

# Three Essays on Strategic and Tactical Issues in Product Design

by

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## **ABSTRACT**

This dissertation consists of three essays on strategic and tactical issues in product design.

The first essay presents a dynamic investment game in which firms that are initially identical develop assets which are specialized to different market segments. The model assumes there are increasing returns to investment in a segment, for example, due to word-of-mouth or learning curve effects. In equilibrium, firms that are only slightly different focus all of their investment in different segments, causing small random differences to expand into large permanent differences. Even though firms do not cooperate and do not make threats to punish each other, in the long run they divide the market, reaching the same outcome that they would if they cooperated to maximize joint profits.

This second essay develops a model in which an incumbent has expertise in an old business format (e.g., running a full service airline), and a new firm enters the market with the possibility of using a new business format (e.g., running a “no frills” airline). Firms play a dynamic investment game in which the incumbent can invest in the new format and the entrant can invest in either format. If brand and format preferences are strong, and if it is easy to implement a format, then a firm already using one format does not invest in the other format, since such an attack would be met with swift retaliation. In this case, the entrant invests in the new format, while the incumbent avoids investing in order to retain the threat of investment as a punishment mechanism.

The third essay shows that improved accuracy in conjoint analysis has important strategic implications. Even if two models provide unbiased part-worths, competitive game theory shows that the more accurate model (with lower error variance in an HB CBC model) implies that differentiation from competitors is more profitable. On the other hand, a less accurate model implies that each firm should forego differentiation and choose feature levels that provide customers the greatest utility (adjusting for marginal cost). I illustrate the theory by varying accuracy in a student-apparel application.

Thesis Supervisor: Birger Wernerfelt  
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## Essay 1:

# How Do Firms Become Different? A Dynamic Model\*

### Abstract

This paper presents a dynamic investment game in which firms that are initially identical develop assets which are specialized to different market segments. The model assumes there are increasing returns to investment in a segment, for example, due to word-of-mouth or learning curve effects. In equilibrium, firms that are only slightly different focus all of their investment in different segments, causing small random differences to expand into large permanent differences. Even though firms do not cooperate and do not make threats to punish each other, in the long run they divide the market, reaching the same outcome that they would if they cooperated to maximize joint profits.

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# 1 Introduction

Casual observation reveals that even firms in the same industry are different from each other. Each firm has unique assets such as its reputation, relationships, and production skills. This is important because a firm’s optimal marketing strategy depends on the differences between its own assets and its competitors’ assets (Wernerfelt 1984; Dutta et al. 1999; Slotegraaf et al. 2003).<sup>1</sup>

This paper proposes an explanation for how persistent differences in firms’ assets arise. In particular, why do all firms in an industry not imitate the most successful firm?<sup>2</sup> The explanation I propose is a dynamic investment model in which firms begin with the same assets, but due to random fluctuations, there are small differences in their performance on various dimensions. In equilibrium, each firm then focuses its investment on the dimension where it has performed best, causing the small random differences to grow into large permanent differences.

Previous dynamic investment models have also shown how random fluctuations, along with firms’ investment decisions, can lead to differences in firms’ asset levels. However, these earlier models do not provide a complete explanation for firm differences, for two reasons. First, they assume each firm invests in a single type of asset. Therefore, firms either repeatedly trade the top position (Aoki 1991; Villas-Boas 1993; Ofek and Sarvary 2003; Hörner 2004), or in the long run one firm has persistently

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<sup>1</sup>Throughout the paper, I use the term “asset” to represent anything that a firm invests in, and builds up over time, that helps it generate higher profits. This term encompasses the constructs known in the strategy literature as “resources” (Wernerfelt 1984), “core competencies” (Prahalad and Hamel 1990), and “dynamic capabilities” (Teece et al. 1997).

<sup>2</sup>In some cases, a simple preemption story answers this question (if one firm is already good at something, it is not profitable for other firms to acquire this skill). However, in other cases this type of story does not provide a satisfactory explanation. For example, imagine a firm is suffering from declining sales because it is worse than its competitor at providing customer service. It seems likely that, at least in some cases, the marginal returns to investing in customer service improvements would be high enough that the laggard should try to catch up along this dimension. This paper presents an explanation for why firms sometimes should *not* try to catch up, even in the absence of strong preemption effects.

higher assets than its competitor (Reinganum 1989; Budd et al. 1993; Athey and Schmutzler 2001; Besanko and Doraszelski 2004; Doraszelski and Markovich 2007; Besanko et al. 2008); but firms do not develop *different* assets. In reality, firms typically invest in various kinds of assets, so they must decide not only how much to invest, but where to allocate that investment.

The second problem is that, like static models of product design, dynamic models can have multiple equilibria. In order to limit the number of possible equilibria, dynamic investment models typically assume firms follow strategies that are Markov (so a firm's investment is a function only of current asset levels) and stationary (so investment functions do not change over time). However, the stationarity assumption in itself is problematic. In reality, firms can and do change their strategies over time. For example, a firm that is currently focusing on a niche customer segment might hire a new chief executive officer (CEO) who adopts the more aggressive strategy of attacking the mainstream segment.<sup>3</sup>

The current paper addresses both of these issues. I develop a dynamic investment model in which firms compete for various market segments. A key assumption of the model is that there are increasing returns to investing in a segment. Depending on the type of asset, this could be true for many reasons, including demand-driven effects such as word-of-mouth and reputation effects, or supply-driven effects such as division-of-labor and learning curves. In equilibrium, firms start with the same assets, but due to small random fluctuations, each is likely to perform slightly better in a different segment. Each firm then focuses investment in the segment where it has performed best, causing its assets in that segment to grow, which makes investment in

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<sup>3</sup>In the model in this paper, as long as firms are sufficiently patient, such strategic changes are always possible in equilibrium. That is, there is always an equilibrium in which the firm that is strong in the niche segment launches a prolonged attack on the mainstream segment until it drives its rival out of this segment. However, as long as firms are not too patient (and other conditions hold), firms stay focused on the segment where they are currently strong.



the segment even more attractive, and so on. Furthermore, once a firm has developed a large asset stock in a segment, it has strong incentives to continue focusing in this segment, where its marginal returns are high. As a result, as long as firms are not too patient, assets do not depreciate too quickly, and random shocks are not too large, firms that develop strengths in different segments must stay focused on their respective strengths in any Pareto optimal subgame perfect equilibrium.<sup>4</sup> Thus, in principle I allow for strategic attacks in which a firm drives its rival out of a segment, but I also derive conditions in which such attacks cannot occur as long as all firms behave rationally.

One interesting property of this model is that permanent differences can arise even if there are no strategic interactions in investment decisions. By contrast, in one-dimensional dynamic investment models, permanent differences can only arise if there are strong “preemption” effects, that is, if one firm having a high asset level compels the other firm to remain at a low asset level. If these preemption effects are sufficiently weak, then a one-dimensional model leads to a perpetual race, whereas in the current paper the presence of a second dimension can induce firms to become permanently focused on different segments.

Another interesting property of this model is that non-cooperative behavior leads to the same long-run outcome as cooperation. Each firm in the model maximizes its own expected discounted profits, and equilibrium behavior does not depend on punishment strategies. Nonetheless, firms eventually divide the market, focusing all of their investment in different segments. Assuming investment costs are sufficiently low (so that the optimal cooperative solution involves firms producing at their investment

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<sup>4</sup>The restriction to Pareto optimal equilibria is needed to rule out mutually harmful behavior that is sustained only by the threat of additional mutual harm. For example, there could be non-intuitive equilibria in which each firm’s private investment incentives are to stay focused, but the firms nonetheless attack each other, and if these attacks are not carried out, then additional attacks follow as a punishment. This type of behavior, which resembles the “self-reinforcing joint cost effects” described by Budd et al. (1993), is ruled out by the restriction to Pareto optimal equilibria.

constraint), this is the same long-run outcome that would occur if firms maximized joint profits. The intuition is that, once firms develop strengths in different segments, each firm's private investment allocation incentives coincide with the interests of its rival. Put differently, we can benefit from our competitor's success in a segment where we are weak, because this discourages them from investing in the segment where we are strong.

In addition to the literature on dynamic investment games cited above, several other streams of literature in economics and marketing address how firms become different.

Product design models such as the Hotelling model predict that firms choose different locations (D'Aspremont et al. 1979; Orhun 2005; Thomadsen 2007; Zhu and Singh 2009; Subramaniam and Gal-Or 2009) or invest in different product attributes (Hauser 1988; Goettler and Shachar 2001; Lauga and Ofek 2009). These models explain why firms want to become different, but they do not explain which firm wins each segment. For example, if one segment is more attractive than another, these models typically have multiple equilibria, and either firm could win the more attractive segment. The current paper proposes an explanation for how firms solve this coordination problem. In particular, I show that small random shocks, along with increasing returns, can determine which firm wins each segment.

If entry is sequential, other papers predict that the first mover acquires more assets, or wins the more attractive segment (Prescott and Visscher 1977; Schmalensee 1982; Moorthy 1988; Lieberman and Montgomery 1988; Sutton 1991). A key difference is that the current paper predicts that random shocks early in the life of a firm determine which segment the firm pursues; therefore, a later entrant could end up with the most attractive segment. In fact, the question of whether first movers actually perform better is a subject of ongoing debate in the empirical literature (e.g.,

Urban et al. 1986; Golder and Tellis 1993; Bronnenberg, Dhar, and Dubé 2009).

Other papers attribute persistent performance differences to firms' inability to optimize, for example, due to principal-agent problems (Gibbons 2006; Chassang 2008; Ellison and Holden 2008) or boundedly rational managers (Barney 1986; Nelson 1991; Goldfarb and Xiao 2009). By contrast, the current paper shows that, even if firms behave optimally, differences in firms' investment incentives can lead to persistent differences in their performance along various dimensions.

Finally, there is also a strategy literature that studies the origins and implications of firm differences (e.g., Wernerfelt 1984; Barney 1986; Prahalad and Hamel 1990; Nelson 1991; Rumelt et al. 1991; Teece et al. 1997; Helfat 2000; Wernerfelt 2003), and an empirical literature in economics and marketing documenting the magnitude and persistence of firm differences (e.g., Mueller 1977, 1986; Mairesse and Griliches 1990; Dutta et al. 1999; Slotegraaf et al. 2003). This paper contributes to this literature by deriving conditions in which permanent differences will or will not arise.

## **1.1 Example: Ben and Jerry's**

I now present an example of how a company became focused on a segment where it had initial good luck, while also benefiting from competitors' success in other segments.

In 1978, Ben Cohen and Jerry Greenfield founded a homemade ice cream shop that sold both chunky flavors (which contained large chunks of chocolate, cookies, or other "mix-ins") and traditional "smooth" flavors, as well as crêpes, brownies, and soups (Lager 1994, p. 24-25). Chunky ice cream soon became their most popular product because they discovered mix-ins that tasted good in ice cream (Ibid., p. 25) and because they decided to use very large chunks (which proved popular), a decision they later attributed to Ben's sinus problems making it hard for him to taste normal flavors (Ibid., p. 22; Dreifus 1994). Also, because they initially decided to incorporate

mix-ins before, as opposed to after, the ice cream hardened, they were able to have a machine package this product into pint containers, which they started selling to local grocery stores (Lager 1994, p. 22, 40).

After this early success, the founders became “obsessed” with “getting the right amount of chunks into every pint,” concerned that “every person who wasn’t satisfied with a product was likely to tell at least ten other people about his or her unfavorable experience, multiplying the consequences of each errant pint” (Ibid., p. 148-149). Therefore, the company modified the machines that dispensed the ice cream to help prevent chunks from getting stuck (Ibid., p. 43), decided to accept the frequent production shutdowns that resulted from clogging rather than reduce the size of their chunks (Ibid., p. 140), and spent “more than five years trying endless variations and countless solutions” to produce a cookie dough mix that would not clog the machinery (Ibid., p. 219).

A somewhat puzzling aspect of this story is that Häagen-Dazs, who was already an established competitor in the superpremium ice cream market, did not mimic Ben and Jerry’s success by creating their own chunky flavors. One plausible explanation, proposed by former Ben and Jerry’s CEO Fred Lager, is that the extensive modifications to production equipment required to produce chunky flavors would have been too much of a distraction from already successful business Häagen-Dazs had selling smooth ice cream flavors (Ibid., p. 140).<sup>5</sup> Thus, Ben and Jerry’s arguably benefited from Häagen-Dazs’s strength in smooth flavors, which prevented the latter from making large investments in the chunky segment of the market. By 1995, Ben and Jerry’s had captured 43 percent of the superpremium market nationwide, roughly equal to the share held by Häagen-Dazs, with the rest of the market shared by many

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<sup>5</sup>Another possible explanation is that Häagen-Dazs did not realize how popular chunky flavors would become. However, from very early on Häagen-Dazs tried to pressure distributors not to carry Ben and Jerry’s ice cream, suggesting that Häagen-Dazs did in fact realize chunky flavors were a potentially large segment (Ibid., p. 106).

small firms (Collis and Conrad 2005, p. 6).

## 1.2 Example: Dell Computer

In 1983, Michael Dell started assembling and selling personal computers from his college dorm room, and a year later he founded Dell Computer Corporation (Dell 1999, p.10-12). Because he only had \$1,000 in start-up capital, he could not maintain a large stock of inventory; instead, he and three employees custom assembled a computer for each order and then shipped it directly to the customer (Ibid., p. 13, 21). As the company grew, they continued investing in this “direct to consumer” business model, building factories that could fill custom orders (Ibid., p. 78), assigning each business unit to a different narrowly defined customer segment so the company stayed informed about which components were important to each type of customer (Ibid., p. 71-76), and developing systems to provide suppliers with up-to-date information on demand for various components (Ibid., p. 174-183). The company “forced all of our people to focus 100 percent on the direct model” by *not* selling through retailers (Ibid., p. 77).<sup>6</sup> As a result, Dell achieved a cost advantage through fast inventory turnover rates. Initially, Hewlett-Packard and Compaq tried to mimic Dell’s direct model; however, they found that by splitting their investment between the retail and direct channels, they performed poorly at both, and so they eventually resumed focusing on the retail channel (Schrage 2002; Townsend 2007).

To summarize, Ben and Jerry’s and Dell each had early good luck on an important dimension. Ben and Jerry’s discovered how to make high quality chunky ice cream,

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<sup>6</sup>In the early 1990s, Dell experimented with retail sales, but found that they could not maintain their fast inventory turnover rates while serving both the retail and direct channels (Thomke et al. 1999, p. 5). More recently, competitors have begun manufacturing in China, eroding Dell’s cost advantage and greatly reducing the value provided by its direct model (Scheck 2008). To try to make up for its reduced profitability, Dell has again begun selling through retail stores (Ibid.). The model in this paper could be interpreted to suggest that this recent diversification by Dell is a strategic mistake.

and Michael Dell’s lack of financial capital forced him to become skilled at filling custom computer orders.<sup>7</sup> Each company then made heavy investments to build on this initial success. Instead of mimicking these innovations, competitors stayed focused on their own areas of expertise. This paper derives conditions in which we should expect to observe this type of behavior, that is, in which small random fluctuations cause firms to become permanently focused in different areas of expertise. Although my main goal is to explain how permanent differences can arise, I also derive conditions in which we might observe other types of behavior, for example, strategic attacks in which a firm drives its rival out of a segment.

The rest of the paper is organized as follows. Section 2 develops a model in which each firm’s investment decisions depend only on its own assets, and so firms make independent investment decisions. This section then derives conditions in which small random events early in the life of a firm have a large permanent effect on the firm’s assets. Section 3 extends the model to a more general profit function in which it is more profitable for a firm to invest in a segment in which competitors are weak, and so there are interactions in different firms’ investment decisions. This section then derives conditions in which: small initial differences determine which firm wins the larger segment, and once differences arise, each firm is guaranteed to stay focused on its core expertise in any Pareto optimal subgame perfect equilibrium. Section 4 concludes.<sup>8</sup>

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<sup>7</sup>Although I model such early innovations as “random shocks,” another interpretation of the model is that companies make different initial decisions, for example, because their founders have different insights or beliefs about the market. Under this interpretation, even if these initial decisions only lead to small early differences between firms, each firm then has an incentive to continue focusing on the segment that it initially pursues, causing the differences between firms to expand.

<sup>8</sup>The paper also includes four appendices. Appendix A presents an example of a model of competition that is consistent with the profit functions used in this paper. Appendix B presents reasons that the key assumptions of the paper (increasing returns and a constraint on investment) are reasonable in many situations. Appendix C shows that an alternative formulation of the model (which assumes increasing returns *across* time periods) produces similar results to the model in the body of the paper (which assumes increasing returns *within* time periods). Appendix D contains all proofs.

## 2 Independent Investment Decisions

This section presents a dynamic investment model in which two firms compete for two market segments. The version of the model in this section assumes that equilibrium profits have an additive functional form, so that there are no interactions in competing firms' investment decisions, and each firm's investments depend only on its own assets (section 3 relaxes this assumption). Although independent investment decisions is not a realistic assumption in most cases, beginning with this version of the model has two advantages. First, it makes the analysis much simpler so that the intuition for the results is relatively clear. Second, it emphasizes that this model proposes an explanation for how firms can become different even if each firm does not explicitly care about being different from its competitor. Appendix A presents an example of a model of competition that is consistent with independent investment decisions. In particular, if prices are fixed or if firms are already very different along another dimension (represented by a Hotelling line), then assets are neither strategic substitutes nor strategic complements, and each firm makes investment decisions based only on its own asset levels.

Assume firms  $i$  and  $j$  compete in two segments, possibly of different sizes. Define  $M_{i,t}$  and  $N_{i,t}$  as firm  $i$ 's assets devoted to the mainstream and niche segment, respectively, at time  $t$ . Firm  $i$ 's profits in each segment are:

$$\pi_{i,M,t} = F(M_{i,t}) - G(M_{j,t}) \tag{1}$$

$$\pi_{i,N,t} = \alpha \left( F(N_{i,t}) - G(N_{j,t}) \right) \tag{2}$$

where  $F$  is twice continuously differentiable with  $F' > 0$ ,  $F'' > 0$ ,  $\alpha \in (0, 1]$ . I assume the functions  $F$  and  $G$  are such that a firm's equilibrium profits in each segment are

positive for all asset levels.

Assets evolve as follows:

$$M_{i,t+1} = \gamma M_{i,t} + m_{i,t} + \epsilon_{m,i,t+1} \quad (3)$$

$$N_{i,t+1} = \gamma N_{i,t} + n_{i,t} + \epsilon_{n,i,t+1} \quad (4)$$

where  $(1-\gamma)$  is the depreciation rate,  $m_{i,t}$  and  $n_{i,t}$  are firm  $i$ 's investment in the mainstream and niche segment, respectively, and  $\epsilon_{.,i,t+1}$  are iid random variables with support on a finite range  $[0, \epsilon_{max}]$ .

Also, assume there is a constraint on a firm's total investment in each period:

$$m_{i,t} + n_{i,t} \leq 1 \quad (5)$$

Finally, assume firms have discount factor  $\delta$  and maximize expected discounted profits. Since the two firms' assets do not interact in the profit functions, it is possible to formulate each firm's maximization problem as an infinite period dynamic program:

$$V(M_{i,t}, N_{i,t}) = F(M_{i,t}) + \alpha F(N_{i,t}) + \max_{m,n:m \geq 0, n \geq 0, (m+n) \leq 1} \{ \delta E[V(M_{i,t+1}, N_{i,t+1})] - Cm - Cn \} \quad (6)$$

where  $C$  is a scaling factor that represents the unit cost of investment.

One key assumption of this model is that in each period a firm has a fixed amount of investment to allocate across all segments ( $m_{i,t} + n_{i,t} \leq 1$ ). This could be true for many reasons, including capital market imperfections (Jensen and Meckling 1976; Myers and Majluf 1984), the need to allocate asset ownership to provide investment incentives (Grossman and Hart 1986; Hart and Moore 1990; Simester and Wernerfelt



2005; Gibbons, Holden, and Powell 2009), and employees' limited time and attention.

Another key assumption is that there are increasing returns to investment ( $F'' > 0$ ). In other words, it is more profitable for a firm to invest in a segment where it has previously performed well, rather than in one where it has previously performed poorly. There are many possible sources of increasing returns, including demand-side effects, such as word-of-mouth incentives (Rob and Fishman 2005), reputation effects (Kreps and Wilson 1982; Diamond 1989), network effects (Arthur 1989), and confirmatory bias in product evaluations (Boulding et al. 1999); as well as supply-side effects, such as division of labor benefits (Smith 1776), learning curves (Argote and Epple 1990; Grenadier and Weiss 1997), and retailer product adoption decisions (Bronnenberg and Mela 2004). Appendix B discusses these assumptions in more detail.<sup>9</sup>

For convenience of exposition, I make the extreme assumptions that each firm has a fixed capacity for investment in each period, and that marginal returns to investment in a segment are always increasing. More realistically, we might expect that a firm's capacity for investment should grow over time, and that firms eventually reach a point of diminishing marginal returns in their core expertise. However, all of my results would hold under less extreme assumptions than the ones I use. All that is necessary is that a firm's capacity for investment cannot grow too rapidly, and marginal returns cannot decrease to the point that the firm wants to shift investment away from its core expertise. These conditions are especially likely to hold for investments with relatively fast depreciation rates, that is, for which firms must constantly replenish

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<sup>9</sup>Whereas the model presented in this section assumes *increasing returns*, meaning that a firm's profits in a segment are convex in its assets, several earlier dynamic investment models involve what I will call *increasing asset sensitivity*, meaning that investment has a larger expected impact on assets if the firm already has a large asset stock (Rob and Fishman 2005), or if it is ahead of its competitor (Hörner 2004). Appendix C develops a model of increasing asset sensitivity, and derives results analogous to those in this section. Because these two models have similar properties, the rest of the paper will continue to focus on the increasing returns model.

their assets by reinvesting in quality, training, or other areas. When this is true, firms are likely to remain on the increasing returns part of the curve in each segment, and so they should stay focused on their core expertise.

## 2.1 Optimal investment behavior

Given the assumptions that there are multiple segments, increasing returns, and a constraint on investment, I will show that, in any given period, a firm either invests nothing or invests as much as possible in one segment. If a firm invests in a segment at a given state, it also invests in the same segment if assets in that segment become larger or assets in the other segment become smaller.<sup>10</sup>

I begin with a series of lemmas. All proofs appear in Appendix D. The first lemma is fairly intuitive. It establishes that two properties of the  $F$  function extend to the value function.

**Lemma 1.** *Firm  $i$ 's value function is strictly increasing and strictly convex in each argument.*

The next result follows from Lemma 1. Given the strict convexity of the value function, a firm either invests nothing or invests as much as possible.

**Lemma 2.** *For any given state, an optimal policy must set total investment equal to 0 or 1.*

The next lemma is not quite so obvious.

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<sup>10</sup>The results in this section bear some resemblance to previous theoretical work on employment search (Jovanovic 1979). In this earlier paper, a worker builds human capital specific to his current job, while also receiving stochastic offers from outside firms. As the worker's firm-specific capital grows, he becomes less likely to leave his job. However, whereas this earlier paper exogenously assumes the worker has exactly one job at a time, the current paper allows firms, in principle, to invest in multiple segments, or to make zero investment (which is in fact optimal when a firm's assets are very low in all segments). Therefore, the following results that characterize optimal investment behavior, and derive conditions that ensure path dependence, are new.

**Lemma 3.** *The marginal value of a firm's assets in a given segment is weakly decreasing in the level of its assets in the other segment.*

The intuition behind Lemma 3 is the following. Imagine a firm's mainstream and niche assets are both low. Because the firm is on the low returns part of the curve in both segments, it does not have anywhere profitable to allocate its investment. An increase in the firm's mainstream assets is very valuable because it gives the firm somewhere profitable to allocate its investment. Now assume, on the other hand, the firm has low mainstream assets but high niche assets. Now the firm has a profitable place to allocate its investment (in the niche segment). An increase in mainstream assets is now not so valuable because this just gives the firm two profitable places to allocate its investment. Given the investment constraint, the firm will have to choose one or the other. Thus, an increase in mainstream assets is very valuable when a firm's niche assets are low, but not so valuable when its niche assets are high. (See Appendix D for a formal proof.)

Lemma 2 showed that the firm either invests nothing, or invests as much as possible. Given the result from Lemma 3 that the interaction effect of increasing assets in different segments is negative, I show that if the firm makes any investment at all, it focuses all of its investment in one segment.

**Proposition 1.** *For any given state, the only possible optimal policies are to invest nothing, to invest as much as possible in the mainstream segment, or to invest as much as possible in the niche segment.*

Proposition 1 says that, for a given value function  $V$ , it is possible to compute the optimal policy function as follows. If firm  $i$ 's current asset levels are  $(M, N)$ , compute the expected return to investing one unit in the mainstream segment:

$$\delta \left( E[V(\gamma M + 1 + \epsilon_m, \gamma N + \epsilon_n)] - E[V(\gamma M + \epsilon_m, \gamma N + \epsilon_n)] \right) - C \quad (7)$$

Then compute the expected return of investing one unit in the niche segment.

$$\delta \left( E[V(\gamma M + \epsilon_m, \gamma N + 1 + \epsilon_n)] - E[V(\gamma M + \epsilon_m, \gamma N + \epsilon_n)] \right) - C \quad (8)$$

If both expressions are negative, do not invest in either segment. If at least one expression is positive, invest one unit in the segment with the higher expected return and zero units in the other segment.

It is also possible to combine Proposition 1 with Lemmas 1 and 3 to further characterize the optimal policy function. Lemmas 1 and 3 imply, respectively, that (7) is increasing in  $M$  and decreasing in  $N$ . Similarly, (8) is decreasing in  $M$  and increasing in  $N$ . This leads to the following corollary.

**Corollary 1.** *If a firm invests in the mainstream segment when its assets levels are  $(M, N)$ , then it also invests in the mainstream segment for all states  $(M_H, N_L)$ , where  $M_H \geq M$  and  $N_L \leq N$ . An analogous result holds for investment in the niche segment.*

This corollary states that a firm becomes more likely to invest in a segment as assets become higher in that segment or lower in the other segment. For the special case in which the two segments are the same size, it is possible to characterize investment behavior more precisely.

**Corollary 2.** *For  $\alpha = 1$ , each firm either invests in the segment where it has greater assets, or does not invest in either segment.*

These results show that optimal investment behavior can be characterized as follows. If a firm's assets are sufficiently low in both segments, then it does not invest in either segment. However, once the firm is sufficiently lucky in one segment, it starts investing as much as possible in that segment, provided assets in the other segment are sufficiently low.

## 2.2 Path dependence

I now show how the investment behavior described above determines the evolution of a firm’s assets. In particular, the following proposition states conditions in which a firm’s asset growth exhibits “path dependence,” meaning that small random events have large permanent consequences. (See Appendix D for a proof).

**Proposition 2.** *If the maximum random shock is smaller than the investment constraint, the niche segment is sufficiently large, and investment costs are neither too high nor too low, then a firm starting out with no assets will, with probability one, end up either in a recurrent class in which it always focuses in the mainstream segment, or in a recurrent class in which it always focuses in the niche segment; furthermore, it has positive probability of ending up in each recurrent class.*

To illustrate the intuition for this proposition, Figure 1 presents the optimal policy function for a numerical example with the following parameter values:<sup>11</sup>  $F(X) = X^2$ ,  $\alpha = .9$ ,  $\delta = .85$ ,  $\gamma = 0.9$ ,  $C = 35$ ,  $\epsilon \sim U[0, 0.5]$ .

The x-axis gives the firm’s mainstream assets; the y-axis its niche assets. When a firm has low assets in both segments, it makes no investment. Once a firm builds up enough assets in a segment, it starts to focus all of its investment in that segment. It then continues to focus in that same segment unless it has a large enough positive shock in the other segment that it switches its investment focus. However, once a firm begins investing in the niche segment (for example), the most likely outcome is that it will move up the y-axis until it reaches a region I call the “niche recurrent class.” Once this occurs, the firm always focuses its investment in the niche segment; its assets fluctuate along both dimensions due to random chance, but never leave the

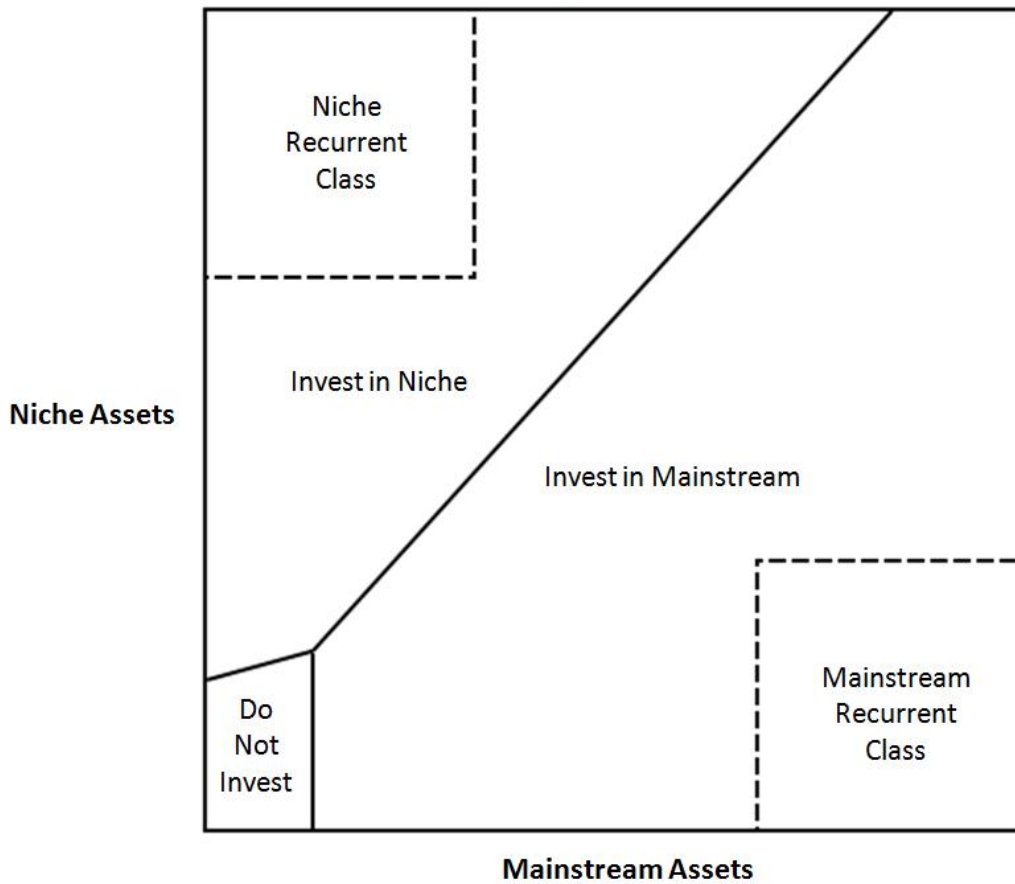
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<sup>11</sup>I calculate the optimal policy function over a 60x60 grid of values in Matlab, using modified policy iteration, with ten value iterations for each policy iteration (see Bertsekas 2007, p. 43). This algorithm converges in about one minute. To account for the asset depreciation rate, expected continuation values must be interpolated during the value function updating step.

recurrent class.

Note that each recurrent class is a perfect square. This is a result of the assumption of iid error terms in each segment. For example, imagine a firm always invests as much as possible in the niche segment. If it repeatedly receives the *worst* possible random shock in each segment, its assets converge to the bottom-left corner of the niche recurrent class. However, if the firm repeatedly receives the *best* possible random shock in the mainstream segment, given the constant depreciation rate, the furthest it can move along the horizontal dimension is to the right boundary of the niche recurrent class. Similarly, if it receives the best possible random shock in the niche segment, the furthest it can move along the vertical dimension is to the top of this recurrent class.

*Figure 1: Optimal policy function and recurrent classes*



On the other hand, note that the upper boundary of the “do not invest” region is not flat, but instead slants upward to the right. The reason is the following. When a firm has very low mainstream assets, it immediately starts to invest in the niche segment as soon as it moves sufficiently far up the increasing returns curve in this segment to offer positive expected returns. However, as the firm’s mainstream assets grow, it is inclined to defer investment in the niche segment in the hope that a positive shock in the mainstream segment in a subsequent period will lead the firm to invest there instead.

For this numerical example, a firm that starts with no assets could end up permanently trapped in either recurrent class, depending on small random changes early in the firm’s life. In other words, asset growth is path dependent. However, if any of the parameters violate the restrictions stated in Proposition 2, then asset growth is *not* path dependent. In particular, if it is possible to have random shocks larger than the investment constraint, then a firm never becomes locked into a segment, and over time it repeatedly changes its focus due to random fluctuations in its assets; if the niche segment is too small, a firm is guaranteed to become focused on the mainstream segment; and if investment costs are too small, a firm immediately becomes locked into investing in the mainstream segment, whereas if investment costs are too large, the firm will never invest in either segment.

In practical terms, the “do not invest” region indicates that a firm should not enter the market unless it already has assets that provide a small advantage in serving one segment. These assets might have been developed through the firm’s activities in another market, or they might result from the founder’s prior experience or useful discovery. To return to the examples from the introduction of the paper, Ben and Jerry’s developed popular chunky flavors while running an ice cream shop, and then entered the grocery store market for ice cream pints; Michael Dell learned to assemble

computers to earn money during college, and then started a full time business that built on this initial expertise. Thus, each firm developed an early small advantage in an area, and then began making heavy investment so that its expertise in this area grew into a large important advantage.

### 2.3 Theoretical examples

I now present three simple theoretical examples, which illustrate possible sources of increasing returns, and which are consistent with the assumptions of the model described above.

**Example: Cost reduction.** Assume that a firm's assets in each segment represent technologies and processes that reduce marginal cost. Furthermore, there are synergies to such investments, so that firm  $i$ 's marginal cost in the mainstream segment is given by:

$$K_{i,M,t} = \widehat{K} - (M_{i,t})^2 \quad (9)$$

where  $\widehat{K}$  is large enough that marginal cost is always positive. A similar equation holds for the niche segment.

There is mass 1 of mainstream customers and mass  $\alpha$  of niche customers, and customers will only purchase a product targeted toward their segment. If we assume all prices are fixed at the same level (for example, because firms maintain high prices through tacit collusion or because consumers treat prices below a certain level as a signal of low quality) then this example satisfies the assumptions of the model.

**Example: Word-of-mouth incentives.** Assume a firm's assets represent investments in product quality. It is convenient (but not essential) to assume prices are



fixed. In each period, a continuum of customers in each segment randomly choose a firm, inspect its product to determine its quality, and then either make a purchase from that firm or make no purchase at all. The number of mainstream customers purchasing firm  $i$ 's product is  $\frac{M_{i,t}}{Q_{max}}$ , where  $Q_{max}$  denotes maximum possible quality level. At this point, a firm's equilibrium profits are a linear function of its quality, so there are constant returns to investment.

Now assume that each person who consumes the product tells one friend about it. That friend then purchases the product with probability  $\frac{M_{i,t}}{Q_{max}}$ . All customers remain in the market for one period. If we assume customers' friends do not overlap, so no one hears about a product from two different sources, then demand does not depend on the competitor's quality, and firm  $i$ 's mainstream profits are proportional to  $\frac{M_{i,t}}{Q_{max}} + \left(\frac{M_{i,t}}{Q_{max}}\right)^2$ . Now there are increasing returns, and the example fits the assumptions of the model.

The intuition is that, when customers learn about products through word-of-mouth, improvements to product quality make more people aware that the product exists, in addition to increasing the fraction of people who purchase it conditional on awareness. The multiplication of these effects leads to increasing returns.<sup>12</sup>

**Example: Marketing vs. engineering.** For this example, we assume there is a single customer segment, and we redefine  $M_{i,t}$  as “marketing” assets (which help the firm understand customer preferences), and  $N_{i,t}$  as “engineering” assets (which help the firm reduce production costs). Assume that success in either activity makes

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<sup>12</sup>In the word-of-mouth model by Rob and Fishman (2005), customers infer that firms with a track record of high quality will continue to make large investments in quality, and in equilibrium, such firms have an incentive to do so. In the preceding example, customers observe quality directly, so the role of word-of-mouth is to generate awareness rather than to provide quality information. A more important difference is that I show how the model changes when firms allocate investment across multiple segments, as opposed to investing in a single segment. In particular, I have shown that even small differences between firms can cause them to focus all of their quality investments on different segments.

additional investment in that activity more effective, for example, due to learning curve or division-of-labor benefits. Customers randomly choose a firm, inspect its product, and are then willing to purchase it at any price up to  $\widehat{K} + M_{i,t}^2$ . A firm's marginal production cost is  $K_{i,t} = \widehat{K} - \alpha(N_{i,t})^2$ . Under these assumptions, a firm's equilibrium profits are proportional to  $M_{i,t}^2 + \alpha N_{i,t}^2$ , so this example satisfies the assumptions of the model presented above. Thus, firms that start off with the same marketing and engineering capabilities can become specialized in different activities based on small random differences in early success at each activity.<sup>13</sup>

To summarize, section 2 has developed a version of the model in which firms make independent investment decisions. One important insight from this version of the model is that permanent differences between firms can arise even on dimensions for which firms do not explicitly care about being different from competitors. The reason is that, under increasing returns, each firm has an incentive to focus on the dimension where it has the most early success, and so firms that are only slightly different might make investments that take them in completely different directions. Another key point is that the results in this section hold under a very general model of increasing returns. In fact, it is important for the model to hold under general conditions, because the observation that firms develop different expertise holds in many different industries and settings. The specific sources of increasing returns will depend on the details of the particular industry in question.

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<sup>13</sup>Academic and popular business literatures have long recognized that some firms are more “marketing-driven,” whereas others are more “engineering-driven” (Cooper 1984; Griffin and Hauser 1992; Hauser and Zettelmeyer 1997; Workman 1998; Day 1998). Recent research has modeled this distinction as a trade-off between property rights allocations that provide stronger incentives for either demand measurement or cost reduction (Gibbons, Holden, and Powell 2009). This example has shown how this trade-off fits into the dynamic model in the current paper.

### 3 Interactions in Investment Decisions

This section extends the model to a more general profit function that allows for interactions in firms' investment decisions. In particular, I assume that a firm's profits are less sensitive to investment in a segment if its competitor has a high asset level in that segment. This assumption holds under many models of competition, for example, if the cost reduction and word-of-mouth examples from the previous section are extended to allow for more realistic competitive interactions. The profit functions used in this section are also consistent with a standard Hotelling model of competition in each segment (see Appendix A).

Firm  $i$ 's profits in the mainstream segment in period  $t$  are:

$$\pi_{i,M,t}(M_{i,t}, M_{j,t}) = F(M_{i,t}) - F(M_{j,t}) + \psi [F(M_{i,t}) - F(M_{j,t})]^2 \quad (10)$$

Its profits in the niche segment are:

$$\pi_{i,N,t}(N_{i,t}, N_{j,t}) = \alpha \left[ F(N_{i,t}) - F(N_{j,t}) + \psi [F(N_{i,t}) - F(N_{j,t})]^2 \right] \quad (11)$$

where  $F' > 0$ ,  $F'' > 0$ ,  $\alpha \in [0, 1]$ , and  $\psi > 0$ .<sup>14</sup>

The cross-partial derivatives are negative:

$$\frac{\partial^2 \pi_{i,M,t}^*}{\partial M_{i,t} \partial M_{j,t}} = -2\psi F'(M_{i,t}) F'(M_{j,t}) \quad (12)$$

$$\frac{\partial^2 \pi_{i,N,t}^*}{\partial N_{i,t} \partial N_{j,t}} = -2\alpha\psi F'(N_{i,t}) F'(N_{j,t}) \quad (13)$$

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<sup>14</sup>Note that for this version of the model I assume the same function,  $F$ , operates on the focal firm's assets and its competitor's assets. However, the following results also hold for more general profit functions. The key properties of this profit function are that a firm's profits are increasing in convex in its own assets, that the cross-partial derivative with the competitor's assets is negative, and that once differences are large enough, a firm would like its competitor to invest where the competitor is strong instead of where the focal firm is strong.

This implies that it is more profitable for a firm to invest in a segment where its competitor is weak. Note that the parameter  $\psi$  represents the strength of competitive preemption effects. For small values of  $\psi$ , each firm bases investment decisions primarily on its own asset levels, without worrying about the competitor's assets. On the other hand, for large values of  $\psi$ , a firm has strong incentives not to invest in a segment where its competitor is strong. For example,  $\psi$  will tend to be high if consumers' search costs are low (Kuksov 2004), or if firms are very similar on dimensions other than the ones represented by the assets in which firms are investing (see Appendix A).

For this more general model, I show how small random initial differences can determine which firm wins the mainstream segment. I also derive conditions in which, once firms become different, they are guaranteed to stay permanently focused on their respective segments, reaching the same long-run solution as if they cooperated in their investment decisions to maximize joint profits. Finally, I show that later entrants can benefit from increasing returns because the incumbent is compelled to stay focused instead of preempting entry.

To keep the analysis tractable, I analyze two phases of the game separately. In the "exploratory" phase firms start with no assets, and the question is which firm builds a strength in each segment. In the "long run" phase, firms have already developed different strengths, and I derive conditions that guarantee each firm stays permanently focused on its respective segment. I then present a numerical example that combines both phases, and also develop a version of the model in which firms make sequential investment decisions to allow for the possibility that the first mover deters investment by the second.

### 3.1 Exploratory phase

For this phase of competition, I assume firms play a finite-horizon game with no discounting ( $\delta = 1$ ) and no depreciation ( $\gamma = 1$ ). Each firm invests a total amount  $K$ , with the following timing:

1. Firms start with no assets. To begin the game, they each receive a random shock to their assets in each segment. The shocks are iid with a distribution that has support on  $[0, \epsilon_{max}]$  and no mass points.<sup>15</sup>
2. In each of  $T$  time periods, firms simultaneously allocate  $\delta$  units of investment, where  $K = \delta T$  (but there are no more random shocks).
3. Profits are realized.

I focus on regions of the parameter space in which, if both firms invest all  $K$  units simultaneously (that is,  $\delta = K$ ), then for the subgame that follows any possible value of the random shocks, there are multiple equilibria (one in which firm  $i$  focuses in the mainstream segment and firm  $j$  focuses in niche, and another in which the opposite happens), and each firm prefers the equilibrium in which it wins the mainstream segment. This is guaranteed to hold as long as  $\psi$  is sufficiently large.<sup>16</sup>

Given this set-up, the question is which firm will in fact win the mainstream segment. I show that making the investment increments sufficiently small guarantees that there is a unique equilibrium, and that even small differences in the initial random shocks determine which firm wins each segment. The intuition is that the firm that knows it would lose a race for the mainstream segment immediately gives up and focuses on the niche segment.

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<sup>15</sup>The assumption of no mass points ensures, for example, that the set of states for which both firms remain exactly the same has measure zero.

<sup>16</sup>I also require that  $\epsilon_{max}$  is not too large (otherwise a firm with a very strong draw in the mainstream segment could have a dominant strategy; or a firm with a very strong draw in the niche segment could prefer the equilibrium in which it wins this segment).

To see why this is true, denote the initial asset levels (following the random shocks) by  $(M_i, N_i, M_j, N_j)$ , and imagine both firms have invested in the mainstream segment for  $t$  time periods. Firm  $i$  then has a dominant strategy of continuing to focus in the mainstream segment if the following condition holds:

$$\int_{x=M_i+\delta t}^{M_i+K} F'(x) \left[ 1+2\psi[F(x)-F(M_j+K)] \right] > \alpha \int_{x=N_i}^{N_i+K-\delta t} F'(x) \left[ 1+2\psi[F(x)-F(N_j)] \right] \quad (14)$$

The left and right side of this inequality represents the incremental returns to firm  $i$  from allocating the remainder of its investment to the mainstream and niche segment, respectively, assuming firm  $j$  continues to focus in mainstream. If this inequality holds for firm  $i$  (but the analogous inequality does not hold for firm  $j$ ), I show that, from this point on, the firms are guaranteed to play an equilibrium in which firm  $i$  always focuses in mainstream and firm  $j$  always focuses in niche.

The rest of the proof depends on investment increments being sufficiently small, which is important for two reasons. First, small investment increments guarantees that (with probability approaching one) there would be a unique winner in the race for the mainstream segment. That is, either firm  $i$  or firm  $j$  would be the first to have a dominant strategy, and they would not both cross the threshold at which they have a dominant strategy in the same period. Second, small investment increments guarantees that a firm is willing to fight for one additional time period to guarantee that it will win the mainstream segment. This is, if firm  $i$  knows that inequality (14) will hold if both firms invest in mainstream for one more period, then firm  $i$ 's dominant strategy is to invest in the mainstream segment in the current period to guarantee victory. The same backwards induction logic then works for the previous period, and so on, guaranteeing that there is a unique equilibrium immediately after the random shocks are realized, and the firm that would win a race for the mainstream

segment focuses in this segment, while its competitor focuses in the niche segment. This is formally stated in Proposition 3 (see Appendix D for a formal proof).

**Proposition 3.** *As the size of the investment increments approaches zero, the investment subgame (which begins after the random shocks are realized) has a unique equilibrium with probability that approaches one.*

This result on equilibrium uniqueness in the “exploratory phase” bears some resemblance to results from deterministic patent races, in which a firm with a small lead is guaranteed to win (Reinganum 1989). However, unlike this earlier result, the result in the current paper does not depend on the type of winner-take-all payoffs observed in patent races. Rather, the result *does* depend crucially on increasing returns, which ensures that a firm that would be the first to have a dominant strategy in a race for the mainstream segment makes investments to ensure that it does, in fact, win this segment. See the proof in Appendix D for more details.

Also, note that Proposition 3 holds for arbitrarily small random shocks. As long as investment occurs in sufficiently small increments, even very small early perturbations determine which firm ends up dominating each segment. The reason is that a firm immediately starts to go after the niche segment once it knows that, in a race for the mainstream segment, its competitor will be the first to have a dominant strategy of always investing in this segment.

It is also possible to characterize which firm wins the mainstream segment as follows. Note that the left side of (14) is increasing in  $M_i$  and decreasing in  $M_j$ , and similarly, the right side is increasing in  $N_i$  and decreasing in  $N_j$ . This leads to the following corollary.

**Corollary 3.** *If there is a unique equilibrium in which firm  $i$  wins the mainstream segment for starting state  $(M_i, N_i, M_j, N_j)$ , then firm  $i$  also wins the mainstream segment for any starting state  $(M_i^U, N_i^D, M_j^D, N_j^U)$ , where  $M_i^U \geq M_i$ ,  $N_i^D \leq N_i$ ,  $M_j^D \leq M_j$ , and  $N_j^U \geq N_j$ .*

This corollary states that a firm becomes more likely to win the mainstream segment as its mainstream assets become higher and its niche assets become lower. One interesting implication is that, at the threshold between the regions where a firm wins and loses the mainstream segment, it would be better off if it could reduce its assets in the niche segment. Intuitively, this lack of an outside opportunity credibly commits the firm to fight for the mainstream segment, so the competitor is more likely to give up and go after the niche segment. Therefore, if one firm has an opportunity to publicly destroy some of its niche assets after the random draws are received, in some cases it would do so. For example, a software firm could spin-off one of its products to prevent its programmers from making further improvements to it, or a consumer products firm could use a new brand name to reduce the appeal of its niche product; either strategy could help entice a competitor to focus on this smaller segment. For similar reasons, a firm could prefer for its competitor to have incremental good luck in the niche segment. This is a significant difference between the current model and most standard (one-dimensional) dynamic investment games, in which a firm is always better off if its assets are higher and its competitor's assets are lower (e.g., Ericson and Pakes 1995).

### 3.2 Long-run phase

I now focus on what happens after firms have developed expertise in different segments. In particular, I derive conditions in which each firm is guaranteed to stay permanently focused on its respective segment. For this phase of the game, I assume



there are an infinite number of periods, and in each period each firm allocates one unit of investment, and also experiences a random shock in each segment. I also assume the distribution of the error terms has no mass points, which guarantees existence of a subgame perfect equilibrium to this infinite period stochastic game (Chakrabarti 1999; Mertens and Parthasarathy 2003).<sup>17</sup>

In general, deriving analytical results for infinite period dynamic investment games is a complicated problem, because of multiple equilibria, and also because counterintuitive phenomena such as “self-reinforcing joint-cost effects” can cause firms to change their investment behavior in certain regions of the state space for reasons that have nothing to do with the underlying model primitives (see Budd et al. 1993). For example, there could be equilibria in which each firm’s private incentives are to stay focused on its own strength, but in equilibrium each firm attacks the other’s strength, and if these attacks do not occur, then further attacks follow as a punishment.

However, I derive conditions that rule out this type of nonintuitive behavior, and guarantee that once firms become sufficiently different there is a unique subgame perfect equilibrium in which each firm stays focused on its strength. The only equilibrium refinement needed for this result is that I do not allow Pareto dominated equilibria. The proof proceeds by finding conditions in which the best possible outcome for both firms is that  $i$  keeps focusing on its strength and firm  $j$  keeps focusing on its strength. Once this is true, this is guaranteed to be an equilibrium, since neither firm would deviate from a strategy that produces the best possible outcome. Furthermore, any other equilibrium must be Pareto dominated. The conditions needed for this result are stated in Proposition 4.

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<sup>17</sup>Note that this game has an uncountable state space, compact action sets, and a countably infinite number of time periods. Given that the distribution of the error term has no mass points, such games are guaranteed to have a subgame perfect equilibrium, but not necessarily a stationary Markov perfect equilibrium (Chakrabarti 1999).

**Proposition 4.** *Once firms become sufficiently different, there is a unique Pareto optimal subgame perfect equilibrium in which they stay permanently focused on different segments if and only if all of the following hold:*

1. *Firms are not too patient.*
2. *Assets do not depreciate too quickly.*
3. *The maximum random shock is not too large.*

The conditions in this proposition are needed to ensure there is not an equilibrium in which the niche firm attacks the mainstream segment to drive its rival out of this segment. Note that this result does not restrict firms to stationary Markov perfect strategies. This is important because, in principle, a firm could change its strategy over time. In fact, the proof of Proposition 4 shows that, if firms are patient enough, from any starting point in the state space there is always an equilibrium in which the niche firm drives its rival out of the mainstream segment. Intuitively, if firms are patient enough, then a firm that is strong in the niche segment is willing to sacrifice short-run profits by shifting its investment to the mainstream segment, until it eventually builds up enough assets to drive its rival out of the mainstream segment; furthermore, this rival's best response to such a strategy is eventually to shift focus to the niche segment. Therefore, if firms are patient enough, then there are equilibria in which they swap segments over time. Similarly, if large enough random shocks are possible, then firms can swap segments just due to random chance, and if assets depreciate very quickly, then they can swap segments because the current state has little bearing on future decisions.

Thus, this model predicts that strategic attacks might occur when one or more of the restrictions in Proposition 4 fails. For example, Sony has recently driven Nintendo out of the segment of hard-cord video game players. A plausible explanation for

why Sony's attack on this segment was successful is that two of the restrictions of Proposition 4 did not hold. First, a large shock occurred due to the invention of CD-ROM technology, which provided a more efficient way of storing large amounts of information; Sony had expertise in this technology, which it used in its PlayStation video game system (see Edge 2009). Second, Sony was patient enough to continue investing in the PlayStation until Nintendo, who had previously been the dominant producer for hard core gamers, eventually decided to stop competing for this segment and to pursue more casual gamers with its Wii system (Ibid.; Mossberg and Boehret 2006).<sup>18</sup>

On the other hand, as long as all of the conditions in Proposition 4 are satisfied, strategic attacks cannot occur in equilibrium.<sup>19</sup> An interesting aspect of this result is that, even though firms do not cooperate and do not make punishment threats, in the long run they play the same strategies as if they cooperated in their investment decisions to maximize joint profits. Intuitively, the incentives of the firms are aligned because, for example, both firms want firm  $i$  to focus on the segment where it is strong and its competitor is weak. This result depends on the assumption that returns are increasing, or at least not too rapidly decreasing. If returns decrease rapidly enough (for example, if marginal returns approach infinity as assets approach zero), then each firm would invest in the segment where it is weak, and Proposition 4 would no longer hold.<sup>20</sup>

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<sup>18</sup>It is commonly believed that Japanese firms are more patient than American firms (e.g., Corbett 1987; Maskin 1995). If this is true, then the model in this paper predicts that Japanese firms are more likely to launch strategic attacks.

<sup>19</sup>Technically, I show that as long as none of the three parameter values named in the proposition is too extreme, strategic attacks cannot occur.

<sup>20</sup>A key difference between Proposition 4 and standard maximum differentiation results for the Hotelling model (D'Aspremont et al. 1979) is that the Hotelling result depends on the exogenous assumption that each firm produces a single product. By contrast, the current paper assumes each firm serves both segments, and allows them to allocate investment between the two. Another key difference is that the previous maximum differentiation result is for a one-shot game, and does not address the possibility of strategic attacks or firms swapping positions, which are allowed for (but ruled out) in the current paper.

### 3.3 Numerical example

In the interest of analytical tractability, the preceding results for the exploratory and long-run phases used different assumptions about the timing of the game. In particular, results for the exploratory phase assumed there were a finite number of periods with a single shock at the start of the game, whereas results for the long-run phase assumed there were an infinite number of periods with a new shock in each period. I now present a numerical example that combines both phases using the timing assumptions previously used in the long run phase (an infinite number of periods and a new shock in each period). The main difference is that, unlike in the analytical results for the exploratory phase, a firm that falls behind does not immediately give up and switch to the niche segment. Intuitively, because there is a new random shock in each period, such a firm knows it still has a chance to win the mainstream segment.

The numerical example uses the following parameter values:  $F(X_i, X_j) = (14 + X_i - X_j)^2$ ;  $(\epsilon_{m,i} - \epsilon_{m,j}) \sim U[-0.5, 0.5]$ ;  $\alpha = .6$ ;  $\gamma = .9$ ,  $\delta = .85$ ; and  $C = 20$ . Given this profit function, which is similar to the one derived from the Hotelling example in Appendix A, a firm's profits in a segment depend only on the difference between its own assets and its competitor's assets.<sup>21</sup>

Figure 2 presents firms' equilibrium policy functions.<sup>22</sup> The x-axis represents the

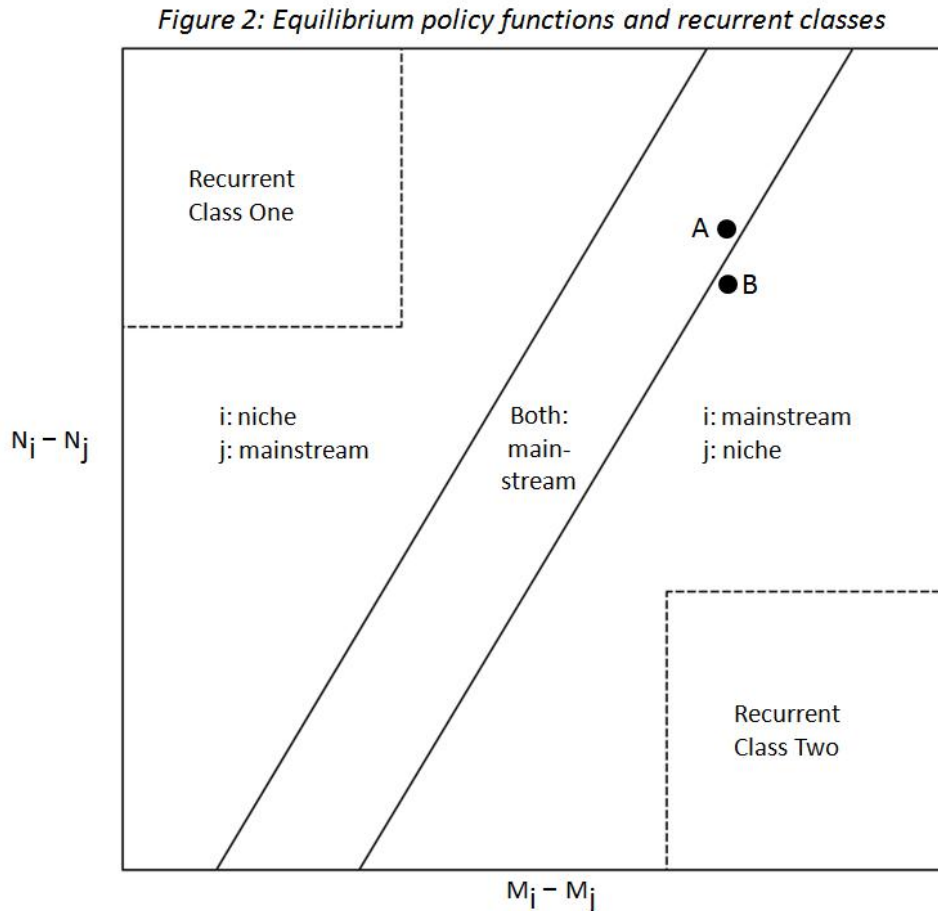
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<sup>21</sup>By expanding the quadratic term, we see that this function is consistent with the profit function used in the model presented above.

<sup>22</sup>I compute equilibrium policy functions over a 121x121 grid of values (with the asset difference in each segment ranging from -15 to 15) using value and policy function iteration (see Pakes and McGuire 1994) in Matlab. The algorithm takes about twenty-five minutes to converge. The following algorithm is used: (1) Initialize each firm's policy function such that it never invests in either segment. (2) Initialize each firm's value function by setting it equal to the firm's profits for the given state; then run fifty iterations to update each value function given the current policy functions. (Due to the constant asset depreciation rate, updating the value function requires estimates of the value of certain points that do not lie on the grid. These estimates are produced using interpolation from the neighboring points.) (3) Compute the optimal policy function for firm  $i$ . (4) Run twenty iterations to update firm  $j$ 's value function. (5) Compute the optimal policy function for firm  $j$ . (6) Run twenty iterations to update firm  $i$ 's value function. (7) Repeat steps 3 to 6 until convergence is achieved.

difference between firms' assets in the mainstream segment, and the y-axis represents the difference in the niche segment. At the center of the figure, both firms have similar asset levels in both segments. In this region, both firms focus their investment in the mainstream segment, so the only net movement is due to random shocks. However, once a firm falls sufficiently far behind in the mainstream segment, or builds up a sufficient lead in the niche segment, it starts investing in the niche segment instead.

This equilibrium has two recurrent classes. In Recurrent Class One, firm  $i$  always invests in the niche segment, and firm  $j$  always invests in the mainstream segment. In Recurrent Class Two, the opposite holds.



In some regions of the asset space, a firm can benefit from its competitor's success. For example, imagine assets are at point A, where firm  $i$  has a lead in both segments.

If firm  $j$  has good luck in the niche segment, so that assets move to point B, then firm  $j$  switches its investment to the niche segment. This makes firm  $i$  better off.<sup>23</sup>

If firms were sufficiently patient ( $\delta$  was very close to 1), then the game would always have multiple equilibria, and it would be possible for firms to move back and forth between the different “recurrent classes” by selecting different equilibria over time. However, for this example, such strategic changes are not possible since a firm starting off with a relatively strong position in the niche segment is not patient enough to drive its competitor out of the mainstream segment.

### 3.4 Investment deterrence

So far this section has assumed investment costs are low enough that each firm always invests at much as possible. I now present an example in which firms make sequential decisions, and the first mover has a chance to deter investment by the second. For this example, I assume firms play a simple two-period game in which firm  $i$  first allocates  $K$  units of investment and firm  $j$  then either allocates  $K$  units or invests nothing.

I assume preemption effects are strong enough that firms do not focus in the same segment, and the niche segment is large enough that firm  $j$  will invest there if firm  $i$  does not make any preemptive investments. Therefore, the only two possible outcomes are either firm  $i$  focuses all of investment in the mainstream segment, and firm  $j$  then focuses in niche; or firm  $i$  allocates positive investment to both segments and firm  $j$  then invests nothing.

To illustrate how increasing returns affects incentives for preemption, I let the parameter  $Z$  represent an index of the extent of increasing returns, and assume:

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<sup>23</sup>Similarly, at point A firm  $i$  would like to *reduce* its own niche assets to entice firm  $j$  to invest in the niche segment. Although the model in this paper does not allow for explicit asset destruction, it would be interesting to extend the model to allow for such decisions.

$$F(X) = \left[ \frac{X}{K} \right]^{1+Z} \quad (15)$$

Note that if a firm invests all  $K$  units in the same segment,  $F(K) = 1$  regardless of the value of  $Z$ . However,  $Z$  determines the extent of increasing returns, with constant returns for  $Z = 0$  and rapidly increasing returns for high values of  $Z$ . Proposition 5 states that the incumbent is less likely to deter investment when there are rapidly increasing returns.

**Proposition 5.** *Firm  $i$  deters investment for all  $Z$  below some cut-off value  $\widehat{Z}$ , that is, if returns do not increase too rapidly.*

The intuition is that, when returns increase rapidly, deterrence is unattractive because it requires the incumbent to shift investment from a segment with high returns to one with low returns. Therefore, contrary to the standard intuition that increasing returns is bad for late entrants, this example shows that increasing returns can make it easier for late entrants to capture a segment, since the incumbent stays focused on its core expertise.

To summarize, section 3 has extended the model to a more general profit function in which it is more profitable for a firm to invest where its competitor is weak. I showed that: (1) if investment occurs in sufficiently small increments, then increasing returns and small random shocks can solve the coordination problem of determining which firm wins the larger segment; (2) a firm would sometimes like to reduce its assets in a niche segment to entice its competitor to focus there; (3) under certain conditions, once firms become different, they each stay permanently focused on their own expertise, which implies that competition leads to the same long-run outcome as cooperation; and (4) rapidly increasing returns can benefit late entrants because incumbents have an incentive to stay focused rather than diversifying to deter entry.

## 4 Conclusion

This paper has developed a model in which firms dynamically compete for different market segments. As long as random shocks are not too large, assets do not depreciate too quickly, and firms are not too patient, this model predicts that each firm becomes permanently focused on the segment where it has the best initial luck.

In markets where some customer segments are more attractive than others, this model could explain which firm ends up winning each segment. An alternative interpretation is that firms invest in various skills or processes (instead of different segments). Under this interpretation, this model could explain why there are persistent differences in firms' performance along various dimensions. For example, firms in an industry often differ in their relative emphasis on marketing, engineering, and R&D (Griffin and Hauser 1992; Hauser and Zettelmeyer 1997).

A key managerial implication is that a firm trying to build a sustainable competitive advantage is more likely to succeed if the following two conditions hold: (1) the firm's point of differentiation from competitors is a dimension with increasing returns; (2) competitors have their own points of differentiation, which also have increasing returns. If these conditions hold, all firms have incentives to stay focused rather than attacking each other.

This model is consistent with evidence that focused firms tend to outperform unfocused firms (Wernerfelt and Montgomery 1988). In particular, a firm's decision to focus suggests that it has formed an advantage on a dimension with increasing returns, and that the firm is rationally exploiting its advantage on this dimension.

The model could also explain why there are persistent regional differences in the market shares of consumer packaged goods — for example, manufacturers founded on the West Coast continue to have higher market share there, whereas those founded on the East Coast continue to have higher market share there, even after many decades of



competition (Bronnenberg, Dhar, and Dubé 2007, 2009). The current paper predicts that these regional differences should be sustained by investments for which firms remain on the increasing returns part of the curve. For example, investments such as acquiring shelf-space and in-store displays need to be constantly replenished, and there are economies of scale to making such investments in a particular region; therefore, firms could stay persistently on the increasing returns part of the curve for the investments. On the other hand, investments such as convincing new stores in a region to carry a product would eventually have decreasing returns, and so firms should eventually diversify and enter new regions. Thus, this model suggests that differences in how firms allocate the former type of investment (in shelf space, displays, etc.) are more likely to sustain persistent differences in regional market shares.

The results in this paper depend on several key assumptions. First, the paper assumes that there are increasing returns to successful investment in a segment. Of course, in some cases there are *decreasing* returns to investment, for example, if customers' utility is a concave function of a particular quality dimension. However, as long as there are increasing returns to investment in at least one of the assets required to serve a segment, the results in this paper apply to investment in this asset. As discussed in Appendix B, previous literature suggests many possible reasons for increasing returns.

This paper also assumes that a firm's profits are more sensitive to investment in a segment where competitors are relatively weak. Although this is true under many models of competition, there are also factors that encourage firms to invest in segments where competitors are *strong*, including technological spillovers and economies of scale from a shared labor pool (Ellison et al. 2008). The results in this paper hold as long as the competitive forces that favor differentiation are stronger than these forces that

favor agglomeration.

Finally, the model assumes there is a constraint on a firm's total per-period investment, for example, because employees have limited time and attention for participating in investment. One possible extension would be to allow firms to invest more as they grow larger. If firms grow so large that they no longer face a binding constraint on investment, or no longer have increasing returns to their core activity, then they should start to diversify. Thus, this model predicts that small to medium sized firms stay focused, whereas large firms are more likely to attack each other.

Future research could extend the model to incorporate several important phenomena that were not addressed in the current paper. For example, if some assets can be used in multiple segments, then success in one segment can make investment in another related segment more profitable (Wernerfelt 1984). In addition, firms often face uncertainty over exogenous factors such as demand for a product (Hitsch 2006), future customer preferences (Bhattacharya, Krishnan, and Mahajan 1998), the value of a new technology (Zhu and Weyant 2003), or the firm's underlying efficiency (Jovanovic 1982); these factors can lead firms to exit the market or reposition a product as new information arrives. Finally, new technologies (known as "radical" or "disruptive" innovations) can reduce or destroy the value of assets based on old technologies (Christensen 1997; Chandy and Tellis 2000; Sood and Tellis 2005; Tellis 2006). It would be interesting to extend the model to explore how these phenomena affect optimal investment behavior.

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## Appendix A: Hotelling Model Example

This appendix shows that the models in sections 2 and 3 are consistent with a model in which competition in each segment is represented by a different Hotelling line.

Assume firms  $i$  and  $j$  compete in two segments. Each firm sells a separate product in each segment, and customers will only purchase a product targeted toward their particular segment. In each segment, firms are (exogenously) located at opposite ends of a Hotelling line of length  $L$ . In the mainstream segment, customers have mass 1, and in the niche segment they have mass  $\alpha$ , where  $\alpha \leq 1$ .

Firms invest in assets that improve their quality on a vertical dimension, which is orthogonal to the Hotelling line. Define  $M_{i,t}$  and  $N_{i,t}$  as firm  $i$ 's assets devoted to the mainstream and niche segment, respectively, at time  $t$ . A mainstream customer located a distance  $d$  from firm  $i$  receives utility  $U - d + F(M_{i,t}) - P_{i,m,t}$  from purchasing firm  $i$ 's product, where  $F$  is a function that translates assets into utility (I assume  $F$  is continuously differentiable and  $F' > 0$ ), and  $P_{i,m,t}$  is firm  $i$ 's price in the mainstream segment.

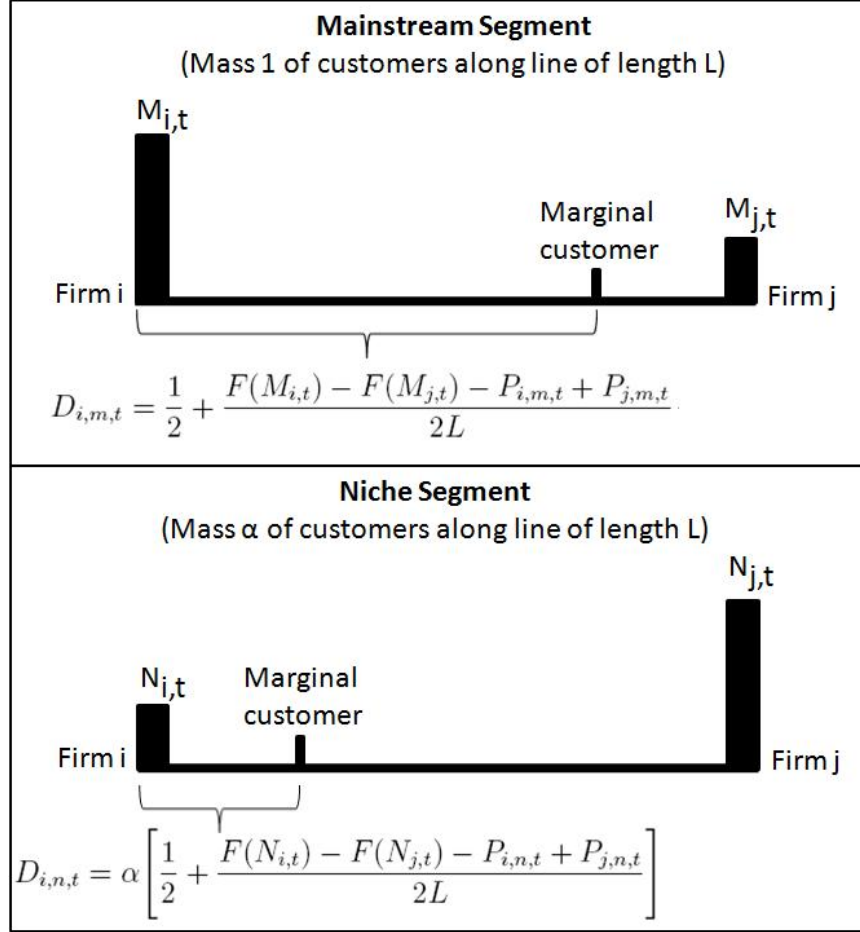
Assuming both firms have positive demand, and the market is covered, at any time  $t$ , demand for firm  $i$ 's product in each segment is given by:

$$D_{i,m,t} = \frac{1}{2} + \frac{F(M_{i,t}) - F(M_{j,t}) - P_{i,m,t} + P_{j,m,t}}{2L} \quad (16)$$

$$D_{i,n,t} = \alpha \left[ \frac{1}{2} + \frac{F(N_{i,t}) - F(N_{j,t}) - P_{i,n,t} + P_{j,n,t}}{2L} \right] \quad (17)$$

This set-up is depicted graphically in Figure A-1.

Figure A-1: An example with Firm  $i$  stronger in the mainstream segment, and Firm  $j$  stronger in the niche segment



Section 2 focuses on cases in which there are no interactions in firms' investment decisions. One case in which this holds is if prices are fixed, for example, because they are set through tacit collusion, or because customers view prices below some minimum level as a signal of low quality. If all prices are fixed at price  $\hat{P}$ , and we assume (without loss of generality) marginal costs are zero, then firm  $i$ 's profits in each segment are given by:

$$\pi_{i,M,t} = \hat{P} \left[ \frac{1}{2} + \frac{F(M_{i,t})}{2L} - \frac{F(M_{j,t})}{2L} \right] \quad (18)$$

$$\pi_{i,N,t} = \alpha \hat{P} \left[ \frac{1}{2} + \frac{F(N_{i,t})}{2L} - \frac{F(N_{j,t})}{2L} \right] \quad (19)$$

Note that there is no interaction between the two firms' assets in these equations.

Another case in which firms make independent investment decisions is if prices are set endogenously, but differentiation along the horizontal dimension (the Hotelling line) is very large relative to potential differences on the vertical dimension (firms' asset levels). To see why this is true, I solve for firm  $i$ 's equilibrium profits when prices are set endogenously:

$$\pi_{i,M,t}^* = \frac{L}{2} + \frac{F(M_{i,t}) - F(M_{j,t})}{3} + \frac{[F(M_{i,t}) - F(M_{j,t})]^2}{18L} \quad (20)$$

$$\pi_{i,N,t}^* = \alpha \left[ \frac{L}{2} + \frac{F(N_{i,t}) - F(N_{j,t})}{3} + \frac{[F(N_{i,t}) - F(N_{j,t})]^2}{18L} \right] \quad (21)$$

Note that firms' assets interact with each other only through the last term in each expression. Because this term has an  $L$  in the denominator, it becomes negligible if the Hotelling line is sufficiently long. In this case, we can approximate firm  $i$ 's equilibrium profits by:

$$\pi_{i,M,t}^* \approx \frac{L}{2} + \frac{F(M_{i,t}) - F(M_{j,t})}{3} \quad (22)$$

$$\pi_{i,N,t}^* \approx \alpha \left[ \frac{L}{2} + \frac{F(N_{i,t}) - F(N_{j,t})}{3} \right] \quad (23)$$

The intuition for this result is as follows. When firms are already very different on a horizontal dimension ( $L$  is large), differentiation along a vertical dimension has a trivial effect on equilibrium prices. Technically, this effect has order  $O(\frac{1}{L})$ . Investments in the vertical dimension also have a small effect on equilibrium demand, also of order  $O(\frac{1}{L})$ . However, because equilibrium prices increase linearly with  $L$ , these changes in demand still have a first order effect on profits. Thus, a firm's equilibrium profits are increasing in its own assets, and decreasing in its competitor's assets, but the *difference* now has a negligible effect.

Finally, note that as long as the Hotelling line is not too long, equilibrium profits with endogenous prices fit the assumptions of section 3, and it is more profitable for a firm to invest in a segment where its competitor is weak. Intuitively, if a firm has larger assets than its competitor, it has higher equilibrium demand, and so incremental improvements in quality allow it to raise its price and collect incremental profits from a larger number of customers.

## Appendix B: Key assumptions

This section provides explanations for the three key assumptions of the model.

### Assumption 1: Increasing returns

I assume firms invest in an asset with increasing returns. This “asset” could represent the accumulated results of investment in product quality, new product development, cost reduction, distribution, or production capacity. Previous literature suggests that increasing returns to such investments are a widespread phenomenon.

*Technology learning curves.* In R&D-intensive industries, past success can improve a firm’s R&D capability, so that additional investments are more productive (Henderson and Cockburn 1996).<sup>24</sup> Similarly, in high tech industries, use of an existing innovations can improve a firm’s ability to make use of future innovations (Grenadier and Weiss 1997).

*Word-of-mouth.* Success in a market can lead to stronger word-of-mouth effects, since it attracts a large customer base who will then inform other customers of how well a firm performs (Rob and Fishman 2005).

*Reputation effects.* Product failures can reveal a firm’s lack of ability, making it impossible to fool customers with additional investments in quality (Kreps and Wilson 1982; Diamond 1989). Cabral and Hortaçsu (2008) present evidence that this causes eBay sellers to reduce investments in quality after their first negative review.

*Network effects.* For products with network effects, there are increasing returns to attracting new customers, since each additional customer benefits all others (Arthur 1989).<sup>25</sup>

*Confirmatory bias.* Customers who have positive expectations spend more time

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<sup>24</sup>Empirical studies of whether there are increasing or decreasing returns to R&D have produced mixed results (Griliches 1990). In the case of pharmaceuticals, there is evidence of increasing returns (Henderson and Cockburn 1996).

<sup>25</sup>However, if network compatibility is endogenous, firms have an incentive to invest more in technologies where competitors are *strong* so that the competitor maintains product compatibility (Chen et al. 2008).

reading any favorable information about a product (Boulding et al. 1999). This implies that it is especially valuable to invest in “excitement” attributes (see Kano et al. 1984) in segments where a firm has previously performed well.<sup>26</sup>

*Customer feedback.* Existing customers can provide helpful user feedback on a firm’s new products, for example, by acting as beta users for new software versions (Hörner 2004).

*Geographic roll-out.* Retailers are more likely to adopt a product if competitors in the same region carry it, and if it is widely distributed in the region (Bronnenberg and Mela 2004). This implies that it is easier for manufacturers to gain additional distribution in regions where they already have a foothold.

*Division of labor.* In *The Wealth of Nations*, Adam Smith (1776) presents evidence that workers in a pin factory become more productive when they focus on narrowly specialized tasks. This implies that manufacturers should invest in large-scale production capacity to allow for narrow specialization.

*Manufacturing learning curves.* Aircraft manufacturers have long recognized that the amount of labor needed to produce each additional airplane declines as the number of planes produced increases (Asher 1956). Many other manufacturing activities also exhibit this type of learning curve effect (Argote and Epple 1990).

## **Assumption 2: Constraint on investment**

The model assumes that in each period a firm has a fixed amount of investment to allocate across all segments. This could be true for any of the following reasons:

*Capital market imperfections.* Asymmetric information about the quality of investment opportunities or how capital is used might constrain firms’ ability to raise capital (Jensen and Meckling 1976; Myers and Majluf 1984).

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<sup>26</sup>On the other hand, customers who have negative prior expectations spend more time reading negative information (Boulding et al. 1999). This implies that it is especially important to correct problems in segments where a firm has performed *poorly* in the past. Thus, when customers exhibit confirmatory bias, there can be increasing or decreasing returns, depending on the type of investment.

*Ownership incentives.* If firms allocate ownership of machinery or other assets as a means of providing investment incentives, this forces them to make trade-offs among investment incentives for various activities (Grossman and Hart 1986; Hart and Moore 1990; Simester and Wernerfelt 2005; Gibbons, Holden, and Powell 2009).

*Limited time and attention.* Because participating in investment depends on tacit knowledge of firms' existing processes, and transferring this knowledge to new employees is costly (Teece et al. 1997), investment is often constrained due to employees' limited time and attention.

### **Assumption 3: Higher returns where competitors are weak**

I assume a firm's profits are more sensitive to its assets in segments where its competitors are weak. This holds under many models of competition (Villas-Boas 1993; Athey and Schmutzler 2001; Doraszelski and Markovich 2007). The intuition is that, if the competitor is strong and the focal firm becomes strong, both earn duopoly profits; whereas if the competitor is weak and the focal firm becomes strong, the latter earns something closer to monopoly profits.

## **Appendix C: Increasing asset sensitivity**

Increasing returns and increasing asset sensitivity are similar concepts, but not necessarily equivalent.<sup>27</sup> Both can give firms an incentive to continue investing in segments where they have previously performed well. However, it is possible to build models of increasing asset sensitivity in which firms do not focus their investment, either because the profit function is concave, or because expected asset growth is a concave function of investment in a particular period. This appendix develops a model of increasing asset sensitivity that assumes both of these functions are either linear or convex, which ensures that firms want to focus their investment.

Profits have the same functional form as before, except that I replace the strict inequality  $F'' > 0$  with the weak inequality  $F'' \geq 0$ , allowing for *constant* returns to assets.

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<sup>27</sup>One special case in which they are equivalent is if profits are linear in assets, and assets evolve as follows:  $M_{t+1} = f(\gamma f^{-1}(M_t) + m_t + \epsilon_{t+1})$ , where  $f', f'' > 0$ . It is straightforward to show that this model of increasing asset sensitivity is equivalent to the model of increasing returns used in the current paper.



Assets evolve as follows:

$$M_{i,t+1} = \gamma M_{i,t} + f(M_{i,t})g(m_{i,t}) + \epsilon_{m,i,t+1} \quad (24)$$

$$N_{i,t+1} = \gamma N_{i,t} + f(N_{i,t})g(n_{i,t}) + \epsilon_{n,i,t+1} \quad (25)$$

where  $f', g' > 0$  and  $f'', g'' \geq 0$ . I also assume  $f, g$ , and  $\gamma$  are such that assets are bounded.

Given this set-up, I prove the following.

**Proposition 6.** *In this model of increasing asset sensitivity, the optimal policy function always involves either zero investment or investing one unit in a single segment, except at a set of points of measure zero.*

The proof of this proposition (which appears in Appendix D) shows that the value function is weakly (not strictly) convex. However, even if the value function is linear in assets, the firm prefers to invest in a segment where it has the larger asset stock, since assets are more sensitive to investment in such a segment. This leads to the following corollaries, which are exactly the same as the corollaries from the increasing returns model.

**Corollary 4.** *If a firm invests in the mainstream segment when its assets levels are  $(M, N)$ , then it also invests in the mainstream segment for all states  $(M_H, N_L)$ , where  $M_H \geq M$  and  $N_L \leq N$ . An analogous result holds for investment in the niche segment.*

**Corollary 5.** *For  $\alpha = 1$ , each firm either invests in the segment where it has greater assets, or does not invest in either segment.*

## Appendix D: Proofs

**Proof of Lemma 1:** I first show convexity in  $M_{i,t}$ . That is, I will show that for any state  $(M, N)$ :

$$V(M, N) < \frac{1}{2}V(M + \omega, N) + \frac{1}{2}V(M - \omega, N) \quad (A1)$$

Let  $\widehat{V}(X, Y)$  denote the value of starting at state  $(X, Y)$ , but using the policy that would be optimal starting from state  $(M, N)$ . Also, let  $\widetilde{M}_{i,t}$  denote a particular

realization of  $M_{i,t}$  starting from state  $(M, N)$  and following the optimal policy for  $t$  periods:

$$\begin{aligned}
& \frac{1}{2}V(M + \omega, N) + \frac{1}{2}V(M - \omega, N) - V(M, N) \\
& \geq \frac{1}{2}\widehat{V}(M + \omega, N) + \frac{1}{2}\widehat{V}(M - \omega, N) - V(M, N) \\
& = E \left[ \sum_{t=0}^{\infty} \delta^t \left( \frac{1}{2}F(\widetilde{M}_{i,t} + \omega\gamma^t) + \frac{1}{2}F(\widetilde{M}_{i,t} - \omega\gamma^t) - F(\widetilde{M}_{i,t}) \right) \right] \\
& > 0
\end{aligned} \tag{A2}$$

The proof for  $N_{i,t}$  is similar. To show that the value function is increasing, one can follow a similar procedure to show that  $V(M, N) < V(M + \omega, N)$ . QED

**Proof of Lemma 2:** Let  $H$  denote firm  $i$ 's objective function at time  $t$ :

$$H(m, n) = \delta E[V(\gamma M_{i,t} + m + \epsilon_{m,i,t+1}, \gamma N_{i,t} + n + \epsilon_{n,i,t+1})] - Cm - Cn \tag{A3}$$

Since convexity is preserved under the expectations operator, we have from Lemma 1 that  $H$  is strictly convex in  $m$  and  $n$ .

Assume there is an optimal policy  $(m^*, n^*)$  with  $m^* > 0$ . By optimality, we have  $H(m^*, n^*) \geq H(0, n^*)$ . Therefore, strict convexity implies that  $H(m^* + \omega, n^*) > H(m^*, n^*)$  for any positive  $\omega$ , so we have a contradiction unless we are bound by the constraint  $m^* + n^* = 1$ . Likewise,  $n^* > 0$  implies that  $m^* + n^* = 1$ . The result follows. QED

**Proof of Lemma 3:** This lemma is equivalent to saying that, for any  $M_H > M_L$  and  $N_H > N_L$ ,

$$V(M_H, N_H) + V(M_L, N_L) \leq V(M_H, N_L) + V(M_L, N_H) \tag{A4}$$

It suffices to show that we can design policies starting from  $(M_H, N_L)$  and  $(M_L, N_H)$  such that the sum of the earnings generated by the policies is at least as high as the sum of earnings generated from optimal policies starting from  $(M_H, N_H)$  and  $(M_L, N_L)$ .

The following policies satisfy this criterion. Let  $\tilde{m}_{H,t}$  and  $\tilde{m}_{L,t}$  denote optimal investments in the mainstream segment starting from states  $(M_H, N_H)$  and  $(M_L, N_L)$ , respectively. Define  $\tilde{n}_{H,t}$  and  $\tilde{n}_{L,t}$  similarly for the niche segment. Starting from state  $(M_H, N_L)$ , always set  $m_{i,t} = \max(\tilde{m}_{H,t}, \tilde{m}_{L,t})$  and  $n_{i,t} = \min(\tilde{n}_{H,t}, \tilde{n}_{L,t})$ . Similarly, from  $(M_L, N_H)$ , always set  $m_{i,t} = \min(\tilde{m}_{H,t}, \tilde{m}_{L,t})$  and  $n_{i,t} = \max(\tilde{n}_{H,t}, \tilde{n}_{L,t})$ .

To see why these policies satisfy the required criterion, note that: (1) In each period the sum of investments in the mainstream segment starting from  $(M_H, N_L)$  and  $(M_L, N_H)$  is the same as the sum under the optimal policies starting from  $(M_H, N_H)$  and  $(M_L, N_L)$ . (2) This implies that the sum of total assets in the mainstream segment is always the same in both cases. (3) Starting from state  $(M_H, N_L)$ , assets in the mainstream segment are always greater than or equal to the maximum of assets in the mainstream segment under optimal policies starting from  $(M_H, N_H)$  and  $(M_L, N_L)$ , whereas starting from  $(M_L, N_H)$  they are less than or equal to the minimum. (4) Since the profit function is convex in assets, (2) and (3) imply that the sum of earnings from the mainstream segment starting from  $(M_H, N_L)$  and  $(M_L, N_H)$  is greater than or equal to the sum starting from  $(M_H, N_H)$  and  $(M_L, N_L)$ .

A similar argument holds for the niche segment. QED

**Proof of Proposition 1:** Again, define  $H(m, n)$  as firm  $i$ 's objective function at time  $t$ . From Lemma 2, we know that any optimal policy must set  $(m^* + n^*)$  equal to 0 or 1. To complete the proof, I assume there is an optimal policy with  $m^*$  and  $n^*$  both positive, and show a contradiction.

By optimality of  $(m^*, n^*)$ , for any sufficiently small, positive  $\omega$ , we have:

$$H(m^* + \omega, n^* - \omega) \leq H(m^*, n^*) \quad (\text{A5})$$

Subtracting the same term from both sides:

$$H(m^* + \omega, n^* - \omega) - H(m^*, n^* - \omega) \leq H(m^*, n^*) - H(m^*, n^* - \omega) \quad (\text{A6})$$

Applying Lemma 3 to both sides, we have:

$$H(m^* + \omega, n^*) - H(m^*, n^*) \leq H(m^* - \omega, n^*) - H(m^* - \omega, n^* - \omega) \quad (\text{A7})$$

By the strict convexity of the value function (Lemma 1), this implies:

$$H(m^*, n^*) - H(m^* - \omega, n^*) < H(m^* - \omega, n^* + \omega) - H(m^* - \omega, n^*) \quad (\text{A8})$$

Adding the same term to both sides, we have:

$$H(m^*, n^*) < H(m^* - \omega, n^* + \omega) \quad (\text{A9})$$

This contradicts the optimality of  $(m^*, n^*)$ . Therefore, there cannot be an optimal policy where  $m^*$  and  $n^*$  are both positive. The result follows. QED

**Proof of Corollary 1:** Follows immediately from Proposition 1 and Lemmas 1 and 3. QED

**Proof of Corollary 2:** Given  $\alpha = 1$ , by symmetry  $V(M, N) = V(N, M)$  for any  $M$  and  $N$ . Suppose it is optimal to invest in the “mainstream” segment at a state  $(M, N)$ , where  $M < N$ . By symmetry, this implies that it is optimal to invest in the “niche” segment at state  $(N, M)$ . However, this contradicts Corollary 1. QED

**Proof of Proposition 2:** I begin by assuming  $\alpha = 1$ , and later relax this assumption. I will show that it is possible to choose a value of  $C$  such that the optimal policy is as follows:

1. At state  $(0, 0)$ , the firm invests nothing.
2. For any state where assets in either segment are greater than or equal to some small positive value  $\omega$ , the firm invests in the segment where it has greater assets.
3. I do not specify how the firm behaves when assets in both segments lie in  $(0, \omega)$ . However, note that Corollary 2 guarantees a firm will only invest in the segment where it has greater assets, if it invests at all.

It suffices to show that we can choose a value of  $C$  such that there is no single profitable deviation from this policy. I must show that the firm does not want to invest in either segment at state  $(0, 0)$ :

$$\delta \left( E[V(0 + 1 + \epsilon_m, 0 + \epsilon_n)] - E[V(0 + \epsilon_m, 0 + \epsilon_n)] \right) < C \quad (\text{A10})$$

I must also show that a firm wants to invest in one of the segments at any state  $(X, X)$ , where  $X \geq \omega$ .

$$\delta \left( E[V(\gamma X + 1 + \epsilon_m, \gamma X + \epsilon_n)] - E[V(\gamma X + \epsilon_m, \gamma X + \epsilon_n)] \right) > C \quad (\text{A11})$$

If (A11) holds, Corollary 1 guarantees the firm invests in the mainstream segment for all points  $(X_H, X_L)$  where  $X_H \geq X$  and  $X_L \leq X$ . Also, since  $\alpha = 1$ , the firm would be willing to invest in the niche segment, and it invests in the niche segment at analogous states  $(X_L, X_H)$ .

I need to show that the left side of (A11) is greater than the left side of (A10). This is equivalent to showing.

$$E[V(\gamma X + 1 + \epsilon_m, \gamma X + \epsilon_n)] - E[V(0 + 1 + \epsilon_m, 0 + \epsilon_n)] > E[V(\gamma X + \epsilon_m, \gamma X + \epsilon_n)] - E[V(0 + \epsilon_m, 0 + \epsilon_n)] \quad (\text{A12})$$

Starting from either of the terms on the left of (A12), as long as the firm continues investing in the mainstream segment, mainstream assets are guaranteed to be larger than  $\omega$  and larger than niche assets. Therefore, given the proposed policy function, for these two terms I can assume the firm invests in the mainstream segment in all future periods.

It is possible to find an upper bound on the right side of (A12) by assuming that, if the firm starts at  $(0 + \epsilon_m, 0 + \epsilon_n)$ , it follows the policy that would be optimal starting at  $(\gamma X + \epsilon_m, \gamma X + \epsilon_n)$ . Note that this upper bound is guaranteed to be less than the left side of (A12) as long as the firm invests in the mainstream segment, or in neither segment, from the terms on the right side. Therefore, the only way the right side could be larger is if the firm starts investing in the niche segment. Assume the random components of assets are most favorable to niche investments, that is, assume  $(\epsilon_m, \epsilon_n) = (0, \epsilon_{max})$ . I would then like to show that the following holds:

$$V(\gamma X + 1, \gamma X + \epsilon_{max}) - V(1, \epsilon_{max}) > V(\gamma X, \gamma X + \epsilon_{max}) - V(0, \epsilon_{max}) \quad (\text{A13})$$

From the terms on the left, the firm always invests in the mainstream segment, but from the terms on the right, it always invests in the niche segment. Furthermore, by symmetry across segments (given  $\alpha = 1$ ), and convexity of the profit function, the value of the left side of the equation is strictly larger than the value of the right side. It can also be shown that the value function, and returns to investment, are continuous in  $C$ . Therefore, this guarantees that there is a range of values of  $C$  for

which (A11) and (A10) are both satisfied, which implies that the proposed policy function is optimal.

Given this policy function, a firm with no assets does not invest in either segment, but due to random chance, the firm eventually reaches a point from which it always invests in the same segment. Because the firm can become locked into either segment, long-run behavior is path dependent. Also, given depreciation rate  $\gamma$ , if a firm repeatedly invests in a segment, with probability one its assets in that segment eventually become confined to a range  $[\frac{1}{1-\gamma}, \frac{1+\epsilon_{max}}{1-\gamma}]$ . Meanwhile, assets in the other segment become confined to the range  $[0, \frac{\epsilon_{max}}{1-\gamma}]$ . Given  $\epsilon_{max} < 1$ , the firm always has greater assets in the segment where it focuses than in the segment where it does not.

Because the profit function and its first two derivatives (with respect to the asset variables) are continuous in  $M_i$ ,  $N_i$ , and  $\alpha$ , the value function and its first two derivatives are also continuous in these variables, and so are the expected investment returns given by (7) and (8). Therefore, as long as  $\alpha$  is sufficiently close to one, investment follows the same basic pattern, and long-run behavior is path dependent. QED

**Proof of Proposition 3:** I first establish that, if the investment increments are sufficiently small, and if both firms continue investing in the mainstream segment, then one firm eventually has a dominant strategy of investing in this segment. To see why this is true, note that:

$$\frac{\partial \pi_{i,M,t}}{\partial M_{i,t}}(K, K) = F'(K) [1 + 2\psi(F(K) - F(K))] = F'(K) \quad (\text{A14})$$

$$\frac{\partial \pi_{i,N,t}}{\partial N_{i,t}}(0, 0) = \alpha F'(0) [1 + 2\psi(F(0) - F(0))] = \alpha F'(0) \quad (\text{A15})$$

By continuity, as long as the random shocks are sufficiently small, and both firms are sufficiently close to being fully invested in the mainstream segment, each firm has a dominant strategy of investing in this segment.

Let  $t_i$  represent the minimum number of periods such that firm  $i$  has a dominant strategy if both firms invest  $\delta$  units in the mainstream segment for  $t_i$  periods, and let  $t_j$  represent the analogous quantity for firm  $j$ . Again by continuity, as the investment increments are made sufficiently small, given that the random shocks have no mass points, the probability that  $t_i = t_j$  approaches zero. Therefore, the probability that one firm will “win the race” and be the first to have a dominant strategy approaches

one.

I next show that, as long as  $\psi$  is sufficiently large, for any value of  $t$ , at the asset state  $(\delta t, \delta t, 0, 0)$  the best possible outcome for either firm is for it to invest all of its remaining capital in the mainstream segment, and for its competitor to invest all of its remaining capital in the niche segment. That is, I will show:

$$\pi_{i,M,t}(K, \delta t) + \pi_{i,N,t}(0, K - \delta t) \geq \pi_{i,M,t}(\delta t + x, \delta t + y) + \pi_{i,N,t}(K - \delta t - x, K - \delta t - y) \quad (\text{A16})$$

for all  $x \in [0, K - \delta t]$  and  $y \in [0, K - \delta t]$  (and in fact the inequality is strict unless both expressions are exactly the same). To see why, note that the component of profits that results from the differentiation term is as large as possible on the left side of this inequality, and given that  $\psi$  is sufficiently large, differentiation is important enough that this must be true in the best possible outcome for either firm. Furthermore, given that the mainstream segment is larger, it is clear that this outcome is better for firm  $i$  than the outcome in which it is the one that invests in the niche segment.

Thus, the best possible outcome for firm  $i$  is that it focuses its remaining investment in mainstream and firm  $j$  focuses its remaining investment in the niche segment. By continuity, this is also true once the random shocks are taken into account. Therefore, once a firm wins the race and has a dominant strategy of investing in mainstream, it will always invest in this segment to ensure that it continues to have a dominant strategy. Knowing this, its competitor will always invest in the niche segment.

Finally, imagine a firm knows it will have a dominant strategy in one more period if both firms continue investing in the mainstream segment. As long as the investment increments are sufficiently small, the firm's dominant strategy is then to focus in the mainstream segment in the current period to ensure that it wins this segment. By backward induction, the firm that would eventually win a race for the mainstream segment immediately focuses in this segment, and the other firm immediately focuses in the niche segment. QED

**Proof of Corollary 3:** This follows immediately from the proof of Proposition 3 and the observation of comparative statics on (14).

**Proof of Proposition 4:** I first show that, given the conditions of the proposition, the best possible outcome for both firms is if each one stays permanently focused on

its respective strong segment. Note that the derivatives of firm  $i$ 's profits with respect to its own assets are:

$$\frac{\partial \pi_{i,M,t}}{\partial M_{i,t}} = F'(M_{i,t}) [1 + 2\psi(F(M_{i,t}) - F(M_{j,t}))] \quad (\text{A17})$$

$$\frac{\partial \pi_{i,N,t}}{\partial N_{i,t}} = \alpha F'(N_{i,t}) [1 + 2\psi(F(N_{i,t}) - F(N_{j,t}))] \quad (\text{A18})$$

Next, note that the derivatives of firm  $i$ 's profits with respect to its *competitor's* assets are:

$$\frac{\partial \pi_{i,M,t}}{\partial M_{j,t}} = F'(M_{j,t}) [-1 - 2\psi(F(M_{i,t}) - F(M_{j,t}))] \quad (\text{A19})$$

$$\frac{\partial \pi_{i,N,t}}{\partial N_{j,t}} = \alpha F'(N_{j,t}) [-1 - 2\psi(F(N_{i,t}) - F(N_{j,t}))] \quad (\text{A20})$$

Assume without loss of generality firm  $i$  is much stronger in the mainstream segment and firm  $j$  is much stronger in the niche segment. It is clear that firm  $i$ 's marginal returns are higher from investing in the mainstream segment. Furthermore, because the second term in (A20) can be made arbitrarily close to zero (or even positive) as firm  $j$  builds up a sufficiently large lead in the niche segment, firm  $i$  would prefer for firm  $j$  to focus on that segment.

Thus, when differences are large enough, each firm has higher marginal returns in the segment where it is stronger, and suffers less from incremental investments by its competitor in the segment where the competitor is stronger. As long as the maximum random shock is not too large, and the depreciation rate is not too fast, this will continue to be true as long as each firm focuses on its respective segment of strength. Now consider an alternative in which at least one firm deviates from these strategies. The first deviation that occurs makes both firms worse off, at least in the short run, and as long as firms are not too patient, they would both prefer an outcome in which this deviation does not occur.

Therefore, each firm staying focused on its strong segment must be a subgame perfect equilibrium because neither firm has an incentive to deviate from the best possible outcome. Furthermore, any other subgame perfect equilibrium must be Pareto dominated.

It is clear that the restriction on the random shocks is necessary, as otherwise firms' respective strengths would change just due to random fluctuations; and the restriction



on the depreciation rate is necessary because at the extreme of total depreciation, the current state does not affect future decisions.

The restriction on the discount factor is necessary for the following reason. Imagine that firm  $j$  plays a strategy in which it always focuses on the mainstream segment. First, consider the case in which the expected long-run average profits to firm  $i$  are higher if it also focuses in the mainstream segment instead of the niche segment. In this case, if firm  $i$  is sufficiently patient, it will leave the niche segment and join firm  $j$  in the mainstream segment. Next, consider the case in which firm  $i$ 's long run profits would be higher if it always focused on the niche segment. If both firms are sufficiently patient, then from any starting point in the state space firm  $i$  always prefers an outcome in which it wins the mainstream segment and firm  $j$  focuses on the niche segment. Furthermore, if firm  $i$  plays a strategy of always investing in mainstream, firm  $j$ 's best response is to switch to the niche segment in finite expected amount of time. Therefore, there are always multiple equilibria from any starting point in the state space, and either firm could win the mainstream segment. The restriction on the discount factor is necessary to rule this out. QED

**Proof of Proposition 5:** I assume that parameters are in the range where, if firm  $i$  focuses all  $K$  units in the mainstream segment, firm  $j$  will focus all  $K$  units in the niche segment. So the only question is whether firm  $i$  would like to allocate some investment toward the niche segment to deter firm  $j$  from investing.

To deter firm  $j$  from investing, firm  $i$  must allocate  $N_i^*$  units to the niche segment, where  $N_i^*$  is the smallest value such that:

$$\pi_{j,N,t}(N_i^*, K) - \pi_{j,N,t}(N_i^*, 0) \leq CK \quad (\text{A21})$$

Note that  $N_i^*$  is increasing in  $Z$ . Intuitively, deterrence requires larger investment in the niche segment when there are rapidly increasing returns (since returns are low at first).

Firm  $i$  will want to deter investment if:

$$\pi_{i,M,t}(K - N_i^*, 0) + \pi_{i,N,t}(0, N_i^*) \geq \pi_{i,M,t}(K, 0) + \pi_{i,N,t}(0, K) \quad (\text{A22})$$

Note that the left side of this equation is decreasing in  $Z$ . Therefore, increases in  $Z$  make deterrence less attractive, both because it requires a larger shift of investment to the niche segment, and because this shift away from the mainstream segment becomes

more costly. QED

**Proof of Proposition 6:** This proof follows the same outline as the proof of Proposition 1. I first show weak convexity in  $M_{i,t}$ . That is, I will show that for any state  $(M, N)$ :

$$V(M + \omega, N) + V(M - \omega, N) \geq 2V(M, N) \quad (\text{A23})$$

Assume that, starting from the terms on the left, the firm follows the policy that would be optimal starting from the term on the right. By the assumption that  $f'' \geq 0$ , we have:

$$f(M + \omega)g(m) + f(M - \omega)g(m) \geq 2f(M)g(m) \quad (\text{A24})$$

Similarly, it is possible to show that, when the same policy is followed starting from all three states, the average of the firm's mainstream assets starting from  $(M + \omega, N)$  and  $(M - \omega, N)$  are always at least as large as the mainstream assets starting from  $(M, N)$ . Because the profit function is increasing and weakly convex, this implies that (A23) holds.

I next show that the interaction effect to increasing assets in the two segments cannot be positive. That is, I will show for any  $M_H > M_L$  and  $N_H > N_L$ :

$$V(M_H, N_H) + V(M_L, N_L) \leq V(M_H, N_L) + V(M_L, N_H) \quad (\text{A25})$$

This can be shown using the same approach as in Lemma 3. That is, starting from state  $(M_H, N_L)$  invest the maximum of the optimal mainstream investments from the states on the left; and starting from state  $(M_L, N_H)$  invest the maximum of the optimal niche investments from the states on the left. This ensures that the sum of the assets in each segment at the states on the right is at least as large as the sum on the left, and also the variance on the right is at least as large. Together, these ensure that total profits on the right side are at least as large.

As shown in the proof of Proposition 1, strict convexity of the value function, along with non-positive interactions, ensures that focused investment is optimal. However, in this case the value function is only guaranteed to be *weakly* convex, so a firm could invest an amount in the interior of the range  $(0, 1)$  at the set of state (of measure zero) where the expected returns to investment are exactly zero, or make a positive investments in both segments at a set of states (also of measure zero) where the

expected returns to investing in the two segments happen to be exactly the same.

These sets have measure zero because the sensitivity of assets to investment in a segment is strictly increasing in the level of assets in that segment (by the assumption that  $f' > 0$ ). Therefore, if the returns to investing in a segment are zero at a given state, they are positive if assets in that segment are made slightly higher. Similarly, if the returns to investing in the two segments are the same at a given state, one segment is strictly preferred if assets in that segment are made slightly higher. QED

## Essay 2:

# A Dynamic Model of Competitive Entry Response\*

### Abstract

This paper develops a model in which an incumbent has expertise in an old business format (e.g., running a full service airline), and a new firm enters the market with the possibility of using a new business format (e.g., running a “no frills” airline). Firms play a dynamic investment game in which the incumbent can invest in the new format and the entrant can invest in either format. If brand and format preferences are strong, and if it is easy to implement a format, then a firm already using one format does not invest in the other format, since such an attack would be met with swift retaliation. In this case, the entrant invests in the new format, while the incumbent avoids investing in order to retain the threat of investment as a punishment mechanism.

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\*Helpful comments were provided by Glenn Ellison, Bob Gibbons, Duncan Simester, Birger Wernerfelt, and seminar participants at MIT.

# 1 Introduction

When a new entrant attacks an incumbent, the incumbent typically has many advantages such as attractive retail locations and expertise in current technology. The entrant might then try to develop its own advantages, for example, by investing in a new distribution channel like the Internet, a new technology, or a lower-cost business model. The problem for the incumbent is how to respond to this threat:

- The incumbent could invest in the new business format. For example, following E-trade’s early success, the largest incumbent discount broker, Charles Schwab, began investing in its own online trading platform (Lal 1996).
- The incumbent could stay focused on the old business format, and cut its price. For example, when Zappos launched a online shoe retailing business with a culture of high-quality customer service (with exceptionally friendly customer support staff), Amazon did not invest in the cultural changes it would have needed to replicate this customer experience, but instead responded by cutting the prices of its own online shoe offerings (Coster 2008).
- The incumbent could stay focused on the old format unless the entrant also attacks this format. For example, when EasyJet first entered the market as a “no frills” airline, British Airways continued to focus on the traditional full service format; however, when EasyJet began moving upscale and serving more business passengers, British Airways retaliated by adopting a “no frills” model for some of its short-haul European routes (Cowell 2002).

This paper develops a dynamic investment model to explore when each of these strategies is optimal. Previous theoretical literature in marketing has also studied optimal defensive strategies (Hauser and Shugan 1983; Kalra, Rajiv, and Srinivasan

1998) and entry strategies (Carpenter and Nakamoto 1990; Narasimhan and Zhang 2000). The current paper contributes to this literature by incorporating competition between an entrant and an incumbent into a dynamic investment game in which firms make repeated investments over multiple time periods. Because successful investment can lead to a series of reactions and counter-reactions, this model generates new insights into how threats of strategic retaliation influence investment behavior.

Competition in the model proceeds as follows. An incumbent monopolist initially has expertise in a particular business format, when a new business format becomes available. This new format might represent a new distribution channel or a lower-cost production model that is now possible, for example, due to exogenous technological progress or changes in customer preferences. After paying a fixed entry cost, an entrant then decides how much to invest in each of the available business formats. Similarly, the incumbent decides how much to invest in the new format. In any given period, each firm has a random probability of successfully adopting a format, with the probability increasing in the amount it invests. If a firm fails to implement a format in a given period, it can try again in the following period. While some customers prefer the original format, others prefer the new format. Customers also vary in their relative brand preferences for the two firms.

Firms' equilibrium strategies in this model depend on two key factors: the strength of customers' format preferences, and the strength of their brand preferences.

When format preferences are weak, then the incumbent can serve most customers by using the old format. This means it has little incentive to adopt the new format, which would mostly just cannibalize sales from the old format. By contrast, the entrant does not need to worry about cannibalization. As long as brand preferences are sufficiently strong (so price competition is not too intense), the entrant adopts the new format. This causes the incumbent to cut its price in response. In the long

run, each firm stays focused on a different format.

When format preferences are strong, the incumbent is less concerned about cannibalization, and both firms have an incentive to adopt the new format. If brand preferences are weak, both firms invest heavily in the new format, each hoping to preempt its rival. In the long run, the first firm to successfully adopt the new format is the only one that uses this format.

On the other hand, if format and brand preferences are both strong, the equilibrium outcome depends on how easy it is to implement a format. Contrary to standard intuition, if it is easy to implement a format, then a firm that is already using one format should not invest in a second format. This is because such an attack would quickly lead the other firm to retaliate by implementing both formats. One implication is that the incumbent does not want to invest in the new format because it would no longer have a weapon to punish the entrant, and so the entrant would implement both formats as well. To avoid this outcome, the incumbent allows the entrant to win the new format, and in the long run each firm stays focused on a different format. By contrast, when implementing a format is difficult, punishment strategies are ineffective because they take so long to implement. In equilibrium, each firm invests a small amount in the new format until it eventually succeeds, and in the long run both firms implement both formats.

Figure 1 summarizes these results.

**Figure 1. Equilibrium Investment Strategies**

	Weak Format Preferences	Strong Format Preferences
Weak Brand Preferences	<i>Neither firm adopts the new format.</i>	<i>Firms have a preemption race for the new format.</i>
Strong Brand Preferences	<i>Only the entrant adopts the new format, and the incumbent cuts its price in response.</i>	<i>If adoption is easy, only the entrant adopts the new format. If adoption is difficult, both firms adopt both formats.</i>

The preceding analysis assumes the two formats are symmetric in all respects. I also present an extension in which the old format (e.g., bricks-and-mortar retailing) requires a recurring fixed expenditure, whereas the new format (e.g., Internet retailing) allows a firm to avoid this expense. If this fixed expense is large enough, the entrant will never invest in the old format, and so the incumbent can safely invest in the new format without fear of retaliation.

Finally, I present an extension in which the new format free-rides on the old format. For example, customers might use sales assistance in a traditional retail store and then purchase through a lower-cost Internet channel. In some cases, the incumbent initially does not invest in the new format in order to avoid damaging its business in the old format, but once the entrant has successfully entered the new format (so the damage to the old format is already done), the incumbent enters the new format as well.

Section 2 discusses related literature. Section 3 presents the formal model. Section 4 concludes.



## 2 Related Literature

In addition to the literature on defensive and entry strategies cited above, this paper relates to several other research streams.

Previous theoretical literature has studied whether incumbents or entrants have stronger incentives to invest in patents for new technologies (Gilbert and Newbery 1982; Reinganum 1983). The current paper focuses instead on situations where patent protection is not possible, and each firm might in principle adopt both formats.

Previous work has also developed dynamic investment models in which there are increasing returns, so firms invest more in areas of current strength than in areas of current weakness (Athey and Schmutzler 2001; Rob and Fishman 2005; Selove 2010). By contrast, the current paper does not rely on increasing returns. Instead, concerns over cannibalization, price competition, and competitive retaliation compel firms to stay focused.

Another related research stream has shown that multi-market contact can facilitate collusion between firms (Karnani and Wernerfelt 1985; Bernheim and Whinston 1990; Bronnenberg 2008). The current paper shows how this concept applies when competing firms attempt to implement different business formats. In particular, I show that successful implementation must be relatively easy in order for firms to sustain mutual punishment threats.

Empirical literature has studied factors that determine whether incumbents invest in new technologies (e.g., Christensen 1997; Chandy and Tellis 1998, 2000; Debruyne and Reibstein 2005) or lower-cost business formats (e.g., Ritson 2009). These papers have identified concerns over cannibalization and competitive price response and key factors that determine whether firms adopt new technologies, and whether defensive strategies are successful. This paper clarifies how these factors determine firms' optimal investment strategies.

### 3 Model

Assume two firms, indexed by  $i \in \{A, B\}$ , compete using two different business formats, indexed by  $j \in \{1, 2\}$ . At any time  $t$ , the state of firms' assets is denoted by  $X_t = (X_{A,1,t}, X_{A,2,t}; X_{B,1,t}, X_{B,2,t})$ , where:

$$X_{i,j,t} = \begin{cases} 1 & \text{if firm } i \text{ uses format } j \text{ at time } t \\ 0 & \text{otherwise} \end{cases}$$

Intuitively, this variable indicates whether a firm has successfully made the necessary investments in physical capital, human resources, and other areas to serve customers using a particular format.

The game begins in state  $(1, 0; 0, 0)$ . That is, firm  $A$  (the incumbent) initially uses only format 1, and firm  $B$  (the entrant) initially does not use either format. Once a firm implements a format, it always uses this format, that is, if  $X_{i,j,t-1} = 1$ , then  $X_{i,j,t} = 1$ . If firm  $i$  has not yet implemented format  $j$ , then the probability that it will successfully implement this format in the current period is given by:

$$P(X_{i,j,t} = 1 | X_{i,j,t-1} = 0; e_{i,j,t}) = F(e_{i,j,t})$$

where  $e_{i,j,t}$  is the amount firm  $i$  invests in format  $j$  at time  $t$ . I assume  $F(0) = 0$ ,  $F' > 0$ ,  $F'' < 0$ , and  $F'$  is bounded so that in some cases a firm might want to invest nothing.

I also make two simplifying assumptions that make analysis of this dynamic investment game more tractable. First, I assume a firm cannot simultaneously invest in both formats in a given period, that is, at any time  $t$  each firm  $i$ 's investment is subject to the constraint  $\min\{e_{i,1,t}; e_{i,2,t}\} = 0$ . Intuitively, due to limitations on managerial time and attention or other internal resource constraints, there are

often diseconomies of scope to investment during a given time period. Limiting the firm to investing in at most one format is one way to operationalize diseconomies of scope while also helping to make the model more tractable. I also assume that firms alternate their investments, so firm  $A$  only invests in odd-numbered periods and firm  $B$  only invests in even-numbered periods.<sup>1</sup> This avoids the need to worry about “ties” in which both firms simultaneously succeed in adopting a format. Although these assumptions make it possible to derive clear analytical results, in principle either or both of these assumptions could be relaxed without substantially changing my results.

There is a unit mass of customers, who vary along two dimensions. First, their brand preferences are represented by a Hotelling line of length  $\beta$ , with firm  $A$  fixed at the left side of the line and firm  $B$  fixed at the right side. The second dimension of customer heterogeneity is format preferences. A fraction  $\alpha$  of customers will only buy using format 1, another  $\alpha$  will only use format 2, and the remaining  $1 - 2\alpha$  are indifferent between the two formats, where  $\alpha \in [0, \frac{1}{2}]$ . Note that a large value of  $\beta$  indicates that customers tend to have strong preferences for one brand or the other, while a large value of  $\alpha$  indicates they have strong preferences for one format or the other.

In each period, a customer purchases at most one product, and the utility derived from a product is computed as follows. Suppose a customer is located a distance  $d$  from the left side of the line. If this customer has either a preference for format 1 or no format preference, he would derive utility  $V - d - P_{A,1}$  if he purchases from firm  $A$  using format 1 at price  $P_{A,1}$ . Similarly, if he has either a preference for format 2 or no format preference, he would derive utility  $V - d - P_{A,2}$  if he purchases from firm  $A$  using format 2 at price  $P_{A,2}$ . However, if he has a preference for format 1, he would

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<sup>1</sup>In most cases, my results do not depend on which firm moves first. In cases for which order does matter, I will also consider what happens if firm  $B$  moves first.

derive utility  $-\infty$  from using format 2.

I assume  $2\beta < V < 3\beta$ , which ensures that the market is always covered in equilibrium, and that the pricing-subgame always has a pure strategy equilibrium.<sup>2</sup> I also assume, without loss of generality, that marginal costs are zero, so that a firm's profits equal price times demand. Each firm has discount factor  $\delta$ , and maximizes expected discounted profits.

**Figure 2. Variables in the Model**

$i \in \{A, B\}$	Index of firms
$j \in \{1, 2\}$	Index of formats
$t \in \{0, 1, 2, \dots\}$	Index of time
$X_{i,j,t} \in \{0, 1\}$	Indicator of whether firm $i$ uses format $j$ at time $t$
$e_{i,j,t}$	Firm $i$ 's investment in format $j$ at time $t$
$F(e_{i,j,t})$	Function that gives the probability firm $i$ will successfully implement format $j$ at time $t$
$\beta$	Strength of brand preferences (length of Hotelling line)
$\alpha \in [0, \frac{1}{2}]$	Strength of format preferences (fraction of customers who will only use format 1; also, fraction who will only use format 2)
$1 - 2\alpha$	Fraction of customers who are indifferent between formats
$V$	Utility customers derive from the product, before price and transportation costs
$P_{i,j,t}$	Price firm $i$ sets for format $j$ at time $t$ (only relevant if $X_{i,j,t} = 1$ )
$\delta$	Each firm's discount factor

At each time  $t$ , the game proceeds as follows:

<sup>2</sup>If the Hotelling line is too short ( $V > 3\beta$ ), then in some cases there is only a *mixed strategy* price equilibrium in which each firm randomizes between setting a high price to serve customers "loyal" to its format and setting a low price to capture the format "switchers," similar to the equilibrium in the model by Varian (1980). However, if the Hotelling line is sufficiently long ( $V < 3\beta$ ), firms always want to capture some of the switchers in equilibrium, and there is always a unique *pure strategy* equilibrium.

1. One firm makes an investment decision (firm  $A$  in odd periods, firm  $B$  in even periods).
2. The outcome of investment is realized, so the firm might succeed in implementing a new format.
3. Firms simultaneously set prices. (Each firm sets a price for each format that it uses.)
4. Customers make purchase decisions, and profits are realized.

I first analyze the pricing subgame (steps 3 and 4 above). I then analyze the infinite-period dynamic investment game that occurs when firms repeatedly engage in all four of these steps. I assume firms always play the one-shot equilibrium in the pricing subgame, and play a Markov perfect equilibrium (MPE) of the dynamic investment game.

### 3.1 Pricing sub-game

Given the assumption that  $V > 2\beta$ , it can be shown that the market is always covered in equilibrium, that is, if either firm uses a format that a customer is willing to consider, then that customer makes a purchase in equilibrium. For most states of the game, it can also be shown that, if a firm is the only one that uses a format, it will set the monopoly price for that format in order to maximize profits from its “loyal” customers. On the other hand, when both firms use a format, equilibrium prices in that format are the same as they would be in a standard Hotelling model.

The only exception to this rule is state  $(1, 0; 0, 1)$ . In this case, equilibrium prices depend on whether the following holds:

**Condition 1.**  $\beta\left(1 + \frac{2\alpha}{1-\alpha}\right) > V - \beta$

This condition ensures there are enough customers with strong format preferences that, when each firm uses a different format, they still want to set the monopoly price for that format. On the other hand, when this condition does not hold, prices are set competitively, in a similar manner to the Hotelling model.

These results, and their implications for equilibrium demand, are stated formally in the following proposition.

**Proposition 1.** *When Condition 1 holds, equilibrium prices are as follows: If only one firm uses a format, its price for that format is  $V - \beta$ . If both firms use a format, they each set price  $\beta$  for that format. In equilibrium, customers with a format preference purchase from the nearest firm that uses their preferred format. Customers who do not have a format preference purchase using a format served by both firms, if one is available, and otherwise they purchase from the nearest firm that offers either format. When Condition 1 fails, equilibrium prices and demand are the same as in the previous case, with the following exception: When each firm uses a different format, they each set price  $\beta(1 + \frac{2\alpha}{1-\alpha})$*

Based on Proposition 1, I compute the equilibrium profits for each firm in each possible state. These are reported in Figure 3.

By using the profit numbers from Figure 3, it is possible to solve the dynamic investment game through backward induction. I first analyze the case of strong format preferences, and then turn to the case of weak format preferences. I also present an extension in which the old format requires a recurring fixed expenditure, which can be avoided in the new format.

**Figure 3. Equilibrium Profits\***

State	Firm A's profits	Firm B's profits
(1, 0; 0, 0)	$(V - \beta)(1 - \alpha)$	0
(1, 1; 0, 0)	$(V - \beta)$	0
(1, 0; 0, 1)*	$(V - \beta)\frac{1}{2}$	$(V - \beta)\frac{1}{2}$
(1, 0; 0, 1)*	$\beta(1 + \frac{2\alpha}{1-\alpha})\frac{1}{2}$	$\beta(1 + \frac{2\alpha}{1-\alpha})\frac{1}{2}$
(1, 1; 0, 1)	$(V - \beta)\alpha + \beta(1 - \alpha)\frac{1}{2}$	$\beta(1 - \alpha)\frac{1}{2}$
(1, 0; 1, 1)	$\beta(1 - \alpha)\frac{1}{2}$	$(V - \beta)\alpha + \beta(1 - \alpha)\frac{1}{2}$
(1, 1; 1, 1)	$\beta\frac{1}{2}$	$\beta\frac{1}{2}$

\* The first set of numbers for state (1, 0; 0, 1) are for the case when Condition 1 holds; the second set are for the case when this condition does not hold. This implies that at this state firms earn the minimum of these two profit numbers. I do not report profits or analyze state (1, 0; 1, 0), in which both firms use the old format. It can be shown that it is not optimal for the entrant to start by investing in the old format, so this state will never occur. I also do not analyze state (1, 1; 1, 0), as it is analogous to state (1, 1; 0, 1).

### 3.2 Strong format preferences ( $\alpha$ large)

This section focuses on the case when customers have strong format preferences. I show that when brand preferences are weak, firms engage in a preemption race for the new format, and in the long run only one firm serves each format. On the other hand, when brand preferences are strong, the equilibrium outcome depends on properties of the function  $F$  which determines how easily a firm can implement a new format.

When implementing a format is easy, this encourages stability at state (1, 0; 0, 1), with each firm serving one format. Neither firm wishes to implement the format in which its competitor is strong, as this would lead to swift retaliation.

However, when implementing a format is moderately difficult, a firm is willing

to invest in attacking its competitor, knowing that when it eventually succeeds, it is likely to take a long time for the competitor to successfully retaliate. In the long run, each firm implements both formats.

I will say that a state is “stable” if neither firm invests once that state is reached. Obviously state  $(1, 1; 1, 1)$  is stable.

To determine whether states  $(1, 1; 0, 1)$  and  $(1, 0; 1, 1)$  are stable, we need to compare the cost of investment with the benefit of implementing a second format. For example, from Figure 3 we see that the per-period profit increase to firm  $B$  of moving from state  $(1, 1; 0, 1)$  to state  $(1, 1; 1, 1)$  is  $\beta\alpha\frac{1}{2}$ . This leads to the following lemma:

**Lemma 1.** *States  $(1, 1; 0, 1)$  and  $(1, 0; 1, 1)$  are stable if and only if:*

$$\left(\frac{1}{1-\delta}\right)\beta\alpha\frac{1}{2} < \frac{1}{F'(0)} \quad (1)$$

The condition in Lemma 1 ensures that it is not profitable to invest the first marginal dollar, and so the firm will invest nothing. (Recall that I assume the probability of success is concave in investment). Note that these states are more likely to be stable (that is, firms are more likely to stop investing) if format and brand preferences are weak. Intuitively, weak format preferences imply that a firm with one format will gain only a small increase in sales by offering the second format, and weak brand preferences imply that equilibrium prices are low if both firms offer the same format.

I next address whether state  $(1, 1; 0, 0)$  is stable. From Figure 3 we see that the per-period profit increase to firm  $B$  of moving to state  $(1, 1; 0, 1)$  is  $\beta(1-\alpha)\frac{1}{2}$ . This leads to the following lemma:



**Lemma 2.** *State  $(1, 1; 0, 0)$  is stable if and only if:*

$$\left(\frac{1}{1-\delta}\right)\beta(1-\alpha)\frac{1}{2} < \frac{1}{F'(0)} \quad (2)$$

This state is more likely to be stable when format preferences are strong. This is because firm  $B$  will not, for example, capture any of the customers with a preference for format 1 if it only implements format 2. (It will capture half of the customers who have either no format preference or a preference for format 2.) Also note that if (2) holds then (16) is guaranteed to hold as well (since  $\alpha \leq \frac{1}{2}$ ), so we do not need to worry whether firm  $B$  would want to implement both formats, as condition (2) guarantees it would not.

The next question is whether state  $(1, 0; 0, 1)$  is stable. Analysis of this state is considerably more complicated because each firm has to account for its competitor's future investment behavior.<sup>3</sup> I focus on deriving conditions in which it is possible for firms to avoid attacking each other, that is, in which there is an equilibrium with state  $(1, 0; 0, 1)$  stable. However, I will later also derive conditions in which such attacks are unavoidable, and state  $(1, 0; 0, 1)$  is unstable in any equilibrium.

From Figure 3 we see that the per-period profit increase to either firm from adding a second format is  $\beta(1-\alpha)\frac{1}{2} - (V-B)(\frac{1}{2}-\alpha)$ . This leads to the following lemma:<sup>4</sup>

**Lemma 3.** *There exists an equilibrium in which state  $(1, 0; 0, 1)$  is stable if:*

$$\left(\frac{1}{1-\delta}\right)\left[\beta(1-\alpha)\frac{1}{2} - (V-\beta)\left(\frac{1}{2}-\alpha\right)\right] < \frac{1}{F'(0)} \quad (3)$$

Unlike the previous lemmas, Lemma 3 provides a sufficient condition but not a

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<sup>3</sup>By contrast, starting at states  $(1, 0; 1, 1)$ ,  $(1, 1; 0, 1)$ , and  $(1, 1; 0, 0)$ , only one firm can possibly invest, which makes these cases easier to analyze.

<sup>4</sup>Lemma 3 assumes  $\alpha$  is large enough that Condition 1 holds, which ensures firms set the monopoly price at state  $(1, 0; 0, 1)$ . If Condition 1 does not hold, a similar lemma would still be true, but the formula would need to be modified to account for the new price at this state.

necessary one. The reason is the following. At state  $(1, 0; 0, 1)$  a firm has to worry that its investment in a second format will prompt its competitor to retaliate. This threat of retaliation can cause this state to be stable even if the condition in Lemma 3 does not hold. However, for now I am only concerned with finding a sufficient condition for stability.

Also note that Lemma 3 only guarantees existence of an equilibrium in which state  $(1, 0; 0, 1)$  is stable. There may also be other equilibria in which this state is unstable. That is, even if the condition in Lemma 3 holds, it is possible that if firm  $A$  attacks, then firm  $B$ 's best response is to make a preemptive attack, and vice versa.

If the condition in Lemma 2 holds, then the condition in Lemma 3 does as well. Therefore, if brand preferences are sufficiently weak ( $\beta$  is sufficiently small) that  $\left(\frac{1}{1-\delta}\right)\beta(1-\alpha)\frac{1}{2} < \frac{1}{F'(0)}$ , then states  $(1, 1; 0, 0)$  and  $(1, 0; 0, 1)$  are both stable. The intuition is that, with weak brand preferences, a firm does not want to use a format that its competitor is already using because this would cause intense price competition.

The only remaining question is whether state  $(1, 0; 0, 0)$  is stable. From Figure 3 we see that firm  $A$  gains  $(V - \beta)\alpha$  and firm  $B$  gains  $(V - \beta)\frac{1}{2}$  by implementing the new format. This leads to the following lemma:

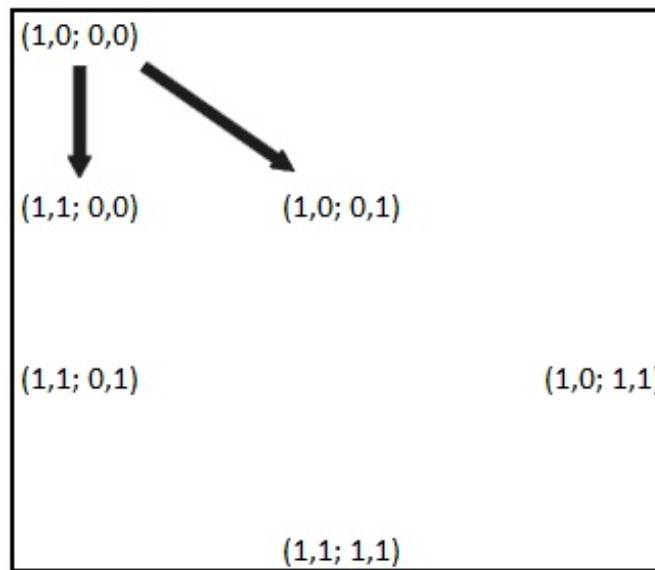
**Lemma 4.** *If states  $(1, 1; 0, 0)$  and  $(1, 0; 0, 1)$  are stable, then both firms make positive investment at state  $(1, 0; 0, 0)$  if:*

$$\left(\frac{1}{1-\delta}\right)(V - \beta)\alpha > \frac{1}{F'(0)} \tag{4}$$

Lemmas 2, 3, and 4 lead to the following proposition:

**Proposition 2.** *If format preferences are sufficiently strong ( $\alpha$  is large enough), and brand preferences are sufficiently weak ( $\beta$  is small enough), such that the conditions in Lemmas 2 and 4 hold, then in equilibrium both firms invest in the new format until one succeeds, at which point neither firm makes any further investment.*

**Figure 4.** When format preferences are strong ( $\alpha$  is large) and brand preferences are weak ( $\beta$  is small), firms have a preemption race until one firm wins the new format. (Arrows indicate possible paths the game state can follow.)



This proposition shows that when format preferences are strong and brand preferences are weak, firms engage in a preemption race.<sup>5</sup> Intuitively, because format preferences are strong, firm 1 knows that implementing the new format would mostly expand the market (as opposed to cannibalizing its current sales), and firm 2 knows that implementing the new format will not lead to intense price competition. Therefore, each firm would like to implement this format as long as the other has

<sup>5</sup>Recall that I assume firms alternate moves, with the incumbent moving first. This implies that if implementation is easy (so investment is likely to succeed in the first period), then the incumbent is likely to win the new format. On the other hand, if implementation is easy and we assume the *entrant* moves first, the entrant is likely to win the new format.

not done so. However, because brand preferences are weak, both firms will never implement the same format, which would lead to intense price competition.

I now turn to the case in which format preferences are strong ( $\alpha$  is large) and brand preferences are also strong ( $\beta$  is large). From Lemmas 1 and 2, we see that states  $(1, 1; 0, 1)$ ,  $(1, 0; 1, 1)$ , and  $(1, 1; 0, 0)$  are not stable when  $\beta$  is sufficiently large. However, for state  $(1, 0; 0, 1)$ , the problem is more complicated. As previously mentioned, at this state each firm must worry that implementing a new format will lead to retaliation by its competitor.

To compute how quickly firm  $B$  is expected to retaliate to an attack by firm  $A$ , let  $e^*$  denote firm  $B$ 's optimal investment level at state  $(1, 1; 0, 1)$ . If the condition in Lemma 1 holds, then  $e^* = 0$ . Otherwise,  $e^*$  must solve the following equality:

$$\left(\frac{1}{1-\delta}\right)\beta\alpha\frac{1}{2} = \frac{1}{F'(e^*)} \quad (5)$$

Because  $F'' < 0$ , this implies that  $e^*$  is increasing in  $\alpha$  and  $\beta$ . Intuitively, when format and brand preferences are strong, firm  $B$  has more to gain by adopting a second format, so it invests more. Also, holding  $F'(0)$  constant, as  $F''$  becomes more negative,  $e^*$  decreases. Intuitively, when the the marginal impact of investment is rapidly decreasing, firm  $B$  does not invest as much. This implies that the probability of firm  $B$  succeeding in a given period, denoted by  $F(e^*)$ , also decreases as  $F''$  becomes more negative.

Imagine firm  $A$  has a choice between staying at state  $(1, 0; 0, 1)$  forever, or implementing the second format and moving to state  $(1, 1; 0, 1)$ . This will lead to a temporary increase in firm  $A$ 's profits; however, once firm  $B$  successfully retaliates the game moves to state  $(1, 1; 1, 1)$ , and firm  $A$ 's profits drop below their initial level.

This is formalized in the following lemma:<sup>6</sup>

**Lemma 5.** *There exists an equilibrium in which state  $(1, 0; 0, 1)$  is stable if and only if:*

$$\left( \frac{\pi_{(1,1;0,1)}^A [1 + \delta(1 - F(e^*))] + [\delta/(1 - \delta)]F(e^*)\pi_{(1,1;1,1)}^A}{1 - \delta^2(1 - F(e^*))} \right) - \left( \frac{1}{1 - \delta} \right) \pi_{(1,0;0,1)}^A < \frac{1}{F'(0)}$$

where  $\pi^A$  represents firm  $A$ 's profits at a given state.

The first term on the left side of this inequality represents the expected discounted profits to firm  $A$  just after reaching state  $(1, 1; 0, 1)$ , accounting for firm  $B$ 's eventual retaliation; the second term represents the expected discounted profits to firm  $A$  of staying permanently at state  $(1, 0; 0, 1)$ . As  $F(e^*) \rightarrow 0$ , the expected time required for successful retaliation grows without bound, and the left side of this inequality approaches  $\frac{1}{1-\delta}(\pi_{(1,1;0,1)}^A - \pi_{(1,0;0,1)}^A)$ . In this case, Lemma 5 relies on the same condition as Lemma 3. Intuitively, when implementing a format becomes sufficiently difficult ( $F''$  is very negative), the expected time required to retaliate becomes so long that firms do not worry about retaliation. Rather, they each invest at least a small amount in the other format as long as the expected discounted profits of a successful attack are enough to justify investing the first marginal dollar.

At the other extreme, as  $F(e^*) \rightarrow 1$ , retaliation occurs in the next period after the attack, and the condition in Lemma 5 becomes:

$$(\pi_{(1,1;0,1)}^A - \pi_{(1,0;0,1)}^A) - \left( \frac{\delta}{1 - \delta} \right) (\pi_{(1,0;0,1)}^A - \pi_{(1,1;1,1)}^A) < \frac{1}{F'(0)} \quad (6)$$

Plugging in the values of  $\pi^A$ , this becomes:

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<sup>6</sup>As was the case with Lemma 3, note that Lemma 5 only guarantees *existence* of an equilibrium in which state  $(1, 0; 0, 1)$  is stable. There could also be another equilibrium in which both firms attack each other because if firm  $A$  knows that firm  $B$  is going to attack in the next period, then firm  $A$  wants to go ahead and attack now, and vice versa.

$$\left[ \beta(1 - \alpha)\frac{1}{2} - (V - \beta)\left(\frac{1}{2} - \alpha\right) \right] - \left( \frac{\delta}{1 - \delta} \right) (V - 2\beta)\frac{1}{2} < \frac{1}{F'(0)} \quad (7)$$

Note that if firms are very impatient ( $\delta \approx 0$ ) or the monopoly price is close to the duopoly price ( $V - \beta \approx \beta$ ), then the threat of retaliation carries little weight, and this condition does not hold. On the other hand, as long as firms are not too impatient and the monopoly price exceeds the duopoly price by enough for this condition to hold, then Lemma 5 will apply as long as  $F(e^*)$  is close enough to one, that is, as long as  $F''$  is not too negative.

I have now characterized the stability of all states except  $(1, 0; 0, 0)$ . The following lemma addresses this state:

**Lemma 6.** *If states  $(1, 1; 0, 1)$  and  $(1, 0; 1, 1)$  are unstable, then state  $(1, 0; 0, 0)$  is also unstable.*

This is true because, if states  $(1, 1; 0, 1)$  and  $(1, 0; 1, 1)$  are unstable, then a firm is willing to make positive investment in a format that will lead to incremental profits of  $\alpha\beta\frac{1}{2}$ . Furthermore, by moving from state  $(1, 0; 0, 0)$  to state  $(1, 0; 0, 1)$ , firm  $B$  guarantees that its profits are always at least  $\alpha\beta\frac{1}{2}$ , so state  $(1, 0; 0, 0)$  cannot be stable.

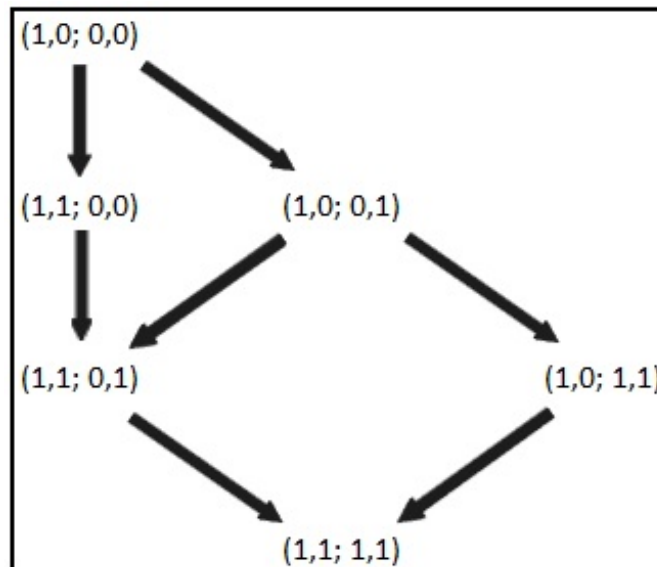
Based on the preceding analysis, the following proposition characterizes conditions in which all states are unstable, so both firms eventually adopt both formats.

**Proposition 3.** *If format and brand preferences are sufficiently strong ( $\alpha$  and  $\beta$  are sufficiently large), and  $F''$  is sufficiently negative, such that the conditions of Lemmas 1 and 5 do not hold, then in the long run both firms implement both formats.*

Intuitively, the condition that format preferences are strong ensures that cannibalization does not prevent a firm from adding a second format, while the condition that brand preferences are strong ensures that price competition is soft enough that

both firms can use the same format. Finally, the condition that  $F''$  is sufficiently negative implies that any retaliation is expected to take a long time to succeed, so firms worry mostly about the immediate profit impact of implementing a new format. When all of these conditions are true, firms make positive investment until they have both implemented both formats. Of course, it will take a long time for this to occur because implementation is so difficult.

**Figure 5. When format and brand preferences are strong ( $\alpha$  and  $\beta$  are large), and implementing a format is difficult ( $F''$  is very negative), both firms implement both formats. (Arrows indicate possible paths the game state can follow.)**



On the other hand, even if brand and format preferences are strong, as long as implementation is sufficiently easy, then firms worry about retaliation. In particular, if the condition in Lemma 1 does not hold, but the condition in Lemma 5 holds, then states  $(1, 1; 0, 1)$ ,  $(1, 0; 1, 1)$ , and  $(1, 1; 0, 0)$  are unstable, but state  $(1, 0; 0, 1)$  is stable. When this is true, I will show that in some cases the incumbent would prefer to let the entrant win the new format, that is, it would prefer the stable state  $(1, 0; 0, 1)$  to the unstable state  $(1, 1; 0, 0)$ .

To derive conditions when this is true, let  $\Lambda^A$  denote firm  $A$ 's continuation value if the game reaches a given state after its investment decision is realized. Firm  $A$ 's continuation value at state  $(1, 1; 0, 1)$  is the same as the first term in the condition of Lemma 5:

$$\Lambda_{(1,1;0,1)}^A = \left( \frac{\pi_{(1,1;0,1)}^A [1 + \delta(1 - F(e^*))] + [\delta/(1 - \delta)]F(e^*)\pi_{(1,1;1,1)}^A}{1 - \delta^2(1 - F(e^*))} \right) \quad (8)$$

Let  $\hat{e}$  denote firm  $B$ 's optimal investment at state  $(1, 1; 0, 0)$ .<sup>7</sup> Firm  $A$ 's continuation value at state  $(1, 1; 0, 0)$  is then given by:

$$\Lambda_{(1,1;0,0)}^A = \left( \frac{\pi_{(1,1;0,0)}^A [1 + \delta(1 - F(\hat{e}))] + \delta F(\hat{e})[\pi_{(1,1;0,1)}^A + \delta\Lambda_{(1,1;0,1)}^A]}{1 - \delta^2(1 - F(\hat{e}))} \right) \quad (9)$$

The following lemma states conditions in which the incumbent prefers to let the entrant win the new format.

**Lemma 7.** *If the condition in Lemma 1 does not hold, but the condition in Lemma 5 holds, firm  $A$  prefers state  $(1, 0; 0, 1)$  to state  $(1, 1; 0, 0)$  if and only if:*

$$\Lambda_{(1,1;0,0)}^A < \left( \frac{\delta}{1 - \delta} \right) (V - \beta) \frac{1}{2} \quad (10)$$

As implementation becomes easier ( $F''$  becomes less negative), so that  $F(e^*) \rightarrow 1$  and  $F(\hat{e}) \rightarrow 1$ , firm  $A$ 's value at state  $(1, 1; 0, 0)$  approaches:

$$\Lambda_{(1,1;0,0)}^A = \pi_{(1,1;0,0)}^A + \delta\pi_{(1,1;0,1)}^A + \delta^2\pi_{(1,1;0,1)}^A + \left( \frac{\delta^3}{1 - \delta} \right) \pi_{(1,1;1,1)}^A \quad (11)$$

This implies that, as long as firms are sufficiently patient ( $\delta$  is not too small) and the monopoly price exceeds the duopoly price ( $\pi_{(1,0;0,1)}^A > \pi_{(1,1;1,1)}^A$ ) by enough that

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<sup>7</sup>It is straightforward to derive an expression for this value by solving the dynamic program faced by firm  $B$  at state  $(1, 1; 0, 0)$  through backward induction. It can be shown that firm  $B$  invests more at state  $(1, 1; 0, 0)$  than at state  $(1, 1; 0, 1)$ , that is,  $\hat{e} > e^*$ .

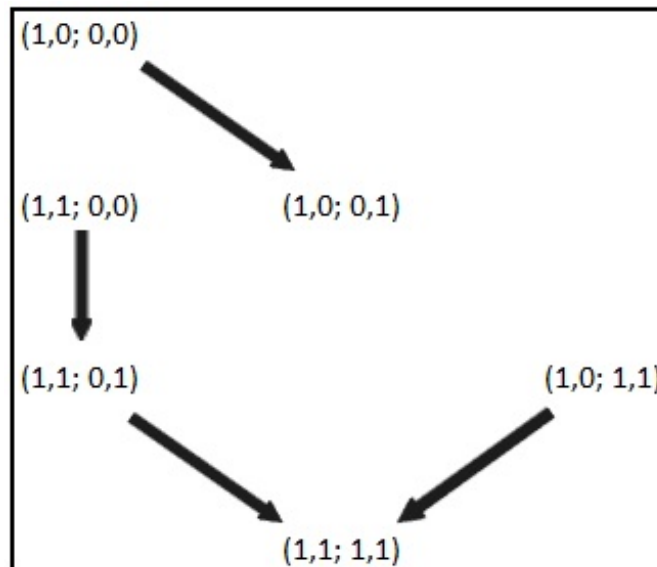


the preceding equation holds, then as long as implementation is easy enough, the incumbent would prefer the stable state  $(1, 0; 0, 1)$  to the unstable state  $(1, 1; 0, 0)$ .

Based on the preceding analysis, the following proposition states conditions in which the entrant wins the new format, and threats of retaliation prevent either firm from adopting a second format.

**Proposition 4.** *If format and brand preferences are sufficiently strong ( $\alpha$  and  $\beta$  are sufficiently large), and  $F''$  is also sufficiently large (that is, not too negative), such that the condition of Lemma 1 does not hold, but the conditions of Lemmas 5 and 7 hold, then in any Pareto optimal equilibrium firm B is the only one that invests in the new format. Once it successfully implements this format, neither firm makes further investment.*

**Figure 6.** When format and brand preferences are strong ( $\alpha$  and  $\beta$  are large), and implementing a format is easy ( $F''$  is not too negative), a firm will not implement a second format because this would lead to swift retaliation. (Arrows indicate possible paths the game state can follow.)



This proposition states that when format and brand preferences are both strong, and implementing a format is easy, firm  $B$  is guaranteed to win the new format. The

intuition is that if firm  $A$  implemented the new format, it would give up its only mechanism for punishing firm  $B$ , which would then lead firm  $B$  to implement both formats. Thus, each firm would prefer to implement only a single format, using the threat of implementing the other format as a way to keep its competitor in check.

### 3.3 Weak format preferences ( $\alpha$ small)

This section focuses on the case when customers have weak format preferences. I show that when brand preferences are weak, neither firm invests in the new format. On the other hand, when brand preferences are strong, the entrant invests in the new format, and the incumbent responds by cutting its price in the old format.

As before, this game can be solved with backward induction. Lemmas 1 still holds, so states  $(1, 1; 0, 1)$  and  $(1, 0; 1, 1)$  are stable when format preferences are sufficiently weak. However, Lemma 3 needs to be modified to account for the new equilibrium prices at state  $(1, 0; 0, 1)$  when Condition 1 does not hold:

**Lemma 8.** *If Condition 1 does not hold, there exists an equilibrium in which state  $(1, 0; 0, 1)$  is stable if:*

$$\left(\frac{1}{1-\delta}\right)\left((V-\beta)\alpha - \beta\frac{1}{2}\left[\alpha + \frac{2\alpha}{1-\alpha}\right]\right) < \frac{1}{F'(0)} \quad (12)$$

This shows that state  $(1, 0; 0, 1)$  is also stable if  $\alpha$  is sufficiently small. Intuitively, when format preferences are weak, any state in which each firm uses at least one format is stable because adding a second format would mostly just cannibalize the firm's sales.

On the other hand, Lemma 2 also still holds, so even when format preferences are weak, the stability of state  $(1, 1; 0, 0)$  depends on the strength of brand preferences. This state is stable if and only if brand preferences are also weak.

The only remaining question is how firms behave at state  $(1, 0; 0, 0)$ . The following lemma addresses this question.

**Lemma 9.** *If Condition 1 does not hold, and states  $(1, 1; 0, 0)$  and  $(1, 0; 0, 1)$  are stable, then state  $(1, 0; 0, 0)$  is stable if and only if both of the following hold:*

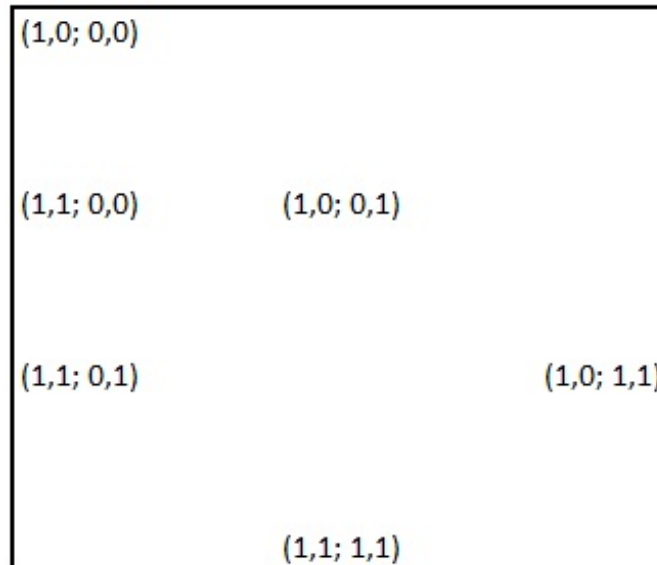
$$\left(\frac{1}{1-\delta}\right)\beta\left(1 + \frac{2\alpha}{1-\alpha}\right)\frac{1}{2} < \frac{1}{F'(0)} \quad (13)$$

$$\left(\frac{1}{1-\delta}\right)(V - \beta)\alpha < \frac{1}{F'(0)} \quad (14)$$

If this condition holds, then the conditions in Lemmas 1, 2, and 8 hold as well. This leads to the following proposition.

**Proposition 5.** *When format and brand preferences are both sufficiently weak ( $\alpha$  and  $\beta$  are sufficiently small), such that the conditions of Lemma 9 hold, then neither firm invests in the new format, and the state  $(1, 0; 0, 0)$  is stable.*

**Figure 7.** When format and brand preferences are weak ( $\alpha$  and  $\beta$  are small), neither firm invests.



The intuition is that, if format preferences are weak, implementing a second format mostly just cannibalizes a firm's sales. Furthermore, if brand preferences are also weak, then even if the entrant adopts a different format than the incumbent, price competition between the incumbent and entrant is intense because many customers will consider both formats. As a result, equilibrium profits are not large enough to justify the cost of entry.

Finally, I address the case in which  $\alpha$  is small enough that state  $(1, 0; 0, 1)$  is stable, but  $\beta$  is large enough that state  $(1, 1; 0, 0)$  is unstable. In this case, the following lemma addresses behavior at state  $(1, 0; 0, 0)$ .

**Lemma 10.** *If state  $(1, 0; 0, 1)$  is stable, but state  $(1, 1; 0, 0)$  is unstable, then at state  $(1, 0; 0, 0)$  firm  $B$  invests in the new format and firm  $A$  does not invest if:*

$$\Lambda_{(1,1;0,0)}^A - \Lambda_{(1,0;0,0)}^A < \frac{1}{F'(0)} \quad (15)$$

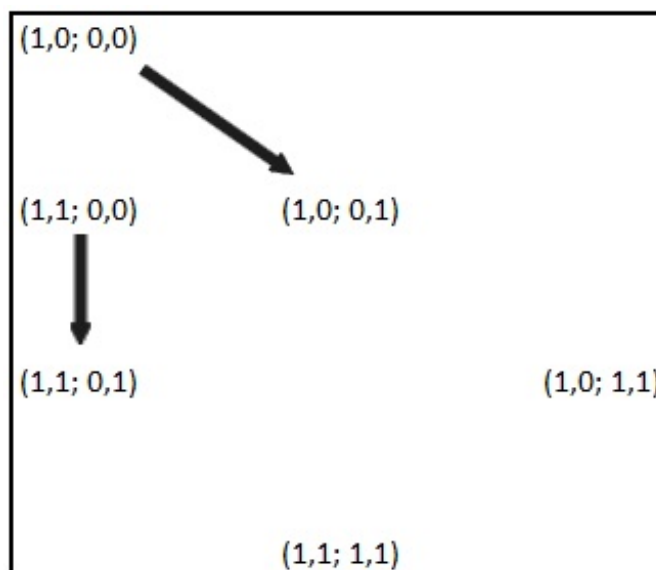
Recall that  $\Lambda^A$  represents firm  $A$ 's continuation value at a given state. In this case, the benefits to firm  $A$  of adopting the new format come both from the immediate increase in its profits and also from the deterrent effect on firm  $B$ 's investment. However, both effects approach zero as  $\alpha \rightarrow 0$ , so the condition in Lemma 10 is guaranteed to hold for  $\alpha$  sufficiently small.<sup>8</sup> This leads to the following proposition.

**Proposition 6.** *When format preferences are sufficiently weak ( $\alpha$  is small enough) and brand preferences are sufficiently strong ( $\beta$  is large enough), such that the conditions in Lemmas 1, 8, and 10 hold, but the condition in Lemma 2 does not hold, firm  $B$  is the only one that invests in the new format. After it successfully adopts this format, firm  $A$  responds by cutting its price.*

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<sup>8</sup>The condition in this lemma can be made more precise by defining a new variable,  $\bar{\epsilon}$ , that represents firm  $B$ 's optimal investment at state  $(1, 0; 0, 0)$  assuming firm  $A$  does not invest, and then comparing the resulting value of  $\Lambda_{(1,0;0,0)}^A$  with the value of  $\Lambda_{(1,1;0,0)}^A$ .

**Figure 8. When format preferences are weak ( $\alpha$  is small) and brand preferences are strong ( $\beta$  is large), the entrant wins the new format. (Arrows indicate possible paths the game state can follow.)**



Intuitively, when format preferences are weak, there is no reason for the incumbent to adopt the new format. This would neither expand the market nor deter the entrant's investment. On the other hand, if the entrant adopts the new format, it captures half of the market. This also leads to price competition, but this is still a profitable investment for the entrant as long as brand preferences are strong enough, which ensures price competition is not too intense.

### 3.4 Fixed expense in the old format

A major appeal of many Internet-based business models is that they allow firms to avoid expenses associated with traditional business models. For example, while bricks-and-mortar retailers like Barnes and Noble incur large real estate expenses due to their physical retail locations, online retailers like Amazon.com have much lower real estate expenses (Ghemawat and Baird 2004). To capture this type of effect, I now present an extension in which there is a recurring fixed expense to operating the

old format, which is avoided in the new format.<sup>9</sup> I show that this fixed expense in the old format can benefit the incumbent and hurt the entrant by allowing the incumbent to adopt the new format without worrying about retaliation.

Because fixed expenses do not affect pricing decisions, the equilibrium prices and demand from Proposition 1 still hold. The only difference is that the profit numbers in Figure 3 need to be modified so that any firm operating in the old format has its per-period profits reduced by a fixed expense  $f$ .

The key effect on investment behavior is that, if the fixed expense is high enough, the entrant will never invest in the old format. Therefore, as long as brand and format preferences are strong enough, the incumbent will invest in the new format, knowing it does not have to worry about retaliation. This leads to the following modification of Proposition 4:

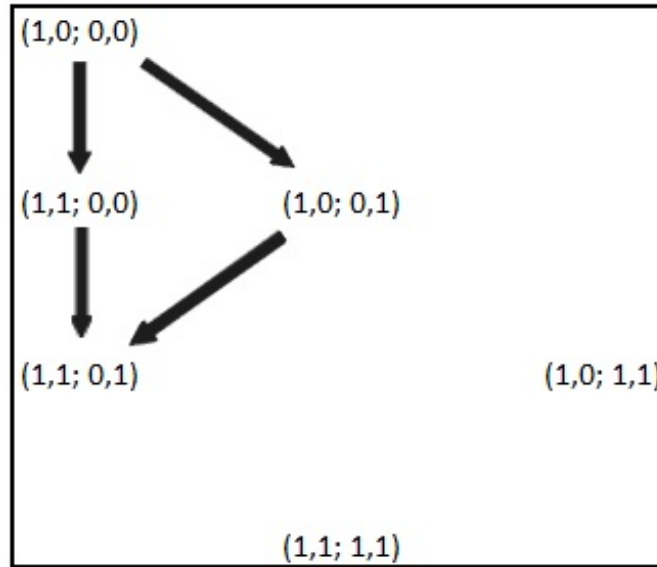
**Proposition 7.** *If format and brand preferences are sufficiently strong ( $\alpha$  and  $\beta$  are sufficiently large), and  $F''$  is also sufficiently large (that is, not too negative), such that the conditions of Proposition 4 hold, then there is a minimum cut-off level of fixed costs in the old format,  $f^*$ , such that incumbent always invests in the new format if and only if  $f > f^*$ .*

Note that the incumbent's profits are higher, and the entrant's profits are lower, at state  $(1, 1; 0, 1)$  than at state  $(1, 0; 0, 1)$ . Therefore, at the threshold level of fixed costs at which the incumbent starts to invest in the new format, an increase in fixed costs in the old format helps the incumbent and hurts the entrant.

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<sup>9</sup>In reality, new formats often reduce both fixed and variable expenditures. I focus on the fixed component because in the Hotelling model variable expenses are passed entirely through to consumers through higher prices, so they would not affect firms' investment incentives.

**Figure 9. Large fixed expenses in the old format allow the incumbent to invest in the new format without fear of retaliation. (Arrows indicate possible paths the game state can follow.)**



### 3.5 The new format free-rides on the old format

When the same product is available through traditional and Internet retailers, customers often visit a retail store for sales assistance but then purchase through the lower-cost Internet channel (Shin 2007). For example, a future bride might visit a wedding dress shop, spend several hours with a sales assistant choosing a dress, and then purchase the same dress (or a similar one) at a much lower price online. This problem can also occur within a single firm that serves both channels. For example, Staples store managers complained about this type of free-riding after the Staples.com website was launched (Anderson et al. 2009).

I now present an extension to show how this type of free-rider problem can lead to delayed investment by the incumbent. Intuitively, the incumbent is initially reluctant to invest in a new format that will damage the value of its investment in the old format. However, once the entrant has already done this damage, the incumbent

might as well invest in the new format too.

To illustrate this idea as simply as possible, I assume that if either firm uses the new format, this destroys the value of old format in such a manner that: (1) all firms earn zero profits in the old format; and (2) if only one firm successfully implements the new format, it earns monopoly profits in that format even if the other firm has implemented the old format. For example, if the free-rider problem is sufficiently severe that a traditional retailer cannot motivate its sales force to make any effort at all, then its retail stores might become worthless.<sup>10</sup>

Figure 10 gives equilibrium profits at each state under these new assumptions:

**Figure 10. Equilibrium Profits**

State	Firm A's profits	Firm B's profits
(1, 0; 0, 0)	$(V - \beta)(1 - \alpha)$	0
(1, 1; 0, 0)	$(V - \beta)(1 - \alpha)$	0
(1, 0; 0, 1)	0	$(V - \beta)(1 - \alpha)$
(1, 1; 0, 1)	$\beta(1 - \alpha)\frac{1}{2}$	$\beta(1 - \alpha)\frac{1}{2}$
(1, 0; 1, 1)	0	$(V - \beta)(1 - \alpha)$
(1, 1; 1, 1)	$\beta(1 - \alpha)\frac{1}{2}$	$\beta(1 - \alpha)\frac{1}{2}$

It is clear that firm  $B$  never wants to invest in the old format, so states (1, 0; 1, 1) and (1, 1; 1, 1) will never be reached. This implies that state (1, 1; 0, 1) is stable. The following lemma characterizes behavior at states (1, 1; 0, 0) and (1, 0; 0, 1).

**Lemma 11.** *In the free-riding game, states (1, 1; 0, 0) and (1, 0; 0, 1) are unstable if and only if:*

$$\left(\frac{1}{1 - \delta}\right)\beta(1 - \alpha)\frac{1}{2} > \frac{1}{F'(0)} \quad (16)$$

<sup>10</sup>Shin (2007) shows that in some cases traditional retailers actually benefit from free-riding because it softens price competition between traditional and Internet retailers. However, if enough customers have low shopping costs, so they will buy online after visiting the traditional retailer, then traditional retailers suffer from free-riding, as is assumed in the current paper.



The next question is what happens at state  $(1, 0; 0, 0)$ . I will derive conditions in which firm  $A$  makes no investment at this state, but starts investing in the new format after this format has been adopted by firm  $B$ . To see how this can happen, let  $e^*$  denote firm  $B$ 's optimal investment at state  $(1, 1; 0, 0)$  and let  $\hat{e}$  denote firm  $B$ 's optimal investment at state  $(1, 0; 0, 0)$  assuming firm  $A$  invests nothing at this state. Clearly  $\hat{e} > e^*$  because the entrant has more to gain by adopting the new format if the incumbent has not yet done so. Again define  $\Lambda^A$  as firm  $A$ 's continuation value at a given state. The following lemma states conditions in which firm  $A$  does not invest at the initial state.

**Lemma 12.** *In the free-riding game, if states  $(1, 0; 0, 1)$  and  $(1, 1; 0, 0)$  are unstable, then at state  $(1, 0; 0, 0)$  firm  $B$  invests in the new format and firm  $A$  does not invest if:*

$$\Lambda_{(1,1;0,0)}^A - \Lambda_{(1,0;0,0)}^A < \frac{1}{F'(0)} \quad (17)$$

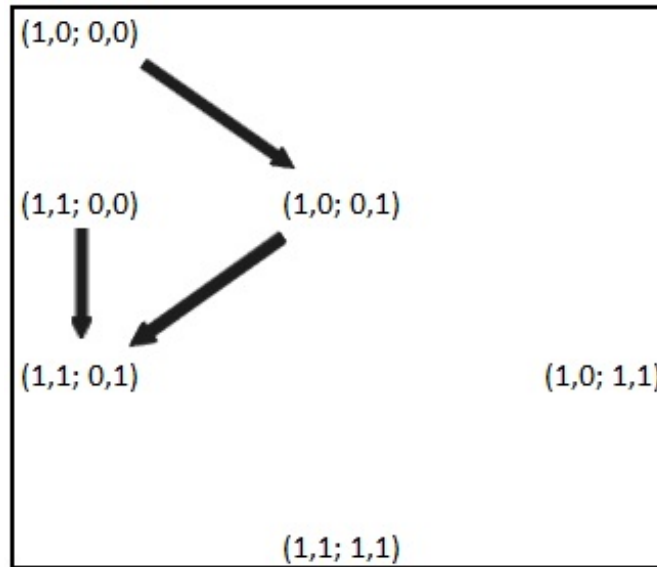
Note that as  $F''$  becomes sufficiently negative,  $e^*$  and  $\hat{e}$  both approach zero. This implies that  $\Lambda_{(1,1;0,0)}^A$  and  $\Lambda_{(1,0;0,0)}^A$  both approach  $\left(\frac{1}{1-\delta}\right)(V-\beta)(1-\alpha)$ , so the condition in Lemma 12 is guaranteed to hold. This leads to the following proposition.

**Proposition 8.** *In the free-riding game, if brand preferences are sufficiently strong ( $\beta$  is sufficiently large), and  $F''$  is sufficiently negative, such that the conditions of Lemmas 11 and 12 hold, then in equilibrium firm  $A$  invests nothing at the initial state, but starts investing in the new format after it has been successfully adopted by the entrant.*

Intuitively, when implementing a new format is difficult, the incumbent initially continues serving customers with the old format, knowing it is likely to take a long time for the entrant to destroy this business by successfully entering the new format. However, once the entrant finally succeeds (so the old format is destroyed), as long

as brand preferences are strong enough, the incumbent begins investing in the new format until it eventually succeeds as well.

**Figure 11. When the new format free-rides on the old format, the incumbent does not invest in the new format until it has been successfully adopted by the entrant. (Arrows indicate possible paths the game state can follow.)**



## 4 Conclusion

This paper has developed a model in which an incumbent and an entrant compete in a market where a new business format has become available. I have shown how equilibrium investment behavior is affected by the strength of brand preferences, the strength of format preferences, and the ease of implementing a format.

When brand and format preferences are both strong, if implementing a format is easy, this leads to an equilibrium in which each firm stays focused on a single format. This outcome depends on managers' ability to look ahead two steps. The entrant knows that if it adopts the new format, the incumbent will not retaliate because the entrant would then retaliate by adopting the old format. On the other hand,

the incumbent knows that if it adopts the new format, it has no more weapons for punishing the entrant, and so the entrant will adopt both formats.

One possible extension would be to assume two incumbents both initially use the old format, and they have the opportunity to invest in the new format. There could be equilibria in which both firms avoid the new format because they know this would cause the competitor to invest there as well. This is consistent with recent empirical work that found “contagion” effects in adoption of online brokerage, that is, when one financial firm began offering online trading, other similar firms were more likely to adopt this new format soon afterwards (Debruyne and Reibstein 2005).

It would also be interesting to study the case in which demand for the new format grows over time. As the new format became more popular, firms would presumably pass through phases where: (1) neither firm invests in the new format; (2) only the entrant invests in the new format (because the incumbent is worried about cannibalization); (3) both firms invest in the new format until one of them succeeds; and (4) both firms invest in the new format until they both succeed. In some cases, the new format would never become popular enough to reach phase 4, so only one firm would adopt. One implication is that if the new format grew slowly, the entrant would probably win because so much time would be spent in phase 2; but if it grew quickly, the incumbent would have a greater chance of winning since it would quickly reach phase 3.

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## Essay 3:

# The Strategic Importance of Accuracy in Conjoint Design<sup>1</sup>

### **Abstract**

Improved accuracy in conjoint analysis has important strategic implications. Even if two models provide unbiased part-worths, competitive game theory shows that the more accurate model (with lower error variance in an HB CBC model) implies that differentiation from competitors is more profitable. On the other hand, a less accurate model implies that each firm should forego differentiation and choose feature levels that provide customers the greatest utility (adjusting for marginal cost). I illustrate the theory by varying accuracy in a student-apparel application.

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<sup>1</sup> I thank John Hauser for his guidance on this project, Sawtooth Software for providing a grant for the software used in the study, and the SloanGear company for assisting with the study design. Helpful comments were provided by seminar participants at MIT.

## 1. Introduction

Academic researchers and practitioners continuously make investments in designing more accurate conjoint studies. For example, previous research has examined the importance of providing incentives for truthful answers (Ding, Grewal, and Liechty 2005; Ding and Huber 2009), and of using a sufficient number of questions, which includes training questions (Johnson and Orme 1996).

As highlighted by Swait and Louviere (1993), improvements in study design might affect both the relative importance of each feature and the amount of uncertainty of an analyst's predictions. Subsequent studies documented instances in which researchers cannot reject the hypothesis that various manipulations affect randomness rather than sensitivity parameters (see Louviere et al. 2002).

There is a common belief that some investments in accuracy are strategically unimportant. For example, if a manager is interested in knowing the most popular set of colors for a product, or the average amount customers would pay for an improvement in a feature, that manager might only care about relative part-worths of different features. As long as a study provides unbiased estimates of relative part-worths, the amount of uncertainty in respondents' behavior does not matter.

However, this paper demonstrates that study design improvements which lead to more accurate models (reduce the error term on product utility in the hierarchical Bayes choice-based-conjoint [HB CBC] analysis) have important strategic implications. When we model competitive price response endogenously, a model with high accuracy implies that differentiation from competitors is the most profitable strategy. On the other hand, a model with low accuracy implies that each firm should forego differentiation and choose feature levels that provide customers the greatest utility (adjusting for marginal cost). Such an inaccurate model which overstates uncertainty in consumer decision-making causes firms to produce products that are too similar to those of competing firms, leading

to destructive price competition.

I derive these results using a framework that incorporates game theory into conjoint analysis, in a similar manner to the approach used in several recent papers (Choi, Desarbo, and Harker 1990; Choi and DeSarbo 1994; Luo, Kannan, and Ratchford 2007; Luo 2009). In this framework, competing firms first set non-price features that depend on manufacturing set-ups that can be changed slowly or at high cost. Firms then adjust prices relatively quickly and at low cost until all prices reach a Nash equilibrium conditional on the non-price features chosen in the first stage.

My theoretical results generalize results from economic theory (de Palma et al. 1985; Irmen and Thisse 1998) to the more realistic demand and cost functions used in conjoint simulators. Whereas these earlier theoretical papers use stylized models in which all products have the same marginal cost, all customers have the same marginal utility of income, and the distribution of preferences is uniform, the current paper uses the more realistic assumptions typical of conjoint simulators, with marginal costs that vary across feature levels, heterogeneity in the marginal utility of income (i.e., the part-worth of price), and a flexible, non-parametric distribution of customer preferences.

To test the practical relevance of these results, I have collected data on conjoint preferences for student apparel. This study uses a 2 x 2 design, with respondents randomly assigned to either incentive compatible or hypothetical conditions, and to either a good design (with careful instructions, training questions, and graphical product profiles) or a less careful “quick-and-dirty” design (with brief instructions, no training, and text-only product profiles). I find that using a less careful design reduces both the internal consistency of a respondent’s answers and their external consistency relative to a hold-out task. According to my theory, this should discourage differentiation. On the other hand, the less careful design also leads to an (apparently spurious) increase in preference heterogeneity. In this case, the latter effect is stronger than the former, and so the less careful design actually leads to greater differentiation. Nonetheless, I illustrate the theory by directly adjusting the random noise parameter to account for variance



between the conjoint and holdout tasks. This adjustment leads to less differentiation, as predicted by my theory.

The rest of the paper is organized as follows. Section 2 discusses related theoretical literature. Section 3 defines the product design game studied in this paper. Section 4 presents analytical results. Section 5 describes a conjoint study that illustrates the theory. Section 6 concludes.

## **2. Related Theoretical Research**

Previous research has addressed existence and uniqueness of price equilibrium in product design models, and has also explored how price competition affects design decisions. The key theoretical contribution of the current paper is to derive results for the more general and realistic models of preference distributions and marginal costs that are used in conjoint analysis.

Choi et al. (1990, page 179) show the existence part of Proposition 1, but not the uniqueness part. Anderson et al. (1992, pages 365-366) show that a Hotelling model with linear transportation costs always has a unique pure strategy price equilibrium if outside differentiation is sufficiently high; this paper extends this result to a more general preferences distribution and cost function.

Caplin and Nalebuff (1991) also derive conditions for existence and uniqueness of price equilibrium for general demand models. Proposition 2 expands on this work in two respects. First, I allow for heterogeneity in customers' marginal utility of income, which is essential in order to build a realistic model with different customer segments. I also derive a restriction of the variance of this parameter ( $\lambda$ ) that ensures a unique equilibrium still exists. Also, whereas the uniqueness results in Caplin and Nalebuff (1991) make use of "dominant diagonal" conditions, which place restrictions on the absolute value of the second derivatives of the profit function, my proof in Appendix A instead derives conditions in which I can use results by Friedman (1990, page 86) to ensure the Jacobian

of the mapping from prices to profit first derivatives is negative quasi-definite, and so there can only be one set of prices that sets these derivatives equal to zero. For the conjoint-based design game, this turns out to allow for a proof of uniqueness under much less strict conditions than would be required for a dominant diagonal argument to hold (see Appendix A for more details).

Proposition 4 shows that de Palma et al.'s (1985) minimum differentiation result for the Hotelling model applies to a much more general demand model, and more general cost function.<sup>2</sup> I also derive a simple formula to determine the design that firms choose in this more general model when the random component of utility is sufficiently large.

### 3. Product Design Game

This section defines a conjoint-based product design game. This approach is similar to the one developed by Choi et al. (1990) and Choi and DeSarbo (1994).

Assume a market includes  $M$  consumers and  $N$  firms. Each firm sells one type of product chosen from the set of possible designs. The game proceeds in three stages.

Stage 1: Firms simultaneously choose product designs.<sup>3</sup>

Stage 2: Firms simultaneously set prices.

Stage 3: Each consumer decides which product to purchase, and payoffs are realized.

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<sup>2</sup> To see the relationship between these results, note that a discrete version of the Hotelling model, with customers and potential products evenly spaced along the line (including one product in the center of the line) is a special case of the conjoint-based model. Proposition 4 implies that all firms would choose the center product when  $\mu$  is sufficiently high.

<sup>3</sup> In some theory papers, firms choose designs sequentially (e.g., Prescott and Visscher 1977; Moorthy 1988). The key results of this paper will hold in either case.

The model uses the following notation:

$i = 1, \dots, M$	index of consumers
$j, k = 1, \dots, N$	indexes of firms
$\beta_i$	vector of consumer $i$ 's part-worths for each possible level of each non-price attribute
$X_j$	vector of 0's and 1's indicating whether firm $j$ selected a given attribute level
$\lambda_i$	customer $i$ 's price sensitivity (part-worth for a one-dollar decrease in price)
$P_j$	firm $j$ 's price
$C_j$	firm $j$ 's marginal cost of production, given its product design decision
$\mu$	magnitude of uncertainty in product utility (this encompasses noise in customer behavior, factors outside the model that influence choice, and random variance in choice behavior across settings)
$Y$	each consumer's income

The net utility to consumer  $i$  from purchasing good  $j$  is:

$$(1) \quad U_{i,j} = \beta_i' X_j - \lambda_i P_j + \mu \varepsilon_{i,j}$$

where the  $\varepsilon_{i,j}$  are assumed to be i.i.d. with a Gumbel distribution. Assuming that no outside good is available and that each consumer purchases one unit of the good that provides the highest utility, the probability of consumer  $i$  choosing product  $j$  is:

$$(2) \quad D_{i,j} = \frac{\exp\left[\frac{1}{\mu}(\beta_i' X_j - \lambda_i P_j)\right]}{\sum_{k=1}^N \exp\left[\frac{1}{\mu}(\beta_i' X_k - \lambda_i P_k)\right]}$$

Assuming zero fixed costs and constant marginal costs, expected profits for firm  $j$  are:

$$(3) \quad \pi_j = (P_j - C_j) \left( \sum_{i=1}^M D_{i,j} \right)$$

Finally, proving equilibrium existence requires an upper bound on the price that firms can charge. This bound can be arbitrarily large, but it is important that prices cannot diverge to infinity. Following the example of previous literature on existence of equilibria in discrete choice models (see Caplin and Nalebuff 1991, page 35; Anderson et al. 1992, pages 162 and 365), I assume that consumers each have an income level  $Y$  which is the maximum amount they can spend on the product, so any firm that sets its price higher than  $Y$  will receive zero demand. Although this is a sufficient technical condition to ensure equilibrium existence, it is not necessary. More realistically, as long as there is some outside good that is weakly substitutable for the product being studied, then prices are bounded, and an equilibrium will exist.

## 4. Analytical Results

This section presents analytical results related to the game described in section 2. Subsection 4.1 derives conditions that guarantee existence and uniqueness of price equilibria, and subsection 4.2 shows how price competition affects product design choices.

## 4.1 Existence and Uniqueness of Equilibria

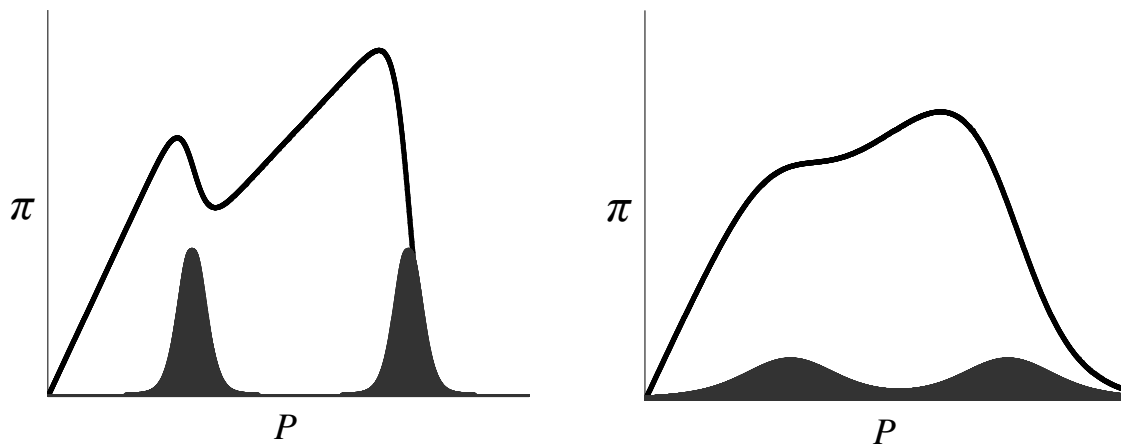
If there is no uncertainty in product utilities, there cannot be a pure strategy price equilibrium in which any firm earns positive profits.<sup>4</sup> However, such an equilibrium is guaranteed to exist if there is enough uncertainty to make each firm's profit function quasi-concave. Figure 1 illustrates this point with two examples of logit-based profits functions with two customer types. In the graph on the left, profits start to fall as the firm raises its price past the valuations of the low types, but start to rise again once it has lost most of these customers. Therefore, this profit function is not quasi-concave. The graph on the right assumes the random component to utility is larger, so the firm loses the low-type customers more gradually, and profits continue to increase until it starts losing high-type customers. Therefore, this profit function is quasi-concave.

When firms have profit functions like the one on the left, their best-response correspondences have discontinuous jumps where changes in competitive prices compel a firm to start serving new customer types. These discontinuities can prevent the correspondences from crossing. However, when firms have quasi-concave profit functions like the one on the right, their best-response functions are continuous, which ensures that they cross and a pure strategy equilibrium exists (see Fudenberg and Tirole 1991, page 34).

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<sup>4</sup> To see why, imagine such an equilibrium exists. If two firms provide exactly the same utility to a customer, so this customer splits demand between the firms, either firm could increase its profits by cutting its price by a small amount  $\epsilon$ . In the absence of such ties, a firm can increase profits by raising its price by a small amount  $\epsilon$  and still retaining the same level of demand. Therefore, no such equilibrium can exist.

**Figure 1. Logit-Based Profit Functions With Two Customer Types**



*Note: Shaded areas represent the pdf of customer valuations.*

Appendix A formally shows that a sufficiently high value of  $\mu$  guarantees quasi-concavity of each firm's profit function (so a pure strategy price equilibrium exists) and also guarantees that the mapping from prices to best responses is a contraction (so there can only be one equilibrium). These statements are summarized in the following proposition.

**Proposition 1.** *There is a  $\mu^*$  such that every pricing sub-game has a unique pure strategy equilibrium for all  $\mu \geq \mu^*$ .*

**Proof.** See Appendix A.

**Discussion.** Proposition 1 assumes that there is an exogenous upper bound on prices. The proposition is no longer true without this bound. For example, a model with two firms and two customer types, one of which is extremely price sensitive and one of which is extremely price insensitive, does not have a pure strategy equilibrium for any level of utility randomness if prices can increase without bound. Rather, this model would only have mixed strategy equilibria of the type characterized by Varian (1980).

Given that equilibrium prices tend to increase with  $\mu$ , once this parameter is large enough, every pricing sub-game has a trivial equilibrium in which all firms charge the maximum price allowed. It would be nice to have assurance that the model can also find interior equilibria. Proving this requires two additional restrictions. First, there needs to be a lower bound on prices, which I denote by  $L$ , in addition to the upper bound  $Y$ .<sup>5</sup> Both of these bounds will grow linearly with  $\mu$ . The second restriction is that the variance in marginal utility of income cannot be too high relative to the mean. More formally:

**Assumption 1:** 
$$\left(\frac{N-2}{N-1}\right)Var[\lambda] < (E[\lambda])^2$$

Assumption 1 ensures that solutions to the first order conditions at the proposed equilibrium are local maxima instead of a local minima, and that optimal prices are sufficiently insensitive to competing prices that the equilibrium is unique (see the proof in Appendix A for more details).

**Proposition 2.** *Under Assumption 1, for any  $\mu$  sufficiently large there is a range of prices  $[L, Y]$  such that every pricing sub-game has a unique interior equilibrium if prices are restricted to this range.*

**Proof.** See Appendix A.

**Discussion.** Proposition 2 guarantees that a unique price equilibrium always exists if the random component of utility ( $\mu$ ) is sufficiently high and the variance of marginal utility of income is sufficiently low. However, note that a large value of  $\mu$  is sufficient, but not necessary. As illustrated by Figure 1, a large value of  $\mu$  is needed to ensure the profit function is quasi-concave only if the market is highly segmented (for example, with one group who strongly prefers one product over the other, and a separate group who is

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<sup>5</sup> To justify the lower bound, we could assume that customers make negative inferences about an unobservable quality attribute if  $P < L$  and make no inference about this attribute if  $P \geq L$ , so there is a pooling equilibrium in which no firm sets price below  $L$ .

roughly indifferent between the products). Because most conjoint estimation is done using hierarchical Bayes methods, which shrink individual estimates toward the mean and thereby reduce the amount of segmentation in the part-worth distribution, the resulting profit function is likely to be quasi-concave even without a large (and unrealistic) value of  $\mu$ .<sup>6</sup>

However, as noted by Andrews, Ansari, and Currim (2002), even hierarchical Bayes methods that assume a normal population distribution can accurately fit segmented markets. The empirically relevant question is whether the shrinkage produced by HB methods reduces segmentation by enough to ensure that a price equilibrium exists. In our empirical application reported later in this paper, this appears to be the case, as we are able to find price equilibria even using the unadjusted  $\mu$  parameter that is estimated using HB methods.

## 4.2 How Price Competition Affects Product Design

This section shows how accounting for price competition affects design choices. I derive conditions in which price competition causes firms to differentiate their products, but I also derive conditions in which all firms choose the same design.

Price competition encourages firms to choose product designs that make customers less price sensitive. For the logit model, price sensitivity is lower when preferences are more extreme. This can be demonstrated by differentiating (2) to find the first derivative of customer type  $i$ 's demand for firm  $j$ 's product, with respect to the firm's own price:

$$(4) \quad \frac{\partial D_{i,j}}{\partial P_j} = \frac{-\lambda_i}{\mu} D_{i,j}(1 - D_{i,j})$$

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<sup>6</sup> A recent survey by Sawtooth Software found that 78% of conjoint studies conducted with their software use hierarchical Bayes methods for estimation (Source: company correspondence).



Equation (4) shows that price sensitivity is greatest when a firm receives one-half of a customer type's demand, and approaches zero as the demand share approaches either zero or one. In other words, price sensitivity is lowest when customers either strongly like or strongly dislike the firm.

Although the precise level at which price sensitivity is greatest depends on the assumption of a logit demand model, similar results would hold for any model in which customers with strong preferences for one product or another are less price sensitive than those with weaker preferences. This seems like an intuitive and reasonable property of a demand function, and it is consistent with experimental evidence that giving customers more information about various products, so that they develop stronger preferences, reduces their price sensitivity (Lynch and Ariely 2000).

If all customer types have the same marginal utility of income, denoted by  $\lambda$ , then the total price sensitivity of these customers is:

$$\begin{aligned} \sum_i \frac{\partial D_{i,j}}{\partial P_j} &= \left( \frac{-\lambda}{\mu} \right) \sum_i [D_{i,j} - D^2_{i,j}] \\ (5) \qquad &= \left( \frac{-\lambda}{\mu} \right) M \left[ \bar{D}_j - (\bar{D}_j)^2 - \text{Var}_i[D_{i,j}] \right] \end{aligned}$$

where  $\bar{D}_j$  is the average demand for firm  $j$ 's product and  $\text{Var}_i[D_{i,j}]$  is the variance across customers. This shows that, conditional on average demand, an increase in the variance of demand across customer types reduces price sensitivity.

*Figure 2. Increased Variance of Demand Reduces Total Price Sensitivity*

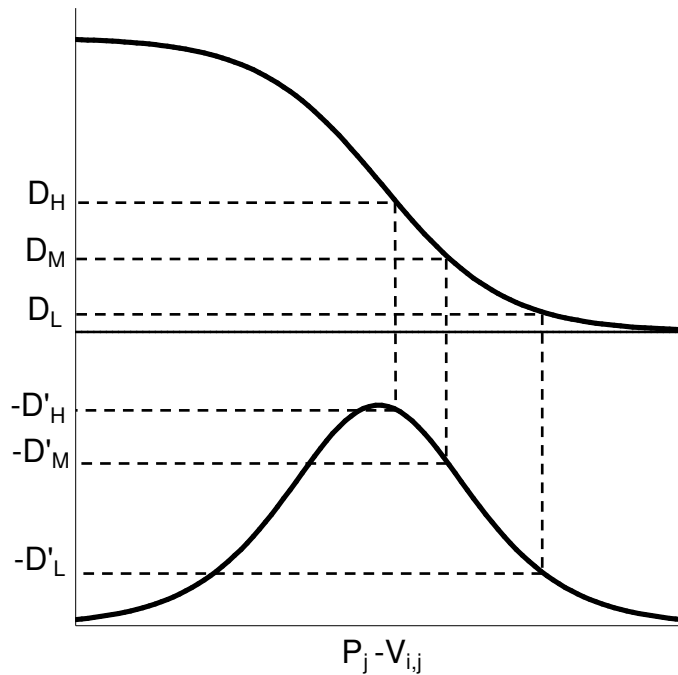


Figure 2 illustrates this point. Assume that, for a given price, two customer types each have a “medium” level of demand for a firm’s product, denoted by  $D_M$ . Now assume that the firm changes its design in such a way that, given the same price, one type’s demand increases to a higher level,  $D_H$ , while the other type’s demand falls by the same amount to a lower level,  $D_L$ . The type with higher demand undergoes an increase in price sensitivity (from  $-D'_M$  to  $-D'_H$ ), but the type with lower demand undergoes an even larger drop in price sensitivity (from  $-D'_M$  to  $-D'_L$ ), so total price sensitivity is reduced.

Assuming the current price was optimal when all customers had demand  $D_M$ , the firm would now like to raise its price after this reduction in average price sensitivity.

These results are formalized by Proposition 3, which shows that if preferences are symmetric across firms, more extreme preferences lead to higher equilibrium prices and profits.<sup>7</sup>

<sup>7</sup> Both of the propositions in this section assume every pricing sub-game has a unique interior equilibrium in a price range constructed as in the proof of Proposition 2.

**Proposition 3.** *If all firms have the same marginal cost and the distribution of preferences is symmetric across firms (meaning that when all firms charge the same price, they all have the same demand), then every firm sets the following price in equilibrium:*

$$(6) \quad P = C + \left( \frac{\mu}{E[\lambda]} \right) \frac{\frac{1}{N}}{\left[ \left( \frac{1}{N} \right) - \left( \frac{1}{N} \right)^2 - \text{Var}_i[D_{i,j}] \right]}$$

where  $\text{Var}_i[D_{i,j}]$  is the variance across customer types of the probability of purchasing firm  $j$ 's product given that all firms set the same price. Total equilibrium demand for each firm is  $\frac{M}{N}$ .

**Proof.** See Appendix A.

**Discussion.** Note that the right side of (6) is increasing in  $\text{Var}_i[D_{i,j}]$ , and so firms earn higher profits when customers have more extreme preferences, that is, when there is a high variance across customers of demand for a given firm's product. Since more differentiation typically leads to greater variance in demand, this proposition provides insight into how price competition encourages differentiation. For example, if two firms each design a product composed of several binary attributes, and preferences for these attributes are symmetric and uncorrelated, Proposition 3 says that profit maximization requires the firms to differentiate along all of the binary attributes.

Whereas Proposition 3 is consistent with the standard intuition that price competition encourages product differentiation, I now derive analytical results that demonstrate a case in which price competition does *not* cause firms to differentiate along the conjoint attributes. In particular, when  $\mu$  is sufficiently high, average part-worth sums and

marginal cost become the only determinants of design choice. One implication is that an inaccurate model that overstates  $\mu$  tends to discourage firms from differentiating their products.

To see why, we take the derivative of price sensitivity, given by (4), with respect to part-worth sums:

$$(7) \quad \frac{\partial^2 D_{i,j}}{\partial V_{i,j} \partial P_j} = \frac{-\lambda_i}{\mu^2} (D_{i,j})(1 - D_{i,j})(1 - 2D_{i,j})$$

As  $\mu \rightarrow \infty$ , every  $D_{i,j}$  converges to  $(1/N)$ . Therefore, (7) converges to the same value for all customer types with a given marginal utility of income,  $\lambda$ :

$$(8) \quad \lim_{\mu \rightarrow \infty} \left[ \frac{\partial^2 D_{i,j}}{\partial V_{i,j} \partial P_j} \right] = \left( \frac{-\lambda_i}{\mu^2} \right) \frac{(N-1)(N-2)}{N^3}$$

Now imagine part-worth sums change for two customer types with the same  $\lambda$ . The asymptotic effect on price sensitivity depends only on the sum of these changes. The effect of differentiation becomes negligible. Another way to see this, in terms of equation (5), is that differentiation affects equilibrium prices by changing the variance of demand. Because variance is the mean *squared* deviation, this is a second order effect, which is insignificant for small changes in demand around the same value.

Even though the demand of each customer type converges to  $(1/N)$ , product design choices remain important in the following sense. Part-worth sums have an effect on demand of order  $(1/\mu)$ . Because equilibrium prices increase linearly with  $\mu$ , this implies an asymptotic impact on equilibrium profits of order one. Marginal cost also has an impact of order one, so the trade-off between popularity and marginal cost determines the asymptotically optimal design. Neither of these values depends on the design choices of

other firms. All firms therefore choose the same design when randomness in product utility is sufficiently high. Formalizing this result requires one additional assumption:

**Assumption 2:**  $Cov_i[\lambda_i, \beta_i] = \vec{0}$

where  $Cov_i[\lambda_i, \beta_i]$  is a vector giving the covariance of marginal utility of income with each element of the part-worth vector, and  $\vec{0}$  is a vector of zeros. This assumption, which says that marginal utility of income is uncorrelated with other part-worths, is not strictly necessary, but it greatly simplifies the formula for the optimal design.

**Proposition 4.** *Under Assumptions 1 and 2, if  $\mu$  is sufficiently high then every firm chooses the same design in equilibrium. For each firm  $j$ , this design maximizes:*

$$(9) \quad F(\bar{V}_j, C_j) = \left[ (E[\lambda])^2 - \left( \frac{N-2}{N-1} \right) Var[\lambda] \right] \left( \frac{\bar{V}_j}{E[\lambda]} \right) - \left[ (E[\lambda])^2 - \left( \frac{N-2}{N} \right) Var[\lambda] \right] C_j$$

**Proof.** See Appendix A.

Expression (9) can be simplified if *at least one* of the following is true: (i) There are only two firms in the market. (ii) There are sufficiently many firms in the market. (iii) All customers have the same marginal utility of income. (iv) Product design choices do not affect marginal cost. When at least one of these is true, then expression (9) reduces to:

$$(10) \quad G(\bar{V}_j, C_j) = \frac{\bar{V}_j}{E[\lambda]} - C_j$$

**Discussion.** Proposition 4 shows that firms do not need to differentiate along the conjoint attributes if the level of “true” uncertainty in product utilities is very high, that is, if there is real randomness in customers’ purchase behavior, or if factors outside of the conjoint model are driving choice. On the other hand, this proposition also shows that if

the true conjoint parameters provide a very accurate prediction of customer behavior, but we overestimate the amount of randomness in behavior due to flaws in the conjoint study, this will prevent firms from differentiating their products when in fact they should. This idea is explored further by the application in the next section.

## **5. Conjoint Study on Student Apparel**

To illustrate the relevance of this theory, I collected data on conjoint preferences for student apparel. I conducted this study in cooperation with a student-run company that sells school-branded clothing to students and alumni. The study focused on their winter clothing product line.

After describing the study design, I present results, including equilibrium product designs assuming two competing firms each offer a single article of clothing. I find that a better study design leads to greater internal and external consistency of responses. According to the theory from the previous section, this should encourage differentiation. However, the less careful design also leads to apparently greater (probably spurious) part-worth heterogeneity, which turns out to be a stronger effect, so that the less careful design actually leads to greater differentiation. Despite this, I show that direct adjustments to the random noise parameter to account for differences between the conjoint and holdout tasks do in fact discourage differentiation, as the theory from the previous section would suggest.

### **5.1 Study Design**

The product features and levels used were as follows:

**Type of clothing:** Hooded sweatshirt; Fleece vest; or Track jacket

**Color:** Red; or Grey

**Logo:** School logo; or No school logo

**Price:** Base level (\$30 for the sweatshirt; \$40 for each of the other two); or Base level plus \$10

To recruit respondents, I e-mailed an MBA marketing class and invited them to visit a website where they could participate in the study. As compensation for participating, the students were promised that one in ten respondents would receive an article of clothing.

The study used a 2x2 design, with respondents randomly assigned to either incentive compatible (IC) or hypothetical conditions (HC) and to either good design or a less careful “quick-and-dirty” design. In the IC condition, I explained that their responses to the conjoint questions would be used to estimate their preferences, and if they won a prize it would be chosen based on their estimated preferences. This mechanism provides an incentive for truthful responses (Ding 2007). The HC simply stated that one in ten respondents would win a prize, but did not say how it would be chosen.

The good design used careful instructions, training questions, and graphical product profiles, whereas the less careful design used brief instructions, no training, and text-only product profiles. Appendix B presents sample conjoint questions and the study layout for each condition.

The study was designed using Sawtooth Software’s SSI Web CBC module. Each respondent answered 16 choice-based-conjoint questions, each consisting of 5 product profiles. Conjoint part-worths were estimated using Sawtooth’s CBC/HB module. All other analysis was conducted in Matlab.

## 5.2 Results

Of the students who received the email invitation, 53 out of 107 (49.5%) began the survey, and 38 out of these 53 (71.7%) successfully completed the survey. The completion rates were 14 out of 18 (78%) in the good/IC condition, 5 out of 6 (83%) in the good/HC condition, 10 out of 15 (67%) in the less careful/IC condition and 9 out of 14 (64%) in the less careful/HC condition. Although these numbers show a trend toward higher completion rates in the good condition, this difference was not statistically significant ( $p > 20\%$ ).

As a manipulation check, each respondent was asked to rate the survey on a scale of 1 to 7 along several dimensions related to the ease of answering questions and their effort level. Figure 3 presents the questions.

Figure 3. Feedback Questions

We are interested in your honest feedback on this survey. Please rate how much you agree with each of these statements on a scale from 1 to 7, where 1 = strongly disagree and 7 = strongly agree.

	Strongly disagree						Strongly agree
	1	2	3	4	5	6	7
The questions were clear and easy to answer.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
I tried my best to give accurate answers to every question.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
The survey was so tedious it was difficult to stay focused.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
I treated each question like a real purchase decision.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
I answered some questions randomly (without even looking at the options).	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Table 1 presents average ratings for each question, broken down by condition. On average, the good design is rated as more clear and easy, and less tedious. It also elicits higher effort and less random responses. Interestingly, the use of incentive compatible design leads respondents to rate the survey somewhat more clear and easy, and less tedious, but it does not affect effort to be accurate, treating the responses like a real



purchase decision, or random responses. One possible explanation is that the respondents knew the survey was for a dissertation project, so they put in high effort even without incentives for accuracy.<sup>8</sup>

Table 1. Survey Feedback (scale of 1 to 7), by condition

	Good/IC	Good/HC	Less/IC	Less/HC
Number of respondents	14	5	10	9
Questions clear and easy	6.5	6.0	5.3	5.0
Tried my best to be accurate	6.7	6.8	5.8	6.3
Survey tedious	2.6	2.4	4.2	5.1
Like real purchase decision	5.5	6.2	5.0	5.1
I answered randomly	1.1	1.0	1.8	1.7

To check the significance of these effects, I run an OLS regression for each question, regressing responses on a constant and dummy variables for whether the respondent was in the good condition and had incentives. Because incentives did not have a significant effect on these responses or other behavior in the study, for the rest of results I pool together the 19 respondents in the two good conditions, and also pool together the 19 in the two less careful conditions.

When the data in Table 1 are pooled in this manner, the differences between the good condition and less careful condition are significant at a level of  $p = 2\%$  or better for ratings of clear and easy questions (average rating of 6.4 in the good condition vs. 5.2 in the less careful condition), effort to be accurate (6.7 vs. 6.1), survey tediousness (2.5 vs. 4.6), and random responses (1.1 vs 1.7); and are marginally significant ( $p=11\%$ ) for treating questions like a real purchase decision (5.7 vs. 5.1).<sup>9</sup>

<sup>8</sup> This explanation is consistent with experimental evidence that social commitment can elicit truthful survey responses, thereby serving as a substitute for financial incentives (Jacquemet 2010).

<sup>9</sup> The good design dummy variable also has similar significance levels in the regressions that control for both good design and incentives.

To compute part-worths, I run separate HB estimation procedures for the good and less careful designs. The estimation was performed Table 2 reports the average part-worths for each level.

Table 2. Average part-worth (relative to the feature’s base level)

	Good	Less Careful
Fleece Vest	-0.7	-0.9
Track Jacket	4.9	2.7
Grey	1.5	0.6
School Logo	4.5	3.0
Plus \$10	-2.7	-1.2

In both conditions, the track jacket has a much higher average part-worth than either of the other clothing types, and the fleece vest has a slightly lower average utility than the sweatshirt.<sup>10</sup> On average, the respondents strongly prefer clothing with a school logo, have a preference for grey over red, and dislike high prices.

Note that in most cases the absolute value of the average part-worth is higher in the good condition than in the less careful condition. Because reported part-worths are equivalent to the part-worth divided by utility randomness (Swait and Louviere 1993), this suggests that randomness is higher in the less careful condition, which is consistent with the responses reported in Table 1. This was also confirmed by a greater U-squared measure of model fit in the good condition (78.2%) relative to the less careful condition (69.7%).

While Table 2 shows that internal (within task) randomness is higher in the less careful condition, previous research suggests that utility parameters should also be scaled to account for external (across conditions) randomness (Sawtooth 2003; Salisbury and Feinberg 2010). In order to perform this type of adjustment, after the conjoint task and an intervening distraction task, I asked respondents to answer a hold-out question in which they ranked their top 5 out of 12 profiles. Regardless of the respondent’s initial

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<sup>10</sup> Average part-worths for clothing types are reported holding price constant. For example, the reported average part-worth for the track jacket (relative to the sweatshirt) would apply if all other features are held constant and both are priced at \$40.

condition, for the hold-out task all respondents saw profiles with pictures, as in the good condition.

Using the approach proposed by Hauser (1978), I compute U-squared measures of how much information the model from the conjoint exercise provides about a respondent's top five choices in the hold-out task.<sup>11</sup> The U-squared is much larger in the good condition (45.2%) than in the less careful condition (23.6%).

The relatively low external validity of the less careful model can be explained as follows. Many respondents change their evaluations of the products after viewing the images in the hold-out task. For example, in the open-ended feedback at the end of the survey, one respondent in the less careful (text) condition wrote, "Just got to say that the products that I thought sounded good actually looked really bad in the pictures to me, so my results changed." Another wrote, "Wish I had seen the pictures before the survey. They grey ones don't look that bad!" As a result, many respondents in the text condition selected as their first choice in the hold-out task a product that the conjoint model predicted they were very unlikely to choose. Because the U-squared measure punishes these very bad predictions, it was relatively low in the less careful condition.

To account for this randomness in product utility across contexts, I hold the conjoint parameters fixed and then use maximum likelihood estimation (MLE) to find the adjustment to the  $\mu$  parameter that leads to the best fit to the hold-out data in each condition. In the good condition I adjust each customer  $i$ 's part-worth vector  $\beta_i$ , as follows:

$$(11) \quad \beta_i^{ADJ} = \frac{\beta_i}{\mu_G^{ADJ}}$$

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<sup>11</sup> To do this, I first compute the information measure for each respondent's first choice. I then remove the first choices from the dataset, and compute the information measure for the second choice, and so on.

In the less careful condition I make a similar computation, but with a different adjustment factor:

$$(12) \quad \beta_i^{ADJ} = \frac{\beta_i}{\mu_L^{ADJ}}$$

Note that dividing the part-worth vectors by  $\mu_G^{ADJ}$  and  $\mu_L^{ADJ}$  is equivalent to multiplying the scale factor representing utility randomness by these same terms (Swait and Louviere 1993).

The MLE estimates for these parameters are  $\mu_G^{ADJ} = 1.55$  and  $\mu_L^{ADJ} = 2.08$ . This implies that the numbers in Table 2 should be reduced by about one-third for the good condition, and cut in half for the less careful condition. To test the significance of the difference between these estimates, I also compute the MLE estimate using data from both conditions under the constraint  $\mu_G^{ADJ} = \mu_L^{ADJ}$  and use a likelihood ratio test to see whether a model with a separate parameter for each condition fits significantly better. This test indicates that the difference between these parameter estimates is significant at a level of  $p = 5\%$ .

Finally, I test the implications of these estimates in a product design game. To keep the exposition simple, I assume two competing firms each offer one track jacket with a logo (that is, I do not allow them to create a sweatshirt, fleece vest, or non-logo product in this game). The firms simultaneously select colors, and then compete on price. The question is whether they will both choose the more popular color (grey) in equilibrium, or one firm will differentiate its jacket with the less popular color (red).

To illustrate the importance of designing products using the correct model of customer behavior, I make the following assumptions:

1. Because firms can easily adjust prices once they observe demand, the price equilibrium always depends on the “true” model of customer behavior. I assume that true customer behavior is represented using the adjusted part-worths from the good design, as computed in equation (11).
2. In some cases, firms make product design choices using an incorrect model of customer behavior. This causes the firms to make incorrect predictions about the equilibrium prices and profits that will result from each design, and might therefore cause them to make design choices that turn out to be sub-optimal.

Table 3 reports “true” equilibrium prices and profits (using the good design adjusted for external validity) for the case in which both firms choose a grey jacket, and also for the case in which one firm chooses grey while the other chooses red.<sup>12</sup>

Table 3. “True” Equilibrium Prices and Profits,  
Computed Using the Good Design Adjusted for External Validity

	Firm A	Firm B		Firm A	Firm B
Color	Grey	Grey	Color	Red	Grey
Marginal Cost	\$40.00	\$40.00	Marginal Cost	\$40.00	\$40.00
Equilibrium Price	\$51.40	\$51.40	Equilibrium Price	\$52.63	\$56.31
Profit Margins	\$11.40	\$11.40	Profit Margins	\$12.63	\$16.31
Demand	9.5	9.5	Demand	8.3	10.7
Profits	\$108.30	\$108.30	Profits	\$104.64	\$174.73

When both firms choose the grey jacket, the equilibrium price for each firm is \$51.40. Each firm is expected to capture half of the 19 customers, so they each earn a profit of \$108.30. If Firm A deviates and produces a red jacket, the Firm B’s equilibrium price increases to \$56.31, while Firm A’s increases to \$52.63. Despite this increase in prices, Firm A ends up worse off because choosing a less popular color causes its expected equilibrium demand to fall to 8.3, and its profits fall to \$104.64. Therefore, in equilibrium both firms choose the color grey.

<sup>12</sup> I compute equilibrium prices using an algorithm similar to the one described by (Choi et al. 1990). Marginal cost numbers are disguised to protect confidentiality of the company’s data, although this does not have a meaningful effect on the results.

Table 4 presents the incorrect prediction of equilibrium prices and profits that results if a firm uses the good design but does not make the adjustment for external validity, that is, if they make predictions using  $\beta_i$  rather than  $\beta_i^{ADJ}$  from equation (11).

Table 4. Incorrect Prediction of Equilibrium Prices and Profits,  
Computed Using the Good Design *Not* Adjusted for External Validity

	Firm A	Firm B		Firm A	Firm B
Color	Grey	Grey	Color	Red	Grey
Marginal Cost	\$40.00	\$40.00	Marginal Cost	\$40.00	\$40.00
Equilibrium Price	\$47.35	\$47.35	Equilibrium Price	\$49.54	\$53.18
Profit Margins	\$7.35	\$7.35	Profit Margins	\$9.54	\$13.18
Demand	9.5	9.5	Demand	8.0	11.0
Profits	\$69.86	\$69.80	Profits	\$76.08	\$145.32

Table 4 indicates that when Firm A switches to the red design, equilibrium prices increase by more than was the case for Table 3. To be precise, Firm B’s price increases by \$5.83 (vs. \$4.91 previously), and Firm A’s price increases by \$2.19 (vs. \$1.23). Furthermore, the “incorrect” predictions in Table 4 suggests that firm A’s profits increase from \$69.86 to \$76.08 if it chooses a red design, while the “true” model in Table 3 indicates that this design decision would actually cause firm A’s profits to fall from \$108.30 to \$104.64.

Recall that the only difference between the models used in Tables 3 and 4 is that the former multiplies the  $\mu$  by an adjustment factor of 1.55. Thus, the “correct” model with greater randomness in customer behavior indicates that firms should focus more on providing utility to customers and less on differentiating from competitors. This is consistent with the theoretical results presented in section 4.2.

Finally, table 5 reports another set of incorrect predictions that result from using the *less careful* design adjusted for external validity, that is,  $\beta_i^{ADJ}$  from equation (12).

Table 5. Incorrect Prediction of Equilibrium Prices and Profits,  
Computed Using the Less Careful Design Adjusted for External Validity

	Firm A	Firm B		Firm A	Firm B
Color	Grey	Grey	Color	Red	Grey
Marginal Cost	\$40.00	\$40.00	Marginal Cost	\$40.00	\$40.00
Equilibrium Price	\$74.45	\$74.45	Equilibrium Price	\$78.53	\$81.50
Profit Margins	\$34.45	\$34.45	Profit Margins	\$38.53	\$41.50
Demand	9.5	9.5	Demand	9.1	9.9
Profits	\$327.32	\$327.25	Profits	\$352.28	\$409.05

Note that the incorrect predictions in Table 5 indicate that equilibrium prices and profits are much higher than the true predictions in Table 3. This is because the bad design implies the customers' behavior is much more random, which in turn implies they are less price sensitive so equilibrium prices are higher.

However, the bad design also predicts that firm A's equilibrium profits increase if it switches from the grey to the red design. Thus, in this case the "incorrect" model with greater randomness (Table 5) suggests that firms should focus more on differentiation than does the "correct" model with less randomness (Table 3). This is contrary to the theory presented in the previous section, and requires an explanation. It turns out that, in addition to increasing randomness in behavior (and perhaps as a result of this), the bad design also leads to a big increase in estimated part-worth heterogeneity. In particular, for the good design only 3 out of 19 (about 16%) of the respondents have a higher part-worth for red than for grey; whereas for the bad design 8 out of 19 (about 42%) do. Similar effects on part-worth heterogeneity were also observed for the other product features. This increase in heterogeneity overwhelms the effect of increased randomness, leading to the unanticipated result that the bad design generates recommendations that firms should differentiate their products, whereas a good design recommends that they should not.

To summarize, comparing Tables 3 and 4 shows that an incorrect model that *understates* the amount of randomness in customer behavior could cause firms to differentiate their products when they should instead focus on providing greater utility to customers. This is

consistent with the theoretical results in the previous section. On the other hand, comparing Tables 3 and 5 leads to the unexpected result that a bad design, which *overstates* randomness, can also cause firms to differentiate their products too much, since the bad design also leads to an increase in estimated preference heterogeneity.

## 6. Conclusion

This paper has shown how improved accuracy in conjoint studies affects product design strategies, when firms select designs accounting for competitive price response. I derived conditions in which a unique price equilibrium is guaranteed to exist. I also showed that price competition encourages firms to differentiate their products unless the randomness in utility is very large, in which case all firms choose the same design, which maximizes a simple function of average utility and marginal cost. Given the increasing trend toward incorporating game theoretic price competition into conjoint simulators, this paper demonstrates the importance of accurately estimating the amount of noise in customers' purchasing behavior.

While the current paper has focused on the importance of designing accurate studies to avoid *overstating* the amount of randomness in product choice, the opposite point holds as well. In particular, if firms *understate* the randomness in product choice, then they will focus too much on differentiating their products at the expense of providing greater utility to customers. For this reason, in cases where there are additional sources of uncertainty in choice decisions beyond those in the conjoint task, it is important to use methods (e.g., Salisbury and Feinberg 2010) that make adjustments to account for this additional randomness.



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## Appendix A. Proofs of Propositions 1 to 4

### Proof of Proposition 1

The profit function defined in equation (3) is continuous in prices, and we can restrict each firm  $j$ 's price to the compact set  $[C_j, Y]$ . To prove existence we need to show that each firm's profit function is quasi-concave in its own price (see Fudenberg and Tirole 1991, page 34). In fact, we will show concavity, which implies quasi-concavity.

For a profit function of the form  $\pi = (P - C)D$  the first derivative is  $\pi' = D + (P - C)D'$  and the second derivative is  $\pi'' = 2D' + (P - C)D''$ . In the case of the CBD game with multiple customer types, the second derivative of firm  $j$ 's profit function is:

$$(A1) \quad \pi_j'' = \sum_i (2D'_{i,j} + (P_j - C_j)D''_{i,j})$$

Taking first and second derivatives of equation (2), we have:

$$(A2) \quad D'_{i,j} = \frac{-\lambda_i}{\mu} D_{i,j} (1 - D_{i,j})$$

$$(A3) \quad D''_{i,j} = \left( \frac{\lambda_i}{\mu} \right)^2 D_{i,j} (1 - D_{i,j}) (1 - 2D_{i,j})$$

We now plug (A2) and (A3) into (A1):

$$(A4) \quad \pi_j'' = \sum_i \left[ -2 \left( \frac{\lambda_i}{\mu} \right) D_{i,j} (1 - D_{i,j}) + (P_j - C_j) \left( \frac{\lambda_i}{\mu} \right)^2 D_{i,j} (1 - D_{i,j}) (1 - 2D_{i,j}) \right]$$

We can divide each term in brackets by  $(\lambda_i / \mu) D_{i,j} (1 - D_{i,j})$  without changing its sign. Also note that  $P_j \leq Y$  and  $(1 - 2D_{i,j}) \leq 1$ . Each term in brackets is therefore negative if:

$$(A5) \quad -2 + (Y - C_j) \left( \frac{\lambda_i}{\mu} \right) < 0$$

which is true for all  $\mu$  sufficiently large. For each firm  $j$ , take a value of  $\mu$  large enough that (A5) holds for each consumer  $i$ , and call the maximum over all these values  $\mu^{**}$ . Every firm's profit function is concave for all  $\mu \geq \mu^{**}$ , which ensures existence of a pure strategy equilibrium.

We have, in fact, shown that profits become *strictly* concave, which implies that each firm has a single-valued best response *function* (rather than a multi-valued *correspondence*). Therefore, to prove uniqueness, it suffices to show that the mapping from prices in  $[C_j, Y]^N$  onto best responses in  $[C_j, Y]^N$  is a contraction (see Friedman 1990, page 84). Let  $P_j^*$  denote firm  $j$ 's best response to competitors' prices. If "distance" is defined as the maximum change in any firm's price, the following condition ensures that the best response mapping is a contraction (Ibid., page 84):

$$(A6) \quad \sum_{k \neq j} \left| \frac{\partial P_j^*}{\partial P_k} \right| < 1 \quad (\forall j)$$

Intuitively, if two distinct equilibria exist, the firm  $j$  with the largest price change between these equilibria must violate (A6). Therefore, (A6) ensures uniqueness. Note that if firm  $j$ 's price is constrained by the upper bound  $Y$ , then (A6) holds trivially because each term is zero. At interior solutions, firm  $j$ 's optimal price must satisfy the first-order condition  $\pi_j' = 0$ . We can evaluate the derivatives in (A6) by applying the implicit function theorem to this first-order condition:

$$(A7) \quad \frac{\partial P_j^*}{\partial P_k} = \frac{-\left(\frac{\partial \pi_j'}{\partial P_k}\right)}{\left(\frac{\partial \pi_j'}{\partial P_j}\right)}$$

The denominator of (A7) is given by equation (A4). The numerator of (A7) is:

$$(A8) \quad \frac{\partial \pi_j'}{\partial P_k} = \sum_i \left( \frac{\partial D_{i,j}}{\partial P_k} + (P_j - C_j) \frac{\partial^2 D_{i,j}}{\partial P_j \partial P_k} \right)$$

To evaluate this expression, we take derivatives of equations (2) and (A2):

$$(A9) \quad \frac{\partial D_{i,j}}{\partial P_k} = \frac{\lambda_i}{\mu} D_{i,j} D_{i,k}$$

$$(A10) \quad \frac{\partial^2 D_{i,j}}{\partial P_j \partial P_k} = -\left(\frac{\lambda_i}{\mu}\right)^2 D_{i,j} D_{i,k} (1 - 2D_{i,j})$$

Plugging (A9) and (A10) into (A8), we have:

$$(A11) \quad \frac{\partial \pi_j'}{\partial P_k} = \sum_i \left[ \frac{\lambda_i}{\mu} D_{i,j} D_{i,k} - (P_j - C_j) \left(\frac{\lambda_i}{\mu}\right)^2 D_{i,j} D_{i,k} (1 - 2D_{i,j}) \right]$$

Next, note that each term  $D_{i,j}$  converges to  $1/N$  as  $\mu \rightarrow \infty$ . In fact, it can be shown that convergence is uniform on the set of possible price vectors in  $[C_j, Y]^N$ . Therefore, as  $\mu \rightarrow \infty$ , (A4) and (A11) become arbitrarily close to:

$$(A12) \quad \frac{\partial \pi_j}{\partial P_j} \approx \frac{-2(N-1)}{N^2} \sum_i \left(\frac{\lambda_i}{\mu}\right) + O\left(\frac{1}{\mu^2}\right)$$

$$(A13) \quad \frac{\partial \pi_j}{\partial P_k} \approx \frac{1}{N^2} \sum_i \left( \frac{\lambda_i}{\mu} \right) + O\left( \frac{1}{\mu^2} \right)$$

where  $O(1/\mu^2)$  represents a term whose absolute value is bounded above by a constant over  $\mu^2$ . Plugging (A12) and (A13) into (A7) we see that as  $\mu \rightarrow \infty$ , (A7) approaches the term given below:

$$(A14) \quad \frac{\partial P_j^*}{\partial P_k} \approx \frac{\frac{1}{N^2} \sum_i \left( \frac{\lambda_i}{\mu} \right)}{\frac{2(N-1)}{N^2} \sum_i \left( \frac{\lambda_i}{\mu} \right)} = \frac{1}{2(N-1)}$$

Because (A14) holds for all  $(N - 1)$  firms  $k \neq j$ , we can make the left-hand side of (A6) arbitrarily close to  $1/2$ . By choosing  $\mu$  high enough that (A6) holds for each firm, we ensure uniqueness of equilibrium. By repeating this entire process for every pricing sub-game, we can find a  $\mu^*$  such that every sub-game has a unique equilibrium for all  $\mu \geq \mu^*$ .

QED

### **Proof of Proposition 2**

This proof shows how to construct a region of prices  $[L, Y]$  in which a unique interior equilibrium occurs for  $\mu$  sufficiently high. The proof relies on Assumption 1's restriction on the variance of  $\lambda$ .

The intuition behind the uniqueness result is as follows. When any firm  $k \neq j$  raises its price, it has two effects on firm  $j$ 's optimal price. First, firm  $j$ 's demand increases, which encourages firm  $j$  to raise its price. Second, the average price sensitivity of firm  $j$ 's customers increases, which encourages firm  $j$  to lower its price. The net effect could go in either direction. When the variance of marginal utility of income is very high, the

second effect is so large that firm  $j$ 's optimal price decreases by more than firm  $k$ 's price increases. Therefore, an interior equilibrium is not stable, and there could be multiple equilibria in which some firms charge the maximum price allowed and others charge the minimum price allowed. Assumption 1 ensures that the variance of marginal utility of income is too low for this to happen. Note that this assumption always holds for  $N = 2$ , and it holds for any number of firms if  $\text{Var}[\lambda] < (E[\lambda])^2$ .

To formally prove this, let  $C_{AVG}$  denote the average marginal cost across all possible product designs. Assume the price bounds have the following form:

$$(A15) \quad L = C_{AVG} - \alpha + \beta\mu$$

$$(A16) \quad Y = C_{AVG} + \alpha + \beta\mu$$

Note that the width of the price range stays constant for all  $\mu$  because both bounds grow at the same rate as  $\mu$  increases. We will first show that for any such price range every sub-game has a unique equilibrium for  $\mu$  sufficiently large. We will next show how to choose the parameters in (A15) and (A16) so that all equilibrium prices lie in the interior of this range.

Proposition 1's proof of existence and uniqueness relied on the assumption that prices were bounded, which is no longer true. We will therefore adopt a different strategy. A sufficient condition for a differentiable profit function  $\pi_j$  to be quasi-concave is that  $\pi_j'' < 0$  whenever  $\pi_j' = 0$ . This is the condition we will use to prove existence. Likewise, to check the uniqueness condition given by (A6), we will evaluate the derivatives of  $\pi_j'$  with respect to  $P_j$  and  $P_k$  at points where  $\pi_j' = 0$ .

We begin by setting  $\pi_j' = D_j + (P_j - C_j)D'_j$  equal to zero and then plugging the resulting expression for the price markup,  $(P_j - C_j) = -(D_j / D'_j)$ , into the expressions for the two derivatives we want to calculate. This yields:



$$(A17) \quad \left. \frac{\partial \pi_j'}{\partial P_j} \right|_{\pi_j'=0} = 2D'_j - \left( \frac{D_j}{D'_j} \right) D''_j$$

$$(A18) \quad \left. \frac{\partial \pi_j'}{\partial P_k} \right|_{\pi_j'=0} = \frac{\partial D_j}{\partial P_k} - \left( \frac{D_j}{D'_j} \right) \frac{\partial^2 D_j}{\partial P_j \partial P_k}$$

If the right-hand side of (A17) is always less than zero, then profit functions are quasi-concave and a pure strategy equilibrium exists. It will simplify the algebra needed to show this result if we multiply both terms in (A17) by  $-(D'_j)$ , which is always positive. This puts the inequality we are trying to demonstrate in the form  $D''_j D_j - 2(D'_j)^2 < 0$ . In the CBD game, firm  $j$ 's profits are quasi-concave if:

$$(A19) \quad \left( \sum_i D''_{i,j} \right) \left( \sum_i D_{i,j} \right) - 2 \left( \sum_i D'_{i,j} \right)^2 < 0$$

If we plug (A2) and (A3) into (A19) and multiply both sides by  $\mu^2$ , the resulting inequality is:

$$(A20) \quad \left( \sum_i \lambda_i^2 D_{i,j} (1 - D_{i,j}) (1 - 2D_{i,j}) \right) \left( \sum_i D_{i,j} \right) - 2 \left( \sum_i \lambda_i D_{i,j} (1 - D_{i,j}) \right)^2 < 0$$

Because we are restricting prices to a range whose width remains constant for all  $\mu$ , each  $D_{i,j}$  converges to  $1/N$  as  $\mu \rightarrow \infty$ . This implies that (A20) holds asymptotically as long as:

$$(A21) \quad \left( \frac{1}{M} \sum_i \lambda_i^2 \right) \frac{M^2 (N-1)(N-2)}{N^4} - \left( \frac{1}{M} \sum_i \lambda_i \right)^2 \frac{2M^2 (N-1)^2}{N^4} < 0$$

Rearranging terms, and noting that  $E[\lambda^2] = \text{Var}[\lambda] + (E[\lambda])^2$ , inequality (A21) becomes:

$$(A22) \quad \frac{M^2(N-1)}{N^4} \left[ (N-2)\text{Var}[\lambda] - N(E[\lambda])^2 \right] < 0$$

which holds under Assumption 1.

We have shown that for  $\mu$  sufficiently large any critical point is a local maximum. This implies that each profit function is quasi-concave and a pure strategy equilibrium exists. To prove that the best response mapping is a contraction, we need to show that absolute value of the ratio of (A18) to (A17) is less than  $1/(N-1)$ , which guarantees that condition (A6) holds. We evaluate (A18) using the same approach as we did for (A17). As before, we multiply the right-hand side of (A18) by  $-D'_j \mu^2$ , and we then plug (A9) and (A10) into the resulting expression. If we let  $\mu \rightarrow \infty$  so each  $D_{i,j}$  converges to  $1/N$ , we end up with the following expression:

$$(A23) \quad \frac{-M^2}{N^4} \left[ (N-2)\text{Var}[\lambda] - (E[\lambda])^2 \right]$$

To apply the implicit function theorem, we take negative (A23) over (A22), which shows that:

$$(A24) \quad \frac{\partial P_j^*}{\partial P_k} \approx \left( \frac{1}{(N-1)} \right) \left( \frac{(N-2)\text{Var}[\lambda] - (E[\lambda])^2}{(N-2)\text{Var}[\lambda] - N(E[\lambda])^2} \right)$$

For (A6) to hold, we need to show that the absolute value of the second term on the right-hand side of (A24) is less than one. Under Assumption 1, the denominator of this expression is always negative. If the numerator is also negative, then the result clearly holds because the denominator is “more negative”.

On the other hand, if the numerator is positive, then an *increase* in firm  $k$ 's price causes a *decrease* in firm  $j$ 's price. When prices become strategic substitutes like this, (A6) is a much stricter condition than necessary to prove uniqueness. Intuitively, if there are two distinct equilibria, some firms must charge higher prices in the second equilibria, while others charge lower prices. Given that (A24) converges to the same value for any pair of firms, the only way two such equilibria can exist is if an increase in firm  $k$ 's price causes a decrease in firm  $j$ 's price that is at least equally large in absolute value. We can therefore replace condition (A6) with the following condition:

$$(A25) \quad \frac{\partial P_j^*}{\partial P_k} > -1 \quad (\forall j, k)$$

Formally, this ensures that the Jacobian of the mapping from prices to profit first derivatives is negative quasi-definite, so there can only be one set of prices that sets these derivatives equal to zero (see Friedman 1990, page 86). By plugging (A24) into (A25), and rearranging terms, we arrive at the following condition:

$$(A26) \quad \frac{N(N-2)}{N(N-1)+1} \text{Var}[\lambda] < (E[\lambda])^2$$

which holds under Assumption 1. (Note that Assumption 1 is slightly stronger than necessary to guarantee existence and uniqueness. This assumption will also be used in Proposition 4 to guarantee that firms prefer *more* popular products instead of *less* popular ones.)

We have shown that for a price range of the form given by (A15) and (A16), Assumption 1 guarantees that a unique equilibrium exists for sufficiently large  $\mu$ . We now show that it's possible to construct a price range such that equilibrium prices lie in the interior of the range. Recall that we assume the lower limit has the form  $L = C_{AVG} - \alpha + \beta\mu$ , and the upper limit has the form  $Y = C_{AVG} + \alpha + \beta\mu$ . We need for the solution to every

firm's first order condition to lie within this range. Firm  $j$ 's first-order condition is solved by setting  $(P_j - C_j) = -(D_j / D'_j)$ . For the CBD game, this implies:

$$(A27) \quad P_j^* = C_j + \frac{\sum_i D_{i,j}}{\sum_i \left[ \frac{\lambda_i}{\mu} D_{i,j} (1 - D_{i,j}) \right]}$$

If  $D_{i,j}$  exactly equals  $1/N$  for all  $i$  and  $j$ , this becomes:

$$(A28) \quad P_j^* = C_j + \left( \frac{N}{N-1} \right) \left( \frac{\mu}{E[\lambda]} \right)$$

This suggests the following value for  $\beta$ , which determines the sensitivity of the price range to  $\mu$ :

$$(A29) \quad \beta = \left( \frac{N}{N-1} \right) \left( \frac{1}{E[\lambda]} \right)$$

Assume all firms have marginal cost equal to  $C_{AVG}$ , and each consumer has the same part-worth sum for all products. We can then set  $\alpha$  arbitrarily small, and for sufficiently high  $\mu$ , there's an equilibrium in which all firms set their price equal to  $C_{AVG} + \beta\mu$ , with  $\beta$  given by (A29). The proof of Proposition 4 shows that, under Assumption 1, the derivatives of equilibrium prices with respect to part-worths and marginal costs converge to constants as  $\mu \rightarrow \infty$ . Rather than providing an explicit expression for the value of  $\alpha$ , we simply state that equation (A56) can be used to set its value large enough that, even after the largest possible perturbations to these parameters, there is still an interior equilibrium for sufficiently large  $\mu$ .<sup>13</sup>

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<sup>13</sup> Equation (A56) relies on an additional assumption that marginal utility of income is uncorrelated with other part-worths, but it's possible to develop an equivalent expression that does not rely on this assumption. In practice, rather than explicitly using this expression to come up with a value for  $\alpha$ , firms could simply choose a reasonable range of prices and check whether interior equilibria exist.

To summarize, we constructed a price region that grows linearly with  $\mu$  while maintaining a fixed width. For any such region, we showed that as  $\mu \rightarrow \infty$  each pricing sub-game has a unique equilibrium. Finally, we showed that for appropriately chosen upper and lower bounds, equilibrium prices lie on the interior of the price range. All of these results depended on Assumption 1's restriction on the variance of marginal utility of income.

QED

### Proof of Proposition 3

At an interior equilibrium, each firm's first order condition is satisfied. For the CBD game, this implies the following, which was also equation (A27):

$$(A30) \quad P_j^* = C_j + \frac{\sum_i D_{i,j}}{\sum_i \left[ \frac{\lambda_i}{\mu} D_{i,j} (1 - D_{i,j}) \right]}$$

To evaluate the denominator of this expression, note the following:

$$(A31) \quad \frac{1}{M} \sum_i \lambda_i D_{i,j} = E[\lambda] E_i[D_{i,j}] + Cov_i[\lambda_i, D_{i,j}]$$

$$(A32) \quad \frac{1}{M} \sum_i \lambda_i D_{i,j}^2 = E[\lambda] E_i[D_{i,j}^2] + Cov_i[\lambda_i, D_{i,j}^2]$$

If we plug (A31) and (A32) into (A30), and note that

$E_i[D_{i,j}^2] = (E_i[D_{i,j}])^2 + Var_i[D_{i,j}]$ , the first-order condition becomes:

$$(A33) \quad P_j^* = C_j + \frac{\mu \bar{D}}{E[\lambda] \left[ \bar{D} - \bar{D}^2 - \text{Var}_i[D_{i,j}] \right] + \text{Cov}_i[\lambda_i, D_{i,j}] - \text{Cov}_i[\lambda_i, D_{i,j}^2]}$$

where  $\bar{D} \equiv E_i[D_{i,j}]$ . Given symmetric preferences, when all firms set the same price the covariance terms in the denominator equal zero and each firm's average demand is  $(1/N)$ . Equation (A33) then becomes:

$$(A34) \quad P_j^* = C_j + \left( \frac{\mu}{E[\lambda]} \right) \frac{\frac{1}{N}}{\left[ \left( \frac{1}{N} \right) - \left( \frac{1}{N} \right)^2 - \text{Var}_i[D_{i,j}] \right]}$$

If all firms have the same marginal cost, then their first order conditions are satisfied when they all set the price given by (A34). Since there are  $M$  consumers and average demand for each firm is  $(1/N)$ , each firm's total demand is  $\frac{M}{N}$ .

QED

#### **Proof of Proposition 4**

Recall that firm  $j$ 's profit function is:

$$(A35) \quad \pi_j = (P_j - C_j) \left( \sum_{i=1}^M D_{i,j} \right)$$

where

$$(A36) \quad D_{i,j} = \frac{\exp\left[\frac{1}{\mu}(\beta_i' X_j - \lambda_i P_j)\right]}{\sum_{k=1}^N \exp\left[\frac{1}{\mu}(\beta_i' X_k - \lambda_i P_k)\right]}$$

Firm  $j$ 's product design choice (choice of  $X_j$ ) affects four inputs into its profit function:

$V_{i,j} \equiv \beta_i' X_j$	each customer $i$ 's part-worth sum for the firm's product
$C_j$	the firm's marginal cost
$P_j$	the firm's own equilibrium price
$P_k$	the equilibrium price of each firm $k \neq j$

By differentiating (A35) and taking limits, we find that as  $\mu \rightarrow \infty$  the first derivative of  $\pi_j$  with respect to each of these variables approaches a constant over the entire allowed price range. This implies that for asymptotic results second-order effects become negligible. Note also that the derivative with respect to the firm's own price is zero at an optimum, so we can ignore this term by a standard envelope theorem argument. We therefore focus on the first order effects of changes in part-worth sums, marginal cost, and competitors' equilibrium prices on firm  $j$ 's profits.

We begin with the effect of changes in part-worth sums:

$$(A37) \quad \frac{\partial \pi_j}{\partial V_{i,j}} = (P_j - C_j) \frac{\partial D_{i,j}}{\partial V_{i,j}}$$

By differentiating the demand equation (A36) with respect to  $V_{i,j}$ , we find:

$$(A38) \quad \frac{\partial D_{i,j}}{\partial V_{i,j}} = \left(\frac{1}{\mu}\right) D_{i,j} (1 - D_{i,j}) \approx \frac{1}{\mu} \left(\frac{N-1}{N^2}\right)$$

By plugging (A28) and (A38) into (A37), we arrive at the following asymptotic approximation:<sup>14</sup>

$$(A39) \quad \frac{\partial \pi_j}{\partial V_{i,j}} \approx \left( \frac{1}{E[\lambda]} \right) \left( \frac{1}{N} \right)$$

Next, the derivative of profits with respect to marginal cost is:

$$(A40) \quad \frac{\partial \pi_j}{\partial C_j} = -\sum_i D_{i,j} \approx \frac{-M}{N}$$

Finally, the effect of the change in the price of any firm  $k \neq j$  on firm  $j$ 's profits:

$$(A41) \quad \frac{\partial \pi_j}{\partial P_k} = (P_j^* - C_j) \sum_i \frac{\partial D_{i,j}}{\partial P_k}$$

By differentiating (A36) with respect to  $P_k$ , we have:

$$(A42) \quad \frac{\partial D_{i,j}}{\partial P_k} = \frac{\lambda_i}{\mu} D_{i,j} D_{i,k} \approx \frac{\lambda_i}{\mu} \left( \frac{1}{N^2} \right)$$

By plugging (A28) and (A42) into (A41), we have:

$$(A43) \quad \frac{\partial \pi_j}{\partial P_k} \approx \frac{M}{N(N-1)}$$

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<sup>14</sup> Recall that (A28) is only an approximation of firm  $j$ 's equilibrium price. However, because this approximation is always within a fixed constant of the true equilibrium price, and equation (A37) multiplies this price by a term (for the derivative of demand with respect to competitors' prices) with  $\mu$  in the denominator, the difference between the two is asymptotically negligible for the purposes of equation (A37).



Note that all  $(N - 1)$  firms  $k \neq j$  change their prices by the same amount (to an asymptotic approximation). Therefore, by combining (A39), (A40), and  $(N - 1)$  times (A43) we see that the net effect on firm  $j$ 's profits of a change in its product design is:

$$(A44) \quad \Delta\pi_j \approx \frac{M}{N} \left[ \left( \frac{1}{E[\lambda]} \right) \left( \frac{1}{M} \right) \sum_i \Delta V_{i,j} - \Delta C_j + \Delta P_k \right]$$

where  $\Delta$  followed by a variable name represents the total change in the variable. We now need to evaluate the change in equilibrium prices. A change in a firm's product design requires prices to adjust so that the first order conditions still hold. The first equation given below ensures firm  $j$ 's first order condition, and the second ensures the first order condition for each firm  $k \neq j$ .

$$(A45) \quad \Delta P_j \approx \frac{\partial P_r^*}{\partial P_s} [(N - 1)\Delta P_k] + \sum_i \frac{\partial P_j^*}{\partial V_{i,j}} \Delta V_{i,j} + \frac{\partial P_j^*}{\partial C_j} \Delta C_j$$

$$(A46) \quad \Delta P_k \approx \frac{\partial P_r^*}{\partial P_s} [\Delta P_j + (N - 2)\Delta P_k] + \sum_i \frac{\partial P_k^*}{\partial V_{i,j}} \Delta V_{i,j}$$

Recall from the proof of Proposition 2 that the derivative of any firm's optimal price with respect to any other firm's price is asymptotically the same for any pair of firms. This derivative is generically represented in these equations by a term for firms  $r$  and  $s$ . These equations also ignore second-order effects, which are asymptotically dominated by first-order effects.

By plugging (A45) into (A46) and rearranging terms, we get:

$$(A47) \quad \Delta P_k \approx \frac{\sum_i \left[ \left( \frac{\partial P_k^*}{\partial V_{i,j}} + \frac{\partial P_r^*}{\partial P_s} \frac{\partial P_j^*}{\partial V_{i,j}} \right) \Delta V_{i,j} \right] + \left( \frac{\partial P_r^*}{\partial P_s} \right) \left( \frac{\partial P_j^*}{\partial C_j} \right) \Delta C_j}{\left[ 1 - (N-1) \left( \frac{\partial P_r^*}{\partial P_s} \right) \right] \left[ 1 + \left( \frac{\partial P_r^*}{\partial P_s} \right) \right]}$$

We now make two substitutions that allow us to simplify (A47). First, note that for any firm  $r$ , a one unit increase in  $V_{i,r}$  is equivalent to a one unit decrease in  $V_{i,s}$  for every firm  $s \neq r$ . Because firms are asymptotically symmetric in the way they respond to changes in part-worth sums, we therefore have:

$$(A48) \quad \frac{\partial P_j^*}{\partial V_{i,j}} \approx -(N-1) \frac{\partial P_k^*}{\partial V_{i,j}}$$

We next evaluate the derivative of firm  $j$ 's optimal price with respect to its marginal cost. Note from (A27) that if firm  $j$ 's marginal cost increases by  $\Delta C_j$  and all firms also raise their price by the same amount, then firm  $j$ 's first order condition continues to hold. But we want to calculate the change in  $P_j^*$  holding competitors' prices fixed. This means that the total change in  $P_j^*$  is equal to  $\Delta C_j$  *plus* the effect if all competing firms *reduce* their price by  $\Delta C_j$ . In other words:

$$(A49) \quad \frac{\partial P_j^*}{\partial C_j} \approx \left[ 1 - (N-1) \left( \frac{\partial P_r^*}{\partial P_s} \right) \right]$$

If we plug (A48) and (A49) into (A47), the first term in the denominator and equivalent terms in the numerator cancel each other, and we are left with:

$$(A50) \quad \Delta P_k \approx \frac{\sum_i \left[ \frac{\partial P_k^*}{\partial V_{i,j}} \Delta V_{i,j} \right] + \left( \frac{\partial P_r^*}{\partial P_s} \right) \Delta C_j}{\left[ 1 + \left( \frac{\partial P_r^*}{\partial P_s} \right) \right]}$$

We now need to find the asymptotic value of the terms in (A50). By applying the implicit function theorem to firm  $k$ 's first order condition, we find:

$$(A51) \quad \frac{\partial P_k^*}{\partial V_{i,j}} \approx \left[ \frac{-1}{M(N-1)} \right] \left[ \frac{(N-2)\lambda_i - (N-1)E[\lambda]}{(N-2)Var[\lambda] - N(E[\lambda])^2} \right]$$

Also, recall that under Assumption 2 there is zero covariance between marginal utility of income and each element of the part-worth vector. This implies that:

$$(A52) \quad \frac{1}{M} \sum_i \lambda_i \Delta V_{i,j} = E[\lambda] \Delta \bar{V}_j$$

By combining (A52) with (A51), we have:

$$(A53) \quad \sum_i \frac{\partial P_k^*}{\partial V_{i,j}} \Delta V_{i,j} \approx \left[ \frac{-1}{(N-1)} \right] \left[ \frac{-E[\lambda]}{(N-2)Var[\lambda] - N(E[\lambda])^2} \right] \Delta \bar{V}_j$$

Now, through a derivation similar to that of (A24), we have:

$$(A54) \quad \frac{\partial P_r^*}{\partial P_s} \approx \left[ \frac{1}{(N-1)} \right] \left[ \frac{(N-2)Var[\lambda] - (E[\lambda])^2}{(N-2)Var[\lambda] - N(E[\lambda])^2} \right]$$

This implies that:

$$(A55) \quad 1 + \frac{\partial P_r^*}{\partial P_s} \approx \left[ \frac{1}{(N-1)} \right] \left[ \frac{[(N)(N-2)]Var[\lambda] - [N(N-1) + 1](E[\lambda])^2}{(N-2)Var[\lambda] - N(E[\lambda])^2} \right]$$

We now plug (A53), (A54), and (A55) into (A50). I also multiply both the numerator and the denominator by -1, which makes the denominator positive, so the equation is easier to interpret. This leads to:

$$(A56) \quad \Delta P_k \approx \frac{-E[\lambda]\Delta\bar{V}_j + ((E[\lambda])^2 - (N-2)Var[\lambda])\Delta C_j}{[N(N-1) + 1](E[\lambda])^2 - [(N)(N-2)]Var[\lambda]}$$

By plugging (A56) into (A44) we get to the total change in firm  $j$ 's profits when it switches to a new product design:

$$(A57) \quad \Delta\pi_j \approx \frac{M}{N} \left[ \frac{\alpha_1 \left( \frac{\Delta\bar{V}_j}{E[\lambda]} \right) - \alpha_2 \Delta C_j}{[N(N-1) + 1](E[\lambda])^2 - [(N)(N-2)]Var[\lambda]} \right]$$

where  $\alpha_1$  and  $\alpha_2$  are defined as follows:

$$(A58) \quad \alpha_1 \equiv [N(N-1)](E[\lambda])^2 - [(N)(N-2)]Var[\lambda]$$

$$(A59) \quad \alpha_2 \equiv [N(N-1)](E[\lambda])^2 - [(N-1)(N-2)]Var[\lambda]$$

Conditional on competitors' product designs, firm  $j$  can therefore maximize its asymptotic profits by choosing the design that maximizes the numerator of (A57). By rearranging terms slightly, we see that this is the same as maximizing:

$$(A60) \quad F(\bar{V}_j, C_j) = \left[ (E[\lambda])^2 - \left( \frac{N-2}{N-1} \right) \text{Var}[\lambda] \right] \left( \frac{\bar{V}_j}{E[\lambda]} \right) - \left[ (E[\lambda])^2 - \left( \frac{N-2}{N} \right) \text{Var}[\lambda] \right] C_j$$











For  $\mu$  sufficiently high, the last mover always chooses the product that maximizes (A60), regardless of what the other firms do. This implies that the next-to-last firm, knowing it cannot affect the last firm's design choice, also chooses the product that maximizes (A60), and by backward induction, all other firms also choose this design.

QED

## Appendix B. Sample Questions and Study Design

### Sample Conjoint Question, Good Design

If these were your only options, which would you choose?  
Choose by clicking one of the buttons below:

<p>Price: \$40 </p>  <input type="radio"/>	<p>Price: \$40 </p>  <input type="radio"/>	<p>Price: \$50 </p>  <input type="radio"/>	<p>Price: \$40 </p>  <input type="radio"/>	<p>Price: \$30 </p>  <input type="radio"/>
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Sample Conjoint Question, Less Careful Design (note the respondent must scroll down to see all five choices)

If these were your only options, which would you choose?  
Choose by clicking one of the buttons below:

Price: \$40  
American Apparel track jacket  
Red  
MIT Sloan logo

Price: \$30  
Champion hooded sweatshirt  
Grey  
MIT Sloan logo

Price: \$40  
Port Authority fleece vest  
Red  
MIT Sloan logo

Price: \$40

Done