## Centralized and distributed power allocation in multi-user wireless relay networks

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# Centralized and Distributed Power Allocation in Multi-User Wireless Relay Networks

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*Abstract*—**Optimal power allocation for multi-user amplify-and-forward wireless relay networks in which multiple source-destination pairs are assisted by a set of relays is investigated.[1] Two relay power allocation strategies based on maximization of either i) the minimum rate among all users or ii) the weighted sum of rates are developed. A distributed implementation of the maximum weighted-sum-rate power allocation strategy is also studied. Numerical results demonstrate the efficiency of the proposed strategies and reveal their interesting throughput-fairness tradeoff in resource allocation.**

*Index Terms–* **Power allocation, relay networks, cooperative diversity, throughput and fairness tradeoff.**

## I. INTRODUCTION

Recently, it has been shown that performance of cellular and/or ad-hoc networks can be improved significantly by exploiting cooperative diversity [1]–[7]. Although various relay models have been studied, the simple two-hop relay model has attracted extensive research attention [1]–[6]. It has been noticed that besides smart cooperative diversity protocol engineering, radio resource management via power allocation also has profound impacts on performance of wireless networks [2]-[6]. In [3], the authors derive closed-form expressions for the optimal and near-optimal relay transmission powers for single and multiple relay scenarios, respectively. In [4], by using either signal-to-noise ratio (SNR) or outage probability as the performance criteria, different power allocation strategies are developed for three-node amplify-and-forward (AF) relay systems by exploiting the knowledge of mean channel gains. Bandwidth allocation problem for a three-node Gaussian orthogonal relay network is investigated in [5] which aims at maximizing a lower bound of capacity. In the case when channel state information (CSI) of wireless links or channel statistics is available, [6] proposes two power allocation schemes to minimize the outage probability. A cross-layer optimization framework, i.e., congestion control, routing, relay selection and power allocation via dual decomposition for multihop networks using cooperative diversity is proposed in [7].

Most of these existing works, however, consider *single-user* scenarios which neglect and simplify many important

[1]Hereafter, the term 'user' refers to a source-destination pair or only the source node depending on the context.

network-wide aspects of cooperative diversity. In this paper, we consider power allocation problems for a more general *multi-user* AF relay network. Each relay is usually delegated to assist more than one users, especially when the number of relays is (much) smaller than the number of users. A typical example of such scenarios is the deployment of few relays in a cellular network to assist mobile users located at the cell edges for both uplink and downlink transmissions. In such scenarios, it is clear that the aforementioned resource allocation schemes for *single-user* relay network cannot be directly applied. Therefore, extending the resource allocation framework to the multi-user relay network presents an open and interesting problem which is the focus of this paper.

This paper considers resource allocation problems for multi-user AF relay networks to maximize either i) the minimum rate of all users (max-min fairness) or ii) the weighted sum of rates (weighted-sum fairness). We show that the corresponding optimization problems are convex; therefore, their optimal power allocation solutions can be obtained by any available convex program algorithms. To reduce overhead involved in a centralized implementation, we propose distributed implementation for the power allocation problem which requires each receiver to collect/estimate CSI for the corresponding transmitter-relay and relay-receiver links. Due to its fully-distributed nature, the proposed distributed algorithm can reduce communications overhead and it can be used in infrastructureless wireless networks such as sensor and ad hoc networks. Besides differences in system modeling and optimization, the main new contribution compared to that in [8] is the derivation and corresponding results on the distributed implementation of the weighted-sum rates maximization. This is an important result which shows that power allocation in large-scale networks is feasible.

## II. SYSTEM MODEL AND ASSUMPTIONS

Consider a multi-user AF relay networkwhere $M$ source nodes $S_i$, $i \in \{1, ...M\}$ transmit data to their corresponding destination nodes $D_i$, $i \in \{1, ...M\}$. There are $L$ relay nodes $R_j$, $j \in \{1, ..., L\}$ which are employed for forwarding the information from source to destination nodes. The set of relays assisting the transmission of $S_i$ is denoted by $\mathcal{R}(S_i)$. The set of sources using the $R_j$ relay is denoted by $\mathcal{S}(R_j)$, i.e., $\mathcal{S}(R_j) = \{S_i \mid R_j \in \mathcal{R}(S_i)\}$. Essentially, one particular relay can forward data to several users. This is a realistic assumption

for wireless systems given the number of relays is usually smaller than the number of users.

We assume *orthogonal transmission* for all users[2] and AF relay, e.g., by time division duplexing (TDD) as follows [1], [6]. Each source $S_i$ transmits data to its chosen relays in the set $\mathcal{R}(S_i)$ in the first time interval and each relay amplifies and forwards its received signal to $D_i$ in the second time interval. Note that the investigated system model can be used in a large variety of applications. Let $P_{S_i}$ denote the power transmitted by $S_i$. The power transmitted by the relay $R_j \in \mathcal{R}(S_i)$ for assisting the source $S_i$ is denoted by $P_{R_j}^{S_i}$. For simplicity, we present the signal model for link $S_i$-$D_i$ only. In the first time interval, source $S_i$ broadcasts the signal $x_i$ with unit energy to the relays $R_j \in \mathcal{R}(S_i)$. The received signal at relay $R_j$ can be written as

$$r_{R_j}^{S_i} = \sqrt{P_{S_i}} a_{R_j}^{S_i} x_i + n_{R_j}, \quad R_j \in \mathcal{R}(S_i) \tag{1}$$

where $a_{R_j}^{S_i}$ denotes the channel gain for link $S_i$-$R_j$, $n_{R_j}$ is the additive circularly symmetric white Gaussian noise (AWGN) at the relay $R_j$ with variance $N_{R_j}$. The channel gain includes the effects of path loss, shadowing and fading. In the second time interval, relay $R_j$ amplifies its received signal and retransmits it to the destination node $D_i$. After some manipulations, the received signal at the destination node $D_i$ can be written as

$$r_{R_j}^{D_i} = \sqrt{\frac{P_{R_j}^{S_i} P_{S_i}}{P_{S_i} |a_{R_j}^{S_i}|^2 + N_{R_j}}} a_{R_j}^{D_i} a_{R_j}^{S_i} x_i + \hat{n}_{D_i}, \quad R_j \in \mathcal{R}(S_i) \tag{2}$$

where $a_{R_j}^{D_i}$ is the channel gain for link $R_j$-$D_i$, $n_{D_i}$ is the AWGN at the destination node $D_i$ with variance $N_{D_i}$, $\hat{n}_{D_i}$ is the modified AWGN noise at $D_i$ with equivalent variance $N_{D_i} + \left( P_{R_j}^{S_i} |a_{R_j}^{D_i}|^2 N_{R_j} \right) / \left( P_{S_i} |a_{R_j}^{S_i}|^2 + N_{R_j} \right)$. Assuming that maximum-ratio-combining is employed at the destination node $D_i$, the SNR of the combined signal at the destination node $D_i$ can be written as [6]

$$\gamma_i = \sum_{R_j \in \mathcal{R}(S_i)} \frac{P_{R_j}^{S_i}}{\alpha_{R_j}^{S_i} P_{R_j}^{S_i} + \beta_{R_j}^{S_i}} \tag{3}$$

where

$$\alpha_{R_j}^{S_i} = \frac{N_{R_j}}{|a_{R_j}^{S_i}|^2 P_{S_i}}, \quad \beta_{R_j}^{S_i} = \frac{N_{D_i} N_{R_j}}{|a_{R_j}^{S_i}|^2 |a_{R_j}^{D_i}|^2 P_{S_i}} + \frac{N_{D_i}}{|a_{R_j}^{D_i}|^2}. \tag{4}$$

It can be shown that the rate $r_i$ for user $i$ defined as $r_i = \log(1 + \gamma_i)$ is concave increasing with respect to (w.r.t.) $P_{R_j}^{S_i}$, $R_j \in \mathcal{R}(S_i)$ which enables us to calculate optimal power allocation using convex optimization techniques.

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[2]For example, by using Time Division Multiple Access (TDMA) or Orthogonal Frequency-Division Multiple-Access (OFDMA), or Orthogonal Code-Division Multiple-Access (OCDMA).

## III. PROBLEM FORMULATIONS AND CENTRALIZED POWER ALLOCATION

### A. Max-Min Rate Based Power Allocation

Given rate $r_i$ of user $i$, the power allocation problem under max-min rate can be mathematically formulated as

$$\max_{P_{R_j}^{S_i} \geq 0} \quad \min_{S_i} r_i \tag{5a}$$

$$\text{subject to:} \sum_{S_i \in \mathcal{S}(R_j)} P_{R_j}^{S_i} \leq P_{R_j}^{\max}, \; j = 1, \ldots, L \tag{5b}$$

where $P_{R_j}^{\max}$ is the maximum power at relay $R_j$. The left-hand side of (5b) is the total power that $R_j$ allocates to its assisted users which is constrained to be less than its maximum power budget. The constraint (5b) on the maximum transmit power can be rewritten equivalently as a constraint on the maximum sum of powers transmitted by the corresponding assisted source nodes. This constraint is required to avoid overloading relays in the network. It can be seen that the set of linear inequality constraints with positive variables in (5a)–(5b) is compact and nonempty. Hence, (5a)–(5b) is always feasible. Moreover, since the objective function $\min_{S_i} r_i$ is increasing function of allocated powers, the inequality constraints (5b) should be met with equality at optimality. Introducing a new variable $t$, the optimization problem (5a)–(5b) can be equivalently rewrite in a standard form as

$$\min_{P_{R_j}^{S_i} \geq 0, \, t \geq 0} \quad -t \tag{6a}$$

$$\text{subject to:} \quad t - r_i \leq 0, \; i = 1, \ldots, M \tag{6b}$$

$$\sum_{S_i \in \mathcal{S}(R_j)} P_{R_j}^{S_i} \leq P_{R_j}^{\max}, \; j = 1, \ldots, L. \tag{6c}$$

It can be shown that the optimization problem (6a)–(6c) is convex so its optimal solution can be calculated using any standard algorithms [9]. In the special case where all users share the same set of relays, we have the following result.

PROPOSITION 1: If all users are assisted by all relays, the rates of all users are equal at optimality.

*Proof:* Suppose that there is at least one user achieving the rate strictly larger than the minimum rate at optimality. Without loss of generality, let $\Omega$ be the set of users achieving minimum rate and suppose that user $l$ has rate larger than that of any user $i \in \Omega$ at optimality. Note that there exists at least one relay $j$ which has nonzero allocated power $P_{R_j}^{S_l} > 0$ at optimality. If we take an arbitrarily small amount of power $\Delta P$ from $P_{R_j}^{S_l}$ and allocate an amount of power equal to $\Delta P / |\Omega|$ to each user $k \in \Omega$ where $|\Omega|$ denotes the cardinality of set $\Omega$, then the resulting rate of user $l$ is still larger than the minimum rate of all users while we can improve the minimum rates for all users in $\Omega$. This is a contradiction to the optimality condition. Hence, the proposition is proved. ∎

### B. Weighted-Sum Rate Maximization Based Power Allocation

Note that the max-min rate based power allocation tends to improve performance of the worst user at the cost of total network throughput degradation. The weighted-sum rate maximization can potentially achieve certain fairness for different

users by allocating large weights to users in unfavorable channel conditions while maintaining good network performance. Moreover, this objective also captures the scenarios in which QoS differentiation has to be performed for users. Let $w_i$ denote the weight allocated to user $i$, the weighted-sum rate power allocation problem can be formally posed as

$$\max_{P_{R_j}^{S_i} \geq 0} \quad \sum_{i=1}^{M} w_i r_i \tag{7a}$$

$$\text{subject to:} \quad \sum_{S_i \in \mathcal{S}(R_j)} P_{R_j}^{S_i} \leq P_{R_j}^{\max}, \ j = 1, \ldots, L. \tag{7b}$$

It can be observed that this optimization problem is also convex and the weighted-sum rate based power allocation does not severely penalize users with bad channel conditions and favor users with good channel conditions. In fact, because the rate $r_i$ for a particular user $i$ is concave increasing w.r.t. allocated powers, that is, the increment in rate is lower when the power is higher. Therefore, instead of allocating more power to "good" users, the optimization problem would allocate power to "bad" users at low SNR to make better improvement in the objective function. Note that the centralized implementation for these power allocation problems may incur high overhead. This is because all network parameters (i.e., channels gains, source powers, receiver noise, user weights) need to be forwarded to a central control point to calculate the optimal power allocation solution which is then disseminated to the corresponding relays.

## IV. DISTRIBUTED IMPLEMENTATION

To reduce communication overhead due to the centralized implementation of the problem under consideration, we propose a distributed algorithm for the problem (7a)–(7b).

### A. Dual Decomposition Approach

The main idea behind dual decomposition theory is to separate the original problem into independent subproblems that are coordinated by a higher-level master dual problem. Toward this end, we first write the Lagrangian function by relaxing the total power constraints for the relays as

$$\mathcal{L}\left(\boldsymbol{\mu}, P_{R_j}^{S_i}\right) = \sum_{i=1}^{M} w_i r_i - \sum_{j=1}^{L} \mu_j \left( \sum_{S_i \in \mathcal{S}(R_j)} P_{R_j}^{S_i} - P_{R_j}^{\max} \right) \tag{8}$$

where $\boldsymbol{\mu} = \mu_j \geq 0$, $j = 1, \ldots, L$ are the Lagrange multipliers corresponding to the $L$ linear constraints on the relay power. Using the fact that

$$\sum_{j=1}^{L} \mu_j \sum_{S_i \in \mathcal{S}(R_j)} P_{R_j}^{S_i} = \sum_{i=1}^{M} \sum_{R_j \in \mathcal{R}(S_i)} \mu_j P_{R_j}^{S_i} \tag{9}$$

the Lagrangian in (8) can be rewritten as

$$\mathcal{L}\left(\boldsymbol{\mu}, P_{R_j}^{S_i}\right) = \sum_{i=1}^{M} \left[ w_i r_i - \sum_{R_j \in \mathcal{R}(S_i)} \mu_j P_{R_j}^{S_i} \right] + \sum_{j=1}^{L} \mu_j P_{R_j}^{\max}. \tag{10}$$

The corresponding dual function of the Lagrangian can be written as

$$g(\boldsymbol{\mu}) = \max_{P_{R_j}^{S_i} \geq 0} \mathcal{L}\left(\boldsymbol{\mu}, P_{R_j}^{S_i}\right). \tag{11}$$

Since the original optimization is convex, strong duality holds, the solution of the underlying optimization problem can be obtained from the corresponding dual problem as

$$\min \quad g(\boldsymbol{\mu}) \tag{12a}$$

$$\text{subject to:} \quad \mu_j \geq 0, \ j = 1, \ldots, L. \tag{12b}$$

The dual function in (11) can be found by solving $M$ separate subproblems corresponding to $M$ different users as

$$\max \ \mathcal{L}_i(\boldsymbol{\mu}, P_{R_j}^{S_i}) = w_i r_i - \sum_{R_j \in \mathcal{R}(S_i)} \mu_j P_{R_j}^{S_i} \tag{13a}$$

$$\text{subject to:} \ P_{R_j}^{S_i} \geq 0, \quad R_j \in \mathcal{R}(S_i) \tag{13b}$$

where $\mathcal{L}_i(\boldsymbol{\mu}, P_{R_j}^{S_i})$ corresponds to the $i$th component of the Lagrangian. Let $\mathcal{L}_i^*(\boldsymbol{\mu})$ be the optimal value of $\mathcal{L}_i(\boldsymbol{\mu}, P_{R_j}^{S_i})$ obtained by solving (13a)–(13b). Then, the dual problem in (12a)–(12b) can be rewritten as

$$\min \quad g(\boldsymbol{\mu}) = \sum_{i=1}^{M} \mathcal{L}_i^*(\boldsymbol{\mu}) + \sum_{j=1}^{L} \mu_j P_{R_j}^{\max} \tag{14a}$$

$$\text{subject to:} \quad \mu_j \geq 0, \ j = 1, \ldots, L. \tag{14b}$$

The distributed power allocation algorithm aims at solving sequentially (13a)–(13b) and (14a)–(14b). It is known as a primal-dual algorithm in optimization theory. The Lagrange multiplier $\mu_j \geq 0$ represents the pricing coefficient for the unit power at relay $j$. Therefore, $\mu_j P_{R_j}^{S_i}$ can be seen as the price which user $i$ must pay for using $P_{R_j}^{S_i}$ at each relay $R_j \in \mathcal{R}(S_i)$. According to the optimization problem (13a)–(13b), user $i$ tries to maximize its (weighted) rate minus the total price that it has to pay given the price coefficients at relays.

### B. Algorithm Implementation

The distributed algorithm requires message passing only between each receiver and its assisting relays. Therefore, it allows each user to calculate the relay powers at its receiver. Since the dual function $g(\boldsymbol{\mu})$ is differentiable, the master dual problem (12a)–(12b) can be solved using the gradient method. The dual decomposition presented in (13a)–(13b) allows each user $i$ to find the optimal relay power $R_j \in \mathcal{R}(S_i)$, for the given $\mu_j$, as

$$P_{R_j}^{S_i}(\boldsymbol{\mu})\Big|_{\text{opt}} = \arg \max \left\{ w_i r_i - \sum_{R_j \in \mathcal{R}(S_i)} \mu_j P_{R_j}^{S_i} \right\} \tag{15}$$

which is unique due to the strict concavity. The individual allocated power can be found in closed-form but the formulas are omitted here for brevity. Due to the fact that the solution in (15) is unique, the dual function $g(\boldsymbol{\mu})$ in the master problem (12a)–(12b) is differentiable which allows us to use the following iterative gradient method to update the dual variables

$$\mu_j(t+1) = \left[ \mu_j(t) - \zeta \left( P_{R_j}^{\max} - \sum_{S_i \in \mathcal{S}(R_j)} P_{R_j}^{S_i}(\boldsymbol{\mu}(t))\Big|_{\text{opt}} \right) \right]^+ \tag{16}$$

where $t$ is the iteration index, $[\cdot]^+$ denotes the projection onto the feasible set of non-negative numbers, and $\zeta$ is the
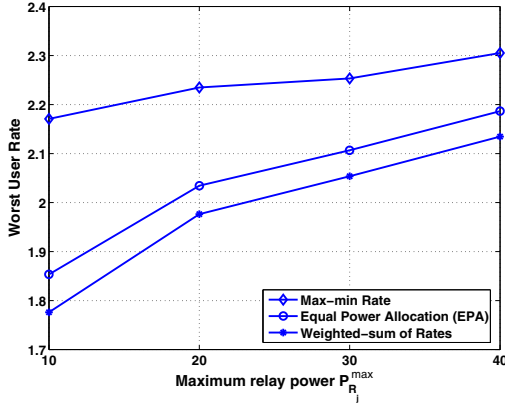
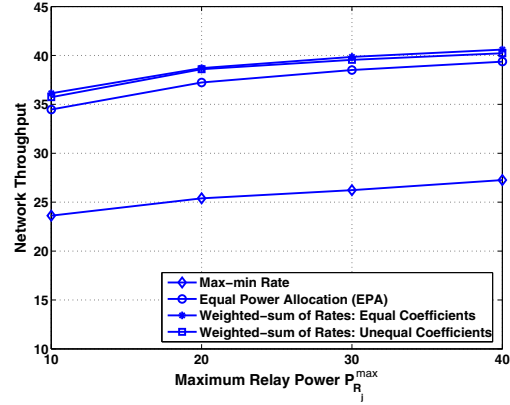Fig. 1. Worst user rate versus $P_{R_j}^{\max}$.



Fig. 2. Network throughput versus $P_{R_j}^{\max}$.

sufficiently small positive step size. Updating $\mu_j(t)$ via (16) can be seen as the relay $R_j$ updates its price depending on the requested levels from its users. The price is increased when the total requested power resource is larger than its maximum limit. Otherwise, the price is decreased. This so-called 'price-based' allocation is also used in wired networks for rate/congestion control in Internet [10]. The distributed power allocation algorithm can be summarized as follows.

**Distributed Power Allocation Algorithm**

- *Parameters:* the receiver of each user estimates/collects its weighted coefficient $w_i$ and channel gains of its transmitter-relay and relay-receiver links.
- *Initialization:* set $t = 0$, each relay $j$ initializes $\mu_j(0)$ equal to some nonnegative value and broadcasts the value.
1) The receiver of user $i$ solves its problem (15) and then broadcast the solution $P_{R_j}^{S_i}(\boldsymbol{\mu}(t))\big|_{\text{opt}}$ to its relays.
2) Each relay $R_j$ receives the requested power levels and updates its prices with the gradient iteration (16) using the information received from receivers of its assisted users. Then it broadcasts the new value $\mu_j(t+1)$.
3) Set $t = t+1$ and go the step 1 until satisfying the stopping criterion.

The convergence proof of the general primal-dual algorithm can be found in [9].

## V. NUMERICAL RESULTS

Consider a wireless relay network with ten users and three relays distributed in a two-dimensional region $14m \times 14m$. The relays are fixed at coordinates $(10,7)$, $(10,10)$, and $(10,12)$. The source and destination nodes are deployed randomly in the area inside the box area $[(0,0), (7,14)]$ and $[(12,0), (14,14)]$, respectively. Each user is assisted by two randomly selected relays. All relays are assumed to have the same maximum transmit power $P_{R_j}^{\max}$. Locations of the source and destination nodes are fixed and each source node has the same transmitted power normalized to 1. The AWGN variance is assumed to be $10^{-5}$. The channel gain[3] for each transmission link consists

of two components: path loss proportional to $d^{-2}$, where $d$ is the path length, and Rayleigh fading with variance 1. The results are averaged over 800 channel instances.

Fig. 1 shows the data rate of the worst user(s) versus relay maximum transmit power $P_{R_j}^{\max}$ for the proposed allocation schemes: max-min rate, and weighted-sum rate with equal weight coefficients. The equal power allocation (EPA) scheme, in which each relay distributes power equally among all relayed sources, is also included as reference. It can be seen that the worst user obtains the best rate under the max-min rate based scheme and the worst rate under the weighted-sum rate based scheme with equal weight coefficients. Over the wide range of maximum relay powers, the best rate offered by the max-min rate based scheme has much smaller variation (about 0.12 b/s/Hz) than the worst rate achieved by the weighted-sum rate based scheme (with variation of about 0.35 b/s/Hz). In other words, as expected, the weighted-sum rate based power allocation scheme can introduce unfairness in terms of the achievable rate of the worst user, especially when relays have low power limits. Moreover, it can be seen that with large power available at the relays, i.e., larger $P_{R_j}^{\max}$, all three schemes provide better performance for the worst users, and thus, for all users.

Fig. 2 shows the network throughput for the aforementioned power allocation schemes. One can see a significant loss in the network throughput for the max-min rate fairness based power allocation scheme. It is because the objective is to improve the performance of the worst users. This confirms that achieving the max-min among users results in a performance loss for the whole system. The weighted-sum rate based scheme results in maximum throughput. Its rate gain as compared to the EPA scheme is about 1.8 b/s/Hz over the range of the relay power limits. This gain comes at the cost of higher system implementation complexity needed to optimize the power levels. The weighted-sum rate based scheme with unequal weights[4] achieves slightly worse performance as compared to its counterpart with equal weights while providing better

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[3]Instantaneous channel fading gains are assumed to be known and not varied during the time required to compute the solutions.

[4]We set the weights as $w_1 = w_2 = 5$, $w_3 = \ldots = w_{10} = 1$ in the optimization problem (7a)–(7b)
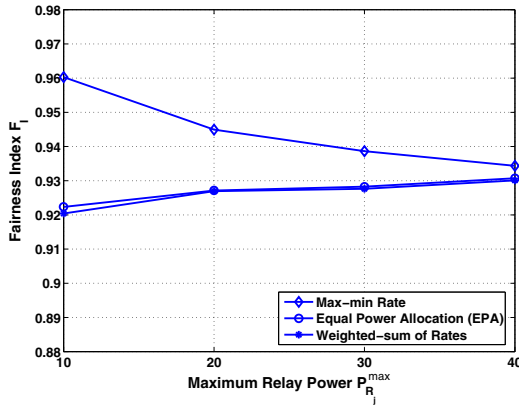
Fig. 3. Fairness index versus $P_{R_j}^{\max}$.



Fig. 5. Evolution of data rate for each user and user sum rate.
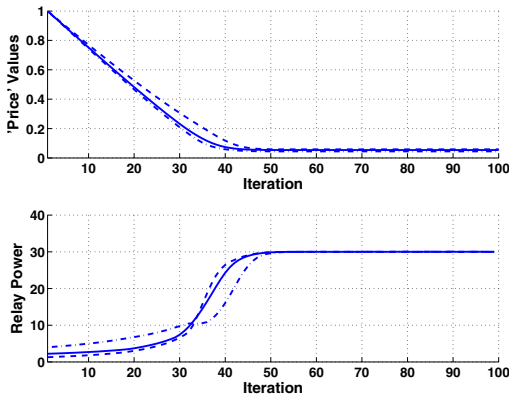


Fig. 4. Evolution of 'price' values and power allocated at each relay.

performance for the high priority users which is not shown here.

We also study the fairness behavior of the aforementioned power allocation schemes in terms of the fairness index which is calculated as $F_I = \left(\sum_{i=1}^{M} r_i\right)^2 / \left(M \sum_{i=1}^{M} r_i^2\right)$ [11]. Specifically, we plot the fairness index versus $P_{R_j}^{\max}$ in Fig. 3. It can be seen that the fairness index is closer to 1 when the power allocation or, equivalently, rate allocation becomes fairer. Clearly, the max-min rate based scheme achieves the best fairness for all users and the weight-sum rate based scheme offers the worst fairness. Note that Figs. 1, 2, and 3 can be used for designing trade-offs between throughput and fairness.

Figs. 4 and 5 show the evolution of different parameters of the proposed distributed algorithm for a specific channel realization. Particularly, Fig. 4 shows the evolution of the price values $\mu_j$, $j = 1, 2, 3$ and powers at each relay. Fig. 5 displays the rates for all ten users and the sum rate of all users. For a update parameter $\zeta = 0.001$, the algorithm converges to the optimal solution obtained by solving the optimization problem centrally after about 50 updates. Typically, it is hard to determine the optimal value of $\zeta$ and small $\zeta$ is usually preferred to ensure convergence.
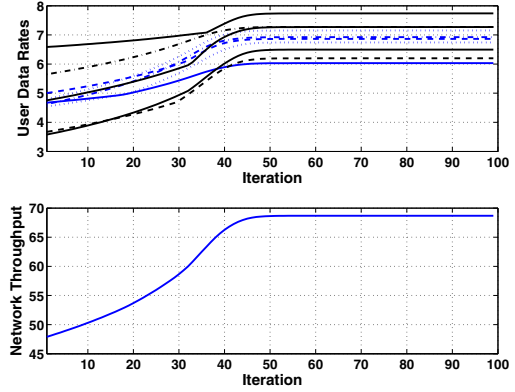
## VI. CONCLUSIONS

In this paper, optimal power allocation schemes under different fairness objectives have been proposed for wireless multi-user AF relay networks. The proposed solutions are based on convex programming, and therefore, are computationally efficient. In particular, the power allocation schemes to i) maximize the minimum rate among all users; ii) maximize the weighted sum of rates have been proposed. For the latter case, the distributed algorithm has been also developed using the dual decomposition approach. Numerical results demonstrate efficiency of the proposed power allocation schemes and reveal interesting tradeoff between throughput and fairness for different schemes.

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