GAS TURBINE BLADE COOLING

by

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Chairman of the Department of the Graduate School

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Dear Sir:

In partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering, I hereby submit a thesis entitled "Gas Turbine Blade Cooling".

Respectfully,

Bertram John Milleville

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Introduction

In the design of any gas turbine there are available to the designer five basic methods for increasing the efficiency, starting from the simple cycle. These are as follows:

1.) Increase in turbine inlet temperature

2.) Use of regeneration

3.) Use of intercooling

4.) Use of reheating

5.) Improvement in machine efficiency

Of these, the first may be considered the most promising from the long term standpoint, offering substantial improvement in thermal efficiency in the simplest cycle as well as in the most complex.

Many papers have been written, both here and abroad, discussing various aspects of the idea of blade cooling. In these, blade cooling is generally presented as a means for making possible higher turbine inlet temperature and correspondingly greater power output for a given size of machine, or else to make possible the use of cheaper and more readily available materials during war-time shortages. In the great majority of these papers, thermal efficiency was given virtually no consideration at all.

Blade cooling is to be considered in this thesis as a means of improving the thermal efficiency of the gas turbine by permitting the use of increased turbine inlet temperature. The improvement obtained in this way is of course not all clear gain, as there will necessarily be thermodynamic losses, possibly power for the operation of auxiliary equipment, greater initial capital investment, etc., all of which must be charged against the nominal gain in cycle efficiency before the net gain can be realized.

The comparatively short history of active development of the gas turbine as a useful source of power has seen a steady increase in turbine inlet temperature, for the most part without resort to blade cooling. It is certain that further progress will be made, with metallurgical improvements in high temperature materials leading the way. Nevertheless, blade cooling must be given due consideration, since it will always make possible the use of gas temperatures beyond those which any blade material can withstand.

In this thesis a simple blade cooling system will be suggested and analysed. It is hoped that the results will give some hint as to the possible merits of blade cooling as a means for improving thermal efficiency. But more important, it is hoped that some of the analytical techniques used may prove useful in future work on this and related subjects.

<u>Bibliography</u>

"Gas Turbines" - L. N. Rowley & B. G. A. Skrotzki

"The Gas Turbine as a Possible Marine Prime Mover" - C. Richard Soderberg & Roland B. Smith

Gains and Losses Resulting from Use of Blade Cooling

In order to justify the use of any blade cooling system which introduces complications in design or operation, it must be shown that a worthwhile net gain in thermal efficiency and/or power output will result. A logical starting point for any related analysis is to determine quantitatively the temperature vs. thermal efficiency and temperature vs. power output characteristics of the cycle or cycles being considered.

The derivations to be outlined below are based on the simplest possible assumption, namely that turbine inlet temperature is the only variable. (exhaust temperature varies as a function of the inlet temperature). Some results of these derivations will be shown, and in addition, characteristic curves will be shown which are based on slightly modified assumptions.

For a simple gas turbine cycle (see Fig. 1a) the thermal efficiency may be written as a simple function of the temperatures as follows: (see page 42 of Appendix for nomenclature)

$$\eta = \frac{(T_3 - T_4) - (T_2 - T_1)}{(T_3 - T_2)} \qquad (1)$$

T_4 may be eliminated from this equation (Appendix, page 44) giving an expression for efficiency as a function of T_3

$$\eta = \frac{(T_2 - T_1)}{(T_3 - T_2)} \left[\frac{\eta_t \eta_c \frac{T_3}{T_1}}{1 + \eta_c \frac{(T_2 - T_1)}{T_1}} - 1 \right] \qquad (2)$$

This may be reduced algebraically to

$$\eta = C - \frac{B}{(T_3 - T_2)} \qquad (3)$$

in which, under the initial assumptions, B, C, and T_2 are constants.

For the regenerative cycle (Fig. 1b) a similar analysis gives a modified version of the above. Here the thermal efficiency is

$$\eta = \frac{(T_3 - T_4) - (T_2 - T_1)}{(T_3 - T_2'')} \qquad (4)$$

Again eliminating T_4 (Appendix, page 47), we obtain

$$\eta = C_r - \frac{B_r}{T_3 - T_2 \frac{E}{D}} \qquad (5)$$

in which T_3 is again the only variable.

In order to demonstrate the way in which these relations may be applied in an actual case, calculations have been made for both the simple cycle and a comparable regenerative cycle, with the following conditions:

		Simple Cycle	Regenerative Cycle
η_c	=	.85	.85
η_t	=	.85	.85
η_r	=	-	.70
T_1	=	60°F	60°F
T_3	=	1150°F	1150°F
p_r (optimum η)	=	6.5	3.3

Evaluating the necessary constants using the above data, we obtain the general expressions for thermal efficiency:

	Simple Cycle		Regenerative Cycle	
$\eta =$	$.351 \dfrac{96.8}{(T_3 - 950)}$	(6)	$.516 \dfrac{369}{(T_3 - 483)}$	(7)
for $T_3 = 1150^{\circ}F$; $\eta =$	.304		.277	
$T_3 = 1250^{\circ}F$; $\eta =$	.324		.297	

From this we see that the gain in thermal efficiency in this particular temperature range is approximately 2% per $100^{\circ}F$ increase in turbine inlet temperature, for both the simple and regenerative cycles.

Fig. 2 illustrates the manner in which thermal efficiency varies with pressure ratio for the simple and regenerative cycles at different turbine inlet temperatures. Figs. 3a and 3b show the principal characteristics of interest to anyone considering blade cooling. These curves are based on pressure ratio for optimum thermal efficiency at every temperature.

With the above picture of the possibilities for improving gas turbine thermal efficiency by increasing the turbine inlet temperature, we will now consider blade cooling as a means to this end. In the balance of the paper a simple blade cooling scheme will be selected for study. The parasitic losses attributable to such a system will be estimated. Then the actual flow of cooling air through the system will be analysed in detail, and an estimate of the effectiveness of such a scheme will be made.

Many schemes have been proposed for cooling turbine blades or protecting turbine blades from the hot gas stream.

A few of the more common are as follows:

1.) Air cooling, with cold air passing through a passage or passages in the blade and discharging into the hot gas stream.

2.) Air cooling, as above, except cooling air discharges into a diffuser in the turbine housing and is recirculated.

3.) Air cooling and boundary layer insulation, with cooling air discharged through slots in the leading edge of blade and maintaining a boundary layer of relatively cool air over surface of blade.

4.) Air cooling, with air passing through U-shaped passages in the blade, returning to disc for recirculation or exhaust to atmosphere.

5.) Water cooling, with water above critical pressure in blind holes in blades, circulating by convection and changing to steam at a point inside the disc.

6.) Water cooling, with water flowing outward through porous blade material, emerging at the surface as superheated steam.

It is not the purpose of this paper to compare blade cooling systems, one with another. Scheme No. 1 above has been selected for analytical purposes because it is reasonably simple, involving only minor modifications in design. Also, because of its relative simplicity, it is reasonably susceptable to theoretical analysis.

Fig. 4 shows such a cooling system in its relation to the various components, as well as the actual physical means by which it is to be accomplished. The blade cooling air is ducted through the shaft and up to the blade roots through a segmented cavity in the disc. Since the radial velocity of the air is relatively low in the disc cavity, a highly efficient compressor stage results, providing a pressure head above the nominal compressor exit pressure. This additional pressure head will make possible a high velocity flow through cooling passages in the blades, and the high velocity flow in turn gives good cooling effectiveness.

To complete the general picture of factors influencing the liklihood of success of a blade cooling system it is necessary to make some quantitative estimate of the thermodynamic losses which will be incurred in the operation of such a system. In the method of Fig. 4, we will consider separately, losses resulting, based on two different assumptions: 1.) irreversible mixing of cooling air with hot gases, and 2.) bleeding off and "throwing away" the high pressure air used for cooling purposes.

In the first of these the analytical approach is difficult because of several factors which are not well understood. We do not know how much stratification of cooling gases will reduce the loss caused by irreversible mixing. The turbine disc does an appreciable amount of work on the cooling air as it passes through, while this in turn reduces the loss due to irreversible mixing. And it is possible that some of the energy in the cooling air in the form of radial and circumferential velocity could be recovered if a well-designed diffuser could be built into the turbine housing.

The following calculations, which are intended to give a reasonable estimate of the thermodynamic losses, are based on the assumption that full irreversible mixing of cooling air with hot gas from the burner takes place before the first turbine stage. That is, instead of entering the hot gas stream at the blade tips of the first stage, the cooling air is assumed to enter the hot gas stream as it leaves the burner.

Thus the compressor thinks the cycle is operating normally, the burner thinks the cycle is operating at a slightly reduced gas flow but normal temperature, while the turbine thinks the cycle is operating at normal gas flow but slightly reduced temperature.

No allowance is made for the enthalpy lost by the cooling air in the cooling air cooler. This will tend to make the result optomistic, but the previous assumption of mixing before the first stage is very conservative, so that the result may be a reasonable estimate of the actual loss.

If we define δ as the ratio

$$\delta = \frac{\text{Weight of cooling air}}{\text{Weight of cooling air plus hot gas}} \tag{7.9}$$

and refer to the simple gas turbine cycle with air cooling as outlined in Figs. 5a and 5b, we may write for the thermal efficiency:

$$\eta = \frac{(T_{3m} - T_{4m}) - (T_2 - T_1)}{(1 - \delta)(T_3 - T_2)} \tag{8}$$

Following virtually the same algebraic steps as in the previous derivations, we may reduce the above expression to

$$\eta = C_m - \frac{B_m}{(1 - \delta)} \tag{9}$$

where C_m and B_m are constants under the given assumptions.

For the regenerative cycle, Figs. 6a and 6b:

$$\eta = \frac{(T_{3m} - T_{4m}) - (T_2 - T_1)}{(1 - \delta)(T_3 - T_2'')} \tag{10}$$

Here, since the cooling air is mixed with the hot gases in the turbine, T_2'' will be a function of $(1 - \delta)$. The relation is obscured by the difference in mass flows of hot and cold gases in the regenerator, which partially compensates for the slight reduction in T_4. Moreover, it would seem reasonable to assume that T_2'' will be affected only slightly by changes in δ. If it is assumed to be independent for small values of δ, Eq. (11) becomes

$$\eta = C_{mr} - \frac{B_{mr}}{(1 - \delta)} \tag{11}$$

where C_m and B_m are constants for small values of δ.

To use these results we assume a set of operating conditions and evaluate the constant terms. The calculations are shown in the Appendix, page 49 and are based on the following:

		Simple Cycle	Regenerative Cycle
η_c	=	.85	.85
η_t	=	.85	.85
η_r	=	-	.70
T_1	=	$60^\circ F$	$60^\circ F$
T_3	=	$1600^\circ F$	$1600^\circ F$
p_r (optimum η)	=	13	5

Evaluating the necessary constants we obtain:

			Simple Cycle	Regenerative Cycle
	η	=	$.428 - \frac{.163}{(1-\delta)}$ (13)	$.455 - \frac{.108}{(1-\delta)}$ (14)
for $\delta = 0$;	η	=	.265	.347
$\delta = .01$;	η	=	.247	.346
$\delta = .10$;	η	=	.2634	.335

Thus the loss in thermal efficiency resulting from the assumption of complete irreversible mixing of cooling air and hot gases as shown in Figs. 5 and 6, is approximately .17% for 1% cooling air flow in the simple cycle, and .10% for 1% cooling air flow in the regenerative cycle.

The second assumption to be considered is that the high pressure air bled off from the compressor is simply "thrown away" and has no further effect on the cycle. This yields readily to analysis, since the various temperatures are now unaffected and the only changes to be considered are in the mass flow. Thus for the simple cycle with defined as before:

$$\eta = \frac{(1-\delta)(T_3 - T_4) - (T_2 - T_1)}{(1-\delta)(T_3 - T_2)} \tag{15}$$

which reduces to (Appendix, page 59)

$$\eta = C_b - \frac{B_b}{(1-\delta)} \tag{16}$$

in which C_b and B_b are constants for a given set of operating conditions.

In the regenerative cycle,

$$\eta = \frac{(1-\delta)(T_3 - T_4) - (T_2 - T_1)}{(1-\delta)(T_3 - T_2'')} \tag{17}$$

which in turn reduces to

$$\eta = C_{br} - \frac{B_{br}}{(1-\delta)} \tag{18}$$

Making use of Eq.(16) and Eq.(18), we assume the same operating conditions as in the preceeding calculations, and arrive at the following results:

		Simple Cycle		Regenerative Cycle	
	$\eta =$	$1.002 - \frac{.721}{(1-\delta)}$	(19)	$.805 - \frac{.444}{(1-\delta)}$	(19.1)
$\delta = 0$;	$\eta =$	.281		.361	
$\delta = .01$;	$\eta =$	.274		.356	
$\delta = .10$;	$\eta =$	.201		.312	

Here we see that the loss in thermal efficiency resulting from the assumption of "throwing away" the cooling air bled from the compressor is approximately .7% for 1% cooling air flow in the simple cycle, and .5% for 1% cooling air flow in the regenerative cycle.

Since the latter assumption gave the more conservative results it would be reasonable to use these results alone. However, we may be even more conservative and use the sum of the losses from both assumptions, giving us .87% loss for 1% cooling air flow in the simple cycle, and .6% loss for 1% cooling air flow in the regenerative cycle.

Referring to Figures 3a and 3b we find that the gain in efficiency per 100°F increase in turbine inlet temperature is approximately 1.5% for the simple cycle and 1.6% for the regenerative cycle. Correlating this with the above figures for losses, we can arrive at a "break-even" ratio of effective cooling to percent of cooling air flow, at which the gain in efficiency made possible by the blade cooling is just enough to make up for the losses in efficiency resulting from the operation of the blade cooling system. This "break-even" rate for the simple cycle is 58°F of effective cooling per percent of cooling air flow, while for the regenerative cycle it is 38°F per percent of cooling air flow.

This is an encouraging result, since it indicates that the most favorable opportunity for successful application of a blade cooling system exists in the cycle which is already the more efficient of the two considered.

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"The Gas Turbine as a Possible Marine Prime Mover" -
C. Richard Soderberg & Roland B. Smith

"Thermodynamic Properties of Air" -
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II

Analysis of Gas Turbine Blade Cooling System

In this section the blade cooling system described in the first section will be analysed. Cooling air flow will be traced through the disc and the cooling passages in the blades, and methods of calculating the changes in temperature, pressure, and velocity will be shown. Then actual calculations will be made, based on certain assumptions of dimensions, proportions, and operating conditions. From the numerical results obtained it will be possible to estimate the heat transfer coefficient on the surface of the cooling passage. The value of this coefficient will then be used to obtain a first estimate of the effectiveness of the blade cooling system.

Referring to Fig. 4 we may observe that it should be possible to keep the pressure drop in the cooling air duct quite low. The duct can be made of generous proportions, and the air at this point is quite dense, being at a high pressure and comparatively cool. The first appreciable change in state experienced by the cooling air occurs as it enters the disc cavity through connecting holes in the shaft. Fig. 7 is a sectional view of the turbine disc, and shows the gas path after it reaches this point.

The pressure distribution in such a rotating space yields to a fairly simple analysis, if radial velocities are assumed to be negligible. The validity of this assumption will be discussed later in this section, when actual dimensions are being considered. For the present it will simply be assumed.

Fig. 8 is a simplified portion of Fig. 7 , identifying the terms used in the following derivation. Considering for simplicity unit thickness perpendicular to the paper, the condition for equilibrium of body force and pressure in the gas is

$$dp = \frac{r \Omega^2 dm}{r \theta} \tag{20}$$

where

$$dm = \frac{\rho}{g} r \theta \, dr \tag{21}$$

Assuming the compression to be isentropic, we may write ρ as a function of p , and the resulting differential equation has as its solution, (Appendix, page 65)

$$p = \left[p_0^{\frac{k-1}{k}} + \frac{k-1}{2k} C \left(r^2 - r_0^2 \right) \right]^{\frac{k}{k-1}} \tag{22}$$

where C is a constant defined in Eq. (21.6)

Thus, if the design proportions are such as to justify the assumption of low radial velocity, we may easily evaluate the pressure available at the root entry to the cooling passage in the turbine blade.

At the exit from the cooling passage at the blade tip, we know that the static pressure of the cooling air must be equal to the static pressure of the hot gas flow in that vicinity. This, in turn, is readily calculated from the turbine inlet conditions and the velocity diagrams used in the first stage.

The problem of determining the flow through the cooling passage thus reduces to a one-dimensional gas flow problem, with entrance and exit pressures known. It will be assumed that the passage is of circular cross-section and of constant area throughout its length. Fig. 9 shows the isolated cooling passage, with the factors influencing the gas flow indicated. The equation for this type of flow, taking into account compressibility effects is

$$(1 - M^2)\frac{dM^2}{M^2} = (1 + kM^2)\left(1 + \frac{k-1}{2}M^2\right)\frac{dT_0}{T_0}$$

$$+ kM^2\left(1 + \frac{k-1}{k}M^2\right)\frac{4f\,dr}{D} - 2\left(1 + \frac{k-1}{2}M^2\right)\frac{dX}{A\,p} \quad (23)$$

where M is the ratio

$$\frac{\text{velocity of cooling air relative to blade}}{\text{velocity of sound in cooling air}}$$

and the other symbols are defined on page 63 of the Appendix.

The usefulness of this equation lies in the fact that by a step-wise integration along the path being studied, the change in Mach Number (M) may be traced, from an assumed value at any point. Since this presupposes a knowledge or assumption of the value of T_0 along the path, the static temperature and consequently the local velocity of sound may be evaluated. The Mach Number then gives the actual velocity of the cooling air relative to the blade. Since the mass rate of flow is constant all along the passage, the density is determined, from which finally we may calculate the corresponding pressure.

The step-wise integration is greatly facilitated by tables of Mach Number functions consisting of combinations of the Mach Number factors appearing in Eq. (23). In the following paragraphs, Eq. (23) will be further explained, and the various influence factors will be evaluated in terms of the particular problem being considered.

Referring again to Eq. (23), we find that the quantities requiring evaluation in terms of the flow conditions are $\frac{dT_o}{T_o}$, $\frac{f\,dr}{D}$, and $\frac{dX}{Ap}$. To determine the value of $\frac{dT_o}{T_o}$ as a function of r, we must take into account the heat that is flowing into the air stream from the cooling passage wall, as well as the work that the tube is doing on the air stream as a result of its angular velocity. The use of Reynold's analogy between heat transfer and friction is convenient, as it replaces the heat transfer coefficient by a function of f, which already appears in Eq. (23). Using Reynold's analogy we find (see Appendix, page 66)

$$\frac{dT_o}{T_o} = \frac{2f\,dr}{D}\,\frac{(T_w - T_o)}{T_o} + \frac{1}{J c_p T_o}\,\frac{\Omega^2 r\,dr}{g} \qquad (24)$$

It is necessary to find T_o as a function of r to carry out the integration of Eq. (23). This can be obtained from Eq. (24) if T_w is known as a function of r. If T_w is assumed to be constant, Eq. (24) is linear in T_o and can be solved directly. The actual solution is long and tedious but straightforward, and the result is shown in the Appendix, page 67. To use this solution, or any other solution giving T_0 as a function of r, it is convenient to plot T_0 against r on a graph, particularly if the function is complicated.

The quantity dX in Eq. (23) represents the body force on the fluid. Neglecting the force of gravity, which is extremely small compared to centrifugal forces, we have

$$dX = r\Omega^2 dm \tag{26}$$

This may be evaluated as a function of T_0(Appendix, page 67) to give finally

$$\frac{dX}{Ap} = \frac{\Omega^2 r\, dr}{RT_0 g}\left(1 + \frac{k-1}{2}M^2\right) \tag{27}$$

Substituting the values found in Eqs. (24) and (27) in Eq. (23) and simplifying (Appendix, page 67) we get the equation which will be used in actual calculations:

$$dM^2 = 2M\,dM = F_\Theta(M)F\frac{(T_w - T_0)}{T_0}dr$$

$$+ F_f(M)2F\,dr + F_A(M)\frac{(k+1)}{(k-1)}BF\frac{r\,dr}{T_0} \tag{28}$$

All symbols in this equation which have not already been defined, are defined in the derivation in the Appendix.

The use of this equation to solve a specific problem in the flow of air through a rotating passage may be facilitated by use of the following procedure:

1.) Obtain all necessary design data and operating conditions, including rotational velocity, hot gas velocity, state at first stage rotor, and all related dimensions.

2.) Determine pressure rise in turbine rotor and corresponding temperature of the cooling air at the root entry to the cooling air passages. Eq. (22) gives pressure rise, and temperature rise may be determined using air tables or perfect gas relations.

3.) Assume a value for Mach Number at tip

4.) Calculate T_0 as a function of the radius r using Eq. (25) with above data and an assumed wall temperature.

5.) Plot T_0 vs. r.

6.) Decide on number of steps to be used in integration, thus determining dr, and evaluate the mean radius r for each step.

7.) Referring to the graph of T_0 vs. r, select a reasonable value for the average temperature, and use that value to calculate an average value of the friction coefficient, f .

8.) For each r in 6.), evaluate the coefficients of the Mach Number functions: $F\,\frac{(T_w - T_0)}{T_0}\,dr$, $2F\,dr$ and $\frac{(k+1)}{(k-1)}\,\frac{BFr\,dr}{T_0}$.

9.) Rule a sheet of paper into as many horizontal spaces as there are to be steps in the integration, and identify each with the mean radius to the corresponding dr. (see Appendix, page 70)

10.) Write the coefficients evaluated in 8.) in a vertical column in the order given, in the appropriate spaces.

11.) Referring to tables, evaluate Mach Number functions for the tip Mach Number chosen in 3.), multiplying immediately by the appropriate coefficients.

12.) Get algebraic sum "S" of the three terms in 11.).

13.) $dM = \frac{S}{2M}$; this is the change in Mach Number corresponding to dr. The calculation should be repeated now for the first step, using the first result to estimate the value of M at r for the first step.

14.) At right margin of sheet and on lines separating the spaces, note the values of M as calculated, and in the middle of the space note the estimated value used as the mean for the step, observing whether this was too high, too low, or about right. This helps in deciding the mean value to use in the next step, as it is desireable to have about the same number of "highs" as "lows" so as to minimize any cumulative error.

15.) Repeat the above for the remaining steps over the entire length of the cooling passage.

The balance of the calculations follow in the order enumerated in the paragraph following Eq. (23). Now the pressure and velocity at the root entrance to the cooling air passage are known. To obtain the pressure in the disc needed to provide this pressure and velocity, we use the relation

$$p_0 - p_1 = \frac{\rho_1 C_{r_1}^2}{2g}\left(1 + K + \frac{M_1^2}{4}\right)^* \qquad (29)$$

which assumes negligible radial velocity in the disc. Here K is an entrance loss coefficient with a maximum value of .5 for an inlet with sharp corners, and zero approach velocity. M_1 is the Mach Number in root entrance.

This gives the value of p_0 in the disc required to maintain the assumed Mach Number at the tip. Now if we use Eq. (22) we can determine the pressure available due to the compressor bleeding pressure and the pressure gain in the disc. Comparison of the two values will show whether the assumed Mach number was too high, too low, or about right.

If it was too high, the step integration should be repeated assuming a lower value. If it was too low, a higher value may be used. If the pressures were in close agreement, or if the available pressure in the disc was found to be somewhat higher than that required for the assumed Mach Number, that Mach Number may be considered to be reasonable, and the following calculations may be made on that basis.

With the flow thus determined it is possible to evaluate the heat transfer coefficient at any point, using any of the several formulas available in the literature.

* "Radiator and Cooling Notes" - Edward S. Taylor

To be consistent with the previous work it is suggested that Reynold's analogy should be used, as given in the Appendix, Eq. (23.2).

To complete the data necessary to set up a heat balance relation, it is necessary to evaluate a heat transfer coefficient for the outside surface of the blade. For this purpose data recommended for use with streamlined shapes* may be used, based on a parameter for a single cylinder, substituting an "equivalent diameter" equal to the perimeter of the section divided by π, for the diameter of the cylinder.

With the heat transfer coefficients for the inside and outside surfaces of the blades it is possible to develop a heat balance through the blade section. Accomplishing this to any degree of accuracy is a project in itself, particularly if heat flow in the third dimension is taken into account. For the purposes of this paper it will be assumed that the blade material is a perfect conductor, i.e. has an infinite thermal conductivity. Since we know the blade temperature which this assumption will lead to must be somewhere between the extremes of temperature which will actually exist in the section, the assumption may be quite a good one for getting an average temperature in the section.

The resulting calculation is extremely simple. The perimeter P_o of the blade section must be determined, as well as the total perimeter P_i of the cooling hole or holes. Then

*"Heat Transmission" - McAdams

neglecting recovery factors, the heat balance is simply

$$(T_g - T_b)h_g P_o = (T_b - T_c)h_c P_i$$

from which $$T_b = \frac{T_g h_g \frac{P_o}{P_i} + T_c h_c}{h_g \frac{P_o}{P_i} + h_c} \qquad (30)$$

The difference between this blade temperature and the stagnation temperature of the hot gas flowing past the blade is our first estimate of the effectiveness of the blade cooling system. Since the total mass rate of flow of cooling air may be calculated, as well as the mass rate of flow of hot gas, we may express the flow of cooling air in terms of percent of the total flow in the compressor. We are then able to refer to the losses accounted for in the first section, charge these against the gain made possible by the amount of effective cooling the system has accomplished, and thus arrive at the desired final result, namely, an estimate of the net gain in thermal efficiency made possible by blade cooling.

A complete set of these calculations has been carried out for an assumed turbine design. Many of the calculations are quite tedious and will not be shown in detail. The principal results will be shown below, and some of the more interesting calculations will be presented in some detail in the Appendix. The sequence of operations will follow that outlined above.

1.) First stage assumed to have proportions indicated in Fig. 7, and for calculations requiring actual dimensions, a mean diameter of 20 inches will be assumed. A complete resume of assumed data is shown on page 69 of the Appendix.

2.) Calculations based on assumed conditions give pressure at radius of root entrance to cooling passages of 184 lbs./in.^{2}, and the corresponding temperature is 192° F

3.) For the first calculation, M at the blade tip (cooling air) is assumed to be .67, which corresponds to a velocity of approximately 1000 ft./sec. at a stagnation temperature of 1000°Fabs.

4.) and 5.) Based on the above assumptions and worked out in accordance with Eq. (25) assuming T_w constant and equal to 1760°Fabs., T_o varies as shown in Fig. 10.

6.) Using a five-step integration, dr will be .045 ft., and mean radii to each of five steps are shown on the calculation sheet, page 70 of the Appendix.

7.) The value of the friction coefficient f based on a temperature of 900°Fabs. is .0052.

8.) and 9.) Values of the required coefficients are listed in the appropriate places in the calculation sheet.

10.), 11.), 12.), 13.), 14.), and 15.) The top space on the calculation sheet identifies the quantities in their respective spaces, and also indicates the successive operations performed in these steps.

The result of the integration is a value of .44 for the Mach Number at the root entrance to the cooling passage. Using this value, we now proceed to calculate the velocity and pressure of the cooling air at the root entrance, as shown in the Appendix, page 71. This indicates a pressure required to sustain the assumed flow, equal to 118 lbs./in.^{2} From the previous calculations we expect to have available a pressure of 184 lbs./in.^{2}

From this favorable result we see that we can either increase our estimate of the Mach Number at the blade tip, or else we can decide to bleed from the compressor at a lower pressure, and thus reduce the losses resulting from the bleeding. For the following calculations it will be assumed that the latter procedure is followed, and that the assumed Mach Number at the blade tip is satisfactory.

Having decided upon a mass rate of flow in the cooling passages, we may now investigate the validity of the assumption of negligible radial velocity of the cooling air in the disc cavities. Assuming 80 blades on the disc, and 3 cooling holes per blade, we have for the total mass rate of flow of cooling air

$$w_{ca} = G \times \text{Area} = 1.74 \text{ LB./SEC.}$$

Now at the holes in the shaft, (see Fig. 7) through which the cooling air enters the disc, we have the greatest restriction in area. Assuming 1 inch diameter holes, the radial velocity at that point is 112 ft/sec., as compared to a tangential velocity of 200 ft./sec. At the radius of the root entrance to the cooling passages, assuming an axial width of the cavity equal to 3/4 in., the average radial velocity of the cooling air is 6 ft./sec., compared to a tangential velocity of the disc at that radius equal to 835 ft./sec. Thus the assumption that radial velocity will be negligible appears to be reasonable.

Calculations following the procedure outlined in the paragraph following Eq. (30) are shown in the Appendix, page 72. The results indicate that the assumed increase in turbine inlet temperature from 1500°F to 1800°F will be permissible, since the calculated average blade temperature is 384°F below the stagnation temperature of the hot gas. The net gain in thermal efficiency, after accounting for thermodynamic losses as indicated in the previous section, is .021 for the simple cycle and .032 for the regenerative cycle. These figures include losses resulting from mixing cooling air with hot gas in addition to those obtained assuming that cooling air is simply bled from the compressor and "thrown away". These represent gains of approximately 8% and 9.4% respectively, based on thermal efficiencies at 1500°F.

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III

Discussion of Results and Suggestions for Further Work

As was stated in the Introduction, it is hoped that the methods and techniques used, plus the methods and techniques they may suggest, are a more important contribution than any of the actual numerical results obtained. On the other hand it is hoped that since these results have proved encouraging, interest in the idea of obtaining increased thermal efficiencies by the use of blade cooling will continue. Much work can and should be done to increase both the scope and the accuracy of the results shown here.

One phase of the problem which will bear a much closer scrutiny than was possible here, is in the evaluation of the cooling air passage wall temperature T_w as used in Eq. (24). This equation is perfectly general, and within the limitations of the error inherent in Reynold's analogy, the accuracy of the values of the cooling air temperature obtained from it will be as good as the assumed values of the wall temperature, but no better.

Here again the assumption used, namely that T_w is constant, is probably conservative. We know that the wall temperature in the root section will be substantially lower than in the blade portion, and therefore the cooling air actually will arrive at the blade portion at a lower temperature, and therefore with a better capacity for cooling, than the calculations of the previous section would indicate.

A point that has not been mentioned previously but is nevertheless quite an important one, is the reason for selecting 300°F as the desired amount of cooling to be required of the blade cooling system. The limitation lies in the fact that the second turbine stage is uncooled, and thus must operate at or below the temperature at which the first stage could operate without a cooling system. It is possible, of course, to cool the second stage also, or even all the stages. But the complication introduced by this would increase the problem much beyond the scope of this paper.

If the cooling is limited to the first stage, therefore, we may increase the cycle temperature only by an amount equal to the temperature drop obtained in the first stage. A reasonable value for this temperature drop is approximately 300°F, but this value can be increased somewhat if the performance of the cooling system seems to justify it.

Another phase of the above calculations in which there is an excellent opportunity for further work, is the heat balance in which the average blade temperature is determined. The problem of determining the actual distribution of temperature in the section is complicated by the fact that the boundary temperature is not known and will in turn be a function of the temperature distribution in the blade. This problem might well be attacked experimentally.

Still another problem, closely related to the above, is that of determining the "mean effective temperature" of the section with a given temperature distribution. This would be the temperature of an uncooled section with equivalent creep strength. This problem is complicated by third dimensional considerations, thermal stress distribution in a multiply connected section, and by the fact that both the coefficient of thermal expansion and the modulus of elasticity are functions of temperature, varying quite markedly in the higher temperature ranges.

No reference has been made in the Bibliography at the end of the previous section, to the tables of functions used in the calculations. These are unpublished, mimeographed tables, and may be available from any organization doing work in compressible flow.

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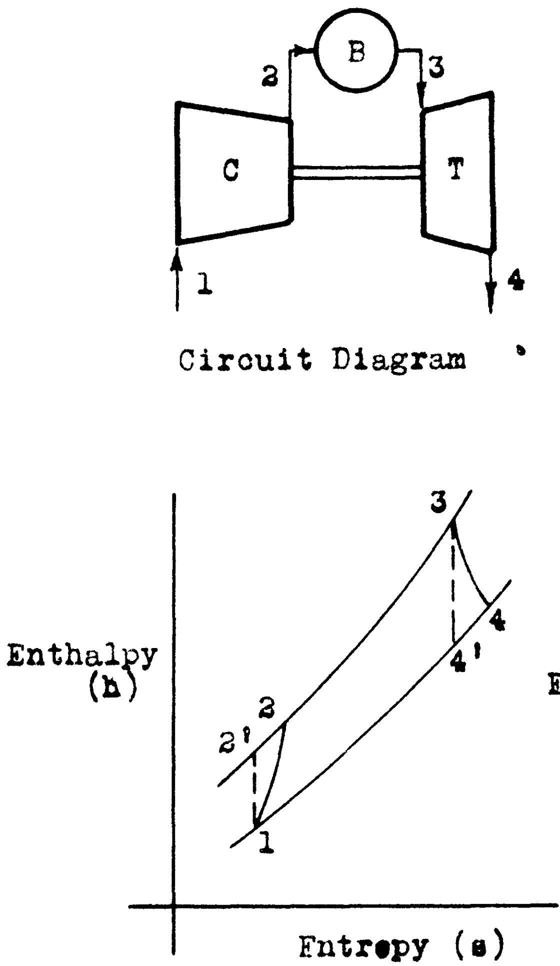


Fig. 1a

Simple Cycle

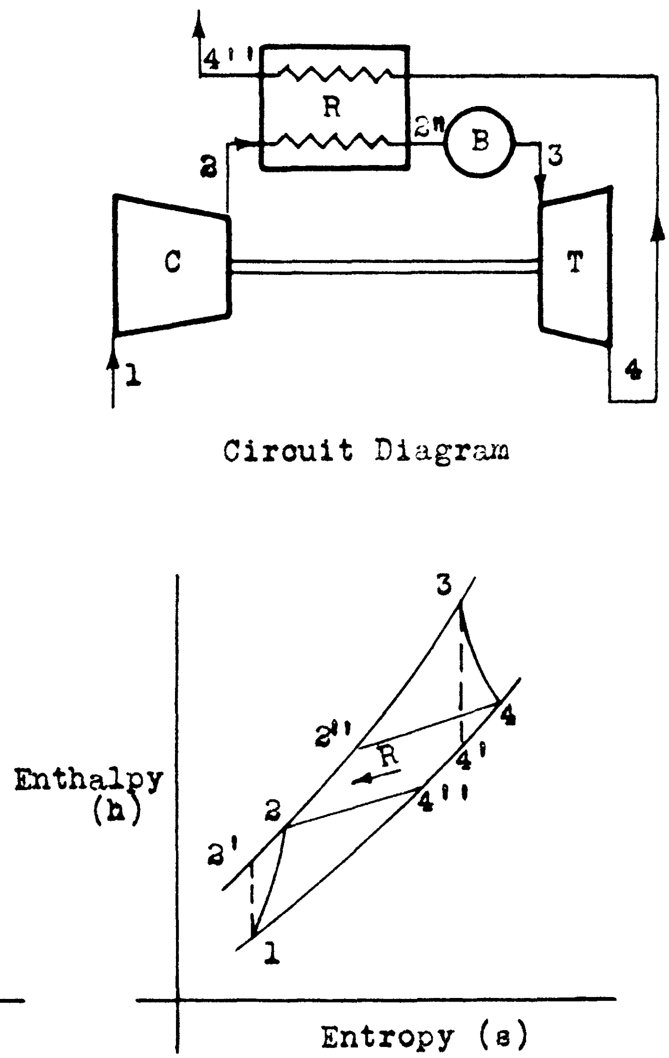


Fig. 1b

Regenerative Cycle

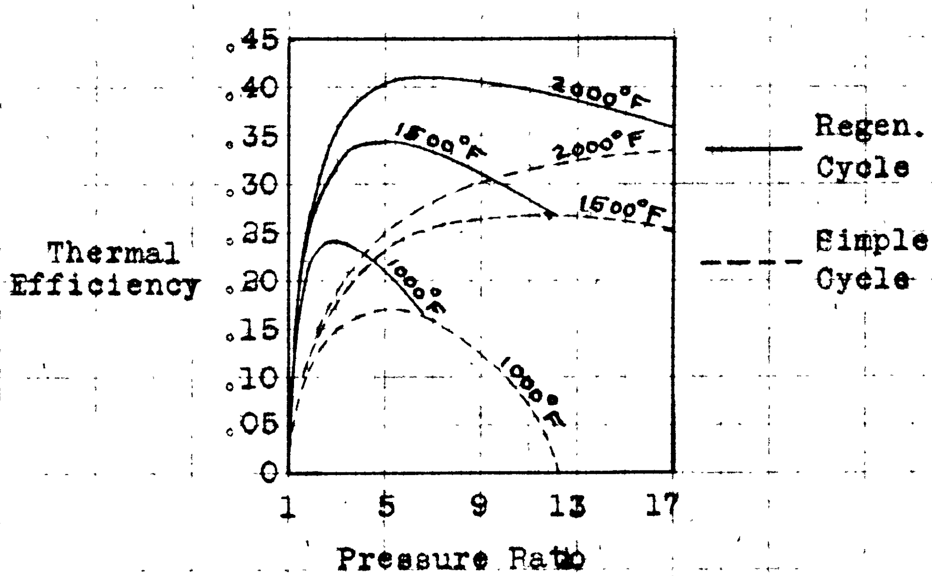


Fig. 2

Gas Turbine Thermal Efficiency vs.

Pressure Ratio

at Various Turbine Inlet Temperatures

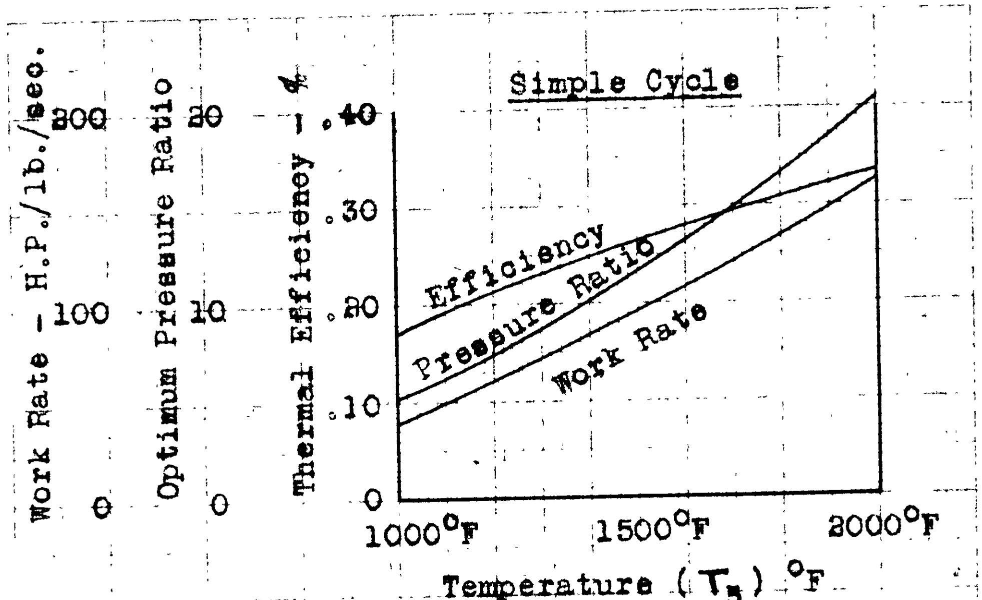

Simple Cycle
Work Rate - H.P./lb./sec.
Optimum Pressure Ratio
Thermal Efficiency - %
200
100
0
20
10
0
.40
.30
.20
.10
0
Efficiency
Pressure Ratio
Work Rate
1000°F
1500°F
2000°F
Temperature (T_3) °F

Fig. 3a

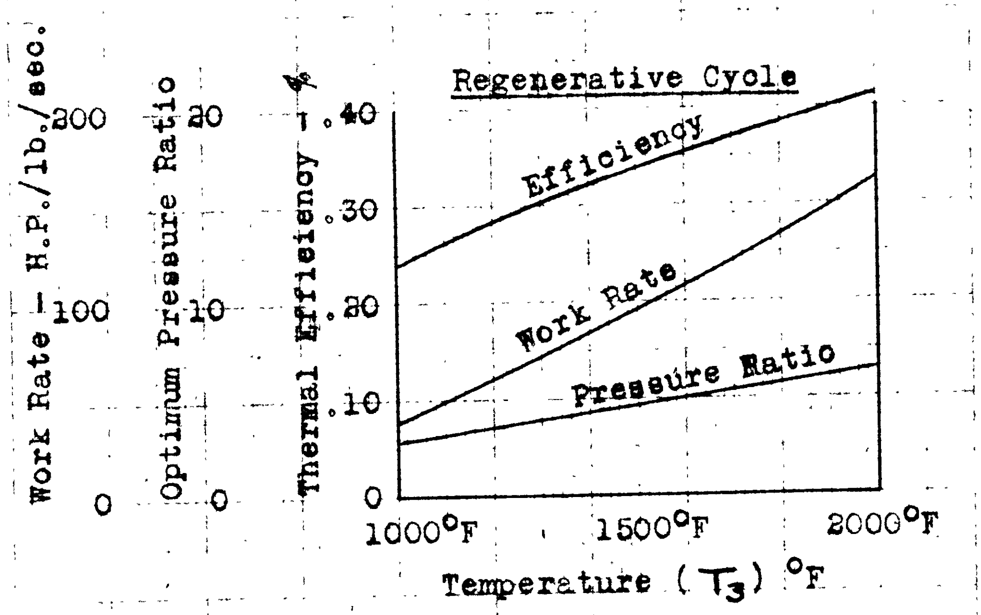

Regenerative Cycle
Work Rate - H.P./lb./sec.
Optimum Pressure Ratio
Thermal Efficiency - %
200
100
0
20
10
0
.40
.30
.20
.10
0
Efficiency
Work Rate
Pressure Ratio
1000°F
1500°F
2000°F
Temperature (T_3) °F

Fig. 3b

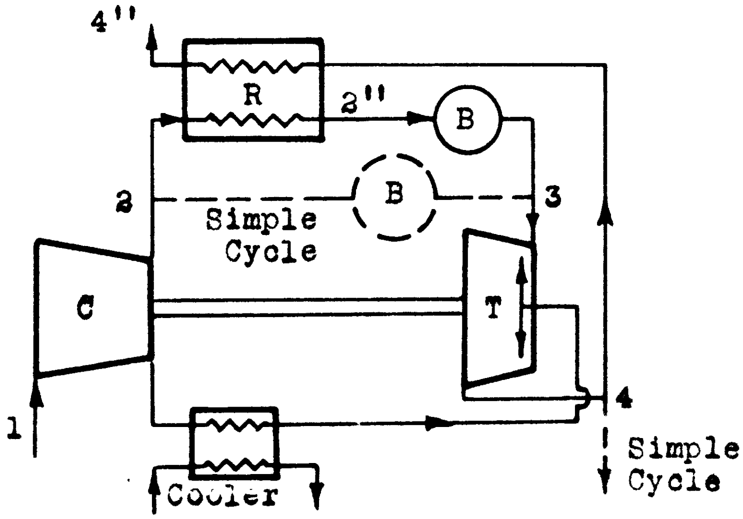


Fig. 4a

Blade Cooling System
for
Simple or Regenerative Cycle

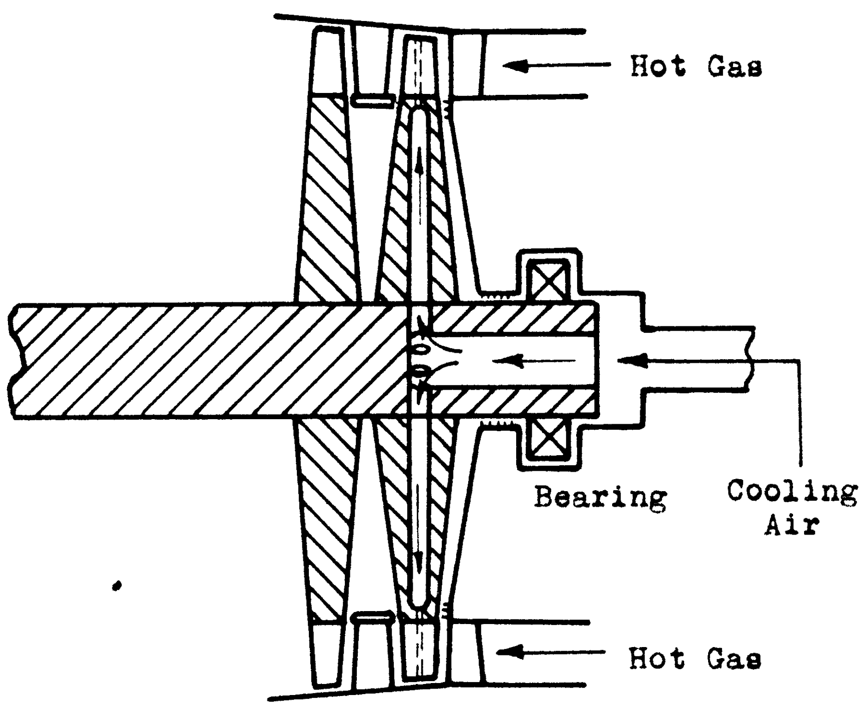


Fig. 4b

Flow of Cooling Air
Through First Stage Rotor

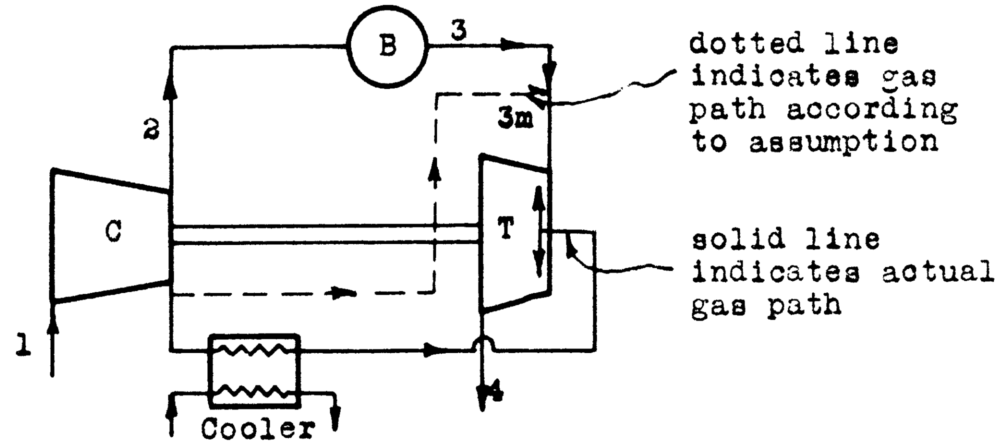


Fig. 5a

Circuit Diagram, Simple Cycle with Air Cooling

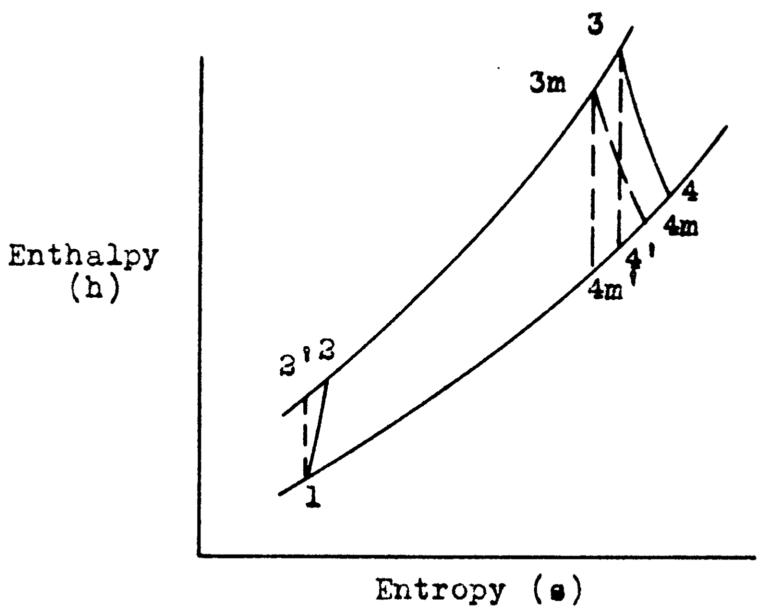


Fig. 5b

Simple Cycle with Air Cooling

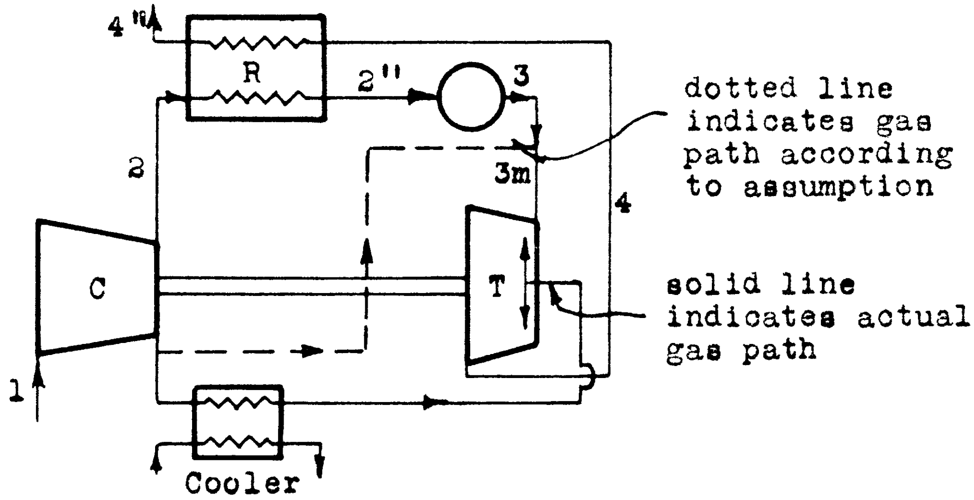


Fig. 6a

Circuit Diagram, Regenerative Cycle with Air Cooling

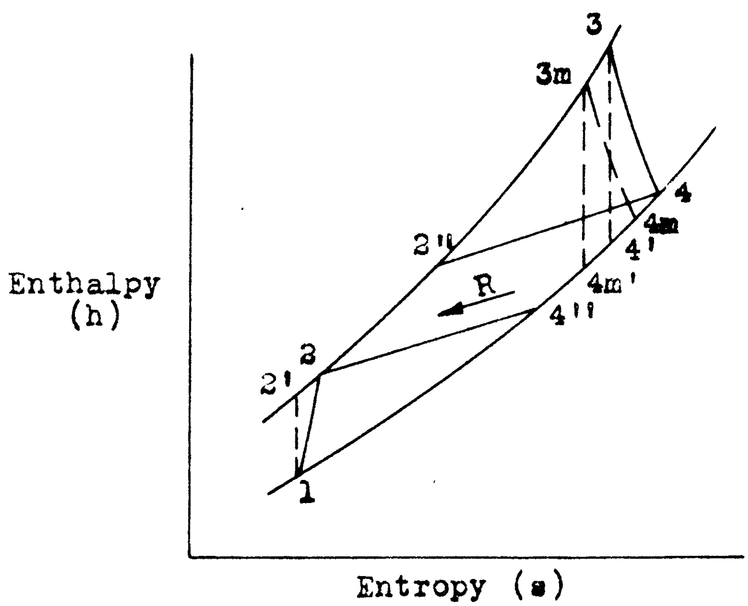


Fig. 6b

Regenerative Cycle with Air Cooling

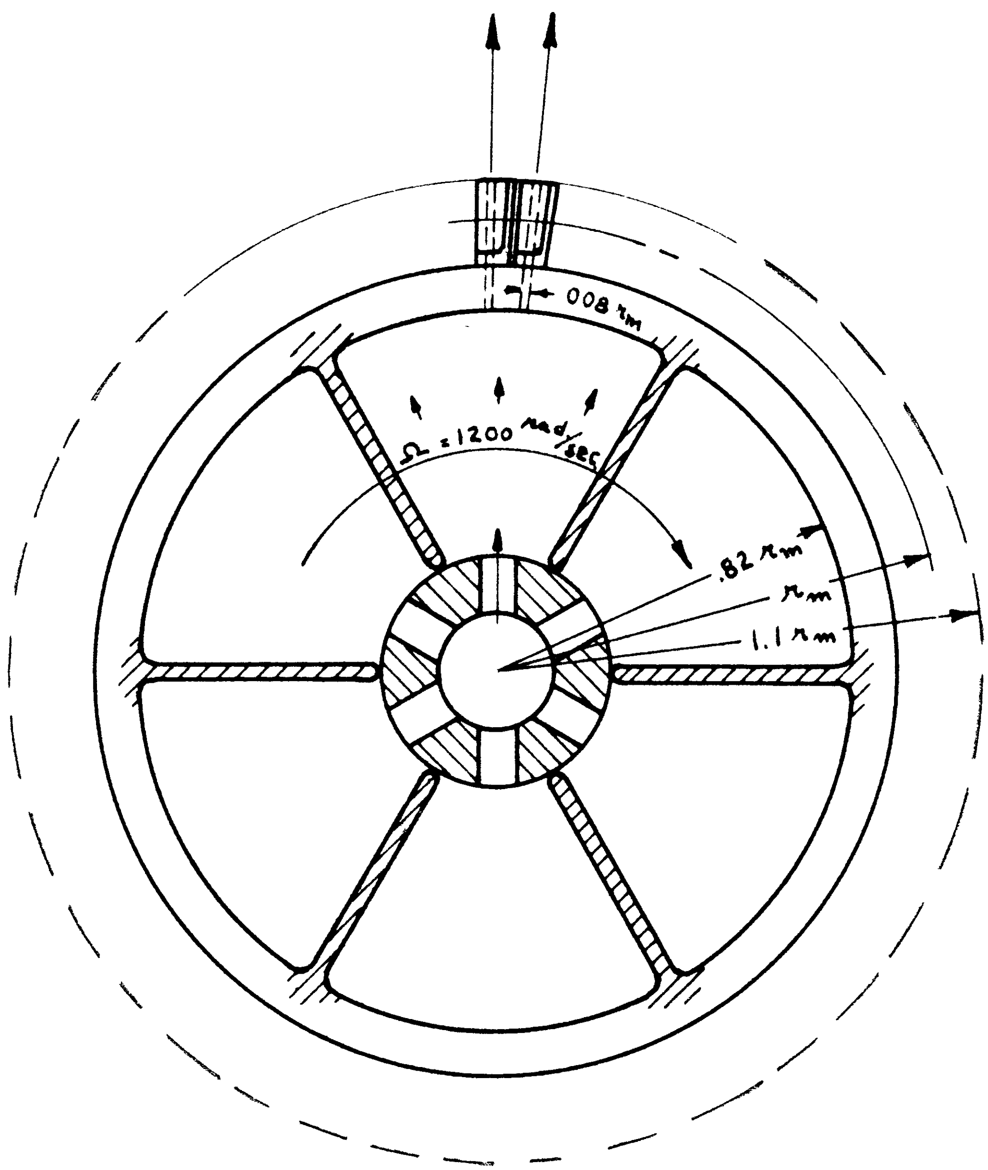


Fig. 7

Turbine Rotor - Showing Cooling Air Passages

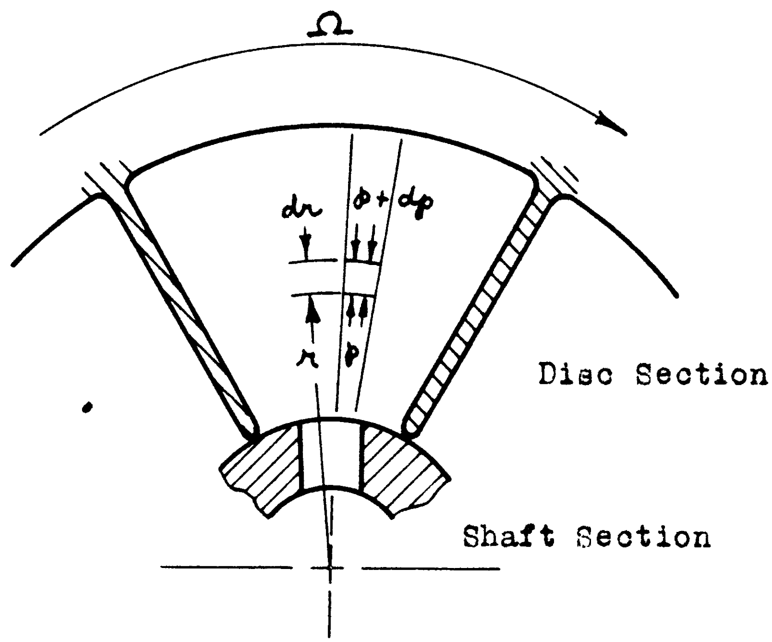


Fig. 8

Identification Sketch
for
Derivation of Equation (22)

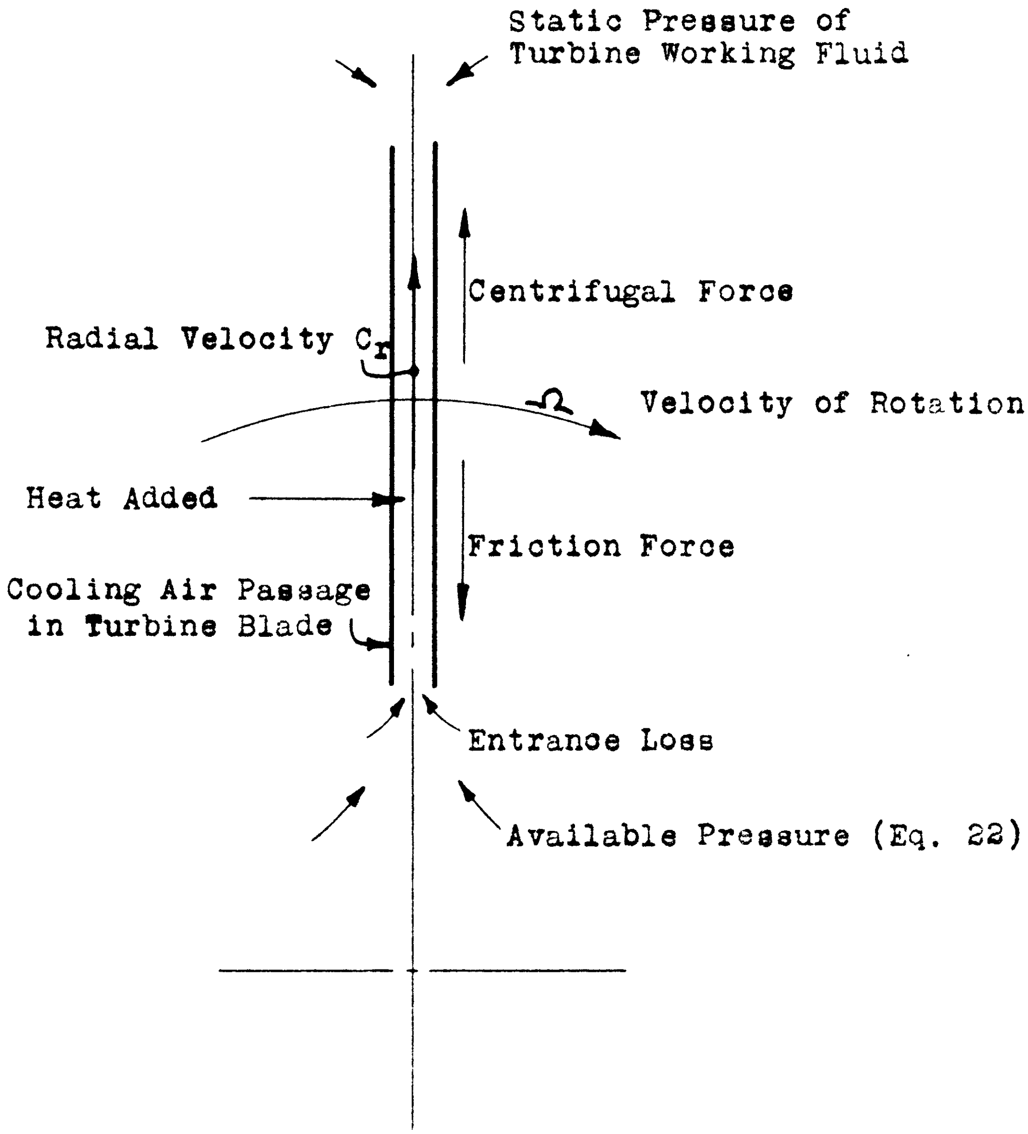
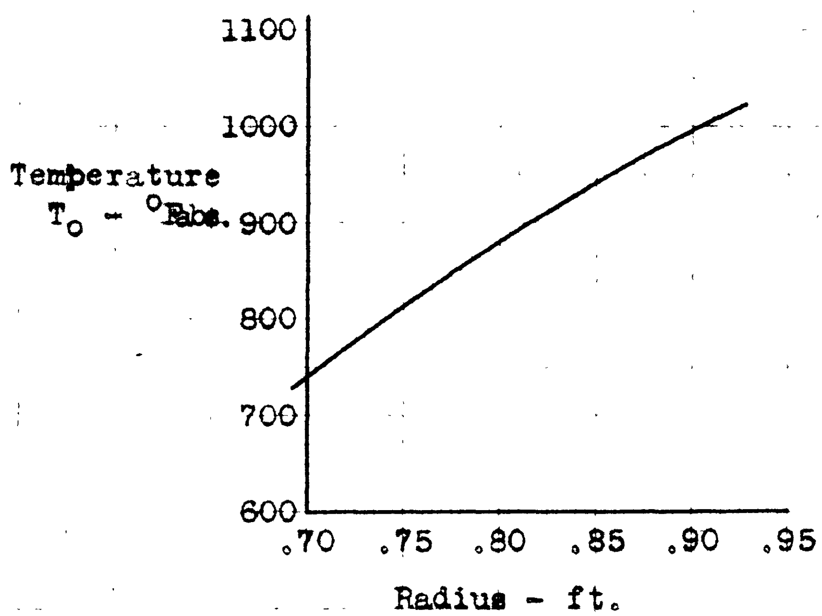


Fig. 9

Factors Influencing Flow of Cooling Air Through

Passage in Turbine Blade



Stagnation Temperature of Cooling Air
vs.
Radial Distance from Center of Disc

Fig. 10

- NOMENCLATURE -
Chapter I

Symbols in the first group below are defined by equations in the Appendix as indicated.

A - Equation (4.7)

B - " (3.2)

B_b - " (16.1)

B_{br} - " (18.1)

B_m - " (8.8)

B_{mr} - " (10.3)

B_r - " (5.1)

C - " (3.1)

C_b - " (16.2)

C_{br} - " (18.2)

C_m - " (8.4)

C_{mr} - " (10.2)

C_r - " (5.2)

D - " (4.8)

E - " (4.9)

c_p = specific heat at constant pressure

c_v = specific heat at constant volume

h = enthalpy, as given in "Thermodynamic Properties of Air" - Keenan and Kaye
Subscripts and primes indicate point in cycle diagram, thus:
h'_2 = enthalpy at point $2'$

k = gas constant $= \frac{c_p}{c_v}$

n = equivalent gas constant used in polytropic change of state.
For expansion,

$$\frac{n-1}{n} = \eta_t \frac{k-1}{k}$$

p = stagnation pressure - numerical subscript indicates point in cycle diagram

pr = cycle pressure ratio - $pr > 1$

pr_1 = relative pressure index as used in "Thermodynamic Properties of Air"

$pr_2 = pr_1 \times pr$

T = stagnation temperature - subscripts and primes indicate points in cycle diagrams, thus: T_2'' = temperature at 2"

δ = cooling air ratio, see Equation (7.9)

Δh = enthalpy rise based on isentropic compression

Δi = actual enthalpy rise in compressor (polytropic compression)

η = thermal efficiency $= \dfrac{\text{net work output}}{\text{heat input}}$

η_c = compressor efficiency $= \dfrac{\text{isentropic enthalpy rise}}{\text{actual enthalpy rise in compressor}}$

η_r = regenerator efficiency, defined in Equation (4.2)

η_t = turbine efficiency $= \dfrac{\text{actual enthalpy drop in turbine}}{\text{isentropic enthalpy drop}}$

APPENDIX - Chapter I

Derivation of Temperature-Efficiency Relations for Simple Gas Turbine Cycle (Fig. 1a)

$$\eta = \frac{(T_3 - T_4) - (T_2 - T_1)}{(T_3 - T_2)} \qquad (1)$$

$$\eta = \frac{\frac{(T_3 - T_4)}{(T_2 - T_1)} - \frac{(T_2 - T_1)}{(T_2 - T_1)}}{\frac{(T_3 - T_2)}{(T_2 - T_1)}} = \frac{\frac{(T_3 - T_4)}{(T_2 - T_1)} - 1}{\frac{(T_3 - T_2)}{(T_2 - T_1)}} \qquad (1.1)$$

by definition:

$$\eta_t = \frac{(T_3 - T_4)}{(T_3 - T_4')} \quad ; \quad (T_3 - T_4) = \eta_t (T_3 - T_4') \qquad (1.2)$$

$$\eta_c = \frac{(T_2' - T_1)}{(T_2 - T_1)} \quad ; \quad (T_2 - T_1) = \frac{T_2' - T_1}{\eta_c} \qquad (1.3)$$

$$\frac{(T_3 - T_4)}{(T_2 - T_1)} = \frac{\eta_t (T_3 - T_4')}{\frac{(T_2' - T_1)}{\eta_c}} = \eta_t \eta_c \frac{(T_3 - T_4')}{(T_2' - T_1)} \qquad (1.4)$$

assuming relations for perfect gas:

$$\frac{T_2'}{T_1} = \frac{T_3}{T_4'} \quad \text{OR} \quad \frac{T_3}{T_2'} = \frac{T_4'}{T_1}$$

$$\therefore \quad \frac{T_3 - T_4'}{T_2' - T_1} = \frac{T_3}{T_2'} \tag{1.5}$$

substituting this in (1.4),

$$\frac{(T_3 - T_4)}{(T_2 - T_1)} = \eta_t \eta_c \left(\frac{T_3}{T_2'}\right) \tag{1.6}$$

from (1.3),

$$T_2' = T_1 + \eta_c (T_2 - T_1) \tag{1.7}$$

$$\therefore \quad \frac{(T_3 - T_4)}{(T_2 - T_1)} = \frac{\eta_t \eta_c T_3}{T_1 + \eta_c (T_2 - T_1)} = \frac{\eta_t \eta_c \left(\frac{T_3}{T_1}\right)}{1 + \eta_c \frac{(T_2 - T_1)}{T_1}} \tag{1.8}$$

applying this to equation (1.1), we have finally:

$$\eta = \left[\frac{\eta_t \eta_c \frac{T_3}{T_1}}{1 + \eta_c \frac{(T_2 - T_1)}{T_1}} - 1\right]\left[\frac{(T_2 - T_1)}{(T_3 - T_2)}\right] \tag{2}$$

now considering all terms as constant except T_3, let

$$A = \frac{\eta_t \eta_c}{\left[1 + \eta_c \frac{T_2 - T_1}{T_1}\right]} \tag{2.1}$$

then,

$$\eta = \left(A \frac{T_3}{T_1} - 1\right) \frac{(T_2 - T_1)}{(T_3 - T_2)} \tag{2.2}$$

add and subtract $A\frac{T_2}{T_1}$ in first factor of (2.2):

$$\eta = \left(A\frac{T_3}{T_1} - A\frac{T_2}{T_1} + A\frac{T_2}{T_1} - 1\right)\frac{(T_2 - T_1)}{(T_3 - T_2)} \quad (2.3)$$

$$= \frac{\frac{A}{T_1}\cancel{(T_3 - T_2)}(T_2 - T_1)}{\cancel{(T_3 - T_2)}} + \frac{\left(A\frac{T_2}{T_1} - 1\right)(T_2 - T_1)}{(T_3 - T_2)}$$

$$\eta = A\frac{(T_2 - T_1)}{T_1} + \frac{\left(A\frac{T_2}{T_1} - 1\right)(T_2 - T_1)}{T_3 - T_2} \quad (2.4)$$

all terms being assumed constant except T_3, we may write this as:

$$\eta = C - \frac{B}{T_3 - T_2} \quad (3)$$

where $$C = A\frac{T_2 - T_1}{T_1} \quad (3.1)$$

and $$-B = \left(A\frac{T_2}{T_1} - 1\right)(T_2 - T_1) \quad (3.2)$$

here, B will be positive, since

$$(T_2 - T_1) > 0 \quad (3.3)$$

$$A\frac{T_2}{T_1} = \frac{\eta_t \eta_c T_2}{T_1 + \eta_c (T_2 - T_1)} = \frac{(\eta_c T_2)\eta_t}{(\eta_c T_2) + T_1(1 - \eta_c)} < 1 \quad (3.4)$$

$$\therefore \left(A\frac{T_2}{T_1} - 1\right) < 0 \quad (3.5)$$

Derivation of Temperature-Efficiency Relations for Regenerative

Gas Turbine Cycle (Fig. 1b)

$$\eta = \frac{(T_3 - T_4) - (T_2 - T_1)}{(T_3 - T_2'')} \tag{4}$$

by manipulation identical to that used for simple cycle, it may be shown that:

$$\eta = \left[\frac{\eta_t \eta_c \frac{T_3}{T_1}}{1 + \eta_c \frac{T_2 - T_1}{T_1}} - 1 \right] \left[\frac{(T_2 - T_1)}{(T_3 - T_2'')} \right] \tag{4.1}$$

in this case, T_2'' is a function of T_3.

by definition, regenerator efficiency

$$\eta_r = \frac{(T_2'' - T_2)}{(T_4 - T_2)} \tag{4.2}$$

$$T_2'' - T_2 = \eta_r (T_4 - T_2)$$

$$T_2'' = T_2 + \eta_r (T_4 - T_2) \tag{4.3}$$

for polytropic expansion,

$$T_4 = T_3 \left(\frac{p_4}{p_3}\right)^{\frac{n-1}{n}} = T_3 \left(\frac{1}{p_r}\right)^{\eta_t \frac{k-1}{k}} \tag{4.4}$$

substituting this in (4.3):

$$T_2'' = T_2 + \eta_r T_3 \left(\frac{1}{p_r}\right)^{\eta_t \frac{k-1}{k}} - \eta_r T_2 \tag{4.5}$$

$$T_3 - T_2'' = T_3 \left[1 - \eta_r \left(\frac{1}{p_r}\right)^{\eta_t \frac{k-1}{k}} \right] - T_2 \left[1 - \eta_r \right] \tag{4.6}$$

in order to avoid an unduly complicated expression,

let
$$A = \frac{\eta_t \eta_c}{\left[1 + \eta_c \frac{T_2 - T_1}{T_1}\right]} \tag{4.7}$$

$$D = \left[1 - \eta_r \left(\frac{1}{p_r}\right)^{\eta_t \frac{k-1}{k}}\right] \tag{4.8}$$

$$E = 1 - \eta_r \tag{4.9}$$

then,
$$\eta = \left(A \frac{T_3}{T_1} - 1\right) \frac{(T_2 - T_1)}{(T_3 D - T_2 E)}$$

$$= \left(\frac{T_3}{T_1} - \frac{1}{A}\right) \frac{\frac{A}{D}(T_2 - T_1)}{\left(T_3 - T_2 \frac{E}{D}\right)} \tag{4.91}$$

adding and subtracting $\frac{T_2}{T_1}\frac{E}{D}$ in first factor,

$$\eta = \left(\frac{T_3}{T_1} - \frac{T_2}{T_1}\frac{E}{D} + \frac{T_2}{T_1}\frac{E}{D} - \frac{1}{A}\right) \frac{\frac{A}{D}(T_2 - T_1)}{\left(T_3 - T_2 \frac{E}{D}\right)}$$

$$\eta = \frac{\cancel{\left(T_3 - T_2 \frac{E}{D}\right)} \frac{A}{D T_1} (T_2 - T_1)}{\cancel{\left(T_3 - T_2 \frac{E}{D}\right)}} + \frac{\left(\frac{T_2}{T_1}\frac{E}{D} - \frac{1}{A}\right) \frac{A}{D}(T_2 - T_1)}{\left(T_3 - T_2 \frac{E}{D}\right)}$$

$$\eta = \frac{A}{D}\frac{(T_2 - T_1)}{T_1} + \frac{\left(\frac{T_2}{T_1}\frac{E}{D} - \frac{1}{A}\right) \frac{A}{D}(T_2 - T_1)}{\left(T_3 - T_2 \frac{E}{D}\right)} \tag{4.92}$$

again all terms are considered constant except T_3, so we may write:

$$\eta = C_r - \frac{B_r}{T_3 - T_2 \frac{E}{D}} \tag{5}$$

where
$$-B_r = \left(\frac{T_2}{T_1} \frac{E}{D} - \frac{1}{A} \right) \frac{A}{D} \left(T_2 - T_1 \right) \tag{5.1}$$

$$C_r = \frac{A}{D} \frac{\left(T_2 - T_1 \right)}{T_1} \quad . \tag{5.2}$$

as in preceding derivations, this gives $B_r > 0$

<u>Sample Calculations - Change in Thermal Efficiency Resulting from Change in Turbine Inlet Temperature.</u>

Simple Cycle:

$$\eta_t = \eta_c = .85$$

$$T_1 = 60^\circ F = 520^\circ F \text{ abs.}$$

$$h_1 = 28.77 \text{ B.T.U./LB.}^{*}$$

$$p_{r_1} = 2.504$$

$$p_r = 6.5$$

$$p_{r_2} = 2.504 \times 6.5 = 16.3$$

$$h_2' = 116.93 \text{ B.T.U./LB.}$$

$$\Delta h = 116.93 - 28.77 = 88.16 \text{ B.T.U./LB.}$$

* Numerical data in this and following calculations taken from "Thermodynamic Properties of Air" - Keenan and Kaye

$$\Delta i = \frac{88.16}{.85} = 104 \text{ B.T.U./LB.}$$

$$h_2 = 104 + 28.77 = 133 \text{ B.T.U./LB.}$$

$$T_2 = 950^\circ F \text{ abs.}$$

$$T_2 - T_1 = 950^\circ - 520^\circ = 430^\circ F$$

using Equation (3),

$$\eta = C - \frac{B}{T_3 - T_2}$$

$$C = A \frac{T_2 - T_1}{T_1}$$

$$A = \frac{\eta_t \, \eta_c}{\left[1 + \eta_c \frac{T_2 - T_1}{T_1}\right]}$$

$$= \frac{.85 \times .85}{\left[1 + .85 \frac{430}{520}\right]} = .424$$

$$C = .424 \times \frac{430}{520} = .351$$

$$B = -\left(A \frac{T_2}{T_1} - 1\right)(T_2 - T_1)$$

$$= -\left(.424 \times \frac{950}{520} - 1\right)(430) = 96.8$$

$$\eta = .351 - \frac{96.8}{T_3 - 950} \qquad (6)$$

$T_3 = 1610 \qquad \eta = .351 - .147 = .204$

$T_3 = 1710 \qquad \eta = .351 - .127 = .224$

Sample Calculation - Change in Thermal Efficiency Resulting from Change in Turbine Inlet Temperature.

Regenerative Cycle:

$\eta_t = \eta_c = .85$

$T_1 = 60^\circ F = 520^\circ F$ abs.; $h_1 = 28.77$ B.T.U./LB.

$pr_1 = 2.504$

$pr = 3.3$

$pr_2 = 3.3 \times 2.504 = 8.27$

$h_2' = 79.2$ B.T.U./LB.

$\Delta h = 79.2 - 28.77 = 50.4$ B.T.U/LB.

$\Delta i = \dfrac{50.4}{.85} = 59.3$ B.T.U./LB.

$h_2 = 59.3 + 28.77 = 88.1$ B.T.U./LB.

$T_2 = 766^\circ F$ abs.

$T_2 - T_1 = 766^\circ - 520^\circ = 246^\circ F$

$$\frac{T_2 - T_1}{T_1} = \frac{246}{520} = .478$$

Equation (5),

$$\eta = C_r - \frac{B_r}{\left(T_3 - T_2 \frac{E}{D}\right)}$$

$$C_r = \frac{A}{D} \frac{(T_2 - T_1)}{T_1}$$

$$A = \frac{\eta_t \eta_c}{1 + \eta_c \frac{T_2 - T_1}{T_1}} = \frac{.85 \times .85}{1 + .85 \times .478} = .515$$

$$D = \left[1 - \eta_r \left(\frac{1}{p_r}\right)^{\eta_t \frac{k-1}{k}}\right] = \left[1 - .7\left(\frac{1}{3.3}\right)^{.85 \frac{.4}{1.4}}\right] = .476$$

$$C_r = \frac{A}{D} \frac{T_2 - T_1}{T_1} = \frac{.515}{.476} \times .478 = .516$$

$$B_r = -\left(\frac{T_2}{T_1} \frac{E}{D} - \frac{1}{A}\right) \frac{A}{D} (T_2 - T_1)$$

$$E = 1 - \eta_r = 1 - .7 = .3$$

$$\frac{E}{D} = \frac{.3}{.476} = .63$$

$$\frac{T_2}{T_1} \frac{E}{D} = \frac{766}{520} \times .63 = .928$$

$$\frac{1}{A} = \frac{1}{.515} = 1.94$$

$$\frac{A}{D} = \frac{.515}{.476} = 1.08$$

$$B_r = -(.928 - 1.99) \times 1.08 \times (246)$$
$$= 269$$

$$T_2 \frac{E}{D} = 766 \times .63 = 483$$

$$\therefore \eta = .516 - \frac{269}{T_3 - 483} \qquad (7)$$

$$T_3 = 1610^\circ \text{ F abs}; \quad \eta = .516 - .239 = .277$$

$$T_3 = 1710^\circ \text{ F abs}; \quad \eta = .516 - .219 = .297$$

<u>Thermodynamic Loss Due to Irreversible Mixing of Hot Gas and Cooling Air</u>.

Simple Cycle: (Figs. 5a and 5b)

$$\delta = \frac{\text{weight of cooling air}}{\text{weight of cooling air plus hot gas}} \qquad (7.9)$$

$$\eta = \frac{(T_{3m} - T_{4m}) - (T_2 - T_1)}{(1-\delta)(T_3 - T_2)} \qquad (8)$$

assuming irreversible mixing,

$$T_{3m} = (1-\delta)T_3 + \delta T_2 \qquad (8.1)$$

assuming behavior of perfect gas,

$$\frac{T_{4m}}{T_{3m}} = \left(\frac{1}{p_r}\right)^{\eta_t \frac{k-1}{k}} \qquad (8.2)$$

since $\quad (T_{3m} - T_{4m}) = T_{3m}\left(1 - \frac{T_{4m}}{T_{3m}}\right)$

we may rewrite (8) using (8.1) and (8.2),

$$\eta = \frac{\left[(1-\delta)T_3 + \delta T_2\right]\left[1-\left(\frac{1}{p_r}\right)^{\eta_t \frac{k-1}{k}}\right] - (T_2 - T_1)}{(1-\delta)(T_3-T_2)} \qquad (8.3)$$

now let

$$\left[1-\left(\frac{1}{p_r}\right)^{\eta_t \frac{k-1}{k}}\right] = C_m \qquad (8.4)$$

then

$$\eta = \frac{\left[(1-\delta)T_3 + \delta T_2\right]C_m - (T_2-T_1)}{(1-\delta)(T_3-T_2)}$$

or

$$\eta = \frac{\delta(T_2-T_3) + T_3 - \frac{(T_2-T_1)}{C_m}}{(1-\delta)\frac{(T_3-T_2)}{C_m}} \qquad (8.5)$$

add and subtract $(T_3 - T_2)$ in the numerator:

$$\eta = \frac{(1-\delta)(T_3-T_2) - (T_3-T_2) + T_3 - \frac{(T_2-T_1)}{C_m}}{(1-\delta)\frac{(T_3-T_2)}{C_m}} \qquad (8.6)$$

$$= \frac{\cancel{(1-\delta)(T_3-T_2)}}{\cancel{(1-\delta)}\frac{\cancel{(T_3-T_2)}}{C_m}} + \frac{-\cancel{T_3} + T_2 + \cancel{T_3} - \frac{(T_2-T_1)}{C_m}}{(1-\delta)\frac{(T_3-T_2)}{C_m}}$$

$$\eta = C_m + \frac{T_2 C_m - (T_2-T_1)}{(1-\delta)(T_3-T_2)} \qquad (8.7)$$

we are interested in the variations in thermal efficiency corresponding to changes in δ , so can consider

$$-B_m = \frac{T_2 C_m - (T_2 - T_1)}{(T_3 - T_2)} \tag{8.8}$$

as a constant, so that (8.7) becomes

$$\eta = C_m - \frac{B_m}{(1-\delta)} \tag{9}$$

here, B_m is always positive.

Thermodynamic Loss Due to Irreversible Mixing of Hot Gas and Cooling Air:

Regenerative Cycle: (Figs. 6a and 6b)

$$\eta = \frac{(T_{3m} - T_{4m}) - (T_2 - T_1)}{(1-\delta)(T_3 - T_2'')} \tag{10}$$

using steps identical with the sequence from (8) to (8.6), we will get:

$$\eta = \frac{(1-\delta)(T_3 - T_2)}{(1-\delta)\dfrac{(T_3 - T_2'')}{C_m}} + \frac{T_2 C_m - (T_2 - T_1)}{(1-\delta)(T_3 - T_2'')} \tag{10.1}$$

here let

$$C_{mr} = C_m \frac{(T_3 - T_2)}{(T_3 - T_2'')} = C_m \frac{(T_3 - T_2)}{(T_3 D - T_2 E)} \tag{10.2}$$

where $D = \left[1 - \eta_r \left(\frac{1}{p_r}\right)^{\eta_t \frac{k-1}{k}}\right]$ (See Equation (4.8))

and $E = (1 - \eta_r)$ (See Equation (4.9))

and $$B_{mr} = \frac{T_2 C_m - (T_2 - T_1)}{(T_3 D - T_2 E)} \tag{10.3}$$

then finally,

$$\eta = C_{mr} - \frac{B_{mr}}{(1-\delta)} \qquad (11)$$

<u>Sample Calculations: Loss of Thermal Efficiency Resulting from Irreversible Mixing of Cooling Air and Hot Gas.</u>

Simple Cycle:

$$\eta_c = .85$$

$$\eta_t = .85$$

$$T_1 = 60^\circ F = 520^\circ F \text{ abs} \;;\; h_1 = 28.77 \text{ B.T.U./LB.}$$

$$pr_1 = 2.504$$

$$pr = 13$$

$$pr_2 = 2.504 \times 13 = 32.6 \;;\; h_2' = 163.34 \text{ B.T.U./LB.}$$

$$\Delta h = 163.34 - 28.77 = 134.57 \text{ B.T.U./LB.}$$

$$\Delta i = \frac{134.57}{.85} = 158.3 \text{ B.T.U./LB.}$$

$$h_2 = 28.77 + 158.3 = 187.1 \text{ B.T.U./LB.}$$

$$T_2 = 1165^\circ F \text{ abs.}$$

$$\eta = C_m - \frac{B_m}{(1-\delta)} \qquad (9)$$

$$C_m = 1 - \left(\frac{1}{pr}\right)^{\eta_t \frac{k-1}{k}} = 1 - \left(\frac{1}{13}\right)^{.85 \frac{.345}{1.345}} \qquad (8.4)$$

(value of k for 1600° F abs.)

$$C_m = 1 - (.077)^{.218} = 1 - .572 = .428$$

$$B_m = \frac{(T_2 - T_1) - T_2 C_m}{(T_3 - T_2)} \qquad (8.8)$$

$$(T_2 - T_1) = 1165° - 520° = 645°$$

$$T_2 C_m = .428 \times 1165° = 499°$$

$$(T_3 - T_2) = 2060° - 1165° = 895°$$

$$B_m = \frac{645° - 499°}{895°} = \frac{146°}{895°} = .163$$

$$\eta = .428 - \frac{.163}{(1-\delta)} \qquad (13)$$

$$\delta = 0 \ ; \ \eta = .428 - .163 = .265$$

$$\delta = .01 \ ; \ \eta = .428 - .1647 = .2633 \ ; \ \Delta\eta = .0017$$

$$\delta = .10 \ ; \ \eta = .428 - .181 = .247 \ ; \ \Delta\eta = .018$$

<u>Sample Calculations: Loss of Thermal Efficiency Resulting from Irreversible Mixing of Cooling Air and Hot Gas.</u>

Regenerative Cycle:

$$\eta_c = .85$$

$$\eta_t = .85$$

$$T_1 = 520° \text{ F abs.} \ ; \qquad h_1 = 28.77 \text{ B.T.U./LB.}$$

$$pr_1 = 2.504$$

$$pr = 5$$

$$pr_2 = 12.52 \ ; \qquad h_2' = 101.55 \text{ B.T.U./LB.}$$

$\Delta h = 101.55 - 28.77 = 72.78$ B.T.U./LB.

$\Delta i = \dfrac{72.78}{.85} = 85.6$ B.T.U./LB.

$h_2 = 85.6 + 28.77 = 114.4$ B.T.U./LB.

$T_2 = 874°$ F abs.

$$\eta = C_{mr} - \frac{B_{mr}}{(1-\delta)} \qquad (11)$$

$$C_{mr} = C_m \frac{(T_3 - T_2)}{(T_3 D - T_2 E)}$$

$$D = 1 - .7\left(\frac{1}{5}\right)^{.85 \frac{.345}{1.345}} = 1 - .485 = .515$$

$$E = 1 - .7 = .3$$

$$C_m = 1 - \left(\frac{1}{5}\right)^{.85 \frac{.345}{1.345}} = 1 - .694 = .306$$

$$C_{mr} = .306 \frac{2060 - 874}{2060 \times .515 - 874 \times .3} = .306 \frac{1186}{798} = .455$$

$$B_{mr} = \frac{.306 \times 874 - (874 - 520)}{798} = \frac{86}{798} = .108$$

$$\eta = .455 - \frac{.108}{(1-\delta)} \qquad (14)$$

$\delta = 0$; $\eta = .455 - .108 = .347$

$\delta = .01$; $\eta = .455 - .109 = .346$; $\Delta\eta = .001$

$\delta = .10$; $\eta = .455 - .120 = .335$; $\Delta\eta = .012$

Loss Due to Bleeding Cooling Air from Compressor

Simple Cycle:

$$\eta = \frac{(1-\delta)(T_3 - T_4) - (T_2 - T_1)}{(1-\delta)(T_3 - T_2)} \tag{15}$$

$$= \frac{(T_3 - T_4) - \frac{1}{(1-\delta)}(T_2 - T_1)}{(T_3 - T_2)}$$

$$= \frac{\frac{(T_3 - T_4)}{(T_2 - T_1)} - \frac{1}{(1-\delta)}\frac{(T_2 - T_1)}{(T_2 - T_1)}}{\frac{(T_3 - T_2)}{(T_2 - T_1)}}$$

$$\eta = \frac{\frac{(T_3 - T_4)}{(T_2 - T_1)} - \frac{1}{(1-\delta)}}{\frac{(T_3 - T_2)}{(T_2 - T_1)}} \tag{15.1}$$

From derivations given earlier,

$$\frac{(T_3 - T_4)}{(T_2 - T_1)} = \frac{\eta_t \eta_c \frac{T_3}{T_1}}{\left[1 + \eta_c \frac{(T_2 - T_1)}{T_1}\right]} \tag{1.8}$$

and $$A = \frac{\eta_t \eta_c}{\left[1 + \eta_c \frac{(T_2 - T_1)}{T_1}\right]} \tag{2.1}$$

so that $$\frac{(T_3 - T_4)}{(T_2 - T_1)} = A\frac{T_3}{T_1} \tag{15.2}$$

then (15.1) becomes

$$\eta = \left[A\frac{T_3}{T_1} - \frac{1}{(1-\delta)}\right]\frac{(T_2 - T_1)}{(T_3 - T_2)} \tag{15.3}$$

assuming δ to be the only variable, we may write:

$$\eta = C_b - \frac{B_b}{(1-\delta)} \tag{16}$$

where $$B_b = \frac{(T_2 - T_1)}{(T_3 - T_2)} \tag{16.1}$$

and $$C_b = AB_b\frac{T_3}{T_1} \tag{16.2}$$

<u>Loss Due to Bleeding Cooling Air from Compressor</u>

Regenerative Cycle:

$$\eta = \frac{(1-\delta)(T_3 - T_4) - (T_2 - T_1)}{(T_3 - T_2'')} \tag{17}$$

by a derivation similar to the above, we can show that:

$$\eta = C_{br} - \frac{B_{br}}{(1-\delta)} \tag{18}$$

where $$B_{br} = \frac{T_2 - T_1}{(T_3 D - T_2 E)} \tag{18.1}$$

and $$C_{br} = A\,B_{br}\,\frac{T_3}{T_1} \qquad (18.2)$$

<u>Sample Calculations: Loss Due to Bleeding Cooling Air from Compressor</u>

Simple Cycle: (Results of previous calculations used wherever possible)

$$\eta = C_b - \frac{B_b}{1-\delta} \qquad (16)$$

$$B_b = \frac{(T_2 - T_1)}{(T_3 - T_2)} = \frac{645^\circ}{895^\circ} = .721$$

$$C_b = A\,B_b\,\frac{T_3}{T_1}$$

$$A = \frac{.85 \times .85}{1 + .85\left(\frac{645}{520}\right)} = .352$$

$$C_b = .352 \times .72 \times \frac{2060^\circ}{520^\circ} = 1.002$$

$$\eta = 1.002 - \frac{.721}{1-\delta} \qquad (19)$$

$$\delta = 0 \;;\quad \eta = 1.002 - .721 = .281$$

$$\delta = .01 \;;\quad \eta = 1.002 - .728 = .274 \;;\quad \Delta\eta = .007$$

$$\delta = .10 \;;\quad \eta = 1.002 - .801 = .201 \;;\quad \Delta\eta = .08$$

Sample Calculations: Loss Due to Bleeding Cooling Air from Compressor

Regenerative Cycle: (Results of previous calculations used wherever possible.)

$$\eta = C_{br} - \frac{B_{br}}{(1-\delta)} \qquad (18)$$

$$B_{br} = \frac{T_2 - T_1}{T_3 D - T_2 E} = \frac{874° - 520°}{798°} = .444 \qquad (18.1)$$

$$C_{br} = A B_{br} \frac{T_3}{T_1}$$

$$A = \frac{.85 \times .85}{1 + .85 \times \frac{354}{520}} = .458$$

$$C_{br} = .458 \times .444 \times \frac{2060}{520} = .805$$

$$\eta = .805 - \frac{.444}{1-\delta} \qquad (20)$$

$\delta = 0 \;;\; \eta = .805 - .444 = .361$

$\delta = .01 \;;\; \eta = .805 - .449 = .356 \;;\; \Delta\eta = .005$

$\delta = .10 \;;\; \eta = .805 - .494 = .312 \;;\; \Delta\eta = .050$

- NOMENCLATURE -
Chapter II

Symbols in the first group below are defined by equations in the Appendix as indicated.

B - Equation (27.8)

C - " (21.6)

F - " (27.9)

$F_A(M)$ - " (27.6)

$F_f(M)$ - " (27.5)

$F_\theta(M)$ - " (27.4)

c_p = specific heat at constant pressure

C_r = radial velocity

c_v = specific heat at constant volume

D = diameter of cooling passage in turbine blade

D_m = mean diameter of turbine blades

dm = mass weight of element of air (Equation 21)

dQ = heat added to cooling air

dW_r = work done by cooling air in distance dr

dX = body force on element of mass (Equation 26)

f = coefficient of friction $= .046 \left(\frac{DG}{\mu}\right)^{-.2}$

g = acceleration of gravity

G = mass rate of flow of air = velocity X density

h = heat transfer coefficient $\begin{cases} h_g & \text{hot gas side} \\ h_c & \text{cooling air side} \end{cases}$

J = ratio $\dfrac{\text{work units}}{\text{equivalent heat units}}$

k = c_p/c_v

M = Mach Number = ratio $\dfrac{\text{radial velocity of air in cooling passage}}{\text{velocity of sound in cooling air}}$

dM = change in Mach Number in distance dr

p = absolute pressure

p_0 = pressure at radius r_0

dp = change in pressure in distance dr

r = distance from center line of turbine shaft

r_1 = radius at which stagnation temperature T_{o_1} is known

R = gas constant for air

T = static temperature (Equation 26.2)

T_o = stagnation temperature relative to bounding surface*

dT_o = change in T_o in distance dr

T_{o_1} = stagnation temperature at r_1

T_w = temperature of wall of cooling passage

v = specific volume

v_o = specific volume at radius r_o

w = mass rate of flow of cooling air (Equation 23.2)

θ = angle of segment (see Fig. 8)

μ = viscosity of air

ρ = weight of unit volume

Ω = velocity of rotation in radians/second

* T_g = stagnation temperature of hot gas

T_c = stagnation temperature of cooling air

T_b = blade temperature

<u>Method of Determining Increase in Pressure of Cooling Air in Turbine Disc.</u>

Radial velocity assumed negligible:

$$dp = \frac{r\,\omega^2\,dm}{r\,\theta} \quad \text{(for unit thickness)} \qquad (20)$$

where
$$dm = \frac{\rho}{g}\,r\,\theta\,dr \qquad (21)$$

assuming isentropic compression,

$$p\,v^{k} = p_0\,v_0^{k} \qquad (21.1)$$

where
$$v = \frac{1}{\rho} \qquad (21.2)$$

$$\therefore \quad \rho = \left(\frac{p}{p_0}\right)^{\frac{1}{k}} \frac{1}{v_0} \qquad (21.3)$$

and
$$dp = \frac{\cancel{r}\,\omega^2\,r\,\cancel{\theta}}{\cancel{r}\,\cancel{\theta}\,g\,v_0}\left(\frac{p}{p_0}\right)^{\frac{1}{k}} dr$$

$$= \frac{r\,\omega^2}{g\,v_0}\left(\frac{p}{p_0}\right)^{\frac{1}{k}} dr \qquad (21.4)$$

this is separable, and may be written:

$$p^{-\frac{1}{k}}\,dp = C\,r\,dr \qquad (21.5)$$

where
$$C = \frac{\omega^2}{v_0\,g\,p_0^{\frac{1}{k}}} \qquad (21.6)$$

the solution of (21.5) is:

$$p = \left[p_0^{\frac{k-1}{k}} + \frac{(k-1)}{2k}\,C\left(r^2 - r_0^2\right)\right]^{\frac{k}{k-1}} \qquad (22)$$

Evaluation of $\dfrac{dT_0}{T_0}$ **and** T_0

From simple heat transfer relations:

$$w \, dQ = h \; \pi D \, dr \left(T_w - T_0\right) \qquad (23.1)$$

where
$$w = \frac{\pi D^2}{4} \rho C_r \qquad (23.2)$$

Using Reynold's analogy between heat transfer and friction,

$$h = \frac{c_p f G}{2} = \frac{c_p f \rho C_r}{2} \qquad (23.3)$$

so that (23.1) becomes:

$$\frac{\cancel{\pi} D^{\cancel{2}}}{4} \cancel{\rho} \cancel{C_r} \, dQ = \frac{c_p f \cancel{\rho} \cancel{C_r}}{2} \cancel{\pi} \cancel{D} \, dr \left(T_w - T_0\right) \qquad (23.4)$$

$$dQ = \frac{2 c_p f \, dr}{D} \left(T_w - T_0\right) \qquad (23.5)$$

within the fluid,

$$dQ = dh + d\left(\frac{C_r^2}{2g}\right) + d\left(W_r\right) \qquad (23.6)$$

here,
$$dh + d\left(\frac{C_r^2}{2g}\right) = c_p \, dT_0$$

and
$$d(W_r) = -\frac{1}{J} d\left(\frac{\Omega^2 r^2}{2g}\right) \qquad (23.7)$$

so that
$$dQ = c_p \, dT_0 - \frac{1}{J} d\left(\frac{\Omega^2 r^2}{2g}\right) \qquad (23.8)$$

combining this with Equation (23.5), we obtain:

$$c_p D \, dT_0 - \frac{1}{J} D \, d\left(\frac{\Omega^2 r^2}{2g}\right) = \frac{2 c_p f \, dr}{D} \left(T_w - T_0\right) \qquad (23.9)$$

from which

$$dT_0 = 2 \frac{f \, dr}{D} \left(T_w - T_0\right) + \frac{1}{J c_p} d\left(\frac{\Omega^2 r^2}{2g}\right) \qquad (23.91)$$

and $$\frac{d T_o}{T_o} = 2 \frac{f\, dr}{D} \frac{T_w - T_o}{T_o} + \frac{\Omega^2 r\, dr}{2 J g c_p} \tag{24}$$

assuming T_w constant, solving Equation (23.91) gives,

$$T_o = T_w - \frac{\Omega^2 D}{4 J g c_p f}\left(r - \frac{D}{2f}\right) + e^{(r_1 - r)}\left[T_{o_1} - T_w + \frac{\Omega^2 D}{4 J g c_p f}\left(r_1 - \frac{D}{2f}\right)\right] \tag{25}$$

Evaluation of Body Force dX

Neglecting gravity, $$dX = r\,\Omega^2\, dm \tag{26}$$

evaluating dm in terms of volume and density, we have

$$dX = \frac{A p \Omega^2 r}{R T g}\, dr \tag{26.1}$$

here, T is the static temperature of the cooling air.

$$T = T_o \left(\frac{1}{1 + \frac{k-1}{2} M^2}\right) \tag{26.2}$$

so that $$dX = \frac{A p \Omega^2 r\, dr}{R T_o g}\left(1 + \frac{k-1}{2} M^2\right) \tag{26.3}$$

or $$\frac{dX}{A p} = \frac{\Omega^2 r\, dr}{R T_o g}\left(1 + \frac{k-1}{2} M^2\right) \tag{27}$$

Development of Final Form of Cooling Passage Air Flow Relation

Combining Equations (23), (24) and (27),

$$\frac{(1 - M^2)}{\left(1 + \frac{k-1}{2} M^2\right)} \frac{d M^2}{M^2} = \left[1 + k M^2\right]\left[2 f \frac{dr}{D} \frac{(T_w - T_o)}{T_o} + \frac{\Omega^2 r\, dr}{J g c_p T_o}\right]$$
$$+ k M^2\, 4 f \frac{dr}{D} - 2 \frac{k}{(k-1)}\left(1 + \frac{k-1}{2} M^2\right) \frac{\Omega^2 r\, dr}{J g c_p T_o} \tag{27.1}$$

$$= (1 + kM^2)\, 2f \frac{dr}{D} \frac{(T_w - T_0)}{T_0} + kM^2 4f \frac{dr}{D} - \frac{(k+1)}{(k-1)} \frac{\Omega^2 r\, dr}{J g c_p T_0} \qquad (27.2)$$

so that

$$dM^2 = \frac{M^2(1+kM^2)(1+\frac{k-1}{2}M^2)}{(1-M^2)}\, 2f \frac{dr}{D}\frac{(T_w - T_0)}{T_0} + \frac{kM^4(1+\frac{k-1}{2}M^2)}{(1-M^2)}\, 4f \frac{dr}{D}$$

$$- \frac{M^2(1+\frac{k-1}{2}M^2)}{(1-M^2)} \frac{(k+1)}{(k-1)} \frac{\Omega^2 r\, dr}{J g c_p T_0} \qquad (27.3)$$

Now we define the following functions, for which tables are available:

$$F_\theta(M) = \frac{M^2(1-\frac{k-1}{2}M^2)(1+kM^2)}{(1-M^2)} \qquad (27.4)$$

$$F_f(M) = \frac{kM^4(1+\frac{k-1}{2}M^2)}{(1-M^2)} \qquad (27.5)$$

$$F_A(M) = \frac{-2M^2(1+\frac{k-1}{2}M^2)}{(1-M^2)} \qquad (27.6)$$

Equation (27.3) then becomes:

$$dM^2 = F_\theta(M)\, 2f \frac{dr}{D}\frac{(T_w - T_0)}{T_0} + F_f(M)\, 4f \frac{dr}{D}$$

$$+ F_A(M) \frac{1}{2} \frac{(k+1)}{(k-1)} \frac{\Omega^2 r\, dr}{J c_p T_0 g} \qquad (27.7)$$

It is convenient, in connection with this and also the calculation of Equation (25), to evaluate these additional factors:

$$B = \frac{\Omega^2 D}{4 J g c_p f} \qquad (27.8)$$

and

$$F = 2\frac{f}{D} \qquad (27.9)$$

so that finally,

$$dM^2 = 2M\,dM = F_\theta(M)\,F\frac{(T_w - T_0)}{T_0}\,dr + F_f(M)\,2F\,dr$$

$$+ F_A(M)\frac{(k+1)}{(k-1)}\,\frac{BFr\,dr}{T_0} \qquad (28)$$

Sample Calculations-Analysis of Blade Cooling System

1.) Assumptions:

Mean Diameter	$= D_m$	$= 1.67$ FT.
Blade Length	$= .11\,D_m$	$= .183$ FT.
Blade Root Depth	$= .025\,D_m$	$= .042$ FT.
Length of Blade Cooling Passage	$= .15\,D_m$	$= .225$ FT.
Diameter of Blade Cooling Passage	$= .004\,D_m$	$= .0067$ FT.
Diameter of Shaft	$= .25\,D_m$	$= .416$ FT.
Rotational Velocity Ω	$= \frac{1000\text{ FT/SEC}}{D_m/2}$	$= 1200$ RADIANS/SEC
Turbine Inlet Temperature		$= 1800°F$
Cooling Air Temperature (entering shaft)		$= 80°F$
Cycle Pressure Ratio		$= 6.5$

2.)

$$p = \left[p_0^{\frac{k-1}{k}} + \frac{k-1}{2k}\,C\,(r^2 - r_0^2)\right]^{\frac{k}{k-1}} \qquad (22)$$

$$C = \frac{\Omega^2}{v_0\,g\,p_0^{\frac{1}{k}}} \qquad (21.6)$$

at root entry to cooling passage,

$p = 26{,}550$ LBS/FT^2 $=$ 184 LBS/IN^2

stage compression ratio $= \frac{184}{95.5} = 1.93$

temperature based on isentropic compression $= 192°$ F

$$F\frac{(T_w-T_o)}{T_o}\,dr \times F_\theta(M) =$$

$$2F\,dr \times F_f(M) = \underline{\qquad}\ \text{SUM OF ABOVE}$$

$$\frac{(k+1)}{(k-1)}\frac{BFr\,dr}{T_o} \times F_A(M) = \underline{\qquad}\ \text{DIFFERENCE}$$

$$\frac{\text{DIFFERENCE}}{2M} = dM$$

R (END OF STEP)	r (MEAN RADIUS FOR STEP)	$F\frac{(T_w-T_o)}{T_o}dr$ / $2F\,dr$ / $\frac{(k+1)}{(k-1)}\frac{BFr\,dr}{T_o}$	$\times F_\theta(M)$, $\times F_f(M)$, sum, $\times F_A(M)$, difference	$\frac{\text{DIFFERENCE}}{2M} = dM$	M (ESTIMATED AVERAGE USED FOR STEP)	M (END OF STEP)
.923						.67
	.900	.0563 .14 .0248	.0666 .0604 .1270 −.0372 .0898	$\frac{.0898}{1.28} = .07$	.64 (HIGH)	
.8775						.60
	.855	.0635 .14 .02495	.0473 .0326 .0799 −.0256 .0543	$\frac{.0543}{1.14} = .0476$	.57 (LOW)	
.8325						.5524
	.810	.0721 .14 .02515	.0441 .0249 .0690 −.0219 .0471	$\frac{.0471}{1.08} = .0436$	.54 (HIGH)	
.7875						.5088
	.765	.0826 .14 .02555	.0365 .0156 .0521 −.0169 .0352	$\frac{.0352}{.96} = .0359$	.49 (LOW)	
.7425						.4729
	.720	.0965 .14 .0262	.0350 .0116 .0466 −.0147 .0319	$\frac{.0319}{.92} = .0347$	.46 (HIGH)	
.6775						.4422

Calculation of Pressure Required to Sustain Assumed Tip Cooling Air Velocity

$$M = .44$$

$$T_0 = 740^\circ \text{ F abs.}$$

$$T = T_0 \left(1 + \frac{(k-1)}{2} M^2\right) = 714^\circ \text{ F abs.}$$

$$G_{\text{ROOT}} = 49\sqrt{T}\ M = 576 \text{ FT./SEC.}$$

at tip, mass rate of flow:

$$G = \rho_{\text{TIP}} G_{\text{TIP}} = 209 \text{ LB./FT.}^2 \text{ SEC.}$$

mass rate of flow at root must be the same,

$$\therefore\ \rho_{\text{ROOT}} G_{\text{ROOT}} = 209 \text{ LB./FT.}^2 \text{ SEC.}$$

$$\rho_{\text{ROOT}} = .368 \text{ LB./FT.}^3$$

$$P_{\text{ROOT}} = \rho_{\text{ROOT}} R T_{\text{ROOT}} = 97 \text{ LB./IN.}^2$$

to get pressure in disc necessary to sustain this pressure and velocity in the cooling passage, Equation (29) gives:

$$P_0 - P_1 = \frac{\rho_1 G_1}{2g}\left(1 + K + \frac{1}{4} M_1^2\right)$$

using $K = .5$, this gives:

$$P_0 = 118 \text{ LB./IN.}^2$$

Calculations - Two Dimensional Heat Balance in Turbine Blade Section

Reynold's Analogy, for inside of cooling passage the heat transfer coefficient

$$h_c = c_p G \frac{f}{2} \qquad (23.2)$$

Assuming same average conditions as in previous calculations, with

$$f = .0052$$

$$h_c = 470 \text{ B.T.U.}/\text{hr.}\,^\circ\text{F FT.}^2$$

For outside of blade, using method recommended for streamlined shapes * the heat transfer coefficient is:

$$h_g = 320 \text{ B.T.U.}/\text{hr.}\,^\circ\text{F FT.}^2$$

With the dimensional proportions assumed at the start of these calculations, and assuming three cooling passage holes per blade, the ratio

$$\frac{\text{circumference of blade section}}{\sum \text{circumference of holes}}$$

will fall in the range 3.5 to 4.5, depending upon the pitch-chord ratio and the shape of the section. Using a value of 4 in this range, and using Equation (30)

$$T_b = 1320^\circ\text{F}$$

The stagnation temperature of the hot gas relative to the blade is

$$T_g = 1674^\circ\text{F}$$

the effective cooling is therefore

$$T_g - T_b = 1674^\circ - 1320^\circ = 354^\circ\text{F}$$

To attain this amountof cooling,tthe cooling air flow, calculated previously, is 1.74 lb./sec. The hot gas flow for a turbine of the assumed dimensions and an axial flow velocity equal to 600 ft./sec. is approximately 63 lb./sec. Thus the cooling air flow is

$$\frac{1.74}{63 + 1.74} = .027, \text{ OR } 2.7\%$$

of the total flow in the compressor. The overall effect of applying the blade cooling system may be summarized as follows:

* "Heat Transmission"- Mc Adams

	Simple Cycle	Regenerative Cycle
η at 1500°F turbine inlet temperature	.265	.340
η at 1800°F turbine inlet temperature	.310	.388
Gross gain due to increase in temperature	.045	.048
Estimated maximum loss due to blade cooling	.024	.016
Net gain resulting from blade cooling	.021	.032