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Network Coding in Optical Networks with O/E/O Based Wavelength Conversion

Ramanathan S Thinniyam, Minkyu Kim, Muriel Médard, Una-May O'Reilly
Massachusetts Institute of Technology, Cambridge, MA 02139
{ram@csail., minkyu@, medard@, unamay@csail.}mit.edu

Abstract: Performing network coding at network nodes with O/E/O wavelength conversion equipment incurs negligible additional cost. Our methodology finds minimum O/E/O equipment for multicast network coding on a minimal wavelength subgraph with one link failure reliability.

OCIS codes: (060.4265) Networks, wavelength routing; (060.4261) Networks, protection and restoration

1. Introduction

Network coding is a process by which messages are encoded and sent across computer networks and then decoded at the receivers to optimize use of bandwidth as well as reliability [1–3]. Optical networks have their own unique properties which affect the way network coding is performed on them. Previous work on network coding for optical networks concentrated on minimization of network resources under the assumption of one link failure reliability (i.e. the connections specified can be maintained even if exactly one of the links in the network fails) [4]. Optical networks make use of wavelength division multiplexing(WDM), where each fibre carries multiple data streams, each of which make use of a different wavelength for transmission. In order to transfer data from one wavelength to another, specific hardware needs to be installed. Previous work [4] assumed cost-free wavelength conversion. We extend this work to the case where Optical-to-Electronic-to-Optical (O/E/O) equipment is required for wavelength conversion. The additional cost of network coding at a node (or wavelength) where O/E/O equipment is present is negligible. Thus the cost of one or both wavelength conversion and network coding is the same. We describe a method using integer linear programming and a genetic algorithm to optimize cost. We test the method on multicast scenarios on the ARPANET, NSFNET and NJLATA networks. We summarize the results from our simulations on these networks to give an estimate of the resource requirement under network coding.

2. Problem Formulation

The algebraic representation of the network coding problem, the single-link fault tolerance condition and assumptions on traffic grooming of optical networks are borrowed from previous work [5]. We now describe the wavelength assignment strategy assumed for O/E/O based wavelength conversion. Consider a node as shown, with two incoming links each carrying 3 and 4 wavelengths and one output link carrying 2 wavelengths. Consider all links connected to a node. The colours of the wavelengths are assigned such that the links containing larger numbers of wavelengths always include the subset of colors that match the wavelengths of the links with fewer ones. In Fig. 1, the solid and dashed lines are the wavelengths of the output link and are also present in the 2 input links, which have larger numbers of wavelengths.

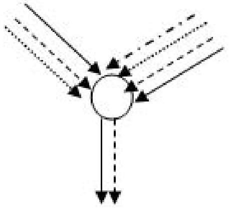

Fig. 1. Wavelength assignment mechanism: links containing larger numbers of wavelengths always include the subset of colors that match the wavelengths of the links with fewer ones.

3. Methodology

Algorithm 1: Network Coding Optimization with Failure Reliability of a Given Optical Network Topology

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   Input: Multicast scenario with required rate,1 source and $n$ receivers, network topology and capacity of links.
   Output: Total number of wavelengths required, number of O/E/O wavelengths required
 1 Solve integer linear program(ILP) for one link failure reliability to obtain minimum wavelength-cost subgraph;
 2 Construct auxiliary graph of obtained subgraph by decomposing merging nodes;
 3 Initialize fraction of population with the all-coding merging strategy with coding of all wavelengths;
 4 Initialize remaining population of chromosomes with different merging strategies of merging nodes;
 5 Evaluate population after converting merging strategy into corresponding subgraph of auxiliary graph;
 6 while generation < maximum number of generations do
 7     Select merging strategies of high fitness which will produce next generation;
 8     foreach parent pair do
 9         Perform crossover to obtain two children with updated merging strategies;
10         Perform mutation on the two children to get two new children;
11         Include the resulting two new children as part of the new population;
12     end
13     Evaluate new population of offspring merging strategies;
14 end
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The ILP in line 1 of algorithm 1 is a modification of the linear program for minimum subgraph construction given by Desmond Lun [6] and is given in reference [7]. Network coding at all nodes and wavelengths is assumed on this subgraph. We now use a genetic algorithm to further minimize the number of wavelengths which have to be coded or converted. A *merging node* is a node with at least one outgoing and at least two incoming wavelengths. Coding need take place only at such nodes. We replace such nodes with equivalent networks as shown in Fig.2 in order to form the auxiliary network of the original network. A binary *coding vector* indicates which of the incoming wavelengths contribute to the flow on each outgoing wavelength of the merging node. Every merging node contributes d_{out} number of *genes* to the chromosome where d_{out} is its number of outgoing wavelengths. Any coding vector which has multiple 1s in it leads to a coding outgoing wavelength and all of these can be represented by one symbol ('-1') in the chromosome. If only one of the input wavelengths is routed then it is indicated by a single number from 1 to k, where k is the number of input wavelengths to the merging node. 0 is used to represent no transmission. Thus there are a total of $k+2$ possible combinations for a gene representing an outgoing wavelength. A given problem instance may be tested on any subgraph of the auxiliary graph using a flow algorithm such as the Ford-Fulkerson algorithm [8] in order to determine if the multicast rate is met (i.e. if the problem instance is solvable). Lines 4 through 13 describe a simple genetic algorithm which adaptively optimizes merging strategies. Tournament selection, uniform crossover and a uniform mutation operator are used [9]. The fitness of a merging strategy z is given by

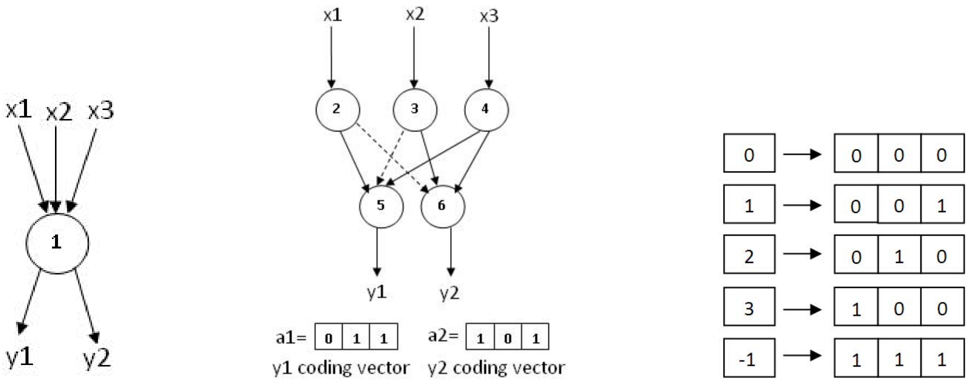
$$F(z) = \begin{cases} \text{\# of O/E/O wavelengths}, & \text{if rate}(z) \geq \text{required rate}, \\ \text{total possible O/E/O wavelengths + \# of infeasibilities}, & \text{if rate}(z) < \text{required rate}. \end{cases} \tag{1}$$

# of O/E/O wavelengths is the sum of the number of coded wavelengths (i.e. number of '-1's in chromosome) and the number of (solely) converted wavelengths (i.e. number of outgoing wavelengths whose wavelength does not match the colour of wavelength of the incoming wavelength which it forwards).Number of infeasibilities is the number of receivers which are unable to obtain the data they require.

4. Results and Conclusion

We used our algorithm on 25 different multicast problem instances of the ARPANET, NSFNET and NJLATA networks with one source, 3 sinks, a target rate of 4 and capacity of 4 per link in each direction (since optical links are bidirectional). Table 1 summarizes the results of our findings. The results show that the fraction of wavelengths which require O/E/O equipment is less than 6% on the average.

Each problem instance requires the placement of O/E/O equipment at different physical nodes. Consider the distribution of physical nodes which required O/E/O equipment: nodes in red in Fig.3 required O/E/O equipment in



(a) Merging Node

(b) Equivalent Network

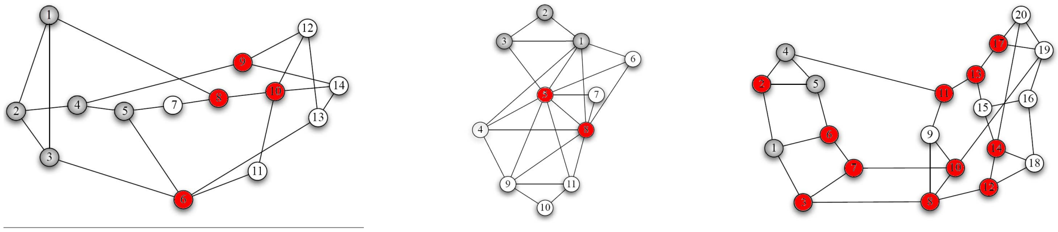
(c) Chromosome to Coding Vector

Fig. 2. Node 1 with 3 incoming and 2 outgoing wavelengths is associated with two coding vectors $a_1 = (a_{11}, a_{21}, a_{31})$ and $a_2 = (a_{12}, a_{22}, a_{32})$.

Table 1. Distribution of the calculated number of O/E/O wavelengths in 25 random multicast scenarios as a percentage of total number of wavelengths. % O/E/O = % coded + % converted

% of Wavelengths	Coded			Converted			O/E/O		
Network	ARPANET	NSFNET	NJLATA	ARPANET	NSFNET	NJLATA	ARPANET	NSFNET	NJLATA
Maximum	5.1	8.3	8.7	2.3	2.4	2.7	6.3	8.9	11.4
Minimum	2.3	1.2	0.0	0.0	0.0	0.0	2.7	1.2	0.0
Mean	3.8	4.6	3.1	0.8	0.8	1.0	4.6	5.4	4.1
Std	0.8	1.6	1.9	0.6	0.6	0.9	0.9	1.9	2.5

more than half of the problem instances. The percentage of such high frequency nodes in ARPANET, NSFNET and NJLATA was 18%, 29% and 55% respectively. We see that the O/E/O requirement gets spread across the network as the network size increases. We conclude that coding and conversion can be effectively combined, as the number of converted wavelengths is at most one per problem instance in each network. We hope to be able to quantify the amount of benefit that can derived out of placing O/E/O equipment at such hotspot nodes in future work.



(a) NSFNET

(b) NJ LATA

(c) ARPANET

Fig. 3. Nodes with high (> 0.5) O/E/O frequency are indicated in red.

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