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# ADAPTIVE-RATE TECHNIQUES FOR FREQUENCY-HOP MULTIPLE-ACCESS PACKET-RADIO NETWORKS

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**Abstract**—The effects of adaptive-rate transmissions and routing on the total throughput of a slow-frequency-hop packet-radio network are considered. Adaptive rates are achieved through the use of error-control coding with perfect side information. Both fixed-length codes and variable-length codes are considered. Performance results are obtained for direct transmission (i.e., no routing), two-hop limited routing, and full routing. Each link metric is a function of the amount of interference caused by using the link. We compare the total network throughput for each of these schemes with both fixed-rate and adaptive-rate coding.

## I. INTRODUCTION

Although ad-hoc networks have been the focus of much research over the past few decades, effectively implementing such networks has proven to be difficult. In contrast to the problem of efficiently using a communication link, which has a clearly-defined Shannon capacity and relatively few design parameters, efficient use of an ad-hoc network has little theoretical guidance and a very large set of interrelated parameters over which to optimize. In addition, most of the theoretical results on optimal performance rely on schemes with high levels of coordination among the nodes, but for some networks, such coordination is infeasible or requires a prohibitive amount of overhead.

We refer to a fully-connected network that employs direct transmission with a fixed code rate as our baseline network, and we consider the effects of changing each of these components; e.g., employing multihop routing in lieu of direct transmission. Instead of evaluating the network performance as the number of users approaches infinity, we emphasize the throughput increases or network size increases that are afforded by changes to our baseline network. This enables us to evaluate how, for example, routing or adaptive coding can improve the performance of ad hoc networks of practical sizes. In fact, a long-standing problem in fully-connected packet radio networks has been whether to transmit directly to the intended receiver or employ multi-hop routing to reach the receiver [1]. In this paper, we briefly review the work that has been done in this area and present new results on the tradeoffs between routing and direct transmission when adaptive-rate links are employed by each transmitter-receiver pair in a slotted, random-access frequency-hop packet-radio network.

There has been a large amount of prior work in the area of ad hoc networks. In [1], spatial reuse in general packet radio networks is investigated. The authors consider

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the relationship between the network topology and routing to find the optimum transmission range for a given traffic load. The conclusion is that to achieve a high throughput, smaller transmission ranges should be used as the network's offered load increases. A practical problem with this approach is that there may be no node to which the packet can be forwarded if a small transmission range is used. The optimum transmission probability is shown to be  $1/d$ , where  $d$  is the expected number of nodes within range of the transmitter. It is remarked that the nodes located near the center of the network are busiest because many routes travel through them. Note that in [2], the authors acknowledge that the center of the network may contain such routing "hot spots," but they nevertheless conclude that throughput is ultimately limited by collisions that happen throughout the network. In [1], the optimal transmission range is stated to be at a level such that six to eight nodes (as in, six is the "magic number" [3]) are within the transmitter's range. The authors also remark that for frequency-hop networks, the power levels of interferers are less important than the number of interferers.

In [4], an optimal fixed transmission range to maximize efficiency is derived, where "efficiency" is defined to be the forward progress of a packet divided by the average area covered by the transmission. Adaptive transmission radii are also considered for increasing efficiency. In this case, the way to maximize efficiency is to choose a radius just large enough to reach the nearest node, which is, in effect, perfect power control. The efficiency can be increased by 85% with adaptive radii.

The authors of [5] consider frequency-hopping networks in which multiple-access hits are treated as interference, i.e., there is no cooperation. The authors state that for synchronous hopping, hits are independent (but not so for asynchronous hopping). Both synchronous and asynchronous hopping are considered, and with each type of hopping, the performance of receivers with and without side information is evaluated. The capacity of such a network is evaluated, and it is shown that there is a certain number of users that maximizes the total network throughput. Moreover, for a transmission probability of unity, perfect knowledge of hits, and no spatial reuse, the optimum number of users approaches the number of frequency channels. Of interest to us is that this implies that the network throughput has a maximum value if routing is not used (whether or not adaptive coding is used), and as the number of users continues to increase, the throughput decreases.

In contrast, it is shown in [2] that the network throughput for routing and optimal scheduling is a nondecreasing function of the number of users. Arbitrary and random networks are

analyzed for both a protocol interference model and a physical interference model. The results are for a network-wide fixed power and fixed rate with perfect scheduling. The authors derive the capacity of the networks they consider, and it is shown that while the network throughput can be large for a large number of users, the throughput of each individual user goes to zero. The authors also consider the addition of pure relay nodes to the network. Such nodes are neither traffic sources or destinations, but they can relay packets for other users in the hopes of allowing smaller transmission ranges and thus larger throughputs. An interesting result is that the number of relay nodes needs to be very large to provide any significant help. For most practical-sized networks of interest, this number is prohibitively large. Insights into the fundamental tradeoffs between range and spatial reuse (i.e., whether to transmit directly to the intended receiver or employ multihop routing to reach the receiver [1]) are provided.

In [6], the optimum transmission parameters (code rate, ALOHA transmission probability, range) to be used by each node of the network are found. Frequency hopping with MFSK under Rayleigh fading is considered. A generalization of Hajek's "efficiency" measure, called "information efficiency," is introduced. Information efficiency is similar to efficiency, but information efficiency additionally considers the amount of information that is conveyed by a transmission, so the spectral efficiencies of different modulations and code rates become a factor.

Adaptive transmission on a single frequency-hop link with partial-band interference is considered in [7], and we employ similar methods for code rate adaptation. In [8], both routing and adaptive-rate coding are considered for a four-node network with partial-band interference. The performance of frequency-hop networks with fixed-rate error-control coding is considered in [9]. We build upon the work in these papers by considering the network-wide performance of combinations of adaptive coding and routing when the primary disturbance is multiple-access interference.

## II. SYSTEM MODEL

We consider a fully-connected slow-frequency-hop packet-radio wireless network. One modulation symbol is transmitted in each dwell interval, but our results also apply to systems with multiple symbols per dwell interval if each symbol in the dwell interval is a member of a different code word or packet [10]. For our results, we assume that each packet contains a single code word. We consider fixed-rate coding, in which each node in the network employs the same fixed code rate, as well as adaptive-rate coding, in which each transmitter-receiver pair can employ a different rate. With adaptive coding, high-rate codes are generally used when there is little multiple-access interference, and lower-rate codes are generally employed to add redundancy when there is a large amount of interference. We employ maximum-distance separable coding, such as Reed-Solomon coding, along with the capability to detect multiple-access hits. In general, the presence of additive white Gaussian

noise (AWGN) can also cause undetected errors when the received signal power is insufficient; however, we assume that our network is interference-limited. Reed-Solomon codes of length  $N$  with  $K$  information symbols have successful decoding if the number of erasures plus twice the number of errors is no more than  $N - K$ .

Slotted Aloha is used for channel access, so scheduling is not considered. With Aloha, multiple nodes can transmit at the same time, which may cause interference at the intended receivers. However, due to the multiple-access capability of frequency-hopping [11] and propagation losses that cause some interference to be weak relative to the packet of interest, some packets can be correctly received even in the presence of other transmissions. Let  $\alpha$  be the probability of an inactive radio deciding to turn on and  $\beta$  be the probability of an active radio deciding to turn off at the end of its code word. The values of  $\alpha$  and  $\beta$  are chosen such that a radio will have a new packet to send approximately 50% of the time.

Nodes are uniformly distributed over a disc-shaped region in the same manner as described in [12], and nodes are not mobile. Each source radio randomly chooses one other radio as its intended receiver. Because the network we consider is fully connected, direct transmissions between the source and the intended destination are possible. However, we also consider two types of multi-hop routing. The first type uses distance vector techniques for route selection. Link metrics are based on the least-interference metric (LIR) [13], which provides a measure of the amount of interference *caused* to the network by employing the link. Here, we use link costs equal to the number of radios in range of the transmitter, so, in general, low-cost routes are those that employ nodes with fewer neighbors so that fewer radios of the network experience interference. Routes found by the distance vector method may contain many hops, which adds delay to the packet. Thus, as an alternative, we consider routes that are limited to at most two hops. Two-hop routing is an attempt at gaining some of the benefits of routing while limiting delay. With this second type, the route with at most two hops that has the lowest cost with the LIR metric is suggested. Note that for both types of routing, some routes are necessarily one-hop; i.e., direct transmission is still employed between some source-destination pairs.

We consider networks using both slot-synchronous transmissions and asynchronous transmissions. For our purposes, a slot-synchronous system is packet-synchronous, which means that packet transmissions can begin only at slot boundaries. For such a system, we assume that each packet is the same length, so variable-rate coding is implemented by adjusting the number of information symbols contained in each packet. For asynchronous networks, on the other hand, transmissions can occur at any time and packets are allowed to have variable length. It is important to note that asynchronous refers to the packets being asynchronous, and that modulation symbol intervals are still modeled as being synchronous. The number of information symbols in each packet is constant, so rates are varied by changing the packet length. Such a scheme mimics

the behavior of rateless coding (e.g., [14]), and, more generally, incremental redundancy, such as hybrid ARQ [15].

We assume the use of perfect power control, which allows the source to transmit at a power level that is just large enough to reach the intended destination with sufficient margin that only multiple-access interference causes symbol loss. It is assumed that each radio knows the path loss to every other radio. Reception in the presence of interference is modeled using the power-based capture considered in [12]. A symbol is erased at the receiver if the total interference power in the *same* FH channel exceeds the received power of the transmission of interest. The power hopping (PH) method of [12] is employed for some of the results in this paper. When PH is used, a power increase of 6 dB is used for the hop with probability 0.5, and the choice of whether to employ a power increase is made independently from hop to hop.

### III. ADAPTIVE CODING TECHNIQUES

Ideal adaptive coding takes on different forms depending on whether the system employs fixed-length transmissions or variable-length transmissions, as well as whether multi-hop routing is in use. For a system employing fixed-length transmissions without routing, the maximum achievable rate is evaluated by counting the number of symbols of the packet that are not hit. When multihop routing is employed, the entire route is considered, and the rate is assumed to be limited by the link that suffers the most hits. Specifically, if the packet length is  $N$ , and link  $i$  in a multi-hop route will have  $m_i$  hits, then the ideal rate for the route is assumed to be  $1 - \max_i \{m_i\} / N$ . Note that this idealized implementation requires future knowledge of the effects of multiple-access interference on each of the hops of the route.

For a system using variable length code words and direct transmission, the maximum rate is based on the number of code symbols that must be transmitted for the receiver to be able to decode. At least  $K$  symbols must be transmitted per code word, since there are  $K$  information symbols per code word. The granularity of each transmission for the ideal system is a single symbol, so individual symbols are transmitted until the packet can be decoded. Feedback is assumed to be perfect and immediate for the ideal case.

We also consider adaptive coding that has no such ideal measurements and information. Instead, the practical adaptive coding that is investigated relies on the number (or fraction) of erasures in the previous code word to decide on the code rate for the next transmission. This approach is known as the E-method in [7], where it was considered for the partial-band interference channel. Similar to the ideal case, the system with fixed-length code words varies the number of information symbols to adapt the rate, and the system with variable-length code words has a fixed number of information symbols, so the rate is varied by adjusting the code word length. However, for the variable-length implementation, a predefined set of lengths are used for adaptation, so incremental redundancy is not used.

In our simulations of practical adaptive coding, the system has  $n_c$  codes,  $C_1, C_2, \dots, C_{n_c}$ , available to it, in order of

increasing rate. The choice of which code to employ is based on the statistic  $\zeta$  that is recorded at the receiver. For our results, we employ the erasure count from the previous reception. It is also possible to form a statistic that is a function of the erasure counts from the previous several transmissions. The statistic is used in an interval test based on  $n_c + 1$  strictly decreasing thresholds  $\gamma_i$ ,  $0 \leq i \leq n_c$ . If  $\gamma_i \geq \zeta > \gamma_{i+1}$ , then  $C_{i+1}$  is chosen. The thresholds  $\gamma_0 = N$  and  $\gamma_{n_c} = -\infty$  are always employed.

For the results presented here, the routing metric does not depend on the link rate, in contrast to the approach in [8] used under the partial-band interference model. Instead, for all routing results, the LIR metric is used. In the future, we intend to investigate alternative routing metrics, such as those that take into account the link rate.

### IV. FIXED-LENGTH CODE WORDS

The source nodes can exploit the fact that the network is fully connected by directly transmitting to their respective destinations. Thus, each transmission can be completed with a single hop. Both fixed-rate (i.e.,  $r = 1/2$ ) error-control coding as well as adaptive-rate error-control coding are considered. For the results in this section, the block length of all error-control codes is the same, so code words of higher-rate codes contain more information symbols than do code words of lower-rate codes.

Assume there are  $q$  channels,  $k$  frequency-hopping interferers within range, and code word length  $N$ . Each user transmits with probability  $p_t$ . It is possible that more than  $k$  other users are transmitting in the network, but lower power or more distant transmitters may not have an effect on some parts of the network due to spatial reuse. The probability of any frequency slot being hit is

$$p_h(k) = 1 - \left(1 - \frac{p_t}{q}\right)^k. \quad (1)$$

If each node uses an error-control code of rate  $r$ , then, with a sufficiently long block length, a correct reception is likely to occur in the presence of  $k$  interferers if  $1 - p_h(k) \leq r$ , and an error occurs otherwise. This expression becomes more accurate as the block length of the error-control code approaches infinity. We define the function

$$C(k) = \begin{cases} 1, & 1 - p_h(k) \leq r \\ 0, & \text{otherwise} \end{cases}$$

to indicate whether a correct packet reception occurs when there are  $k$  interferers. An approximation of the total network throughput with  $k$  interferers (and thus  $k + 1$  total users) is

$$T_1(k) = (k + 1) p_t r C(k). \quad (2)$$

Another more accurate approximation for the total network throughput suitable for finite block lengths is based on the probability distribution of the number of hits rather than the expected value of the number of hits. The probability of  $i$  hits

given that there are  $k$  active interferers within range is

$$\Pr(i|k) = \binom{N}{i} p_h(k)^i (1 - p_h(k))^{N-i}. \quad (3)$$

A correct reception occurs with probability

$$P_c(k) = \sum_{i=0}^{N-K} \Pr(i|k),$$

and the corresponding total network throughput is

$$T_2(k) = (k + 1) p_t r P_c(k). \quad (4)$$

Now consider the use of an ideal adaptive coding system that transmits at the maximum rate at which successful decoding will result each time, which requires knowledge of the number of hits that are to be experienced during the next transmission. For most networks, this is impractical, but it provides an upper bound on performance.

The expected value of the rate when  $k$  interferers are present can be found using equation (1) as  $\hat{r}(k) = 1 - p_h(k)$ , and the resulting throughput for the network is

$$T_3(k) = (k + 1) p_t \hat{r}(k). \quad (5)$$

This is described in [5] as the capacity of a slot-synchronous frequency-hopping system with side information. In [5], it is also pointed out that  $T_3(k)$  has a maximum value as a function of  $k$ ; that is, the total network throughput is maximized for some finite number of users. The same is true for  $T_1(k)$  and  $T_2(k)$ . This implies that when routing is not employed, total throughput is not a strictly increasing function of the number of users. However, code adaptation may increase the value of  $k$  for which the maximum throughput occurs. In addition, based on these approximations, the value of the maximum throughput with adaptive coding is at least as large as for fixed-rate coding.

For our evaluations of practical adaptive coding with fixed-length code words,  $n_c = 6$ . The code word length is  $N = 32$ . Let  $K_i$  be the number of information symbols of code  $C_i$ . Then,  $K_1 = 4$ ,  $K_2 = 8$ ,  $K_3 = 12$ ,  $K_4 = 16$ ,  $K_5 = 20$ , and  $K_6 = 26$ . The interval thresholds are  $\gamma_1 = 19$ ,  $\gamma_2 = 15$ ,  $\gamma_3 = 11$ ,  $\gamma_4 = 7$ , and  $\gamma_5 = 2$ .

Results are given in Figure 1 for different variations on the baseline network with  $q = 20$  channels. Direct transmission with a fixed rate 1/2 code ( $N = 32, K = 16$ ) provides the largest throughput when the number of radios is between 18 and 40. Direct transmission with the erasure-count adaptive coding scheme has slightly larger throughput when the number of radios is less than 18, because there is less interference and higher rates can be used. The throughput of the fixed-rate direct transmission scheme quickly decreases as the network size exceeds 40 radios. Two-hop routing provides the largest throughput for networks having between 40 and 70 radios. For larger networks, it appears that full (i.e., distance vector) routing provides the largest total throughput. Note that for the results in this paper, the choice of routing metric parameters was not extensively studied. In a companion paper [16], additional routing alternatives for this network are considered.

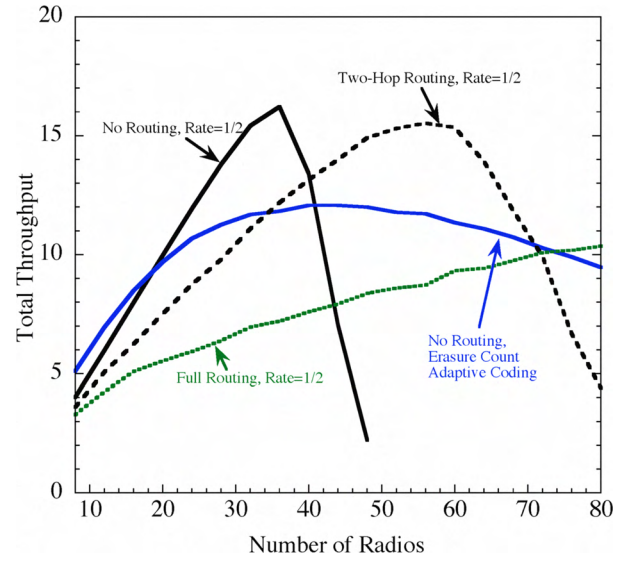


Fig. 1. Performance of systems employing fixed-length code words.

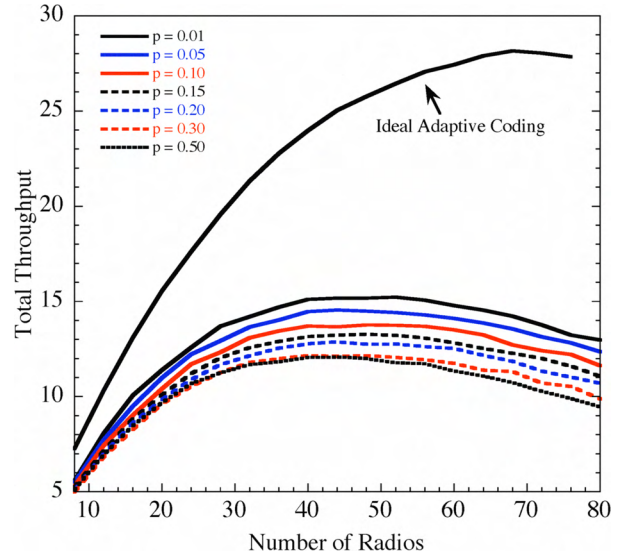


Fig. 2. Performance of practical adaptive coding for different probabilities of transmitters turning on and off.

Consider a slot-synchronous system without routing, in which a decision on whether to transmit is made every  $N$  symbols (i.e., at each slot boundary). With probability  $1 - \beta$  there is no gap between successive transmissions. For  $n \geq 1$ , an inactive period has length  $Nn$  with probability

$$\Pr(Nn) = (1 - \alpha)^{n-1} \alpha. \quad (6)$$

The expected length of an inactive period is thus

$$E[\text{off}] = \sum_{n \geq 1} Nn \Pr(Nn) \quad (7)$$

$$= N\alpha \sum_{n \geq 1} n(1 - \alpha)^{n-1} \quad (8)$$

$$= \frac{N\alpha}{1 - \alpha} \frac{1 - \alpha}{\alpha^2} = N/\alpha. \quad (9)$$

The probability that the total length of a transmission is  $nN$  modulation symbol durations (and thus comprises  $n$  packets),  $n \geq 1$ , is

$$\Pr(Nn) = (1 - \beta)^{n-1}\beta. \quad (10)$$

The expected number of modulation symbol durations that a radio is on is thus  $N/\beta$ . Because we employ  $\alpha = \beta$ , the probability of a radio being on is  $1/2$ . Note that this is independent of the number of users in the network.

The performance of adaptive coding with the erasure count is shown in Figure 2 for different transmission probabilities. The values of  $p$  shown in the figure are such that  $\alpha = \beta = p$ . Note that the probability that a radio is on is  $1/2$  in each case. However, as  $p$  approaches zero, the state of each radio tends to stay the same from slot to slot. Notice that the network throughput for adaptive coding with the erasure count increases as  $p$  decreases. This is because the network activity tends to stay the same from slot to slot, which makes the erasure count a more accurate predictor of the interference to be experienced in the next slot. Also shown is the ideal throughput of the network, which has exact knowledge of the highest code rate that can be employed by each radio in the next slot. The performance of the erasure count is ultimately limited by the randomness of the number of hits from slot to slot, which depends on the hopping patterns in use by each radio. As  $p$  changes, the adaptive coding thresholds are constant for all the results in Figure 2, but if the thresholds were optimized for each specific value of  $\alpha$  and  $\beta$ , then the margin between the throughputs of the practical adaptive protocol and the ideal adaptive protocol would decrease. In addition, employing longer code words not only provides better code word performance, but the resulting erasure count statistics are also more accurate. For the remaining results on fixed-length codes in this paper,  $\alpha = \beta = 1/2$ , so networks with less-dynamic transmission patterns would generally have better performance with adaptive coding than what is presented here.

## V. VARIABLE-LENGTH CODE WORDS

The use of variable-length code words is of interest for some applications. In such a scheme, the number of information symbols per packet is a constant  $K$ , and the number of transmitted code symbols,  $N_v$ , varies. Thus, a high rate is achieved if  $N_v$  is only slightly larger than  $K$  and the packet is successfully received. Multihop routing with adaptive coding is more easily implemented if  $K$  is fixed, since, for example, the information symbols do not have to be fragmented and reassembled as code rates change from hop to hop.

If a system can transmit and acknowledge as few as  $Y$  symbols at a time, we say that the system has a resolution of  $Y$ . For example, rateless coding over a physical layer link allows for a transmission granularity as small as a single symbol. If acknowledgments can be done on a symbol-to-symbol basis, then the overall system has a resolution of one. A more general incremental redundancy scheme transmits multiple redundancy symbols as a group and receives a single acknowledgment for

the entire group. Such a system has a resolution larger than one.

As in previous sections, we refer to a system as ideal when the system has perfect future knowledge of the channel. Consider a system that employs ideal adaptive coding with variable-length code words and a resolution of one. For a rate of unity to be achieved,  $N_v$  must be equal to  $K$ . Let  $\Pr(N_v)$  be the probability of successful reception after exactly  $N_v$  symbols have been received,  $N_v \geq K$ . Then,  $\Pr(N_v) = \Pr(r = K/N_v)$ , where  $r$  is the rate. If the probability of a hit is  $p_h$ , then the probability of achieving a rate of unity is  $\Pr(K) = (1 - p_h)^K$ . In general, the expression for  $\Pr(N_v)$ ,  $N_v \geq K$ , is given by

$$\Pr(N_v) = \binom{N_v - 1}{K - 1} (1 - p_h)^{K-1} p_h^{N_v - K} (1 - p_h), \quad (11)$$

which is the probability that only  $K - 1$  of the first  $N_v - 1$  transmitted symbols are successfully received, and symbol  $N_v$  is correct. Let  $N'$  be the expected value of  $N_v$ . For purposes of the present analysis, we will assume that each code word is of length  $N'$ .

For the results presented in this section, the number of information symbols is fixed at 16 for all code rates and all schemes. Because the packet lengths are variable, we consider an unslotted system in which packets from different radios can start and end at different times. The individual modulation symbols, however, are assumed to be synchronous. PH is used for all the results in this section.

For such an unslotted system, an inactive radio decides with probability  $\alpha$  to turn on at each symbol start time. An active radio remains active until the code word has finished transmission, at which point it can decide with probability  $\beta$  to turn off. With probability  $1 - \beta$ , there is no gap between successive transmissions. For  $n > 1$ , there is an idle interval of size  $n$  symbol durations with probability  $\Pr(n) = (1 - \alpha)^{n-1}\alpha$ . The expected length in modulation symbols of the transmission gap is thus

$$E[\text{off}] = \sum_{n \geq 1} n \Pr(n) \quad (12)$$

$$= \alpha \sum_{n \geq 1} n (1 - \alpha)^{n-1} \quad (13)$$

$$= \frac{\alpha}{1 - \alpha} \frac{1 - \alpha}{\alpha^2} = 1/\alpha. \quad (14)$$

For our results,  $\beta = 1/2$  and  $\alpha = 1/64$ , so  $E[\text{off}] = 64$ . The probability that the length of a transmission containing multiple length  $N'$  code words is  $nN'$  modulation symbol durations is

$$\Pr(N'n) = (1 - \beta)^{n-1}\beta. \quad (15)$$

The expected number of modulation symbol durations that a radio is on is thus  $N'/\beta$ . As stated previously,  $\beta = 1/2$ , so  $E[\text{on}] = 2N'$ . In contrast to the system employing fixed code word lengths of  $N$ , the length  $N_v$  is a random variable. This is because the expected length per code word varies with the amount of traffic in the network, which means that the probability of a transmitter being active is also dependent on

the amount of traffic. Thus,  $N'$  depends on the number of network users.

Numerical results are presented for three coding schemes with  $K = 16$ . Ideal adaptive coding transmits information symbols until 16 symbols are successfully received. This is an idealization of the code rate selection process, and it also provides a wide variety of code rates because a resolution of one is employed. However, rateless codes (on a per-link basis) can provide performance very close to what is shown for ideal adaptive coding, if overhead for ACKs is ignored. Fixed-rate coding uses a perfect erasure code of rate 1/2. The third scheme is a practical adaptive coding scheme which uses the erasure count from previous transmissions to select the rate for the next transmission. The selection process for this practical adaptive coding scheme is similar to that presented in the previous section. The number of available codes is  $n_c = 4$ , and  $K_i = 16$  for each value of  $i$ . Let  $N_i$  denote the length of code words when code  $C_i$  is used. Then,  $N_1 = 24$ ,  $N_2 = 32$ ,  $N_3 = 48$ , and  $N_4 = 64$ . Because code word lengths are allowed to vary, the statistic in use is  $\xi$ , the *normalized* erasure count. The nominal code word length is defined to be  $N_2$ . If the length of the previous code word was  $M$  and there were  $E$  erasures, then  $\xi = N_2 E / M$ . It is straightforward to define  $\xi$  based on functions of erasure counts of several previous code words. The thresholds are  $\gamma_1 = 19$ ,  $\gamma_2 = 12$ , and  $\gamma_3 = 6$ .

Results are presented in Figure 3 for direct transmission with an adaptive coding protocol using the erasure count. The performance of rate-1/2 coding with direct transmission, two-hop routing, and full routing is also illustrated. For a small number of users, both direct transmission methods perform well. As the number of radios increases, the erasure count adaptive protocol continues to provide high throughput even as the fixed-rate system's throughput quickly falls to zero. The erasure count adaptive protocol even outperforms two-hop routing up to approximately 65 radios. Because code word lengths are allowed to increase, the number of erasures in each code word approaches the average, so the measure of channel quality provided by the erasure count becomes more accurate. Also, long code words of rate  $r$  are more reliable than shorter ones. For networks exceeding 80 radios, full routing provides the best performance.

Figure 4 provides an illustration of the performance improvements that can be obtained by combining routing with erasure count adaptive coding. In general, the adaptive coding protocol increases the throughput of both two-hop routing and full routing. Adaptive coding causes a slight decrease in throughput for two-hop routing for networks containing between 40 and 65 nodes, but it improves performance otherwise (i.e., throughput does not undergo a drop-off).

Figure 5 includes results on the upper bounds of performance when ideal adaptive coding is used with direct transmission, two-hop routing, and full routing. For reference, the performance of the erasure count adaptive protocol is also provided. These results suggest that improved adaptive coding schemes could further increase throughput.

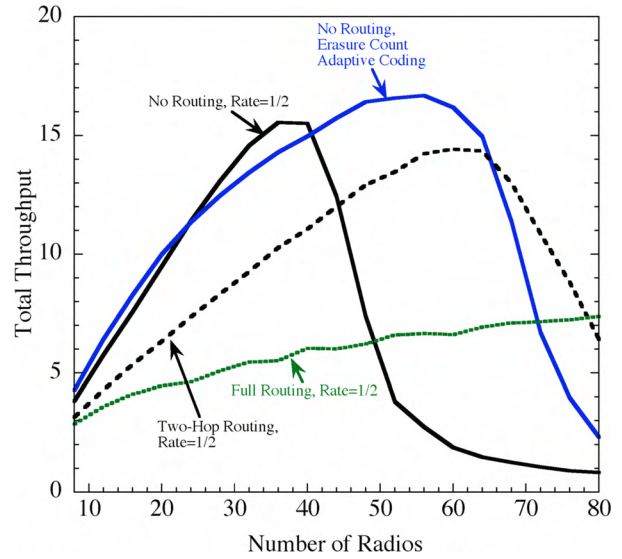


Fig. 3. Performance of variable-length code words.

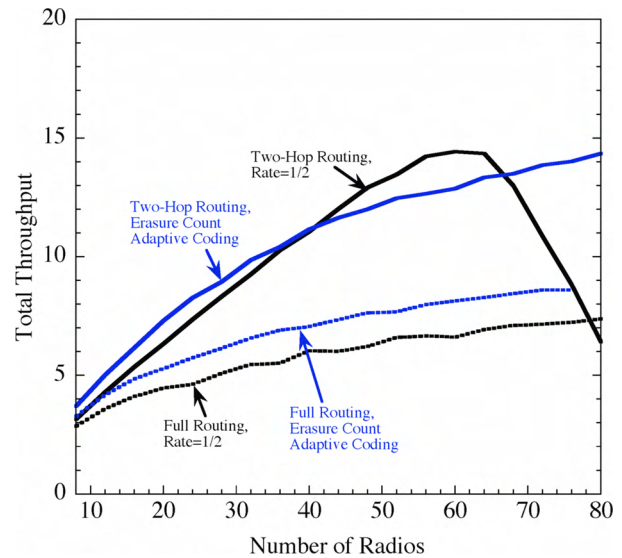


Fig. 4. Performance of practical adaptive coding with variable-length code words.

## VI. DISCUSSION OF RESULTS

Consider a network employing direct transmission with fixed-rate coding. If only a few transmitters are active, all transmissions are successful because few hits occur and the code is able to correct the erasures. As more transmitters become active, the channel becomes more crowded until it can no longer support the transmissions. If adaptive-rate coding is employed in the same network, the channel can be used much more efficiently, so the total throughput is larger and more users can be supported. However, as more users enter the network, it is inevitable that adaptive-rate coding will eventually suffer from crowding, and the network throughput approaches zero.

A network that employs routing with power control and fixed-rate coding is expected to have a strictly increasing throughput as the number of users increases [2], and our

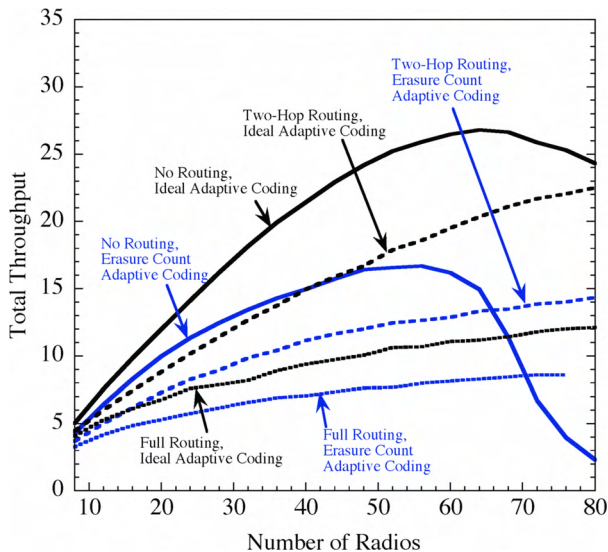


Fig. 5. Performance of ideal and practical adaptive coding with variable-length code words.

results are consistent with this. Routing allows lower values of transmitter power, which reduces interference to the total network. As more users become active, routing hops can become shorter and transmitter power can be further decreased.

The combination of power control, routing, and adaptive coding provides the best performance as the number of users becomes large. This scheme not only enjoys the reduction in interference due to short hops, but it also benefits from the efficiency of adaptive-rate techniques.

Two-hop routing is expected to have a maximum throughput for a particular number of users (because of fundamental limitations derived in [5]), but the maximum number of users is larger than the maximum number of users for direct transmission and no routing. Essentially, the power reductions afforded by two-hop routing results in support for a larger number of users than when routing is not used at all.

## VII. CONCLUSION

We have considered the use of adaptive-rate coding, two-hop routing, and full routing as alternatives to improving throughput or increasing the supportable number of users in frequency-hop packet-radio networks. Adaptive coding or two-hop routing can be used individually as relatively simple means to enable small to moderate sized networks to support additional users compared with direct transmission and fixed-rate coding. If these increases in network size are sufficient for the application, then more complex schemes need not be considered. If even more users need to be supported, then combinations of adaptive coding and routing should be employed.

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