ENERGY LABORATORY

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

INTEGRATION OF DECENTRALIZED GENERATORS WITH THE ELECTRIC POWER GRID

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MIT ENERGY LABORATORY REPORT NO. MIT-EL 81-011

April 1981

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ABSTRACT

This report develops a new methodology for studying the economic interaction of customer-owned electrical generators with the central electric power grid. The purpose of the report is to study the reciprocal effects of the operation and expansion plans of the utility, and the resulting price of electricity, and the demand patterns and expansion plans of customers. The system is modeled in an open-loop feedback mode that allows both the utility and the customers to update their plans and expectations for the next time period based on the other's actions in the current time period and based on any new information such as the current price of oil. The utility and the customers solve similar operation and expansion problems, except that each has control over different variables. In addition, each may have different expectations about the future. A complete methodology encompassing these ideas is developed and implemented.

ACKNOWLEDGMENT

To Miss Ruth Dean
who taught me Anglo-Norman and Paleography

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1.I Introduction

Of the total energy consumed in the United States, twenty-five percent is consumed by electric power utilities.[1] Utilities traditionally have generated electricity at central stations and then sent it through a grid of transmission and distribution lines to industrial, commercial, and residential consumers. Electricity has many advantages over other source of power. It is versatile and can be used to produce work from motors or to produce heat from resistive devices. It causes no pollution in its end use. Reliable electrical devices are widely available on the market. To the consumer, the supply of electricity has always appeared to be unlimited: A flip of a switch and electricity is delivered instantly. Until recently, the real price of electricity declined steadily over time as shown in Figure 1.1.

Counterbalancing the advantages of electrical energy to the consumer are disadvantages which occur elsewhere in the system: to produce one BTU of electrical energy requires three BTUs of thermal energy. (The maximum thermal efficiency computed using thermodynamics is on the order of thirty-five percent.) Thermal energy is produced by burning fossil fuel which creates air pollution or by controlling nuclear reactions which create radioactive wastes. The thermal energy that cannot be converted into electricity is lost as thermal pollution. In addition, economic inefficiencies result from the current price structure of

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[1]Erikson, L.E., "A Review of Forecasts for U.S. Energy Consumption in 1980 and 2000," [21], p. 19.

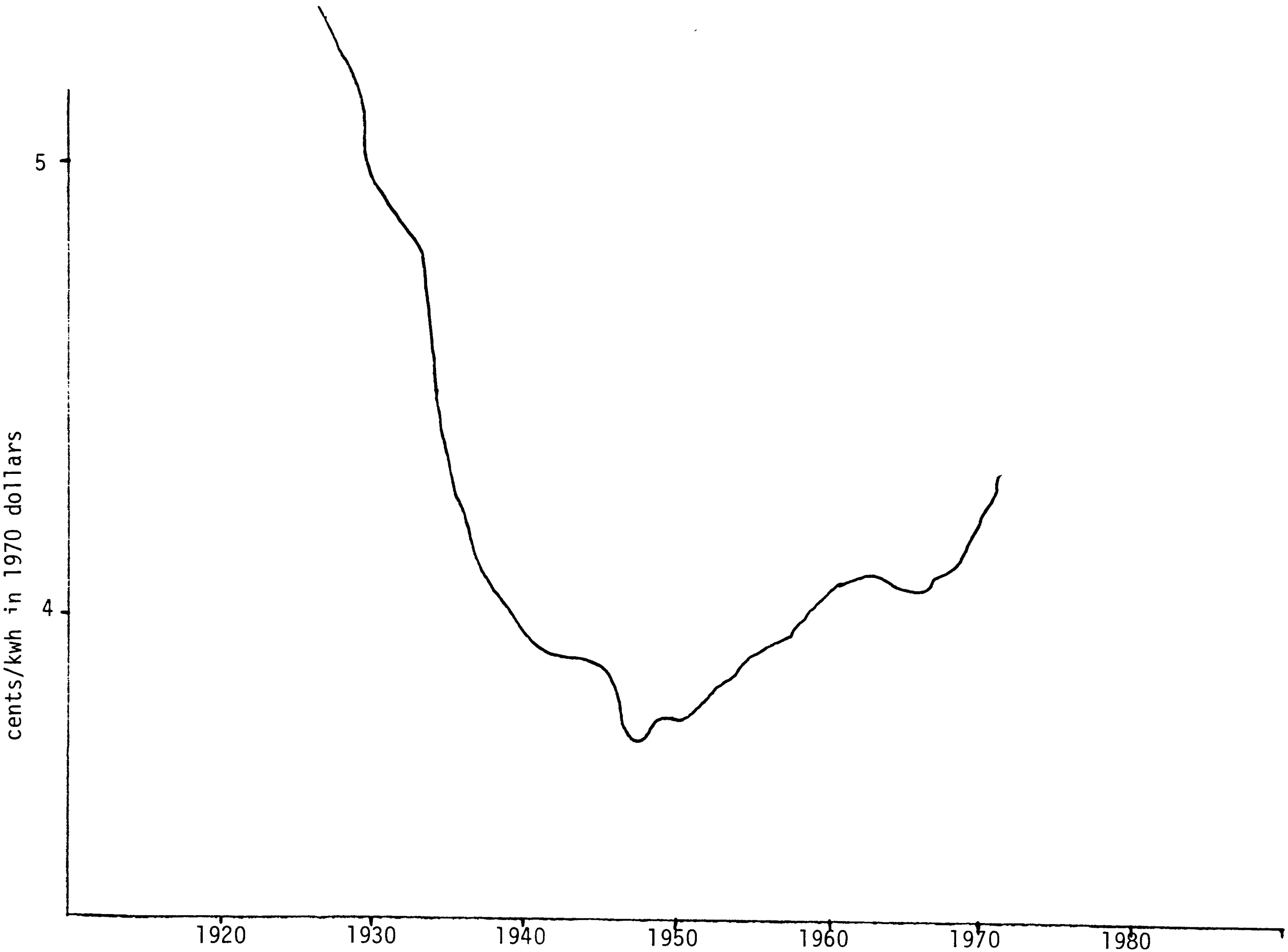

Figure 1.1 Real Price of Electricity, Source[29]

electricity which gives consumers misinformation about the cost of the power they use.

For many years, technical innovations kept increasing the maximum plant size and the cost per installed megawatt rose slowly. However, as these scale economies were completely exploited the cost per megawatt in constant dollars began to increase rapidly as illustrated in Figure 1.2. It was also in the late sixties that environmental problems became national concerns. With the passage of air and water pollution regulations, utilities were forced to add pollution abatement equipment to new and existing power plants and were encouraged to switch from burning coal to burning low-sulfur oil. As a result of the oil embargo, new laws have been passed requiring utilities to switch back to coal while maintaining environmental standards. Safety-related equipment has driven up the cost of nuclear power plants, so that a unit, that in the fifties was expected to produce electricity too cheap to meter, now costs billions of dollars before it even begins to produce electricity.

Awareness of environmental pollution has also made siting power plants more difficult. People are now conscious of the potential long-term effects of having a nuclear unit, a coal-fired unit, a high-tension transmission line, or even a hydroelectric reservoir in their neighborhood. Consequently, utilities have fewer sites available and the costs of securing sites and rights of way have risen.

As plants have become larger, and new regulations have been passed, the time between the announcement of a new plant and the startup of

Figure 1.2 Real Price per Kilowatt of Generation, Source[29]

that plant has lengthened. It can take fifteen years to bring a nuclear power plant on-line. These long lead times have two major effects. The first effect is to increase the financial burden on the utility. In most states, utilities cannot collect revenues on plants until the plants start to generate electricity. So, a utility may be required to carry a project that produces no revenue for many years. The second effect is to reduce the planning flexibility of the utility. A utility may find itself committed to building a plant which is no longer needed, and be unable to change its plans without severe penalties.

All of these factors have combined so that the cost to a utility of building a new base load unit can be greater than the utility's total prior investment. As the financial performance of utilities has declined, so have their bond ratings, making it even more expensive for them to raise the necessary capital.

As the price of electricity has risen, consumers have begun to pay more attention to the rate hearings at which utilities request price increases. Consumer groups have been organized to keep utilities from collecting money for units that are not yet in service and from passing fuel costs directly through to consumers without rate hearings. Many regulatory commissions that used to hear unopposed rate cases now must hear long contested rate cases. Their rulings are highly visible and political. Lately, more and more rulings have favored the consumers over the utilities.

What has saved the utilities from having to build ever more, ever larger, ever more expensive plants is that in response to higher prices and governmental urgings, consumers have cut back on their use of electricity. The leveling off of demand growth can be seen from Figure 1.3. After the depression and before 1973, electrical load grew steadily at about eight percent per year. Then, the load growth began to drop to between one and three percent per year depending on the region of the country. Utilities were completely unprepared for this sudden drop in demand and most refused to recognize that demand patterns were changing in response to higher prices. For a while, utilities continued to plan as if demand were going to return to its previous levels. Now, utilities have begun to recognize that demand patterns have changed in response to higher prices and have canceled or postponed many plants originally scheduled for the early eighties.

So far, most of the price response by consumers has been short-run response. That is, many wasteful uses of electricity have been eliminated through conservation efforts, but there has not been time for consumers to change their capital stock markedly. In the long run, consumers can buy more efficient refrigerators as the current stock wears out, they can replace resistive heaters with heat pumps or wood stoves, and, in general, they can change their capital stock to more effectively use the electricity supplied by the utility. They can also begin to supply their own electricity.

Recent government policies and regulations encourage the installation of small, privately owned generators that operate as part of the utility system. The Public Utilities Regulatory Policies Act [73] (PURPA) requires that utilities create fair rates for the purchase of

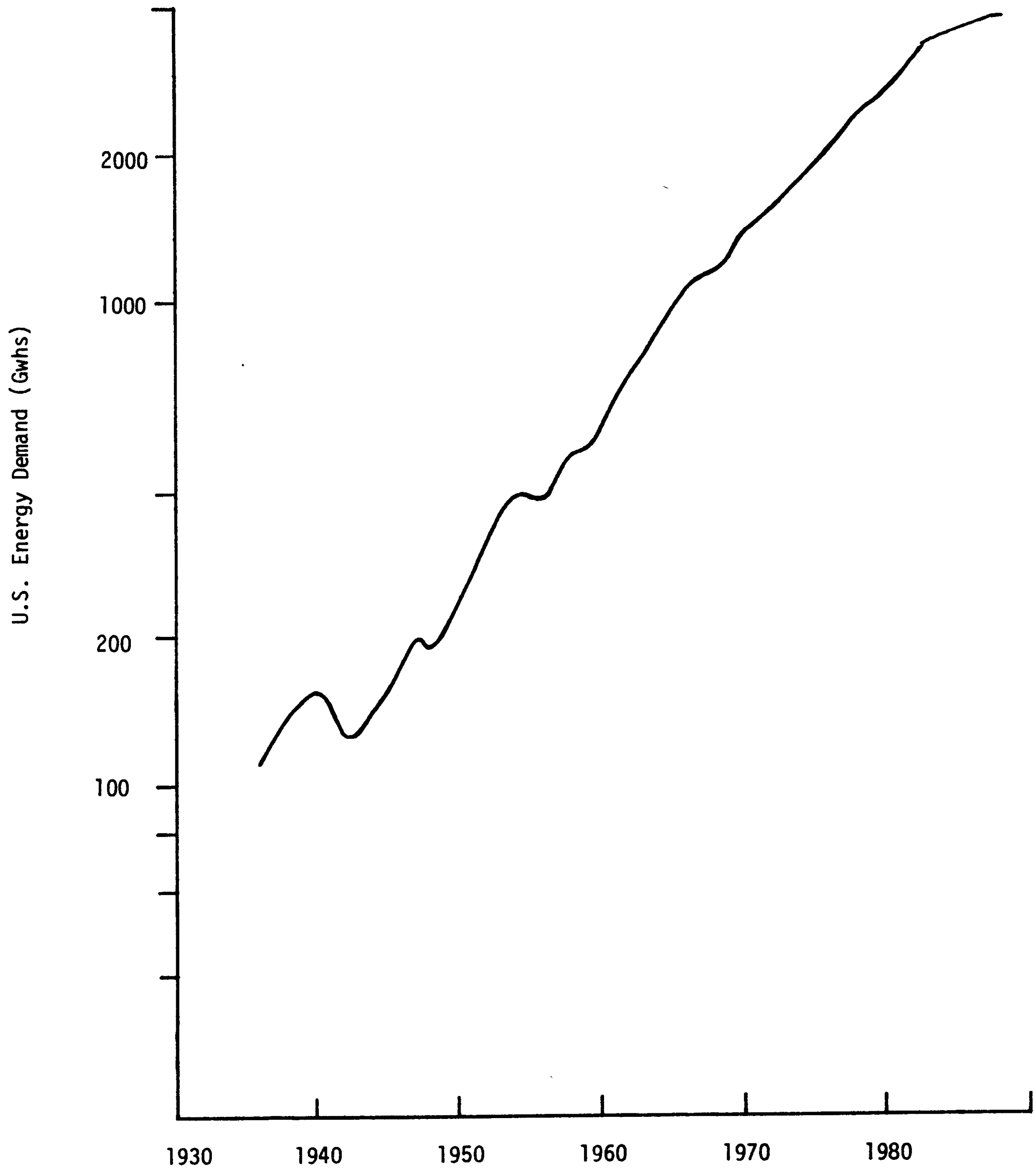

Figure 1.3 Growth in Electrical Demand
Source [29]

electricity from privately owned generators. The installation of decentralized generators will affect the operations and planning of electric utilities in many ways.

Among the technologies currently available to consumers are: solar space and water heating, electrical and mechanical power generation from the wind, and industrial co-generation of process steam and electricity. Photovoltaic cells have the potential to become a common source of decentralized power because they produce electricity directly from sunlight, are highly reliable, and require almost no maintenance; although at the moment, photovoltaics are too expensive to compete with other energy sources in most applications.

Those technologies using renewable resources, such as wind or solar insolation, as a fuel have many advantages. A long-term supply of free fuel is assured even though the short-term supply may be uncertain. (One can never be positive that it will not be cloudy or calm the next day.) Reducing consumption of fossil fuel by replacing it with renewable resources eliminates pollution due to the combustion process. Matching the energy source to the demand characteristics improves the efficiency. For example, it is more energy efficient to use solar radiation directly for heating than to burn fossil fuel to produce steam to drive a turbine to drive a generator to produce electricity to be sent over transmission lines to be run through a resistive coil for heating. The technology for co-generation also can improve energy efficiency by utilizing the waste thermal energy which is a by-product of electricity production. Co-generation can improve the efficiency by more than a factor of two, from about 34 percent to 80 percent,[2] thereby reducing the fuel burned and the pollution produced per unit of output.

In an economically perfect world, the prices charged by the utility would reflect the marginal cost of energy and capacity and would result in the optimal level of centralized and decentralized generation. However, neither the utility nor the customer knows exactly what the other will do, so each must base decisions on what the other is expected to do. Also, neither knows the future prices of fuel or generating capacity with certainty. In addition, there are market imperfections resulting from facts such as that utilities are regulated monopolies and that consumer mortgages are indirectly subsidized through the federal and state tax structure.

It is important to note that even without the problems discussed above, centralized and decentralized generation are not equivalent. Firstly, some types of generation exhibit economies of scale while other do not. Secondly, the electrical distribution system that connects the central generators to customers affects the reliability of the delivered energy and, through its losses, increases the amount of centralized electricity that must be generated. So that, with perfect markets, the ownership might not matter, but the size and location still would.

1.II Problem Definition

Utilities must now plan in an environment that is changing rapidly,

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[2]Gyftopoulos, [43], p. 25.

but the techniques currently available do not recognize this. The hypothesis of this report is that the plans made by utilities would be quite different from their present plans if they could more accurately account for factors such as demand response to price, lead times, and decentralized generation.

This report develops a new methodology for long-range planning for electric utilities. Most long-range planning models construct the optimal plan over twenty or thirty years based on current knowledge. These models require that the utility specify the customer demand, fuel prices, and capital costs, for thirty years into the future. To study the effect of an increase in the price of oil, a sensitivity run could be made changing the price that utilities pay for oil; however, it would be unlikely that the exogenously specified demand would be changed too since that would require that a separate model be rerun. Because different groups use the supply and demand models, it is rare to see a study that links them. This new methodology explicitly accounts for the dynamics of electricity supply and demand, allowing both the utility and its customers to react to changing prices and changing expectations over time.

The result of this report is a methodology that allows planners to model aspects of the system that have been ignored previously. The methodology is flexible and allows the planner to substitute models of more or less detail for any of those described here. The purpose is to develop a new way of looking at long-range planning, rather than to develop a better algorithm for modeling some part of the total system. Because the system is complex, explicit assumptions have been made about the interactions between the utility and its customers:

1) that the utility and its customers influence each other only through a fixed set of signals. For the utility, the signals sent are the price and reliability of the electricity provided. For the customers, the signals sent are the peak power demand, the energy consumption and the demand pattern (load shape);

2) that neither the utility or its customers can change the decision criteria or the choices available to the other. This assumption means that factors such as ad campaigns in which a utility attempts to induce customers to switch to electric heat or to conserve energy for other than purely economic reasons can not be modeled;

3) that both the utility and its customers plan for the future based on uncertain information and that as time passes decisions about actions in the near future become fixed while those further in the future may change based on new information. The new information may be a signal from the other or a changed factor from an exogenous source. An example of the former would be an increase in the price of electricity while an example of the latter would be an increase in the price of oil;

4) that the long range plans of the utility have little influence on customers' decision except through their expectations of the future price of electricity. The converse of this assumption is not true, the next assumption being:

5) that the expected long range behavior of customers can influence

the plans of the utility. That is, the utility can take into account the expected response of demand to price when making its long range forecast;

6) that the system need not be in equilibrium. That is, at any time, it would be possible to iterate to an equilibrium solution in which the price charged by the utility exactly matched what customers were willing to pay. The assumption, however, is that due to regulations, lead times, and other factors, that the system continues to evolve over time without ever necessarily reaching an equilibrium state; and

7) that adjustments are made at the beginning of discrete time steps. This assumption could be relaxed, allowing, for example, new units to be installed or price increases to take effect in the middle of time steps. This assumption is made for ease in exposition.

1.III General Formulation

In broad outline, the interactions of the utility and its customers are as follows: the utility announces a price for the current year. Customers respond to that price in the short run by changing the patterns of their consumption and, if they own their own generators, of their supply as well. Both the customers and the utility make plans that commence in the following year based on their current knowledge. For either the utility or the customer it may not be possible to alter plans instantly. Because of the lead times required for some projects, the decisions made in the current time step may not take effect until some time in the future. However, it is assumed that in each time step some decisions are made fixed while others are allowed to float. For example, it may be necessary to make firm commitments today in order to have a base load generator seven years from now, but the commitments for a second unit on the same site, needed in twelve years, may be minimal. When the time step advances, the plans of the utility and its customers are fixed and a new price is announced based on the customer demand and the installed units in the previous time period.

Figure 1.4 gives a schematic representation of the interactions. The inner loop of utility operation, rate setting, and customer demand represent the direct interactions of the utility and its customers. The peripheral loop of utility and customer expansion planning represents the decision processes whose effects filter down to the direct interactions. This is made more precise in Figure 1.5 which illustrates the signals received and sent by each actor, using the notation developed in the report.

Each box in Figure 1.5 can be thought of as a function that transforms the input signal to the output signal. Most of the remaining chapters of the report will be devoted to making the function forms of these transformation explicit, although they can be written in general form now.

First, the utility announces a price for electricity for the next time period, based on the installed capacity and the system operating cost in the current time period. In some areas, the price may include

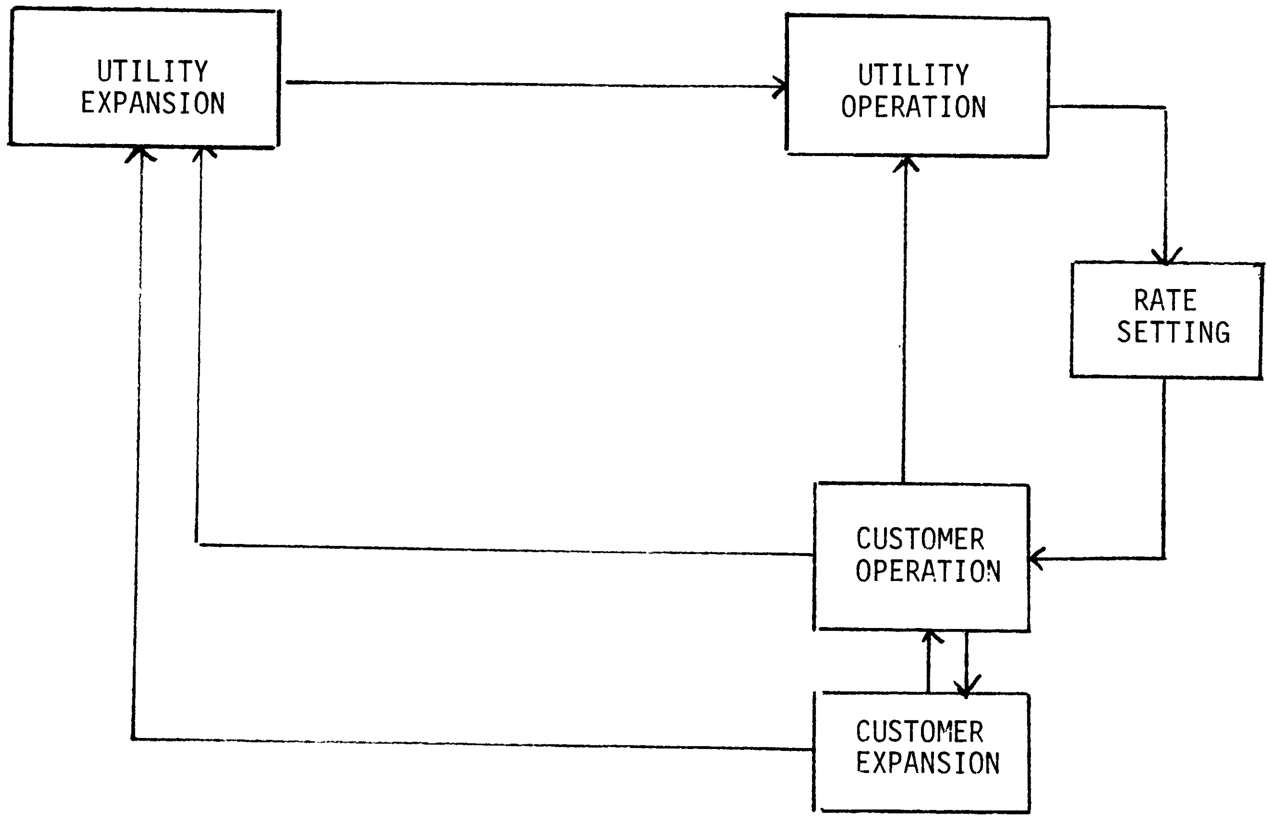

Figure 1.4 Interaction of Utility and Customer Expansion Plans

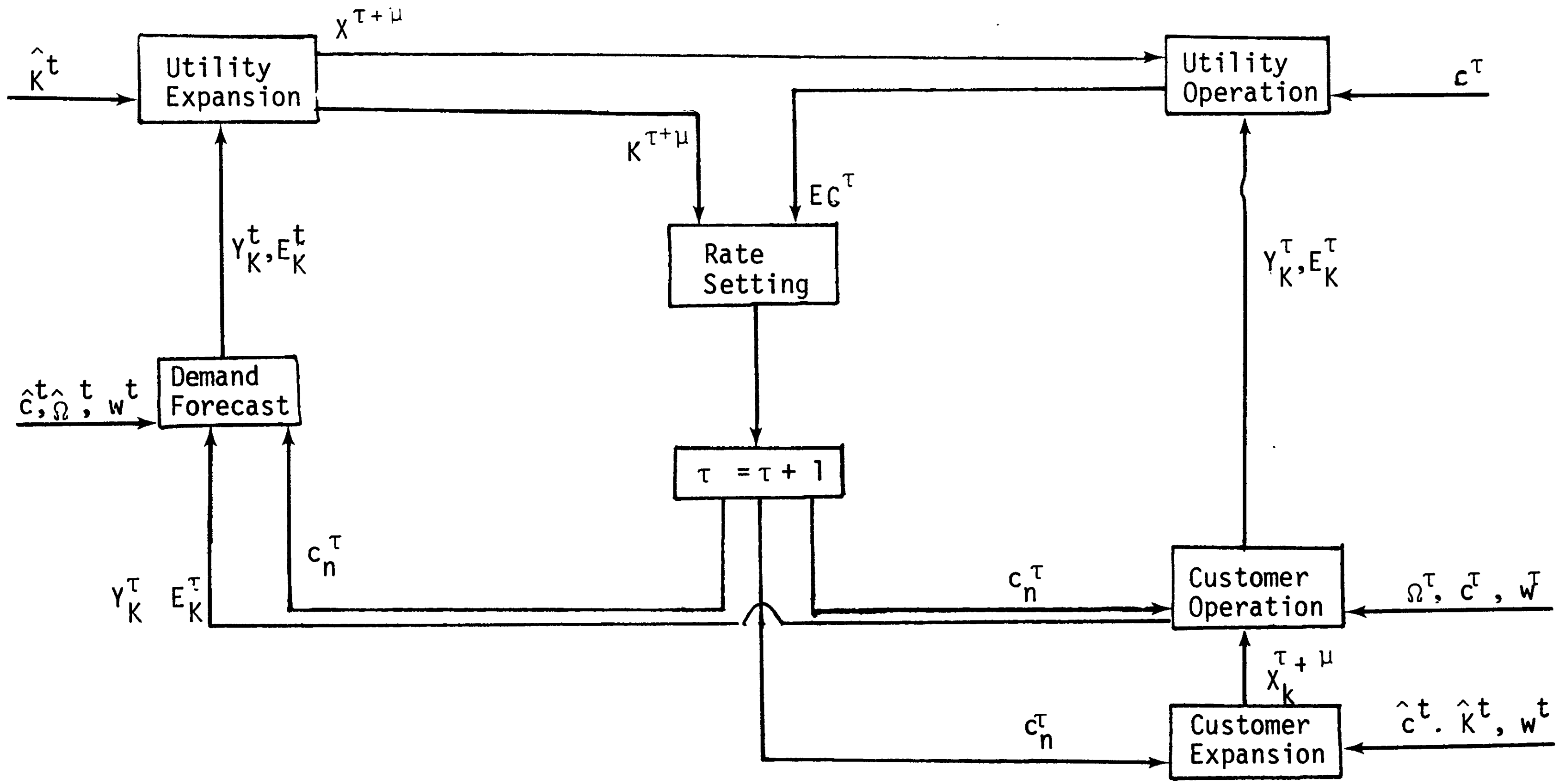

Figure 1.5 Information Flow among Subsystems
(Notation is explained in accompanying text)

anticipated costs, but the price is always announced prior to the time period in which it takes effect. (For an interesting discussion of taking this principal to its limit by announcing a new price every five minutes, see the paper on homeostatic utility control, reference [79].) The price announced by the utility can be computed as some function of the capital and operating costs of the system:

$$c_n^{\tau+1} = f(EC^{\tau}, CX^{\tau}) \qquad (1.1)$$

where $c_n^{\tau+1}$ = price of electricity in time period $\tau+1$. (The subscript n indicates that the variable is a property of the utility) ($/MBTU)

EC^{τ} = total cost of operating the utility system in time period τ ($) (see equation (1.5))

CX^{τ} = historical or replacement cost of system capacity in time τ ($) (see equation (1.12)).

This price becomes effective for the next year and the next year becomes the current time period:

$$\tau = \tau + 1. \qquad (1.2)$$

Given the new price, customers may change their demand patterns making short-run adjustments:

$$Y_K^{\tau} = f(c_n^{\tau}, X_k^{\tau}, c^{\tau}, \Omega^{\tau}, w^{\tau}) \qquad (1.3)$$

$$E_K^{\tau} = f(c_n, X_k, c, \Omega^{\tau}, w^{\tau}) \qquad (1.4)$$

where

Y_K^{τ} = peak power demand of all customers in time period τ (MW)

E_K^{τ} = energy demand from customers in time period τ (MBtu)

c_n^τ = price of electricity in time period τ computed using equation (1.1)($/MBtu)

X_K^τ = vector of installed capacities owned by customers in time period τ (MW)

c^τ = vector of prices for competing sources of energy available to customers in time τ ($/MBtu) (exogenous)

Ω^τ = vector of demographic variables describing customers in time τ (exogenous)

w^τ = vector of meteorological variables for time τ (exogenous).

And, the utility must adjust its operating schedule to meet the new demand at minimum cost, given the price of available fuels:

$$EC^\tau = f(Y_K^\tau, E_K^\tau, X^\tau, c^\tau) \tag{1.5}$$

where

EC^τ = expected cost of meeting demand in time τ ($)

Y_K^τ = customer demand as computed in equation (1.3)

E_K^τ = customer energy demand as computed in equation (1.4)

X^τ = vector of installed capacities in time τ (MW)

c^τ = vector of prices for fuels in time τ ($/MBtu) (exogenous)

However, both the customers and the utility can change their capital stock in the long run, although neither can change it instantly. For the customer, this can mean buying appliances to match the rate structure. For example, under some rate structures it would be worthwhile to buy an oversized air conditioner that could be cycled to run only off-peak, while under other rate structures, it would be worthwhile to buy an undersized air conditioner that would run constantly. In addition, customers can buy appliances that use alternative fuels, for example, gas stoves. And, customers can install their own source of power using conventional fuels, e.g., diesel generators, or nonconventional fuels, e.g., solar insolation. For any appliance or generator, the customer can compute the breakeven capital cost:

$$BECC_{ik}^{t} = f(\hat{Y}_k, \hat{E}_k, \hat{c}, w) \qquad (1.6)$$

where

$BECC_{ik}^{t}$ = breakeven capital cost of system i installed in year t by customer k

$\hat{Y}_k$ = vector of estimates of customer k's power demands over the lifetime of system i

$\hat{E}_k$ = vector of estimates of customer k's energy demands over the lifetime of system i

$\hat{c}$ = estimates of power costs from all sources over the lifetime of system i (exogenous)

w = vector of meteorological parameters (exogenous)

The breakeven capital cost is the cost that the system must sell for so that a purchaser would be indifferent between that system and the best alternative. If the purchase price is less than the breakeven cost, then one should buy the system and save money. However, since there are many factors that influence a purchase besides cost, only a fraction of predicted purchases will be made:

$$\hat{X}_{ik}^{t} = f(\hat{BECC}_{ik}^{t}, \hat{K}_{i}^{t}) \qquad t = \tau + \mu_i \ldots T \qquad (1.7)$$

where

X_{ik}^{t} = planned capacity of system i to be installed in year t by customer k

$BECC_{ik}^{t}$ = estimate of the breakeven capital cost for system i for customer k in time t

K_{i}^{t} = estimate of the purchase price of system i in time t

μ_i = minimum number of time steps to install system i.

Then, for all systems with

$$X_{ik}^{t} > 0 \tag{1.8}$$

and $t = \quad + \mu_i$,

$$X_{ik}^{t} = X_{ik}^{t}.$$

That is, if one plans to have a system operating in time t and the decision must be made in time τ in order to have it in time t, then the decision variables are fixed in time τ for time t even though the plans are based on estimates of the future. Thus, when time t becomes the current time period, the capital stock of the customer is known.

The breakeven capital cost can be computed in one of two ways. The usual way is to run a simulation of the customer demand computing the difference in the cost of meeting demand with and without the system in question. The cost saving is the total amount one would be willing to pay to have the system installed. An alternative is to perform an optimization to find the capital stock that will meet the demand at minimum cost. From the optimal solution, one can perform sensitivity analysis to find the cost at which a system just enters the solution.

If properly performed, the optimization and the simulation should yield the same results. However, because of the many nonlinearities in demand, formulating realistic solvable optimization problems is difficult. The optimization technique has the advantages of being elegant and of ensuring that the breakeven cost of the system is always relative to the best alternative. The simulation technique has the disadvantages of requiring a lot of data and computer time and the advantage of allowing almost any form of constraints.

At the same time that the customer is making decisions about the future, so is the utility. First, the utility must know how much demand there will be in order to decide how much to build. The demand projection must take into account the expected price of electricity relative to the expected price of other fuels and the expected customer behavior:

$$\hat{Y}_k^t = f(\hat{Y}_k^{t-1}, \hat{E}_k^{t-1}, \hat{X}_k^t, \hat{c}^t, \Omega^t, w) \tag{1.9}$$

$$\hat{E}_k^t = f(\hat{Y}_k^{t-1}, \hat{E}_k^{t-1}, \hat{X}_k^t, \hat{c}^t, \Omega^t, w)$$

where

$\hat{Y}_k^t$ = estimate of customer power demand in time t (MW)

$\hat{E}_k^t$ = estimate of customer energy demand in time t (MBtu)

$\hat{X}_k^t$ = estimate of customer's capital stocks in time t (MW)

$\hat{c}^t$ = vector of estimates of fuel prices in time t ($/MBtu) (exogenous)

Ω^t = vector of demographic variables for customers in time t (exogenous)

w = vector of meteorological variables (exogenous).

Using the demand estimate from equation (1.9), the utility can plan on how much capacity to build in each year. The long-range planning problem for utilities has been studied for years, and there are many alternative formulations available. Usually, the utility's objective function is assumed to be cost minimization, although other criteria can be used. In general form, the optimal capacity can be found using:

$$X_k^{t*} = f(\hat{Y}_k^t, \hat{E}_k^t, X^{t-1}, \hat{c}^t, \hat{K}^t) \qquad t = \tau + \mu_i, \ldots T \qquad (1.10)$$

where

X_k^{t*} = optimal capacity of type i to be installed in year t (MW)

$\hat{Y}_k^t$ = estimate of the total customer power demand in year t (MW)

$\hat{E}_k^t$ = estimate of the total customer power demand in year t (MBtu)

X^{t-1} = vector of capacity installed prior to year t (MW)

$\hat{c}^t$ = vector of estimates of fuel prices for time t ($/MBtu) (exogenous)

$\hat{K}^t$ = vector of estimates of installment costs for time t ($/MBtu) (exogenous)

μ_i = minimum number of time steps between the decision to build a plant of type i and its beginning operation.

For all those decisions that must be made in the current time period τ to have a plant in time t, that is, for:

$$X_i^{t*} > 0 \qquad (1.11)$$

and $t = \tau + \mu_i$,

then $X_i^t = X_i^{t*}$.

And the capital cost of the system is updated:

$$CX^t = f(X^t, K^t, CX^{t-1}) \qquad (1.12)$$

where

CX^t = capital cost of the system in time t ($)

X^t = capacity installed in time t (MW)

K^t = capital cost of new capacity in time t ($/MW)

Just as for the customer, when t becomes the current time period, the installed capacity is known, although it may no longer be optimal since it was planned based on estimates of future customer demand and prices.

The operating and capital costs computed in equations (1.5) and (1.12) are then used to compute rates for the next time period using equation (1.1) and the cycle repeats.

1.IV Implementation

In order to study the system outlined in Section 1.III. it is necessary to make the functional forms explicit. One option is to specify very simple functional forms to find the precise mathematical relationships among variables and to find the sensitivity of the model to changes in certain variables. However, to get beyond such basic conclusions as if the price of electricity goes up, then the demand decreases and less capacity is needed, more complex formulations are needed. So, the other option is to use detailed models of each of the subsystems. This approach virtually eliminates the ability to write closed-form solutions for the relationships among variables in different submodels.

The advantage of the second approach is that each subsystem can be modeled according to the requirements of the problem under study. In many ways, the precise functional forms used to describe different subsystems are arbitrary. Any long-range forecasting model can be used as long as the user believes either that the model accurately depicts demand growth or that it accurately depicts the way that the utility would forecast load growth.

This report uses the second approach, giving the model descriptions that follow an air of impermanence. But, in order to demonstrate how the methodology works it is necessary to assume some functional form. And, any model that was substituted for those described below would have to solve the same problem as discussed in each model description.

Each of the next six chapters of this report describe one of the subsystems in Figure 1.4. The entire system is summarized in Chapter 8. An example is given in Chapter 9. The models described are state-of-the-art models, some of which are still under development. Because of this, not all of the models have been fully implemented as computer models. Therefore, in the example given in Chapter 9, which demonstrates how the modules fit together, some models have been run on a hand calculator, while other models have been replaced by less advanced, but available, computer models. So, the example in Chapter 9 is there as a concrete guide, but the derived numbers should not be interpreted as results.

The author has contributed to most of the models described in this thesis, particularly, the long-range planning model, the customer operation model, the time-dependent generation model, and the rate-setting model. The author is completely responsible for the production costing and reliability models, and for the code which links the models.

I.V Overview

Each subsystem in Figure 1.4 affects and is affected by the other subsystems. Rather than attempting to model the entire system as a single entity, each subsystem has been modeled separately explicitly showing the interactions among them. A common data base has been used for consistency.

Chapter 2 projects the long-run demand for electrical energy based on fuel price forecasts. Then, the total demand is modified by consumer response to time-of-day pricing and by customer-owned generating units, resulting in a projection of the total demand on the electric utility.

Chapter 3 finds the distribution of the net load on the utility based on correlation of the response to time-of-day pricing and of the output of customer-owned generators with the fluctuations in the original electricity demand.

Chapter 4 develops a methodology for finding the minimum cost of operating a set of electrical generators to meet the demand as specified in Chapters 2 and 3, using the fuel price projections. This methodology also finds the reliability with which the generators meet the demand.

Chapter 5 uses the methodology of Chapter 4 as part of an optimization algorithm that finds the minimal cost expansion plan for a utility subject to a reliability constraint. In addition to fuel price projections and demand specificatons, this methodology also requires capital cost projections and new technology specifications.

Chapter 6 uses the rates, along with fuel price projections, capital cost projections, and technology specifications to find the amount of capacity that customers would install for themselves.

Chapter 7 uses the capital and operating costs from the optimal expansion plan to compute new electricity rates.

The new electricity rates and new customer installations are then used to project electricity demand starting in the next time period, thus closing the loop.

Chapter 8 summarizes Chapters 2 through 7 and can be read in parallel with the example in Chapter 9.

For those unfamiliar with electric utilities, Appendix A contains a basic description of how utilities operate.

2. Electrical Demand

The demand forecasting component of the model estimates the future demand for electric energy based on the current demand (Chapter 3), the current capital stock of customers (Chapter 6), and the current price of electricity (Chapter 7). The demand forecasting model also requires exogenously specified estimates for the projected prices of competing fuels and exogenously specified estimates for variables that describe the general state of the economy. The demand forecast is used only by the utility long-range planning model.

2.I. Introduction

People do not need energy for itself. They need warm houses, cooked food, conveyance from here to there, and so on. For the most part, the type of energy used to produce the desired result is not of great importance to consumers. When installing a new energy conversion system or buy a new energy-consuming appliance, consumers made trade-offs between capital and operating costs subject to the way in which they expect to use it and to their own preferences. Of course, expectations are not always met. If prices rise or the weather is extreme, consumers may revise their needs or find alternative ways to meet the same needs. For example, a homeowner who chose electric heat when utilities were offering promotional rates might now find that double-glazed windows pay for themselves in fuel savings and that 68° is comfortable given the cost of keeping the house at 75°.

When studying electrical power systems, it is important to know how the price of electricity and the prices of competing fuels will affect the overall demand for electricity both through changes in capital stock and through changes in operating characteristics. It is also important to know how the price of electricity will affect the shape of the demand curve. For example, with flat electrical rates, it may be worth it to consumers to keep their air conditioners off until late in the afternoon when it gets hottest. Then, since they have their air conditioners on for only a short time, their electricity bills are low and they are comfortable. Although for the utility, even though the energy demand is reduced, the peak power demand is the same or higher. This in turn may lead to even higher electricity prices. The obvious solution to this problem is to institute a price schedule that reflects the time-varying cost of generation. However, the point is that the price of electricity and its rate structure, the price of alternative fuels, and environmental factors will all affect the peak demand, the load shape, and the total energy demand. These in turn affect the cost of operating the electrical generators and affect new capacity requirements. It is worth noting that in long-range planning models used by utilities the demand is exogenously specified and is not linked to the cost of generating power. See, for example, the survey of models in Anderson [2].

There are two distinct camps in modeling the demand for electricity. Economists model the aggregate demand for electricity based on the historical relationship of electricity consumption with factors such as income level, regional fuel prices, and capital costs. Engineers model the demand for electricity by simulating the use of appliances in response to the weather, time-of-day, and other environmental factors.

Economists tend to ignore the time variation in demand and its physical causes while engineers tend to ignore the price response of demand. Of course, there are many models that combined elements of both, but the underlying bias toward physical causality or toward statistical correlation is usually apparent.

Both types of models will be used for this report. A two-level economic model will be used to predict shifts in demand in response to different rate structures and to preduct long-run changes in demand in response to the average price of electricity. An engineering model will be used to simulate the joint dependence of electrical demand and electrical supply for homeowners who elect to build weather-dependent generators. These models are only loosely linked. An integrated approach is currently under development by Hartman [45] at the MIT Energy Laboratory.

2.II. Long-Run Electrical Demand

The economic literature is full of long-run energy demand models, each with a different emphasis. For this report, the Baughman-Joskow demand model [7,8,9,14,54], a relatively simple, well-documented, regional model, will be used. In this model, the total residential and commercial energy demand for each state is calculated and then the relative fuel split is calculated. For the industrial demand, the total national energy demand is estimated, then allocated by state, then split by fuel type.

The Baughman-Joskow demand model does not explicitly account for trade-offs between the prices of fuel and the prices of the equipment associated with burning each fuel; however, it does account for trade-offs among fuels based on their relative prices. Because the estimates of the way people make choices are based on historical data, if one is willing to assume that the choices do not change drastically, then the projections are valid. So that, as long as no inventor comes up with a small, efficient, inexpensive cogenerator that can supply heat and electricity to a single-family house, or as long as the electric car does not become suddenly popular, then the Baughman-Joskow model is sufficient for the purposes of this study. A critique of economic demand models, including the Baughman-Joskow model, can be found in references [35] and [45].

Changes in the long-run demand due to installations of customer-owned generation will be addressed in Chapter 7. There, an optimization model will be used to model the trade-offs between capital and fuel in selecting new technologies. For now, the capital stocks, including customer-owned generators, are assumed to be fixed and known.

The Baughman-Joskow demand model presented here uses the equations and coefficients in the latest computer model. These are similar but not identical to those presented in reference [54]. Also, the coefficients have been adjusted so that the variables are given in the standard units used in the rest of this report. Unfortunately, a statistical analysis of the coefficients is not available.

2.II.A Residential-Commercial Demand

In the Baughman-Joskow model, the total residential energy demand for a particular region is estimated by:

$$ET^t_{Ra} = POP^t_a \exp(X1) \tag{2.1}$$

where

$$X1 = A1 + B1(PI^t_a/POP^t_a) + C1\ Temp^{min}_a + D1(POP^t_a/AREA_a)$$

$$+ E1\ \log(\bar{c}^t_{Ra} + F1\ \log(E^{t-1}_a/POP^{t-1}_a)$$

where

t = time period

ET^t_{Ra} = total residential and commercial energy consumption in region a at time t (MBtu)

POP^t_a = population in region a at time t

PI^t_a = personal income in region a at time t (1970 constant dollars/person)

$Temp^{min}_a$ = minimum temperature in region a (°F)

$AREA_a$ = area of region a (square miles)

$\bar{c}^t_{Ra}$ = weighted average residential-commercial energy price in region a (1970 constant dollars/MBtu)

The coefficients in equation (2.1) have been estimated on historical data and are given in Table 2.1.

Coef	Value
A1	.668
B1	2.69e-5
C1	-.0012
D1	9.36e-6
E1	-.134
F1	0.842

Residential-Commercial Total Demand Coefficients
Table 2.1

Dropping the regional subscript, the residential gas consumption in region a, relative to the regional residential electricity consumption, E_R^t, is given by:

$$EG_R^t = E_R^t \exp(X2) \qquad (2.2)$$

where

$$X2 = A2 + C2 \log (c_{RG}^t / c_{Rn}^t) + D2 \, Temp^{max}$$

$$+ F2 \, Temp^{min} + H2 \log(EG^{t-1}/E^{t-1}).$$

The residential oil consumption, relative to the residential electricity consumption is:

$$EO_R^t = E_R^t \exp(X3) \qquad (2.3)$$

where

$$X3 = B2 + C2 \log (c^t_{Ro}/c^t_{Rn}) + E2\ Temp^{max}$$

$$+ G2\ Temp^{min}_a + H2 \log(EO^{t-1}_R/E^{t-1}_R).$$

and where

E^t_R = electrical energy consumed by residential-commercial customers in region a at time t (MBtu)[1]

EG^t_R = gas energy consumed by residential-commercial customers in region a at time t (MBtu)

E^t_R = electrical energy consumed in region a at time t (MBtu) in region a at time t (MBtu)

EO^t_R = oil energy consumed by residential-commercial customers

c^t_{Rg} = residential gas price in region a at time t (1970 constant dollars/MBtu)

c^t_{Rn} = average residential electricity price in region a at time t (1970 constant dollars/MBtu)

c^t_{Ro} = residential oil price in region a at time t (1970 constant dollars/MBtu)

$Temp^{max}$ = maximum temperature in region a (°F)

The coefficients for equations (2.2) and (2.3) are given in Table 2.2.

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[1]These are Btu's of delivered energy, and do not include the energy lost in producing the electricity.

Coef	Value
A2	0.082
B2	0.415
C2	-0.207
D2	-0.00177
E2	-0.00429
F2	-0.00363
G2	-0.0102
H2	0.839

Residential-Commercial Fuel Split Coefficients

Table 2.2

Finally, the electrical energy consumed by residential and commercial customers in region a at time t is:

$$E_R^t = ET_R^t - EG_R^t - EO_R^t \qquad (2.4)$$

or

$$E_R^t = ET_R^t / \lfloor 1 + \exp(X2) + \exp(X3) \rfloor \qquad (2.5)$$

2.II.B <u>Industrial Demand</u>

For the industrial energy demand, first the national demand is estimated, then it is broken down by state. The total demand is:

$$ET_I^t = \exp\lfloor(A3 + B3 \log(\bar{c}_I^t) + C3 \log(VADD^t)) \qquad (2.6)$$

where

ET_I^t = national industrial energy demand in time t (MBtu)

$\bar{c}_I^t$ = national weighted average price of industrial energy in time t (1970 constant dollars/MBTU)

$VADD^t$ = value added to industrial goods in time t (1970 constant dollars)

The coefficients for equation (2.6) are given in Table 2.3.

Coefficient	Value
A3	26.48
B3	-0.219
C3	0.688

Industrial Energy Demand Coefficients
Table 2.3

The relative energy consumed by each state is found using

$$ET_{Ia}^t = ET_{1a}^t \exp[(A4 \log (\bar{c}_a^t/\bar{c}_1^t) + B4 \log (POP_a^t/POP_1^t) + C4 \log(ET_{Ia}^{t-1}/ET_{I1}^{t-1}) \qquad (2.7)$$

where the variables are the same as those defined in equation (2.6) except that they are given by region. In equation (2.7), all of the energies are computed relative to region 1. For the coefficients given in Table 2.4, region 1 is California. The absolute energy consumption for any region can be found from the relative weights and the total consumption.

COEF	VALUE
A4	-0.156
B4	-0.47
C4	0.927

Industrial State Allocation Coefficients

Table 2.4

The amount of gas consumed by industrial customers relative to the amount of electricity consumed by industrial customers is:

$$EG^{t}_{Ia} = E^{t}_{Ia} \exp\lfloor A5 + D5\log (c^{t}_{Iga}/c^{t}_{Ina}) + E5\log(EG^{t-1}_{Ia}/E^{t-1}_{Ia})\rfloor. \quad (2.8)$$

The amount of oil consumed relative to amount of electricity consumed is:

$$EO^{t}_{Ia} = E^{t}_{Ia} \exp\lfloor B5 + D5\log(c^{t}_{IOa}/c^{t}_{Ina}) + E5\log (EO^{t-1}_{Ia}/E^{t-1}_{Ia})\rfloor. \quad (2.9)$$

The amount of coal consumed relative to the amount of electricity consumed is:

$$EC^{t}_{Ia} = E^{t}_{Ia} \exp\lfloor C5 + D5\log(c^{t}_{ICa}/c^{t}_{Ina}) + E5\log (EC^{t-1}_{Ia}/E^{t-1}_{Ia})\rfloor. \quad (2.10)$$

Finally, the electrical energy consumed in a region a time t is:

$$E^t_{Ia} = ET^t_a - EG^t_{Ia} - EC^t_{Ia} - EO^t_{Ia} \qquad (2.11)$$

where

ET^t_{Ia} = total energy consumed by industry in region a in time t (MBtu)

E^t_{Ia} = Electrical energy consumed by industry in region a in time t (MBtu)

EG^t_{Ia} = Gas consumed by industry in region a in time t (MBtu)

EO^t_{Ia} = oil consumed by industry in region a in time t (MBtu)

EC^t_{Ia} = coal consumed by industry in region a in time t (MBtu)

c^t_{Ina} = electricity price for industry in region a in time t (1970 constant dollars/MBtu)

c^t_{IGa} = gas price for industry in region a in time t (1970 constant dollars/MBtu)

c^t_{IOa} = oil price for industry in region a in time t (1970 constant dollars/MBtu)

c^t_{ICa} = coal price for industry in region a in time t (1970 constant dollars/MBtu).

The coefficients for equations (2.8) to (2.10) are given in Table 2.5.

COEF	VALUE
A	-0.231
B5	-0.354
C5	-0.540
D5	0.301
E5	0.856

Industrial Fuel Split Coefficients

Table 2.5

Equations (2.1) through (2.11) can be used to estimate the total electrical demand, residential, commercial, and industrial, based on projections of population growth, personal income, value added, cost of capital, and fuel prices. Within the model, the fuel prices are linked so that the price the utility pays for fuel is based on the price of fuel to industry. Thus, the computed price for electricity is consistent with the other fuel prices.

2.III. Short-Run Electrical Demand

In the short run, consumers can not adjust their capital stock to take the most advantage of changing fuel prices. At best, they can change their overall consumption using the same capital stock or they can change their patterns of usage to take advantage of time-differentiated rates. This section will study changes in patterns of usage due to time-of-day rates and to customer-owned generation.

The changes in overall consumption are measured relative to the forecast from the long range model described above. Changes in patterns of use will be measured relative to a base case load shape, for example, from the last year of historical data. Marginal changes to the base load shape will be computed based on an economic model of consumer reponse to time-of-day rates and on an engineering model of customer-owned generation.

This section will discuss only residential customers. Many industrial electricity users have had time-of-day rates for many years, but there is very little data available. Acton and Mitchell [1] have studied industrial time-of-day rates in Britain and the United States. However, because of the difficulty of getting matched cross-sectional data, most of their evidence is anecdotal. When models of industrial response to time-of-day rates become available, they can be fit into the structure described here.

For convenience, residential customers have been divided into two classes: those who own their own electrical generators and those who do not. This section will deal only with those customers who do not own generators. The next section will deal with those who do.

2.III.A. Load Shifting

Much has been written about how time-of-day rates should cause customers to shift their demand away from peak demand periods and much has been written about how time-of-day rates should be calculated, but little is known about how customers in the United States respond to time-of-day rates. Since 1974, the Federal Energy Administration (FEA) has sponsored residential time-of-day pricing experiments throughout the country. A great deal of data has been collected, but it has only begun to be analysed.

Hausman, Kinnucan, and McFadden [48] have analysed the data from the Connecticut peak load pricing experiment and have developed a model to predict how residential users respond to time-of-day rates. The coefficients for their model have been estimated only on the Connecticut data, but the structure of the model is general and could be expanded when more data becomes available. In particular, because there was not enought variation in the prices, it was not possible for them to estimate cross-price effects. That is, effects from people changing the time at which they do something in order to take advantage of a price differential. The cross-price terms have been dropped from the presentation given here. They can be found in the original reference.

The model has two demand levels. On the first level, the total electrical demand is computed based on its price relative to other fuels. Then, since the total demand is set, the demand in each time period is computed based on the relative electrical prices. The Baughman-Joskow model will be used for the first level and Hausman, Kinnucan, and McFadden model for the second level. Under their structure, if the price of electricity were to double the overall consumption would be reduced, but as long as the relative prices remained the same, then the relative consumption in each time period would remain the same. This assumption is not overly restrictive for our purposes.

The electrical energy demand for customer k in subperiod s is found relative to consumption in a period in which the demand and price are both known, e.g. a corresponding historical time period:

$$e_k^s = A6^s + \sum_{i=1}^{I} B6_i^s \ App_{ik} + \sum_{i=2}^{I_1} C6_i^s \ App_{ik} \ App_{1k}$$

$$+ \sum_{j=1}^{J} D6_k^s \ Soc_{jk} + E6^s c_k^s$$

$$+ \sum_{i=1}^{I} F6_i^s \ c_k^s \ App_{ik} + \sum_{i=2}^{I_1} G6_i^s \ c_k^s \ App_{ik} \ App_{1k} \tag{2.12}$$

$$+ \sum_{j=1}^{J} H6_j^s \, c_k^s \, Soc_{jk} + I6^s \, Temp^s$$

$$+ J6^s \, c_k^s \, Temp^s + \sum_{i=I_2}^{I_3} K6_i^s \, App_{ik} \, Temp^s$$

$$+ \sum_{i=I_2}^{I_3} L6_i^s \, c_k^s \, App_{ik} \, Temp^s + \sum_{n=1}^{7} M6_n^s \, Day_n$$

where

s = subperiod

c_k^s = price of electricity to customer k in subperiod s relative to the price in the base period

App_{ik} = 0-1 variable indicating whether customer k owns appliance i

- App_{1k} = hot water heater
- $App_{2 \text{ to } I_1, k}$ depend on the hot water heater
- $App_{I_2 \text{ to } I_3, k}$ depend on the temperature

Soc_{jk} = sociological factors about customer k such as number of people in the household and the income level.

$Temp^s$ = temperature in subperiod s (°F).

Day_n = 0-1 variable indicating the day of the week (Day_1 = Sunday).

For any class of customers, i.e., those having the same appliance stock and sociological factors, equation (2.12) can be simplified to just those terms that depend on the temperature and time:

$$e_k^s = A7_k^s + B7_k^s\, c_k^s + C7_k^s\, Temp^s \tag{2.13}$$

$$+ D7_k^s\, c_k^s\, Temp^s + \sum_{n=1}^{7} M6_n^s\, Day^m$$

where $$A7_k^s = A6_k^s + \sum_{i=1}^{I} B6_i^s\, App_{ik} + \sum_{i=2}^{I_1} C6_i\, App_{ik}\, App_{ik}$$

$$+ \sum_{j=1} D6_j^s\, Spc_{jk}$$

$$B7_k^s = E6^s + \sum_{i=1}^{I} F6_i^s\, App_{ik} + \sum_{i=2}^{I_1} G6_i^s\, App_{ik}\, App_{1k}$$

$$+ \sum_{j=1} H6_j^s\, Soc_{jk}$$

$$C7_k^s = I6^s + \sum_{i=I_2}^{I_3} K6_i^s\, App_{ik}$$

$$D7_k^s = J6^s + \sum_{i=I_2}^{I_3} L6_i^s\, App_{ik}$$

and the rest of the terms are as defined in equation (2.12).

Table 2.6 defines the appliances and sociological factors that were included in the study. Since the data is for winter in Connecticut, air conditioning load is not included. Table 2.7 gives the estimates of the coefficients in equation (2.12). The statistical analysis of the coefficients, omitted for simplicity, can be found in the original reference.

App_1:	electric hot water heater
App_2:	dishwasher (depends on water heater)
App_3:	clothes washer (depends on water heater)
App_4:	electric heat (depends on temperature)
App_5:	electric range
App_6:	freezer
App_7:	electric clothes drier
Soc_1:	home during day (0-1 variable)
Soc_2:	home size (square feet)
Soc_3:	home type (single home or multiple dwelling)
Soc_4:	people in home
Soc_5:	income (1000's dollars)

Appliances and Sociological Factors in the Connecticut Peak Load Pricing Test

Table 2.6 from reference [48], p. 276

Using equation (2.12), the coefficients in Table 2.7, and statistics on the number of residential customers and the types of appliances they own, shifts in the residential load curve can be computed.

The change in the electrical energy demand can be written as:

$$\Delta E_c^{st} = E^{st} - E_c^{st} \qquad (2.14)$$

where

E_c^{st} = energy demand after price shifting (MBtu).

Winter Weekday Load Shift Coefficients

Table 2.7 from reference [48], p. 276

Coefficient		Peak Period	Intermediate Period	Off-Peak Period
A6		-2.230	-2.370	-1.380
$B6_1$	(water heater)	.010	.545	.058
$B6_2$	(dishwasher)	-0.064	-0.215	.152
$B6_4$	(heat)	.039	-0.163	-0.356
$B6_5$	(range)	.166	.374	-0.126
$B6_6$	(freezer)	-0.283	-0.407	-0.462
$B6_7$	(dryer)	.222	.450	.705
$C6_2$	(dish x water)	.307	.266	.192
$C6_3$	(clothes x water)	-0.079	-0.899	-0.411
$D6_1$	(at home)	.257	.079	-0.228
$D6_2$	(size)	.084	.114	-0.076
$D6_3$	(type)	-0.395	-0.206	-0.316
$D6_4$	(number)	.070	.038	.065
$D6_5$	(income)	.0058	.0074	.0020
E6	(price c)	.011	.137	.019
$F6_1$	(c x water)	-0.018	-0.208	-0.200
$F6_2$	(c x dish)	.014	.062	.066
$F6_4$	(c x heat)	-0.012	-0.283	-0.021
$F6_5$	(c x range)	-0.020	-0.031	-0.073
$F6_6$	(c x freezer)	.018	.123	-0.036
$F6_7$	(c x dryer)	-0.022	-0.181	-0.141
$G6_2$	(c x dish x water)	-0.027	-0.017	-0.067
$G6_3$	(c x clothes x water)	.043	.379	.266
$H6_1$	(c x at home)	-0.005	.061	.118
$H6_2$	(c x size)	-0.010	-0.084	.019
$H6_3$	(c x type)	.021	-0.024	.030
$H6_4$	(c x number)	-0.002	.0098	-0.025
$H6_5$	(c x income)	-0.0006	-0.0065	-0.0028

continued on next page

Table 2.7 (continued)

Coefficient		Peak Period	Intermediate Period	Off-Peak Period
I6	(temp)	-0.0006	.0024	-0.012
J6	(c x temp)	.0002	-.0002	.004
$K6_4$	(temp x heat)	.037	-0.047	.034
$L6_4$	(c x temp x heat)	.0006	.024	-0.020
$M6_3$	(Tuesday)	-0.011	-0.107	.002
$M6_4$	(Wednesday)	-0.043	-0.079	.051
$M6_5$	(Thursday)	-0.011	-0.109	.032
$M6_6$	(Friday)	-0.029	-0.153	.050

By definition, e_k^{st} is:

$$e_k^{st} = E_c^{st}/E_k^{s\tau} \qquad (2.15)$$

where

E_k^{st} = demand in subperiod s of the current time period by customer k (MBtu)

The change in energy demand can be written as:

$$\Delta E_c^{st} = \sum_s \sum_k (E_k^{st} - e_k^{st} E_k^{s\tau})$$

$$= E^t - E^\tau \sum_s \sum_k e_k^{st}, \qquad (2.16)$$

so the net energy demand is known as a function of the demand in the current time period and the load shift coefficients.

2.III.B Customer-Owned Generation

The economic demand models discussed above cannot account for new technologies, nor can they account for the specific choices made by particular customers. Since the purpose of this report is to study the customers' options for generating their own electricity, a different type of model is required. In a Chapter 6, a model of how customers decide what to build and how much to build will be developed. In this chapter, it is assumed that the amount and type of customer-owned generation is known. The problem is to find how customer-owned generation, particularly time-dependent generation, affects the energy demand on the utility.

The output from an electrical generator can be described in great detail by modeling the steam cycle, harmonics of the stator and the rotor, the mechanical and electrical inertia, the power angle and so on. For studies of the power conditioning and safety equipment required for small generators, such detail would be necessary. For a discussion of some of the technical issues of integrating small electric generators with a utility see reference [94]. A much simpler model will be used here both for the central and decentral generators.

For conventional generators, that is, those that are not time-dependent, the only necessary parameters are the capacity, the heat rate (efficiency), and the forced outage rate. In Chapter 3, it will be shown how the generators and load are modeled together. For the time being, it is only necessary to define the operating capacity of a generator. The operating capacity is a random variable given by:

$$\tilde{Y}_{ik} = \tilde{\Psi}_i X_{ik} \tag{2.17}$$

where $\tilde{Y}_{ik}$ = operating capacity of the unit i owned by customer k (MW)

$\tilde{\Psi}_i$ = zero-one random variable describing forced outages for unit i

X_{ik} = rated capacity of unit i owned by customer k (MW).

The distribution of the forced outage component is:

$$P[\tilde{\Psi}_i = y] = \begin{cases} p_i & y = X \\ q_i & y = 0 \end{cases} \tag{2.18}$$

$$p_i + q_i = 1$$

where q_i = probability of forced outage of unit i.

For time-dependent generators, additional information on the nature of the time dependence is needed. Detailed models of different types of generators are available. Within this study, it is sufficient to know that the output of a nominally sized generator can be written as:

$$\tilde{n}_i^s = f_i(w_s) \tag{2.19}$$

where s = subperiod (e.g., hour)

w_s = vector of meteorological data (e.g., wind speed, ambient temperature, solar insolation)

f_i = function that transforms meteorological data into electrical output for generator i

$\tilde{n}_i^s$ = normalized output for generator type i in subperiod s.

In equation (2.19), it is implicitly assumed that the function f_i is independent of the size of the generator. That is, if a one megawatt generator can produce x megawatts, then, under the same conditions, a ten megawatt generator can produce 10x megawatts. If this assumption is violated for a particular generation type, for example, wind, then each size range can be labeled as a distinct technology.

The operating capacity of a time-dependent generator is written:

$$\tilde{Y}^s_{ik} = \tilde{n}^s_i \tilde{\Psi}_i X_{ik} \tag{2.20}$$

where

$\tilde{Y}^s_{ik}$ = operating capacity of unit i owned by customer k in superiod s (MW).

For computing the long-run energy demand, it is only necessary to know the total energy produced by customer-owned generators. Their effect on power demand will be discussed in Chapter 3.

The total energy supplied by a time-dependent generator is the integral over time of the operating capacity:

$$E^s_{ik} = \sum Y^s_{ik} h_s = \sum \tilde{n}^s_i \tilde{\Psi}_i X_i h_s = h_s \tilde{\Psi}_i X_{ik} \sum \tilde{n}^s_i \tag{2.21}$$

where h_s = hours in subperiod s.

Define the total nominal energy output from a unit to be:

$$\tilde{H}_s \equiv \sum \tilde{\eta}_s \tag{2.22}$$

Given that the mechanical failures of the customer-owned generators are independent, with the additional assumption that all generators of the same type have the same binomial failure rate no matter who owns them, and there are enough of them, then the sum of their outages is normally distributed:

$$\tilde{\Psi}_i = \sum_{k=1}^{K} \tilde{\Psi}_{ik} \quad (2.23)$$

$$\tilde{\Psi}_i \sim N\,(n_i p_i,\ n_i p_i q_i) \quad (2.24)$$

where n_i = number of installations of type i.

As a rule of thumb, the normal approximation to the binomial distribution is valid in the region where:

$$E(\Psi_i) > 3\lfloor VAR(\Psi_i)\rfloor^{1/2} \quad (2.25)$$

where $E(\Psi_i) = n_i p_i$

$VAR(\Psi_i) = n_i p_i q_i$ for the binomial distribution

So, for outage rates on the order of .1, there would have to be more than 81 installations to use the approximation.

Assuming there are enough customer owned generators on the system,, then the expected energy generated by units of type i is:

$$\Delta E_i^t = n_i p_i X_i \quad (2.26)$$

where ΔE_i^t = change in electricity demand due to system i in time t (MBtu)

X_i = installed capacity for system i (MW)

If we then assume that all the electricity generated by the time dependent units displaces electricity that otherwise would have been produced by the grid, then the net reduction in energy demand can be computed.

In computing the change in energy, it was implicitly assumed that the energy from the time-dependent units would be used whenever it was available. This assumption may not be true for conventional generators, such as diesels, which may be used only under special circumstances. For this type of generator, the function described in equation (2.16) might require more complicated inputs.

2.IV. Summary

From equations (2.1) and (2.11), estimates of the total electrical demand in area a in time t can be found using the current price and demand for electricity along with projections of their future values. These equations can be expressed in functional form as:

$$E_a^t = f(E_a^{t-1}, \hat{c}_a^t, \hat{\Omega}_a^t, w_a) \qquad t = \tau + 1, \ldots T \qquad (2.27)$$

where

E_a^t = electrical energy consumption in region a in time t

$\hat{c}_a^t$ = vector of estimates of fuel prices for region a in time t

$\hat{\Omega}_a^t$ = vector of estimates of demographic and economic factors for region a at time t

w_a = vector of weather parameters for region a.

From equation (2.13), the consumption in a subperiod, relative to a known subperiod, can be estimated for each customer class in a given region:

$$e_{ak}^{st} = E_{ak}^{st}/E_{ak}^{s} = f(E^{t-1}, \hat{c}_{nk}^{st}, \hat{\Omega}_{ak}^{t}, w_a^{st}) \qquad t = \tau + 1, \ldots T \qquad (2.28)$$

and

$$\Delta E_a = E_a^t - E_a^\tau \sum_s \sum_k e_k^{st} \qquad t = \tau + 1,\ldots T \qquad (2.29)$$

where

ΔE_a^t = change in energy demand due to price of shifting in region a in time t

$\hat{c}_{nk}^{st}$ = estimated price of electricity for customer k in subperiod s, time period t relative to the price in subperiod s, time period

$\hat{\Omega}_k^t$ = vector of estimates of demographic and economic factors for customer k in time period t

w_a^{st} = vector of weather parameters for region a for subperiod s, time period t.

From equation (2.26), the change in electrical demand due to output of custom-owned generators of type i can be found:

$$\Delta E_i^t = p_i \, X_{ia}^t \, H_{ia} \qquad (2.30)$$

where

p_i = forced outage rate of unit type i

X_{ia}^t = installed capacity of type i in region a at time t (MW)

H_{ia} = sum of annual normalized output of generator a in region i.

3. Utility Load Shape

To find the effect on the utility of changes in demand, it is necessary to know not just the change in the energy consumption, but also the change in the patterns of consumption. The load reduction model uses the current customer capital stock (Chapter 6), the current price of electricity (Chapter 7), and the exogenously specified weather and prices for competing fuel to compute the shape of the net load duration curve.

The net load duration curve is used in Chapter 4 to compute the cost of operating the central station units to meet demand. If desired, the net load shape can also be projected for future years and used instead of the base case load shape in the utility expansion model described in Chapter 5.

3.I. Introduction

The instantaneous demand on the central generators of a utility is the sum of the demands from all its customers, residential, commercial, and industrial, plus the demand due to losses in the transmission and distribution system, plus any demand from other utilities due to purchase agreements. The transmission lines that link utilities are monitored closely, so the external demand is known. However, without costly metering and monitoring an electric utility cannot determine which customers are demanding power at any instant. The only accounting is in monthly energy metering which does not reveal the variations in power demand over time.

A utility cannot tell whether customers have decided to consume less electricity due to a peak price or whether customers are using an alternative source to supply some of their electricity or whether the load is lower than expected due to some unknown factor. This suggests that load shifts and decentralized generators can be modeled by appropriately modifying the projected demand on the utility. This chapter will show how the energy demand, load shifting, and customer-owned generation are combined to find the net load duration curve, that can then be used in the analysis of the effect of load shifting and customer-owned generation on the electric utility.

First, the total demand for the utility is calculated based on current and projected prices using the long-run demand model described in Chapter 2. This results in projections for the annual energy demand for each of the remaining years of the study, ignoring load shifts and customer-owned generation.

Then, the marginal changes in the load shape due to time-of-day pricing are computed. If flat rates assumed, this step is skipped because the current model would predict no changes. With a more sophisticated model, load shifts could be computed if the relative price of electricity rose or fell even though the rates were flat.

Finally, the electricity supplied by the decentralized generators is subtracted from the load leaving the net load to be supplied by the central station generators. There are, however, several complications with this procedure. One is that the decentralized electrical generation may vary with the time of day and may be correlated with the original demand. Another is that the output of different decentralized generators may be correlated with one another complicating the subtraction procedure. Also, the electricity from decentralized generators is consumed close to the point of generation and so bypasses much of the distribution system and the losses incurred therein.

The load shifting and customer owned generation can be modeled independently because it was assumed that owners of their own generation shifted their loads based on the expected price of electricity in a given time period. The expected price includes both grid electricity and user-generated electricity. Of course, load shifting would occur based on the actual price in that hour, not the expectation. That is, a customer might expect the photovoltaic generator to be working on a sunny noonday and plan to run a load of wash; however, if the generator failed to work, the wash would be canceled and the repair person called. In a future model, it may be possible to combine the load shifting within the decentralized generation model.

In this chapter, a procedure for modeling decentralized generators as negative demands will be developed. Using this methodology, the utility system can be studied with and without decentralized generators and changes in the installed capacity of decentralized generators can be modeled easily.

3.II Load Representation

From historical data, one realization of the random variable of the electricity demand can be observed. The underlying process is assumed to have the form:

$$P[Y^{st}] = x[w_s, s, c_n^{st}] \tag{3.1}$$

where

Y^{st} = electrical demand at subperiod s of time period t

w_s = vector of meteorological data for time s

s = subperiod

c_n^{st} = price of electricity in subperiod s, time period t.

Let $E(Y^{st} | c) = g^{st}(c)$ (3.2)

where $g^{st}(c)$ = demand response function to price c in subperiod s, time period t

$E(Y^{st}|c)$ = expected electrical demand given the price in subperiod s, time period t.

A typical demand response function is illustrated in Figure 3.1.

Assume also that the distribution of the load, as a function of weather, is independent of the price. That is, the mean of the distribution moves along the demand response curve, but the shape of the distribution remains unchanged as shown in Figure 3.2. So, from historical data, the function

$$P[Y^{st} = x | w_s, s] \quad (3.3)$$

can be estimated by grouping observations of load taken at a particular time of day under similar weather conditions. If observations of load under similar weather conditions, but with different pricing schedules, were available, it would be possible to test the validity of the assumption that the distribution remained constant. When further data from peak load pricings studies become available, it may be possible to test this assumption.

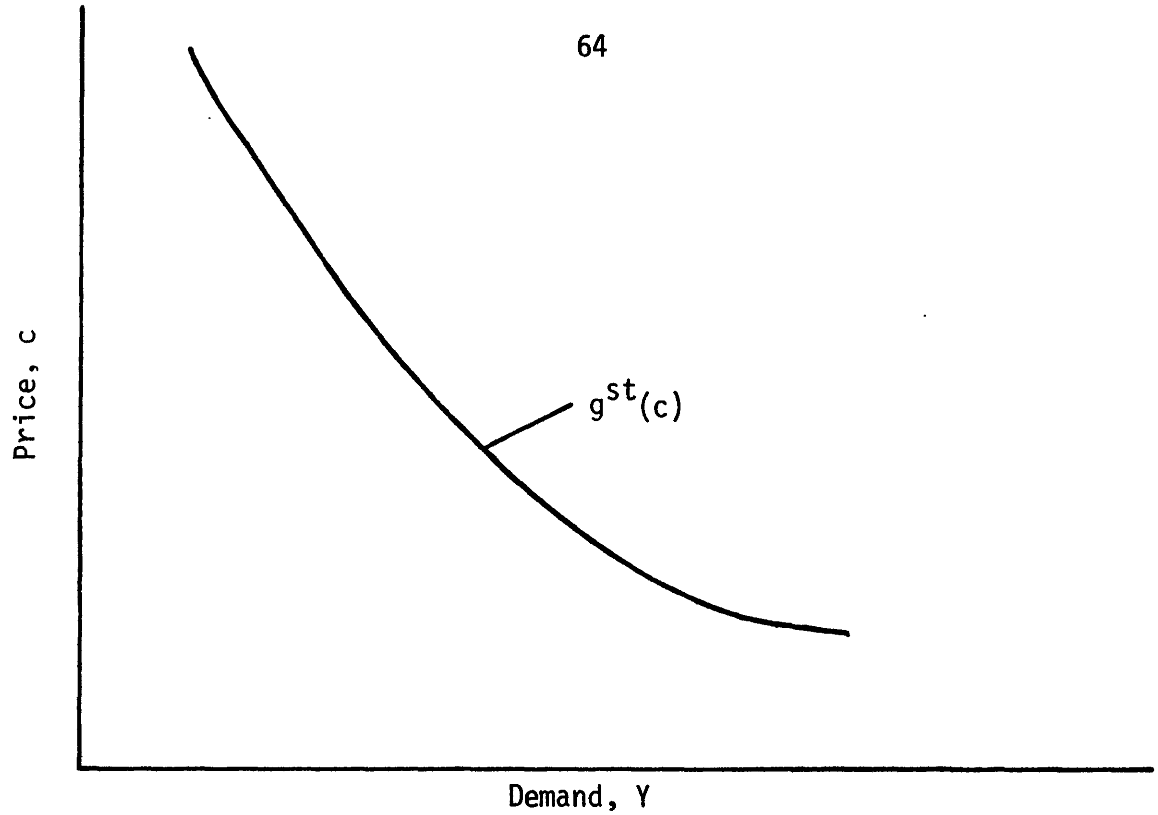


Figure 3.1 Demand Response Function

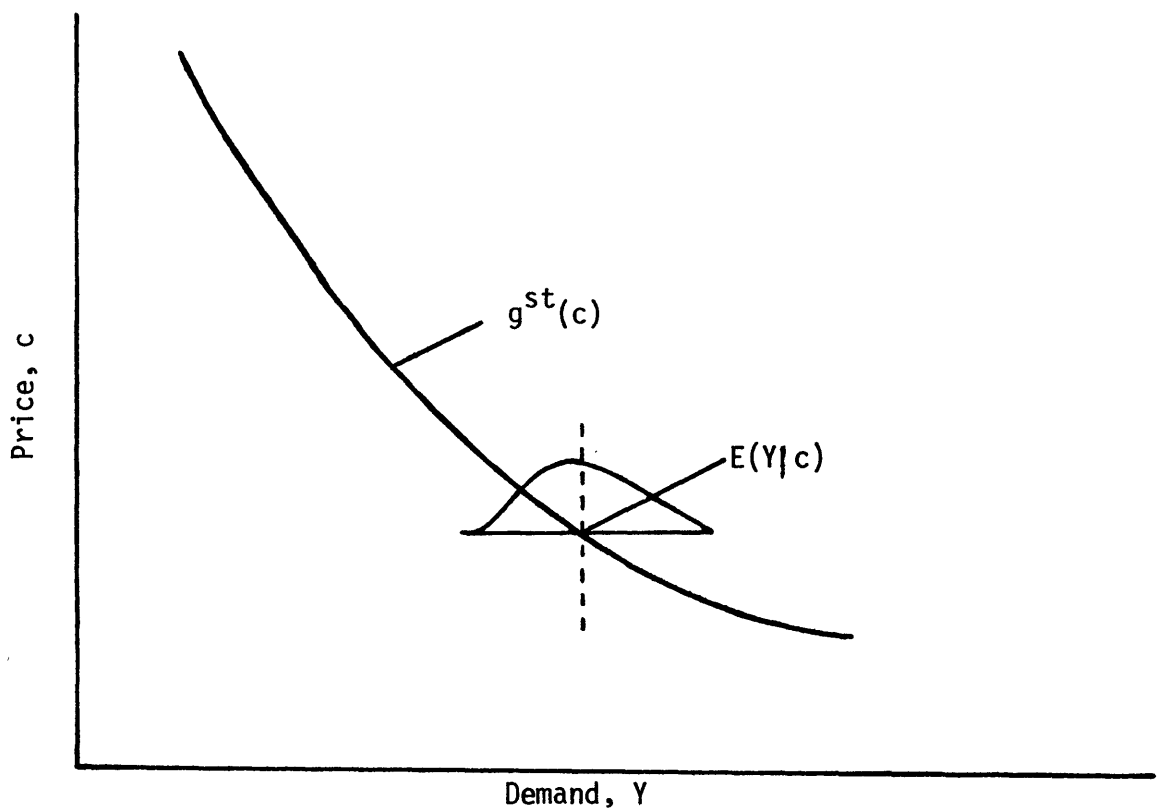


Figure 3.2 Demand Response with Random Component

The total instantaneous demand for electricity by a particular customer can be represented as a random variable that is a function of many variables including weather, time of day, and lifestyle. Customer may modify this demand because of time varying prices. In addition, customers may meet some of their demand with an alternative source to the electric utility. The net demand on the utility can then be written as:

$$\tilde{Y}^{st}_{nk} = \tilde{Y}^{st}_{ok} - \Delta\tilde{Y}^{st}_{k} \tag{3.4}$$

where $$\Delta\tilde{Y}^{st}_{k} = \Delta\tilde{Y}^{st}_{ck} + \sum_{i=1}^{N} \tilde{Y}^{st}_{ik} \tag{3.5}$$

and $\tilde{Y}^{st}_{nk}$ = net electrical demand on the utility from customer k in subperiod s, time period t(MW)

$\tilde{Y}^{st}_{ok}$ = original electrical demand from customer k in subperiod s, time period t(MW)

$\Delta\tilde{Y}^{st}_{k}$ = reduction in electric demand by customer k(MW)

$\Delta\tilde{Y}^{st}_{ck}$ = reduction in electric demand by customer k due to the price in subperiod s, time period k(MW)

$\tilde{Y}^{st}_{ik}$ = output from generator type i owned by customer k in subperiod s, time period t(MW).

N = number of generator types available to customers (e.g. windmills or photovoltaics).

For notational simplicity, let

$$\tilde{\bar{Y}}^{st}_{k} = \sum_{i=1}^{N} \tilde{Y}^{st}_{ik} . \tag{3.6}$$

The probability distribution of the original demand from a customer is difficult to model or estimate for several reasons. One is that there is not much data to work from. Another is that individual households vary greatly in their consumption and in their consumption patterns. In addition, the demand from a single household has a great deal of variance. For example, the demand may be from just an electric clock until someone comes home to cook dinner, then the demand may jump suddenly to include lights, a stereo, and the oven.

So that, if the net demand on the utility is represented as the sum of the demands from all customers, it must be recognized that some demands are random while others have causative factors. For example, refrigerators go on and off all day and from a central limit argument, one would expect only a percentage of them to be on at any time. This type of load is called diversified load. On the other hand, people turn on lights when it gets dark and tend to eat meals at about the same times. This type of load is called co-incident load.

We shall see, however, that with the methodology developed here the original demand from an individual customer is never required. Returning to equation (3.4), to find the total net demand on the utility, the sum over all customers must be taken. Suppressing the time period superscript, yields:

$$\tilde{Y}_n^s = \sum_{k=1}^{K} \tilde{Y}_{nk}^s = \sum_{k=1}^{K} (\tilde{Y}_{ok}^s - \Delta\tilde{Y}_k^s) \qquad (3.7)$$

where $\tilde{Y}_n^s$ = net demand on the electric utility in subperiod s (MW)

K = total number of customers

Under certain conditions, the original demand and the reductions in demand are independent random variables, so equation (3.7) can be written:

$$\tilde{Y}_n^s = \tilde{Y}_o^s - \sum_{k=1}^{K} \Delta\tilde{Y}_k^s \qquad (3.8)$$

where $\tilde{Y}_o^s$ = original demand on the electric utility in subperiod s (MW)

The probability distribution of the original customer demand can be inferred from historical data, as will be done here, or it can be derived from a more sophisticated demand model if one is available. In any case, this chapter will focus on finding the distribution of the changes in electrical demand due to time of day pricing and customer owned generation.

3.II.A Price - Dependent Demand

In Chapter 2, the energy demand in one time period was found relative to that in a corresponding base case time period as a function of price , appliance stock, sociological factors, temperature and time. Assuming that the average power demand is proportional to the energy demand, the relative energy demand from equation (2.13) can be written in terms of the variables in equation (3.4):

$$e_k^s = (\tilde{Y}_{ok}^s - \Delta\tilde{Y}_{ck}^s)\ /\tilde{Y}_{ok}^s\ . \qquad (3.9)$$

Defining the price modified demand, $\tilde{Y}_{ck}^s$, as:

$$\tilde{Y}_{ck}^s = \tilde{Y}_{ok}^s - \Delta\tilde{Y}_{ck}^s, \qquad (3.10)$$

then $\tilde{Y}_{ck}^s = e_k^s\ \tilde{Y}_{ok}^s,$ (3.11) and the total price modified demand is:

$$\tilde{Y}_c^s = \sum_{k=1}^{K} \tilde{Y}_{ck}^s = \sum_{k=1}^{K} e_k^s \; \tilde{Y}_{ok}^s \qquad (3.12)$$

$$= \tilde{Y}_o^s \sum_{k=1}^{K} e_k^s \qquad (3.13)$$

The economic demand model does not derive a probability distribution for the changes in demand due to price. For lack of a better assumption, the original demand and the price modified demand are assumed to have the same distribution, shifted by a constant:

$$P_{Y_c}[\tilde{Y}_c^s \leq x] = P_{Y_o}[\tilde{Y}_o^s \leq x + k^s] \qquad (3.14)$$

where the subscript on the probability distribution, P, indicates the random variable that is being described. So, from equations (3.13) and (3.14), the change in demand due to time varying prices and the distribution of the modified demand can be found.

3.II.B Customer-Owned Generation

Modeling the operation of a small generator running in parallel with the utility system can be complicated. One must know the characteristics of all the electrical appliances, when they are likely to be used and whether or not this demand can be delayed. One must know which appliances are used in response to what weather conditions. One must know the relatively costs of buying and selling electricity at different times of day and the customer's strategy for using storage or other load shifting techniques. One must also know the characteristics of the generator: its size, efficiency, and response to differing weather conditions. Since all of these data are required on an hourly basis, models of small power generators can become rather large. Several theses have been written at the MIT Energy Laboratory on this problem alone [17,27,87].

The major reason for using hour by hour simulators for small generators is that the electrical demand and electrical generation both depend on the weather and on the time of day. This correlation must be modeled to get a good estimate of the value of the system. For example, air conditioning load and solar insolation are highly positively correlated, so a photovoltaic array would have a high value for meeting air conditioning demand. On the other hand, a wind turbine would probably have a low value for air conditioning demand, but a high value for space heating demand. Another reason for using hourly simulators is that there are frequently inter-hour dependencies. Some demands, for example, clothes washing, can be performed earlier than planned to take advantage of excess energy or can be delayed to take advantage of lower rates.

For this study, a simplified generation model will be used. This model will ignore many of the complications mentioned above, although the structure of the study allows a more complex model to be used if it is required.

The random variable representing the output of a customer-owned generator has two components. One component represents the variable output from a generator due to, for example, fluctuations in solar insolation or wind intensity. The other component represents the changes in output due to mechanical failures. The latter component is assumed to be independent of time. For simplicity, two further assumptions will be made. The first is that the output of a machine is a linear function of its size. That is, if a 100 MW solar array produces x megawatts, then, under identical conditions, a 200 MW solar array produces 2x megawatts. The second assumption is that mechanical failures always result in zero output. That is, a generator cannot run at reduced capacity if there are mechanical failures.

With these simplifying assumptions, the output of generation type i for customer k at time s is given by:

$$\tilde{Y}_{ik}^{s} = X_k \; \tilde{\Psi}_i \; \tilde{\eta}_i^{s} \qquad (3.15)$$

where

X_{ik} = installed capacity of generation type i for customer k

$\tilde{\psi}_{ik}$ = zero-one random variable representing mechanical failure for generation type i, owned by customer k

$\tilde{\eta}_i^{s}$ = random variable representing fluctuations in output due to time or weather for generation type i $[0 \leq \eta \leq 1]$.

The distribution of $\tilde{\psi}_i$ is:

$$P_X[\tilde{\psi}_i = x] = \begin{cases} p_i & x = 1 \\ q_i & x = 0 \end{cases} \qquad (3.16)$$

and $\quad p_i + q_i = 1$

where q_i is the probability of mechanical failure for generation type i.

Rewriting equations (3.4) and (3.5) using equations (3.14) and (3.15) yields:

$$\tilde{Y}^s_{nk} = \tilde{Y}^s_{ck} - \sum_{i=1}^{N} X_{ik}\, \tilde{\Psi}_{ik}\, \tilde{n}^s_i \tag{3.17}$$

or, summing over all customers:

$$\tilde{Y}^s_{n} = \tilde{Y}^s_{c} - \sum_{i=1}^{N} \tilde{n}^s_i \sum_{k=1}^{K} \tilde{\Psi}_{ik} X_{ik}. \tag{3.18}$$

If the total installed capacity for each generator type is known and all generators of a given type behave the same way irrespective of ownership then equation (3.18) becomes:

$$\tilde{Y}^s_{n} = \tilde{Y}^s_{c} - \sum_{i=1}^{N} \tilde{n}^s_i\, X_i \sum_{k=1}^{K} \tilde{\Psi}_{ik} \tag{3.19}$$

where X_i = installed capacity of generator type i (MW).

To find the distribution of the net load we need to combine the distributions of the price modified load and the customer owned generation. That is, we must look at all possible combinations of load and generation and weight them by their probability of occurrence. If the load and generation were independent, this would be a relatively straight forward procedure; however, we know that the load and generation are not independent for time and weather dependent generators. But, it is reasonable to assume that conditioned on time and weather that the load and generator output are independent and that generator outputs are independent of one another. That is to say, given that it is dark and windy, the output of a wind turbine will not affect the output from a photovoltaic array nor will it affect the customer's desire to have a light on. And, illustrating a previous assumption, the mechanical failure of the photovoltaic array is independent of its output, the output of the wind turbine and the customer demand.

The conditional distribution for the output of generator i is:

$$P_{Y_i}\lfloor \tilde{Y}_i^{\,s} = xX_i \mid s, w_s \rfloor = \begin{cases} p_i \; P_\eta\lfloor \tilde{\eta}_i^{\,s} = x \mid s, w_s \rfloor, & x > 0 \\ q_i \; P_\eta\lfloor \tilde{\eta}_i^{\,s} > 0 \mid s, w_s \rfloor \\ \quad + P_\eta\lfloor \tilde{\eta}_i^{\,s} = 0 \mid s, w_s \rfloor, & x = 0 \end{cases} \tag{3.20}$$

Equation (3.20) can be rewritten as:

$$P_{Y_i}\lfloor \tilde{Y}_i^{\,s} = xX_i \mid s, w_s \rfloor = \sum_y P \lfloor \tilde{\Psi}_i = \frac{x}{y} \rfloor \; P_\eta\lfloor \tilde{\eta}_i^{\,s} = y \mid s, w_s \rfloor \tag{3.21}$$

where the distribution of $\tilde{\Psi}_i$, the mechanical failure, as defined in (3.16) with the additional definitions:

$$\begin{aligned} P \lfloor \tilde{\Psi}_i = \frac{x}{y} \rfloor &\equiv 1 \qquad \text{for } x = y = 0 \\ P \lfloor \tilde{\Psi}_i = \frac{x}{y} \rfloor &\equiv 0 \qquad \text{for } x = y \neq 0 \end{aligned} \tag{3.22}$$

If the output from different types of generators are assumed to be independent conditional on the time and weather, then the conditional distribution of the sum of their outputs is given by:

$$\begin{aligned} P_{Y_i}\lfloor \tilde{Y}_i^{\,s} = xX_i \mid s, w_s \rfloor &= P_{Y1} * P_{Y2} * \cdots \\ &* P_{YN}\lfloor \tilde{Y}_N^{s} = xX_i \mid s, w_s \rfloor \end{aligned} \tag{3.23}$$

where the symbol '*' represents convolution.

Defining the multiplicative convolution of (3.21) by 'o', equation (3.23) becomes:

$$P_{y_i}[\tilde{\tilde{Y}}^s = xX_i \mid s, w_s] = (P^1 \text{ o } P_\eta^1)*(P^2 \text{ o } P_\eta^2)$$

$$* \dots * (P^N \text{ o } P_\eta^N[\tilde{\eta}_N^s = \frac{x}{X_i} \mid s, w_s]) \tag{3.24}$$

Since the distribution of the total customer demand is known, the distribution of the net demand can be found using equation (3.18) and equation (3.22):

$$P[\tilde{Y}_n^s = x \mid s, w_s] = \sum_y P[Y_c^s = x + y \; s, w_s] \; P[\tilde{\tilde{Y}}^s = y \mid s, w_s] \tag{3.25}$$

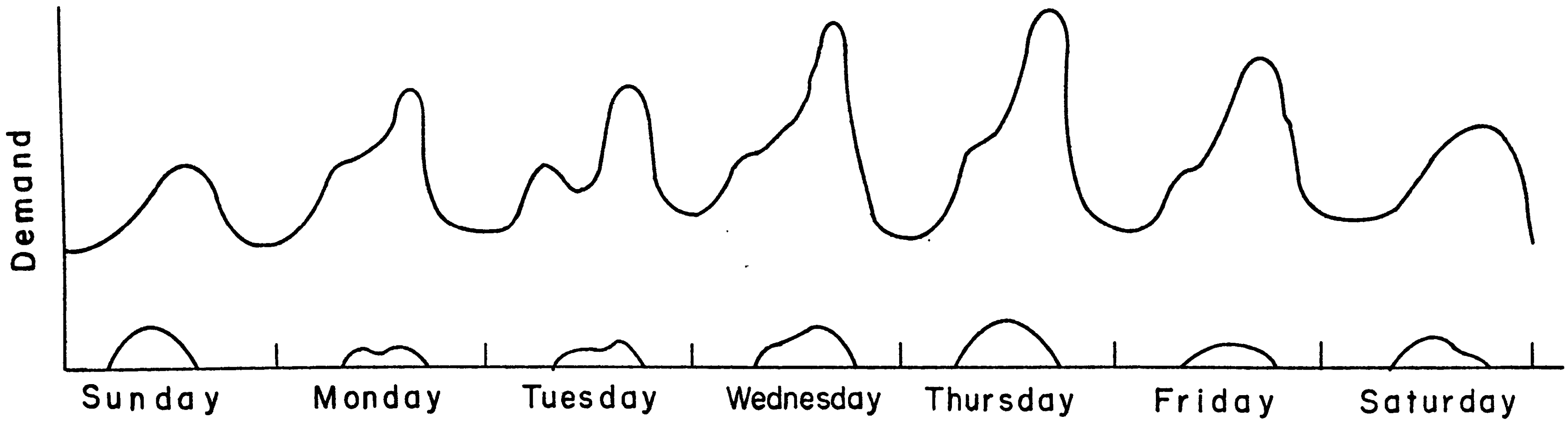
where $\tilde{\tilde{Y}}^s = \sum_{i=1}^{N} X_i \, \tilde{\psi}_i \, \tilde{\eta}_i^s$.

The conditional distribution as written in (3.25) can be computed for each time period and then the distribution for the net load could be found by summing out over time:

$$P[\tilde{Y}_n = x] = \sum_s P[\tilde{Y}_n^s = x] P[s] \tag{3.26}$$

where P[s] = probability of time s, e.g., the number of observations made at time s and weather w_s weighted by the total number of observations.

One way to perform the computation of equation (3.18) is to have observations of customer load and weather matched for time and location as illustrated in Figure 3.3. The load can be assumed to have an error distribution around the observed value and the output of the weather



Weekly time dependent demand curve. The output of a solar generator is plotted at the bottom of the curve

Figure 3.3 Time-dependent Load and Generation Curves dependent generators can be calculated from the weather data. With the mechanical failure rate superimposed on the generator output, the distribution of the net load for each hour can be found using equation (3.25). This procedure can be time consuming although it has the advantage of implicitly modeling the dependence of load and weather without requiring additional models. This methodology has been implemented and used in several studies of the breakeven cost of photovoltaics [84,85,86] and is documented in reference [39].

This methodology is not well suited to planning because it depends on historical data and uses substantial computer time. An alternative method is to create models that study the causal relationship between weather and electrical demand [27,62,77,87]. However, these models are more detailed than are necessary for the current study.

Another alternative is to study the statistical correlation of weather and load. This approach is currently being studied by Michael Caramanis [16] at the MIT Energy Laboratory. The following section outlines his work.

Given a series of historical observations of load and weather, a set of orthogonal vectors, γ, can be constructed using least squares:

$$\tilde{d}^{\sigma} = \sum_{j=n+1}^{N+1} b'^{0\sigma}_{N+1_j} \tilde{\gamma}^{\sigma}_j + \beta'^{\sigma}_{N+1} \qquad (3.27)$$

$$\tilde{n}^{\sigma}_i = \sum_{j=n+1}^{N} b'^{\sigma}_{ij} \tilde{\gamma}^{\sigma}_j + \beta'^{\sigma}_i \qquad i = 1, \ldots N$$

where σ = index for a set of subperiods with the same characteristics, e.g., sunny breezy summer noontimes

$\tilde{d}^{\sigma}$ = observed load at time σ, normalized by the peak demand as a function of prices for the time period

$\tilde{n}_i^{\sigma}$ = output of generation type i at time σ, with no mechanical failures, normalized by the installed capacity X_i

$[b_{ij}^{'\sigma}]$ = matrix of regression coefficients

$\beta_i^{'\sigma}$ = error terms

$\tilde{\gamma}^{\sigma}$ = equal orthogonal vectors.

Substituting (3.27) into (3.15):

$$\tilde{Y}_n^{\sigma} = \sum_{j=n+1}^{N} X_{N+1}[b_{N+1}^{'s}{}_j\ \tilde{\gamma}_j^{\sigma} + \beta_{N+1}^{'\sigma}]$$

$$- \sum_{i=n+1}^{N} X_i\ \tilde{\psi}_i [\sum_{j=n+1}^{N} b_{ij}^{'\sigma}\ \tilde{\gamma}_j^{\sigma} + \beta_j^{'\sigma}]. \tag{3.28}$$

Letting

$$\begin{aligned} b_{ij}^{'\sigma} &= -b_{ij}^{\sigma} & i \neq N+1 \\ b_{ij}^{'\sigma} &= b_{ij}^{\sigma} & i = N+1 \\ \beta_j^{\sigma} &= -\beta_j^{\sigma} & i \neq N+1 \\ \beta_j^{'\sigma} &= \beta_j^{\sigma} & i = N+1 \\ \tilde{\psi}_{N+1} &= 1, \end{aligned} \tag{3.29}$$

then (3.28) can be written compactly as:

$$\tilde{Y}_n^{\sigma} = \sum_{j=n+1}^{N} \sum_{i=n+1}^{N+1} X_i\ \tilde{\psi}_i\ b_{ij}^{\sigma}\ \tilde{\gamma}_j^{\sigma} + \sum_{i=n+1}^{N+1} X_i\ \tilde{\psi}_i\ \beta_j^{\sigma}. \tag{3.30}$$

Simplifying, and using the fact that the matrix B is lower triangular:

$$\tilde{Y}_n^{\sigma} = \sum_{j=n+1}^{N+1} \left[\sum_{i=j}^{N+1} X_i \tilde{\psi}_i b_{ij}^{\sigma} \tilde{\gamma}_j^{\sigma} + X_j \tilde{\psi}_j \beta_j^{\sigma} \right] \tag{3.31}$$

By construction, the random variables, $\tilde{\gamma}$, are mutually linearly independent. That is, even though the variables are not statistically or probabilistically independent, they have the following properties:

$$\xi_1(\gamma_i + \gamma_j) = \xi_1(\gamma_j) + \xi_1(\gamma_j)$$

$$\xi_2(\gamma_i + \gamma_j) = \xi_2(\gamma_j) + \xi_2(\gamma_j) \tag{3.32}$$

where $\xi_1(x)$ = ith cumulant of the random variable x.

The cumulants of a distribution as used in equation (3.32) have many useful invariance properties that the moments of a distribution do not have. The cumulants can be derived from the characteristic function of a distribution and can be written in terms of the moments. A discussion of the cumulants and a derivation of their properties can be found in reference [58]. For this discussion, it is only necessary to know the definition of the first four cumulants in terms of the moments:

$$\xi_1 = m_1$$

$$\xi_2 = m_2 - m_1^2 \tag{3.33}$$

$$\xi_3 = m_3 - 3m_2 m_1 + 2m_1^3$$

$$\xi_4 = m_4 - 4m_3 m_1 - 3m_2 + 12m_2 m_1 - 6m_2^4$$

where $m_i = E(x^i)$.

One very useful property of the cumulants is that the cumulant of the sum of two independent random variables is equal to the sum of the individual cumulants. In addition, the distribution of the sum can be found from the new cumulants. So, to find the distribution of the net load, the cumulants of the load and the cumulants of the generation can be computed separately, added together, and then operated on to yield the distribution of the net load.

However, this operation can only be performed in the two random variables are independent. So, Caramanis assumes that property (3.32) holds for all cumulants, even though it only holds necessarily for the first two.

The first two cumulants of the net load can be written as:

$$\xi_1(\tilde{Y}_n^\sigma) = \sum_{j=1}^{N} \sum_{i=j}^{N+1} \lfloor X_i \; b_{ij}^\sigma \; E(\tilde{\psi}_i)E(\tilde{\gamma}_j^{\sigma 2}) + X_j \; \beta_j^\sigma \; E(\tilde{\psi}_j) \rfloor$$

and

$$\xi_2(\tilde{Y}_n^\sigma) = \sum_{j=1}^{N} \sum_{i=j}^{N+1} [\, X_i \; b_{ij}^\sigma \; E(\psi_i^2)E(\tilde{\gamma}_j^{\sigma 2}) + X_j \; \beta_j^\sigma \; E(\tilde{\gamma}_j^2 \,)] \tag{3.34}$$

The cumulants found above in (3.34) are conditional on time. As in equation (3.26), the unconditional moments can be found by multiplying by the probability of each time increment and summing over time:

$$\xi_1(\tilde{Y}_n) = \sum_\sigma \xi_1(\tilde{Y}_n{}^\sigma) P_\sigma[\sigma]$$

$$\xi_2(\tilde{Y}_n) = \sum_\sigma \xi_2(\tilde{Y}_n^\sigma) P_\sigma[\sigma] \tag{3.35}$$

where $P_{\sigma}[\sigma]$ = Probability of set σ.

If the moments of the random variables $\tilde{\Psi}$ and $\tilde{\gamma}$ are known and the coefficients b and β are known, then the moments of $\tilde{Y}_n$ can be generated as a function of the peak load and the installed capacity of the generators.

There are several ways to generate a probability distribution given the moments of the distribution. Following Rau and Schenk [74], Caramanis uses a truncated Gram-Charlier series to approximate the distribution:

$$P_y[z] = N(z) - \frac{G_1}{3} N^3(z) + \frac{G_2}{4} N^4(z) + \frac{10}{6} G_1^2 N^6(z) \qquad (3.36)$$

where $N^j(z) = j^{th}$ derivative of the standardized normal distribution

$$G_1 = \xi_3 / \xi_2^{3/2} \qquad (3.37)$$

$$G_2 = (\xi_4 - 3\xi_4^4 + 3\xi_2)^{1/2} / \xi_2^2 - 3 \qquad (3.38)$$

For ease of computation, it is not necessary to derive the distribution itself. Rather, it is easier to find the transformation of the distribution from the moments and to perform all the convolutions in transform space. Since the use of transforms only eases the computational burden and does not change the theory, a discussion of transforms is omitted from here. See Caramanis [16] or Rau and Schenk [74].

3.III Transmission and Distribution

Transmission and distribution have two effects important to this study. First, the demand on the central station generators is higher than the sum of the customer demands due to losses in the transmission lines. Second, the failures in the transmission system reduce the reliability of the electricity delivered to consumers and reduce the net load on the system.

The study of transmission and distribution (T+D) systems is a large and complicated field in itself. An overview of current areas of study in T+D reliability can be found in reference [32]. To model it properly requires data on the topography of the system and specific information on the characteristics of each line. Rather than attempt a less than adequate model of the T+D system, a simple proxy will be used.

Equation (3.4) gives the net load as a function of the original demand and the reductions in demand. The original demand includes line losses, so any reductions in losses caused by reductions in demand must be accounted for. Equation (3.4) becomes:

$$\tilde{Y}_n = Y_o - \sum_k \Delta\tilde{Y}_k \, L_k(Y_n) \tag{3.39}$$

where $L_k(Y)$ = loss function for demand from customer k when the total demand is Y.

Or, using equations (3.7) and (3.13):

$$\tilde{Y}_n = \tilde{Y}_o \sum_k e_k - \sum_k \tilde{\tilde{Y}}_k L_k \, (Y_n) \tag{3.40}$$

The price-dependent demand was found as a fraction of the original demand using equation (3.11). Depending on where the metering is done, this fraction, e_k, may or may not account for the change in losses. If the metering is done at the customer's meter, the most likely place, then e_k does not include losses. Defining a new e_k to include losses yields:

$$e_k' = e_k \; L_k(Y_o)/L_k(Y_o) \qquad (3.41)$$

Finally, for simplicity, a piecewise linear loss function is assumed and the same loss function is assumed for all customers. Equation (3.40) becomes:

$$\tilde{Y}_n = L(Y_c)/L(Y_o) \; \tilde{Y}_c - L(Y_n) \sum_i X_i \; {}_i n_i \qquad (3.42)$$

These loss multipliers must be included when finding the load correlations in equations (3.27) and (3.28).

The T+D system also affects the reliability of grid electricity. In chapter 7, it will be seen that some customers may install their own generation if the grid reliability is not high enough. So, the end reliability to customers must be found. From historical data on the frequency and duration of outages for different types of customers, one can estimate a failure rate for the transmission and distribution system. In Chapter 4, the loss-of-load probability for the generation system will be found. The net reliability to customer k is:

$$p_{nk} = 1 - q_G - q_{Tk} \qquad (3.43)$$

p_{nk} = reliability of grid electricity to customer k

q_G = LOLP = loss of load probability for the generation system

q_{Tk} = failure rate of the T+D system for customer k losses of power

Equation (3.43) assumes that all are due either to the generation system or to the transmission system, but never both at once.

3.IV Summary

The changes in power demand for the current time period have been found as a function of the current price of electricity, the price of competing fuels, the customer capital stock, demographic variables, and the weather. For future time periods, the changes in power demand can be estimated based on the projections of these variables.

From equation (3.9) estimates of the change in demand due to time-of-day prices can be made. From equation (3.15) estimates of the change in demand due to customer owned generation can be made and from equation (3.42) estimates of the change due to changes in T+D losses can be made. Combining these in functional form yields:

$$\Delta Y_{ak}^{st} = f(c_k^{st}, X_k^{t}, \Omega_k^{st}, w_a^{st}, L_k) \qquad t = \tau,\ldots T \quad s = 1,\ldots S \qquad (3.44)$$

$$Y_{an}^{st} = Y_{ao}^{st} - \Delta Y_a^{st} \qquad (3.45)$$

$$P_{yn}[Y_{an}^{st} \leq x] = P_{\Delta Y_{ac}} * P_{Y_a} * P_{Y_{ao}}[Y_{ao}^{\tau} \leq x] \qquad (3.46)$$

$$\Delta E_a^{st} = \sum_k h_s \Delta Y_{ak}^{st} \qquad t = \tau,\ldots T \quad s = 1,\ldots S \qquad (3.47)$$

where

ΔY_{ak}^{st} = net change in demand by customer k in region a in subperiod s, time period t.

Y_{na}^{st} = net demand on the utility in region a in subperiod s, time period t.

ΔE_a^t = net change in grid electricity demand in area a, time period t.

c_k^{st} = vector of fuel prices for customer k in subperiod s, time period t.

X_k^t = vector of installed capacities of generation owned by customer k in time period t.

Ω_k^{st} = vector of demographic and economic variables for for customer k in subperiod s, time period t.

L_k = T+D loss function for customer k

ΔY_{ac}^{st} = change in demand due to price effects in region a in subperiod s time period t.

Y_a^{st} = change in demand due to customer owned generation in area a in subperiod s, time period t

Y_{ao}^{st} = original electrical demand in region a, subperiod s, time period t

h_s = number of hours in subperiod s

4. Utility Operation[1]

The production costing model uses the net load duration curve (Chapter 3), the capital stock of the utility (Chapter 5), and exogenously specified prices for fuel to compute the operating cost and generator reliability of a utility system. The operating cost is used in setting rates (Chapter 7) and the reliability is used in the customer expansion model (Chapter 6). The generating cost model is also used with the long-range planning model (Chapter 5) to find the cost of each potential system chosen during the optimization process.

4.I Introduction

Electric power systems are operated to meet the fluctuating power demand at minimum cost. Electric utilities monitor the power flow throughout the system to decide what the power output from each generator should be. These decisions are based on economic criteria, but are constrained by electric stability requirements imposed by the transmission network. A complete model of the cost of operating a power system requires detailed models of, and data on, each generator and each transmission line. Such models are too complex to be used for planning studies, so many simplifying assumptions must be made. For example, most production costing models, including the one presented here, do not consider transmission or stability constraints.

This chapter discusses a standard production costing methodology that

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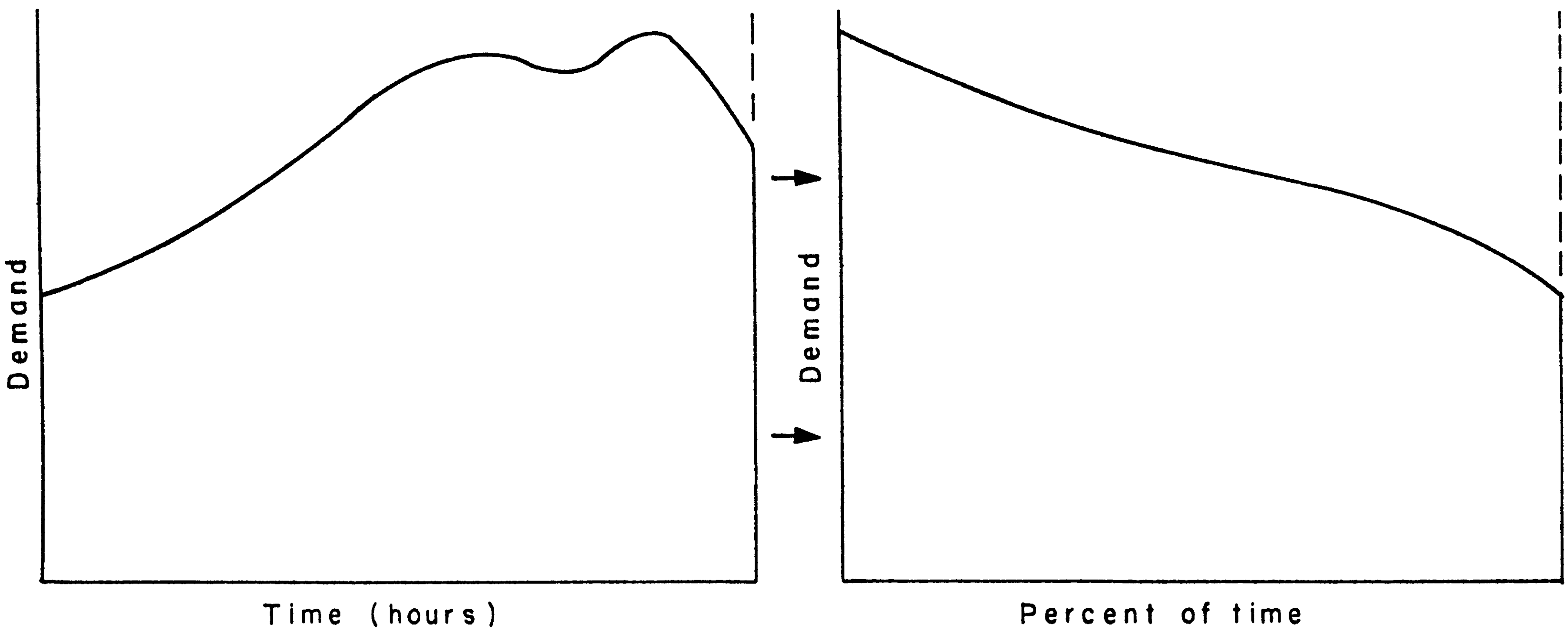
[1]This chapter has been extracted from Finger [37]. Reference [37] includes addditional material on multiple block units, frequency and duration, limited energy units, storage units, and time-dependent units.

models the average generator output. The framework of the model is first presented as a deterministic model in which the customer demand is fixed and plants do not fail. Then, the model is expanded to a probabilistic model in which the customer demand and plant failures are random variables.

4.II Deterministic Production Costing

Electric power systems are operated with the goal of meeting the electric demand at minimum cost. For a fixed set of generators, the dispatch strategy that results in the minimum operating cost is to use the generators in order of increasing marginal cost. In practice, this strategy may be modified to account for operating constraints such as spinning reserve requirements, high startup or shutdown costs and transmission constraints. The final ranking of generators is called the merit order or the economic loading order.

The power demand on an electric utility varies with the season and the time of day. Figure 4.1a shows a typical daily variation in power demand. Although the overall pattern is predictable, there is a large random component that makes hourly predictions difficult. For this reason, most planning studies use load duration curves that give just the percent of time that each demand level occurs. Figure 4.1 shows how a time-dependent curve can be converted into a load duration curve. Although detail is lost in the conversion, the load duration curve is easier to work with for time periods longer than a day and for future time periods for which there is not enough information to create hourly



4.1a. Time-dependent load curve for a typical day. 4.1b. Load duration curve of 1a.

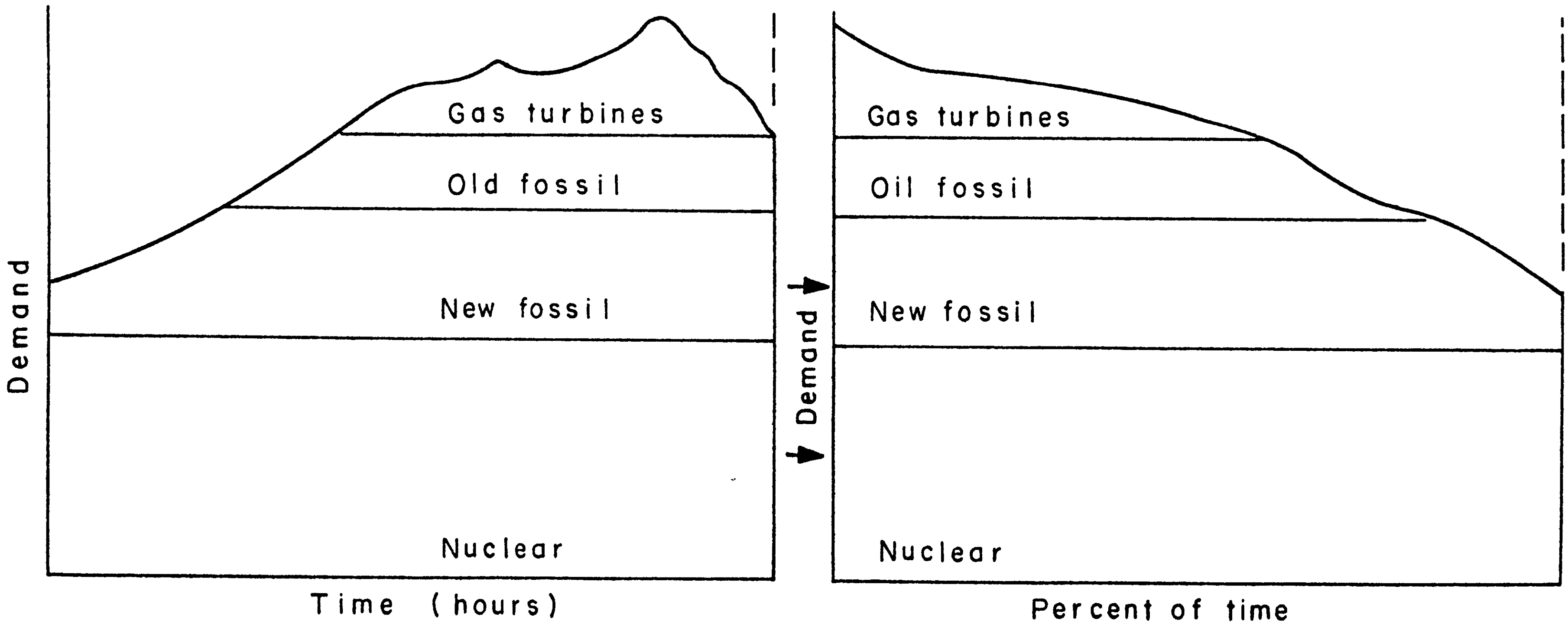
Figure 4.1. Conversion of time dependent curve to load duration curve.

curves.

The operation of the power system can be modeled by plotting the capacity of the generators, in merit order, along the vertical axis of the customer demand curve as shown in Figure 4.2a. The demand level at which a unit starts to generate is called its loading point. The energy that a unit generates is the area under the customer demand curve between its loading point and the loading point of the next unit. Converting the time-dependent curve into a load duration curve, as shown in Figure 4.2b, leaves the loading point and the energy the same as in 4.2a.

Conventional central station power plants are plants that can generate power at full capacity at any time, except when they are on maintenance or forced outage. These plants are much easier to model than hydro, storage, or solar plants that have limited energy and time-dependent power output. Nonconventional power generation will be discussed in later section.

In the deterministic model, the conventional power plant with the lowest marginal cost is loaded under the customer demand curve at a derated capacity that reflects the plant's availability. For example, a 1000 MW plant with an 80 percent availability factor would be brought up to 800 MW. This plant generates as much energy as it can to meet the customer demand. Since there is still unmet demand, the unit with the next lowest marginal cost is brought on line. This process continues until all the area under the load duration curve has been filled in. The total cost of the system operation can be computed by multiplying each plant's total megawatt hours by the cost per megawatt-hour for that plant



4.2a. Typical operating schedule.

4.2b. Equivalent schedule on a load duration curve.

Figure 4.2 Deterministic operating schedule.

and then summing the costs over all plants.

4.III Probabilistic Production Costing

Two major factors affecting system operating costs are uncertainties in demand and random failures of plants. There are several models available that take these factors into account. The simplest is a deterministic model with heuristic calibration coefficients added to account for plant failures. Slightly more complicated is the method developed by Baleriaux and Jamoulle [6] which combines the probability distributions of customer demand and of plant failures to find the expected value of the energy produced by each plant and the probability that the customer demand will not be met. There is also a frequency and duration method developed by Ringlee and Wood [75] that models both the load and plant failures as Markov chains. Recently,Ayoub and Patton [5] have developed a method that includes frequency and duration in the Jamoulle-Baleriaux model and that requires fewer assumptions than the Ringlee-Wood model. The combined method of Ayoub and Patton and several extensions that allow the model to treat plants with limited energy and time-dependent power output are described in reference [37]. This chapter describes only this basic model necessary for long range planning and price setting.

The main difference between the deterministic model and the probabilistic model is that the electrical demand and electrical generation are treated as random variables in the probabilistic model. In the deterministic model, a plant's capacity is derated to reflect random outages of the plant during its operating period. This assumes that the plant is always available at its derated capacity, or equivalently, that it has a forced outage rate of zero at its derated capacity. In fact, the plant is not always available. When a plant fails, more expensive generation must be brought on line to replace it. Since the deterministic model assumes that units never fail, the energy supplied by more expensive plants is underestimated. The deterministic model also assumes that the electrical demand is fixed. In the probabilistic model, uncertainty in the demand can be included in its probability distribution.

In the probabilistic model, the electrical demand and power plant failures are modeled as random variables with memory. That is, a power plant has a probability of failure and an expected time that it remains in a failure state. The electrical demand has a probability of being at a given level and an expected time that it remains at that level.

4.III.A. Electrical Demand Representation

The probability distribution for the net electrical demand, Y_C, was found in Chapter 3. Throughout this chapter, the following notation will be used:

$$f_Y(x)dx = \text{Probability } [x \leq Y_n \leq x + dx]$$

$$G_Y(x) = \text{Probability } [Y_n \leq x] = \int_0^x f_C\,(y)\,dy \tag{4.1}$$

$$F_Y(x) = 1 - G_Y(x) = \Pr\,[Y_n \geq x] = \int_x^\infty f_Y(y)\,dy.$$

where f_Y is the probability density function, G_Y is the cumulative probability function, and F_Y will be referred to as the reverse cumulative probability function.

The superscripts for subperiod and time period have been suppressed throughout this chapter.

4.III.B Conventional Power Plants

In the probabilistic model, the equivalent demand on a unit is defined to be the sum of the demand due to customers plus the demand due to failures of plants lower in the merit order. The equivalent demand Y_E is the sum of two random variables:

$$Y_E = Y_n + Y_F \tag{4.2}$$

where Y_n is the net customer demand as derived in Chapters 2 and 3. Y_F is the demand due to forced outages of units already dispatched. Using the formulas for the convolution of two independent random variables, the cumulative distribution of the equivalent demand becomes:

$$\begin{aligned} G_E\ (d) &= \int_0^d f_F\ (Y_F)\ G_Y\ (d - Y_F)\ dY_F \\ &= \text{Probability } [\text{load + outages} \leq d]. \end{aligned} \tag{4.3}$$

The distribution of the equivalent demand is central to the probabilistic model. As will be shown below, the expected energy generated by each unit can be computed from it, as can the loss of load probability .

For the case in which the forced outage rate of each plant is a discrete random variable, the integral over the probability density function $f_F(Y_F)$, can be replaced by the sum over the probability mass function. For a plant with forced outage rate, q, and capacity, X, this probability mass function is given by:

$$P_F\ (Y_F) = \begin{cases} p & \text{if } Y_F = 0 \\ q & \text{if } Y_F = X \end{cases} \tag{4.4}$$

where p + q = 1. That is, there is a probability, q, that the plant will not perform and the demand on plants higher in loading order due to its failure will be the capacity of the plant. There is a probaility, p, that the plant will perform and the demand due to forced outage will be zero.

Replacing the integral with the sum, equation (4.3) becomes:

$$G_E(d) = pG_Y(d) + qG_Y(d - X)$$

or since p + q = 1 and $G_E = 1 - F_E$:

$$F_E(d) = pF_Y(d) + qF_Y(d - X). \tag{4.5}$$

With these basic questions, the probabilistic analysis proceeds in much the same way as the deterministic analysis. Units are loaded starting at the left of the equivalent load duration curve. The demand on the first base-loaded unit to be brought up is the entire customer demand. There are no outages from previous units, so

$$Y_{E1} = Y_n \tag{4.6}$$

where Y_{E1} = equivalent demand on the first unit

Y_n = total net customer demand.

Because the two random variables, Y_{E1} and Y_n, are equivalent, their distribution function is the same:

$$F_{E1}(d) = F_Y(d). \qquad (4.7)$$

In the deterministic model, a unit is loaded onto the system by filling in the area under the load duration curve. The area gives the energy generated. To load a unit in the probabilistic model, the area is again filled in. The vertical axis, instead of being the percent of time that a unit operates at a given capacity, is now the probability that a unit operates at that capacity at any given time. Taking the integral over the capacity gives the expectation of the operating capacity[2] for the unit at any given time. The expected capacity for the first unit is:

$$E(Y_1) = \int_0^{X_1} F_Y(x)dx \qquad (4.8)$$

where X_1 = capacity of the first unit

Y_1 = random variable describing the running capacity of the first unit.

$E(Y_1)$ is the expected capacity required to meet the equivalent load, without considering the availability of the unit. The total expected energy from the first unit, taking outages into account, is:

$$MW_1 = p_1 \, h_s \, E(Y_1) \qquad (4.9)$$

where p_1 = availability of unit one

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[2]The operating capacity is a continuous variable which takes on values between zero and the unit's capacity in response to the customer demand. This does not violate the assumption that plant outages occur in discrete blocks.

h_s = length of the time period in hours.

The capacity factor, α, the ratio of operating capacity to nameplate capacity, is given by:

$$\alpha_1 = p_1 \; E(Y_1)/X_1 \tag{4.10}$$

and the cost of running the system with unit 1 loaded is:

$$E(X_1) = H_{f1} \; c_f \; MW_1 \tag{4.11}$$

where H_{f1} = full load heat rate for unit 1 burning fuel f (MBtu/MWH)

c_f = cost of fuel f ($/MBtu)

The equivalent demand on the second unit to be brought up is the customer demand plus the demand due to the outages of the first unit:

$$Y_{E2} = Y_n + Y_{F1} \tag{4.12}$$

Because of the way the equivalent load is defined, the loading point of the second unit on the equivalent load duration curve is the same whether or not the first unit fails. If the first unit fails, it creates a demand, X_1, so the second unit is loaded when the equivalent demand is X_1. If the first unit does not fail, there is no demand due to outage. The first unit supplies the demand until the demand exceeds X_1, at which point the second unit is loaded. The loading point, U, for the rth unit is just the sum of the capacities of the previously loaded units:

$$U_r = \sum_{i=1}^{r-1} X_i$$

and

$$U_1 = 0 \tag{4.13}$$

Equation (4.5) gives the equivalent load curve for Y_{E1}:

$$F_{E2}(d) = p_1 F_{E1}(d) + q_1 F_{E1}(d - X_1) \quad (4.14)$$

Having found the equivalent load curve for the second unit, the expected capacity, capacity factor, and the energy generated can be obtained:

$$E(Y_2) = \int_{U_2}^{U_3} F_{E2}(x)dx$$

$$\alpha_2 = p_2 E(Y_2) X_2 \quad (4.15)$$

$$MW_2 = p_2 h_s E(Y_2)$$

$$EC(U_3) = \sum_{i=1}^{2} H_{fi} c_f MW_i$$

For the third unit, the equivalent load is given by:

$$Y_{E3} = Y_n + Y_{F1} + Y_{F2} \quad (4.16)$$

Using the definition of Y_{E2} in equation (4.12):

$$Y_{E3} = Y_{E2} + Y_{F2} .$$

Then,

$$F_{E3}(d) = p_2 F_{E2}(d) + q_2 F_{E2}(D - X_2)$$

$$E(Y_3) = \int_{U_3}^{U_4} F_{E3}(x)dx \quad (4.17)$$

$$\alpha_3 = p_3 E(Y_3) X_3$$

$$MW_3 = p_3 h_s E(Y_3)$$

$$EC(U_4) = \sum_{i=1}^{3} H_{fi} c_f MW_i$$

In general,

$$Y_{Er} = Y_C + \sum_{i=1}^{r-1} Y_{Fi} = Y_{Er-1} + Y_{Fr-1}$$

$$F_{Er} = p_{r-1}\, F_{Er-1}\,(d) + q_{r-1}\, F_{Er-1}\,(d - X_{r-1})$$

$$E(Y_r) = \int_{U_r}^{U_{r+1}} F_{Er}\,(x)\, dx \tag{4.18}$$

$$\alpha_r = p_r\, E(Y_r)\, X_r$$

$$MW_r = p_r\, h_s\, E(Y_r)$$

$$EC(U_{r+1}) = \sum_{i=1}^{r} H_{fi}\, c_f\, MW_i$$

where r = loading order of the plant.

4.III.C Reliability Measures

After the last unit has been loaded, the final curve is the equivalent load curve for the entire system. Since the loss of load probability is defined to be the percent of time that the customer demand cannot be met, its value can be read directly from the final curve. The energy demand that cannot be supplied is given by:

$$EF(U_{n+1}) = h_s \int_{U_{I+1}}^{\infty} F_{EI}(x)\, dx \tag{4.19}$$

where I = number of plants.

The loss-of-load probability is given by:

$$LOLP = F_{EI}(U_{I+1}) \tag{4.20}$$

where U_{I+1} is the total installed capacity of the system. Figure 4.3 shows the final system configuration.

An other measure of the reliability of a power system is its loss of energy probability, LOEP. The LOEP is not a probability, but an expected value for the fraction of the original demand that cannot be met. It is defined as:

$$LOEP = \frac{\int_{U_{I+1}}^{\infty} F_{EI}(x)dx}{\int_{0}^{Q} F_{Y}(x)dx} \tag{4.21}$$

where U_{I+1} = total installed capacity

Q = peak customer demand.

4.IV Summary

From basic information about the generating units and the load, it is possible to produce a great deal of information about the operation of each unit and about the operation of the system. The information about each unit is used within the long-range planning model (Chapter 5) and is not presented here.

Equations (4.17) and (4.20) give the functions for the system operating cost and for the unserved energy demand as a function of the operating capacities:

$$\begin{aligned} EC_a{}^t(Y^t\ ) &= f\ (E_a^t,\ P_a^t,\ p,\ X_a^t\ ) \\ EF_a{}^t(Y^t) &= f\ (E_a^t,\ P_a^t,\ p,\ X_a^t,\ c^t{}_a) \end{aligned} \tag{4.22}$$

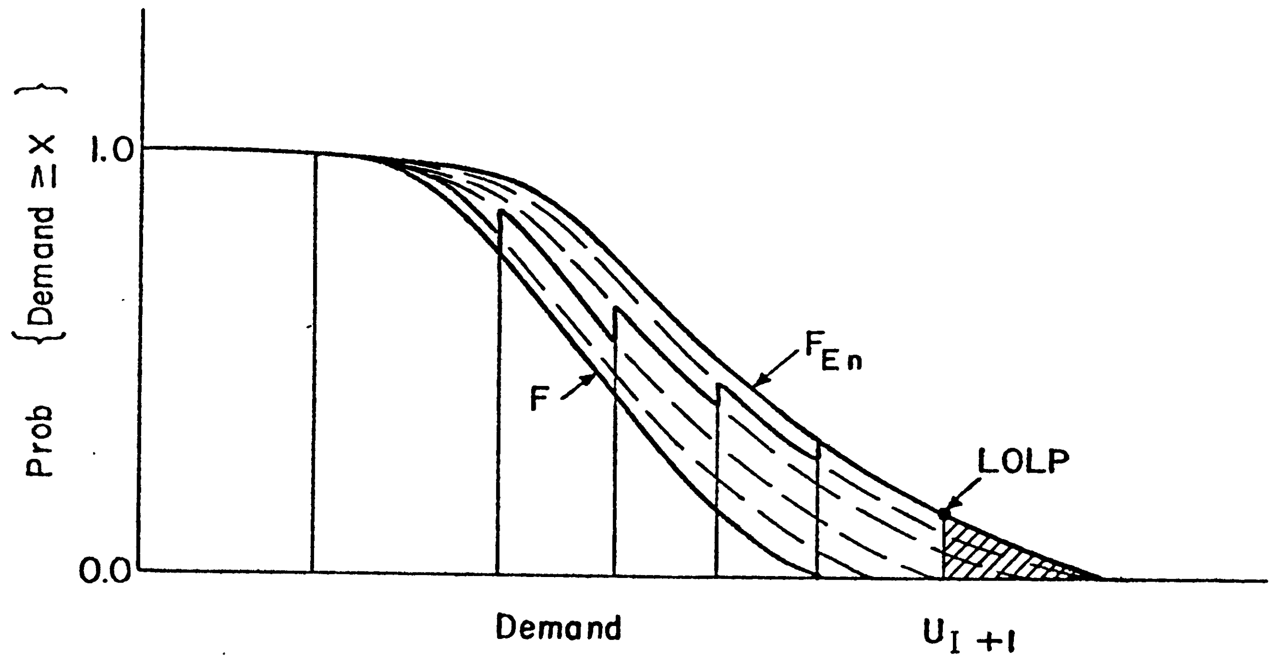


Figure 4.3 **Final equivalent demand curve**

$$EC_a^t(Y^t) = f(E_a^t, P_a^t, p, X_a^t)$$

$$EF_a^t(Y^t) = f(E_a^t, P_a^t, p, X_a^t, c_a^t) \qquad (4.22)$$

$$Y^t = f(E_a^t, P_a^t, p, X_a^t)$$

where EF_a^t = expected unserved energy as a function of operating capacity in region a at time t

EC_a^t = expected operating cost as a function of operating capacity in region a at time t

Y^t = vector of operating capacities for units operating in time t

E_a^t = total net electrical demand in region a at time t

P_a^t = probability distribution for the net load in region a at time t

p = vector of availabilities for units installed in region a

X_a^t = vector of unit capacities in region a at time t.

5. Utility Expansion

The utility expansion model uses the energy forecasts (Chapter 2), the net load curves (Chapter 3), exogenously specified capital and fuel costs and exogenously specified available technologies to find the optimal expansion path. The utility operation model (Chapter 4) is used within the long-range planning model, and decisions from previous expansion model runs (Chapter 5) are used to specify the existing capital stock of the utility. The output of the expansion model is the optimal installment plan over the specified time horizon. Based on the optimal plan, decisions are made to begin construction of some new units. Only those units for which construction must begin in the current time period in order to have them when planned, are considered to be firm units. These units are then included as committed units in the next time period when the expansion model is run. And, when their installment date comes, they are used in the utility operation model to meet demand (Chapter 4) and are placed in the rate base by the rate-setting model (Chapter 7). (For some types of rates, these units may appear in the rate base before they are installed.)

5.I. Introduction

Capacity expansion models are a central element of the planning process of electric utilities. Most are based on optimization models that search for the capacity plan with the minimum capital and operating cost that reliably meets the expected customer demand over a time horizon of twenty to thirty years. Objectives other than minimizing cost can be considered. Some planning models have financial objectives such as maximizing the cash flow while others have social objectives such as maximizing total welfare. While the latter objective is in keeping with the tenor of this report, in order to maximize social welfare one must know the social value of energy, power, short run reliability and long run sufficiency and one must also have a consistent and defendable method to measure consumers' surplus. Rather than descending into this morass, a simple engineering cost minimization objective will be used.

Anderson [2] has published a survey paper on capacity expansion models. All of these models require input data on the cost and performance of potential generating plants, the expected customer demand for each year in the planning period, and a reliability criterion for meeting the demand. Within these models, the system operating costs and reliability are usually calculated using linear approximations. None of the available models include decentralized or weather-dependent generation as potential capacity additions. Nor do they allow anyone except the utility to install new capacity.

In each time step, the utility must make a decision on whether or not to begin construction of a new unit or delay a unit in construction based on new information on customer demand, fuel prices, and capital costs. This problem is solved by performing the optimization, using the new information, starting with the current year and comparing the new plan to the plan produced in the preceding time step and adjusting the new unit schedule. In adjusting the new unit schedule, the utility would take into account whether the cost of the adjustment was greater than any savings due to the adjustment before making any changes. For simplicity, it will be assumed that the new schedule will always be adopted. However, with additional data on the cost of delays and speedups, it would be possible to incorporate these tradeoffs. The next plants to be built are found using a modification of a utility planning model developed by Bloom [11] that uses Benders' decomposition. The use of decomposition allows the utility operation constraint to be nonlinear, allows a realistic reliability constraint and allows the inclusion of customer-owned and time-dependent generators.

For simplicity, it will be assumed that new units can be built in any size. In fact, one can not build one half of a 500 MW unit. One must build a 250 MW which may have characteristics quite different from a 500 MW unit due to technologies and economies of scale. It is possible to use Benders' decomposition to solve an integer program allowing only certain plant sizes to be built [22, 23, 70]; however, the MIT EGEAS program [65] which will be used here has not yet been expanded to allow only integer solutions. In addition, it will be assumed that the planning is done for aggregated time periods, rather than explicity including constraints, for all subperiods. This is a reasonable assumption since annual data projected for more than ten years is at most a good guess. Monthly data would require omniscience. So, throughout this chapter, the superscript for the subperiod s will be dropped, although constraints for subperiods in the near future could be added.

5.II. General Formulation

A general formulation of the apacity expansion problem is given by:

$$\min_{X_{jv}\, Y_{its}} \sum_{j=1}^{J} \sum_{v=\tau}^{T} K_{jv} X_{jv} + \sum_{t=\tau}^{T} \sum_{i=N+1}^{I^t} H_{fi} c_f^t Y_i^t \qquad (5.1)$$

subject to

$$\sum_{i=N+1}^{I^t} Y_i^t \geq E^t \qquad t = \tau, \ldots T \qquad (5.2)$$

$$EF^t(Y^t) \leq \varepsilon^t \qquad t = \tau, \ldots T \qquad (5.3)$$

$$0 \leq Y_i^t \leq \sum_j \sum_v \delta_{jv}^{it} X_{jv} \qquad t = \tau, \ldots T \qquad (5.4)$$

$$X_{jv} \geq 0 \qquad (5.5)$$

where τ = starting time period

t = time period

v = plant vintage

j = plant type

i = plant's place in the economic operating order

δ_{jv}^{it} = 0 – 1 variable that gives a plant's place in the operating order as a function of its age and type

K_{jv} = per unit capital cost for plant type j installed in year v ($/MW)

c_f^t = cost of fuel f in time period t ($/MBtu)

H_{fi} = full load heat rate for unit i burning fuel f (MBtu/MWH)

ε^t = minimum unserved energy requirement for time t (MWHS)

X_{jv} = installed capacity for lant type j in year v (decision variable) (MW)

Y_i^t = operating capacity for plant i in time t, (decision variable) (MW)

E^t = energy function for time period t

EF^t = unserved energy function for time period t

I^t = number of operating plants in the system at time t.

The objective function (5.1) is the sum of the capital costs for plants built during the planning horizon plus the cost of operating all the existing plants to meet the demand. Constraint (5.2) requires that the demand be met in each subperiod of the planning horizon. Constraint (5.3) requires that the unserved energy in each time period be less than the specified reliability level. Constraints (5.4) and (5.5) assure that plants are never operated above their capacity and that the capacities are never negative.

All of Chapter 4 was devoted to solving constraints (5.2) and (5.3). The Y_i's, the operating capacities, and EF, the unserved energy, are outputs of the probabilistic simulation. Chapters 2 and 3 studied E^t, the net energy demand on the central station generators. Using the results of these chapters, constraint (5.2) can be written as:

$$\sum_{i=N+1}^{I^t} Y_i^t \geq E^t(c_n^t, Y_n^t, \ldots Y_N^t) \qquad (5.6)$$

and

$$c_n^t = f(CX^t, Y_1^t, Y_N^t \ \ldots, Y_I^t, M) \tag{5.7}$$

where

c_n^t = price charged for grid electricity at time t

CX^t = capital cost (historical or replacement) of installed capacity at time t (\$/MW)

Y_n^t = electricity purchases in time t (MW) subperiod s

$Y_1^t \ldots Y_n^t$ = operating capacity of generators owned by customers in time t (MW)

M = fixed costs due to e.g., metering and billing, transmission and distribution (\$).

The price function of equation (5.7) will be discussed in Chapter 6.

5.III. Benders' Decomposition

Bloom in his report [11] has solved the optimization of (5.1)-(5.5) using Benders' decomposition. A general discussion of Benders' decompostion algorithm can be found in Lasdon [59].

Benders' decomposition is used when a hard-to-solve optimization problem can be broken down into two, or more, not-so-hard-to-solve otpimization problems with distinct sets of decision variables. The master problem is used to solve for the primary decision variables which have the property that once these primary variables are fixed then the optimal secondary decision variables of the subproblems can be found. In addition, from the optimal solution to the subproblem, shadow prices on the fixed primary variables can be calculated. These shadow prices can then be used to generate new linear constraints, Benders' cuts, to the master problem. The Benders' cuts serve to successively restrict the feasible region of the master problem to the feasible region of the original problem.

In this case, the master problem chooses the amount of each type of capacity to build. Then, once the unit capacities are known, the production costing algorithm described in Chapter 4 can be used to find the cheapest way to meet demand with that set of units. So, if the master problem chose all nuclear plants in the first iteration, the shadow prices from the subperiod would indicate that there were cheaper and more reliable ways to meet load.

A detailed description of the generation of Benders' cuts and the calculation of the dual multipliers is given in reference [11] and an analysis of convergence properties is given in reference [44].

Using Benders' decomposition for (5.1)-(5.5) the master problem is:

$$\min_{X} z$$

$$\text{subject to} \quad z \geq \sum_{j=1}^{J} \sum_{v=\tau}^{T} K_{jv} X_{jv} + \sum_{t=\tau}^{T} \left[EC^{t}(Y_m^t) + \pi_m^t \, \delta^t (X_m - X) \right] \qquad m = 1, \ldots M \tag{5.8}$$

$$EF(Y_m^t) + \theta_m^t \, \delta^t (X_m - X) \leq \varepsilon^t \qquad t \in \Gamma_m \qquad m = 1, \ldots M \tag{5.9}$$

where m = iteration number

X = vector of installed capacities (decision variable)

X_m = vector of installed capacities chosen in iteration m

Y_m^t = operating capacities for central units in time period t in iteration m

π_m^t = shadow price associated with the energy constraint in time period t, subperiod s, in iteration m

θ_m^{st} = shadow price associated with the reliability constraint for time period t, subperiod s, in iteration m

$_m$ = set of time periods in iteration m in which the reliability constraint is not met

At each iteration m, an upper bound and lower bound for z, the value of the objective function, can be generated. The algorithm halts when the upper and lower bounds are within some prespecified tolerance.

5.IV. Relaxed Formulation

The purpose of the model developed by Bloom was to find the optimal expansion path for a utility over a time horizon of twenty to thirty years. Since this report focuses on the plants to be built in the current time period, less detail is needed for time periods further in the future. For this reason, the reliability constraint is retained for only the next T_1 years, where T_1 is some number less than the number of years remaining in the study. In addition, for time periods greater than T_1, the nonlinear operating constraint is replaced by a linear approximation, thus reducing the computational effort to find the optimal solution for each time step.

Thus, for time periods further in the future, constraints (5.2) and

(5.3) are replaced by:

$$\delta^t \alpha X \geq E^t \qquad T_1 < t \leq T \qquad (5.10)$$

$$\delta^t X \geq Q^t(1 + RM) \qquad T_1 < t \leq T \qquad (5.11)$$

where T_1 = last time period modeled with nonlinear constraints

T = last time period

RM = reserve margin requirement

α_{jv} = design capacity factor for plant type j of vintage v.

Constraint (5.10) requires that the average expected energy production meet the average demand. Constraint (5.11) requires that the installed capacity exceeds the peak power demand by a specified margin.

Retaining constraints (5.2) and (5.3) for time periods less than T_1, the master problem becomes:

$$\min z + \sum_{j=1}^{J} \sum_{v=T_2}^{T} K_{jv} X_{jv} + \sum_{t=T_2}^{T} \sum_{i=N+1}^{I_t} H_{fi} c_f^t \; \delta^t \; \alpha X \qquad (5.12)$$

$$\text{s.t.} \qquad z \geq \sum_{j=1}^{J} \sum_{v=\tau}^{T_1} K_{jv} X_{jv} + \sum_{t=\tau}^{T_1} [EC^t (Y_m^t) + \pi_m^t \delta \; (X_m - X)]$$

$$m = 1, \ldots M \qquad (5.13)$$

$$\delta^t \alpha X \geq E^t \qquad T_1 < t \leq T \qquad (5.14)$$

$$\delta^t X \geq \theta^t(1 + RM) \qquad T_1 < t \leq T \qquad (5.15)$$

$$EF^t(Y_m^t) + \theta_m^t \delta^t(X_m - X) \leq \epsilon \qquad t \in \Gamma_m \qquad (5.16)$$

Solving the optimization of (5.12)-(5.16) yields a solution, X*, Y*, that gives the installed capacity and operating capacity for all plants over the planning horizon. However, only the capacity installed in the first year of the optimization is needed for the rest of the study. This new capacity is included in the plant operation model to determine the energy costs and its capital is placed in the rate base.

5.V Summary

Based on the projections of demand, capital costs, fuel costs, and available technologies, the expansion planning model finds the optimal plan over a long time frame. Because only the decisions in the near future are made firm, the near time periods are modeled with more precise nonlinear constraints while those far in the future are modeled with less precise, but computationally simple, linear constraints.

From equations (5.12) through (5.16) the optimal expansion plan for a given region can be found based on current estimates of costs and demand:

$$X_a^{t*} = f(\hat{E}_a^t,\ P_a^t,\ \varepsilon_a^t,\ \hat{c}_a^t,\ \hat{K}_a^t,\ X_a^{t-1}) \qquad t = \tau, \ldots T \qquad (5.17)$$

where X^{t*} = vector of optimal capacities for region a

$\hat{E}_a^t$ = estimate of total electricity demand in region a at time t

P_a^t = distribution of the electrical demand in region a at time t

ε_a^t = reliability requirement for region a at time t

$\hat{c}_a^t$ = vector of estimtes of fuel cost in region a at time t

$\hat{K}_a^t$ = vector of estimates of capacity costs for region a at time t

X_a^{t-1} = vector of installed capacity in region a.

6. Customer Expansion Planning

The customer in making decisions about whether or not to install a generator considers the current price of grid electricity (Chapter 7), the reliability of grid electricity (Chapter 4), the current and estimated price of alternative fuels, and the current and estimated costs for alternative generators. The result of the customer expansion model is a series of decisions about what to build in each time period. Presumably, most customers would not install more than one generator during the planning period and most would rely on the grid as their only source of electricity. Just as for the utility, decision by customers cannot be implemented instantly so lead times must be considered, although lead times are more on the order of one year than ten years for small generators.

6.I Introduction

For an electricity consumer, the decision of whether or not to build an electrical generation system, what kind to build and when to build it can be modeled in much the same way as it is modeled for an electric utility as discussed in Chapter 5. However, consumers do not run optimization models every year to decide what to do next. At most, they may make a rough approximation of the breakeven value or the payback period for some particular system. Usually, only one or two systems are feasible for a site and the final decision usually factors in many non-economic considerations that would be difficult to include in an optimization model. The following sections will discuss a breakeven model, optimization model, and a marketing model and the fusion of these that will be used in this report.

6.II Breakeven Cost

The breakeven cost is the amount that one would be willing to pay for a particular system so that one was indifferent between that system and the next best alternative. If you were offered a system that would supply all your electricity needs for the next twenty years at a cost exactly equal to the present value of your expected electricity bill over the next twenty years, you should be indifferent between the two systems. This tradeoff can be made more precise using decision analysis which takes uncertainties into account. For example, if you are risk averse, and you suspect that the alternative system may last only ten years although it could last as many as thirty, but you are positive that electricity rates will rise no faster than inflation, you would require a lower breakeven cost than someone who was risk indifferent or who had different expectations about the relative risks of the alternatives. For this report, money will be used as a proxy for consumer satisfaction while acknowledging the limitations of this assumption.

The usual way to find the breakeven cost for a time dependent generator is to run a simulation model like the one described in Chapter 2. If one assumes that the generator operation for the simulation year is typical, then one can compute the total savings due to the time-dependent generator over its lifetime. A complete derivation of the breakeven cost of a photovoltaic system is given by Carpenter and Taylor

[17]. A generalized version of their basic formula is, suppressing the subscript k for the customer and assuming grid electricity is always the best alternative:

$$BEC_i = \sum_{t=\tau}^{T} [\sum_{s=1}^{S} (Y_n^s c_n^{st} - Y_i^s c_i^{st}) h_s] \frac{\Delta_i^t}{(1 + \rho)^t} \quad (6.1)$$

where BEC_i = total breakeven cost for system i installed in year including capital, variable, and fixed cost ($)

Y_n^s = original grid electricity demand in subperiod s (MW)

Y_i^s = electrical output of unit i in subperiod s (MW)

h_s = number of hours in subperiod s

c_n^{st} = price of grid electricity in subperiod s, time period t($/MWH)

c_i^{st} = cost of operating unit i in subperiod s, time period t ($/MWH)

Δ_i^t = cumulative degradation factor for unit i in time period t

ρ = discount factor.

The total breakeven cost can be broken down to yield the breakeven cost per installed megawatt, or as it is called, the breakeven capital cost:

$$BECC_i^t = \frac{BEC_i^t - FIX_i^t}{X_i} - VC^t{}_i \quad (6.2)$$

where $BECC_i^t$ = breakeven capital cost for system i installed in year ($/MW)

FIX_i^t = initial fixed cost for system i installed in year t, e.g., power conditioning, lightening protection ($)

VC_i^t = initial variable cost for system i installed in year t, e.g., insurance, taxes ($/MW)

X_i = installed capacity of system i (MW).

For each customer, the breakeven cost for each system could be found and compared with the current selling price of that system. If any system's current selling price was lower than its breakeven cost, it would be purchased. If more than one system met this criterion then the system with the largest net benefit would be chosen, assuming there was no synergy between systems. (If synergy was suspected to be an important factor, then a hybrid, for example a wind-photovoltaic system, could be considered as a separate technology.)

6.II.A Statistical Method

The hourly simulation implied in equation (6.1) can be simplified using the statistical techniques that were used in Chapter 3. Looking at the inner summand of equation (6.1), we have:

$$B_i^{st} = (Y_n^s \; c_n^{st} - Y_i^s \; c_i^{st}) h_s \tag{6.3}$$

where B_i^{st} = net benefit from system i in subperiod s, time period t ($).

As in Chapter 3, the original demand and the output of generator i are random variables. Rewriting the generator output as the product of its capacity, outage rate, and time-dependence, yields:

$$Y_i^s = X_i \, \tilde{\psi}_i \, \eta_i^\sigma \tag{6.4}$$

where σ, X_i, $\tilde{\Psi}_i$, and η_i are defined in equation (3.2). Assuming that there is only one generator at this site, substituting equations (3.18) and (3.20) into equation (6.3) yields:

$$B_i^{st} = h_s\ c_n^{st}\ X_n\ \tilde{\Psi}_n(b_{ni}^{\sigma}\ \gamma_i^{\sigma} + b_{nn}^{\sigma}\ \gamma_n^{\sigma} + \beta_n^{\sigma})$$

$$+ h_s\ c_i^{st}\ X_i\ \tilde{\Psi}_i(b_{ii}^{\sigma}\ \gamma_i^{\sigma} + \beta_i^{\sigma}). \qquad (6.5)$$

Since the diagonal elements of the b matrix are equal to one, equation (6.5) simplifies to:

$$B_i^{st} = h_s[\sum_{j=i,n} c_j^{st}\ X_j\ \psi_j(\gamma_j^{\sigma} + \beta_j^{\sigma}) + c_n^{st}\ X_n\psi_n\ b_{ni}\ \gamma_i]. \qquad (6.6)$$

Assuming that the subperiods are partitioned such that all s in the set with σ index have the same cost structure, then equation (6.1) can be written as:

$$BEC_i = \sum_{t=\tau}^{T} (\sum_{\sigma=1} B_i^{\sigma t})\ \frac{\hat{A}_i^{t}}{(1 + \rho)^t} \qquad (6.7)$$

Whereas before the inner sum was over all the hours of a time period, now the number of elements in the sum has been reduced to the number of sets into which the hours are partitioned.

Equation (6.7) simplifies the computation of the breakeven cost greatly. Once the orthogonal vectors, γ, and the regression coefficients, b, have been computed for a particular customer, then the size of the generator, the peak load, the outage rate and the price structure can be varied and a new breakdown cost calculated without requiring large amounts of additional work.

The breakeven model still presents problems because it evaluates only a particular proposed system rather than allowing an easy choice among systems. The following section discusses an alternative.

6.III.B Optimization Method

As mentioned in the introduction to this chapter, optimization models do not necessarily give a good representation of the decision process of a homeowner deciding whether or not to install an electrical generator. In addition, for computational simplicity, most optimizations are run with linear constraints. Because it is difficult to represent the operation of time-dependent generators using linear equations, optimization models are almost never used to model them. In this section, an optimization model will be presented which overcomes these objections.

In general, the optimization problem for an electricity customer can be written as:

$$\min \sum_{j=1}^{J} \sum_{v=\tau}^{T} K_{jv} X_{jv} + \sum_{t=\tau}^{T} \sum_{s=1}^{S} \sum_{i=1}^{n} c_i^{st} \; Y_i^{st} \qquad (6.8)$$

subject to

$$\sum_{i=1}^{n} Y_i^{st} \geq Y_n^{st} \qquad s = 1, \ldots S \quad t = \tau, \ldots T \qquad (6.9)$$

$$EF(Y^t) \leq \epsilon_t \qquad t = \tau, \ldots T \qquad (6.10)$$

$$0 \leq Y_i^{ts} \leq \sum_j \sum_v \delta_{jv}^{it} X_{jv} \qquad i = 1, \ldots n \quad (6.11)$$

$$s = 1, \ldots S$$

$$t = \tau, \ldots T$$

$$X_{jv} \geq 0 \qquad j = 1, \ldots J \quad (6.12)$$

where the subscript k, for the customer, has been suppressed. All of the quantities are the same as those defined for the utility in Chapter 5.II, except that one of the choices available to customers is to meet their demand with grid electricity. This option requires no capital investment, assuming that the customer is already served by the utility. If customers without electrical service, for example those in remote locations are considered, then grid electricity will also have a capital investment component. In any case, the current source of electricity can be treated just as existing generators were treated in the utility optimization.

In Chapter 3.II, it was shown that time-dependent changes in the load on a utility could be modeled using statistical techniques. In Chapter 4, it was shown that the resulting curve could be used, along with generating unit characteristics, to find the total energy generated by each unit. By analogy, the time-dependent generation of a customer can be modeled using statistical techniques to yield the customer's net load duration curve. The single central utility unit, as it appears to the customer, can be operated against the load duration curve giving the total grid energy supplied and the reliability of electricity supplied. This looks exactly like the operating submodel of the utility optimization problem.

If the probability distribution of the customer's total demand for electricity is known and the generators available to meet the demand are given (chosen by the master problem), then the same model used for utility operation can be used for the customer. The net demand on the electricity utility, Y_i^{st}, is given by:

$$Y_n^{st} = Y_o^{st} - \sum_{i=1}^{n-1} Y_i^{st} \tag{6.14}$$

where Y_0^{st} is the original demand and Y_i^{st} is the output from customers' generators. Assume for simplicity that the customer has only one generator, call it i, and as before its output is given by:

$$Y_i^{st} = X_i \, \psi_i \, \eta_i^{\sigma} \tag{6.15}$$

where X_1 is the capacity, ψ_i the mechanical failures, and η_i the time-dependent fluctuations.

From the methods discussed in Chapter 3, the probability distribution of Y_n can be found using statistical techniques. That is, assuming that the distribution for the total demand, at a given electricity price structure, is known and the correlation of time-dependent generation with load is known, then equation (3.40) can be used to find the distribution of the net demand on the utility. The electricity from the grid is supplied to the customer with a reliability that is the product of the generation system reliability and the transmission and distribution system reliability. The maximum capacity that can be delivered is the fuse rating on the house. Therefore, the grid electricity looks like a generating unit with the following properties:

$$P[Y_{nk}^{st} = x] = \begin{cases} p_{nk}^{t} & x = X_{nk} \\ q_{nk}^{t} & x = 0 \end{cases} \qquad (6.16)$$

where $p_{nk}^{t} + q_{nk}^{t} = 1$

q_{nk}^{t} = probability that grid electricity cannot be supplied to customer k (power outage)

X_{nk} = maximum power that customer k can draw from the grid (MW).

The probability of power outage is given by:

$$q_{nk}^{t} = 1 - (1 - LOLP^{t})(1 - q_{Tk}) \qquad (6.17)$$

where q_{Tk} = T + D outage rate for customer k.

The loss-of-load probability (LOLP) is found from the current run of the power plant operation model. The T + D outage rate is assumed to be constant and is taken from historical data for different types of customers (e.g. urban versus rural or residential versus industrial). As discussed in Chapter 3, it would be possible to replace the assumption that the T + D system remains constant by using a model similar to that used for generator reliability.

Then, following the logic of Chapter 5, the reverse cumulative distribution of the net load is:

$$F_{k}'^{t}(x) = p_{nk}^{t} F_{k}^{t}(x) + q_{nk}^{t} F_{k}^{t}(x - X_{nk}) \qquad (6.18)$$

where $F_{k}'(x)$ = distribution of the equivalent load after the central utility "unit" has been loaded.

$F_k(x)$ = distribution of the equivalent loau after the customer k's generator has been loaded.

The operating capacity of the utility is

$$Y_{nk}^{st} = \int_0^{X_{nk}} F_k^{st'}(x)dx, \tag{6.19}$$

and the total energy supplied by the utility to customer is:

$$MW_{nk}^{st} = p_{nk}^{st} h_s Y_{nk}^{st} \,. \tag{6.20}$$

The cost function for electricity for customer k is:

$$EC(Y_k^{st}) = h_s(p_i^{st} \; Y_{ik}^{st} \; c_i^{st} + p_{nk}^{st} \; Y_{nk}^{st} \; c_{nk}^{st}) \tag{6.21}$$

The overall reliability of electricity supply (central plus decentral generators) for customer k is:

$$LOLP_k^{t} = F_k^{t'}(X_{nk}) \tag{6.22}$$

The unserved energy is:

$$EF(Y_k^{t}) = \int_{X_{nk}}^{\infty} F_k^{t'}(x)dx. \tag{6.23}$$

Since the form of the function EC in the objective tunction (6.8) is known, and the form of the function EF in constraint (6.10) is known, the optimization (6.8)-(6.12) can be formulated using Benders' decomposition just as it was for the central utility. The optimization becomes:

$$\min_{X} Z$$

$$\text{s.t.} \quad z \geq \sum_{j=1}^{J} \sum_{v=\tau}^{T} K_{jv} X_{jv} + \sum_{t=\tau}^{T} \sum_{s=1}^{S} [EC(Y_{nk}^{st})$$

$$+ \sum_{i=1}^{n} \pi_{im}^{st} \delta_{jv}^{st}(X_{mjv} - X_{jv})] \qquad m = 1, \ldots M \qquad (6.24)$$

$$EF(Y_m^{st}) + \theta_m^{st} \delta^{st}(X_m - X) \leq \varepsilon_k^t \qquad s,t \in \Gamma_m$$

$$m = 1, \ldots M \qquad (6.25)$$

plus constraints (6.11) and (6.12). The only difference between this formulation and that for utilities is that the shadow price on the capacity for the time dependent unit can not be computed with the formula given in reference [11]. Bloom has expanded this formula so that the shadow price can be calculated, but the methodology has not yet been implemented.

It should be pointed out, that with Bloom's revisions, hydro-electric power plants, storage plants, and time-dependent generators could be included in the utility optimization. For simplicity of explanation, these types of units have been omitted from the discussion.

This discussion of the use of Benders' decomposition has been included even though Bloom has not yet completed his work in order to emphasize the symmetry of the utility and customer planning problems. The breakeven methodology will be used n the sample problem given in Chapter 8. When the time-dependent methodology is complete, it will be possible to use sensitivity analysis to find the breakeven cost for each alternative technology. Thus, the optimization will be able to answer both questions: given the current price, how much should I buy and how much is that system worth to me

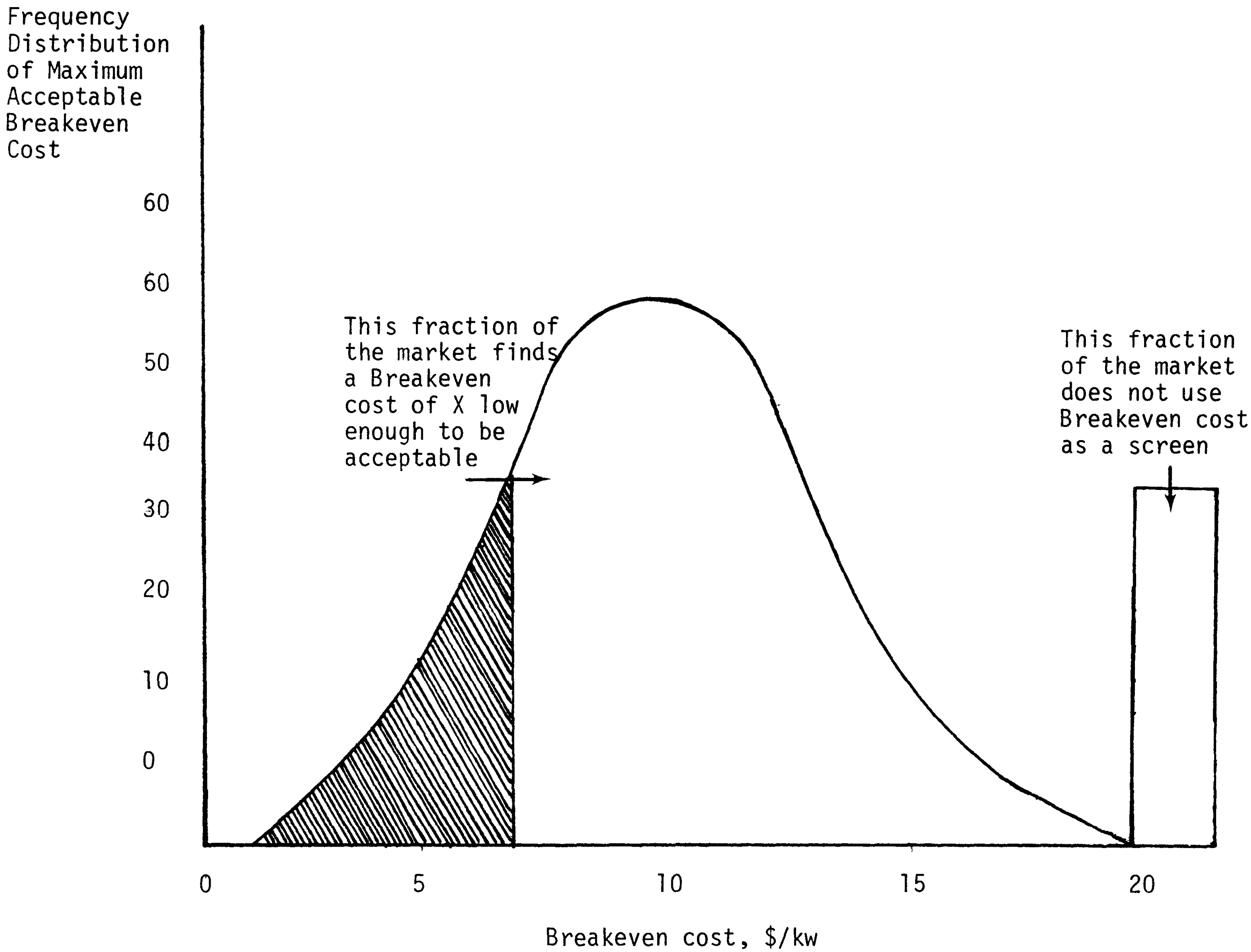
6.III Market Penetration

Even though a product may appear to be economically attractive, it does not necesarily follow that everyone will go out and buy one. There are many reasons that consumers may not buy something that economists think they should. The reasons range from lack of information to incompatible color schemes. Lilien and Wulfe [60] at MIT have studied market acceptance of photovoltaics. Only a small portion of their work will be used here and reference should be made to the original work for more detail. Basically, before someone buys something, the product must pass a number of screening tests. Lilien and Wulfe have developed screening curves from survey reslts that give the fraction of the market that finds a product acceptable at a given value of one of its attributes. The screening curve for breakeven cost for photovoltaics is given in Figure 6.1.

6.IV Summary

The customer decision process has been shown to be similar to the utility decision process. It is simpler in that each customer makes far fewer active decisions than the utility, and it is more complex in that each customer considers many more criteria than simple cost minimization.

From either the breakeven methodology or the optimization



Note: Survey results are used to develop this distribution.

Figure 6.1 Screening curve for the breakeven cost for photovoltaics. From reference [60].

methodology, the breakeven capital cost for each system can be computed:

$$\hat{BECC}_i^t = f(\hat{c}^t, \hat{K}^t, w^t) \quad t = \tau,\ldots,T \tag{6.26}$$

where $\hat{c}^t$ = vector of current and projected fuel prices (\$/MBtu)

$\hat{K}^t$ = vector of capital costs in time t (\$/MW)

w^t = vector of meteorological variables

From the screening curves, the number of customers who would buy system i at time t at its projected price can be found:

$$\hat{X}_i^t = f(\hat{BECC}_i^t, \hat{K}_i^t) \tag{6.27}$$

where $\hat{K}^t$ = estimated capital cost of the system i at time t (\$/MW).

Then, the decisions that must be made now in order to have the capacity in time t are made firm:

$$X_i^t = \hat{X}_i^{\tau+\mu_i} \tag{6.28}$$

where τ = current time period

μ_i = lead time for generator type i.

7. Price Setting

The rate-setting component of the model uses the operating cost from Chapter 4 and the capital cost from Chapter 5 to compute the rate structure for electricity consumers for the next time period. Because rate-setting procedures are not standard, several methodologies are presented. Doing so allows one to answer questions about the effect of different rate structures on the overall efficiency of the system.

7.I. Introduction

Rate setting procedures vary from state to state depending on the practices of the local regulatory agency. These agencies are usually mandated by the state to oversee the monopoly granted to the utilities. Until recently, any control that the agencies did exert was through the rate setting hearings. There used to be little controversy in rate setting since utilities were able to exploit technological advances in generation and economies of scale so that the real price of electricity fell for many years. (See Figure 1.1). Now, however, the real price of generating equipment is rising rather than falling, as shown in Figure 1.2, and fuel prices have escalated rapidly. In addition, electricity demand has ceased to grow at eight percent per annum as it did for so many years. Growth now seems to be about two or three percent per year (see Figure 1.3). Regulatory agencies have begun to look more closely at the rates proposed by utilities and some agencies have even begun to question seriously utilities' expansion plans. This section will outline the major issues in rate setting. There is a large literature on rate setting and its regulatory, economic and engineering aspects [10,12,18,19,24,25,32,53,55,56,66,68,69,72,78,80,91,92,93,95]. Reference should be made to this literature if more depth is required.

7.II. Customer Classes

Utilities have been allowed to divide their customers into classes and to charge different rates to these classes based on differences in the cost of service. The standard classifications are residential, commercial and industrial. Usually even these classes are subdivided so that, for example, residential customers with electric heat are a separate class from residential customers with non-electric heat.

The typical rate structure in the United States is a declining block rate. An example is given in Table 7.1. The first large block is designed to recover fixed costs such as hook-up costs, billing costs and metering costs. The price declines thereafter because of the assumption that the more electricity is consumed, the lower the marginal cost will be. The assumed shape of the demand curve and the declining block structure are illustrated in Figure 7.1.

10 kwhr or less	$1.90
next 40 kwhr	$0.0487 per kwhr
next 50 kwhr	$0.0356 per pwhr
next 200 kwhr	$0.0281 per kwhr
next 300 kwhr	$0.0225 per kwhr
additional kwhrs	$0.0206 per kwhr

plus $0.03839 per kwhr fuel adjustment charge.

Table 7.1: Residential Declining Block Tariff

from Cambridge Electric Light Company Bill (1980)

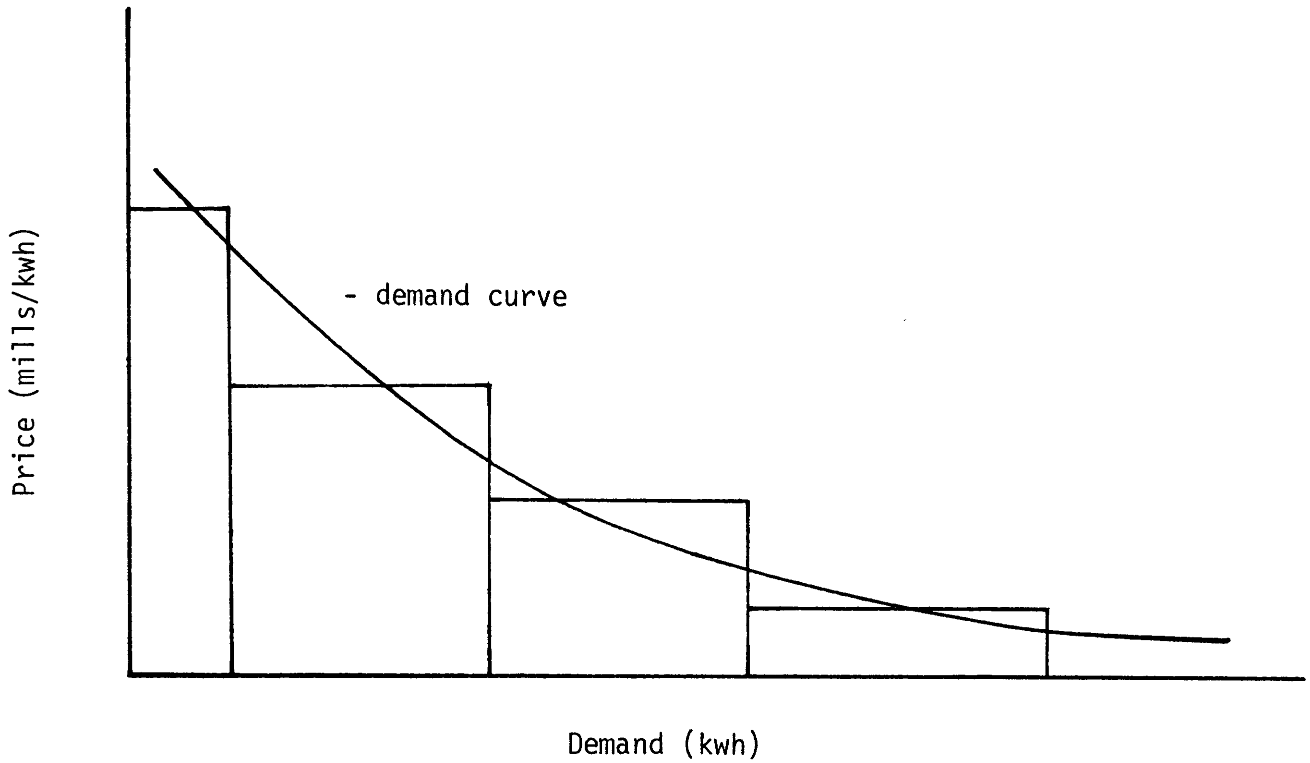


Figure 7.1 Electrical Demand and the Declining Block Rate Structure

The rates to different classes of consumers vary by the height and length of the declining block. For example industrial customers tend to receive power at higher voltages and supply more of their own power conditioning equipment, like step-down transformers. So the investment by the utility required for these customers is lower per kilowatt hour consumed. Also, because industrial customers consume larger amounts of energy, the fixed costs are spread over a large base and hence are lower. However, in the process of consuming energy, large users can also place higher power demands on the utility. That is, a utility may have to install capacity that sits idle much of the time in order to provide energy when a customer wants it. Both to cover these costs and to prevent spikes in demand, many utilities charge industrial users demand charges based on their maximum power demand during the billing cycle. High rates to residential users are usually justified by the argument that residential users cause the peaks in demand, but that it is too expensive to put demand meters on residences. The issue of allocating capital costs among customer classes is central to many of the debates on how rates should be set. There are insufficient data available to make definitive statements, but a great deal has been written on the subject.

In the ERATES model [24] used in this study, only two customer classes are considered: industrial, and residential-commercial. The capital cost for generation and transmission equipment is simply apportioned to each class by its relative share of electrical energy consumption. While it is known that capital costs are more directly related to power consumption than to energy consumption, lack of data forces this simplifying function.

7.III. Rate Setting

Rate structures for electricity are currently under increased scrutiny by the federal government, state utility boards, and public interest groups. It is difficult to describe how rates are set since the procedures vary greatly from state to state. It is impossible to describe how they should be set since there is no consensus among regulators, economists, and engineers about the proper methods.

The first division of opinion comes over whether embedded or marginal costs should be charged. One group argues that the utilities should only be allowed to recover their actual cost of producing and distributing electricity. The other group argues that customers should be charged the cost of producing the last unit of electricity demanded. In this way, customers pay the amount that it would cost the utility to increase production. If the electricity is worth that much to them, they will be willing to pay that amount to the utility and the utility will be willing to supply the electricity. This is marginal cost pricing.

The second division of opinion comes over flat rates versus time of day rates. One group argues that, even though the cost of producing electricity varies over time, the investment in metering required to implement time varying rates will not pay off because most customers would be unwilling to change their habits. The other group argues that customers will respond to time of day rates and that the potential savings to utilities in fuel and capital savings are substantial.

The following sections outline some of the ways in which rates can be set.

7.III.A. Embedded Rates

In the United States utilities are regulated to prevent monopoly prices. A basic concept is that a utility should be allowed to charge prices such that it earns a fair rate of return on its capital investments. The capital investments are commonly called the rate base. What is allowed to be counted in the rate base varies from state to state, but it is basically the undepreciated stock of generation and transmission equipment plus an allowance for fuel inventory. The revenue requirements for a utility are computed by multiplying the rate base by the allowed rate of return and then adding in variable costs such as fuel, operating and maintenance, and wages. How the required revenues are collected depends on the type of rate structure, e.g. flat versus time of time rates, but the underlying premise is that the revenues are based on the historical cost to the utility. For more detail on how the required revenues are computed see [10] or [56].

For this study, the ERATES model [24] method for computing the required revenues will be used. In this model, the required revenues are given by:

$$RR^t = rr(CX_d^t + W^t) + EC^t + D^t \tag{7.1}$$

where

RR^t = required revenue in time t

rr = allowed rate of return on investment

CX_d^t = undepreciated stock of capital in time t ($)

W^t = allowance for working capital in time t ($)

EC^t = system operating cost in time t ($)

D^t = taxes, depreciation and other expenses ($)

7.III.B. Marginal Rates

For many years, economists have been writing in journals about the desirability of marginal cost rates for electricity. The basic argument is that if customers were charged the true cost of electricity, then they could make more rational decisions about when and how much electricity to use. The problem, however, is to determine what the true cost of electricity is.

The marginal cost for an incremental unit of energy demanded in subperiod s, time period t can be written as:

$$MC_E^{st} = \frac{\partial(CX^t + EC^{st})}{\partial E^{st}} \tag{7.2}$$

where

MC_E^{st} = marginal energy cost in subperiod s, time period t ($/MWH)

CX^t = cost of installing new capacity at time t ($)

EC^{st} = system operating cost at subperiod s, time period t ($)

∂E^{st} = marginal change in the energy demand in subperiod s, time period t (MWH).

The marginal cost for an incremental unit of power demanded in subperiod s, time period t can be written as:

$$MC_p^{st} = \frac{\partial(CX^t + EC^{st})}{\partial Y^{st}} \qquad (7.3)$$

where

MC_p^{st} = marginal power cost in subperiod s, time period t ($/MW)

∂Y^{st} = marginal change in the energy demand in subperiod s, time period t (MW).

The partial derivatives of equations (7.2) and (7.3) can be computed explicitly within a capacity optimization model as discussed by Bloom [11]. However, since this methodology has not been implemented, the partial derivatives of (7.2) and (7.3) will be approximated by a difference equation:

$$MC^{st} = \frac{(TC_1^{st} - TC_2^{st})}{E_1^{st} - E_2^{st}} \qquad (7.4)$$

where

$TC_i^{st} = CX_i^t + EC_i^{st}$

= total minimum system cost to produce energy E_i^{st} in subperiod s, time period t ($)

7.III.C. Flat Rates

The term flat rate refers to rates that do not vary with the time of day, and depend only on the amount of energy consumed. Thus, declining block rates are flat rates even though the price is not constant. In theory, a flat rate could be based on either embedded costs or marginal costs.

To compute embedded flat rates, the cost of service to each customer class is computed as a function of the energy consumed. To do so requires assumptions about the capital investments made for each type of customer, and about the consumption patterns of each type of customer. Utilities have developed many ways of allocating costs from scant data. We will make the simplifying assumption that costs are directly related to energy consumption. So, the required revenue for class k is:

$$RR_k^t = RR^t \; E_k^t / E^t \tag{7.5}$$

where

RR_k^t = required revenue from class k in time t (\$)

E_k^t = energy consumed by class k in time t (MWH)

E^t = total energy consumed by all customers (MWH).

Then, the imbedded flat price of electricity is:

$$c_{nk}^t = RR_k^t / E_k^t \tag{7.6}$$

where

c_{nk}^t = price of grid electricity to customer k in time t (\$/MWH)

Under a marginal flat rate, the marginal cost in equation (7.4) would be computed for the entire time period and the rate would be given by:

$$c_{nk}^t = MC_k^t + M_k/\hat{E}_k^t = \frac{(TC_1^t - TC_2^t)}{\Delta E_k^t} + M_k^t/\hat{E}_k^t \tag{7.7}$$

where

ΔE_k^t = change in energy demand in time period t by customer, class k (MWH)

M_k^t = fixed costs attributable to customer k in time t ($)

$\hat{E}_k^t$ = estimated energy consumption by customer k in time t (MWH)

7.III.D. Time of Day Rates

Because the cost of producing electricity varies with the time of day, it seems only reasonable to charge rates that vary with the time of day. Many European countries have successfully introduced time of day rates after some experimentation. Electric utilities in the United States are only beginning to consider time of day rates and usually only because of pressure from the federal government or state regulatory boards.

Time of day rates can be either embedded rates or replacement rates. For embedded time of day rates, one computes the historical cost of the capital and fuel required to supply electricity at that time. For the replacement rate, one computes the cost of increased consumption at the given time using equations (7.2) and (7.3). The increased cost has both a capital and a fuel component, since an increase in demand requires additional generating and transmission equipment as well as an increase in fuel consumption. In theory, a cutomer should be charged both a capital and energy cost in each time period. Since this is not possible with present metering, the capacity charge is subsumed in the energy charge:

$$c_{nk}^{st} = MC_E^{st} + MC_p^{st} Z_k^{st} + M_k/\hat{E}_k \tag{7.8}$$

where

c_{nk}^{st} = cost of grid electricity to customer k in subperiod s, time period t ($/MWH)

Z_k^{st} = ratio of power demand to energy demand in subperiod s, time period t for customer k.

There has always been some controversy over whether all capital costs should be allocated to the peak demand periods. However, a simple argument illustrates that the base periods should have capital costs included. Suppose a system had an intermittent demand as illustrated in Figure 7.2a. For this type of demand, a utility would build only peaking units. An increase in demand on the peak would result only in a slightly larger peaking unit being built, and the marginal cost for the peak period would be the marginal capital cost of a peaking unit plus the fuel cost. Now suppose there is an increase in demand in the periods in which there was formerly no demand, as illustrated in Figure 7.2b. The utility would then build a small base load unit to meet this new demand, and the marginal cost for the base periods would be the marginal capital cost of a base load unit plus its fuel cost. Therefore, because of the choices of technology available to a utility, increases in base demand do have a capital cost component. This component can be computed directly using the partial derivatives of equation (7.2) and (7.3) and using the historical cost of capacity, rather than its replacement cost. The formula for the marginal embedded cost is then the same as equation (7.8) with the marginal costs redefined.

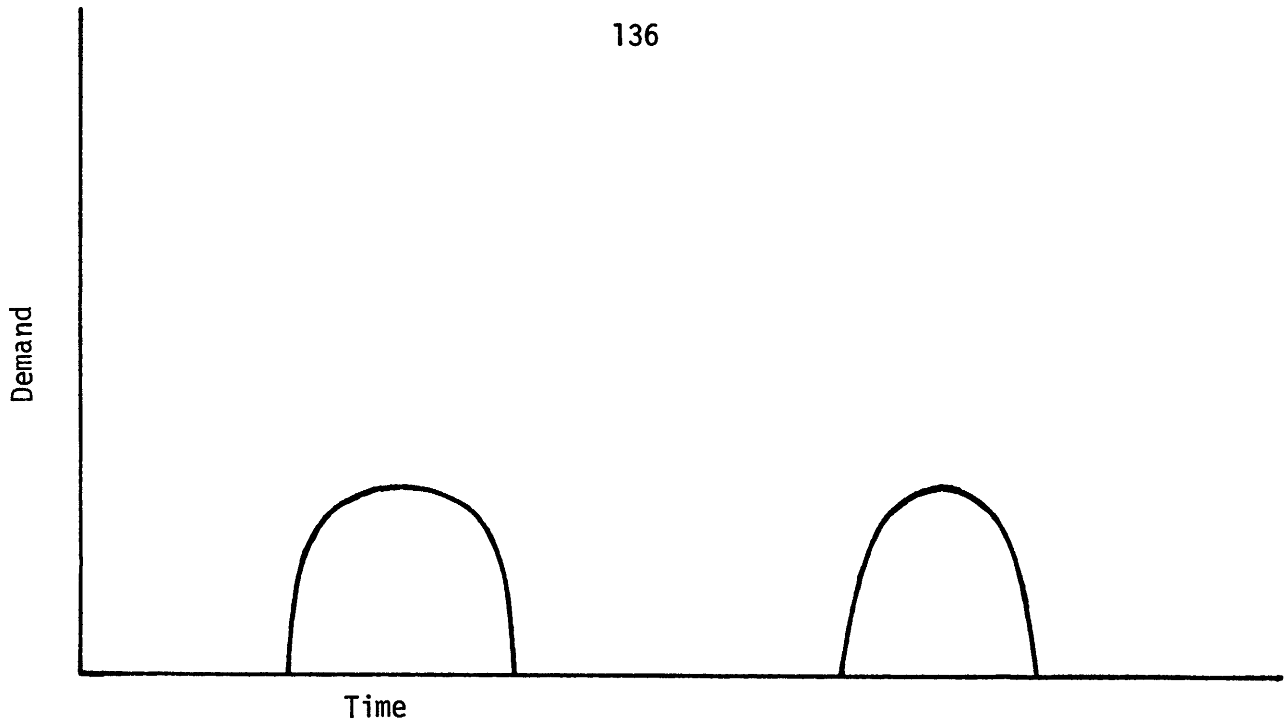


Figure 7.2a Intermittant Demand. Optimal mix = peaking units

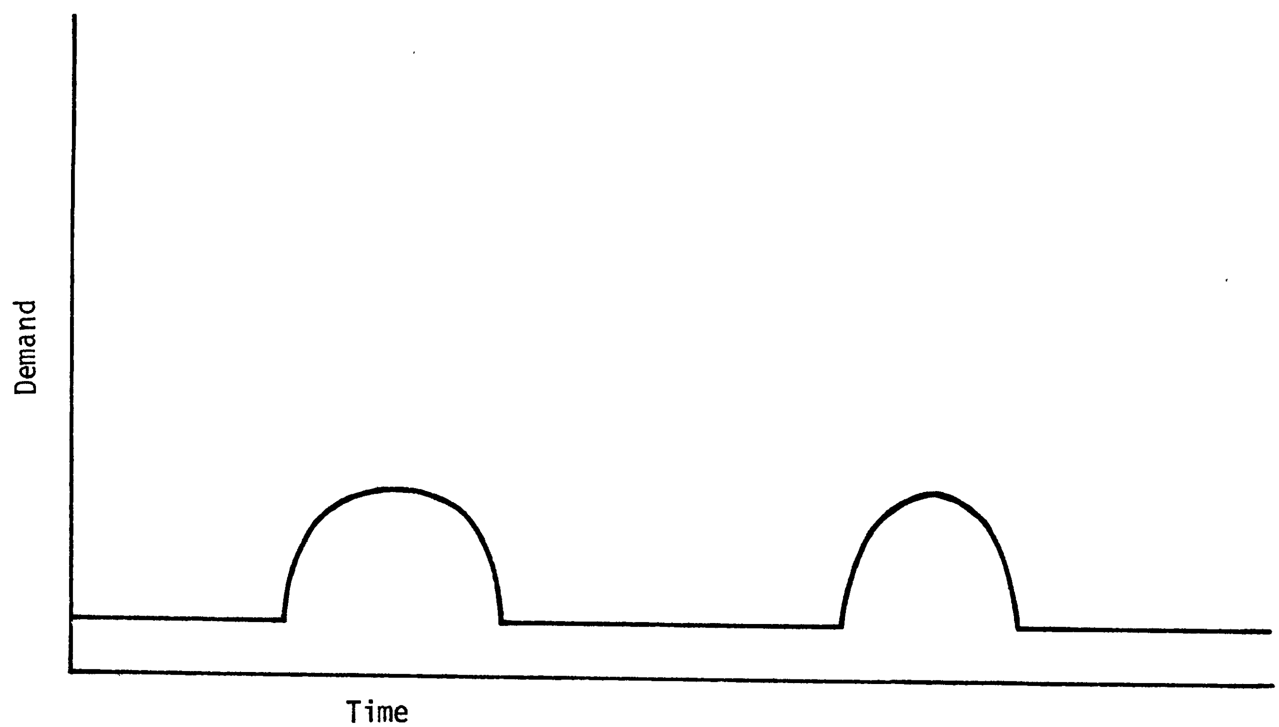


Figure 7.2b Same demand as in Figure 6.5a with constant component added. Optimal mix = peaking units + base unit

Figure 7.2 Effect of Demand patterns on the generation mix

7.IV. Summary

The price of electricity can be computed using embedded or marginal cost principles given the operating and capital cost of the system as computed in Chapters 4 and 5. The rate may be either a flat rate or a time of day rate depending on the costing principle.

The rates for the next time period are set using one of the methodologies described here. These rates are then sent to the customers and influence both their short-run and long-run demand as described in Chapters 3 and 6. The new rates are also used by the utility to update their demand forecast as described in Chapter 2. The basic formulation of the electrical rates is:

$$c_{nk}^{st} = f(EC^{st}, CX^{t}, M_k) \tag{7.9}$$

where

EC^{st} = expected cost of operating the system in subperiod s, time period t ($)

CX^{t} = capital cost (historical or replacement) in time t ($)

M_k = fixed costs attributable to customer k ($)

8. Summary

Chapters 2 through 7 have described in detail specific algorithms that can be used to implement the general methodology presented in Chapter 1. This chapter will summarize the algorithms and the flow of information among them. Chapter 9 gives an example that can be studied in parallel with this chapter.

The general methodology assumes that the process of planning is iterative and that plans are changed as new information becomes available and as the future becomes the present. This chapter describes one iteration, throughout which the exogenous variables remain constant. Once the decisions for that time period have been made, based on both endogenous and exogenous variables, then the data base is updated and the process begins for the next time period. Each iteration depends on the previous iterations in that current decisions may be limited by previous decisions that have restricted or eliminated certain choices. The flow of data between decision points and over time is illustrated in Figure 1.5.

To begin the planning process, one must have a data base containing the current prices of fuels, the capital costs of large and small scale generators, characteristics of the existing electrical generating system, meteorological parameters, socio-economic factors, previous demand patterns, and, where appropriate, the changes expected in these parameters by each of the decision makers. These parameters must be specified for every region to be studied.

The demand forecasting model, described in Chapter 2, is used to project the future energy demand in order to plan for the size and installation dates of new generating units. Knowledge of future demand is particularly important for units with long lead times which must be begun before there is an apparent need for them. Since the demand for electricity is sensitive to the price of electricity and to the price of alternative fuels, in a time when fuel prices are changing rapidly and unpredictably, the demand forecasts must be constantly updated. And, the expansion plans which are based on them must also be updated and revised.

In general, the future demand for electricity depends on the price of all fuels, including electricity, socio-economic factors, meteorological factors, and the previous demand. Of these parameters, the socio-economic factors, the meteorological factors, and the price of all fuels except electricity are specified exogenously. The price of electricity is found within the model based on the demand for electricity and the cost of meeting that demand as computed in the previous iteration.

In this report, the Baughman-Joskow demand model [9] was used to forecast long-run energy demand. In this model, a set of log-linear regression equations is used to define the relationship between the demand for electricity and the price of fuels. The coefficients for these equations are estimated based on historical data on energy consumption and energy prices for different fuels in different economic sectors.

The Baughman-Joskow model assumes that utility rates are flat (not time-differentiated) and that a central utility supplies all the electricity demanded. Since we are interested in studying both new rate structures and alternative ownership of generators, the Baughman-Joskow model has been modified by two models to account for these factors.

The first is a load shifting model, developed by Hausman, Kinnucan, and McFadden, which estimates the energy demand in one time period relative to another based on the price differential, the appliance stock, weather conditions, and sociological factors. The net change could be either positive or negative. Some demands, like lighting demand cannot usually be deferred, so the demand is permanently reduced when someone decides to turn off a light to save on the electricity bill. Other demands, such as heating and cooling may only be delayed and their delay can result in a greater total demand than if they had been met when originally required. If the off-peak prices are low enough, consumers may actually increase the total electricity they consume in response to the low price.

The second model modifies the demand projection to account for energy that is produced by customer-owned generators rather than by the utility's generators. The total demand for electricity is assumed to be the demand as calculated by the Baughman-Joskow model and modified by the Hausman model. However, the net energy demand on the utility's generators is lowered by the total amount of energy generated by customers, adjusted for losses. The adjustment for losses is computed using a simple, linear assumption about the additional energy that must be generated to the utility to make up for losses in the transmission lines. In this model, it is assumed that the number and type of generators owned by customers is known. We will see that this information is computed within the larger model and is updated at every time step.

The combined forecasting method described above projects only the total energy demand. To plan for new units, it is also necessary to know the load shape in order to take advantage of the generating technologies available. Because it is extremely difficult to construct a load shape from scratch, the load shape from the last year is used as a basis and then the modifications due to time-of-day pricing and to customer-owned generation are superimposed on it. The same models are used to modify the load shape as were used to modify the energy demand. For the time-of-day pricing model, it is assumed that the change in power demand is proportional to the change in energy demand within a pricing period. While this assumption lacks refinement, there is not enough data to warrant any other assumption. Once the demand in each time period has been modified for the price response, the time-varying customer generation is subtracted from it. Because the output from renewable resource generators is frequently highly correlated with electricity demand, it is necessary to take this correlation into account when performing the subtraction. One way to do this is to use a statistical method that finds the correlation of the demand with meteorological variables, as developed by Caramanis [16] and described in Chapter 3. An alternative way is to match hourly historical electricity demands with historical weather data as is done in the example of Chapter 9.

The essential outputs of the demand model are the new forecasts of the peak power demand, the total electricity demand and the load shapes for each time period in the study. These forecasts are used in the capacity planning model in determining the number and type of new generators to be built. The demand model can also be used to compute the net demand on the central generators for the current time period. This information is passed to the plant operation model to determine the costs of meeting the demand in the current time period.

The plant operation model uses the load shape and energy demand, the physical characteristics of the utility's generators, and the fuel prices for the current time period to compute the expected cost to the utility of meeting the customer demand and the reliability with which the utility meets the customer demand can be found. In this report, the SYSGEN model [38] developed by the author was used for the production costing and reliability analysis.

The plant operation model takes into account the random nature of the customer demand and the availability of generators using a technique developed by Baleriaux [6]. As described in Chapter 4, this technique makes it possible to compute the amount of energy that each plant would be expected to generate taking into account that other plans may fail requiring it to generate more electricity or that the demand may be unusually high or low requiring it to generate more or less electricity. This technique does not compute all possible combinations of plant failures and load levels. Instead, it uses convolution to find the probability distribution of the demand plus the plant outages. This distribution can then be used to compute the expected value of the energy generated by each plant. It can also be used to find the loss-of-load probability and the expected unserved energy of the system as illustrated in Figure 4.3.

The plant operation model produces detailed information about the operation of each unit. This information is used within the long range planning model in making trade-offs between capital and operating costs for different plants. The long range planning model also uses the reliability measures computed in the plant operation model to ensure that the new system meets the reliability requirement of the utility.

The total cost of operating the system is passed on to the rate setting model to be included in the required revenues when setting the rates for the next time period. The reliability of the system is sensed directly by the customers and is one of the inputs to their decisions of whether or not to build their own generators.

One of the assumptions of the system operating model is that it is known what units have been installed and are available to generate power. Once the current projections for the net load shape and energy demand are known, the utility expansion plan can be found based on current estimates of construction costs and fuel costs for new generators. The expansion plan is usually made for about twenty years into the future, but only the plans for the near future are made definite. In fact, only those decisions which must be made in the current time period to have a new unit when needed are made firm. All other decisions are allowed to float until some future time period when they are made firm, postponed, or abandoned depending on new conditions.

The model used for long-range planning, the EGEAS model developed at

MIT, is described in Chapter 5. The model, based on Bloom's thesis [11], uses Benders' decomposition to solve the optimization problem. Using Benders' decomposition allows the use of a nonlinear model of the power system operation, but retains the advantage of having a linear model for the optimization. The master problem generates a capacity expansion plan based on the criteria of minimizing total cost. The plant operation model then computes the cost of operating that set of plants to meet the demand. It also computes the shadow price on the capacity of each plant in the current plan. The shadow price on the capacity indicates to the master problem how the total cost of operating the system would change if the plant were made a little bigger or a little smaller. Using this information, the master problem computes a new expansion plan. This process continues until the total cost is arbitrarily close to the upper bound generated by the Benders' technique.

For this report, a relaxed version of the EGEAS model was used since only the early years of a study are critical in deciding the next unit to be built. In the relaxed version, the early years are modeled using the complete Benders' structure and the later years are modeled using a linear, rather than a non-linear, plant operation model. This allows for faster running times during computation.

Once a unit has been committed to construction that information is passed on to the rate setting model so that its capital and carrying costs will be included in the rates set for customers. The information of committed units is also passed to the system operation model since, at some point, the unit will come on line and can be used to meet customer demand.

In the demand model, it was assumed that the number of customers owning their own generators was known. In order to find this number out, one more model must be used. This model describes how customers respond to the electricity rates and other factors in deciding whether or not to build their own power generators. In terms of the larger model, customers are given the price of electricity by the utility for the current year and have expectations about the future price based on the current price and the prior behavior of the price. In addition, the customers perceive directly the reliability of the grid electricity to their households.

Customers, in deciding what kinds of equipment to install and whether or not to generate their own power, make the same kinds of decisions as the utility although usually in a far less sophisticated way, and usually with many more criteria besides cost minimization. Because of this, the model is structured to answer the question of the worth of the system to the owner, rather than the question of the number of systems that would be installed under a minimum cost criterion. In optimization theory, these two questions are closely linked, one being the dual of the other. However, for our purposes, the first question is more useful. Since the available computer version of EGEAS [30] does not yet treat time-dependent generators, the example in Chapter 9 uses a breakeven analysis model developed specifically for time-dependent generators and described in reference [84].

The breakeven cost of a system is dependent on the price of the should change with the time of day. Rather than attempting to justify using one rate or another, the ERATES model [24] was used because it can calculate both flat and time of day rates on either an embedded or marginal cost basis. The option of computing different types of rates allows one to study the effect that different types of rates will have on the demand.

The new rates for the next year are announced to the consumers and the electriciy rates for the demand model and the customer choice model are updated. All the exogenous variables are also updated and the process begins again.

These interconnected models allow a planner to work through a full utility planning structure incorporating the sequential and interdependent nature of decisions. The following chapter works through an example showing how the models fit together. The example demonstrates method and logic while giving reasonable results, although the results would not be applicable to any particular utility.

9. Example

This chapter presents results from hand calculations and computer runs. Several of the computer modules were unavailable because they were still under development. For these modules, simplifying assumptions were made so that the model could be run on a pocket calculator, or else another documented, available, but less advanced model was substituted.

A time frame of eight years was chosen, starting in 1975, with five time periods modeled exactly and three approximately. The data on fuel prices, socio-economic conditions, and weather conditions are based on New England data for 1975. The example should be taken as an example of the mechanics of the methodology, and not as the results of a study.

9.I. Demand Model

9.I.A. Long-Run Demand

The basic inputs to the long-run demand model are the estimates of fuel prices, demographic factors and the energy consumption by economic sector for the base year. Table 9.1 gives fuel prices for 1975 and Table 9.2 lists the demographic and economic factors and their assumed growth rates. The population figure has been scaled so that the total demand matches the capabilities of the test utility in the EGEAS data base. Because the only version of the Baughman-Joskow model available was estimated prior to 1973, the elasticities were too low for current price levels. Therefore, in order to have demand grow so that the utility would build new plants, it was necessary to assume that the population in the test region grew at 10 percent per year.

Table 9.1

Fuel Price Data 1975 prices in 1970 dollars

Industrial Fuel

	$/MBTU		Annual Real Escalation Rate (percent)
Electricity	4.0176*	$.014/kWh	3.0
Oil	.6313	$3.66/barrel	3.0
Coal	.5092	$13.24/ton	3.0
Nuclear	.500	$62.50/gram	4.0
Gas	.8461	$909.56/mcf	4.0
No. 6 Oil	1.198	$7.57/barrel	3.0

Residential and Commercial Fuel

	$/MBTU		
Electricity	4.754*	$.016/kWh	3.0
No. 6 Oil	1.260	$.25/gallon	3.0
Gas	2.344	$.0018/hundred cf	4.0

*assumes direct conversion rate of 1kwh = .003412 MBTU

Table 9.2

Demand Model Input Assumptions

1970 dollars

		Annual growth rate (percent)
Personal Income ($/person)	3,000	0.0
Population	803,127	10.0
Value Added ($) (national)	3.46e11	5.0
Real Discount Rate (percent)	3	
Average Density (people/sq mi)	300	10.0
Minimum Temperature	15°F	
Maximum Temperature	85°F	
Average Residential energy consumption (MBtu/capita)	117	
Average industrial consumption (MBtu/capita)	166	
Average residential electrical consumption (MBtu/capita)	16.65	
Average industrial electrical consumption (MBtu/capita)	31.69	

The demand equations, as given in Chapter 2, were solved on a hand calculator. The resulting energy demand projections of the Baughman-Joskow model are given in Table 9.3.

As can be seen by the declining per capita energy consumption, customers respond very strongly to the real increases in the price of electricity. With more recent estimates of elasticities, one would not expect such a sharp reduction in demand; however, the trend would be in the same direction.

The long-run demand projections from the Baughman-Joskow model are passed on to the short-run demand model in order to find the change in load shape due to time-of-day pricing and customer owned generation. The long-run demand projections will ultimately be used by the long-range planning model as the given demand that must be met by building and generating units.

9.I.B. Short-Run Demand

9.I.B.1. Load-Shifting

Because the Baughman-Joskow model predicts only the total energy demand, without any other information, one would assume that the load shape for the base year, as given in Table 9.4, remained constant over time. However, two models have been presented that change both the total energy demand and the load shape. The first model predicts changes due to a price response to time-differentiated rates and the second predicts changes due to customer-owned generation. Because both models compute changes in energy demand and changes in load shape, the two functions are not described separately.

Table 9.3

Energy Demand Projections

	Electrical Energy Demand (MBtu/capita) Residential	Industrial	Total Electric Energy Demand (MBtu)*	Electric Power Demand (MW)
1975	16.65	31.69	38,750,530	2100
1976	15.39	30.42	40,700,014	2193
1977	14.17	30.42	42,243,058	2289
1978	13.44	29.25	44,564,624	2415
1979	13.01	27.35	47,457,644	2572
1980	12.37	26.62	50,431,381	2733
1981	11.79	26.05	53,838,317	2917
1982	11.39	25.61	57,907,486	3138
1983	11.03	25.37	62,665,291	3396

*assumes direct conversion rate of 1 MWh = 3.412 MBtu

Table 9.4

Initial Reverse Cumulative Distribution of the Customer Demand

x = percent of peak demand

x (percent)	P[demand > x]
30	1.000
35	.980
40	.925
45	.820
50	.750
55	.628
60	.548
65	.441
70	.324
75	.213
80	.122
85	.109
90	.040
95	.001
100	0.0

Due to problems with the published values of the coefficients for the load shifting model, a proxy model was used for the test case. The relative demand for all residential customers was assumed to have the form:

$$\log e^s = M1^s \log c^{base} + M2^s \log c^{shoulder} + M3^s \log c^{peak} \qquad (9.1)$$

where e^s = relative demand in subperiod s

c^s = relative price in subperiod s

$M1^s$ = elasticity of base period consumption with respect to the price in subperiod s

$M2^s$ = elasticity of shoulder period consumption with respect to the price in subperiod s

$M3^s$ = elasticity of peak period consumption with respect to the price in subperiod s.

Table 9.5 gives the hypothetical elasticities used in the test case. These elasticities can be interpreted rather simply. The negative elasticities along the diagonal mean that when the price goes up in that time period, the consumption decreases. The effect is largest on the peak and smallest in the base period. The off-diagonal elements indicated the relative change in the demand in a time period when the price in another time period changes. So, if the peak period price rises relatively, one would expect the peak demand to fall and to have the change in demand shifted mostly to the shoulder period and some to the base periods, with some demand being lost altogether.

Table 9.5

Inter-hour Price Elasticities

Elasticities/Price	C^{base}	$C^{shoulder}$	C^{peak}
C^{base}	-.05	.05	.02
$C^{shoulder}$	.01	-.15	.06
C^{peak}	.01	.05	-.20

Table 9.6 gives, for three subperiods, the energy consumption and prices in the previous time period and the projection of relative consumption for the next year of the study. These values were found by substituting the elasticities from Table 9.5 into equation (9.1). The prices in the subperiods are assumed to escalate at the same rate, so the relative consumption in subperiods remains constant although the absolute level of consumption changes. The new load distribution is given in Table 9.7. Chnges in each hour were computed as well, since the hourly curve is needed for the time-dependent generation model. As expected, some demand has shifted to the base time periods. The probabilities of being found in low load states are now higher. And, the probabilities of being found in higher load states is lower. It should be noted that the peak demand doesn't change since there is always some probability that the original peak will still occur.

It should also be noted that the load shifting reduces the total energy demand. Computing the area under the curves tabulated in Tables 9.4 and 9.7, given that the peaks are the same for each curve, one finds

Table 9.6

Price-Sensitive Demand

	Base	Shoulder	Peak
Price relative to previous time period	.5	3	10
Residential consumption relative to previous equivalent time period	1.1453	.7335	.6620
Length of time period (hours/day)	12	8	4

Table 9.7

Reverse Cumulative Distribution after Residential Price Shifting

x = percent of peak demand

x (percent)	P[demand > x]
30	1.000
35	.985
40	.935
45	.860
50	.750
55	.650
60	.500
65	.295
70	.165
75	.080
80	.010
85	.005
90	.001
95	.0005
100	0.0

that the overall reduction in demand is approximately six percent.

The load shape, modified by the price effect, is passed on to the customer-owned generation model so that the effects of time-dependent generation can be incorporated.

9.I.B.2 Customer-Owned Generation

There were initially assumed to be fifty identical wind turbines, fifty identical photovoltaic generators, and fifty identical diesel generators on the system with characteristics as given in Table 9.8. The T+D loss function was assumed to be piecewise linear and is given as a function of peak demand in Table 9.9.

The hourly load reductions for the wind and solar generators were

Table 9.8

Customer Generator Data

Unit Type	Fuel	Capacity (kw)	Mechanical Forced Outage Rate	Installment Cost ($/kw)	Heat Rate (BTU/KWH)
Photovoltaic	Insolation	5	.01	500	---
Wind Turbine	Wind	5	.10	400	---
Diesel	No. 6 Oil	5	.10	200	17,000

Table 9.9

T+D Loss Function

	base	shoulder	peak
percent of peak demand	50	75	100
loss multiplier	1.041	1.092	1.163

computed using the OESYS computer model [27] using a Boston weather tape as input. These load reductions were written to a computer file and then input to the ELECTRA [39] computer model. The new distribution for the net demand on the utility, as computed by ELECTRA, is based on an hourly analysis in order to capture the correlation of the output of the solar and wind generators with the demand on the utility. For ELECTRA, the price modified loads were computed on an hourly basis and then converted to a cumulative distribution as reported in Table 9.7.

Table 9.10 gives the new reverse cumulative distribution of the load including price effects, customer owned generation, and T+D losses. Tables 9.7 and 9.10 are compared graphically in Figure 9.1.

Table 9.11 gives the final net energy and net power demand projections for the first time step in the study. The net energy was found by computing the area under the load curve using linear interpolation between the points in Table 9.10. The projection for the current year, 1975, is the demand that will be used in the plant operation model as the demand that the utility must serve.

Table 9.10

Final Reverse Cumulative Distribution of Customer Demand

x = percent of peak demand

x (percent)	P[demand > x]
30	1.000
35	.941
40	.864
45	.787
50	.710
55	.629
60	.551
65	.481
70	.342
75	.243
80	.134
85	.058
90	.027
95	.007
100	0.0

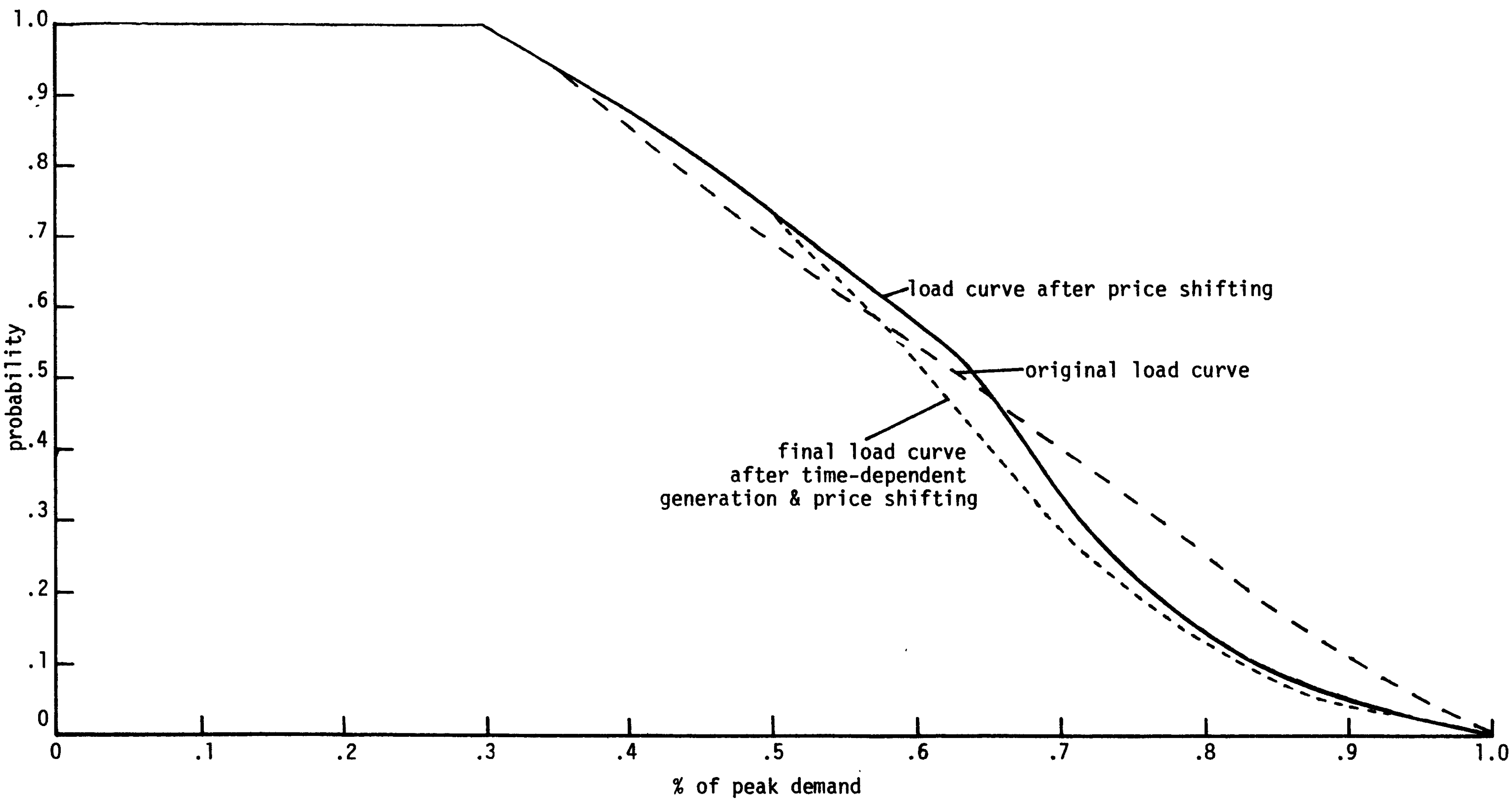


Figure 9.1 Initial, Intermediate, and Final Load Distribution Curves

Table 9.11

Net Energy and Power Demand on the Utility

	Energy Demand (MBTU)*	Power Demand (MW)
1975	38,403,534	2100
1976	40,107,626	2193
1977	41,864,797	2289
1978	44,165,576	2415
1979	47,032,687	2572
1980	49,979,800	2733
1981	53,356,225	2917
1982	57,388,955	3138
1983	62,104,159	3396

*Assumes a direct conversion rate of 1 MWh = 3.412 MBtu

9.II. Utility Model

The expansion plan for the example was found using a prototype of the EGEAS computer model [30] which uses SYSGEN [38], the production costing model, as a submodel. The basic plant data are given in Table 9.12. Also included in Table 9.12 are data on the historical and replacement capital cost which will be used in the rate setting model. The cost of fuel is taken from the cost of fuel for industries given in Table 9.1. The data on demand from Tables 9.10 and 9.11 were converted to the EGEAS format and input to the program.

The total installed capacity is 2300 MW, giving a reserve margin of approximately 10 percent over the initially projected power demand. In the optimization, the unserved energy constraint is that at least 99.1 percent of the original energy demand must be met. These, and other relevant, figures are summarized in Table 9.13.

The number of new alternatives is restricted to five to keep the optimization from becoming too large. Since a basic alternative can be installed in any year, there are actually forty alternatives within the optimization. Data on the alternative units is given in Table 9.14.

The capacity expansion plan and the operating cost in each time period are given in Table 9.15. In the sample case, lead times are ignored, so that the unit listed for installation in 1976 is assumed to be installed then.

The operating costs are computed as part of the long range planning model and so are not considered separately here. The operating cost for 1975 is passed on to the rate setting model as part of the required revenues.

Table 9.12

Installed Unit Data

(1970 dollars)

Unit	Type	Install-ment Year	Name-plate Capacity (MW)	Heat Rate (MBTU/MWH)	Forced Outage Rate	Historical Cost ($/kw)	Replace-ment Cost ($/kw)
Nuclear	Base	1971	600	10.400	.20	109	189
Oil	Base	1974	800	9.300	.13	130	137
Oil	Cycling	1963	800	9.400	.13	74	137
Gas Turbine	Peaking	1968	50	14.000	.24	48	95
Gas Turbine	Peaking	1970	50	14.000	.24	56	95

*Units built before 1958 are fully depreciated.

Data from reference [71] and [24].

Table 9.13

Optimization Data

Number of years, T	=	8
Discount rate, ρ	=	.03
Reliability requirement, ϵ	=	0.9 percent of energy demand
Reserve margin, RM	=	20 percent of peak power demand
Capital escalation rate	=	3 percent
Allowed rate of return	=	14 percent

Table 9.14

New Unit Data

Unit Type		Fuel	Capacity (MW)	Forced Outage Rate	Installment Cost ($/MW)	Full Load Heat Rate (MBtu/MWh)
Nuclear	Base	Nuclear	1000	.35	50,000	10.400
Coal	Base	Coal	800	.25	40,000	9.750
Oil	Intermediate	Oil	500	.20	30,000	9.400
Coal	Intermediate	Coal	600	.20	35,000	9.000
Gas Turbine	Peak	Oil	150	.15	13,000	14.000

Table 9.15

Utility Capacity Expansion Plan

1970 Dollars

Year	Unit Type	Capacity (MW)	Net Present Value Operating Cost (Million $)
1975			51.7
1976	Nuclear	208	44.1
1977			37.5
1978			32.4
1979			29.1

9.III. Customer Expansion Model

Because the prototype of EGEAS could not handle time dependent plants, the customer expansion model was run using the breakeven methodology described in reference [84]. The OESYS computer model [27] was used again to find the annual energy savings. Only one class of customers was considered. Their peak load and energy requirements are given in Table 9.16. The embedded flat rate for residential customers in Table 9.1 was used for the price of electricity. The costs for each potential new generating system are given in Table 9.8, along with other operating characteristics of the generators. Each generator type was assumed to have an expected useful life of 20 years for the purpose of computing the breakeven capital cost. The payback period was computed by dividing the capital cost of the system by the annual energy savings to give the number of years required to recover the investment. No discounting was used since Lilien and Wulfe [60] found that most consumers did not use discounting when computing the payback time on which they based decisions.

The marketing curves were only available as a function of payback time rather than breakeven cost, as illustrated in Figure 6.1. From the information on the amount of energy that the system provides annually, and from the expected fuel costs, both the payback time and the breakeven

Table 9.16

Customer Characteristics

Peak power demand	6 kw
Annual energy demand	5256 kwh
Reliability requirement	95 percent of energy demand
Discount rate	3 percent real

cost can be computed. These figures are included in Table 9.17. From Figure 6.1, the number of installations of each type can be found assuming that customers use the same criteria for all systems. The number of new installations is given in Table 9.18, as read from Figure 6.1.

For the residential customers in the test case, the reliability constraint was not binding. If industrial customers with higher reliability requirements had been included, some diesel generators might have been installed to meet power rather than energy requirements.

Table 9.17

System Breakeven Costs

(1970 dollars)

	$/KW Installed	Payback Time (years)
Photovoltaics	$352.00	14
Wind Turbine	$465.00	11
Diesel	-$ 73.00	∞

Table 9.18

New Customer Installations

	New Installations	Total Installations
Photovoltaics	20	70
Wind Turbine	30	80
Diesel	—	50

9.IV. Price Setting Model

The ERATES computer model [24] was used to compute the new rates based on the operating and capital costs of the system and to project the rates for the entire time horizon based on the expected operating and capital costs. The input data necessary to run the price setting model have been given in Tables 9.1, 9.2, 9.12, 9.13, and 9.15. These are: the price of fuels, the number of ratepayers, the average consumption, the existing capital structure and its depreciated and replacement value, the allowed rate of return, and the newly installed capital. The data were converted to ERATES format and input to the model.

Table 9.19 gives rates for 1976 for four different rate structures. Instead of running the rate model for all the remaining years of the study, a simple annual escalation rate of 5 percent was assumed. It should be noted that the prices computed are considerably higher than

Table 9.19

Electricity Rates for 1976

1970 Dollars

	Residential ($/kwh)	Industrial ($/kwh)
Flat embedded	.0479	.0511
Flat replacement	.0735	.0730
Time of day embedded		
peak	.0658	.0787
off-peak	.0638	.040
Time of day replacement		
peak	.0866	.0940
off-peak	.0645	.0649

those assumed in Table 9.1, and therefore one would expect the demand to be less than projected for the next time period.

For the next time period, the new rates would replace the flat rate given in Table 9.1 and the other fuel costs would be updated as would the socio-economic factors, expected growth rates, and capital costs for both centralized and decentralized generation. The demand for 1976 and beyond would be projected and the planning process would start again.

10. Conclusion

The author has developed a dynamic, non-equilibrium methodology for long-range planning of electric power systems. The methodology synthesizes the models used in electricity planning in a way which captures the common dependence of the utility and its customers on exogenous factors. The methodology also captures the influence that the customers have on the utility's decisions and that the utility has on the customers' decisions. This new methodology does not assume the existence of a long-term equilibrium solution. The general methodology presented in this report allows one to assume that as exogenous and endogenous factors change over time, decision makers can modify previous plans in order to track an ever-changing optimum. Thus, in a rapidly changing environment a stable equilibrium solution would not be expected using this methodology. But, if exogenous variables behaved as expected and if both the utility and its customers had the same expectations, and if each could predict what the other would do, then the solution would be equivalent to the equilibrium solution.

The methodology assumes that in each time period new information becomes available and that old decisions are revised and new decisions are made. The explicit inclusion of the time parameter allows the utility and its customers to change their expectations of the other's future behavior based on the new set of signals and to change their expectations of the future behavior of exogenous variables, such as price of oil, based on their latest values. The inclusion of the time parameter also allows factors such as lead times for new units and short-versus long-run responses to be modeled accurately.

The methodology focuses on near-term decisions based on the assumption that a decision is made only when the lead time for a project becomes critical. That is, even though plans may be made for the next thirty years, only those decisions within the first ten years or so require that commitments be made immediately. This allows the use of simplifying assumptions for time periods further in the future that reduce the computational effort required to find a solution.

One underlying assumption of the methodology is that the utility and its customers interact only through a small set of quantifiable signals. This assumption makes the lines of communication as illustrated in Figure 1.5 clean and well-defined. However, the utility may influence demand in many ways besides through the price they charge. They may promote new appliances or they may promote conservation. They may discourage non-utility generators through entangling rules or they may encourage them through educational programs on alternative resource technology. Of course, customers can influence the decisions of the utility other than through their demand patterns. They may file lawsuits to prevent specific projects or through referenda they may direct their legislations to encourage or discourage the use of particular fuels. In addition, the anticipated political response to new rates may influence how the rates are set by a utility, and rates are frequently substantially changed in response to consumer intervention during rate hearings. Interactions such as these could be incorporated into the general structure if they could be quantified as model inputs and outputs. However, the difficulty of quantifying such variables makes their inclusion in any utility model quite impractical.

Another limitation is that, since the boundary defining the system must be drawn somewhere, many potential interactions are ignored. For example, the electric utilities by their choices of generation technologies may affect the price of those technologies particularly since they are the only buyers of large power plants. A case in point of this principle is photovoltaic technology for which the price to all buyers is expected to drop rapidly if sales increase to a substantial level. Within the methodology, effects such as this must be treated exogenously because they did not directly affect the interaction of the utility and its customers.

By its very nature, the general methodology by trying to include everything can become so unwieldy as to be impractical to use. However, it can always be broken out into its components and then used as utility planning models have always been used. Just the exercise of looking at the planning process as a whole should help utility planners to see the many interactions within the system and allow them to construct internally consistent scenarios when planning.

To implement the general methodology fully would require computer programs for each model described in Chapters 2 through 7, a common data base, and linking programs. Of the models described above, some are complete while others are still under development. In particular, the demand models presented here require the most developed since the demand drives the entire proces. Either the econometric models must be re-estimated or they must be replaced with end-use models. And, for most of these models, there are other just as acceptable available models that could replace them in the general methodology. Thus, most of the work in implementing the methodology is in developing the data base and the linking programs. In the example given in Chapter 9, the data were reworked for each model's particular input requirements. For a large study, this would not be practical. A common data base would have to be developed with the high linking programs interfacing between the main programs and the data base.

As with any model that attempts to prescribe the actions of decision makers, there is no test which allows one to accept or reject the underlying hypotheses of the model. One possible experiment would be to run the model for a particular utility which experienced sharp demand reductions after 1973. Using this model, one could perform a planning study starting in 1973 and updating the plan each year based on the new information available in that year. By comparing the utility's actual plan to the computed plan one could get an estimate of the value of this type of planning methodology. Of course, the savings would be overestimated since utilities always operate with more constraints than are possible to model. The ultimate test will be its reasonableness and usefulness to those who must plan for new electricity supply.

Appendix A Electric Utilities

In the United States, electric power is supplied by public utilities which are regulated monopolies. A utility must supply power to all customers who wants it, whenever they want it, at a price approved by regulators. For many years, supplying demand was not a problem because fuel prices were stable, advances in technology steadily decreased the capital requirements for new plants, and demand growth was steady. Under these conditions, a price struction was set up which encouraged increased consumption. A typical declining block tariff of this kind was shown in Table. 7.1.

The price charged for a kilowatt of electricity under this tariff does not necessarily represent the cost of its production so that some customers subsidize others, but the rate are set so that the total revenues balance the total costs plus the allowed profit. The price of electricity did not become an issue until the late sixties when the trends mentioned above began to reverse: fuel prices increased, capital requirements increase, and demand growth declined unsteadily.

A.I. Electricity Demand

Electricity demand varies with the time of day as people wake up, go to work, eat, turn on lights and the television, and go to sleep. The demand also varies with the season as the length of the day changes and people require heating or cooling. Figures A.1. and A.2. show typical daily and annual load curves for a utility. Demand patterns vary markedly among utilities, depending on their location, the industries in

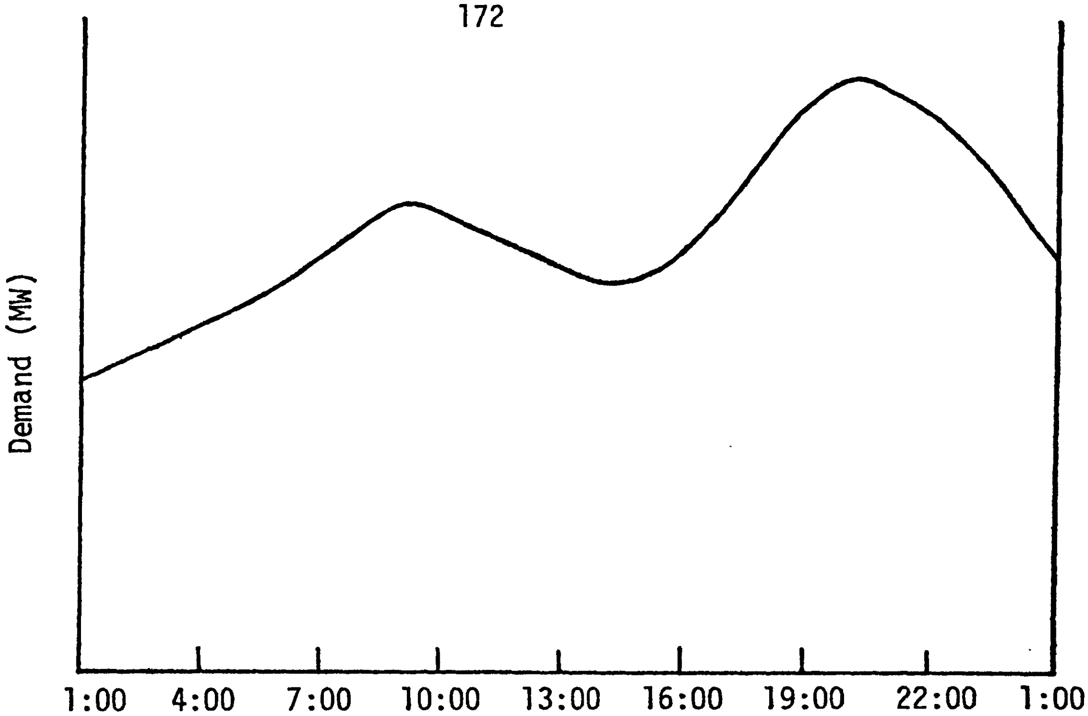


Figure A.1 Typical Daily Load Curve

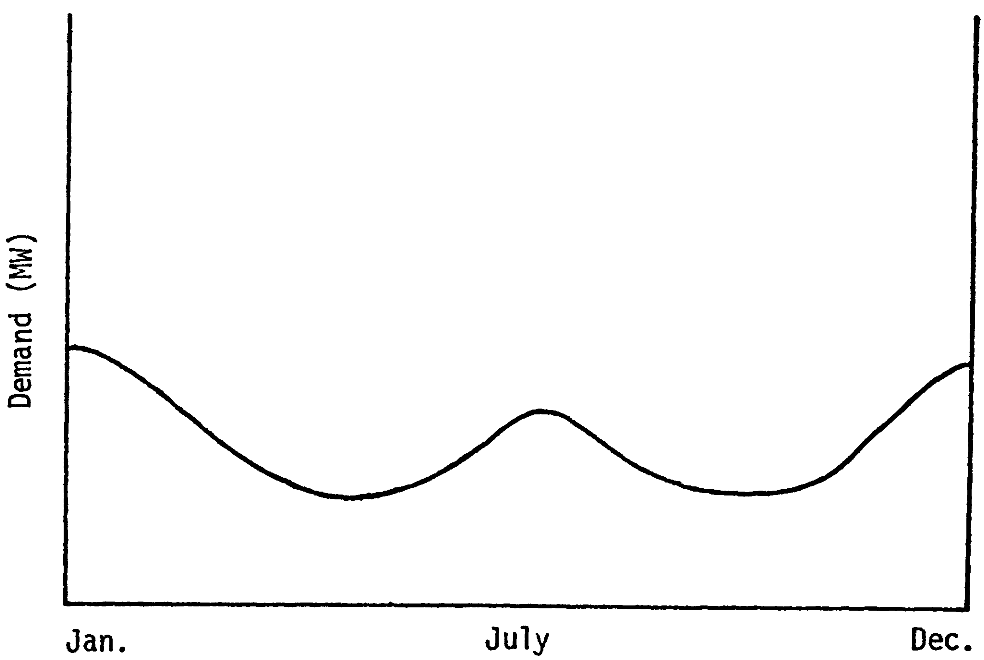


Figure A.2 Typical Annual Load Curve

the area, and the lifestyle of the poplulation. Most utilities, except those in the northernmost states, now experience their peak demand in the summertime from the air conditioning load. In the north, the peak demand occurs in the winter from the heating load and from the lighting load due to the short days.

Electricity consumers can conveniently be divided into three categories: residential, commercial, and industrial. The classes are distinguished by the end use of the power, the volume and pattern of demand, and the supply voltage of the power. Under the current rate structure, each customer class has a different tariff based on the utility's perception of the relative costs of supplying power and on its perception of the relative demand elasticities. Although utilities justify lower industrial rates on the basis that high voltage power is cheaper to supply, it has been suggested[1] that the lower prices are attributable to the higher industrial price elasticity. That is, industrial users are more sensitive to the price of electricity than other users.

A.II. Electricity Supply

For stable operation, an electric utility must balance the power it generates with the power that is demanded. If too little power is generated then the electrical frequency drops below the standard 60 cycles per second causing, in extreme cases, brown-outs, slowing of

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[1]Cicchetti, [18], p. 37.

electrical clocks, and damage to motors. If too much power is generated, the frequency rises, causing clocks to speed up and again causing damage to motors.

Because the demand for electricity is not uniform over time, utilities build three different types of power plants: peaking units, cycling (or intermediate) units, and base load units. A peaking unit has low capital cost and high fuel cost and can be started up or shut down rapidly. Keeping the capacity of a peaking unit available has a low cost, but there is a high cost to generate energy with it. A base load plant by contrast has high capital cost and low fuel cost. Base load plants are usually large (500 to 1000 megawatts) and have lengthy start up or shut down times. A cycling plant has capital and fuel costs between those of a peaking and base load plant. The particular combination of base load, cycling and peaking units owned by a utility is called the generation mix.

A totally different type of plant is a storage unit which generated no energy of its own. Storage units are used when there is not enough customer demand to run a base load unit at full capacity. The extra energy is saved in the storage unit until the customer demand rises and the extra energy is needed. Not all energy is recovered, however, because there are losses incurred in storing and in retrieving the energy. The most common type of storage used by utilities is pumped hydroelectric storage in which the excess energy is used to pump water up into a reservoir where it remains until the energy is required. The water is then released to drive a turbine which drives an electrical generator.

Figure A.3. shows trade-offs between initial cost and operating costs for base, cycling, and peaking plants. Storage units are not included because their energy cost is a function of the base power cost and the demand pattern. The curve shown in Figure A.3 is called a screening curve and can be used to make rough tradeoffs between different plant types. In Chapter 5, a long range planning model is described which makes the same trade-offs between capital and operating costs, but with much more sophistication.

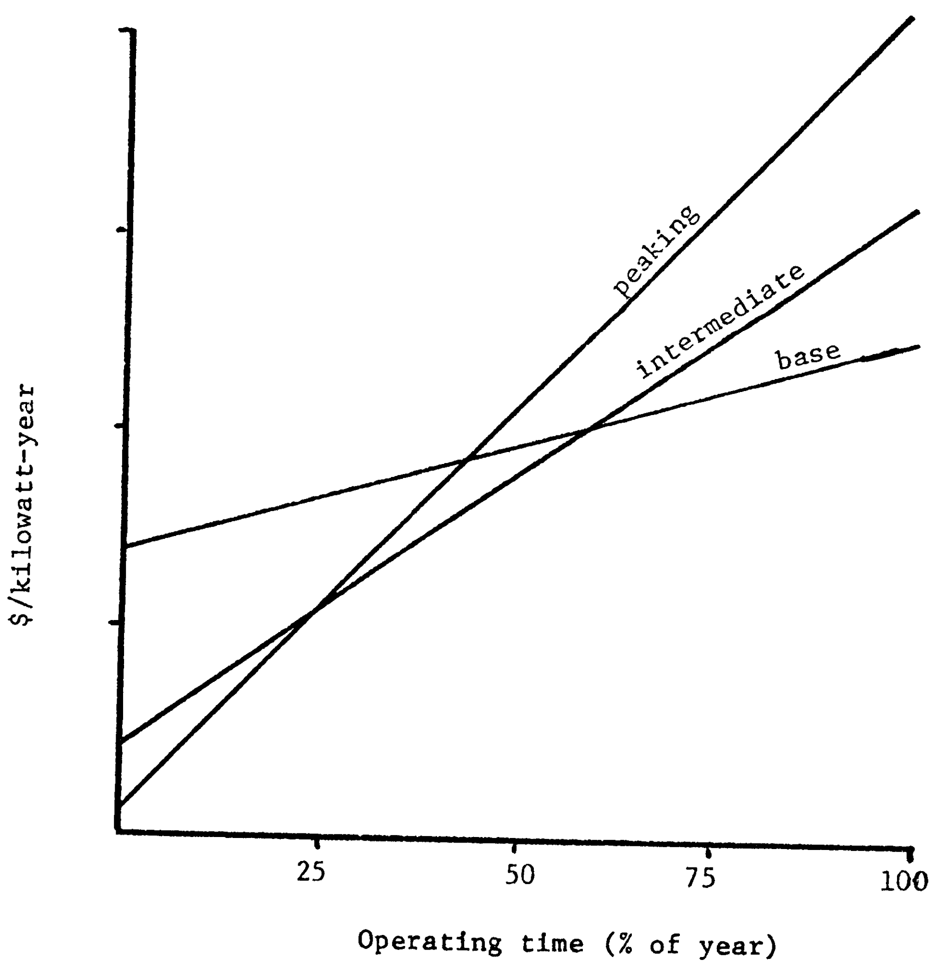

Figure A.3 Yearly Fixed Versus Operating Costs for Power Plants

When operating its plants to meet the customer demand, it is easy to show that the utility should bring the plants on-line in order of increasing cost. That is, if the demand increases, and the utility has to start up another plant, then it should start up the one that is the least costly to run among the plants that are not currently generating. In this way, the utility minimizes the total cost of operating the system. In fact, there are frequently reasons why the utility cannot bring up the next cheapest unit: there are physical constraints on some large generators that make it costly to start them up and shut them down; there are operating constraints on some hydroelectric generators because the water is used for many purposes or because reservoir size is limited; there are reliability constraints that dictate that a certain amount of capacity be kept in ready reserve; there are transmission constraints that can effect the order in which plants are used due to their geographical location.

In Chapter 4, a production costing model was presented that takes most of the constraints into account. The purpose of this model is to answer the question: How much does it cost to run the utility system given the demand and the fuel costs

A.III. Electric Reliability

From the discussion so far, it would appear that a utility would build peaking units only if high demands were expected for short perids of time; however, other factors make extra capacity necessary. One is that a utility does not know what the peak demand will be. Extra capacity must be built as a hedge against excessive demand. Another factor is that power plants cannot operate one hundred percent of the time. Plants must be shut down for preventative maintenance at least once a year. The utility needs extra capacity to make up for any plant that is being serviced. Also, a power plant generating electricity can fail suddenly requiring other units to be brought up quickly. Utilities usually operate with enough plants idling, ready to start generating, so that if the largest unit were to fail, the electricity it was generating could be replaced immediately.

All of these factors combine to require utilities to build more capacity that would at first seem necessary. For planning purposes, a rule of thumb is that the installed capacity should exceed the expected peak demand by twenty percent. Most regulations, though, are written in terms of the reliability of the system. For example, a utility may not lose load for more than one day in ten years. In order to compute the reliability at any one instant, one must know which plants are available for generation, the probability of failure for each machine, and the probability distribution of demand. These factors can be combined to give the loss of load probability (LOLP). Another measure of the reliability of a generating system is the expected unserved energy. Expressed as a fraction of the total energy demand, the unserved energy is referred to as the loss-of-load expectation, LOLE. The derivation of these reliability measures will be discussed in Chapter 4.

Another aspect of system reliability is the reliability of the transmission and distribution (T+D) network. T+D failures cause most outages experienced by consumers, but these failures are not included in the reliability measures of the LOLP or LOLE. Measuring the reliability of the transmission and distribution system is much more difficult than measuring the reliability of the generating system. The models require more data and there is no closed form solution. Load flow models must be run for each possible combination of available transmission lines, generating units, and demand levels. In addition, the reliability of the distribution system can vary greatly within a single network, so one must distinguish between, for example, the reliability to an urban customer served by underground lines and a rural customer served by long over-head lines.

Notation

$A1, \ldots A6$
$M1, \ldots M6$ = regression coefficients for the demand equations

a = regional index

App_{ik} = 0-1 variable indicating whether customer k owns appliance i

$AREA_a$ = area of region a (square miles)

$[b^{\sigma}_{ij}]$ = matrix of regression coefficients for time dependent generator outputs and customer demand.

B^{st}_i = net benefit from system i in subperiod s, time period t (\$)

BEC^t_i = Total breakeven cost for system i installed in year including capital, variable, and fixed cost (\$)

$BECC^t_i$ = breakeven capital cost for system i installed in year t (\$/MW)

$\bar{c}^t_I$ = national weighted average price of industrial energy in time t (\$/MBtu)

c^t_{Ifa} = cost to industrial customers for fuel f in region a in time t. Fuel n = electricity (\$/MBTU)

$\bar{c}^t_{Ra}$ = weighted average cost of energy for residential and commercial cutomers in region a in time t (\$/MBtu)

c^{st}_{Rfa} = cost to residential and commercial customers for fuel f in region a in subperiod s of time period t
Fuel n = electricity (\$/MBTU)

CX^t_i = capital cost of system iu in time t (\$)

$\tilde{d}^{\sigma}$ = observed load at time σ, normalized by the peak demand

D^t = taxes, depreciation, and other utility expenses in time t (\$)

Day_1 = 0-1 variable indicating the day of the week (Day_1=Sunday)

e_k^s = energy demand by customer k in subperiod s relative to demand in a known period.

E_{ka}^t = electrical energy consumed in area a by customer class k in time t (MBTU)

EC_{ka}^t = coal consumed in area a by customer class k in time t (MBtu)

EG_{ka}^t = gas consumed in area a by customer class k in time t (MBtu)

EO_{ka}^t = oil consumed in area a by customer class k in time t (MBtu)

ET_{ka}^t = total energy demand in area a by customer class k in time t (MBtu)

$E(\tilde{x})$ = expected value of random variable $\tilde{x}$

$EC(Y^t)$ = expected system operating cost as a function of plant operating capacities in time period t ($)

$EF(Y^t)$ = expected unserved energy as a function of plant operating capacities for time period t (MWH)

f = fuel type

$f_c(\tilde{x})dx$ = Probability $[\tilde{x} \leq Y_C \leq \tilde{x} + dx]$

$F_c(\tilde{x})$ = $1 - G_C(\tilde{x}) = Pr\,[Y_C \geq \tilde{x}] = \int_x^\infty f_C(y)\,dy$.

= reverse cumulative distribution of the net customer load

FIX_i^t = initial fixed cost for system i installed in year t, e.g., power conditioning, lightening protection ($)

$g^{st}(c)$ = demand response function to price c in subperiod s time period t

G_i = coefficients for Gram-Charlier expansion

$G_C(\tilde{x})$ = Probability [net customer load $\leq$ x]

$G_E(\tilde{d})$ = Probability [load + outages $\leq$ d].

h_s = number of hours in subperiod s

H_{if} = full load heat rate for unit i burning fuel type f (MBtu/MWH)

I^t = total number of units in the loading order in time period t

K_{jv} = capital cost of installing a unit of type j and vintage v ($/MW)

$L_k(Y)$ = transmission loss function for energy from customer k when the total demand is Y

$LOLP^{st}$ = loss of load probability in subperiod s, time period t

$LOEP^{st}$ = ratio of expected unmet demand to total expected demand in subperiod s, time period t

m = iteration number in Benders' decomposition algorithm

M^t_k = fixed costs attributable to customer k in time t ($)

MBtu = mega Btu (10^6 Btu)

MC^{st}_E = marginal energy cost in subperiod s, time period t ($/MWH)

MC^{st}_p = marginal power cost in subperiod s, time period t ($/MWH)

MW^{st}_r = energy produced by generator r in subperiod s, time period t (MWH)

n_i = number of installations of customer owned generator type i

$N^j(z)$ = j^{th} derivative of the standardized normal distribution

p_i = availability of generator type i

p_{nk} = reliability of grid electricity to customer k

$P_y[x]$ = Probability $[\tilde{y} = x]$

PI^t_a = personal income in region a at time t ($/person)

POP^t_a = population in region a at time t

q^t_{nk} = probability that grid electricity cannot be supplied to customer k in time period t

q_{Tk} = failure rate of the T+D system for customer k

Q^{st} = peak customer demand in subperiod s, time period t (MW)

r = loading order of a central generator

RM^t = reserve margin required in time period t as a percent of peak demand in time period t

RR^t = required revenue in time t ($)

rr = allowed rate of return on investment

s = subperiod

S = total number of subperiods

Soc_{jk} = sociological factors about customer k such as number of people in the household and the income level

t = time period

T = total number of time periods

T_1 = last time period with non-linear constraints

$TC(Y^t)$ = total cost function in time period t as a function of unit operating capacities ($)

TC^{st} = total cost of the system in subperiod s, time period t ($)

$Temp^s$ = temperature in subperiod s (°F)

$Temp_a^{max}$ = maximum temperature in region a (°F)

$Temp_a^{min}$ = minimum temperature in region a (°F)

U_r = loading point of unit r (MW)

U_{I_t} = total installed capacity in time period t (MW)

$VADD^t$ = value added to industrial goods in time t ($)

$Var(\tilde{x})$ = variance of random variable x

VC_i^t = initial variable cost for system i installed in year t ($/MW)

w_a^s = vector of meteorological variable for subperiod s in region a

X_i = installed capacity of generation type i (MW)

$\tilde{Y}_{ER}^{st}$ = equivalent demand on unit r in subperiod s, time period t (random variable) (MW)

$\tilde{Y}_i^{st}$ = expected operating capacity of unit i in subperiod s, time period t (random variable) (MW)

$\tilde{Y}_{nk}^{st}$ = net electrical demand on the utility from customer k in subperiod s, time period t (random variable) (MW).

$\tilde{Y}_{ok}^{st}$ = original electrical demand from customer k in subperiod s, time period t (random variable) (MW)

$\Delta\tilde{Y}_k^{st}$ = reduction in electric demand by customer k (random variable) (MW)

$\Delta\tilde{Y}_{ck}^{st}$ = reduction in electric demand by customer k due to the time-of-day prices (random variable) (MW)

z_k = ratio of power demand to energy demand for customer k in subperiod s, time period t (hour-1)

α_r^{st} = capacity factor for unit r in subperiod s, time period t. Ratio of average operating capacity to nameplate capacity.

$\beta_i^{'\sigma}$ = error term in the linear regression

$\tilde{\gamma}^s$ = equal orthogonal vectors for demand transformation

Γ_m = set of time periods in iteration m of the optimization algorithm for which the reliability constraint is not met

δ_{rjv}^{st} = 0-1 variable that converts the indeces of unit type j installed in year v into a loading order index, r, for subperiod s, time period j.

Δ_i^t = cumulative degradation factor for unit type i after t years of operation.

ϵ_t = maximum unserved energy allowed in time period t (MWH)

$\ddot{\eta}_i^s$ = normalized output for generator type i in subperiod s given that the unit has not failed mechanically $[0 \leq \eta \leq 1]$ (random variable)

H_t^i = sum of normalized outputs for system i for period t

θ_m^t = dual multiplier associated with a time period t in which there is insufficient capacity to meet the reliability requirement in iteration m.

π_m^t = shadow price associated with the energy constraint in time period t, iteration m

ρ = discount factor

σ = index for a set of subperiods with the same characteristics, e.g., sunny breezy summer noontimes.

τ = first time period in the current time set

$\tilde{\Psi}_{ik}$ = zero-one random variable representing mechanical failure for generation type i, owned by customer k

Ω_{ak}^{st} = set of demographic and economic variables for customer k in region a in subperiod s of time period d.

$\wedge$ = estimate

$\sim$ = random variable

$*$ = optimal

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