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The Fragmented Warehouse: Location Assignment for Multi-item Picking

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Abstract - **Warehouse operations typically aggregate identical stock keeping units (SKU) into the same storage bin for easier bookkeeping and organizing of goods. With the emergence of automatic identification and tracking technologies like RFID, free-form storage of goods becomes a viable alternative. We consider the strategy of storing identical copies of an SKU in a fragmented manner and evaluate the operational characteristics that benefit from fragmented storage. Fragmented storage of identical SKUs creates a greater number of feasible pick-list opportunities -- with greater choice, greater optimization follows. We present an abstract warehouse model to evaluate the location assignment problem in warehouse systems. Specifically, we investigate picking operations that retrieve multiple items during a warehouse pass. We provide an analytical result for operations using a 'hybrid-cost' cost metric, and a brute force analysis for operations using the more common 'maximum-cost' picking metric. We show that fragmentation is more favorable when the number of copies picked for each SKU is small.**

Keywords - **location assignment, order picking, warehouse storage planning**

I. INTRODUCTION

Optimization of warehouse picking operations falls into two main categories: location assignment and optimal routing. Location assignment refers to the placement of items within the warehouse such that retrieval (picking) is more efficient. Optimal routing addresses the problem of determining the movement path through a warehouse to retrieve the necessary items in the most efficient way (*e.g.*, lowest distance traveled). Typically, research and warehouse operations assume fixed assignment of stock keeping unit (SKU) to storage location and cluster identical items, which establishes a one-to-one correspondence between storage location and SKU. In some operations, the storage location holds a single unit (storage size one); in others, storage locations hold many copies of a single SKU. However, both cases create a one-to-one mapping between location and SKU. In other words, given a need for a specific SKU, one warehouse storage location holds the item necessary to satisfy the need.

Van den Berg [1] outlines several approaches to address location assignment including randomized storage, class-based storage, and dedicated storage. Class-based storage [3] is a common strategy that assigns stored items to classes based on criteria such as turnover frequency or cube-per-order index (COI) [4]. These classes then map to regions of the warehouse into which class members may be stored such that more frequently accessed SKU are located in more easily accessed locations. For example, Hausman [2] and Graves [3] study unit load, class-based automatic warehouse systems.

Our previous study on fragmentation [5] focused on unit-load order picking where single items are retrieved from a warehouse one-by-one. Unit-load picking analysis benefits from decoupling each pick into independent operations with independent costs. More complex picking processes that pick multiple items during one pass through the warehouse introduce additional complications for optimal order picking. For example, with multiple item picking, the ordering of storage location visitation yields different traversal distances and is an example of the traveling salesman problem [6]. When considering the location assignment problem with multi-pick operations, the traveling salesman problem remains an implicit problem to the overall assignment problem. Manzini [7] investigates order picking including the effect of class-based storage with multiple picks per pass through a warehouse and found class-based storage performed better than random (and non-fragmented) storage.

Frazelle [8] notes that improvements to warehouse order-picking efficiency offers the greatest potential for productivity improvements. The bulk of research in the area of optimizing picking falls under improving optimal picking routes. Ratliff [9] and Roodbergen [10, 11] optimize order picking with a solution the warehouse version of the traveling salesman problem. Ratliff determined an optimal solution to the warehouse-TSP for a warehouse with no cross-aisles. Roodbergen showed the optimal routing solution for a warehouse with cross-aisles.

Optimization of routes takes the warehouse configuration as a static given quantity then seeks to determine the optimal picking route for a pick order. Research that addresses route optimization generally do not address the location assignment problem; those that do consider location assignment are

limited to evaluating the effectiveness of different storage policies (class-based storage) rather than overall location assignment.

Most related literature addresses warehouse operations that expect a one-to-one mapping between SKU and storage location. We relax this constraint with the understanding that modern identification technologies like RFID and tracking technologies like Wi-Fi will enable warehouse systems to keep track of items even if they are distributed non-contiguously across a warehouse. Ehrenberg [12], for example, describes a system which uses an RFID reader attached to an autonomous mobile robot which maps the locations of tagged items in a warehouse.

Changing the traditional one-to-one correspondence between SKU and storage location such that the warehouse stores a SKU in multiple locations can improve the picking routes for fulfilling pick orders [13]. In other words, purposefully storing identical items in disparate locations within a warehouse offers cost savings opportunity. To this end, understanding the characteristics of optimal location assignment is a key step in understanding warehouse fragmentation for improved material storage.

To better understand location assignment in situations other than unit-load picking, we evaluate different multi-pick policies and observe the prevailing characteristics of optimal location assignment configurations. We begin in section II by defining *fragmentation* and placing our current study into context. In section III we explain the warehouse configuration problem we seek to solve. We explain the various picking cost metrics that affect the optimal configuration in section IV. We discuss single row optimization results in section V and multiple row results in section VI. We conclude with a discussion in section VII.

II. FRAGMENTATION

Our previous study [5] on fragmentation focused solely on a simple picking cost metric where the cost of picking was the sum cost of visiting each warehouse location individually. This metric is analogous to unit-load picking in warehouse operations where the picking mechanism can only hold one item at a time. In this paper, we expand the discussion of fragmentation in material storage and retrieval to multi-pick retrieval operations.

Our study focuses on the impact that fragmentation of material storage has on warehouse operations. We define fragmentation as the "scattering" of identical stored items throughout the warehouse. Most warehouse operators and academic researchers take contiguousness (collection) of identical items as a given. We observe that new technologies such as RFID break down the necessity of contiguous warehouse storage and offer additional opportunities for optimization and study.

Storing multiple identical items in a large storage bin constitutes another common warehouse framework. Many warehouse operations must address multiple copies of the same item stored within the warehouse. For example, retail warehouses and stores stock many duplicates of the items sold to consumers. These warehouse situations typically manage multiple copies by storing identical items in the same storage bin.

Many previous studies assume aggregated storage of items [2, 3, 14, 7]. Maintaining aggregated storage of products simplifies the management of inventory by concentrating identical products into a single location. Historically, this one-to-one mapping between item identity and storage location facilitates critical management tasks such as manual inventory counting. With all identical items in one place, counting involves visiting one location per SKU. In a fragmented warehouse, many more locations would need to be visited to count the copies of a SKU. Without organization of products, inventory management is difficult to control and verify manually.

Fragmented storage occurs when an SKU occupies multiple locations within the warehouse, and the mapping between SKU and storage location is no longer one-to-one. Freedom from contiguous storage restrictions allows location assignment to focus on optimal assignment based on picking needs rather than unrelated operational considerations such as inventory counting. Furthermore, multiple locations lead to additional choice when selecting which locations to visit to fulfill a pick-order and added opportunity for optimization.

Multi-picking refers to the picking of several items from multiple warehouse locations between visits to the i/o point. For multi-picking problems, order of visitation becomes important and adds the traveling salesman problem to warehouse picking analysis and optimization [9, 10]. Furthermore, fragmented storage with multi-pick processes requires optimization of which warehouse locations to visit in addition to visitation order [13].

For this study we focus on fragmented storage in a warehouse with multi-picking, unit-sized storage locations, and multiple copies of items. By using unit-sized storage bins, we design the study to freely allow fragmentation while not requiring it. If contiguous storage were the optimal arrangement, then an optimal unit-sized storage warehouse should demonstrate contiguousness of identical products. Conversely, if fragmentation is optimal, then the optimal unit-sized warehouse should demonstrate fragmentation of identical items.

III. PROBLEM STATEMENT

We ignore the costs associated with organizing and placing items and focus on the picking component of the material handling cost. In other words, we seek to identify what arrangement of warehouse items best facilitates order

picking. We assume a linear warehouse of storage locations. Each storage location has space for exactly one item of any SKU type. We also assume that the warehouse configuration may use any SKU location assignment with zero cost. In other words, we ignore the costs associated with placing items into the warehouse and maintaining the desired configuration. We seek the location assignment that yields the best picking performance given demand probability distributions for each SKU.

Given a set of independent probability distributions P, where $P_{i,j}$ indicates the probability that the pick-order requires j copies of the ith SKU; determine the arrangement of items that leads to minimal expected cost.

We calculate expected cost by enumerating all possible pick-order combinations, the associated probability of each pick-order based on the given probability distributions for each SKU, and the cost to pick each pick-order combination. For a warehouse system with three SKUs, the probability of an order needing a copies of SKU 1, b copies of SKU 2, and c copies of SKU 3 is

$$p(a, b, c) = P_{1, a} \cdot P_{2, b} \cdot P_{3, c}. \tag{1}$$

The cost associated with the pick-order depends on the cost function under consideration. The expected cost (EC) for the warehouse configuration is the summation of pick-order probability times the pick-order cost.

$$EC \sum_{a=0}^{n_a} \sum_{b=0}^{n_b} \sum_{c=0}^{n_c} p(a,b,c) \cdot pickcost(a,b,c) \tag{2}$$

where $p(a,b,c)$ is the probability of an order needing exactly a copies of SKU 1, b copies of SKU 2, and c copies of SKU 3; and $pickcost(a,b,c)$ is the cost of retrieving a copies of SKU 1, b copies of SKU 2, and c copies of SKU 3.

We seek to minimize the expected cost of retrieving a pick-order from the warehouse by determining the optimal configuration of items. In general, locating items with higher probability of being present within the pick-order in lower picking cost warehouse locations will contribute to better solutions. However, since multiple copies of each SKU are to be considered, the solution must determine how each set of SKUs should be distributed across the warehouse storage locations. If the highest probability SKU occupies all of the least cost warehouse locations, then the remaining SKUs will incur higher cost. The optimal solution must balance the probabilities of all SKUs and account for the distribution of those probabilities.

IV. COST METRICS

In [5] we addressed the warehouse location assignment problem for unit-load picking which corresponds to a sum-cost metric illustrated in Figure 1. The single row warehouse model consists of a linear array (row) of storage locations numbered sequentially 1 through n. Each location is able to store one copy of any SKU. We define the cost of picking as equal to the location number of the storage location.

With sum-cost, the cost of picking equals the addition of the cost value of each location visited regardless of which other locations are visited in the warehouse. This sum-cost metric models warehouse operations where picking processes handle single units only. These unit-load operations include forklift pallet retrieval, automated systems, and other specialized cases where single picking is required. These picking systems require a return to the input/output (i/o) point between individual visits to a storage location. The sum-cost metric describes these picking situations by including the individual costs of all locations from which items are picked.
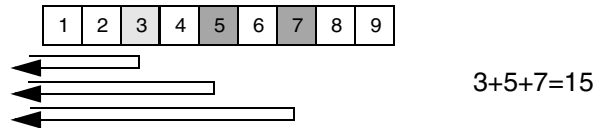

Fig. 1. Sum cost metric models unit-load picking

For this study, we consider two cost metrics: maximum-cost and hybrid-cost. Maximum-cost (Figure 2) is the value of the highest cost location visited during the picking operation; this cost metric corresponds to a picking process where all needed items are retrieved during a single pass through the warehouse. Furthermore, the distance traveled (cost) corresponds to how deep into the warehouse the operation must go to access all needed items. Shorter distances required yield lower values for the maximum-cost metric.
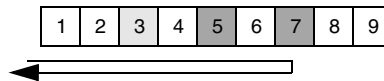

Fig. 2. Maximum-cost models single pass multi-pick operations)

In addition to maximum-cost, we consider a hybrid-cost metric as well. The hybrid metric combines attributes of the maximum-cost metric and the sum-cost metric [5], which models unit-load retrieval of items. In unit-load picking, only one item may be retrieved during each pass into the warehouse; therefore the cost is the sum of the values of each storage location visited to fulfill the pick-order. Hybrid-cost models a picking operation where multiple items may be retrieved during each pass through the warehouse, but the items must be the same SKU. Figure 3 illustrates the hybrid-cost metric.
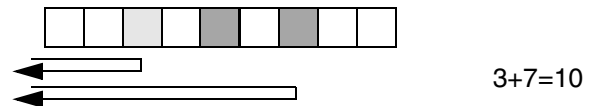

Fig. 3. Hybrid-cost metric combines aspects of unit-load and multi-pick

In warehouses that enforce a one-to-one correspondence between SKU and storage location, the hybrid-cost metric is equivalent to the sum-cost metric.

A. Hybrid Cost

Equation 2 describes the expected picking cost for any cost metric. For the specific hybrid-cost metric, the expected cost expression simplifies due to the independence of picking each SKU. In other words, the cost contribution of picking SKU i has no effect on the cost contribution of picking SKU j. Equation 3 expresses SKU independence for a case with three SKUs. The cost to pick x copies of SKU #1 has no effect on the cost to pick y copies of SKU #2 or z copies of SKU #3:

$$hybrid\ x, y, z$$
$$= (hybrid\ x, 0, 0\ + hybrid\ 0, y, 0\ + hybrid\ 0, 0, z\ ) \quad (3)$$
$$= (hybrid_1(x) + hybrid_2(z) + hybrid_3(z))$$

where $hybrid_i(a)$ is the hybrid-cost of picking a copies of the ith SKU.

Since hybrid-cost picking of each SKU is independent of the others, equation 2 simplifies to:

$$EC_{hybrid} \quad \sum_{i}^{n_{sku}} \sum_{1\,a}^{n_i} P_{i,\,a} \cdot hybrid_i\ a \quad (4)$$

where $P_{i,a}$ is the probability of the pick-order needing a copies of the ith SKU; n_i is the maximum number of the ith SKU potentially needed; and n_{sku} is the total number of SKU types.

The hybrid-cost on an SKU-by-SKU basis is simply the location cost of the ath copy of the SKU.

$$hybrid_i\ a \quad location_i\ a \quad (5)$$

where $location_i(a)$ is the warehouse location value of the ath copy of the ith SKU. In other words, SKU i has exactly a-1 locations with lesser values than $location_i(a)$.

$$EC_{hybrid} \quad \sum_{i}^{n_{sku}} \sum_{1\,a}^{n_i} P_{i,\,a} \cdot location_i\ a \quad (6)$$

With a hybrid-cost metric we minimize the expected cost by assigning the lowest warehouse value locations $location_i(a)$ to the corresponding quantity, $P_{i,a}$ with the greatest value. However, the definition of $location_i(a)$ imposes some restrictions on the assignments:

$$location_i(a) < location_i(a+1) \quad (7)$$

In other words, the ath copy of SKU i, must be a location with lower cost than the $a+1^{th}$ copy.

Equation 6 shows that if all probabilities $P_{i,a}$ are equal, then all configurations yield the same expected hybrid picking cost. When the probability of needing a copies of an SKU is less than the probability if needing $a+1$ copies ($P_{i,a} < P_{i,a+1}$), then equation 6 shows that assigning a lower cost location to $P_{i,a+1}$ is best; however doing so violates equation 7: the ath copy of the ith SKU must be in a lower cost location than the $a+1^{th}$ copy. Therefore, when $P_{i,a} < P_{i,a+1}$, the ath copy and $a+1^{th}$ copy will be adjacent in the 1-D warehouse (aggregation).

When the probability of needing a copies of an SKU is greater than the probability if needing $a+1$ copies ($P_{i,a} > P_{i,a+1}$), the ath copy and $a+1^{th}$ copy of SKU i will be adjacent if there are no other probabilities $P_{j,b}$ such that $P_{i,a} > P_{j,b} > P_{i,a+1}$. If there exists a $P_{j,b}$ such that $P_{i,a} > P_{j,b} > P_{i,a+1}$, then equation 6 indicates that the optimal solution could have a copy of SKU j between the ath and $a+1^{th}$ copy of SKU i (interleaving).

Probability distributions with a rising probability (positive slope) will result in identical SKU in adjacent warehouse locations (aggregation) while probability distributions with a declining probability (negative slope) will permit interleaving (fragmentation).

B. Maximum Cost

The maximum-cost metric assigns a picking cost equal to the visited location with the greatest cost for a pick-order regardless of SKU type. Figure 2 illustrates the maximum-cost picking metric. Unlike simpler cost metrics such as hybrid-cost or sum-cost, the maximum cost metric does not allow for independent calculation of subcosts based on the contribution of picking each SKU separately. In other words, the number of items picked for SKU i effects the cost of picking SKU j, and equation 3 does not apply in the maximum-cost case.

Due to the complexity of calculating the maximum cost for a particular pick-order, analysis of the maximum cost metric must rely on optimization via brute force simulation rather than equation-based analysis.

V. SINGLE ROW OPTIMIZATION

We employed a hill-climbing algorithm for optimization of warehouse configurations for hybrid-cost picking and maximum-cost picking. Beginning with a random configuration of SKU items in the warehouse, the hill-climbing method compares the expected cost order picking a warehouse configuration with the same configuration modified with a few randomly swapped items. The search method keeps the configuration with the lower expected cost then repeats the process with a new modified configuration involving different swapped items.

Figure 4 illustrates the results for 2 SKUs. Fig 4a shows the probability distribution of the first SKU. The probability of the pick order requiring a small number of copies of SKU#1 is small, while the probability of the pick order requiring a larger number of SKU #1 copies is higher. Fig 4b shows the probability distribution for the second SKU. The probability of the pick-order requiring a small number of SKU #2 is high, while the probability of needing a larger number of SKU #2 is low. Fig. 4c shows the single-row low-cost solution with the hybrid-cost picking metric. The x-axis shows the positions for each item in the solution. The items on the left are in low-cost

locations, and items on the right are in higher-cost locations. As the mathematics showed in Section (IV)A, an SKU with an increasing probability (higher probability of needing a greater number of copies) leads to full aggregation of the SKU. Fig. 4d shows the low-cost solution with the maximum-cost picking metric. In this case, the behavior of the low-cost solution shows favor towards putting SKU#1 near the lower cost end of the warehouse and placing SKU#2 near the higher cost end. Unlike the hybrid-picking case, maximum-cost picking permits picking different SKU during the same pass through the warehouse. The result shows greater favor towards interleaving in the maximum-cost picking case, but the tendency towards aggregation as found in the hybrid-case remains.
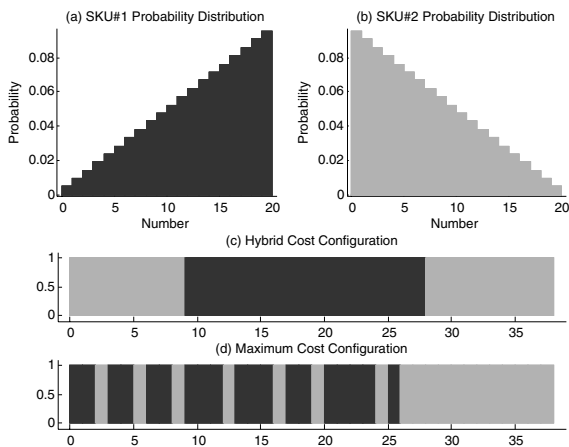

Fig. 4. Two SKU warehouse placement

Figure 5 illustrates the results for 4 SKU, two with identical increasing probability distribution and two with identical decreasing probability distribution. The hybrid result (Figure 5c) is consistent with the analysis and the SKU with increasing probability (higher probability of needing more copies of the SKU) show strong aggregation of the items. The SKU with decreasing probability show interleaving is more cost effective.
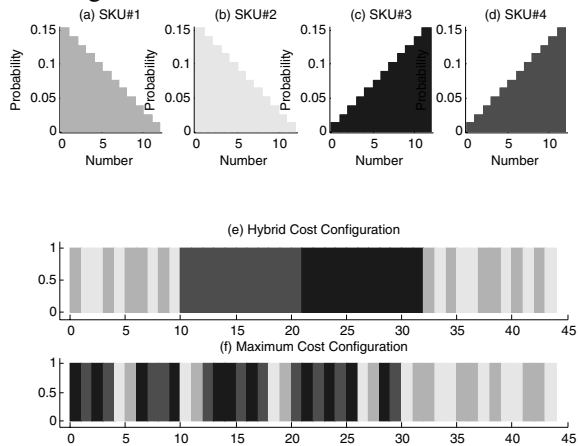

Fig. 5. Four SKU warehouse placement

MULTIPLE ROW LOCATION ASSIGNMENT

Unlike the sum-cost picking cost case, the introduction of additional warehouse rows changes the optimal configuration for hybrid-cost and maximum-cost picking metrics. The hybrid-cost metric shows a strong tendency for configuring identical copies of an SKU in the same row. The maximum location subcost for each SKU as the definition of hybrid-cost explains the tendency for same row configurations. With a single copy of an SKU in one of the rows, the incremental cost for picking another copy shifts based on the location of the first item. In other words, previously 'expensive' locations become less expensive. Figure 6e illustrates the tendency for same-row SKU aggregation for the hybrid-cost case.

Figure 6f shows the low-cost solution to the four SKU case with two rows in the warehouse. Unlike the hybrid cost case, all items of the same SKU do not appear in the same row. Therefore, despite the benefits of keeping identical SKU in the same row, the maximum-cost picking method benefits from separating identical items. This result is additional evidence for interleaving when using maximum-cost picking.
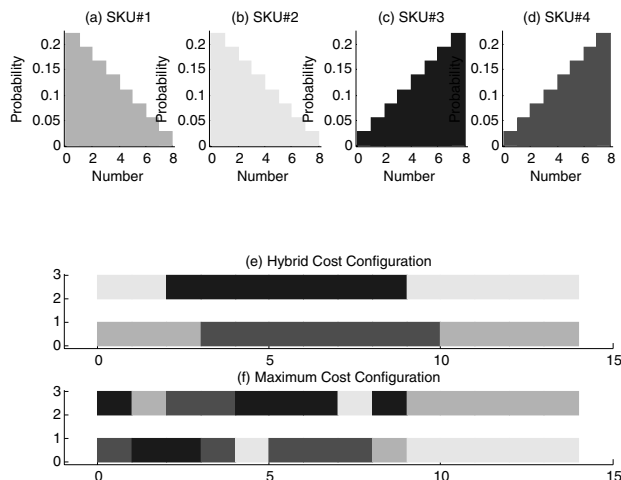

Fig. 6. Four SKU in a Two Row Warehouse

Table I compares solutions for six different two-row warehouse problems with 4 SKU found through a hill-climbing search, randomly generated solutions, and a heuristic solution. The heuristic is based on the optimal solution to the sum-cost location assignment problem [5]. On average (of 100 trials), the local optimal solution is 20 percent lower than a randomly generated solution; however, the local optimal solution is only 5-10 percent lower than the most efficient of the 100 randomly generated solutions.

TABLE I: Normalized picking costs

	Hill-climbing	Random Min	Random Ave	Heuristic
1	1.00	1.053	1.196	1.021
2	1.00	1.030	1.120	1.004
3	1.00	1.093	1.210	1.024
4	1.00	1.024	1.104	1.009
5	1.00	1.093	1.151	1.045
6	1.00	1.064	1.158	1.023

A heuristic-based solution to the warehouse location assignment yields solutions that are on average 13 percent better than a randomly selected configurations and average only 2 percent worse than the hill-climbing solution.

Determining optimal solutions is calculation intensive and intractable for larger problems (larger number of SKU and/or larger warehouse). A heuristic, such as the optimal solution to the sum-cost picking metric offer very good solutions to the computationally difficult maximum-cost metric case. Furthermore, the maximum-cost picking case represents a large portion of warehouse operations so computationally feasible approaches are necessary.

VII. CONCLUSION

Mathematical analysis of hybrid-cost metrics shows interleaving items (*i.e.*, fragmentation) leads to favorable warehouse configurations when the probability of needing more copies of an item decreases and aggregation of identical items is optimal when the probability of needing more copies of an item increases. In other words, when the probability of needing a small number of an item is low and needing a large number of an item is high, we favor aggregation; but when the probability of needing a small number of an item is high and needing a large number of an item is low, we favor fragmentation.

Maximum-cost picking represents a large portion of the picking methods used in warehouse systems, yet analytical methods are not available for evaluating warehouse picking with the maximum-cost picking metric. The complexity of determining optimal picking routes for maximum-cost picking is a well-known and makes the determination of an optimal warehouse item arrangement difficult. By studying simplified cases of the warehouse configuration problem, we have shown that optimal arrangements for maximum-cost picking tend to encourage mixing of different SKU. Randomized configurations of the warehouse are not significantly more costly than the local optimal solution, and a simple heuristic produces results very close to the local optimal solution. Due to ease of implementation, random solutions offer greater opportunity for practical applications. In fact, in [13] we studied random configurations and note

situations where random location assignment performs very well.

When considering the multiple row warehouse case, a keeping identical items in the same row seems like a natural way to optimize picking. In the hybrid-cost case, keeping items of the same type in the same row is advantageous. However, when considering the maximum-cost picking metric, the best solution shows items of the same type in different rows. Mixing across rows demonstrates the advantage of fragmented storage for the maximum-cost picking case.

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