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Peak-to-Average Power Ratio (PAR) Reduction for Acoustic OFDM Systems

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Abstract—A major drawback of orthogonal frequency division multiplexing (OFDM) is its large peak-to-average power ratio (PAR). Techniques for PAR reduction have been extensively studied for radio communication systems. While these techniques are applicable to acoustic systems, we take a different approach that aims to capitalize on the fundamental differences between the acoustic and radio systems, namely the fact that acoustic transmissions are inherently band-limited. We extend the tone reservation technique to the out-of-band carriers, and design efficient methods for constructing OFDM signals with lower PAR. Two approaches are investigated, one based on a gradient algorithm, and another that uses random sequences. Simulation results show that our technique can provide PAR reduction without the loss in data rate.

Index Terms—Peak-to-average power ratio, PAR, underwater communications, OFDM, tone reservation, out-of-band tone insertion.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been considered for underwater communications as a low-complexity alternative to single carrier broadband modulation. It has proven to be an effective technique to combat multipath dispersion without the need of complex time-domain equalizers [1], [2].

One of the major drawbacks of OFDM modulation is its high peak-to-average power ratio (PAR). Large PARs occur when symbol phases line up so as to constructively form peaks in the time-domain signal. Since the peak transmission power is limited, either by regulatory or hardware constraints, the average power must be reduced, leading to a loss in performance relative to the constant amplitude modulation techniques.

Furthermore, an OFDM signal is very sensitive to non-linear distortion, which causes spectral growth of the multicarrier signal in the form of intermodulation products among subcarriers. Non-linearity is present in the power amplifier at transmitter, and to avoid it, the signal should be kept within the linear region. However, this may require a large back-off, calling for a compromise between the average power and a distortion that can be tolerated.

Digital signal processing techniques can be applied to reduce the high amplitude peaks. PAR reduction techniques have been extensively studied for radio systems (a good overview can be found in [3]), but there have been no solutions developed specifically for acoustic systems.

In this work we address a PAR reduction technique suited exclusively to underwater acoustic communication systems. The technique is based on inserting a set of tones outside of the nominal transducer bandwidth, so as to cancel the high peaks in the time-domain signal. The out-of-band tones are inserted before the signal is D/A converted and amplified. These tones will subsequently be removed by filtering before transmission. Filtering occurs naturally in the transducer, although additional (analog) filtering is also possible at low frequencies used in typical acoustic systems.¹

The paper is organized as follows. In Sec. II, the PAR problem is defined. An overview of existing PAR techniques is offered in Section III. The differences between acoustic and radio systems are highlighted in Sec. IV. Sec. V presents the the out-of-band tone insertion (OTI) technique, and outlines different algorithms for its efficient implementation. Simulation results are presented in Sec. VI. Sec. VII concludes the paper.

II. PROBLEM STATEMENT

A multicarrier signal is the sum of many independent signals modulated onto carriers of equal bandwidth. In the case of OFDM, these carriers are orthogonal, with spacing $\Delta f = 1/T$, where T is the OFDM symbol duration. The complex baseband representation of an OFDM signal consisting of K carriers is given by

$$x(t) = \sum_{k=0}^{K-1} d_k e^{j2\pi k \Delta f t}, t \in [0, T] \quad (1)$$

The PAR is defined as the ratio between the maximal power and the average power,

$$PAR = \frac{\max_{0 \leq t < T} |x(t)|^2}{\frac{1}{T} \int_0^T |x(t)|^2 dt} \quad (2)$$

Since the signal is generated digitally, the PAR can be computed using discrete-time values $x_l = x(lT_s)$, $l = 0, \dots, N_s$. To accurately account for all the amplitude values, it is necessary to oversample the signal. An oversampling factor

¹Acoustic spectrum regulation does not impose explicit requirements on sidelobe suppression.

$L = N_s/K = 4$ is considered to be sufficient, since the error due to sampling can be bounded by [4]

$$\left| \max_t |x(t)| - \max_l |x_l| \right| \leq K [\cos^{-1}(\pi/2L) - 1] \quad (3)$$

An OFDM signal is a random process, and its PAR is commonly characterized by the complementary cumulative distribution function (CCDF). The CCDF is defined as the probability that PAR exceeds a certain threshold, $\Pr(\text{PAR} > \text{PAR}_0)$. We will use this metric when we discuss the performance of various PAR reduction techniques.

III. OVERVIEW OF EXISTING TECHNIQUES

Over the last decades, a number of techniques have been developed for PAR reduction in OFDM radio systems. Below, we briefly summarize the basic principles used in some of these techniques.

Clipping and filtering. This is the simplest technique, in which the signal amplitude is clipped to a predetermined value [5], [6]. The distortion caused by clipping is seen as both in-band and out-of-band noise. The latter can be filtered out, but this may cause some peak regrowth, i.e. the signal after clipping and filtering may exceed the clipping level at some point. Repeated clipping-and-filtering iterations reduce the regrowth effect. However, the in-band distortion cannot be reduced by filtering, resulting in performance degradation.

Coding. Coding can also be used to reduce the PAR of an OFDM signal [7]. For each data block, a codeword with minimal PAR is selected. This approach requires an exhaustive search and storage of codewords in a large lookup table.

Interleaving. Scrambling with a set of interleavers is another technique for PAR reduction [8], [9]. An interleaver simply permutes the data symbols in a given block. The information in the permuted data block is the same as in the original, but the resulting waveform in time is different, and may exhibit a lower peak amplitude. Thus, if several interleavers are used, the one that yields the lowest PAR will be chosen. The corresponding signal will be transmitted along with the interleaver's code. The amount of PAR reduction depends on the number of interleavers, but so does the overhead needed to transmit the side information.

Selected Mapping. In this technique, the phases of data symbols are altered in order to avoid the alignment that produces high amplitude peaks [10]. The transmitter uses P sets of K phases $\varphi_{k,p} \in [0, 2\pi]$, to generate candidate signals

$$x_p(t) = \sum_{k=0}^{K-1} d_k e^{j\varphi_{k,p}} e^{j2\pi k\Delta f t}, p = 1, \dots, P \quad (4)$$

The signal with the lowest PAR is then selected for transmission. This technique also requires transmission of side information to indicate which phase set was used.

Partial Transmitted Sequences. This technique is also based on phase manipulations [11]. An input data block of K symbols is first partitioned into M disjoint sub-blocks, and the carriers of each sub-block are then weighted by phase

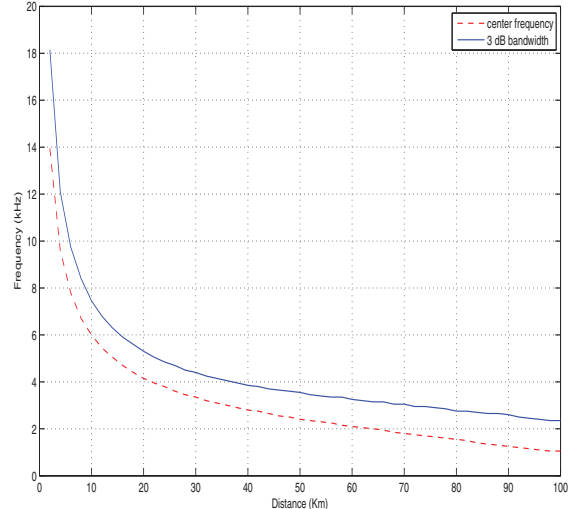


Fig. 1. Optimal center frequency and the 3 dB acoustic bandwidth as functions of distance [14].

factors. The phase factors are selected such that the PAR of the resulting signal is minimized. The selection of phases is limited to sets with finite number of elements to keep the search complexity manageable.

Active Constellation. In this technique, outer constellation points in the data block are dynamically extended further out such that the resulting signal has a lower PAR than the original [12]. Note that unlike with the previous techniques, this technique requires additional power to regulate the PAR.

Tone Reservation. In this technique, a set of carriers is reserved for control tones which are inserted among the carriers so as to reduce the time-domain peaks [13]. Since the carriers are orthogonal, these additional tones do not cause distortion to the data-bearing carriers. In wireline systems, those carriers whose SNR is too low for reliable information transmission are used for PAR reduction. In wireless systems, it is more difficult to identify the low SNR carriers, since the channel is time-varying, and limited or no feedback may be available from the receiver. Therefore, a set of carriers must be reserved a-priori, resulting in a reduction of the useful information bandwidth. The performance of this scheme depends on the number of control tones and their allocation.

IV. ACOUSTIC VS. RADIO SYSTEMS

The major difference between acoustic and radio systems is in the frequency band that they occupy. Acoustic propagation occurs at frequencies that are much lower than those used for typical radio communications, as illustrated in Fig. 1. The bandwidth is fundamentally limited by absorption, but also by the transducer technology, which imposes strict additional limitations.

It is also interesting to note that unlike radio spectrum, acoustic spectrum usage is not legally regulated. This is not to say that one should create interference to neighboring acoustic

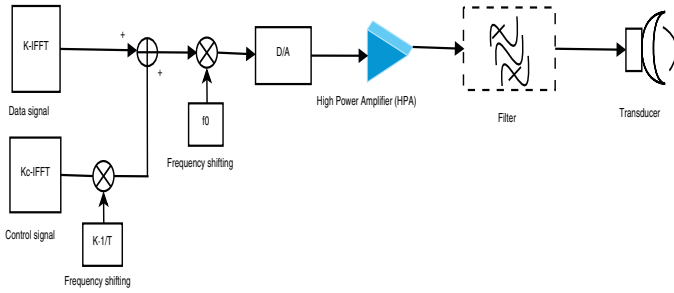


Fig. 2. Block diagram of the transmitter using out-of-band tone insertion.

systems, but simply that acoustic emissions outside of the nominal bandwidth are left to the designer's best effort. When designing a system, it is also important to keep in mind that higher acoustic frequencies attenuate faster with distance, and, hence, the interference spectrum measured at the transmitter will not be the same as that measured at the receiver.

V. OUT-OF-BAND TONE INSERTION (OTI) TECHNIQUE

The proposed technique is based on adding a data-block-dependent control signal to the original multicarrier signal. The control signal is outside of the useful bandwidth, and is given by

$$y(t) = \sum_{k=0}^{K_c-1} c_k e^{j2\pi(K+k)\Delta f t}, t \in [0, T] \quad (5)$$

where K_c is the number of control tones. The control tones are here placed immediately above the useful bandwidth, but other arrangements are possible as well. The coefficients c_k are chosen so as to reduce the PAR at the input to the non-linear amplifier. The inserted tones are removed after amplification, either by the transducer alone, since it has a limited bandwidth, or by explicit filtering, as illustrated in Fig.2. An efficient implementation of the post-amplifier (analog) filter is deemed possible at frequencies used in typical acoustic communication systems.

The main advantage of the OTI technique is that no side information needs to be transmitted, and, hence, there is no trade-off between the data rate loss and the PAR reduction capability.

Although filtering is applied before transmission, there is some amount of power lost in amplifying the inserted signal. Therefore, it is important to maintain the number of out-band inserted tones as low as possible while aiming for a certain PAR reduction.

While the set of reserved tones is chosen in advance, the coefficients c_k are selected depending on the data vector to be transmitted. These coefficients can be chosen optimally (to minimize the PAR) but the computational demands of optimization are high. We will thus investigate other approaches that are sub-optimal, but offer manageable complexity.

Specifically, we want to answer the following questions:

- 1) Where should the tones be, so that they provide the best performance for all the signals?
- 2) How many tones are needed in order to achieve a certain improvement?
- 3) Given a properly chosen number and placement of the inserted tones, how can we efficiently compute the coefficients c_k ?

A. OTI optimal formulation

Mathematically, we can formulate the problem as

$$\min_{\mathbf{c}} \max_{l=0, \dots, N_s-1} |x_l(\mathbf{d}) + y_l(\mathbf{c})| \quad (6)$$

where x_l and y_l are the samples of the information-bearing signal (1) and the control signal (5), which depend on the vector of data symbols \mathbf{d} and the vector of control coefficients \mathbf{c} , respectively. The samples of the control signal can also be grouped into a vector,

$$\mathbf{y} = \Phi \mathbf{c} \quad (7)$$

where Φ is the matrix of $N_s \times K_c$ FFT coefficients

$$\phi_{l,k} = e^{j2\pi l(K+k)/N_s}, l = 0, \dots, N_s-1, k = 0, \dots, K_c-1 \quad (8)$$

Denoting by Φ_l the l -th row of Φ , the optimization problem can be expressed as

$$\min_{\mathbf{c}} \max_{l=0, \dots, N_s-1} |x_l + \Phi_l \mathbf{c}| \quad (9)$$

Fortunately, this proves to be a convex problem, which can be solved numerically using quadratically constrained quadratic programming (QCQP) [13]. Namely, since minimizing an absolute value $|a|$ is the same as minimizing its square $p = |a|^2 = aa^*$, the problem can be re-formulated as

$$\min_{\mathbf{c}} p, \text{ subject to } [x_l + \Phi_l \mathbf{c}][x_l + \Phi_l \mathbf{c}]^* \leq p, \quad (10)$$

$$\text{for } l = 0, \dots, N_s - 1$$

This formulation involves minimization of a linear function over a set of quadratic constraints, which is a convex problem [13]. In what follows, we will refer to this solution as the optimal solution, and use it as a benchmark to compare the performance of other techniques whose computational demands are conducive to practical implementation. Two such techniques are discussed below.

B. Gradient technique

This technique substitutes the (optimal) criterion of PAR minimization by the (suboptimal) minimum mean squared error (MMSE) criterion applied to the clipping noise. A gradient algorithm is then applied to solve the MMSE optimization in a fast and computationally efficient manner.

To arrive at this algorithm, let us first define the passband signals

$$\begin{aligned} \tilde{x}(t) &= \text{Re}\{x(t)e^{j2\pi f_0 t}\} \\ \tilde{y}(t) &= \text{Re}\{y(t)e^{j2\pi f_0 t}\} \end{aligned} \quad (11)$$

as well as the composite signal $\tilde{z}(t) = \tilde{x}(t) + \tilde{y}(t)$, which is input to the power amplifier. The amplifier non-linearity is modeled as

$$\bar{z}(t) = g[\tilde{z}(t)] = \begin{cases} \tilde{z}(t), & |\tilde{z}(t)| \leq A \\ A, & \tilde{z}(t) > A \\ -A, & \tilde{z}(t) < -A \end{cases} \quad (12)$$

The resulting error, i.e. the clipping noise, is given by

$$\tilde{e}(t) = \tilde{z}(t) - \bar{z}(t) \quad (13)$$

and the corresponding MSE is defined as

$$D = \int_0^T |\tilde{e}(t)|^2 dt \quad (14)$$

Taking the derivative of D with respect to the control coefficients c_k , we obtain

$$\frac{\partial D}{\partial c_k} = 2 \int_0^T \tilde{e}(t) \frac{\partial \tilde{e}(t)}{\partial c_k} dt \quad (15)$$

The integration interval can be split into two complementary parts: \mathcal{T} , in which $|\tilde{z}(t)| \leq A$, and $\bar{\mathcal{T}}$, in which clipping occurs. Since the error is zero in the first part, only the second part will contribute to the MSE. In that part, the error is given by $\tilde{e}(t) = \tilde{z}(t) \pm A$, and, hence,

$$\frac{\partial \tilde{e}(t)}{\partial c_k} = \frac{\partial \tilde{e}(t)}{\partial \tilde{z}(t)} \frac{\partial \tilde{z}(t)}{\partial c_k} = \frac{\partial \tilde{z}(t)}{\partial c_k} \quad (16)$$

The remaining term is obtained as

$$\frac{\partial \tilde{z}(t)}{\partial c_k} = \frac{1}{2} \phi_k^*(t) e^{-j2\pi f_0 t} \quad (17)$$

where

$$\phi_k(t) = e^{j2\pi(K+k\Delta f)t} \quad (18)$$

We thus finally have the MSE gradient,

$$\frac{\partial D}{\partial c_k} = \int_0^T \tilde{e}(t) \phi_k^*(t) e^{-j2\pi f_0 t} dt \quad (19)$$

In this expression, we have switched the integration bounds from $\bar{\mathcal{T}}$ back to the original ones, as this does not affect the result since $\tilde{e}(t) = 0$ outside of $\bar{\mathcal{T}}$.

Further simplification of the above expression is also possible if we express the passband error as

$$\tilde{e}(t) = \text{Re}\{e(t)e^{j2\pi f_0 t}\} \quad (20)$$

where $e(t)$ is the complex equivalent evaluated with respect to f_0 . Substituting this expression into the gradient (19), the high-frequency terms at $2f_0$ vanish under integration, leaving

$$\frac{\partial D}{\partial c_k} = \frac{1}{2} \int_0^T e(t) \phi_k^*(t) dt \quad (21)$$

The complex envelope $e(t)$ can be related to an equivalent baseband non-linearity, described by the AM/AM and AM/PM characteristic g_0 corresponding to the nonlinearity g , such that $e(t) = z(t) - g_0[z(t)]$. For the hard limiter model (12), this function is given by [15]

$$\bar{z}(t) = g_0[z(t)] = \begin{cases} z(t), & |z(t)| \leq A \\ \frac{4}{\pi} A e^{j \arg[z(t)]}, & |z(t)| > A \end{cases} \quad (22)$$

Once the gradient (21) is known, the least mean squares (LMS) algorithm can be applied to calculate the coefficients c_k . The gradient will be calculated in discrete time, giving way to the coefficient update

$$c_k(i+1) = c_k(i) - \mu \sum_{l=0}^{N_s-1} e_l(i) \phi_{l,k}^* \quad (23)$$

where μ is the step size, $e_l(i)$ is the clipping error in the i -th iteration, and $\phi_{l,k}$ are as given in the expression (8). Using the notation of Sec.V-A, the vector update is given by

$$\mathbf{c}(i+1) = \mathbf{c}(i) - \mu \sum_{l=0}^{N_s-1} e_l(i) \mathbf{\Phi}'_l \quad (24)$$

Instead of generating the control signal from the coefficients after the algorithm has converged, the signal itself can be updated [13]. The LMS algorithm will then operate in the time domain, directly generating the signal vector $\mathbf{z} = \mathbf{x} + \mathbf{y}$. The corresponding update equation is obtained by multiplying both sides of the expression (24) by the FFT matrix $\mathbf{\Phi}$, and adding the information bearing signal:

$$\mathbf{z}(i+1) = \mathbf{z}(i) - \mu \sum_{l=0}^{N_s-1} e_l(i) \mathbf{\Phi} \mathbf{\Phi}'_l \quad (25)$$

Since the error depends only on the signal, $e_l(i) = z_l(i) - \bar{z}_l(i)$, the control coefficients never need to be computed explicitly. The algorithm is initialized by $\mathbf{z}(0) = \mathbf{x}$. The vectors $\mathbf{\Phi} \mathbf{\Phi}'_l$, $l = 0, \dots, N_s - 1$, can be pre-computed and stored, which accounts for the very low computational complexity of the algorithm. The LMS convergence time, nominally $20N_s$ iterations, amounts to $80K$ iterations with the oversampling ratio of 4. These iterations need to be completed within the duration of one block, $T = K/B$, in order for a real time implementation to be possible. If we take as an example a 160 MHz processor, this will be possible so long as $160 \text{ MHz}/80B = 2/B[\text{MHz}]$ is greater than the number of instructions required per iteration. As we shall see from the simulation results, it suffices to perform only a few iterations.

C. Random insertion

In this technique, the out-of-band tones are generated from a finite modulation alphabet, which can be the same as that of the information-bearing signal, or different. Thus, there is a finite number of possible selections for the control sequence, but this number may be large (M^{K_c} for the modulation level M), making it impractical to conduct an exhaustive search.

Instead of performing a systematic search, the selection is made from a finite set of randomly generated control sequences. The search for the best sequence is conducted until a certain improvement in the PAR is reached, or until a pre-determined number of trials have been exhausted (after which the best candidate sequence is retained).

Random tone insertion aims to reduce the implementation complexity by sacrificing some improvement in the PAR reduction capability. This technique is similar to interleaving

[9] in that the transmitter only needs a random generator for the out-of-band tones and a module that computes the PAR. Note, however, that the two techniques are conceptually different, and can even be combined.

Two specific questions are to be addressed with this technique. The first question refers to the size of the control sequence alphabet. The greater the alphabet size, the better the PAR reduction (the optimal solution described in Sec.V-A can in fact be regarded as a modulation with an infinite constellation size). However, an increase in the modulation alphabet implies a greater number of candidate sequences, which complicates the search. The second question refers to the number of trials needed to achieve a certain performance. Obviously, the more trials, the better, but we would like to know how much can the performance be improved with a relatively small number of trials, suitable for a practical implementation. These questions will be addressed in the following section.

VI. SIMULATIONS RESULTS

A simulation analysis was conducted for an OFDM signal with 512 carriers employing QPSK in the 8-28 kHz band. A total of 10,000 randomly generated OFDM blocks were used to assess the performance of the proposed OTI techniques. The results are contrasted with the original signals' statistics (no PAR reduction method employed) and the optimal case, which was evaluated according to the principles of Sec.V-A.

A. Control bandwidth allocation

The out-of-band tones can be placed either below or above the useful bandwidth, and the question is which is better. Fig. 3 shows the system performance under different allocation policies. Indicated in the figure is the bandwidth occupied by the control signal consisting of 64 tones. Clearly, allocating the control signal above the useful bandwidth results in a better performance. Moreover, the control tones are best placed as close as possible to the upper edge of the data bandwidth. Ideally, no separation between the two bands should be used, although some separation may be required to keep the radiated out-of-band acoustic power below a pre-specified level.

B. Number of control tones

Given the control signal placement at the upper edge of the useful bandwidth, we now want to determine the minimum number of control tones needed to ensure a certain PAR reduction. Note that there is a trade-off here, as more tones enable better control of the peak power, but increase the overall average power. The relationship between the number of control tones and the achievable improvement is depicted in Fig. 4. The average achievable improvement is defined as the average PAR reduction achieved using the optimal control signal. This result quantifies the effect of diminishing returns that takes place as the number of control tones is increased, but it also demonstrates that an improvement of several dB is available from the OTI technique with a reasonably small number of control tones.

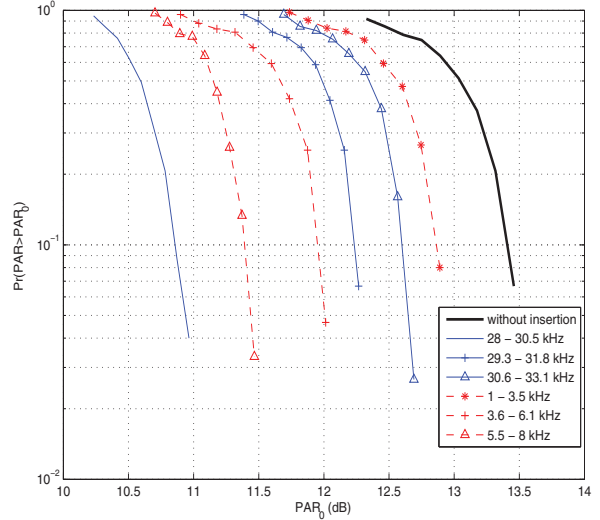


Fig. 3. CCDF of the PAR when control tones are inserted above the useful bandwidth (solid) and below the useful bandwidth (dashed). Legend indicates the bandwidth occupied by the control signal.

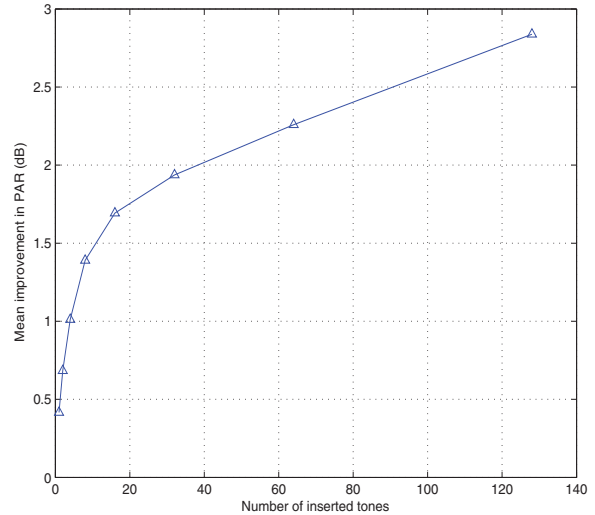


Fig. 4. PAR improvement vs. the number of out-of-band tones.

Below, we address the performance of practical OTI techniques, namely the gradient technique and the random insertion technique.

C. OTI-Gradient technique

The gradient algorithm described in Sec.V-B was applied to each incoming data block until its original PAR was reduced by 4 dB, or a pre-determined number of trials has been reached, whichever came first. For each incoming OFDM block, the step size was chosen as $\mu = 2/K$.

Fig. 5 shows the performance of the gradient technique, obtained after a varying number of LMS iterations. The PAR

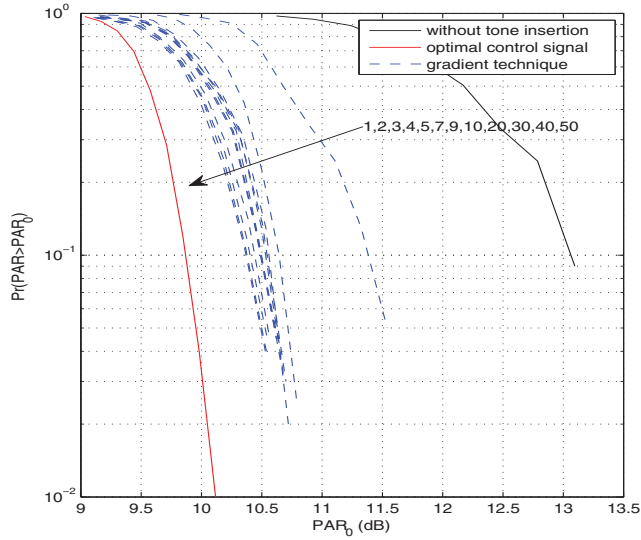


Fig. 5. CCDF of the PAR resulting from the gradient technique after a varying number of iterations.

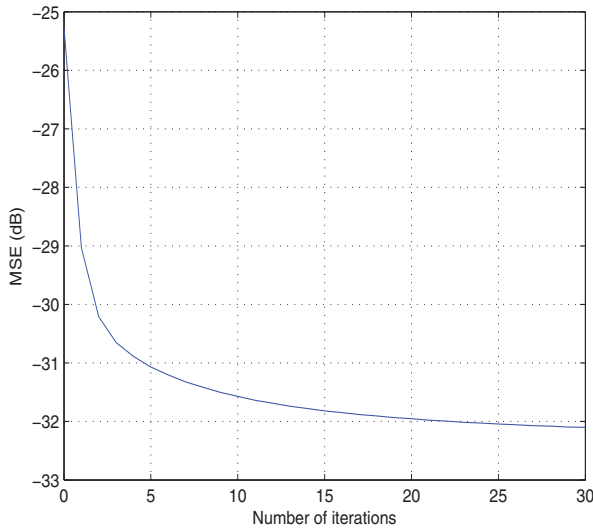


Fig. 6. Normalized MSE of the gradient technique.

reduction achieved after the first few iterations is outstanding, with diminishing improvement thereafter. For example, a 2.5 dB reduction is obtained after only three iterations when the original PAR is 13 dB.

Performance of the gradient technique is further illustrated in Fig.6, which shows the normalized MSE, i.e. the variance of the clipping error obtained after a given number of iterations, averaged over all data blocks. These results demonstrate that a considerable improvement is available from the OTI technique at a very low computational cost.

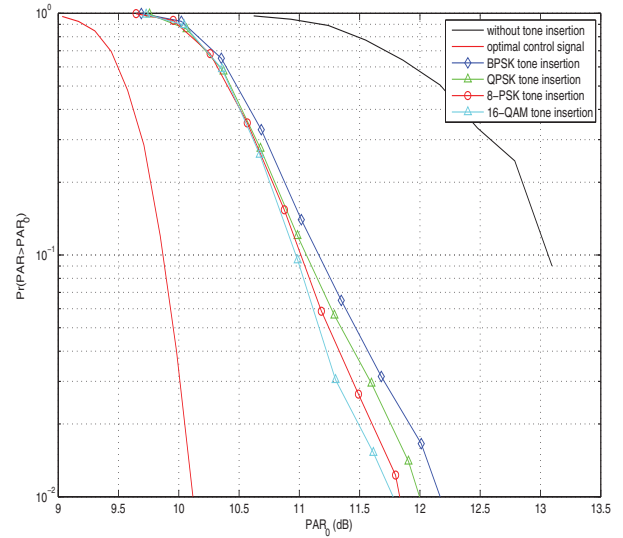


Fig. 7. CCDF of the PAR resulting from random insertion technique. The data sequence is modulated using QPSK, and the number of trials is limited to 100.

D. OTI–Random insertion

Fig. 7 shows the results obtained using the random insertion technique described in Sec.V-C. The maximal number of trials (randomly generated control sequences) is set to 100, and the modulation method (alphabet size) used for the control signal is varied. The data sequence is modulated using QPSK. Interestingly, there is not much to be gained by increasing the modulation level from 2 to 16. This fact justifies the use of simple control sequences, such as BPSK or QPSK. We also note that the overall PAR reduction is comparable to that obtained using the gradient technique.

The question of the number of trials needed to achieve a certain performance is addressed in Fig.8. Similarly as with the number of iterations in the gradient technique, we observe an effect of diminishing returns with the number of trials here. However, the results are somewhat less encouraging, since at least a few tens of trials are needed to achieve a substantial improvement. At a (hypothetical) 1000 trials, the performance deviates from the optimal by about 0.75 dB. In comparison, the gradient technique achieves this in about ten iterations.

VII. CONCLUSIONS

Out-of-band tone insertion has been proposed as a PAR reduction technique for underwater acoustic OFDM systems. A set of tones is inserted outside of the nominal signal bandwidth prior to D/A conversion. The control tones are digitally optimized to provide PAR reduction before the signal is D/A converted and fed to the (nonlinear) power amplifier. The tones are subsequently removed by the transducer which acts as a filter, or by explicit filtering. The main advantage of the technique proposed is that PAR improvement comes at no reduction in the data rate.

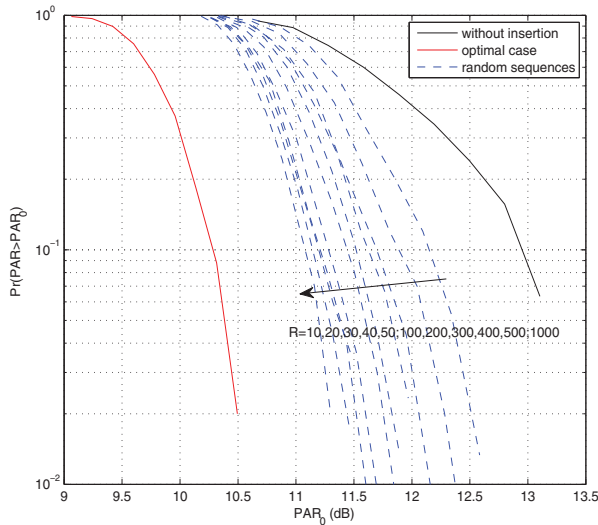


Fig. 8. CCDF of the PAR resulting from random insertion technique with a varying number of trials. The data and the control sequence are modulated using QPSK.

Two approaches were considered for computationally-efficient design of the control signal: a gradient technique which minimizes the mean-squared clipping error, and a random insertion technique in which the selection of control signal is made from a finite set of randomly generated symbols. The performance of these techniques, as well as the number and placement of control tones, were studied via a numerical analysis.

Results show that the best tone placement is at the high end of the useful signal bandwidth. The improvement in PAR reduction grows with the number of tones, but there is an effect of (exponentially) diminishing returns, which justifies the use of a relatively small number of tones (not more than what is used for the information signal). Both the gradient technique and random insertion offer non-negligible PAR improvements. The gradient technique exhibits fast convergence, yielding a close-to-optimal solution in only a few LMS iterations. Random insertion offers a comparable, albeit slightly inferior performance, using control symbols from the same alphabet as the data symbols (QPSK), and a search limited to about a hundred sequences. Future work in this area should target the design of low-complexity systematic search methods for the random insertion technique, as well as integration of the OTI principle with other PAR reduction techniques such as interleaving or phase randomization techniques.

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