Generative Morphologies of Architectural Organization in Matter Force Field

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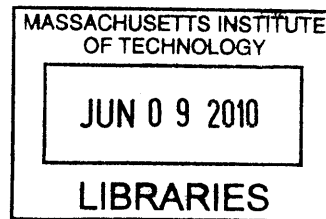
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Abstract

This thesis investigates generative methods of architectural
form finding in matter force fields that produce spatial subdivi-
sion and organizational variation. Unlike the style driven con-
temporary free-form architecture or decorative computational
form making processes, this thesis is interested in inventing
methods of informing architectural forms with constraints of
matter realities, namely mechanics of matter. The consider-
ation of matter mechanics in a conventional design process is
only a post-rationalization design input. The initial form is as-
sumed to be the datum to work with and not re-configured
after the engineering input beyond thickening material. This
approach resembles the mindset of the modern era architect
who desires to shape the world with their own ideas of how
the world should be like rather than incorporating material re-
alities in making forms. On the other hand, in a pure material
efficiency driven design process, the designer generates form
that is only able to provide a single shell space of a certain
span distance and height. The latter process is neither able
to provide organizational variation nor programmatic subdivi-
sions.

Given the advancements in computational tools, the designer
is now able to create his own tools to evaluate both material
and visual performance while thinking of organizational prin-
ciples. This thesis investigates opportunities that work with
the constraints of material force fields to generate organiza-
tional rules for spatial constraints by inventing its own compu-
tational procedures. Topology formations, pattern formations
within topological boundaries and aggregated topology forma-
tions are three main categories of form finding methods being
explored throughout the thesis. The goal of this particular
thesis is not to find ways to achieve optimum structural ef-
ficiency with minimum material, but rather to attain the me-
dium between the two while generating new aesthetics and
organizational rules.

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Robert Maillart, Antonio Gaudi, Felix Candela and
Heinz Isler, Axel Killian and Mutsuro Sasaki
_ Formations within Topological Boundaries
Luigi Nervi, Reisser + Umemoto, Mutsuro Sasaki,
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RESEARCH BEGININGS

The driving force for this thesis is the interest to invent methods of informing architectural forms with constraints of matter realities. This work is also a reaction to my experience at Zaha Hadid Architects and SOM, also the blobitecture and parametrically fetishized projects that has been coming out of both academia and practice in general.

Since the digital tools, that enable organic architectural form making, have become available to wide architecture public, there has been less distinction between the work of some architects or even students and professional architects. With the flux of capital to architecture before the financial crisis, major concern for such architects was style. Some of those changed their strategy to attract clients by tagging sustainability concept to their projects. Offices like Zaha's have been successful in keeping the image driven design approach as their motto. For Zaha's office, the fluidity of concrete or lately the formal flexibility of fiber reinforced panels and the advanced manufacturing techniques that make these panels at any cost have been the justification for any free-from making therefore eliminating any material constraint issue until the construction stage. The process of making the free form curves is neither informed by structure nor any other performance criteria except achieving sculptural effects *(figure_1)*. This totalitarian approach to form making, has shown its failure in the mobile pavilion for Chanel. The idealized predefined continuous skin fails to do so when there is a transition from the FRP panels
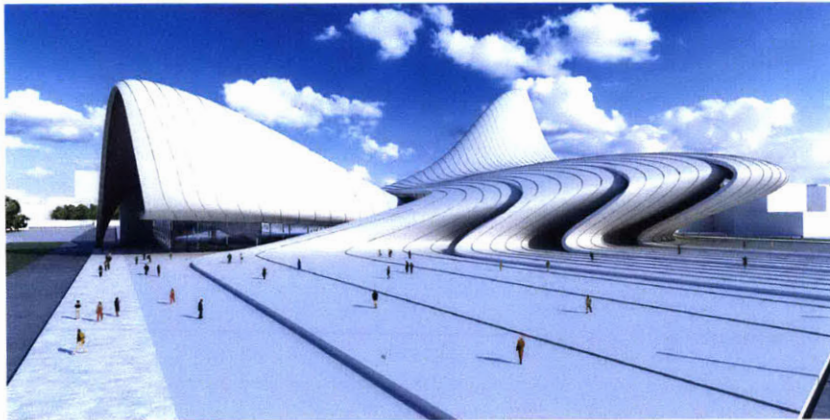
figure_1
Baku Hayder Aliyev Cultural Center, Zaha Hadid Architects
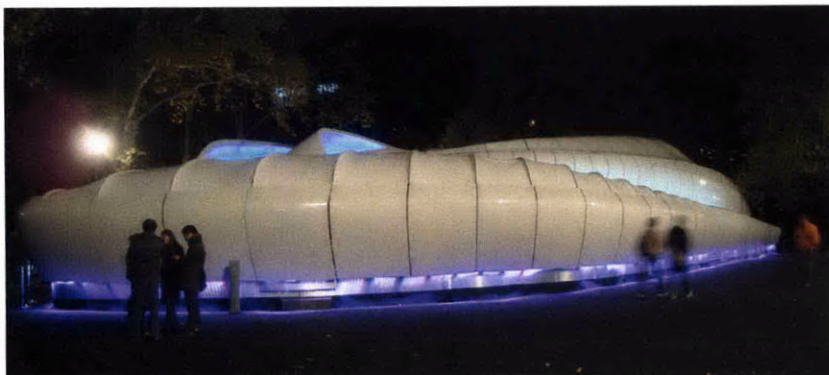
figure_2
Chanel Mobile Pavillion, Zaha Hadid Architects

panels to ETFE. Had they considered the fabric like material behavior of ETFE and worked rigorously with this material to achieve the desired 'fluidity', the form would have been different than the final output *(figure_2)*.

One observation about the image or style driven design offices such as Zaha and Frank Gehry, is that they do not have any significant former employee who is now running an internationally renowned design office. On the other hand OMA, who generates form as an output of the constraints of any design process, has had 'graduates' like Foreign Office Architects, MVRDV, BIG, REX and WorkAc. Based on the success of the OMA 'education' versus Zaha, one can make the argument that training architects with constraints of matter and culture can help them create variety of architectural outputs rather than producing a single effect. It is the success of these younger firms that proves this point.

This stylistic approach reminds us of the traditional architects whose sole inspirations were the utopian ideas. The modern era architects derived their designs not from reality, but from a fixed world of ideality. Their approach was to shape the world with the ideas of what the world should be like rather than incorporating material realities in making forms. However, in the contemporary world the reality has become much more complex than what the idealistic visions entail. The question then arises - how should designers keep abreast of the dynamic and unpredictable movements of reality?

When talking about Manhattan in Delirious New York, Rem Koolhaas suggested that the success of this city relied on the fact that its architecture had surrendered itself to the needs

of this metropolis. This kind of architecture has a similar relationship with the forces of the contemporary trends, like a surfer does with waves. To follow the movements of reality is synthesizing observations from the real world in making design decisions. Without collaborating with actuality, the designer will no doubt get lost in a world of abstract visions which are irrelevant to what is demanded from him. In order to surf the wave, any contemporary design practice needs to derive its aspiration from available opportunities which require a comprehensive knowledge of the constantly evolving market. Each available opportunity –or hybridization of opportunities- becomes a design instrument from which the designer can claim a strong position to develop ideas. Our skills as designers come from being able to design with what is already out there , rather than proposing ideas and forms that are derived from our fantasies of a controlled utopian world.

The last two decades have created an ever-growing wave of Information Technology for designers. Both on the engineering and architectural sides, the digital tools have been very much part of their design processes for the purpose of generating or evaluating design. When the first computer aided software packages became commercial, the draftsman was able to insure his work since all the drawing information could be stored and replicated many times. This opportunity has led many practices to give up on drafting with conventional methods. The fact that a lot more information can be produced faster in the digital space has been one of the driving reasons for these professionals' interest in riding this wave of IT. The awareness of the power of this IT wave has influenced the opportunistic designers to guide this wave into a more malleable state for
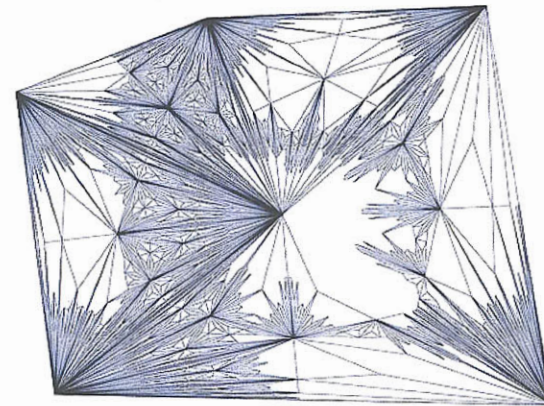
figure_3
Tooling, Aranda /Lasch

design flexibility where the design rules are laid out as codes of information. The designer's ability of abstracting the design of a building into geometric information is now taken to a higher level of abstraction where the geometry is now defined as lines of scripts. This ability has allowed a re-consideration of the part to whole relationship in architecture. Parametric modeling and scripting methods can generate parts of the same DNA with differences to one another and still be sufficient to make the whole. However the logics behind variation have often been arbitrarily imposed or generated with a mindset of computational thinking rather than architectural. Therefore we often see voronoi, recursion or circle packing type high level computational problems forced to be used as decorative surfaces . This superficial usage of the information technology wave is a missed opportunity *(figure_3 and _4)*.

The mass production of organic architectural forms and the superficiality of the computational form making have to be approached critically. Those issues pushed this research to find ways and which form becomes an emergent phenomenon of a materially conscious design process. Deuleze and Guattari suggest that a precondition for form making is to be formless, to delay the state of having form, so that a new possibility can emerge. Whether it is mechanics, constructability, acoustical or optical behaviors of matter, there is an open flux of possibilities around us that can become driving forces to form finding. Given the amount of time for a master's thesis and the amount time that each of those matter behaviors require to comprehend enough to use in a design research, one has to narrow the topics. My interest has been towards researching the mechanics of matter which is the fundamental condition

figure_4
Wondelgem Office Building, ConixArchitecten

to make form that those other behaviors develop with it. We couldn't be speculating on constructability, optics, or acoustics of space if the mechanics of matter didn't provide enough substance to work with.

In the past the 'master builder' was able to comprehend all the knowledge to construct an idea. When designing a building, he would know what materials needed to be used in what form, how the loads would be distributed in the structure, how the public would engage with the space. Since all the necessary knowledge for designing the artifact was contained in one mind, the process of design was already established with these constraints of materiality from the starting point. However, in the contemporary world it is not possible for one design practice or practitioner to comprehend a meta-knowledge of construction technology, material science, structures, urbanism, information technology and other such fields that would entail as constituents for a design to materialize. The Industrial Revolution introduced new building materials like iron that was a new concept for the traditional master builder. In order to surf the wave of time, the master builder/architect had to formulate his design knowledge about the new way of constructing forms by collaborating with experts. It is no coincidence that this was also when the first building engineering, structural, profession emerged.

Since the Industrial Revolution, in a traditional design process the architect will develop a formal concept of his design solution to the given problem that often lacks relevancy to real material issues. It is not until the designer completes the concept that the building engineers start rationalizing the initial form. Among the critical avant-garde architecture practices, it was OMA who first started working closely with an engineer, Cecil Balmond of Arup. For them, the desire for finding the opportunities of reality led them into collaborations with engi-

neers who are already knowledgeable about the potentials of materiality and also aware of the industry-standard construction and fabrication techniques. This collaboration enabled the projects to be conceptualized with real material and allowed construction issues to be taken into account from the beginning of the design processes. For instance, in the Maison À Bordeaux project, it was this early collaboration and dual thinking of architecture and engineering that enabled OMA to perforate the floating mega concrete beam –which also acts as a façade- to create windows for the rooms inside *(figure_5)*.

The case of Maison À Bordeaux is an exception to the mainstream practice within which there is a disconnect between programmatic organization and built form. In a conventional design process: once finished with the programmatic thinking of a project, designer generates a building form, then gets engineering input to post-rationalize the initial form. The initial form is assumed to be the datum to work with and not re-configured after the engineering input beyond thickening material. The Rolex Learning Center designed by SANAA architects is an example of such design process. The plinth that is lifting itself from ground at certain locations to allow public to walk underneath is also hollowed out with round courtyards around which different programs are allocated. This design concept is only refined by the engineers with some manipulations of the bottom and top plinth surfaces in the z dimension and also defining thicknesses of these concrete surfaces *(figure_6)*. This effort to post-rationalize the pre-conceived form could have been used in a more instrumental way that finds a medium between form and organization. The locations of the courtyards could have been reconfigured as a part of the material
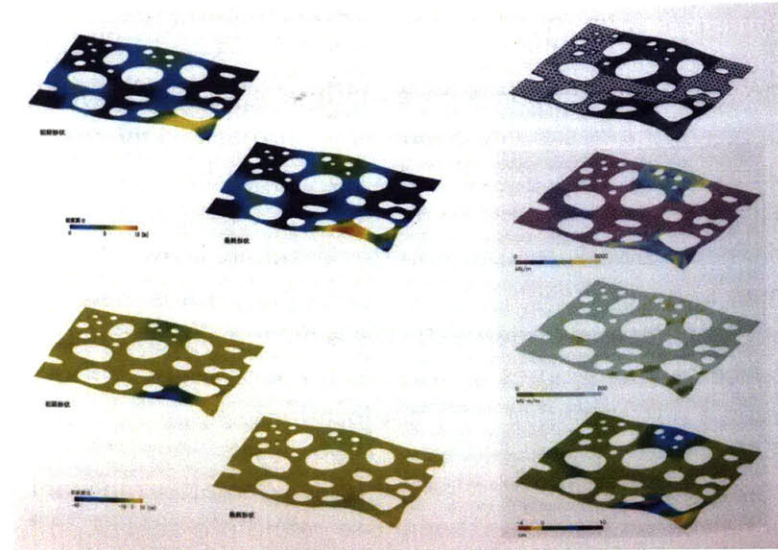
figure_6
Rolex Learning Center, SANAA

figure_5
Maison À Bordeaux, OMA

optimization process and eventually this would generate an emergent form that is a result of this dual thinking.

On the other hand, in a pure material efficiency driven design process, the designer generates form that is only able to provide a single shell space of a certain span distance and height. The process is not able to provide any organizational variation or a framework for programmatic subdivisions. This disconnect between building form and its program is clear in many shell examples. Underneath the similar shell geometries, the space can sometimes be used as an auditorium or a chapel or even a gas station at different locations. *(figure_7)*.

In the mainstream practice the forces of matter mechanics are either used as an afterthought to refine pre-conceived forms, or as a stubborn form generation process that doesn't care about the relationship between form and its spatial consequences. When Deleuze and Guattari talk about the "plateau'", they refer this concept as a state of creativity where preconceptions are set aside. It is this stable state where internal forces interact with one another before a design takes shape without interference from outside. Conditions may change, but the changes will be worked out from within to generate emergent phenomena. This "plateau" can only be stable to generate architectural forms only if the forces of matter mechanics and architectural organization constraints are worked together internally rather than imposing either of them as an external force after a form is already emerged.

This idea of working simultaneously with material systems and organizational logics is not an easy task. It requires a tight collaboration between the architect and the engineer like in the Bordeaux house project. In a small scale project like a house, engineer's intuition can predict any load distribution and this will be an indispensable input during the conceptual design stage. However as scale gets bigger and geometries become non-orthogonal, engineer has to rely on computational tools to evaluate architect's design. This is not a smooth process. Because the tools of architects create design data that is often not recognized by engineer's tools, this data has to be remade inside the engineer's tool. It is this reason that the collaboration between the engineer and architect often results in post-rationalization. Because this process is neither smooth nor fast, it doesn't allow for iteration and therefore an opportunity to re-configure the 'pre-conceived' form is missed. Whoever wants to tackle this problem have to make a new tool or develop the existing ones to allow for this dual thinking. People like Axel Killian and Philippe Block have done studies to achieve real time visual feedback for analysis as designing. However their tools are customized to very specific geometries and construction methods: shells and masonry. From an architectural point of view it is still unclear what kind of morphologies can emerge from matter mechanics and programmatic complexity.

figure_7
Motorway Service Station, Deitingen Sud, 1968, Heinz Isler

RESEARCH OBJECTIVES

When talking about the work of Rem Koolhaas, Sanford Kwinter compares his 'extreme' architecture with a pilot flying a jet plane. The pilot, instead of being flesh and blood, is part of the mechanic realm of the plane. Only if the pilot is fully cognizant of the physical tolerances of the aircraft, would this machine suddenly be able to be maneuvered successfully to different directions that would offset the opponent within the physical limitations. The designer on the other hand similarly grasps and utilizes the intuition of material continuity in order to find what is unseen as a source of novelty and creativity. Similarly this thesis is investigating opportunities that work with the constraints of material force fields to generate organizational rules for spatial constraints by inventing its own computational procedures. It is not an obsession with structures or computational power that drives this research, but the curiosity to explore unknown territories out of which novelty or creativity can emerge. Therefore the main objective is not to find ways to achieve optimum structural efficiency with minimum material, but rather to find the medium between structural efficiency and programmatic complexity while generating new aesthetics and organizational rules.

visual feedback which requires fast computation, the analysis model used in this thesis is a truss analysis model which has its limitations in terms of applicability to provide accurate evaluation for some conditions (shell analysis and bending moments). However, it is sufficient enough to give an intuition of structural performance of any given case which is what this thesis is aiming for.

Force flow in a structure is irrelevant of the material type but purely related to geometry. As discussed earlier this stems from matter mechanics being the fundamental matter constraint that constructability or other behaviors build on to it. Although the driving force of this research is using material constraints to design, material types and constructability are issues that are omitted to allow for full comprehension of structural mechanics. So throughout the experiments of the research, material properties are default values for each of the analysis case meaning that using steel vs. concrete doesn't affect the configuration of form but can change member thicknesses.

SCOPE OF RESEARCH

Background research for such thesis topic requires acquiring advanced structural analysis knowledge and an ability to transfer this knowledge into computational procedures for the matter mechanics part of the thesis agenda. The knowledge I had acquired from Professor Jurgen Bathe's Finite Element Analysis, and Professor Jerome Connor's Analysis and Control courses at MIT have set up the basis for the technical side of the research. The other half of the agenda that deals with architectural organization feeds itself from my 6 years of architectural education and practice experience.

PRACTICES OF FORM FINDING

This chapter analyzes the work of practices of form finding from Swiss engineer Robert Maillart to contemporary engineers and architects. These works are classified into two categories: topology formations and formations within topological boundaries. Topology formations deal with global scale form finding such as a change in the height of a surface structure. On the other the proposed category of formations within topological boundaries deals with local scale form finding within a pre-defined surface space or volume.

Dune formations in nature are optimized topology formations that are resultant of continuous erosion or aggregation of sand particles towards a state of equilibrium with the forces of nature. Surface of dune formations is also subjected to a natural formation process which happens at a local scale and works within the global topology of dune formations *(figure_8)*. In the case of dune formations the overall topology defines the intensity of local formations depending on exposure to various conditions. This relationship of global defining local conditions reverse in the case of crystal or bubble formations where local formations affect the formations of global topology.

The form finding methods of Mailart and Gaudi evolved from linear topology formations to surface topologies in the work of Heinz Isler and Candela. These surface topology formations then became the basis of computational procedures in the work of Axel Killian and Mutsuro Sasaki who both also advanced the limited precedent shell geometries to free-form topology formations. The work of Frei Otto and the Lightweight Institute are also worth mentioning but is not explained in this book in order not to create repetition of concepts.

Given the detail and intricacy it requires figuring out structural formations in a given topological space, it was difficult to anticipate load paths within a volume or even in a surface until the age of computers. However Nervi was able to speculate on force path directions within slabs. Recently, the works of Reisser + Umemoto, Mutsuro Sasaki, Ney & Partners and AKT P.art have become examples of such formations.
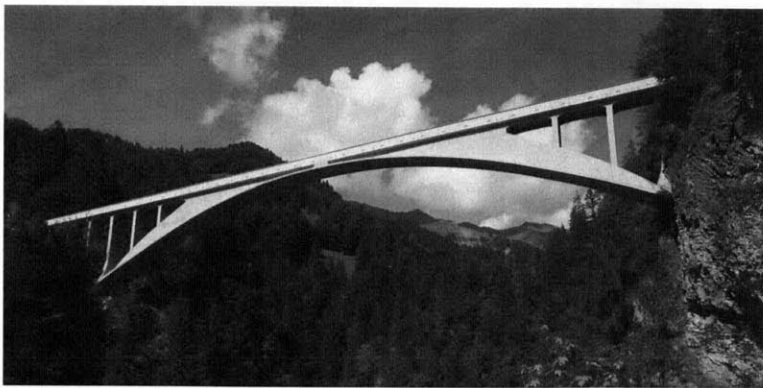

figure_8
Dune Formations

Robert Maillart:

The relationship between optimum force paths and form was first explored with the invention of the arch by Ancient Romans. It is no doubt that this geometry was an inspiration from rock formations in nature (refer to image). The horseshoe arch (semicircular arch) evolved its geometry to parabolic and catenary in the 17th century. If a cable is suspended at its end points, the resultant curve geometry is the catenary curve formation. This geometry is then flipped to reverse the tensile forces in the cable to compressive forces to act as an optimum arch geometry. In the 18th century, Karl Cullman developed a method called graphic statics which tries to represent graphically the force directions and magnitudes in a structure.

It was those techniques that helped the Swiss engineer Robert Maillart to form find his bridge geometries in the early twentieth century. The Salginatobel Bridge in Switzerland depicts his mastery in finding efficient structural systems. The geometric difference between bottom and top arch curves, the distribution of ties between the deck and the arch, the varying member thicknesses are clear outcome of an ambition to generate form that follows optimum force paths (figure_9).

figure_9
Robert Maillart, The Salginatobel Bridge in Switzerland, 1930

Antonio Gaudi:

In his early career Gaudi was first influenced by the work gothic revivalists such as the historian Viollet-le-Duc who tried to explain all gothic architecture in terms of structural rationality. In his later career, Gaudi questioned the rationality of gothic architectures. He criticized the flying buttress for being an extremely inadequate structure as an oblique column should essentially extend to the earth's surface. This was an observation after him studying cracks in the structure of Parma Cathedral in Mallorca. Gaudi's interest in nature and its formations, and the imperfectness of the gothic style has led him to experiment with new ways of defining spatial formations.

The catenary curve had been used before Gaudi. Gaudi's achievement was to bring multiplicity to this method which allows for application to more complex buildings. The design of the Sagrada Familia church was developed based on a model built with hanging chains whose geometry was then reversed to become centenary arches (figure_10). The process of designing this building required 10 years of experimenting with trials and errors. This idea of using physical models to test formal performance influenced figures like Heinz Isler, Frei Otto.

Among all the spatial form finders, Gaudi's approach in the Sagrada Familia church has been the most architecturally interesting. His experiments involved consideration of adjusting form until it accommodated various programmatic functions and constraints. Unlike the shell structures Gaudi was able to subdivide a globally form found space that creates various spaces for various programs to fit in. Given these consideration for both programmatic usage and structure, it is no surprise that the design process took 10 years to finalize. Although Gaudi's conception of space and structure is very compelling for his era, there are some drawbacks of his design process. The catenary forms can only provide a limited amount of geometric configuration. These formations create only stretched dome like spaces or aggregations of them. The chain formations are only optimized for single gravity load case. Consideration of horizontal load conditions could have affected the final member configuration beyond thickening the geometries extracted from the chain model.
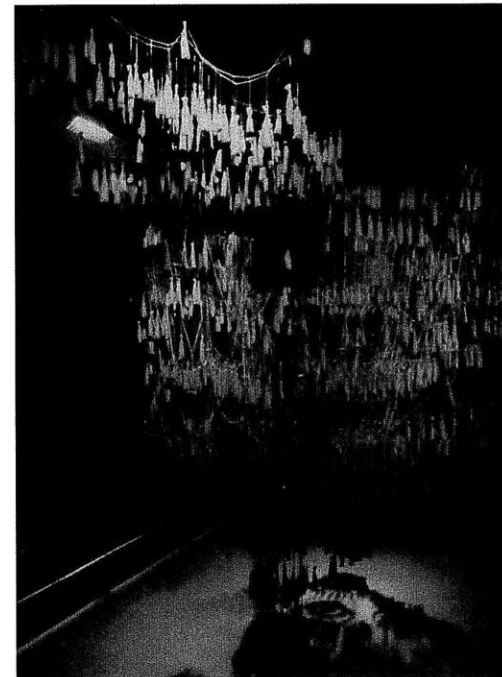
figure_10
Antonio Gaudi, Hanging Chains model for the Sagrada Familia Church, 1882

Felix Candela and Heinz Isler:

Candela who is seen as the master builder of shell structures of his time had deep interest for lightness and elegance in his work. He was an architect, engineer and contractor all at the same. The combination of these skills was essential for what he achieved. In order to achieve lightness and thinness he sought a mathematical way. The application of hyperbolic surface geometries enabled his concrete shell structures to be as thin as 1.5" thick and also allowed an ease of constructability. Since hyperbolic surface geometries can be defined with a series of straight contour lines, it was easy to translate that geometric input into material terms. These straight contour lines became wooden planks for the formwork. Constructability and the structural performance of these geometries were the success of this master builder *(figure_11)*.

A generation after Candela civil engineer Heinz Isler took a different methodology in form finding for shell structures. Influenced by Gaudi, Isler sought for manual shape creation process through physical model experiments. It was initially pneumatically shaped then hanging cloth geometries shaped by gravity methods that Isler explored. While Gaudi's work produced linear formations, Isler's hanging models form found surface geometries.

The shells that Candela and Isler produced are pure structural forms. Every inch square of the built form works to transfer loads to supports. There is no redundancy in the system that these structural shapes also have to perform as architectural surfaces like roofs or slanted walls. Both builders were able to create cracks and openings on the shell surfaces to allow light to penetrate. Since Candela's work relied on the geometric possibilities of hyperboloids, the boundaries of his shells could change proportionally by scaling, or additively by aggregation. Since Isler used a physical process, he was able to re-configure boundary conditions depending on site constraints and let gravity to work with those constraints to shape the hanging cloth.

In the works of both builders, resultant single spaces of shell geometries depict an architectural limitation of these surfaces. Whether a hyperboloid surface is aggregated or undulated to create variation to form a shell structure, space underneath is still singular unless there are additional non-structural surfaces for subdividing this single space. These shell geometries are generic forms and doesn't correspond to a program type. The same shell geometry can be built in different locations of the world to be used as a restaurant or as an aquatic center or even as a gas station. The genericness of these geometries does not register a typological identity.
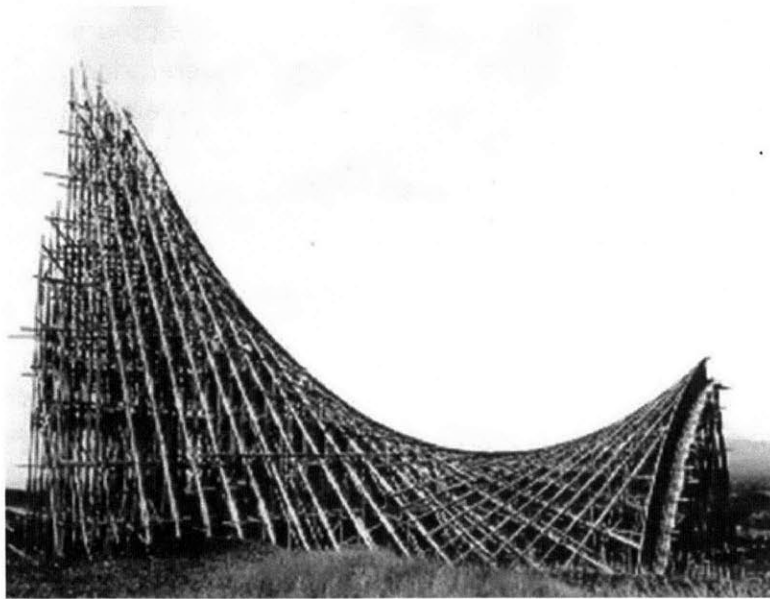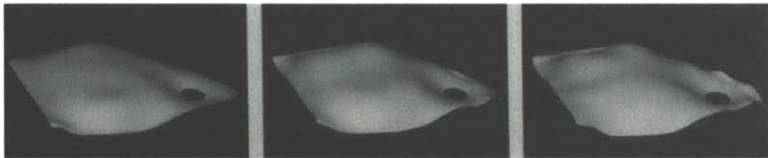
Axel Killian and Mutsuro Sasaki:

Inspired by the work Gaudi, Axel Killian developed a computational tool that generates digital hanging models. This tool uses a particle spring system which is well-known in computer science for creating physical simulations . Particle systems are networks of particles/points which are connected by virtual linear elastic springs with initial damping coefficients. By assigning weights to each particle, the particles as a network searches for a formal state until the system reaches to balance. This convergence to balance creates an approximation of hanging chain models of Gaudi. The required parameters for this form generation are the two-dimensional boundary of the design space, the number of supports and their locations, and the amount of applied load. The long process of generating the chain model for the Sagrada Familia Church can be achieved with this computational form finding method in minutes. Although this model is simplistic in terms of its analysis, it is an intuitive way of generating structural topologies (figure_12).

Mutsuro Sasaki is a Japanese engineer who has been involved in many innovative architectural structure projects including the Sendai Mediatheque, Rolex Learning Center and many others. The firm is aware of the power of computation and utilizes it to push the limits of conventional engineering. One of the many computational experiments of Sasaki's firm was working with the optimization method called Sensitivity Analysis which is a method of shape analysis and topology finding from an initial geometry. Shell structures have to be in a state of minimal stress and deformation in order to accommodate gravity loads in a very thin section. Strain energy which is the potential energy of a form when it deforms is the performance criteria for

the Sensitivity Analysis Method. This algorithm divides a surface into finite elements. By pushing and pulling these nodes of each element in the z dimension, the algorithm converges to a minimum state of strain energy. Although this algorithm is used as a post-rationalization tool in the Rolex Center in collaboration with SANAA, initial investigations used predefined geometries as catalysts for emergent structural topologies like in the projects of the National Grand Theater Proposal in Beijing in collaboration with Toyo Ito and the Kitagata Community center in Gifu Prefecture in collaboration with Arata Isozaki. (figure_13).
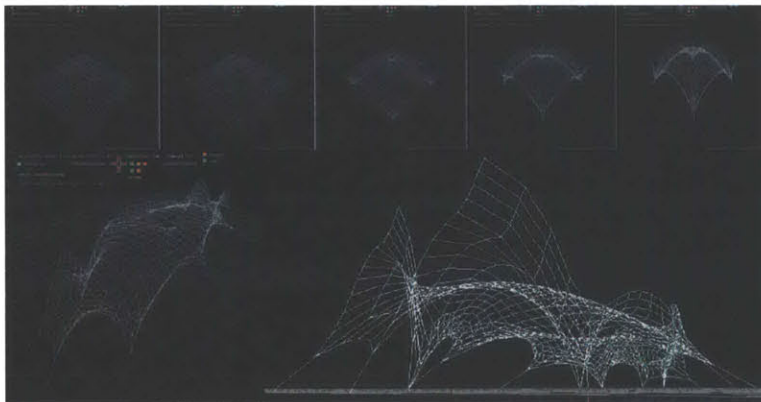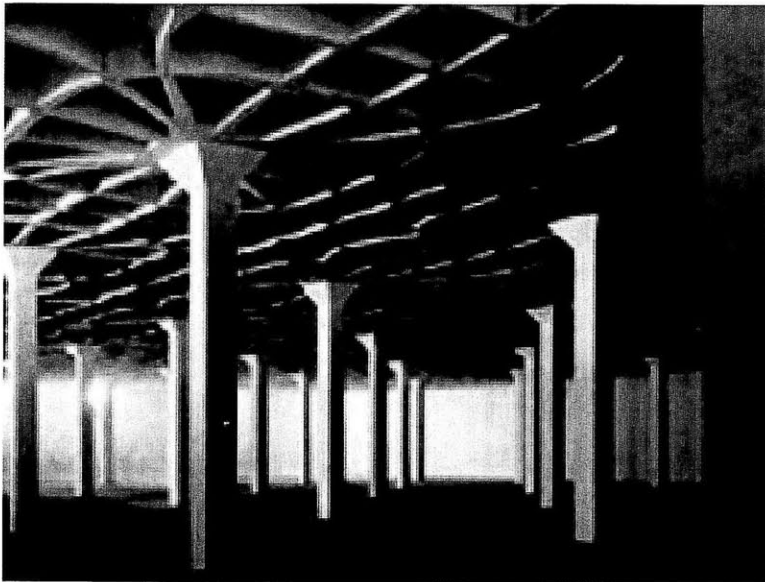
The shift from physical experiments of form making to digital experiments makes the process of design evaluation much faster as real time visual feedback is achieved in the case of Axel's model which only considers axial loads. The Sasaki method is more sophisticated from a mechanical engineering point of view since their tool considers both axial forces and bending moments. This sophistication brings in the burden of heavy computation of hours for both iterative analysis and topology generation. Given the goals and the ambitions of this thesis, a method similar to the former is more suitable. these computational approaches advance the work Isler and Candela by enabling free-form structurally optimized surface generation. However, there is still a lack of responsiveness to the given program under these surfaces.

figure_13
Mutsuro Sasaki, Kitagata Community center in Gifu Prefecture in collaboration with Arata Isozaki

figure_12
Axel Killian, Digital Hanging Model

FORMATIONS within TOPOLOGICAL BOUNDARIES

figure_14
Pier Luigi Nervi, Underside of floor slab of the Gatti wool factory, Rome, 1951

Luigi Nervi:

Nervi's work is a reflection of the awareness that form should follow the qualities of a material's nature. He stated that the reinforced concrete beams lose the rigidity of wooden beams or of metal shapes and ask to be molded accordingly to the line of the bending moments and the shearing stress. The beam structure of floor slab of the Gatti wool factory is the materialization of this interest (figure_14). The optimum force paths define the major areas of stress in the slab prior to form definition. These isostatic stress lines are then materialized as concrete beams that transfer load to the columns.

Beyond expressing the flow of forces through the slab struc-ture, the Gatti project creates a visually dynamic pattern, monolithic appearance due to its materialization, efficiency of material given that the slab thickness is reduced with this type of structural formation. According to Nervi requirements of construction are functional, economical and aesthetics. The functional and aesthetic requirements are achieved in this project as a blended outcome of this design process. To eco-nomically build such structure that follows the isostatic stress lines, Nervi fabricated special ferro-cement forms that could be used repetitively.

The functionality that Nervi was aiming for is only a structural functionality where these curved ribs function as beams. The system is so uniform and rigid that it wouldn't allow for any inflections due to an architectural function response. The func-tionality claim would have been justified in architectural terms if this slab pattern would have allowed a staircase to puncture through and mutate the isostatic lines accordingly.
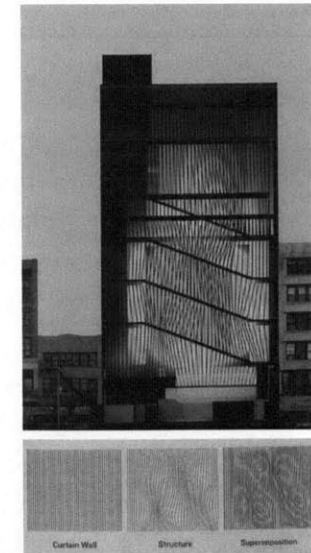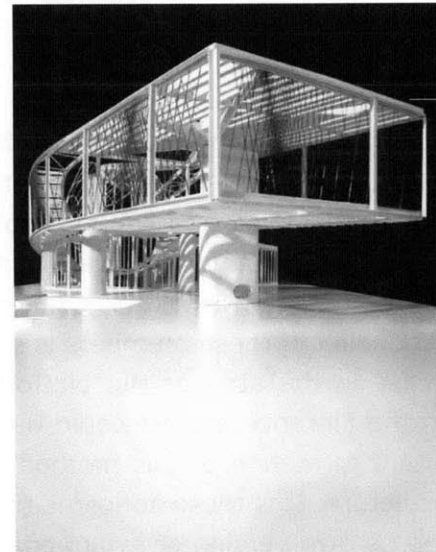
Reisser + Umemoto:

When Reisser+Umemoto is talking about the work of Frank Gehry, they criticize how he piles steel in order to achieve the forms he wants, ignoring the behavior of forces within the project. Gehry is optimizing his projects towards pure form. On the other hand, engineering purity tries to optimize toward the behavior of force only such as a suspension bridge. They claim to situate their practice in between these two approaches by navigating a range between the minimizing athletics of pure forces an maximizing of structures required by unrestrained of form, keeping both in play rather than extending into one at the expense of the other.

The conceptual work of Reisser+Umemoto feeds from the writings of Gilles Deleuze. Deleuze defines three types of geometry exact, inexact and anexact. Exact geometry is the regular or standard manifestation of form. Inexact is an accidental or an approximation of the exact form. The anexact is neither produced by an idealist nor essentialist mentality. The anexact is assumed to play out in real space rather than in the ideal space of abstract geometry. The scientific vagueness of anexact geometry and its linkage to spatial field is explored in the work Reisser+Umemoto.

The house project in Sagaponac uses an exact architectural geometric genotype of a diagrid meshwork. This pattern is mutated strategically to correspond to load paths. The densification of pattern forms local conditions of column like behaviors but yet not becoming exact column geometries. The façade of the New Museum proposal works with similar intentions of mutating an existing typology towards structural performance.

The standard vertical extrusions bend to support the loads of the circulatory system on the façade. It is the intentions to work with both material field and architectural genotypes that produces these patterns in both cases *(figure_15 and _16)*.

The intentions in the work of this office, is similar to the interest of this thesis. However their work is exploring this anexact condition at a 2d dimensional level. The spatial quality of the patterning in their work almost ends up as decoration and there is not a clear proposal for how the pattern mutations affect spatial organization.
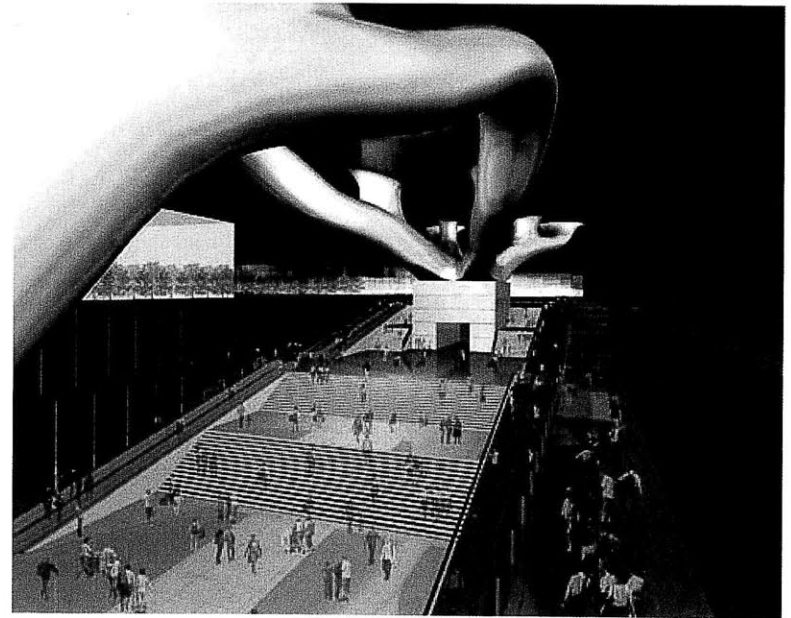

figure_15
Reisser+Umemoto, House in Sagaponac, New York

figure_16
*Reisser+Umemoto, New Museum Proposal, New York*

Mutsuro Sasaki:

The firm has reconsidered a topology optimization method called Evolutionary Optimization, which is often used in the field of mechanical engineering, at an architectural scale. Given a design space whose spatial boundaries are definite this computational procedure divides this space into finite cubic elements. The stress level of each element is evaluated. Based on the evaluation, the least stressed elements are removed from the work space. The method is trying to converge to a formation of equally stressed aggregation of elements. This is taking the work of Nervi to a three dimensional state. The resultant geometry is the following the major force/stress paths similar to the Gatti slab. The two-dimensional curved ribs of the Gatti slab evolved to become spatial structures to support the roof in both the Florence New Station proposal and the Qatar Education City projects *(figure_17)*.

This computational process is only constrained with boundaries and minimum height for the central space in both projects. There is not a relationship between form and organization. The process is creating a single space within which anything can happen. They have taken the work of Nervi to a 3 dimensional level however the functionality of these members is still pure structural. Grand staircase in the atria or the platform structure above the trains in the Florence project could have been used to push the resultant formation of this method to integrate more with the architecture. It is these concerns that inspired this thesis to develop its own version of evolutionary optimization where there are spatial constrained embedded n the form finding process *(See Chapter 4)*.

figure_17
Florence New Station Proposal, Arata Isozaki (architecture)-Mutsuro Sasaki (engineering), 2003
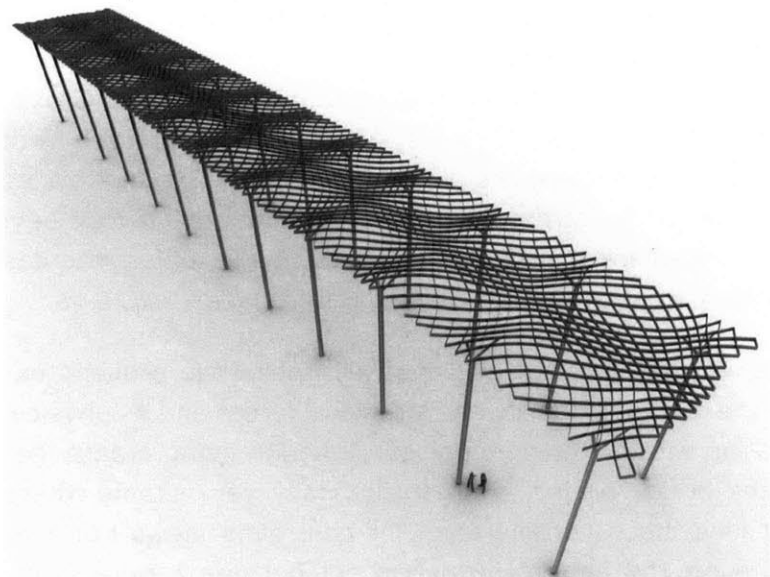
Laurent Ney:

In his book called 'Freedom of Form Finding' Belgian engineer discusses about how hierarchy has been a central theme in both structural and architectural projects and this being an obsolete method in a manufacturing age that requires more integration. The method of hierarchy in building industry simplifies a complex problem into parts. 19th century truss bridges within which each member has a function to carry a particular load. A collective behavior of multiple members acting on multiple load conditions is never part of the discussion. Ney criticizes this approach and projects their practice to be more similar to production processes of cars, airplanes and computers, all of which require an integrated thinking that blurs the distinction between structure, secondary elements and finish.

A clear example of his concepts and the logic of optimum force paths defining morphologies of a construct is the Kiel Canopy project in Antwerp *(figure_18)*. Structural optimization of the grid geometry is done by introducing variation in the grid geometry instead of changing member depths. The densification of the diagrid pattern towards the columns is an expression of the force flows that works within the pre-defined rectangular boundary. The canopy is a product of integrated thinking since the design goal of slender aesthetics that the architect was looking is achieved via formation of structural members.

AKT P.art:

P.art is a computational research group within the engineering firm of Adams Kara Taylor (AKT). In their articles called Simplexity, Sawako Kajima and Michalatos Panagiotis of P.art research group, stand for the emergency of simplicity out of intricate and complex sets of rules. They criticize the trend to exploit algorithmic design to produce complex forms by implementing simple easy formulas. Although visually complicated, the Land Securities Bridge project done in collaboration with Future Systems explores this idea. The algorithm in this project operates on the stress distribution diagram of the input bridge topology which was given by the architects. The complexity of the algorithm attempts to simplify and equalize the stress diagram. *(figure_19).*

Another project by AKT P.art which explores a non-directional densification of stress pattern is the Ren Building in Shanghai which was a product of collaboration with Bjarke Ingles Group. The façade, which also acts as structure, is punctured with circles of various sizes. The distribution and the scale of circles correspond to the stress diagram of the initial tower topology. High stressed areas are punctured with smaller holes whereas low stress areas are punctured with larger holes *(figure_20).*
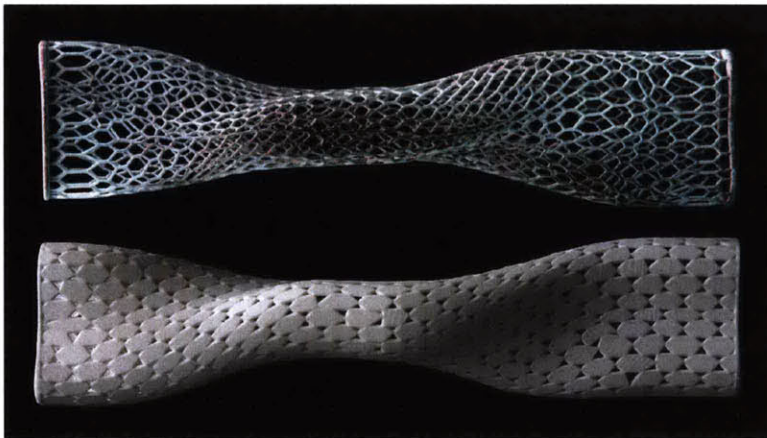
Similar to the work of Nervi and Ney, these two projects explore the relationship between structural forces and its physical formation within a given topology. Although being a passageway the bridge project is organizationally very simple, there could have been consideration for controlled views out that could allow the pattern formation not become a pure form-finding output. The tower in China is truly ignoring the rela-

figure_19
Land Securities Bridge, Future Systems (architecture), AKT (engineering), London

tionship between the structural skin and the programs behind it. One cannot register any scale in the project from outside. Due to the stress distribution there is less area of openings towards the base. Ideally to achieve a uniform lighting condition in every floor, the floors near base need more openings than top which is the opposite of what the structural form-finding emerges to. By thickening the skin near the base and allowing larger openings could have solved this issue.

figure_20
Ren Building, BIG (architecture), AKT (engineering), Beijing

OBSERVATIONS and DIRECTIONS for RESEARCH

After studying these precedents, it is clear that the relation-
ship between physical form which is constrained with matter
forces and organization is not explored except the work of
Gaudi who did this to a certain level. What is also surprising is
that there is no precedent work which -similar to dune forma-
tions- deal with first global topology optimization then form
finding within this topology. Each case study either deals with
topology formation which usually generates single spaces or
pattern formation within a topology which generates decora-
tion rather than architecture. Those observations set up the
basis for what should be done in the methodology of the re-
search. Blending the topology formations and pattern forma-
tions within topologies can create emerging formal and organi-
zational conditions. Any form finding procedure has to involve
organizational parameters; otherwise the generated form be-
comes irrelevant to the architectural function of the physical
space. The work Axel Killian and Mutsuro Sasaki's office are
great examples to set the computational direction of the re-
search. One has to understand the mathematics and physics
behind force flow patterns in order to be able to program both
matter logics and spatial logics. The next chapter looks at the
math and physics behind force patterns. In the fourth chapter
these scientific logics are tested with spatial parameters.

PRE-FORMATION STATE: GEOMETRY EVALUATION METHOD

The stiffness method of matrix analysis used in this research is a displacement based analysis. When computers were first introduced in the field of structural engineering, engineers needed to reconsider their classical methods for analysis. Matrix notations of mathematics which were first used in electrical engineering to solve for complex multi-parameter electricity distribution problems were introduced in structural engineering. Since matrix notations are the natural language of computation, a systematical reformulation was necessary for the classical displacement methods. This chapter will explain the procedures for analyzing a single member, matrix notations and how with matrix notations we can analyze larger groups of members.

In a force method analysis structural reactions are first solved then deflections, whereas in a displacement based analysis deflections are first calculated then reactions. Displacement based methods have advantages over force based methods. One of them is with displacement methods both determinate and indeterminate structures can be analyzed. When structural configurations are indeterminate, it means that the number of static equilibrium equations is not sufficient to solve for the internal forces and reactions on the given structure. This is not a sign of instability, but proves that force based analysis methods are not capable of analyzing any configuration.

Application of the displacement method, often called as the stiffness method, can show the deformation of one single node when a force is applied. Hooke's law of elasticity, which is an approximation that states the extension of a spring when load is applied, is the basis of this method. According to Hooke's law we can find the amount of displacement at the end of

Because of the complexity of data transfer from design tools to analysis tools, having a tool that does both will increase efficiency of work pace. In addition, this will also allow for emerging formations to happen which couldn't come directly from intuition. The designers' intuition of how to position structural members in a symmetrical or rectilinear form may work, but when the form is not symmetrical and have multiplicity of various organizational conditions, his/her own intuition may not help. The limitation of intuition also wouldn't allow for the possibility of blending structural and organizational forms.

In this chapter a pre-state of form finding method is explained. There is an initial geometry or structural configuration that needs to be fed to the analysis process, but this initial geometry is not a final form but rather a catalyst to find emerging forms by manipulating the configuration as getting simultaneous feedback.

$$F = k * u$$

(F= applied force, k= stiffness constant, u=displacement)

figure_21
Single Dimension member

spring from its equilibrium by dividing the applied force with a 'k' spring stiffness constant (figure_21).

Similar to the spring example, this formulation can be applied to any linear member. For a linear member 'u' is the displacement at the end node where force is applied. 'k' is the geometric stiffness coefficient which changes due to the relative Euclidean positioning of both end nodes. Multiplication of u and k will result in the axial force F. For any structural optimization process, the sole criteria is, for a given load condition, to minimize u by increasing the geometric stiffness of the structural configuration. It seems like a simple optimization goal but gets complicated when dealing with a network of members.

When a network of nodes of the same continuum structure is analyzed the relative relationships between each node has to be formulated and solved all together at one single operation. This is only possible with matrix notations which will be explained later in this chapter. When a larger structure is analyzed it has to be subdivided into discrete finite elements. If the discrete element is a line member, the relationship between end points of each member has to be solved with all the other end node couple geometries. So the stiffness value for each element has to be considered collectively in order to define the global stiffness "K" of the whole structure.

How to formulate a matrix?

Given a set of algebraic equations, unknown parameters and their scalar values can be grouped separately in rectangular matrix formats as in the example. By doing so the given set of information is concisely formulated into one equation. The scalar values for x, y and z are listed respectively in columns. If one of the unknown parameters in the system is not present in a equation, the corresponding scalar value is registered as zero in the matrix. The unknown parameters and the results constitute single column matrices. The multiplication of the 3 x 3 matrix of scalar values and the 3 x 1 column matrix of unknown parameters gives a 3 x 1 column matrix of resultant values. Multiplication of matrices will be explained.

$$2x + 3y + z = 12$$
$$x + 2y \quad\;\; = 4$$
$$3x + 12y - 2z = 8$$

$$
\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 0 \\ 3 & 12 & -2 \end{bmatrix}
*
\begin{bmatrix} x \\ y \\ z \end{bmatrix}
=
\begin{bmatrix} 12 \\ 4 \\ 8 \end{bmatrix}
$$

Row, Column and Square Matrix:

If the matrix consists only of elements in a single row, it is called a row matrix. For example, a 1 x n row matrix is written as:

$$A = [x_1 \; x_2 \; x_3 \; .... \; x_n]$$

*_MATRIX NOTATIONS AND OPERATIONS_*

The notation of matrix algebra is essentially a representation of grouping scalar values of multiple numeric equations. The practicality of matrices lies on the ability to represent a whole group of equations in one single symbol. This way not only an economy of symbols is achieved, but also procedures to solve these equations are simplified to solving a single unknown parameter problem vs. solving a problem with many unknown parameters. By definition a matrix is a rectangular array of elements arranged in rows and columns. Given that computers store data as arrays, the matrix formulation of a structural analysis problem can easily be programmed

If the elements of a matrix are stacked in a single column it is called a column matrix. Displacement values and forces on each node need to be formulated in column matrices. For example, a n x 1 row matrix is written as:

$$A = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

When the number of rows in a matrix equals the number of columns in a matrix, it is called a square matrix. Both member stiffness and global stiffness matrices have to be formulated as square since each node has both a corresponding index on the rows and columns which create an equally sized matrix. An n x n square matrix is formulated as:

$$A = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{21} & \cdots & x_{2n} \\ \vdots & & & \\ x_{n1} & x_{n2} & \cdots & x_{nn} \end{bmatrix}$$

Addition and Subtraction:

To formulate the global stiffness matrix all the local stiffness matrices have to be added to each other with this operation. If the dimensions of two matrices are same, values in each corresponding cell can be added or subtracted. If they are not the same, the dimensions of the smaller matrix are increased by adding zeros to the new cells.

$$A = \begin{bmatrix} 12 & 3 & 0 \\ 3 & 4 & -7 \\ -6 & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 6 & 2 \\ 9 & 0 & 0 \\ -2 & -4 & 1 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 17 & 9 & 2 \\ 12 & 4 & -7 \\ -8 & -3 & 1 \end{bmatrix} \qquad A-B = \begin{bmatrix} 7 & -3 & -2 \\ -6 & 4 & -7 \\ -4 & 5 & -1 \end{bmatrix}$$

Multiplication by a Scalar:

This operation is used when a material type or the section of structure changes. If strength is increased by a scalar value, each element in the stiffness matrix is multiplied by the given scalar.

$$A = \begin{bmatrix} 7 & 1 \\ 3 & 4 \end{bmatrix} \qquad k = 2$$

$$k*A = \begin{bmatrix} 14 & 2 \\ 6 & 8 \end{bmatrix}$$

Matrix Multiplication:

This operation is used to satisfy the F= k*u equation of Hooke's Law. In that case k is a square matrix and u and F are column matrices. In order to multiply a matrix with another matrix, the matrices have to be conformable. If the number of columns of an A matrix equals the number of rows in a B matrix,

then this condition is satisfied. A*B will have a solution. The order of multiplication is important Unless A and B are square matrices, B*A is not possible since the dimensions are shifted. Also multiplication of more than two matrices is not possible to do at once. However multiplication in groups of two is possible.

$$\begin{array}{ccc} A & * & B & = & C \\ (m \times n) & & (n \times t) & & (m \times t) \end{array}$$

After checking if matrix dimensions are conformable, multiply the elements in each row of A by corresponding elements with each corresponding column of B, then add the results to the rows of C respectively.

$$A = \begin{bmatrix} 11 & 9 \\ 4 & 6 \\ -3 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & -1 \\ 2 & 7 \end{bmatrix}$$

$$\begin{array}{ccc} A & * & B & = & C \\ (3 \times 2) & & (2 \times 2) & & (3 \times 2) \end{array}$$

$C_{11} = (A_{11} * B_{11}) + (A_{12} * B_{21}) = 11 * 5 + 9 * 2 = 73$
$C_{12} = (A_{11} * B_{21}) + (A_{12} * B_{22}) = 11 * (-1) + 9 * 7 = 62$
$C_{21} = (A_{21} * B_{11}) + (A_{22} * B_{21}) = 4 * 5 + 6 * 2 = 32$
$C_{22} = (A_{21} * B21) + (A_{22} * B_{22}) = 4 * (-1) + 6 * 7 = 38$
$C_{31} = (A_{31} * B_{11}) + (A_{32} * B_{21}) = (-3) * 5 + 1 * 2 = -13$
$C_{32} = (A_{31} * B_{21}) + (A_{32} * B_{22}) = (-3) * (-1) + 1 * 7 = 10$

Then the resultant matrix becomes:

$$C = \begin{bmatrix} 73 & 62 \\ 32 & 38 \\ -13 & 10 \end{bmatrix}$$

Matrix Partitioning:

Matrices can be subdivided into sub matrices by portioning. This is very helpful if some of the data in a matrix are not necessary for further matrix operations. Support nodes in a structure are assumed to have zero displacement and removing those nodes from the matrix not only saves computation time but also allows the possibility to find a solution.

$$A = \left[\begin{array}{c|ccc} x_{11} & x_{12} & x_{13} & x_{14} \\ \hline x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \end{array} \right] = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

Where;

$$A_{11} = [\; x_{11} \;] \qquad A_{12} = [\; x_{12} \quad x_{13} \quad x_{14} \;]$$

$$A_{21} = \begin{bmatrix} x_{21} \\ x_{31} \end{bmatrix} \qquad A_{22} = \begin{bmatrix} x_{22} & x_{23} & x_{24} \\ x_{32} & x_{33} & x_{34} \end{bmatrix}$$

figure_22
Two Dimensional Member

How to establish a stiffness matrix for a single line planar member will be discussed in this section by using local co-ordinate axes defined as x' and y'. In order to find a relative relationship between geometry and material properties, in one case we need to assume one node to be fixed and load applied on the other end, and in the second case flip the load node and the fixed node, then set up the relationships *(figure_22)*.

$$f1' = \frac{A*E}{L} * u1$$

$$f2' = -\frac{A*E}{L} * u1$$

$$f1'' = -\frac{A*E}{L} * u2$$

$$f2'' = \frac{A*E}{L} * u2$$

by superposition, we can combine these four equations into two:

$$f1 = \frac{A*E}{L} * (u1 - u2)$$

$$f2 = \frac{A*E}{L} * (u2 - u1)$$

(f=local forces, A=cross sectional area of member, E= Young Module"s of Elasticity, L= length of the member)

Referring to the matrix multiplication operation technique, these two equations can be written in matrix form as follows:

$$
\begin{matrix} f_1 \\ f_2 \end{matrix} = \frac{A*E}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} u_1 \\ u_2 \end{matrix}
$$

Therefore the local stiffness k' matrix becomes:

$$
\underline{k'} = \frac{A*E}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad , \quad \underline{f} = \underline{k'} * \underline{u}
$$

(the underscore notation for f, u or k' refer to those symbols being a matrix rather than having a single real value)

Each member in a structure network can have different local axes than any other member in the system. These local directions have to be translated to global coordinate system so the evaluation process can operate in one unified coordinate system. The assumed x' and y' local axes will be translated to x and y global axes, and the angle between each local and global axes are notated as ax and ay. To make the notations even simpler it is assumed that $\cos(a_x)$ is A_x and $\cos(a_y)$ is A_y. The figures show the amount of displacements on both x and y axes which are respectively equal to Δ_1*A_x and Δ_2*A_y for Node1 and $\Delta_3 * A_x$ and $\Delta_4 * A_y$ for Node2 (figure_23).

$$
u_1 = \Delta_1 * A_x + \Delta_2 * A_y \quad , \quad u_2 = \Delta_3 * A_x + \Delta_4 * A_y
$$

$$
\begin{matrix} u_1 \\ u_2 \end{matrix} = \begin{bmatrix} A_x & A_y & 0 & 0 \\ 0 & 0 & A_x & A_y \end{bmatrix} \begin{matrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \end{matrix} \text{x form:}
$$
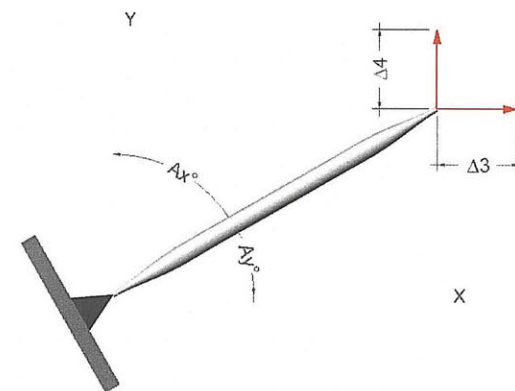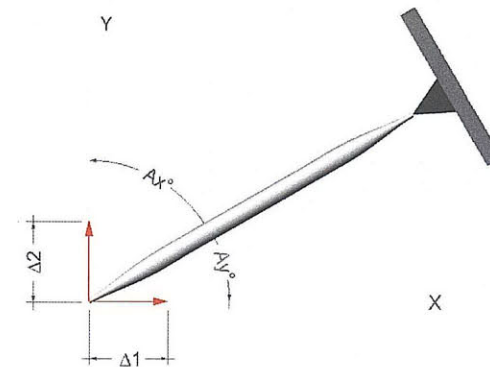
figure_23
Translation of local displacements to global coordinate system

$\underline{u} = \underline{T} * \underline{\Delta}$

The "T" matrix is what transforms the global coordinate displacement values (Δ) to local displacements (u). This matrix is only composed of cosine values of ax and ay. Similarly there has to be a relationship set between global and local forces. Given that the forces and displacements operate on the same axes but with negative directions to each other, the transformation matrix to translate local forces to global forces has the same values as the displacement transformation matrix except that the matrix is rotated. If an 'M' matrix is rotated it is called a transpose of the original matrix and denoted as 'M^T'. So the T displacement transformation matrix becomes T^T force transformation matrix. The local stiffness matrix k' is already defined, now the relationship between the k' and k global member stiffness matrix need to be set so as to define what k is.

$F_1 = f_1 A_x, \qquad F_2 = f_1 A_y, \qquad F_3 = f_2 A_x, \qquad F_4 = f_2 A_y$

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \begin{bmatrix} A_x & 0 \\ A_y & 0 \\ 0 & A_x \\ 0 & A_y \end{bmatrix} * \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

$\underline{F} = \underline{T}^T * \underline{f}$

Since $\underline{f} = \underline{k}' * \underline{u}$ and $\underline{u} = \underline{T} * \underline{\Delta}$; therefore: $\underline{f} = \underline{k}' * \underline{T} * \underline{\Delta}$
This will give us: $\underline{F} = \underline{T}^T * \underline{k}' * \underline{T} * \underline{\Delta}$

$\underline{F} = \underline{k} * \underline{\Delta}$
$\underline{k} = \underline{T}^T * \underline{k}' * \underline{T}$

With this last equation, by performing the matrix multiplication operations, we can find the global member stiffness matrix.

$$\underline{k} = \frac{A*E}{L} \begin{bmatrix} A_x^2 & A_x*A_y & -A_x^2 & A_x*A_y \\ A_x*A_y & A_y^2 & -A_x*A_y & -A_y^2 \\ -A_x^2 & -A_x*A_y & A_x^2 & A_x*A_y \\ -A_x*A_y & -A_y^2 & A_x*A_y & A_y^2 \end{bmatrix}$$

This process was to demonstrate how to find the stiffness matrix of a planar member whose coordinates are only on XY plane. Given the complexity of the explanation of establishing the global member stiffness matrix of a 3d truss member whose coordinates can lie on any 3d axes, is omitted in this chapter and just the matrix itself is given below:

$$\underline{k} = \frac{A*E}{L} \begin{bmatrix} A_x^2 & A_x*A_y & A_x*A_z & -A_x^2 & -A_x*A_y & -A_x*A_z \\ A_x*A_y & A_y^2 & A_y*A_z & -A_x*A_y & -A_y^2 & -A_y*A_z \\ A_x*A_z & A_y*A_z & A_z^2 & -A_x*A_z & A_y*A_z & -A_z^2 \\ -A_x^2 & -A_x*A_y & -A_x*A_z & A_x^2 & A_x*A_y & A_x*A_z \\ -A_x*A_y & -A_y^2 & -A_y*A_z & A_x*A_y & A_y^2 & A_y*A_z \\ -A_x*A_z & -A_y*A_z & -A_z^2 & A_x*A_z & A_y*A_z & A_z^2 \end{bmatrix}$$
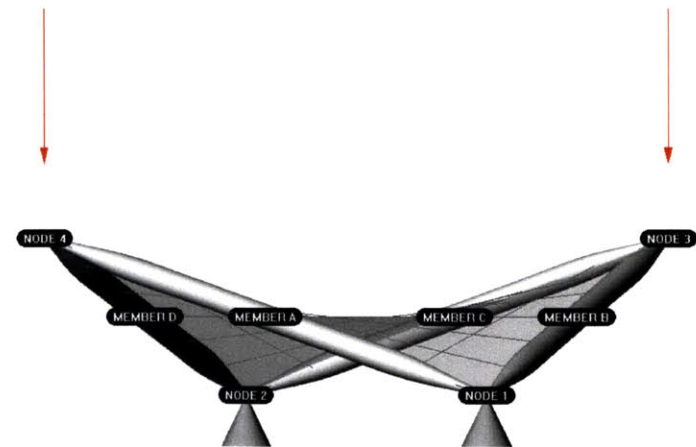
$A_x = \cos(a_x), A_y = \cos(a_y), A_z = \cos(a_z)$

These A_x, A_y and A_z cosine values of each node is found by using the coordinate locations of each node. Rhinoscript can easily give the x,y and z coordinate values of a point node. These values can be programmed as:
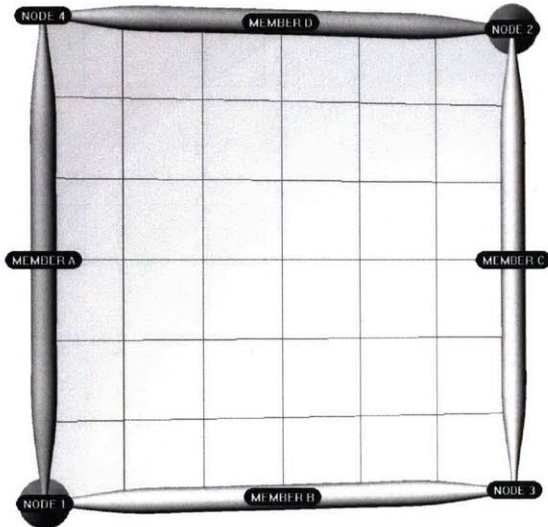
$$A_x = \cos(a_x) = \frac{N_x^A - N_x^B}{Length} = \frac{N_x^A - N_x^B}{\sqrt{(N_x^A - N_x^B)^2 + (N_y^A - N_y^B)^2 + (N_z^A - N_z^B)^2}}$$

$$A_y = \cos(a_y) = \frac{N_y^A - N_y^B}{Length} = \frac{N_y^A - N_y^B}{\sqrt{(N_x^A - N_x^B)^2 + (N_y^A - N_y^B)^2 + (N_z^A - N_z^B)^2}}$$

$$A_z = \cos(z) = \frac{N_z^A - N_z^B}{Length} = \frac{N_z^A - N_z^B}{\sqrt{(N_x^A - N_x^B)^2 + (N_y^A - N_y^B)^2 + (N_z^A - N_z^B)^2}}$$
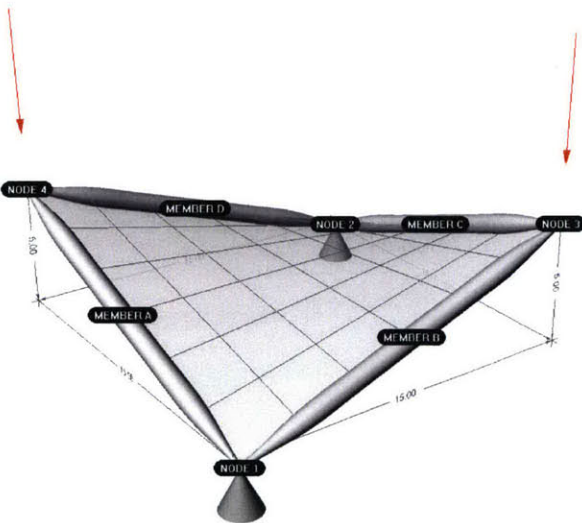
figure_24
member assembly of four with two support nodes

GLOBAL STIFFNESS MATRIX FORMULATION

How to formulate a member stiffness matrix is explained in the previous section. In order to constitute a global stiffness matrix of a given structure, each discrete member of the structure has to be identified as well as the specific nodes of connections to supports. The order of labeling nodes (or the intersection points between members) is crucial as it affects the solvability of the mathematical problem. The indexing of the matrix is based on nodal indexing. So values from the member stiffness matrix of the first node will be on the top left whereas values of the last node will be on the bottom right. It is important that labeling starts from support nodes. This will allow easy partitioning later to remove those nodal values from the problem solving process since the support nodes don't have impact on that.

How to formulate a global stiffness matrix for a given structure will be explained through an example. The example has only four members and four nodes two of which are connected to ground. There is load applied on the free nodes *(figure_24)*. The process is to first set up a stiffness matrix for each member. Then by matrix addition method, add corresponding values from the member matrices to the global stiffness matrix. If there is no corresponding value coming into some of the cells, those cells will remain as zero.

MEMBER A;

	Node 1			Node 4		
	x	**y**	**z**	**x**	**y**	**z**
	2.84	0.07	0.94	2.84	-0.07	-0.94
Node 1	0.07	0	0.02	-0.07	0	-0.02
	0.94	0.02	0.31	-0.94	-0.02	0.31
	2.84	-0.07	-0.94	2.84	0.07	0.94
Node 4	-0.07	0	-0.02	0.07	0	0.02
	-0.94	-0.02	0.31	0.94	0.02	0.31

MEMBER B;

	Node 1			Node 3		
	x	**y**	**z**	**x**	**y**	**z**
	0	0	0	0	0	0
Node 1	0	2.76	0.88	0	2.76	-0.88
	0	0.88	0.28	0	-0.88	0.28
	0	0	0	0	0	0
Node 3	0	2.76	-0.88	0	2.76	0.88
	0	-0.88	0.28	0	0.88	0.28

MEMBER C;

	Node 2			Node 3		
	x	**y**	**z**	**x**	**y**	**z**
	2.84	-0.07	-0.94	2.84	0.07	0.94
Node 2	-0.07	0	0.02	0.07	0	-0.02
	-0.94	0.02	0.31	0.94	-0.02	0.31
	2.84	0.07	0.94	2.84	-0.07	-0.94
Node 3	0.07	0	-0.02	-0.07	0	0.02
	0.94	-0.02	0.31	-0.94	0.02	0.31

MEMBER D;

	Node 2			Node 4		
	x	**y**	**z**	**x**	**y**	**z**
	0	0	0	0	0	0
Node 2	0	2.87	-0.97	0	2.87	0.97
	0	-0.97	0.33	0	0.97	0.33
	0	0	0	0	0	0
Node 4	0	2.87	0.97	0	2.87	-0.97
	0	0.97	0.33	0	-0.97	0.33
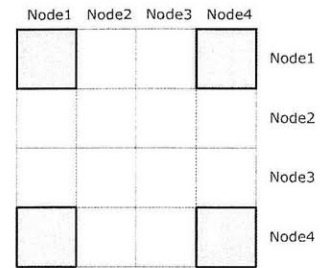
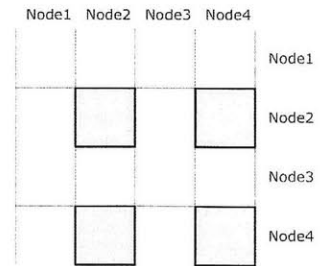☐ MEMBER A
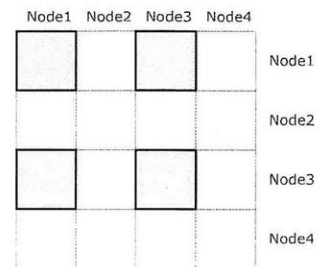
☐ MEMBER B
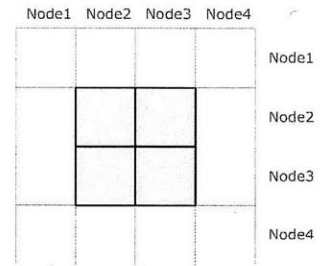
☐ MEMBER C

☐ MEMBER D

These four matrices are then compiled to constitute the global stiffness matrix.

		N.1			N.2			N.3			N.4	
	2.84	0.07	0.94	0	0	0	0	0	0	2.84	-0.07	-0.94
Node 1	0.07	2.77	0.91	0	0	0	0	2.76	-0.88	-0.07	0	-0.02
	0.94	0.91	0.6	0	0	0	0	-0.88	0.28	-0.94	-0.02	0.31
	0	0	0	2.84	-0.07	-0.94	2.84	0.07	0.94	0	0	0
Node 2	0	0	0	-0.07	2.88	-0.94	0.07	0	-0.02	0	2.87	0.97
	0	0	0	-0.94	-0.94	0.64	0.94	-0.02	0.31	0	0.97	0.33
	0	0	0	2.84	0.07	0.94	2.84	-0.07	-0.94	0	0	0
Node 3	0	2.76	-0.88	0.07	0	-0.02	-0.07	2.77	0.91	0	0	0
	0	-0.88	0.28	0.94	-0.02	0.31	-0.94	0.91	0.6	0	0	0
	2.84	-0.07	-0.94	0	0	0	0	0	0	2.84	0.07	0.94
Node 4	-0.07	0	-0.02	0	2.87	0.97	0	0	0	0.07	2.88	-0.94
	-0.94	-0.02	0.31	0	0.97	0.33	0	0	0	0.94	-0.94	0.64

This global matrix multiplied by the displacement matrix will give the force matrix

$k*u = F$

N.1   N.2   N.3   N.4

0	F_1
0	F_2
0	F_3
0	F_4
0	F_5
0	F_7
u_7	F_7
u_8	F_8
u_9	F_9
u_{10}	F_{10}
u_{11}	F_{11}
u_{12}	F_{12}

$*$ ... $=$

K^1	K^2
K^3	K^4

$*$

U^1
U^2

$=$

F^1
F^2

If we recall matrix partitioning method, by subdividing these above matrices as shown, the support nodes will be eliminated from the equation.

By partitioning we get two equations:

$K^1 * U^1 + K^2 * U^2 = F^1$ *(Equation 1)*

$K^3 * U^1 + K^4 * U^2 = F^2$ *(Equation 2)*

The values in U^1 are known and they are all zeros since there is not displacement on a support node. The values in the F^1 sub matrix are not known since they are the reaction forces on the support nodes. Values in the F^2 sub matrix are already given and we are trying to solve to find the values in U^2.

Since all the values in the U^1 column matrix are zeros, the first term of both *equations 1* and *2* become zero after matrix multiplication. Therefore the resultant equations are:

$K^2 * U^2 = F^1$ *(Equation 3)*

$K^4 * U^2 = F^2$ *(Equation 4)*

With these conditions *Equation 3* is not solvable since U^2 and F^2 are not known. So we only deal with *equation 4* where K^4 is:

Node 3			Node 4		
2.84	-0.07	-0.94	0	0	0
-0.07	2.77	0.91	0	0	0
-0.94	0.91	0.6	0	0	0
0	0	0	2.84	0.07	0.947
0	0	0	0.07	2.88	-0.94
0	0	0	0.94	-0.94	0.64

To solve for U^2 , The F^2 matrix has to be divided by the K^4 matrix. These types of problems can be solved with a method called Gauss Elimination which is an algorithm for solving inter-related linear equations by finding the inverse of a given matrix then using that to solve for the unknowns. The algorithm has two parts. The first part reduces the coefficient matrix of the equations to a triangular matrix form (meaning the other half reaches to zeros). This reduction is done by dividing each row with a value such that when each successive row is subtracted all the values on the left of the diagonal line become zero. The second part of the algorithm, by back-substitution, calculates the unknown displacements. In a upper triangular matrix form there is only one non-zero value in the last row which allows for finding the corresponding unknown displacement value since all the other unknown displacements have the coefficient of zero for that row. This found displacement value is then substituted for the row above and the second unknown displacement is solved. This operation is repeated until every value in the U matrix is found.

Gauss Elimination Pseudo Code:

Part 1: Forward Elimination

+ start from the second row
+ For every row
 + find the factor value by dividing the value in the first column of the current row with the first column value of the row above
 + multiply each value in the row above by the factor value
 + subtract all the multiplied values from the corresponding values in the current row
 + multiply the Force value of the row above by the factor then subtract this value from the current row's force value
 + go the next row below
 + loop until it reaches the last row

Part 2: Back Substitution

+ Start from the last row last column value.
+ For every row
 +multiply all the column values with the corresponding known U values
 +add the multiplied values
 +subtract the sum value from the corresponding force value
 +divide this resultant value by the diagonal element of the matrix on the current row to find the corresponding unknown U
 + go to the next row above
 + loop until it reaches the top row

With this method, for a given load value of 0.01 U2 column matrix is solved:

Node3 X axis displacement: 0.68
Node3 Y axis displacement: -0.66
Node3 Z axis displacement: 2.10
Node4 X axis displacement: -0.51
Node4 Y axis displacement: 0.51
Node4 Z axis displacement: 1.51

PROCEDURES FOR PROGRAMMING an ANALYSIS TOOL

Rhino 3d is one of the most widely used three dimensional architectural software packages. In comparison to other software packages, it allows for organic as well as rectilinear form modeling while keeping high precision of geometries. Therefore it is quite popular among architects. Digital modeling processes have their fallacies. One of which is: there is no sense of understanding weight and physical constraints of matter. Instead of programming a separate analysis tool for this research, making a plug-in code for Rhino allowed the opportunity to evaluate form while designing.

The procedures to analyze a network of a certain number of structural members were explained in the previous section. Recent practices in architecture tested the single surface concept both for form making and organizing space. For instance the idea of one single surface behaving both as a wall and a ceiling is proposing an integrated systems approach where it introduces simplicity of formalization which emerges from a more complex organization. Therefore the continuous surface has always a strong position in any architectural discourse. From an analysis point of view, there has to be discrete amount elements for analysis to happen. The necessity to decompose a continuum structure to discrete elements sometimes brings the burden of excessive information to analyze. So the number of elements to be subdivided from the continuum structure should be neither minimal in a way that it doesn't ignore any topological details by having less elements nor too much that there is excessive information. The custom tool made for this research, uses a parametric value for the amount of subdivision, so this can be adjusted depending on the topology of the continuum matter. The nurbs modeling features of Rhino for surfaces already have embedded parametric space definitions (u and v). So it is a fairly easy process to define the subdivision points on a surface. Multiplicity of surfaces at different levels will allow volumetric subdivisions.

Once a continuous surface or volume is discretized to nodal points, these points have to be connected to each other with straight member lines. It is crucial that the members are drawn such that there is a continuous flow of possible forces over the network of these newly drawn members. Since the locations of all the nodes and members are defined after dis-

cretization, support nodes can be selected by user depending on where the continuum surface or volume is touching the ground or in other words: transferring its loads out. The labeling of those nodes and members are important for assembling the global stiffness matrix. As explained in the example of the previous section, by listing support nodes first we can then remove those values from the matrix in order solve for the unknown displacements.

For all the discrete members, member stiffness matrices have to be formed. In a separate array the information of which member stiffness matrix goes which part of the global matrix has to be stored. Using this array of indexical information, values from the member stiffness matrix can be added to the corresponding cells on the global stiffness matrix. Since some members share nodes, there will be overlaps of member stiffness matrices on the global matrix. Those overlapping values have to be summed up.

As explained in the previous section, the global matrix need to be trimmed by three times the number of support nodes both on the left and top sides before gauss elimination can be processed. The Gauss elimination is coded in the pseudo code available in this book. After the displacements are solved, the magnitude of the sum of the x, y and z displacement vectors for each node can be visualized by extruding normal lines out the input surface whose length is proportional with this magnitude. Spheres of proportional scales at nodes or gradient color mapping are other ways of visualizing the displacement values.

Work Flow Diagram:

SUPPORT NODES

NUMBER OF DIVISIONS

SURFACE TO BE ANALYZED

FORCE MATRIX

DIVIDE SURFACE TO GRID POINTS

NODES

MEMBERS

SORTED NODES

FORMULATE MEMBER STIFFNESS MATRICES

ASSEMBLE GLOBAL STIFFNESS MATRIX

GAUSS ELIMINATION

VISUALIZE DISPLACEMENT DATA

INPUTS

PROCESSING

OUTPUT

Analysis Tool Psuedo Code:

_Input:
_surface/volume,
_subdivision numbers (size: m x n),
*_force column matrix (size: 1 x (m*n))*

_For the given subdivision number
*_Divide the surface/volume to points (number of points: m*n)*
_Generate members between these points
(number of members: m(n+1 + n*(m+1))*

*_Select Support nodes (has to be less then m*n)*

_For each node
*   _Check if the node is a support node then append to the new*
array of "sorted nodes"

_For each node
*   _Check if the node is not a support node then append to the*
sorted nodes array

_Use the indexing of sorted nodes array to initialize a global matrix
of zero values
* (matrix size: 3*m x 3*n)*

_For each member
*   _Formulate a member stiffness matrix*
(using the procedures in the previous sections)
*   _For each value in the stiffness matrix*
_Find the corresponding cells in the global matrix and add to existing
values in the cells (initially zero)

_Perform matrix partitioning by removing columns of 3 times the
number of support nodes on the left and rows of 3 times the number
of support nodes on the top of the global matrix

_Using the given force matrix solve for the displacement values on
each node with Gauss Elimination method
_To reflect this data of solved displacements visually either extrude
lines from nodes, or add spheres at nodes, or do a color mapping
gradient over the surface/volume

TOPOLOGY AND PATTERN FORMATIONS

TOPOLOGY FORMATIONS

The precedent projects of topology formation types show ways of finding optimum shell geometries of single space conditions. Because of this spatial singularity of those form generation methods, we do not see them as being used in contemporary practice. Contemporary architecture requires mixed usage and multiplicity of spatial conditions. One of the objectives of the thesis is to tackle this issue and find ways of subdividing a non-deterministic topology to form find.

Form is the final shape outcome of a process. If a point on the surface changes its coordinate, the form of the surface changes whereas topology is still the same as long the surface does not tear or is not patched with other surfaces. So the term topology is a better fit for a geometric re-configuration process which aims to find a form at the end. Topologies are non-deterministic in their nature since their geometric configuration is by definition subject to change due to forces or any other mathematical rules. This concept also works parallel with the idea that to find a form requires to be formless, to delay the state of having form, so that a new possibility can emerge.

There will be two types of topologies examined in this section: continuous and discontinuous. From a computational point of view, continuous surfaces are simpler to subdivide and analyze versus discontinuous surfaces. Because of holes or bifurcations on the surface, discontinuity of geometry requires additional procedures to subdivide the surface to meaningfully sorted analysis nodes. In this section, the continuous topology formations will be explained first followed by the presentation of discontinuous surface and volumetric topologies.

The first example presented here, looks at how to manipulate a continuous surface geometry from a flat condition to define a volume while considering the structural performance. Initial inputs are loads (vertical loads only in this case), number of analysis nodes, number of support nodes and their locations. The bar to the right of the figure shows total amount of deformation on the surface *(figure_25)*.

As the surface becomes to have arch like sections from a shallow flatter sectional configuration, the amount of deformation starts to decrease. In this case vertical loads are applied only, and these loads can be transferred smoother when the volume created by the surface has more depth in the z direction. The loads can be transferred to the support nodes through each truss member axially that is connecting the analysis nodes. Because truss members assume no fixed rotation at the joints, flat alignment and vertical load application to such joints will result in infinite displacement values. As explained in the previous section the stiffness matrix is composed of cosine values. Mathematically cosine values are proportional with the z height change. If Z coordinate value of a node increases, its cosine value will increase therefore the configuration becomes stiffer. This is not to say that the higher the surface height, the better structural configuration it is, since after certain height increase deformation change increases on a given surface.

Once a desired height is achieved, the stresses on the surface structure can be relaxed by dipping the topology to touch the
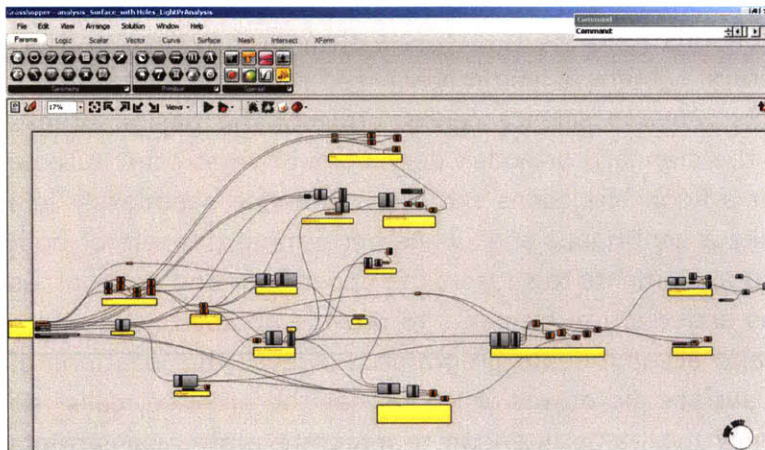
figure_26
screen shot, analysis tool in grasshopper format, plug-in for Rhino 3D
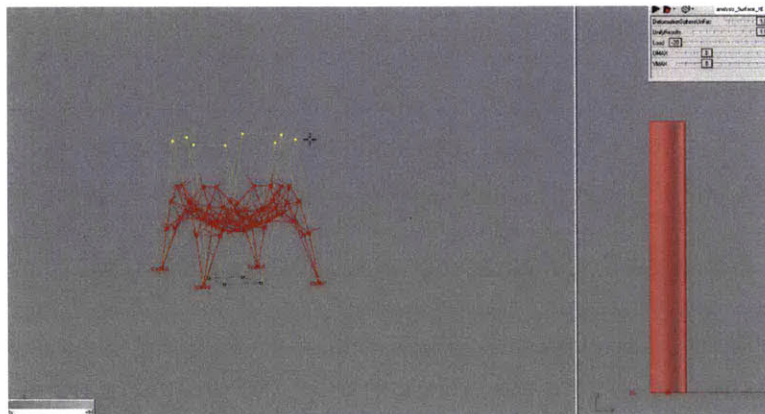
figure_25
screen shot, a continuous topology formation
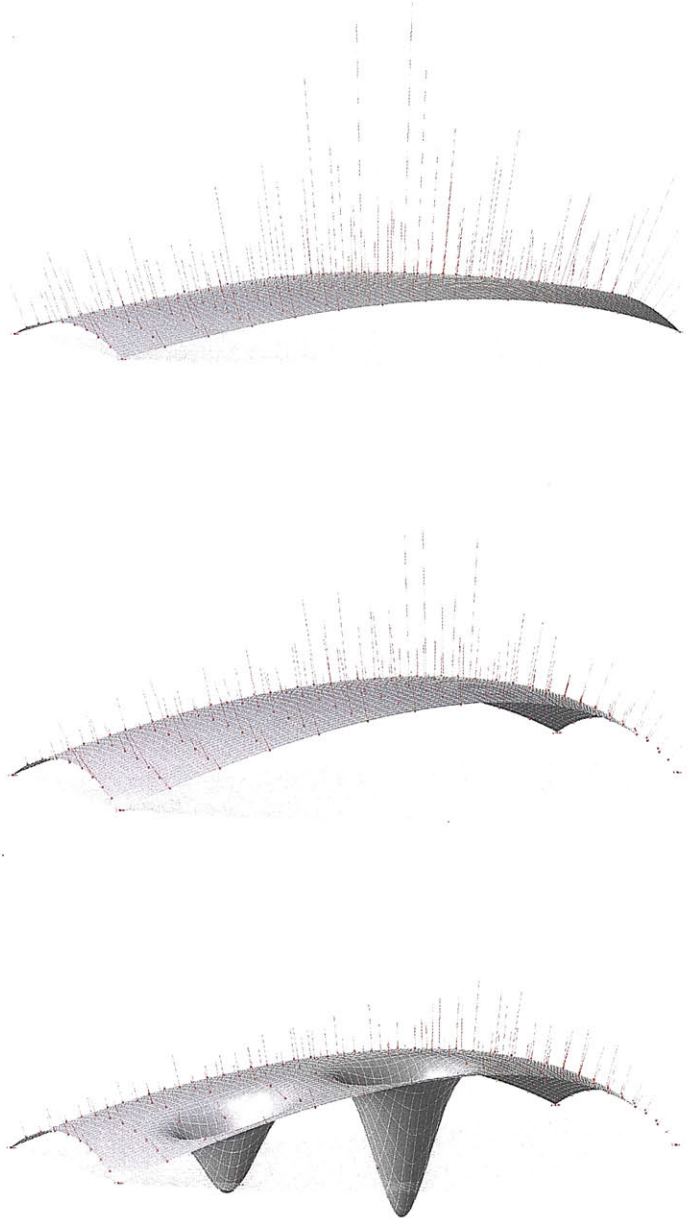
ground and transfer loads at these new locations. These dip downs act almost like columns but cannot simply be identified as one *(figure_27)*. Column slab structures are not integrated architectural elements. Slab is an element that defines the boundary of a space both on the bottom and top. However columns are often intrusions to space and function as pure structural elements rather than architectural. Manipulating a continuous topology to act as a structural system and architectural system is also materialized in one of Zaha's early projects: Pheano science center project. The design breaks down the standard orthodox definitions of what constitutes a wall or a floor. The cones and the waffle slab seamlessly join to make a continuous shell while performing the role of both structural elements to support the upper floor and the roof, as well as architectural elements to provide access to the upper level and accommodate programs. However the locations of these surface dip downs or 'cones'- as the architect calls- do not show this same intention to integrate matter constraints and architectural organization. The design concept proposes that the locations of the cones are being constrained by only site paths and views.

The next case study looks at a similar architectural form making process like the Pheano but considering both matter constraints and programmatic definitions as organization generator *(figure_28)*. Given a number of private programs of specific areas and predefined adjacency relationships, the goal is to allocate each program pocket within a field of public zone. This allocation process searches for ideal positioning for spots where the roof surface structure can meet the ground to transfer its loads. At those intersections where the roof structure

figure_27
continuous topology formation, flat- shell - column formations
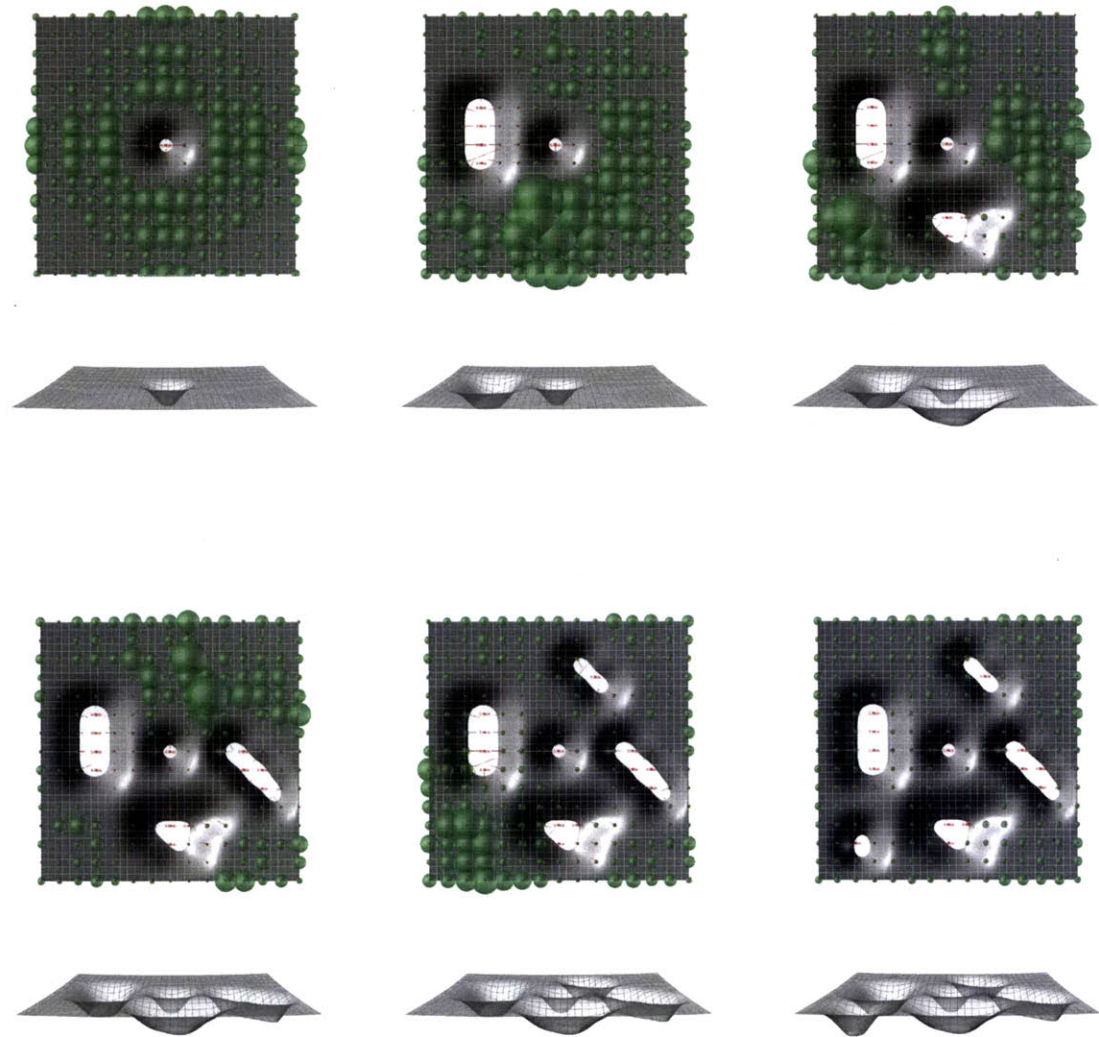
figure_28
continuous topology formation of program pockets

becomes part of the public field, there are the pockets of private programs.

The initial stage has a dip down at the center of the roof surface which is a seed condition to start off the process of this evolutionary growth. This central alteration creates a symmetrical deformation affect on four different directions. Therefore the second alteration is also a non-materially constrained selection, though the largest program is created in this step to relax the geometry as early as possible in the process. In the next steps, a new program emerges gradually in the field where the roof structure is more stressed. Depending on the relative amount of stress a different size of program is formed. As topology is altered the stress distribution also changes and evolves to a state of equilibrium. This process is a result of the compromise between forces of gravity and organizational requirement to create programs by manipulating a given topology. Although there is certain architectural formal typology, single surface, the end configuration is an emergent condition of both matter and organizational logics.
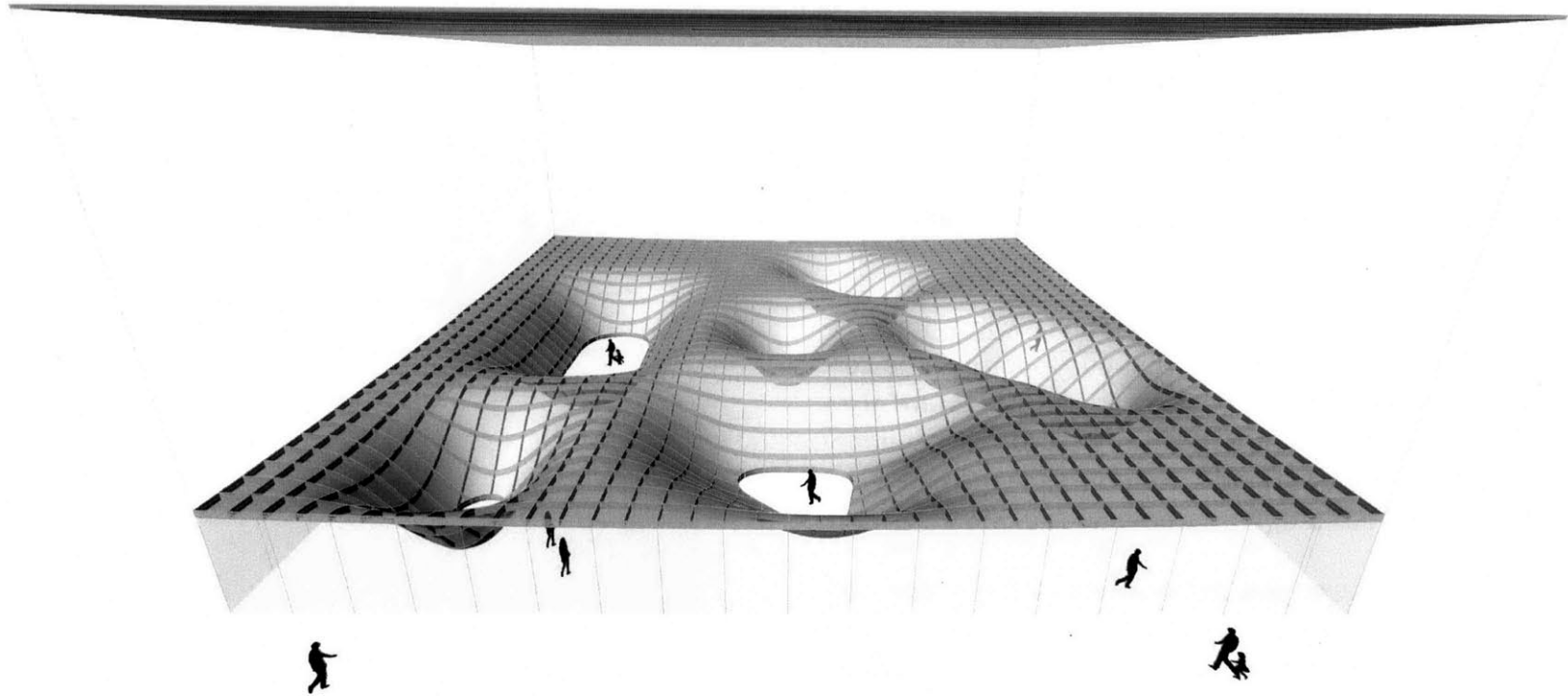
figure_29
continouos topology formation, private programs with in a field of public zone

Any cuts or bifurcations within in a surface space, voids or aggregations of other spaces with in a volume are examples of topological discontinuity. In mixed use buildings, spaces like auditoriums or atriums are size-wise anomalies in the whole grain of program assembly. If we assume that the short and mid span programs are part of the same uniform grain, those large span programs create discontinuity in the overall program topology. Continuous topology analysis model assumes a uniform grain of programs. So these anomalies have to be embedded in the analysis model for a given discontinuous topology *(figure _30 and _31).*

When analyzing a given discontinuous topology, it is first subdivided to analysis nodes like a continuous topology. The corresponding nodes to volume voids or surface cuts are then removed. A separate algorithm, which defines the boundary of the cut or void that ties the disconnected line member ends whose nodes were deleted, has to be modeled.

The surface topology example demonstrates a roof scape condition where surface undulation happens at support nodes and other spots to define various height conditions. The cuts on the surface are courtyard conditions that allow for light access in this deep space. This study is also a take on to the Rolex Learning Center design by SANAA. As discussed in the earlier chapters, matter forces only had post rationalization impact to the form of the plinth. This model shows how a similar design could be abstracted to its principal elements like the roof
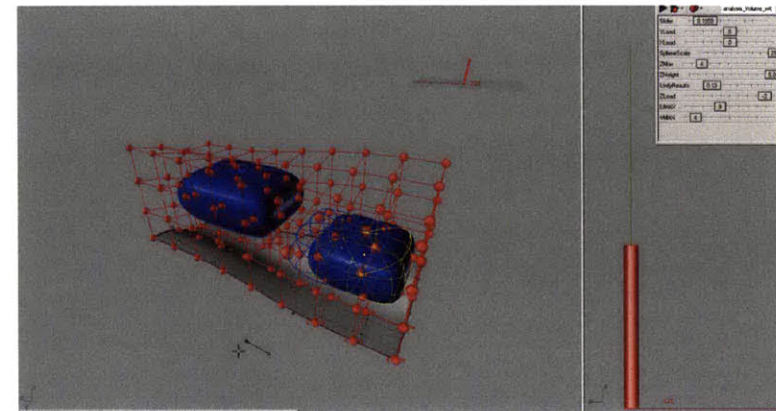
figure_30
discontinouos topology formation, volumetric analysis
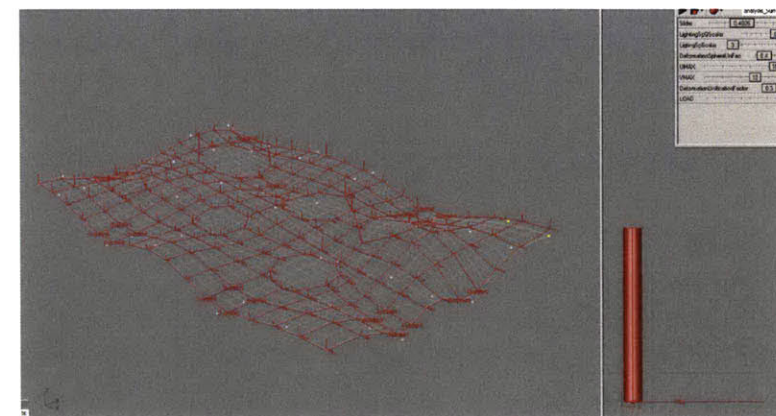
figure_31
discontinouos topology formation, surface analysis

surface and courtyard cuts which are parametrically related to each other. Any kind of re-configuration of the cut locations and surface topology alteration can affect the structural performance. Since these courtyards bring in light to their edges, these cuts create programmatic amalgamation around them. *(figure_32)*. So this parametric model creates a condition where structural performance and organization can be interrelated and evaluated together.

The 3D topology example also depicts a relationship between matter force impact and organization. The voids and the solid topology are parametrically defined, therefore when changing either the topology boundaries or void size or locations a visual feedback for evaluating structural and organizational performance is achieved. The algorithm for the 3D topology analysis needs more procedures to set up a three-dimensional analysis node matrix which covers all the topology. The intersection of the voids with the matrix of members are sought and from those additional nodes new members which define the three dimensional boundary of the void are generated. Similar to the surface topology example the nodes within the voids are removed from the analysis model.

figure_32
discontinouos topology formation , programmatic subdiviision due to proximity to light

PATTERN FORMATIONS within TOPOLOGICAL BOUNDARIES

Precedent pattern formation examples are form finding investigations within continuous topological boundaries. Although those continuous topologies form patterns due to force flows, they lack organizational constraints. In this section ways of constraining both continuous and discontinuous topologies with programmatic discontinuities will be investigated. The first investigation method deals with evolution of a given topology to a stable state of pattern by a removal process. The second method requires a cartesian pattern input which is then mutated by forces and user constraints. The third method is non-cartesian and works with both continuous and discontinuous topologies.

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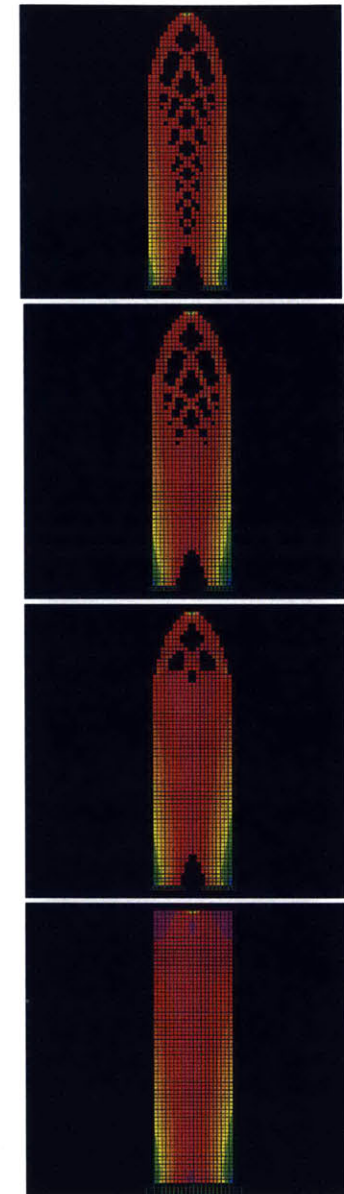
figure_33
material removal, case study 1

Material Removal:

As with all the form finding methods investigated in this research, this method also seeks to optimize constraints of both material limitations and architectural spatial requirements. The method investigated in this section is an application of Evolutionary Structural Optimization method in architectural scale. ESO (Evolutionary Structural Optimization) is an iterative form generation method that has been explored by various engineers in pursue of optimum structural forms. An extended version of the ESO method is explored in this section that -given boundary conditions and prescribed architectural programs- can generate structural frames around the required spaces while being able to re-allocate the lay-out of these spaces in search for material efficient forms. So the method aims to generate forms as a product of a hybrid design process of both architectural and engineering knowledge, rather than considering pure material optimization like in the precedent examples. This method is applied in three case examples with different load, boundary and spatial constraints.

The material removal process is based on the local stress levels. The aim is to obtain more efficient designs by creating a more uniform stress level throughout the whole structure. Once a topology is defined with boundary conditions and loads applied on, the given design space is divided into finite elements. It is often that part of the material is under-stressed in comparison to the rest of the structure. Using a criterion for rejection such under-stressed elements can be removed from the system (in Case Two and Three there are additional spatial/boundary constraints where the elements cannot be removed at specific locations). The stress level at each element node is measured with von Mises stress values which gives an average value of all stress components of the given plane stress problem in each case example.

$$\sigma vm = (\sigma x2 + \sigma y2 - \sigma x\, \sigma y + 3\,\sigma xy2)\ \tfrac{1}{2}$$

At each iteration step all the elements which satisfy the following condition will be removed:

$$(\sigma evm\ /\ \sigma maxvm\ ) < \text{Rejection Ratio}$$

This rejection ratio can be increased as the system reaches to a stable condition when the user wants to optimize the geometry more. In case two, the removal operation has to work with the constraint of the pre-defined void boundaries and in case three it has to accommodate the given floor to ceiling height, which are the extended spatial constraints to the ESO method.

The first case study demonstrates a pure force driven form generation process so as to compare the result of this approach with both a material and organization constrained
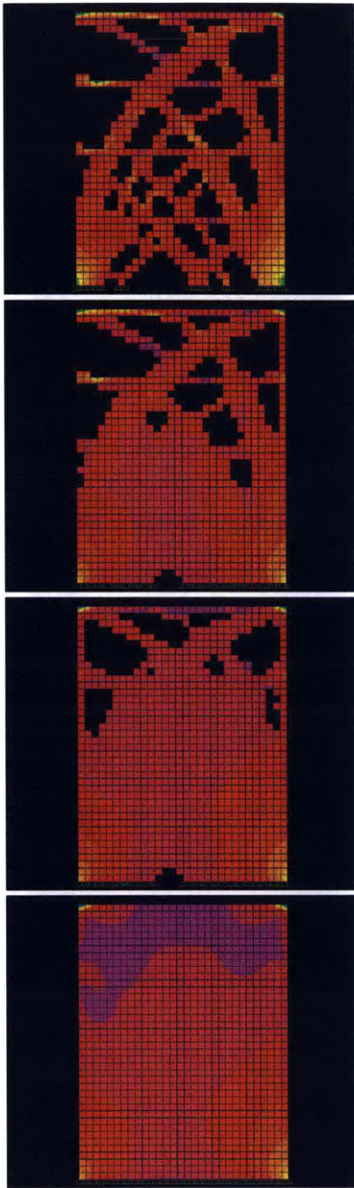
finding process. In the first case study a rectangular topology is divided into 20x70 elements. A horizontal load of P is applied. This case is imagined to be a wind load affecting a high-rise building. Since the resolution of the model is very low the iteration process couldn't refine the bottom left and right of the model space. After 60 steps iterative removal process converges to uniformly stressed overlapped parabolic curve geometries *(figure _33)*.

In the second case study a discontinuous rectangular topology with voids is the initial topological boundary that the iteration process starts from. Again an initial 20x70 mesh is used but the elements that correspond to the internal void spaces are deleted before the iteration process started. These void spaces are imagined to be potential large public spaces in a high-rise building (lobby, auditorium, restaurant...). The rest of the structure is thought to be carrying hanging floor slabs from the generated structure. A horizontal load of P is applied. Also there is load applied from the inner void peripheries to the overall structure. In this case example the removal process has to also check each time that no void periphery element will be deleted. With those spatial constraints and forces impacting on the topology, the pattern evolution converges to a relatively more uniform condition after 45 iterations *(figure _34)*.

The third case study is an example of how a continuous topology can be constrained with non-force factors for pattern form making. In this case the design space is divided into a 40x60 mesh. A triangular distributed horizontal load is applied on both sides of the design space. There is also distributed gravity load applied at every floor level. This case is imagined to be an earthquake load applying to a low-rise building. Differ-

P
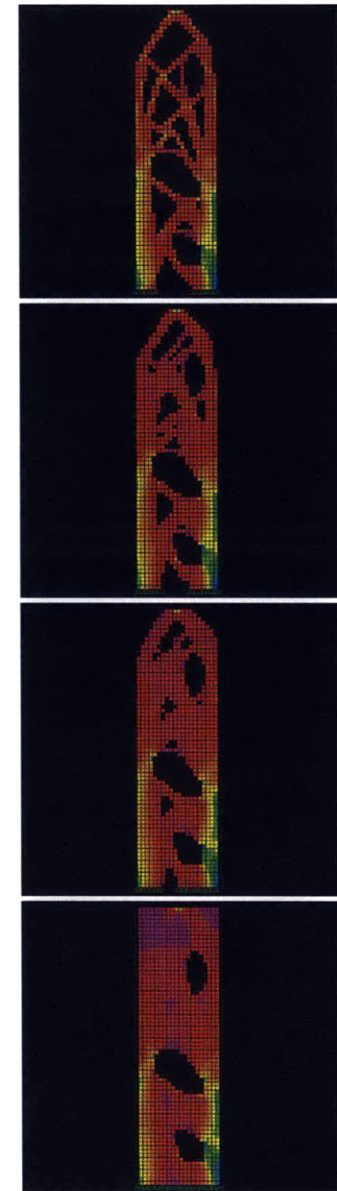
figure_34
material removal, case study 2

ent from the first two cases, here there is a spatial constraint of keeping a certain floor to ceiling height where the removal process has to check each time that no floor slab element will be deleted *(figure _35)*.

These studies are done by using existing finite element analysis software. The cumbersomeness of the engineering analysis tools in terms of geometry modeling and the advanced analysis which takes too much time prevent them to be used as quick intuition based form generation tool. It is this reason that these studies are not taken to a more advanced level and explored in three dimensions.

figure_35
material removal, case study 3
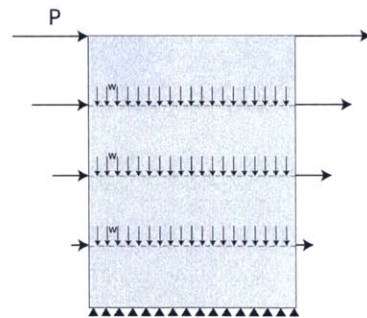
Genotype Mutation

This section will explain how to materialize an analysis data of structural deformation into architectural patterns. Whether it be a floor slab, skin, or a roof canopy, the topology of the structural geometry need to be defined. What is called genotype patterns here are uniform grids, dia-grids, honeycomb or voronoi like patterns which can be mutated based on force distribution. The state of zero force impact is the genotype condition. When the genotype pattern starts mutating, then a new phenotype emerges. Phenotype patterns can be formed by structural force directions and by other user constraints. To map the scalar changes in forces on a given topology vector fields are used.
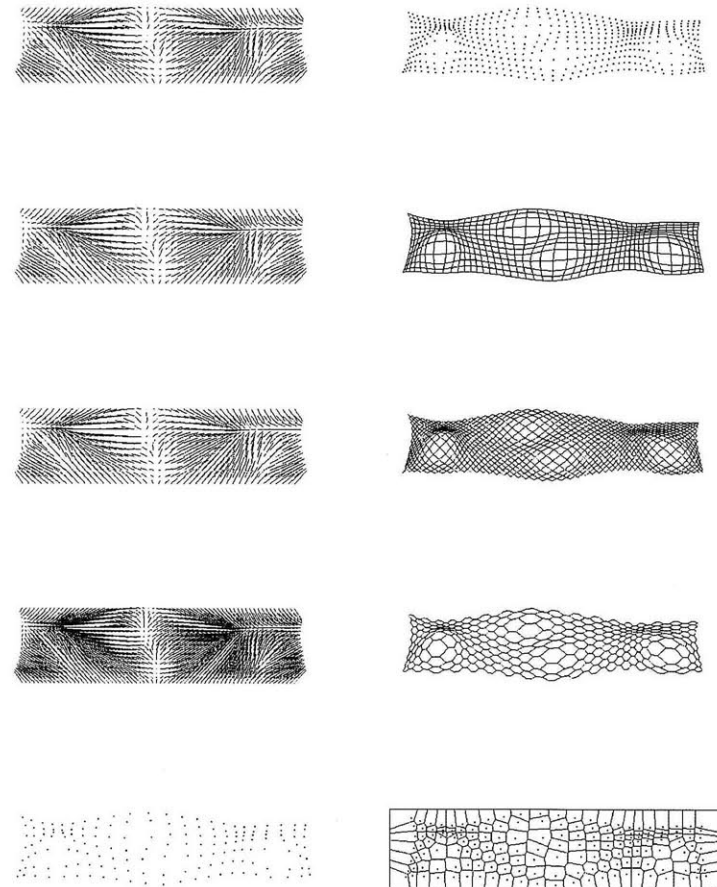
figure_36
vector fields to mutated patterns

In mathematics, a vector is an object with line representation which has a certain direction and magnitude. The vector magnitude depicts the amount of attraction to given forces in the vector space, and vector direction is defined by the positions of these active forces. A vector field is a group of vectors that are constructed from a set of points in a topology. The vectors of a vector field are exposed to the same forces. However since proximity to forces change, each vector can have a different direction or magnitude *(figure_36)*.

For the genotype mutation method, each point that defines the lines of a genotype pattern is assigned a vector. These points are called pattern nodes and they are not to be confused with the analysis nodes. After an analysis is performed on the given topology, each analysis node will have a value of deformation. The higher these values are the weaker the topology is structurally at those spots. In order to strengthen the weak spots, the pattern has to be mutated such that weak spots will get more pattern densification therefore more material. The strong spots (this mentioned strength is due to topological formation) will get more open pattern therefore less material. This way the stresses on the surface or inside a volume will be distributed evenly *(figure_37)*.

After a topology is subdivided to a specific number of analysis nodes, the number of pattern nodes can be a subdivision of higher numbers. Since each pattern node will not have a corresponding deformation value, those in between pattern nodes will need to have interpolated values. From a computational perspective, pattern formations require less computing time than analysis. This method requires a minimum number of analysis nodes that is sufficient enough to define the geom-

figure_37
mutated patterns on topologies
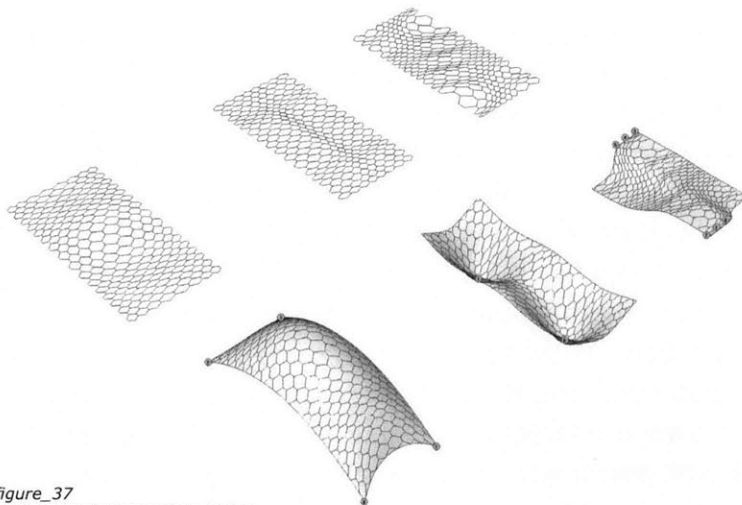
etry of the given topology. This procedure is a fast approximation that can be an intuition based alternative to advanced finite element analysis methods which require mathematical integration of enormous computing time.

For each pattern node the algorithm formulates a magnitude equation to each analysis node. The amount of deformation at the analysis node multiplied by the inverse of distance to the pattern node gives a scalar factor for the vector magnitude. The found vector magnitudes are then averaged to define the final vector magnitude at the pattern node. To find the vector direction of the evaluated pattern node, lines starting from the node towards the direction of each analysis node are drawn at a length of corresponding scalar magnitude. These lines are then added to the end of each other. This procedure is also called vector addition. The line that connects the evaluated pattern node and the end of the last added line will give the final vector direction at the pattern node.

After a vector field is generated, each pattern node will have a corresponding vector line of a defined direction and length. By displacing the pattern nodes from their original locations, start point of the corresponding vectors, to the corresponding vector end points, the initial genotype pattern is mutated. In the slab studies these phenotype conditions are the reflections of pure matter force fields. Where there are columns, the stresses around them attract patterns to create local densifications. The slab studies show resemblance to the Nervi's Gatti wool factory slab structure. However, Nervi did not have the opportunity to compute forces via computers. So now, it is possible to work with asymmetric column positioning and the locations of the columns are parametrically defined therefore allowing
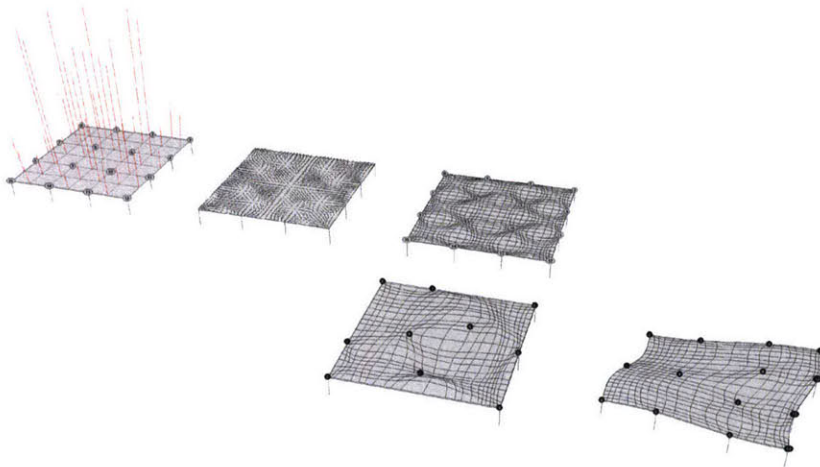
figure_38
slab pattern mutations

GENOTYPE

MATTER
FORCE FIELD

CUSTOM
CONSTRAINTS

PHENOTYPE
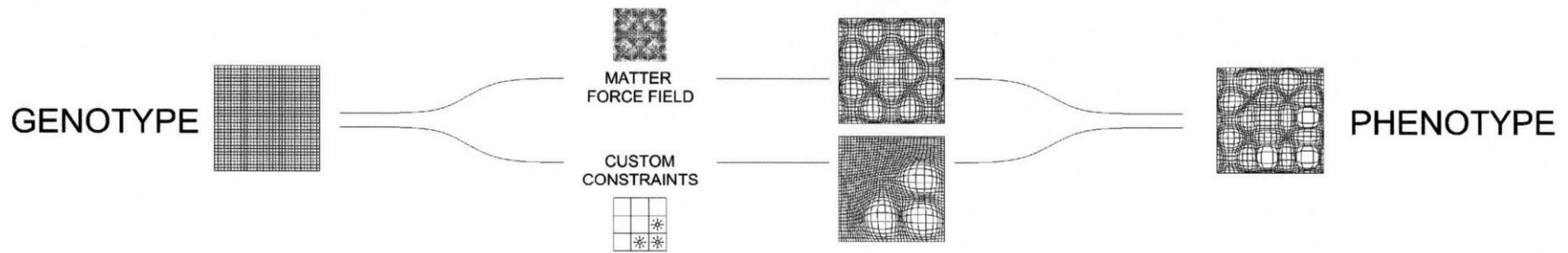
Pseudo code for pattern mutations

For a given surface or volume topology:
_subdivide the topology with a predefined number of analysis nodes
_perform analysis and extract deformation values for each analysis
node
For the same given topology:
_subdivide the topology with a predefined number of genotype pat-
tern nodes
_for each pattern node:
_evaluate proximity and attraction to deformation values of analysis
nodes
_evaluate the impact of user constraints (repelling or attraction)
_generate a vector field reflecting both matter and user forces
_move the pattern node to the tip of each corresponding vector
_with in the set of repositioned pattern nodes redraw lines to connect
the pattern nodes and visualize the mutated pattern

figure_39
phenotype generation

visual feedback upon any change in the system *(figure_38)*.

These genotype patterns can also be mutated by user constraints. Certain locations of a given pattern can be required to have more porosity or density. Such changes in the pattern organization can be related to architectural organization. Certain programs will need more privacy so pattern densification can do that while maintaining the continuity of the pattern system. The opposite can happen by increasing porosity in the pattern, spatial interaction between what is behind the pattern and outside the pattern can increase. For slab patterns, the uniformity of slab rib structures can be inflected by architectural function inputs. Relatively larger increase of porosity within the grain of the same pattern system can allow for circulation to penetrate through. One can also use these user constraints with the constraints of matter field. So the phenotype that is created is an average of both *(figure_39)*.

Grids, dia-grids or honeycomb type patterns have to work within the framework a consistent logic. The mutation process via shifting pattern nodes to vector ends is trying to maintain the continuity of the genotype pattern, therefore this process struggles when applied to discontinuous topologies. Additional constraints to the matter force constraints cannot create actual anomalies in the grain but be able to disturb the patterns locally. This problem becomes more apparent in the 3D grid mutation examples *(figure_40)*. The mutated pattern still has uniform grain that speculating about richness of architectural organization is not possible. The next pattern formations method looks at how to form patterns within discontinuous topologies which require a non-Cartesian logic of form making.
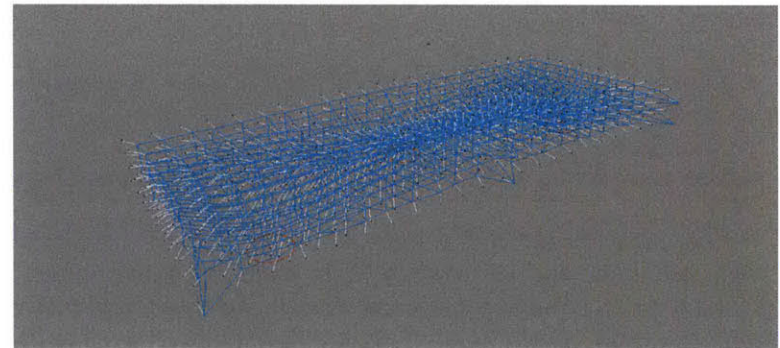
figure_40
3D grid mutation

Force Flow Pattern Formations

The Force Flow pattern formation method looks at a more emergent formation which does not need a genotype pattern. The results of the genotype patterns are not unexpected. It is a formation process that can only change the distances between each pattern node, but it does not reconfigure the connection lines between the pattern nodes. In a visually stable rectangular grid, after any number of mutation iteration a corner node will always be connected to the same two nodes adjacent to it. This is similar to the concept of topology formations. The genotype mutation method doesn't break the continuous topology of the pattern but it works with it. However with this force flows pattern formation method, the goal is to define procedures that can generate pattern topologies rather than simply manipulate a given pattern topology.

Similar to the procedures of pattern mutation method, the force flow method works within a vector field that is generated by structural analysis data. This field of vectors when perceived as a group reflects the flow of forces on the topology. There has to be such algorithm modeled that is able to search for the best paths to connect vectors within this visually continuous but structurally discontinuous vector field map. Application to continuous or discontinuous topologies does not matter, since formation happens by searching the best paths within the vector field that follow the major force directions *(figure_41)*.

Once a vector field of a given topology is generated, for each vector tip point the algorithm searches for the closest next vector node. Then the new vector's tip point is used to search for the next closest vector node. The algorithm iterates until a predefined max number is reached or the process hits the boundary of the topology. These tip points are stored until the iteration terminates, and after termination these non-orthogonally related nodes are connected to define one of the path curves of the force flow pattern formation. Since vectors of a larger vector assembly often locally converge to weak stress spots or support nodes on a topology, we see converging or branching of force paths. Because it works with a search algorithm based on distance not dimension or order, the algorithm to define force paths in two dimensional spaces or three dimensional is the same *(figure_42..*

The roof-scape case study being explained in this section is an extension of the case study from the discontinuous topology formation section. A continuous topology formation is first used to test this pattern formation method. Then the force flow
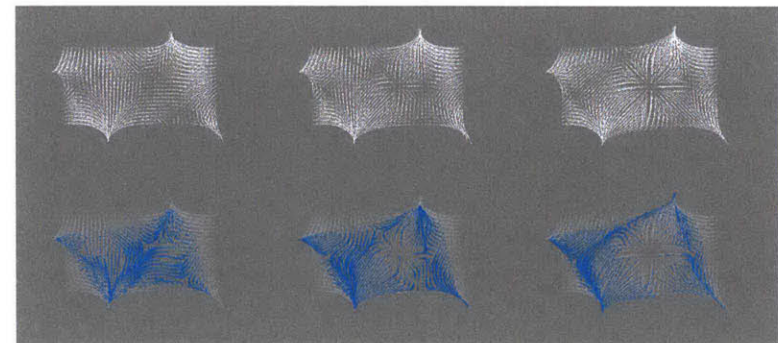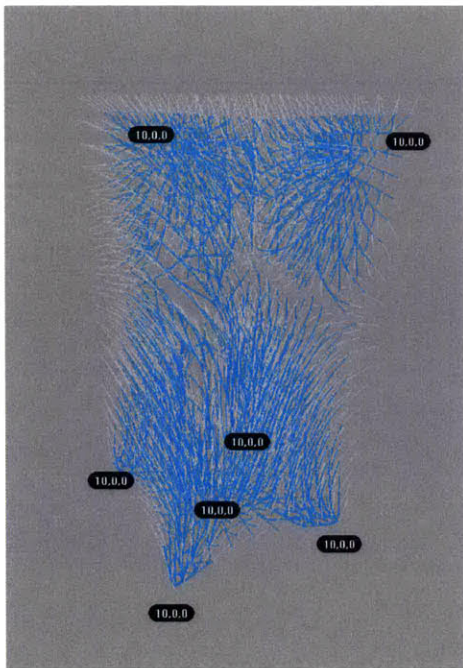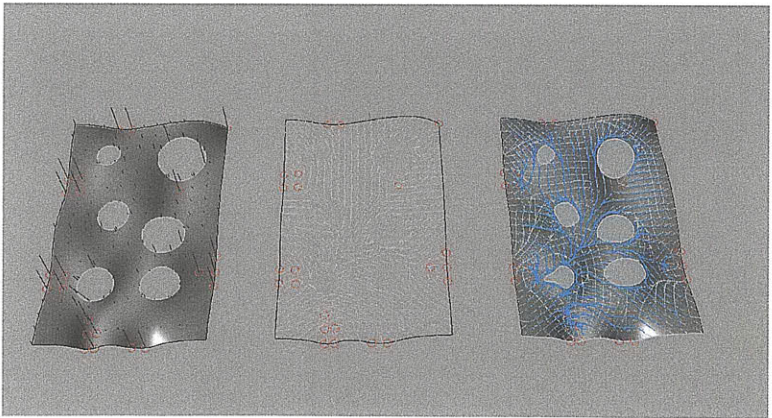
figure_42
three dimensional force flow pattern formation

pattern formation algorithm is executed again on the same topology except that now there are holes cut from it. These two topologies are analyzed separately, and due to the weak spots around the cuts the discontinuous geometry has more deformation at those spots. While being attracted by these weak spots, the force flows converge towards the support nodes to transfer the applied loads to ground. Therefore, there is a clear pattern densification of these linear elements around the support nodes and around the edges of the cuts. These linear elements which connect various nodes on the topology to the support nodes are tied together with cross members which are generated by the horizontally applied loads. Since these linear major force paths do most of the work, their cross sections are modeled larger than the laterally running members. *(figure_43, _44, and_45).*
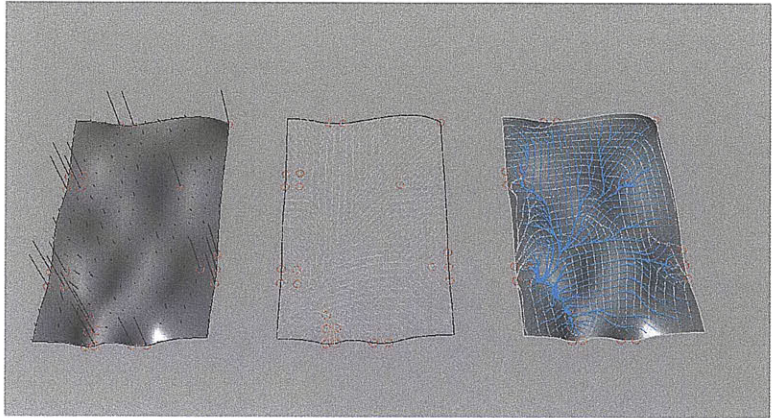
figure_43
force flow pattern formations on a continuous topology

figure_44
*force flow pattern formations on discontinous topology*
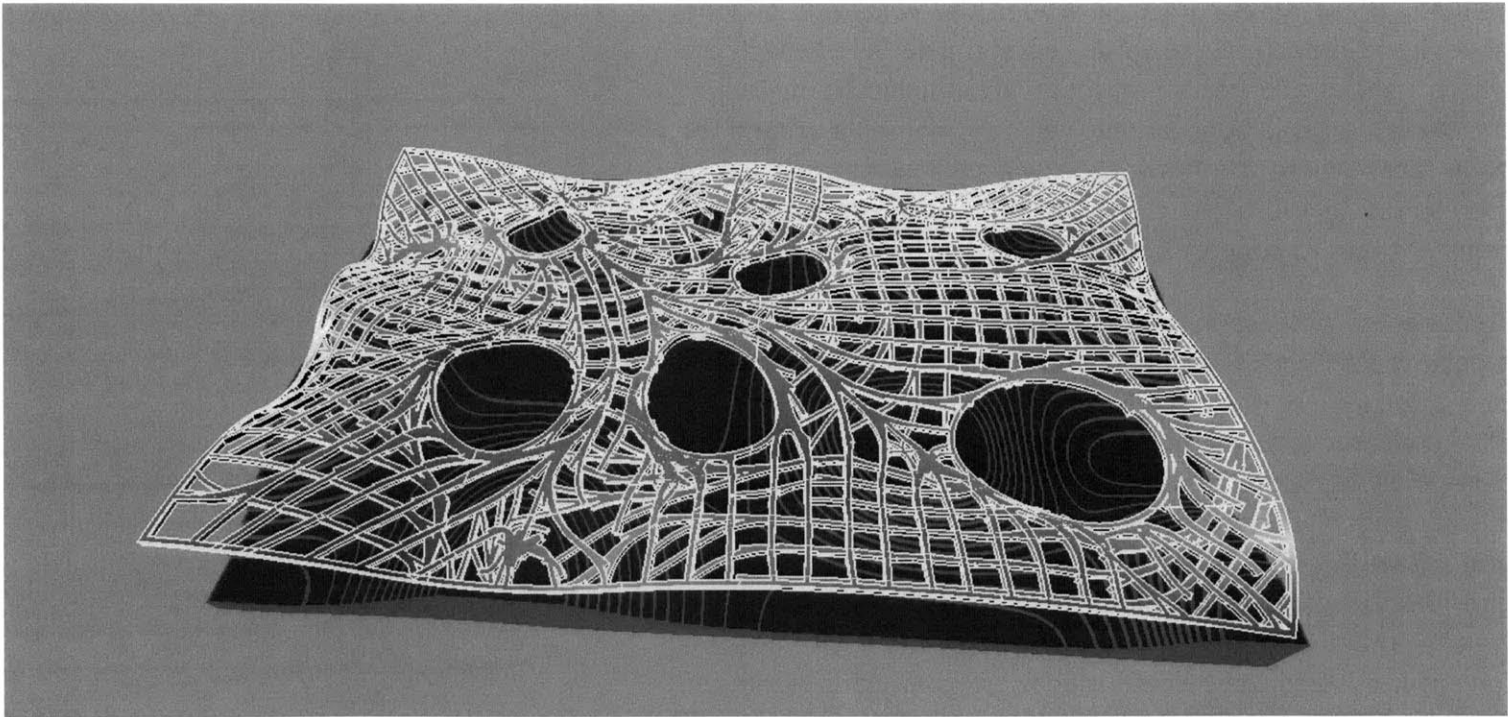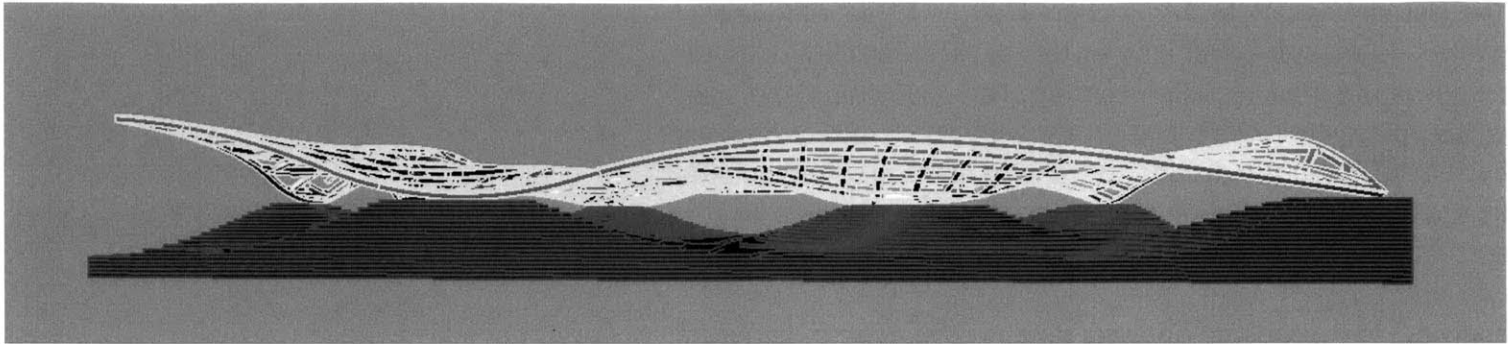
figure_46
materialization of force flow patterns as roof members

The last case study in this section looks at three dimensional pattern formation condition. In this case a discontinuous topology is the medium of processing. The positioning and sizes of the voids within the topology and how much of the topology base is touching the ground to transfer loads are manipulated by the topology formation method as explained in the previous sections *(figure_46)*. Given the topology, voids, support nodes and applied load conditions, the process is then to perform analysis. The extracted analysis data is then used to generate a three dimensional vector assembly. These vectors become the infrastructure for the force flow pattern formation algorithm. The algorithm generates major force flows within the volume. These flows converge around the voids and support nodes; and branch out towards top and side faces which are where vertical and horizontal loads are applied from. Due to this horizontal load application, there are moments around the edge faces where the flows are very shallow or even flat to correspond to the horizontal force directions and be able to transfer them to support points *(figure_47)*.

The analysis process subdivides the volumetric topology into a point matrix grid. Every level of horizontal grid corresponds to a possible floor slab. Since each analysis node has been applied the same load conditions and due to the limitations of the truss analysis model, assuming the slabs being at those levels is a justified hypothesis and works to stiffen the overall system. However once the force flow pattern formation occurs the gird infrastructure for the slabs is irrelevant. Slabs can work with the grain of the force flows. Wherever the force flows are too shallow, slabs can be deformed to transfer loads within the surface. Architecturally speaking, these slanted surfaces can
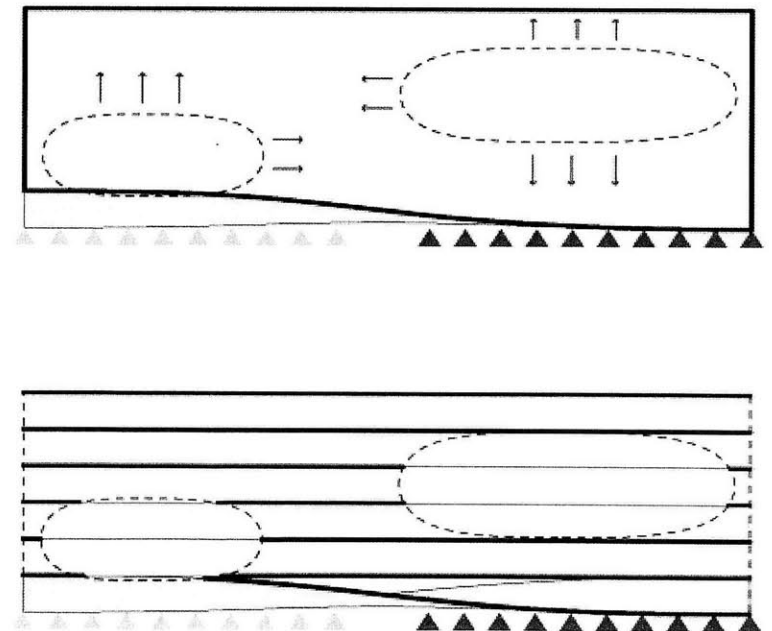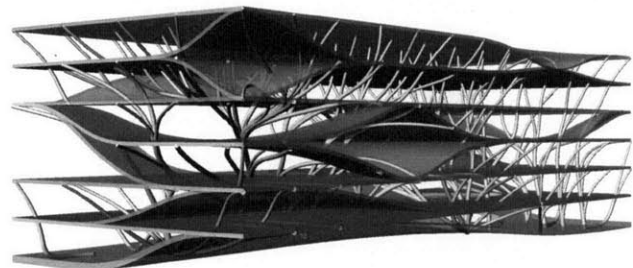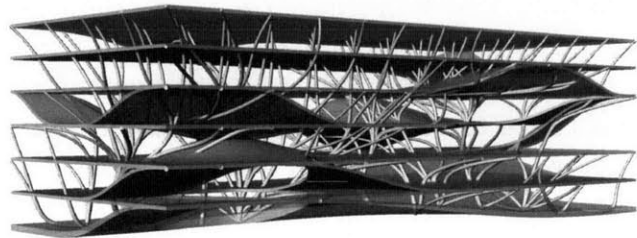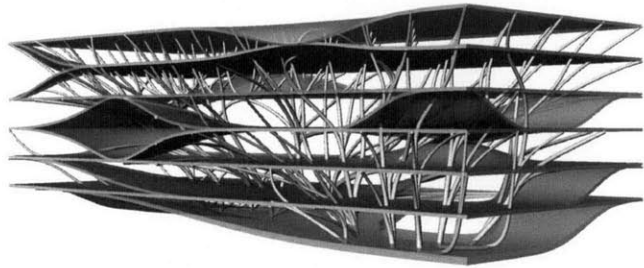

figure_46
topology formation state
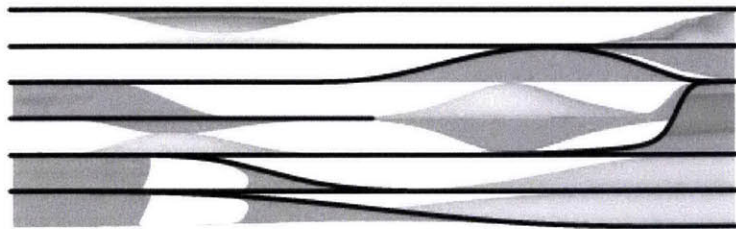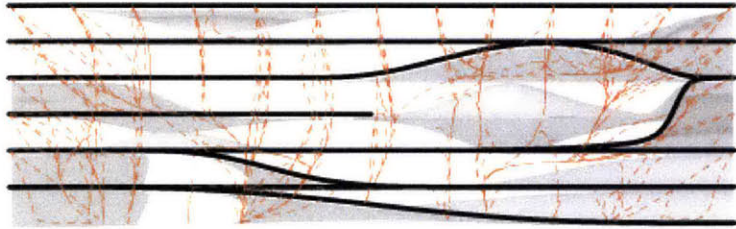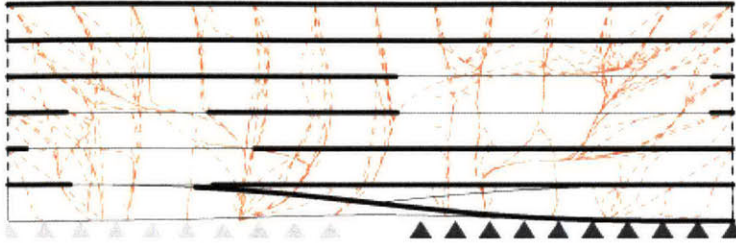
figure_47
force path generations, slab mutations
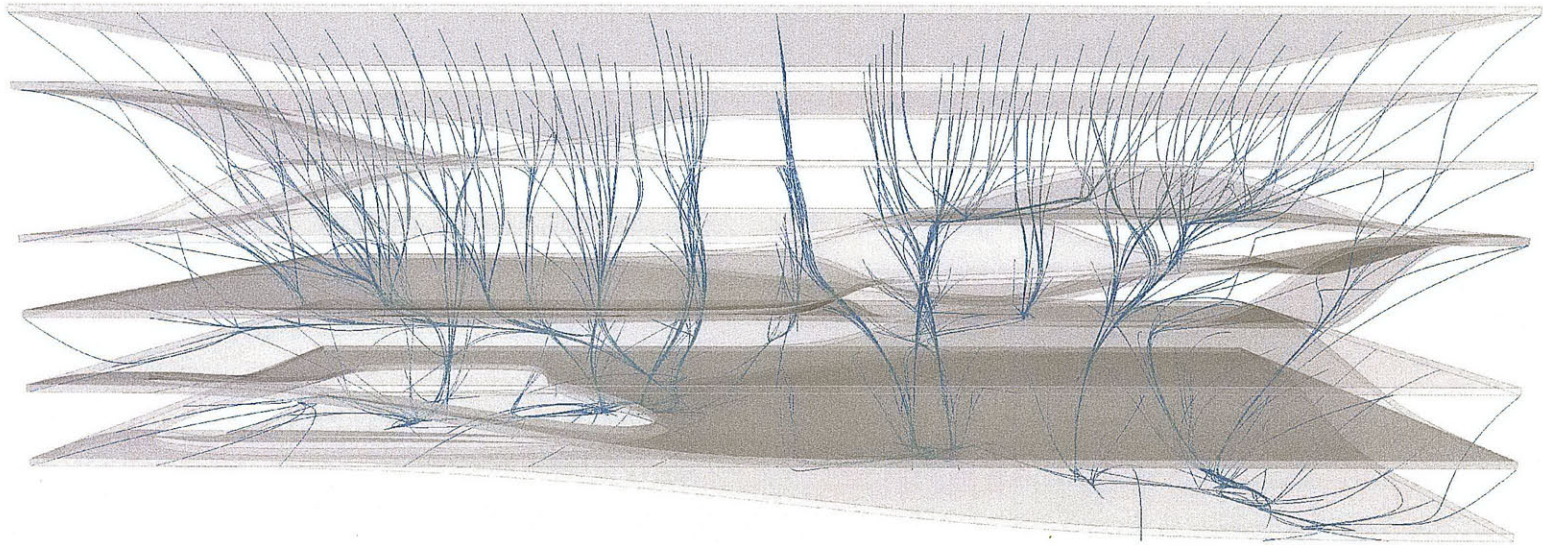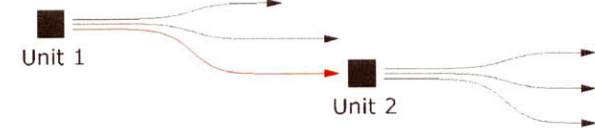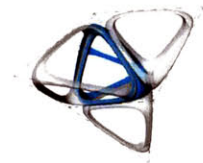
figure_48
force paths altering slab topologies

be used for circulation, or auditorium type spaces for gathering or like in the Rolex Center, become open activity fields of undulating topologies *(figure_48)*. When the slabs cannot accommodate to become vertical unless the goal is to subdivide the space into finer grain, the force flows are materialized as columns. These columns change cross section and branch out as they go higher since the amount of load being transferred decreases at higher levels *(figure_49)*.

The slab building example shows a process of pattern formation which first works with constraints of larger public spaces, which are modeled as voids in the topology to generate the force flows. These optimized paths then create an infrastructure to allow sectional changes in the building. What is typically achieved with slab buildings is staggering of programs with no relationship to one another. This infrastructure of force paths accommodate inflections in the slab structure that allows for spatial, programmatic and organizational interconnectivity between different levels.

figure_49
materialization of force paths and slabs

AGGREGATED TOPOLOGIES

The precedent practices of form finding show a separation and hierarchy between topology formations and pattern formations within topological boundaries. Therefore the materialization of the grain is only a resolution of a larger system. There is not a case study where the grain can generate a topology from no definition. The force flow pattern formation method exemplifies a generative form making process which can generate pattern topologies rather than simply manipulate a given pattern topology. However being linear geometries, by nature the force flow patterns require associative elements like slab or roof-scape surfaces to connect within it. Only then these formations can have spatial contribution to architectural organization. Aggregated topology studies strive for how can a unit , of a spatial definition, by its aggregation with matter force constraints taken into account define architectural spaces. It is a reverse study of pattern formations within topological boundaries. This study has a definite grain but indefinite topology which emerges from the interactions of forces, unit assembly constraints, and goals to define spaces.

Unit 1

Unit 1

Unit 1   analyze

Unit 1   analyze

Unit 1   Unit 2

figure_50
*aggregation path selection*

79

The unit of exploration is four sided hollow pyramid geometry. The edges of the pyramid are the truss members for the analysis. Aggregation of this unit is expected to define spatial boundaries which are vaguely defined by two target points in the first example. The first target and second target are positioned such that a dome like space of uncertain boundary can be generated. The goal is to aggregate units towards the direction of targets respectively which will produce space underneath.

The pyramid unit always has one face connected to the ground or to another unit. In this manner, each unit has only three directions to grow. Each path will have different proximity to the targets. Of the three paths, the farthest path towards the target is eliminated. The remaining two configuration options are subjected to structural analysis and stronger configuration survives to continue growing *(figure_50)*. This process is iterated until the first target is in within a defined radius. The target is then switched and aggregation happens towards a new direction with the same process of path selection. Once the main spine is formed, from weak spots new branches of aggregation emerge to buttress and distribute the stresses in the global structure evenly to a more relaxed state of equilibrium. The path selection for branch aggregation at each iteration eliminates the closest path direction to the targets in order to span out as much possible to define a larger spatial condition. Then the remaining two configurations are subject to analysis tests similar to the spine aggregation. *(figure_51)*
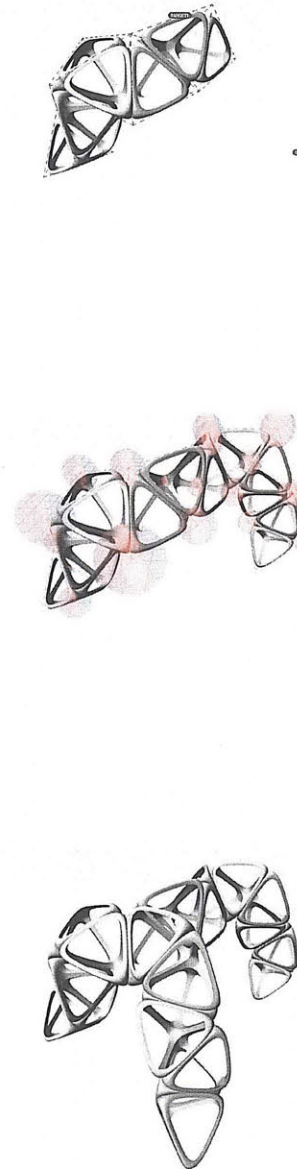
figure_51
single space definition, branching
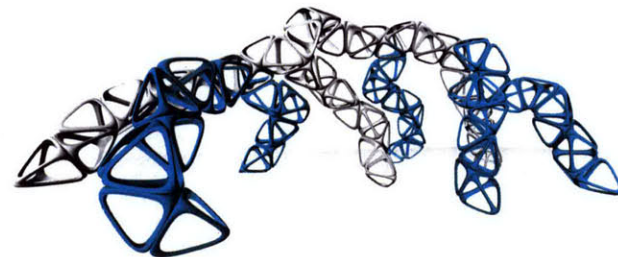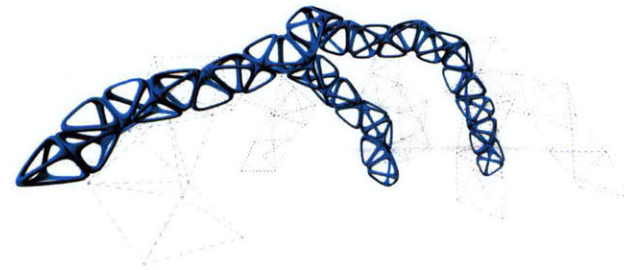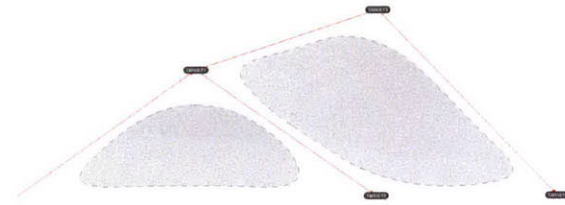
In the first study a single space condition is created with the aggregation of units. The next example looks at how to define multiple spaces with such aggregation logic by defining multiple target points. Given that the system is larger it requires more branches to emerge of the main spine in order to relax the overall structure *(figure_52)*.

figure_52
multiplicity of spatial conditions, spine and branch aggregation
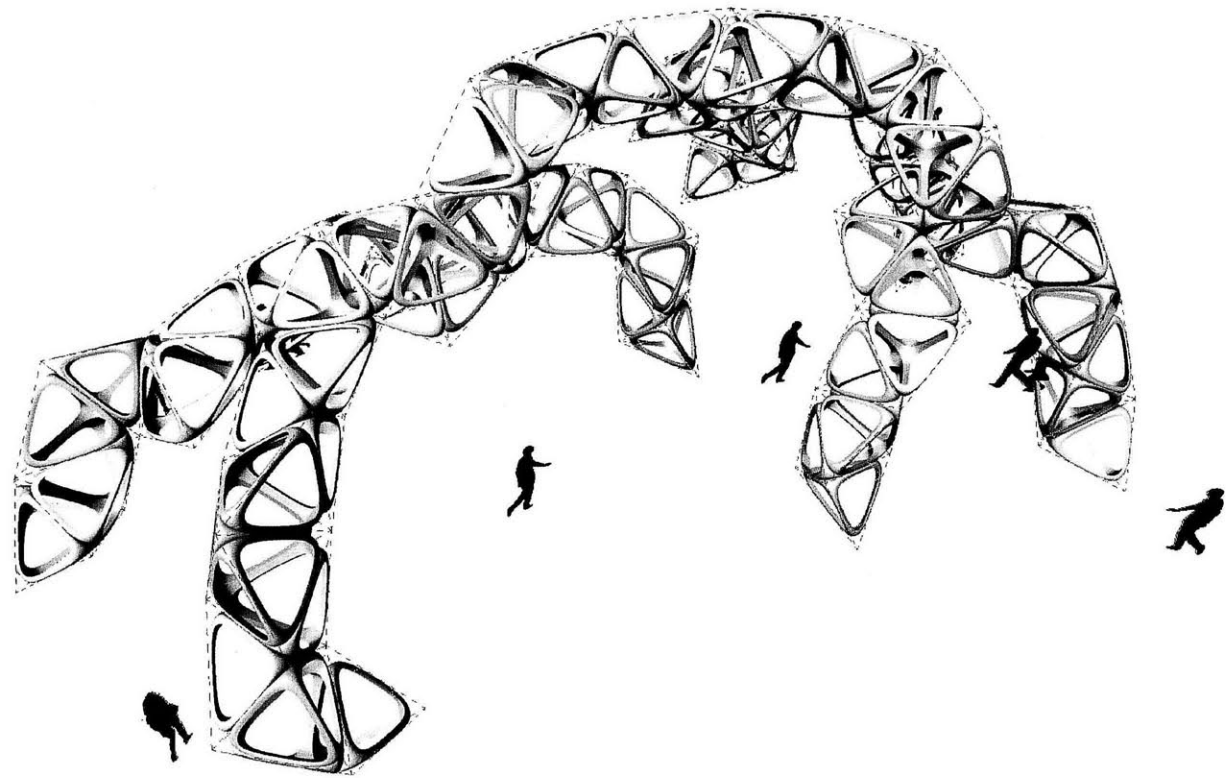
figure_53
aggregated topologies

CONCLUSIONS

In his essay 'Arguing for Elegance', Schumacher promotes formal elegance as an articulation of complexity that achieves a reduction of visual complexity while preserving an underlying organizational complexity. For him elegant compositions are highly integrable systems that cannot be easily decomposed into independent subsystems like natural systems where all forms are the result of the lawful interaction of physical forces. The products of this thesis similarly promotes an integrated process where the shared knowledge from both architecture and engineering, is highly blended to create an artifact that is the resultant of an interplay between different forces in adaptation to material and spatial performances. This integrated design process creates an ambiguity in defining of what category such work fits into: form finding or traditional post rationalization. It is not neither a pure force driven nor organization driven form finding process, but rather a collaboration of both ways of thinking. This dual thinking was the underlying principle that produced all the studies of topology formations, formations within topological boundaries and aggregated topologies.

This process of working is like riding the 'waves' of reality as Koolhaas had stated. Riding this matter wave is to work together with it and to both direct and be directed by its constants. The constraints of matter are not obstacles for design to progress but rather opportunities to discover what is unseen. Even though material reality in design seems to be rigid and constant, it is dynamic and continuously updating itself with new compounds and hybrid opportunities as in the examples of slab building and roof scape case studies which blend the rational with irrational. Only if the designers conform to the waves of matter, can they manipulate the direction of its flow to new and unchartered territories.

Designers have to be aware of the potential of the digital tools, so that they direct this flow of evolving information technologies while avoiding its fallacies. The capacity of Information Technology can propagate the process, but this itself doesn't generate ideas. For the opportunistic designer the digital information processing is a test field for ideas in which the feedback between design and evaluation is much faster than without it, as Cecil Balmond explains his process of design: '...speculating an idea, probing and sketching it... do simple hand calculations and then I use the computer to prove the point' . The word 'computing' refers to the procedure of calculating; determining something by mathematical or logical methods. Therefore 'computer' is a tool for processing logics which are defined by the user but not a tool of generating logics for processing. The logics and the constraints which need to be processed have to come from the designer in the form of a speculated idea, probe or sketch, in relation to real matter. The question of what the digital can't perform needs to be critically considered in the design process. Contemporary design process simply cannot rely on the power of computational tools. Within the finite number of possibilities that exist, a need to search for the most appropriate conditionals before riding the wave of Information Technologies has to be comprehensively carried out. Throughout all the case studies, the role of computation is not to generate random forms. The analysis code essentially originates from physical laws. Pattern formation methods are designed within the code to perform the way they are. The voids and other spatial or programmatic discontinuities are established by organizational principles not computation. However with computation the process of synthesizing both spatial and matter forces is enabled in this thesis.

Unlike the utopian designer who is always after the same form and style, the opportunistic designer, through any available means (Information Technology, computer aided manufacturing, collaboration... etc.), uses these constraints of materiality as a potential to be innovative and creative. Their abilities are not reserved by any trend or style but rather their design inspiration come from the dynamic essence of reality. Even though trends change, the opportunistic designer will still have an active role in shaping his design agenda as he is surfing the wave of material reality. This research is conducted with a mindset of an opportunistic designer. The issues being tackled pushed this research to find ways and which form becomes an emergent phenomenon of a materially conscious design process. In order to find form, within a state of formless condition, forces of matter and space are worked together to allow new formations to emerge.

This research focused on using matter mechanics as its fundamental form making criteria coupled with spatial constraints. Although the proposed forms are ideal forms for force flows within the spatial limitations, these forms may not be the constructible form. Aggregated topologies studies scratches the surface of how assembly process can constraint both how building blocks are put together and in what order they will assembled. The ideal or the virtual form explored in this book is the realm of pre-possible. Any constructible form has to come out of within the possibilities of this realm by either fully following the ideal path or applying its own constraints like the aggregation studies. Future work should consider the constraints of constructability as part of form finding process. Only then these virtual forms will be able to materialize into built forms.

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