DETECTION OF INDUCED POLARIZATION

by

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>BACKGROUND WORK</td>
<td>6</td>
</tr>
<tr>
<td>SYSTEM ANALYSIS</td>
<td>12</td>
</tr>
<tr>
<td>MEASURING SYSTEM</td>
<td>17</td>
</tr>
<tr>
<td>COMPONENTS IN SYSTEM</td>
<td>19</td>
</tr>
<tr>
<td>CALIBRATION</td>
<td>26</td>
</tr>
<tr>
<td>NOISE EFFECT ON THE OUTPUT</td>
<td>30</td>
</tr>
<tr>
<td>ERRORS ASSOCIATED WITH SQUARE WAVE GENERATION</td>
<td>37</td>
</tr>
<tr>
<td>RECOMMENDATION FOR ADDITIONAL WORK</td>
<td>47</td>
</tr>
<tr>
<td>APPENDIX I    DIODE CHARACTERISTICS</td>
<td>49</td>
</tr>
<tr>
<td>APPENDIX II   FOURIER ANALYSIS</td>
<td>51</td>
</tr>
<tr>
<td>APPENDIX III  MODIFICATIONS TO GENERAL RADIO VTVM</td>
<td>53</td>
</tr>
</tbody>
</table>
LIST OF FIGURES AND TABLES

FIGURE 1 APPROXIMATE ELECTRICAL CIRCUIT (TEXT)................................. 9
2 INPUT WAVE FORM............................................................. 13
3 SIGNAL INPUT TO CLAMP CIRCUIT........................................ 13
4 RESULTANT SIGNAL FROM CLAMP CIRCUIT................................. 13
5 FIRST TWO STAGES TO TRANSISTOR AMPLIFIER (TEXT).................. 5
6 SCHEMATIC OF AMPLIFIER.................................................. 23
7 EFFECT OF NOISE ON WAVE FORMS........................................ 32
8 ERROR DUE TO MISBALANCE.................................................. 35
9 INPUT WAVE FORM WITH DEAD TIME........................................ 40
10 MEASURED WAVE FORM WITH DEAD TIME................................... 40
11 EXPONENTIAL APPROXIMATION TO INPUT WAVE FORM................... 42
12 MEASURED WAVE FORM, EXPONENTIAL APPROXIMATION................... 42
13 ERROR GRAPH DUE TO DEAD TIME EXPONENTIAL APPROXIMATION........ 46
14 FOURIER ANALYSIS OF MEASURED WAVE FORM............................ 52
15 DIODE CHARACTERISTICS AT LOW VOLTAGES.............................. 50
16 MODIFICATION TO GENERAL RADIO VTVM................................ 53
17 CALIBRATION OF GENERAL RADIO VTVM................................ 28, 29

TABLE 1 ERROR DUE TO SQUARE WAVE SWITCHING.............................. 38
2 ERRORS DUE TO DEAD TIME MODEL 1........................................ 41
3 ERRORS DUE TO DEAD TIME EXPONENTIAL APPROXIMATION................ 44
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Abstract of Thesis: Detection of Induced Polarization

by John McAllister

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Work done on induced polarization effects in the earth have shown that the decay of a voltage, called the over voltage, resulting from the interruption of a current passing through the earther hold useful information regarding the physical properties of the earth. It is shown that the decay voltages when integrated can be correlated with the amount of mineralization in the earth. The integrated decay voltages are proportional to the capacitive coupling between mineralized and non-mineralized grains. In the past the ways of measuring this over voltage and integrating it depended on direct connection with the transmitter of the current, then using relays to switch in an integrating circuit when the primary current flow was interrupted. This system developed a self contained, needs no direct connection with the sender, battery operated that enables the user to directly read the integrated decay voltages. The measuring system consists of primarily an amplifier, a diode bridge for rectification of the signal, a clamp circuit for removing the d.c. voltages that appear from the diode bridge. A modified General Radio VTVM is used to measure the integrated voltage. The system has been built into a chassis approximately 1" X 5' X 10' and has been calibrated. Some of the possible sources of reading errors have been investigated, the effects of random noise evaluated with respect to its effect on the output reading.

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BACKGROUND WORK

In the past few years detection of sulfide ore zones has been augmented, by measuring certain transient or frequency dependent electrical parameters of these ore zones. These methods, known as induced polarization methods, appear to have a greater resolving power in areas of low conductivity contrast. There has been much work done on induced polarization methods, both here at M.I.T. and elsewhere*. While many of the techniques have been developed, all can stand improvement in equipment. Components can be reduced in size and weight by new developments in transistors, and there by reducing power requirements. This thesis develops a possible system for use in measuring the induced polarization effect, without being dependent on an external connection back to the generator. This small self contained package enables the geophysicist to make unencumbered measurements faster, with satisfactory accuracy and not be burdened by a large amount of equipment.

* Madden, T. H., et. al., Background Effects in Induced Polarization Methods of Geophysical Prospecting; A.E.C. AT(05-1)-718, June 1957.
Friche and Butler, Theoretical Studies of Electrical Polarization; Geophysics XXII, July 1957.
Vacquier, et. al., Prospecting for Underground Water by Induced Electrical Polarization; Geophysics, XXII, July 1957.
Bleil, D. F., Induced Polarization; Geophysics XVII, 1953, p. 636
Prospecting for mineralization in the earth demands the geophysicist to find some parameter in his measurements indicative of the mineralization. In electrical prospecting the conductivity and the conductivity contrast delineate between the sulfides and metallic minerals and most of the other rock which is relatively non-conducting. The method breaks down when the mineralization is unconnected, or too finely divided to allow spotting of the anomalies by conductivity measurements. For this reason, it was desirable to develop another method that would be sensitive to a different parameter, enabling detection of the anomalous area, even though the contrasts are negligible. This was the motivating force behind the development of induced polarization measurements.

The term induced polarization is coined by analogy with the polarized electrode phenomena. Each mineralized A is postulated to act like an electrode polarized by current passed through the system. This polarized grain is surrounded by potential and electro chemical barriers that impede the flow of current through the polarized electrode solution interface. To force current across this barrier requires an additional voltage drop greater than required by the ohmic losses in the system. This voltage is termed the over voltage.

In a rock composed of some mineralized sections along with other unmineralized sections, there is a network of interconnecting paths; some of the paths in the pore fluid are blocked by mineralized grains, metal or metal sulfides. This block requires that extra over voltage be required to drive current flow past this interfacial barrier. The mineralized grains conduct electricity by electron flow, whereas the principle mechanism of conduction in rocks is generally accepted to be ion convection;
ions travelling through the pore fluids. Most mineralization prospecting is done for sulfides, native metal or metal oxides, which are better conductors than ionic conductors and are excellent conductors compared to other rock forming minerals. The properties of the barrier between the mineralized grain and pore solutions is the important facet of the induced polarization method. This barrier between the grains and fluid acts capacitively responsible discharge flow of current in the rock after the primary charging current is turned off, and the increased conductivity with increased frequency. The measuring of either or both of these two phenomena is the basis for most induced polarization prospecting measurements.

The usual procedure for induced polarization is to drive current through the ground from a motor generator set into two electrodes, thereby inducing a polarization on all of the mineralized grains. Upon switching off the primary current, relays are actuated to switch in some measuring device to record or integrate the over voltage decay between two other electrodes. This integrated voltage is then the parameter indicative of mineralization. This set up requires the sender and reciever be connected for relay actuation. A simpler way of measuring these over voltages is the experimental heart of this work. But before going into the electronics, a careful background in what is being measured is necessary.

Translating the simple phenomenon into electrical circuit elements we arrive at a circuit like Figure 1. $R_1$ represents the ionic conduction paths unblocked by mineralized grains; $R_2$, the part of the ionic conduction path blocked; $R_3$, the electronic path resistance. The interface impedance
is roughly representable by a large capacitance $C$, in parallel by a large resistance, $R_4$.

![Approximate Electrical Equivalent Circuit](image)

**FIGURE 1  APPROXIMATE ELECTRICAL EQUIVALENT CIRCUIT**

The parameter we are trying to locate is some thing to give more indication of the amount of mineralization in the sample. Since $R_1$ has nothing to do with the metal in the rock, representing the ionic paths, a simple way of emphasizing the relative importance of the metal in the sample would be to subtract off the low frequency conductivity. The change in conductivity with frequency is thus only the change of conductivity of the blocked conduction paths.

Conductance $Y = \left[ \frac{1}{R_1} + \frac{1}{Z} \right]$

Where $Z = R_2 + R_3 + \frac{R_4}{R_4C_S + 1}$

Subtracting the d.c. conductivity

$Y(f) - Y(o) = \frac{1}{R(f)} - \frac{1}{R(o)} = \frac{R(o) - R(f)}{R(o) \cdot R(f)}$

Where $R(o)$ is the d.c. impedance, and $R(f)$ is the impedance at frequency $f$,

Since $R_4$ is large; $Y(o) \approx \frac{1}{R_1}$

then; $Y - Y(o) = \left[ \frac{1}{R_1} + \frac{1}{Z} \right] - \frac{1}{R_1}$.
the parameter \( \frac{R_y - R_o}{R_y} \times 10^5 \) is called the metal factor since it is assumed that the metal in the rocks causes this change of impedance. Since in most applications \( Z_f \ll Z_0 \) for low frequency the metal factor is

\[
\text{m.f.} = \frac{1}{Z(f)} \times 10^5
\]

The more metal the rock has, the more paths will be blocked off, and as the metal content increases, the individual paths will have higher a.c. conductivities. This two fold effect will rapidly increase the value of the metal factor.

This thesis will not be directly measuring the secondary over voltage decay after the primary current is turned off. Instead of using a +, off - square wave, a simple +, - square wave will be used. The operation will then charge up the grain boundary capacitance first one way, then as the current reverses direction, this capacitor will discharge and be recharged in the reverse direction. Working out what the resultant waveform would be using the model of Figure 1, when driven by square pulses of current \( I_{in} \),

\[
e = e_{primary current} - e_{secondary current}
\]

where the secondary current is the discharge of the capacitor. By straightforward analysis \( e \) is found to be

\[
\frac{e_{in}}{R_{in}} = \frac{R_1 (R_2 + R_3)}{R_1 + R_2 + R_3} e^{-t/T_1} + \frac{-(R_2 + R_3 + R_4) R_1 e^{-t/T_2}}{R_1 + R_2 + R_3 + R_4} - \frac{R_1}{R_1 + R_2 + R_3} \left( \frac{R (R_2 + R_3 + R_4)}{R_1 + R_2 + R_3 + R_4} \right) e^{-t/T_2}
\]

where

\[
\mathcal{C} = \frac{(R_1 + R_2 + R_3) R_4 \kappa}{R_1 + R_2 + R_3 + R_4}
\]
Simplifying this

\[ \frac{C_0}{I_{in}} = \frac{R_i(R_2+R_3+R_4)}{R_i+R_2+R_3+R_4} - \frac{R_i^2(2R_4+R_i+R_3)}{(R_i+R_2+R_3+R_4)(R_i+R_2+R_3+R_4)} \]

where \( I_{in} \) is magnitude of input square wave. If we neglect \( R_4 \) as much larger than \( R_1 + R_2 + R_3 \), this simplifies to

\[ \frac{C_0}{I_{in}} = \frac{R_i}{R_i+R_2+R_3+R_4} e^{-t/\theta'} \]

Where \( \theta' \) is given by \( \theta' = (R_1+R_2+R_3)C \), a step then an exponential rise to a higher value.

The measurement to be made is that involving the missing area. This is easily expressed as, from figure 2,

\[ \text{Area} \approx \frac{\int_0^2t R_2 R_4 C (2R_4 + R_i + R_3)}{(R_i + R_2 + R_3 + R_4)^2} \]

Normalizing this by some factor such as the net voltage after the exponential has died off,

\[ \frac{\text{Area}}{\text{Height}} \approx \frac{R_i R_4 C (R_i + R_3 + 2R_4)}{(R_i + R_2 + R_3 + R_4)(R_i + R_3 + R_4)} \]

assuming \( R_4 \) is much larger than the other resistors in the model,

\[ \frac{\text{Area}}{\text{Height}} \approx \frac{\text{Height}}{R_1} \]

Thus, the normalized area gives a measure of the capacitive coupling between the grain solution interface, since \( R_1 \) is approximately the d.c. conductivity of the rock.
SYSTEM ANALYSIS

The measurement that is possibly the most indicative of mineralization in the rocks is the decay voltages, or the integrated decay of the over voltages in the rocks. This thesis tells how to detect this change in wave form. Using a square wave generator to send rectangular pulses of current into the ground, the resulting wave form is similar to Figure 2. The area to be detected is the missing corners of the square wave.

To produce a signal proportional to the missing area, we pass the wave through a full wave bridge, composed of four diodes. The bridge rectifies the wave form, inverting the negative half and adding it to the + half, so all the area is positive. If a perfect square wave was passed through the bridge, there would result in only an even d.c. voltage. If there are parts of the square wave missing, then there will be a slightly lower d.c. voltage produced, along with an a.c. component. After we pass the signal through the bridge, made of IN 34 crystal diodes, the resulting wave form will be similar to Figure 3.

This wave form has a high constant d.c. level, almost equal to the peak value of the square wave. To find out the distortion we now want a circuit to always hold the zero level at the peak value of the wave form, or the same thing, to subtract off a d.c. voltage equal to the peak of the wave form. This can be done with a diode, capacitor clamp circuit. The output is taken off the diode, and thus this circuit clamps the peak value of the wave form to zero. The signal from the diode is proportional to the integrated voltage decay curves. After the input value has been normalized to be a unit height, then the area, or equivalently the d.c. value is proportional to
Figure 2 Input waveform

Figure 3 Signal input to clamp circuit

Figure 4 Resultant signal from clamp circuit
the distortion in the input. The diode clamp circuit and the resultant voltage are illustrated in Figure 4.

This circuit cannot be used as it is however, because of two major disadvantages, first it will not have high enough input impedance, second, there is no really perfect device as an ideal diode, at low voltages diodes look less like diodes more like resistors with only slightly different forward and reverse resistances. For some of the low voltage characteristics, see Appendix 1, Diode Characteristics.

Both of these two problems can be overcome with a suitable amplifier. The amplifier must have high input impedance, it must pass essentially undistorted all frequencies in the signal and must have sufficient gain to raise the input signal from the levels encountered in prospecting to a level where a diode can operate closer to an ideal diode. The design of the amplifier poses a very serious problem, especially in the frequency response area. The lowest fundamental of the square wave is on the order of 1 to 2 cycles per second. This means that this amplifier will have to be direct coupled or have very large coupling capacitors. Since the effect we are trying to measure can be considered a 1% change in the waveform, the RC coupling would have to pass the waveform, with less than a 1% fall off, a square wave of 1 to 2 cps. If vacuum tubes were to be used, in the amplifier with a grid leak resistor of one meg-ohm, the coupling capacitor would have to be large enough so that the RC decay falls off less than 1% in 1 to 2 seconds, or C would have to be on the order of 100 UF. This would be an almost impossible large capacitor, especially since electrolities, which come that large, are unsuitable for the job, they have too high a leakage current. If this is the limit with vacuum tubes, with their associated
high input impedance, it is completely impossible with transistors with
their low input impedance. Hence, the amplifier will have to be direct
coupled. Low gain d.c. amplifiers are not hard to design and build, but
high gain amplifier are very subject to drift in operating points of the
individual transistors over a length of time, both due to the aging and more
important, slight changes of operating temperature. The only recourse short
of making a chopper stabilized amplifier, is to make higher gain d.c. amp-
liifiers and then plunge the gain back in the device in the form of negative
feed back to stabilize the gain, reduce drift, improve linearity and dis-
tortion.

Since transistors are small, readily suitable for prospecting work,
they were chosen as the basic building block for this amplifier. Transistors,
however, will not solve one condition, very high input impedance, which was
placed one amplifier. High input impedance can be engineered, but not as
easily as with vacuum tubes. So as it was visualized there would be a tube
in the first stage, used to present the earth with a high input impedance;
this tube would transform this high impedance to a lower impedance to match
the input impedance of the transistor. Rather than a common tube requiring
a large B+ supply, around 9 watts of filament power, a sub-minature tube
requiring only 45 volts B+ and .0125 watts filament power, a Raytheon
CK 547 DX. With an anticipated signal on the order of a few millivolts,
the gain of the amplifier has to be on the order of 300 to 600 to bring the
signal up to a reasonable level. This is necessary since the actual rounding
is a very low fraction of the total voltage, on the order of millivolts/
volt input. With a 1 volt (p-p) square wave entering, having a 1% rounding
of the leading edge, the peak of the spikes will be 0.01 volts. The
resultant d.c. signal to be measured is approximately

\[ E_{\text{d.c.}} = \frac{(\text{height}) \times (?)}{\pi} \]

where (?) is the approximate time constant, \( T \) the length of the square wave on or off cycle. The value to be measured only a few tens of millivolts under usual operation, even with the indicated gain. This amplifier is the heart of the electronics, it can not be too good.
MEASURING SYSTEM

Interrelated with the amplifier design is the design of the system to measure the resultant d.c. component that is illustrated in Figure 4. Any system that is to be used must be able to meter signals on the order of few millivolts and have a suitably high impedance so as not to effect the operation of the diode clamp circuit. This is especially true with the filter that must be used to remove the ripple from the d.c., this can not reflect back into the input of the measuring system. There are two main alternatives in trying to measure this voltage.

One satisfactory method is envisioned to use a chopper such as a Ballantine Sensitive inverter, chop up the wave form, so that the signal can then be sent to an a.c. VTVM. This process would use the filters in the Ballantine to produce essentially a d.c. level which modulates the height of the 60 cycle output which is produced. This system has the advantage that the Ballantine has some amplification in it, and the available Hewlett Packard VTVM is very sensitive. This eliminates some of the d.c. amplification, substituting in its place amplifiers capable of passing only 60 cps, essentially much easier to design. This system is present in the lab now and needs only slight modification to make it useful. The disadvantages are both instrument take 110 V a.c. line power, not readily available when prospecting, also they have one side of the input grounded to the chassis. This is undesirable because everything past the diode bridge is floating. Hence, there is much trouble with the grounds in the system, and consequently the noise and pickup problem is very bad.

As an alternative to this system, some less complex was tried. This is essentially building a d.c. VTVM. Most passive meters, 0-50 Ua or 0-100 Ua take 0.1 volts for full scale deflection, much too insensitive.
A d.c. VTVM therefore was constructed by modifying an existing General Radio A.C. Vacuum tube volt meter. For details in the modification see Appendix II. At best this device could be improved to have approximately 40 millivolts full scale deflection. Since the device is battery operated, and has its own filtering system, built in, it seemed much more advantageous than the chopper system.

To smooth the ripples out a 2000 UF capacitor was inserted across the 750 meter movement, giving a time constant of 1.5 seconds. With its own power supply, and small size it has many advantages over the layer heavier systems, even though they are able to handle lower voltage levels and have higher sensitivities. The grounding difficulties negate the higher sensitivities. Even building all the equipment into a common case for shielding was considered but it is still better to get good grounds to the most sensitive part of the circuit, the first stages and float the rest. The noise problem will be explored more fully under the section, Noise Effect on the Output.
COMPONENTS IN SYSTEM

After one can see how the system operates as a whole, then one examines the operation of the individual components. In the heart of the complete system is a high gain amplifier capable of passing the frequencies used. Since the equipments used at low frequencies, the amplifier was designed as a d.c. coupled transistorized amplifier. As required the system has high input impedance, which at maximum sensitivity is 11 meg-ohms, while at lower sensitivities it is less. This impedance is achieved by using a sub-minature tube in the input, a Raytheon CK 547 DX with 45 plate voltage in cathode follow design. An input impedance of several meg ohm is difficult to obtain with transistors. Cathode follow design while its gain is always less than unity, is stable and allows the output impedance to be reduced to a low value, \( R_{\text{in}} \approx \frac{1}{1 + j \omega R_{\text{in}}} \), which allows coupling into transistor circuitry with minimum power loss. The cathode resistor is made variable to allow for variations in the tubes, and to allow adjustment of the bias of the first transistor.

The transistor amplifier is of the basic form of grounded emitter. It is composed of four transistors, the last two are ingrounded emitter, the first two are basically grounded emitter, with variation to increase the input impedance so as not to load the cathode follow stage previous to these transistors.

The grounded emitter has low input impedance, unless there is a resistance in the emitter circuit. The impedance can be shown to be

\[
R_{\text{in}} = r_b + \frac{r_e}{1 - \alpha}
\]

where \( r_b \) is the net resistance in the base, and the
resistance in the emitter circuit. If this resistance is large it moves the resistance up from about 8000hms, the value without an external emitter resistor to any value that can be successfully inserted and the bias of the transistor kept satisfactory. The solution was to put another transistor in the emitter circuit and use its input resistance as the external emitter resistance of transistor T1.

![FIGURE 5](image)

Therefore the first part of the circuit is like Figure 5. The current gain of the two stages is \( \frac{1}{2} \) where \( 1 \) and \( 2 \) are the current gains of each stage respectively. The voltage gain can be shown to be approximately to be

\[
\frac{E_o}{E_{in}} = \frac{\kappa R_o}{r_e + (r_e + r_b)(1-\alpha) + r_b(1-\alpha)^2}
\]

and the input resistance is

\[
R_{in} = r_b + \left( \frac{1}{1-\alpha} \right) \left( r_e + r_b + \frac{r_e}{1-\alpha} \right)
\]

assuming that both the transistors have the same parameters. This assumption is only fair even though they are called the same transistors. Plugging in some numbers the gains and resistances turn out to be

\[
\frac{E_o}{E_{in}} = 0.014 R_o
\]

\[
R_{in} = 7750 \, \Omega
\]
Where it was assumed \( \alpha = 0.90; \ R_b = 250 \Omega \ R_e = 25 k \Omega \) the output resistance \( R_O \) on the second transistor is roughly the input impedance of the next stage, in parallel with the load resistance on T2, which is approximately,

\[
R_o = R_{in} \approx 500 \Omega
\]

The voltage gain is rather small, but the current gain is \( \beta_1 \beta_2 \) or on the order of 100, and this is on the conservative side since \( \beta \) is normally 22 instead of 10. The value 10 was used because that was the experimentally determined value of these transistors.

The next two transistors are hooked up in the common emitter design, the gain of each stage is

\[
\frac{E_o}{I_{in}} = \frac{\alpha R_e}{R_{in}} \quad \frac{I_o}{I_{in}} = \beta
\]

where the last stage has as large a resistor as practicable and still allowing for correct bias. The overall voltage gain of the circuit is measured at 3000, however, as with any circuit that is a d.c. circuit it is very subject to drift. Hence, feed back was used to reduce gain and improve the stability.

A feedback resistor, from the collector on the output was returned to the base of the first transistor. However, associated capacitance made the circuit oscillate at a very high frequency, hence, a small capacitor was placed on the output to shunt high frequencies out to ground. That capacitor rounded just a little the leading edge of a square wave at the output. To counteract this, the feedback resistor was split into half and a small mica capacitor inserted from there to ground. This delayed the feedback enough so the rise time was faster, before feedback had a chance to bring the level back down.

The drift stability would be considered satisfactory by any but the
most exacting standards, however, it is being used to measure small d.c. signals so this remaining drift is still a major problem. The drift results from variations of the transistor parameters mostly due to variations of temperature. Touching a transistor is enough to make it warm up and change characteristics. The whole circuit takes a long time to come to temperature equilibrium. The temperature change could possibly be reduced by using thermistors in the circuit, but this was not attempted. Even after a length of time suitable for the temperature to come to equilibrium, the amplifier still has a tendency to drift, careful replacement of the batteries often helps, but the trouble soon all came back to the transistors.

For any exacting use, much work remains to be done to get a drift free amplifier. For the complete schemetric see Figure 6.

The net purpose of the amplifier is to increase the signal up to a point where it can be applied to the diode circuit. The operation of the diode bridge is simple, any wave form is full wave rectified. When the net current through the bridge is zero, as set by the 0-100mA meter, then the wave form will be rectified about its zero d.c. level. Therefore, the output of the bridge is an almost constant d.c. value with slight dips in the value. The whole bridge was found to operate better when it had a load resistor of 47K ohms across it. The output of the bridge is applied to a clamp circuit directly. The clamper circuit operates as follows; the voltage across the diode equals the input voltage minus the voltage across the capacitor.

\[ E_d = E_{in} - E_c \]

Since the voltage across the diode can not be positive, the voltage across
TUBE CK517DX
TRANSISTORS CK732
DIODES 1N34
M1 0-100μA
M2 GR. VTVM
M3 0-10 Volts

Figure 6
Schematic of Amplifier
the capacitor charges up to equal the peak value of the input wave form. On the first + swing of the input the diode conducts, there for charging the capacitor. So after a few cycles, the d.c. component appears across the capacitor, the ripple is clamped to zero and appears as a net negative voltage across the diode. The charged capacitor tends to discharge through the back resistance of the diode giving a higher net negative voltage than there should be. This can be circumvented highly by putting a resistor in parallel with the capacitor to shunt this current around the diode. Then one must use a larger capacitor to increase the $R' C$ discharge time, where $R'$ is the parallel combination of the back diode resistance and the resistor across $C$.

The diode in the clamp circuit is forced to operate in unfavorable conditions, i.e. near its break point, hence, the best diode available should be used in that place. Since CK 706 and IN 34 were available, the best of the seven available was used.

The output signal, which is the spikes appearing across the clamp diode are measured by a modified General Radio A.C. voltmeter. Before modification, see Appendix II, this meter was essentially a d.c. coupled system, with a diode in the first stage for rectification of any a.c. wave form. The modifications involved removing the diode an changing some resistors in the circuitry to improve sensitivity. The metering was improved until the sensitivity was approximately a millivolt per unit deflection. A large capacitor was placed across the meter movement to increase the time constant up to 1.5 sec. (2000 microfarads in parallel with 750 ohm meter movement). As this meter has self contain battery power, and is small, and as sensitive as necessary for all foreseeable contingencies, this was used in preference to the
chopper and a.c. metering system that was first proposed as a possibility. The self powering makes it much preferable for field usage. The complete amplifier schematic is illustrated in Figure 6.

The amplifier, diode bridge, and clamp circuit are all assembled on plastic sheets with solder lugs on the sheets; then these are fitted into a chassis approximately 14 X 5" X 10". The input, output, are on the side of the chassis, along with the bias adjustment, and bridge balance variable resistors. The switch for the batteries are mounted on the side, even though the batteries are all carried externally, except for one small 1.5 volt battery. The gain control is inside the chassis, however, and in the prototype there is no easy way to measure the net voltage applied to the input tube.

Operating the device is easy; after allowing the system to settle down after turn on, meter 3 is set to 7 to 8 volts by changing the bias adjustment, then the amplifier is operating at its optimum point. The bridge is set to zero micro amps into it; the VTVM is set to zero by shorting the input and adjusting the fine adjustment knob. Then with every thing ready, the bridge balance knob is rocked around zero while watching the VTVM, the minimum reading is balanced on the bridge and is hence, the net integrated decay voltage, with the lowest possible error.
CALIBRATION

When completed, the amplifier diode circuits were packaged into a small box approximately 1" X 5" X 10" with the controls on one edge. This had to be done to cut down on stray 60 cycle pickup that seriously deteriorated the signal otherwise. With the device assembled, a signal approximately 2mv RMS volts as measured on a Hewlett Packard 400H meter was fed into the amplifier. This square wave was produced by a Krohn-hite low frequency generator, paralleled by a variable capacitor, divided by a voltage divider and the two millvolt output of the divider fed into the amplifier. As seen in the schematic, meter 1 was a Simpson on 0-100μ amp scale used to balance the bridge, meter 2 was the General Radio VTVM measuring the output, an additional meter, M3 had to be used to monitor the amplifier and keep it operating in the optimum range. Meter 3 was a small 10,000 ohm per volt Triplette Meter.

With a given square wave input voltage set at two millvolts, the capacitor was varied while holding the operating point of the amplifier constant, balancing out any slight drift with the variable resistor in the bridge circuit. The readings were made on the General Radio VTVM on its most sensitive scale. Since the General Radio VTVM scales are not calibrated in divisions near zero, for low readings, zero was set to be 10 on the top most scale, then all readings correspondingly corrected. For larger capacitors and hence, higher time constants, the zero setting was placed on zero, on the top scale. This way there is a larger range with higher accuracy. The device was calibrated using a simple exponential decay, then this exponential was measured. However, the rectification property of the diodes does not allow the meter to read all the
area under the exponential. With reference to the Figure 12, it can be seen that the system only measures a part of an exponential. The total area of an exponential

\[ A = H e^{(1 - e^{-\theta})} \]

the area measured

\[ A_m = H \left[ \theta + R_e^{\theta} e^{-\theta} - e \right] \]

the ratio Am/A = .693, a constant. What is plotted in the calibration graphs is the true corrected area.

For changes in the input level, another correction has to be made, the correspondence between change in output reading for a change in input level. While all the measurements were taken at 10 cycles per second, they can easily be corrected for any other frequency, providing the approximation that the exponential will have completely decayed by the end of the half cycle is upheld. If this is not the case then a new calibration must be undertaken.

The calibrations are presented in graphical form, part A was run with the zero set at 10 for low levels, part B was at higher levels with the zero set at zero on the top scale. The readings were made by rocking the bridge balance not through zero micro amp position, the output was watched for the associated minima indicating a balanced bridge. This minimum was the attained. This also has the advantage that only one meter, the C7.R. VTVM, need be watched, where the minima was reached on the VTVM, then the bridge must have been balanced. During one series of readings, the bias knob did not need adjusting, however, after each reading the C.R. VTVM, had to be reset to zero for the meter drifts noticibly.
Volts
Second
Millivolt

True integrated area = 1.943 Scale Reading
by exponential
Low Read

Figure 17 A

Deflection Top Scale
Figure 1703

Calibration of VTVM
By Exponential
Meter Set to Zero

True Integrated Area = \(1.443\) Scale Reading

Volt-Second
MilliVolt

Deflection
Top Scale
NOISE EFFECT ON THE OUTPUT

One of the primary difficulties encountered in prospecting and the use of equipment to measure induced polarization is noise, random voltages caused by 60 cycle, spontaneous potentials, and many other spurious causes, all of which help to drown out the signal. For convenience the noise signals will be divided into noise which has a frequency higher than twice the fundamental of the square wave; and noise of a frequency less than twice the fundamental. This distinction will be made clear. In any case, the amplitude of the superfluous noise must be limited; it always must be of a lower amplitude than the incoming signal, at no time can the noise be great enough to cause the net voltage to have an axis crossing. This is because of the rectification properties of the diodes. Consider the following case, a net d.c. signal and one cycle of sinusoidal noise. Upon integration, there will be no net contribution to the signal due to the noise unless there is axis crossing and rectification. If rectification, then there is non-equal positive and negative areas, non-cancelling, and a net contribution to the signal level. So therefore, the first basic limitation, the amplitude of the signal must always be greater than the noise. This also eliminates the case where the signal is boosted up by spontaneous potential to where there is no axis crossing. If this is the case, the device will not work unless by some method, a system is devised to remove this large slowly varying self potential. For some of the alternatives upon this problem, see section, Suggestions for Continued Work.

Now considering some of the problem inherent in the case where the period of the noise is less than the period of the fundamental square wave.
With the square wave input and the usual polarization effect, noise voltage is added; this noise will be amplified along with the rest. Spikes of noise will finally appear at the clamp circuit. A positive spike with the signal will be greater than the diode bias, and will charge the capacitor \( C \) to a slightly higher voltage than it had formerly across it, and consequently, the output is forced to a slightly lower voltage than it was at before. A negative voltage spike will not affect the charge on the capacitor, since the capacitor can not be discharged because of the back biased diode. In Figure 7, the wave forms are illustrated showing the relationship. The output in the presence of positive spikes will read slightly high until this extra voltage on the capacitor is discharged, and resumes its earlier level as determined by the peak value of the wave form. For the average noise which consists of positive and negative spikes, the effect is such less. Consider the case illustrated by Figure 7. The signal has some noise associated with it. All the areas average out with the exception of the very tail which tends to charge \( C \) and depress the wave form, increasing the area between the wave form and zero volts. This effect can be decreased as small as desired however, by a simple modification, but only at the expense of the signal output voltage. Either by increasing the internal resistance of the previous stage, or increasing the size of the capacitor \( C \). All of these methods effectively increase the charging time constant of the circuit so as to filter out some of the more rapid changes. The whole device is acting more like an RC filter system. Since the trouble with spikes is more disturbing than most, it pays to have some control of the discharge rate on \( e_C \) the voltage on the capacitor, so that this voltage can be made to decay back to the quiescent value, i.e. A volts as in Figure 7. For this reason a resistor \( R' \) was put
Figure 7
Effects of noise on Waveforms

noise spike

Discharge of $E_c$

Charging of $E_c$

Due to finite forward resistance of diode
in parallel with C. The additional positive voltage that appears in the output due to a charging current is almost undetectable due to the relatively low forward resistance of the diode in the clamp.

Hence, the effect of the noise voltage of a short period adds very little to the signal out, which is the integrated voltage appearing across the diode in the clamp. If the incoming noise is an exact multiple of the incoming signal then it will fundamentally change the waveform, and cannot be differentiated from the effect of the earth. However, for its phase to hold constant too is implausible and it too will soon average out like all the rest.

Low frequency noise is another much harder problem. Included under low frequency noise is drift and spontaneous potentials that occur at the electrodes. As analyzed before, the circuitry amplifies the signal and the noise, the voltage to be impressed on the clamp circuit is not symmetrical about zero, however. The drift voltages or low frequency noise unbalances the bridge and after being clamped there is a net increase in area measured. This is illustrated in Figure 8. This out of balance seriously limits the usefulness of the device, should a small imbalance completely obscure the polarization. Therefore, there will be closer examination of the effect of the unbalance to see its effect on the design and measurement.

Calling the length of the fundamental 2T, then approximating the distortion to the input square wave by an exponential of time constant \( \tau \), and calling the height contributed by misbalance \( \delta \), then the area is equal to

\[
A = \int_0^{\tau} [(A-\delta)e^{-\frac{t}{\tau}} + \delta] \, dt + \int_0^{2\tau} Ae^{-\frac{t}{\tau}} \, dt
\]

\[
A = 2Ae^{-\frac{2\tau}{\tau}} + \delta \tau + \delta \tau (e^{-\frac{2\tau}{\tau}} - 1)
\]
the anomalous area = √\(\delta\)

This has been plotted in Figure 8 for various values of the parameters.

To keep the circuit in balance within 1% would take careful and constant adjustment of the balance controls. Since drift and very low frequency noise cannot be expected to remain constant very long, a reading will have to be made quickly when balance is achieved and voltages have become stabilized.

The balance problem is one of the most perplexing in the design; amplifiers, batteries, everything changes with time often rather quickly, feedback and better components seem the only answer. To increase the gain using more transistors and more feedback is one solution but basically this is satisfactory only up to a point where the extra equipment adds more in size than it does in extra stability. While transistors were used in this amplifier because of their inherent size advantages, they do not perform as well as vacuum tubes in d.c. amplifiers. Either better transistors will be needed in more advanced models or some other means of stabilization, such as a chopper system. The main source of drift or low frequency noise is in the temperature variation of the transistor parameters, which are very sensitive to temperature variations. While thermistors or back biased diodes could be used for compensation, they are essentially an adjustment device; they do not sense their adjustment and compensate accordingly as with feedback, they are passive and hence, not as effective as feedback where economy is not exceedingly important, to make any more advances in the direction of better amplifiers, chopper stabilization will have to be used, it affords more stability without excessive complexity.

This changing the mechanics of the amplifier will help the internal drift problem, still to be solved is how to account for variations in the
ERROR ANALYSIS
LOW FREQUENCY NOISE
FIGURE 10

\[ T = \frac{1}{f} \text{ PERIOD} \]
\[ \delta = \text{MISBALANCE} \]

INPUT

\[ \text{Measured} \]

\[ \text{Time Constant} \quad \text{Seconds} \]

- Error Percent
- Time Constant
- \( T = 2 \text{ sec} \)
- \( T = 1 \text{ sec} \)
- \( T = 2.5 \text{ sec} \)
- \( T = 1.5 \text{ sec} \)

\[ \delta = 3\% \]
\[ \delta = 5\% \]
\[ \delta = 10\% \]

- 22
- 16
- 14
- 12
- 8
- 6
- 4
- 2
- 0
spontaneous potentials, magneto telluric currents and other very low frequently noise. Filtering these out with either a RC filter or some electronic system is one alternative. With the available high input impedance of the first stage, it would not be difficult to obtain simple RC filters with time constants of up to 50 sec. This would allow the square wave to pass without any attenuation but would filter out any spontaneous potentials. This would be a large improvement, since it allows the system to be used in a situation that would have previously rendered it useless. To find the required R,C, we first limit R, the grid leak resistor to the CK 547 DX, to a maximum of 6.8M , then for a maximum of 1%. The longest square wave use will probably be about 1 sec., half period is half second.

\[ e^{-\frac{70}{50}} = 0.99 \]

therefore;

\[ 1 - 0.5 = 0.99 \]

\[ RC = 50 \text{ sec} \]

therefore \[ C = 7.5 \text{ MF} \]

This RC filter can not be shortened to help attenuate more rapid variation in spontaneous potentials without damaging the square wave form.
ERRORS ASSOCIATED WITH SQUARE WAVE GENERATION

In order to be able to accurately make measurements of the induced polarization in an area, the input square wave must meet certain requirements. The deviation from a true square wave input is limited by the accuracy with which we wish to measure over voltage decay. With the refinement of motor generators, the pulses of current can be kept with a flat top, no droop or rounding to any appreciable percent. However, in the timing of the square waves, there are two factors which may affect the measurements: These are, first, a variation in the length of the square pulse of current, and second, the possibility that there is some dead time, when the generator is switching from a positive pulse to a negative pulse.

Considering first the variation in the length of the pulses, we assume that \( \alpha \) seconds are clipped of the end of a rectangular pulse of \( T \) seconds, that then is this effect. Assuming an exponential for the decay, we have

\[
\text{true area} = T \left( 1 - e^{-\frac{T \alpha}{T}} \right)
\]

\[
\text{missing area} = \alpha \left[ e^{-\frac{T \alpha}{T}} - e^{-\frac{T \alpha}{T}} \right]
\]

\[
\text{the error} \quad \left( e^{\frac{T \alpha}{T} + \frac{1}{2} e^{-\frac{T \alpha}{T}}} \right) / \left( 1 - e^{-\frac{T \alpha}{T}} \right)
\]


approximating \( e^{-\frac{T \alpha}{T}} \approx \frac{1}{1 + \frac{T}{T \alpha}} \)

for \( \alpha \% \) error:

\[
\frac{e^{-\frac{T \alpha}{T}} + \frac{T}{T \alpha}}{1 - e^{-\frac{T \alpha}{T}}} \approx \alpha
\]

\[
\delta = \frac{\alpha}{1 + \frac{T}{T \alpha} \left( e^{\frac{T \alpha}{T}} - 1 \right)}
\]
Summarizing these results in a table

<table>
<thead>
<tr>
<th>Error (%)</th>
<th>Time Constant (sec)</th>
<th>Allowable $\delta$ when $T = 1$ sec (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>.05</td>
<td>greater than $\frac{7}{4}$ sec</td>
</tr>
<tr>
<td>5</td>
<td>0.1</td>
<td>greater than $\frac{3}{4}$ sec</td>
</tr>
<tr>
<td>5</td>
<td>0.2</td>
<td>greater than $\frac{1}{2}$ sec</td>
</tr>
<tr>
<td>5</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>.05</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>.05</td>
<td>greater than $\frac{1}{2}$ sec</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.3</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE I  ERROR DUE TO SQUARE WAVE SWITCHING**

The clipping off of the last tenth of a second of one half the square wave at worse will cause an error of 3%, however, the time average, which the meter measures will be higher.

\[
\text{meter reading} = \frac{.99 \text{ (Area)}}{.9 \text{ Time}} = 1.1 \text{ (old reading)}
\]

Thus the clipping of 10% off the wave form will cause the reading to be approximately 10% high.
This is also applicable to changes of frequency of the sender. A 10% increase in the fundamental will cause approximately a 10% increase in the reading, unless compensated for in the calibration or data taking. The error in frequency is not magnified by the receiving apparatus, there is almost a one to one correspondence. Hence, for 1% accuracy there should be at least a control of the frequency with in 1/2%.

Associated with the transmitting of the square wave is the dead time, when the relays are switching from plus to zero and then to minus. If there were no induced polarization, then there still would be a reading due to the large gaps in the wave form, gaps due to the dead time in the transmission.

When there is some dead time, the received wave form will be of the form of Figure 9. Upon rectification the negative half of the wave form is inverted and the gap indeed makes a large contribution to the area, as in Figure 10.

The area measured with dead time

\[ A_m = \int_0^\Delta (1 - e^{-ct}) dc + \int_{\Delta}^{\frac{\pi}{\omega}} A(\omega + \beta)e^{-ct} dt \]

Where \( B = \omega e^{-\frac{\Delta}{\omega}} \)

Therefore \( A_m = \Delta + \omega \left( e^{-\frac{\Delta}{\omega}} - 1 + (1 + \omega e^{-\frac{\Delta}{\omega}}) (e^{-\frac{\Delta}{\omega}} - e^{-\frac{\pi}{\omega}}) \right) \)

When \( \omega \) is less than one half, as expected it will be, the error introduced by the dead time can be given by

\[ Error = \frac{true\ area - Measured}{true\ area} \times 100\% \]

Making the approximation that \( 1 >> e^{-\frac{\pi}{\omega}} \)

\[ Error = 1 - \frac{\Delta}{2\omega} - \frac{1}{\omega} \left( 2e^{-\Delta/\omega} - 1 + \omega e^{-2\Delta/\omega} \right) \]
Figure 9 Input waveform with dead time

Figure 10 Measured waveform with dead time
This estimation of the error is accurate as long as the true received waveform can be approximated by the equation of the form

\[ V(t) = A - B e^{-\frac{t}{\Delta}} \]

Where \( A \) is the asymptotic value of the signal after time has elapsed; \((A-B)\) is the initial value of the signal immediately after switching. This is probably not the exact picture, for times very close to the switching time, the voltage waveform is probably an exponential. In any case some numerical figures will be worked out for this approximation to the switching error.

A table of the errors has been compiled.

<table>
<thead>
<tr>
<th>( \Delta )</th>
<th>0.001</th>
<th>0.01</th>
<th>0.1</th>
<th>0.05</th>
<th>0.1</th>
<th>0.2</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \overline{C} )</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>( \Delta \overline{C} )</td>
<td>0.0025</td>
<td>0.005</td>
<td>0.01</td>
<td>0.02</td>
<td>0.1</td>
<td>0.05</td>
<td>0.2</td>
</tr>
<tr>
<td>Error %</td>
<td>25.1</td>
<td>25.2</td>
<td>25.4</td>
<td>26.0</td>
<td>26.2</td>
<td>26.2</td>
<td>26.2</td>
</tr>
<tr>
<td>( H = 1/\overline{C} )</td>
<td>37.3</td>
<td>37.1</td>
<td>41.8</td>
<td>41.3</td>
<td>35.9</td>
<td>33.6</td>
<td>27.3</td>
</tr>
<tr>
<td>( H = 1/4 )</td>
<td>43</td>
<td>42</td>
<td>41</td>
<td>38</td>
<td>37</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>( H = 1/2 )</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

*see text*

**TABLE 2** ERRORS DUE TO DEAD TIME. MODEL 1.
Figure 11. Exponential approximation to input waveform

Figure 12. Measured waveform, exponential approximation
A word of explanation, the readings are all low by the given percentage except the * ones which are high. The approximation to the wave form obviously breaks down for small values, where it reaches assymptotic values of error for vanishingly small values of the dead time. The assumption that the wave form always drops some fixed fraction \( h \) of its height as illustrated in figures 9 and 10 leads to some interesting errors, first too little then too much. Once the error goes to positive, too high readings, the error soon skyrockets.

Although this approximation had some weak points, it does show that for the usual cases, the gap smaller than the decay constant, no wild error result.

Approximation by Exponential. Making a new approximation to the wave form briefly after switching, an exponential decay will be used. The input wave form is now of the form of Figure 11 and after rectification it will have the form of Figure 11.

As one can calculate the area measured will be

\[
Am = \int_0^\Delta (1-e^{-\frac{T}{2}}) \, dt + \int_\Delta^\infty (1-e^{-\frac{T}{2}}) \, dt + \int_0^\infty \frac{p-\Delta}{e^{-\frac{T}{2}}} \, dt
\]

Where the symbols are defined in conjunction with Figure 11 continuing;

\[
\text{Area Measured} = \Delta + 2\delta + (1+\beta) \left( 2 e^{-A/\delta} \right) + (1-e^{-\frac{A}{\delta}}) + \delta(e^\frac{-A}{\delta} - 1)
\]

Solving for \( B \)

\[
l = (1+\beta) e^{-\frac{\delta}{2}}
\]

or

\[
\beta = e^{-\frac{A}{\delta}}
\]

therefore

\[
\text{Area Measured} = \Delta + 2\delta + (1+\beta) \left( 2 e^{-A/\delta} \right) + (1-e^{-\frac{A}{\delta}}) + \delta(e^\frac{-A}{\delta} - 1)
\]

Even as \( \Delta \) goes to zero, i.e. no dead time, there is a fixed finite error introduced by the rectification of the signal when it passes through zero.
The area measured with no dead time can be solved. With reference to the figure, when $\phi$ is zero,

$$A = \int_0^\phi \left( 2(1 - e^{-\frac{t}{\lambda}}) d\tau + \int_\phi^\infty \frac{\phi}{e^t} dt \right)$$

$$= \phi^2 \left( 2 - e^{-\frac{\phi}{\lambda}} - e^{-\frac{\phi}{\lambda/2}} - 1 \right)$$

Assuming that $1 > > e^{\frac{\phi}{\lambda}}$

then

$$\phi = \lambda \ln \frac{\phi}{\lambda}$$

then the measured area with no dead time becomes

$$A = \lambda \left( 2 - e^{-\frac{\phi}{\lambda}} - e^{-\frac{\phi}{\lambda/2}} - 1 \right)$$

For no dead time this becomes a constant to calibrate all readings by, but with dead time the error becomes

$$\text{Error} = \frac{\text{Area Measured with dead time} - \text{Area no dead time}}{\text{Area no dead time}}$$

for various values of $\phi$ and $\phi/\lambda$

$$\phi = 1, 5, 10 \times 10^{-3} \text{ seconds}$$

$$\phi/\lambda = 5, 50, 100 \times 10^{-3} \text{ seconds}$$

then we have

<table>
<thead>
<tr>
<th>$\frac{\phi}{\lambda}$</th>
<th>Error Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>132%</td>
</tr>
<tr>
<td>1.0</td>
<td>75%</td>
</tr>
<tr>
<td>0.2</td>
<td>32.3</td>
</tr>
<tr>
<td>0.1</td>
<td>12.5</td>
</tr>
<tr>
<td>0.05</td>
<td>6.85</td>
</tr>
<tr>
<td>0.02</td>
<td>2.14</td>
</tr>
<tr>
<td>0.01</td>
<td>1.4</td>
</tr>
<tr>
<td>0.00</td>
<td>0</td>
</tr>
</tbody>
</table>

**TABLE 3**  **ERROR DUE TO DEAD TIME, EXPONENTIAL APPROXIMATION**
The results of these approximations to the wave form and the associated errors that are involved are plotted together in Figure 13.
Figure 13

Error due to Dead Time

Exponential Approximation
RECOMMENDATIONS FOR ADDITIONAL WORK

Early in the development, a Fourier analysis of the wave form that appeared to the meter was made. The results of this analysis are now in Appendix III. The crux of this is that, while the d.c. component has a large magnitude, some of the a.c. harmonics also have a reasonable value, and should possess considerable information. Since this was essentially a time domain device, there is no reason why with some sharp R.C. filtering these harmonics could not be used to add to the signal energy. Hence, recommendation number one, the design and construction of suitable filters to include the first few harmonics, and measure their resultant RMS value. This would also require different meter to read this wave form. Care would have to be taken to filter out 60 cps effectively since a system measuring R.C. would be much more subject to a.c. noise.

Although always more work can be done on any system, most of the other recommendations fall into the class of improvement. The drift in the amplifier is now mostly due to thermal effects; compensation for this, in addition to the feed back, could be worked out. Chopper stabilizing the amplifier is a possibility. The use of better diodes and transistors should make a satisfying improvement, especially in the diodes of the clamp circuit.

Metering will be a problem with this, due to the low levels and the requirement of d.c. coupling. The General Radio VTVM is now only fair, more stable meter would cut down the number of setting that have to be reset measurement.

This work circumvents the problem of self potential by the use of a large capacitor in the input circuit; at present there is no provision for
measuring this potential across the capacitor. With no more trouble a series of switches could be incorporated to allow a single meter to measure this potential, the operating point voltage that is now measured with meter 3, and the output of the diode clamp. This meter would have to be more versatile than the 6J.R. VTVM to allow for the wider range.
APPENDIX I

DIODE CHARACTERISTICS

The diodes such as IN34 and CK 706 do not exhibit their extreme non-linear behavior at low voltages and currents. At low ranges of current and voltage, their response deviates far from ideal, at levels below approximately 5 milli volts. The diode looks almost like a simple resistance of about 2000 ohms. For a graphical demonstration of the characteristics see accompanying graph.
Appendix L
Diode Characteristics
APPENDIX II

FOURIER ANALYSIS

A fourier analysis of the expect wave form that would appear to the meter after the clamp circuit. The wave form was assumed to be that of a series of exponential decays of length \( L \), time constant \( \tau \), height \( A \), appearing one after another every \( L \) seconds. The results will be summarized in closed form and a few of the first harmonics plotted in Figure 14.

In standard notation,

\[
A_0 = A \frac{2}{L} \\
A_n = \frac{1}{L} \left( \frac{1}{(\frac{n}{L})^2 + (\frac{n\pi}{L})^2} \right) \left( \frac{1 + \cos n\pi}{2} \right) \\
B_n = \frac{1}{L} \left( \frac{1}{(\frac{n}{L})^2 + (\frac{n\pi}{L})^2} \right) \left( \frac{\sin n\pi}{\pi} \right) \left( 1 + \cos n\pi \right)
\]

Two cases are examined more carefully in the graph,

Case 1; \( \tau = L/3 \)  
Case 2; \( \tau = L/4 \)

For the difference see Figure 14.

The frequency of the input square wave on figure 14 is at the \( n=1 \) point on the graph. Hence when measuring the effect of any of the higher harmonics, the effect of any inbalance in the input signal will be of the same frequency as the square wave, not affecting the harmonics; there fore it can be filtered out.
Relative Amplitude

Figure 14. Fourier Analysis of Measured waveform

Case 1: \( f = \frac{1}{3} \)

Case 2: \( f = \frac{1}{4} \)

CODE

\[ E_n \sin \left( \frac{2\pi n x}{L} \right) \]

\[ E_n \cos \left( \frac{2\pi n x}{L} \right) \]

Frequency of input square wave at \( n \) L

d.c. 2 4 6 8 \( \Rightarrow \) d.c. 2 4 6 8
APPENDIX III

MODIFICATIONS TO THE GENERAL RADIO AC VACUUM TUBE VOLTMETER TYPE NUMBER 727A

This meter was originally designed to measure a.c. signals of a few volt up to approximately 300 volts. For a a.c. meter it has a very strange circuit indeed, it is all direct coupled, capable of reading deflection due to a d.c. signal except for an input capacitor. The operation was as follows; the first tube acted as half wave diode, the rest of the circuit amplified this half wave rectified signal and fed it to a meter. The modification included removing the input capacitor to take advantage of the direct coupled circuit; removing the rectifier, to raise the input impedance. The tube was not necessary anyway since only d.c. was to be measured and no rectification of any spurious a.c. signals was desired. A few changes in resistances values improved the sensitivity. After the modification the meter satisfactorily measures low d.c. voltages, full scale deflection is accomplished with about 40 millivolts. The scale is poorly marked at the lower (zero) and, and hence difficult to read accurately. The sensitivity of the meter changes with position, being much more sensitive in the right hand side of the dial. Hence, for low reading, it is more satisfactory to set zero to be some where in the middle of the dial and make all readings relative to the new zero. A schematic with the noted changes is included, it is listed as Figure 15.
Figure 16. Modifications to the General Radio VTVM

Modifications dotted and indicated by arrows