Hough, S. S., 1898: The application of harmonic analysis to the dynamical theory of the tides, Part II. On the general integration of Laplace's dynamical equations, Phil. Trans. Roy. Soc. A, 191, 139-185.

IBM, 1968: System/360 Scientific Subroutine Package (360A-CM-03X) Version III, Programmer's Manual, Fourth Ed.

Jacchia, L. G. and Kopal, Z., 1952: Atmospheric oscillations and the temperature profile of the upper atmosphere, J. Meteorol. 9, 13-23.

Kuo, J. T. et al., 1970: Transcontinental tidal gravity profile across the United States, Science, 168, 968-971.

Lamb, H., 1932: Hydrodynamics, 6th ed., Cambridge University Press, Cambridge, England.

Laplace, P. S. (Later Marquis de la Place), 1775, 1776: Recherches sur quelques points du systeme du monde, Mem. de l'Acad. Roy. des Sciences.

Lindzen, R. S., 1967: Thermally driven diurnal tide in the atmosphere, Quart. J. Roy. Meteorol. Soc., 93, 18-42

Longuet-Higgins, M. S., 1968: The eigenfunctions of Laplace's tidal equations over a sphere, Phil. Trans. Roy. Soc. A 262, 511-607.

Love, A. E. H., 1911: General theory of earth tides, in Some Problems of Geodynamics, Cambridge University Press.

Margules, M., 1893: Luftbewegungen in einer rotierenden Sphäroidschale, Sitzber. Akad. Wiss. Wien, 11a, 102 11-56, 1369-1421.

Matsushita, S., 1967: Lunar tides in the ionosphere, in Handbuch der Physik, XLIX/2, Springer-Verlag, 547-602.

Melchior, P., 1966: Earth Tides, Academic Press, New York.

Müller, H. G., 1966: Atmospheric tides in the meteor zone, Planet. Space Sci., 14, 1253-1272.

Müller, H. G., 1968: Meteor winds and ionospheric drifts, J. Atmosph. Terr. Phys., 30, 701-706

Müller, H. G., 1970: The Sheffield meteor wind experiment, Quart. J. Roy. Meteorol. Soc., 96, 195-213.

Pekeris, C. L., 1937: Atmospheric oscillations, Proc. Roy. Soc., A 158, 650-671.

Pekeris, C. L., and Accad, Y., 1969: Solution of Laplace's equation for the M_2 tide in the world oceans, Phil. Trans. Roy. Soc. A 265, 413-436.

Press, F., 1962: Long period waves and free oscillations of the earth, in Research in Geophysics Vol. II, M.I.T. Press, Cambridge, Mass., 1-26.

Ramana, K. V. V. and Rao, B. R., 1962: Lunar daily variation of horizontal drifts in the ionosphere at Waltair, J. Atmosph. Terr. Phys., 24, 220-221.

Rawer K. (ed), 1968: Winds and Turbulence in Stratosphere, Mesosphere and Ionosphere, North Holland Pub. Co., 421 pp., op. cit.

Sawada, R., 1965: The possible effect of oceans on the atmospheric lunar tide, J. Atmos. Sci., 22, 636-643.

Sawada, R., 1966: The effect of zonal winds on the atmospheric lunar tide, Arch. Meteorol. Geophys. Biokl., A 15, 129-167.

Siebert, M., 1957: Tidal Oscillations in an Atmosphere with Meridional Temperature Gradients, Sci. Rept. No. 3, Project 429, N. Y. Univ., Dept. of Meteorol. Oceanogr.

Siebert, M., 1961: Atmospheric tides, in Advances in Geophysics, Vol. 7, Academic Press, New York, pp. 105-182.

Taylor, G. I., 1929: Waves and tides in the atmosphere, Proc. Roy. Soc. A 126, 169-183.

Taylor, G. I., 1936: The oscillations of the atmosphere, Proc. Roy. Soc. A 156, 318-326.

Wallace, J. M. and Hartrauft, F. R., 1969: Diurnal wind variations, surface to 30 km., Monthly Weather Review, 96, 446-455.

Wilkes, M. V., 1949: Oscillations of the Earth's Atmosphere, Cambridge University Press, 1949, 74 pp.

BIOGRAPHICAL SKETCH

I was born and reared in Dublin. They tell me the day was July 6, 1943, a Tuesday. I went to school to the Christian Brothers in James' Street from 1950 to 1960 and I did fairly well there. I got a cadetship in the Irish Meteorological Service in the year 1961 and I went to University College in Cork to study Mathematics, Mathematical Physics and sundry other topics of worth. I must have done fairly well there too because they gave me a first class honors degree in 1964. I went forecasting in Shannon Airport in 1965 and it was there I met and courted Breda Cunningham. We got married in 1967. That same year M.I.T. gave me a Jonathan Whitney Fellowship and so in September we packed our bags and came here. We've been here since.