AN INVESTIGATION OF THE RELATIVE

EFFICIENCIES OF THE STRATIFORM AND

CELLULAR MODES OF PRECIPITATION

by

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ABSTRACT

Methods are devised for studying the water vapor budget of a storm in order to calculate the efficiency of the storm in precipitating the available water. The efficiency is defined as the ratio of total precipitation to total available water. In order to determine the total available water, the flux of water vapor across the boundaries of a fixed volume in space is calculated and initial static water vapor content of the volume is evaluated. The sum of these two quantities integrated over the period of the storm is assumed to be the total available water. The total precipitation is determined from both rain gauge and radar data. The errors involved in all computations are estimated and it appears that for typical values of available water and precipitation, the efficiencies can be expected to be accurate to within ± 2 to 4 per cent.

These methods are applied to three stratiform and two cellular storms using a volume above southern New England. Stratiform storms are those which exhibit uniform, large-scale lifting, and the cellular storms, those which exhibit small-scale lifting. Efficiencies of approximately 5 per cent are computed for each of the cellular storms, although one was an organized line of cells, while the other was scattered showers and thundershowers. Two of the stratiform storms studied are snowstorms and have efficiencies of approximately 17 per cent. The final storm, a November rainstorm which was stratiform for all but the last few hours of the period studied, has an efficiency of approximately 8 per cent. Although these results indicate that the stratiform mode of precipitation is more efficient than the cellular, more storms of both types must be studied before definite conclusions can be drawn.

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I. INTRODUCTION

A quantitative study of the efficiencies of the two basic lifting processes in precipitating the available water from the atmosphere involves two fundamental problems. The first is the definition and computation of the available water in a storm; the second, the determination of the total amount of precipitation over the area of the storm. Then:

$$\text{Efficiency} = \frac{\text{precipitation}}{\text{total available water}} \qquad (1)$$

Previous studies of the distribution and transport of water vapor have been either on a large scale (Starr and White, 1954; Benton and Estoque, 1954) or have been seasonal means (Huff and Stout, 1951; Hutchings, 1957). The precipitable water calculations made by the United States Weather Bureau are currently used to represent the water vapor in the atmosphere. However, this analysis depicts only a static situation and not the water vapor available for precipitation in a given storm. The available water vapor is the sum of the water vapor initially present in the volume under consideration and the water vapor advected into the volume during the time period in question. Thus:

$$W(t) = V_1 + A(t) \qquad\qquad (2)$$

where $W(t)$ is the total available water, V_1 is the
initial water vapor content of the volume of the storm,
and $A(t)$ is the inflow of water vapor into the volume dur-
ing the time period involved. The initial water content
can be computed with relative ease and accuracy from the
available data and requires little comment. The computa-
tion of the transport of water vapor into the storm is
subject to much greater uncertainties and much of this
thesis is devoted to the methods of computation, estima-
tion of errors, and consistency checks of the inflow of
water vapor.

In order to determine the available water for a
storm, it is necessary to define the boundaries of the
storm in space. A volume is chosen to represent the storm.
Ideally, the volume should move with the storm; but for
the sake of simplicity, a fixed volume was used in this
study. In order to obtain some idea of the changes of a
storm, one storm was studied at two stages in its life-
time, using two fixed volumes.

When the available water for a storm is found, it
is necessary to determine the actual amount of water
precipitated by it. This reduces to the problem of
determining the representativeness of the rain gauge data

which was studied by means of the M.I.T. weather radar. In addition, the radar measurements of total rainfall were used to supplement the rain gauge data for the area within sixty miles of M.I.T.

The final step in the investigation of the relative efficiencies of the two principal types of lifting is the choice of the storms to be studied.

The stratiform mode is defined as the precipitation mechanism which involves large-scale lifting and produces generally uniform precipitation over a large area. The cellular mode involves small, convective type lifting and produces intense but brief showers. The main requirement is that the structure of each storm exhibit predominantly one mode of overturning. The structure of a storm was determined from the radar data, both PPI and RHI films (especially the latter). Thus, the availability of good radar data also became a factor in the choice.

Besides the type of lifting, the over-all synoptic situation was considered. This led to interesting speculations as to the effect of the synoptic features on the efficiency of precipitation.

II. THEORY

The determination of the relative efficiencies of various storms is simple in theory. For each storm it is necessary to apply equation (1). The determination of the total available water and the precipitation become the actual problems.

The total available water was defined by equation (2). The initial water content is merely the precipitable water as evaluated at the beginning of the storm. This is calculated from radiosonde data as described by Solot (1939). Briefly, from the definition of specific humidity, $q = \dfrac{\rho_w}{\rho}$, where ρ_w is the density of the water vapor and ρ the density of moist air, we find, using the hydrostatic approximation, the total mass of water in a column of height z and unit cross section:

$$M_w = \int_0^z \rho_w \, dz = \int_p^{P_o} \frac{q}{g} \, dp \qquad (3)$$

where M_w is the precipitable water, g is the acceleration due to gravity, and p is pressure. Since this yields the water content of the atmosphere above a "point" on the surface of the earth, it is necessary to use the average from several radiosonde stations over the area of the storm in order to approximate the initial water content of the storm.

The inflow of water vapor into the storm is the inward flux of water vapor across the boundaries of the selected volume. This can be found using the volume with its vertical dimensions expressed in terms of pressure through the hydrostatic approximation. Then, assuming a right prism whose base is an area on the surface of the earth:

$$\text{Water vapor convergence} = -\int \nabla \cdot \vec{c}\, \rho_w \, dv$$

$$= \int \rho_w \, \vec{c} \cdot d\vec{A}$$

$$\text{or:} \quad = \oint_B \int_0^z \rho_w \vec{c}\, dz \cdot d\vec{s} = \int_p^{P_o} \left(\oint_B \frac{q\vec{c}}{g} \cdot d\vec{s} \right) dp \quad (4)$$

where $d\vec{s}$ is an increment of the boundary, B, (positive normal chosen inward), p is the pressure, q the specific humidity, $\vec{c}$ the horizontal vector wind, and g the acceleration due to gravity. The inflow is then the positive part of the convergence so that the integrand is defined to be zero for $\vec{c} \cdot d\vec{s} < 0$ and $\frac{q\vec{c}}{g} \cdot d\vec{s}$ for $\vec{c} \cdot d\vec{s} \geqslant 0$. The total inflow of water during the storm involves only integration in time. Thus, if t is the period of the storm, then:

$$A(t) = \begin{cases} \int_0^t dt \int_p^{P_o} dp \oint_B \frac{q\vec{c}}{g} \cdot d\vec{s}, & \vec{c} \cdot d\vec{s} \geqslant 0 \\ 0 & \vec{c} \cdot d\vec{s} < 0 \end{cases} \quad (5)$$

The remaining step is to determine the total amount of water precipitated by the storm. This can be done by

considering the conservation of water vapor:

$$V_1 + A(t) = V_2 + O(t) + R \qquad (6)$$

where V_2 is the final water vapor content of the volume,
$O(t)$ is the outflow of vapor during the time period t, and
R is the rainfall over the region considered. It is more
satisfactory, however, to determine the rainfall directly,
either from the rain gauge network or by means of radar
and to use equation (6) as a check of the over-all consis-
tency of the estimates. Unfortunately, there are errors
in both methods.

If rainfall is determined by considering the conser-
vation of water vapor, it appears as a small difference
between two large quantities, as can be seen in equation
(6). The quantities to be subtracted are of the order of
10 g/cm^2, whereas the precipitation is of the order of
1 g/cm^2. The percentage error in the estimate of pre-
cipitation by this means would thus be some ten times the
error of the estimate of the available water.

Direct measurement of the rainfall is more accurate
but is not free from error. If rain gauges are used, it is
assumed that the collected water can be accurately measured,
but there are two types of sampling error. The rain gauge
may not measure the true amount of rain which is falling

at that place because of wind or poor location of the
rain gauge, and second, the rain which does fall at the
rain gauge is not necessarily representative of the rain-
fall in the surrounding region. The first error is
negligible compared to the second, for in a cellular type
storm, especially the widely scattered showers of some
summer days, the average of the rain gauge measurements
can be markedly different from the true average rainfall.
In the more uniform storms, an average of the rain gauge
measurements is usually satisfactory.

To determine the extent to which rain gauge data
represent the areal distribution of rainfall, radar was
used to measure total rainfall in an area 85 by 60 miles
covering eastern Massachusetts. The area was divided into
5 by 5 mile squares, and the total rainfall was computed
by integrating the radar iso-echo contours for each square.
The results, shown in figures 1 and 2, show a difference
between the isohyets drawn using all the radar data (upper
left) and those drawn using only the data from the squares
for which rain gauge measurements were available (lower
left). The accuracies of the radar data are approximately
± 3 db in signal strength or ± 60 per cent in rainfall for
absolute measurements. However, some of the error will
be the same for all measurements at a given time. Thus,
the relative measurements for the individual squares are
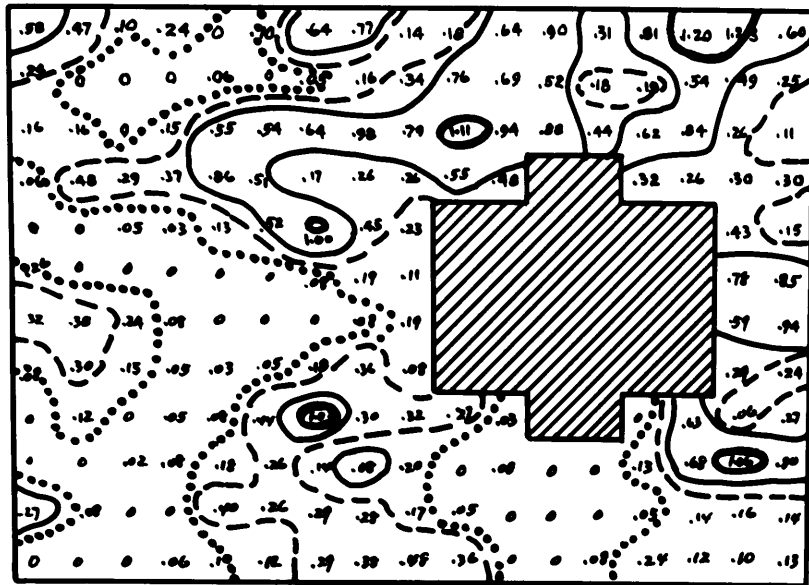
Complete radar data

Rain gauge data

85 miles

60 miles
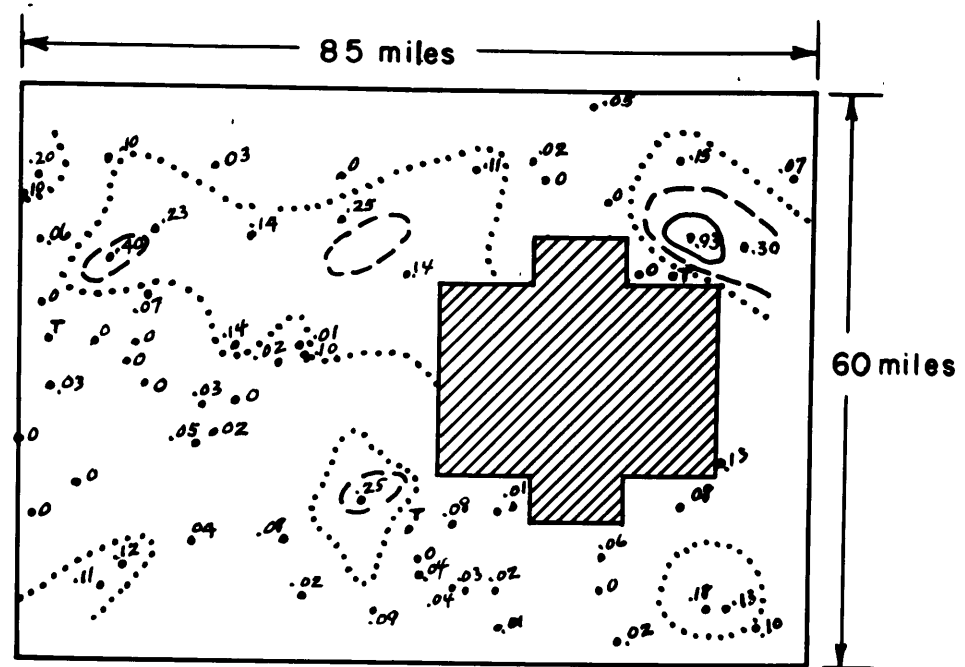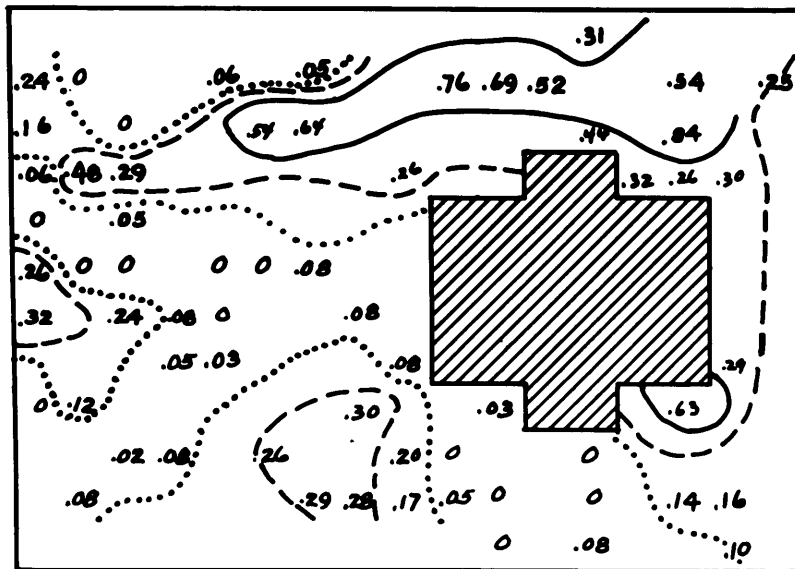
Selected radar data

Code :

dotted = .10 inch
dashed = .25 "
solid = .50 "
heavy solid = 1.00 "

note:
cross hatched area
covers ground clutter

Fig. 1
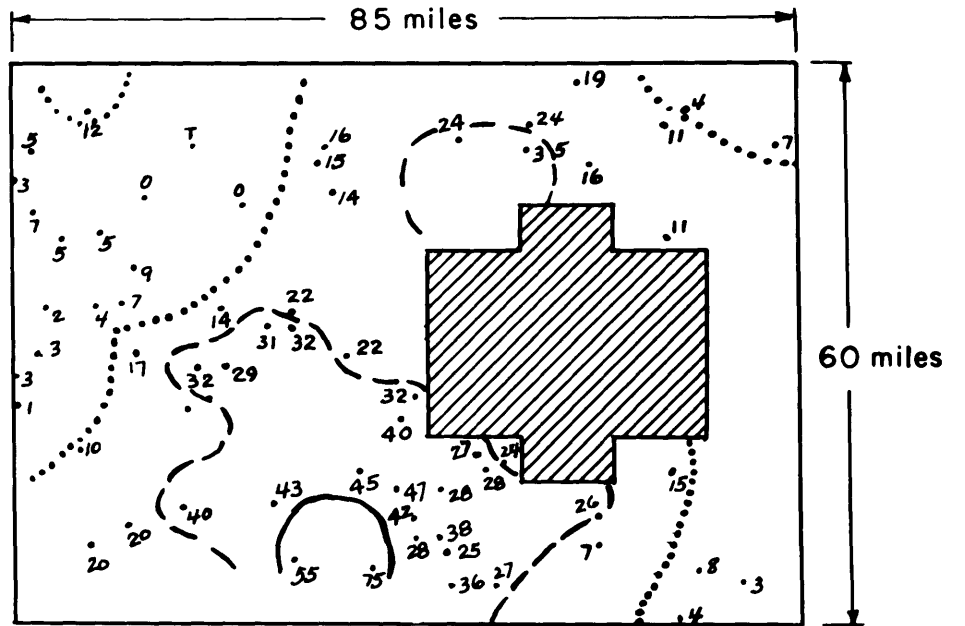Total rainfall 10 July 1961   1230-2130 EST

Complete radar data

Selected radar data

Rain gauge data

85 miles

60 miles

Code:

dotted = .10 Inch
dashed = .25 "
solid = .50 "

Note:
Cross hatched area covers ground clutter.
Numbers on charts represent hundredths of an inch.

Fig. 2
Total rainfall 16 May 1961

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accurate to approximately ± 1 db in signal strength or
± 20 per cent in rainfall. Average rainfall values, which
even more clearly indicate the problem, were computed from
the two sets of data. On July 10, the average rainfall
in the selected squares was .29" and for all squares was
.20". The May 16 precipitation was relatively uniform,
and the two averages were essentially the same, being
.132" for the selected squares and .128" for all squares.
From this, it was obvious that rain gauge measurements
were not reliable in cellular storms, and even the strati-
form storms were variable, as is well-known from the
appearance of such precipitation on the PPI scope. In
this study, the average rainfall in Massachusetts was
determined using both rain gauge and radar data; for the
rest of the area, the rain gauges had to be used alone.

The above procedures provide all the information
necessary for the determination of the efficiency of a
storm. In order to assess the relative efficiencies of
the stratiform and cellular modes of precipitation, it
is necessary to make a careful selection of the storms to
be studied. Stratiform storms are those which involve
relatively slow and large-scale lifting. They are easily
distinguished on the radar scopes, where they show up as
generally solid, uniform precipitation covering a large
area. Between the clear-cut stratiform and cellular types,

there are many storms which exhibit both wide-spread
precipitation and smaller, more intense areas of vertical
motion. The extent of this small-scale development is
what has been loosely termed the cellularity of a storm,
but the category of cellular storms includes many different
types of small-scale development. The storms chosen to
illustrate the cellular mode of precipitation have been
selected from different types of cellular activity. More
will be said about this selection later.

In conclusion, the relative efficiencies of the
stratiform and cellular modes of precipitation can be
determined by solving equation (2) for a variety of storms
chosen to be representitive of the two modes. The solution
of equation (2) involves the computation of the average
rainfall over the area being considered, as well as the
water vapor content of the atmosphere at the start of the
storm and the inflow of water vapor during the storm.
Practical methods of carrying out these calculations are
considered in the next section.

III. METHOD

The choice of the volume within which this study was carried out was based on several considerations. It was decided to use a fixed volume since it was more difficult to apply these methods to one moving with the storm. The volume was taken to be a right prism and thus, is defined by choosing an area for its base and limiting its height. In the volume, there must be upper air data on both the wind, $\vec{c}$, and the specific humidity, q, in addition to surface data on rainfall. Radar data are required in order to determine the structure of the storm and also to supplement the rain gauge data. In New England, the volume satisfying these requirements was defined by using the radiosonde stations at Idlewild Airport; Albany, New York; Portland, Maine; and Nantucket, Massachusetts as the verticies of the base of a quadrilateral prism. A volume largely in Ohio was used in order to study one of the storms at an earlier stage in its lifetime. This volume was defined by using the radiosonde stations at Dayton, Ohio; Pittsburgh, Pennsylvania; and Flint, Michigan as the vertices of the base of a triangular prism. This discussion of method will use the southern New England quadrilateral prism as its example.

One other choice that must be made is the time period

to be considered. The period studied began with the last
twelve-hourly radiosonde observation before the storm
entered the area and ended with the first radiosonde
observation after the storm left the area. In general,
the time periods would be of varying lengths, but as it
turned out, four of the six storms studied lasted just
one day.

Once these space and time boundaries have been
established, attention may be turned to the computation
of the terms in the expression for the water budget of the
storm. As indicated earlier, this requires an evaluation
of the initial water content of the chosen volume, the in-
flow of water vapor into the volume, and the average rain-
fall. Computations of the outflow of water vapor and of
the final water content are necessary to check the consis-
tency of the calculations by making use of the conservation
of water vapor. Finally, it is useful to compute the large-
scale vertical velocities involved in the storm since these
would be expected to have an effect on the efficiencies.
Even in the cellular storms, where it is conceivable that
there could be precipitation when the large-scale vertical
motion was downward, it has been observed that showers
are substantially more probable with large-scale upward
motion (see Curtis and Panofsky, 1958). It would then
be expected that the efficiency would increase with

increasing large-scale upward motion in both cellular and stratiform type storms. With all this information available, it should be possible to draw some conclusions about the relative efficiencies of the stratiform and cellular modes of precipiation.

The initial water content can be determined from equation (3), using the radiosonde data published by the U.S. Weather Bureau in the Daily Upper Air Bulletins. In order to evaluate the integral in equation (3), the following approximation is used:

$$ M_w \doteq \sum_{i=1}^{N-1} \frac{q_i}{g} \left(\frac{P_{i-1} - P_{i+1}}{2} \right) \tag{7} $$

Here, N is the number of levels of observation, and P_i and q_i the pressure and specific humidity at these levels. In practice, the first N - 1 levels were taken at the mandatory and significant levels of the radiosonde observation below the level where the specific humidity became negligible (between 300 and 400 mb usually), and the Nth level is that level where q can be considered to be zero. With g in cm/sec^2, q in g/kg, and p in mb, M_w will be in units of g/cm^2, or, since one gram of water is one cubic centimeter, M_w is often given in centimeters of water, the depth of water which would be measured by a rain gauge if all the water in the atmosphere were precipitated. The total water content of the quadrilateral prism was approx-

imated by the average value of M_w at the four radiosonde stations. This average, evaluated at the beginning of the storm, is the initial water content.

The next step in the determination of the available water is the computation of the inflow of water into the quadrilateral prism. This involves calculating the total water vapor flux across the faces of the prism at each observation and integrating over the period of the storm. [See equation (5).] The flux of water vapor across a surface is merely the normal component of the water vapor transport integrated over the area of the surface. The water vapor transport was computed at each station at consecutive levels in the atmosphere. Then the components of the transport normal to the faces of the quadrilateral prism were found and integrated over the surface of the prism and over the period of the storm. In order to obtain the inflow, the positive direction of the normal is chosen inward and all negative values of flux are omitted from the calculations.

The water vapor transport, w , is defined by:

$$\vec{w_i} = \vec{c_i} \, \rho_{w_i} \, \Delta z_i \tag{8}$$

which, with the aid of the hydrostatic equation, may be written as:

$$\vec{w_i} = \frac{q_i \, \vec{c_i}}{g} \, \Delta p_i \tag{9}$$

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Integration over pressure up to the level of negligible
q yields the vertically integrated total transport, $\vec{W}$.
This integral can be approximated by the summation:

$$\vec{W} = \sum_{i=1}^{N} \vec{w}_i = \frac{1}{g} \sum_{i=1}^{N} \left(\overline{q\vec{c}} \right)_i \Delta P_i \qquad (10)$$

There are two methods of determining the mean value
of the product of specific humidity and wind for each layer,
$\left(\overline{q\vec{c}} \right)_i$. It is possible to determine both the mean wind
and the mean specific humidity directly from the soundings
and take the product of the means as an approximation to
the mean of the product. The other method is to determine
the actual product, $q\vec{c}$, at consecutive pressure levels and
take a linear average between levels. The first method
has the advantage that true mean values can be determined
for both q and $\vec{c}$, although each of the averaged quantities
varies irregularly within the layer. This is offset by
the fact that the mean value of the product, $q\vec{c}$, is not
necessarily equal to the product of the mean wind and
mean specific humidity since they vary independently. There
is also the disadvantage that, in order to determine the
mean values of winds and specific humidity accurately, one
must go back to the original computation sheets of the
soundings. This is both costly and time consuming. The
second method has the disadvantage that it is impossible
to tell just what is going on between the selected levels,

and it is very likely that significant variations in the transport profile will be smoothed out. It does have the great advantage of being easy to calculate from the data given in the Upper Air Bulletins. In this study the second method has been used in all cases, while the first method was used in only one storm purely for comparison.

The method of computing the mean wind and the mean specific humidity involves only one new concept. The mean specific humidity was determined by means of the equal area method on a plot of q versus p. The mean winds were calculated from the original computation sheets for the wind observations by subtracting the position vector of the balloon at the time it entered the layer from the position vector at the time it emerged from the layer, and dividing the resultant vector by the time the balloon was in the layer. Then it was assumed that:

$$\overline{\overline{W_i}} \doteq \frac{\overline{q_i}\,\overline{\vec{c_i}}}{g}\,\Delta P_i \qquad (11)$$

where $\overline{\overline{W_i}}$ is the mean transport in the ith layer, and $\overline{q_i}$ and $\overline{\vec{c_i}}$ the mean specific humidity and mean wind respectively, in that layer. Convenient units for $\overline{\overline{W_i}}$ are g/(cm 100mb sec) and are obtained by using q in g/kg, g in m/sec^2, $\vec{c_i}$ in m/sec. In carrying out these computations, it was found to be necessary to use four 50mb layers plus five 100mb layers, since deeper layers led to unacceptable errors.

The final layers used were: sfc-950 (a variable thickness layer), 950-900, 900-850, 850-800, 800-700, 700-600, 600-500, 500-400, and 400-300 mb.

The second method of computing the mean water vapor transport in a layer was to take a linear average of the transports at the top and bottom of the layer:

$$\vec{W_i} \doteq \frac{q_i c_i + q_{i+1} \vec{c}_{i+1}}{2g} \Delta P_i \qquad (12)$$

The levels used were: sfc, 1000, 950, 900, 850, 800, 700, 600, 500, 400, and 300 mb. Occasionally there was no 1000 mb level and usually when the water vapor transport curve was extrapolated upwards, it reached a level of zero transport around 300 mb. Therefore, the extrapolated level of zero transport was used in place of the 300 mb level. Since it is less time consuming, this method of computing the mean transport was used in the storms considered.

The computation of the net inflow of water vapor into the quadrilateral prism was accomplished by summing all the positive (inward directed) values of normal transport, multiplied by the pressure thickness of the layer represented, or:

$$I = \frac{1}{g} \sum_{i=1}^{N-1} q_i c_{i_n} \left(\frac{P_{i-1} - P_{i+1}}{2} \right) \qquad (13)$$

Here I is the vertically integrated inflow of water and is the inward directed normal component of $\vec{W}$ with negative

values of $\vec{w}_i$ omitted. c_{ii} is the inward directed normal component of the wind vector, at the ith level, and if not positive, is set equal to zero. (The negative values are used to compute the outflow of water vapor.) When $i = 1$, $P_{i-1} = P_1$, and when $i = N-1$, P_{i+1} is the pressure at the level of zero transport.

In computing the components of the transfer normal to the sides of the quadrilateral prism, the curvature of the earth was taken into account in a sufficiently accurate way by measuring the normal directions separately from each station. The resulting angles at the corners of the quadrilateral then add up to more than 360°, as they must for a spherical quadrilateral surface, and for a due south wind at all stations, there will be a net convergence of air in the quadrilateral.

With the vertical integral of the inflow computed at each station for each observation time, it is necessary to integrate both in space, around the quadrilateral and in time, over the period of the storm. If the linear average of the computed inward directed components of the transport is taken in both space and time, then the total inflow through a side of length 1, $\emptyset_j$, will be given by:

$$\emptyset_j = \frac{l \Delta t}{4 n - 4}\left[I_1 + 2I_2 + 2I_3 + \ldots + 2I_{n-1} + I_n + \\ + I_1' + 2I_2' + \ldots + 2I_{n-1}' + I_n' \right] \quad (14)$$

Here I_1, I_2, ... I_n represent the vertically integrated inflow values at observations 1,2, ... n in time at one radiosonde station, and I_1', I_2', ... I_n' the vertically integrated inflow values at the station at the opposite end of the side of the quadrilateral being considered. For I in units of g/cm sec; l, the length of the side, in cm; and ▲t , the time interval of the observations in sec., the net inflow will be in grams. Dividing by the total area of the quadrilateral in cm^2 gives the inflow in g/cm^2 or simply depth of water in cm as before. The total inflow is the sum of the inflows through each of the four sides of the quadrilateral prism; i.e.:

$$\text{Total inflow} = \phi_1 + \phi_2 + \phi_3 + \phi_4 \qquad (15)$$

where $\phi_1, \phi_2, \phi_3, \phi_4$ are the inflows across the four sides of the quadrilateral. The total available water is this total inflow plus the initial water content.

The next step is the determination of the precipitation. As stated in the section on theory, the average of the rain gauge data was used though this was modified by the radar measurements in the Massachusetts area. A second method of determining the total precipitation, not really independent of the first, is the measurement of the areas enclosed by successive isohyets drawn from the rain gauge data. A planimeter was used to measure the areas,

and the total rainfall was computed by multiplying the area obtained by the average value of rain contained in it. The average rain was taken as the mean of the bounding isohyets if there was a smaller isohyet contained within a given isohyet. If there was no smaller isohyet, the mean of the highest rainfall recorded and the bounding isohyet was used. This method is also subject to errors, and being more difficult, was used for only two storms. The accuracy of the two methods is considered further in the section on errors.

It is desirable at this stage to make use of equation (6) as a consistency check. For this purpose the final water content of the volume and the outflow are computed in the same way as the initial water and inflow were evaluated. Though this is a check on all computations, it should be considered primarily a check on the inflow and outflow calculations; the accuracies of the computations of the static water contents and rainfall are both substantially greater than that of the computations involving the flux of water vapor.

The success of the investigation depends largely on the choice of the storms. For the present purpose, the storm had to be markedly cellular or markedly stratiform. Radar data were used extensively in order to determine the nature of the storm. Once a storm had been selected and classified as stratiform or cellular, its efficiency was

determined by following the procedures outlined above.

To give real meaning to these results it is nec-
essary to attempt to sort out the various factors affecting
the efficiencies so that the effect of the mode of precip-
itation can be singled out. One of the most important
effects to be taken into consideration is the vertical motion.
The magnitude of the lifting is directly related to the
amount of precipitation in a stratiform storm. Large scale
lifting acts to trigger convective lifting, but it is not
certain that the effect is the same for cellular as for
stratiform storms. Therefore, it is helpful to evaluate
the large-scale vertical velocity. This can be done using
the following relationship:

$$\omega_p = \frac{1}{\rho_p} \iiint conv(\rho \vec{c})\, dV = \frac{1}{\rho_p} \int \rho \vec{c} \cdot d\vec{A} = \frac{1}{g\,\rho_p} \int_{p_o}^{p_s} dp \oint \vec{c} \cdot d\vec{s} \quad (16)$$

where ω_p is the vertical velocity at the pressure level p,
ρ_p the density of the air at this level, and ρ the var-
iable density of air throughout the atmosphere. Note that
the positive direction of the normal is chosen inward.
This integral was evaluated from the summation:

$$\omega_p \doteq \frac{1}{\rho_p} \sum_{i=1}^{m} conv(\rho \vec{c})_i \left(\frac{P_{i-1} - P_{i+1}}{2} \right) \quad (17)$$

where the summation is over the m levels of observation
below the pressure level p. In practice, the level p was
chosen as 600 mb, to approximate the level of zero diver-

gence where the vertical velocities should be greatest. The convergence was determined from:

$$
\text{conv}(\rho \vec{c})_i = \frac{1}{3A} \left[\frac{C_{ab\perp} + C_{ba\perp}}{2} \ell_{ab} + \frac{C_{bc\perp} + C_{cb\perp}}{2} \ell_{bc} + \frac{C_{cd\perp} + C_{dc\perp}}{2} \ell_{cd} + \frac{C_{da\perp} + C_{ad\perp}}{2} \ell_{ad} \right]
$$
(18)

Here, $c_{ab\perp}$, $c_{bc\perp}$, $c_{cd\perp}$, $c_{da\perp}$ etc., are the normal, positive inward components of the wind at the four radiosonde stations: a, b, c, and d; ℓ_{ab}, ℓ_{bc}, ℓ_{cd}, ℓ_{da}, are the lengths of the sides of the quadrilateral; and A is its horizontal area.

Using the methods outlined above, five storms were investigated in order to determine their relative efficiencies. Initial water and inflow were computed to determine the total available water. Average precipitation was estimated and efficiencies calculated. The consistencies of these calculations were checked by computing the final water content and the outflow of water using equation (6). Finally, an estimate of the large-scale vertical velocities was made in order to try to isolate the effect of the mode of lifting on the efficiencies.

IV. DISCUSSION OF ERRORS

There are two types of error to consider in this
study: 1) the instrumental error and 2) the sampling error.
The instrumental errors are relatively easy to evaluate with
confidence, but the errors due to sampling are more difficult
to assess. In order to obtain a general idea of the nature
of the sampling error, the synoptic charts for the periods
in question were studied.

The instrumental errors in the radiosonde data have
been thoroughly discussed by the Air Weather Service (1955).
In addition to the random errors, there are systematic errors,
especially in the humidity element. These errors have been
considered by Hutchings (1957) who found a small systematic
over-estimation of relative humidity at levels above 700 mb.
It has also been observed that, even when the radiosonde
is in the clouds, where the air must be saturated, it seldom
reads 100 per cent relative humidity. The net effect of
these two errors on the precipitable water is negligible
since both are small compared with the others involved.

Other insignificant errors are those due to the
liquid water and ice content of the atmosphere and those
due to evaporation. The liquid water and ice content is,
even under extreme conditions, less than 10 per cent of the
total water content of the atmosphere. There are too

few clouds and too little water in the clouds to significantly alter the calculations. Evaporation, though it must be considered in a long period study, is also entirely negligible in a study such as this.

The mean vector error in the wind was of the order of 1 meter per second at 900 mb, approximately 2 m/sec at 500 mb, and somewhat less than 3 m/sec at 300 mb. The temperature error is of the order of 1° Celsius at all levels considered in this study, and the humidity errors due to the instrument itself are approximately 1 per cent relative humidity at 950 mb, 5 per cent at 700 mb, 4 per cent at 500 and 300 mb. An additional error is introduced by the motorboating condition which occurs at low relative humidities. In this case, the relative humidity may lie between zero and an upper limit set by the characteristics of the humidity element and the circuitry. Hutchings (1961) has studied this effect over Australia by evaluating the total water content and the water vapor transport under the two extreme assumptions. He found that in summer the maximum error was only 2 to 4 per cent of the precipitable water and in winter, 10 to 20 per cent. In the present study, a linear extrapolation of the known humidity values was made in order to reduce this error further. It would be expected that there would be more cases of motorboating in the seasonal means considered by Hutchings than in the storms

considered here where the humidities were generally high. For this reason, the error due to motorboating was taken to be of the order of 1 per cent for the summer storms and 5 per cent for the winter.

The error introduced by the linear averaging between stations and between observation times has been studied by Len hard (1959). In his study, he found that the error in a linear average between two stations was of the order of the instrumental error when the stations were separated by approximately 100 miles for the levels of interest in the present study. Thus, for the distances involved in the triangle and the quadrilateral, the interpolation errors were of the order of four times the instrumental error. There has been little study of the error due to linear time averaging between successive observations, and it is assumed that it is the same as that due to spatial averaging. In order to obtain an estimate of the total error due to the time and space averages of water vapor transport and the space averages of static water content, it is satisfactory to take the square root of the sum of the squares of the individual errors. This is roughly one standard deviation in the resulting random sample of measurements of a given phenomenon. The net interpolation error is, therefore, approximately six times the instrumental error. In other words, the total error in relative humidity is $\pm$ 7, $\pm$ 35, and $\pm$ 28

per cent at the 900, 700, and 300 mb. levels respectively;
while the mean vector errors in the winds are $\pm$ 6, $\pm$ 12,
and $\pm$ 18 m/sec at the same levels. If typical values of
specific humidity are taken at these levels, the resulting
errors in the water vapor transfer are $\pm$ .25 g/(cm mb sec)
at 900 mb, $\pm$ 1.0 g/(cm sec mb) at 700 mb, and $\pm$ 0.4 g/(cm
sec mb) at 300 mb. The error in the total vertically
integrated transport is then $\pm$ 330 g/(cm sec ➤) or approxi-
mately $\pm$ 1.0 g/cm^2 in the final inflow of water. In the
static water content, there is no error due to time aver-
aging and the wind error does not enter. In this case, the
net error in the precipitable water is approximately $\pm$ 0.5
g/cm^2. The resulting error in the total available water is
therefore, approximately $\pm$ 1.2 g/cm^2.

The accuracy of the average rainfall calculations
was somewhat difficult to determine. In the first place,
it was assumed that the rain gauge did measure the actual
precipitation which occured at that spot. The errors in
averaging were investigated using the radar data, plani-
meter measurements of isohyets drawn for rain gauge data,
and the simple averaging technique. For two stratiform
storms, the average precipitation was computed using both
the planimeter method and a simple average of the rain gauge
data. In each case, the results of the two methods differed
by less than .08 cm. This was taken to be the error for

the stratiform storms. The radar data for the cellular
storm on July 10 indicated that the rain gauges did not
give a satisfactory areal representation of rainfall in
the Massachusetts area. For this reason, corrections were
made using the radar data. However, no corrections could
be made for the remainder of the area. In this case, it
appeared that the error could be as large as $\pm$ 0.25 cm and
this was taken to be the error for the cellular storms.

If an available water content of approximately
10 g/cm^2 and a precipitation of approximately 1 g/cm^2 is
assumed, the resulting error in efficiency is less than
$\pm$ 2 per cent for the stratiform storms and about $\pm$ 4 per
cent for the cellular. The efficiency calculations for
storms having greater available water and producing pro-
portionately larger rainfalls are more accurate by as much
as a factor of two. In this study, the final error was
estimated for each storm separately. In each case, the
RMS value was computed; and thus, for a large number of
storms one would expect nearly forty per cent of the
computations to exceed this error, approximately five per
cent to exceed twice this error, and less than one-half
per cent to exceed three times it.

The above estimates of error are somewhat on the
high side in order to make some allowance for those small
errors which were neglected. Some of the estimates have

no really satisfactory basis so that further study may reveal that the errors are not as large as estimated here.

V. RESULTS

Computations were carried out for five different storms, one of them at two stages in its lifetime. A relatively uniform rainstorm on November 10, 1960, was studied first in Ohio, and then later as it crossed New England. This storm was of the stratiform type, except in the last two or three hours in New England where it showed the cellular structure often observed in connection with a dying storm. The other two winter storms were snowstorms occurring January 1, 1961, and December 28, 1961. Unfortunately, there were no RHI data available for the January storm, but from the PPI data it seemed to be stratiform in nature. The RHI data for the storm of December 28 showed it to be definitely stratiform.

The two obviously cellular storms which were studied both occurred in July, 1961. The first, on July 2, was an intense squall line, while the second, on July 10, was a disorganized group of showers and thunderstorms, some of which produced hail.

The various parameters of the storms which were calculated are shown in Table 1. First, it is interesting to look into the differences in the various parameters of the individual storms. The individual results can be relied upon to within the experimental error, but general conclusions regarding the differences between the strati-

TABLE 1

Storm	Initial water content g/cm^2	Inflow of water vapor g/cm^2	Total available water g/cm^2	Precipi- tation g/cm^2	Efficiency per cent	Accuracy per cent	Average large-scale lifting cm/sec	Mean 850 mb temperature $^\circ C$
November 10, 1960 Ohio	1.4	14.7	16.2	1.17	7.2	±1.1	+ 9.0	- 1.1
November 10, 1960 New England	1.7	15.5	17.2	1.50	8.8	±1.1	- 2.4	+ 4.0
January 1, 1961	1.8	9.9	11.7	2.10	18.0	±2.8	+14.5	- 2.0
July 2, 1961	3.0	16.2	19.2	0.87	4.5	±1.9	- 0.1	+16.2
July 10, 1961	2.5	7.5	10.0	0.51	5.1	+3.5 -2.2	+14.5	+ 9.4
December 28, 1961	1.4	7.2	8.6	1.42	16.6	±3.6	- 9.8	- 3.1

form and cellular modes of precipitation cannot be drawn from such a limited sample.

In the individual results, it is interesting to note the differences, both expected and unexpected, between the two summer-type cellular storms. The storm of July 10, which was connected with no obvious large-scale disturbance, had an inflow of only 7.5 g/cm^2 of water vapor as compared to 16.2 g/cm^2 in the squall line associated with a definite frontal system. This is not unexpected, but it seems somewhat surprising to find the large-scale vertical motions so much greater in the disorganized case, being - 0.1 cm/sec on July 2 and + 14.5 cm/sec on July 10. The maximum values of the large-scale vertical motion were only slightly different, being 23.5 cm/sec for July 10 and 17.8 cm/sec for July 2.

The difference in inflow of water vapor created a rather large difference in available water for the two storms, but as can be seen, the difference in average precipitation was almost exactly proportional, so that the two storms had essentially the same efficiency. The consistency of the computations involved in these two storms was checked on the basis of the conservation of water vapor, equation (6). This check suggests that the calculations for July 10 are the more accurate, for there was a net surplus of only 1.4 g/cm^2 in the water vapor calculations, while

for the July 2 storm there was a net deficiency of 2.4 g/cm^2.

There is a great difference between the two types of winter storms, as would be expected from the synoptic data. The November storm remained much the same in character throughout the time it was studied (over Ohio and over New England) with the exception of the mean large-scale vertical motion. Again, the maximum values agreed more closely than the mean values. The maximum values were 19.8 and 13.7 cm/sec for the Ohio and New England areas of study respectively. Furthermore, it seems likely that some of this difference is due to errors, since a consideration of the conservation of water vapor shows that there is a net deficiency of approximately 2.6 g/cm^2 in the calculations of water vapor transport and static water vapor for the New England area and a net surplus of 2.8 g/cm^2 for the Ohio area. These discrepancies must be attributed to instrumental and averaging errors. From all the data presented, it seems that the water vapor budget of the storm did not change much in the time it took to travel from Ohio into southern New England.

The calculations for the last two storms, both completely stratiform snowstorms, were interesting in that the efficiencies computed were in both cases greater than in any other case by approximately a factor of two. The calculations for the storm of January 1 were apparently the more accurate since there was a net excess of only 0.8 g/cm^2

in water vapor. The calculations for the storm of December
28 showed a net deficiency of 2.4 g/cm^2 in available water.
The large-scale vertical velocity in the storm of December
28 was surprisingly small at all observation times, and the
level of non-divergence was unusually low, being at about
850 mb at the times of the soundings. In other ways, these
storms were much alike, as would be expected from the
synoptic situation.

In order to make the final results consistent, the
discrepancies in the water vapor computations and the rain-
fall were used to obtain a new estimate of the total available
water. The discrepancies are very sensitive to the errors
in the calculations of the inflow and outflow of water vapor,
since they are essentially a small difference in two large
quantities. Because both the inflow and the outflow were
calculated with the same accuracy, a revised estimate of the
actual available water was made by subtracting one-half of
the net excess of water vapor from the previously calculated
available water. These revised estimates are shown in Table
2. They make only two changes worthy of note, and these
may or may not be significant.

First, in the summer storms, the revision seems to
imply that, to some extent, the close agreement in the
efficiencies of the two storms was fortuitous since the
corrections place the calculated values farther apart. On

Table 2

	Total Available Water--g/cm^2	Precipitation g/cm^2	Efficiency Per cent
10 Aug 1960	10.1	1.63	16.1
10 Nov 1960 (Ohio)	14.75	1.17	7.9
10 Nov 1960 (New England)	18.5	1.50	8.1
1 Jan 1961	11.3	2.10	18.6
2 July 1961	20.4	0.87	4.3
10 July 1961	9.3	0.51	5.5
28 Dec 1961	9.8	1.42	14.4

the other hand, the corrections seem to imply that the squall line, or more organized case, was in fact less efficient than the somewhat random array of storms on July 10, and that this appearance was not the result of the errors involved. Second, it is interesting to see that this makes the two periods of the November 10 storm even more alike. The resulting efficiencies of 7.9 per cent and 8.1 per cent in the Ohio and New England areas respectively show no significant difference. The expected difference in the available water in the coastal areas of New England and the inland areas of Ohio is shown.

It should be understood that these corrections are simply an empirical change to make the calculations consistent and may or may not represent an actual improvement in the accuracy of the estimates. It does provide a basis for speculation on the meaning of the implied results.

Since six storms is by no means a statistically significant sample, it is only possible to suggest relationships in the relative efficiencies of the stratiform and cellular modes of precipitation. The fact that there are very few storms which do not exhibit some of the characteristics of both modes of precipitation makes it even more difficult to determine the relative efficiencies of the two modes. However, if enough storms were studied, the results obtained here could be made more significant.

With this caution in mind, the results can be taken
for what they are worth. Obviously from figure 3, the
cellular storms were less efficient than the stratiform ones.
This difference cannot be attributed to large-scale vertical
velocities since they were not greatly different in the
several cases. It is possible that there is a temperature
effect since the most efficient were the snowstorms and the
two summer storms were least efficient. To examine this
further, a plot of efficiency versus mean 850 mb. temperature
is shown in figure 4, but with so few cases, no real con-
clusions can be drawn from it. Cellular winter storms will
be studied as well as some of the stratiform warm weather
storms in order to clear up this question of temperature
dependance.

The major consideration here is the effect of the
mode of lifting. If it is assumed that this is the primary
factor in determining the efficiency of precipitation, then
the few storms studied suggest that the efficiencies decrease
as the cellularity of the storms increases since the two
snowstorms are the least cellular and the two summer storms,
most cellular. From these few storms, it is impossible to
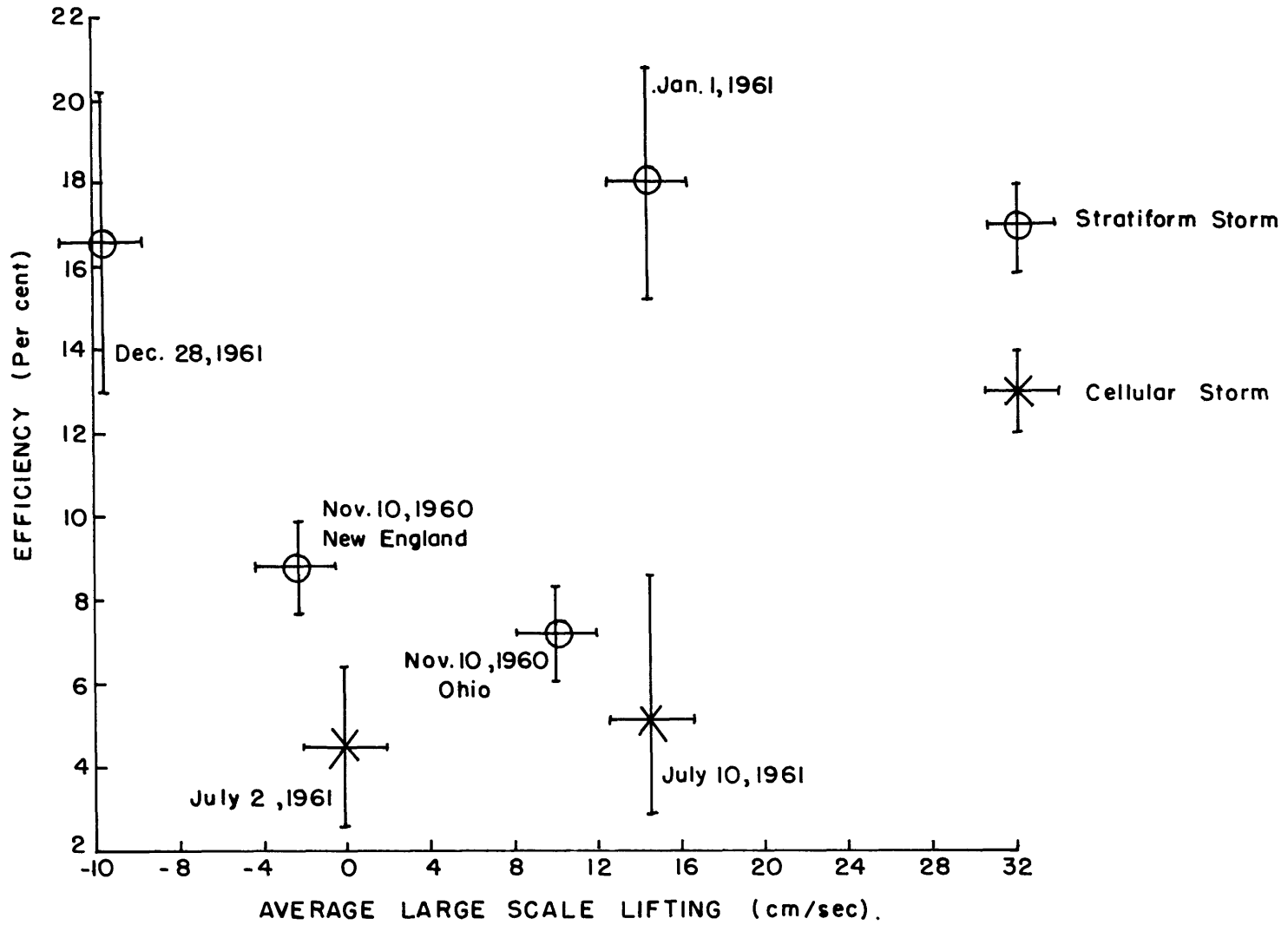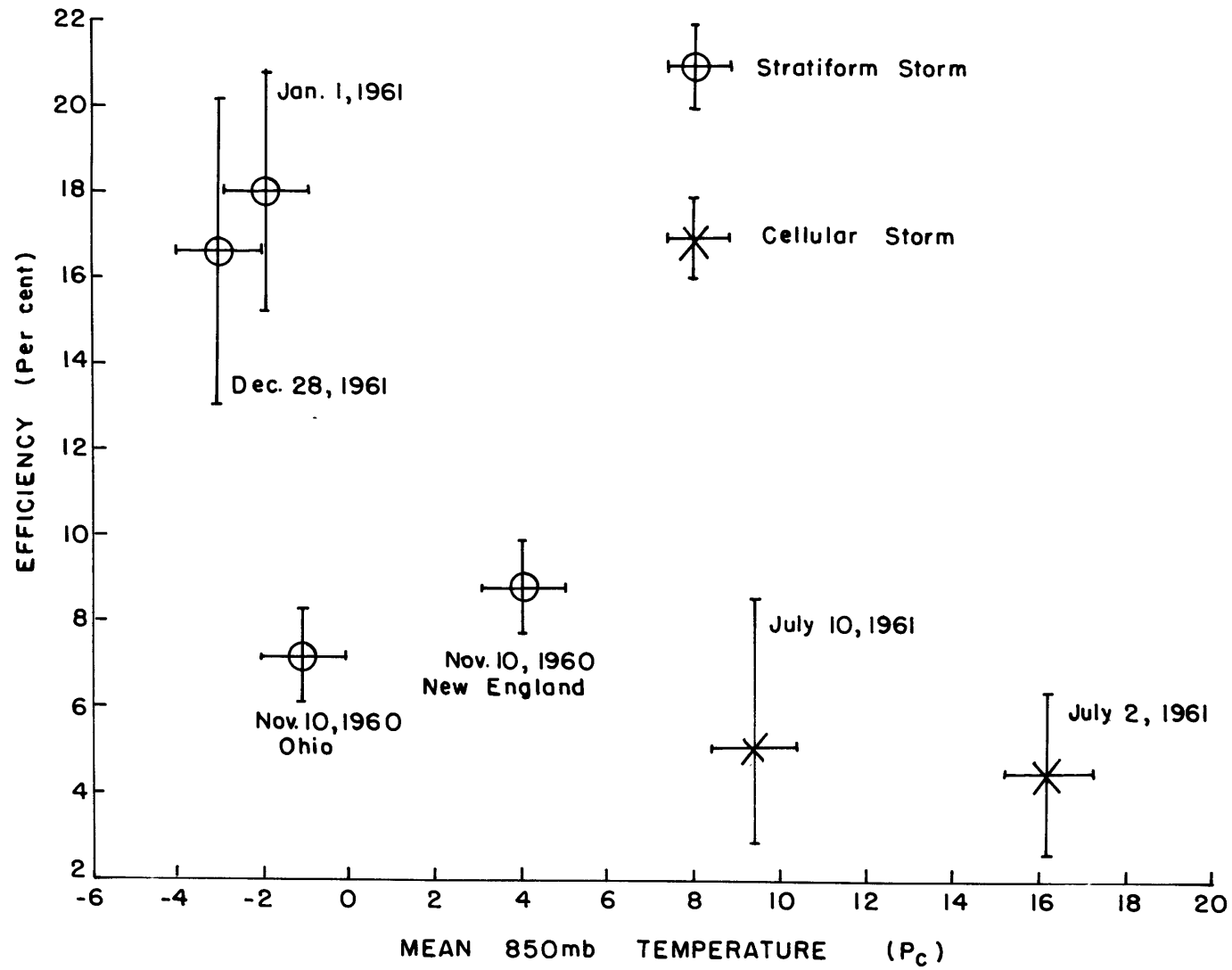say that there is more than the suggestion of such an
effect.

Fig. 3.

Fig. 4.

VI. SUMMARY AND CONCLUSIONS

This study must be taken as no more than a pre-
liminary attempt to determine the water budget of an
individual storm. Only a very few storms, typical of
those encountered in the Northeastern United States, were
studied. The results obtained are interesting, but such
a small sample cannot lead to any definite conclusions as
to the relative efficiencies of the cellular and stratiform
modes of precipitation.

Methods were devised to study the water budget in a
storm. By taking specific humidity observations at each
of the mandatory and significant levels up to 300 mb and
summing their values weighted by the pressure layer they
represented, it was possible to determine the static water
content of the atmosphere. Then, by a similar integration
of the water vapor transport directed into the chosen volume
of space, it was possible to determine the inflow of water
vapor into the storm. A sum of the initial static water
and total inflow during the period of the storm was taken
as the water available to the storm. Then, the total rain-
fall was approximated by averaging the rain gauge measure-
ments in the area, and the efficiency, defined as the ratio
of rainfall to available water, was computed.

Once the method had been devised, it was necessary

to determine the accuracy of the results. It was shown
that the errors involved came from two main sources; the
instruments used and the averaging methods required. From
a consideration of these errors, it was shown that the
accuracy, though it varied slightly with the particular
storm, was in general, adequate to show up differences
greater than 2 to 3 per cent in efficiency. Because of the
variability of the storms themselves, it was not possible
to demonstrate any significant differences in efficiency
between the two primary modes of lifting.

In the few storms which were studied, the stratiform
ones were observed to be more efficient than the cellular
in precipitating the available water. The two summer cellu-
lar storms had efficiencies of about 5 per cent. One winter
rainstorm, which became cellular for the last few hours,
had an efficiency of approximately 8 per cent both in Ohio
and later in its lifetime, in New England. The two snow-
storms, which were completely stratiform, had by far the
highest efficiencies. A tendency for the efficiency to
increase with decreasing temperature was observed and should
be studied before positive conclusions can be drawn. One
expected fact which this study does substiantiate is that
storms with similar synoptic features have similar efficien-
cies, although the water budgets are not necessarily the same.

Though the few storms studied showed the stratiform

ones to have greater efficiencies than the cellular, much more data must be compiled before any definite conclusions can be drawn.

Plans are now being made to continue these calculations with the aid of a program written for a high speed computer. It is hoped that it will then be possible to study many storms so that definite conclusions can be reached. Also, more of the various types of cellular and stratiform storms will be studied.

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