Two Sided Markets and Efficiency in the Internet

by

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Abstract

Competition between network providers is believed to be a necessary condition for an efficient functioning Internet industry. It spurs technology innovation as network providers compete to provide better quality and enhanced services; improving consumers' welfare in the process. Given the impact of competition, it is important for policy makers to understand how the institutional structures, interconnection agreements, market and regulatory environment in the Internet affect it. This will help in crafting of policies that enhance investments and deployment of new technologies in networks, foster development of new applications by content providers as well as safeguard consumer interests. This thesis investigates how access charges and market regulations affect competition, and consequently technological investments and welfare.

The first part of this thesis investigates competition between network providers under an interconnection agreement in which networks charge each other a reciprocal access charge (in the presence of congestion) using a two-sided market framework. Primarily, we look at how the access charge, congestion costs and two-sidedness of the market features in the strategic decision that the network providers take into account in setting their prices. In addition, we study how the access charge affects consumer enrollment and consequently its effect on social welfare. In particular, we investigate whether the current "bill and keep" practice employed in the Internet is an efficient interconnection agreement.

The second part of the thesis studies network neutrality and its effects on competition and network quality investments. Defining a neutral network as one in which network providers only charge content providers connected to them and a non-neutral network as one in which network providers are allowed to charge all content providers that deliver content to the network's consumers, we investigate the effect of these different pricing structures on platform investment patterns and consequently social welfare. Current debate is composed of two camps: On one hand are content and application providers who would like to see the Internet regulated and on the other are network providers who would like it to stay de-regulated. The former claim that the price differentiation by network providers, which is allowed under a de-regulated market, would not only erect barriers to new entrants but would also reduce innovation by current content and application providers. In contrast, network providers oppose such regulation on the grounds that it will hinder investment in network infrastructure because they will have no incentive to invest. The implications of net neutrality regulation on network provider investment incentives, social welfare and market structure is not well understood. We develop a competition model that aims to investigate the above economic issues under both a regulated(neutral) and de-regulated(non-neutral regime). Again the competition framework is based on two-sided markets.

Our competition model also contributes to the literature on two-sided markets by considering competition in interconnected platforms (network providers are abstractly viewed as platforms). Classical two sided markets usually consider closed platforms, i.e., platform end users only benefit from subscribers to that platform. However, in both our competition models, participants of one platform benefit from the presence of participants on their platforms as well as from the off-net platforms (because of the interconnection). In addition, we also consider investment decisions by the platforms, by showing the mechanism by which investment decisions in interconnected platforms are made.

Thesis Supervisor: Asuman E. Ozdaglar
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Chapter 1

Introduction

From its origin more than quarter of a century ago as a military research project, the Internet has become a core component of our society. Businesses have moved part or whole of their commercial activities online, communication and dissemination of information is, almost by default, carried out through the Internet, and Internet applications such as viral videos have permeated facets of everyday life. A special feature of the Internet that makes it appealing to consumers and content providers, is its universal connectivity which gives it a global outreach. A computer anywhere in the world can potentially communicate with a server or another computer connected to the Internet. In particular, content providers (CPs)[1] on one network are able to distribute their content to other networks, and consumers on the same network are able to access content on other networks.

Internet connectivity is provided by Internet (network) backbone providers. In the current market these consist of firms such as MCI/Worldcom, Sprint, AT&T, and UUNET. Competition between network providers is seen as a necessary condition for the efficient functioning and growth of the Internet industry [34]. Moreover, it also spurs technology innovation as network providers compete to provide better quality and enhanced services [60]; improving consumers' welfare in the process. Given the impact of competition, it is important for policy makers to understand how the institutional structures, interconnection agreements, market and regulatory environment in the Internet

[1] We shall use the terms content providers and websites interchangeably.

affect it. This will help in crafting policies that enhance investments and deployment of new technologies in networks, foster development of new applications by content providers as well as safeguard consumer interests. This thesis investigates the effect of net-neutrality regulation and reciprocal access charges on competition. In particular we study the effect of network neutrality on ISPs investment incentives and the effect of access charges on welfare.

The first part of this thesis investigates competition between network providers under an interconnection agreement in which networks charge each other a reciprocal access charge. Primarily, we look at how the access charge features in the strategic factors that the network providers[2] take into account in setting their prices. Moreover, we study how the access charge affects consumer enrollment and social welfare.

The second part of the thesis analyzes how network neutrality regulation affects competition and network quality investments. Currently competition between network providers is de-regulated. However, content and application providers prefer to see the Internet regulated. They claim that the price differentiation by network providers, which is allowed under a de-regulated market, would not only erect barriers to new entrants but would also reduce innovation by current content and application providers. In contrast, network providers oppose such regulation on the grounds that it will hinder investment in network infrastructure because they will have no incentive to invest. With a few exceptions current literature lacks systematic quantitative models for understanding the implications of such regulation on network provider investment incentives, social welfare and market structure. In the second part of the thesis we develop a competition model that aims to investigate the above economic issues under both a regulated(neutral) and de-regulated(non-neutral) regime.

[2]We use network providers, Internet backbone providers, ISPs, and broadband access providers interchangeably.

1.1 Competition in Interconnected Internet Platforms

Since the commercialization of the Internet industry, network backbone operators have been competing with each other to offer connection services to both consumers and content providers. However, in order to provide universal Internet connectivity, backbone providers usually enter into interconnection agreements with each other. There are several types of such agreements, see Bailey [4], but a common pricing agreement that is carried out between backbones is a type of interconnection agreement called *Bill and Keep* peering. In this type of agreement the two participating backbones agree to handle each others traffic at no cost, i.e., they do not charge each other access charges.

In the first part of this thesis we use a two-sided market framework to study price competition between two interconnected Internet backbones in the presence of congestion costs and an interconnection agreement that allows reciprocal access charges. A two-sided market[3] is one where two groups of agents interact with each other through an intermediary known as a platform [37, 50]. This type of market is characterized by cross group externalities which can be positive or negative. More specifically, if value is created when the two groups interact then the externality is positive and both groups prefer there to be more of the other group. An example of a two-sided market, in which value is created when the two groups interact, is the credit card market. The credit card issuing company is the platform while the merchants and the customers form the group of agents. The more customers that hold a particular card the more merchants will want to serve that card and vice versa.

When there is value generated by a consumer and content provider interaction, more usage of the Internet by consumers and more posting of content by content providers is preferred by the society. Indeed, the Internet can be considered as a platform, through which consumers and content providers interact; the utility of being connected to the Internet increases for the consumers as the number of content providers increases and vice versa. Thus, the Internet is subject to cross-group externalities and can be viewed

[3]Alternative definition: A market is two-sided if a platform can affect the volume of transactions by charging more to one side of the market and reducing the price paid by the other side by an equal amount, see Tirole and Rochet [48].

as a two-sided market. But increasing Internet usage means that congestion in the networks increase.

The prices offered by network providers to both content providers and consumers determines their subscriptions, the volume of traffic generated in the Internet, and consequently the level of congestion. The different network providers would like to offer services at prices that maximize their profits taking into account congestion effects and access charges. Key questions that arise in this scenario are;

1. How do competing network providers set their prices in the presence of access charges[4] and congestion cost so as to maximize profits? What strategic factors do they consider ?

2. How does competition affect consumer enrollment vis a vis enrollment at a social optimum, where a social optimum is defined as the allocation to internet backbones of consumers by a social planner?

3. What role do access charges play? More specifically does *bill and keep* peering maximize social welfare?

To answer the above questions we develop a stylized game-theoretic model where network providers are represented as profit maximizing two-sided interconnected platforms which compete in prices for both websites and consumers. While there is much work on competition models between two-sided platforms (see for example, [37, 45, 23, 50]), existing work does not address interconnection between platforms, access charges and congestion effects together. In this thesis we consider a model of platform competition that addresses these three effects in tandem. Moreover, existing work on access charges in interconnected networks in the telecommunications industry seem to advocate *Bill and Keep* peering, see [9, 15, 6] as a suitable interconnection agreement. Our objective is to provide an endogenous model of oligopoly competition that helps understand the pricing strategy employed by the platforms, how this affects consumer enrollment as

[4]Access charges are also referred to as terminal fees in the literature. As the name suggests these are charges levied by a terminating network upon a another network which tries to access its subscribers.

well as investigate the effects of the access charge on welfare; specifically we look into whether *Bill and Keep* is welfare maximizing.

Our model consists of two interconnected platforms, a mass of websites, and a mass of consumers. Platforms provide connection services to consumers and websites and charge transaction prices to both. Moreover, they are interconnected and charge each other a non-negative reciprocal access fee, a, per transaction. Consumers face convex congestion costs in form of latency costs when they join a platform. Consumer demand is elastic while each website has an inelastic demand with a reservation utility v. For simplicity, we assume that v is high enough such that the website market is covered. In our model, platforms play a two-stage game. Assuming an exogenously set access price, a, they pick transaction prices for both consumers and websites in the first stage. In the second stage, the websites and consumers simultaneously determine which platforms to locate to according to a Hotelling and Wardrop Equilibria, which will be defined in Chapter 2. Our framework is similar to the Laffont *et al.* [34] baseline model but it incorporates a more realistic assumption of congestion costs faced by consumers which the previous work does not consider.

1.2 Investments in Two-Sided Markets and the Net Neutrality Debate

The second part of the thesis focuses on net neutrality and its concomitant issues. Since 2005, when the Federal Communications Commission (FCC) changed the classification of Internet transmissions from "telecommunication services" to "information services," Internet Service Providers (ISPs) are no longer bound by the non-discrimination policies in place for the telecommunications industry, see [13]. This has led to the so called net neutrality debate. While there is no standard definition of what a net neutral policy is, it is widely viewed as a policy that mandates broadband access providers[5] to provide open-access, preventing them from any form of discrimination against Content Providers (CPs) or traffic flowing across their links. Content and application providers

[5]Used interchangeably with ISPs.

have coalesced together to form a lobby whose aim is to maintain the non-discrimination policy through net neutrality regulation. There are three main themes that net neutrality regulation seeks to enforce, see [57, 20]. These are:

(a) Broadband access providers should transmit all Internet packets without discrimination.

(b) Content and application providers should not have to pay off-net broadband access providers termination (access) fees for delivering content to end users, i.e., they do not have to pay any other network provider other than the one they are directly connected to.

(c) Broadband access providers should not vertically integrate into applications and contents.

To understand the effects of a net neutral policy demands a more precise clarification of the theme that the policy addresses. In this work, we focus on the second theme, i.e., whether broadband access providers should be allowed to charge access fees to content providers who are not directly connected to them. Under current practice, ISPs charge only CPs who are directly connected to them. Looking at net neutrality from a pricing perspective raises the question of what limits, if any, should be placed on ISPs pricing policies. Explicitly, should ISPs be allowed to charge off-network[6] CPs who want to deliver content to consumers or should the status quo remain (and be mandated by law)?

Net neutrality proponents (who are mostly composed of content/application providers) argue that the non-discrimination principle has been responsible for the explosive innovation in Internet applications that is seen today. They argue that if network providers were to charge content/application providers for accessing end users this would reduce investment incentives for content providers, see [31, 57]. Moreover, they argue that network access providers will have market power due their sole access to subscribers and therefore the access fees will be high, increasing the barriers of entry and resulting in a more concentrated content provider market, [20].

[6]These refers to CPs who are not directly connected to the ISP.

On the other hand the opponents (network providers) maintain that to meet the traffic needs of the content/application providers they need to make substantial investments in the network infrastructure which they cannot recover from consumers and/or content providers who directly connect to them. In order to attract consumers and content providers to their networks, network providers generally set low access fees which are lower than the average communication costs [31]. Whilst content providers are able to justify their investments from advertising sales, network providers argue that they have no such revenue streams. This argument is perhaps best exemplified by the former CEO of AT&T, Ed Whitacre, who made the following claim in a "Business Week" interview regarding CPs: *"Now what they would like to do is use my pipes free, but I ain't going to let them do that because we have spent this capital and we have to have a return on it. So there's going to have to be some mechanism for these people who use these pipes to pay for the portion they're using"*, see [63]. The upshot of the above argument is that if network access providers cannot charge for the last-mile broadband they will not get enough funds to invest in the network quality. This, they argue, causes network quality to degrade leading to less user demand and consequently to less investments by both content providers and network providers, [29, 30].

Unfortunately, the above debate has mostly been of a qualitative nature, see for example [20, 57, 64, 66, 21, 25]: With some notable exceptions notwithstanding, see for example [39, 16], not much formal economic analysis has been done to shed light on the validity, or lack thereof, of these arguments. This work develops a game theoretic model based on a two-sided market framework to investigate net neutrality as a pricing rule, i.e, whether there should be a mandate to preserve the current pricing structure. In order to understand the effects of such a policy on the internet, we study its effect on investment incentives of ISPs and its effects on social welfare, consumer and CP surplus and market coverage by CPs.

Our analysis involves two models; a neutral and a non-neutral model. In both, two ISPs are represented as profit maximizing two-sided interconnected platforms that choose quality investment levels and then compete in prices for both CPs and consumers. The difference between the two models is the pricing structure employed. In a neutral model

CPs pay only once to access the Internet and thus all consumers who are subscribed to the platforms, whilst in the non-neutral model they pay additional fees to reach off-network subscribers. To illustrate, Comcast and Verizon, who are ISPs, will demand that CPs such as Yahoo and Google, who are not on their network, pay them to reach their subscribers in a non-neutral model. In contrast no such payments are demanded in a neutral model. Our work complements, and in some cases challenges, current literature on net neutrality. At the same time it provides useful insight on how investment incentives of ISPs, which are important drivers for innovation and deployment of new technologies, differ under the two policies.

More specifically, our models consist of two interconnected ISPs (represented by platforms), a mass of CPs that are heterogenous in content quality, and a mass of consumers. Moreover, CPs make revenue from advertising. We model the interaction between ISPs and end-users[7] as a six-stage game. In the first stage, platforms simultaneously invest in a quality level. Second, they simultaneously compete in CP prices. Third, the CPs decide which platforms, if any, to connect to. Fourth, the platforms simultaneously compete in consumer prices. In the fifth stage, consumers decide what platforms to join. In the last stage consumers decide which CPs to patronize.

1.3 Outline of Thesis and Contributions

We conclude the introduction with an outline of the rest of the thesis and a summary of the results. In Chapter 2 we consider a model of oligopoly competition between two interconnected network providers that are represented as platforms.

- We first define and characterize the Wardrop and Hotelling equilibria according to which consumers and content providers locate to the different platforms given platform prices. We then consider the price competition between the platforms. We show that when congestion costs are linear the price competition game has a pure strategy Oligopoly Equilibrium (OE). We characterize using system parameters the prices that the platform charges to both consumers and content providers.

[7]The term end-users refers to both CPs and Consumers

- Our first major result shows that the transaction price charged to a consumer can be decomposed into three parts;(i) the opportunity cost of servicing a marginal consumer, (ii) a Pigovian tax which internalizes the congestion costs and (iii) a switching cost markup due to the market power that the congestion externality induces. This result shows that there are two competing effects on the price charged to consumers as a result of the two-sidedness of the market, the interconnection of the platforms, and the congestion costs. Due to the cross externality that a marginal consumer exerts on the websites, his price is discounted by the revenue he creates on the website side. Moreover, due to the cross net traffic he generates from the other platform his price is further discounted by the revenue earned from access fees levied on this traffic. On the other hand, a platform marks up the price to a marginal consumer with both a switching cost and Pigovian tax. The switching cost occurs because a consumer switching platforms incurs a congestion cost on the link of the platform he moves to. Therefore, a platform can raise its price by this congestion cost without losing market share. The Pigovian tax, internalizes the congestion cost on its link. The price charged to a website can be decomposed to the off-net-cost, see Laffont *et al.* [34] and a transport cost; the transport cost is a standard result arising from the hotelling model, see Tirole [59].

- Our next result shows that the enrollment of consumers to platforms under the Oligopoly Equilibrium (OE) is generally less than that under the social optimum. Consequently we show that, in general, the social welfare at the OE is also less than that at the social optimum. We give conditions under which the allocation of consumers at the OE and the social optimum are equal.

- Our last result shows that under some mild conditions *Bill and Keep* peering is not a welfare maximizing interconnection agreement. In particular, we show that when there is no full coverage on the consumer market at the OE, and congestion costs are linear, then a social planner would prefer a non zero access charge. This is an important result because it suggests that under a particular market structure, and for a certain class of demand and latency cost functions, regulation may have a role in enforcing efficient reciprocal access charges. In the case where there is full coverage, welfare is neutral to the access charge. In particular, the access charge distributes communication costs between the consumers and websites. For instance, high access charges result in higher prices for websites extracting website surplus which is used to subsidize the consumers.

In Chapter 3, we develop a two-sided market model that employs vertical differentiation on both sides of the market to investigate the effects of network neutrality regulation on investments and welfare in a duopoly competition between broadband providers.

- We provide an explicit characterization of equilibrium investment levels, and market coverage levels under both the neutral and non-neutral regime. We show how these values depend on the average CP quality $\overline{\gamma}$, the consumer mass f and CP heterogeneity a; which refers to how diverse the CP market is in terms of quality.

- Under some mild assumptions on f, we show that the investment levels are driven by the trade-offs platforms make in softening price competition on the consumer side and increasing CP surplus on the CP side from which they expropriate revenue. This trade-off is determined by the regime and also by the mass of consumers f in the market as explained in the next two bullets.

  - In the neutral model platforms maximally differentiate to corner different consumer and CP niches in the markets. More precisely, one platform opts not to invest whilst the other picks the highest quality permitted by investment costs. We refer to former as a low-quality platform and the latter as a high-quality platform. Essentially, by not investing, the low-quality platform trades-off making revenue on the CP side to making revenue on the consumer side. In contrast, the investment by the high-quality platform differentiates it from the low quality platform enabling it to make revenue from the CP side as well as consumers. This result is similar to the maximal differentiation result in Njoroge et al. [46]: This paper investigates a competition model with the same pricing structure but quality is considered costless and congestion effects are also considered. In Section 3.3.7, we further discuss the relationship between the investment level of the high-quality platform with both the average CP quality and the heterogeneity of the CPs content.

  - In the non-neutral model each platform has a monopoly over access to its consumer base. As a result, the investment patterns at equilibrium differ from those in the neutral regime. For a large consumer base and low values of average CP quality we have maximal differentiation (for similar reasons to those alluded to in the neutral model), in all other cases we have partial differentiation. In particular, both platforms invest in positive qualities. We refer to the one with a higher (lower) quality as a high-quality (low-quality) platform. Moreover, in this regime the difference in levels of investment between the two platforms is a function of the consumer mass. Indeed, as the consumer base decreases, the trade-off between softening price competition on the consumer side and increasing CP revenue, for the low-quality platform, shifts towards the latter; which leads to more investment in platform quality and more revenue from the CP side.

- We establish, under the assumptions of our model, that the non-neutral regime is superior for overall social welfare to the neutral regime. This result is primarily driven by the fact that the low-quality platform has an incentive to invest in the non-neutral regime increasing the aggregate level of investment in this regime. Investment increases the overall social welfare value by increasing consumer and CP gross[8] surplus. In addition, we show that the difference in social welfare between the two regimes increases with the average CP quality and decreases with CP heterogeneity. This reflects the effect of these two factors on the investment level of the low-quality platform and the linkage effect of this investment on gross consumer and CP surplus.

- Third, contrary to popularly held opinion in the policy debate, we note that CPs

[8]The term gross refers to surpluses that include the payments to be made to the platforms. Equivalently the total utility earned before the prices are deducted.

Table 1.1: Summary of Relationship of difference in social welfare with $\overline{\gamma}$ and a

Relationship of difference in social welfare with $\overline{\gamma}$ and a		
Average CP quality	$\overline{\gamma}\uparrow$	Social Welfare difference= $W_{nn}-W_n\uparrow$
Heterogeneity	$a\uparrow$	Social welfare difference= $W_{nn}-W_n\downarrow$

and consumers' surplus are higher in the non-neutral regime. Again these results are driven by the change in investment incentives of the low-quality platform. For CPs the larger investment on the low-quality platform leads to more revenue from advertisers. For consumers a larger investment on the low-quality platform has two major effects:

- First, it increases price competition between platforms and thus lowers prices which means consumers keep more of the value generated by their interaction with CPs.
- Secondly, the higher investment results in a higher utility for consumers when they interact with CPs since CP quality is enhanced by platform quality.

- Finally, although the low-quality platform prefers a non-neutral policy, we find that the high-quality platform prefers a neutral policy. For the high-quality platform a neutral network involves maximal differentiation in quality. Therefore it makes maximum revenue from both sides of the market, recall that platforms are viewed as substitutes on both sides. On the other hand, for the low-quality platform, the investment it makes in the non-neutral model enables it to gain CP revenue due to its monopoly access over its consumer base. In spite of the loss on the

Table 1.2: Consumer, CP surplus and Platform profits: Preference Under both regimes

	Surplus		Platform profits	
Regime	Content Provider (CP)	Consumer	High-quality	Low-quality
Non-Neutral	✓	✓		✓
Neutral			✓	

consumer side, due to the increased competition caused by this investment, the revenue gained on the CP side is much higher than this loss. These results are summarized in table 1.2; the check mark shows which regime has a higher value of the attribute on top of the columns

We conclude and give future directions for research in Chapter 4.

Chapter 2

Competition in Interconnected Platforms

In this chapter we study price competition between two interconnected Internet backbones in the presence of congestion costs and access charges using a two-sided market framework. We develop a game-theoretic model where Internet backbones are represented as profit maximizing two-sided-interconnected platforms which compete in prices for both websites and consumers. We show that the transaction price charged to a consumer can be decomposed into three parts;(i) the opportunity cost of servicing a marginal consumer, (ii) a pigovian tax which internalizes the congestion costs and (iii) a switching cost markup due to the market power that the congestion externality induces. We further show that market penetration under the Oligopoly Equilibrium is strictly less than that under the social optimum. Consequently we show that the welfare under competition is also strictly less than that at the social optimum. Our last result shows that if under the Oligopoly Equilibrium there is no full coverage on the consumer market and consumers face a concave inverse demand and linear latencies then *Bill and Keep* peering is not a welfare maximizing interconnection agreement.

2.1 Related Literature

Our work is most related to work by Shneorsen and Mendelson [55]. They adopt the Laffont *et. al.* [34] model and add consumer delay costs to investigate congestion effects on market structure using a queueing model. However, in their model, website prices are given exogenously. In particular, they are set at the off-net cost given in [34]. This implies that platforms make no profit on the website side and the model effectively considers only competition on the consumer side. Moreover, this competition is modeled as a Cournot game where each network provider determines how many consumers to acquire given the number of consumers acquired by its competitors. In our model, the allocation of websites and their pricing are done endogenously. This leads to a richer price characterization because platforms have to consider the effects of consumer enrollment on website profits. In addition, Bertrand competition is used in our two stage game because in practice network providers compete in prices. In a different piece of work, Shrimali and Kumar [56] investigate situations in which networks having access to transit providers decide to peer. Using a simple economic model they show that *bill and keep peering*, where access charge is set to zero, is a fair and efficient outcome under symmetric transit and peering costs. Their work focuses on the decision faced by networks on whether to peer or not, and is not set on finding whether bill and keep peering is efficient for already interconnected platforms. Issues of consumer and website pricing do not arise in their work.

The remaining part of this chapter is organized as follows. Section 2.2 introduces the model together with the notation and basic assumptions that will be used throughout this chapter. The next section characterizes both the Wardrop equilibria which consumers use to locate to the different platforms and the Hotelling equilibria by which website shares on the platforms are determined. Section 2.4, introduces the notion of Oligopoly equilibrium that results following price competition between platforms. It also characterizes prices charged by the platforms to both sides of the market. The final section, analyzes the effects of access charges on welfare. Section , contains our concluding remarks.

2.2 Model

In this section we formulate the model that is used to investigate competition between interconnected backbones [1]. The backbones are represented as two-sided interconnected platforms with consumers and websites on different sides. We define the utility functions of the consumers and websites together with our assumptions on those functions. We also describe the costs incurred when a connection is made between a website and a consumer. We finally provide the timing of events in the model.

We consider a duopoly of two-sided interconnected platforms. Let $\mathcal{I} = \{1, 2\}$ denote the set of platforms. Each platform provides a connection service to both consumers and websites who are on different sides of the platform. Let c, w represent these sides, where c stands for the consumer side and w for the website side. As in Laffont *et al.* [34], we assume a continuum of mass 1 of both consumers and websites on each side of the platform.

We denote the unit transaction prices charged by a platform $i \in \mathcal{I}$ to a consumer and website, who connect (transact) through it, as p_i^c and p_i^w respectively. Let $\mathbf{p_i} = [p_i^m]_{m \in \{c,w\}}$ be the vector whose components are the connection prices that a platform i charges to the two sides of the platform. We also define the vector $\mathbf{p^c} = [p_i^c]_{i \in \mathcal{I}}$ to be the connection prices offered by both platforms on side c to consumers. Similarly we define the vector $\mathbf{p^w} = [p_i^w]_{i \in \mathcal{I}}$ to represent the connection prices offered by both platforms on side w to consumers. We denote the mass of consumers and websites on a platform $i \in \mathcal{I}$ as x_i^c and x_i^w respectively. We define the vector of consumers $\mathbf{x^c} = [x_i^c]_{i \in \mathcal{I}}$ on side c of the platform to represent the mass of consumers locating on the different platforms. The total mass of consumers on side c is given by $x^c = \sum_{i \in \mathcal{I}} x_i^c$ and the total mass of websites is given by $x^w = \sum_{i \in \mathcal{I}} x_i^w$.

Traffic is assumed to flow from websites to consumers. Each connection between a consumer and a website generates a unit of traffic flow. We assume that consumers' interests in a website are unrelated to the platform hosting it. A consumer on side c is equally likely to access any web site on side w irrespective of the platform. We

[1] We use the term backbone operators and network providers interchangeably

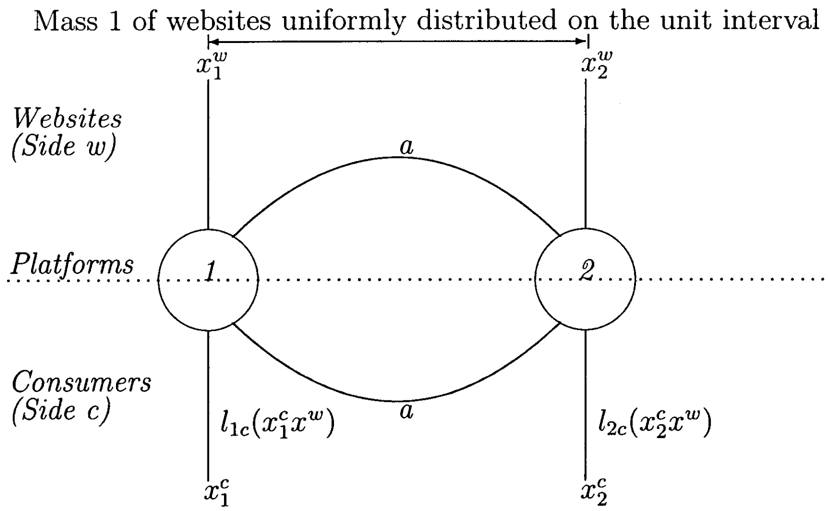

Figure 2-1: Two interconnected platforms with links to consumers and websites.

also assume that platforms are not allowed to price discriminate between on-platform and off-platform connections: A platform charges a consumer the same for accessing a website on its' platform as it does for accessing a website on the other platform. This is a similar assumption to the *balanced calling pattern* referred to in Laffont *et al.* [34]. It implies that the fraction of traffic flow through a platform from the website side to a platform's consumers, is proportional to the platform's mass of consumers and the mass of websites on the other side, i.e., if a mass x^w of websites is available, a mass x_i^c of consumers on platform i generates a traffic flow of $x_i^c x^w$ from the website side.

Each platform has two links; one link connects it to the consumers and the other to the websites, see Figure 2-1. A link to consumers, on side c of a platform i, has a flow dependent latency function $l_{ic}(x_i^c x^w)$, which measures the travel time as a function of the total traffic flow given by $x_i^c x^w$. In this model we assume the latency of links on the website side are negligible. This assumption is a good approximation to communication networks where most congestion happens at the connection between consumers and the access providers; see Zehra and Moghe [67].

The platforms are assumed to be interconnected by high capacity access links, therefore we assume no congestion on these links either. In our interconnection model, each platform charges the other for access. We denote the access fee for accessing one platform from another as a and assume it to be non-negative. For example in Figure 2-1, platform 2 delivering content to platform $1's$ consumers has to pay platform 1 an access fee of a for each unit of traffic. This fee is assumed to be exogenous and reciprocal. Currently most interconnection agreements set access charges at zero which is a reciprocal arrangement. However, there have been calls to set positive reciprocal charges, see [24].

When a platform originates a unit of traffic it incurs a non-negative originating cost c_o and when it terminates a unit of traffic it incurs a non-negative terminating cost c_t. We define the off-platform [2] consumer cost as $c^c = c_t - a$ and the off-platform website cost as $c^w = c_o + a$. Referring to Figure 2-1, platform 2 delivering content to platform $1's$ consumers incurs an origination cost c_o and an access fee a for each unit of traffic. Thus it ends up incurring an off-platform website cost c^w. On the other hand platform 1 terminating traffic from platform 2 incurs a termination cost c_t but gets paid an access fee a for each unit of traffic. Thus it ends up incurring an off-platform consumer cost $c^c = c_t - a$. These off-platform costs appear in the later sections of our work and we use them to draw comparisons between the price characterizations of our model and that of Laffont*et al.* [34]. Note that the cost of a unit of traffic between a unit consumer and a unit website is given by $c = c_o + c_t$. Since a consumer generates a unit of traffic flow from a website, this cost can be thought of as the marginal cost of traffic.

A unit consumer derives utility m drawn from a distribution $F_M(m)$ whose support ranges from 0 to $\overline{m}$ when connected to any unit website. Consumer preferences are represented by an aggregate utility function $V_c(x^c, x^w)$ which represents the amount of utility gained when a mass x^c and x^w of consumers and websites respectively, join each side of the platform. Given a distribution $F_M(m)$, the aggregate gross utility of a mass x^c of consumers who are connected to x^w websites is given by

$$\begin{aligned} V_c(x^c, x^w) &= \int_{F_M^{-1}(1-x^c)}^{\overline{m}} m x^w dm, \\ &= \frac{1}{2}\left(\overline{m}^2 - (F_M^{-1}(1-x^c))^2\right) x^w. \end{aligned}$$

Defining $u(x^c)$ as $\frac{1}{2}\left((\overline{m})^2 - (F_M^{-1}(1-x^c))^2\right)$, we have $V_c(x^c, x^w) = u(x^c)x^w$. We note

[2] These are the same costs that appear in the base model of Laffont *et al.* [34] but they are labeled as Off-Net-Costs.

that $u(x^c)/x^c$ represents for side c the gross utility of a representative user who connects to a website. We make the following assumptions on the gross utility function $u(x^c)$.

Assumption 1.

- *The utility function* $u : [0,1] \to [0,\infty)$ *is concave, continuously differentiable, and increasing.*
- *The derivative of the utility function is concave, decreasing and continuously differentiable.*

The above assumptions on $u(x^c)$, for instance, will be satisfied under the assumption

$$\frac{3(F''_M(m))^2}{F'_M(m)} \leq F'''_M(m) \qquad \text{for all } \ m \in [0, \overline{m}].$$

The conditions imposed on the derivative of $u(x^c)$ are similar to those adopted by Engel, Fisher and Galetovic [17], Hayrapetyan, Tardos, Wexler [26] and Ozdaglar [43].

The gross utility of a marginal user on side c per transaction is determined by the elastic inverse demand curve,

$$\frac{1}{x^w}\frac{\partial V_c(x^c, x^w)}{\partial x^c} = u'(x^c).$$

Thus a marginal consumer on side c has a marginal utility $u'(x^c)$ and will only join as long as the effective cost they experience is less than this amount. In the parlance of two-sided platforms, a marginal consumer enjoys a benefit $u'(x^c)$ from interacting with each user on side w. Given the assumptions on the utility function we see that this benefit is platform independent and diminishes with the number of users on side c. This is intended to capture the notion that earlier users place a higher value to transacting with websites. Each unit website on side w gains utility v for each connection made with a unit consumer on side c. The website preferences on side w of the platform can therefore be represented by an aggregate utility function $V_w(x^c, x^w)$ which is the amount of utility gained when a mass x^c and x^w of consumers and websites join each side of the platform respectively, i.e., the aggregate utility function is quadratic in the mass of consumers and websites.

Assumption 2. *For side w the gross utility of a representative website is given by,*

$$V_w(x^c, x^w) \;=\; vx^c x^w \;\; 0 \leq x^c, x^w \leq 1.$$

In this instance the gross utility of a marginal user per transaction is determined by the inelastic inverse demand curve,

$$\frac{1}{x^c}\frac{\partial V_w(x^c, x^w)}{\partial x^w} = v.$$

The benefit per transaction enjoyed by each website is thus v and is also platform independent. We make following assumption on the marginal benefit of marginal consumers and websites.

Assumption 3. $0 \leq c_t < u'(1)$ *and* $0 \leq c_o < v$.

This is a feasibility condition and ensures that a platform has the incentive to participate in the market, i.e. $u'(x) + v \geq c$ for all $x \in [0, 1]$.

We assume that consumers are delay sensitive and they do not consider their individual decisions as having an effect on the prices charged by the platforms or congestion on the links. Faced with the connection price vector $\mathbf{p}^c$ offered by the platforms and a vector of websites $\mathbf{x^w}$, we assume that consumers locate to different platforms according to a Wardrop equilibrium which will be defined in Section 2.3. As discussed in Acemoglu and Ozdaglar [1], the Wardrop equilibrium is used extensively in modeling traffic behavior in communications networks.

We assume that websites locate to different platforms under a standard Hotelling model of user choice. The platforms are located at each end of a unit interval and the websites are uniformly located along this unit interval. Given the vector of prices $\mathbf{p^w}$ for side w and a vector of consumers $\mathbf{x^c}$ on side c of the platform users on side w locate to the different platforms according to a Hotelling equilibrium which is also defined in Section 2.3.

The payoff functions for the platforms are given by their profit functions. Given an access charge a, platforms compete in prices where the objective of each platform is to maximize his profit. The profit for a platform $i \in \mathcal{I}$ charging a connection price p_i^c and p_i^w to a consumer and a website respectively is given by

$$\begin{aligned}\Pi_i &= x_i^c x_i^w (p_i^c + p_i^w - c) + x_i^c x_j^w (p_i^c - (c_t - a)) + x_j^c x_i^w (p_i^w - (c_o + a)), \\ &= (p_i^c - c^c) x_i^c x^w + (p_i^w - c^w) x_i^w x^c.\end{aligned}$$

With an exogenously determined access charge, the model described above refers to a two stage game. In the first stage, connection prices for both websites and consumers are simultaneously set by the profit maximizing platforms. In the second stage, given the platform prices, the websites and consumers simultaneously determine which platforms to locate to according to the Wardrop and Hotelling equilibria which is defined in Section 2.3. The subgame-perfect equilibrium of the above game can be characterized using backward induction.

2.3 Wardrop and Hotelling Location Equilibrium

In this section we consider the second stage of the game described in Section 2.2, i.e., consumer and website behavior on both sides of the platforms given platform prices. We formally define the Wardrop and Hotelling equilibrium that determine the mass of consumers and websites joining the two sides of the platforms which consequently determines the flows allocated to the platforms. We characterize this equilibria and establish their existence and properties.

We will restrict the strategy space of the prices offered by platforms as follows $-\infty < p_i^c \leq u'(0)$ and $-\infty < p_i^w \leq v$ for all $i \in \mathcal{I}$. The upper bounds can be justified by the fact that offering prices higher than $u'(0)$ on the consumer side will yield 0 consumers on the platforms and offering prices higher than v on the website side will yield 0 websites on the platforms; this can be directly inferred from Lemma 1 and 2 that follow shortly.

We are interested in how consumers and websites locate to the different platforms. We first look at the problem of how consumers choose between the two platforms. We assume, given the allocation of websites and the choice of consumer prices, they do so according to Wardrop's principle, see [61]. Consumers join the platform that has the least effective cost defined as the sum of the congestion cost they experience when the join the platform and the price they pay for each unit of traffic. Formally we assume that given a price vector $\mathbf{p^c}$ and a vector of users $\mathbf{x^w}$ on side w of the platform, users on side c locate to the different platforms according to a Wardrop equilibrium defined which will be defined shortly. Regarding notation, all vectors are viewed as column vectors and inequalities are to be interpreted component wise. We denote by $\mathbb{R}^I_+$ the set of non-negative $\mathcal{I}$-dimensional vectors.

Definition 1. For a given price vector $\mathbf{p^c}$ and vector $\mathbf{x^w} \in \mathbb{R}^I_+$, a vector $\mathbf{x^c} \in \mathbb{R}^I_+$ is a Wardrop Equilibrium (WE) if

$$\mathbf{x^{cWE}} \in \arg \max_{\mathbf{x^c} \geq 0 \sum_i x_i^c \leq 1} \left\{ u(x^c)x^w - \sum_{i \in \mathcal{I}} \left(l_{ic}(x^w x_i^{cWE}) + p_i^c \right) x_i^c x^w \right\}. \tag{2.1}$$

We denote the set of Wardrop Equilibria at a given price vector $\mathbf{p^c}$ and $\mathbf{x^w}$ by $W(\mathbf{p^c}, \mathbf{x^w})$. Using optimality conditions for problem (1) we see that if $\mathbf{x^c} \in W(\mathbf{p^c}, \mathbf{x^w})$, then we have,

$$\begin{aligned} u'(x^c)x^w - l_{ic}(x^w x_i^{cWE})x^w - p_i^c x^w &\leq \lambda^c \quad \text{if } x_i^{cWE} = 0, \\ &= \lambda^c \quad \text{if } x_i^{cWE} > 0. \end{aligned}$$

We therefore have the following characterization of a WE.

Lemma 1. For a given price vector $\mathbf{p^c}$ and a vector of users $\mathbf{x^w} \in \mathbb{R}^I_+$ and $\mathbf{x^w} \neq 0$, a vector $\mathbf{x^c} \in \mathbb{R}^I_+$ is a Wardrop Equilibrium (WE) if and only if

$$\begin{aligned} l_{ic}(x_i^c x^w) + p_i^c &\leq u'(x^c), & \forall\, i \text{ with } x_i^c > 0, \\ l_{ic}(x_i^c x^w) + p_i^c &\leq \min_n \{ l_{nc}(x_n^c x^w) + p_n^c \}, & \forall\, i \text{ with } x_i^c > 0, \end{aligned}$$

$$\sum_{i \in \mathcal{I}} x_i^c \;\leq\; 1,$$

with $\sum_{i\in\mathcal{I}} x_i^c = 1$ if $\min_n\{l_{nc}(x_n^c x^w) + p_n^c\} < u'(x^c)$.

Thus, for each interaction with a website, a consumer faces a disutility equivalent to the congestion on his link and the price he pays to the platform. It is worth mentioning that when $\mathbf{x^w} = 0$, any feasible solution is Wardrop equilibrium.

We next proceed to show the existence and continuity properties of a WE. The proof relies on a standard argument in the traffic equilibrium literature based on establishing the equivalence of a WE and the optimal solution of a convex optimization problem (see [14], [1]). We adopt the following assumption on the latency functions.

Assumption 4. *For each $i \in \mathcal{I}$, the latency function $l_{ic} : [0,1] \to [0,\infty)$ is convex, continuously differentiable, strictly increasing and has $l_{ic}(0) = 0$.*

The strictly-increasing and convexity condition imposed on the latency functions captures the congestion effect experienced by consumers when more traffic flows on a platforms' link. The zero latency flow assumption is a good approximation to communication networks where queueing delays are more substantial than propagation delays.

Proposition 1. *(Existence and continuity.)* Let Assumptions 1 and 4 hold. For a price vector $\mathbf{p^c}$ and location vector $\mathbf{x^w} \in \mathbb{R}_+^I$, the set $W(\mathbf{p^c}, \mathbf{x^w})$ is non-empty. Moreover, the correspondence $W(\mathbf{p^c}, \mathbf{x^w}) : \mathbb{R}_+^I \rightrightarrows {\mathbb{R}_+}^I$ is upper semicontinuous.

Proof. Given any price vector $\mathbf{p^c}$ and vector $\mathbf{x^w}$, consider the following optimization problem,

$$\max_{\mathbf{x^c} \geq 0,\; x^c \leq 1} \quad \varphi_c(\mathbf{x^w}, \mathbf{x^c}, \mathbf{p}), \tag{2.2}$$

where $\varphi_c(\mathbf{x^w}, \mathbf{x^c}, \mathbf{p^c}) \equiv u_c(x^c)x^w - \sum_{i\in\mathcal{I}} p_i^c x_i^c x^w + \int_0^{x_i^c x^w} l_{ic}(y)dy$.

Whenever $\mathbf{x^w} \neq 0$, from Assumptions 1 and 4, we can deduce that the objective function is strictly concave over a constraint set which is compact and convex; this follows because we restrict ourselves to the set of vectors, $X^c = \{\mathbf{x^c} \mid \sum_i x_i^c \leq 1, x_i^c \geq$

0 for all $i \in \mathcal{I}\}$, which form a simplex. Therefore the first order optimality conditions are also sufficient for optimality. Moreover due to the strict concavity there exists a unique vector $\mathbf{x^c}$ which achieves the maximum. Since, the first order optimality conditions are identical to those of the Wardrop Equilibrium, a vector $\mathbf{x^c} \in W(\mathbf{p^c}, \mathbf{x^w})$ if and only if it is a solution to problem 2.2. Since problem 2.2 is continuous over a compact constraint set a solution exists. Therefore the set $W(\mathbf{p^c}, \mathbf{x^w})$ is also non-empty and unique whenever $\mathbf{x^w} \neq 0$. If we allow $\mathbf{x^w} \in [0,1] \times [0,1]$ then $W(\mathbf{p^c}, \mathbf{x^w})$ is an upper semicontinuous correspondence from the Theorem of the Maximum [58]. ■

On the website side we assume a standard Hotelling model of user choice. This model of horizontal differentiation enables us to assign website platform shares to the different platforms. In this model platforms are located at each end of a unit interval and the websites are uniformly located along this unit interval. Websites have a transportation cost t per unit length. Given that platforms i and n charge prices p_i^w and p_n^w respectively, the effective price of going to platform i for a website located at coordinate x_i^w is $p_i^w + x_i^w t$ and that of going to platform n is $p_n^w + (1 - x_i^w)t$. Since a connection with a consumer yields a benefit of v, the net utility gained by a website located at a coordinate x_i^w transacting with a unit consumer is given by

$$V_w = \begin{cases} v - p_i^w - tx_i^w & \text{if website joins platform } i, \\ v - p_n^w - tx_n^w & \text{if website joins platform } n. \end{cases}$$

Given the vector of prices $\mathbf{p^w}$ for side w, websites locate to the different platforms according to the Hotelling Equilibrium which is defined below. Lemma 2 shows that the following definition of a Hotelling Equilibrium yields the standard characterization for a Hotelling model [59].

Definition 2. For a given price vector $\mathbf{p^w}$, a vector $\mathbf{x^w} \in \mathbb{R}_+^I$ is a Hotelling Equilibrium HE if

$$\mathbf{x^{wHE}} \in \arg \max_{\mathbf{x^w} \geq 0,\ \sum_i x_i^w \leq 1} \left\{ \sum_i \int_0^{x_i^w} (v - tx - p_i^w) dx \right\}. \tag{2.3}$$

We denote the set of Hotelling Equilibria at a given price vector $\mathbf{p^w}$ by $H(\mathbf{p^w})$.

Using optimality conditions for problem (2.3) we see that $\mathbf{x^w} \in \mathbb{R}^I_+$ is a HE if and only if there exists $\sum_i x_i^w \leq 1$ and there exists some $\lambda_w \geq 0$ such that $\lambda_w(\sum_i x_i^w - 1) = 0$ and for all i,

$$\begin{aligned} v - tx_i^w - p_i^w &\leq \lambda_w \quad \text{if } x_i^w = 0, \\ &= \lambda_w \quad \text{if } x_i^w > 0. \end{aligned}$$

In view of Assumption 2.2, we have the following characterization of a HE.

Lemma 2. For a given price vector $\mathbf{p^w}$, a vector $\mathbf{x^w} \in \mathcal{R_+}^I$ is a Hotelling Equilibrium (HE) if and only if,

$$\begin{aligned} p_i^w + x_i^w t &= \min_n\{p_n^w + x_n^w t\} \text{ for all } i \text{ with } x_i^w > 0, \\ p_i^w + x_i^w t &\leq v \text{ ,for all } i \text{ with } x_i^w > 0, \\ \sum_i x_i^w &\leq 1, \end{aligned} \tag{2.4}$$

with $\sum_i x_i^w = 1$ if $\min_n\{p_n^w + x_n^w t\} < v$.

The existence and continuity properties of $H(\mathbf{p^w})$ are proved in a similar way to those of $W(\mathbf{x^w}, \mathbf{p^c})$ by considering the optimization problem,

$$\max_{\mathbf{x^w} \geq 0, x^w \leq 1} \varphi_w(\mathbf{x^w}, \mathbf{p^w}), \tag{2.5}$$

where $\varphi_w(\mathbf{x^w}, \mathbf{p^w}) \equiv \sum_i \int_0^{x_i^w} (v - tx - p_i^w) dx$. It can also be shown using similar arguments to those in Proposition 1 that the Hotelling Equilibrium is unique given $\mathbf{p^w}$. In particular, we note that, given $\mathbf{p^w}$, $\varphi_w(\mathbf{x^w}, \mathbf{p^w})$ is a strictly concave function over a convex and constraint set. Therefore a maximum exists and its unique.

In this section we have described the mechanism by which consumers and websites locate to platforms given the price vectors $\mathbf{p^c}$ and $\mathbf{p^w}$. We define the Location Equilibrium as a vector whose components consist of the allocations of consumers and websites on both platforms given the price vectors $\mathbf{p^c}$ and $\mathbf{p^w}$. This definition helps us use simpler notation in the expressions that we develop in the next section. Given the vector prices offered by the other platform, $\mathbf{p_{-i}} = [\mathbf{p_n}]_{n \neq i}$, the Location Equilibrium is defined as follows:

Definition 3. Given the vector of prices $(\mathbf{p_i}, \mathbf{p_{-i}})$, a vector $(\mathbf{x^{wLE}}, \mathbf{x^{cLE}})$ is a Location Equilibrium (LE) if $\mathbf{x^{cLE}} \in W(\mathbf{p^c}, \mathbf{x^{wLE}})$ and $\mathbf{x^{wLE}} \in H(\mathbf{p^w})$. We denote the set of LE at given price vectors $(\mathbf{p_i}, \mathbf{p_{-i}})$ by $LE(\mathbf{p_i}, \mathbf{p_{-i}})$.

Since, given $\mathbf{p^w}$, the set $H(\mathbf{p}^w)$ is non-empty and a singleton, it is immediate from Proposition 1 that the set $W(\mathbf{p^c}, \mathbf{x^w})$, where $\mathbf{x^w} \in H(\mathbf{p}^w)$, is also non-empty and a singleton. Therefore the set $LE(\mathbf{p_i}, \mathbf{p_{-i}})$ is also non-empty and a singleton.

2.4 Price Competition and Nash Equilibrium

In this section we analyze the first stage of the game; platform competition in the presence of congestion effects and the access (interconnection) charge. We show that a Nash equilibrium exists for affine latency functions. We present price characterizations for both sides of the platform in terms of the system parameters and give a qualitative description of these prices. In particular we show how the platforms factor congestion costs and access charges in their pricing strategies.

Given the vector prices offered by the other platforms and the access charge a, $\mathbf{p_{-i}} = [\mathbf{p_n}]_{n \neq i}$, the profit of platform i is defined as

$$\Pi_i(\mathbf{p_i}, \mathbf{p_{-i}}) = \underbrace{(p_i^c - c^c)x_i^c x^w - a x_i^w x_i^c}_{\pi_i^c} + \underbrace{(p_i^w - c_w)x_i^w x^c + a x_i^w x_i^c}_{\pi_i^w},$$

where π_i^c is the revenue from the consumer side of the platform, π_i^w is the revenue from the website side and $\mathbf{x} = (\mathbf{x^c}, \mathbf{x^w})$ is the Location equilibrium at a price vector $\mathbf{p} = (\mathbf{p_i}, \mathbf{p_{-i}})$. The objective of each platform is to maximize its overall profits. Since the demand and profit depends on prices set by the other platform, each platform conjectures about the actions of other platform as well as behaviors of consumers and websites. We assume that they do this according to the Nash equilibrium notion.

Definition 4. A vector $(\mathbf{p^{OE}}, \mathbf{x^{OE}})$ is a pure strategy Oligopoly Equilibrium (OE) of the price competition game if $\mathbf{x^{OE}} \in \mathbf{LE}(\mathbf{p_i^{OE}}, \mathbf{p_{-i}^{OE}})$ and for all $i \in \mathcal{I}$,

$$\Pi_i(\mathbf{p_i^{OE}}, \mathbf{p_{-i}^{OE}}, \mathbf{x^{OE}}) \geq \Pi_i(\mathbf{p_i}, \mathbf{p_{-i}^{OE}}, \mathbf{x}), \quad \forall\, \mathbf{p_i}, \text{ and } \mathbf{x} \in LE(\mathbf{p_i}, \mathbf{p_{-i}^{OE}}).$$

We refer to $\mathbf{p^{OE}}$ as the OE price.

We will refer to the game described above as the *price competition game.* We proceed to show that in the case of full coverage on the website side and for affine latency functions this game has a pure strategy OE. We first prove a series of Lemmas, which we use to show the existence of a pure strategy OE.

Assumption 5. *There is full coverage on the website side, i.e.* $x^w = 1$.

The above Assumption ensures that the website side is covered. For instance, it will be satisfied when all websites have a high enough reservation price that they participate in the platforms. From Lemma 2, this condition is fulfilled whenever $min_n \, \{p_n^w + x_n^w t\} < v$.

Lemma 3. Let Assumption 1 and 4 hold. Consider a vector $\mathbf{x^w} \in \mathbb{R}_+^{\mathbb{I}}$ such that $\mathbf{x^w} \neq 0$ and two price vectors $\mathbf{p^c}$ and $\overline{\mathbf{p}}^{\mathbf{c}}$ such that $\overline{p}_n^c = p_n^c$ and $\overline{p}_i^c > p_i^c$ for $n \neq i$ and $n, i \in \mathcal{I}$. Let $\mathbf{x^c} \in W(\mathbf{p^c}, \mathbf{x^w})$ and $\overline{\mathbf{x}}^{\mathbf{c}} \in W(\overline{\mathbf{p}}^{\mathbf{c}}, \mathbf{x^w})$. If $x_i^c > 0$ then $\overline{x}_i^c < x_i^c$, $\overline{x}_n^c > x_n^c$ and $\overline{x}^c \leq x^c$.

Proof.

a) We first show that $\overline{x}_i^c < x_i^c$. We consider two cases, when $x^c < 1$ and $x^c = 1$.

Case 1: $x^c < 1$.

Suppose that $\overline{x}_i^c \geq x_i^c$. By Lemma 1, the fact that $l_{ic}(\cdot)$ is strictly increasing and $\overline{p}_i^c > p_i^c$, we have the following inequalities,

$$\begin{aligned} u'(0) &> u'(\overline{x}^c) \\ &\geq l_{ic}(\overline{x}_i^c x^w) + \overline{p}_i^c, \\ &> l_{ic}(x_i^c x^w) + p_i^c, \\ &= u'(x^c). \end{aligned} \tag{2.6}$$

This implies, by the decreasing property of $u'(\cdot)$, that $\overline{x}^c < x^c$. Consider now the other link $l_{nc}(\cdot)$ and assume it has positive flow at price $\mathbf{p^c}$; if it has no positive flow we immediately obtain a contradiction. If it has positive flow it follows that,

$$\begin{aligned}
l_{nc}(x_n^c x^w) + p_n^c &= u'(x^c), \\
&= l_{ic}(x_i^c x^w) + p_i^c \\
&< l_{ic}(\overline{x}_i^c x^w) + \overline{p}_i^c, \\
&= l_{nc}(\overline{x}_n^c x^w) + \overline{p}_n^c. \qquad (2.7)
\end{aligned}$$

We obtain the first two equalities from Lemma 1. The third inequality arises from our assumption and the fact that $\overline{p}_i^c > p_i^c$ and the fourth from Lemma 1. Since $\overline{p}_n^c = p_n^c$ for $n \neq i$ it follows that $\overline{x}_n^c > x_n^c$. Therefore, $\overline{x}^c > x^c$ implying that $u'(x^c) > u'(\overline{x}^c)$ which yields a contradiction.

Case 2: $x^c = 1$.

Suppose that $\overline{x}_i^c \geq x_i^c$. We obtain the following set of inequalities,

$$\begin{aligned}
l_{nc}(x_n^c x^w) + p_n^c &= l_{ic}(x_i^c x^w) + p_i^c \\
&< l_{ic}(\overline{x}_i^c x^w) + \overline{p}_i^c, \\
&= l_{nc}(\overline{x}_n^c x^w) + \overline{p}_n^c. \qquad (2.8)
\end{aligned}$$

The first and third inequality follow from Lemma 1, the second arises from the assumptions that $\overline{x}_i^c \geq x_i^c$ and $\overline{p}_i^c > p_i^c$. It follows that $\overline{x}_n^c > x_n^c$. This yields a contradiction since $\overline{x}^c \leq 1$ and $\overline{x}_i^c \geq x_i^c$ imply that $\overline{x}_n^c \leq x_n^c$.

b) We next show that $\overline{x}_n^c > x_n^c$. Suppose that $\overline{x}_n^c \leq x_n^c$. We first assume $\overline{x}_n^c < x_n^c$. Since $\overline{p}_n^c = p_n^c$ for $n \neq i$, then by Lemma 1 and the fact that $l_{ic}(\cdot)$ is strictly increasing,

$$u'(\overline{x}^c) = l_{nc}(\overline{x}_n^c x^w) + \overline{p}_n^c,$$

$$
\begin{aligned}
&< l_{nc}(x_n^c x^w) + p_n^c, \\
&\leq u'(x^c). \qquad (2.9)
\end{aligned}
$$

We next assume that $\overline{x}_n^c = x_n^c$. It follows, from the proof in part a), that $\overline{x}^c < x^c$. Therefore we get the following set of inequalities,

$$
\begin{aligned}
u'(\overline{x}^c) &= l_{ic}(\overline{x}_i^c x^w) + \overline{p}_i^c \\
&= l_{nc}(\overline{x}_n^c x^w) + \overline{p}_n^c, \\
&= l_{nc}(x_n^c x^w) + p_n^c, \\
&\leq u'(x^c). \qquad (2.10)
\end{aligned}
$$

The above inequalities follow from Lemma 1 and the fact that $\overline{x}^c < x^c \leq 1$. Thus $u'(x^c) \geq u'(\overline{x}^c)$, which implies that $\overline{x}^c \geq x^c$ yielding a contradiction.

c) We now show that $\overline{x}^c \leq x^c$. This follows from directly applying the results from part b). The following inequalities follow from Lemma 1 and the fact that $\overline{x}_n^c > x_n^c$,

$$
\begin{aligned}
u'(\overline{x}^c) &\geq l_{nc}(\overline{x}_n^c x^w) + \overline{p}_n^c, \\
&> l_{nc}(x_n^c x^w) + p_n^c, \\
&= u'(x^c). \qquad (2.11)
\end{aligned}
$$

We deduce that $\overline{x}^c \leq x^c$ since $u'(x^c) \leq u'(\overline{x}^c)$.

■

We next define the best response vector $\mathbf{p_i^*}$ for a platform $i \in \mathcal{I}$ given the price vector $\mathbf{p_{-i}}$ offered by the other platform. This definition will be used in Lemma 4 as well as in the proof of Proposition 2.

Definition 5. A price vector $\mathbf{p_i^*}$ is a best response to a price vector $\mathbf{p_{-i}}$ if

$$
(\mathbf{p_i^*}, \mathbf{x}) \in \arg \max_{\mathbf{p_i},\, \mathbf{x} \in LE(\mathbf{p}_i, \mathbf{p_{-i}})} (p_i^c - c^c) x_i^c x^w + (p_i^w - c^w) x_i^w x^c. \qquad (2.12)
$$

Lemma 4. Let Assumptions 1 through 5 hold; let $\mathbf{p_i^*}$ be a best response price vector to a price vector $\mathbf{p_{-i}}$. If $\Pi_i(\mathbf{p_i^*}, \mathbf{p_{-i}}) = 0$ then $\Pi_{-i}(\mathbf{p_{-i}}, \mathbf{p_i^*}) < 0$. Moreover, we have $\mathbf{x}^* = 0$ where $\mathbf{x}^* \in LE(\mathbf{p_{-i}}, \mathbf{p_i^*})$.

Proof. Let $\rho_i^w = (p_i^w - c^w)x_i^w x^c$ and $\rho_i^c = (p_i^c - c^c)x_i^c x^w$ then $\Pi_i(\mathbf{p_i^*}, \mathbf{p_{-i}}) = \rho_i^w + \rho_i^c = 0$. We claim that $\rho_i^w = 0$ and $\rho_i^c = 0$. We then show that $\mathbf{x}^* = 0$. We first show that cases (i) and (ii) are not possible.

i) $\rho_i^w > 0$ and $\rho_i^c < 0$,

ii) $\rho_i^w < 0$ and $\rho_i^c > 0$,

Assume to arrive at a contradiction that the first case holds. Consider setting price p_i^{c*} to c^c and leaving p_i^{w*} unchanged. We denote this new price vector as $\tilde{\mathbf{p}}_\mathbf{i}^*$. We claim that $\tilde{x}^c$ is positive. For if $\tilde{x}^c = 0$ then by Lemma 1, we have $\tilde{p}_i^c = c^c \geq u'(0)$ which violates Assumption 3. It follows that $\Pi_i(\tilde{\mathbf{p}}_\mathbf{i}^*, \mathbf{p_{-i}})$ is positive since, by Lemma 2, the allocation x_i^w is not affected by the price change. This contradicts the hypothesis that $\mathbf{p_i^*}$ is a best response.

Assume to arrive at a contradiction that case (ii) holds. Consider setting p_i^{w*} to c^w. We denote the new price vector as $\hat{\mathbf{p}}_\mathbf{i}^*$. Because of the full coverage condition, see Assumption 5, setting p_i^{w*} to c^w and leaving p_i^{c*} unchanged does not affect x_i^{c*}. This implies that ρ_i^c remains positive. Therefore $\Pi_i(\hat{\mathbf{p}}_\mathbf{i}^*, \mathbf{p_{-i}})$ becomes positive contradicting the hypothesis.

Therefore $\rho_i^w = 0$ and $\rho_i^c = 0$. We first show that $x_i^{c*} = 0$ and $x_i^{w*} = 0$. Assume to arrive at a contradiction that $x_i^{c*} > 0$. It follows that $p_i^{c*} = c^c$. Let $\bar{p}_i^c = K - \epsilon > c^c$ where $u'(x^c) \geq K = c^c + l_{ic}(x_i^{c*})$ for some small $\epsilon > 0$. It can be seen that, at the price vector $(\bar{p}_i^c, p_{-i}^c)$, the corresponding WE consumer mass assignment on platform i, $\bar{x}_i^c$, is positive. Therefore platform i has an incentive to deviate to price, $\bar{p}_i^c$, and make positive profit contradicting that $\mathbf{p_i^*}$ is a best response . Similarly if $x_i^{w*}x^{c*} > 0$ then $p_i^{w*} = c^w$. Let $\bar{p}_i^w = K - \epsilon > c^w$ where $K = c^w + x_i^{w*}t$ for some small $\epsilon > 0$. At the price vector $(\bar{p}_i^w, p_{-i}^w)$, the corresponding HE website mass assignment on platform i, $\bar{x}_i^w$, is positive. Thus platform i has an incentive to deviate to price, $\bar{p}_i^w$, and make positive profit contradicting that $\mathbf{p_i^*}$ is a best response.

Therefore both $x_i^{c*} = 0$ and $x_i^{w*}x^{c*} = 0$. This implies that either $x_i^{w*} = 0$ or $x_n^{c*} = 0$. We show that $x_n^{c*} > 0$. If $x_n^{c*} = 0$ then $p_i^{c*} \geq u'(0)$ since $x_i^{c*} = 0$. Let $\bar{p}_i^c = K - \epsilon > c^c$ where $K = u'(0)$ for some small $\epsilon > 0$. Note from Assumption 3 that $K > c^c$. It can be seen that at the price vector $(\bar{p}_i^c, p_{-i}^c)$ that the corresponding WE consumer mass assignment on platform i, $\bar{x}_i^c$, is positive. This contradicts the fact that $\mathbf{p_i^*}$ is a best response. It follows that $x_i^{w*} = 0$.

By Assumption 5, $x_n^w = 1$. Let $K = p_n^w + x_n^w t$. We claim that $K \leq c^w$ for if $K > c^w$ then platform i can set $\bar{p}_i^w = K - \epsilon > c^w$ for a small $\epsilon > 0$ which will result in $\bar{x}_i^w$, the corresponding HE website mass assignment on platform i, being positive as previously seen above. Thus platform i will have an incentive to deviate to price, $\bar{p}_i^w$, and make positive profit contradicting that $\mathbf{p_i^*}$ is a best response. Therefore $p_n^w < c^w$. In a similar manner one can show that $p_n^c < c^c$. Therefore, it follows that $\Pi_{-i}(\mathbf{p_{-i}}, \mathbf{p_i^*}) = (p_{-i}^c - c^c)x_{-i}^c x^w + (p_{-i}^w - c^w)x_{-i}^w x^c < 0$ where $\mathbf{p_{-i}} = \mathbf{p_n}$, $x_{-i}^c = x_n^c$ and $x_{-i}^w = x_n^w$. ■

In the following proposition we show we show that if the congestion costs are linear then there exists a pure strategy Oligopoly Equilibrium (OE).

Proposition 2. Let Assumptions 1 through 5 hold. Assume further that the latency functions are affine. Then the price competition game has a pure strategy Oligopoly Equilibrium (OE).

Proof. We define $B(\mathbf{p}*) = [B_i(\mathbf{p_{-i}^*})]_{i\in\mathcal{I}}$. Where $B_i(\mathbf{p}_{-i}^*)$ is defined as the set of best response vectors given $\mathbf{p}_{-i}^*$, i.e., the set of all vectors $\mathbf{p}_i^*$ that meet definition (2.12).

By the Theorem of the Maximum [58], it follows that $B(\mathbf{p}*)$ is an upper semi-continuous correspondence. We proceed to rewrite the profit of platform i as follows,

$$\Pi_i(\mathbf{p_i}, \mathbf{p_{-i}}) = \underbrace{(p_i^c - c^c)x_i^c}_{\rho_i^c} + \underbrace{(p_i^w - c^w)x_i^w x^c}_{\rho_i^w}, \tag{2.13}$$

where, $\rho_i^{w*} = (p_i^w - c_w)x_i^w x^c$ and $\rho_i^{c*} = (p_i^c - c^c)x_i^c$. We now show that this correspondence is convex-valued. Let $\mathbf{\bar{p}_i^*} \in B_i(\mathbf{p_{-i}}^*)$ and $\mathbf{p_i^*} \in B_i(\mathbf{p_{-i}^*})$ be two best response vectors and

$\bar{\Pi}_i(.)$ and $\Pi_i(\cdot)$ their respective profits. Suppose $\bar{\Pi}_i(\cdot) = \Pi_i(.) = 0$ then

$$(p_i^c - c^c)x_i^c + (p_i^w - c^w)x_i^w x^c = (\bar{p}_i^c - c^c)\bar{x}_i^c + (\bar{p}_i^w - c_w)\bar{x}_i^w \bar{x}^c = 0.$$

Consider a price $\mathbf{p_i^\delta}$ such that $\mathbf{p_i^\delta} = \delta \mathbf{p_i^*} + (1-\delta)\bar{\mathbf{p}}_\mathbf{i}^*$ for $\delta \in [0,1]$. Let $\mathbf{x}^\delta \in LE(\mathbf{p}_i^\delta, \mathbf{p}_{-\mathbf{i}}^*)$. It follows that $(p_i^{\delta c} - c^c)x_i^{\delta c} + (p_i^{\delta w} - c^w)x_i^{\delta w}x^{\delta c} = 0$ which implies that $\mathbf{p_i^\delta} \in B_i(\mathbf{p}_{-i}^*)$. The equality follows from Lemma 4, in particular, the fact that $x_i^{c*} = \bar{x}_i^{c*} = 0$ and $x_i^{w*} = \bar{x}_i^{w*} = 0$. It follows that $p_i^{c*} \geq p_n^{c*} + l_{nc}(x_n^{c*}) = K$ and $\bar{p}_i^{c*} \geq p_n^{c*} + l_{nc}(x_n^{c*}) = K$, therefore any linear combination of the two prices also gives a price larger or equal to K which implies $x_i^{\delta c} = 0$. In a similar way one can show that $x_i^{\delta w} = 0$.

Next we consider the case when $\bar{\Pi}_i(\cdot) = \Pi_i(\cdot) > 0$. We first show that $\bar{\rho}_i^{c*} = \rho_i^{c*}$. Without loss of generality, we assume to arrive at the contradiction that $\bar{\rho}_i^{c*} < \rho_i^{c*}$. It follows that $\bar{\rho}_i^{w*} > \rho_i^{w*} \geq 0$ since $\bar{\Pi}_i(\cdot) = \Pi_i(\cdot)$. The second inequality follows from the fact that if $\rho_i^{w*} < 0$ then the platform could set its website price to c^w and increase its profit since there is full coverage and a change in the price of p_i^{w*} does not affect x_i^{c*}. It is also the case that $p_i^{c*} \neq \bar{p}_i^{c*}$ since if they were equal $\bar{\rho}_i^{c*} = \rho_i^{c*}$.

Since $\bar{\rho}_i^{w*} > \rho_i^{w*} \geq 0$, it follows that $(\bar{p}_i^{w*} - c^w)\bar{x}_i^{w*} \leq (p_i^{w*} - c^w)x_i^{w*}$. For if $(\bar{p}_i^{w*} - c^w)\bar{x}_i^{w*} > (p_i^{w*} - c^w)x_i^{w*}$ then platform i can charge the same price p_i^{c*} on the consumer side and change the price on the website side to $\bar{p}_i^{w*}$. This would increase profit for platform i on the website side since $(\bar{p}_i^{w*} - c^w)\bar{x}_i^{w*}x^{c*} > (p_i^{w*} - c^w)x_i^{w*}x^{c*}$ contradicting the fact that $\mathbf{p_i^*}$ is a best response vector. Since $\bar{\rho}_i^{w*} > \rho_i^{w*} \geq 0$ and $(\bar{p}_i^{w*} - c^w)\bar{x}_i^{w*} \leq (p_i^{w*} - c^w)x_i^{w*}$ it follows that $\bar{x}^{c*} > x^{c*}$.

To help us with the characterization of best response prices we show that given our assumptions $\bar{\Pi}_i(\cdot) = \Pi_i(\cdot) > 0$ and $\bar{\rho}_i^{c*} < \rho_i^{c*}$ the following hold $\bar{x}_i^{w*} > 0$, $x_i^{w*} > 0$, $x_i^{c*} > 0$ and $\bar{x}_i^{c*} > 0$. Note that $x_i^{c*} > 0$ since we assume $\bar{\rho}_i^{c*} < \rho_i^{c*}$. We next show that $\bar{x}_i^{c*} > 0$. Assume that $\bar{x}_i^{c*} = 0$ then the price $\bar{p}_i^{c*} \geq u'(0)$. If this price is lowered to $\hat{p}_i^c = u'(0) - \epsilon$, where $\epsilon > 0$, then by Lemma 1, $\hat{x}_i^{c*} > 0$. It then follows, by Lemma 3, that $\hat{x}^{c*} \geq \bar{x}^{c*}$. This implies that $\hat{\Pi}_i > \bar{\Pi}_i$ contradicting that $\bar{\mathbf{p}}_\mathbf{i}$ is a best response. Therefore $\bar{x}_i^{c*} > 0$. From our assumption, $\bar{\rho}_i^{w*} > \rho_i^{w*}$, it is immediate that $\bar{x}_i^{w*} > 0$.

Now we show that $x_i^{w*} > 0$ is also positive. We have shown, given our assumptions

$\bar{\Pi}_i(\cdot) = \Pi_i(\cdot) > 0$ and $\bar{\rho}_i^{c*} < \rho_i^{c*}$, that the following hold $\bar{x}_i^{w*} > 0$ and $x_i^{c*} > 0$. Assume $x_i^{w*} = 0$. Consider setting p_i^{w*} to $\bar{p}_i^w$ it follows that $x_i^{w*} = \bar{x}_i^{w*} > 0$. Since $x_i^{c*} > 0$ it follows that $x^{c*} > 0$ and $\rho_i^{w*} > 0$. The profit $\Pi_i(\cdot)$ increases since ρ_i^{c*} is unchanged which contradicts that $\mathbf{p_i}$ is a best response.

Therefore $\bar{x}_i^{c*} > 0$, $\bar{x}_i^{w*} > 0$, $x_i^{c*} > 0$ and $x_i^{w*} > 0$ are positive and the first order conditions for the following maximization problem ,

$$\text{maximize}_{\mathbf{p_i},\, \mathbf{x} \in LE(\mathbf{p_i}, \mathbf{p^*_{-i}})} \{(p_i^c - c^c)x_i^c x^w + (p_i^w - c^w)x_i^w x^c.\} \tag{2.14}$$

yield the following price characterizations for the two prices p_i^{c*} and $\bar{p}_i^{c*}$(cf. proof of Proposition 3)[3],

$$\bar{p}_i^{c*} = c^c + \bar{x}_i^{c*}\left(a_{ic} + \frac{1}{\frac{1}{a_{nc}} - \frac{1}{u''(\bar{x}^{c*})}}\right) + (\bar{p}_i^{w*} - c^w)\bar{x}_i^{w*}\left(\frac{\frac{1}{a_{nc}}\left(1 - \frac{\phi}{(\bar{p}_i^{w*} - c^w)\bar{x}_i^{w*}}\right)}{\frac{1}{a_{nc}} - \frac{1}{u''(\bar{x}^{c*})}} - 1\right) + \phi,$$
$$p_i^{c*} = c^c + x_i^{c*}\left(a_{ic} + \frac{1}{\frac{1}{a_{nc}} - \frac{1}{u''(x^{c*})}}\right) + (p_i^{w*} - c^w)x_i^{w*}\left(\frac{\frac{1}{a_{nc}}}{\frac{1}{a_{nc}} - \frac{1}{u''(x^{c*})}} - 1\right).$$

Since $\bar{x}_i^{c*} > 0$, $\bar{x}^{c*} > x^{c*}$ we deduce from the contrapositive of Lemma 3 that $\bar{p}_i^{c*} < p_i^{c*}$. This also implies that $\bar{x}_i^{c*} > x_i^{c*}$. Applying the following inequalities, $\bar{x}_i^{c*} > x_i^{c*}$, $(\bar{p}_i^{w*} - c^w)\bar{x}_i^{w*} \leq (p_i^{w*} - c^w)x_i^{w*}$, $\phi \geq 0$ and $\bar{x}^{c*} > x^{c*}$, in the two price characterizations above, yields $\bar{p}_i^{c*} > p_i^{c*}$. This contradicts the assumption that $\bar{\rho}_i^{c*} < \rho_i^{c*}$, therefore $\bar{\rho}_i^{c*} = \rho_i^{c*}$.

We now show that $p_i^{c*} = \bar{p}_i^{c*}$. Since $\bar{\rho}_i^{c*} = \rho_i^{c*}$ and $\bar{\Pi}_i(.) = \Pi_i(.)$, it follows that $\bar{\rho}_i^{w*} = \rho_i^{w*}$. Assume without loss of generality that $p_i^{c*} > \bar{p}_i^{c*}$. By Assumption 5, the market is covered and thus $\mathbf{x^w} \neq 0$. Therefore, we can apply Lemma 3 to conclude $\bar{x}_i^{c*} > x_i^{c*}$ and $\bar{x}^{c*} \geq x^{c*}$. These inequalities, together with the following relation $\bar{\rho}_i^{w*} = \rho_i^{w*}$, imply that $(\bar{p}_i^{w*} - c^w)\bar{x}_i^{w*} < (p_i^{w*} - c^w)x_i^{w*}$. Applying these inequalities to the price characterizations derived previously for the two prices yields $\bar{p}_i^{c*} > p_i^{c*}$, a contradiction.

Having established that $\bar{p}_i^{c*} = p_i^{c*}$, it follows that $\bar{x}^{c*} = x^{c*}$. The first order conditions of problem (2.14) give us the following expressions for prices on the website side (cf proof

[3]The only modification is the addition of the constraints $\bar{x}_i^{c*} + \bar{x}_n^{c*} \leq 1$ which is assigned the lagrange multiplier ϕ.

of Proposition 3),

$$\bar{p}_i^{w*} = 2\bar{x}_i^{w*}t + c^w,$$

$$p_i^{w*} = 2x_i^{w*}t + c^w.$$

Using the above price characterizations for the website side and the fact that $\bar{\rho}_i^{w*} = \rho_i^{w*}$ we determine that $\bar{x}_i^{w*} = x_i^{w*}$, which in turn implies $\bar{p}_i^{w*} = p_i^{w*}$. This shows $B(\mathbf{p}*)$ is convex valued. We can now use Kakutani's fixed point theorem [8] to assert the existence of $\mathbf{p}*$ such that $B(\mathbf{p}*) = \mathbf{p}*$. ■

We need the following two Lemmas for our price characterization. The first shows in an OE all the platforms make positive profit. The subsequent Lemma shows that at the OE consumers and websites join both platforms.

Lemma 5. Let Assumptions 1 through 5 hold; let $(\mathbf{p^{OE}}, \mathbf{x^{OE}})$ be a pure strategy Oligopoly Equilibrium OE. Then $\Pi_i^{OE}(\cdot) > 0$ for all $i \in \mathcal{I}$.

Proof. Since $(\mathbf{p^{OE}}, \mathbf{x^{OE}})$ is an OE, the price vectors $\mathbf{p_i^{OE}}$ and $\mathbf{p_n^{OE}}$ are best responses to each other. Assume there exists an $i \in \mathcal{I}$ such that $\Pi_i^{OE}(\cdot) = 0$. Since the price vector $\mathbf{p_i^{OE}}$ is a best response to the price vector $\mathbf{p_n^{OE}}$, it follows that $\Pi_n^{OE}(\cdot) < 0$ by Lemma 4. But platform n can set the following prices $p_n^c = c^c$ and $p_n^w = c^w$ increasing profits for platform n from negative to zero. This contradicts that $(\mathbf{p^{OE}}, \mathbf{x^{OE}})$ is an Oligopoly Equilibrium. ■

Lemma 6. Let Assumption 1 through 5 hold. Let $(\mathbf{p^{OE}}, \mathbf{x^{OE}})$ be a pure strategy Oligopoly Equilibrium. Then for all $i \in \mathcal{I}$, x_i^{wOE} and x_i^{cOE} are positive.

Proof. By Lemma 5, $\Pi_i(.)^{OE} > 0$ for all $i \in \mathcal{I}$, therefore $x^{cOE} > 0$ and by Assumption 5, there's full coverage so that $x^{wOE} = 1$. If $x_i^{cOE} = 0$ and $x_i^{wOE} = 0$ then $\Pi_i^{OE}(.) = 0$ contradicting Lemma 5. We will show that both x_i^{cOE} and x_i^{wOE} are positive for all $i \in \mathcal{I}$. Assume to arrive at a contradiction that there exists an i such that $x_i^{cOE} = 0$. Then $x_n^{cOE} = x^{cOE} > 0$. From Lemma 1 and the fact that $\mathbf{p^{OE}}$ is a pure OE we have ,

$$p_n^{cOE} + l_{nc}(x_n^{cOE}) = K \leq u'(x^{cOE}).$$

By Assumption 3, $u'(x^{cOE}) > c^c$. Consider setting price p_i^{cOE} to $\tilde{p}_i^c = K - \epsilon > c^c$ for some small $\epsilon > 0$. At the price vector $(\tilde{p}_i^c, p_n^{cOE})$ the corresponding WE results in $\tilde{x}_i^c > 0$. Therefore platform i has an incentive to deviate to price $\tilde{p}_i^c$ contradicting that $\mathbf{p^{OE}}$ is an OE.

In a similar manner we show that $x_i^{wOE} > 0$. Assume that $x_i^{wOE} = 0$, by the full coverage assumption $x_n^{wOE} = 1$. Let $k = p_n^{wOE} + x_n^{wOE}t$, it follows that $k < c^w$. If it were larger we can set p_i^{wOE} to $\tilde{p}_i^w = k - \epsilon$ for some small $\epsilon > 0$. From Lemma 2 we have $\tilde{x}_i^w > 0$ which increases platform's i's profit implying $\mathbf{p^{OE}}$ is not an OE. Therefore $p_n^{wOE} < c^w$ which implies $\rho_n^{wOE} < 0$. But by setting the price p_n^{wOE} to c^w we have $\rho_n^{wOE} = 0$ and $\Pi_n^{OE}(.)$ increases since ρ_n^{cOE} doesn't change. This contradicts the fact that $\mathbf{p^{OE}}$ is an Oligopoly Equilibrium. ∎

We next provide an explicit characterization of the OE prices. This characterization will help us in decomposing the pricing strategy of the platforms at the OE.

Proposition 3. Let $(\mathbf{p}^{OE}, \mathbf{x}^{OE})$ be an OE. Let Assumptions 1 through 5 hold.

a) Assume that $l_{ic}(x_i^c) + p_i^c = u'(x^c)$, then the consumer side Oligopoly Equilibrium price for a platform $i \in \mathcal{I}$ is given by,

$$\begin{aligned} p_i^{cOE} &= c^c + x_i^{cOE}\left(l'_{ic}(x_i^{cOE}) + \frac{1}{\frac{1}{l'_{nc}(x_n^{cOE})} - \frac{1}{u''(x^{cOE})}}\right) \\ &\quad + (p_i^{wOE} - c^w)x_i^{wOE}\left(\frac{\frac{1}{l'_{nc}(x_n^{cOE})}}{\frac{1}{l'_{nc}(x_n^{cOE})} - \frac{1}{u''(x^{cOE})}} - 1\right). \end{aligned} \tag{2.15}$$

b) Assume that $l_{ic}(x_i^c) + p_i^c < u'(x^c)$, then the consumer side Oligopoly Equilibrium price for a platform $i \in \mathcal{I}$ is given by,

$$p_i^{cOE} = c^c + x_i^{cOE}\left(l'_{ic}(x_i^{cOE}) + \frac{1}{\frac{1}{l'_{nc}(x_n^{cOE})}}\right) \tag{2.16}$$

c) The Oligopoly Equilibrium price for i on the website side is given by,

$$p_i^{wOE} = t + c^w. \tag{2.17}$$

Proof. For a platform i the vector $(\mathbf{p_i^{OE}}, \mathbf{x_i^{OE}})$ is an optimal solution to the maximization problem (2.14). Note that since $\Pi_i^{OE} > 0$ we can rewrite the optimization problem as follows,

$$\underset{p_i^w,\, p_i^c,\, \mathbf{x}\geq 0}{\text{maximize}} \qquad (p_i^c - c^c)x_i^c + (p_i^w - c^w)x_i^w x^c \tag{2.18}$$

$$\begin{aligned} \text{subject to} \quad & h1: \; l_{ic}(x_i^c) + p_i^c = l_{nc}(x_n^c) + p_n^{c*} \quad n \neq i\,, n \in I, \\ & h2: \; l_{ic}(x_i^c) + p_i^c \leq u'(x^c), \\ & h3: \; x_i^w t + p_i^w = x_n^w t + p_n^{w*} \quad n \neq i\,, n \in I, \\ & h4: \; \textstyle\sum_i x_i^w = 1. \end{aligned}$$

Note that by Lemma 6 we have $x_i^{wOE} > 0$ and $x_i^{cOE} > 0$ for all $i \in \mathcal{I}$. By Assumption 5 we have full coverage on the website side, i.e. $x^w = 1$.

We first assume that $l_{ic}(x_i^c) + p_i^c = u'(x^c)$ and $x^c < 1$. The prices p_i^{cOE}, p_i^{wOE} and the masses of consumers and websites x_i^{cOE}, x_n^{cOE} and x_i^{wOE}, x_n^{wOE} form a regular feasible vector f, i.e., $\nabla h_1(f), \cdots, \nabla h_4(f)$ are linearly independent. The linear independence of the constraint vectors can be verified by considering the matrix H consisting of the constraint gradient vectors evaluated at vector f. We define H as follows:

$$H \equiv \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ l'_{ic}(x_i^{cOE}) & l'_{ic}(x_i^{cOE}) - u''(x^{cOE}) & 0 & 0 \\ -l'_{nc}(x_i^{cOE}) & -u''(x^{cOE}) & 0 & 0 \\ 0 & 0 & t & 1 \\ 0 & 0 & -t & 1 \end{bmatrix}.$$

Each column i represents the constraint gradient $\nabla h_i(f)$ where the differentiation for each row has been done with respect to $p_i^c, p_i^w, x_i^c, x_n^c, x_i^w, x_n^w$. It can be verified that the above columns are linearly independent. This follows since there does not exist a non-zero vector $\mathbf{z}$ such that $\mathbf{Hz} = \mathbf{0}$. To see this note that $z_3 = 0$ since this is the only way the second row can sum to 0 in the system **Hz**. This further implies that z_4 is also 0 so that rows 5 and 6 sum to zero. This leaves us to verify that columns 1 and 2 are linearly independent. Since the entries of column 1 and 2 are the same these two columns are linearly dependent if and only if all their entries are equal. But it is clear from the matrix that this is not the case. Therefore H has linearly independent columns.

We can now use Karush Kuhn Tucker(KKT) conditions to derive the necessary conditions for the above maximization problem, see [7]. We assign the Lagrange multipliers λ_n^c to constraint h_1, μ to constraint h_2, γ_n^w to constraint h_3 and α to constraint h_4. We can write the Lagrangian of problem (2.18) as follows,

$$
\begin{aligned}
& L(p_i^c, p_i^w, x_i^c, x_i^w, x_n^c, x_n^w, \lambda_n^c, \mu, \gamma_n^w, \alpha, \theta) = \\
& \quad (p_i^c - c^c)x_i^c + (p_i^w - c^w)x_i^w x^c - \lambda_n^c(l_{ic}(x_i^c) + p_i^c - l_{nc}(x_n^c) - p_n^{c*}) \\
& \quad -\mu(l_{ic}(x_i^c) + p_i^c - u'(x^c)) - \gamma_n^w(x_i^w t + p_i^w - x_n^w t - p_n^{w*}) \\
& \quad -\alpha(\sum_i x_i^w - 1).
\end{aligned}
$$

We drop the arguments in the Lagrangian function and refer to it as simply L. The KKT conditions are derived as follows,

$$
\begin{aligned}
\frac{\partial L}{\partial p_i^c} &: x_i^{c*} - \lambda_n^c - \mu = 0, && (2.19) \\
\frac{\partial L}{\partial p_i^w} &: x_i^{w*} x^{c*} - \gamma_n^w = 0, && (2.20) \\
\frac{\partial L}{\partial x_i^c} &: (p_i^{c*} - c^c) + (p_i^{w*} - c^w)x_i^{w*} + (-\mu - \lambda_n^c)l'_{ic}(x_i^{c*}) + \mu u''(x^{c*}) = 0, && (2.21) \\
\frac{\partial L}{\partial x_n^c} &: (p_i^{w*} - c^w)x_i^{w*} + \lambda_n^c l'_{nc}(x_n^{c*}) + \mu u''(x^{c*}) = 0, \quad n \neq i, && (2.22) \\
\frac{\partial L}{\partial x_i^w} &: (p_i^{w*} - c^w)x^{c*} - \gamma_n^w t - \alpha = 0, && (2.23) \\
\frac{\partial L}{\partial x_n^w} &: \gamma_n^w t - \alpha = 0 \;\; n \neq i. && (2.24)
\end{aligned}
$$

From Eq. (2.22) we obtain,

$$\lambda_n^c = \frac{-(p_i^{w*} - c^w)x_i^{w*} - \mu u''(x^{c*})}{l'_{nc}(x_n^{c*})}. \tag{2.25}$$

Using the above expression for λ_n^c together with Eq. (2.19), we obtain the following,

$$-\mu = \frac{\frac{1}{u''(x^{c*})}\left[\frac{(p_i^{w*}-c^w)x_i^{w*}}{l'_{nc}(x_n^{c*})} + x_i^{c*}\right]}{\frac{1}{l'_{nc}(x_n^{c*})} - \frac{1}{u''(x^{c*})}}. \tag{2.26}$$

Plugging μ into Eq. (2.21) and then using Eq. (2.19) we derive p_i^{c*}.

$$\begin{aligned} p_i^{c*} = c^c + x_i^{c*} &\left(l'_{ic}(x_i^{c*}) + \frac{1}{\frac{1}{l'_{nc}(x_n^{c*})} - \frac{1}{u''(x^{c*})}} \right) \\ + (p_i^{w*} - c^w)x_i^{w*} &\left(\frac{\frac{1}{l'_{nc}(x_n^{c*})}}{\frac{1}{l'_{nc}(x_n^{c*})} - \frac{1}{u''(x^{c*})}} - 1 \right). \end{aligned}$$

If $x^c = 1$ then it follows that $p_i^{c*} = u'(x^{c*}) - l_{ic}(x_i^{c*})$.

We next assume that $l_{ic}(x_i^c) + p_i^c < u'(x^c)$. From Lemma 1 this implies that

$$x_i^c + x_n^c = 1.$$

If we solve the maximization problem (2.18) with the above constraint added we get the following price characterization,

$$p_i^{c*} = c^c + x_i^{c*}\left(l'_{ic}(x_i^{c*}) + \frac{1}{\frac{1}{l'_{nc}(x_n^{c*})}} \right) \tag{2.27}$$

We next derive the expression for p_i^{w*}. In this instance the expression follows directly from Eq. (2.23), Eq. (2.24) and Eq. (2.20). By solving for p_i^{w*} we find that the website price is given by,

$$p_i^{w*} = 2x_i^{w*}t + c^w.$$

From Lemma 6, we have that $x_i^{w*} > 0$ for all $i \in \mathcal{I}$. This implies via Lemma 2 that $2x_n^{w*}t + c^w + x_n^{w*}t = 2x_i^{w*}t + c^w + x_i^{w*}t$. Therefore, $x_i^{w*} = x_n^{w*}$. By Assumption 5, the full coverage condition we have that $x_i^{wOE} = 1/2$ for all $i \in \mathcal{I}$ and $p_i^{w*} = t + c^w$ as presented in the proposition. ■

The analysis of the Karush Kuhn Tucker conditions for the oligopoly problem analyzed above yields the following price characterizations on the consumer side:

Corollary 1. Let $(\mathbf{p}^{OE}, \mathbf{x}^{OE})$ be an OE; let Assumptions 1 through 5 hold. Then for all $i \in \mathcal{I}$ the consumer side prices are given by,

$$p_i^{cOE} = \begin{cases} c^c + x_i^{cOE}\left(l'_{ic}(x_i^{cOE}) + \frac{1}{\frac{1}{l'_{nc}(x_n^{cOE})} - \frac{1}{u''(x^{cOE})}}\right) \\ +(p_i^{wOE} - c^w)x_i^{wOE}\left(\frac{\frac{1}{l'_{nc}(x_n^{cOE})}}{\frac{1}{l'_{nc}(x_n^{cOE})} - \frac{1}{u''(x^{cOE})}} - 1\right), & \text{if } \sum_{i\in\mathcal{I}} x_i^{cOE} < 1, \\ \min\left\{u'(1) - l_{ic}(x_i^{cOE}),\ c^c + x_i^{cOE}\left(l'_{ic}(x_i^{cOE}) + \frac{1}{\frac{1}{l'_{nc}(x_n^{cOE})}}\right), \right. & \text{if } \sum_{i\in\mathcal{I}} x_i^{cOE} = 1. \end{cases}$$

We next show that we can interpret the constraint $l_{ic}(x_i^c x^w) + p_i^c = u'(x^c)$ for all $i \in \mathcal{I}$ as defining $\mathbf{x}^c \in \mathbb{R}_+^I$ as a function of $\mathbf{p}^c$. In other words the inverse function $\mathbf{x}^c$ as a function of $\mathbf{p}^c$ is well defined. We will use the inverse function and its differentiability properties to rewrite the equilibrium price characterization.

Lemma 7. Let Assumptions 1 and 4 hold. Then $l_{ic}(x_i^c x^w) + p_i^c = u'(x^c)$ for all $i \in \mathcal{I}$ implicitly defines $\mathbf{x}^c$ as a function of $\mathbf{p}^c$ and this function is continuously differentiable.

Proof. Let $p(\mathbf{x}^c) = (p_i^c(x_i^c, x_n^c), p_n^c(x_i^c, x_n^c))$ be a function of I variables from R_+^I to R_+^I. Where $p_i^c(x_i^c, x_n^c) = u'(x^c) - l_{ic}(x_i^c x^w)$ for all $i \in \mathcal{I}$. By Assumption 1 and 4, $p(\mathbf{x}^c)$ is continuously differentiable. Therefore the Jacobian of $p(\mathbf{x}^c)$ is given by matrix M defined as follows,

$$M \equiv \begin{bmatrix} u''(x^c) - l'_{ic}(x_i^c x^w)x^w & u''(x^c) \\ u''(x^c) & u''(x^c) - l'_{nc}(x_n^c x^w)x^w \end{bmatrix}.$$

The determinant of M is given by,

$$\det M = l'_{ic}(x_i^c x^w) l'_{nc}(x_n^c x^w)(x^w)^2 - u''(x^c)(l'_{nc}(x_n^c x^w)x^w + l'_{ic}(x_i^c x^w)x^w).$$

The above determinant is positive because by Assumption 4 the latency functions are strictly increasing and therefore their derivatives are positive and by Assumption 1, the second derivative of the utility function is negative. Having shown that the Jacobian is non-singular, we can now apply the Inverse Function Theorem [3] to conclude that the inverse function is continuously differentiable. ■

We now proceed to give a natural interpretation of the price characterization in Proposition 3. After some calculations we can derive the following relations,

$$\begin{aligned}\frac{1}{\frac{1}{l'_{nc}(x_n^c)} - \frac{1}{u''(x^c)}} &= -l'_{nc}(x_n^c)\frac{\partial x_n^c/\partial p_i^c}{\partial x_i^c/\partial p_i^c}, \\ &= -l'_{nc}(x_n^c)\frac{\varepsilon_{in}x_n^c}{\varepsilon_{ii}x_i^c},\end{aligned}$$

where $\varepsilon_{in} = -\frac{\partial x_n^c/\partial p_i^c}{x_n^c/p_i^c}$ is the cross-elasticity of demand on link n with respect to the price on link i. Similarly $\varepsilon_{ii} = -\frac{\partial x_i^c/\partial p_i^c}{x_i^c/p_i^c}$ is the own elasticity of demand of link i. It can also be shown that,

$$\frac{\frac{1}{u''(x^c)}}{\frac{1}{l'_{nc}(x_n^c)} - \frac{1}{u''(x^c)}} = -\frac{\partial x^c/\partial p_i^c}{\partial x_i^c/\partial p_i^c}.$$

Using the above relations we can rewrite the price characterization on the consumer side, when $l_{ic}(x_i^{cOE}x^{wOE}) + p_i^{cOE} = u'(x^{cOE})$, as follows,

$$p_i^{cOE} = \underbrace{c_t - ax_n^{wOE} - \frac{\partial \pi_i^w/\partial p_i^c}{\partial x_i^c/\partial p_i^c}}_{\text{Opportunity cost}} + \underbrace{x_i^{cOE}l'_{ic}(x_i^{cOE})}_{\text{Pigovian tax}} - \underbrace{x_n^{cOE}l'_{nc}(x_n^{cOE})\frac{\varepsilon_{in}}{\varepsilon_{ii}}}_{\text{Switching cost}}$$

Opportunity Cost

The first three terms in the price characterization constitute the opportunity cost of servicing a marginal unit consumer. The first term represents the cost saving to a platform when a marginal unit mass leaves platform i; a platform no longer has to pay the termination cost c_t for all the traffic flow generated by that consumer. The second term is the revenue lost from the cross traffic emanating from the other platform n; the marginal unit consumer causes a traffic flow of x_n^{wOE} from platform n and each unit flow earns a. The third term represents the loss of website profit. To see this note that a price increase on platform i that causes a marginal unit consumer mass to leave causes a marginal increase on platform n of $\frac{\partial x_n^c/\partial p_i^c}{\partial x_i^c/\partial p_i^c} = f$ due to substitution effects, see Lemma 3. Therefore the decrease in website profit can be written as follows,

$$\begin{aligned}
(p_i^{wOE} - c^w)x_i^{wOE} - (p_i^{wOE} - c_o)x_i^{wOE}f &= -(p_i^{wOE} - c^w)x_i^{wOE}(f+1) - ax_i^{wOE} \\
&= -(p_i^{wOE} - c^w)x_i^{wOE}(\frac{\partial x_n^c/\partial p_i^c}{\partial x_i^c/\partial p_i^c} + \frac{\partial x_i^c/\partial p_i^c}{\partial x_i^c/\partial p_i^c}) - ax_i^{wOE} \\
&= -(p_i^{wOE} - c^w)x_i^{wOE}(\frac{\partial x^c/\partial p_i^c}{\partial x_i^c/\partial p_i^c}) - ax_i^{wOE} \\
&= -\frac{\partial \pi_i^w/\partial p_i^c}{\partial x_i^c/\partial p_i^c}.
\end{aligned}$$

The third term adjusts the consumer price downwards. It represents the additional loss to platform i caused by a price change which causes a marginal unit consumer to leave the platform. Thus the price charged to the consumer is discounted by the external benefit or the marginal increase in website profit that they cause. Therefore the first three terms summed together are the opportunity cost to platform i of lowering platform's i price so that a marginal unit consumer connects to the platform.

Pigovian Tax

The fourth term in the characterization is the Pigovian tax which internalizes the congestion cost. It corrects for the negative externality a consumer exerts on the link; the consumer is charged for the loss of revenue its congestion causes on the platform's link.

Switching Costs

Each platform has market power on the consumer side because of the switching costs induced on the consumer by the congestion externality. To see this, consider a consumer switching from link i to n, he causes a negative externality on link n equivalent to $x_n^{cOE} l'_{nc}(x_n^{cOE}) \frac{\varepsilon_{in}}{\varepsilon_{ii}}$. Everything else held equal, the rational consumer will not switch to the other platform if the markup is less than the switching cost. Thus the switching cost term measures the extra revenue that a platform can extract on its users without losing platform share. A platform, therefore, manages to lock-in consumers and raises its prices by this congestion cost. The magnitude of the switching cost depends on the latency function of the link on the other platform as well as the cross-elasticity and own-elasticity of demand. For instance if link n has high latency, high cross-elasticity of demand, and a low own-elasticity of demand then platform i will charge a higher markup. This term is positive because the value of $\varepsilon_{in}/\varepsilon_{ii}$ is negative. This follows from the fact that the links are substitutes. In particular ε_{ii} is negative and this can be inferred from Lemma 3. Similarly, ε_{in} can be shown to be positive.

The interaction price of the website represents the standard hotelling result with the off-net cost added to it, i.e., $p_i^{wOE} = t + c^w$, see [59]. We can also see from Proposition 3 that if the latency functions tend to 0 we get the offnet pricing in [34] when there is full coverage on both sides. We next define the social problem and the social optimum,

which is the consumer and website mass allocation that would be chosen by a social planner that has full information and control over the two sided market structure to maximize welfare. Welfare is defined as the total utility gained by the consumers added to the total utility gained by the websites less congestion and communication costs.

Definition 6. A vector $\mathbf{x^s}$ is a social optimum if it is an optimal solution of the social problem

$$\begin{aligned} \text{maximize}_{\mathbf{x}\geq 0} \quad & u(x^c)x^w - \sum_{i\in\mathcal{I}} l_{ic}(x_i^c x^w)x_i^c x^w - cx^c x^w + \cdots \\ & \cdots + \left(\sum_{i\in\mathcal{I}} \int_0^{x_i^w} v - xtdx\right) x^c, \qquad (2.28) \\ \text{Subject to} \quad & \sum_{i\in\mathcal{I}} x_i^c \leq 1, \\ & \sum_{i\in\mathcal{I}} x_i^w \leq 1. \end{aligned}$$

In view of Assumptions 1, 2 and 4, the above function is continuous over a compact constraint set, therefore a maximum exists. A vector $\mathbf{x^{cS}}$ that is a social optimum satisfies the following necessary conditions,

$$u'(x^{cS})x^{wS} - l'_{ic}(\cdot)x_i^{cS}(x^{wS})^2 - l_{ic}(\cdot)x^{wS} + vx^{wS} - cx^{wS} - \frac{t}{2}\sum_{i\in\mathcal{I}}(x_i^{wS})^2 \leq \theta^c, \text{ if } x_i^{cS} = 0,$$
$$= \theta^c, \text{ if } x_i^{cS} > 0. \tag{2.29}$$

Similarly a vector $\mathbf{x^{wS}}$ that is a social optimum satisfies the following conditions,

$$u(x^{cS}) - \sum_{i\in\mathcal{I}}(l'_{ic}(\cdot)(x^{cS})^2 x^{wS} + l_{ic}(\cdot)x^{cS}) + vx^{cS} - cx^{cS} - x_i^{wS}tx^{cS} \leq \theta^w, \text{ if } x_i^{wS} = 0,$$
$$= \theta^w, \text{ if } x_i^{wS} > 0. \tag{2.30}$$

For future reference we denote the value of the objective function in the social problem,

$$W(\mathbf{x}) = u(x^c)x^w - \sum_{i\in\mathcal{I}} l_{ic}(x_i^c x^w)x_i^c x^w - cx^c x^w + \left(\sum_{i\in\mathcal{I}}\int_0^{x_i^w} v - xtdx\right)x^c, \tag{2.31}$$

as social welfare.

We compare the mass of consumers that join the platforms at the social optimum to those that join at the OE. We show that the total mass of consumers that enroll in the two platforms under price competition is less than that at the social optimum if the consumer market is not fully covered at the social optimum [4]. Consequently we show that the duopoly competition is not always welfare maximizing. A similar effect is also noticed by Engel et al in the context of toll competition on congested roads, see [17].

Proposition 4. Let $(\mathbf{p}^{OE}, \mathbf{x}^{OE})$ be an OE and $\mathbf{x^s}$ be a social optimum. Let Assumptions 1 through 4 hold. In addition, assume that $x^{cS} < 1$. Then $x^{c0E} < x^{cS}$.

Proof. Assume to arrive at a contradiction that $x^{cOE} \geq x^{cS}$. This implies that there exists some $i \in \mathcal{I}$ such that $x_i^{c0E} \geq x_i^{cS}$. By concavity of u we have $u'(x^{c0E}) \leq u'(x^{cS})$.

[4] If the consumer market is fully covered at the social optimum, it can also be fully covered at the OE.

Using Lemma 1 and the necessary conditions for the social optimum we obtain,

$$l_{ic}(x_i^{cOE}) + p_i^{cOE} \leq l'_{ic}(x_i^{cS}x^{wS})x_i^{cS}x^{wS} + l_{ic}(x_i^{cS}x^{wS}) + c - v + \frac{t}{2}\sum_{i\in\mathcal{I}}\frac{(x_i^{wS})^2}{x^{wS}} + \theta^c. \quad (2.32)$$

Since from the proposition statement we have $x^{cS} < 1$, it follows from the KKT conditions that $\theta^c = 0$. From Assumption 1, we know that $\frac{1}{u''(x^{cOE})} < 0$ and from Assumption 3, $l'_{ic}(\cdot) > 0$, for all $i \in \mathcal{I}$. These imply that,

$$-1 < \frac{\frac{1}{u''(x^{cOE})}}{\frac{1}{l'_{nc}(x_n^{cOE})} - \frac{1}{u''(x^{cOE})}} \leq 0.$$

Moreover, as a consequence of Lemma 2, $p_i^{wOE} - c^w \leq v - c^w - x_i^{wOE}t$. This implies the following,

$$-(v - c^w - x_i^{wOE}t) < (p_i^{wOE} - c^w)x_i^{wOE}\frac{\frac{1}{u''(x^{cOE})}}{\frac{1}{l'_{nc}(x_n^{cOE})} - \frac{1}{u''(x^{cOE})}} \leq 0.$$

From the OE price characterization in Proposition 3 and the above inequality we deduce that,

$$p_i^{cOE} > c - v + x_i^{wOE}t + x_i^{cOE}l'_{ic}(x_i^{cOE}).$$

Plugging this inequality in Eq. 2.32 yields,

$$\begin{aligned} &c - v + x_i^{wOE}t + x_i^{cOE}l'_{ic}(x_i^{cOE}) + l_{ic}(x_i^{cOE}) \\ &< l'_{ic}(x_i^{cS}x^{wS})x_i^{cS}x^{wS} + l_{ic}(x_i^{cS}x^{wS}) + c - v + \frac{t}{2}\sum_{i\in\mathcal{I}}\frac{(x_i^{wS})^2}{x^{wS}}. \end{aligned} \quad (2.33)$$

From Proposition 3, we have that $x_i^{wOE} = 1/2$. We note that $\frac{t}{2}\sum_{i\in\mathcal{I}}\frac{(x_i^{wS})^2}{x^{wS}} - \frac{t}{2} \leq 0$; we therefore get the following inequality,

$$c - v + x_i^{cOE}l'_{ic}(x_i^{cOE}) + l_{ic}(x_i^{cOE}) < l'_{ic}(x_i^{cS}x^{wS})x_i^{cS}x^{wS} + l_{ic}(x_i^{cS}x^{wS}) + c - v.$$

Via Assumption 4, the above implies $x_i^{cS}x^{wS} > x_i^{cOE}$. It immediately follows that, $x_i^{cS} \geq x_i^{cS}x^{wS} > x_i^{cOE}$ yielding a contradiction. ■

We next compare welfare at the social optimum and at the OE. We show as a consequence of Proposition 4 that social welfare at the social optimum is higher than that at the OE under Assumption 5 and some mild conditions. We first show by example that, it is possible for social welfare at the social optimum and the OE to be equal. Note that for comparison purposes, we also assume that the social planner allocates websites so that their is full coverage on the website side at the social optimum.

Example 1. Consider two competing interconnected platforms where the latency functions to the consumers are given by, $l_{ic} = x$, and $l_{nc} = x$, the aggregate utility function is $u(x) = -x^3/3 + 5x/2$, the reservation price of each website be $v = 2$, the transport parameter $t = 0.5$ and the origination, termination costs and access charge are respectively set to zero. By symmetry, the unique allocation of websites and consumers by the social planner is $\mathbf{x^{wS}} = (1/2, 1/2)$ and $\mathbf{x^{cS}} = (1/2, 1/2)$ respectively. The unique allocation at the OE for websites and consumers is given by $\mathbf{x^{wOE}} = (1/2, 1/2)$ and $\mathbf{x^{cOE}} = (1/2, 1/2)$ respectively. Therefore the welfare is the same in both cases.

Corollary 2. Let Assumption 1 through 5 hold. Let $(\mathbf{p}^{OE}, \mathbf{x}^{OE})$ be an OE and $\mathbf{x}^{\mathbf{s}}$ be a social optimum. In addition, assume either (i) $x^{cS} < 1$ or (ii) $x^{cOE} = 1$ and $l_{ic}(x_i^{cOE})/x_i^{cOE} \neq l_{nc}(x_n^{cOE})/x_n^{cOE}$ then $W(x^{OE}) < W(x^s)$.

Proof. Assume that $x^{cOE} < 1$. For a fixed value of the access charge, the social welfare function is as defined in Eq. (2.31). From Assumption 5, the full coverage condition on the website side, we have $x^w = 1$. From proposition 3, we have $x_i^{wOE} = 1/2$ for all $i \in \mathcal{I}$. One also deduces from the optimality conditions in Eq. (2.29) that $x_i^{ws} = 1/2$ for all $i \in \mathcal{I}$. The Jacobian of $W(\mathbf{x})$ is given by matrix M which is defined as follows,

$$M \equiv \begin{bmatrix} u''(x^c) - 2l'_{ic}(x_i^c) - l''_{ic}(x_i^c)x_i^c & u''(x^c) \\ u''(x^c) & u''(x^c) - 2l'_{nc}(x_n^c) - l''_{nc}(x_n^c)x_n^c \end{bmatrix}.$$

Since $u''(x^c) - 2l'_{ic}(x_i^c) - l''_{ic}(x_i^c)x_i^c < 0$ and $u''(x^c) - 2l'_{nc}(x_n^c)l''_{nc}(x_n^c)x_n^c < 0$ are less than 0 and $u''(x^c)^2 > 0$, M is negative definite which implies that $W(\mathbf{x})$ is strictly concave. Since $W(\mathbf{x})$ is continuous over a compact and convex constraint set the maximization problem (2.28), where $x^w = 1$, yields a unique maximum. Since $x^{cOE} \neq x^{cS}$, by Proposition

4, we deduce that $W(x^{OE}) < W(x^s)$.

Assume that $x^{cOE} = 1$ and $l_{ic}(x_i^{cOE})/x_i^{cOE} \neq l_{nc}(x_n^{cOE})/x_n^{cOE}$. From optimality conditions in Eq. 2.29 and the fact that $x_i^{cS} > 0$ for all $i \in \mathcal{I}$ we have,

$$l'_{ic}(x_i^{cS})x_i^{cS} + l_{ic}(x_i^{cS}) = l'_{nc}(x_n^{cS})x_n^{cS} + l_{nc}(x_n^{cS}).$$

From the price characterization in Proposition 3 and Lemma 1, we have

$$(l'_{ic}(x_i^{cOE}) + l'_{nc}(x_n^{cOE}))x_i^{cOE} + l_{ic}(x_i^{cOE}) = (l'_{ic}(x_i^{cOE}) + l'_{nc}(x_n^{cOE}))x_n^{cOE} + l_{nc}(x_n^{cOE}).$$

Since $l_{ic}(x_i^{cOE})/x_i^{cOE} \neq l_{nc}(x_n^{cOE})/x_n^{cOE}$, the result follows. ■

2.5 Access Charge

In this section we investigate the effect of the reciprocal access charge a on welfare. The baseline model in [34] is incapable of analyzing the effect of the access charge on welfare since demand is fixed on both sides of the market. In our model, consumers face an elastic demand and respond to prices. We show that if the latency functions are linear and there's no full coverage on the consumer side then the number of consumers that join both platforms is increasing with the access charge. Consequently, we show that welfare is also an increasing function of the access charge. This is important because it suggests that 'Bill and Keep' peering in this setting is not a welfare maximizing interconnection agreement. In the case where consumer market is covered at the OE, the access charge is welfare neutral, i.e., increasing the access charge does not increase welfare.

We first show that we can express $\mathbf{x^{cOE}}$ as a function of the access charge a. From Lemma 1 and 6, if the consumer market is not covered, we have the following equation satisfied at the Oligopoly Equilibrium for all $i \in \mathcal{I}$; we have explicitly put the dependency on the access charge a in the equation,

$$a_{ic}x_i^{cOE} + p_i^{cOE}(a) \quad = \quad u'(x^{cOE}). \tag{2.34}$$

For a fixed a, this equation implicitly defines x^{cOE} as a function of p^{cOE}, by Lemma 7. Similarly we show that this equation implicitly defines $\mathbf{x^{cOE}}$ as a function of a.

Lemma 8. Let Assumption 1, 4 and 5 hold. Then $l_{ic}(x_i^c) + p_i^c(a) = u'(x^c)$ for all $i \in \mathcal{I}$ implicitly defines $\mathbf{x}^c$ as a function of a and this function is continuously differentiable.

Proof. Let $f_i(a, x_i^c, x_n^c) = a_{ic}x_i^c + p_i^c(a) - u'(x^c) = 0$ for all $i \in \mathcal{I}$ be a function of $I+1$ variables from $\mathbb{R}_+^{I+1}$ to $\mathbb{R}_+^I$. $f(a, \mathbf{x}^c)$ is continuously differentiable. Therefore, the Jacobian of $f(a, \mathbf{x}^c)$ is given by matrix M defined as follows,

$$M \equiv \begin{bmatrix} \partial f_i/\partial x_i^c & \partial f_i/\partial x_n^c \\ \partial f_n/\partial x_i^c & \partial f_n/\partial x_n^c \end{bmatrix},$$

We rewrite M as follows,

$$M \equiv \begin{bmatrix} A+B & B \\ D & C+D \end{bmatrix}.$$

where ,

$$\begin{aligned}
A &= 2a_{ic} - u''(x^c) + \frac{1}{\frac{1}{a_{nc}} - \frac{1}{u''(x^c)}}, \\
B &= \frac{-1}{\left(\frac{1}{a_{nc}} - \frac{1}{u''(x^c)}\right)^2} \frac{u'''(x^c)}{u''(x^c)^2} \left(x_i^c + \frac{t}{2a_{nc}}\right), \\
C &= 2a_{nc} - u''(x^c) + \frac{1}{\frac{1}{a_{ic}} - \frac{1}{u''(x^c)}}, \\
D &= \frac{-1}{\left(\frac{1}{a_{ic}} - \frac{1}{u''(x^c)}\right)^2} \frac{u'''(x^c)}{u''(x^c)^2} \left(x_n^c + \frac{t}{2a_{ic}}\right).
\end{aligned}$$

The determinant of M is given by

$$\det M = AC + AD + BC.$$

The above determinant is positive because A, B, C and D are all positive. Having shown that the Jacobian is non-singular, we can now apply the Implicit Function Theorem [7] to conclude that $\mathbf{x^{cOE}}$ is a continuously differentiable function of a. ■

Lemma 9. Let $(\mathbf{p}^{OE}, \mathbf{x}^{OE})$ be an Oligopoly Equilibrium such that $x^{cOE} < 1$. Let Assumptions 1 through 5 hold. Assume further that the latency functions are linear with $l_{ic} = a_{ic}x_i^c$ and $l_{nc} = a_{nc}x_n^c$. Then $x^{cOE}(a)$ is an increasing function of a.

Proof. From Lemma 1 and 6 , we have the following equation satisfied at the Oligopoly Equilibrium for all $i \in \mathcal{I}$; we have explicitly put the dependency on the access charge in the equation,

$$a_{ic}x_i^{cOE} + p_i^{cOE}(a) \quad = \quad u'(x^{cOE}). \tag{2.35}$$

From Lemma 8, this equation implicitly defines $\mathbf{x^{cOE}}$ as a function of a. After implicitly differentiating Eq. 2.35 with respect to the access charge, where $p_i^{cOE}(a)$ and $p_i^{wOE}(a)$ are given in Proposition 3, we have, for all $i \in \mathcal{I}$, the following equation,

$$\underbrace{(2a_{ic} + A_{nc})}_{d_i} \frac{\partial x_i^{cOE}}{\partial a} - 1 = \frac{\partial x^{cOE}}{\partial a} \underbrace{\left(u''(x^{cOE}) + \left(A_{nc}^2 \frac{u_c'''(x^{cOE})}{u''(x^{cOE})^2}\right)\left(x_i^{cOE} + t\frac{1}{a_{nc}}\right)\right)}_{b_i},$$

where $A_{mc}(x^{cOE}) = \frac{1}{\frac{1}{l'_{mc}(x_m^{cOE})} - \frac{1}{u''(x^{cOE})}}$ for $m \in I$. From Assumption 1, $u''(x^{cOE}) < 0$ and $u'''(x^{cOE}) < 0$, and from Assumption 4, $a_{ic} > 0$ for all $i \in \mathcal{I}$. These assumptions imply that $b_i < 0$ and $d_i > 0$ for all $i \in \mathcal{I}$. Using the following relation, $\frac{\partial x_i^{cOE}}{\partial a} + \frac{\partial x_n^{cOE}}{\partial a} = \frac{\partial x^{cOE}}{\partial a}$, we obtain the following inequality,

$$\frac{\partial x^{cOE}}{\partial a} = \frac{\sum_{i \in \mathcal{I}} \frac{1}{d_i}}{1 - \left(\sum_{i \in \mathcal{I}} \frac{b_i}{d_i}\right)} > 0.$$

Therefore $x^c(a)$ is an increasing function of a. ■

We now show that welfare is an increasing function of the access charge. We rewrite the definition of social welfare explicitly showing the dependency of the consumer allocations across the platforms to the access charge. Note that Assumption 5 holds for this definition.

Definition 7. Welfare as a function of the access charge at the Oligopoly Equilibrium is defined as follows;

$$W(a) = u(x^{cOE}(a)) - \sum_{i \in \mathcal{I}} l_{ic}(x_i^{cOE}(a))x_i^{cOE}(a) - cx^{cOE}(a) + \left(\sum_{i \in \mathcal{I}} \int_0^{1/2} v - xt \; dx \right) x^{cOE}(a).$$

Proposition 5. Let $(\mathbf{p}^{OE}, \mathbf{x}^{OE})$ be an Oligopoly Equilibrium such that $x_i^{cOE} + x_n^{cOE} < 1$. Let Assumption 1 through 5 hold. Assume further that the latency functions are linear. Then $W(a)$ is an increasing function of the access charge a.

Proof. We differentiate the welfare function with respect to the access charge and obtain the following ,

$$\frac{\partial W(a)}{\partial a} = u'(x^c(a))\frac{\partial x^c(a)}{\partial a} + \sum_{i \in \mathcal{I}} \frac{\partial x_i^c(a)}{\partial a} \left(l'_{ic}(x_i^c(a))x_i^c(a) - l_{ic}(x_i^c(a)) \right) - c\frac{\partial x^c(a)}{\partial a} - (v - t/4)\frac{\partial x^c(a)}{\partial a}.$$

Let $K_i = x_i^c A_{nc}$ and $F_i = A_{nc}/u''(x^c)$ for all $i \in \mathcal{I}$ where $n \neq i$. Using these together with the following relation $u'(x^c) - l_{ic}(x_i^c) = p_i^c$ we can rewrite the above differentiation as follows,

$$\frac{\partial W(a)}{\partial a} = \left(K_i + \frac{1}{2}tF_i \right) \frac{\partial x_i^c(a)}{\partial a} + \left(K_n + \frac{1}{2}tF_n \right) \frac{\partial x_n^c(a)}{\partial a} + (v - t/4 - c^w)\frac{\partial x^c(a)}{\partial a}.$$

One can establish that K_i is positive for all $i \in \mathcal{I}$ and similarly F_i is negative for all $i \in \mathcal{I}$. Moreover, $| F_i |< 1$, therefore it follows that,

$$\begin{aligned} \frac{\partial W(a)}{\partial a} &> \left(\min\{K_i, K_n\} + \frac{1}{2}t \min\{F_i, F_n\} + (v - t/4 - c^w) \right) \frac{\partial x^c(a)}{\partial a}, && (2.36) \\ &> \left(\min\{K_i, K_n\} + \frac{1}{2}(-1) + (v - t/4 - c^w) \right) \frac{\partial x^c(a)}{\partial a}, && (2.37) \\ &> 0. && (2.38) \end{aligned}$$

The last inequality follows from the positivity of K_i and K_n, from Assumption 5 which implies $v - \frac{3t}{2} - c^w \geq 0$, and from Lemma 9 which shows that $x^c(a)$ is an increasing function of a. Thus welfare is an increasing function of the access charge. ■

From Proposition 3 we see how access charges feed into consumer prices. In particular, they decrease the prices charged to consumers. Higher access charges benefit consumers because they lower the prices that platforms charge. This increases the enrollment. From Corollary 2 we infer that given any access charge, social welfare is higher at the social optimum if the consumer market is not covered at the OE. Thus, so long as the consumer market is not covered at the OE, increasing the access charge increases welfare. In the instance where the market is covered demand is fixed. Lowering the access charge does not change the allocation of consumers on both platforms. Therefore, the access charge allocates costs between the websites and consumers.

2.6 Chapter summary

In this chapter, we have studied competition between interconnected platforms in presence of access prices and congestion effects on the consumer side. We have characterized, using system parameters, the prices that a platform would charge to both consumers and websites when there's full coverage on the website side. In particular, we note that the price charged to consumers consists of the opportunity cost of attracting the consumers to the platform and markups resulting from a pigovian tax and the switching cost. We have also shown that under mild conditions less consumers join the platforms under price competition than at the social optimum. We have also analyzed the effect of access charges on welfare. In particular, if at the Oligopoly Equilibrium there's no full coverage on the consumer side then welfare is increasing in the access charge in the presence of linear latencies and concave inverse demand.

Chapter 3

Net Neutrality

3.1 Introduction

In this chapter we present and solve the two-sided market models that represent both the neutral and non-neutral regime. Our principal context is competition between two interconnected ISPs that serve both CPs and consumers who are heterogenous in their tastes. We show that the different pricing structures between the regimes determines the investment patterns. Moreover, we show the trade-offs platforms make in softening price competition on the consumer side and increasing CP surplus on the CP side from which they expropriate revenue, is the mechanism by which investment levels are determined. Finally we show comparative statics of consumer, CP and social welfare in both models and explain the role of investments.

3.1.1 Related Literature

As initially mentioned, much of the net neutrality debate has been qualitative; mostly from the law and policy sphere. In addition to the papers cited in the introduction, the following also discuss various policy aspects of the net neutrality debate [47, 28, 42, 18, 36, 41]. Recently, a few publications have formalized some of the issues around net neutrality. Conceptually, it is useful to classify these emerging work into two broad classes categorized by the working definition of net neutrality that they adopt. One group views abandoning of net neutrality as a licence to introduce differentiated service classes in the Internet, or establishment of priority lanes (see for example [13, 32, 27, 11, 51, 52]). In contrast, the other group views abandoning net neutrality as abolishing the current pricing structure in the Internet, see Economides and Tag [16], Cañón [10], , and Musacchio, Schwartz and Walrand [39].

This second group is more related to our work. Economides and Tag [16] use a two-sided market framework to investigate the effect of net neutrality regulation (defined as setting zero access fee to CPs) in both a monopoly and duopoly setting. In general they find that total welfare is higher in the neutral regime under both scenarios. However, their model does not include investment decisions by platforms, a key driver of our results. Cañón [10] also investigates the effect of net-neutrality under various pricing regulations in the presence of investment decisions. He finds that the neutral regime is superior in terms of total welfare. Unlike our setup, though, his model considers only a single monopoly ISP. Musacchio et al. [39] is the closest to our work. They develop a two-sided market model and compare economic welfare under a neutral and non-neutral regime. The former corresponds to "one-sided" pricing where only consumers are charged and the latter corresponds to "two-sided" pricing where both consumers and CPs are charged. The authors find that either regime can be superior for overall welfare and even for each of the CPs and ISPs; a detailed summary can be found in [53]. Although we use a similar definition of net neutrality, our models differ to theirs in a significant number of ways. In particular, the novel features of our models are:

- CPs are not assumed to be homogenous and atomic, but are heterogeneous in quality with a scaled standard deviation given by a and an average given by $\overline{\gamma}$. Moreover, advertising rates increase with CP and platform quality. In addition the market coverage is also endogenously derived.

- Consumers are also heterogeneous in their tastes or income and distributed over the interval $[0,f]$, see [40]. Here f is a fraction representing the mass of consumers in the market.

- We highlight the impact of quality difference between the platforms on consumer's experienced QoS with a bottleneck effect. Specifically, if a consumer is on a low-quality platform and accesses content on a higher quality platform he experiences low quality. Similarly, if he is on a high-quality platform and accesses content on a low-quality platform he experiences low quality.

Moreover, they assume that ISPs have local market power and that the consumer base is split equally amongst the ISPs. This lack of competition on the consumer side coupled with the above listed differences explains the dissemblance of our results from theirs.

Our work complements previous research by explicitly considering quality investment by platforms in the two regimes. We do so by building on two main strands of literature in Industrial Organization: Price competition and quality choice in vertically differentiated markets, [22, 62, 12, 38, 54], and two-sided markets [44, 2, 50, 23, 49]. These help us model platform quality investment endogenously in the presence of a bottleneck.

Equally important, we are able to capture the effects of CP heterogeneity and average quality on the investments made in both regimes. Moreover, we are also able to address how CP market coverage, which is not addressed in most of the literature, and surplus (both proxies for innovation) compare under the two regimes. This research adds to the growing body of economic analysis that will help inform policy on the net neutrality debate.

We also contribute to the two sided market literature by considering a model where the participants of one platform benefit from the presence of participants of another platform (because of the interconnection). This is in contrast to most two sided market models in which platform end users only benefit from subscribers to that platform. In addition, we also consider investment decisions by the platforms, an area that has received little attention. Farhi and Hagiu [19] consider investment as a strategic variable in a two-sided duopoly market model. However, their analysis investigates how investment strategies of an incumbent platform may help it accommodate or deter entry of another platform. In our models, both platforms simultaneously compete in the investment stage and platform participation is an endogenous result of the game.

The rest of this Chapter is organized as follows. In Section 3.2, we present the neutral model. In Section 3.3, we analyze the model solving for the subgame perfect equilibrium (SPE) of this game as well as discussing our findings. Section 3.4 introduces the non-neutral model. In Section 3.5, we analyze the model and discuss the results. In Section 3.6, we perform a comparison of various welfare metrics between both regimes. We conclude in Section 3.7 by summarizing our results and providing insight for policy makers. To improve readability all proofs have been relegated to the Appendix.

3.2 Neutral Model

We consider two platforms denoted by α and β, and a continuum of consumers and CPs with the former having a mass f, with $f \in [0,1]$, and the latter a mass of unit volume. Let $y_\alpha \in \mathbb{R}_+$ and $y_\beta \in \mathbb{R}_+$ be the quality-of-service chosen by platforms α and β, respectively. Without loss of generality we assume $y_\alpha \geq y_\beta \geq 0$. Let γ_j be the quality of CP j where $j \in [0,1]$. Here, γ_j is uniformly distributed with support $[\overline{\gamma} - a, \overline{\gamma} + a]$ and $0 < a < \overline{\gamma}$. We assume γ_j are independent identically distributed random variables across the population of CPs. Let $\phi : [0,f] \to \{\alpha, \beta\}$ and $\widehat{\phi} : [0,1] \to \{\alpha, \beta\}$ be mappings from the space of consumers and CPs respectively to the set of platforms. Let r_α and r_β (q_α and q_β) be the masses of CPs (consumers) that join platform α and β respectively. Platform $z \in \{\alpha, \beta\}$ has its own services and content that enhance or complement those of CPs that connect to it denoted by k_z: a random variable whose average is the same as that of γ_j and support lies on the positive interval. This content and service is only available to consumers who enroll on the platform.

Consumer Utility: A consumer i on a platform $\phi(i) \in \{\alpha, \beta\}$ connecting to a CP j on platform $\widehat{\phi}(j) \in \{\alpha, \beta\}$ receives utility,

$$u_{ij}(y_{\phi(i)}, y_{\widehat{\phi}(j)}, \gamma_j, k_{\phi(i)}, r_{\widehat{\phi}(j)}) = \min\{y_{\phi(i)}, y_{\widehat{\phi}(j)}\} \left(\frac{\gamma_j}{r_{\widehat{\phi}(j)}} + k_{\phi(i)} \right). \tag{3.1}$$

The quality of a CP j is divided by the mass of the CPs that connect to $\hat{\phi}(j)$, denoted by $r_{\widehat{\phi}(j)}$, to model the congestion whose effect is to lower the quality of the CP content: the more CPs that join a platform the higher the congestion on that platform lowering the quality of CPs. k_z is not affected by the congestion because, unlike CPs content, it does not have to traverse congested links to get to the ISP as it is hosted at the ISP's servers.

Consumer utility gained both from the platform content and CP content is affected by the minimum of the quality-of-service of the platforms on which the consumer and CP are located. This implies that a consumer on a high-quality platform, connecting to a content provider present on a high-quality platform, receives higher utility than if he connected to a content provider of the same quality on the lower-quality platform. In essence, consumer utility depends on the platform that acts as a bottleneck.

A consumer i on platform $\phi(i)$ connects with any CP j subscribed to either platform since $u_{ij} \geq 0$. Let $F_i(y_{\phi(i)}, y_{\phi(-i)}, \overline{\gamma}, a, r_\alpha, r_\beta)$ be the quality perceived by a consumer i when he joins platform $\phi(i)$. Formally,

$$F_i(y_{\phi(i)}, y_{\phi(-i)}, \overline{\gamma}, a, r_\alpha, r_\beta) = \int_0^1 E\left[u_{ij}(y_{\phi(i)}, y_{\widehat{\phi}(j)}, \gamma_j, k_{\phi(i)}, r_{\widehat{\phi}(j)})\right] dj. \tag{3.2}$$

We assume that consumers have heterogenous preferences represented by parameter θ_i which is uniformly distributed in the interval $[0, f]$. A consumer i perceives the quality of platform $\phi(i)$ as his expected utility, $F_i(y_{\phi(i)}, \cdot)$. In addition, each consumer has a reservation utility R. The prices charged by the platforms to consumers α and β are p_α and p_β for platforms α and β respectively. Each consumer connects to at most one platform but once connected has access to all content due to the interconnection of the platforms. Therefore, the net utility of a consumer i connecting to platform $\phi(i)$ is given by

$$U_i(\phi(i)) = \max\{R + \theta_i F_i(y_{\phi(i)}, y_{\widehat{\phi}(j)}, \overline{\gamma}, a, r_\alpha, r_\beta) - p_{\phi(i)}, 0\}. \tag{3.3}$$

Consumers prefer the platform with the higher perceived quality, ceteris paribus.

Content Provider Utility: CPs make revenues by selling advertising and pay platform $\alpha(\beta)$ a fixed connection fee w_α (w_β) if they join it. The utility v_j of a CP j is defined to be his profit

$$v_j = V_j(\gamma_j, y_\alpha, y_\beta, q_\alpha, q_\beta) - w_{\widehat{\phi}(j)}, \tag{3.4}$$

where the gross revenue earned by a CP j is given by

$$V_j(\gamma_j, y_\alpha, y_\beta, q_\alpha, q_\beta) = \begin{cases} g(\gamma_j, y_\alpha)q_\alpha + g(\gamma_j, y_\beta)q_\beta & \text{if } \widehat{\phi}(j) = \alpha, \\ g(\gamma_j, y_\beta)q_\alpha + g(\gamma_j, y_\beta)q_\beta & \text{if } \widehat{\phi}(j) = \beta. \end{cases}$$

Here, $g(\gamma_j, y_{\widehat{\phi}(i)})$ is a function that represents the advert price and is increasing in both parameters; CP j gets a higher ad price for having a higher content quality and also for locating on a platform with higher quality. Note that $V(.)$ depends on which platform the CP joins and the number of consumers on the other side of the market. In particular, if a CP j joins the higher quality platform, it is able to charge a higher advert price for connections arising from consumers on that platform. If a CP joins the lower quality platform its advert price is constant across the two platforms, i.e, the advert price depends on the platform that acts as the bottleneck. Figure 3-1 shows a depiction of the model.

Platform Payoffs: Finally we consider the payoff functions of the platforms: we assume that platforms incur a cost when investing in quality. The payoff of platform α, which we denote by π_α, is given by

$$\pi_\alpha = p_\alpha q_\alpha + w_\alpha r_\alpha - I(y_\alpha). \tag{3.5}$$

where q_α is the mass of consumers attached to platform α and r_α is the mass of CPs attached to platform α as mentioned earlier. There is a convex investment cost $I(y_\alpha)$ associated with quality y_α resulting in a decreasing return to investment. The payoff for platform β is similar. The model we have outlined corresponds to a dynamic game with the following timing of events.

1) Quality Investment Stage: Platforms α and β simultaneously choose quality-of-service from the interval $[0, \infty)$.

2) Pricing Decisions: Platforms simultaneously choose connection fees w_α and w_β.

3) Connection Decisions: CPs decide which platform to join.

4) Pricing Decisions: Platforms simultaneously choose prices p_α and p_β.

5) Connection Decisions: Consumers decide which platform to join.

6) Consumption Decisions: Consumers decide which CPs to connect.

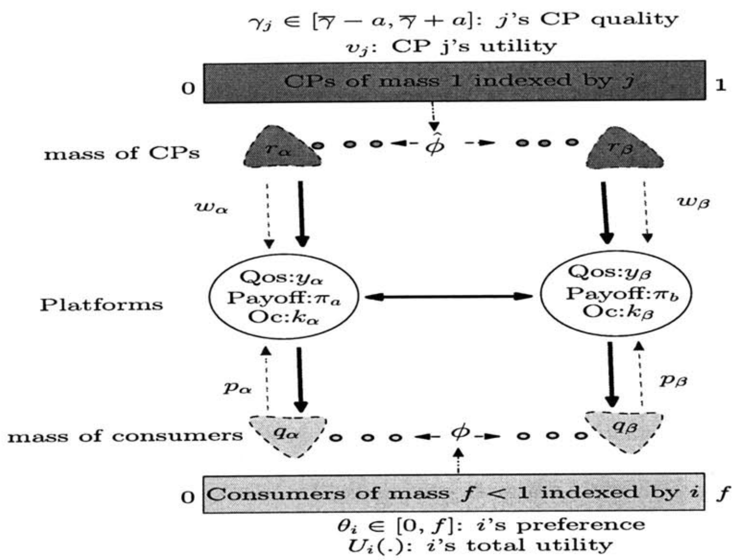

Figure 3-1: An illustration of the model, where the red (dashed) lines show movement of fees and prices from CPs and consumers respectively whilst the blue(solid) line shows the movement of content. Oc refers to own content. The bubbles show dissipation of CPs and consumers to the various platforms according to the mappings $\hat{\phi}$ and ϕ.

The timing of the extensive game is predicated on the view that prices adjust more quickly than investments. The latter is viewed as a medium to long-term decision whereas the former is a short run decision, see [5, 33]. Thus investment is the first stage of the game. We solve this game by considering its subgame perfect Nash equilibrium (SPE)[1], which we find using backward induction. Steps 4-7 are similar to a pricing game with vertical differentiation; steps 1-3 are similar to a quality choice and pricing game with vertical differentiation.

3.3 Model Analysis

Let $\mathcal{I} = \{\alpha, \beta, [0,1]_j, [0,f]_i\}$ denote the set of players in the multi-stage game, where α and β are the platforms, $[0,1]_j$ and $[0,f]_i$ are the continuum of content providers and

[1]We focus on optimal actions/decisions along the equilibrium paths .

consumers respectively. We denote the information set at stage k of the game for a player $i \in \mathcal{I}$ by h_i^k. Let the set of actions available to a player i at stage k and information set h_i^k be denoted as $A_i(h_i^k)$. The consumer price SPE, follows from the standard vertical differentiation model, see [59]. The main challenges of our model arise from solving the CP price and the platform investment SPE. For the former, we first identify the candidate price CP price equilibrium pairs in all the market configurations that may arise. Then by construction we show that these pairs are also best replies on the whole domain of strategies, i.e., a candidate price pair not only consists of prices that are mutual best responses in a particular market configuration but across all market configurations. For the latter we identify sets in which the best response lie and consequently find candidate SPE pairs; we then show that these pairs are indeed SPE.

3.3.1 Consumption Decisions

We begin by analyzing the last stage of the game, i.e, the consumption decisions of the consumers. Only the consumers make a move in this stage. A consumer i on a platform $\phi(i) \in \{\alpha, \beta\}$ accessing content of a CP j on platform $\widehat{\phi}(j) \in \{\alpha, \beta\}$ receives utility represented in (3.2).

As previously discussed, this implies that a consumer connecting to a higher quality platform gets more utility when he accesses CPs on that platform, compared to when he connects to the same content providers while connected to the lower quality platform. Consumer i on platform $\phi(i)$ connects with CP j whenever $u_{ij} \geq 0$ which implies that i connects with CP j if $\gamma_j \geq -k_{\phi(i)} r_{\widehat{\phi}(j)}$. Since γ_j is positive for all j and the term on the right hand is always non-positive, when a consumer joins any of the platforms he will connect to all CPs on both platforms.

3.3.2 Consumer Platform Connection Decisions

In this stage the consumers are the only movers and they decide which platforms to join. The choice set of a consumer i given any h_i^k is $A_i(h_i^k) = \{\alpha, \beta\}$. Through his information set, a consumer has knowledge of the number of CPs on each platform, the prices that the platforms charge and the quality level of each platform. Each consumer i maximizes his net utility given by Eq. (3.3) to determine what platform to connect to.We proceed next to give the demand functions addressed to each platform based on consumer choices in this stage whenever $y_\alpha > y_\beta$. We will show in the next stage that if $y_\alpha = y_\beta$ then any allocation of demand across platforms is possible at the resulting price equilibrium. We first make the following assumption on the reserve price which will apply throughout this paper.

Assumption 6. *R is large enough that the consumer market is covered.*

The above assumption results in a covered consumer market because for large values of R, $\theta_i > (p_{\phi(i)} - R)/F_i(y_{\phi(i)}, .)$ which implies that every consumer derives positive utility upon joining one of the platforms [cf.Eq. (3.3)]. If $y_\alpha > y_\beta$, we consider two cases in determining the demands: $p_\alpha < p_\beta$ and $p_\alpha \geq p_\beta$. In the former case, the consumers always join the platform with the highest perceived quality since $U_i(\phi(i) = \alpha) > U_i(\phi(i) = \beta)$, this follows directly from applying Lemma 10, see Appendix A.1.1. Demand addressed to the platforms in this case is therefore given by $q_\alpha = 1$ and $q_\beta = 0$. When $p_\alpha \geq p_\beta$ the demand derivation is more involved. Let $\tilde{\theta} \equiv (p_\alpha - p_\beta)/(F_i(y_\alpha, \cdot) - F_i(y_\beta, \cdot))$. Consumers with a taste parameter $\theta_i \geq \tilde{\theta}$ join the platform with the higher perceived quality, $F_i(y_\alpha, \cdot)$, since $\theta_i F_i(y_\alpha, \cdot) - p_\alpha \geq \theta_i F_i(y_\beta, \cdot) - p_\beta$ if and only if $\theta_i \geq \tilde{\theta}$. Those whose taste parameter $\theta_i < \tilde{\theta}$ will join platform β. One can show that the demands in the consumer market are characterized as follows $q_\beta(p_\alpha, p_\beta) = \left(\frac{p_\alpha - p_\beta}{F_i(y_\alpha, \cdot) - F_i(y_\beta, \cdot)}\right)$ and $q_\alpha(p_\alpha, p_\beta) = \left(f - \frac{p_\alpha - p_\beta}{F_i(y_\alpha, \cdot) - F_i(y_\beta, \cdot)}\right)$

3.3.3 Platform Pricing Decisions on Consumer Side

In this stage of the game, the platforms are the only movers and they decide what prices to charge to the consumers. The choice set of platform $i \in \{\alpha, \beta\}$, given any h_i^k, is $A_i(h_i^k) = p_i \in \mathbb{R}$. Thus the platforms simultaneously decide what prices p_α and p_β to charge to consumers. Through his information set, a platform has knowledge of the number of CPs on each platform and the quality level of each platform. Profit for platform z is given by the expression in Eq. (3.5). The Nash equilibrium in this price subgame depends on the information set h_i^k. In particular, if h_i^k is such that $y_\alpha > y_\beta$ it can be shown that, $p_\beta = f(F_i(y_\alpha, \cdot) - F_i(y_\beta, \cdot))/3$ and $p_\alpha = 2f(F_i(y_\alpha, \cdot) - F_i(y_\beta, \cdot))/3$, and the consumer demands addressed to the platforms at this equilibrium are $q_\alpha = 2f/3$ and $q_\beta = f/3$. If h_i^k is such that $y_\alpha = y_\beta$ then $F_i(y_\alpha, \cdot) = F_i(y_\alpha, \cdot)$. A Bertrand competition ensues with $p_\alpha = p_\beta = 0$ the resulting subgame Nash equilibrium. The consumer demands addressed to the platforms at this equilibrium price are indeterminate, i.e, any allocation such that $q_\alpha + q_\beta = f$ is a solution. In this case we make the standard assumption that consumers are evenly split between the platforms.

3.3.4 Content Provider Connection Decisions

Given the quality of service offered by platforms y_α and y_β and the prices w_α and w_β, the content providers decide on which platform to locate. The choice set of a CP j given any h_j^k is $A_j(h_j^k) = \{\alpha, \beta\}$. As mentioned in Section 3.2, γ_j is uniformly distributed with a support $[\overline{\gamma} + a, \overline{\gamma} - a]$ where $\overline{\gamma} \geq a$. The utility v_j gained by a content provider when he joins a platform is given by (3.4). A CP's utility is zero if he does not join any platform. In this stage, CPs take the investment(choice) in quality as given. Moreover, they anticipate the mass of consumers on each platform q_α and q_β. Let $g(\gamma_j, y_{\widehat{\phi}(j)}) = \gamma_j y_{\widehat{\phi}(j)}$, a CP j perceives the quality of platform α to be $y_\alpha q_\alpha + y_\beta q_\beta$ and that of platform β to be $y_\beta q_\alpha + y_\beta q_\beta$.

A CP j maximizes the utility v_j and is indifferent between the two platforms if and only if $\gamma_j(y_\alpha q_\alpha + y_\beta q_\beta) - w_\alpha = \gamma_j(y_\beta q_\alpha + y_\beta q_\beta) - w_\beta$. Let $\widetilde{\gamma}_j \equiv w_\alpha - w_\beta / q_\alpha(y_\alpha - y_\beta)$, then the CPs with quality exceeding $\widetilde{\gamma}_j$ join the high quality platform α. Those whose content quality is lower than $\widetilde{\gamma}_j$, but larger than $w_\beta/(y_\beta(q_\beta + q_\alpha))$, join the lower quality platform β. The others do not join any platform. If $y_\alpha > y_\beta$, there is a possibility of platform α preempting the market with a limit price $w_\alpha = w_\beta + (\overline{\gamma} - a)(q_\alpha(y_\alpha - y_\beta))$. It follows that given the tuple $(\overline{\gamma}, a, y_\alpha, y_\beta, w_\alpha, w_\beta)$, there are four possible market configurations that may arise reflecting the demands addressed to the platforms. We next describe the market configurations and demand functions on the CP side that arise at different CP prices. The mass of CPs $r_\alpha(w_\alpha, w_\beta)$ or $r_\beta(w_\alpha, w_\beta)$ is defined by those CPs who maximize v_j when they join platform α or β).

1. Uncovered Market: $r_\alpha(w_\alpha, w_\beta) < 1$, $r_\beta(w_\alpha, w_\beta) = 0$. We denote this configuration as CI. It has only platform α participating in the market with only a fraction of the content providers being served.

2. Uncovered Market: $r_\alpha(w_\alpha, w_\beta) + r_\beta(w_\alpha, w_\beta) < 1$, $0 < r_\alpha(w_\alpha, w_\beta) < 1$, $0 < r_\beta(w_\alpha, w_\beta) < 1$. We denote this configuration as CII. In this configuration, both platforms participate in the market but there are CPs who are not served by either platform.

3. Covered market: $r_\alpha(w_\alpha, w_\beta) + r_\beta(w_\alpha, w_\beta) = 1, r_\alpha(w_\alpha, w_\beta) > 0$ and $r_\beta(w_\alpha, w_\beta) > 0$. The third configuration is denoted as $CIII$. In this configuration, both platforms participate in the market and all CPs are served.

4. Preempted covered market: $r_\alpha(w_\alpha, w_\beta) + r_\beta(w_\alpha, w_\beta) = 1, r_\alpha(w_\alpha, w_\beta) = 1$, and $r_\beta(w_\alpha, w_\beta) = 0$. The fourth configuration is denoted as region CIV; in this configuration only one platform participates in the market and all the CPs are served.

3.3.5 Platform Pricing Decision for the Content Provider Side

In this stage of the game the platforms are the only movers and they decide what prices to charge to the CPs. The choice set of platform $z \in \{\alpha, \beta\}$ given any h_i^k is $A_i(h_i^k) = w_i \in \mathbb{R}$. Thus the platforms simultaneously decide what prices w_α and w_β to charge to CPs. Before proceeding we make the following definition of a subgame price equilibrium. At the price SPE each platform z maximizes its own profit, $\pi_z = p_z q_z + r_z w_z - I(y_z)$, given the other platform's price strategy and has no incentive to deviate to another price.

In this section, we provide results showing that given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ such that $y_\alpha > y_\beta > 0$, there exists a pure strategy price SPE pair (w_α^*, w_β^*)[2]. In addition we characterize the market configurations that result. Specifically, we show the conditions under which particular market configurations arise depending on the parameters in the tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$

Our results show that the uncovered market configuration CI does not occur at a

[2]The actual price characterizations can be found in the appendix .

SPE. In this configuration no CPs join the low-quality platform even though it has positive quality. We show that there exists a profitable price deviation by the low-quality platform that involves CPs joining this platform. On the other hand, we show that given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ one of the other configurations, CII, $CIII$ or CIV, will emerge. In doing so, we determine the set of parametric values $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ for which these different configurations exist.

We prove the existence of the price SPE by a construction argument. The proofs, which are in the appendix, involve first identifying candidate equilibrium price pairs in each possible market configuration.[3] We then check to see whether these price equilibrium pairs are indeed Nash equilibria of the price subgame.[4] We do so by verifying that the equilibrium price candidates are best replies on the whole domain of strategies: That is, not only are they best responses in their respective market configurations but that they are also best replies if the other market configurations are taken into account.

For ease of presenting our first theorem that summarizes the above results we define the following sets of prices which we use in the theorem,

$$\begin{aligned}
\mathcal{R}_{\mathcal{I}} &= \{(w_\alpha, w_\beta) | r_\alpha(w_\alpha, w_\beta) + r_\beta(w_\alpha, w_\beta) < 1,\ r_\alpha(w_\alpha, w_\beta) > 0,\ r_\beta(w_\alpha, w_\beta) = 0\}, \\
\mathcal{R}_{\mathcal{II}} &= \{(w_\alpha, w_\beta) | r_\alpha(w_\alpha, w_\beta) + r_\beta(w_\alpha, w_\beta) < 1,\ r_\alpha(w_\alpha, w_\beta) > 0,\ r_\beta(w_\alpha, w_\beta) > 0\}, \\
\mathcal{R}_{\mathcal{III}} &= \{(w_\alpha, w_\beta) | r_\alpha(w_\alpha, w_\beta) + r_\beta(w_\alpha, w_\beta) = 1,\ r_\alpha(w_\alpha, w_\beta) > 0,\ r_\beta(w_\alpha, w_\beta) > 0\}, \\
\mathcal{R}_{\mathcal{IV}} &= \{(w_\alpha, w_\beta) | r_\alpha(w_\alpha, w_\beta) + r_\beta(w_\alpha, w_\beta) = 1,\ r_\alpha(w_\alpha, w_\beta) = 1,\ r_\beta(w_\alpha, w_\beta) = 0\}.
\end{aligned}$$

The sets $\mathcal{R}_{\mathcal{I}}$, $\mathcal{R}_{\mathcal{II}}$, $\mathcal{R}_{\mathcal{III}}$ and $\mathcal{R}_{\mathcal{IV}}$ consists of price pairs (w_α, w_β) that result in configuration CI ,CII, $CIII$ and CIV respectively. We next present a theorem showing that for any tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ a price subgame Nash equilibrium exists and only one market configuration is feasible. In addition, for market configurations CII and $CIII$, the price characterizations are unique.

Theorem 1. *Let Assumption 6 hold and $y_\alpha > y_\beta > 0$. Given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ there exists a Nash equilibrium pair (w^*_α, w^*_β) in the quality-subgame. Moreover, the*

[3]This is done in appendix A.1.2.

[4]Computations for this are in Appendix A.1.3.

resulting market configuration is unique and the following hold:

1. *If* $1 < \frac{\overline{\gamma}}{a} < \frac{2f(y_\alpha - y_\beta)+30y_\alpha - 3y_\beta}{2f(y_\alpha - y_\beta)+18y_\alpha+9y_\beta}$, *then the equilibrium price pair is unique and* $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{II}$.

2. *If* $\frac{2f(y_\alpha - y_\beta)+30y_\alpha - 3y_\beta}{2f(y_\alpha - y_\beta)+18y_\alpha+9y_\beta} \leq \frac{\overline{\gamma}}{a} < \min\left\{\frac{2f(y_\alpha - y_\beta)+18y_\alpha+9y_\beta}{2f(y_\alpha - y_\beta)+6y_\alpha+21y_\beta}, \frac{4f(y_\alpha - y_\beta)+18y_\alpha - 9y_\beta}{4f(y_\alpha - y_\beta)+6y_\alpha+3y_\beta}\right\}$ *then the equilibrium price pair is unique and* $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{III}$.

3. *If* $\frac{2f(y_\alpha - y_\beta)+18y_\alpha+9y_\beta}{2f(y_\alpha - y_\beta)+6y_\alpha+21y_\beta} < \frac{\overline{\gamma}}{a} < \frac{5f+18}{5f+6}$ *then the equilibrium price pair is unique and* $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{III}$.

4. *If* $\max\left\{\frac{5f+18}{5f+6}, \frac{4f(y_\alpha - y_\beta)+18y_\alpha - 9y_\beta}{4f(y_\alpha - y_\beta)+6y_\alpha+3y_\beta}\right\} \leq \frac{\overline{\gamma}}{a}$ *then* $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{IV}$.

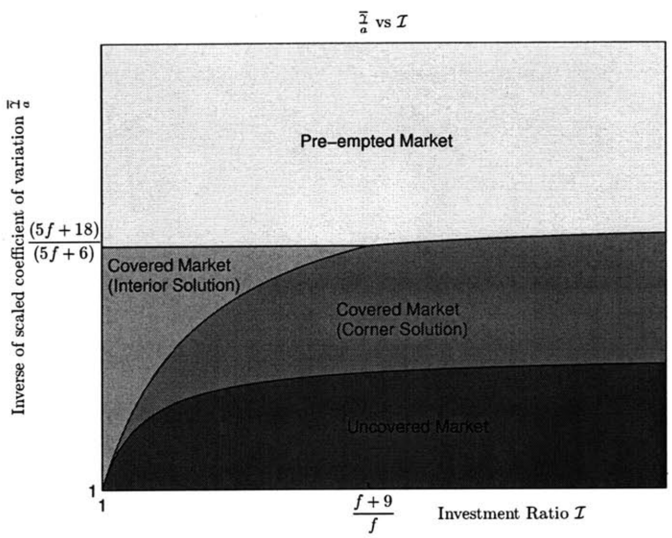

Figure 3-2: Inverse of the scaled coefficient of variation $\overline{\gamma}/a$, versus the investment ratio $y_\alpha/y_\beta = \mathcal{I}$, and resulting market configurations in a neutral regime.

Figure 3-2 shows the resulting market configurations for different values of the investment ratio, $y_\alpha/y_\beta = \mathcal{I}$, the inverse of a scaled coefficient of variation, $\overline{\gamma}/a$ and a fixed mass f of consumers. In particular, given an $\mathcal{I}$, $\overline{\gamma}/a$ and f, Figure 3-2 shows the distinct resulting market configurations. For a fixed $\mathcal{I}$, as $\overline{\gamma}/a$ increases the covered market is more likely. At the extreme, when $\overline{\gamma}/a$ is high, the coefficient of variation, which measures the dispersion of the CP qualities, is low. Therefore CPs are relatively close to each other and less distinguishable from each other; a decision made by a CP will be mirrored by the other close CPs and a covered market is likely. On the other hand, for a fixed value of low $\overline{\gamma}/a$, as the investment ratio increases the two platforms become more differentiated; price competition becomes less intense. This softening of price competition results in an uncovered market because less CPs can afford to join the platforms. However, for a fixed high $\overline{\gamma}/a$, the relative closeness of the content providers quality, dominates the differentiation effects of the platforms and a preempted covered market is realized as all CPs flock to one platform.

So far the results from Subsection 3.3.5 have shown that given any tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ such that $y_\alpha > y_\beta > 0$ a pure strategy subgame equilibrium occurs. We now show that any tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ where $y_\alpha > y_\beta = 0$ also has a SPE. Moreover we show that in this case only configurations CI and CIV exist. Note that under this restriction on platforms qualities, CPs do not join the low-quality platform since they make no revenue, hence only configuration CI and CIV exist.

Theorem 2. *Let $y_\alpha > y_\beta = 0$ hold. Given a tuple $(\overline{\gamma}, a, f, y_\alpha)$ there exists a unique Nash equilibrium pair (w_α^*, w_β^*) in the price subgame. Moreover, the resulting market configuration is unique and the following hold:*

1. *If $1 < \frac{\overline{\gamma}}{a} < \frac{9+2f}{3+2f}$, then the equilibrium price pair $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{\mathcal{I}}$.*
2. *If $\frac{9+2f}{3+2f} \leq \frac{\overline{\gamma}}{a} < \infty$, then the equilibrium price pair $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{\mathcal{IV}}$.*

Theorems 1 and 2, give a complete characterization of the quality subgame given any tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ where $y_\alpha > y_\beta \geq 0$. Results of Theorem 2 show we naturally get a tipping equilibrium with all CPs locating to the platform with the highest quality when the low-quality platform does not invest. This occurs for all values of $\overline{\gamma}/a$.

3.3.6 Quality Choice

In this section, we consider quality investment by the platforms. We assume that the cost of quality is increasing. Platforms are the only movers and they decide what quality level to set. The choice set of platform $j \in \{\alpha, \beta\}$ given any h_j^k is $A_j(h_j^k) = y_j$ where

$y_j \in [0, \infty)$. Thus the platforms simultaneously decide what quality level to invest in. We show that a subgame perfect Nash equilibrium exists. In addition, we show that this equilibrium involves maximal differentiation subject to investment costs, i.e., one platform choosing the highest quality possible taking cognizance of investment cost whilst the other chooses not to invest. Moreover, we characterize the investment levels in terms of the system parameters and give an interpretation to these expressions. We find the equilibrium quality choices by considering the best reply responses of the two platforms. We find the set that contains platform β's best replies to platform α's choices and vice versa. We then analyze the points where these sets intersect and show that they indeed are the subgame perfect equilibria. We make the following standard assumption on the investment cost to simplify the discussion and characterization of the investment level.

Assumption 7. *The investment cost function $I : [0, \infty) \to [0, \infty)$ is strictly convex, differentiable, nondecreasing and satisfies $I'(0) = I(0) = 0$.*

The following Theorem shows the necessary conditions for a subgame perfect Nash equilibrium to exist.

Theorem 3. *Let Assumption 6 and 7 hold, and $f \geq \frac{3}{5}$. If a subgame perfect Nash equilibrium (SPE) exists in the quality investment game then one platform does not invest any quality and the other invests in some positive quality $y^*(\overline{\gamma}, a)$. Moreover, the following holds:*

$$\begin{aligned}
\textit{If } \tfrac{\overline{\gamma}}{a} < \tfrac{9+2f}{3+2f} \textit{ then } I'(y^*) &= j(\overline{\gamma}, a, f) \textit{ where,} \\
j(\overline{\gamma}, a, f) &= (4(\overline{\gamma} - a)^2 f^3 + 12(\overline{\gamma} + a)(\overline{\gamma} + 3a) f^2 + 9(\overline{\gamma} + a)^2 f)/108a, \\
\textit{If } \tfrac{\overline{\gamma}}{a} \geq \tfrac{9+2f}{3+2f} \textit{ then } I'(y^*) &= h(\overline{\gamma}, a, f) \textit{ where,} \\
h(\overline{\gamma}, a, f) &= 2f(\overline{\gamma}(3 + 4f) - 3a)/9.
\end{aligned}$$

The above theorem enables us to identify candidate equilibrium investment pairs given an investment cost function $I(y)$ that satisfies Assumption 7 and a consumer mass

$f \geq 3/5$[5]. The next theorem shows that for quadratic investment functions of the form $I(y) = cy^2$ that the above characterization is indeed a SPE in the investment stage.

Theorem 4. *Let Assumption 6and 7 hold. Further, let the investment cost function be of the form $I(y) = cy^2$. Given a tuple $(\overline{\gamma}, a, f, c)$ where f is sufficiently large, there exists a SPE in the quality investment game.*

In general, the above results suggest that the platforms differentiate in platform quality to soften price competition. If the platforms are undifferentiated both platforms earn zero profits due to Bertrand price competition on both sides of the market. Therefore, platforms have incentive to invest in different quality levels in equilibrium. In particular, one platform invests in a positive quality whilst the other opts not to invest.

3.3.7 Investment and Market Coverage at the SPE

In this section we discuss the investment levels and market coverage at the SPE in the neutral model. On both sides of the market the platforms are viewed as substitute products by both consumers and CPs. Thus platforms make higher profits when they are more differentiated. The high-quality platform gains by investing more and the low-quality platform by investing less. For the low-quality platform the differentiation not only gives it market power on the consumer side but also reduces its investment cost. Even though investment in quality by the low-quality platform would generate revenue on the CP side, this gain would be offset by the loss of revenue on the consumer side. Indeed, investment by the low-quality platform increases competition on the consumer side in addition to increasing investment cost, resulting in lower consumer prices and consequently platform profit.

The investment level of the high-quality platform increases with CPs average quality. This increases the revenues that CPs earn; recall that the advert price is increasing in platform quality. Thus the surplus from which the high-quality platform can extract revenue also increases. In contrast, the relationship between the investment level and the

[5]This consumer mass is lower than the percentage of consumers in the U.S. who use the internet, which is around 74%. See http://www.internetworldstats.com/stats14.htm

heterogeneity is unimodal and convex. An increase in heterogeneity generally decreases the price elasticity of demand of the CPs. Hence, the high-quality platform prefers to make revenue directly by raising prices rather than through investment which is more costly. However, as heterogeneity increases beyond a critical point the platform prefers to invest in quality. This occurs for two reasons. First, due to the high prices the CP market becomes progressively uncovered. To gain revenue from the diminishing CP base, the high-quality platform invests to increase the surplus from which it can expropriate revenue. Second, due to the two-sided nature of the market, an increase in investment increases the value consumers gain when they interact with CPs. Consequently, it is able to charge a higher price to consumers and thus extract some of the gain generated, see Figure 3-3. We next present a corollary from theorem 3 that shows the market coverage

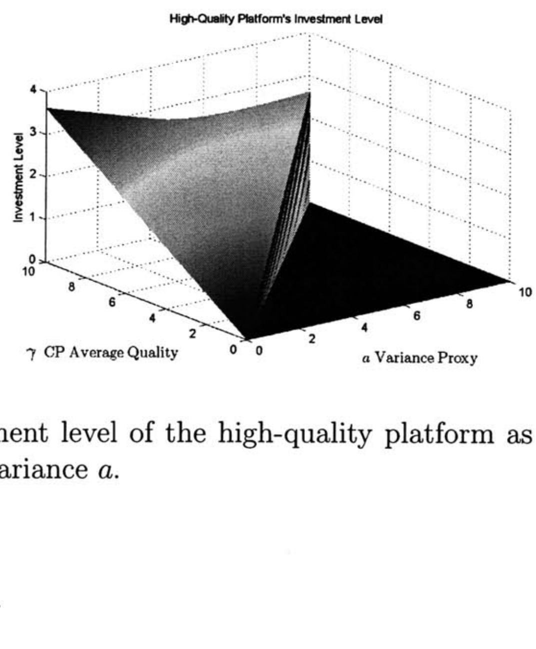

Figure 3-3: Investment level of the high-quality platform as a function of average CP quality $\overline{\gamma}$ and CP variance a.

by CPs at the SPE.

Corollary 3. *Let Assumption 6and 7 hold, and* $f \geq \frac{3}{5}$. *If a SPE exists then the following hold:*

(a) If $\frac{\overline{\gamma}}{a} < \frac{9+2f}{3+2f}$, *then the market is uncovered and one platform has all the market share in the content provider market. The market share is an increasing function of the dispersion measure* $\frac{\overline{\gamma}}{a}$.

(b) If $\frac{\overline{\gamma}}{a} \geq \frac{9+2f}{3+2f}$, then the market is covered and one platform has all the market share in the content provider market.

When $\overline{\gamma}/a$ is low, then an uncovered market with all CPs flocking to the high quality platform results. Either a is high which implies that the price elasticity of demand is low which leads to higher prices for the CPs and less enrollment or $\overline{\gamma}$ is low which implies that low quality CPs do not earn enough revenues to join the platform.For high values of $\overline{\gamma}/a$ a covered market results. In this case, either $\overline{\gamma}$ is high or the variance, represented by a, is low. In the former case CPs earn high advertising revenues, recall that the advert price increases with $\overline{\gamma}$, and thus all CPs can afford to join the platform. In the case of a low variance, CPs price elasticity of demand is high. Therefore prices charged to CPs are low encouraging high enrollment. Moreover, the high-quality platform prices out the low-quality platform leading to a pre-empted market.

3.4 Non-Neutral Model

The model structure of the non neutral regime is the same as that in the neutral regime except for the pricing rule on the CP side. Recall that, whilst CPs pay only once and have access to all consumers in the neutral regime, they have to pay each platform separately to access consumers on both platforms. Specifically, once a CP pays to be connected to a platform he gains access to those consumers connected to the same platform. However, should he also want to access consumers on the other platform he also has to pay access fees to the other platform.

The connection service offered by a platform to CPs can be viewed as an indivisible good that can be consumed separately (if a CP connects to consumers on one platform only) or jointly (if a CP connects to consumers on both platforms). Platform α (β) charges a fixed connection fee $w_\alpha(w_\beta)$ to CPs that connect to them regardless of what platform they are located on. CPs make revenues by selling advertising as before. The game's timing of events is same as that presented in the neutral model: We solve this game by considering its subgame perfect equilibria (SPE), which we find using backward induction.

3.5 Model Analysis

3.5.1 Consumption Decisions

We begin by analyzing the last stage of the game, i.e, the consumption decisions of the consumers. Only the consumers make a move in this stage and their choice sets are same as those defined in Section 3.3.1. A consumer i on a platform $\phi(i) \in \{\alpha, \beta\}$ connects to a CP j only if the CP has access to $\phi(i)$. Due to the pricing structure, CPs that access both platforms do so through the high-quality platform. This is shown in Section 3.5.4. This implies that a consumer i on the high-quality platform does not connect to CPs on the low-quality platform. Thus the utility gained by a consumer i connecting to CP j is given by $u_{ij}(y_{\phi(i)}, k_{\phi(i)}, r_{\phi(i)}, \gamma_j) = y_{\phi(i)} \left(\gamma_j / r_{\phi(i)} + k_{\phi(i)}\right)$. The quality perceived by a consumer i when he joins platform $\phi(i)$ is given by $F_i(y_{\phi(i)}, r_{\phi(i)}, a, \overline{\gamma}) = \int_0^1 E\left[u_{ij}(y_{\phi(i)}, k_{\phi(i)}, r_{\phi(i)}, \gamma_j)\right] dj$.

3.5.2 Consumer Platform Connection Decisions

A consumer i has a choice set $A_i(h_i^k) = \{\alpha, \beta\}$ and picks a platform which maximizes his utility, $U_i(\phi(i)) = \max\{R + \theta_i F_i(y_{\phi(i)}, r_{\phi(i)}, a, \overline{\gamma}) - p_{\phi(i)}, 0\}$. Given an information set h_i^k one of the following three relations hold; $(i) F_i(y_\alpha, r_\alpha, a, \overline{\gamma}) > F_i(y_\beta, r_\beta, a, \overline{\gamma})$, $(ii) F_i(y_\alpha, r_\alpha, a, \overline{\gamma}) < F_i(y_\beta, r_\beta, a, \overline{\gamma})$, $(iii) F_i(y_\alpha, r_\alpha, a, \overline{\gamma}) = F_i(y_\beta, r_\beta, a, \overline{\gamma})$. Platforms demands are derived in a similar manner to those in Section 3.3.2. They are based on consumer choices in this stage which in turn depend on which of the above relations holds and prices offered by platforms.

3.5.3 Platform Pricing Decisions On the Consumer Side

In this stage of the game the platforms simultaneously decide what prices to charge to the consumers. The choice set of platform i given any h_i^k, is $A_i(h_i^k) = p_i \in \mathbb{R}$. Information sets in this stage can be classified into three types depending on which of the three relations in Section (3.5.2) holds. We characterize for each type prices along the equilibrium path.

For information sets such that the first relation holds, the equilibrium prices and consumer allocations on the platforms are given by $q_\beta(F_i(y_\alpha,\cdot)-F_i(y_\beta,\cdot))$ and $q_\alpha(F_i(y_\alpha,\cdot)-F_i(y_\beta,\cdot))$ and the consumer demands addressed to the platforms at this equilibrium are $q_\alpha = 2f/3$ and $q_\beta = f/3$. When the second relation holds a symmetric characterization applies. If the third relation holds then $p_\alpha = p_\beta = 0$, and we make the standard assumption that consumers are evenly split. The analysis is similar to that in Section 3.3.3.

3.5.4 Content Provider Connection Decisions

In this stage CPs simultaneously decide which platforms to join, and their aggregate choices determine the mass of CPs connecting to each platform. The utility v_j gained by a CP is characterized as follows:

$$v_j = \begin{cases} g(\gamma_j, y_\alpha)q_\alpha + g(\gamma_j, \min\{y_\beta, y_\alpha\})q_\beta - w_\alpha - w_\beta & \text{if } \widehat{\phi}(j) = \alpha \text{ and CP also connects to } \beta, \\ g(\gamma_j, \min\{y_\beta, y_\alpha\})q_\alpha + g(\gamma_j, y_\beta)q_\beta - w_\alpha - w_\beta & \text{if } \widehat{\phi}(j) = \beta \text{ and CP also connects to } \alpha, \\ g(\gamma_j, y_\alpha)q_\alpha - w_\alpha & \text{if } \widehat{\phi}(j) = \alpha, \\ g(\gamma_j, y_\beta)q_\beta - w_\beta & \text{if } \widehat{\phi}(j) = \beta. \end{cases}$$

Similar to the neutral model, we focus on the special case where $g(\gamma_j, y_{\widehat{\phi}(j)}) = \gamma_j y_{\widehat{\phi}(j)}$. CPs decide on which platform to locate given the quality of service (y_α, y_β) and prices (w_α, w_β) offered by the platforms. To determine which platform(s) a CP joins, we can view a CP as having an option to buy four types of connection services. In particular, a CP j maximizes the utility function v_j to determine whether to connect to both platforms (joint consumption), a single platform (exclusive consumption) or no platform at all.

We first look at the joint consumption cases: A CP j is willing to join platform α and also connect to platform β if $\gamma_j \geq (w_\alpha + w_\beta)/(y_\alpha q_\alpha + \min\{y_\beta, y_\alpha\}q_\beta)$. Similarly, a CP j is willing to join platform β and also connect to platform α if $\gamma_j \geq (w_\alpha + w_\beta)/(\min\{y_\beta, y_\alpha\}q_\alpha + y_\beta q_\beta)$. For the exclusive consumption cases, a CP j is willing to join platform α if $\gamma_j \geq w_\alpha/(y_\alpha q_\alpha)$ and to join platform β if $\gamma_j \geq w_\beta/(y_\beta q_\beta)$. Given a pair (y_α, y_β) the utility gained by joint consumption through platform α dominates that gained through platform β if $y_\alpha \geq y_\beta$. Thus, given a pair (y_α, y_β), a CP is effectively choosing between three connection options. Maximizing v_j involves CP j picking the connection service that gives it maximum surplus.

Given a price pair (w_α, w_β), together with the tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$, the resulting CP demand on each platform is an aggregate of the mass of CPs who choose to join that platform. We refer to this demand as the CP allocation equilibrium. A CP allocation equilibrium not only characterizes demand faced by each platform but also determines which of the relations in section (3.5.2) will hold on the equilibrium path. In Appendix B, we derive the sets of prices $\mathcal{W}_{R(i)}$, $\mathcal{W}_{R(ii)}$ and $\mathcal{W}_{R(iii)}$ for which the CP allocation equilibrium leads to relations (i), (ii) and (iii) holding on the equilibrium path. Note that, if a price pair lies on the intersection of any of the sets $\mathcal{W}_{R(i)}$, $\mathcal{W}_{R(ii)}$, and $\mathcal{W}_{R(iii)}$, then more than one CP allocation equilibrium exists. Therefore, CP demand faced by platform $z \in \{\alpha, \beta\}$ is given by $r_z = \max\left\{\min\left\{1, \frac{1}{2a}\left(\overline{\gamma} + a - \frac{w_z}{q_z y_z}\right)\right\}, 0\right\}$; where q_z depends on which of the relations holds on the equilibrium path.

3.5.5 Platform Pricing Decisions On Content Provider Side

In this section we analyze the price competition on the CP side of the market. Platforms are the only movers and they decide what prices to charge to CPs. Given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$, there maybe more than one price game to analyze. Observe that if $y_\alpha \in \Lambda \cup \Delta$, here Λ and Δ are as defined in A.2.1, some price pairs (w_α, w_β) result in multiple CP allocation equilibria. This implies that for $y_\alpha \in \Lambda \cup \Delta$ more than one reduced extensive form game exists and consequently more than one price game exists. In the case $y_\alpha \notin \Lambda \cup \Delta$, only one price game exists since given any price pair (w_α, w_β) only one CP allocation equilibrium exists. We first show in Appendix A.2.2 that if a SPE exists in one of the CP price games then the CP allocation equilibrium on the equilibrium path does not yield relation (ii). In the remaining sections, we focus only on price games that result when the CP allocation equilibria selected (if multiple equilibria exist in the price subgames) are such that either relation (i) or (iii) hold. We formalize this in the following assumption.

Assumption 8. *Given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta, w_\alpha, w_\beta)$ such that multiple CP allocation* *equilibria exist in the price subgame we select the equilibrium that yields relation (i) or (iii).*

We then show in Appendix A.2.3 that without loss of generality we can consider the price game that results when the CP allocation equilibria chosen at the price subgames, when multiple equilibria exists, are the ones that yields relation (i). We refer to this game as the baseline CP price game. All other price games that have a SPE have the same unique pure strategy subgame-perfect Nash equilibrium pair (w_α^*, w_β^*). This follows because the CP games that arise from the reduced extensive form games are almost identical. In particular, the payoffs for both platform α and β given a strategy profile (w_α, w_β) are the same everywhere for all the games except for a set of measure zero. To prove that the baseline CP price game has a unique SPE we show that the best responses have a unique intersection point. Finally, we show that the market configuration depends only on the heterogeneity parameter a, the average CP quality $\overline{\gamma}$ and consumer mass f. We now present the Theorem showing that equilibrium CP prices exist given the tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ such that $y_\alpha > y_\beta$. The proof, price characterizations and conditions for various market configurations to exist are given in Appendix A.2.3.

Theorem 5. *Let Assumptions 6 and 8 hold. Given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ such that $y_\alpha > y_\beta$ there exists a unique subgame-perfect Nash equilibrium pair (w_α^*, w_β^*) in the price-subgame. Moreover, the resulting market configuration is unique.*

3.5.6 Quality Choice

In this stage of the game the platforms are the only movers and they decide what quality to invest in. We assume quadratic investment costs of the form cy^2 and $c \geq 1$. The choice set of platform $i \in \{\alpha, \beta\}$ given any h_i^k is $A_i(h_i^k) = y_i$ where $y_i \in [0, \infty]$. Thus the platforms simultaneously decide what quality to choose. We find the equilibrium quality choices by considering the best reply responses of the two platforms. We find the set that contains platform β's best replies to platform α's choices and vice versa. We establish that the best reply functions intersect at a unique point proving the existence of a unique subgame perfect equilibria (SPE) in the investment game. For ease of presenting the Theorem that characterizes the SPE of the quality investment game, and the corollaries that characterize the resulting market configurations, we make the following classifications given the tuple $(\overline{\gamma}, a, f)$:

R.1 $1 < \overline{\gamma}/a \leq \min\left\{(9+2f)/(3+2f), (f^2+12f-9+4\sqrt{3}\sqrt{f^3})/(-6f+9+f^2)\right\}$,

R.2 $\max\left\{1, (f^2+12f-9+4\sqrt{3}\sqrt{f^3})/(-6f+9+f^2)\right\} < \overline{\gamma}/a < (9+2f)/(3+2f)$,

R.3 $(9+2f)/(3+2f) < \overline{\gamma}/a < (f^2+12f-9+4\sqrt{3}\sqrt{f^3})/(-6f+9+f^2)$,

R.4 $\max\left\{(9+2f)/(3+2f), (f^2+12f-9+4\sqrt{3}\sqrt{f^3})/(-6f+9+f^2)\right\}$
$\leq \overline{\gamma}/a \leq (9-f)/(3-f)$,

R.5 $(9-f)/(3-f) < \overline{\gamma}/a \leq \infty$.

These regions are broadly classified according to the market configurations (as defined in Appendix A.2.3) that arise at the SPE given the tuple $(\overline{\gamma}, a, f)$. Qualitatively, the partitions represent regions in which the dispersion of CP content quality is either low (R.1,R.2), medium (R.3,R.4) or high (R.5). . We now present the theorem that characterizes the investment levels followed by an exposition of this result. The proof is given in Appendix A.2.5:

Theorem 6. *Let Assumption 6 and 8 hold. Given a tuple $(\overline{\gamma}, a, c)$ and $f > 0.47$ there exists a subgame perfect Nash equilibrium (SPE) in the quality investment game. Moreover, the following hold:*

1. *If **R.1** holds then the SPE entails one platform investing in positive quality $y^*(\overline{\gamma}, a, f, c) = j(\overline{\gamma}, a, f)/2c$ and the other not investing in any quality.*

2. *If **R.2** holds then the SPE entails both platform investing in positive qualities where one invests in a higher quality $y_h^*(\overline{\gamma}, a, f, c) = j(\overline{\gamma}, a, f)/2c$ and the other investing in a lower quality $y_l^*(\overline{\gamma}, a, f, c) = ((\overline{\gamma}-a)^2 f^3 - 6(\overline{\gamma}+a)(\overline{\gamma}+3a)f^2 + 9(\overline{\gamma}+a)^2 f)/(432ca)$.*

3. *If **R.3** holds then the SPE entails one platform investing in positive quality $y^*(\overline{\gamma}, a, fc)$ and the other not investing in any quality. In particular, $y^*(\overline{\gamma}, a, f, c) = h(\overline{\gamma}, a, f)/2c$.*

4. *If **R.4** holds then the SPE entails one platform choosing a higher positive quality, $y_h^*(\overline{\gamma}, a, f, c) = h(\overline{\gamma}, a, f)/2c$, and the other one choosing a lower positive quality $y_l^*(\overline{\gamma}, a, f, c) = ((\overline{\gamma}-a)^2 f^3 - 6(\overline{\gamma}+a)(\overline{\gamma}+3a)f^2 + 9(\overline{\gamma}+a)^2 f)/(432ca)$.*

5. *If* $\boldsymbol{R.5}$ *holds then the SPE entails one platform choosing a higher positive quality,* $y_h^*(\overline{\gamma}, a, f, c) = h(\overline{\gamma}, a, f)/2c$, *and the other one choosing a lower positive quality* $y_l^*(\overline{\gamma}, a, f, c) = f(3\overline{\gamma} - 2f\overline{\gamma} - 3a)/18c$.

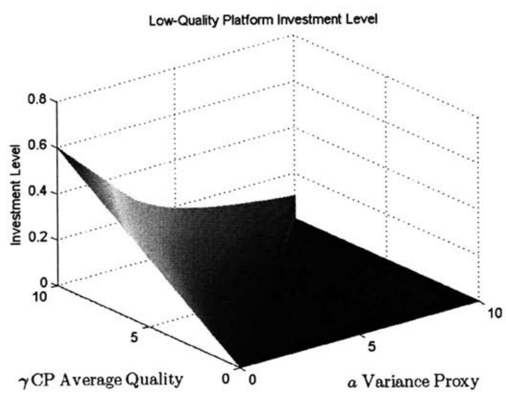

Figure 3-4: Investment level of the low-quality platform as a function of average CP quality $\overline{\gamma}$ and CPs heterogeneity/Variance Proxy a.

We impose the condition $f > 0.47$ since a SPE in the quality investment game does not exist in all the regions, for smaller values of f.

3.5.7 Investment and Market Coverage at the SPE

In the non-neutral regime the platforms are substitutes only on the consumer side of the market. On the CP side of the market, a CP makes a decision to join one platform independently from its decision to join the other. Hence there is no competition between platforms to attract CPs. In this sense a platform acts as monopoly on the CP side since its only through them that CPs can connect to consumers who subscribe to it. Thus investment decisions observed in this regime are driven by the trade-off platforms make between differentiating in quality to make revenue of the consumer side and exerting their monopoly power to extract revenue from the CP side.

The level of investment of the high-quality platform is the same as that in the neutral regime and varies with average CP quality, and variance in a similar way. Quality investment in the low-quality platform also increases with the average CP quality. An increase in average CP quality increases the advertising rate paid to CPs. Therefore, the low-quality platform increases its investment to enhance that of the CPs which in turn earns the CPs more advertising revenue enabling the platform to extract more revenue.

As the heterogeneity of CPs quality increases the value of investment decreases, this is in contrast to the behavior exhibited by the high-quality platform. An increase in heterogeneity leads to CPs becoming less price sensitive, hence the platform charges more; it prefers to extract CP revenue through price than invest in quality which is costly. Despite a shrinking CP base, the low-quality platform does not increase its investment level as is the case for the high-quality platform. Even though the low-quality platform can gain from increased CP surplus with more quality investments, the increase in competition on the consumer side would offset this gain. The resulting higher competition on the consumer side from this investment would lower prices significantly limiting the only instrument via which the platform can extract these gains.

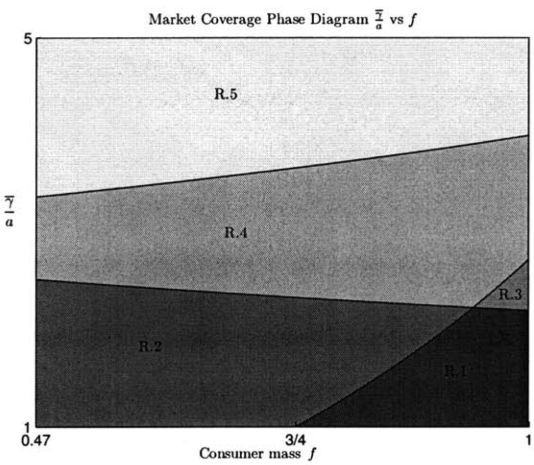

Figure 3-5: Phase Diagram Showing Market Coverage as a function of $\overline{\gamma}/a$ and f .

We now explore the investment patterns as a function of the consumer mass and the dispersion measure, see Figure 3-5. When $\overline{\gamma}/a$ has a low value and f is high, i.e. in the regions denoted by $R.1$ and $R.3$ the platforms differentiate as much as possible. One platform invests in a positive quality whilst the other opts not to invest. Similar to the neutral case, both platforms make more profit when they are more differentiated. A low value of $\overline{\gamma}/a$ is primarily driven by low average CP quality. Therefore the advertising revenue gained by CPs is also low. Consequently, profits expropriated from CPs if the low-quality platform invested, are not enough to cover the loss of revenue due to the resulting competition intensity and the costs of investment. Moreover, since f is large, the mass of consumers joining the platforms is higher which further increases the revenue made from the consumer side and dissuades the platform from investing.

In contrast when f and $\overline{\gamma}/a$ are both low, platforms invest albeit in different levels of quality. This region is denoted by $R.2$. In this region, the mass of consumers joining the low-quality platform is low. As before, the low-quality platform faces a trade-off between investing in quality to increase CP surplus from which it extracts revenue or not investing so as to gain more market power which enables it to gain more revenue from consumers through higher prices. However, since f is small the platform gains more from the CP side when it invests than it loses in revenue from competition on the consumer side; the low f reduces the volume of subscribing consumers and in turn revenues that the platform would make by maximally differentiating from the high quality platform.

Where $\overline{\gamma}/a$ has a medium to high value, the regions denoted by $R.4$ and $R.5$, the platforms partially differentiate. As in the previous case, both platforms invest; one invests in a larger quality than the other. If the high value of $\overline{\gamma}/a$ is primarily caused by a high average CP quality, the investment choice can be explained in a similar manner to the previous case, i.e. the lower quality platform trades-off maximal differentiation on the consumer side for the opportunity to make profits on the CP side. If the coefficient of variation is low due to low variance we also obtain similar results but through a different mechanism. In particular, for low values of a, the CPs become less distinguishable and the demand is highly elastic. Therefore, decisions made by one CP are mirrored by the others. To be able to distinguish between these CPs, and take advantage of their monopoly power, both platforms investment in quality. Note that CPs revenues increase with platform quality since advert price depends on it. However, the increase in quality is tempered down by the fact that the platforms would still like to differentiate and make profits on the consumer side. So the differentiation is partial.

Corollary 4. *If* ***R.1*** *or* ***R.3*** *holds CPs exclusively serve the high quality platforms.*

*Moreover, if **R.1** (**R.3**) holds then an uncovered (covered) market is the outcome.*

Corollary 5. *If **R.2**, **R.4**, or **R.5** holds, then CPs multi-home. In particular,*

1. *If **R.2** (**R.4**)holds then only the high quality CPs multi-home and market is uncovered (covered).*

2. *If **R.5** holds then all CPs multi-home.*

In region $R.1$ CPs exclusively join the high quality platform and the market is uncovered, i.e, they single-home. Observe that in this region the low-quality platform does not invest. Therefore there is no value to be gained by a CP joining the lower quality platform. Region $R.2$ also represents an uncovered market but CPs patronize both platforms with the lower-quality CPs being exclusive to the high-quality platform and the high-quality CPs joining both. Only the high-quality CPs multi-home since they earn enough advertising revenue to cover the cost of connecting to both platforms.

In the remaining regions the market is covered by the high quality platform, i.e., all the CPs in the market serve their content to the high-quality platform. This is because for medium to high values of $\overline{\gamma}/a$ CPs earn higher advertising rates on the high-quality platform which also happens to have a higher number of consumers (eyeballs). Lack of investment, in region $R.3$, by the low-quality platform leads to CPs not joining it since they will not make any advertising revenue. In contrast, high-quality CPs also serve the low-quality platform in region $R.4$. Investments by the low-quality platform are attractive to high-quality CPs who can command higher advertising prices to offset costs of joining two platforms and still make revenue. Finally, in region $R.5$ all CPs patronize both platforms, i.e, they all multi-home. The average CP quality is high enough and the variation of CP quality low enough that the advertising prices the CPs command enable them to gain more value when they connect to both than when they connect to only one platform.

3.6 Comparative Statics

In this section we compare social welfare, consumer and CP surplus, and profits between the two regimes at the SPE's. We first define social welfare and its constituent parts. We then show that given a tuple $(\overline{\gamma}, a, f)$ the non-neutral regime results in a larger social welfare compared to the neutral model. Next, we show that gross CP and consumer surplus in the non-neutral model are at least equal to, if not, superior to that in the neutral model. Finally, we show a dichotomy of preferences for the two regimes by platforms. Low-quality platforms prefers the non neutral regime whilst the higher quality platform prefers the neutral regime.

3.6.1 Social Welfare, Consumer and CP surplus, Platform profits

Social welfare is defined as the sum of consumer surplus, platform profits and content provider surplus. We define each of these terms below:

1. Consumer surplus: A consumer $i \in [0, f]_i$ subscribing to platform $\phi(i)$ has an expected consumer surplus given by, $E[U_i] = E[R + \theta_i F_i(y_{\phi(i)}, \cdot) - p_{\phi(i)}]$. Aggregate consumer surplus is given by $\int_0^f E[U_i]di = \sum_{\phi(i)\in\{\alpha,\beta\}} \left(R + E[\theta_i|\phi(i)]F_i(y_{\phi(i)}, \cdot) - p_{\phi(i)}\right) q_{\phi(i)}$.

2. Platform profits: Platform $z \in \{\alpha, \beta\}$ has a profit function π_z given in (3.5). The total profit across both platforms is given by $\sum_{z\in\{\alpha,\beta\}} \pi_z = \sum_{z\in\{\alpha,\beta\}} p_z q_z + r_z w_z - cy_z^2$.

3. Content provider surplus: A CP $j \in [0, 1]_j$ on platform $\hat{\phi}(j)$ has an expected surplus given by $E[v_j]$. The CP surplus depends on what regime is being considered. In the neutral regime it is given by $\int_0^1 E[v_j]dj = \int_0^1 \sum_{\hat{\phi}(j)\in\{\alpha,\beta\}} E[v_j|\hat{\phi}(j)]P[\phi(j)]dj$. The content provider surplus in the non-neutral regime is represented as follows $\int_0^1 E[v_j]dj = \int_0^1 \sum_{\hat{\phi}(j)\in\{\alpha,\beta,(\alpha\cap\beta)\}} E[v_j|\hat{\phi}(j)]P[\phi(j)]dj.$,

3.6.2 Social Welfare Comparison

Comparison of social welfare at the SPE under both models shows that in general the non-neutral model is superior, see Appendix A.3.1. Figure 3-6, shows the difference in welfare between the non-neutral regime and the neutral regime when ($f = 0.6, c = 1$). For a fixed variance, the difference in welfare is increasing as a function of $\overline{\gamma}$. This follows from the fact that the difference between the two regimes is the level of investment by the low-quality platform. Indeed, an increase in $\overline{\gamma}$ increases the investment level of the low-quality platform (in the non-neutral regime) as discussed in Section 3.5.7. This in turn results in an increase in CP surplus as well as consumer surplus for those CPs and consumers on the low-quality platform in the non-neutral regime. Although investing is costly the resulting increase in both CP and consumer surplus is higher. On the other hand, an increase in heterogeneity a results in a lower welfare difference. This again reflects the effects of heterogeneity on investments in the low-quality platform. Recall from Section 3.5.5 that the low-quality platform prefers to make revenue from raising prices rather than investing in quality as CP heterogeneity increases. Therefore CPs that join the low-quality platform, in the non-neutral regime, receive a lower surplus for an increasing a, and this is reflected by the welfare difference between the two models being lower.

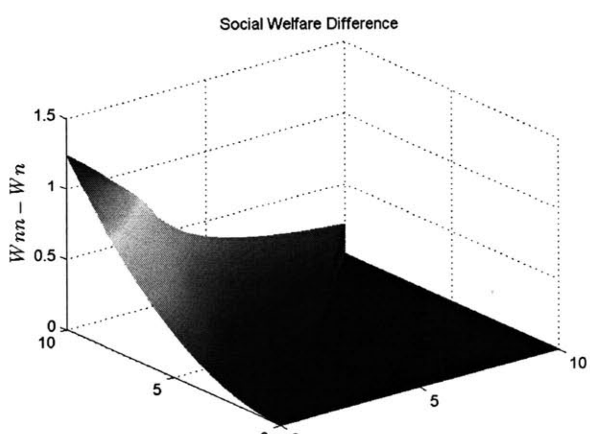

Figure 3-6: Welfare difference between Non-neutral and neutral regime.

3.6.3 CP and Consumer surplus Comparison

CP surplus is generally higher in the non-neutral model compared to the neutral model, see Appendix A.3.2. Investments by the low-quality platform increase the revenue earned by CPs who multi-home since they command more advertising revenue from advertisers. In contrast, since the low-quality platform does not invest in the neutral model the CP surplus is less. Consumer surplus is also higher in the non-neutral regime because the low-quality platform invests in quality, see Appendix A.3.3. This investment increases the consumer surplus for two reasons. First, an increase in the low-quality platform investment level intensifies price competition on the consumer side resulting in lower prices. Therefore consumers are able to keep more of their surplus. Second, an increase in platform quality increases the value gained by consumers who join the low-quality platform due to the cross externality caused by CPs whose quality is enhanced.

3.6.4 Platform profits Comparison

Aggregate profit in the neutral regime is generally higher than that in the non-neutral regime, see Appendix A.3.4. Competition induced by the investment of the low-quality platform on the consumer side reduces the platforms profits in the non-neutral regime. Even though there is a gain in CP revenue by the low-quality platform it is not large enough to cover the loss of consumer profit due to the increased competition. Profits of the high-quality platform are also higher in the neutral regime, see Appendix A.3.4. In this regime the platforms are maximally differentiated and the high-quality platform serves high-quality consumers and CPs. Thus it is able to extract more revenue from both due to the resulting market power that arises from the differentiation. In the non-neutral regime, the investment by the low-quality platform results in a higher price competition on the consumer side that reduces the high-quality platform's overall revenue. In contrast, the low-quality platform's profits are superior in the non-neutral regime, see Appendix A.3.4. Note that the low-quality platform makes revenue only on the consumer side in the neutral regime. In the non-neutral regime it makes revenue from both sides of the market. Although the investment by the low-quality platform intensifies competition on the consumer side reducing revenue, it enables the platform to gain more CP revenue which offsets losses due to this competition.

3.7 Chapter Summary

We contribute to the net neutrality debate by analyzing investment decisions of ISPs, under both a neutral and non-neutral model. In addition we explore their concomitant effects on social welfare, consumer and CP surplus, and platform profits. First, we find that the pricing structure in the non-neutral model leads to a higher aggregate level of investment. This is because the low-quality platform invests in the non-neutral model but not in the neutral model. More precisely, in the neutral model, an increase in investment by the low-quality platform makes it more attractive to CPs but at the same time intensifies competition on the consumer side. The loss of consumer revenue due to this competition is much higher than the gain of revenue from the CP side, hence it decides not to invest. In contrast, due to the monopoly over access to consumers that platforms have, the gain in revenue by low-quality platform, made from expropriating CP surplus in the non-neutral model, is higher than the loss of revenue due to intensification of competition on the consumer side; therefore it decides to invest. Contrary to qualitative arguments, see for example [65, 35], our results suggest that access fees[6] could positively impact investment incentives leading to upgrades of existing network infrastructure.

Second, and in contrast to some results in the literature, e.g., [16, 10], we find social welfare is generally superior in the non-neutral regime compared to the neutral regime. This follows because the aggregate level of investment is higher in the former regime increasing both CP and consumer surplus. CP surplus is higher in the non-neutral regime because CPs that multi-home earn more revenue from advertising which increases aggregate CP surplus. On the other hand, consumer surplus is higher because competition is more fierce in the non-neutral regime reducing prices and enabling consumers to keep more of their surplus.

Finally, we show that the platforms prefer different regimes; the low-quality platform

[6] Payments paid by off-net CPs to ISPs in order to access consumers.

prefers the non-neutral regime while the high-quality platform prefers the neutral regime. In the non-neutral regime, despite the intense competition on the consumer side, the high revenues gained on the CP side by the low-quality platform offset the losses due to the competition. Conversely, the high-quality platform makes more revenue on the consumer side in the neutral regime because the platform qualities are maximally differentiated.

Chapter 4

Conclusion

In this thesis we sought to understand the role of interconnection agreements and market regulation on competition in the internet. In Chapter 2, we investigated the role of access charges, in the presence of congestion, on pricing, consumer enrollment and social welfare and established the following;

- There are two opposing effects on the price charged to consumers as a result of the two-sidedness of the market, the interconnection of the platforms and the access charge, and the congestion costs. Due to the cross externality that a marginal consumer exerts on the websites, his price is discounted by the revenue he creates on the website side. Moreover, due to the cross net traffic he generates from the other platform his price is further discounted by the revenue earned from access fees levied on this traffic. On the other hand, a platform marks up the price to a marginal consumer with both a Pigovian tax, which internalizes the congestion cost on its link, and a switching cost. A consumer switching platforms incurs a congestion cost on the link of the platform he moves to. Therefore a platform can raises its price by this congestion cost without losing market share.

- Consumer enrollment in an oligopoly equilibrium is lower than that at the social optimum but it is increasing in the access charge. The access charge discounts the price charged to consumers which increases the number of consumers that join the platform.

- If the consumer market is uncovered, social welfare is increasing in the access charge; this follows because the number of consumers also increases with the access charge.

These results suggest that a zero access charge or the so called *Bill and Keep* peering needs further scrutiny if it is to be accepted as a standard interconnection agreement that enhances social welfare. We next briefly describe possible paths to pursue in this research. Our model assumed a non-discriminatory pricing policy for consumers, i.e., consumers were priced the same for both on-net and off-net content. Interesting follow up to this work would allow price discrimination depending on the origin of the content. Secondly, we model websites as homogenous agents whose reservation price is common. Introducing heterogeneity on the website side would present a more realistic model of content providers' utilities. Another interesting direction would involve endogenously determining the access charge as well as removing the reciprocal constraint. These would help shed light on the bargaining process of the interconnection agreements. Finally, the current model assumes full coverage assumption on the website side. An important generalization would do away with this assumption and analyze a model that endogenously determines market structure on both sides of the market.

In chapter 3 we investigated the net neutrality issue from a "pricing rule" perspective. We added new insights to the mechanism via which broadband providers invest and showed the effects of these investments on consumer, CP and social welfare. More precisely we showed:

- Investment levels are driven by the trade-offs platforms make in softening price competition on the consumer side and increasing CP surplus on the CP side from which they expropriate revenue.
  - In the neutral model the platforms are viewed as substitutes by both CPs and consumers. Hence at the equilibrium, platforms maximally differentiate to corner different consumer and CP niches in the markets.
  - In the non-neutral model platforms are viewed as substitutes only by consumers. Due to the pricing structure, each platform has a monopoly over

access to its consumer base. Investment patterns at equilibrium differ from those in the neutral regime. Even though for a large consumer base and low values of average CP quality we have maximal differentiation, in all other cases we have partial differentiation. Moreover, in this regime the difference in levels of investment between the two platforms is a function of the consumer mass.

- Social welfare is superior in the non-neutral regime compared to the neutral regime. This result is primarily driven by the fact that the low-quality platform has an incentive to invest in the non-neutral regime increasing the aggregate level of investment in this regime.

- We show that CPs and consumers' surplus are higher in the non-neutral regime. This result runs counter to the popularly held opinions on net neutrality in the policy debate. Again these results are driven by the increase in the aggregate level of investment in the non-neutral regime. For CPs the larger investment on the low-quality platform leads to more revenue from advertisers. For consumers a larger investment on the low-quality platform has two major effects. First, it increases price competition between platforms and thus lowers prices which means consumers keep more of the value generated by their interaction with CPs. Secondly, the higher investment results in a higher utility for consumers when they interact with CPs since CP quality is enhanced by platform quality.

- Finally, we show that the low-quality platform prefers a non-neutral policy, whilst the high-quality platform prefers a neutral policy. For the high-quality platform a neutral network involves maximal differentiation in quality. Therefore it makes maximum revenue from both sides of the market. On the other hand, for the low-quality platform, the investment it makes in the non-neutral model enables it to gain CP revenue due to its monopoly access over its consumer base. In spite of the loss on the consumer side, due to the increased competition caused by this investment, the revenue gained on the CP side is much higher than this loss.

These results suggest that price regulation, mandating the current pricing structure, could possibly be an inapt policy for increasing value in the internet because it would limit the investment incentives of smaller ISPs or network providers. Low investments by these ISPs would decrease CPs and consumer utility directly through low QoS and indirectly through bottleneck effects. Therefore, if creating more value is desired, policies that foster investments should be pursued as this would lead to higher social welfare benefitting both consumers and CPs. However, a number of concluding remarks are useful. One simplifying feature of our analysis is the lack of transaction costs in the non-neutral regime. These present new analytical challenges but are an important area to explore because they would reduce the revenue earned by the platforms and thus likely temper (perhaps in a negative way) the investment incentives of ISPs. Another direction of future research is the explicit modeling of quality investment by CPs. An interesting modification would have CP quality determined endogenously and investigate CP incentives under both models. Nevertheless, the results show that if average CP quality is high a non-neutral regime is preferred by CPs due to the larger surpluses they gain. This suggests that in a framework where CP quality is endogenously determined, CPs are likely to prefer the non-neutral model if CP quality investment costs are not large. We leave these ideas and extensions for future research.

Appendix A

Appendix

A.1 Proofs

A.1.1

The following lemma shows that if a platform has a higher platform quality in the neutral regime then consumers joining it perceive it to be of a higher value.

Lemma 10. *Let $y_\alpha > y_\beta$, then $F_i(y_\alpha, \cdot) > F_i(y_\alpha, \cdot)$.*

Proof. After some algebraic manipulations the explicit expression for $F_i(y_{\phi(i)}, y_{\widehat{\phi}(j)}, \overline{\gamma}, r_\alpha, r_\beta)$ in terms of average CP content quality, platform quality and the mass of content providers on both platforms is as given below for both $\phi(i) = \alpha$ and $\phi(i) = \beta$;

$$\begin{aligned}
F_i(y_{\phi(i)}, y_{\widehat{\phi}(j)}, \overline{\gamma}, r_\alpha, r_\beta) &= \int_0^1 E\left[\max\{u_{ij}(y_{\phi(i)}, y_{\widehat{\phi}(j)}, \gamma_j, c_{\phi(i)}, r_{\widehat{\phi}(j)}), 0\}\right] dj, \\
&= y_\alpha(\overline{\gamma}(r_\alpha + 1) + a(1 - r_\alpha)) + y_\beta(\overline{\gamma}(r_\beta + 1) + a(1 - r_\alpha - r_\beta)), \\
F_i(y_{\phi(i)}, y_{\widehat{\phi}(j)}, \overline{\gamma}, r_\alpha, r_\beta) &= \int_0^1 E\left[\max\{u_{ij}(y_{\phi(i)}, y_{\widehat{\phi}(j)}, \gamma_j, c_{\phi(i)}, r_{\widehat{\phi}(j)}), 0\}\right] dj, \\
&= y_\beta(\overline{\gamma}(r_\alpha + 1) + a(1 - r_\alpha)) + y_\beta(\overline{\gamma}(r_\beta + 1) + a(1 - r_\alpha - r_\beta)).
\end{aligned}$$

It immediately follows from above that $F_i(y_\alpha, \cdot) > F_i(y_\alpha, \cdot)$. □

A.1.2 Candidate Equilibrium Prices for Different Markets.

Uncovered Market - CI. In this case we suppose *ex ante* that the market is uncovered with only the high quality platform serving the market. We identify the equilibrium prices for this market configuration and the conditions on $(\overline{\gamma}, a)$ for which this market configuration is feasible. We first derive the best price responses of each platform to the price set by its rival. The condition for an uncovered market where only the high-quality platform participates in the market is given by,

$$\frac{w_\beta}{(q_\beta + q_\alpha)y_\beta} \geq \frac{w_\alpha}{y_\beta q_\beta + y_\alpha q_\alpha} > \overline{\gamma} - a. \tag{A.1}$$

In this configuration, both platform's profit do not depend on w_β. Thus given w_α that satisfies $\frac{w_\alpha}{y_\beta q_\beta + y_\alpha q_\alpha} > \overline{\gamma} - a$, any w_β that satisfies the following condition

$$w_\beta \geq y_\beta \frac{w_\alpha(q_\alpha + q_\beta)}{y_\beta q_\beta + y_\alpha q_\alpha}$$

is a best response by platform β. This follows from condition (A.1). On the other hand given $w_\beta > (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$ the best response is given by the optimal solution of the following problem

$$\begin{aligned} &\max && \pi_\alpha^{ui}(w_\alpha, w_\beta) \\ &s.t. && w_\alpha \in \left((\overline{\gamma} - a)(y_\beta q_\beta + y_\alpha q_\alpha), \tfrac{w_\beta(y_\beta q_\beta + y_\alpha q_\alpha)}{y_\beta(q_\alpha + q_\beta)}\right]. \end{aligned} \tag{A.2}$$

From the first order conditions of Eq. (A.2) we infer that the best response is characterized as follows,

$$w_\alpha = \begin{cases} w_\alpha^* & \text{if } w_\beta \geq \frac{w_\alpha^* y_\beta(q_\alpha + q_\beta)}{(y_\beta q_\beta + y_\alpha q_\alpha)} \\ \frac{w_\beta(y_\beta q_\beta + y_\alpha q_\alpha)}{y_\beta(q_\alpha + q_\beta)} & \text{if } w_\beta < \frac{w_\alpha^* y_\beta(q_\alpha + q_\beta)}{(y_\beta q_\beta + y_\alpha q_\alpha)}. \end{cases}$$

where $w_\alpha^* = \frac{f}{9}(5a + \overline{\gamma})y_\alpha - \frac{f}{18}(a - 7\overline{\gamma})y_\beta$ and is the unrestricted solution to problem A.2. Thus any price combination such that,

$$\frac{w_\alpha^{ui}}{w_\beta^{ui}} = \frac{y_\beta q_\beta + y_\alpha q_\alpha}{y_\beta(q_\alpha + q_\beta)}, \tag{A.3}$$

where $(\overline{\gamma} - a)(q_\alpha y_\alpha + y_\beta q_\beta) < w_\alpha^{ui} \le w_\alpha^*$ and $(\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta < w_\beta^{ui} \le \frac{w_\alpha^* y_\beta (q_\alpha + q_\beta)}{(y_\beta q_\beta + y_\alpha q_\alpha)}$, is an equilibrium price pair in this configuration. In addition, when $w_\alpha = w_\alpha^*$ any price combination such that,

$$w_\alpha^{ui} = w_\alpha^* \tag{A.4}$$
$$w_\beta^{ui} \ge \frac{w_\alpha^* y_\beta (q_\alpha + q_\beta)}{(y_\beta q_\beta + y_\alpha q_\alpha)}, \tag{A.5}$$

is an equilibrium price too. It remains to specify the necessary condition for this configuration to occur. From condition (A.1), configuration CI occurs only if $(\overline{\gamma} - a)(y_\alpha q_\alpha + y_\beta q_\beta) - w_\alpha^{ui} < 0$. This results in the following necessary condition,

$$\frac{\overline{\gamma}}{a} < \frac{4f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta}{4f(y_\alpha - y_\beta) + 6y_\alpha + 3y_\beta}. \tag{A.6}$$

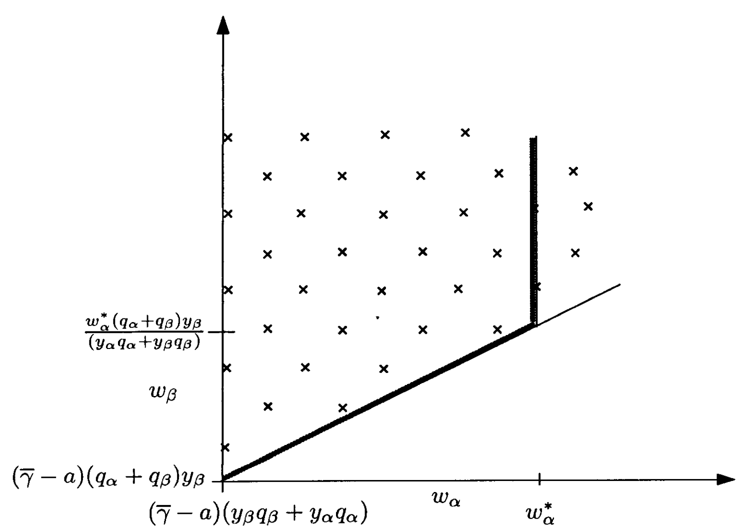

Figure A-1: The reaction correspondence of platform β given $w_\alpha > (\overline{\gamma} - a)(y_\beta q_\beta + y_\alpha q_\alpha)$ and the reaction curve of platform α given $w_\beta > (\overline{\gamma} - a)$. Their intersection points give the equilibrium price pairs in this market configuration. These are depicted by the red (dark) line.

Uncovered Market - ***CII*****.** In this case we suppose *ex ante* that the market is uncovered with both platforms serving the market. We first identify the equilibrium prices and then the values of $(\overline{\gamma}, a)$ for which this market configuration is feasible. The condition for an uncovered market in which both the high-quality and low-quality platforms serve the market is given by,

$$\overline{\gamma} - a < \frac{w_\beta}{(q_\alpha + q_\beta)y_\beta} < \frac{(w_\alpha - w_\beta)}{q_\alpha(y_\alpha - y_\beta)} < \overline{\gamma} + a. \tag{A.7}$$

The best reply functions of the respective platforms are obtained from the first order conditions of the platforms profit functions and are given below.

$$w_\alpha(w_\beta) = \frac{5f}{9}(y_\alpha - y_\beta) + \frac{f}{9}\overline{\gamma}(y_\alpha - y_\beta) + \frac{1}{2}w_\beta, \tag{A.8}$$

$$w_\beta(w_\alpha) = \frac{1}{6}\frac{y_\beta(f(\overline{\gamma} - a)(y_\alpha - y_\beta) + 9w_\alpha)}{(2y_\alpha + y_\beta)}. \tag{A.9}$$

Note that the above functions are linear. Solving the above two simultaneous equations yields the following unique equilibrium prices,

$$w_\alpha^u = \frac{f((8\overline{\gamma} + 40a)y_\alpha^2 - (23\overline{\gamma} + a)y_\beta y_\alpha - (17a + 7\overline{\gamma})y_\beta^2)}{9(y_\beta + 8y_\alpha)}, \tag{A.10}$$

$$w_\beta^u = \frac{4fy_\beta(\overline{\gamma} + 2a)(y_\alpha - y_\beta)}{3(y_\beta + 8y_\alpha)}. \tag{A.11}$$

The reaction functions and their intersection point are shown in Figure A-2. Since the market is not covered the lowest quality content provider does not join the lower quality platform. Therefore a necessary condition for the above configuration to hold is $(\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta - w_\beta^u < 0$. Substituting for w_β^u the above condition can be rewritten as,

$$\frac{\overline{\gamma}}{a} < \frac{2f(y_\alpha - y_\beta) + 30y_\alpha - 3y_\beta}{2f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta}. \tag{A.12}$$

Covered market- *CIII* **.** We now suppose *ex ante* that the market is covered with both platforms serving the market. We again identify the equilibrium prices and then the values of $(\overline{\gamma}, a)$ for which this market configuration is feasible. Proceeding as we did in the previous market configurations, to derive the equilibrium prices, we first derive best response prices of each platform to the price set by the other platform. The

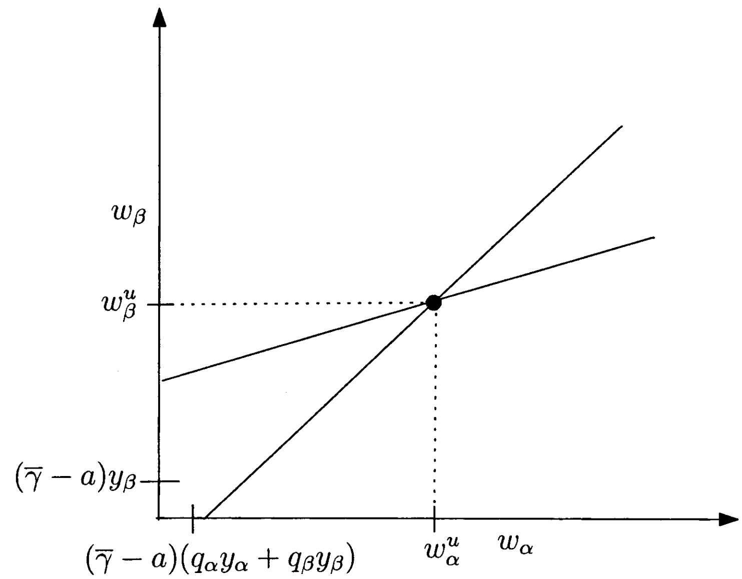

Figure A-2: The reaction curve of platform β given $w_\alpha > (\overline{\gamma}-a)(y_\beta q_\beta + y_\alpha q_\alpha)$ and the reaction curve of platform α given $w_\beta > (q_\alpha + q_\beta)(\overline{\gamma}-a)y_\beta$ intersect at the price pair (w_β^u, w_α^u).

condition for a covered market in which both the high-quality and low-quality platforms serve the market is

$$\frac{w_\beta}{(q_\alpha + q_\beta)y_\beta} \leq \overline{\gamma} - a < \frac{(w_\alpha - w_\beta)}{q_\alpha(y_\alpha - y_\beta)} < \overline{\gamma} + a. \tag{A.13}$$

The first order conditions associated with the profit functions for both platforms yield the following best reply functions,

$$w_\alpha(w_\beta) = (y_\alpha - y_\beta)(\tfrac{1}{3}f(\overline{\gamma}+a) - \tfrac{2}{9}f^2(\overline{\gamma}-a)) + \tfrac{1}{2}w_\beta,$$
$$w_\beta(w_\alpha) = \begin{cases} (y_\alpha - y_\beta)(\tfrac{1}{3}f(a-\overline{\gamma}) - \tfrac{1}{18}f^2(\overline{\gamma}-a)) + \tfrac{1}{2}w_\alpha & \text{if } w_\alpha < w_\alpha^* \\ (\overline{\gamma}-a)(q_\alpha + q_\beta)y_\beta & \text{if } w_\alpha \geq w_\alpha^* \end{cases}$$

where $w_\alpha^* = y_\alpha(\frac{2}{9}f^2(a-\overline{\gamma}) + \frac{1}{3}f(\overline{\gamma}+a)) - y_\beta(\frac{2}{9}f^2(a-\overline{\gamma}) + \frac{1}{6}f(5a-\overline{\gamma}))$.

Interior Solution. From the above best response functions we get the following unique equilibrium prices in the case of an interior solution.

$$w_\alpha^{ci} = (y_\alpha - y_\beta)\frac{1}{27}f(7f(\overline{\gamma} - a) - 6(3a - \overline{\gamma})), \tag{A.14}$$

$$w_\beta^{ci} = (y_\alpha - y_\beta)\frac{2}{27}f(f(\overline{\gamma} - a) + 3(3a - \overline{\gamma})). \tag{A.15}$$

A market is covered with an interior solution in the price subgame if the price charged by the lower quality platform is lower than the value derived by the lowest quality content provider, i.e., $(\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta - w_\beta^{ci} > 0$. In this market configuration the lowest quality content provider prefers the lowest quality platform, otherwise we have a preempted market. Moreover, the lowest-quality content provider's net utility must also be positive. Thus the following condition has to hold in equilibrium,

$$(\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta - w_\beta^{ci} > \max\{w_\alpha^{ci} - (\overline{\gamma} - a)(y_\beta q_\beta + y_\alpha q_\alpha), 0\}.$$

By plugging the equilibrium prices in Eq. (A.14) and Eq. (A.15) into the above inequality, we obtain the following necessary conditions on the tuple $(y_\alpha, y_\beta, \overline{\gamma}, a)$ for this configuration to exist.

$$\frac{2f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta}{2f(y_\alpha - y_\beta) + 6y_\alpha + 21y_\beta} < \overline{\gamma} < \frac{5f + 18}{5f + 6}. \tag{A.16}$$

Corner solution. We denote the content provider market to be covered with a corner solution in the price subgame if the lower quality platform quotes a price that is just sufficient so that the lowest quality content provider joins the platform. In this case, a corner solution occurs and we have the following price charged by platform β,

$$w_\beta^{cc} = (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta. \tag{A.17}$$

From the first order conditions of the high quality profit function we deduce that the equilibrium price is given by,

$$w_\alpha^{cc} = \frac{1}{18}(f4f(a - \overline{\gamma}) + 6(a - \overline{\gamma}))y_\alpha + \frac{1}{18}(4f(\overline{\gamma} - a + 3(\overline{\gamma} - 5a))y_\beta. \tag{A.18}$$

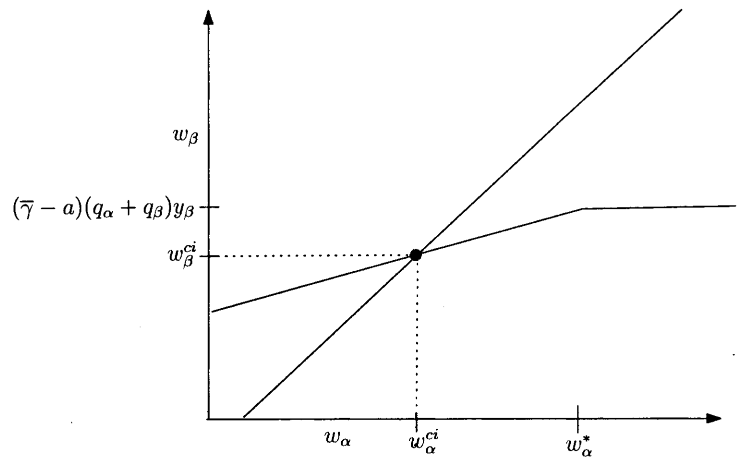

Figure A-3: The best response functions of platform β and α given $\frac{2f(y_\alpha-y_\beta)+18y_\alpha+9y_\beta}{2f(y_\alpha-y_\beta)+6y_\alpha+21y_\beta)} < \frac{\overline{\gamma}}{a} < \frac{5f+18}{5f+6}$. These curves intersect at $(w^{ci}_\beta, w^{ci}_\alpha)$.

For configuration $CIII$ to occur with a corner solution the following three conditions need to hold,

$$\begin{aligned}(\overline{\gamma}-a)(q_\alpha+q_\beta)y_\beta - w^{cc}_\beta &> (\overline{\gamma}-a)(y_\beta q_\beta + y_\alpha q_\alpha) - w^{cc}_\alpha, \\ w^{ci}_\beta &\geq (\overline{\gamma}-a)(q_\alpha+q_\beta)y_\beta, \qquad \text{(A.19)} \\ w^{u}_\beta &\leq (\overline{\gamma}-a)(q_\alpha+q_\beta)y_\beta. \qquad \text{(A.20)}\end{aligned}$$

The above inequalities yield the following necessary and sufficient conditions on $\overline{\gamma}, y_\alpha, y_\beta$ for the above equilibrium prices to yield Configuration $CIII$,

$$\frac{2f(y_\alpha-y_\beta)+30y_\alpha-3y_\beta}{2f(y_\alpha-y_\beta)+18y_\alpha+9y_\beta} \leq \frac{\overline{\gamma}}{a} \leq \min\left\{\frac{2f(y_\alpha-y_\beta)+18y_\alpha+9y_\beta}{2f(y_\alpha-y_\beta)+6y_\alpha+21y_\beta}, \frac{4f(y_\alpha-y_\beta)+18y_\alpha-9y_\beta}{4f(y_\alpha-y_\beta)+6y_\alpha+3y_\beta}\right\} \qquad \text{(A.21)}$$

Covered Preempted market CIV **.**

In this case we suppose *ex ante* that the market is covered with only the high quality platform serving the market. We identify the equilibrium prices for this market configuration and derive the best price responses of each platform in the usual way. The condition for a covered market where only the high-quality platform participates in the

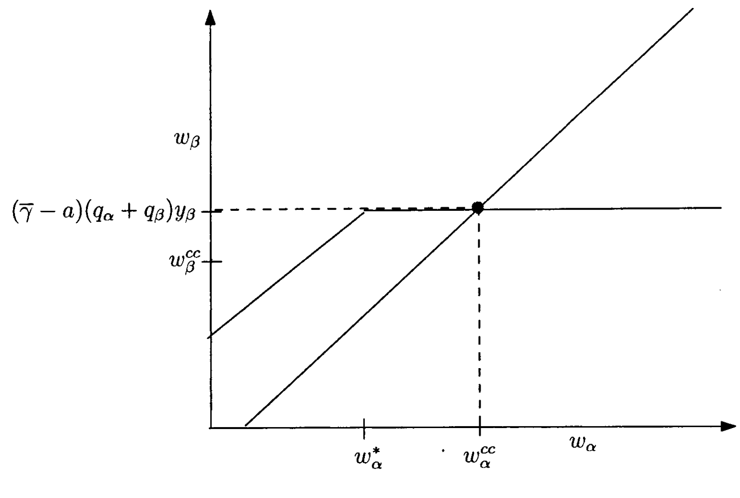

Figure A-4: The best response functions of platform β and α given $\frac{2f(y_\alpha-y_\beta)+30y_\alpha-3y_\beta}{2f(y_\alpha-y_\beta)+18y_\alpha+9y_\beta} \leq \frac{\overline{\gamma}}{a} \leq \min\left\{\frac{2f(y_\alpha-y_\beta)+18y_\alpha+9y_\beta}{2f(y_\alpha-y_\beta)+6y_\alpha+21y_\beta}, \frac{4f(y_\alpha-y_\beta)+18y_\alpha-9y_\beta}{4f(y_\alpha-y_\beta)+6y_\alpha+3y_\beta}\right\}$. These curves intersect at $(w_\beta^{cc}, w_\alpha^{cc})$.

market is,

$$(\overline{\gamma}-a)(y_\beta q_\beta + y_\alpha q_\alpha) - w_\alpha \geq \max\{0, (\overline{\gamma}-a)(q_\alpha+q_\beta)y_\beta - w_\beta\}. \tag{A.22}$$

The profit functions for platforms α and β given y_α and y_β are

$$\pi_\alpha^p = \frac{8}{9}f^2\overline{\gamma}(y_\alpha - y_\beta) + w_\alpha, \tag{A.23}$$
$$\pi_\beta^p = \frac{2}{9}f^2\overline{\gamma}(y_\alpha - y_\beta). \tag{A.24}$$

We note that in this configuration, given w_α, platform β's profit does not depend on w_α. Thus any w_β that meets the condition specified by Eq. (A.22) is a best response. Given w_β, it follows from the condition specified by Eq. (A.22) and the first order conditions of Eq. (A.23), that

$$w_\alpha = \begin{cases} (\overline{\gamma}-a)(q_\alpha y_\alpha + q_\beta y_\beta) & \text{if } w_\beta \geq (\overline{\gamma}-a)(q_\alpha+q_\beta)y_\beta, \\ (\overline{\gamma}-a)q_\alpha(y_\alpha - y_\beta) + w_\beta & \text{if } w_\beta < (\overline{\gamma}-a)(q_\alpha+q_\beta)y_\beta. \end{cases} \tag{A.25}$$

Thus the above characterizes the price equilibrium combinations for configuration CIV.

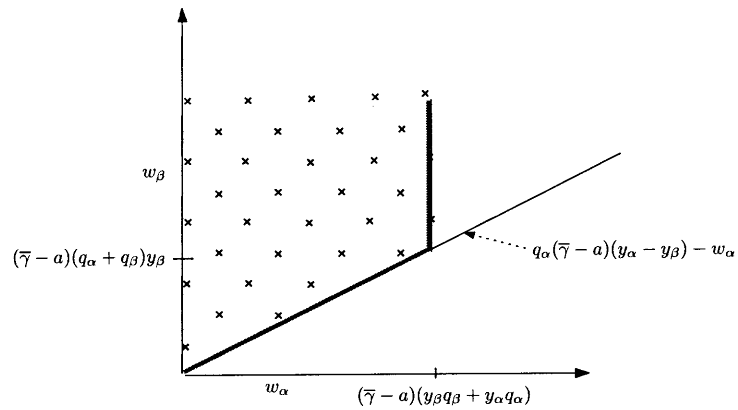

Figure A-5: The reaction correspondence of platform β given $w_\alpha \leq (\overline{\gamma}-a)(y_\beta q_\beta + y_\alpha q_\alpha)$ and the reaction curve of platform α given w_β. Their intersection points give the equilibrium price pairs in this market configuration. These are depicted by the red (dark) line.

A.1.3 Nash Equilibrium in the Price Subgame

In this section, we show the existence of pure strategy Nash equilibrium in the price-subgame. We look at the equilibrium price pairs derived in the pervious section and determine if they are best replies across all the configurations. We characterize the price subgame equilibria in terms of the tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ and give the conditions for their existence. Specifically, we give the conditions for these price equilibria to yield their corresponding market configurations.

We show that the uncovered market configuration, (CI), does not occur at a subgame price equilibrium. We then show that market configurations CII, $CIII$ and CIV exist. In doing so, we determine the set of parametric values $(\overline{\gamma}, y_\alpha, y_\beta, a)$ for which these different configurations exist and characterize the prices in each configuration using the same parameters.

In the remaining part of this section, we show that if Assumption 6 holds configuration CI does not exist whilst CII, CIII and CIV exist. We also give the conditions for their existence given the tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ and their accompanying equilibrium prices.

Lemma 11. *Let Assumption 6 hold. Any equilibrium price pair $(w_\alpha^{ui}, w_\beta^{ui}) \in \mathcal{R}_\mathcal{I}$ is not a pure strategy Nash equilibrium in the price subgame.*

Proof. We assume to arrive at a contradiction that their exists a pair of equilibrium prices $(w_\alpha^{ui}, w_\beta^{ui}) \in \mathcal{R}_\mathcal{I}$ that are a pure strategy Nash equilibrium in the price subgame. To prove our Lemma we show that given a subgame $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ such that condition in Eq. (A.6) is met, the prices in the pair $(w_\beta^{ui}, w_\alpha^{ui})$ are not best reply pairs on the whole domain of strategies, i.e, there exists for at least one platform the incentive to deviate to a price that will yield a different configuration and higher profits. In particular, we show that w_β^{ui} does not beat all price strategies in the projection of $\mathcal{R}_{\mathcal{II}} \bigcup \mathcal{R}_{\mathcal{III}} \bigcup \mathcal{R}_{\mathcal{IV}}$ against w_α^{ui}.

As shown in section A.1.2 there are two possible characterizations for the equilibrium price pair that holds if configuration CI is exogenously imposed. We show that prices satisfying both characterizations are not best reply pairs on the whole domain.

Case I. Equilibrium price pair $(w_\alpha^{ui}, w_\beta^{ui})$ in Eq. (A.4) and Eq. (A.5).

As previously discussed in section A.1.2, the above price characterizations yield configuration CI only if the condition in Eq.(A.6) is met. We denote the profit for platform β under the price pair $(w_\alpha^{ui}, w_\beta^{ui})$ as π_β^{ui} and that under the pair $(w_\alpha^{ui}, \overline{w}_\beta)$ as $\overline{\pi}_\beta$. We also denote the difference between the two profits, $\pi_\beta^{ui} - \overline{\pi}_\beta$, as $d(\overline{\gamma})$. Let $\overline{\gamma}^*$ denote the upper bound value of γ such that configuration CI is possible. We now show that there exists configurations with price pairs $(w_\alpha^{ui}, \overline{w}_\beta)$ such that $\pi_\beta^{u} < \overline{\pi}_\beta$ for $\overline{\gamma} < \overline{\gamma}^*$, which implies that these price characterization cannot be a subgame equilibrium. For this purpose we fix w_α^{ui} and consider profits of platform β under configurations $CIII$ and CIV.

Let $\overline{w_\beta} = (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$, then configuration $CIII$ will arise whenever $\overline{\gamma}^{**} < \overline{\gamma} < \overline{\gamma}^*$, where $\overline{\gamma}^{**} = \frac{a(4f(y_\alpha - y_\beta) + 33y_\beta - 6y_\alpha)}{4f(y_\alpha - y_\beta) + 6y_\alpha + 3y_\beta}$. The function $d(\overline{\gamma})$ is convex since $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} > 0$. Moreover, $d(\overline{\gamma})$ has two roots,

$$\begin{aligned} \overline{\gamma}_1 &= a\frac{4f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta}{4f(y_\alpha - y_\beta) + 6y_\alpha + 3y_\beta}, \\ \overline{\gamma}_2 &= a. \end{aligned}$$

It follows that whenever $\overline{\gamma}_2 \leq \overline{\gamma} < \overline{\gamma}^*$ then $d(\overline{\gamma}) < 0$. This implies that platform β would prefer to deviate to a covered market with a corner solution. However, this configuration is possible for all values of $\overline{\gamma} > a$ when $\overline{\gamma}^{**} < \overline{\gamma}_2$. And this occurs when $\frac{y_\alpha}{y_\beta} \geq \frac{5}{2}$. So we now proceed to show that when $\frac{y_\alpha}{y_\beta} < \frac{5}{2}$ that platform β would prefer to deviate to configuration CIV where all masses of CPs join it.

Let $\overline{w_\beta} = (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$ and consider the case when $\frac{y_\alpha}{y_\beta} < \frac{5}{2}$. It follows that $\overline{\gamma}^{**} > a$, so for $a < \overline{\gamma} \leq \overline{\gamma}^{**}$ a covered preempted market results. The difference $d(\overline{\gamma})$ under this configuration is convex since $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} > 0$. The roots of $d(\overline{\gamma})$ are, $r1$[1] and a. Moreover, $r1 > \overline{\gamma}^{**}$ whenever $1 < \frac{y_\alpha}{y_\beta} < \frac{9+f}{f}$. Since $\frac{5}{2} \leq \frac{9+f}{f}$ for $f \in (0, 1]$) platform β prefers to deviate to configuration CIV whenever $\frac{y_\alpha}{y_\beta} < \frac{5}{2}$.

Case II. Equilibrium price pair $(w_\alpha^{ui}, w_\beta^{ui})$ in Eq. (A.3).

We show that if platform β picks the price $\overline{w}_\beta = (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$ then it makes a higher profit in the resulting configuration. We denote the profit of platform β under the price pair $(w_\alpha^{ui}, w_\beta^{ui})$ as π_β^{ui} and that under the price pair $(w_\alpha^{ui}, \overline{w}_\beta)$ as $\overline{\pi}_\beta$. Note that when platform β picks the price $\overline{w_\beta}$ the market becomes covered and configuration $CIII$ emerges. The function $\overline{\pi}_\alpha - \pi_\alpha = d(w_\alpha^{ui})$ is increasing in w_α^{ui} since $\frac{\partial d(w_\alpha^{ui})}{\partial (w_\alpha^{ui})} > 0$. Moreover, it has a single root at $w_\alpha^{ui*} = (\overline{\gamma} - a)(y_\alpha q_\alpha + y_\beta q_\beta)$. Thus for given any $w_\alpha^{ui} > w_\alpha^{ui*}$ platform β would prefer to deviate to price $\overline{w}_\beta$. Therefore an equilibrium price pair $(w_\alpha^{ui}, w_\beta^{ui})$ for which the characterization in Eq. (A.5) holds is not a subgame Nash equilibrium. □

We now show that the equilibrium price pair characterized for configuration CII in Section A.1.2 is a subgame equilibrium price pair and give the conditions on $\overline{\gamma}$, a y_β, and y_α for this to hold.

Lemma 12. *Let Assumption 6 hold. Given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$, there exists a unique equilibrium price pair $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{II}$ only if,*

$$1 < \frac{\overline{\gamma}}{a} < \frac{2f(y_\alpha - y_\beta) + 30y_\alpha - 3y_\beta}{2f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta}.$$

[1] $r1 = a(4y_\beta^2 f^2 + 4f^2 y_\alpha^2 + 3f y_\alpha y_\beta + 216 y_\alpha y_\beta - 8f^2 y_\alpha y_\beta + 108 y_\beta^2 + 3f y_\beta^2 - 6f y_\alpha^2)/f/(4f y_\alpha^2 + 6y_\alpha^2 - 3y_\alpha y_\beta - 8f y_\alpha y_\beta + 4f y_\beta^2 - 3y_\beta^2)$.

Proof. From section A.1.2 we know that the prices in the pair (w_α^u, w_β^u) are unique and mutual best replies in the restricted domain $\mathcal{R}_{II}$; which corresponds to the market configuration CII. Therefore this price pair is our only candidate for the price equilibrium that falls in $\mathcal{R}_{II}$. To show that the candidate pair (w_α^u, w_β^u) is a price subgame equilibrium, we need to show that the prices in these pair are also mutual best replies on the whole domain of strategies, i.e, given price w_α^u, platform β does not have an incentive to change to price $\overline{w}_\beta$ which will result in another configuration and a higher profit. Formally, we have to show that w_β^u beats any strategy w_β in the projection $R_I \cup R_{III} \cup R_{IV}$ against w_α^u and vice versa. We denote the equilibrium price candidate (w_α^u, w_β^u) as (w_α^*, w_β^*).

We first fix w_β^* and show that platform α has no incentive to deviate to any price $\overline{w}_\alpha$ in any configuration. We denote the profit under the price pair (w_α^*, w_β^*) as π_α^* and that under the pair $(\overline{w}_\alpha, w_\beta^*)$ as $\overline{\pi}_\alpha$. We denote the difference $\pi_\alpha^* - \overline{\pi}_\alpha$ as $d(\overline{\gamma})$.

1. *Platform α has no incentive to deviate to configuration CI.*

   We find platform $\alpha's$ best reply given w_β^* under market configuration CI and show that the profit realized is less than that under configuration CII at price w_α^*. Let $\overline{w}_\alpha$ be the best reply of platform under configuration CI. It is given as the solution to the following maximization problem,

$$\begin{aligned} &\max \ \pi_\alpha(w_\alpha, w_\beta^*), \\ &\text{s.t. } w_\alpha \leq \frac{w_\beta^*}{y_\beta} \frac{q_\alpha y_\alpha + q_\beta y_\beta}{q_\alpha + q_\beta}. \end{aligned}$$

   The constraint in the above problem arises from the necessary conditions expressed in (A.1) for market configuration CI to hold. The profit function π_α is concave in w_α since $\frac{\partial^2 \pi_\alpha(w_\alpha)}{\partial^2 w_\alpha} < 0$. The unconstrained optimal solution to the above maximization problem is lager than the constraint. Therefore the constraint binds and it is the best reply.

   We now compare the two profits under both configurations. After evaluating the difference $d(\overline{\gamma}) = \pi_\alpha^* - \overline{\pi}_\alpha$, we obtain that $d(\overline{\gamma})$ is a convex function in $\overline{\gamma}$, because,

$\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} > 0$. In addition, $d(\overline{\gamma}) \geq 0$ since $d(\overline{\gamma})$ is a quadratic function in $\overline{\gamma}$ with a single root at $\frac{a4(1+f)y_\alpha+(2+f)y_\beta}{4(f-1)y_\alpha+(f-2)y_\beta}$. Therefore, given w^*_β platform α has no incentive to deviate to a price $\overline{w}_\alpha$ that would result in configuration CI.

2. *Platform α cannot deviate to configuration $CIII$.* Form section A.1.2 we know that w^*_β is defined only if the condition in Eq.(A.12) is satisfied. This implies that $w^*_\beta > (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$. Therefore, it is not possible to have a covered market with content providers patronizing the two platforms when platform $\beta's$ price is fixed at w^*_β.

3. *Platform α has no incentive to deviate to configuration CIV.*

   We proceed in a similar manner to the first case. We find platform $\alpha's$ best reply given w^*_β under market configuration CIV and show that the profit realized is less than that under configuration CII at price w^*_α. Let $\overline{w}_\alpha$ be the best reply of platform under configuration CIV. It is given as the solution to the following maximization problem

$$\max \pi_\alpha(w_\alpha, w^*_\beta),$$
$$\text{s.t. } w_\alpha \leq (\overline{\gamma} - a)q_\alpha y_\alpha + q_\beta y_\beta$$

   Since π_α is linear and increasing in w_α, the constraint binds and is the best response. Under this price $d(\overline{\gamma})$ is a convex function in $\overline{\gamma}$, because $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} > 0$. The function $d(\overline{\gamma})$ has two roots $\overline{\gamma}_1$ and $\overline{\gamma}_2$, these have been defined in the proof of the previous Lemma. Since configuration CII is defined outside these two roots it follows that $d(\overline{\gamma})$ is positive. Therefore, platform α has no incentive to deviate to configuration CIV.

We now fix w^*_α and show that platform β has no incentive to deviate to any price $\overline{w}_\beta$ that will yield another configuration. We denote the profit of platform β under the price pair (w^*_α, w^*_β) as π^*_β and that under the pair $(\overline{w}_\beta, w^*_\alpha)$ as $\overline{\pi}_\beta$. We denote the difference $\pi^*_\beta - \overline{\pi}_\beta$ by $d(\overline{\gamma})$.

1. *Platform β has no incentive to deviate to configuration CI.*

   We proceed in a similar fashion to the previous parts. We find platform $\beta's$ best reply given w_α^* under market configuration CI and show that the profit realized is less than that under configuration CII at price w_β^*. Let $\overline{w}_\beta$ be the best reply of platform under configuration CI. It is given as the solution to the following maximization problem,

$$\max \pi_\beta(w_\alpha^*, w_\beta),$$
$$\text{s.t. } w_\beta \geq \frac{w_\alpha^* y_\beta(q_\alpha + q_\beta)}{q_\alpha y_\alpha + q_\beta y_\beta}.$$

   The constraint in the above problem arises from the necessary conditions expressed in (A.1) for market configuration CI to hold. The profit function π_β is concave in w_β since the $\frac{\partial^2 \pi_\beta(w_\beta)}{\partial^2 w_\beta} > 0$. Through computation one can show that the optimal solution is at the boundary since the constraint binds.

   We now compare the two profits. The difference, $d(\overline{\gamma})$, is a convex function in $\overline{\gamma}$, because, $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} > 0$. Moreover, this function is a quadratic function in $\overline{\gamma}$ with a single root at $\gamma = \frac{a(4y_\alpha+4fy_\alpha+2y_\beta+fy_\beta}{4fy_\alpha+fy_\beta-4y_\alpha-2y_\beta)}$. Therefore, for all $\overline{\gamma}$ the difference $d(\overline{\gamma}) \geq 0$. Thus given w_α^*, platform β has no incentive to deviate to a price that results in CI.

2. *Platform β has no incentive to deviate to configuration $CIII$.*

   We show that platform β makes more profit under configuration CII than if it changed its price and deviated to configuration $CIII$. Let $\overline{w}_\beta$ be the best response price under configuration $CIII$ given w_α^*. It is defined below,

$$\begin{aligned}\overline{w}_\beta &= \text{argmax } \pi_\beta(w_\alpha^*, w_\beta),\\ &\quad \text{s.t. } w_\beta \leq (\overline{\gamma} - a)(q_\alpha + q_\beta) y_\beta. \end{aligned} \tag{A.26}$$

   The above profit function is concave in w_β since $\frac{\partial^2 \pi_\beta}{\partial^2 w_\beta} < 0$. Moreover one can show through computation that the constraint in problem (A.26) binds at the optimum.

We now compare profits under CII and those resulting in $CIII$ under the deviation price $\overline{w}_\beta$. The difference in profits given by $d(\overline{\gamma})$ is a convex function in $\overline{\gamma}$, because $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} > 0$. In addition, the function $d(\overline{\gamma})$ has a single root at $\overline{\gamma} = a\frac{2f(y_\alpha - y_\beta)+30y_\alpha - 3y_\beta}{2f(y_\alpha - y_\beta)+18y_\alpha + 9y_\beta}$ therefore $d(\overline{\gamma}) \geq 0$. Thus given w^*_α, platform β has no incentive to deviate to a price that results in configuration $CIII$.

3. *Platform β has no incentive to deviate to configuration CIV.*

   If platform β chooses to deviate to a configuration where all CPs subscribe to it, the best price it can offer is given by $\overline{w}_\beta = w^*_\alpha + (k + a)q_\alpha(y_\beta - y_\alpha)$. Platform β has no incentive to deviate in this case. Therefore we consider cases where $\overline{w}_\beta$ is positive. Proceeding in a similar manner to the previous cases we can show that $d(\overline{\gamma}) \geq 0$ for $\overline{\gamma} > a$. This implies that platform β has no incentive to deviate to configuration CIV.

□

We have shown that the equilibrium price pair (w^u_α, w^u_β) for which condition $(A.12)$ holds is a pure strategy Nash Equilibrium in the price subgame. We next show that configuration $CIII$ with a corner solution exists and give both the necessary and sufficient conditions under which this configuration exists.

Lemma 13. *Let Assumption 1 hold. Given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$, there exists a unique equilibrium price pair $(w^*_\alpha, w^*_\beta) \in \mathcal{R}_{III}$ such that $w^*_\beta = (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$ only if,*

$$\frac{2f(y_\alpha - y_\beta) + 30y_\alpha - 3y_\beta}{2f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta} \leq \frac{\overline{\gamma}}{a} \leq \min\left\{\frac{2f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta}{2f(y_\alpha - y_\beta) + 6y_\alpha + 21y_\beta}, \frac{4f(y_\alpha - y_\beta) + 18y_\alpha - 9y_\beta}{4f(y_\alpha - y_\beta) + 6y_\alpha + 3y_\beta}\right\}.$$

Proof. In section A.1.2 we determined that the prices, in the unique equilibrium pair $(w^{cc}_\alpha, w^{cc}_\beta)$, are unique and mutual best replies in the restricted domain $\mathcal{R}_{III}$ if a covered market configuration with a corner solution[2] was assumed. Thus this price pair is our only price subgame equilibrium candidate.

[2]A corner solution refers to the instance when $w^*_\beta = (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$.

Since our only candidate pair is $(w_\alpha^{cc}, w_\beta^{cc})$, we need to show that the prices in these pair are also mutual best replies on the whole domain of strategies, i.e, given price w_α^{cc}, platform β does not have an incentive to change to price $\overline{w_\beta}$ which will result in another configuration and a higher profit. We show that w_β^{cc} beats any strategy $\overline{w_\beta}$ in the projection $R_I \cup R_{II} \cup R_{IV}$ against w_α^{cc} and vice versa. We denote $w_\alpha^* = w_\alpha^{cc}$ and $w_\beta^* = w_\beta^{cc}$.

We first fix w_β^* and show that given this price, platform α has no incentive to deviate to a price that would result in configuration CI, CII or CIV. We note that under the price w_β^* it is not possible to have the uncovered configurations CI or CII since all content providers have an incentive to participate. So we only look at the possibility of deviating to configuration CIV. We denote the profit under the price pair (w_α^*, w_β^*) as π_α^* and that under the pair $(\overline{w}_\alpha, w_\beta^*)$ as $\overline{\pi}_\alpha$. We denote the difference $\pi_\alpha^* - \overline{\pi}_\alpha$ as $d(\overline{\gamma})$.

1. *Platform α has no incentive to deviate to configuration CIV.* We find platform $\alpha's$ best reply given w_β^* under market configuration CIV and show that the profit realized is less than that under configuration $CIII$ at price w_α^*. Let $\overline{w}_\alpha$ be the best reply of platform under configuration CIV. It is given by,

$$\begin{aligned} \overline{w}_\alpha &= \text{argmax } \pi_\alpha(w_\alpha, w_\beta^*), \\ &\text{s.t. } w_\alpha \leq (\overline{\gamma} - a) q_\alpha y_\alpha + q_\beta y_\beta. \end{aligned}$$

   The constraint in the above problem arises from the necessary condition expressed in Eq. (A.22) for market configuration CIV to hold. The profit function π_α is linear and increasing in w_α. Therefore the constraint binds and it is the best reply. We now compare profit at the price pair (w_α^*, w_β^*) in configuration $CIII$ to that under the pair $(\overline{w}_\alpha, w_\beta^*)$ in configuration CIV. The difference in profits is given by $d(\overline{\gamma})$ which is a convex function in $\overline{\gamma}$ because $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} > 0$.[3] Moreover, $d(\overline{\gamma})$ has a single root at $\gamma = \frac{a(4f(y_\alpha - y_\beta)+18y_\alpha - 9y_\beta)}{4f(y_\alpha - y_\beta)+6y_\alpha+3y_\beta}$, therefore $d(\overline{\gamma}) \geq 0$. Consequently platform α has no incentive to deviate to configuration CIV.

We now fix w_α^* and show that platform β has no incentive to deviate to any price $\overline{w}_\beta$.

[3] $\frac{1}{216a(y_\alpha - y_\beta)}(-(y_\alpha - y_\beta)^2 16f^3 + (y_\alpha - y_\beta)^2 32f^2 + (12y_\alpha y_\beta + 84y_\alpha^2 - 15y_\beta^2)f)$.

We note that it is not possible for platform β to come up with prices which will result in configuration CIV where all CP's flock to platform α, because w_α^* is defined only for $\overline{\gamma} \leq \min\left\{\frac{2f(y_\alpha - y_\beta)+18y_\alpha+9y_\beta}{2f(y_\alpha - y_\beta)+6y_\alpha+21y_\beta}, \frac{4f(y_\alpha - y_\beta)+18y_\alpha-9y_\beta}{4f(y_\alpha - y_\beta)+6y_\alpha+3y_\beta}\right\}$, where as configuration CIV results only if $\overline{\gamma} > \frac{2f(y_\alpha - y_\beta)+18y_\alpha+9y_\beta}{2f(y_\alpha - y_\beta)+6y_\alpha+21y_\beta}$. We denote the profit of platform β under the price pair (w_α^*, w_β^*) as π_β^* and that under the pair $(\overline{w}_\beta, w_\alpha^*)$ as $\overline{\pi}_\beta$. We denote the difference $\pi_\beta^* - \overline{\pi}_\beta$ as $d(\overline{\gamma})$.

1. *Platform β has no incentive to deviate to configuration CI* We show that the best response given w_α^*, such that configuration CI emerges, will yield a lower profit. Let $\overline{w}_\beta$ denote the best response under CI given w_α^*. It is given by,

$$\begin{aligned} \overline{w}_\beta &= \text{argmax } \pi_\beta(w_\alpha^*, w_\beta), \\ & \quad \text{s.t. } w_\beta \geq \frac{w_\alpha^*(q_\alpha + q_\beta)y_\beta}{q_\beta y_\beta + q_\alpha y_\alpha}. \end{aligned}$$

For this configuration to occur we need the condition in Eq. (A.1) to be satisfied hence the constraint in the above maximization problem. Since π_β is independent of w_β we have the best response satisfying the constraint inequality i.e., $\overline{w}_\beta \geq \frac{w_\alpha^*(q_\alpha+q_\beta)y_\beta}{q_\beta y_\beta + q_\alpha y_\alpha}$. The function $d(\overline{\gamma})$ is a concave function in $\overline{\gamma}$, because, $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} < 0$.[4] Moreover, $d(\overline{\gamma})$ has two roots at

$$\gamma_1 = a, \tag{A.27}$$

$$\gamma_2 = \frac{a(4f(y_\alpha - y_\beta) + 18y_\alpha - 9y_\beta)}{4f(y_\alpha - y_\beta) + 6y_\alpha + 3y_\beta}. \tag{A.28}$$

Thus for all $\gamma_1 \leq \overline{\gamma} \leq \gamma_2$, we have $d(\overline{\gamma}) \geq 0$. In Section A.1.2 the equilibrium pair $(w_\alpha^{cc}, w_\beta^{cc})$ is defined only if $\overline{\gamma} \in [\gamma_1, \gamma_2]$. Therefore platform β has no incentive to deviate to a price that results in CI.

2. *Platform β has no incentive to deviate to configuration CII.*

This follows from the fact that the maximization problem given below has no

[4] $\frac{1}{108}\frac{f(-12y_\alpha^3 - 108y_b^2 y_\alpha - 33y_\beta^3 - 90y_\beta y_\alpha^2 - 48fy_\alpha^2 y_\beta + 3fy_\alpha y_\beta^2 + 4fy_\alpha^3 + 41fy_\beta^3 + 8y_\alpha^3 f^2 + 4y_\beta^3 f^2 - 12y_\alpha^2 f^2 y_\beta)}{a(y_\alpha - y_\beta)(2y_\alpha + y_\beta)}$

solution.

$$\begin{aligned} \text{Let } \overline{w}_\beta &= \text{argmax } \pi_\beta(w_\alpha^*, w_\beta), \\ & \quad \text{s.t. } w_\beta > (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta. \end{aligned}$$

We note that the supremum to the this problem is given by $\overline{w}_\beta = (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$. Therefore any price w_β satisfying the maximization constraint will yield a lower profit.

3. *Platform β has no incentive to deviate to configuration CIV.*

If platform β chooses to deviate to a configuration where all CPs subscribe to it, the best price it can offer is denoted by $\overline{w}_\beta$ and is given by,

$$\begin{aligned} \overline{w}_\beta &= \text{argmax } \pi_\beta(w_\alpha^*, w_\beta), \\ & \quad \text{s.t. } w_\beta \leq w_\alpha^* + (\overline{\gamma} + a)q_\alpha(y_\beta - y_\alpha). \end{aligned}$$

The profit function is increasing in w_β, therefore the constraint binds and we have $\overline{w}_\beta = w_\alpha^* + (k + a)q_\alpha(y_\beta - y_\alpha)$. [5] The difference $d(\overline{\gamma})$ between the profits under the price pair (w_α^*, w_β^*) in configuration $CIII$ and that under price pair $(w_\alpha^*, \overline{w}_\beta)$ in configuration CIV is concave whenever $y_\alpha \leq y_\beta \frac{9+f}{f}$ and convex vice versa. [6]. Moreover, $d(\overline{\gamma})$ has two roots at

$$\gamma_1 = \frac{a((3 - f)y_\beta + (-12 + f)y_\alpha)}{((-9 - f)y_\beta + f y_\alpha)}, \tag{A.29}$$

$$\gamma_2 = \frac{a((15 - 4f)y_\beta + (-6 + 4f)y_\alpha)}{((-4f + 3)y_\beta + (6 + 4f)y_\alpha)}. \tag{A.30}$$

One can show that when $y_\alpha \leq y_\beta \frac{9+f}{f}$ the interval in which config CIII is defined lies between the interval defined by the two roots. Since in this case $d(\overline{\gamma})$ is concave the difference is positive implying that platform β has no incentive to deviate. In

[5] The constraint directly arises from the utility maximization by the CPs. In particular, all CPs have to prefer joining the low quality platforms including those with highest quality $(\overline{\gamma} + a)$.

[6] $\frac{\partial^2 d(\overline{\gamma})}{\partial^2 \overline{\gamma}} = \frac{1}{108} \frac{(33 f y_\beta^2 - 27 y_\beta^2 - 8 y_\alpha f^2 y_\beta + 6 f y_\alpha^2 - 54 y_\alpha y_\beta - 39 y_\alpha f y_\beta + 4 y_\alpha^2 f^2 + 4 y_\beta^2 f^2) f}{a(y_\alpha - y_\beta)}$

the case where For the region in which configuration $CIII$ is defined $d(\overline{\gamma}) > 0$ since previous cases we can show that $d(\overline{\gamma}) \geq 0$ for $\overline{\gamma} > a$. This implies that platform β has no incentive to deviate. In the case when $y_\alpha \geq y_\beta \frac{9+f}{f}$ the roots given by Eq. (A.29) and Eq. (A.30) above are negative. Since $d(\overline{\gamma})$ is convex and configurations $CIII$ is defined only for positive $\overline{\gamma}$ we have that platform β has no incentive to deviate.

□

We now show that configuration $CIII$ with an interior solution exists and give both the necessary conditions under which this configuration exists.

Lemma 14. *Let Assumption 6 hold. Given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$, there exists a unique equilibrium price pair $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{III}$ such that $w_\beta^* < (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$ only if,*

$$\frac{2f(y_\alpha - y_\beta) + 9y_\beta + 18y_\alpha}{2f(y_\alpha - y_\beta) + 6y_\alpha + 21y_\beta} < \frac{\overline{\gamma}}{a} < \frac{5f + 18}{5f + 6}.$$

Proof. We follow the same line of proof applied in the previous two lemmas. From section A.1.2, we know that the prices in the pair $(w_\alpha^{ci}, w_\beta^{ci})$ are unique and mutual best replies in the restricted domain $\mathcal{R}_{III}$; if a covered market configuration was assumed and an interior solution resulted.[7]. Thus this price pair is our only candidate for the price equilibrium pair that falls in $\mathcal{R}_{III}$ (with an interior solution). Moreover, it is also shown in the same section that for $(w_\alpha^{ci}, w_\beta^{ci})$ to be in $\mathcal{R}_{III}$ it is necessary and sufficient that the condition expressed in Eq. (A.16) holds.

We now show that the prices in the equilibrium price pair $(w_\alpha^{ci}, w_\beta^{ci})$ are also mutual best replies on the whole domain of strategies, i.e, given price w_α^{ci}, platform β does not have an incentive to change to price $\overline{w}_\beta$ which will result in another configuration and a higher profit, and vice versa. Formally, we show that w_β^{ci} beats any strategy w_β in the projection $R_I \cup R_{II} \cup R_{IV}$ against w_α^{ci} and vice versa.

[7]An interior solution refers to the instance when $w_\beta^* < (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$

We first fix $w_\beta^* = w_\beta^{ci}$ and show that platform α has no incentive to deviate to any price $\overline{w}_\alpha$. We note that it is not possible for platform α to come up with prices which will result in either configuration CI or CII because $w_\beta^* < (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$ [8] . We therefore check to see if platform α deviates to a covered but preempted market, i.e, configuration CIV. We denote the profit of platform α under the price pair (w_α^*, w_β^*) as π_α^* and that under the pair $(\overline{w}_\alpha, w_\beta^*)$ as $\overline{\pi}_\alpha$. We denote the difference $\pi_\alpha^* - \overline{\pi}_\alpha$ as $d(\overline{\gamma})$.

1. *Platform α has no incentive to deviate to configuration CIV.*

   If platform α chooses to deviate to a configuration where all CPs subscribe to it, the best price it can offer is denoted by $\overline{w}_\alpha$ and is given by,

$$\begin{aligned} \overline{w}_\alpha &= \text{argmax } \pi_\alpha(w_\alpha, w_\beta^*), \\ & \quad \text{s.t. } w_\alpha \leq q_\alpha(\overline{\gamma} - a)(y_\alpha - y_\beta) + w_\beta^*. \end{aligned}$$

   The constraint in the above maximization problem reflects the fact that all content providers should prefer platform α to platform β for configuration CIV to occur. Since π_α is linear and increasing in w_α, $\overline{w}_\alpha = (\overline{\gamma} - a)q_\alpha(y_\alpha - y_\beta) + w_\beta^*$. Under this price $d(\overline{\gamma})$ is a convex function in $\overline{\gamma}$, because, $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} > 0$.[9] Moreover $d(\overline{\gamma})$ has a single root at $\overline{\gamma} = a\frac{5f+18}{5f+6}$. Thus for all values of $\overline{\gamma}$, the following inequality holds, $d(\overline{\gamma}) \geq 0$. Consequently platform α has no incentive to deviate to configuration CIV.

We now fix $w_\alpha^* = w_\alpha^{ci}$ and show that platform β has no incentive to deviate to any price $\overline{w}_\beta$ in any other configuration. We denote the profit of platform β under the price pair (w_α^*, w_β^*) as π_β^* and that under the pair $(\overline{w}_\beta, w_\alpha^*)$ as $\overline{\pi}_\beta$. We denote the difference $\pi_\beta^* - \overline{\pi}_\beta$ as $d(\overline{\gamma})$.

1. *Platform β has no incentive to deviate to configuration CI.*

   We show that the best response given w_α^*, such that configuration CI emerges, will yield a lower profit. Let $\overline{w}_\beta$ denote the best response under CI given w_α^*. It

[8] The fact that w_β^* is an interior solution implies a covered market will result for any value $\overline{w}_\alpha$.

[9] $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} = \frac{1}{486a}(6+5f)^2(y_\alpha - y_\beta)f$.

is given by,

$$\overline{w}_\beta = \text{argmax } \pi_\beta(w^*_\alpha, w_\beta),$$
$$\text{s.t. } w_\beta \geq \frac{y_\beta w^*_\alpha(q_\alpha + q_\beta)}{(q_\alpha y_\alpha + y_\beta q_\beta)}.$$

For this configuration to occur the lowest quality content provider should not join platform α, which implies $w^*_\alpha > (\overline{\gamma} - a)(q_\alpha y_\alpha + q_\beta y_\beta)$. This implies that the configuration is possible only if $\frac{\overline{\gamma}}{a} < \frac{7f(y_\alpha - y_\beta)+36y_\alpha - 9y_\beta}{7f(y_\alpha - y_\beta)+12y_\alpha+15y_\beta}$. We denote this bound by $\tilde{\gamma}$. Moreover, from section A.1.2 we know that w^*_α is defined only if $\frac{\overline{\gamma}}{a} > \frac{2f(y_\alpha - y_\beta)+18y_\alpha+9y_\beta}{2f(y_\alpha - y_\beta)+6y_\alpha+21y_\beta}$. We denote this upper bound by $\hat{\gamma}$. Therefore, configuration CIV can occur only if $\hat{\gamma} < \overline{\gamma} < \tilde{\gamma}$. The function $d(\overline{\gamma})$ is a convex function in $\overline{\gamma}$, because, $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} > 0$.[10] Moreover $d(\overline{\gamma})$ has two roots at γ_1 and γ_2. These are given explicitly below,

$$\gamma_1 = \frac{a(Q(f, y_\beta) + \sqrt{(8f^2 + 216 + 96f)y_\alpha + 36(18fy_\beta^2 + 6f^2y_\beta^2 - 6y_\alpha f^2 y_b + 36y_\alpha f y_\beta))}}{((36 - 30f + 67f^2)y_\beta + (8f^2 + 72 + 48f)y_\alpha)} \tag{A.31}$$

$$\gamma_2 = \frac{a(Q(f, y_\beta) + \sqrt{(8f^2 + 216 + 96f)y_\alpha - 36(18fy_\beta^2 + 6f^2y_\beta^2 - 6y_\alpha f^2 y_\beta + 36y_\alpha f y_\beta))}}{((36 - 30f + 67f^2)y_b + (8f^2 + 72 + 48f)y_\alpha)} \tag{A.32}$$

where $Q(f, y_\beta) = (67f^2 + 102f + 108)y_\beta$. Thus for $\overline{\gamma} \leq \gamma_2$, we have $d(\overline{\gamma}) \geq 0$. It is also the case that $\gamma_2 \geq \tilde{\gamma} \geq \hat{\gamma}$ when $\frac{y_\alpha}{y_\beta} \leq \frac{f+9}{f}$. Therefore for $\frac{y_\alpha}{y_\beta} \leq \frac{f+9}{f}$ platform β has no incentive to deviate. For $\frac{y_\alpha}{y_\beta} > \frac{f+9}{f}$, $\tilde{\gamma} < \hat{\gamma}$ which implies that configuration CIV is not possible. Thus given w^*_α, platform β has no incentive to deviate to a price that results in CIV.

2. *Platform β has no incentive to deviate to configuration CII.*

   For this configuration to occur the lowest quality content provider should not join platform β, which implies $w_\beta > (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$. Therefore, platform $\beta's$ best price under this configuration is formally given by,

$$\overline{w}_\beta = \text{argmax } \pi_\beta(w^*_\alpha, w_\beta),$$

[10] $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} = \frac{(y_\alpha - y_\beta)f((36-30f+67f^2)y_\beta+(8f^2+72+48f)y_\alpha)}{486(y_\beta+2y_\alpha)a}$

$$\text{s.t. } w_\beta > (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta.$$

The profit function π_β is concave in w_β. An interior solution to the above maximization problem exists only if $\overline{w}_\beta > (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$. One can show that this happens only if $\overline{\gamma} < \check{\gamma}$ where $\check{\gamma} = a\frac{(20f(y_a - y_b) + 9y_\alpha + 18y_\beta)}{(20f(y_\alpha - y_\beta) - 3y_\alpha + 30y_\beta)}$. But configuration CIII with an interior solution is only defined for $\hat{\gamma} < \overline{\gamma} < a\frac{5f+18}{5f+6}$. Since $\hat{\gamma} > \check{\gamma}$, a maximum does not exist and the supremum of the profit function under this configuration is that given under the price $\overline{w}_\beta = (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$. The function $d(\overline{\gamma})$, under this price, is a convex function of $\overline{\gamma}$, because $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} > 0$.[11] Moreover, $d(\overline{\gamma})$ has a single root at $\hat{\gamma}$. Therefore $d(\overline{\gamma}) > 0$ for all $\hat{\gamma} < \frac{\overline{\gamma}}{a} < \frac{5f+18}{5f+6}$. This implies that given w^*_α, platform β has no incentive to deviate to a price that results in configuration CII.

3. *Platform β has no incentive to deviate to configuration CIV where all CPs migrate to platform α..*

   For this configuration to occur the lowest quality content provider should not join platform β but platform α. This implies $w_\beta \geq (\overline{\gamma} - a)(y_\beta - y_\alpha) + w^*_\alpha$. Therefore, platform $\beta's$ best price under this configuration is formally given by,

$$\begin{aligned} \overline{w}_\beta &= \text{argmax } \pi_\beta(w^*_\alpha, w_\beta), \\ &\quad \text{s.t. } w_\beta \geq (\overline{\gamma} - a)(y_\beta - y_\alpha) + w^*_\alpha. \end{aligned}$$

   For this configuration to occur $\overline{\gamma} \geq \tilde{\gamma}$[12]. The function $d(\overline{\gamma})$ is a convex function of $\overline{\gamma}$, because $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} > 0$. [13]. Moreover, $d(\overline{\gamma})$ has a single root at $a\frac{5f+18}{5f+6)}$. Therefore $d(\overline{\gamma}) \geq 0$ for all $\overline{\gamma}$, and in particular when $\overline{\gamma} \geq \tilde{\gamma}$. This implies that given w^*_α, platform β has no incentive to deviate to a price that results in configuration CIV.

4. *Platform β has no incentive to deviate to configuration CIV where all CPs migrate*

[11] $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} = \frac{(((f+3)((f+3/2)^2-63/4))^2 y_\beta^2 + ((f+3)((f+3/2)^2-63/4))y_\alpha y_\beta + 4f^2(f+3)^2 y_\alpha^2)}{486fa(y_\alpha - y_\beta)}$.

[12] If $\overline{\gamma} < \tilde{\gamma}$ then we cannot have a covered market where all CP's patronize platform α.

[13] $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} = \frac{1}{486a}(25f^2 + 60f + 36)(y_\alpha - y_\beta)f$

to platform β*.*

For this configuration to occur the highest quality content provider should join platform β . This implies $w_\beta \leq (\overline{\gamma}+a)(y_\beta - y_\alpha) + w_\alpha^*$. Therefore, platform $\beta's$ best price under this configuration is formally given by,

$$\begin{aligned} \overline{w}_\beta &= \text{argmax } \pi_\beta(w_\alpha^*, w_\beta), \\ & \quad \text{s.t. } w_\beta \leq (\overline{\gamma}-a)(y_\beta - y_\alpha) + w_\alpha^*. \end{aligned}$$

The function $d(\overline{\gamma})$ is a convex function of $\overline{\gamma}$, because $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} > 0$. Moreover, $d(\overline{\gamma})$ has a single root at $a\frac{5f-18}{5f+6)}$. Therefore $d(\overline{\gamma}) \geq 0$ for all $\overline{\gamma}$, and in particular when this configuration occurs. This implies that given w_α^*, platform β has no incentive to deviate to a price that results in configuration CIV.

□

We finally show that configuration CIV exists. We give the necessary conditions for its existence together with the possible price characterizations in this configuration.

Lemma 15. *Let Assumption 6 hold. Given a tuple* $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$*, there exists an equilibrium price pair* $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{\mathcal{IV}}$ *such that*

1. $w_\beta^* > (\overline{\gamma}-a)y_\beta$, $w_\alpha^* = (\overline{\gamma}-a)(q_\beta y_\beta + q_\alpha y_\alpha)$ *only if,*

$$\frac{\overline{\gamma}}{a} \geq \frac{4f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta}{4f(y_\alpha - y_\beta) + 6y_\alpha + 3y_\beta} \text{ and } y_\alpha \geq \frac{f+9}{f} y_\beta.$$

2. $w_\beta^* = (\overline{\gamma}-a)y_\beta$, $w_\alpha^* = (\overline{\gamma}-a)(q_\beta y_\beta + q_\alpha y_\alpha)$ *only if,*

$$\frac{\overline{\gamma}}{a} \geq \frac{4f(y_\alpha - y_\beta) + 18y_\alpha - 9y_\beta}{4f(y_\alpha - y_\beta) + 6y_\alpha + 3y_\beta} \text{ and } y_\alpha \geq \frac{f+9}{f} y_\beta.$$

3. $w_\beta^* = (\overline{\gamma}-a)y_\beta - c(\overline{\gamma}-a)y_\beta$, $w_\alpha^* = \frac{1}{3}(\overline{\gamma}-a)(q_\beta y_\beta + q_\alpha y_\alpha) - c(\overline{\gamma}-a)y_\beta$ *only if,*

$$\frac{\overline{\gamma}}{a} \geq \frac{4f(y_\alpha - y_\beta) + 18y_\alpha - 9y_\beta - 9y_\beta c}{4f(y_\alpha - y_\beta) + 6y_\alpha + 3y_\beta - 9y_\beta c} \text{ and } y_\alpha \geq 10y_\beta - 9cy_\beta,$$

$$\textit{where } 0 < c < 1 \textit{ and } y_\alpha \geq \frac{f+9-9c}{f} y_\beta.$$

4. $w_\beta^* = 0,\ w_\alpha^* = \frac{2}{3}(\overline{\gamma} - a)(y_\alpha - y_\beta)$ *only if,*

$$\frac{9+2f}{3+2f} \leq \frac{\overline{\gamma}}{a} < \infty.$$

Proof. From section A.1.2, the condition in Eq. (A.25) characterizes the equilibrium price pairs that exist if configuration CIV is exogenously assumed. We show a subset of this characterization is a subgame Nash equilibrium for the range of values of $\overline{\gamma}$ stated in the Lemma.

Proving case 1 : $w_\beta^* > (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta,\ w_\alpha^* = (\overline{\gamma} - a)(q_\beta y_\beta + q_\alpha y_\alpha)$.

Let (w_α^*, w_β^*) be a price pair that satisfies the condition in Eq. A.25 where $w_\beta^* > (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$. We fix w_α^* and check whether platform β has an incentive to deviate to configuration $CIII$. Note that this is the only configuration that platform β can deviate too; since $w_\alpha^* = (\overline{\gamma} - a)(q_\alpha y_\alpha + q_\beta y_\beta)$ the lowest quality content provider will join at least one platform. Thus platform β can only deviate to a covered market configuration.

We denote the profit of platform β under the price pair (w_α^*, w_β^*) as π_β^* and that under the pair $(\overline{w}_\beta, w_\alpha^*)$ as $\overline{\pi}_\beta$. We denote the profit difference $\pi_\beta^* - \overline{\pi}_\beta$ as $d(\overline{\gamma})$. Platform β maximizes its profit function to find the best price $\overline{w}_\beta$ that will yield configuration $CIII$ given the tuple $(y_\alpha, y_\beta, \overline{\gamma}, a)$. The maximization problem has a constraint which ensures that the price is less than the value gained by the lowest quality CP.

$$\begin{aligned} \text{Let } \overline{w}_\beta &= \text{argmax } \pi_\beta(w_\alpha^*, w_\beta), \\ &\quad \text{s.t. } 0 \leq w_\beta < (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta. \end{aligned}$$

It follows that $0 \leq \overline{w}_\beta < (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$ only if $\overline{\gamma} > a$ and $\frac{y_\alpha}{y_\beta} < \frac{f+9}{f}$. Consequently, market configuration $CIII$ is possible with this price only if $\frac{y_\alpha}{y_\beta} < \frac{f+9}{f}$ since we assume in the problem formulation that $\overline{\gamma} > a$. Under the price $\overline{w}_\beta$ the function $d(\overline{\gamma})$ is a concave function of $\overline{\gamma}$, because $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} < 0$.[14] Moreover, $d(\overline{\gamma})$ has a single root a. Therefore for all $\overline{\gamma} > a$ and $y_\alpha < y_\beta \frac{f+9}{f}$ platform β will deviate to configuration $CIII$. This suggests that we potentially could have a preempted solution when $y_\alpha \geq y_\beta \frac{f+9}{f}$.

We fix w^*_β and check whether platform α has an incentive to deviate to configuration CI or CII when $\frac{f+9}{f}$. We only consider those two configurations because configuration $CIII$ is not possible given $w^*_\beta > (\overline{\gamma} - 1)y_\beta$. We denote the profit of platform α under the price pair (w^*_α, w^*_β) as π^*_α and that under the pair $(\overline{w}_\alpha, w^*_\beta)$ as $\overline{\pi}_\alpha$. We denote the difference $\pi^*_\alpha - \overline{\pi}_\alpha$ as $d(\overline{\gamma})$.

$$\begin{aligned} \text{Let } \overline{w}_\alpha &= \text{argmax } \pi_\alpha(w^*_\beta, w_\alpha), \\ &\quad \text{s.t. } w_\alpha \leq w^*_\beta(\frac{q_\beta y_\beta + q_\alpha y_\alpha}{(q_\alpha + q_\beta)y_\beta}). \end{aligned}$$

Let $\overline{w}^{ur}_\alpha$ be the interior solution. It follows that this solution exists whenever $w^*_\beta \geq \frac{(q_\alpha+q_\beta)y_\beta \overline{w}^{ur}_\alpha}{y_\beta q_\beta + q_\alpha y_\alpha}$. We denote this bound by w^{**}_β. Therefore an interior solution $\overline{w}^{ur}_\alpha$ results in market configuration CI only if $w^*_\beta \geq w^{**}_\beta > (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$. One can show that this occurs only if $\overline{\gamma} < \frac{a(4f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta)}{(4f(y_\alpha - y_\beta) + 6y_\alpha + 3y_\beta)}$. We denote this bound as $\overline{\gamma}^*$. The function $d(\overline{\gamma})$ under this price is a concave function of $\overline{\gamma}$, because $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} < 0$.[15] Moreover $d(\overline{\gamma})$ has a single root at $\overline{\gamma}^*$. Therefore, for all $\overline{\gamma} < \overline{\gamma}^*$ platform β will deviate to configuration CI.

We now investigate the case when $w^*_\beta \in ((\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta, w^{**}_\beta)$. In this case $\overline{w}_\alpha = \frac{w^*_\beta(y_\beta q_\beta + q_\alpha y_\alpha)}{y_\beta(q_\beta + q_\alpha)}$. We denote the difference $\pi^*_\alpha - \overline{\pi}_\alpha$ as $d(\overline{w}^*_\beta)$.. This difference is convex in w^*_β since $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} = \frac{q_\alpha y_\alpha + q_\beta y_\beta}{a y_\beta} > 0$.. This difference has two roots at r_1[16] and r_2.[17] Whenever $w^*_\beta \in (r1, r2)$ platform α has an incentive to deviate. It follows that this occurs whenever $r_2 > r1$ which in turn results whenever $\overline{\gamma} < \overline{\gamma}^*$. Therefore if $w^*_\beta \in ((\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta, w^{**}_\beta)$, which occurs only if $\overline{\gamma} < \overline{\gamma}^*$ platform α has an incentive to deviate. This implies that there is a potential for a preempted market if $\overline{\gamma} \geq \overline{\gamma}^*$.

We now check whether there is an incentive for platform to deviate to configuration CII or CIII where $r_\alpha \in (0,1)$. This can occur whenever $w^*_\beta \in [(\overline{\gamma} - a)(q_\alpha +$

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14 $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} = \frac{-1(y_\alpha f - (9+f)y_\beta)^2 f}{216a(y_\alpha - y_\beta)}$.

15 $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} = \frac{-1/108f(36y_\alpha^2 + 16y_\alpha^2 f^2 + 48y_\alpha^2 f - 24y_\alpha f y_\beta - 32y_\alpha f^2 y_\beta + 36y_\beta y_\alpha + 9y_\beta^2 + 16f^2 y_\beta^2 - 24f y_\beta^2)}{a(2y_\alpha + y_\beta)}$.

16 $r_1 = (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$,

17 $r_2 = \frac{2}{3}\frac{f y_\beta((2f(\overline{\gamma} - a) + 3a)y_\beta + (2f(a - \overline{\gamma} + 6a)y_\alpha))}{2y_\alpha + y_\beta}$.

$q_\beta)y_\beta, 2/9(-\overline{\gamma}(2f+3)+a(2f-3))(y_\alpha - y_\beta)f]$. Moreover, this interval is non-empty whenever $\frac{\overline{\gamma}}{a} < \frac{(4f(y_\alpha - y_\beta)+18y_\alpha - 9y_\beta)}{(4f(y_\alpha - y_\beta)+6y_\alpha + 3y_\beta)}$. This implies that platform α would have an incentive to deviate to whenever the former applies since $d(\overline{w}^*_\beta) > 0$ is positive in this range.

Putting all the above results together implies that configuration CIV with the price pair given above is possible only if $\overline{\gamma} > \overline{\gamma}^*$.

Proving case 2: $w^*_\beta = (\overline{\gamma} - a)y_\beta$, $w^*_\alpha = \frac{1}{3}(\overline{\gamma} - a)(q_\beta y_\beta + q_\alpha y_\alpha)$

The first part of the proof where we fix w^*_α and check whether platform β has an incentive to deviate to configuration $CIII$ is exactly the same as in the previous case. We fix w^*_β and check whether platform α has an incentive to deviate to configuration $CIII$ when $\frac{y_\alpha}{y_\beta} \geq \frac{f+9}{f}$. We only consider these configuration because configuration CI and CII are not possible given $w^*_\beta = (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$.

We denote the profit of platform α under the price pair (w^*_α, w^*_β) as π^*_β and that under the pair $(\overline{w}_\alpha, w^*_\beta)$ as $\overline{\pi}_\alpha$. We denote the difference $\pi^*_\alpha - \overline{\pi}_\alpha$ as $d(\overline{\gamma})$.

$$\begin{aligned} \text{Let } \overline{w}_\alpha &= \text{argmax } \pi_\alpha(w^*_\beta, w_\alpha), \\ &\quad \text{s.t. } w_\alpha > (\overline{\gamma} - a)(q_\beta y_\beta + q_\alpha y_\alpha). \end{aligned}$$

It follows that $\overline{w}_\alpha$ exists whenever $\overline{\gamma} < \overline{\gamma}^*$. It is also the case that function $d(\overline{\gamma})$ is a concave function of $\overline{\gamma}$, because $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} < 0$.[18] Moreover, $d(\overline{\gamma})$ has a single root $\overline{\gamma}^*$. Therefore for all $\overline{\gamma} \neq \overline{\gamma}^*$ platform β will deviate to configuration $CIII$. In particular, when $\overline{\gamma} < \overline{\gamma}^*$ platform α will always deviate since configuration $CIII$ is defined for that range. This implies that a preempted market with prices (w^*_α, w^*_β) occurs only if $\overline{\gamma}^* \leq \overline{\gamma}$ and $\frac{y_\alpha}{y_\beta} \geq \frac{f+9}{f}$.

Proving case 3: $w^*_\alpha = (\overline{\gamma} - a)(q_\alpha y_\alpha + q_\beta y_\beta) - c(\overline{\gamma} - a)y_\beta$ *and* $w^*_\beta = (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta - c(\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$, *where* $c \in [0, 1)$.

We fix w^*_α and check whether platform β has an incentive to deviate to configuration $CIII$. Note that this is the only configuration that platform β can deviate too since

[18] $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} = \frac{-1(y_\alpha f - (9+f)y_\beta)^2 f}{216a(y_\alpha - y_\beta)}$.

$w^*_\alpha < (\overline{\gamma} - a)(q_\alpha y_\alpha + q_\beta y_\beta)$ which implies that the lowest quality content provider will join at least one platform.

We denote the profit of platform β under the price pair (w^*_α, w^*_β) as π^*_β and that under the pair $(\overline{w}_\beta, w^*_\alpha)$ as $\overline{\pi}_\beta$. We denote the difference, $\pi^*_\beta - \overline{\pi}_\beta$ as $d(\overline{\gamma})$. Platform β maximizes the following profit function to find the best price $\overline{w}_\beta$ that will yield configuration $CIII$ given the tuple $(y_\alpha, y_\beta, \overline{\gamma})$.

$$\begin{aligned} \text{Let } \overline{w}_\beta &= \text{argmax } \pi_\beta(w^*_\alpha, w_\beta), \\ & \quad \text{s.t. } 0 < w_\beta < (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta(1-c). \end{aligned}$$

It follows that $\overline{w}_\beta \in (0, (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta(1-c))$ whenever $\frac{y_\alpha}{y_\beta} \leq \frac{(9-9c+f)}{f}$. It is also the case that function $d(\overline{\gamma})$ is a concave function of $\overline{\gamma}$, because $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} < 0$.[19] Moreover, $d(\overline{\gamma})$ has a single root a. Therefore, for all $\overline{\gamma} > a$ and $\frac{y_\alpha}{y_\beta} < \frac{(9-9c+f)}{f}$ platform β will deviate to configuration $CIII$. This suggests that we potentially could have a preempted solution when $\overline{\gamma} > a$ and $\frac{y_\alpha}{y_\beta} > \frac{(9-9c+f)}{f}$.

We now fix $w^*_\beta = (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta - c(\overline{\gamma} - a)y_\beta$ where $c \in [0, 1)$ and check whether platform α has an incentive to deviate to configuration $CIII$. We again consider only this configuration because configuration CI and CII are not possible given $w^*_\beta < (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$.

$$\begin{aligned} \text{Let } \overline{w}_\alpha &= \text{argmax } \pi_\alpha(w^*_\beta, w_\alpha), \\ & \quad \text{s.t. } w_\alpha > (\overline{\gamma} - a)\frac{q_\beta y_\beta + q_\alpha y_\alpha}{3} - c(\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta. \end{aligned}$$

It follows that $\overline{w}_\alpha > (\overline{\gamma} - a)(q_\beta y_\beta + q_\alpha y_\alpha) - c(\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$ and results in configuration $CIII$ only if $\frac{\overline{\gamma}}{a} \geq \frac{4f(y_\alpha - y_\beta) + 18y_\alpha - 9y_\beta c - 9y_\beta}{4f(y_\alpha - y_\beta) + 6y_\alpha - 9y_\beta c + 3y_\beta}$. Therefore platform α deviates only in the above case. Putting all the above results together we find that this price configuration holds whenever, $\frac{\overline{\gamma}}{a} < \frac{4f(y_\alpha - y_\beta) + 18y_\alpha - 9y_\beta c - 9y_\beta}{4f(y_\alpha - y_\beta) + 6y_\alpha - 9y_\beta c + 3y_\beta}$ and $\frac{y_\alpha}{y_\beta} \geq \frac{9+f-9c}{f}$.

Proving case 4:$w^*_\beta = 0,\ w^*_\alpha = q_\alpha(\overline{\gamma} - a)(y_\alpha - y_\beta)$.

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[19] $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} = \frac{-1}{216a} \frac{f(f(y_\alpha - y_\beta) + 9y_\beta(c-1))^2}{a(y_\alpha - y_\beta)}$.

We proceed in a similar way to that used in proving case 3. We fix w_α^* and check whether platform β has an incentive to deviate to configuration $CIII$. Note that this is the only configuration that platform β can deviate too since $w_\alpha^* < (\overline{\gamma} - a)(q_\alpha y_\alpha + q_\beta y_\beta)$. This means that platform β maximizes its profit function to find the best price $\overline{w}_\beta$ that will yield configuration $CIII$ given the tuple $(y_\alpha, y_\beta, \overline{\gamma}, a)$.

$$\begin{aligned} \text{Let } \overline{w}_\beta &= \text{argmax } \pi_\beta(w_\alpha^*, w_\beta), \\ & \quad \text{s.t. } w_\beta < 0. \end{aligned}$$

However any price $w_\beta < 0$ is dominated by $w_\beta = 0$ thus platform β has no incentive to deviate for any $\overline{\gamma}$.

We fix $w_\beta^* = 0$ and check whether platform α has an incentive to deviate to configuration $CIII$. We again consider these configuration because configuration CI and CII are not possible given $w_\beta^* < (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$.

$$\begin{aligned} \text{Let } \overline{w}_\alpha &= \text{argmax } \pi_\alpha(w_\beta^*, w_\alpha), \\ & \quad \text{s.t. } w_\alpha > q_\alpha(\overline{\gamma} - a)(y_\alpha - y_\beta). \end{aligned}$$

It follows that $\overline{w}_\alpha > q_\alpha(\overline{\gamma} - a)(y_\alpha - y_\beta)$ only if $\frac{\overline{\gamma}}{a} < \frac{2f+9}{2f+3}$. This implies that when $\frac{\overline{\gamma}}{a} \geq \frac{2f+9}{2f+3}$ this price structure and market configuration are possible .

□

In this Appendix we have shown that there exists equilibrium price pairs that are Nash equilibrium in the price subgame. Moreover, we have shown the market configurations in which they occur and the conditions for them to occur. In particular, we have shown that each of the configurations CII, $CIII$, and CIV exists. We next present a proof for Theorem 1 which shows that given any tuple $(\overline{\gamma}, a, y_\alpha, y_\beta)$ only one market configuration is feasible in the price subgame Nash equilibrium. In addition, for market configurations CII and $CIII$, the price characterizations are unique.

Proof of Theorem 1

Proof. Given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$, we know from Lemma 11 through 15 that an equilibrium pair (w_α^*, w_β^*) exists. Moreover, cases 1, 2, and 3 directly follow from Lemma 12 through 14. In particular,

1. If $1 < \frac{\overline{\gamma}}{a} < \frac{2f(y_\alpha - y_\beta)+30y_\alpha-3y_\beta}{2f(y_\alpha-y_\beta)+18y_\alpha+9y_\beta}$, then the equilibrium price pair is unique and $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{II}$. This follows from Lemma 12.

2. If $\frac{2f(y_\alpha - y_\beta)+30y_\alpha-3y_\beta}{2f(y_\alpha-y_\beta)+18y_\alpha+9y_\beta} \leq \frac{\overline{\gamma}}{a} < \min\left\{\frac{2f(y_\alpha-y_\beta)+18y_\alpha+9y_\beta}{2f(y_\alpha-y_\beta)+6y_\alpha+21y_\beta}, \frac{4f(y_\alpha-y_\beta)+18y_\alpha-9y_\beta}{4f(y_\alpha-y_\beta)+6y_\alpha+3y_\beta}\right\}$ then the equilibrium price pair is unique and $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{III}$. This follows from Lemma 13.

3. If $\frac{2f(y_\alpha-y_\beta)+18y_\alpha+9y_\beta}{2f(y_\alpha-y_\beta)+6y_\alpha+21y_\beta} < \frac{\overline{\gamma}}{a} < \frac{5f+18}{5f+6}$ then the equilibrium price pair is unique and $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{III}$. This follows from Lemma 14.

4. If $\max\left\{\frac{5f+18}{5f+16}, \frac{4f(y_\alpha-y_\beta)+18y_\alpha-9y_\beta}{4f(y_\alpha-y_\beta)+6y_\alpha+3y_\beta}\right\} \leq \frac{\overline{\gamma}}{a} < \infty$ then $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{IV}$. This follows from Lemma 15.

□

Proof of Theorem 2

We first show that without loss of generality we can assume platform β chooses a price $w_\beta \geq 0$. This will enable us to show that no content provider joins platform β and consequently enable us to rule out the existence of configuration CII and $CIII$.

Lemma 16. *Platform β charges $w_\beta \geq 0$.*

Proof. We first show that $w_\beta \geq 0$ dominates any price $w_\beta < 0$. Assume that $w_\beta < 0$ and $r_\beta > 0$, then platform β makes negative revenue on the content provider side. By raising its price to $w_\beta = 0$ it increases its total revenue. This is because the revenue from the content provider side becomes non-negative and the profits on the consumer side increase: This happens across all configurations because r_α is non-decreasing in w_β. □

Since by Lemma 16, $w_\beta \geq 0$, it follows that any content provider j joining platform β will get utility $v_j \leq 0$. Therefore, no content provider has incentive to join platform β. This implies that market configurations CII and $CIII$ where content providers patronize both platforms do not exist. We now show that there trivially exists pure strategy subgame equilibrium price pairs when one platform has zero investment. We show that these prices result in configurations CI and CIV and give the conditions on $\overline{\gamma}$ for these to occur. We now proceed to prove Theorem 2.

Proof. We first derive the demand function $r_\alpha(w_\alpha)$. Given $y_\beta = 0$, $y_\alpha > 0$ and Lemma 16, the content provider decisions are as if only one platform is on offer. Therefore, the demand addressed to platform α is equal to the mass of content providers with content quality γ_j such that $\gamma_j y_\alpha q_\alpha \geq w_\alpha$ and is given by,

$$r_\alpha(w_\alpha) = \frac{1}{2a}\left(\overline{\gamma} + a - \frac{w_\alpha}{q_\alpha y_\alpha}\right).$$

The value $\overline{w}_\alpha$ that maximizes platform alpha's profit problem for platform α is represented as,

$$\begin{aligned} \overline{w}_\alpha &= \text{argmax } \pi_\alpha(w_\alpha), \\ & \quad \text{s.t. } w_\alpha \geq (y_\alpha q_\alpha)(\overline{\gamma} - a). \end{aligned}$$

The profit function does not depend on w_β so platform α maximizes the above function with respect to w_α and ensuring that $w_\alpha \geq y_\alpha q_\alpha(\overline{\gamma} - a)$. This last constraint reflects the fact that when the constraint binds all content providers are on board; a price lower than this yields no more content providers and results in a loss of revenue. The interior solution for the above maximization is $w_\alpha^* = \frac{1}{9} f y_\alpha(2f(a - \overline{\gamma}) + 3(\overline{\gamma} + a))$, and occurs whenever $1 < \overline{\gamma} < (9 + 2f)/(3 + 2f)$. In this case since $w_\alpha^* > q_\alpha y_\alpha(\overline{\gamma} - a)$ the resulting configuration is CI. The constraint binds when $\overline{\gamma} \geq (9 + 2f)/(3 + 2f)$. In this instance the resulting configuration is CIV since all content providers join platform α. $\square$

A.1.4 Best reply in the domain $[0, y_h)$

In order to avoid confusion when platform β is the high quality firm we will change notation as follows; we label the high(low) quality platform as $h(l)$ and the quality associated with it as $y_{h(l)}$. Given y_h, we will compute firm l's best reply. We will show that the profit for the low quality firm is decreasing in y_l across all configurations which are possible given $(\overline{\gamma}, y_h, a)$. This will help us infer that the low quality platform chooses 0 as its best response. Since the choice of y_l by the low quality firm determines the market configuration we define the critical limits for which the various configurations exist given y_h.

- Market is uncovered, with positive masses of consumers on both platforms, in the in the price subgame whenever,

$$y_l < 2y_h \frac{a(f+15) - (f+9\overline{\gamma})}{(9-2f)\overline{\gamma} + a(2f+3)}. \tag{A.33}$$

- Market is covered and a corner solution applies in the price subgame whenever,

$$y_l \in \left[2y_h \frac{a(f+15) - (f+9\overline{\gamma})}{(9-2f)\overline{\gamma} + a(2f+3)}, 2y_h \frac{a(f+9) - (f+3)\overline{\gamma}}{(21-2f)\overline{\gamma} - (9-2f)a}\right], \quad \text{if } 1 < \tfrac{\overline{\gamma}}{a} \leq \tfrac{f+15}{f+9}, \tag{A.34}$$

$$y_l \in \left[0, 2y_h \frac{(f+9)a - (f+3)\overline{\gamma}}{(21-2f)\overline{\gamma} - (9-2f)a}\right], \quad \text{if } \tfrac{f+15}{f+9} < \tfrac{\overline{\gamma}}{a} \leq \tfrac{5f+18}{5f+6}. \tag{A.35}$$

$$y_l \in \left[0, 2y_h \frac{(2f+9)a - (2f+3)\overline{\gamma}}{(3-4f)\overline{\gamma} + (9+4f)a}\right], \quad \text{if } \tfrac{5f+18}{5f+6} < \tfrac{\overline{\gamma}}{a} \leq \tfrac{2f+9}{2f+3}. \tag{A.36}$$

- Market is covered and an interior solution applies in the price subgame whenever,

$$y_l \in \left(2y_h \frac{(-(f+3)\overline{\gamma} + (f+9)a)}{(21-2f)\overline{\gamma} - (9-2f)a}, y_h\right), \quad \text{if } 1 < \tfrac{\overline{\gamma}}{a} < \tfrac{5f+18}{5f+6}. \tag{A.37}$$

- Market is preempted whenever,

$$y_l \in \left[2y_h\frac{(2f+9)a-(2f+3)\overline{\gamma}}{(3-4f)\overline{\gamma}+(9+4f)a}, y_h\right), \text{ if } \tfrac{5f+18}{5f+6} \leq \tfrac{\overline{\gamma}}{a} < \tfrac{9+2f}{3+2f}. \quad \text{(A.38)}$$

$$\frac{\overline{\gamma}}{a} \geq \frac{9+2f}{3+2f}. \quad \text{(A.39)}$$

We now show that given the tuple $(\overline{\gamma}, y_h, a)$ the profit function π_l is decreasing in every configuration that it is defined.

Lemma 17. *Given $(y_h, \overline{\gamma}, a), f \geq 3/5$ and $y_l \in [0, y_h)$, the profit function $\pi_l(y_l, y_h)$ is decreasing in y_l for all market configurations for which it is defined.*

Proof. We show that for each configuration the revenue function $r_l = \pi_l(y_l, y_h) + c(y_l)$ is decreasing in y_l.

Uncovered Configuration CI: Let the revenue function in this configuration be defined by r_{ui}, one can by show that for $1 < \frac{\overline{\gamma}}{a} < \frac{f+15}{f+9}$ and y_l in the set defined in (A.33), $\frac{\partial r_{ui}}{\partial y_l} < 0$. Hence the revenue function in this configuration is decreasing in y_l.

Covered Configuration with interior solution CIII: Let the revenue function in this configuration be defined by r_{ci}. One can also show that for $1 < \frac{\overline{\gamma}}{a} < \frac{5f+18}{5f+6}$ and y_l in the set defined in A.37, the derivative of the above function, $\frac{\partial r_{ci}}{\partial y_l} < 0$. Therefore the profit function is decreasing in y_l when y_l lies in the set specified in (A.37).

Covered Configuration with corner solution CIII: Let the revenue function in this configuration be defined by r_{cc}. One can show that for $1 < \frac{\overline{\gamma}}{a} < \frac{2f+9}{2f+3}$ and y_l in the sets defined in (A.34), (A.35) and (A.36) the derivative of the above function, $\frac{\partial r_{cc}}{\partial y_l} < 0$.

Pre-empted Configuration CIV: Let the revenue function in this configuration be defined by r_p. The derivative of the above function, $\frac{\partial r_p}{\partial y_l} = -2/9f^2\overline{\gamma}$. The above derivative is negative therefore the profit function is decreasing in y_l when this configuration is defined. ☐

Lemma 18. *Given the tuple $(y_\alpha,\ \overline{\gamma}, a, f)$, $f \geq \frac{3}{5}$ and the domain $[0, y_\alpha)$, then $B_\beta(y_\alpha) =$ 0.*

Proof. We show that given $(y_h, \overline{\gamma}, a)$ platform l will prefer not to invest. Specifically, we show that the profit function π_l is continuous in y_l over the domain $[0, y_h)$. This coupled with Lemma 17 implies that platform l picks the lowest quality as the Lemma claims. We split the domain in which $\frac{\overline{\gamma}}{a}$ lies into four sections depending on the number and type of market configurations that are possible. We show that in each section the profit function is continuous.

Case I. $1 < \frac{\overline{\gamma}}{a} < \frac{f+15}{f+9}$

Four market configurations are possible when $1 < \frac{\overline{\gamma}}{a} < \frac{f+15}{f+9}$; these are uncovered (CI), uncovered (CII) with both platforms participating, covered with a corner solution and covered with an interior solution (both of which are in configuration $CIII$). Given a $\frac{\overline{\gamma}}{a}$ in the above range, the domain $[0, y_h)$ in which y_l lies can be partitioned into three sets, each of which corresponds to one of the latter three market configurations. These partitions are captured in (A.33), (A.34) and (A.37). By Lemma 17, we know that profits are decreasing in y_l for each partition. We will first show that the value of the profit function in the partition defined in (A.33), is larger than any profit attained in the partition defined in (A.34). Similarly, we show that any profit attained when y_l lies in the partition defined by the constraint in (A.37) is not greater than that attained when y_l lies in the partition specified by (A.34). Lastly we show that the profit of a platform in configuration CII tends to that in configuration CI as $y_l \to 0$ and is in fact equal at the limit.

To show the first result we compare the infimum value of the profit function in the uncovered configuration to the highest possible profit attained when platform l chooses y_l such that a covered market with a corner solution results (i.e, y_l is in the set specified in (A.34)). Let $y_l^{cc} = 2y_h \frac{a(f+15)-(f+9\overline{\gamma})}{(9-2f)\overline{\gamma}+a(2f+3)}$, it follows that $\lim_{y_l \to y_l^{cc}} \pi_l^u(y_l, y_h) = \pi_l^{cc}(y_l^{cc}, y_h)$ (Since π_l^u is right continuous, the limit exists). Since $\pi_l^u(y_l, y_h) > \pi_l^u(y_l^{cc}, y_h)$ when y_l satisfies the inequality in (A.33), it also follows from Lemma 17 that $\pi_l^u(y_l, y_h) > \pi_l^{cc}(\tilde{y}, y_h)$ when $\tilde{y}$ lies in the set specified in (A.34).

To show the second result, we compare the lowest value of the profit function in the covered configuration with a corner solution to the supremum profit value attained when platform l chooses y_l such that a covered market with an interior solution results. The interval over which the covered configuration with an interior solution, $CIII$, is defined is open. Let $y_l^{ci} = 2y_h\frac{-(3\overline{\gamma}+(f+9)a)}{(21-2f)\overline{\gamma}-(9-2f)a}$, we define the supremum of $\pi_l^{ci}(y_l, y_h)$ over the range in which this configuration is defined as $\pi_l^{ci}(y_l^{ci}, y_h)$. We note that y_l^{ci} is the infimum of the interval over which this configuration is defined, therefore $\lim_{y_l \to y_l^{ci}} \pi_l^{ci}(y_l, y_h) = \pi_l^{ci}(y_l^{ci}, y_h)$ since $\pi_l^{ci}(y_h, y_l)$ is left continuous. By plugging in $y_l = y_l^{ci}$ into $\pi_l^{cc}(y_l, y_h)$ we note that $\pi_l^{cc}(y_l^{ci}, y_h) = \pi_l^{ci}(y_l^{ci}, y_h)$. Therefore, it follows from Lemma 17, that $\pi_l^{cc}(y_h, y_l) > \pi_l^{ci}(\tilde{y}, y_h)$ when y_l satisfies the constraint in (A.34) and $\tilde{y}$ satisfies the constraint in (A.37).

Finally, since $\pi_l^{u}(y_l, y_h)$ is left continuous by plugging $y_l = 0$ to the function $\pi_l^{u}(y_l, y_h)$ we show that $\lim_{y_l \to 0} \pi_l^{u}(y_l, y_h) = \pi_l^{ui}(0, y_h)$. Where $\pi_l^{ui}(0, y_h)$ is the profit function when configuration CI is defined.

Case II. $\frac{f+15}{f+9} \leq \frac{\overline{\gamma}}{a} < \frac{5f+18}{5f+6}$.

In this instance three market configurations are possible depending on the value of y_h and y_l; these are uncovered CI, covered with a corner solution, $CIII$, and covered with an interior solution, $CIII$. Given a $\frac{\overline{\gamma}}{a}$ in the above range, the domain $[0, y_h)$ in which y_l lies can be partitioned into two sets each of which corresponds to the latter two of the three market configurations. These partitions are captured in (A.34) and (A.37). We proceed in a similar manner as we did for the previous case. By Lemma 17 we know that profits are decreasing in y_l for each partition. We claim that any profit attained in the partitions defined in (A.37) is less than that attained by the minimum profit in the partition defined in (A.34). The proof is exactly the same as that described in case I. We then show that as $y_l \to 0$ the profit of a platform in configuration $CIII$ with a corner solution approaches that of the profit in configuration CI and in the limit when $y_l = 0$ they are equal. Since $\pi_l^{cc}(y_l, y_h)$ is left continuous by plugging $y_l = 0$ to the function $\pi_l^{cc}(y_l, y_h)$ one can show that $\lim_{y_l \to 0} \pi_l^{cc}(y_l, y_h) = \pi_l^{ui}(0, y_h)$. Where $\pi_l^{ui}(0, y_h)$ is the profit function when configuration CI is defined.

Case III. $\frac{5f+18}{5f+6} \leq \frac{\overline{\gamma}}{a} < \frac{2f+9}{2f+3}$.

In this section we need only show that the profit function is continuous across configuration $CIII$ with a corner solution and a pre-empted market configuration CIV. Indeed these are the only two configurations possible when $y_l > 0$. (We already showed in the previous case that $\lim_{y_l \to 0} \pi_l^{cc}(y_l, y_h) = \pi_l^{ui}(0, y_h))$. To show this result we compare the infimum value of the profit function in the covered configuration with a corner solution to the profit value attained when platform l chooses a y_l such that a pre-empted market results. Note the interval over which the covered configuration with a corner solution, $CIII$, is defined is open on its upper limit. Let $y_l^p = 2y_h \frac{(2f+9)a-(2f+3)\overline{\gamma}}{(3-4f)\overline{\gamma}+(9+4f)a}$, we define the infimum of $\pi_l^{cc}(y, y_l)$ over the range in which this configuration is defined as $\pi_l^{cc}(y_l^p, y_h)$. Since $\pi_l^{cc}(y, y_h)$ is right continuous $\lim_{y_l \to y_l^p} \pi_l^{cc}(y_l, y_h) = \pi_l^{cc}(y_l^p, y_h)$. Note y_l^p is the infimum of the range. By plugging in $y_l = y_l^p$ into the profit functions under a covered market (with a corner solution) and a pre-empted market, we find that $\pi_l^{cc}(y_l^p, y_h) = \pi_l^p(y_l^p, y_h)$. This implies that the profit function is continuous across these two market configurations at this point.

Case IV. $\frac{2f+9}{2f+3} \leq \frac{\overline{\gamma}}{a} < \infty$.

When $\overline{\gamma}$ falls in the above range only market configuration CIV is possible when $y_l > 0$. We showed in Lemma 17 that the profit function $\pi_l(y_l, y_h)$ is decreasing in y_l in this configuration. So we only need show that $\lim_{y_l \to 0} \pi_l^p(y_l, y_h) = \pi_l^{uic}(0, y_h)$, where $\pi_l^{uic}(0, y_h)$ is the profit function in CI. Since $\pi_l^p(y, y_h)$ is left continuous $\lim_{y_l \to 0} \pi_l^p(y_l, y_h) = \pi_l^p(0, y_h)$. Via simple algebra, one can show that $\pi_l^p(0, y_h) = \pi_l^{uic}(0, y_h)$ which shows that π_l is continuous. □

A.1.5 Proof of Theorem 3

We first show that a symmetric equilibrium is not feasible. Let $j, i \in \{\alpha, \beta\}$ and $B_i(y_j)$ be the set of $y_i^* \in [0, \infty]$ such that,

$$y_i^* \in \arg\max_{y_i \in [0,\infty]} \pi_i(y_i, y_j).$$

Lemma 19. *Let Assumption 6 and 7 hold. If $y_j \in [0, \infty]$ then $y_j \notin B_i(y_j)$.*

Proof. We show that given y_j, platform i never chooses $y_i = y_j \geq 0$ and therefore a symmetric equilibrium is not possible. A symmetric argument applies for the other platform. Assume $y_j \in B_i(y_j)$ so that $y_i = y_j > 0$, then both platforms would make zero profits because of Bertrand competition on both sides of the market. We now check if platforms would prefer $y_i = y_j = 0$ and show that there exists a profitable deviation for platform i. There are two cases to consider, when $\overline{\gamma}/a < (2f+9)/(2f+3)$ and $\overline{\gamma}/a \geq (2f+9)/(2f+3)$. The arguments for both cases are very similar so we present only one.

Case I. $\frac{\overline{\gamma}}{a} < \frac{2f+9}{2f+3}$.

Let platform i increase its quality by a small $\epsilon > 0$; platform i becomes the high quality platform. Results from Theorem 2 imply that the resulting equilibrium profit π_i for the high quality platform given the subgame $(\overline{\gamma}, a, \epsilon, y_j)$ can be expressed as follows using a Taylor series expansion, $\pi_i(\epsilon, 0) = Q(\overline{\gamma}, f, a)\epsilon - \frac{1}{2}I''(0)\epsilon - o(|\epsilon)|^2|$, where $Q(\overline{\gamma}, f, a)$ is a positive number. There exists an ϵ^* such that for all $\epsilon \in (0, \epsilon^*)$ the above quantity is positive. Thus platform i would prefer to set quality $\epsilon \in (0, \epsilon^*)$ instead of 0. □

We now proceed to find the sets in which the best replies lie given each platform's investment level. Given quality choice y_α, platform β can choose a best reply that depends on whether it acts as a high-quality or a low-quality platform. In the former case it chooses a reply in the domain (y_α, ∞) and in the latter case it chooses a reply in the domain $[0, y_\alpha)$. Lemma 18 shows that if platform α invests in a positive quality and platform β acts as a low quality platform, then platform β prefers not to invest. By symmetry a similar claim exists for platform α given platform $\beta's$ quality choice. We now proceed to prove Theorem 3.

Proof. Given that platform β invests in $y_\beta > 0$ platform α can choose to be a low quality or a high quality platform. The best response for platform α given it acts as a low-quality (high-quality) platform is given by $B_\alpha(y_\beta) = 0$ ($B_\beta(y_\beta) \in (y_\beta, \infty)$). The former follows from Lemma 18 and the latter from Lemma 19. The overall best response is the maximum of these two best responses, i.e., the value for which the profit function is highest. Let $\bar{y} = \{y_\alpha | y_\alpha > y_\beta\}$. Given $y_\beta > 0$ then $B_\alpha(y_\beta) \in \{0 \bigcup \bar{y}\}$. If $y_\beta = 0$ then the best response is given by $y^* = \{y_\alpha | I'(y_\alpha) = r'_\alpha(y_\alpha)\}$; where $r_\alpha(y_\alpha)$ is the revenue made by the high quality platform. Note that given the tuple $(\overline{\gamma}, a, f)$, $r_\alpha(y_\alpha)$ is a linear function in y_α. It follows that y^* is a singleton since the profit function is concave in y_α. Therefore given $y_\beta = 0$, $B_\alpha(y_\beta) = y^*$. Since the explicit form of the revenue function depends on whether $\overline{\gamma}/a < (9+2f)/(3+2f)$ or $\overline{\gamma}/a \geq (9+2f)/(3+2f)$ we have two implicit characterizations of this singleton. The sets in which platform $\alpha's$ best response lies is similar by symmetry. Consequently, the only points of intersection are $[y^*, 0]$ and $[0, y^*]$. □

A.1.6 Proof of Theorem 4

Proof. We show that for a $c \geq 1$ and f large enough the pair $(y^*, 0)$, as defined in the Theorem statement is a SPE. Some of the expressions involved are too large to put in the paper. Where this is the case we state the importance of the results for the proof. In Theorem 3, we showed that the pair $(0, y^*)$ is a candidate equilibrium pair. In particular y^* is the best response of one platform given the other platform chooses not to invest. We proceed to show that for a quadratic investment function when one platform invests in y^* the other opts not to invest concluding that a SPE exists for this investment function. To analyze this response we partition the space in which $\overline{\gamma}/a$ lies into three regions corresponding to the types of market configurations that exits in each of the region. Note that the revenue[20] function is of a different form in each of these regions, hence the different analysis.

Case I. $1 < \frac{\overline{\gamma}}{a} < \frac{5f+18}{5f+6}$:

If the platform acts as a high quality platform, i.e., chooses $y > y^*$ there are three possible revenue functions that may result depending on the choice of y. Let $r(y, y^*)$ denote the revenue function of the platform that is responding to an investment level of y^* by the other platform. This revenue function is made up of a concatenation of three other revenue functions. These are,

$$\begin{array}{ll} r^{ci}(y,y^*) & \text{if } y \in \left(y^*, y^* \frac{1}{2}\frac{(21-2f)\overline{\gamma}+(2f-9)a}{(f+9)a-(f-3)\overline{\gamma}}\right) \\ r^{cc}(y,y^*) & \text{if } y \in \left[y^* \frac{1}{2}\frac{(21-2f)\overline{\gamma}+(2f-9)a}{(f+9)a-(f-3)\overline{\gamma}}, y^* \frac{1}{2}\frac{(9-2f)\overline{\gamma}+(2f+3)a}{(f+15)a-(f+9)\overline{\gamma}}\right] \\ r^{ui}(y,y^*) & \text{if } y \in \left(y^* \frac{1}{2}\frac{(9-2f)\overline{\gamma}+(2f+3)a}{(f+15)a-(f+9)\overline{\gamma}}, \infty\right) \end{array}$$

The restrictions over which these functions are defined are derived from the market

[20] $Revenue = Profit + Investment\ cost$

configurations in Theorem 1. The first refers to the revenue function when the market is covered with an interior solution, the second refers to the revenue function when the market is covered with a corner solution and the last refers to when the market is uncovered with masses present in both configurations.

We find a differentiable upper-bound of $r(y, y^*)$ and show that the best response when the platform acts as a high-quality platform, under this function, is dominated by the best response when it acts as a low-quality platform. Let this upper-bound be denoted by r^{est}. Lemma 20 shows that $r^{ci}(y, y^*)$ over the domain $y > y^*$ is an upper-bound of $r(y, y^*)$. So we find the best response under this function and compare it with the best response when the platform acts as a low-quality platform and opts not to invest. Let the maximum profit value when the platform acts as a high-quality platform under the upper-bound revenue be denoted by $\pi^{est}(y^{est}, y^*)$, where

$$\begin{aligned} y^{est} &= \text{argmax } \pi^{est}(y, y^*), \\ &\quad \text{s.t. } y > y^*. \end{aligned} \tag{A.40}$$

Let the maximum profit value when the platform acts as a low-quality platform be denoted by π^{low}. From Lemma 18 this occurs at $y = 0$. One can show that $\pi^{low} > \pi^{est}$[21]

Case II. $\frac{5f+18}{5f+6} \leq \frac{\overline{\gamma}}{a} < \frac{9+2f}{9+2f}$:

In this region two market configurations are possible if the platform picks $y > y^*$. These are a preempted market and a covered market with a corner solution. Since in the preempted market several price equilibria exist we pick the one that yields the highest price and use that to calculate an upper-bound for the profit function. We denote it by π^{est}. We compare this solution against π^{low}, which is the profit of the platform when it chooses to be the low-quality platform. In a similar manner to the first case we show $\pi^{low} > \pi^{est}$. In particular $\pi^{est} - \pi^{low} = -2/27f^2(3a - 4f\overline{\gamma} - 3\overline{\gamma})(a - \overline{\gamma} + f\overline{\gamma}) < 0$ whenever $f > 1 - a/k$.

Case III. $\frac{5f+18}{5f+6} < \frac{\overline{\gamma}}{a}$

In this interval, when the platform decides to act as a high quality platform, only

[21]The expression showing that the difference of the two terms is positive has many terms and is omitted for the sake of clarity.

the pre-empted market exists. Moreover, there are multiple price equilibria. So we use the price equilibria that yields the highest possible profit and use it to derive an upper-bound for the profit function. The analysis then proceeds in exactly the same manner as that in case II because the upper-bound for the profit function when the platform chooses to act as a high-quality is the same as that in case II. $\square$

Lemma 20. *If* $y \in \left[y^* \frac{1}{2} \frac{(21-2f)\overline{\gamma}+(2f-9)a}{(f+9)a-(f-3)\overline{\gamma}}, \infty\right]$ *then* $r^{ci}(y, y^*) \geq r(y, y^*)$.

Proof. $r(y, y^*)$ is continuous since $\lim_{y \to \bar{y}} r^{ci}(y, y^*) = r^{cc}(\bar{y}, y^*)$ and $\lim_{y \to \hat{y}} r^{cc}(y, y^*) = r^{ui}(\hat{y}, y^*)$ where $\bar{y} = y^* \frac{1}{2} \frac{(21-2f)\overline{\gamma}+(2f-9)a}{(f+9)a-(f-3)\overline{\gamma}}$ and $\hat{y} = y^* \frac{1}{2} \frac{(9-2f)\overline{\gamma}+(2f+3)a}{(f+15)a-(f+9)\overline{\gamma}}$. The revenue functions $r^{cc}(y, y^*)$ and $r^{ui}(y, y^*)$ are increasing in y since the derivatives are positive.

Moreover, the former is convex in y, whilst the later is concave y. The difference $r'^{ci}(\bar{y}, y^*) - r'^{cc}(\bar{y}, y^*)$[22] is positive whenever $1 < \frac{\overline{\gamma}}{a} < \frac{f+15}{f+9}$. Furthermore, $r'^{ci}(\hat{y}, y^*) - r'^{ui}(\hat{y}, y^*)$[23] and $r'^{ui}(\hat{y}, y^*) - r'^{cc}(\hat{y}, y^*)$ [24] are also positive in this interval. This coupled with the fact that the revenue functions are increasing implies that $r^{ci}(y, y^*) > r^{cc}(y, y^*)$ in the domain of y where $r^{cc}(y, y^*)$ is defined. Moreover, the concavity of $r^{ui}(y, y^*)$ also implies that $r^{ci}(y, y^*) > r^{ui}(y, y^*)$ in the domain of y where the latter is defined. Therefore, whenever $1 < \frac{\overline{\gamma}}{a} < \frac{f+15}{f+9}$ then $r^{ci}(y, y^*) \geq r(y, y^*)$. In the case where $\frac{f+15}{f+9} < \frac{\overline{\gamma}}{a} < \frac{18+5f}{6+5f}$ only two market configurations exist, a covered market with an interior solution and an uncovered market with a corner solution. The difference between the revenue functions over the region where the covered market with a corner solution is defined is concave in y and increasing. This follows because i) the second derivative of the difference, $r'^{ci}(y, y^*) - r'^{cc}(y, y^*)$, is negative[25] whenever $y > y^*$; ii) the difference is increasing in y since the derivative is positive in this range; iii) for $y > \bar{y}$, the range in which the covered market with corner solution is defined, the difference is positive. $\square$

[22] $\frac{1}{486a} f((f+9)a - (f+3)k)((5f+6)k + (18-5f)a)$
[23] $\frac{1}{27} f(a - \overline{\gamma})(5f^2a^2 + 32fa^2 - 261a^2 - 10f^2\overline{\gamma}a - 58fa\overline{\gamma} + 120\overline{\gamma}a + 5f^2\overline{\gamma}^2 + 26f\overline{\gamma}^2 - 3\overline{\gamma}^2)/a((f+3)a + (3-f)\overline{\gamma})$
[24] $\frac{1}{486}(-5fa + 5f\overline{\gamma} + 24a)(fa + 15a - f\overline{\gamma} - 9\overline{\gamma})^2 fa((f+3)a + (3-f)\overline{\gamma})$
[25] $-\frac{3}{8} \frac{fy^{*2}(\overline{\gamma}-a)^2}{(y-y^*)^3 a}$

A.2 Non-neutral Model

A.2.1 Sets of Prices that yield relations (i),(ii) and (iii) as defined in section (3.5.2)

We now define sets $\mathcal{W}_{R(i)}$, $\mathcal{W}_{R(ii)}$ and $\mathcal{W}_{R(iii)}$ that contain price pairs (w_α, w_β) that may yield relations (i), (ii) and (iii), as defined in sections 3.5.2, respectively. First we define set $\mathcal{W}_{R(i)}$ by solving for the price pairs for which $F_i(y_\alpha, \cdot) > F_i(y_\beta, \cdot)$. We characterize this set below;

$$\mathcal{W}_{R(i)} = \begin{cases} \{(w_\alpha, w_\beta) | w_\alpha < \overline{w}_\alpha, w_\beta > \overline{w}_\beta \text{ and } w_\alpha < q_\alpha f A + q_\alpha / q_\beta w_\beta\} & \text{if } y_\alpha \in \Lambda, \\ \{(w_\alpha, w_\beta) | w_\alpha < \overline{w}_\alpha, w_\beta > \overline{w}_\beta\} & \text{if } y_\alpha \in \Delta, \\ \{(w_\alpha, w_\beta) | w_\alpha \geq 0, w_\beta \geq 0\} & \text{if } y_\alpha \in \Theta. \end{cases}$$

Here $(q_\alpha, q_\beta) = (2/3, 1/3)$, $A = (y_\alpha - y_\beta)(\overline{\gamma}+a)^2/(\overline{\gamma}-a)$, $\Lambda = \left(y_\beta, y_\beta \frac{f(\overline{\gamma}+a)^2-(\overline{\gamma}-a)^2}{f(\overline{\gamma}+a)^2-(\overline{\gamma}^2-a^2)}\right)$, $\Delta = \left[y_\beta \frac{f(\overline{\gamma}+a)^2-(\overline{\gamma}-a)^2}{f(\overline{\gamma}+a)^2-(\overline{\gamma}^2-a^2)}, 2y_\beta \frac{\overline{\gamma}}{\overline{\gamma}+a}\right)$, $\Theta = \left[2y_\beta \frac{\overline{\gamma}}{\overline{\gamma}+a}, \infty\right)$. If $y_\alpha \in \Lambda$ then $\overline{w}_\alpha = q_\alpha f A + q_\alpha(\overline{\gamma}-a) f y_\beta$, and $\overline{w}_\beta = (\overline{\gamma}+a) f y_\alpha - f A q_\beta$. If $y_\alpha \in \Delta$ then $\overline{w}_\alpha = (\overline{\gamma}+a)2/3 f y_\alpha$ and $\overline{w}_\beta = (\overline{\gamma}-a)1/3 f y_\beta$.

We similarly characterize set $\mathcal{W}_{R(ii)}$ by solving for price pairs for which $F_i(y_\alpha, \cdot) < F_i(y_\beta, \cdot)$;

$$\mathcal{W}_{R(ii)} = \begin{cases} \{(w_\alpha, w_\beta) | w_\alpha > \underline{w}_\alpha, w_\beta < \underline{w}_\beta \text{ and } w_\alpha > q_\alpha f A + q_\alpha / q_\beta w_\beta\} & \text{if } y_\alpha \in \Lambda, \\ \{(w_\alpha, w_\beta) | w_\alpha > \underline{w}_\alpha, w_\beta < \underline{w}_\beta\} & \text{if } y_\alpha \in \Delta, \\ \{(w_\alpha, w_\beta) \in \emptyset\} & \text{if } y_\alpha \in \Theta. \end{cases}$$

Here $(q_\alpha, q_\beta) = (1/3, 2/3)$, A, Λ, Δ, and Θ are as previously defined. In addition, if $y_\alpha \in \Lambda$ then $\underline{w}_\alpha = q_\alpha f A + q_\alpha(\overline{\gamma} - a) f y_\beta$, and $\underline{w}_\beta = (\overline{\gamma} + a) f y_\alpha - f A q_\beta$. If $y_\alpha \in \Delta$ then $\underline{w}_\alpha = (\overline{\gamma} + a)1/3 f y_\alpha$ and $\underline{w}_\beta = (\overline{\gamma} - a)2/3 f y_\beta$.

We similarly characterize set $\mathcal{W}_{R(iii)}$ by solving for price pairs for which $F_i(y_\alpha, \cdot) = F_i(y_\beta, \cdot)$;

$$
\mathcal{W}_{R(iii)} = \begin{cases} \{(w_\alpha, w_\beta)\} | w_\alpha = \underline{w}_\alpha & \text{if } w_\beta \leq (\overline{\gamma} - a)1/2fy_\beta \text{ and } y_\alpha \in \Lambda, \\ \{(w_\alpha, w_\beta)\} | w_\beta = \underline{w}_\beta & \text{if } w_\alpha \geq (\overline{\gamma} + a)1/2fy_\beta \text{ and } y_\alpha \in \Lambda, \\ \{(w_\alpha, w_\beta)\} | w_\alpha = q_\alpha f A + q_\alpha / q_\beta w_\beta & \text{if } w_z \in ((\overline{\gamma} - a)y_z/2, (\overline{\gamma} + a)y_z/2) \\ & \text{ for } z \in \{\alpha, \beta\} \text{ and } y_\alpha \in \Lambda, \\ \{(w_\alpha, w_\beta) | w_\alpha \geq \underline{w}_\alpha, w_\beta \leq \underline{w}_\beta, & \text{if } y_\alpha \in \Delta \bigcup 2y_\beta \frac{\overline{\gamma}}{\overline{\gamma}+a}, \\ \{(w_\alpha, w_\beta) \in \emptyset, & \text{if } y_\alpha \in \Theta / 2y_\beta \frac{\overline{\gamma}}{\overline{\gamma}+a}. \end{cases}
$$

Here $(q_\alpha, q_\beta) = (1/2, 1/2)$, A, Λ, Δ, and Θ are as previously defined. In addition, if $y_\alpha \in \Lambda$ then $\underline{w}_\alpha = q_\alpha f A + q_\alpha(\overline{\gamma} - a) f y_\beta$, and $\underline{w}_\beta = (\overline{\gamma} + a) f y_\alpha - f A q_\beta$. If $y_\alpha \in \Delta$ then $\underline{w}_\alpha = (\overline{\gamma} + a)1/2fy_\alpha$ and $\underline{w}_\beta = (\overline{\gamma} - a)1/2fy_\beta$.

We note that if a price pair lies on the intersection of any of the sets $\mathcal{W}_{R(i)}$, $\mathcal{W}_{R(ii)}$, and $\mathcal{W}_{R(iii)}$, then more than one equilibrium allocation exists.

A.2.2 Relation (ii) does not hold on the equilibrium Path

We first present a lemma showing that if a price subgame results in multiple CP allocation equilibria, such that relations (i) and (ii) hold, the CP allocation equilibrium for which relation (i) holds yields the highest profit for platform α. Let $\overline{\pi}_\alpha(w_\alpha, w_\beta)$ ($\widehat{\pi}_\alpha(w_\alpha, w_\beta)$) denote platform α's profit whenever relation (i) ((ii)) holds.

Lemma 21. *Given a price pair (w_α, w_β) such that the CP allocations which yield relations (i) and (ii) can occur, then $\overline{\pi}_\alpha(w_\alpha, w_\beta) > \widehat{\pi}_\alpha(w_\alpha, w_\beta)$.*

Proof. Let $\overline{r}_\alpha, \overline{r}_\beta$ ($\widehat{r}_\alpha, \widehat{r}_\beta$) denote the CP demand when relation (i) ((ii)) holds. It follows that $\overline{r}_\alpha \geq \widehat{r}_\alpha$ and $\overline{r}_\beta \leq \widehat{r}_\beta$ since $\overline{q}_\alpha > \widehat{q}_\alpha$ and $\overline{q}_\beta < \widehat{q}_\beta$. Moreover, $\overline{p}_\alpha \overline{q}_\alpha > \widehat{p}_\alpha \widehat{q}_\alpha$. To see this note that,

$$
\begin{aligned}
\overline{p}_\alpha &= \overline{q}_\alpha (y_\alpha((\overline{\gamma} + a) + (\overline{\gamma} - a)\overline{r}_\alpha) - y_\beta((\overline{\gamma} + a) + (\overline{\gamma} - a)\overline{r}_\beta)), \\
\widehat{p}_\alpha &= \widehat{q}_\alpha (y_\beta((\overline{\gamma} + a) + (\overline{\gamma} - a)\widehat{r}_\alpha) - y_\beta((\overline{\gamma} + a) + (\overline{\gamma} - a)\widehat{r}_\beta)).
\end{aligned}
$$

Therefore, $\overline{\pi}_\alpha(w_\alpha, w_\beta) = \overline{r}_\alpha w_\alpha + \overline{p}_\alpha \overline{q}_\alpha > \widehat{r}_\alpha w_\alpha + \widehat{p}_\alpha \widehat{q}_\alpha = \widehat{\pi}_\alpha(w_\alpha, w_\beta)$. □

We now show in the next lemma that if an SPE exists then the CP allocation that holds in the equilibrium path does not yield relation (ii).

Lemma 22. *If* (w_α^*, w_β^*) *is an SPE of the quality subgame then the CP allocation on the equilibrium path does not yield relation* (ii).

Proof. From appendix A.2.1, there are three cases to consider; $y_\alpha \in \Lambda$, $y_\alpha \in \Delta$ and $y_\alpha \in \Theta$. In the last case, a CP allocation equilibrium that yields relation (ii) does not exist. Therefore, we only consider the first two cases.

(a) $y_\alpha \in \Lambda$

Let $\widehat{\pi}_\alpha(w_\alpha^*, w_\beta^*) = \widehat{r}_\alpha w_\alpha^* + p_\alpha^* \widehat{q}_\alpha$ denote the revenue under the equilibrium price pair (w_α^*, w_β^*). Here, $\widehat{r}_\alpha$ ($\widehat{q}_\alpha$) refers to the mass of CPs (consumers) at equilibrium and p_α^* is the price offered to the consumer at equilibrium. Let $\overline{w}_\alpha = w_\alpha^* - \epsilon$, where $\epsilon > 0$. Observe that since $y_\alpha \in \Lambda$ and $(w_\alpha^*, w_\beta^*) \in \mathcal{W}_{R(ii)}$, it is always possible to choose ϵ such that $(\overline{w}_\alpha, w_\beta^*) \in \mathcal{W}_{R(ii)}$. Let $\pi_\alpha(w_\alpha, w_\beta^*)$ be the profit function for platform α generated by choosing the CP allocation equilibrium which yields relation (ii), whenever more than one CP allocation equilibrium is possible. This profit function is quadratic and concave in w_α over the interval $\mathcal{I} = (\max\{\underline{w}_\alpha, 1/3fA + 1/2w_\beta^*\}, 1/3fy_\alpha)$. Here, $\underline{w}_\alpha$ is as defined in appendix A.2.1, where $(q_\alpha, q_\beta) = (1/3f, 2/3f)$. The unrestricted maximum $w_\alpha^u = arg\max \pi(w_\alpha, w_\beta^*) < \max\{\underline{w}_\alpha, (\overline{\gamma} - a)1/3fy_\alpha\}$. Therefore, $\pi(w_\alpha, w_\beta^*)$ is decreasing in the interval $\mathcal{I}$. If we pick an ϵ small enough, then $\overline{w}_\alpha \in \mathcal{I}$. This implies that $\pi(\overline{w}_\alpha, w_\beta^*) > \widehat{\pi}_\alpha(w_\alpha^*, w_\beta^*)$. Note also that at price $(\overline{w}_\alpha, w_\beta^*)$, the revenue arising under the CP allocation equilibrium which yields relation (i) is higher, see lemma 21. Therefore price $\overline{w}_\alpha$ dominates price w_α^* and platform α has an incentive to deviate.

(b) $y_\alpha \in \Delta$

Let $\overline{w}_\alpha = \underline{w}_\alpha - \epsilon$, where $\epsilon > 0$ such that $(\overline{w}_\alpha, w_\beta^*) \in \mathcal{W}_{R(i)} \cap \mathcal{W}_{R(iii)}^c$. Here, $\underline{w}_\alpha$ is as defined in appendix A.2.1, with $(q_\alpha, q_\beta) = (1/3f, 2/3f)$. Denote the revenue under the equilibrium price pair (w_α^*, w_β^*) by $\widehat{\pi}_\alpha(w_\alpha^*, w_\beta^*) = \widehat{r}_\alpha w_\alpha^* + p_\alpha^* \widehat{q}_\alpha$. At the price pair $(\overline{w}_\alpha, w^*_\beta)$ only one CP allocation equilibrium is possible; the one that yields relation (i). Let $\overline{\pi}_\alpha(\overline{w}_\alpha, w^*_\beta) = \overline{r}_\alpha \overline{w}_\alpha + \overline{p}_\alpha \overline{q}_\alpha$ represent the revenue at this price.

We next show that $\overline{\pi}_\alpha(\overline{w}_\alpha, w^*_\beta) > \widehat{\pi}_\alpha(w^*_\alpha, w^*_\beta)$. First we note that revenue made on the CP side by platform α is higher under the new price since $\overline{r}_\alpha > \widehat{r}_\alpha = 0$. This follows from the fact that $\overline{w}_\alpha < \underline{w}_\alpha < (\overline{\gamma} + a)2/3fy_\alpha$. Therefore, a positive mass of CPs will patronize the platform since they gain positive utility upon joining. Revenue on the consumer side is also higher under this new deviation. To see this, note that since relation (i) holds, $\overline{q}_\beta < \widehat{q}_\beta$. This implies $\overline{r}_\beta < \widehat{r}_\beta$ which further implies that $\overline{p}_\alpha > p^*_\alpha$. Observe that $\overline{q}_\alpha > \widehat{q}_\alpha$ (since the CP allocation equilibrium yields relation (i)), $y_\alpha \geq y_\beta$ and,

$$
\begin{aligned}
\overline{p}_\alpha &= \overline{q}_\alpha(y_\alpha((\overline{\gamma} + a) + (\overline{\gamma} - a)\overline{r}_\alpha) - y_\beta((\overline{\gamma} + a) + (\overline{\gamma} - a)\overline{r}_\beta)), \\
p^*_\alpha &= \widehat{q}_\alpha(y_\beta((\overline{\gamma} + a) + (\overline{\gamma} - a)\widehat{r}_\alpha) - y_\beta((\overline{\gamma} + a) + (\overline{\gamma} - a)\widehat{r}_\beta)).
\end{aligned}
$$

□

A.2.3 Proof of Theorem 5

In this Appendix, we show that given the tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ such that $y_\alpha > y_\beta$ a unique SPE exists in the CP price game. We define the baseline CP price game as the game induced by selecting the CP allocation equilibrium which yields relation (i), whenever multiple equilibria exist. We then show that this game has a unique SPE. In addition, we show that all reduced extensive form games, for which an SPE exist, have this same unique SPE. Thus without loss of generality we may only consider the baseline CP price subgame. Recall we are considering reduced extensive form games in which the CP allocation equilibria chosen, whenever multiple equilibria exist, are those that yield either relation (i) or (iii).

Proof. The proof involves the following two steps.

Step. 1 Baseline-CP price game has a unique SPE

Given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ such that $y_\alpha > y_\beta$ and price w_β, we denote platform

$\alpha's$ profit function by $\pi_\alpha(w_\alpha, w_\beta)$. This profit function is quadratic in w_α over the range $\mathcal{I} = [(\overline{\gamma} - a)2/3fy_\alpha, min\{(\overline{\gamma} + a)2/3fy_\alpha, 2/3fA + 2w_\beta\}]$. It is linear and decreasing for $w_\alpha < (\overline{\gamma} - a)2/3fy_\alpha$. Let $w_\alpha^u = \text{argmax } \pi_\alpha(w_\alpha, w_\beta)$ be the unrestricted maximum of the quadratic function. This value is given by $w_\alpha^u = \frac{1}{9}((3-2f)\overline{\gamma} + (3+2f)a)fy_\alpha$. Whenever $y_\alpha > y_\beta$ then $w_\alpha^u < min\{(\overline{\gamma} + a)2/3fy_\alpha, 2/3fA + 2w_\beta\}$, where $w_\beta \geq (\overline{\gamma} - a)1/3fy_\beta$. Therefore, given w_β, the best response is given by $w_\alpha^* = \max\{w_\alpha^u, (\overline{\gamma} - a)2/3fy_\alpha\}$. Given another w'_β, the profit function $\pi'_\alpha(w_\alpha, w'_\beta) = \pi_\alpha(w_\alpha, w_\beta) + k(w_\beta, w'_\beta)$ over the range $[0, 2/3fA + 2((\overline{\gamma} - a)1/3fy_\beta)]$. Therefore $w_\alpha^{u\prime} = w_\alpha^u = \text{argmax } \pi_\alpha(w_\alpha, w'_\beta)$ and the best response w_α^* is also the same. Thus given any w_β the best response is a constant w_α^*. We can similarly show that given any w_α the best response by platform β is given by $w_\beta^* = \max\{w_\beta^u, (\overline{\gamma} - a)1/3fy_\beta\}$ where $w_\beta^u = \frac{1}{18}((3+f)\overline{\gamma} + (3-f)a)fy_\beta$. Thus the pair (w_α^*, w_β^*) form a unique SPE.

If $w_\alpha^u \leq (\overline{\gamma} - a)2/3fy_\alpha$ then all CPs will connect to platform α. Following some algebra the former holds when $\frac{\overline{\gamma}}{a} \geq \frac{9+2f}{3+2f}$. On the other hand, only a fraction of the CPs join the platform whenever $\frac{\overline{\gamma}}{a} < \frac{9+2f}{3+2f}$. If $w_\beta^u \leq (\overline{\gamma} - a)1/3fy_\beta)$ then all CPs will connect to platform β. Following some algebra, one can show the former holds when $\frac{\overline{\gamma}}{a} \geq \frac{9-f}{3-f}$. Therefore, only a fraction of the CPs join the platform whenever $\frac{\overline{\gamma}}{a} < \frac{9-f}{3-f}$.

We define the following sets of prices which we use to characterize market configurations that hold at the SPE.

$$
\begin{aligned}
\mathcal{R_I}^n &= \{(w_\alpha, w_\beta) | r_\alpha(w_\alpha, w_\beta) < 1,\ r_\beta(w_\alpha, w_\beta) < 1\}, \\
\mathcal{R_{II}}^n &= \{(w_\alpha, w_\beta) | r_\alpha(w_\alpha, w_\beta) < 1, r_\beta(w_\beta, w_\alpha) = 1;\ \text{or } r_\alpha(w_\alpha, w_\beta) = 1, r_\beta(w_\alpha, w_\beta) < 1\}, \\
\mathcal{R_{III}}^n &= \{(w_\alpha, w_\beta | r_\alpha(w_\alpha, w_\beta) = 1,\ r_\beta(w_\alpha, w_\beta) = 1)\}.
\end{aligned}
$$

The set $\mathcal{R_I}^n$ consists of prices (w_α, w_β) such that only a fraction of the CPs in the market subscribe to the platforms. Set $\mathcal{R_{II}}^n$ consists of prices $(w_\alpha w_\beta)$ such that the market is covered; all CPs patronize either platform α or β but not both. Lastly set $\mathcal{R_{III}}^n$ consists of a pair of prices such that the market is covered with all CPs patronizing both platforms. We summarize results of the previous paragraph below.

a) If $1 < \frac{\overline{\gamma}}{a} < \frac{9+2f}{3+2f}$, then $(w_\alpha^*, w_\beta^*) \in \mathcal{R_I}^n$.

b) If $\frac{9+2f}{3+2f} \leq \frac{\overline{\gamma}}{a} < \frac{9-f}{3-f}$ then $(w^*_\alpha, w^*_\beta) \in \mathcal{R}_{II}{}^n$.

c) If $\frac{9-f}{3-f} \leq \frac{\overline{\gamma}}{a} < \infty$ then $(w^*_\alpha, w^*_\beta) \in \mathcal{R}_{III}{}^n$.

Step. 2 CP price games that have a SPE, have the same SPE as the baseline CP price game Given a price game and any pair $(w_\beta, w'_\beta) \in \mathbb{R}^+$, the profit value $\pi_\alpha(w_\alpha, w_\beta) = \pi_\alpha(w_\alpha, w'_\beta) + k(w_\beta, w'_\beta)$ for all $w_\alpha \in [0, 2/3fA + 2((\overline{\gamma} - a)1/3fy_\beta)]$ except possibly at $w_\alpha = 1/2fA + w'_\beta$ and $w_\alpha = 1/2fA + w_\beta$. Therefore the best response for platform α given platform β charges w_β is given by w^*_α as defined in the previous step if $w^*_\alpha \neq 1/2fA + w_\beta$. In the case $w^*_\alpha = 1/2fA + w_\beta$ then a best response does not exist. Similarly, we can show that the best response given any w_α is given by w^*_β, as defined in *step 1*, if a best response exists. Thus if the best responses intersect they only do so at the price pair (w^*_α, w^*_β).

□

A.2.4 The case when $y_\alpha = y_\beta$

Lemma 22 implies that neither relation (i) or (ii) hold at the SPE when $y_\alpha = y_\beta$. Therefore, if an SPE exists it must be that relation (iii) holds. We bound the maximum revenue value that can result in instances for which an SPE exists. Given a tuple $(\overline{\gamma}, a, f)$, we show later in the investment stage that this pair is not an SPE because either platform has an incentive to deviate.

We now provide an upper-bound for the revenue gained by the platforms if an SPE exists. Revenue for both platforms is derived only from the CP side. This follows because only relation (iii) can hold in equilibrium; due to Bertrand competition, platforms earn no revenue from the consumer side. As discussed in section 3.5.3, the allocation of consumers is evenly divided when relation (iii) holds, i.e, $q_\alpha = q_\beta = 1/2f$. Moreover, if relation (iii) holds then $r_\alpha = r_\beta$ which further implies $w_\alpha = w_\beta$. Let revenue for platform α and β be represented by $\pi_\alpha(w^*_\alpha, w^*_\beta)$ and $\pi_\beta(w^*_\alpha, w^*_\beta)$ respectively at the SPE price (w^*_α, w^*_β). Then $\pi_\alpha(w^*_\alpha, w^*_\beta) = \pi_\beta(w^*_\alpha, w^*_\beta)$ where $w^*_\alpha \in ((\overline{\gamma} - a)(q_\alpha y_\alpha), (\overline{\gamma} + a)(q_\alpha y_\alpha))$. We consider only prices in this range because other prices are dominated and will not be picked in equilibrium. Consider the function $\pi(w_\alpha) = r_\alpha w_\alpha$ where $r_\alpha = 1/2a(\overline{\gamma} + a -$

$w_\alpha/q_\alpha y_\alpha)$. This function is concave and quadratic in w_α and at the value w^*_α we have $\pi(w^*_\alpha) = \pi_\alpha(w^*_\alpha, w^*_\beta)$. Let,

$$\begin{aligned} \hat{w}_\alpha &= \text{argmax } \pi(w_\alpha), \\ & \quad \text{s.t. } w_\alpha \in ((\overline{\gamma} - a)(q_\alpha y_\alpha), (\overline{\gamma} + a)(q_\alpha y_\alpha)). \end{aligned}$$

If an SPE exists platform $\alpha's$ profits are bounded by $\pi(\hat{w}_\alpha)$. Since platform $\beta's$ profits are the same as $\alpha's$ they are also bounded from above by the same value.

A.2.5 Proof of Theorem 6.

Given a quality choice y_β, we derive platform's α best response and vice versa. We then find the intersection points that form the SPE. We will give Lemmas that define the best responses and then we will be able to infer the SPE's from these responses.

Lemma 23. *Let Assumptions 6 and R.1 hold. Then*

$$B_i(y_\beta) = \begin{cases} y^*(\overline{\gamma}, a, f, c) & \text{if } y_\beta < \overline{y}, \\ 0 & \text{if } y_\beta \geq \overline{y}, \end{cases}$$

where

$$\begin{aligned} y^*(\overline{\gamma}, a, f, c) &= \frac{a^2 f}{216c}\left(\frac{(2f+3)^2}{a^3}\overline{\gamma}^2 + \frac{90 - 8(f-3)^2}{a^2}\overline{\gamma} + \frac{((2f+9)^2 - 36)}{a}\right), \\ \overline{y}(\overline{\gamma}, a, f, c) &= \frac{48\overline{\gamma} fa + 36fa^2 + 12f\overline{\gamma}^2 + 4f^2\overline{\gamma}^2 - 8f^2\overline{\gamma}a + 4f^2a^2 + 9\overline{\gamma}^2 + 18\overline{\gamma}a + 9a^2)^2}{-432ac(-60\overline{\gamma}a - 45a^2 - 15\overline{\gamma}^2 + 2fa^2 + 2f\overline{\gamma}^2 - 4\overline{\gamma}fa)}. \end{aligned}$$

Proof. We first find the best response given $y_\beta = 0$. Let $y^* = \text{argmax } \pi_\alpha(y_\alpha, y_\beta)$, s.t. $y_\alpha \in \mathbb{R}_+$. In region R.1 the market is uncovered as shown in Appendix A.2.3. The profit function $\pi_\alpha(y_\alpha, y_\beta)$ is quadratic and concave in y_α in this market configuration. The best response, $B_\alpha(y_\beta) = y^*(\cdot)$ exists since $\pi_\alpha(y_\alpha, y_\beta)$ is coercive and its value is that given in the Lemma statement.

Next, we find the best response when $y_\beta > 0$. The price equilibria that holds depends on whether platform α acts as the high-quality or the low-quality platform. Indeed, given $y_\beta > 0$ platform $\alpha's$ choice of investment y_α will determine which of the following three relations defined in section 3.5.2 will hold on the equilibrium path.

If y_α is higher (lower) than y_β then relation $(i)((ii))$ will hold. When $y_\alpha = y_\beta$, relation (iii) may hold if an SPE exists. Therefore, we partition the domain $[0, \infty)$ depending on whether platform α acts a high quality or low quality platform. These partitions are defined as, $I_1 = [0, y_\beta)$, $I_2 = (y_\beta, \infty)$. In interval I_1 (I_2) only $(ii)((i))$ holds. In contrast, at the point $y_\alpha = y_\beta$ relation (iii) holds if an SPE exists. In order to find the best reply given y_β we proceed as follows. We find the best response of platform α in partitions I_1 and I_2. We pick the best reply amongst these choices and show it dominates the maximum possible choice given $y_\alpha = y_\beta$ as calculated in the previous Appendix.

$$\begin{aligned} y^*_{\alpha 1} &= \text{argmax } \hat{\pi}_\alpha(y_\alpha, y_\beta) \\ & s.t. \;\; y_{\alpha 1} \in I_1. \end{aligned} \tag{A.41}$$

Where $\hat{\pi}_\alpha$ is the profit function in the interval I_1. In a like manner we denote the best reply in interval I_2 by $y^*_{\alpha 2}$. Formally,

$$\begin{aligned} y^*_{\alpha 2} &= \text{argmax } \pi_\alpha(y_\alpha, y_\beta) \\ & s.t. \;\; y_{\alpha 2} \in I_2. \end{aligned} \tag{A.42}$$

Where π_α is the profit function in the interval I_2. The profit function $\hat{\pi}_\alpha$ is concave in y_α. Let $y^{ur}_{\alpha 1}$ be the unrestricted solution. This value is less than zero in region R.1. Therefore, the lower constraint in the maximization problem A.41 binds and we have $y^*_{\alpha 1} = 0$. On the other hand , the unrestricted maximization of problem A.42 yields $y^*_{\alpha 2} = y^*(\overline{\gamma}, a, f, c)$ as defined in the statement of the Lemma.

Next we compare the profit values at the solutions in both intervals. Let $\pi^*_\alpha = \pi_\alpha(y_\alpha, y_\beta)|_{y_\alpha = y^*_{\alpha 2}}$ and $\hat{\pi}^*_\alpha = \hat{\pi}_\alpha(y_\alpha, y_\beta)|_{y_\alpha = 0}$. One can show that the difference $\pi^*_\alpha - \hat{\pi}^*_\alpha$ is decreasing in y_β. Moreover, the two are equal at $y_\beta = \overline{y}$; the value presented in the Lemma statement. Hence whenever $y_\beta < \overline{y}$, $y^*_{\alpha 2}$ dominates $y^*_{\alpha 1}$ and vice versa. In addition, $y^*_{\alpha 2} > y_\beta$ whenever $y_\beta < \overline{y}$. Therefore this solution lies in the interior of I_2.

To complete this proof we now show that the profit attainable when $y_\alpha = y_\beta$ is less than $\max\{\hat{\pi}^*_\alpha, \pi^*_\alpha\}$. We denote the upper-bound profit value when $y_\alpha = y_\beta$ by $\overline{\pi}^*_\alpha$. It suffices to show that this value is less than π^*_α when $y_\beta > \overline{y}$ and less than $\hat{\pi}^*_\alpha$ when $y_\beta < \overline{y}$. We first show the former, i.e, the difference $\pi^*_\alpha - \overline{\pi}^*_\alpha$ is positive whenever$y_\beta > \overline{y}$. This difference[26] is convex and quadratic in y_β. Moreover it has roots at 0 and $\overline{y}$. Therefore the difference is positive whenever $y_\beta > \overline{y}$. Next we show $\hat{\pi}^*_\alpha - \overline{\pi}^*_\alpha$ is positive. The difference is quadratic and convex in y_β. Moreover, the roots are imaginary[27] thus we infer that the difference is positive. □

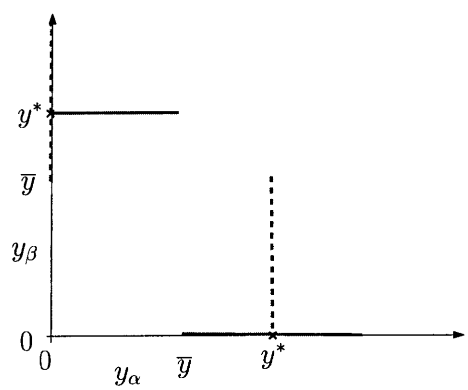

Figure A-6: The best reply responses of both platforms in region R.1. The intersection points, $(y^*, 0)$ and $(0, y^*)$, give the equilibrium investment levels in region $R.1$.

A similar analysis follows for platform β. The best responses of the platforms intersect at points $(y^*, 0)$ and $(0, y^*)$. Consequently this points form an SPE, see Figure A-6.

We next state a number of Lemmas that yield the other results in Thereof 6. We omit the proofs because they are very similar. Where there's significant diversion we add comments.

[26] $\frac{1}{432a} y_\beta (48 f^2 \overline{\gamma} a + 36 f^2 a^2 + 12 f^2 \overline{\gamma}^2 + 8 f^3 \overline{\gamma}^2 - 16 f^3 \overline{\gamma} a + 8 f^3 a^2 - 27 f \overline{\gamma}^2 - 54 \overline{\gamma} f a - 27 f a^2 + 432 c y_\beta a)$

[27] $root = \frac{-1}{864ca}(-27a^2 + 16f^2a^2 - 144fa^2 - 32f^2\overline{\gamma}a - 48f\overline{\gamma}^2 - 192\overline{\gamma}fa + 16f^2\overline{\gamma}^2 - 27\overline{\gamma}^2 - 54\overline{\gamma}a \pm \sqrt{(-3(\overline{\gamma} + a) * (96fa + 21a + 21\overline{\gamma} + 32f\overline{\gamma})(32f^2a^2 + 9a^2 - 64f^2\overline{\gamma}a + 18\overline{\gamma}a + 32f^2\overline{\gamma}^2 + 9\overline{\gamma}^2)}))f$

Lemma 24. *Let Assumptions 6 and 8, $f > \frac{1}{3}$ and R.2 hold. Then*

$$B_i(y_\beta) = \begin{cases} y_h^*(\overline{\gamma}, a, f, c) & \text{if } y_\beta < \overline{y}, \\ y_l^*(\overline{\gamma}, a, f, c) & \text{if } y_\beta \geq \overline{y}, \end{cases}$$ [28]

where

$$\begin{aligned} y_h^*(\overline{\gamma}, a, f, c) &= \frac{a^2 f}{216c}\left(\frac{(2f+3)^2}{a^3}\overline{\gamma}^2 + \frac{90 - 8(f-3)^2}{a^2}\overline{\gamma} + \frac{((2f+9)^2 - 36)}{a}\right), \\ y_l^*(\overline{\gamma}, a, f, c) &= \frac{a^2 f}{432c}\left(\frac{(f-3)^2}{a^3}\overline{\gamma}^2 - \frac{-90 + 2(f+6)^2}{a^2}\overline{\gamma} + \frac{((f-9)^2 - 72)}{a}\right). \end{aligned}$$

Lemma 25. *Let Assumptions 6, 8 and R.3 hold. Then*

$$B_i(y_\beta) = \begin{cases} y^*(\overline{\gamma}, a, f, c) & \text{if } y_\beta < \overline{y}, \\ 0 & \text{if } y_\beta \geq \overline{y}. \end{cases}$$

where

$$\begin{aligned} y_h^*(\overline{\gamma}, a, f, c) &= \frac{f}{9c}\left(4f\overline{\gamma}^2 + 3\overline{\gamma} - 3a\right), \\ y_l^*(\overline{\gamma}, a, f, c) &= 0, \\ \overline{y}(\overline{\gamma}, a, f, c) &= \frac{(-4f\overline{\gamma} - 3\overline{\gamma} + 3a)^2 a}{c3(18\overline{\gamma}a + 9a^2 + 3\overline{\gamma}^2 - f\overline{\gamma}^2 + 2f\overline{\gamma}a - fa^2)}. \end{aligned}$$

Lemma 26. *Let Assumption 6, 8 and R.4 hold. Then*

$$B_i(y_\beta) = \begin{cases} y_h^*(\overline{\gamma}, a, f, c) & \text{if } y_\beta < \overline{y}, \\ y_l^*(\overline{\gamma}, a, f, c) & \text{if } y_\beta \geq \overline{y}, \end{cases}$$ [29]

where

$$y_h^*(\overline{\gamma}, a, f, c) = \frac{f}{9c}\left(4f\overline{\gamma}^2 + 3\overline{\gamma} - 3a\right),$$

$$y_l^*(\overline{\gamma}, a, f, c) \quad = \quad \frac{af}{432c}\left(\frac{(f-3)^2}{a^3}\overline{\gamma}^2 - \frac{-90+2(f+6)^2}{a^2}\overline{\gamma} + \frac{((f-9)^2-72)}{a}\right).$$

Lemma 27. *Let Assumption 6, 8 $f > 0.47$ and R.5 hold. Then*

$$B_i(y_\beta) = \begin{cases} y_h^*(\overline{\gamma}, a, f, c) & \text{if } y_\beta < \overline{y}, \\ y_l^*(\overline{\gamma}, a, f, c) & \text{if } y_\beta \geq \overline{y}, \end{cases}$$

where

$$\begin{aligned} y_h^*(\overline{\gamma}, a, f, c) &= \frac{f}{9c}\left(4f\overline{\gamma} + 3\overline{\gamma} - 3a\right), \\ y_l^*(\overline{\gamma}, a, f, c) &= \frac{f}{18c}\left(-2f\overline{\gamma} + 3\overline{\gamma} - 3a\right), \\ \overline{y}(\overline{\gamma}, a, f, c) &= \frac{1}{120\overline{\gamma}c}(20f^2\overline{\gamma}^2 + 36f\overline{\gamma}^2 - 36\overline{\gamma}fa + 9\overline{\gamma}^2 - 18\overline{\gamma}a + 9a^2). \end{aligned}$$

Proof. We first find the best response given $y_\beta = 0$. Similar to the proof of Lemma 23 we let $y^* = \text{argmax } \pi_\alpha(y_\alpha, y_\beta), \text{s.t. } y_\alpha \in \mathbb{R}^+$. In region R.5 the market is covered as shown in Appendix A.2.3. The profit function $\pi_\alpha(y_\alpha, y_\beta)$ is quadratic and concave in y_α in this market configuration. The best response, $B_\alpha(y_\beta) = y^*(\cdot)$ exists since $\pi_\alpha(y_\alpha, y_\beta)$ is coercive and the solution is that given by $y_h^*(\overline{\gamma}, a, f, c)$ in the Lemma statement.

Next we find the best response when $y_\beta > 0$. Given $y_\beta > 0$ a platform α decides to have a quality that is the same as platform β or to be either the high or low quality platform. The choice made will determine which of the three relations defined in section 3.5.2 will hold.If $y_\alpha > (<) y_\beta$ then relation (i)((ii)) results. If $y_\alpha = y_\beta$ and an SPE exists then relation (iii) holds. Therefore we partition the domain $[0, y_\alpha)$ into two regions that do not include the point y_β. These partitions are defined as $I_1 = [0, y_\beta)$,$I_2 = (y_\beta, \infty)$.

In order to find the best reply we proceed as follows. We find the best response of platform α in partitions I_1 and I_2. We pick the best reply amongst these choices and show it dominates the choice $y_\alpha = y_\beta$. Let

$$y^*_{\alpha 1} = \text{argmax } \hat{\pi}_\alpha(y_\alpha, y_\beta) \quad s.t. \quad y_{\alpha 1} \in I_1. \tag{A.43}$$

Where $\hat{\pi}_\alpha$ is the profit function in the interval I_1. In a like manner we denote the best reply in interval I_2 by $y^*_{\alpha 2}$. Formally,

$$y^*_{\alpha 2} = \text{argmax } \pi_\alpha(y_\alpha, y_\beta) \quad s.t. \quad y_{\alpha 2} \in I_2. \tag{A.44}$$

Where π_α is the profit function in the interval I_2. The profit function $\hat{\pi}_\alpha$ is concave in y_α. Let $y^*_{\alpha 1}$ be the unrestricted solution of problem A.43. Its value is given by $y^*_{\alpha 1} = y^*_l(\overline{\gamma}, a, f, c)$. This value is greater than zero in region R.5. The unrestricted maximization of problem A.44 yields $y^*_{\alpha 2} = y^*_h(\overline{\gamma}, a, f, c)$ as defined in the statement of the Lemma.

Next we compare the profit values at these solutions. Let $\pi^*_\alpha = \pi_\alpha(y_\alpha, y_\beta)|_{y_\alpha = y^*_{\alpha 2}}$ and $\hat{\pi}^*_\alpha = \hat{\pi}_\alpha(y_\alpha, y_\beta)|_{y_\alpha = y^*_{\alpha 1}}$. One can show that the difference $\pi^*_\alpha - \hat{\pi}^*_\alpha$ is decreasing in y_β. Hence whenever $y_\beta < \overline{y}$, $y^*_{\alpha 2}$ dominates $y^*_{\alpha 1}$ and vice versa. Moreover, the two are equal at $y_\beta = \overline{y}$; this is the value presented in the Lemma statement. In addition, one can show that $\overline{y} \in (y^*_{\alpha 1}, y^*_{\alpha 2})$ whenever $f > 3(2\sqrt{19} - 1)/50 \approx 0.47$.

To complete this proof we now show that the highest profit attainable when $y_\alpha = y_\beta$, and a SPE in the quality subgame exists, is less than $\max\{\hat{\pi}^*_\alpha, \pi^*_\alpha\}$. We denote the upper-bound profit value when $y_\alpha = y_\beta$ by $\overline{\pi}^*_\alpha$. It suffices to show that this value is less than π^*_α. From previous Appendix it follows that $\overline{\pi}^*_\alpha < \pi^*_\alpha$. To see this note that when $y_\alpha = y_\beta$, and an SPE results, platform α makes no revenue on the consumer side. □

The best responses for platform β are similarly derived. These responses intersect at the investment pairs (y^*_h, y^*_l) and (y^*_l, y^*_h). Thus these form the SPE as stated in the Theorem, see Figure A-7.

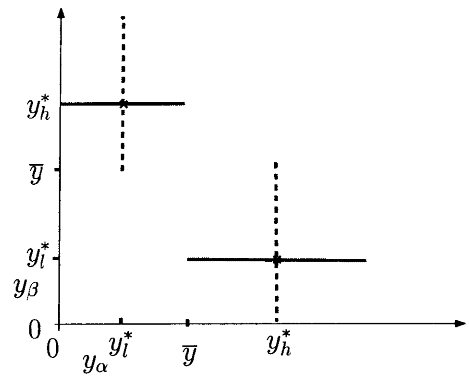

Figure A-7: The best reply responses of both platforms in region R.5. The intersection points, $(y^*, 0)$ and $(0, y^*)$, give the equilibrium investment levels in region $R.5$.

A.3 Social Welfare, CP and Consumer Surplus Comparison

A.3.1 Social Welfare Comparison

In this section we provide comparison of social welfare at the SPE under both models. First, we characterize the difference between welfare of the non-neutral and neutral regime in terms of the following exogenous parameters $(\overline{\gamma}, a, f, c)$. We show that this difference is non-negative and that in general, the non-neutral regime is favored to the neutral regime. Since the SPE for both models have been characterized for $f > 3/5$ the comparisons are also based for the same range of f.

We denote the difference between the non-neutral and neutral welfare by dw. The welfare functions at the SPE's have different forms in both models depending on whether the market is covered or uncovered. The restrictions on the tuple $(\overline{\gamma}, a, f, c)$ that define the limits in which the CP market is uncovered and covered coincide in both models. So we compare the welfare between the two regimes for each of the regions defined in Section 3.5.5. After some algebra, the difference in welfare for the different regions are given below:

Regions	Difference in welfare dw
$R.1$ and $R.3$	dw=0.
$R.2$ and $R.4$	$dw = \frac{1}{93312ca^2}((\overline{\gamma}-a)^2f^2 - 6(\overline{\gamma}+a)(\overline{\gamma}+3a)f + 9(\overline{\gamma}+a)^2).\cdots$ $\times(-(\overline{\gamma}-a)^2f^2 + (60a^2 - (\overline{\gamma}-7a)^2)f + 6(\overline{\gamma}+a)(2\overline{\gamma}+3))f^2.$
$R.5$	$dw = \frac{1}{324c}f^2(3a+5\overline{\gamma}+2\overline{\gamma}f)(3\overline{\gamma}-3a-2\overline{\gamma}f).$

For $f \geq 3/5$ the value dw is positive in regions $R.2$, $R.4$ and $R.5$. In these regions, the low-quality platform makes an investment that increases the gross value of CPs and Consumer surplus compared to their values in the neutral regime. On the other hand, in regions $R.1$ and $R.3$ the welfare in both regimes is the same because the investment levels are the same.

A.3.2 CP surplus comparison

In this section we compare the CP surplus in both regimes. Let dcp denote the difference in CP surplus between the two regimes. The following table shows this difference in the regions defined in Section 3.5.5.

Regions	Difference in welfare dcp
$R.1$ and $R.3$	dcp=0.
$R.2$ and $R.4$	$dcp = \frac{1}{186624ca^2}f^2(-6(\overline{\gamma}+a)(\overline{\gamma}+3a)f + (\overline{\gamma}-a)^2f^2 + 9(\overline{\gamma}+a)^2)\cdots$ $\times(3(\overline{\gamma}+a) - f(\overline{\gamma}-a))^2.$
$R.5$	$dcp = \frac{1}{54c}f^2(\overline{\gamma}(3-2f)-3a)a.$

For regions $R.1$ and $R.3$ the CP surplus is the same in both regimes. In these regions the investments across both platforms are the same. Therefore, the aggregate utility gained by the CPs is the same across both regimes. In regions $R.2$, $R.4$ and $R.5$ the value dcp is positive.

A.3.3 Consumer surplus Comparison

In this subsection we compare the consumer surplus in both regimes. Let dc denote the difference in consumer surplus between the two regimes. The following table shows the consumer surplus difference in the regions defined in Section 3.5.5.

Regions	Difference in welfare dc
$R.1$ and $R.3$	dc=0.
$R.2$ and $R.4$	$dc = \frac{1}{93312ca^2} f^2(-6(\overline{\gamma}+a)(\overline{\gamma}+3a)f + (\overline{\gamma}-a)^2 f^2 + 9(\overline{\gamma}+a)^2)\cdots$ $\times(1+10f)(3(k+a)(k+3a)-(k-a)^2 f)f^2.$
$R.5$	$dc = \frac{1}{162c} f^2(\overline{\gamma}(3-2f)-3a)(\overline{\gamma})(1+10f).$

In regions $R.1$ and $R.3$ consumer surplus is the same under both regimes because both platforms invest in the same qualities. However, in regions $R.2$, $R.4$ and $R.5$ the value dc is positive.

A.3.4 Platform profits Comparison

In this subsection we compare aggregate and individual platform profits in both regimes. Let dtp denote the difference in aggregate platform profit between the two regimes. The following table shows this difference in the regions defined in Section 3.5.5.

Regions	Difference in aggregate *dtp*
$R.1$ and $R.3$	dtp=0.
$R.2$ and $R.4$	$dtp = \frac{1}{186624ca^2} f^2(-6(\overline{\gamma}+a)(\overline{\gamma}+3a)f + (\overline{\gamma}-a)^2 f^2 + 9(\overline{\gamma}+a)^2)\cdots$ $\times(-54(k+a)(k+3a)f + 17(k-a)^2 f + 9(k+a)^2)$.
$R.5$	$dtp = -\frac{1}{108c} f^2(\overline{\gamma}(3-2f) - 3a)(\overline{\gamma})(k(6f-1)+a)$.

In regions $R.1$ and $R.3$ the profits of the platforms are the same in both the neutral and non-neutral regimes. This follows because the investments across platforms are equal in both regimes. In regions $R.2$, $R.4$ and $R.5$ the aggregate profit in the neutral regime is higher than that in the non-neutral regime since *dtp* is positive.

Next we show that profit of the high-quality platform is larger in the neutral regime in regions $R.2$, $R.4$ and $R.5$. Let *dtph* be the difference between the high-quality platform's profit in both regimes.

Regions	Difference in profits *dtph*
$R.1$ and $R.3$	$dtph$=0.
$R.2$ and $R.4$	$dtph = \frac{1}{11664ca^2} f^3(-6(\overline{\gamma}+a)(\overline{\gamma}+3a)f + (\overline{\gamma}-a)^2 f^2 + 9(\overline{\gamma}+a)^2)\cdots$ $\times(-3(k+a)(k+3a)f + 17(k-a)^2 + 9(k+a)^2 f)$.
$R.5$	$dtph = -\frac{4}{81c}\overline{\gamma} f^3(\overline{\gamma}(3-2f) - 3a)$.

Given the restrictions that define regions $R.2$, $R.4$ and $R.5$, *dtph* is positive whenever $f > 3/5$.

Next we show that in general the low-quality platforms prefer the non-neutral regime. Let the difference between the low-quality platform in the neutral and non-neutral profit be denoted by *dtpl*. The table below shows the value of this difference in the different regions.

Regions	Difference in profits *dtpl*
$R.1$ and $R.3$	$dtpl$=0.
$R.2$ and $R.4$	$dtpl = \frac{1}{186624ca^2} f^2 \left((-6(\overline{\gamma}+a)(\overline{\gamma}+3a)f + (\overline{\gamma}-a)^2 f^2 + 9(\overline{\gamma}+a)^2)\right)^2$
$R.5$	$dtpl = \frac{1}{324c} f^2 \overline{\gamma} f^3 (\overline{\gamma}(3-2f) - 3a)^2$.

The value *dtpl* is non-negative. In regions $R.1$ and $R.3$ the profits are the same because the investments across both platforms are the same. In contrast, the low-quality platform's profits in regions $R.2$, $R.4$ and $R.5$ are superior in the non-neutral regime because *dtpl* is positive.

Bibliography

[1] Daron Acemoglu and Asuman Ozdaglar. Competition and efficiency in congested markets. *Mathematics of Operations Research*, 32(1):1–31, 2007.

[2] Mark Armstrong. Competition in two-sided markets. Industrial Organization 0505009, EconWPA, May 2005.

[3] Michael Artin. *Algebra.* Prentice Hall, 1991.

[4] Joseph P. Bailey. Economics and internet interconnection agreements. *Journal of Electronic Publishing*, 1995.

[5] Jean-Pierre Benoit and Vijay Krishna. Dynamic duopoly: Prices and quantities. *The Review of Economic Studies*, 54(1):23–35, 1987.

[6] Ulrich Berger. Bill-and-keep vs. cost-based access pricing revisited. *Economic Letters*, 86:107–112, 2005.

[7] D. Bertsekas. *Nonlinear Programming.* Athena Scientific, 1999.

[8] Kim C. Border. *Fixed Point Theorems with Applications to Economics and Game Theory.* Cambridge University Press, 1985.

[9] Carlo Cambini and Tommaso M. Valleti. Network competition with price discrimination:'bill-and-keep' is not so bad after all. *Economic Letters*, 81:205–213, 2003.

[10] Carlos I. Caon. Regulation Effects on Investment Decisions in Two-Sided Market Industries: The Net Neutrality Debate. *SSRN eLibrary*, 2009.

[11] Hsing K. Cheng, Subhajyoti Bandyopadhyay, and Hong Guo. The Debate on Net Neutrality: A Policy Perspective. *Information Systems Research, Forthcoming*, 2008.

[12] Chong-Ju Choi and Hyung Song Shin. A comment on a model of vertical product differentiation. *Science*, 40(2):229–231, June 1992.

[13] Jay Pil Choi and Byung-Cheol Kim. Net neutrality and investment incentives. CESifo Working Paper Series CESifo Working Paper No. 2390, CESifo Group Munich, 2008.

[14] S. Dafermos and Sparrow F.T. The traffic assignment problem for a general network. *Journal of Research of the national Bureau of Standards-B. Mathematical Science*, 73(2):91–118, 1969.

[15] DeGraba. Reconciling the off-net cost pricing principle with efficient network utilization. *Information Economics and Policy*, 16:475–494, 2004.

[16] Nicholas Economides and Joacim Tag. Net neutrality on the Internet: A two-sided market analysis. Technical report, 2007.

[17] Eduardo Engel, Ronald Fischer, and Alexander Galetovic. Toll competition among congested roads. *The B.E. Journal of Economic Analysis and Policy*, 4(1), 2004.

[18] Farell and Weiser. Modularity, vertical integration, and open access policies: Towards a convergence of antitrust and regulation in the internet age. *Harvard Journal of Law & Technology*, 17(1), 2003.

[19] Emmanuel Farhi and Andrei Hagiu. Strategic Interactions in Two-Sided Market Oligopolies. *SSRN eLibrary*, 2008.

[20] Gerald R. Faulhaber. Network neutrality: The debate evolves. *International Journal of Communication*, pages 680–700, 2007.

[21] Rob M. Frieden. A primer on network neutrality. http://works.bepress.com, 2007.

[22] Jaskold Gabszewicz and J Thisse. Price competition, quality and income disparities. *Journal of Economic Theory*, 20:340–359, 1979.

[23] Jean J. Gabszewicz and Xavier Y. Wauthy. Two-sided markets and price competition with multi-homing. CORE Discussion Paper No. 2004/30, May 2004.

[24] Lixin Gao. On inferring autonomous system relationships in the internet. *IEEE/ACM Transactions On Networking*, 9(6), 2001.

[25] Robert Hahn and Scott Wallsten. The economics of net neutrality. *The Economists' Voice*, 3(6), June 2006.

[26] Ara Hayrapetyan, Eva Tardos, and Tom Wexler. A network pricing game for selfish traffic. to appear in Distributed Computing, 2005.

[27] Benjamin E. Hermalin and Michael L. Katz. The economics of product-line restrictions with an application to the network neutrality debate. *Information Economics and Policy*, 19(2):215–248, June 2007.

[28] Christian Hogendorn. Broadband internet: Net neutrality versus open acess. *International Economics and Economic Policy*, 4, 2007.

[29] Mark A. Jamison and Janice A. Hague. Getting what you pay for: Analyzing the net neutrality debate. Working Paper Series, August 2007.

[30] Byung-Cheol Kim Jay Pil Choi. Net neutrality and investment incentives. NET Institute Working Paper No. 08-03., September 2008.

[31] Galina Schwartz John Musacchio and Jean Walrand. Network neutrality and provider investment incentives. Submitted to Review of Network Economics, November, December 2007.

[32] Jan Krämer and Lukas Wiewiorra. Innovation through discrimination!? a formal analysis of the net neutrality debate. MPRA Paper 16655, University Library of Munich, Germany, August 2009.

[33] David M. Kreps and Jose A. Scheinkman. Quantity precommitment and bertrand competition yield cournot outcomes. *Bell Journal of Economics*, 14(2):326–337, Autumn 1983.

[34] Jean-Jacques Laffont (deceased), J. S. Marcus, Patrick Rey, and Jean Tirole. Internet Interconnection and the Off-Net-Cost Pricing Principle. *The Rand Journal of Economics*, 34:370–390, 2003.

[35] Robin S. Lee and Tim Wu. Subsidizing Creativity Through Network Design: Zero Pricing and Net Neutrality. *SSRN eLibrary*, 2009.

[36] David Levinson. Network neutrality: Lessons from transportation. *Review of Network Economics*, 8(1):13–21, June 2009.

[37] Mark Armstrong Author-Email: mark.armstrong@ucl.ac.uk. Competition in two-sided markets. *RAND Journal of Economics*, 37(3):668–691, Autumn 2006.

[38] Sridhar Moorthy. A comment on a model of vertical product differentiation. *Marketing Science*, 7(2):141–168, 1988.

[39] John Musacchio, Galina Schwartz, and Jean Walrand. A two-sided market analysis of provider investment incentives with an application to the net-neutrality issue. *Review of Network Economics*, 8(1):22–39, March 2009.

[40] Michael Mussa and Sherwin Rosen. Monopoly and product quality. *Journal of Economic Theory*, 18(2):301–317, August 1978.

[41] Jonathan E. Nuechterlein and Philip J. Weiser, editors. *Digital crossroads: American telecommunications policy in the internet age.* MIT Press, Cambridge, first edition, 2005.

[42] Andrew Odlyzko. Network neutrality, search neutrality, and the never-ending conflict between efficiency and fairness in markets. *Review of Network Economics*, 8(1):40–60, June 2009.

[43] Asuman Ozdaglar. Price competition with elastic traffic. *Netw.*, 52(3):141–155, 2008.

[44] Geoffrey G. Parker and Marshall W. Van Alstyne. Two-sided network effects: A theory of information product design. *Management Science*, 51(10), October 2005.

[45] Geoffrey G. Parker and Marshall W. Van Alstyne. Two-sided network effects: A theory of information product design. *Manage. Sci.*, 51(10):1494–1504, 2005.

[46] Njoroge Paul, Ozdaglar Asuman, Stier Nicolas, and Weintraub Gabriel. Competition, market coverage, and quality choice in interconnected platforms. *NetEcon Conference*, 2009.

[47] Chong R. The 31 flavors of the net neutrality debate: Beware the trojan horse. *Advanced Communications Law and Policy Institute*, 2007.

[48] Jean-Charles Rochet and Jean Tirole. Platform competition in two-sided markets. *Journal of the European Economic Association*, 1(4):990–1029, 06 2003.

[49] Jean-Charles Rochet and Jean Tirole. Platform competition in two-sided markets. *Journal of the European Economic Association*, 1(4):990–1029, 06 2003.

[50] Roberto Roson. Two-sided markets: A tentative survey. *Review of Network Economics*, 4(2):142–160, June 2005.

[51] Galina Schwartz, Nikhil Shetty, and Jean Walrand. Impact of qos on internet user welfare. *International Workshop On Internet And Network Economics (WINE).*, 2008.

[52] Galina Schwartz, Nikhil Shetty, and Jean Walrand. Net neutrality and ISPs property rights: Why internet lacks differentiated services? *Submitted for Publication Rand Journal.*, 2009.

[53] Marius Schwartz and Philip J. Weiser. Introduction to a special issue on network neutrality. *Review of Network Economics*, 8(1):1–12, March 2009.

[54] Avner Shaked and John Sutton. Relaxing price competition through product differentiation. *Review of Economic Studies*, 49(1), January 1982.

[55] Haim Mendelson Shiri Shneorsen. Babel, a multilingual style-option system for use with latex's standard document styles. *Stanford Graduate School of Business Working Paper*, June 2003.

[56] Gireesh Shrimali and Sunil Kumar. Bill and keep peering. Lecture notes in computer science ISSN 0302-9743, August 2006.

[57] J. Gregory Sidak. A consumer-welfare approach to network neutrality of the internet. *Journal of Competition Law and Economics*, 2006.

[58] Nancy Stokey, Lucas Robert, Jr., and Edward Prescott. *Recursive Methods In Economic Dynamics.* Harvard University Press, 1989.

[59] Jean Tirole. *The Theory of Industrial Organization.* The MIT Press, 1988.

[60] Andrew Wan. How competition among network providers can produce better services for users. Reinventing The Internet, November 2006.

[61] J. G. Wardrop. Some theoretical aspects of road traffic research. In *Proceedings of the Institute of Civil Engineers*, pages 325–378, 1952.

[62] Xavier Wauthy. Quality choice in models of vertical differentiation. *The Journal of Industrial Economics*, 44:345–353, 1996.

[63] Interview With Edward Whitcare. At sbc, it's all about scale and scope. Business Week Online, November 2005. Interview with Edward Whitcare.

[64] Tim. Wu. Network neutrality, broadband discrimination. *Journal On Telecommunications and High Technology Law*, 3:141–176, 2003.

[65] Tim Wu and Christopher S. Yoo. Keeping the Internet Neutral?: Tim Wu and Christopher Yoo Debate. *Federal Communications Law Journal, Vol. 59, No. 3, 2007.*

[66] Christopher S. Yoo. Network neutrality and the economics of congestion. *Georgetown Law Journal*, 94:1847–1908, 2006.

[67] Cataltepe Zehra and Prat Moghe. Characterizing nature and location of congestion on the public internet. In *Proceedings of the eighth IEEE international Symposium on Computers and Communication (ISCC)*, 2003.