16.07 Flight Simulation Laboratory

Lab III

Issued: Oct 15, 2004
Due: Nov 05, 2004

This laboratory assignment concentrates on relative motion and the differences between inertial and non-inertial reference frames. You will be extending your code from Lab II. We define two coordinate systems as shown in Figure 1, one with center at the radar station O and one with center at the aircraft B. The first one is the fixed inertial Oxy coordinate system you have been using until now and the second one is a non-inertial coordinate system with basis vectors $\boldsymbol{t}$ and $\boldsymbol{n}$. These vectors are defined according to

$$\boldsymbol{t} = \frac{\boldsymbol{v}_B}{|\boldsymbol{v}_B|}, \qquad \boldsymbol{n} = \boldsymbol{k} \times \boldsymbol{t} \ .$$

Here, $\boldsymbol{k} = \boldsymbol{i} \times \boldsymbol{j}$ is the unit vector in the direction normal to the plane containing the aircraft trajectory.

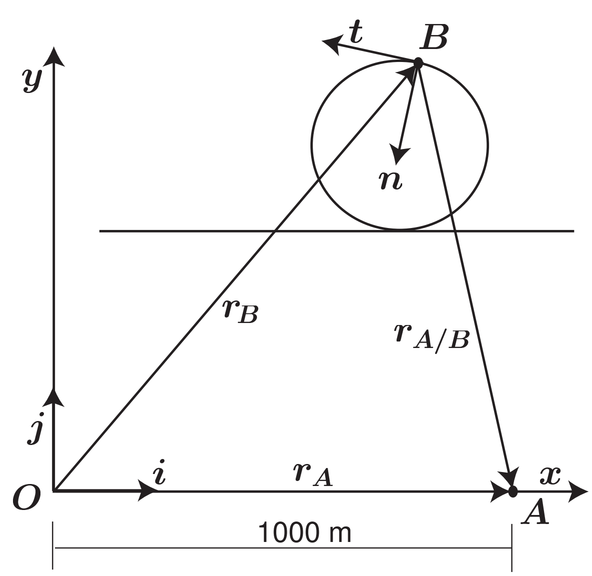

Figure 1: Radar tracking station A and aircraft B

The objective of this assignment, is to verify the expressions that relate the velocities and accelerations observed in the two frames,

$$\boldsymbol{v}_A = \boldsymbol{v}_B + (\boldsymbol{v}_{A/B})_{tn} + \boldsymbol{\Omega}_B \times \boldsymbol{r}_{A/B} \quad (1)$$

$$\boldsymbol{a}_A = \boldsymbol{a}_B + (\boldsymbol{a}_{A/B})_{tn} + 2\boldsymbol{\Omega}_B \times (\boldsymbol{v}_{A/B})_{tn} \quad (2)$$

$$+ \dot{\boldsymbol{\Omega}}_B \times \boldsymbol{r}_{A/B} + \boldsymbol{\Omega}_B \times (\boldsymbol{\Omega}_B \times \boldsymbol{r}_{A/B}). \quad (3)$$

The observed point A is on the x-axis at a distance of 1000 m from O. We know that the velocity and the acceleration of A as seen in the inertial frame are zero, i.e., $\boldsymbol{a}_A = \boldsymbol{v}_A = \boldsymbol{0}$. We are interested in verifying that the application of equations (1) and (3) leads to the same result. We will do this in several steps:

1.- Using your code from Part II in which the aircraft B flew a circular loop, find the coordinates of the vector $\boldsymbol{r}_B$, the inertial position of B, expressed in the stationary coordinate system Oxy. Use the `derivative` function to differentiate this vector and find the inertial velocity, $\boldsymbol{v}_B$, and the inertial acceleration, $\boldsymbol{a}_B$.

2.- Find the coordinates of the vector $\boldsymbol{r}_{A/B}$ in the stationary coordinate system and then convert them to coordinates in the non-inertial coordinate system with basis vectors $\boldsymbol{t}$ and $\boldsymbol{n}$. Differentiate the coordinates to find the velocity $(\boldsymbol{v}_{A/B})_{tn}$ and the acceleration $(\boldsymbol{a}_{A/B})_{tn}$ of the aircraft as seen in the non-inertial coordinate systems. Convert $(\boldsymbol{v}_{A/B})_{tn}$ and $(\boldsymbol{a}_{A/B})_{tn}$ into the stationary frame Oxy and plot the resulting x and y components of $(\boldsymbol{v}_{A/B})_{tn}$ and $(\boldsymbol{a}_{A/B})_{tn}$ vs. time. If the velocity and acceleration of A observed by B zero? Explain why not.

3.- Find the angular velocity $\boldsymbol{\Omega}_B$ of the non-inertial reference frame. Differentiate the angular velocity to obtain $\dot{\boldsymbol{\Omega}}_B$. Use the following relationships for the computation[1]:

$$\Omega_B = \frac{d\beta}{dt} = \frac{d\beta}{ds}\frac{ds}{dt} = \frac{v}{\rho} = \frac{1}{v_B}\frac{v_B^2}{\rho} = \frac{\boldsymbol{a}_B \cdot \boldsymbol{n}}{v_B}$$

4.- Compute the term $\boldsymbol{\Omega}_B \times \boldsymbol{r}_{A/B}$; plot the x and y components vs. time[2].

[1] Note that this calculation is most easily performed in the inertial coordinate system using the inertial coordinates of the vectors

[2] Again we recommend doing the multiplication in terms of coordinates in the stationary frame

5.- Verify the relation (1) by summing the three terms on the right hand side (convert all terms into stationary reference frame if you have not done so yet) and plotting the x and y components vs. time. Is the result zero? Why isn't it exactly zero?

6.- Compute centrifugal acceleration term $\boldsymbol{\Omega}_B \times (\boldsymbol{\Omega}_B \times \boldsymbol{r}_{A/B})$. Do the calculation using the coordinates from the stationary frame. In what direction does this term point? Turn in a y versus x plot of the answer, as well as the time history of the x and y components.

7.- Compute the Coriolis acceleration $2\boldsymbol{\Omega}_B \times (\boldsymbol{v}_{A/B})_{tn}$. Again, be sure to use the terms expressed in the stationary coordinate frame. In what direction does this term point? Turn in a y versus x plot of the answer and a time history of the x and y components.

8.- Compute the term due to the angular acceleration $\dot{\boldsymbol{\Omega}}_B \times \boldsymbol{r}_{A/B}$. In what direction does this term point? Turn in a time history plot of the x and y components.

9.- Finally, verify the equation (3) by summing all five terms on the right hand side expressed in the stationary reference frame and plot the x and y time histories. Is the result zero? If not, why not?

Warning: *Make sure that your vectors are expressed in the same coordinate frame before doing any additions, dot products or cross products.*

You must turn in electronic versions of your MATLAB code on the server (for all parts); no paper copies of the code will be accepted and the server will not accept electronic turn-ins after the due date and time. Your code must produce all requested figures in a single run without any manual changes in it (consult the spring-mass example). All calculations, derivations and plots must be turned in on paper by the due date.

Hints :

- You will find it easier if you create your arrays as 3D vectors in column format. That is, create an array that has dimensions of 3 (rows) by number-of-times-steps (columns). Remember that the MATLAB array indexing format is rows first, columns second. For example:

$$x = a(3, 1);$$

assigns the value at row 3, column 1 of the vector a to x.

$$x = a(3, :);$$

makes a vector x containing the numbers at every column of row 3 in the vector a.

- To carry out the basis transformations, create a 3D (3×3) transformation matrix. Then, write a loop that steps through your arrays and transforms each vector individually. To check that you are doing this correctly, write simple code that transforms vectors from one coordinate frame to another and then transforms the vectors back to the original frame. Clearly, you should end up with vectors virtually identical to what you started with. Compare them by plotting their x vs. y components on a plot (don't use the MATLAB comparison operator "==" due to rounding error).

- When doing your plots use the `'x'` or `'o'` symbols in the MATLAB plot command option section instead of having the plot command make solid lines.