AN INVESTIGATION OF THE TIME SERIES OF VISIBILITY AND PRECIPITATION INTENSITY FLUCTUATIONS

by

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Signature of Authors. . .

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Chairman, Departmental Committee on Graduate Students

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ABSTRACT

Visibility is an extremely important factor in flight operations, and some work has been done in the past in attempting to find a correlation between the fluctuations of visibility and the fluctuations of precipitation rate. In this paper records of visibility and precipitation rate, recorded by transmissometer and radar, respectively, are subjected to spectrum and cross spectrum analyses. Cross spectrum analysis intimates that the long period fluctuations of the two variables are more closely related than the shorter period fluctuations. At these longer periods, the variations of precipitation rate tend to lead the variations of visibility by one to four minutes. Grouping of storms by ceiling height and synoptic situation indicates a tendency for the cross spectral patterns to show characteristics dependent on meteorological conditions. Further work in visibility might prove more fruitful if other factors which affect the visibility were also studied.

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An Investigation of the Time Series of Visibility and Precipitation Intensity Fluctuations

I. Introduction

Visibility and ceiling are critical factors in the operation of aircraft. Landings and take offs are severely limited during those periods when ceiling and visibility are low. Hence, it is of the utmost importance that the meteorologist be able to forecast those weather elements. In general, one of the problems of forecasting is to be able to relate the behavior of one variable to that of others. Accordingly, the forecasting of visibility and ceiling would be facilitated if more information were available on the manner in which the fluctuations of other weather elements were related to them.

For the past few years the Weather Radar Project at the Massachusetts Institute of Technology has been interested in determining the correlation between precipitation rate, as determined by radar, and the fluctuations in visibility and ceiling. A pilot study was performed by Austin and Glass (1). They used iso-echo contours from the radar and USAF WBAN Forms 10A to obtain reports of ceiling and visibility. In the majority of cases studied, it was observed that increased precipitation was accompanied by a decrease in visibility and ceiling height. However, any relation which might prove useful to the forecaster was far from obvious from the results they obtained. In 1959, Roylance (2) made a study, concerning himself only with the behavior of ceiling and visibility following peaks of precipitation intensity. Roylance had the advantage of use of the pulse integrated radar return, as well as a continuous visibility record from the transmissometer. He was able to conclude that ceiling and visibility falls were related to increases in the precipitation rate, especially when the peaks of precipitation rate were long duration changes. Roylance was able to obtain good graphical correlations, especially when he stratified by storm types. Austin (3), in a paper given at the Eighth Weather Radar Conference, corroborates what he presented earlier, but also points out that ceiling and visibility are determined by many factors. In the case of ceiling, short term precipitation bursts might not increase the saturation of the air below the cloud base to a sufficient extent to lower the base. The wind speed and direction also play an important part in determining the ceiling. It is not infrequently observed that the ceiling height increases with an increase in precipitation rate. Visibility is certainly related to precipitation rate. However, it is also correlated to drop size, and drop size need not necessarily be related to precipitation rate. Perhaps an even more important factor than precipitation intensity in determining the visibility is fog. Fog with a light rain has been observed to bring the visibility as low as intense precipitation without fog. All of these factors only contribute to the confusion when an attempt is made to correlate precipitation rate with ceiling and visibility.

The purpose of this paper is th delve further into the relationship between visibility and precipitation rate. The availability of continuous records of these two variables and a high speed digital computer allow the use of a different technique with which to investigate the problem. By utilizing the spectra and cross spectra of the records of the precipitation rate and visibility, it is hoped that some information will be uncovered concerning the correlation at different frequencies between the two variables. It is known that, by itself, an increase in the precipitation rate will cause the visibility to drop. Perhaps the spectra and cross spectra of these two variables will uncover certain frequencies in their behavior which predominate. Also, a study of the spectra and cross spectra of the records of storms which may be classified according to the condition of other variables recorded during the storm might reveal a difference in the relationship between visibility and precipitation rate dependent on these other variables.

Eleven precipitation cases were chosen to be studied. Six storms were observed at Logan Airport in East Boston, Massachusetts, and five storms were observed at Bedford Airport in Lexington, Massachusetts. Since one of the storms was also utilized to make a study of the correlation between visibility and ceiling, twelve spectrum and cross spectrum analyses were done.

II. Measurements of Rainfall, Visibility, and Ceiling

A. Rainfall intensity measurements

The variability and intensity of precipitation over Bedford and Logan Airports was estimated from signal intensity measurements as recorded by the AN/CPS-9 and SCR-615-B radars at the Massachusetts Institute of Technology which operate on 3 and 10 centimeter wavelengths, respectively. Radar measurements of precipitation rate have the advantage of providing a continuous record over any desired point. However, these measurements are not as accurate as those obtained from a rain guage. Fortunately, the inaccuracy lies in the magnitude of the precipitation rate. Since this report is concerned with fluctuations rather than magnitudes of precipitation rate, no harm is done by using this less accurate measuring device. The instruments which recorded the intensity of the radar signal were pulse intergrating devices which measured the time-averaged signal intensity in decibels with respect to a milliwatt.

Rainfall measurements over Bedford Airport were made with the AN/CPS-9 directed along an azimuth of 305°, at a range of 13.1 miles and an elevation of 0.5°. The volume of air sampled by the radar was approximately 1200 feet in both vertical and horizontal extent with the center at an altitude of about 1200 feet.

Over Logan Airport, the AN/CPS-9 radar was aimed at an azimuth of 92°, elevation of $2\frac{1}{2}$°, and a range of 4.1 miles. These coordinates enabled the rainfall to be measured as accurately as possible directly over transmissometer at the airport without interruption of the signal due to ground clutter. At an elevation angle of $2\frac{1}{2}$°, the radar beam is sampling the rainfall approximately 1000 feet above the ground elevation of the transmissometer. Because of the location of the SCR-615-B radar on the roof of Building 24, here at M.I.T., the necessary elevation of the radar beam was 4° to enable the signal to be free from interference from ground clutter. At an elevation of 4°, the radar beam is sampling the rainfall approximately 1800 feet above the ground elevation of the transmissometer. The azimuth and range of the SCR-615-B were the same as the AN/CPS-9 when measuring rainfall over Logan Airport.

The data from the rainfall measurements were taken over the longest time intervals possible in order to make a logical time series of points. Sampling was performed at given time intervals with amplitude resolution to the nearest decibel below a milliwatt (dbm). Some minor breaks are present in the records for times less than 10 minutes due to malfunctioning of the radar and the interruption of the record to make other measurements. If the period of interruption was longer than 10 minutes, the time series was terminated and a new series started. If the interruption was less than 10 minutes in length, straight line interpolation was performed on the record between the value of the rainfall intensity at the start of the interruption and the first value of signal intensity after the measurements started again.

Radar intensity measurements from Bedford Airport have been analyzed in decibels below a milliwatt, while those from Logan Airport have been converted to rainfall rates in millimeters per hour and normalized to one mile. The conversion of rainfall intensity in decibels below a milliwatt to millimeters per hour has been discussed by Austin (4), and is as follows:

$$P_r(\text{dbm}) - Q = -60 + 16 \log_{10} R - 20 \log_{10} r$$

where (P) is the power returned in decibels below a milliwatt, (Q) is the cable correction of the individual radar set, (R) is the precipitation rate in millimeters per hour, and (r) is the distance at which the measurements are taken. The purpose of using both forms for representing the radar signal intensity will be discussed in a later section.

Over Logan Airport, because of its closeness to M.I.T., it was observed that the radar would often saturate during periods of heavy rain intensity. For this reason the maximum value of rainfall that could be measured was 3.2 millimeters per hour corresponding to 0 decibels below a milliwatt. No instances of saturation were reported from measurements over Bedford Airport.

Both AN/CPS-9 and SCR-615-B radar sets are capable of recording rainfall intensities above an arbitrary noise level. This noise level is designated where one gets some signal return even though there is no precipitation occurring, or if occurring, the values of rainfall are insignificant for the purpose of study of this particular problem. The noise level, therefore, arbitrarily selects the lowest level of signal intensity measurement. For Logan Airport, the noise level was placed at -40 (dbm) corresponding to .01 millimeter of rainfall per hour; while at Bedford Airport the noise level was placed at -50 (dbm) corresponding to .04 millimeter of rainfall per hour.

B. Visibility Measurements

Instantaneous measurements of visibility were made at Logan and Bedford Airports with a transmissometer, an instrument used for estimating the runway visual range. The accuracy of this device for determining the visibility is limited. Such an instrument measures the visual range in a sample of air along the runway only, and hence, the estimates of visibility in other directions can not be determined. Whether an estimate of visibility along the runway is representative of the column of air sampled by the radar beam is uncertain.

The transmissometer defines daytime visibility in terms of the distance at which an observer can see a dark object against the horizon sky. At night the instrument defines the visibility as the distance at which an observer can see a low intensity light source. Because of the difference of physical principles involved between day and night, the instrument requires two separate calibrations. Both calibrations are designed to emphasize the lower visual ranges, (under 3 miles).

During the daytime the relationship between the visibility and the actual transmissometer reading is given by:

$$E_o = T^{V/L}$$

where (E_o) is the assumed observer contrast threshold, (.055 lumens per kilometer); (L) is the length of the baseline between the light source and the photoelectric detecting device; (T) is the transmissometer reading; and (V) is the visibility. For are purposes, this function was best represented by Figure 1.

During the nighttime the relationship between the visibility and transmissometer reading is given by:

$$S = \frac{IT^{V/L}}{V}$$

where (S) is numerically equal to the assumed observer illumination threshold, (.052 lumens per kilometer); and (I) is the assumed candle power of the light source, (25 candle power). This function is best represented by Figure 2.

Because of the above difference in calibration, the transmissometer readings must be modified during the times of dawn and dusk, or at least

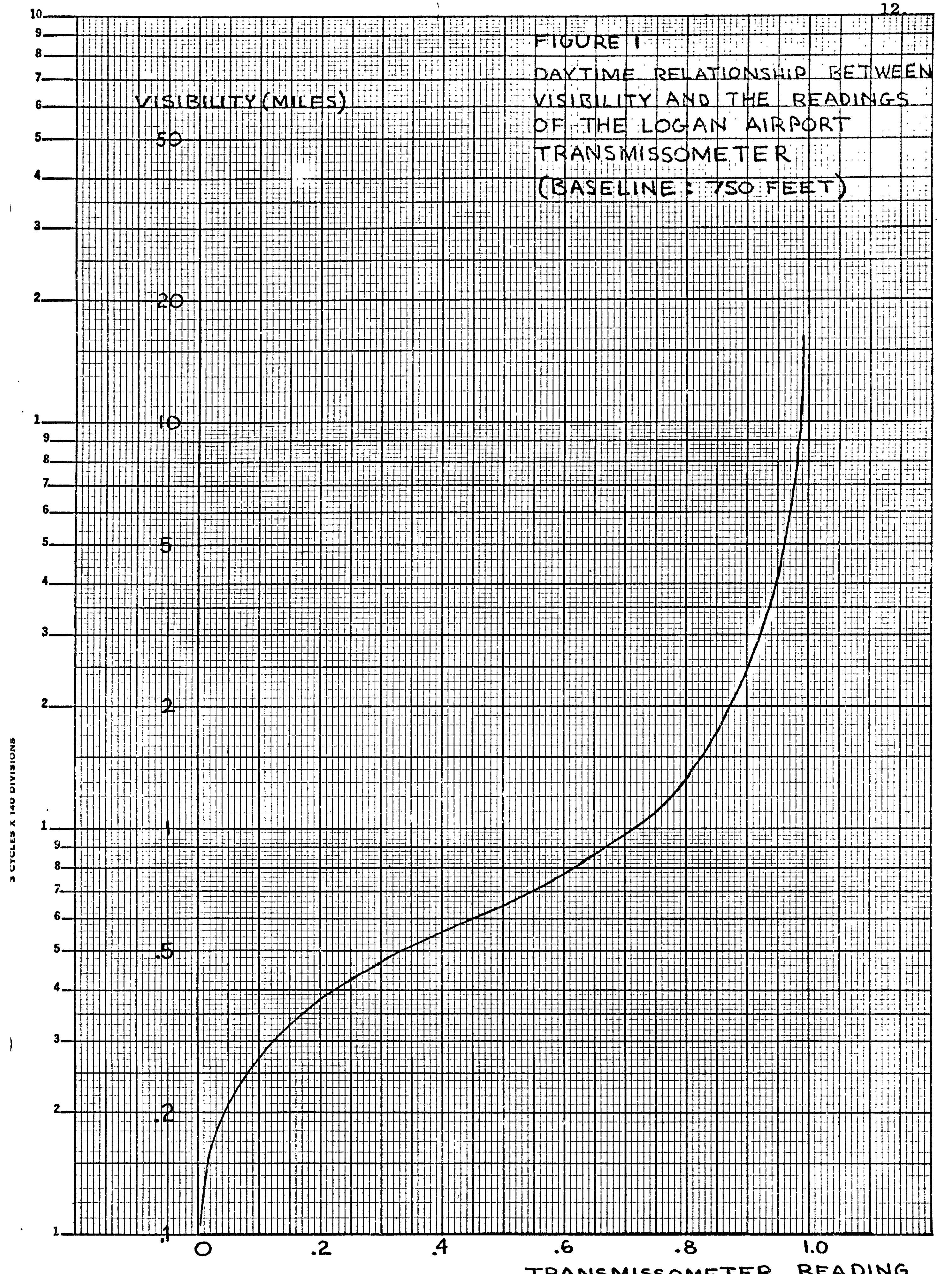
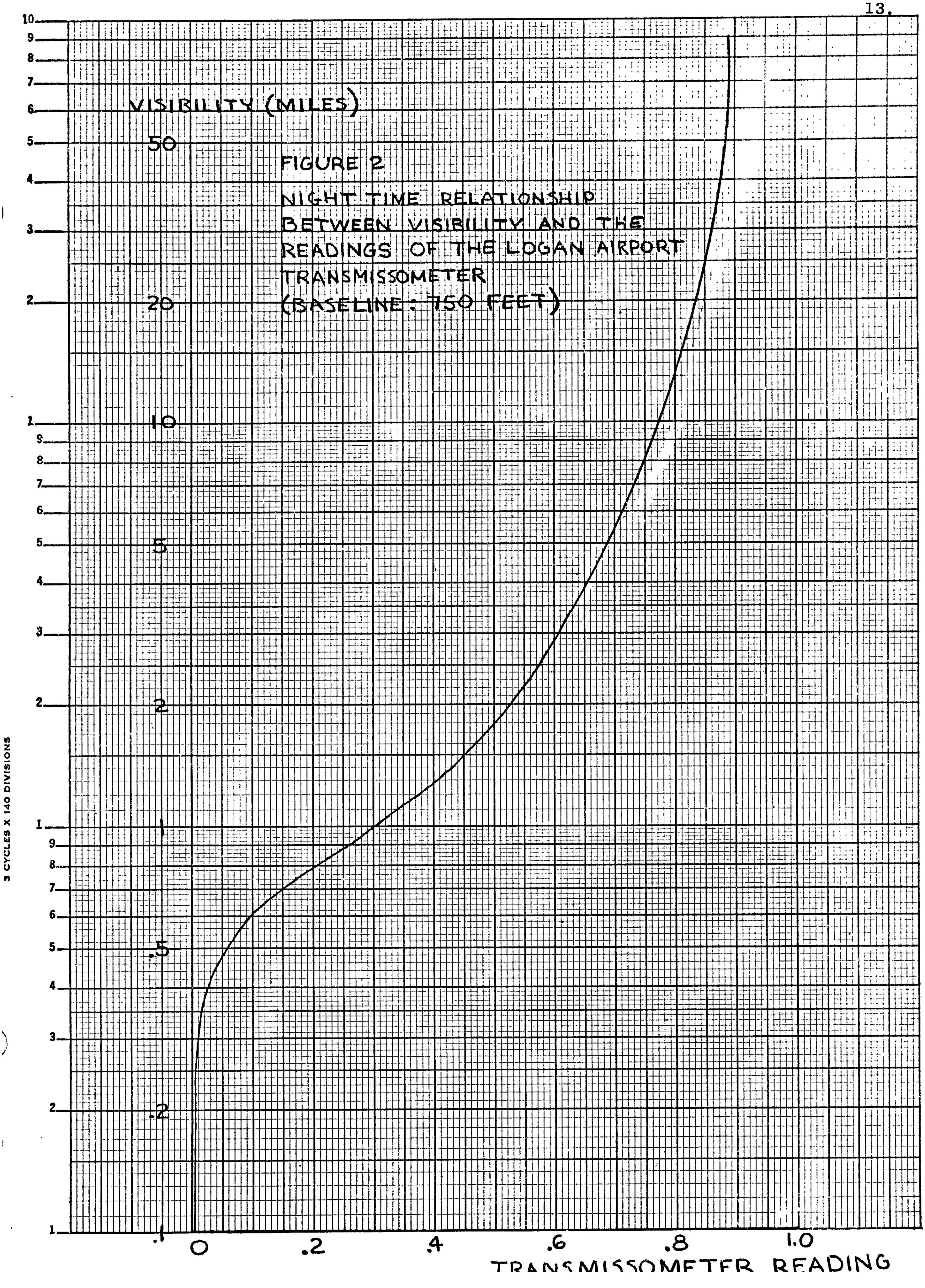


FIGURE I
DAYTIME RELATIONSHIP BETWEEN VISIBILITY AND THE READINGS OF THE LOGAN AIRPORT TRANSMISSOMETER (BASELINE: 750 FEET)
VISIBILITY (MILES)
50
20
10
5
2
1
.5
.2
.1
0
.2
.4
.6
.8
1.0
TRANSMISSOMETER READING
3 CYCLES X 140 DIVISIONS


VISIBILITY (MILES)
50
20
10
5
2
1
.5
.2
.1
FIGURE 2
NIGHT TIME RELATIONSHIP
BETWEEN VISIBILITY AND THE
READINGS OF THE LOGAN AIRPORT
TRANSMISSOMETER
(BASELINE: 750 FEET)
0
.2
.4
.6
.8
1.0
TRANSMISSOMETER READING
3 CYCLES X 140 DIVISIONS

the presence of such a transition period must exist in the record. For the purpose of our investigation, we did not try to modify the transmissometer readings during such a transition period. If measurements were made in the daytime through the transition period to night, the daytime calibration was used over the length of the entire record. If measurements began after sundown, the nighttime calibration was used.

Observers at Bedford and Logan Airports have also noticed minor fluctuations in the transmissometer record due to air traffic along the runway on which the instrument is located. In the daytime, the exhaust of jet aircraft during take-off and landing operations sometimes affects the transmissometer readings; depending on the wind direction. At night, when aircraft use lights for take-off and landing procedures, there was also some variation.

It can be seen for a number of reasons that the visibility as measured by the transmissometer may be quite different from that in the column of air sampled by the radar beam. The advantage of using a transmissometer for our measurements was that it was the only known instrument which provided a means of instantaneously recording the variation of atmospheric transmission characteristics.

C. Ceiling Measurements

Values of the ceiling height are available from the individual WBAN records of each storm. It was thought promising to look at instantaneous values of ceiling height in the form of a time series for certain select storms. Because of the lack of measuring facilities at Bedford Airport during the period in which the data were taken, it was impossible to obtain records of the instantaneous values of ceiling height. At Logan Airport, however, instantaneous values of ceiling height were available for certain select times during which visibility and rainfall rates were recorded. As a result of the uncertain ceiling height at times in which fog prevailed, and because of mechanical malfunctioning of the ceiling measuring device, few storms comprised enough continuous data points to make up a time series that could be analyzed.

Instantaneous values of the ceiling height were recorded at Logan Airport when possible with a rotating beam ceiliometer. The projector and and detector of the device are located far enough apart to permit accurate measurement of the ceiling, if the cloud height is less than 5,000 feet.

For the few cases analyzed, little information could be determined from the instantaneous record of the ceiling. It was thought profitable to abandon the time series analyses of ceiling, compared with rainfall and visibility, due to the difficulties in comprising a continuous record of ceiling and also because of lack of information in such a record. One select storm, (January 28, 1960), which afforded the best available instantaeous ceiling record has been included for the purpose of completeness of discussion of the variability of the ceiling measurements.

III. Tukey Spectrum Estimation

It was decided that spectrum and cross spectrum analyses might prove an interesting and profitable approach to a study of the correlation between visibility and precipitation. The spectra of each variable would indicate the important frequency components of the behavior of that variable and cross spectrum analysis of the two variables would point out which frequencies share a common importance in both variables. Furthermore, the method of collecting the data in a continuous record lends itself to the establishment of evenly spaced time series. As in the case of all statistical studies of this nature, however, the time series are truncated. Thus, the general trend of the series will have the appearance of a section of a very long period, and will be interpreted as such by the numerical estimation process of the program. This causes a great deal of variance to be associated with the first two or three periods.

The program used was the Tukey Spectrum Estimation, developed by Convair of San Diego. Given two series, (X) and (Y), the primary and secondary series, this program computes the following things: (after first determining the mean of each series and subtracting it out)

1. The auto correlation of each series, by the equation:

$$A(L) = \frac{1}{n-L}\sum_{i=L+1}^{n} x_{i-L}\, x_i - \left[\frac{1}{n-L}\right]^2 \sum_{i=L+1}^{n} x_{i-L} \sum_{i=L+1}^{n} x_i$$

where n = the number of points in the series

L = 0,1,2,....,m

m = the number of lags taken

2. The in-phase correlation between the two series, by the equation:

$$E(L) = \frac{1}{2}\left[C(L) + D(L)\right]$$

where C(L), the positive part of the cross correlation is given by:

$$C(L) = \frac{1}{n-L}\sum_{i=L+1}^{n} x_{i-L}\, y_i - \left[\frac{1}{n-L}\right]^2 \sum_{i=L+1}^{n} x_{i-L} \sum_{i=L+1}^{n} y_i$$

and D(L), the negative part of the cross correlation is given by:

$$D(L) = \frac{1}{n-L}\sum_{i=L+1}^{n} x_i\, y_{i-L} - \left[\frac{1}{n-L}\right]^2 \sum_{i=L+1}^{n} x_i \sum_{i=L+1}^{n} y_{i-L}$$

3. The out-of-phase correlation between the two records, given by:

$$F(L) = \frac{1}{2}\left[D(L) - C(L)\right]$$

4. The energy estimation of the spectrum of each series, which gives the variance as a function of frequency, is given by the equation:

$$S(k) = \frac{\delta_k}{m}\left[\sum_{L=1}^{m} 2e(L)\cos\frac{kL\pi}{m} A(L) + A(0)\right]$$

where k = 0,1,2,.....,m

δ_k = for k = 0 or k = m, = 0 for all other k

2e(L) = 1 $\cos\frac{L\pi}{m}$ is the hanning factor, and acts to smooth the series.

5. The co-spectrum, or in-phase energy spectrum, which gives the covariance as a function of frequency, is given by:

$$Z(k) = \frac{\delta_k}{m}\left[\sum_{L=1}^{m} 2e(L)\cos\frac{kL\pi}{m} E(L) + E(0)\right]$$

6. The quadrature spectrum or out-of-phase energy, which gives the covariance as a function of frequency, is given by:

$$W(k) = \frac{\delta_k}{m}\left[\sum_{L=1}^{m} 2e(L)\sin\frac{kL\pi}{m} F(L)\right]$$

7. In addition, the program gives the respective normalized co-spectra and quadrature spectra, expressed by:

$$P(k) = Z(k) / \sqrt{S_x(k)\, S_y(k)}$$

$$Q(k) = W(k) / \sqrt{S_x(k)\, S_y(k)} \quad ,$$

the coherence, which is analagous to the square of the correlation coefficient, as a function of frequency, given by:

$$R(K) = \sqrt{P(K)^2 + Q(K)^2}$$

the coherence squared, and finally, the phase lead of the secondary series over the primary series, as a function of frequency, given by:

$$PH(K) = \tan^{-1} \frac{W(K)}{Z(K)}$$

The frequencies to be examined are related to the number of lags chosen by the following equation.

$$f = \frac{L}{2m\Delta t}$$

where Δt is the interval between points in the time series. It should be noted here that the 20 frequencies chosen for observation are frequently referred to in this paper as points, implying points on the spectral curves.

When choosing how many lags to consider, two points must be kept in mind. If one chooses a very small number of lags, much of the detail of the power spectrum may be missed. If, on the other hand, considering the length (n) of the series, too many lags are chosen, the confidence that can be placed on each point of the spectrum is small.

Although the lengths of the various series considered in this study varied between 143 and 521 data points taken at one minute intervals, a standard number of 20 frequencies were investigated in each case. Through experimentation with different lags, it was observed that a greater number of lags did not really yield any more information, even in the cases of the longest storms. On the other hand, fewer lags did obscure a certain amount of detail.

IV. Discussion of Individual Storms

The purpose of this section is to discuss, individually, the eleven storms considered in this study. A brief paragraph, describing the weather elements, and several graphs, which include the record of the storm itself, the spectra of each of the variables considered in the storm, and their co-spectrum, quadrature spectrum and coherence are contained in the discussion of the storm. No attempt will be made in this section to group or stratify the various storms.

The first paragraph in each written discussion will discuss the synoptic situation and the state of those meteorological variables which pertain to visibility and precipitation. The general pressure pattern influencing Boston is obtained from the M.I.T. hemispheric charts prepared by the Department of Meteorology and the U.S. Weather Bureau daily weather maps. The remaining information is obtained from records kept at Logan and Bedford Airports. The meteorological variables considered are wind speed and wind direction, present weather, visibility, ceiling, temperature and dew point. No attempt is made to analyze these data and try to guess what caused the precipitation or fog inhibiting the visibility. However, in another chapter storms will be grouped according to the state of some of these elements, to see if thereby any light is shed on the spectra and cross spectra of the storms. If particular relations are obvious from the observational data such as between ceiling and visibility, these relations will be pointed out.

The second paragraph considers the graphs accompanying each storm. The record is discussed first. The values of the variables considered for each storm were obtained from similar graphs plotted earlier, in connection with another study. These earlier graphs were not all plotted from data sampled at one minute intervals. Values at one minute intervals were interpolated from them and used in plotting the present graphs and in determining the spectra and cross spectra. This introduces a certain amount of smoothing, but the smoothing would be too insignificant to cause any harm. The records, spectra and cross spectra of the Logan Airport storms deal with rainfall rate and reciprocal visibility as variables. Reciprocal visibility, rather than visibility was used because the relationship between visibility and rainfall rate is an inverse one and it was felt that a direct relationship was to be preferred both from the point of view of observing the record and from computing the spectra and cross spectra. Also, the reciprocal tends to place a good deal more emphasis on the fluctuations of the lower visibilities, which physically are more meaningful. The variables used in the records, spectra and cross spectra of the Bedford Airport storms are visibility and radar signal intensity, taken directly from the pulse integrator. This also gives the desired direct relationship, since the decibel values of radar signal intensity increase as the rainfall rate decreases, and vice versa. The spectra of the records, which show variance versus frequency x $2m\Delta t$, are plotted on log paper to emphasize the shorter periods. Previous plots on linear paper completely hid the features of the curve when the variance was small. It was decided that values of variance less than 0.1 would be considered meaningless. This was because the variance at the first two points on the spectral curve lay between 10 and 20, and values more than three orders of magnitude smaller could be attributed to round off error in the numerical computations.

Thus the abscissa of the spectral curves lies along the line variance = 0.1. The cross spectra, consisting of the co-spectra and the quadrature spectra shows normalized covariance versus frequency x $2m\Delta t$ and the coherence shows the correlation coefficient squared versus frequency x $2m\Delta t$. The dashed horizontal lines on the coherence curves are the 5% and 10% confidence limits, calculated from the equation:

$$\beta = \sqrt{1 - p^{1/(df - 1)}} \qquad (6)$$

where β = confidence limit

df = degrees of freedom, given by

$$df = \frac{2n - m/2}{m}$$

p = probability level

Points on the coherence curve below these limits were not considered significant, nor were the corresponding points on the co-spectrum and quadrature spectrum. Also, points on the co-spectrum and quadrature spectrum whose corresponding points on the spectrum of either variable showed variance of less than 0.1 were considered meaningless. For even if a certain frequency component of the two records had a value above the confidence limits on the coherence curve, if it turned out that that frequency had only a negligible component in either one of the records or both, little meaning could be attached to it. One exception to this rule must be considered. If it transpires that the shape, or trend in a particular section of the co-spectra or quadrature spectra of several different storms exhibit certain similarities, especially if these storms are stratified in the same class, even though the several points comprising the shape of these curves do not lie above the confidence limits on the coherence curve, these trends might still be considered significant.

October 1, 1959 (1534-1800) Logan Airport (Figures 3-5)

A low pressure center, located over central Pennsylvania and moving due east, with an associated warm front to the north of Boston and a cold front to the southwest forms the synoptic picture for the period under consideration. Light rain and fog accompanied by light winds from the ESE are recorded throughout the interval in which the observations were taken. Low visibilities, ($2-\frac{1}{4}$ miles) and low ceilings, (200 feet) characterize this particular time. The temperature and dew point remain constant throughout the period.

The records of the reciprocal visibilities and rainfall rates in millimeters per hour consist of 153 data points commencing at 1628 and terminating at 1900. The record, itself, contains more major fluctuations in the lower frequencies for the case of rainfall than for the case of reciprocal visibility. This can be borne out by looking at the individual spectra of the two variables. Periods shorter than 8 minutes can be generally disregarded due to the lack of variance in the high frequencies. Because of this lack of variance in the high frequencies and the absence of significant points denoted by the confidence limits on the coherence curve, there can be little meaning attached to the cross spectrum curves. The possibility of a significant co-spectral peak at point 0 is probably the only thing of consequence in the cross spectrum record.

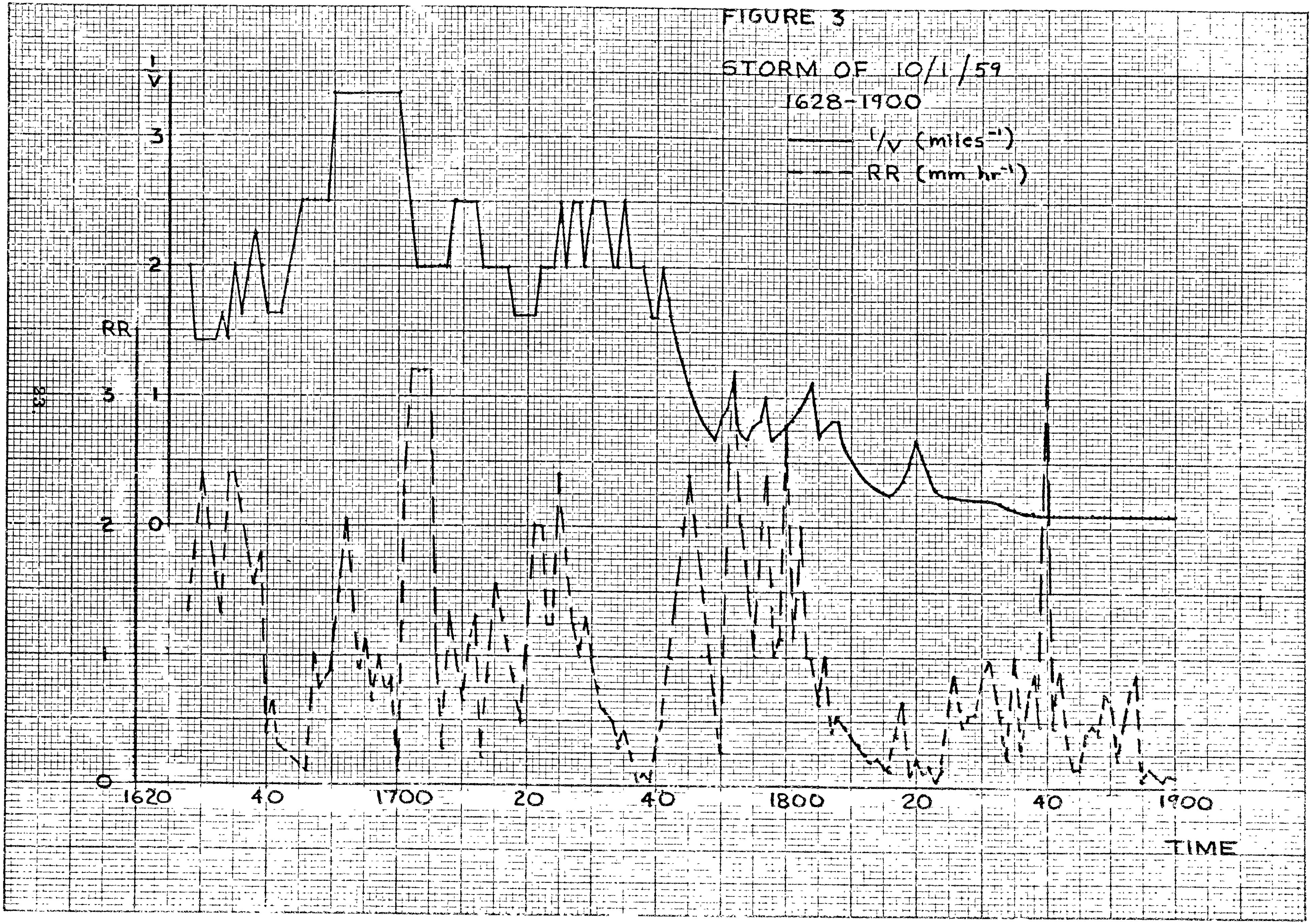


FIGURE 3

STORM OF 10/1/59

1628-1900

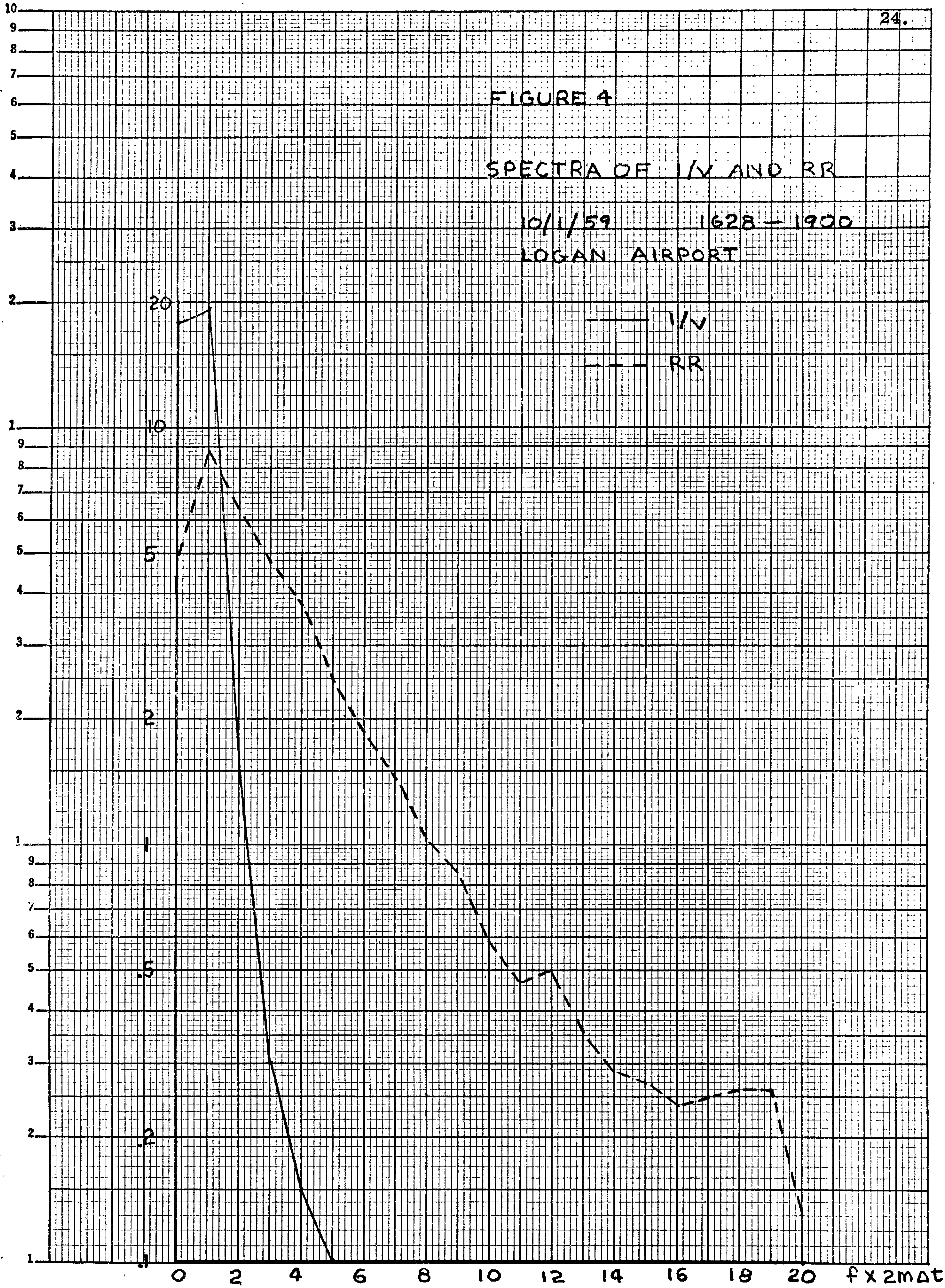


FIGURE 4
SPECTRA OF 1/V AND RR
10/1/59 1628 – 1900
LOGAN AIRPORT
1/V
RR
20
10
5
2
1
.5
.2
.1
0 2 4 6 8 10 12 14 16 18 20 f x 2mΔt

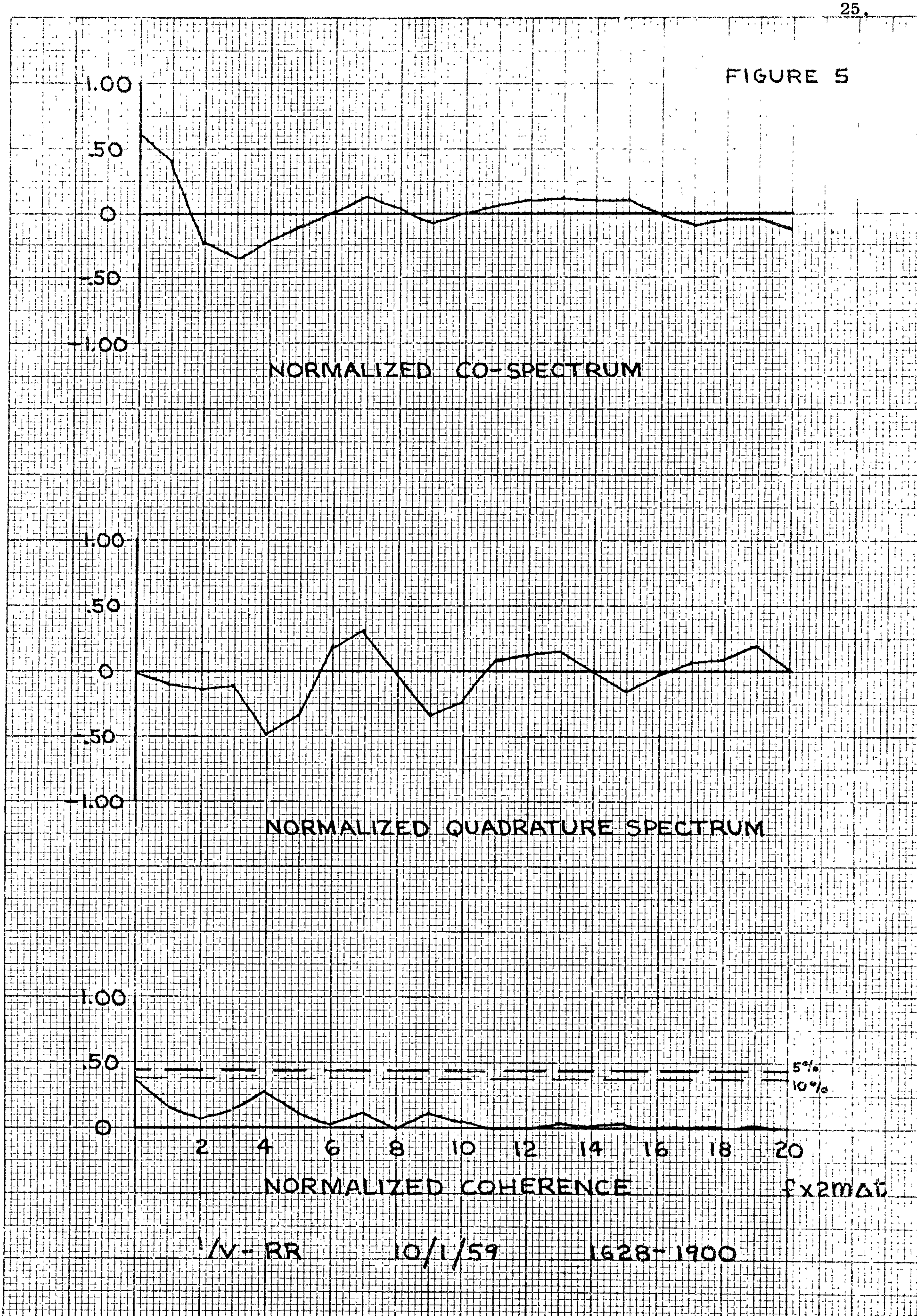


FIGURE 5
1.00
.50
0
-.50
-1.00
NORMALIZED CO-SPECTRUM
1.00
.50
0
-.50
-1.00
NORMALIZED QUADRATURE SPECTRUM
1.00
.50
0
5%
10%
2
4
6
8
10
12
14
16
18
20
NORMALIZED COHERENCE
f x 2mΔt
1/V - RR
10/1/59
1628-1900

<u>November 24, 1959 (1422-1930) Logan Airport (Figures 6-8)</u>

Boston lies in the warm air ahead of a cold front extending from a low pressure center in eastern Ontario, and at the same time, to the north of a warm front extending from a low pressure center off the New Jersey coast. The low center associated with the cold front lies over the eastern Great Lakes region throughout the period. The wind speeds are moderate, and the wind direction shifts from the SSE to the WSW. Light rain, light moderate and heavy rain showers, light and moderate thunderstorms, and fog are all reported during the period under consideration. The visibility ranges from $\frac{1}{8}$ to 8 miles with the lowest values reported at times of light rain showers and fog. When the wind shifts from south to southwest, the fog clears and the visibility rises, dropping again later when moderate and heavy rain and thunder showers are reported. The ceilings are variable between 700 and 2500 feet, generally showing lower values when the visibilities are lower. The temperature rises several degrees at the time the wind shifts to the west, and then drops diurnally at the end of the period.

The record of reciprocal visibilities and rainfall rates consists of 237 data points starting at 1516 and terminating at 1912. It is to be noted that toward the end of the period there is a correlation between the two variables, such that a rise in the rainfall rate is usually followed by a rise in the reciprocal visibility. Earlier, the rather small scale fluctuations in the rainfall rate have no effect on the reciprocal visibility (except for the peak at 1545). The spectra of the two variables are in keeping with their records, the rainfall rate showing somewhat more variance in the higher frequencies and the reciprocal visibility ceasing to show significant variance after a period of four minutes. We assume then, that nothing above point 10 is significant in the cross spectra.

The co-spectrum has its maximum positive value at point 0 and then tapers down to insignificant values. The quadrature spectrum exhibits a peak at point 3, which, according to the confidence limits on the coherence curve, is significant.

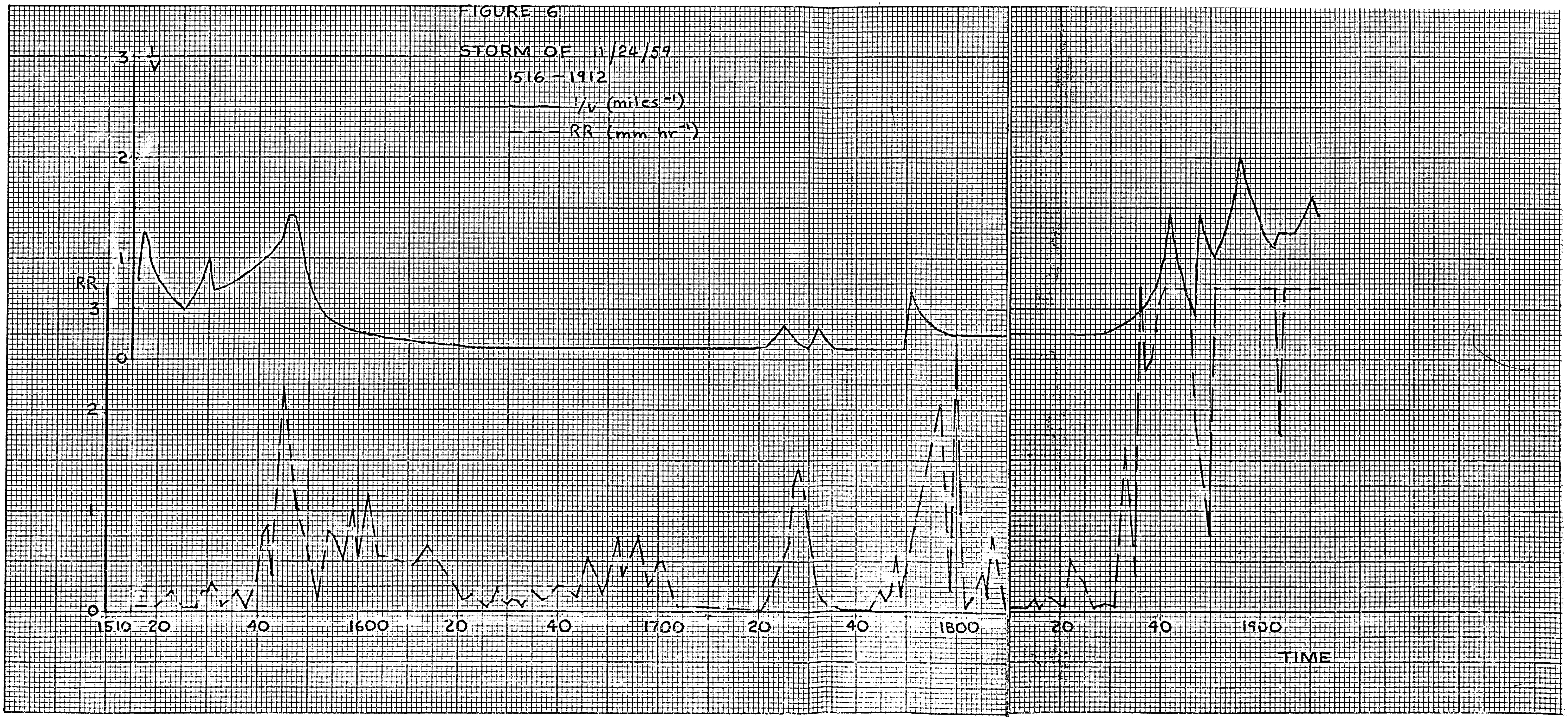


FIGURE 6
STORM OF 11/24/59
1516 – 1912
1/v (miles⁻¹)
RR (mm hr⁻¹)
3 1/V
2
1
0
RR
3
2
1
0
1510 20 40 1600 20 40 1700 20 40 1800 20 40 1900
TIME

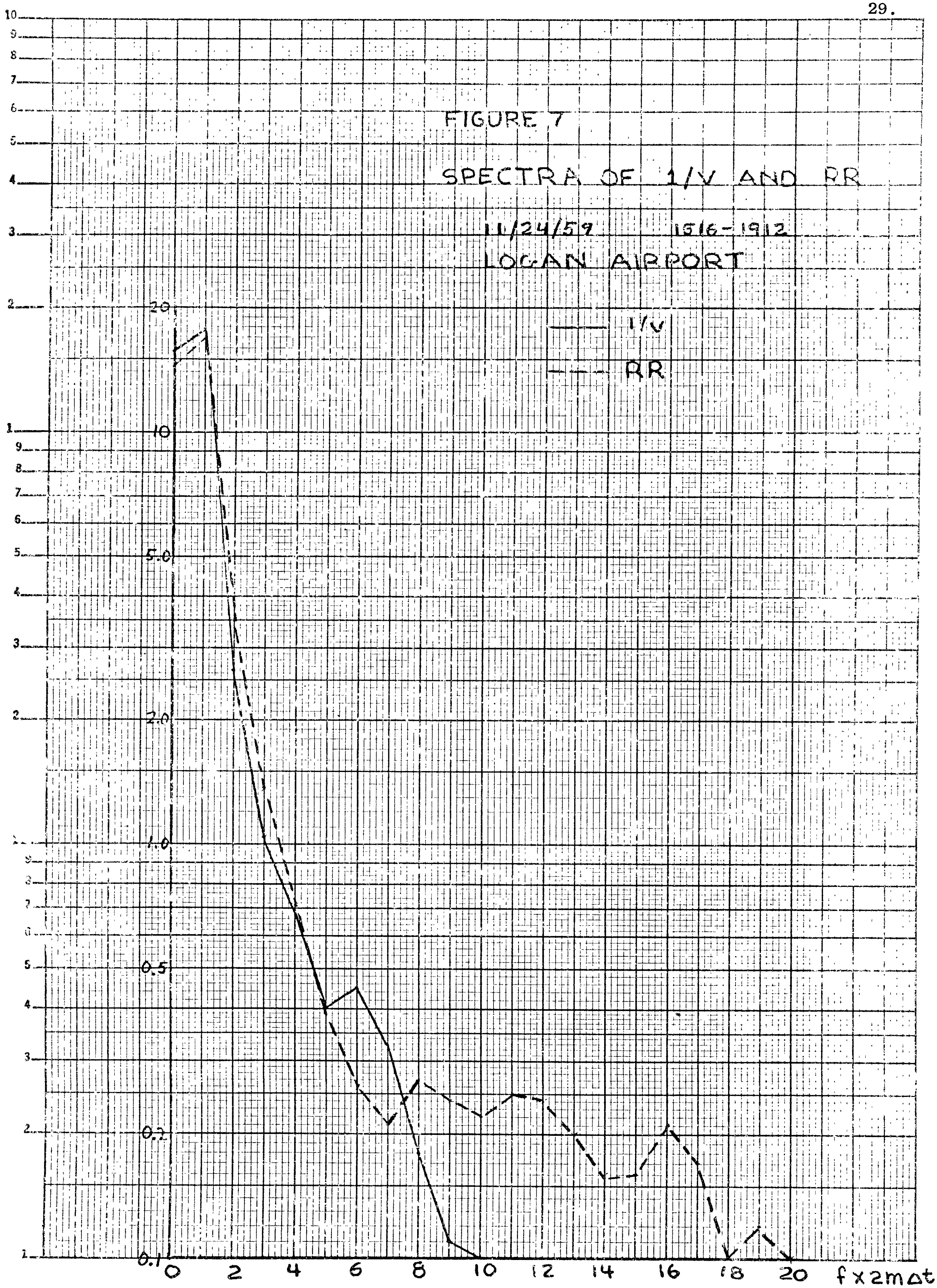
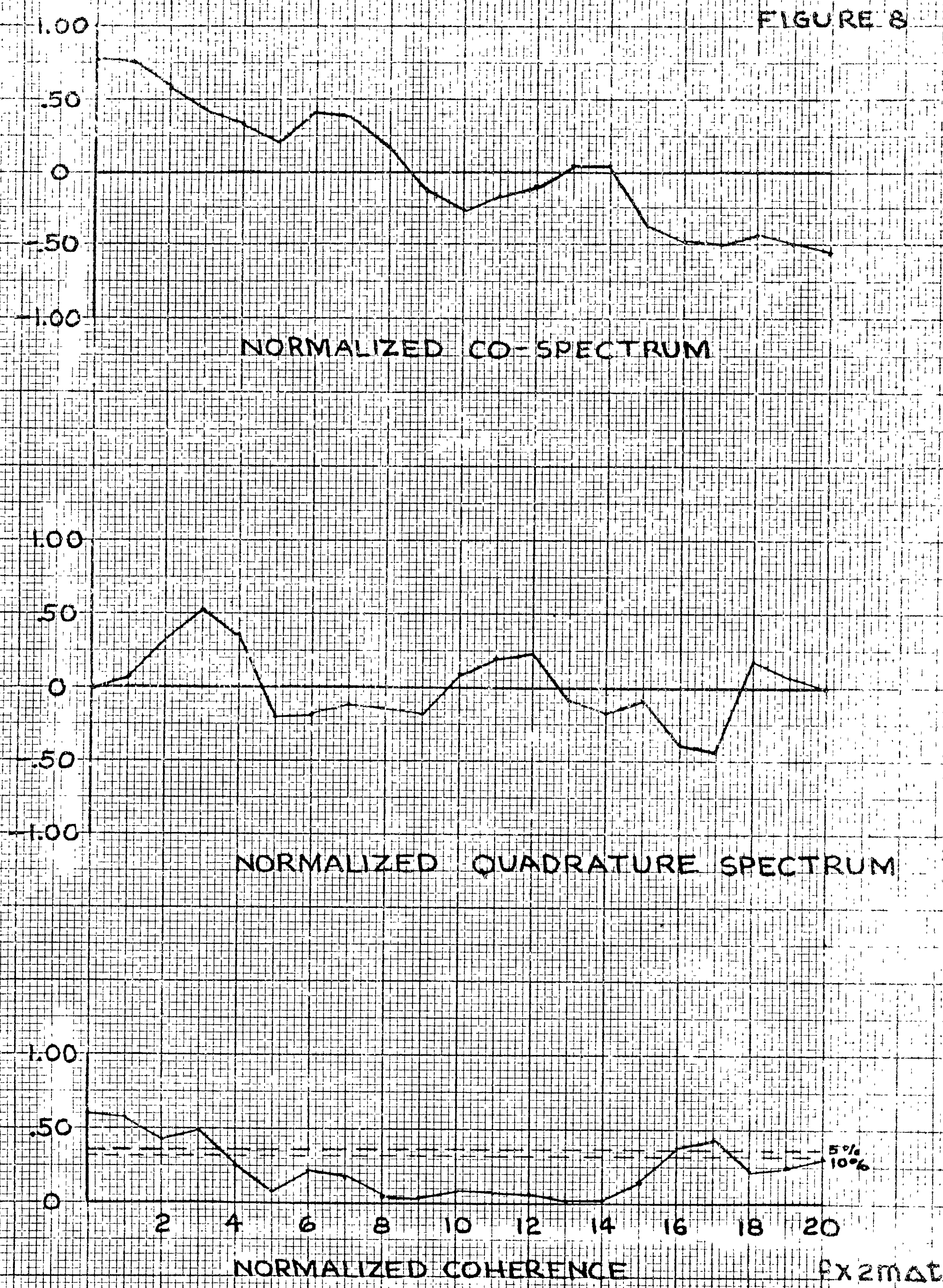


FIGURE 7
SPECTRA OF 1/V AND RR
11/24/59 1516-1912
LOGAN AIRPORT
1/V
RR
20
10
5.0
2.0
1.0
0.5
0.2
0.1
0
2
4
6
8
10
12
14
16
18
20
f x 2mΔt

FIGURE 8



1/V - RR 11/24/59 1516-1912

<u>December 28, 1959 (1008-1742) Logan Airport (Figures 9-11)</u>

Boston lies in the cold air ahead of an advancing warm front, located south of the city. Light rain, light sleet and fog are reported. The wind is from the east during most of the period and ranges from 10 to 23 knots, gradually increasing from the beginning to the end of the period. The visibility varies from 2 to 8 miles, and is lowest during the first three hours of the storm. The highest visibilities are reported early in the period. The ceiling slowly lifts to 1200 feet. The temperature and dew point remain fairly constant.

The records of the rainfall rate and the reciprocal visibility begin at 1008 and end at 1742. They contain 515 data points. Only in two or three instances does there appear any correlations at all between the peaks in rainfall rate and the peaks in reciprocal visibility. During the most interesting part of the rainfall rate record, the reciprocal visibility record is devoid of fluctuations. It is only due to the fluctuations in the first half of the reciprocal visibility record that its spectrum shows any variance except at the longest periods. The spectrum of the rainfall rate indicates variance through point 19, owing to the much greater variability of that record. Except at point 0 and possibly point 1, the coherence curve indicates no frequency of any significance. At these two points, all of the covariance shows up negatively on the co-spectrum, indicating an inverse relationship between the reciprocal visibility and the rainfall rate.

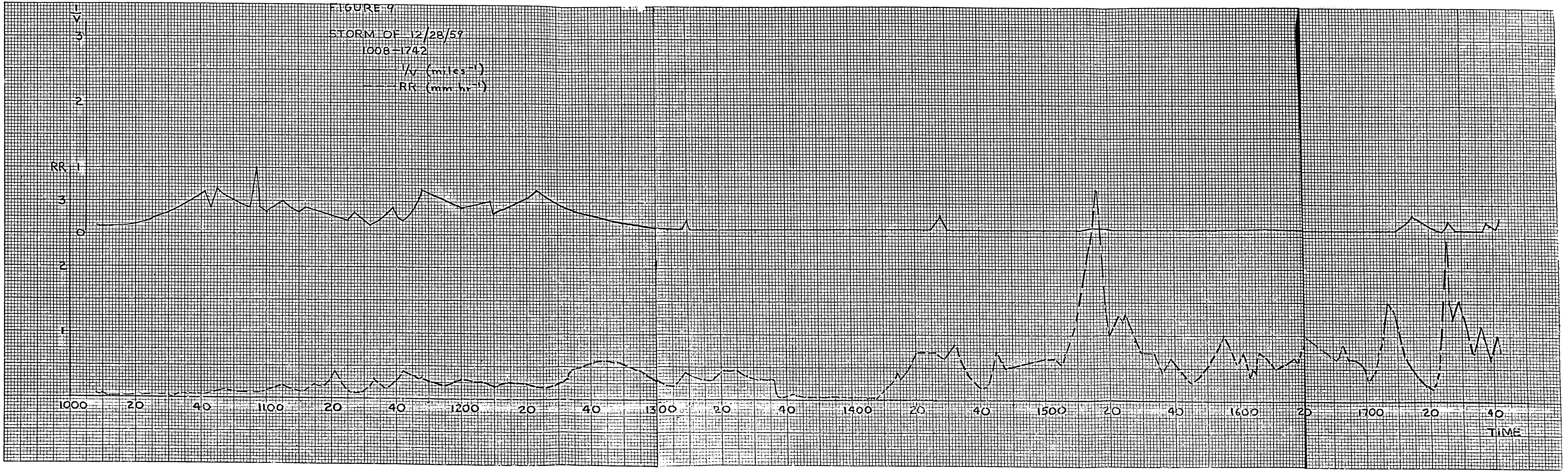


FIGURE 9
STORM OF 12/28/59
1008-1742
1/V (miles⁻¹)
RR (mm hr⁻¹)
1/V
3
2
1
RR
3
0
2
1
1000
20
40
1100
20
40
1200
20
40
1300
20
40
1400
20
40
1500
20
40
1600
20
1700
20
40
TIME

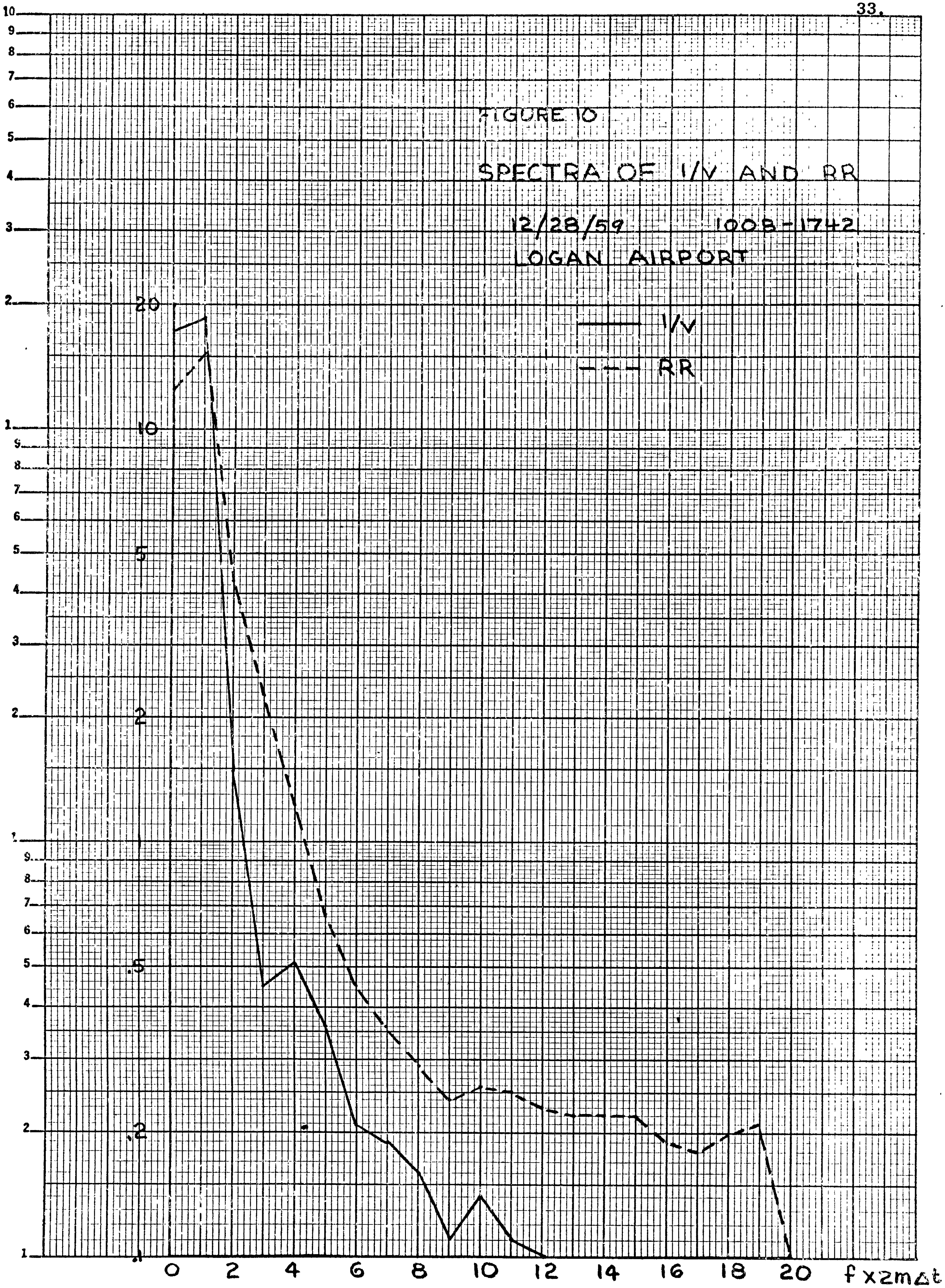


FIGURE 10
SPECTRA OF 1/V AND RR
12/28/59 1008-1742
LOGAN AIRPORT
1/V
RR
20
10
5
2
1
.5
.2
.1
0
2
4
6
8
10
12
14
16
18
20
f x 2m∆t

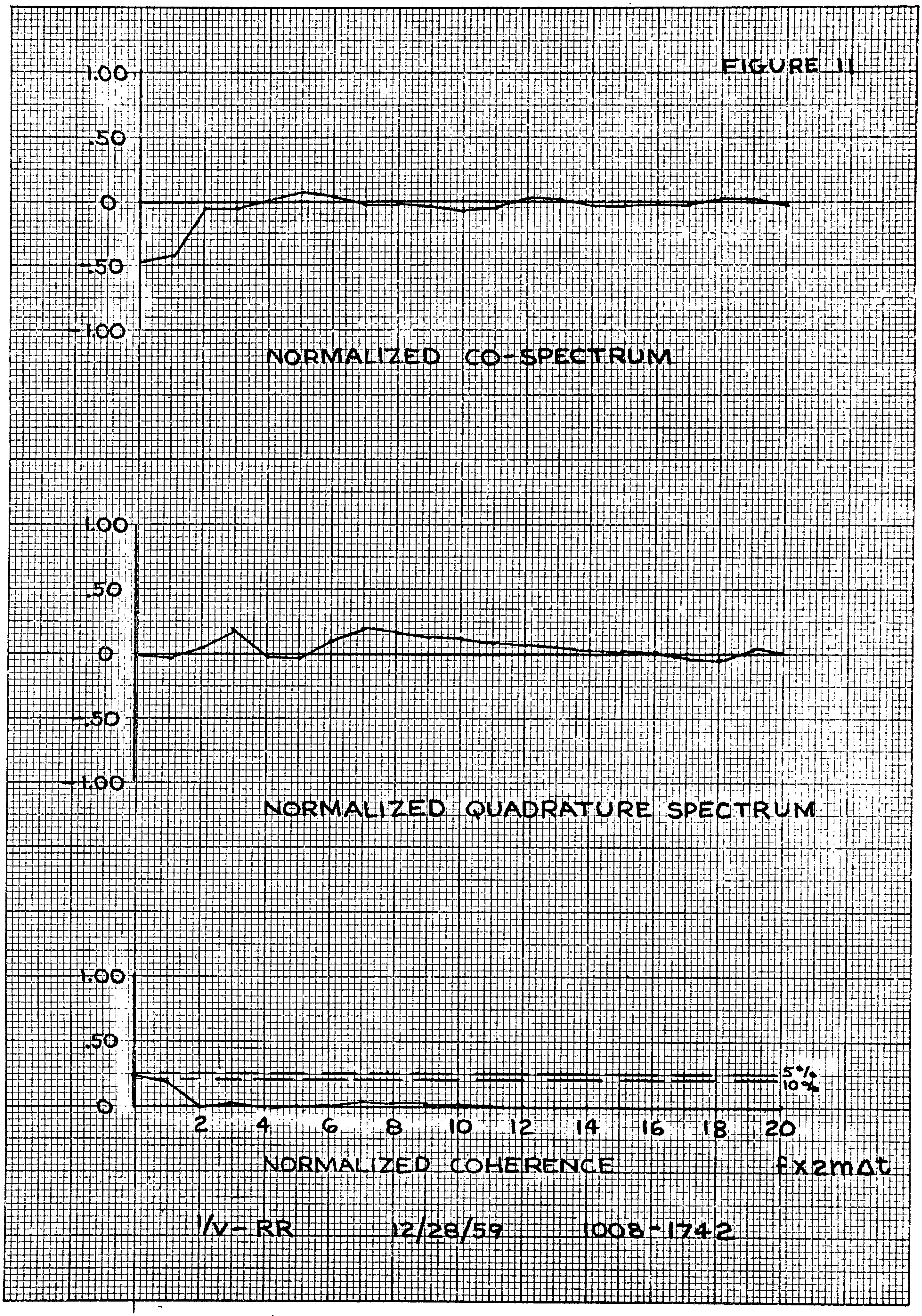


FIGURE 11
1.00
.50
0
-.50
-1.00
NORMALIZED CO-SPECTRUM
1.00
.50
0
-.50
-1.00
NORMALIZED QUADRATURE SPECTRUM
1.00
.50
0
5%
10%
2
4
6
8
10
12
14
16
18
20
NORMALIZED COHERENCE
fx2mΔt
1/v-RR
12/28/59
1008-1742

December 28, 1959 (1918-2140) Logan Airport (Figures 12-14)

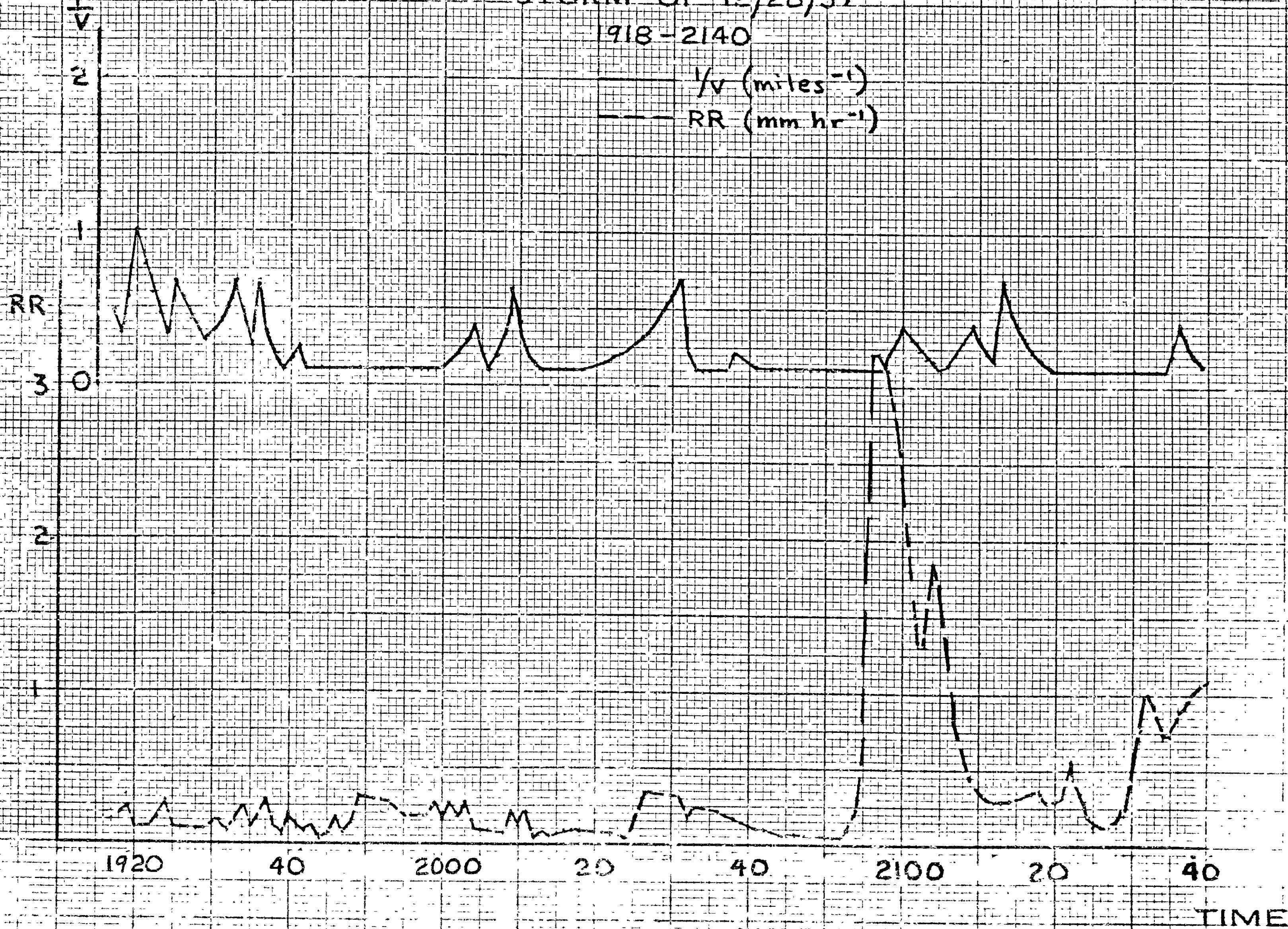
Boston is in the cold air of a stationary front, which lies to the south of the city. Light rain, light sleet, and fog are reported throughout the period. The visibility varies between 3 and 5 miles, showing no trend. The wind is from the east with speeds ranging from 17 to 25 knots. The ceiling fluctuates between 500 and 1600 feet, with the lowest ceilings roughly corresponding to the lowest visibilities. The temperature and dew point remain stationary throughout the period.

The records of the reciprocal visibility and the rainfall rate commence at 1918 and end at 2140. There appears to be some correlation between rainfall rate and reciprocal visibility, noticeable especially at the rainfall rate peaks around 2100 and the peak at 2132. The corresponding reciprocal visibility peak lags the rainfall rate peak by about 4 minutes. Both records show considerable short frequency variability. This shows up on the spectra, especially in the case of the reciprocal visibility which indicates appreciable variance all the way to a period of 2.1 minutes. Unfortunately, all points on the coherence curve lie well below the confidence limits, indicating that no importance can be attached to the peaks on the co-spectrum and quadrature spectrum curve.

FIGURE 12

STORM OF 12/28/59

1918-2140



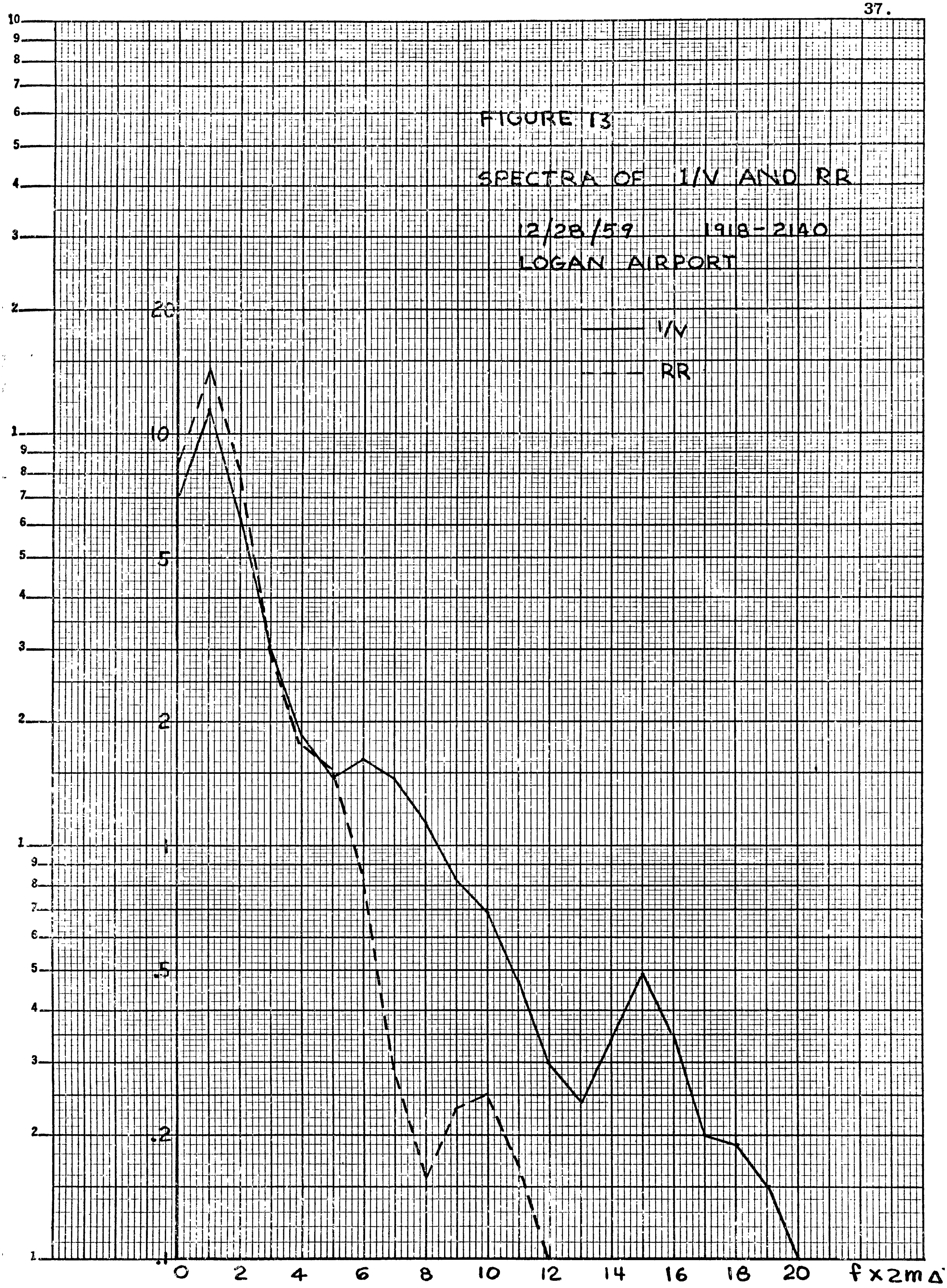


FIGURE 13
SPECTRA OF 1/V AND RR
12/28/59 1918-2140
LOGAN AIRPORT
1/V
RR
20
10
5
2
1
.5
.2
.1
0
2
4
6
8
10
12
14
16
18
20
f x 2mΔ

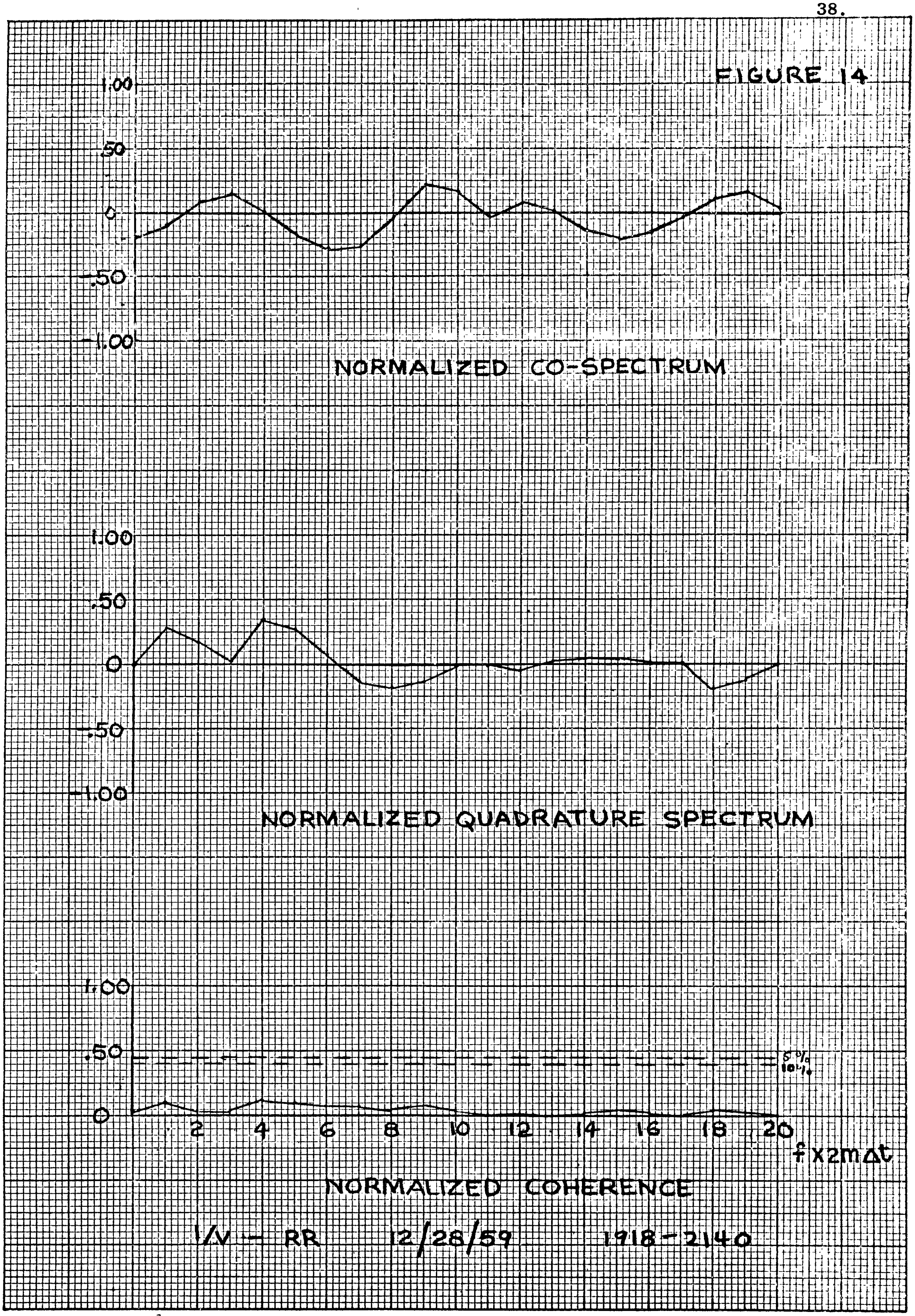


FIGURE 14
1.00
.50
0
-.50
-1.00
NORMALIZED CO-SPECTRUM
1.00
.50
0
-.50
-1.00
NORMALIZED QUADRATURE SPECTRUM
1.00
.50
0
2
4
6
8
10
12
14
16
18
20
f x2mΔt
NORMALIZED COHERENCE
1/V - RR 12/28/59 1918-2140

January 28, 1960 (0839-1615) Logan Airport (Figures 15-20)

A cold front lies to the east of Boston and is moving out to sea. The wind speeds range from 15 to 22 knots, and the direction is from the NE for most of the period, with a shift to the N toward the end of the period. Light, moderate, and heavy rain, along with fog is reported. The visibility varies between 4 and $1\frac{1}{2}$ miles, and there is noted a tendency for the heavier rains to bring the lower visibilities. The ceilings range from 400 to 1000 feet with a gereral tendency for the lower ceilings to accompany lower visibilities. The temperature and dew point remain essentially constant for the entire period.

For this storm, two records were considered; and two sets of spectra and cross spectra were determined. The first pair of records were the visibility and ceiling, and the second were the reciprocal visibility and radar determined rainfall rate. Both records commence at 0911 and terminate at 1610.

It is difficult to observe much correspondence in the records of visibility and ceiling. Although there are times when a low ceiling occurs around the same time as low visibility, at other times high ceilings are seen to accompany low visibilities. On the spectrum curves, the variance of the ceiling spectrum becomes insignificant after a period of four minutes and the variance of the visibility spectrum ceases to be significant at a period of 3.6 minutes. Although the quadrature spectrum shows a maximum at point 2, the co-spectrum is zero; indicating a phase difference of exactly 90°, or a phase lag of 5 minutes. A look at the coherence shows the peak at that period, as at all other periods, to be well below the significance level.

There is evident in the records of reciprocal visibility and rainfall rate, a tendency for peaks in one to be associated with peaks in the other. However, the lengths of time by which the maxima of rainfall rate lead the maxima in reciprocal visibility are variable, between 0 and 8 minutes. Both records show short period fluctuations. The spectrum of the reciprocal visibility shows appreciable variance all the way down to a period of 2 minutes, while the spectrum of rainfall rate becomes insignificant after a period of 3.3 minutes. Nothing significant appears on the cross spectrum curves. The coherence curve remains well below the significance level at all periods.

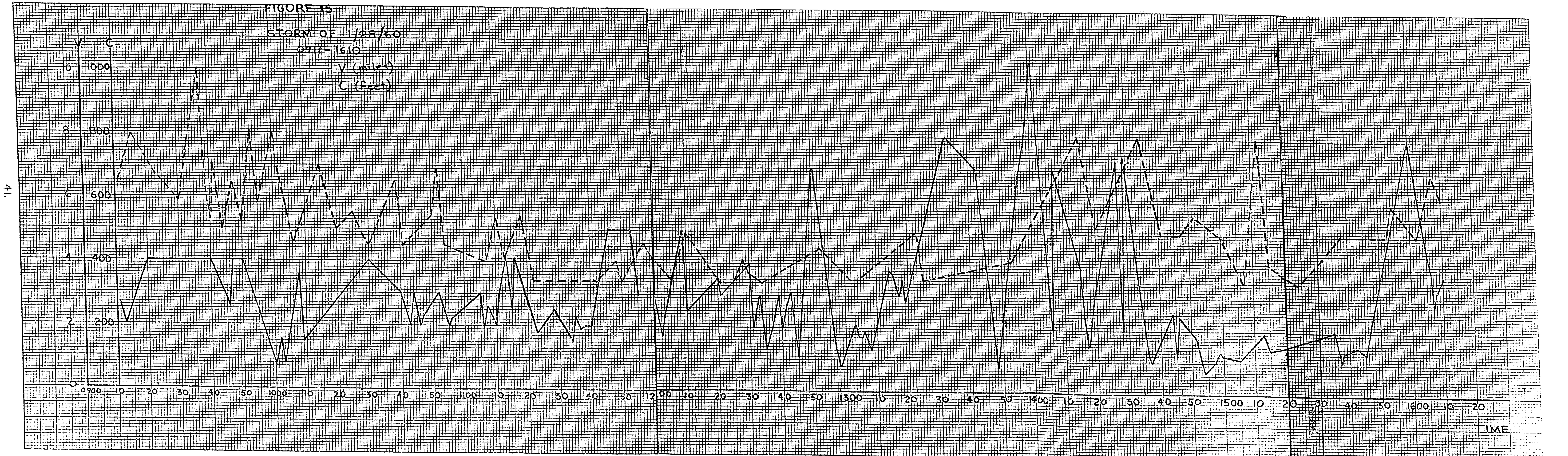


FIGURE 15

STORM OF 1/28/60

0911-1610

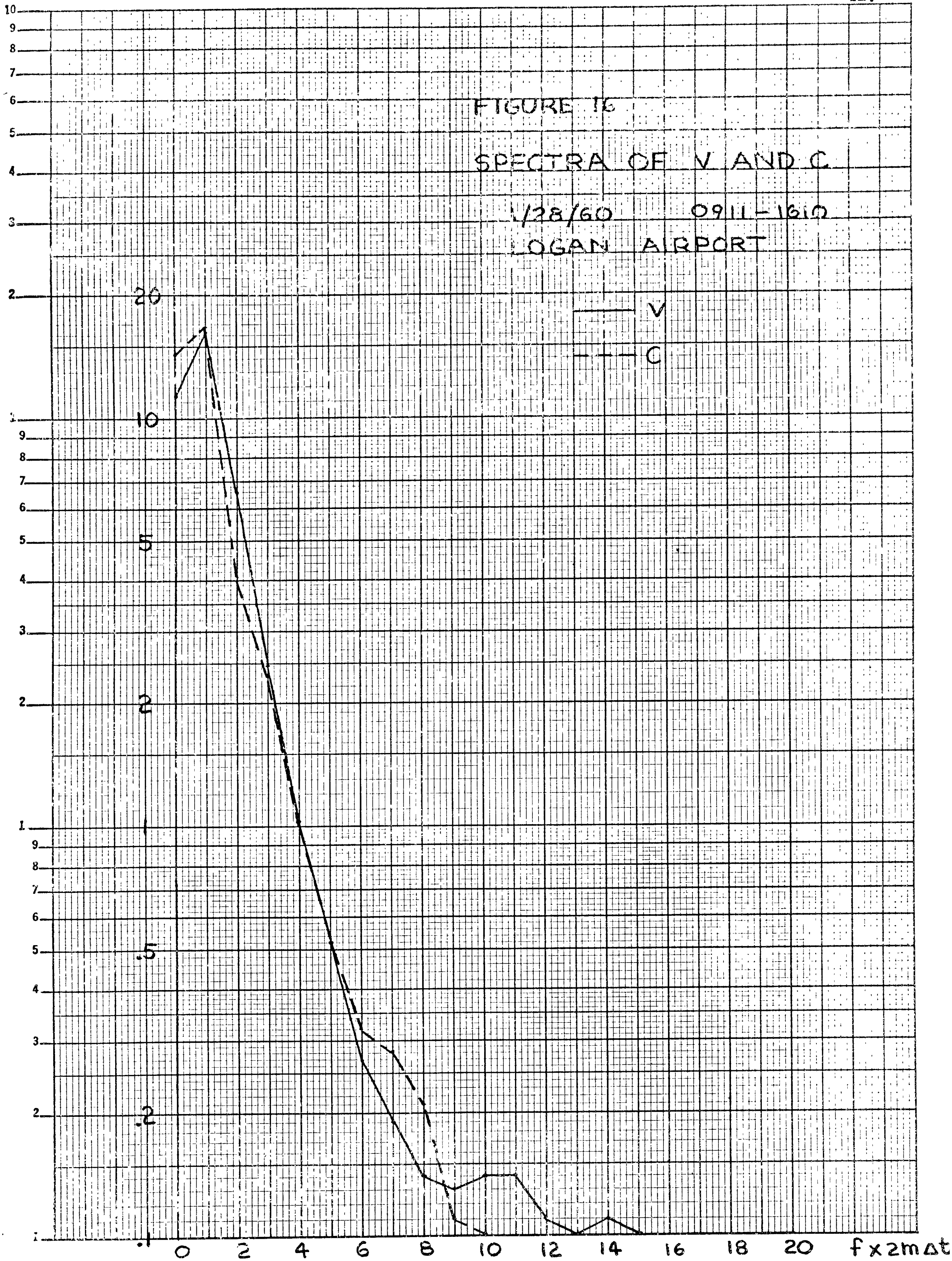


FIGURE 16
SPECTRA OF V AND C
1/28/60 0911-1610
LOGAN AIRPORT
V
C
20
10
5
2
1
.5
.2
.1
0
2
4
6
8
10
12
14
16
18
20
f x 2mΔt

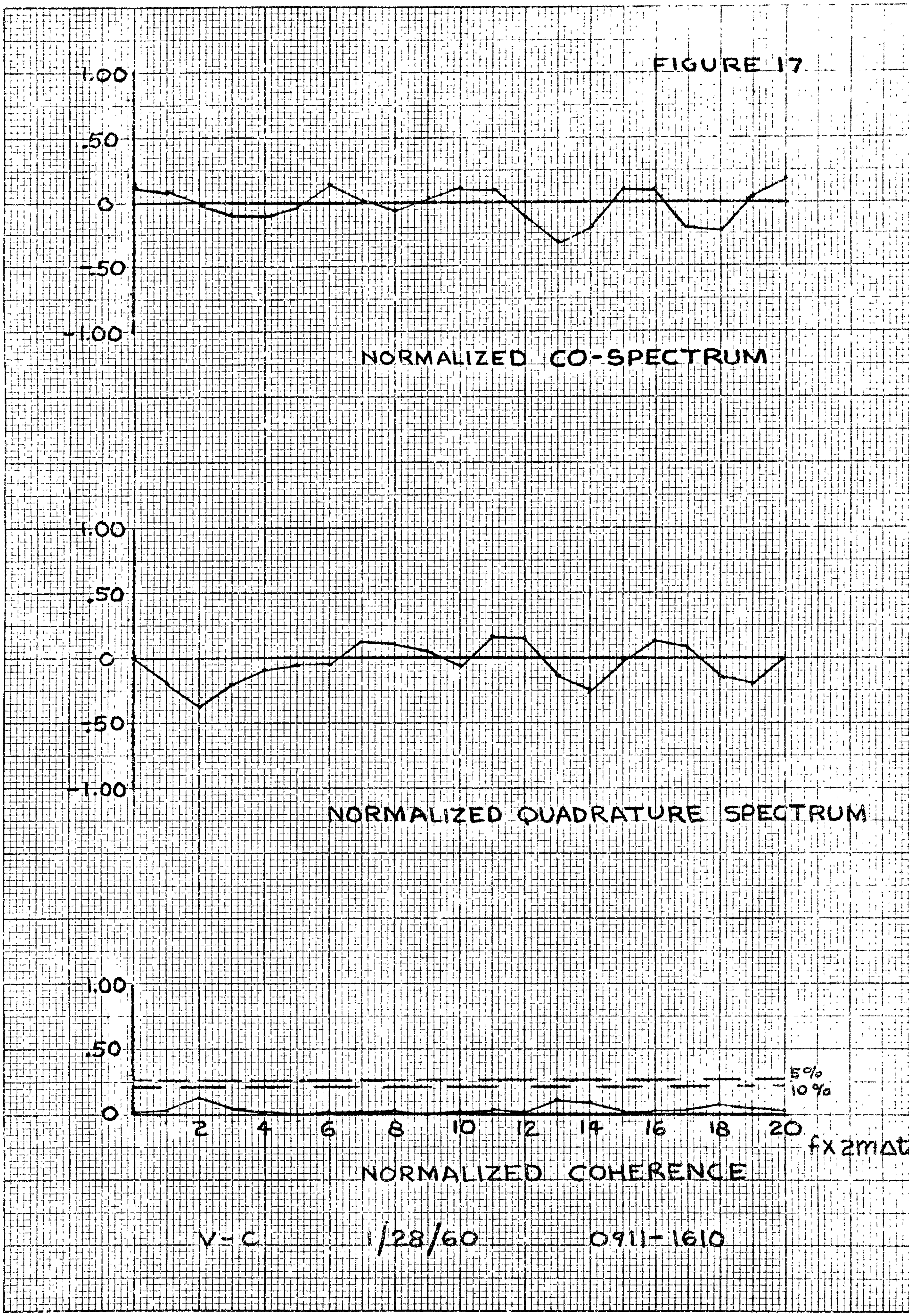


FIGURE 17
1.00
.50
0
-.50
-1.00
NORMALIZED CO-SPECTRUM
1.00
.50
0
-.50
-1.00
NORMALIZED QUADRATURE SPECTRUM
1.00
.50
0
5%
10%
2
4
6
8
10
12
14
16
18
20
f x 2mΔt
NORMALIZED COHERENCE
V-C
1/28/60
0911-1610

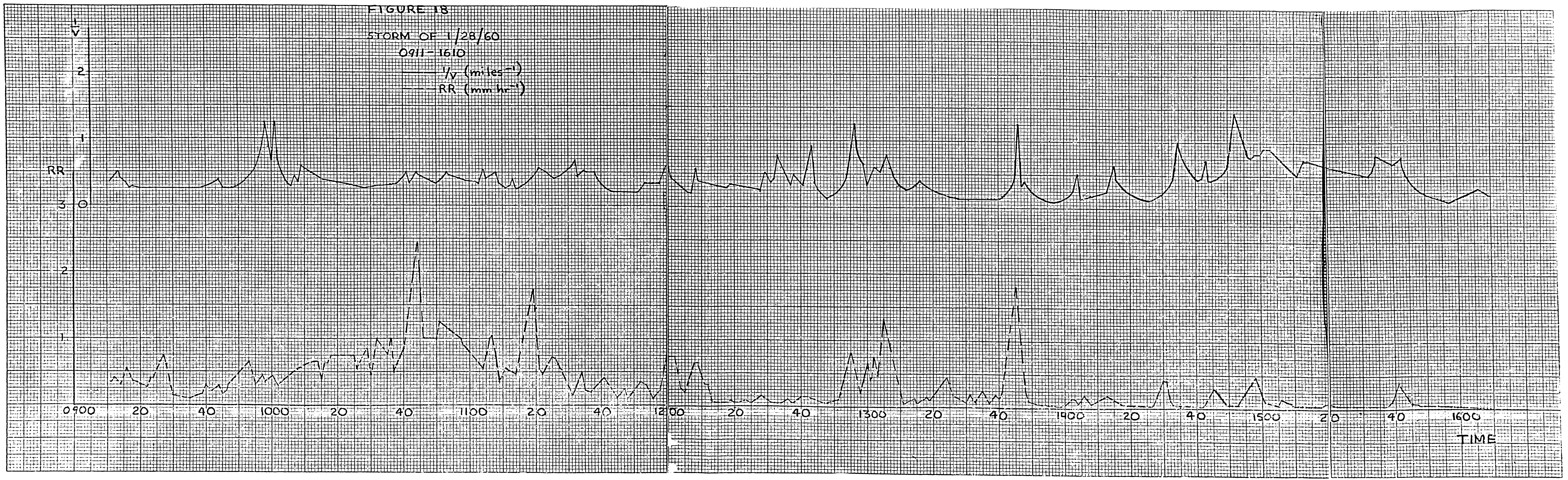


FIGURE 18
STORM OF 1/28/60
0911-1610
1/V (miles⁻¹)
RR (mm hr⁻¹)
1/V
RR
2
1
3
0
2
1
0900
20
40
1000
20
40
1100
20
40
1200
20
40
1300
20
40
1400
20
40
1500
20
40
1600
TIME

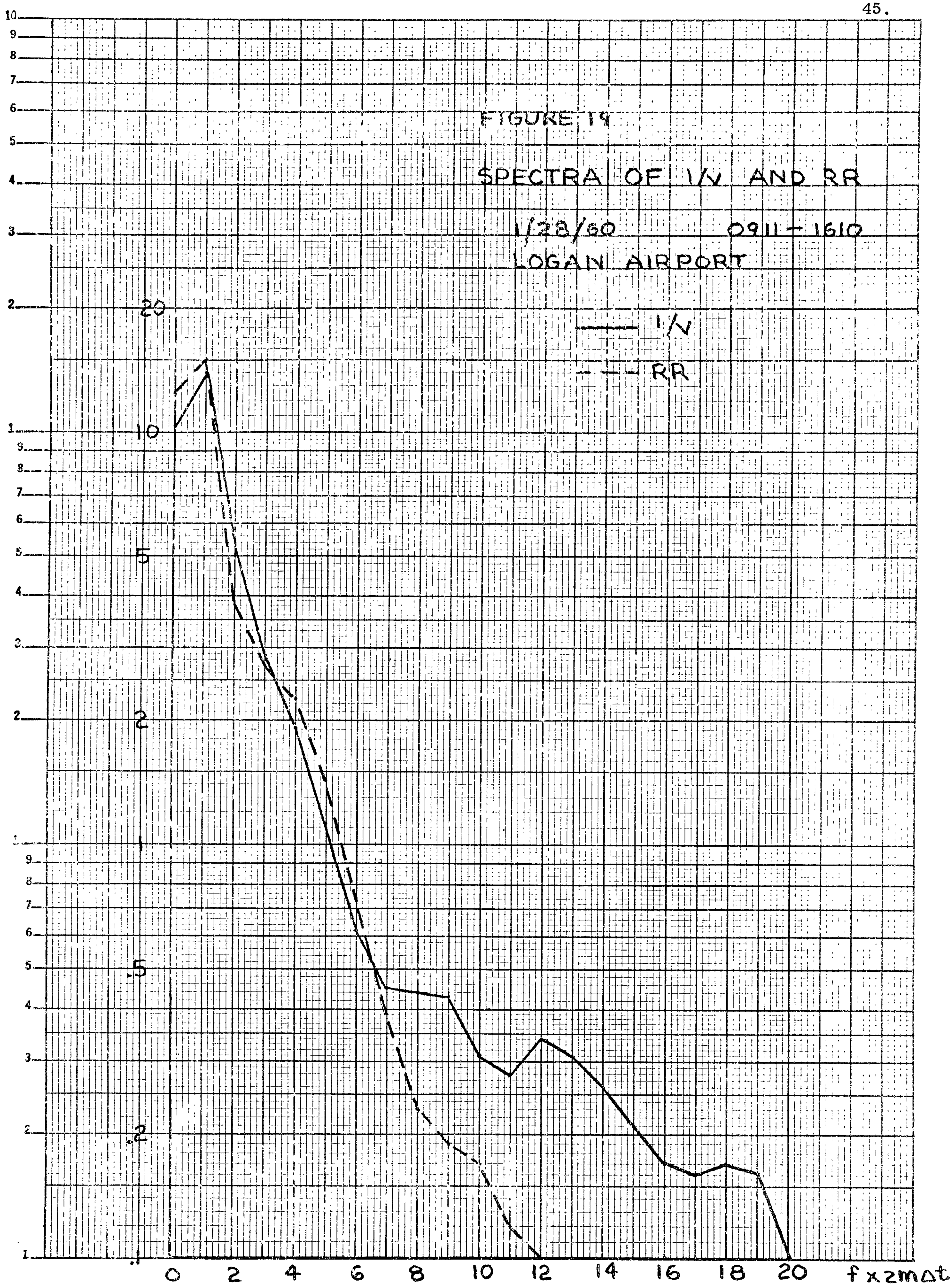


FIGURE 19
SPECTRA OF 1/v AND RR
1/28/60 0911 - 1610
LOGAN AIRPORT
1/v
RR
20
10
5
2
1
.5
.2
.1
0
2
4
6
8
10
12
14
16
18
20
f x 2mΔt

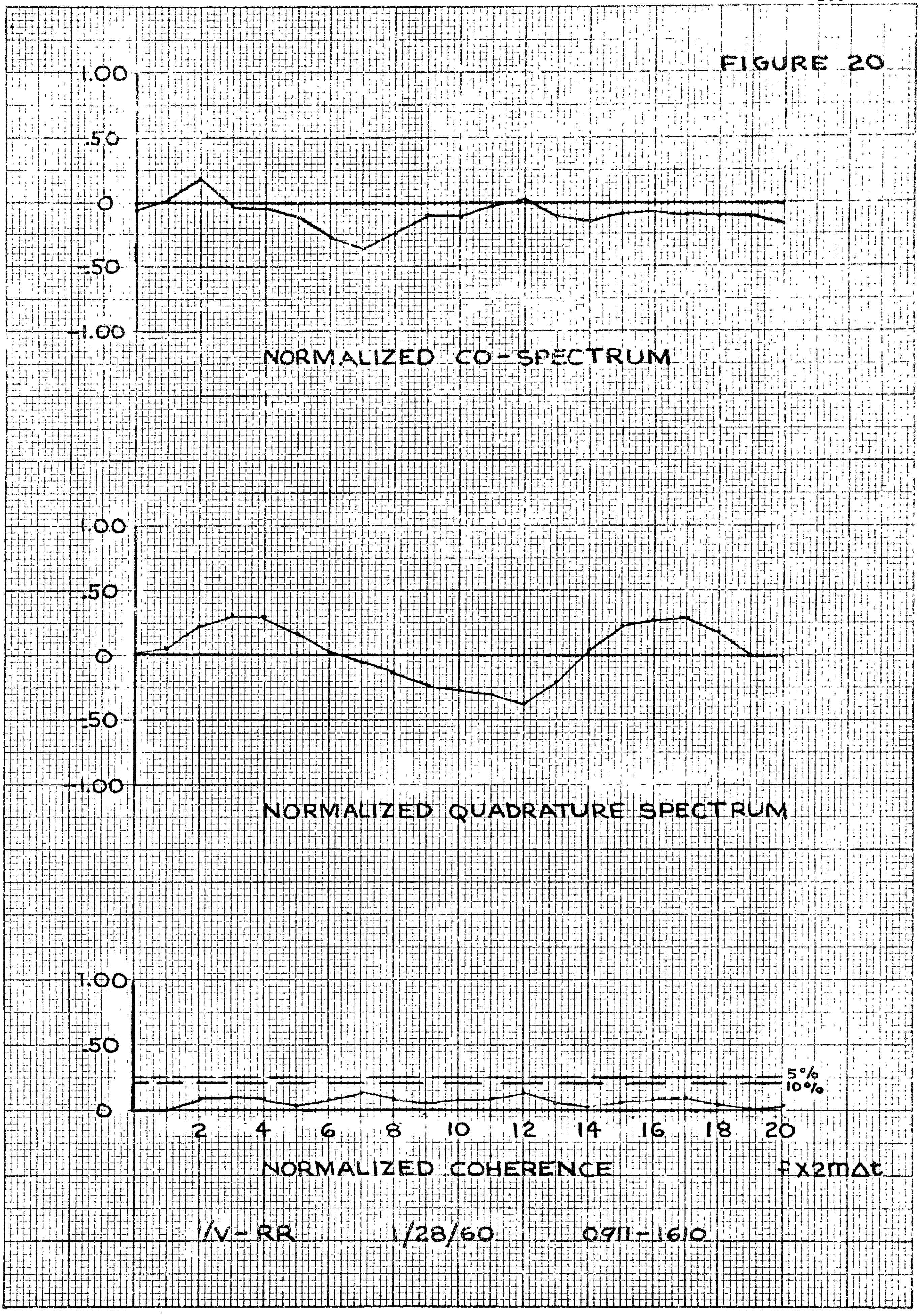


FIGURE 20
1.00
.50
0
-.50
-1.00
NORMALIZED CO-SPECTRUM
1.00
.50
0
-.50
-1.00
NORMALIZED QUADRATURE SPECTRUM
1.00
.50
5%
10%
0
2 4 6 8 10 12 14 16 18 20
NORMALIZED COHERENCE
F X 2mΔt
I/V-RR 1/28/60 0911-1610

May 24, 1960 (1112-1520) Logan Airport (Figures 21-23)

There is no frontal activity in the vicinity of Boston. The city is in a low pressure region. Very light rain, light rain, and moderate rain showers are reported, as well as fog throughout the period. At 1100, Logan Airport was reporting SE winds; but during the coarse of the period, the wind gradually shifts to the southwest; and finally when the observations have ceased, the wind is blowing from the NW. The speed varies between 12 and 4 knots, gradually decreasing as the direction changes. The visibility fluctuates between 3 and $1\frac{1}{2}$ miles, the lowest visibilities roughly corresponding to the heaviest rainfall. The ceilings fluctuate between 400 and 1100 feet. Low ceilings occur at lowest visibilities, although equally low ceilings are reported when the visibilities are higher. The temperature and dew point slowly rise through most of the record, and then drop at the end.

The records of the rainfall rate and the reciprocal visibility contain 250 data points, commencing at 1112 and terminating at 1521. Although the general period of greater rainfall rate corresponds to that of greater reciprocal visibility, it is quite difficult to recognize any shorter frequency correspondence. The spectrum of reciprocal visibility shows a relatively great amount of variance at the higher frequencies. The variance is greater after a period of 3.7 minutes, in fact, than any of the other records considered. This is explained by the fact that although for a long period of time the record shows practically no variance, in the intervals in which it does fluctuate, these fluctuations are of higher frequency than is generally observed in the other records. Unfortunately, from the point of view of gleaming any information concerning these higher frequencies from the cross spectra, the rainfall rate spectrum indicates no appreciable variance after a period of 5 minutes. The coherence curve indicates possible significance at points 0 and 1, and significance at point 3. At the frequency intervals corresponding to points 0 and 1, almost all the covariance is observed as a positive peak on the co-spectrum, indicating an in-phasedness of the longest periods, while at point 3, a positive peak appears on the quadrature spectrum, with little covariance appearing on the cospectrum. The phase lead of the rainfall rate over the reciprocal visibility at this period is 103°, which corresponds to a time of four minutes.

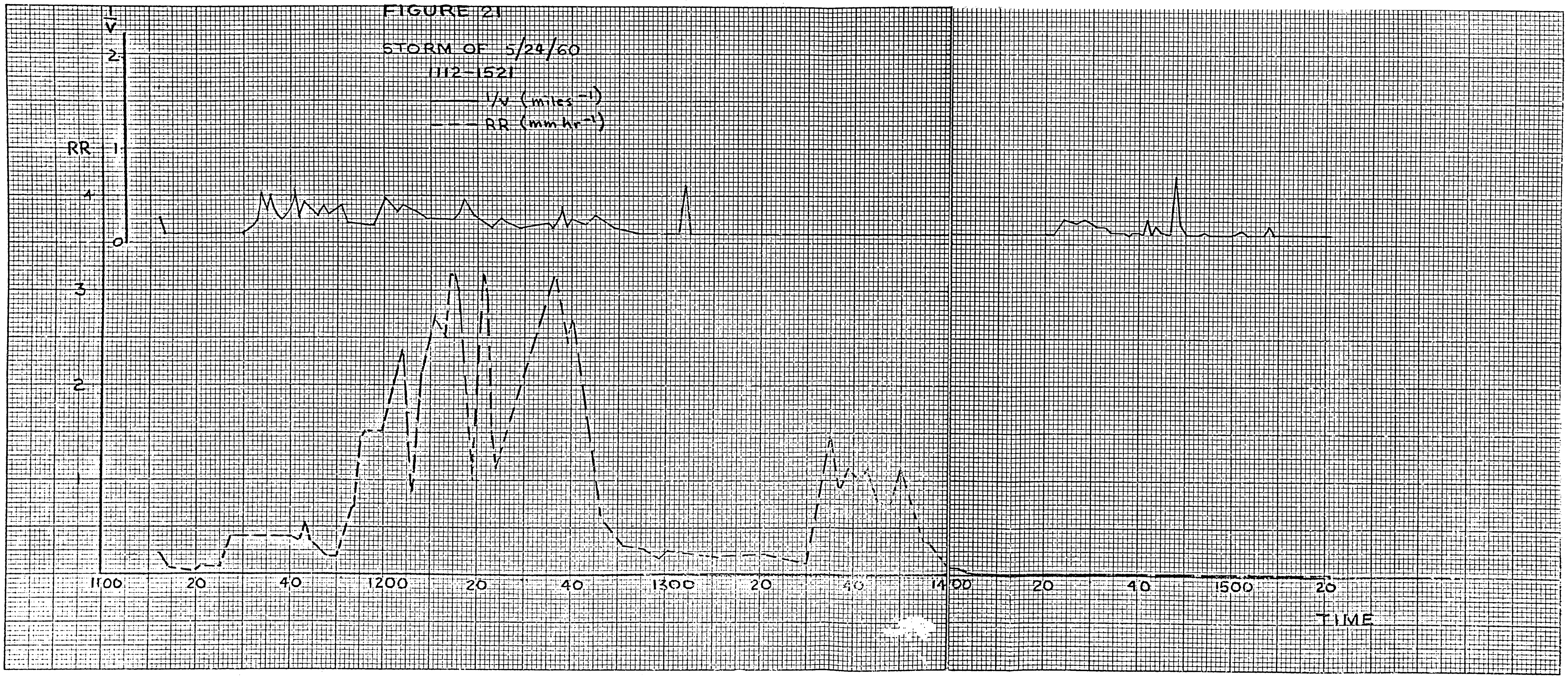


FIGURE 21
STORM OF 5/24/60
1112-1521

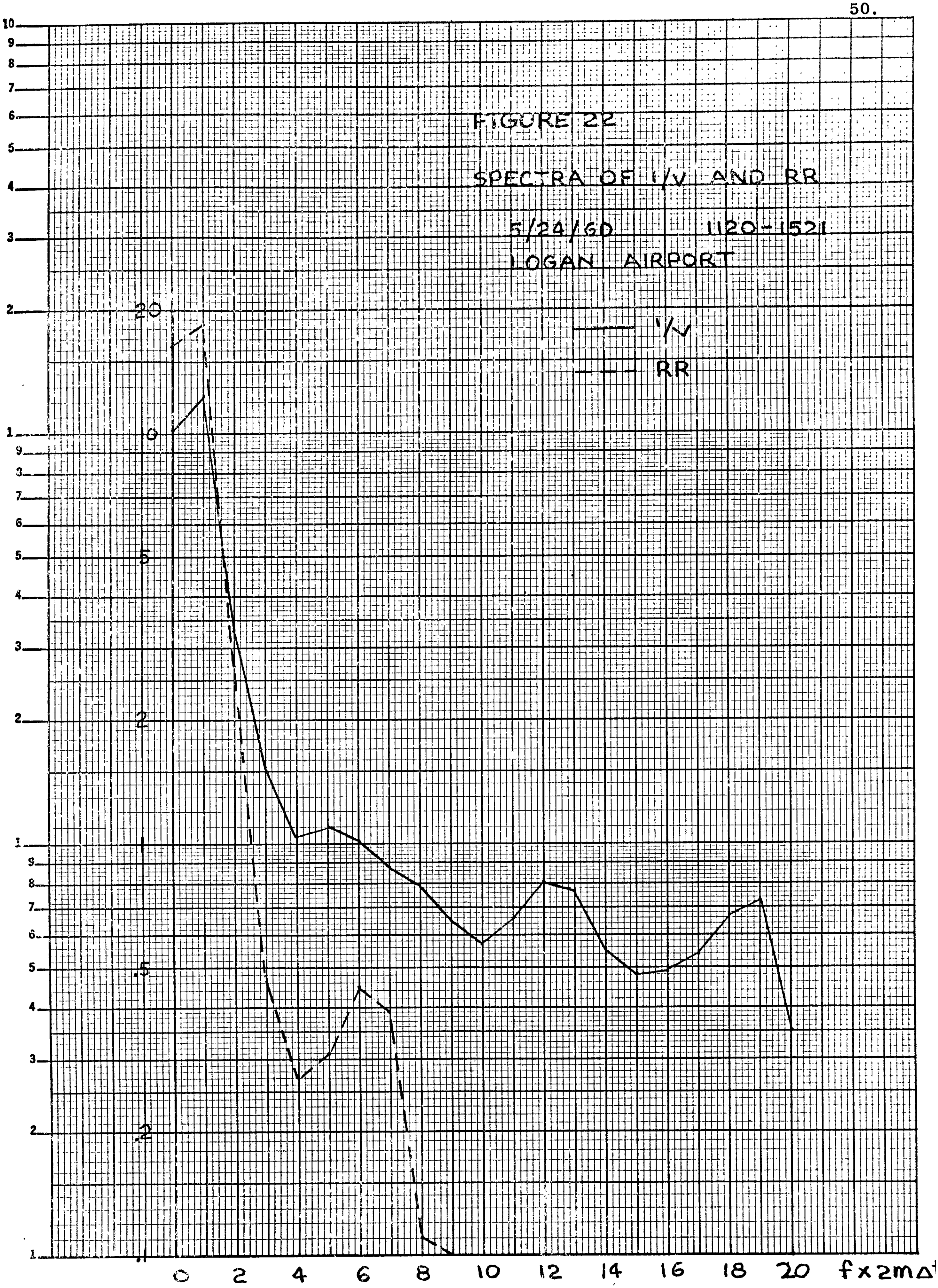


FIGURE 22
SPECTRA OF 1/V AND RR
5/24/60 1120-1521
LOGAN AIRPORT
1/V
RR
20
10
5
2
1
.5
.2
.1
0
2
4
6
8
10
12
14
16
18
20
f x 2mΔt

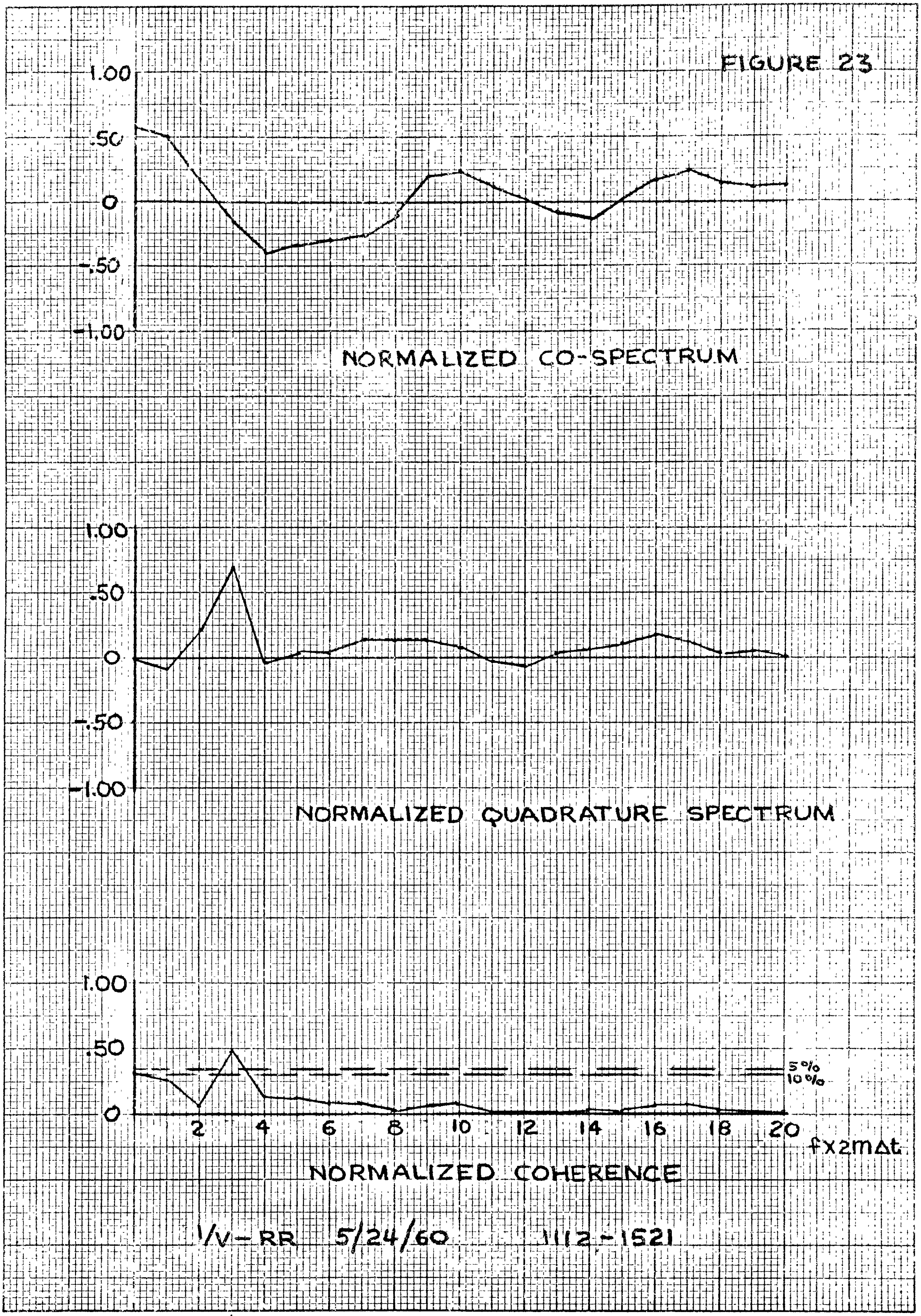


FIGURE 23
1.00
.50
0
-.50
-1.00
NORMALIZED CO-SPECTRUM
1.00
.50
0
-.50
-1.00
NORMALIZED QUADRATURE SPECTRUM
1.00
.50
0
2
4
6
8
10
12
14
16
18
20
fx2mΔt
5%
10%
NORMALIZED COHERENCE
1/V-RR 5/24/60 1112-1521

February 14, 1959 (2158-0143) Bedford Airport (Figures 24-26)

A warm front is located south of Boston, extending from a low pressure center which lies over the Great Lakes region and is moving to the ENE. Very light winds, light rain, and fog are reported during the period under consideration. The visibility is relatively high, (5 miles) in the beginning, but gradually drops to $\frac{1}{2}$ mile, with the onset of fog. The ceiling gradually drops from 700 to 400 feet over the length of the period. The temperature and dew point remained constant.

The records of the transmissometer visibilities and radar signal intensities consist of 159 data points commencing at 2251 and terminating at 0129. There are higher frequency fluctuations present in the radar signal intensity record than in the transmissometer record. This is apparent from the spectra of the two records. The shortest period in the transmissometer spectrum containing any appreciable variance is 13.3 minutes, while it is 4 minutes in the radar signal intensity spectrum. Thus, in the co-spectrum and quadrature spectrum very little significance should be attached to periods of less than 13.3 minutes. The co-spectrum for point 0 is negative and approaches zero at point 3. The quadrature spectrum remains zero through a period of 20 minutes and has a significant positive value at a period of 13.3 minutes as indicated by the confidence limits on the coherence curve. This implies the rainfall leads the visibility by approximately 3 minutes at this frequency.


FIGURE 24
STORM OF 2/14/59
2251-0129
V (miles)
PI (db)
V
0
2
4
6
8
10
PI
0
10
20
30
40
50
2240
2300
20
40
2400
20
40
0100
20
TIME

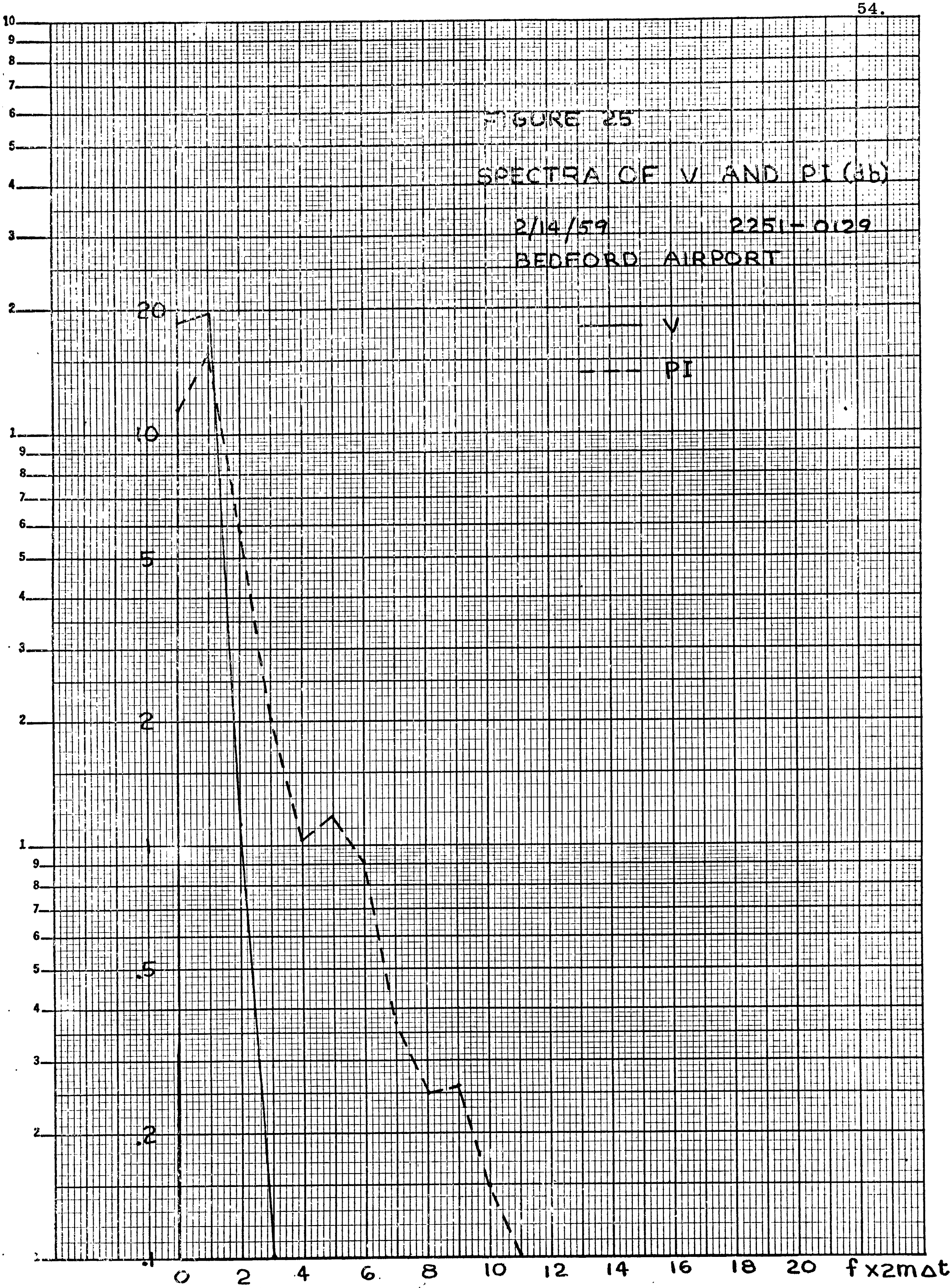
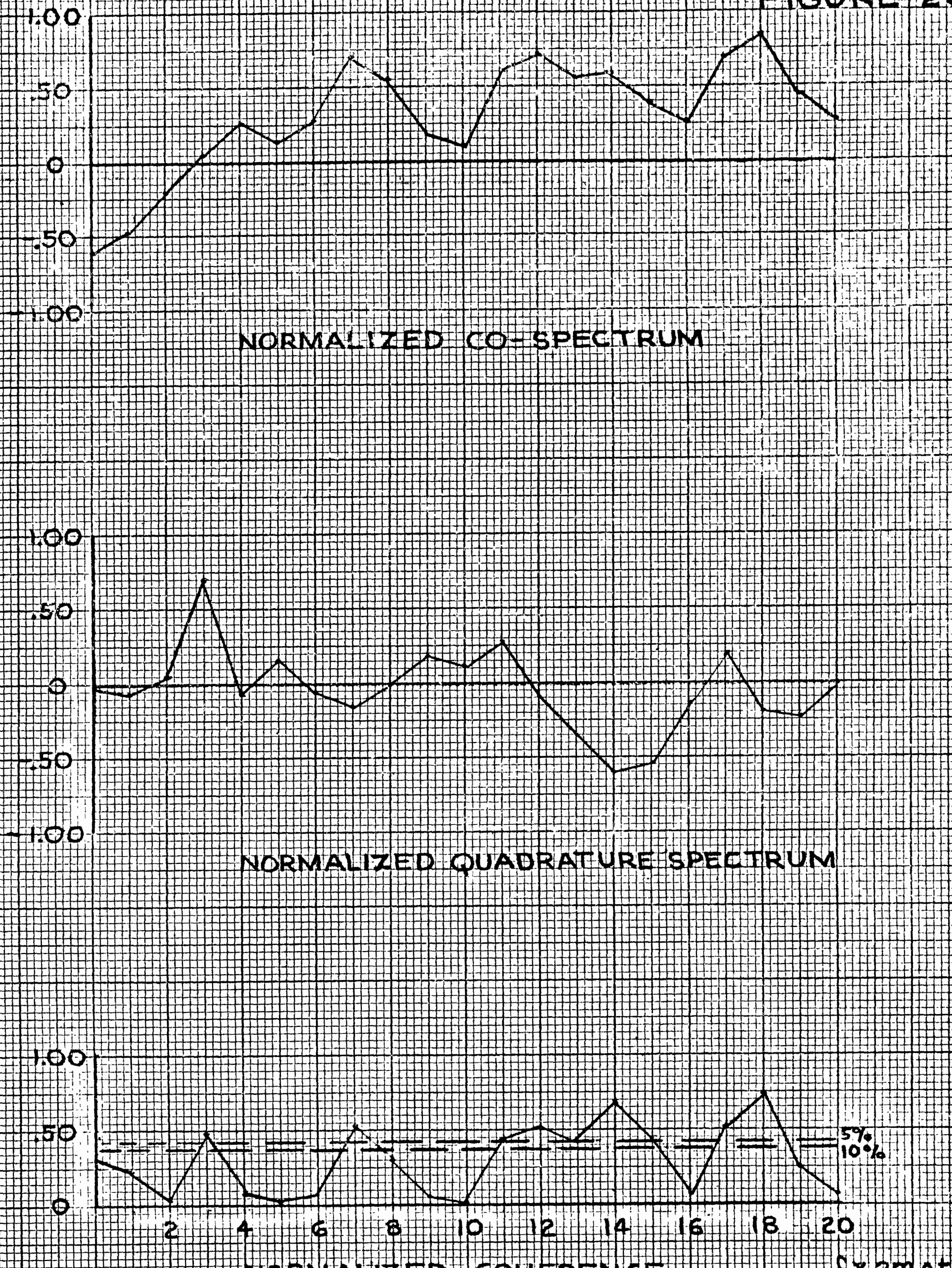


FIGURE 25
SPECTRA OF V AND PI (db)
2/14/59 2251-0129
BEDFORD AIRPORT
V
PI
20
10
5
2
1
.5
.2
.1
0
2
4
6
8
10
12
14
16
18
20
f x2mΔt

FIGURE 26



V-PI(db) 2/14/59 2251-0129

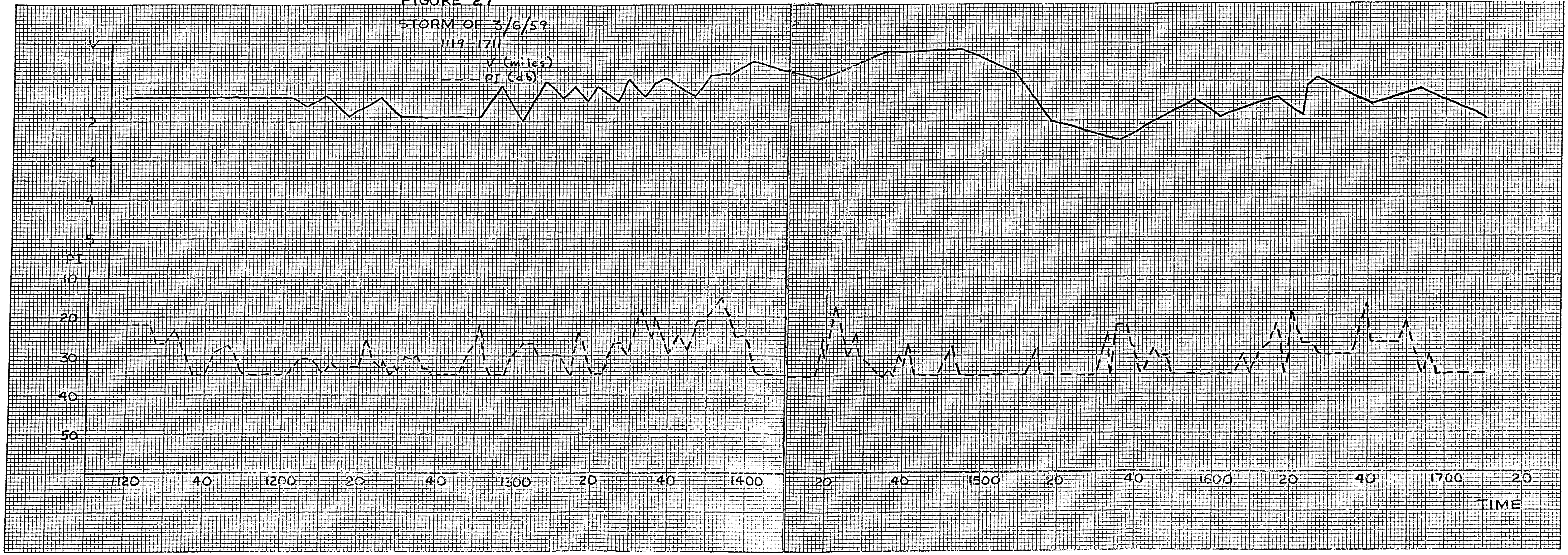
March 6, 1959 (1028-1813) Bedford Airport (Figures 27-29)

At the beginning of the period, a warm front is located a few miles to the south of Boston, extending from a low pressure region which is centered over the Great Lakes and moving towards the ENE. During this time, Bedford Airport was reporting light rain and fog, low visibilities, ($1\frac{1}{4}$ miles) and moderate SE winds. The ceiling fluctuates sporadically between 400 and 700 feet. At approximately 1300, the WBAN record indicates the passage of the front which is accompanied by a marked temperature rise and a wind shift from the SE to the SW. At this time, moderate rain and fog are reported, after which light rain and very light rain and fog are reported for the remainder of the period. An increase in the visibility, (1 to 7 miles) and an increase in ceiling height, (500 to 2500 feet) are noted after the passage of the warm front.

The records of the visibilities and radar signal intensities consist of 353 data points commencing at 1119 and ending 1711. The individual spectrum of the visibility and radar signal intensity records indicate that the transmissometer record contains fewer high frequency fluctuations than that of the rainfall. The shortest period in the transmissometer spectrum containing any significant variance is 6.6 minutes. Thus, in the co-spectrum and quadrature spectrum, very little significance should be attached to periods less than 6.6 minutes. The co-spectrum at point 0 is positive but is zero at points 1 and 2, and reaches a minimum at points 5 and 6. The quadrature spectrum is positive from point 0 to point 4, and has a possible significant peak at point 2 as indicated by the confidence limits on the coherence curve. This implies the rainfall leads the visibility by approximately 5 minutes at this frequency. At points

5 and 6, corresponding to periods of 6 to 8 minutes, the confidence limits on the coherence curve indicate possible significance to the co-spectrum and quadrature spectrum. Both the co-spectrum and quadrature spectrum are negative for this frequency range.

FIGURE 27



STORM OF 3/6/59
1119-1711
V (miles)
PI (db)
V
1
2
3
4
5
PI
10
20
30
40
50
1120
40
1200
20
40
1300
20
40
1400
20
40
1500
20
40
1600
20
40
1700
20
TIME

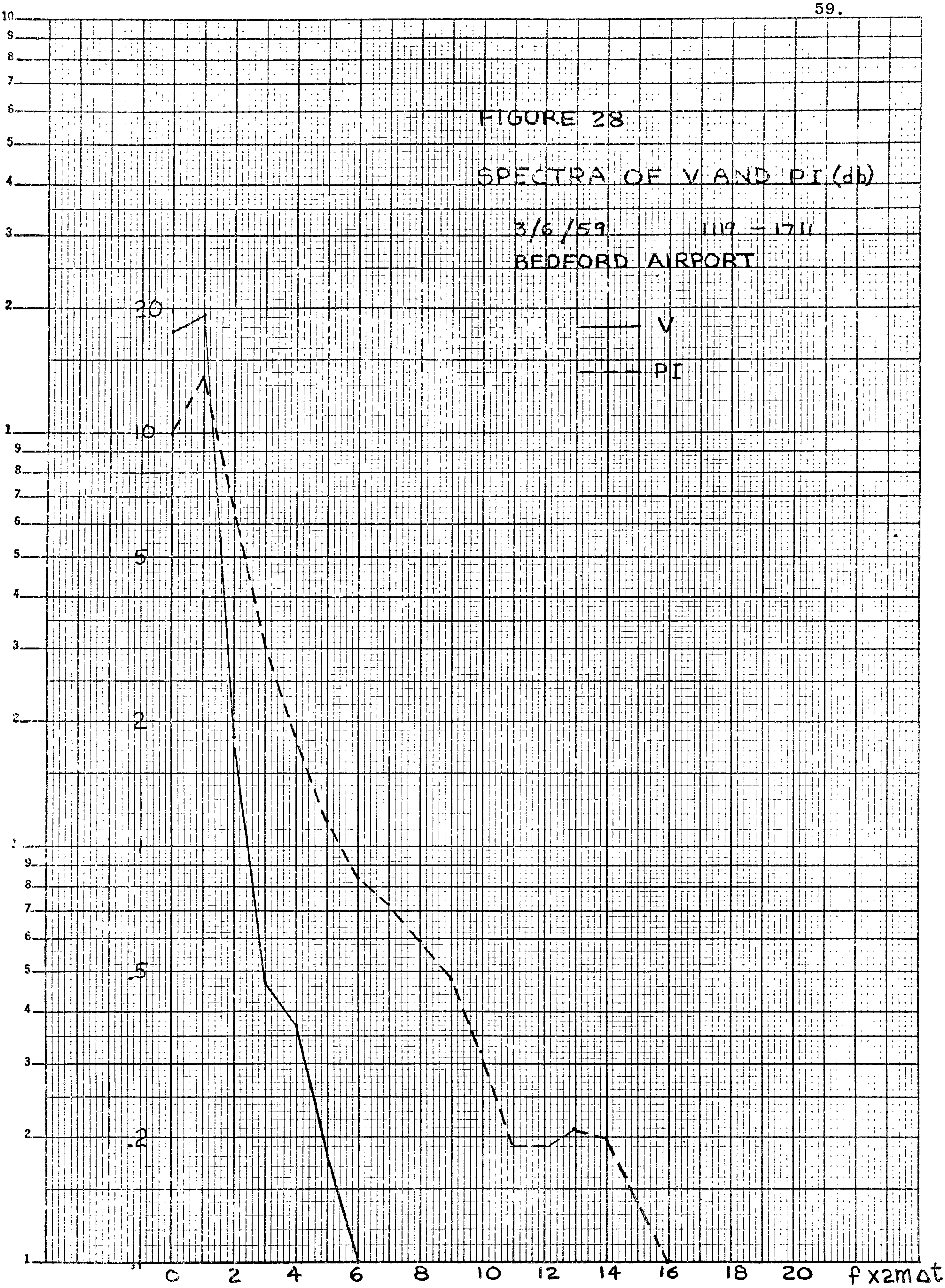


FIGURE 28
SPECTRA OF V AND PI (db)
3/6/59 1119 - 1711
BEDFORD AIRPORT
V
PI
20
10
5
2
1
.5
.2
0
2
4
6
8
10
12
14
16
18
20
f x 2mΔt

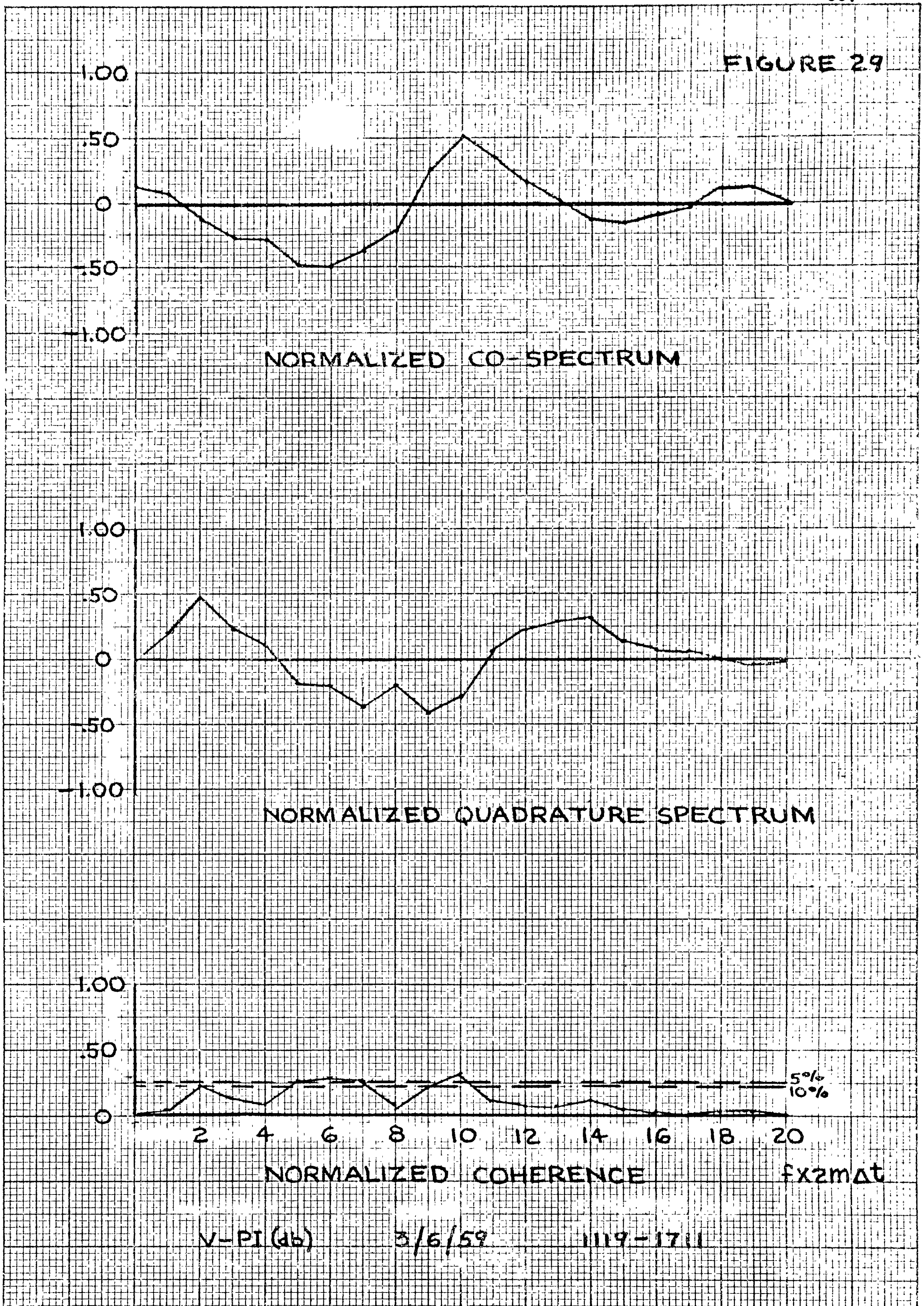
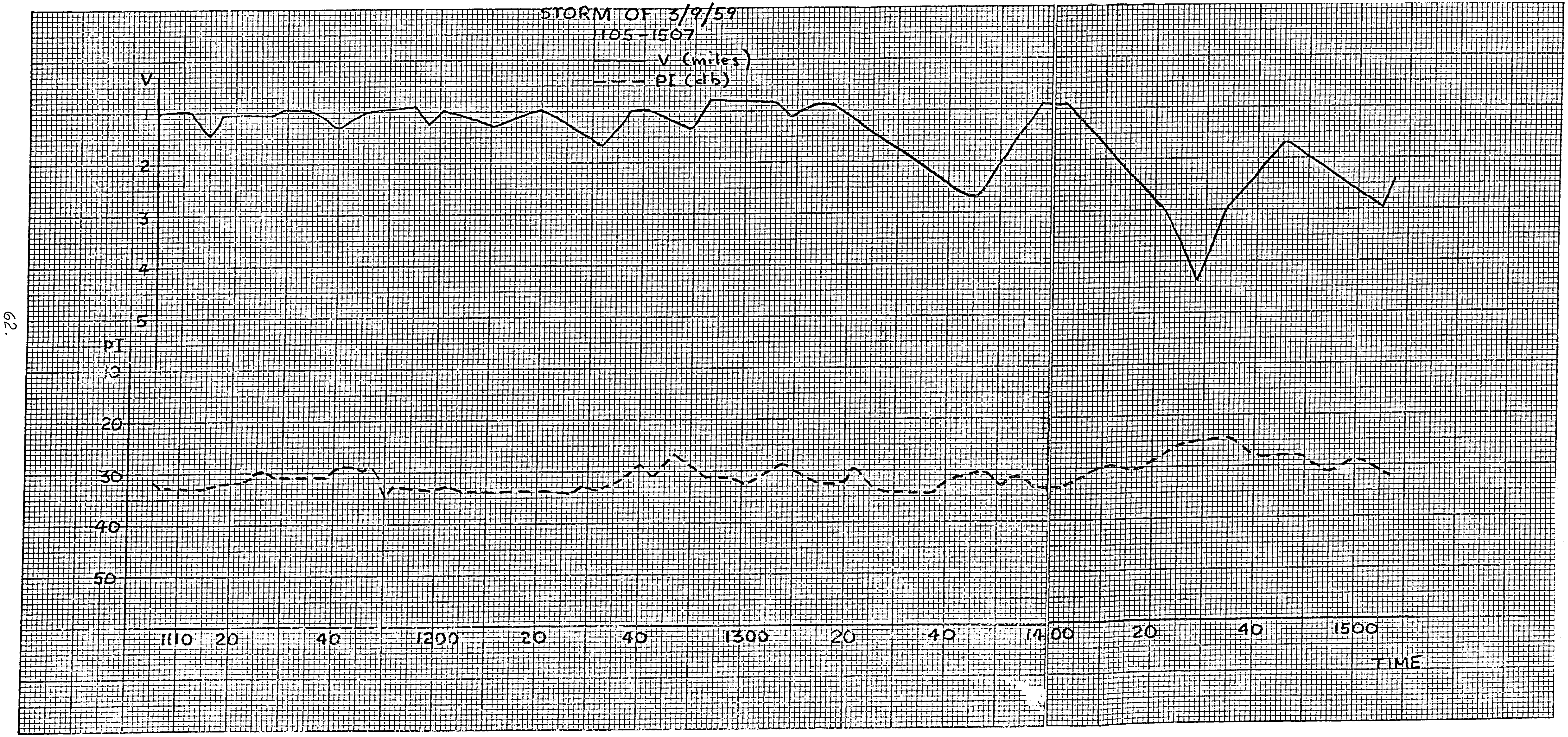


FIGURE 29
1.00
.50
0
-.50
-1.00
NORMALIZED CO-SPECTRUM
1.00
.50
0
-.50
-1.00
NORMALIZED QUADRATURE SPECTRUM
1.00
.50
0
5%
10%
2 4 6 8 10 12 14 16 18 20
NORMALIZED COHERENCE
fx2mΔt
V-PI (db)
3/6/59
1119-1711

March 9, 1959 (1013-1513) Bedford Airport (Figures 30-32)

A stationary warm front is located south of Boston extending from a low pressure center which moves due east over Southern New England during the coarse of the period. Light snow and fog accompanied by light north and northeast winds are reported throughout the interval of observation. The visibility exhibits irregular fluctuations between $\frac{1}{2}$ and 1 mile, while the ceiling gradually drops from 1000 to 400 feet over the length of the period. There is a gradual decrease in the temperature during the time under consideration.

The records of the visibilities and radar signal intensities consist of 243 data points commencing at 1105 and terminating at 1507. Neither the radar signal intensity record nor the visibility record exhibit high frequency fluctuations. The shortest period of the transmissometer spectrum containing significant variance is 8 minutes, while 4.4 minutes is the shortest significant period in the radar signal intensity spectrum. No significant co-spectrum or quadrature spectrum variations should then be attributed to periods of less than 8 minutes. The co-spectrum at point 0 is negative and is zero at point 5. Possibly some significance might be attributed to the large negative values in the lowest frequencies as indicated by the confidence limits on the coherence curve. The quadrature spectrum has a maximum at point 2, is zero at point 3, and has a minimum at point 5. The confidence limits on the coherence curve at point 5 seem to indicate some significant information in the cross spectrum at this frequency.


STORM OF 3/9/59
1105-1507
V (miles)
PI (db)
V
1
2
3
4
5
PI
10
20
30
40
50
1110 20 40 1200 20 40 1300 20 40 1400 20 40 1500
TIME

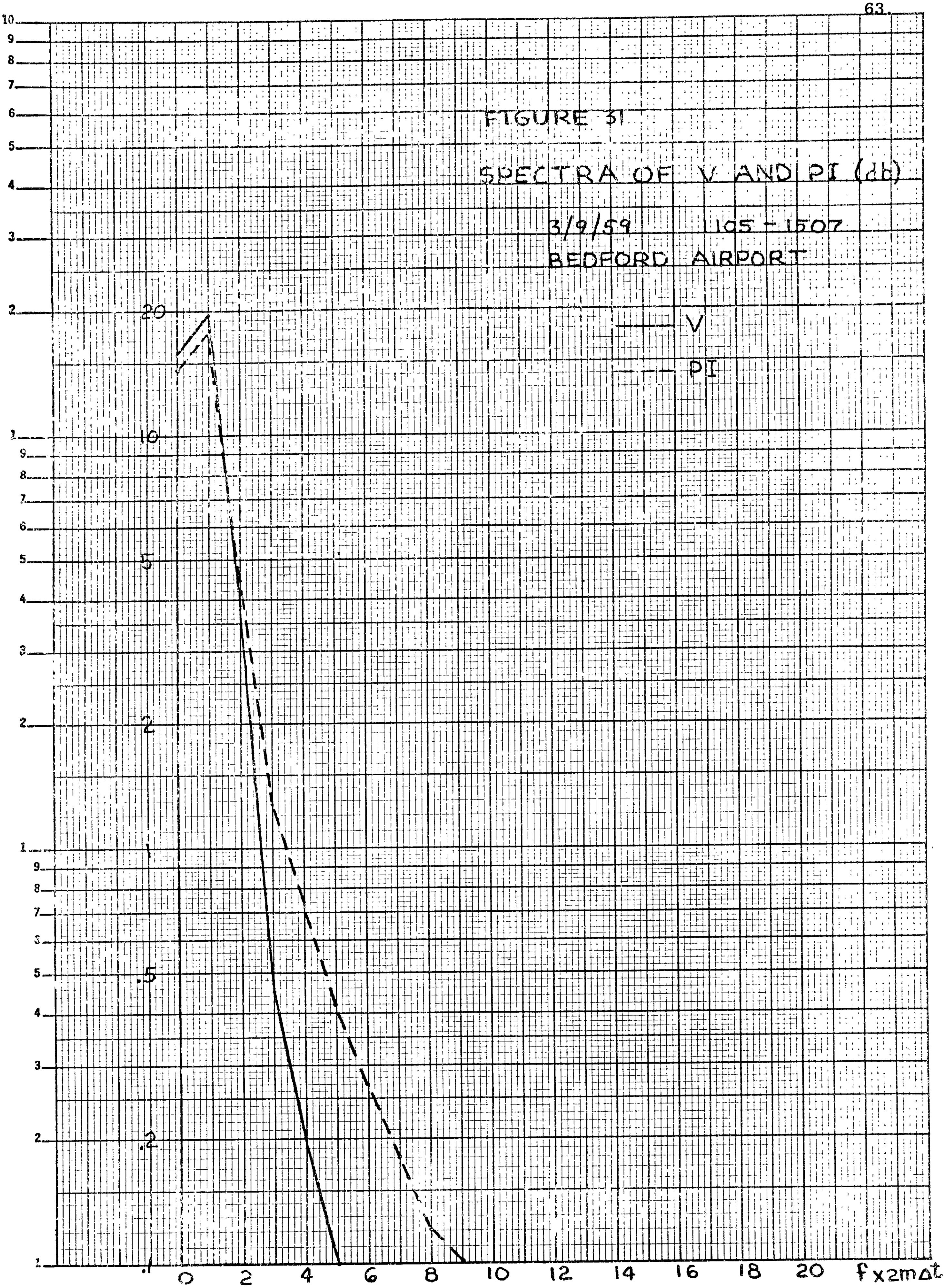


FIGURE 31
SPECTRA OF V AND PI (db)
3/9/59 1105 - 1507
BEDFORD AIRPORT
V
PI
20
10
5
2
1
.5
.2
.1
0 2 4 6 8 10 12 14 16 18 20 f x 2mΔt

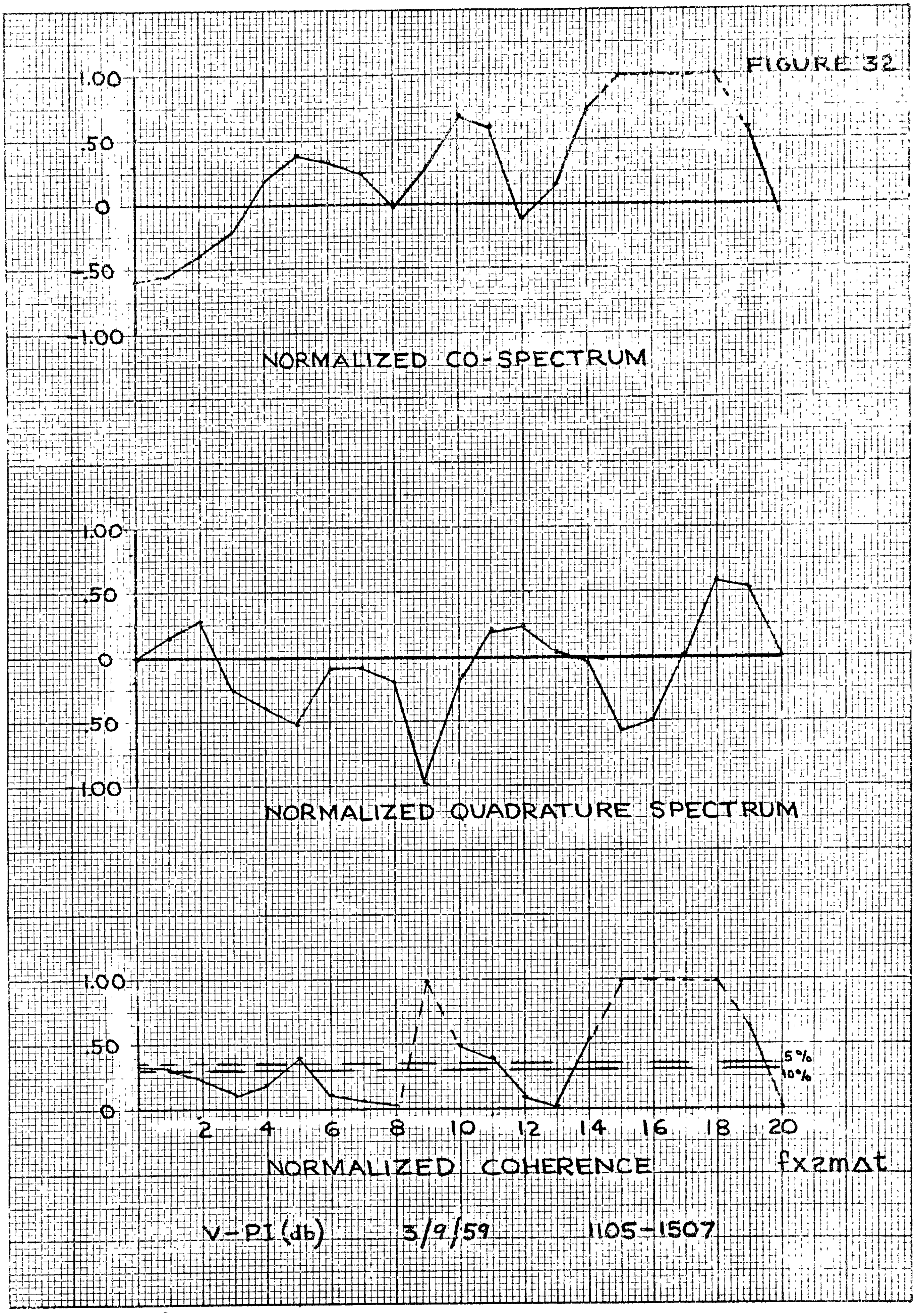


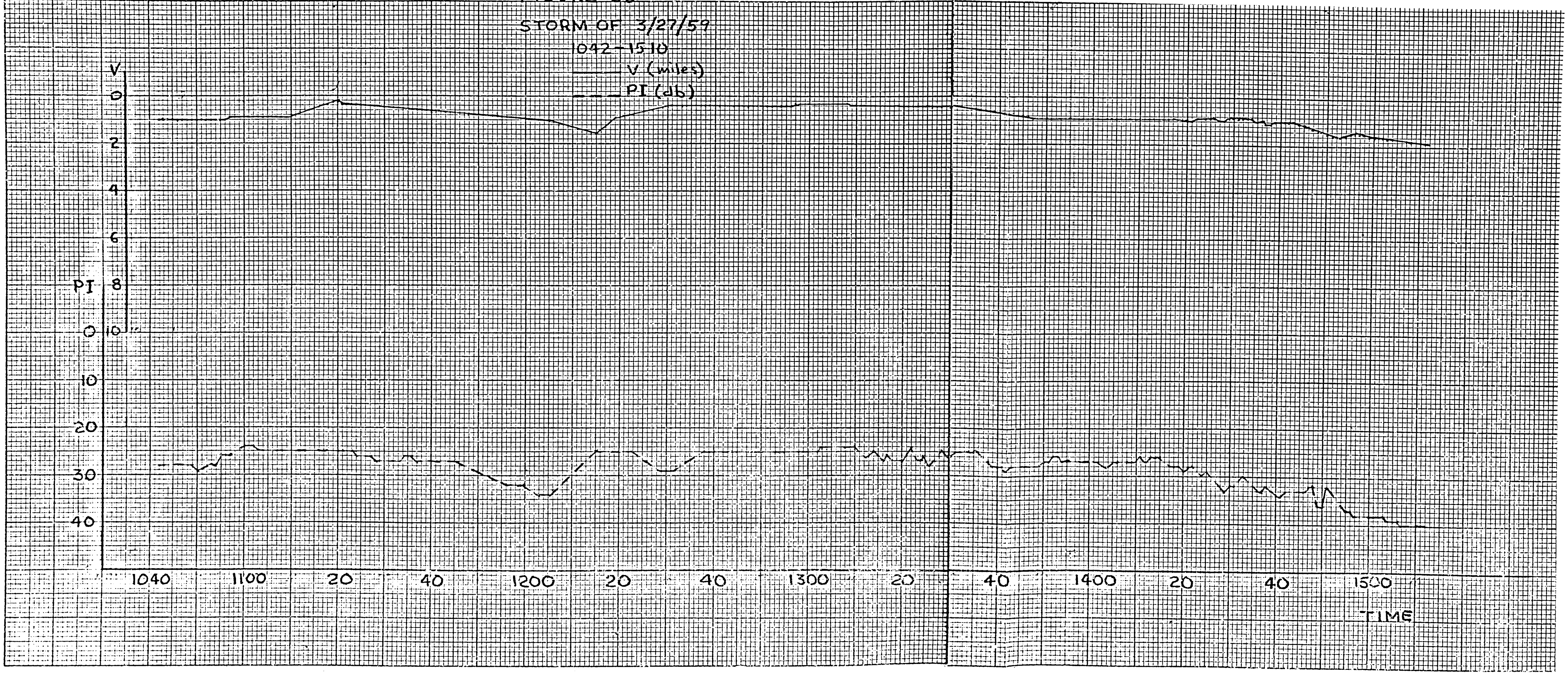
FIGURE 32
1.00
.50
0
-.50
-1.00
NORMALIZED CO-SPECTRUM
1.00
.50
0
.50
1.00
NORMALIZED QUADRATURE SPECTRUM
1.00
.50
0
5%
10%
2 4 6 8 10 12 14 16 18 20
NORMALIZED COHERENCE
fx2mΔt
V-PI(db) 3/9/59 1105-1507

March 27, 1959 (1042-1510) Bedford Airport (Figures 33-35)

A stationary warm front is located south of Boston, extending from a low pressure center which lies over the Great Lakes region and is moving to the east. Heavy snow, moderate snow, and light snow and fog accompanied by moderate SE and E winds are reported during the period under consideration. The visibility is low, ($\frac{1}{16}$ mile) in the beginning but increases to $2\frac{1}{2}$ miles at the termination of the period. The ceiling also gradually increases from 200 to 1000 feet. The temperature and dew point remain constant.

The records of the visibilities and radar signal intensities consist of 269 data points commencing at 1042 and terminating at 1510. The individual spectrum of the radar signal intensity and transmissometer visibilities show no appreciable variance in the high frequencies. The shortest period in the transmissometer spectrum containing any significant variance is 6.6 minutes, while it is 4.4 minutes in the radar signal intensity spectrum. Thus, in the co-spectrum and quadrature spectrum, very little significance should be attributed to periods of less than 6.6 minutes. The co-spectrum from point 0 to 6 remains nearly significantly positive except for points 2 and 3, as indicated by the confidence limits on the coherence curve. During the same frequency intervals, the quadrature spectrum is practically zero except for a positive peak at point 2. Comparison of the two forms of the cross spectrum indicate that the visibility and radar intensity measurements are approximately in phase and could be expected to be well correlated with each other. At points 2 and 3, one would expect the rainfall to lead the visibility by a small phase angle.

FIGURE 33


STORM OF 3/27/59
1042-1510
V (miles)
PI (db)
V
PI
0
2
4
6
8
10
0
10
20
30
40
1040
1100
20
40
1200
20
40
1300
20
40
1400
20
40
1500
TIME

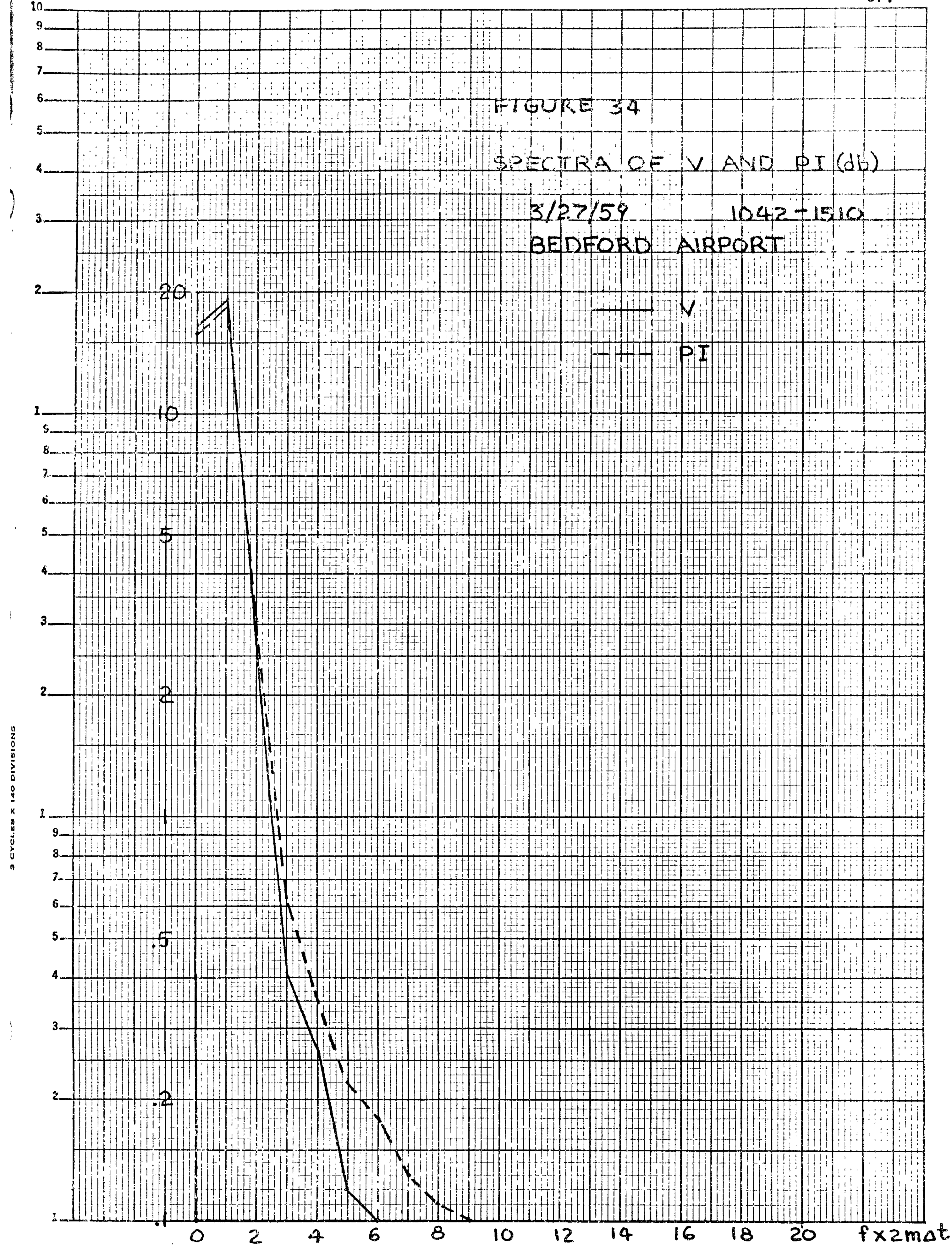


FIGURE 34
SPECTRA OF V AND PI (db)
3/27/59 1042-1510
BEDFORD AIRPORT
V
PI
20
10
5
2
1
.5
.2
.1
0
2
4
6
8
10
12
14
16
18
20
f x 2mΔt
3 CYCLES X 140 DIVISIONS

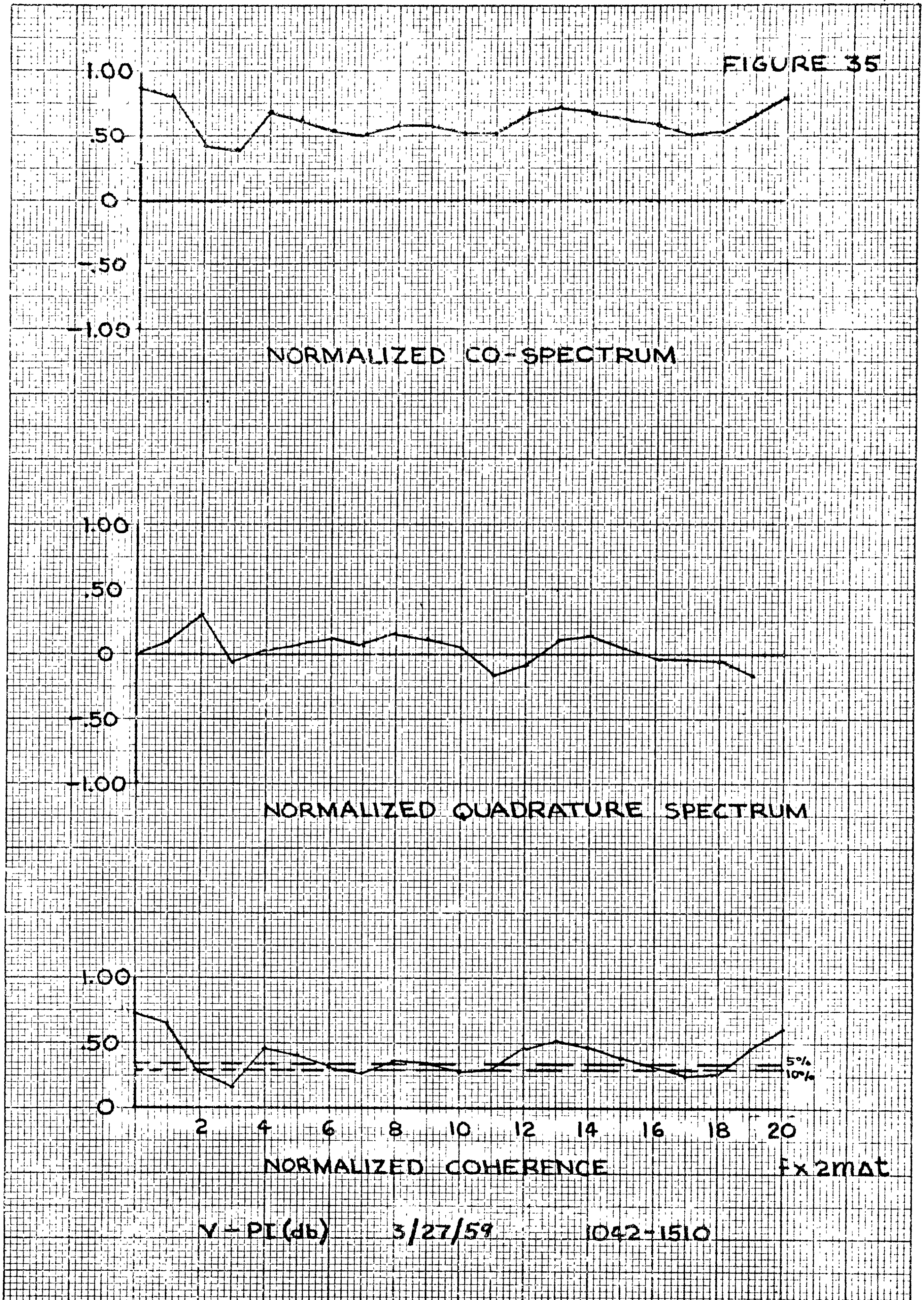


FIGURE 35
1.00
.50
0
-.50
-1.00
NORMALIZED CO-SPECTRUM
1.00
.50
0
-.50
-1.00
NORMALIZED QUADRATURE SPECTRUM
1.00
.50
0
5%
10%
2
4
6
8
10
12
14
16
18
20
NORMALIZED COHERENCE
f x 2mΔt
V - PI (db)
3/27/59
1042-1510

June 19, 1959 (0658-1642) Bedford Airport (Figures 36-38)

A weak low pressure system with no associated frontal activity is present over New England for the duration of the period. The low center, itself, passes over Boston at approximately 1300, within the interval under consideration. Light rain and fog and very light rain and fog accompanied by light variable winds are associated with the low pressure system. The visibility remains low, ($\frac{1}{2}$ to 2 miles) with irregular fluctuations throughout the period as indicated by the transmissometer record. The WBAN observations of the visual range show the visibility to be a great deal higher but still with the irregular fluctuations. The ceiling is noted to increase quite rapidly at approximately 1030 and then fall suddenly at 1500. Variability in the ceiling height ranges between 500 and 1300 feet. The temperature remains practically constant throughout the interval in which the data were taken.

The records of the transmissometer visibilities and radar signal intensities consist of 521 data points commencing at 0752 and terminating at 1632. Upon examination of the individual spectrum of the visibility and radar signal intensity, it is apparent that there are no significant high frequency fluctuations in either of the records. The shortest period in the visibility spectrum containing any appreciable variance is 8 minutes, while the shortest significant period in the radar signal intensity spectrum is 4 minutes. Therefore, very little significance should be placed in the co-spectrum and quadrature spectrum of the two variable for periods less than 8 minutes. The co-spectrum for the frequencies under consideration shows nothing significant. The quadrature spectrum at point 3 shows a minimum, but its significance is doubtful as determined by the coherence.

FIGURE 36
STORM OF 6/29/59
0752 - 1632
—— V (miles)
- - - PI (db)

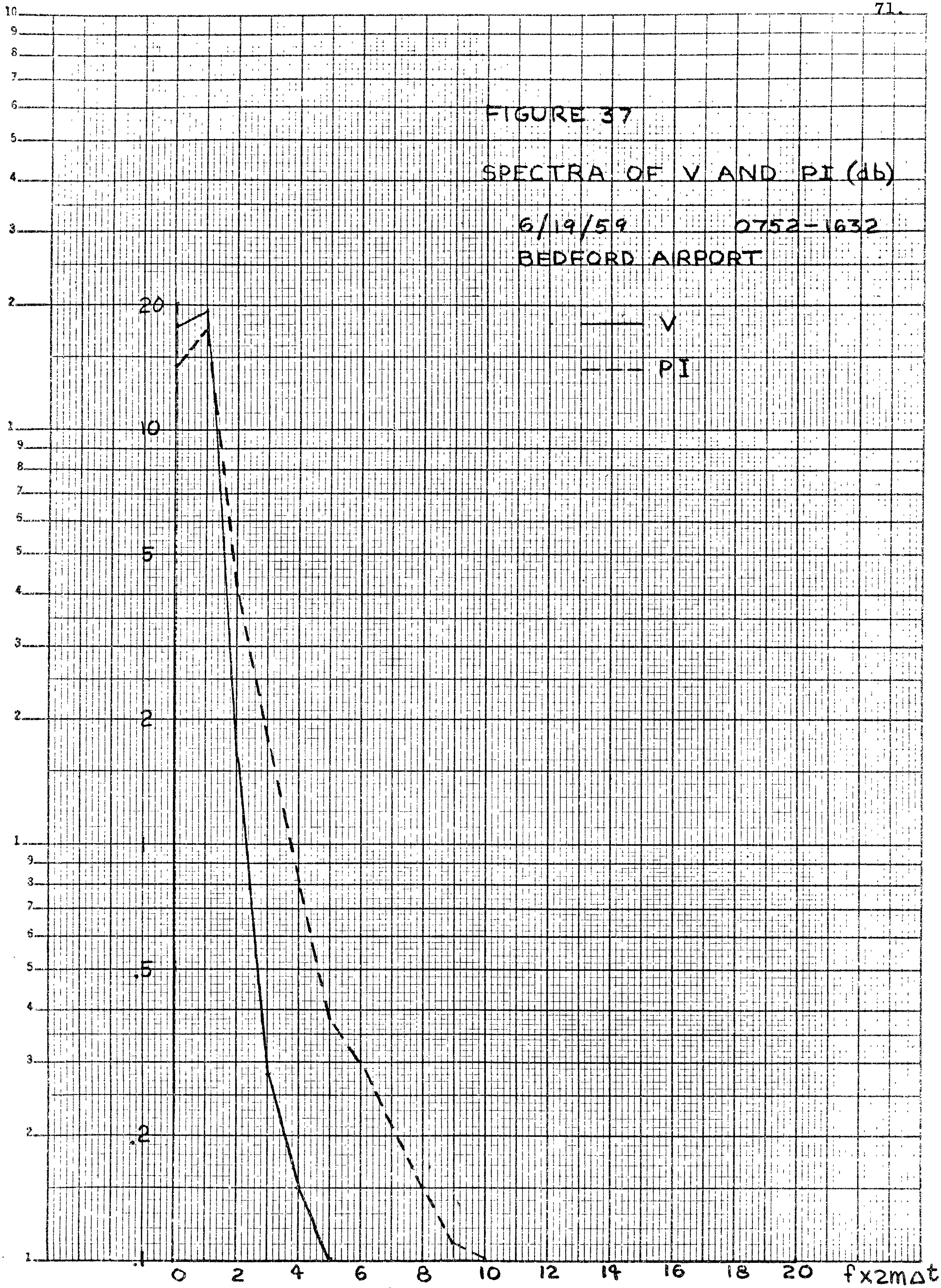
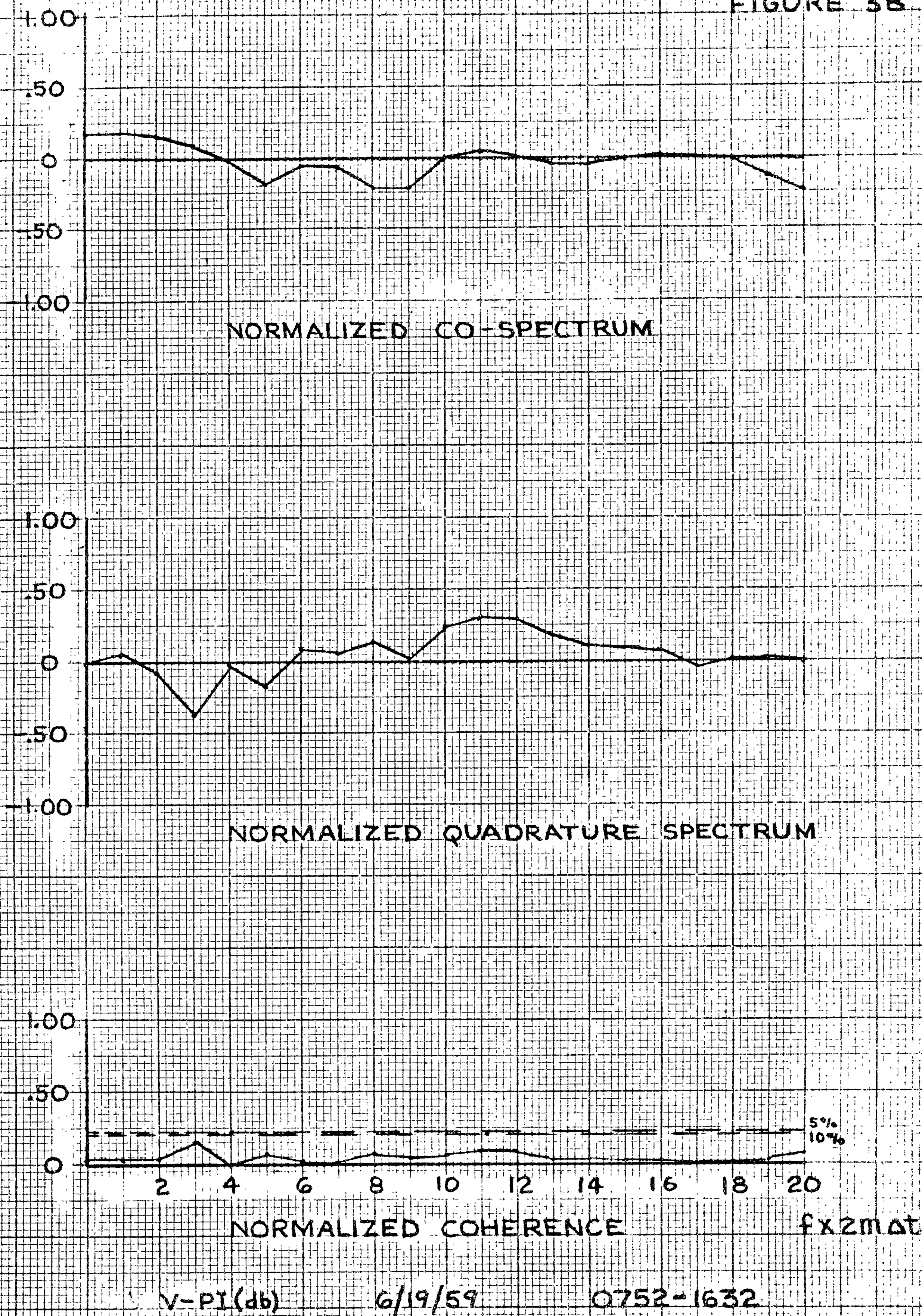


FIGURE 37
SPECTRA OF V AND PI (db)
6/19/59 0752-1632
BEDFORD AIRPORT
V
PI
20
10
5
2
1
.5
.2
.1
0
2
4
6
8
10
12
14
16
18
20
fx2mΔt

FIGURE 38


1.00
.50
0
-.50
-1.00
NORMALIZED CO-SPECTRUM
1.00
.50
0
-.50
-1.00
NORMALIZED QUADRATURE SPECTRUM
1.00
.50
0
5%
10%
2
4
6
8
10
12
14
16
18
20
NORMALIZED COHERENCE
fx2mΔt
V-PI(db)
6/19/59
0752-1632

V. General Spectral and Cross Spectral Results

It is very difficult to look at the graphic record of a time series of the nature of those studied in this paper and pick out any given frequency which appears to predominate. If such a frequency does exist, the technique of spectrum analysis will uncover it. The most striking thing about the spectra of the variables studied in these 12 cases is the preponderance of variance in the first two periods. This, however, is usually the case when the time series exhibits a good deal of persistence and when, as mentioned before, one is dealing with a truncated series. The difference in the several spectra lies in the rapidity with which the variance becomes insignificant with increasing high frequencies, and the shape of the spectral curve after the initial peak in the first period which is present in each spectrum. The graphic representation of the time series often indicates, in a qualitative manner, roughly what the spectrum will look like. If there are many short period fluctuations in the record, one can usually expect to find appreciable variance at the higher frequencies. This is borne out by the spectrum of the reciprocal visibility during the storm of May 24, 1960 where the fluctuations, when any are present at all, are of quite high frequency. Note also the spectrum of the precipitation rate during the record of October 1, 1959. The same effect is present. On the other hand, the visibility spectrum of the storm of February 14, 1959 indicates practically all the variance in the first three periods. This also is to be expected from the remarkably flat record. This process occasionally, however, forms some surprising results. For instance, the spectrum of the radar signal return from March 27, 1959 shows practically no variance at the higher frequencies, a somewhat unexpected result from the appearance of the record, which appears to have reasonably high frequency fluctuations.

It is interesting to note the spectra of the visibilities for all five of the Bedford Airport storms. In every case extremely little high frequency variance is observed. Accordingly, the records show practically no high frequency fluctuations. No systematic difficulty of this nature is observed in the Logan Airport data. This could be due to: (a) lack of sensitivity in the Bedford Airport transmissometer, (b) lack of concern for detail on the part of those who originally converted the data to visibilities, (c) a more steady behavior of those factors which restrict visibility in the Bedford Airport region, or (d) coincidence.

Unfortunately, nothing is discernable from the spectra which directly aids in the search for a correlation between visibility and precipitation rate. At best the spectra are useful in picking out those frequencies in the individual variables which make negligible contribution to the total record.

It was the hope of the authors that the cross spectra of the 12 cases considered would yield valuable information toward a better understanding of the problem. With regard to the dual criteria for significance of the different frequencies on the cross spectra alluded to earlier, the results are somewhat disappointing. Out of all 12 sets of cross spectra, only 15 frequencies satisfy the criteria. (The single study done on visibility and ceiling had no significant points on its cross spectrum.) Figure 39 shows the distribution of these 15 significant frequencies. It is immediately noted that the longer periods are the only contributors. This would lead to the conclusion that it is only in these longer periods that the behaviors of the two variables share anything significant in common. It is difficult

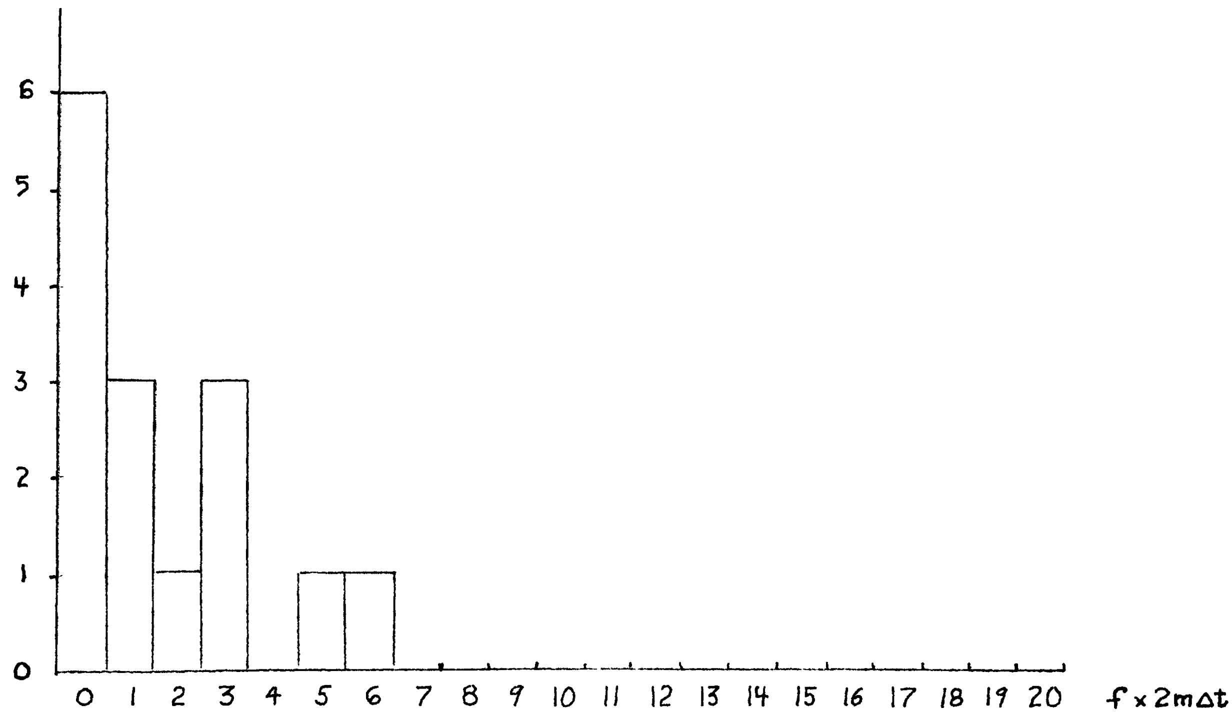


Figure 39.--Frequency distribution of significant periods in the cross spectrum of visibility and rainfall.

to speak of the period represented by point 0, which has infinite length. Perhaps one might suggest that the two variables tend to persist in the same manner. Point 1 represents a period of 40 minutes, point 2 a period of 20 minutes and point 3 a period of 13.3 minutes. In all but one of the seven significant cases represented by these three frequency intervals, the phase angle shows that the visibility tends to lead the precipitation rate by one to four minutes. The information presented by the cross spectra show an agreement with the conclusions of Roylance, that it is the fluctuations of longer duration that tend to be related to one another to a greater extent than the fluctuations of short duration.

An attempt was made to stratify according to ceiling cross spectral relationships of the visibility and signal intensity, along with those of the reciprocal visibility and rainfall rate in millimeters per hour. It was hoped that similar cross spectral relationships might hold for those storms in which the ceiling remained between defined limits. It was noticed that the ceilings for the storms under consideration could be subdivided into two groups: those storms where the ceiling remained below 500 feet, and those storms where the major contribution to ceiling height was between 500 and 1000 feet.

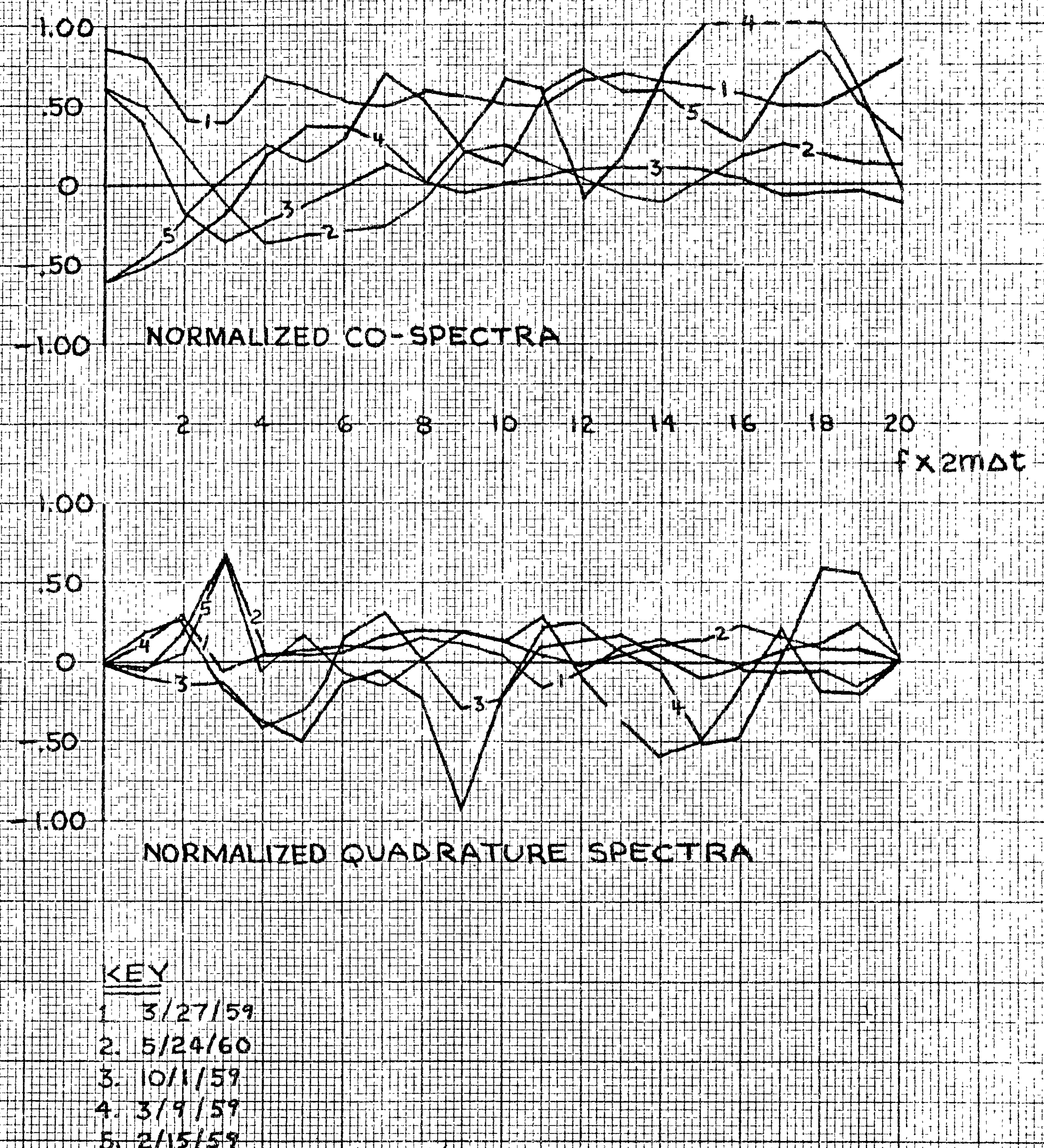
Five storms could be classified as having a major portion of the visibility and rainfall records occuring when the ceiling was less than 500 feet. (Figure 40) Three storms have weather associated with pre-warm frontal activity, one storm is associated with a low pressure trough, and the other is associated with an occluded system to the west of Boston.

The normalized co-spectra for these five individual storms do not exhibit much similarity in periodicity. If we notice the amplitudes at the various frequency intervals, there is a trend for the co-spectra to be more positive than negative. This is primarily due to the positive contribution to the co-spectra at all frequencies for the March 27, 1959 storm. There are also large positive contributions by other storms in the mid and high frequency range but these are thought to be in some instances, round-off errors in the Tukey program caused by the lack of variance in these frequency ranges.

The quadrature spectra also do not show much similarity. All the storms except October 1, 1959 show a peak in the quadrature spectrum at either point 2 or 3. This would indicate that for periods in the data between 13 and 20 minutes, the visibility lags the rainfall from 2 to 8 minutes, respectively. The rest of the quadrature spectrum is highly irregular, although one storm may be comparable to another

FIGURE 40

CROSS SPECTRA OF RECORDS OF STORMS WITH CEILINGS REPORTED BELOW 500 FEET



at particular frequencies, but bears little or no relationship to other storms at the same frequencies.

Five storms could also be classified as having ceilings ranging between 500 and 1000 feet. The weather associated with these storms is varied; consisting of a pre-warm frontal situation, a warm front passage, a post cold frontal situation, a weak low pressure system, and a broad frontal trough.

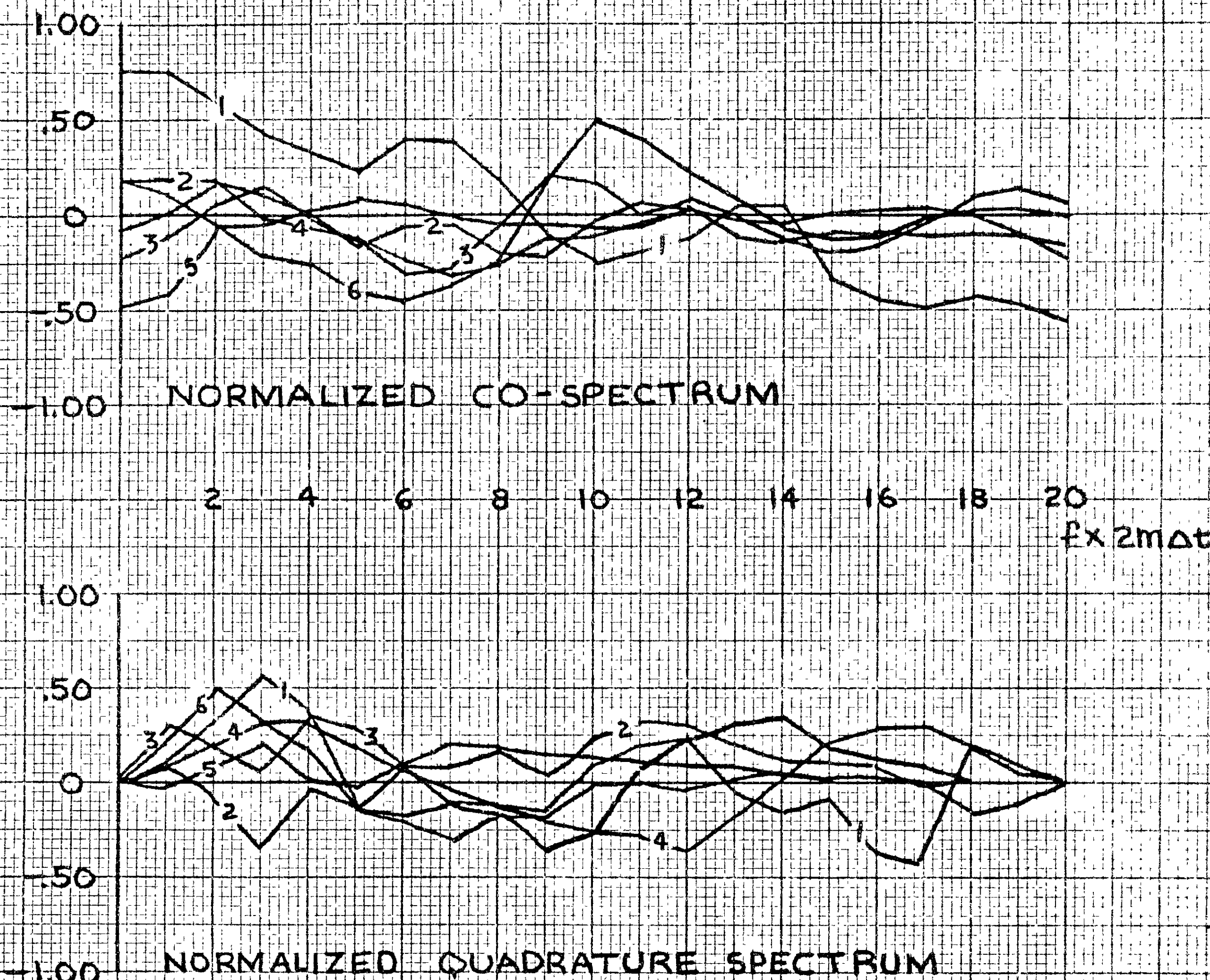
The normalized co-spectra show predominantly neutral or slightly negative values. (Figure 41) If we exclude the storm on November 24, 1959, we notice some similarities in the periodicity of the co-spectra. At point 0 the co-spectra of the remaining storms are slightly negative or neutral; at points 6 and 7, they each have a minimun value, and at points 9 and 10 have a maximum value. Since values of the co-spectra are small in magnitude at these frequencies, the significance of the possibility of a trend as mentioned above is questionable.

In the quadrature spectrum, if we exclude the June 19, 1959 storm, we again see the characteristic positive maximum at points 2,3, and 4. For periods of 10 to 20 minutes, the visibility lags the rainfall 2 to 8 minutes, respectively, for the above criteria. No similarities in the remaining quadrature spectra are noticeable.

An investigation was undertaken to see if similarities in the cross spectra of visibility and rainfall existed for similar synoptic situations. The synoptic situations associated with the eleven storms subjected to investigation were as follows: Six cases were observed where the pre-valent synoptic activity was associated with warm fronts; three cases were found where low pressure regions or troughs played an important part in the weather; and one case each was associated with the passage of a

FIGURE 41

CROSS SPECTRA OF RECORDS OF STORMS WITH CEILINGS REPORTED BETWEEN 500 FEET AND 1000 FEET



KEY

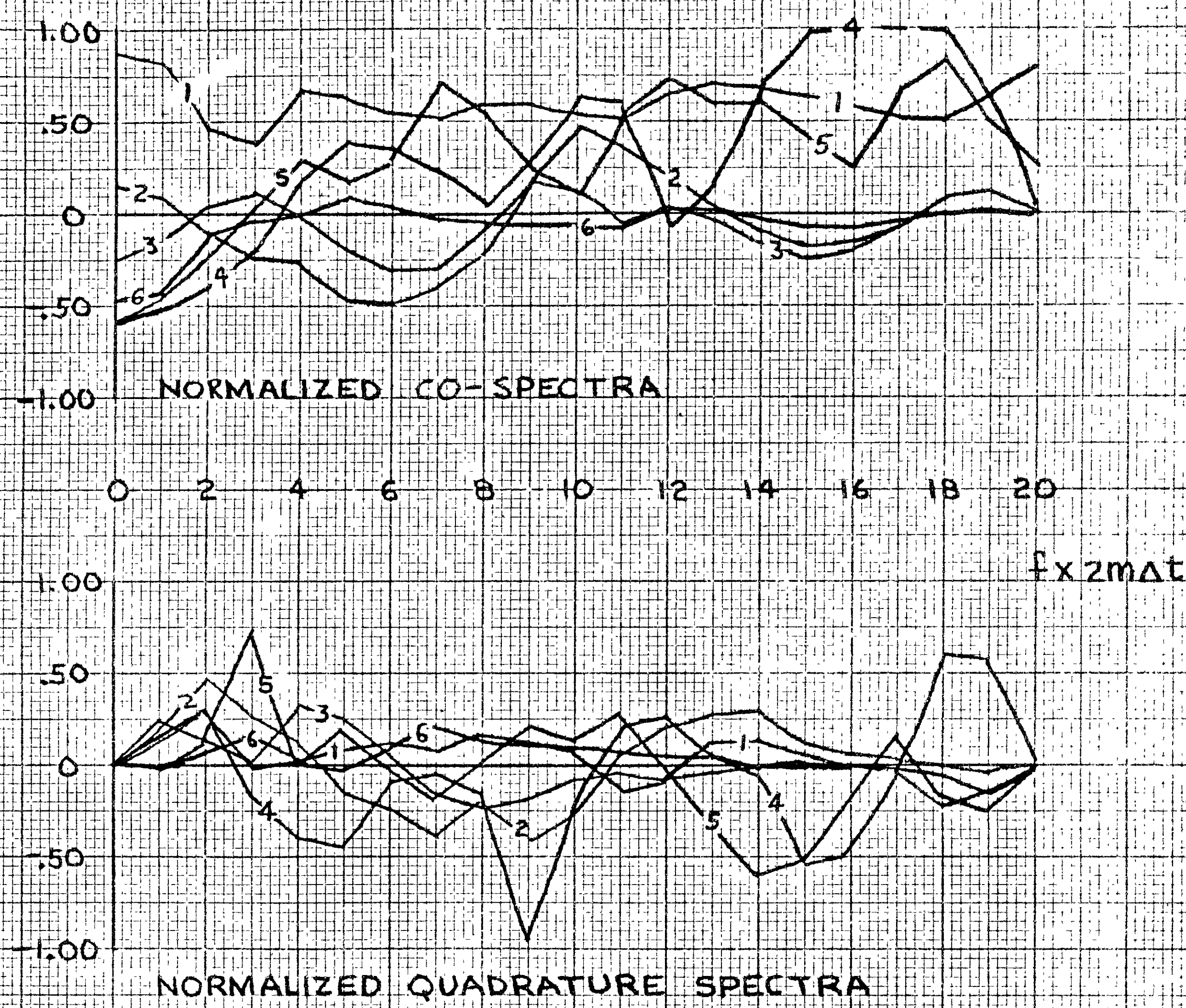
1. 11/24/59
2. 6/19/59
3. 12/28/59 (1918-2140)
4. 1/28/60
5. 12/28/59 (1008-1742)
6. 3/6/59

cold front and the presence of an occluded system near Boston.

Examination of the co-spectra of the warm frontal situations indicates a similarity in the first four periods of the individual time series with the exception of March 6, 1959 and March 27, 1959. However, if one examines the storms individually, it is observed that all storms, with the exception of March 6, 1959, are pre-warm frontal situations. (Situations where during the period of observation Boston was never in the warm air.) During the March 6, 1959 storm, passage of the warm front was indicated by the WBAN record, accompanied by a marked wind shift and a temperature increase. The passage of the warm front occurred in the early part of the record so this particular storm is unique in the respect that the cross spectra might not be expected to be similar to those of the other warm frontal situations, although this is not particularly evident from the actual records of rainfall and visibility for that date. The storm on March 27, 1959 is unique in the respect that although it is a pre-warm frontal situation, snow was reported during the entire length of the record. One might expect that the precipitation in the form of snow would have a greater effect on the visibility than would precipitation in the form of rain. This is suggested by the large positive values of the co-spectra for that date. However, this presents another problem: During the March 9, 1959 storm, snow was also reported over the length of the record. The similarities in the WBAN record and the synoptic pre-warm frontal situation for the March 9, 1959 and March 27, 1959 storms are remarkable except for the wind direction and the geographical location of the storm center. Still the co-spectra exhibit no similarities. The March 27 storm was located over the Great Lakes region and Bedford Airport was reporting SE winds,

FIGURE 42

CROSS SPECTRA OF RECORDS OF STORMS ASSOCIATED WITH WARM FRONTS



KEY
1. 3/27/59
2. 3/6/59
3. 12/28/59 (1918-2140)
4. 3/9/59
5. 2/14/59
6. 12/28/59 (1008-1742)

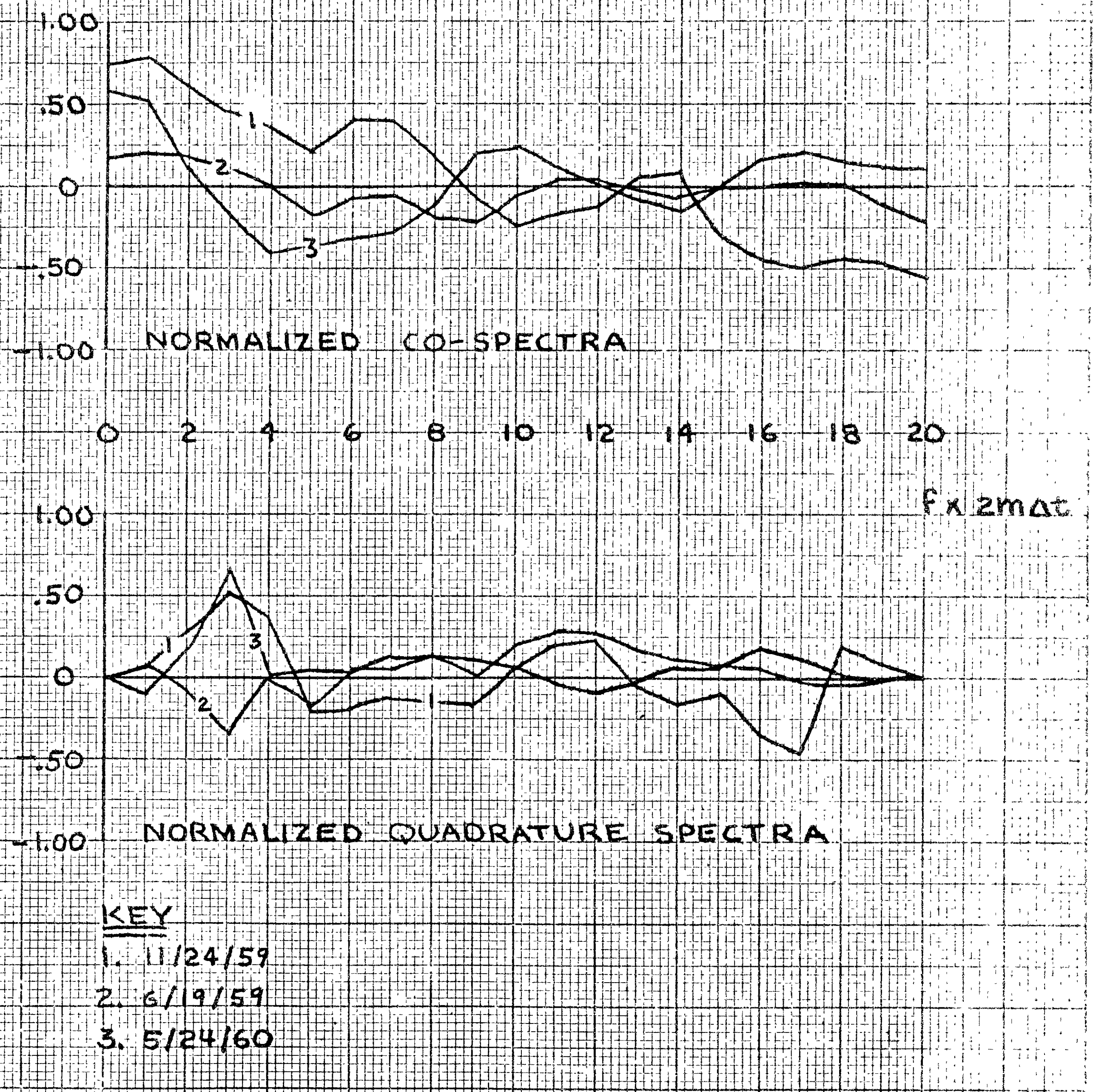
while the March 9 storm lay over Southern New England and Bedford reported NE winds. Perhaps the previous trajectory of the air associated with the two storms might account for the difference in the co-spectra between the first four periods. The trajectory of the air from the storm on March 9 probably originated along the Atlantic coast and underwent modification over the ocean. This was not the case for the March 27 storm.

There are now four pre-warm frontal situations that exhibit co-spectra similar to the co-spectrum of the March 9, 1959 storm. These co-spectra exhibit negative values at point 0 and gradually become positive or zero at either points 3 or 4. The co-spectra of the pre-warm frontal situations show no similarity for the intermediate and high frequencies. The contribution to the variance of the record at these frequencies is very small compared with the variance of the first four periods. The quadrature spectra show the positive values which were indicated in the ceiling study for either of the first three periods. The quadrature spectra for the intermediate and high frequencies show no similarity where the variance of the record was considered insignificant.

A listing of the phase relationships of the visibility and rainfall for the warm frontal situations for the first five periods, (Table 1), indicates the probability of a great deal more out-of-phase correlation than in-phase correlation. If the storms of March 6, 1959 and March 9, 1959, which were exceptional cases, are neglected, even better out-of-phase relationships are found. This would tend to indicate that for the cases of long period fluctuations of rainfall and visibility in pre-warm frontal situations, the inverse of the suspected relationship might be expected. Table 1 also suggests that there is a greater tendency for

FIGURE 43

CROSS SPECTRA OF RECORDS OF STORMS ASSOCIATED WITH LOW PRESSURE REGIONS



Date	Point	0	1	2	3	4
2/14/59	(2251-0129)	180	-173	167	86	- 13
3/6/59	(1119-1711)	0	70	105	140	158
3/9/59	(1105-1507)	180	165	145	-130	- 64
3/27/59	(1042-1510)	0	7	34	- 8	2
12/28/59	(1008-1742)	180	-178	136	107	- 22
12/28/59	(1918-2140)	180	110	66	2	88

Table 1.--Cross spectrum phase relationships between visibility and rainfall for warm frontal situations. (Phase angles -90 to 0 to +90 indicate probability of in-phase relationships, phase angles +90 to ± 180 to -90 indicate probability of out-of-phase relationships) (Phase angles 0 to +90 to +180 indicate rainfall leads visibility, phase angles 0 to -90 to -180 indicate rainfall lags visibility)

the visibility fluctuations to lag the precipitation fluctuations in most instances.

The three storms associated with cases of low pressure regions or troughs have similar co-spectra for the longer period fluctuations. Generally the co-spectra starts out as a maxima at point 0 and decrease to a minima between points 4 and 10. There does not seem to be any similarity in the co-spectra for the intermediate and high frequencies. The quadrature spectra with the exception of June 19, 1959 show the characteristic peak at point 3. For the remainder of the frequencies, the quadrature spectra oscillate rather irregularly about zero.

A listing of the phase relationships of the visibility and rainfall for the low pressure situations for the first five periods, (Table 2), indicates that the probability of in-phase relationships is greater than the probability of out-of-phase relationships. There is a tendency for the visibility to lag the rainfall for the storms with no associated

Date	Point	0	1	2	3	4
6/19/59	(0952-1632)	0	16	- 27	- 77	-147
11/24/59	(1516-1912)	0	6	30	51	46
5/24/60	(1120-1521)	0	- 9	53	102	-175

Table 2.--Cross spectrum phase relationships between visibility and rainfall for low pressure regions. (Phase angles -90 to 0 to +90 indicate probability of in-phase relationships, phase angles +90 to ± 180 to -90 indicate probability of out-of-phase relationships) (Phase angles 0 to +90 to +180 indicate rainfall leads visibility, phase angles 0 to -90 to -180 indicate rainfall lags visibility)

frontal activity.

Precipitation from the January 28, 1960 storm follows the passage of a cold front. Nothing of any significance is indicated in the cross spectral records, other than the characteristic peak in the quadrature spectrum at point 3.

The occluded system of October 1, 1959 also does not exhibit any significant cross spectral results, other than the possible significant peak in the co-spectrum between points 0 to 1. A normal relationship between rainfall and visibility might be expected for very long periods.

VI. Conclusions

As has been brought out earlier in this paper, there are many factors which make this problem a difficult one, not the least of which is fog. A look at any observational records of storms reveals several instances where the presence of fog causes the visibility to act in a manner contrary to what would be considered by precipitation considerations alone. A good example is the storm of November 24, 1959, where visibilities below one mile are reported along with light rain showers and fog, early in the period. Two hours later light rains are reported, but the fog has disappeared and the visibilities have increased to over five miles. Another two hours later, thunder storms and heavy rain showers are observed, without fog, and the visibility still is not as low as it was with light rain and fog. Thus, although increased precipitation does bring about visibility drops, fluctuations in the intensity of fog have a similar effect. The radar has no way of observing fog. This situation has an unfortunate effect upon the cross spectra, especially the cross spectra of long records where fog comes and goes and the precipitation varies from light to heavy. Fog is seldom observed with very heavy precipitation, but often with moderate or light precipitation. Thus, two precipitation situations exist which can be responsible for visibility drops. Obviously this is going to have a deleterious effect on any attempts at classification made through cross spectrum analysis.

This study is certainly not what can be termed a controlled experiment. For one thing, data collection is an uncertain problem, depending on the simultaneous operation of and availability of data from the radar and the transmissometer. Further, precipitation does not always behave in a manner which is easy to study. To often heavy bursts of rain will be followed by practically no rain at all, which is satisfactory if one is attempting to deal with the effects of such bursts upon the visibility, but not good if one is attempting to set up a reasonably long time series for spectrum and cross spectrum analyses. The visibility is being measured along only one direction which in some instances may not give an accurate picture of true conditions. Also, the radar measures the precipitation between 1000 and 2000 feet above the transmissometer. This creates a false time delay between precipitation rate and visibility.

These are the factors which make it difficult to draw very many positive conclusions about the relationship between visibility and precipitation intensity. The study has failed to uncover any definite frequencies of behavior in the records. The cross spectra show little similarity, especially at the higher frequencies. Although the spectra are useful in interpreting the cross spectra, a study of them alone reveals nothing.

This study does, however, indicate several interesting tendencies, some of which are in agreement with previous work done on the problem. On all 12 cross spectra studied, only 15 points are significant when considered by themselves. These 15 points are represented by six different periods. The fact that each of these six periods are long suggests that it is in the longer periods that there exists a correlation between the precipitation rate and the visibility. There is also an indication that in the important longer periods the precipitation tends to lead the visibility one to four minutes. A general trend which expresses this phase lead is observed in most of the storms studied. The tendency is for the co-spectrum to approach zero covariance between points 1 and 4, and at the same time, for the quadrature spectrum to take on intermediate or high values of covariance, indicating a phase lead of the precipitation rate over the visibility. Although many of the points comprising these trends are not by themselves significant, the fact that this behavior recurs so often points to the fact that the trends are meaningful. It would be improbable that a random distribution of points on the cross spectra would fall in this pattern so frequently.

There is a tendency for better relationships to exist between the precipitation and visibility during periods when the ceiling is low. The positive contributions to the co-spectra by the storms in which the ceilings were predominantly under 500 feet indicate this.

There appears to be a tendency for different relationships between the visibility and precipitation dependent on the synoptic situation. For pre-warm frontal conditions there is a tendency for increases in prcipitation rate to be associated with increases in the visibility rather than with decreases, as would be expected. The physical processes that might possibly be ascribed for such an other than normal relationship are related to pre-warm frontal fogs. (George, 1940) (7) In such cases we may think of increases in precipitation as actually washing out the fog and allowing the visibility to increase. Similarly, after a decrease in precipitation the fog would reform and the visibility would fall. In pre-warm frontal conditions it was observed that, for the longer period fluctuations, the precipitation tended to lead the visibility by approximately a quarter of a period. For low pressure patterns with no associated frontal activity, the cross spectrum analyses showed a normal relationship between rainfall and visibility. The trend for the rainfall to lead the visibility, however, was not particularly distinguishable. The post cold frontal situation and the occluded system that were studied indicated no significant cross spectral relationships.

Although some interesting trends and relationships were obtained from this study, future time might be more profitably spent in looking not only at the precipitation, but at the many other causes of visibility and ceiling fluctuations as well. Should such studies be undertaken in the future, utilizing the tools of spectrum and cross spectrum analyses, considerable care should be taken in collecting good data. Because of the statistical nature of these processes, time series in sufficient number should be used.

List of Tables and Figures

Tables

Figures

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