16.07 - Background Material Test Answers

September 09, 2004

The following questions cover some basic concepts from 18.02, 18.03, and 8.01. Complete familiarity and understanding of these concepts is vital for your performance in this course. This test is anonymous and has duration of 30 minutes.

1. Point **A** has cartesian coordinates $x = 2$ and $y = 1$. What are its polar coordinates r and θ?

$$r = \sqrt{x^2 + y^2} = \sqrt{5}$$
$$\theta = \tan^{-1} \frac{y}{x} = \tan^{-1}(0.5)$$

2. Given the two vectors $\mathbf{a} = (1, 1, 1)$ and $\mathbf{b} = (1, -2, 1)$ answer the following two questions:

   (a) What is $\mathbf{a} \cdot \mathbf{b}$?

$$\mathbf{a} \cdot \mathbf{b} = 1 \times 1 + 1 \times (-2) + 1 \times 1 = 0$$

   (b) What is $\mathbf{a} \times \mathbf{b}$?

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix} = 3\mathbf{i} + 0\mathbf{j} - 3\mathbf{k} \;\; or \;\; (3, 0, -3)$$

3. What is the most general solution to the following equation:

$$\ddot{x} + 100x = 0$$

$$x = A \sin(10t) + B \cos(10t),$$

   where A and B are arbitrary constants.

4. What is the most general solution to the following equation:

$$\ddot{x} - 100x = 1$$

$$x = Ae^{10t} + Be^{-10t} - .01,$$

where A and B are arbitrary constants.

5. A vector $\mathbf{r}(t)$ is such that its modulus, r, is always equal to unity. That is, $r^2 = \mathbf{r}(t) \cdot \mathbf{r}(t) = 1$. Determine whether the time derivative of $\mathbf{r}(t)$, $\dot{\mathbf{r}}(t)$, is

   - zero,
   - parallel to $\mathbf{r}(t)$, or,
   - perpendicular to $\mathbf{r}(t)$

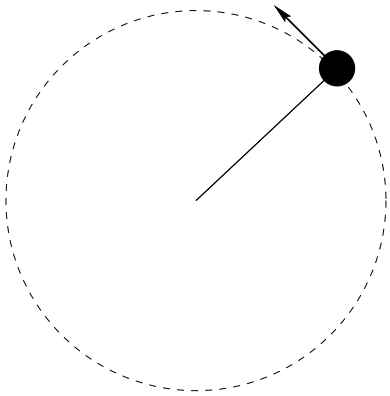
   Explain why.
   Differentiation gives $2\mathbf{r}(t) \cdot \dot{\mathbf{r}}(t) = 0$ therefore $\dot{\mathbf{r}}(t)$ is perpendicular to $\mathbf{r}(t)$.

6. The position of a particle moving in space is given by $\mathbf{r} = (\sin(t^2),\ \cos(t^2), 4)$ (in cartesian coordinates) where the parameter t represents time. What is the praticle velocity $\mathbf{v}$? What is the trajectory described by the particle? What is the angle between the position and velocity vectors, $\mathbf{r}$ and $\mathbf{v}$?

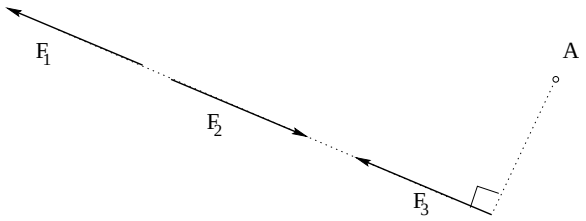
$$\mathbf{v} = (2t\cos(t^2), -2t\sin(t^2), 0)$$

The trajectory is a circle and the angle is 90° because $\mathbf{r} \cdot \mathbf{v} = 0$.

7. A student is spinning in circle (counterclockwise) a ball tied on a rope. If she releases the rope at the instant shown which direction will the ball continue moving on? Draw an arrow pointing in the direction of the answer.



8. The three forces F_1, F_2 and F_3 on the figure are equal in magnitude. What are the relationships between their moments M_1, M_2 and M_3 about point **A**? Possible choices are $=$, $<$ and $>$. If more informative relationships exist what are they?



$$M_1 = M_3 = -M_2$$

9. A bullet with mass $m = .01\ kg$ and velocity $v_b = 100\ m/s$ hits a wooden block lying at rest on a frictionless table. The mass of the block is $M = .1\ kg$ and after the impact the bullet becomes embedded in the block. What is the final velocity of the block?
   Momentum conservation implies

$$v_f(M + m) = mv_b.$$

   Therefore $v_f = \frac{.01 \times 100}{.01 + .1} = 9.09$ m/s.

10. A ball having mass $m = .1\ kg$ is thrown vertically downwards from the top of Green Building with velocity $v_0 = 10\ m/s$. Determine the velocity of the ball when it reaches the ground. The height of Green Building is 90 m. Neglect aerodynamic drag effects.
    Energy conservation implies

$$\frac{mv_0^2}{2} + mgh = \frac{mv_f^2}{2}$$
$$v_f = \sqrt{v_0^2 + 2gh} \approx 43.17\ m/s$$