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Calculation and Measurement of Higher Order Mode Losses in ITER ECH Transmission Lines

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***Abstract*—The ITER transmission lines (TLs) must be designed to deliver 20 MW from a 24 MW, 170 GHz gyrotron system. Miter bends are the main source of loss for these highly overmoded, corrugated, cylindrical waveguide TLs. Previous estimates have used only a pure HE_{11} mode for the analysis of the loss due to a miter bend, however higher order modes (HOMs) must be considered for a practical analysis. For the linearly-polarized, Gaussian-like beam from a gyrotron, the LP_{mn} mode basis set should be used to describe the fields in the corrugated waveguide. The HOM content greatly affects the propagation of HE_{11} content in a miter bend, with a large emphasis placed on the percentage of HOMs and the phase difference between HE_{11} and each HOM. By considering LP modes, a complete basis set is used to investigate the HOM effects on HE_{11} loss in a miter bend. We also present a new conservation theorem relating the power centroid offset and propagation angle due to any two LP_{mn} modes propagating in the corrugated waveguide.**

I. Introduction

The ITER170 GHz ECH system specifies that 20 MW be delivered from the 24 MW gyrotron system. To satisfy the low loss requirement, a 63.5 mm corrugated cylindrical waveguide will be used with a diameter to wavelength ratio of $2a/\lambda = 36$. This waveguide has been designed to operate with a pure HE_{11} mode input, and the Ohmic loss in straight sections of the waveguide is negligible. The majority of loss originates from other components in the TLs, mainly miter bends.

II. LP Modes

With the need for completeness in theory and the knowledge that a gyrotron beam is inherently linearly polarized, the LP_{mn} mode basis is described and is used to analyze the gap model described in Section III. LP modes allow a complete analysis of mode mixture combinations when considering the linearly polarized restriction on the input beam. They can be expressed in odd and even mode forms:

$$\begin{aligned} \vec{E}^{\perp}_{mn} &= \frac{X_n}{a} J_m\left(X_n \frac{r}{a}\right)\cos(m\phi)\hat{y} \quad (\mathrm{LP}_{mn} \text{ odd}) \\ \vec{E}^{\perp}_{mn} &= \frac{X_n}{a} J_m\left(X_n \frac{r}{a}\right)\sin(m\phi)\hat{y} \quad (\mathrm{LP}_{mn} \text{ even}) \end{aligned} \tag{1}$$

Note that the LP_{01} is the same as the HE_{11} mode. Figure 1 shows the electric field and polarization of the lowest order LP modes.

The Gaussian-like beam from a gyrotron can be matched to the modes of a corrugated waveguide using either the LP_{mn} mode basis set or a basis set of HE_{mn}, EH_{mn}, TE_{0n}, and TM_{0n} modes. If the latter basis set is used, care must be taken to use the correct mode combinations to produce linear polarization.

III. Transmission in Gaps and Miter Bends

The power loss due to a miter bend is estimated by calculating the equivalent problem of the loss due to a gap in the waveguide with a length of $L=2a$, following the approach used previously for the HE_{11} mode by Doane and Moeller[1]. The electric field after a gap of length $L=2a$ is derived using Fresnel Diffraction, such that, for LP_{mn} odd modes,

$$\vec{E}^{\perp}_{mn} = \hat{y}\frac{2\pi k X_n j}{aL} e^{jm\frac{\pi}{2}} e^{j\frac{kr^2}{2L}} \cos(m\phi)\int_0^a f(r')dr', \tag{2}$$

$$\text{where } f(r') = \mathrm{J}_m\left(X_n \frac{r'}{a}\right)\mathrm{J}_m\left(\frac{krr'}{L}\right)e^{j\frac{kr'^2}{2L}} r'.$$

Even modes follow the same equation with $\cos(m\phi)$ replaced by $\sin(m\phi)$. Loses are divided into two parts: field lying outside of the output waveguide and field within the output waveguide. The former couples to very higher order modes that do not propagate through the miter bend and produce Ohmic loss, and the latter couples to high order LP_{mn} modes (HOMs) that, for large a/λ, will propagate down the TL. This gap loss calculation can also be done for a combination of LP mode inputs by calculating the field of each mode traversing the gap, then combining the electric fields after the gap to find the new mode content in the waveguide. The loss due to a miter bend is estimated as half of the loss due to a gap.

The phase difference between certain input mode combinations causes variations in the power loss and mode content after the gap. As has been previously established with a combination of HE_{11} and HE_{12} modes, the power loss in an

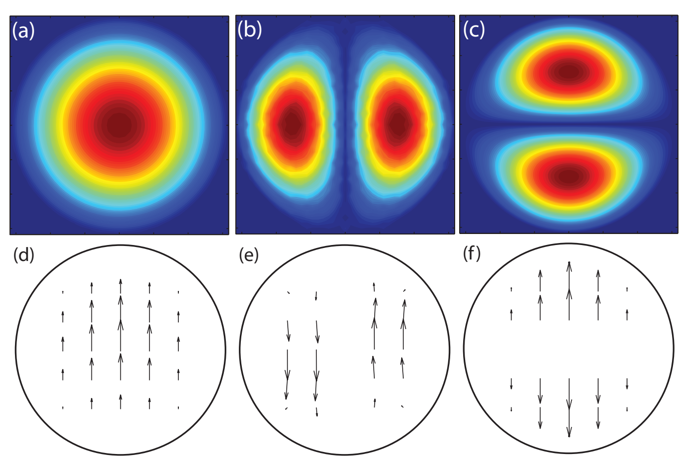

Figure 1: Illustrations of the electric field magnitude and vector plots for: (a) HE_{11}, (b) $LP_{11}^{(o)}$, and (c) $LP_{11}^{(e)}$ modes.

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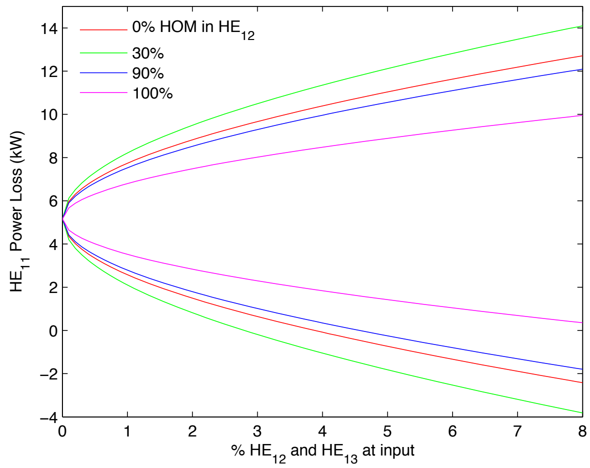

Figure 2: Maximum and minimum HE_{11} power loss vs. percent of HOMs at the input for varying percentages of the HOMs split into HE_{12} and HE_{13}. The total input power is 1 MW. The phase differences between HE_{11} and HE_{12}/HE_{13} for maximum loss is $120^o/300^o$ and for minimum loss is $300^o/120^o$

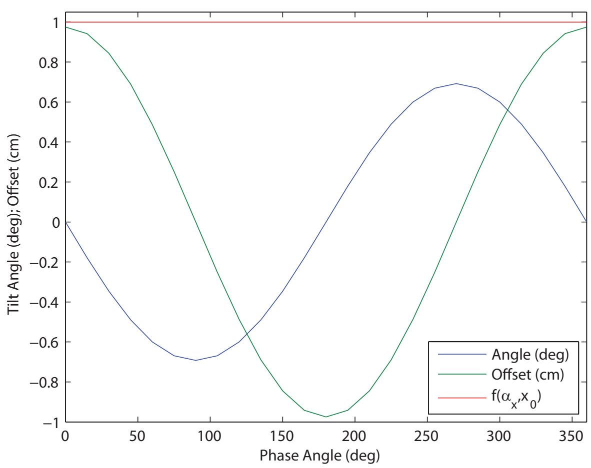

Figure 3: For 68% HE_{11} and 32% LP_{11}, the angle (in degrees) and offset (in cm) vs. phase difference between modes. The normalized result of Eq. 3 is shown in red. For this example, the waveguide has a diameter of 63.5 mm and the frequency is 170 GHz.

HE_{11} mode through a miter bend is dependent on the phase of the HE_{12} mode.[2] Figure 2 shows the phase dependence of power loss due to a gap for a combination of HE_{11}, HE_{12}, and HE_{13} modes. In Fig. 2, the x-axis represents the total power in the combination of the HE_{12} and HE_{13} modes. The HE_{11} power loss is dependent on the phases of both modes as well as the percentage of power in each mode.

With LP_{mn} modes, this phase dependence holds for combinations with the same azimuthal (m) index. Mode combinations with different m indices do not couple to each other after the gap, due to the two-dimensional symmetry of the problem. Therefore, variations in power loss due to phase in HOMs can be more easily isolated using the LP mode basis set.

IV. Conservation Theorem

When a single mode propagates in a waveguide, the centroid of the mode power is always on axis and the mode has a constant flat phase front. However, when two modes propagate, the power centroid will generally be off center from the axis by an amount, x_0 and y_0, and the phase front will be tilted by an angle, α_x and α_y, in the x and y direction, respectively. In this Section, we derive a conservation theorem expressing the relationship between tilt and offset for two propagating LP_{mn} modes. For the two modes, the electric field is defined as $E(x,y) = C_1u_1(x,y) + C_2u_2(x,y)$. Here, C_m is a constant and u_m is the normalized field pattern of each mode. With this electric field, the offset and propagation angle in the x direction can be expressed as:

$$x_0 = \langle x \rangle = 2\,\mathrm{Re}\left(C_1 C_2^*\right) b_{12},$$

$$\alpha_x = \frac{\langle k_x \rangle}{k} = 2\,\mathrm{Im}\left(C_1 C_2^*\right) d_{12}.$$

The variables b_{12} and d_{12} are mode-specific integrals where

$$b_{12} = \int x u_1 u_2 \,dxdy, \; d_{12} = \frac{-1}{k}\int u_2 \frac{du_1}{dx} dxdy.$$

The offset and angle in the y direction is similarly found.

The offset and angle change with the beating, or phase difference, between modes as the field propagates. However, these expressions can be combined such that there is no dependence on phase:

$$\left(\frac{x_0}{b_{12}}\right)^2 + \left(\frac{\alpha_x}{d_{12}}\right)^2 = 4\left|C_1 C_2^*\right|^2. \tag{3}$$

Figure 3 shows these results for a combination of HE_{11} and $LP_{11}^{(o)}$. In this case, b_{12} and d_{12} are calculated as

$$b_{12} = \frac{\sqrt{2}}{a^2 J_1(X_0) J_0(X_1)} \int_0^a J_0\left(\frac{X_0}{a} r\right) J_1\left(\frac{X_1}{a} r\right) r^2 dr,$$

$$d_{12} = \frac{\lambda X_0 X_1}{\sqrt{2}\pi a \left(X_1^{\,2} - X_0^{\,2}\right)},$$

where X_0=2.405 and X_1=3.832. Other two-mode combinations that result in a centroid offset will behave similarly. Figure 3 shows that due to interference effects the power in the two modes propagates down the waveguide with sinusoidal oscillations in both tilt and offset. The oscillations are out of phase by 90º and combine (using Eq. 3) to form a constant of the motion.

V. Conclusions

When considering ITER applications, the LP_{mn} modes are a useful and complete basis set to describe the fields in a corrugated cylindrical waveguide. These modes have been used to describe the phase dependence of loss in a miter bend for multi-mode inputs. A conservation theorem also relates the power centroid offset to the propagation angle of a field for any two LP_{mn} modes in a corrugated waveguide.

References

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