PERFORMANCE MEASUREMENT FOR BOND PORTFOLIOS

by

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ABSTRACT

Performance measurement is well developed for equity portfolios. The same techniques, when applied to portfolios restricted to bond holdings, do not yield particularly enlightening results. A methodology for measuring the performance of such portfolios is advanced, and is applied to a group of mutual bond funds.

Although the results are such that there are no indications of superior management capabilities, the analysis holds strong implications for the management of fixed income portfolios. In particular, the regression coefficients used to find the naive benchmark portfolios are excellent proxies for market risk and interest rate risk. As such, the methodology provides valuable insight into proper portfolio structuring.

Thesis Supervisor: Fischer Black
Professor of Finance

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Chapter I

INTRODUCTION

In the past, actively managed bond portfolios were a rarity. The growing importance of pension funds and the competition among investment advisers for these funds have led to an increasingly large number of actively managed bond portfolios.

The rewards for successful management can be great because new business is often awarded on the basis of past performance. Therefore, it is important to devise a system that can recognize superior performance when it exists.

Chapter II examines the theoretical justification for measuring performance on a risk adjusted basis. Both the static single period capital asset pricing model and the continuous time formulation are considered. It is shown that the inclusion of a term relating to interest rate risk may be introduced without biasing results, and with the possibility of eliminating a bias in the original formulation.

Chapter III examines standards of measurement, where the assets are restricted to debt instruments. In this case, the standard measures of performance may not successfully differentiate good bond management from bad bond management, since the performance vis-a-vis the market will dominate any differentials between funds. Using continuous time analysis an alternative measure of performance is derived that is consistent with either of the models presented in Chapter II.

Chapter IV presents the empirical results for the managed funds, and an analysis of some of the methodological problems.

Chapter V examines some of the implications of the method used for measuring the performance of a bond portfolio. In particular, implications for portfolio management are discussed and a measure of interest rate elasticities for bond prices is presented.

Chapter II

FOUNDATIONS

A. Performance and the Capital Asset Pricing Model

The theoretical justification generally used in performance studies of managed portfolios [1] is provided by the work of Sharpe, Litner, and Mossin [2] on capital asset pricing. Their results, derived under rather restrictive assumptions about capital markets and investor behavior, indicate that all assets should be priced such that the expected return on asset i may be described as follows:

$$E[R_i] = R_F + \beta_i(E[R_m]-R_F) \qquad (1)$$

where: $E[R_i]$ = expected return on security i

R_F = return on a riskless asset

$E[R_m]$ = expected return on the market

β_i = correlation between returns on security i and the market

Furthermore, for an investor operating under the assumptions of the model, the optimal strategy is to invest a portion of his assets in the market portfolio, and the remainder in the riskless asset. Relative levels would be determined by the degree of risk aversion for each individual.

Since the analysis of the Capital Asset Pricing Model

(CAPM) of Sharpe, Litner and Mossin leads to a naive strategy as the optimal one, we are provided with a natural naive portfolio against which to measure returns. Returns on portfolio i are assumed to be generated by the following process:

$$R_{pi} = \alpha + R_F + \beta[R_{Mi} - R_F] + e_i \tag{2}$$

where: R_{pi} = return on portfolio p in time i

α = excess return

R_{Mi} = return on market in period i

e_i = random error term with mean equal to zero

In general, when the performance of a managed portfolio is measured,[3] a regression is used to determine the values of α and β. Although methods differ on the manner in which performance is rank ordered, a significant positive α is considered to show superior performance.

Jensen[4] tested this standard against a large number of mutual funds and found that there was little indication of superior management abilities. These results seem consistent with the efficient market hypothesis. However, it is important to examine possible biases in the methodology. Dissatisfaction with the assumptions used in the derivation of the CAPM have been well documented,[5] in particular the two major assumptions of homogeneous expectations and a single one period horizon for all investors. Relaxation of these assumptions leads to pricing models that may be substantially different from the traditional CAPM. Nevertheless, even if the assumptions are too stringent, assets could be priced as though the model held.

Black, Jensen and Scholes [6] tested this hypothesis using careful portfolio grouping techniques. The results suggest that the CAPM does not hold. In particular, the findings indicate that returns on low beta securities were in excess of those predicted by the model.

A possible misspecification of the model may introduce significant biases in measurements of performance. If the use of the CAPM tends to favor low beta portfolios in a systematic manner, then it might be better to propose a naive portfolio based on a different model of capital asset pricing.

B. Performance and the Intertemporal Capital Asset Pricing Model

There are several possible explanations for excess returns on low beta portfolios. [7] One explanation involves recognition of possible intertemporal effects that the single period CAPM does not capture: [8]

> "Due to intertemporal effects not considered in the one period CAPM, other sources of uncertainty besides market risk are significant in portfolio choice and hence, the expected return on an asset will depend on more than its covariance with the market."[9]

Merton demonstrates that in a continuous time framework with homogeneous expectations, costless trading, and a constant investment opportunity set, there exists a continuous time analog to the single period CAPM:

$$e[R_i] = (1-\beta)R_F + \beta(e[R_M]) \tag{3}$$

where: $e[R_i]$ = instantaneous expected return on i

$e[R_M]$ = instantaneous expected return on the market

The above result suggests that the CAPM framework is only valid under the condition of a constant investment opportunity set. This is particularly relevant in that we know that one element in the opportunity set is changing; namely the interest rate. Since the interest rate is changing, it is certainly possible that investors in the aggregate would be willing to pay a premium so that they were protected from unfavorable interest rate fluctuations. In the instantaneous framework a long-term default free bond is perfectly negatively correlated with a change in interest rate. This is because interest rates are the sole determinant of default free bond pricing. Merton solves the model for allowance of a changing interest rate and derives the following relationship:

$$e[R_i] = \frac{\sigma_i[\rho_{iM} - \rho_{iL}\rho_{LM}]}{\sigma_M(1-\rho^2_{LM})}(e[R_M]-r)$$

$$+ \frac{\sigma_i[\rho_{iL} - \rho_{iM}\rho_{LM}]}{\sigma_L(1-\rho^2_{ML})}(e[R_L]-r) + r \qquad (4)$$

where: ρ_{xy} = correlation coefficient for x and y

$e[R_L]$ = instantaneous expected return on long term default free bond

If it seems more reasonable to accept the specifications of the intertemporal CAPM with changing interest rates than to accept the standard CAPM, we might want to specify returns on a portfolio as:

$$R_{pi} = \alpha + (1-\beta-\gamma)R_F + \beta R_{Mi} + \gamma R_{Li} + e_i \qquad (5)$$

Using regression, the coefficients β and γ could be estimated, and the performance measure α could be determined. This model might eliminate a possible bias inherent in the original formulation. Even if there is no bias in the original formulation, the addition of the long term bond will not unfairly evaluate portfolios that were fairly evaluated before.

The introduction of the long term bonds allows for a capture of possible effects due to certain positionings with respect to interest rate exposures. The criteria for a valid measure of performance is that the portfolio have a naive selection process, and that it be of similar risks. The addition of the long term bond does not violate the naive selection process, yet it does allow for the possibility of a better matching of risks.

Chapter III

MEASURING BOND PERFORMANCE

A. Standards for Measuring Mutual Funds

A mutual fund, or any managed portfolio, may perform several functions for the individual investor. It provides an opportunity for diversification that might not otherwise be possible. Furthermore, it might be true that a full-time money manager may be more adept at managing funds than the individual who can only devote a limited amount of time to that function. If so, he should be able to outperform a naive portfolio of the relevant risk.

Assume for the moment that the single period capital asset pricing model holds. Furthermore, assume that all investors hold their assets in varying proportions in one mutual fund and the risk free asset. In such a world it would make sense to measure performance against the standard mixture of the risk free asset and the market portfolio.

However, in a world where assets are held outside of mutual funds the diversity in types of mutual funds allows for more effective hedging of risks. This means that bond funds serve a function that is independent of the fact that bonds might outperform stocks over a given period. Because of this it makes sense to run funds that are basically restricted to a certain type of asset. However, if fund assets are restricted to a certain type of security, it does not make sense to measure the performance of a manager using the market as a benchmark.

For example, assume that there is a demand for a mutual fund of automobile stocks. Clearly most of the movement of the fund will be due to general movements of the car industry against the fund. However, common sense tells us that a manager restricted to holding only automobile stocks would have performed very well if he only lost 5 percent to the market while the index of automobile stocks lost 15 percent. This suggests that we should perhaps use a mixture of the riskless asset and a portfolio indexed to the assets available to the fund. In general, for portfolio i whose holdings are restricted to type j assets, performance might be measured against

$$R_{ij} = \alpha + (1-\beta)R_F + \beta R_J + e_{ij} \qquad (6)$$

where: R_{ij} = return on fund i whose assets are restricted to type j assets

R_J = return on weighted index of type j

Such logic appears to argue for comparing returns on a bond portfolio against the return of a naive portfolio constructed from a mixture of the risk free asset and a bond index.

There are, however, significant problems with this approach. The fundamental principle underlying performance measurement for managed portfolios is that the returns can be compared with the returns from a naively constructed portfolio of similar risk. The problem with using a mixture of the riskless asset and a bond index is that in many cases we cannot use the two to construct a portfolio of similar risk.

It is generally recognized that there are two major sources of risks in fixed income securities. There is the risk of default on the issue, and there is the risk of unfavorable interest rate fluctuations. One index cannot possibly capture both effects.

Without advancing theories as to why this might be the case, assume that there exists a bond that is riskless in terms of default and a bond that is risky in terms of default but so short-lived that it is almost immune to interest rate fluctuations. Furthermore, assume that because of risks involved, both had the same expected return. Clearly, there is only one combination of the riskless asset and the index that can have the same expected return, but it is clear that if the portfolio has similar risks as one bond it can't have similar risks for the other.

Although this example is obviously an extreme one, its implications should be clear. A single index is not sufficient for performance measurement of fixed income portfolios.

B. Options Pricing and the Risk Structure of Corporate Bonds

The preceding analysis suggests that we need at least three securities in our naive portfolio: the riskless asset, a security that will capture interest rate risks, and a security that will capture default risk. Although there are several possible candidates, the work of Merton[10] on the pricing of corporate debt suggests a particular approach. The principle involved is that at any given instant in time the expected return on the debt instrument of a firm over the next instant in time can be duplicated by investing in a combination of that firm's equity and riskless (in terms of default) debt. The proof of this comes from a continuous time arbitrage argument and will be discussed shortly. However, the importance of this prospect should be recognized. Since it is possible to exactly duplicate the expected return on a bond with a combination of risk free debt and equity, then it must be true that such a combination will have the same risk structure. This result suggests that our naive portfolio should be contructed so that it contains some combination of the naive portfolios that would be used to evaluate performance for separate equity and riskless debt portfolios.

The proof that one can duplicate the instantaneous returns of a bond with some combination of equity and risk-free debt follows the same lines as the options pricing formula of Black and Scholes.[11] It stems from the observation that if the term structure of interest rates is known, then at any given instant in time the values of the firm's debt and equity will be perfectly correlated with any change in the value of the firm. There will, however, be a difference in the magnitude of the changes. This difference will be a function of riskiness of the firm, the interest rate, the time until maturity, as well as the value of the firm and the payment schedule of the bond. However, at any single instant in time there will always be some combination of riskless debt and equity that will duplicate the expected return over the next instant.

Unless one is familiar with the work on continuous time arbitrage models for pricing, these results are probably not intuitively obvious. A simple example may be of some value. Consider a firm that appears to be almost certain to default. Since the bond holders are going to be able to recover only the assets of the firm at default, the value of the debt will fluctuate almost precisely with the value of the firm. In this case, the debt will behave exactly as though it were equity. Furthermore, imagine that this company strikes gold and there is now no chance of default. In this case the value of the debt will behave as though it were default free debt. Clearly, these two cases represent the extremes. However, one can imagine that anything in between can be approximated by a combination of equity and risk free debt.

This does not mean that debt is simply a combination of equity and risk free debt. The claim is simply that at any time the return over a sufficiently short period of time can be duplicated by some combination of the two. In fact, the continuous time analysis suggests that the combination will change over time, making risky debt a unique financial claim. However, even though the combination will change over time, we are provided with a strong motivation for measuring performance against a naive portfolio that will account for price movements in risk free debt and equity.

C. Measuring Performance for Equity Portfolios

Several methods have been developed to test the performance of equity portfolios.[12] In general the methodology used the following regression in determining the naive portfolio and the excess return alpha:

$$R_{pi} = \alpha + (1-\beta)R_F + \beta R_{Mi} + e_i \qquad (7a)$$

or

$$R_{pi} = \alpha + (1-\beta-\gamma)R_F + \beta R_{Mi} + \gamma R_{Li} + e_i \qquad (7b)$$

Although the first model is probably sufficient for equity portfolios where beta is close to one, it may not be an adequate description for very low betas or very high ones.[13] In any case, the benchmark portfolio for bond measurement will be consistent with the choice of either model, so the problem is not significant here.

D. Measuring Performance for Bonds with no Default Risk

There are two ways in which a portfolio of risk free debt might be managed. Passive management would entail very low turnover and would provide a service by maintaining a known level of average maturity length. This policy would allow for investors to hedge against interest rate risks. The second management style would emphasize the anticipation of interest rate shifts. It would attempt to be in long bonds when interest rates are falling, and short bonds when interest rates are rising.

The first management strategy is a "naive" strategy, while the second strategy could well result in performances that were significantly better or worse than expected. Since we are trying to measure performance for an actively managed portfolio restricted entirely to government bonds, we would like to construct a performance measurement that would indicate neutral performance for a buy and hold strategy, yet would recognize superior interest rate forecasting. One could use the following measure of performance:

$$R_j = \alpha + x_o R_F + x_1 R_1 + x_2 R_2 + \ldots + x_n R_n + e_i \qquad (8)$$

where: $\sum_{o}^{n} x_i = 1$

R_j = return on ith default free bond

n = number of default free bonds

This would clearly indicate neutral performance on a buy and hold strategy yet recognize superior interest rate forecasting if it existed. In practice this method would have the problem of having too many variables, and would suffer from high correlation between explanatory variables. There is another method that would most likely give a buy and hold strategy a neutral performance measurement and recognize forecasting ability yet would not suffer very much from the two problems mentioned above. The method involves reducing the number of variables to a few bonds or indexes representing short, medium and long term maturities. Furthermore, correlation is reduced somewhat by only looking at excess returns.

$$R_j = \alpha + x_1 R_F + x_2(R_S - R_F) + x_3(R_M - R_S) + x_4(R_M - R_L) + e_i \qquad (9)$$

where: $\sum_{1}^{4} x_i = 1$

R_S = return on short term default free bond

R_M = return on medium term default free bond

R_L = return on long term default free bond

Although the above is an adequate measure of performance, we can derive a simpler one if we make the following assumptions. These are the standard continuous time efficient market assumptions, and the additional assumption that the returns on all maturities of default free bonds are instantaneously perfectly correlated with the risk free rate. If so, to avoid arbitrage they must be priced so that their expected returns follow

$$E[R_i] = (1-\gamma)R_F + \gamma E[R_j] \tag{10}$$

where: $E[R_i]$ = instantaneous expected return on default free bond i

$E[R_j]$ = instantaneous expected return on default free bond j

$$\gamma = \frac{\sigma_i}{\sigma_j}$$

However, since the σ_i's will change over time the γ will not remain constant. Nonetheless, if the observation period is short enough the gamma will remain fairly stable. This suggests that a reasonable measure of performance for default free debt would be

$$R_{ji} = \alpha + (1-\gamma)R_{Fi} + \gamma R_{Li} + e_i \tag{11}$$

where: R_L = return on long term default free bond

Unfortunately, there was no data available on an actively managed portfolio of government bonds. However, the regression was run against three bonds using monthly data from June 1974 to May 1976. The results are documented in Table 1. As expected, the alphas were not significantly different from zero. Furthermore, the γ increased with time to maturity. This indicates that, as expected, as maturities increase bonds behave less like the riskless asset and more like a long term bond. These results do not indicate, however, that there is necessarily a larger expected return for long bonds as compared to short. However, the fact that the alphas were not significantly different from zero indicates that to a first approximation, expected returns are described by the linear relationship in equation [10]. The R^2 are somewhat smaller than might be hoped for, although three years is a long time to expect the gammas to remain constant. The results do suggest that weekly or even daily observations over a shorter time period might be better.

E. Measuring Portfolios of Risky Bonds

A method has been presented for measuring performance on actively managed portfolios of equity and riskless (in terms of default) bonds. Furthermore, it was suggested that a portfolio of risky bonds will behave similarly to a combination of equity and default free debt. In principle, then,

TABLE 1

Summary of Government Bond Performance Using Regression Equation [11]

$$R_{ji} = \alpha + (1-\gamma)R_{Fi} + \gamma R_{Li} + e_i$$

Coupon	Maturity Date	α	γ	R^2
6 1/4	February 1978	.00033 (0.319)	.441 (9.149)	.73
7	August 1981	-.00008 (-0.055)	.555 (8.101)	.68
6 3/8	August 1984	.00017 (0.103)	.647 (9.373)	.71

we could divide the return on the risky bonds into two parts: one that corresponds to the return associated with interest rate changes, and one that corresponds to changes in the value (credit worthiness) of the companies, and measure the performance on each part.

$$x_1R_j = x_1[\alpha_1+(1-\gamma_1)R_F+\gamma R_L] + e_i \qquad (12)$$

and

$$x_2R_j = x_2[\alpha_2+(1-\beta)R_F+\beta R_M] + e_i \quad \text{or} \qquad (13a)$$

$$x_2R_j = x_2[\alpha_2+(1-\beta-\gamma_2)R_F+\beta R_M+\gamma_2R_L] + e_i \qquad (13b)$$

where: x_1 = fraction of return R_j contributed to by bond effects

x_2 = fraction of return R_j contributed to by equity effects

α_1 = excess returns on bond segment

α_2 = excess returns on equity segment

This method would require some method for determining the proper x_1's and x_2's. Unfortunately, this cannot be easily done.

This problem can be sidestepped by combining the two measures of performance and regressing against the actual returns on the bond portfolio. The only loss is that there is only one alpha and it is no longer possible to attribute performance to either superior selection of individual bonds or superior selection or maturity lengths.

$$R_p = \alpha + (1-\beta-\gamma)R_F + \beta R_M + \gamma R_L + e_i \qquad (14)$$

This will be the basis for measuring superior returns on bond portfolios.

The derivation of this measure was based in part on a continuous time capital asset pricing model with changing opportunity set. Furthermore, the notion that a funds performance should be compared to the returns on the type of assets it is restricted to, coupled with a continuous time analysis of risky bond returns, leads to the same performance standard.

However, an appeal can be made on a strictly intuitive level. A risky bond has two kinds of risk: an interest rate risk and a market risk. Any regression that incorporates variables that can capture both risks will provide an adequate naive portfolio, since the process of regression guarantees a similarity in portfolio risks. The reason the market and long term default free bonds were chosen is because they are less correlated than other measures, and therefore better capture the type of risk exposure inherent in a given portfolio. In a later section the implications of this separation of risk will be discussed.

Chapter IV

THE DATA AND EMPIRICAL RESULTS

A. Data

The sample consisted of the monthly returns on nine mutual funds specializing in bonds for the three year period from June 1973 to May 1976. The data were obtained from the monthly Wiesenberger's Current Price and Dividend Record [1] for those months. The restriction to that three year period was made for pragmatic reasons. Many of the funds did not begin until 1973 and a few left the sample after 1976. The figures for Treasury Bills, stock market index, long term government bonds, and corporate bond index were all taken from Ibbotson-Sinquefield tapes. [15]

The mutual funds used in the sample are listed in Table 1 with some summary data. For a more detailed profile of each fund, see Appendix 1.

B. The Measurement of Performance

The body of this paper has urged a specific measure of performance based on a naive portfolio constructed of Treasury Bills, long term government bonds, and the market portfolio. For comparative purposes, two other naive portfolios were constructed, both of which have been used to measure performance for bond portfolios. The first is the standard naive

TABLE 2

Mutual Bond Funds and Abbreviations

Name	Abbreviation
Channing Bond Fund	CH
Delchester Bond Fund	DL
Keystone B1 (high-grade bonds)	B1
Keystone B2 (medium-grade bonds)	B2
Keystone B4 (discount bond fund)	B4
Lord Abbett Bond-Debenture Fund	LA
National Bond Fund	NA
Northeast Investors Trust	NE
United Bond Fund	UB

portfolio composed of Treasury Bills and the market portfolio. This measure is relevant under the dual assumptions that the CAPM holds and bond portfolios should be measured on the same terms as other portfolios. The second is a naive portfolio constructed of Treasury Bills and a corporate bond index. This might be relevant if mutual bond funds tend to have the same default risk of the corporate index. Tables 3, 4, and 5 have summary statistics for all three performance measures.

Regardless of the naive portfolio used, the alphas tended to be slightly negative. However, in most cases they were not significantly different from zero. Although one should be careful about making strong statements about the results, in every instance the regression using the corporate bond index gave higher alphas than the other two. This could be a quirk of the sample, a downward bias in the other two methods, or a problem with the use of the corporate bond index. It seems that the latter may be the case since, as was mentioned previously, the corporate bond index fails to simultaneously capture both interest rate risk and default risk.

The beta coefficient, particularly in the equation with both the market portfolio and the long term bond, illustrates that market related risks are significant for a bond portfolio. This should have implications for portfolio management. Since investment in corporate bonds implies a market risk equivalent to holding one sixth of the assets in the

TABLE 3

Summary of Estimated Regression Statistics for Nine Mutual Bond Funds

Using Monthly Observations from June 1973 to May 1976

$$R_{jt} = \alpha_j + (1-\beta_j-\ell_j)R_{Ft} + \beta_j R_{Mt} + \gamma_j R_{Lt} + e_{jt}$$

Item	Mean Value	Median Value	Minimum Value	Maximum Value
α	-.030%	-.103%	-.237%	.437%
β	.183	.156	.086	.362
γ	.398	.376	.203	.703
R^2	.486	.482	.250	.703

TABLE 4

Summary of Estimated Regression Statistics for Nine Mutual Bond Funds

Using Monthly Observations from June 1973 to May 1976

$$R_{jt} = \alpha_j + (1-\beta_j)R_{Ft} + \beta_j R_{Mt} + \tilde{e}_{jt}$$

Item	Mean	Median	Min	Max
α	-.054%	-.125%	-.260%	.419%
β	.217	.188	.124	.387
R^2	.320	.297	.182	.545

TABLE 5

Summary of Estimated Regression Statistics for Nine Mutual Bond Funds

Using Monthly Observations from June 1973 to May 1976

$$R_{jt} = \alpha_j + (1-\phi_j)R_{Ft} + \phi R_{CBt} + \tilde{e}_{jt}$$

Item	Mean	Median	Min	Max
α	-.009%	-.076	-.224	.466
ϕ	.592	.596	.394	.704
R^2	.442	.409	.269	.749

R_{CBt} = Corporate Bond Index

market, diversification procedures should recognize this.

The R^2 on all three regressions were fairly low. This is somewhat of a disappointment, but is primarily a function of the time between observations. It seems plausible that the R^2 would have been higher if yearly observations had been made, or if daily observations had been made over a much shorter period. The reasoning behind those thoughts is that the principle problem is that the betas and the gammas are not stationary over time. Although this point will be discussed in greater detail later, it is important to recognize that the low R^2 are not necessarily indicative of a failure to explain the process generating returns. The problem is that over monthly intervals there are great shifts in the types of assets held, from very risky to less risky, from short term to long term. However, on a yearly basis it might be fair to assume that some average level is maintained. In this case, the regression would have a much better fit. By the same token, if daily returns were taken for periods in which the make-up did not change dramatically, we would also get a better fit.

The individual bond funds exhibited a fairly wide range of performance. In no case can the hypothesis $\alpha = 0$ be rejected at a reasonable confidence level. The Lord Abbett Bond-Debenture Fund, coded LA, did better than the naive portfolio, but this could be attributed as much to luck as to superior management. On the other hand, the Delchester Bond Fund (DL) and Keystone B1 (B1) Fund did worse than the naive portfolio over this three year period. Table 6 summarizes the performance measures.

The betas range from .091 for Keystone B1 up to .362 for Lord Abbett. In all cases the coefficients are significantly different from zero at the 95 percent confidence interval. Furthermore, examination of the progressively higher betas from Keystone B1 to Keystone B2 to Keystone B4 is encouraging. The B1 fund is basically a blue chip bond fund. The analysis based on the continuous time framework suggests that these should behave less like equity than the riskier bonds. Keystone B2 is an intermediate grade bond fund and, as expected, it has a higher beta than B1. Keystone B4 is composed of "low" quality bonds and behaves, as predicted, more like equity than the other two.

One other point to mention is that the two funds with the highest beta were the only funds with a positive alpha. This may be an indication that high risk bonds were a good investment during this period or it might indicate some biases in the methodology. On the other hand, the lower beta portfolios did not under-perform relative to the others. Further empirical work might resolve this problem.

TABLE 6

Summary of Individual Fund Performance

Using Regression Equation [14]

$$R_{jt} = \alpha + (1-\gamma-\beta)R_{Ft} + \beta_j R_{Mt} + \gamma_j R_{Lt} + e_{jt}$$

ID	α	T-statistic
CH	-.025%	-.077
DL	-.208%	-.845
B1	-.103%	-.765
B2	-.037%	-.122
B4	.136%	.362
LA	.437%	1.089
NA	-.113%	-.206
NE	-.237%	-.638
UB	-.116%	-.465

TABLE 7

Estimated Beta Coefficients for Regression [14]

$$R_{jt} = \alpha_j + (1-\gamma-\beta)R_{Ft} + \beta_j R_{Mt} + \gamma_j R_{Lt} + e_{jt}$$

ID	α	β	T-statistic for beta
CH	-.025%	.156	3.140
DL	-.208%	.141	3.752
B1	-.103%	.091	4.397
B2	-.037%	.129	2.834
B4	.136%	.255	4.461
LA	.437%	.362	5.899
NE	-.113%	.242	2.902
NA	-.237%	.187	3.296
UB	-.116%	.086	2.263

The results for gamma are indicative of two things. First, for all the funds except the United Bond Fund the gamma level was such that on average the funds' exposure to interest rate fluctuations were somewhat equivalent to the exposure of a government bond with about five years to maturity, as can be seen from Table 1. This may be a relevant consideration if the bond fund was being used to help hedge against interest rate fluctuations.

The second point is that the t-statistics are not as significant as might be expected. There are at least two possible reasons for this. First, fluctuations in government bonds are not a significant explanatory variable in the fluctuations of corporate bonds. Although this may be possible, it does seem a little farfetched. A more likely explanation is that the managers of the fund are constantly changing the maturity of the fund. In this case gamma is non-stationary and we should expect low t-statistics. In a sense then, we might look at the low t-statistic for gamma to be more an indication of active management than a misspecification of the model.

Table 9 shows the estimated coefficients and R^2 for the nine regressions.

TABLE 8

Estimated Gamma Coefficients for Regression Equation [14]

$$R_{jt} = \alpha_j + (1-\beta_j-\gamma_j)R_{Ft} + \beta_j R_{Mt} + \gamma_j R_{LT} + e_{jt}$$

ID	α	γ	T-statistic for γ
CH	-.025	.371	2.470
DL	-.208	.491	4.320
B1	-.103	.392	6.280
B2	-.037	.382	2.763
B4	.136	.354	2.040
LA	.437	.299	1.614
NA	-.113	.203	.805
NE	-.237	.376	2.190
UB	-.116	.712	6.173

TABLE 9

Summary of Regression Coefficients for Equation [14]

$$R_{jt} = \alpha_j + (1-\beta_j-\gamma_j)R_{Ft} + \beta_j R_{Mt} + \gamma_j R_{LT} + e_{jt}$$

ID	α	$1-\beta-\gamma$	β	γ	R^2
CH	-.025%	.473	.156	.371	.393
DL	-.209%	.368	.141	.491	.571
B1	-.103%	.517	.091	.392	.703
B2	-.037%	.489	.129	.382	.390
B4	.136%	.391	.255	.354	.482
LA	.437%	.339	.362	.299	.578
NA	-.113%	.555	.242	.203	.250
NE	-.237%	.437	.187	.376	.386
UB	-.116%	.202	.086	.712	.597
Corporate Bond Index	-.029%	-.148	.099	1.049	.842

C. Methodological Problems

This paper has sought to provide a strong rationale for using a certain naive portfolio in the measurement of bond portfolios. When the model was applied to nine managed funds, the descriptive value of the return generating process

$$R_{jt} = \alpha + (1-\gamma_j-\beta_j)R_{Ft} + \beta_j R_M + \gamma_j R_L + e_{jt}$$

was less than might have been expected. Moreover, none of the funds had an alpha that was significantly different from zero. Both of these are results of the same phenomenon.

In order for the estimation of the alpha and any statistical inference concerning that measure to be robust, the systematic risk and the interest rate risk as measured by gamma must be constant over the period. This is clearly violated since the average maturity of the portfolio and the average riskiness of the bonds are important decision variables. Any actively managed fund will violate these necessary statistical conditions.

If we know the assets in the fund we can handle this problem. A new regression can be run each time the risk level is changed, and statistically robust estimations of the parameters may be obtained. The alpha measure would then indicate security selection skills, but not market forecasting skills. However, since we know the time of the risk change and the direction, we can recognize timing skills by looking at the direction of the *ex post* market movements.

Even though we can do this if we know the portfolio holdings, we might still choose to measure performance as before. The changing risk levels tend to make the equation a poorer fit and therefore yield lower t-statistics for alpha. If performance is measured by significant t-statistics, the changing risk levels will lower measured performance.

In theory, the mutual fund can hold any risk level and the investor can adjust his holdings so that the optimal risk level is obtained. However, the investor does not know what the risk levels are at all times and is therefore exposed to suboptimal risk levels. A risk measure that penalizes against changing risk levels is therefore not undesirable.

There are methodological problems with the approach used in the paper. To a large extent, however, they are unavoidable. Since risk levels change over time, it might be wise to make the measurement period fairly short. If this were the case, it is clear that shorter observation periods would be necessary. Daily observations would be valuable in any case. The larger the number of observations, the more likely that superior performance will be recognized as statistically significant.

Chapter V

γ, EXPECTED RETURNS, DURATION, AND PORTFOLIO MANAGEMENT

From the analysis it was shown that every bond has a β which approximates returns related to market effects, and a γ which approximates returns related to interest rate changes. In the standard CAPM framework, expected return is a linear function of β and γ has no effect on the expected return. In such a framework long term government bonds have the same expected return as Treasury Bills. However, Ibbotson and Sinquefield [16] find that over a 50 year period long term bonds received a premium of approximately 1.6 percent yearly return over Treasury Bills.

If this *ex post* observation is reflective of *ex ante* expectations, then one must consider the γ of a security when determining expected returns. This is true for equities as well as bonds. For a firm, the expected return is the weighted average of the expected returns on the debt and equity.

$$E[R_{FIRM}] = x_1 \; E[R_E] + x_2 \; E[R_D]$$

$$= x_1 R_F + x_1 \beta_E (E[R_M] - R_F) + x_1 \gamma_E (E[R_L] - R_F)$$

(continued)

$$+ x_2 R_F + x_2 \beta_D (E[R_M] - R_F) + x_2 \gamma_D (E[R_L] - R_F)$$

$$= R_F + (x_1 \beta_E + x_2 \beta_D)(E[R_M] - R_F)$$

$$+ (x_1 \gamma_E + x_2 \gamma_D)(E[R_L] - R_F) \qquad (15)$$

where: x_1 = market value of equity divided by market value of firm

x_2 = market value of debt divided by market value of firm

In the Miller-Modigliani [17] framework, the expected returns on the firm is independent of financial makeup. Therefore, even if there were no debt outstanding, the equity might have a γ significantly different from γ on the market. This is an interesting result since it implies that there is a significant γ on an instrument that is very much unlike riskless debt.

The previous analysis demonstrated possible effects of γ on expected returns. This, however, hinged on the assumption that long term government bonds received a premium over short term bonds. Although the Ibbottson-Sinquefield results are suggestive of such a phenomenon, there is no strong empirical evidence that implies magnitudes that make this a significant effect in most uses.

In spite of the conclusion that γ is relatively insignificant in determining expected returns, it does provide a useful function in portfolio management. The reason for this is that it is an excellent measure of relative interest rate risks.

If the CAPM holds, then expected returns depend on R_F, $E[R_M]$, and β_i. Nonetheless, with every portfolio there may be some risk that is systematic to interest rate changes. If there is to be no increase in expected returns to compensate for this risk, then portfolio management should be such that an interest rate exposure target level is set and maintained. This would allow for users of the fund to "immunize" themselves against interest rate fluctuations. In other words, if a person has liabilities with a certain interest rate exposure, then in order to be immune to interest rate fluctuations he would pick an asset that had the exact same exposure to interest rates. A trivial example would be the case where $1000 was due one year hence. A discount bond (riskless in terms of default) promising to pay exactly $1000 in one year would have the same interest rate exposure. If the assets and liabilities came due on different days, there would be some exposure to interest rate fluctuations. Matters are further complicated if any intervening coupons are paid out.

Forty years ago Macaulay[18] attempted to solve this problem by deriving a relationship he called duration. Duration was to be a measure of "average" length of the bond. In general duration is defined as

$$D \equiv \Sigma tC(t)P(t)/\Sigma C(t)P(t) \tag{16}$$

where: $C(t)$ = time stream of payment

$P(t)$ = present value of one dollar to be received at time t.

Other authors championed the use of duration as a measure of interest rate exposure, since under some simplifying assumptions it can be shown that interest rate elasticity is linear with respect to duration.[19]

It turns out, however, that these simplifying assumptions lead to absurd pricing mechanisms and are therefore inappropriate.[20] In general, duration is dependent on the stochastic process by which interest rates are determined. Specfications of such a function is the necessary anticedent to computation of interest rate elasticity. Furthermore, the measure would also apply only to riskless in terms of default debt. The determination of duration for risky debt would be different, although it would almost assuredly be of a shorter length. The existence of call provisions would certainly complicate things. The measurement of equity duration is even more problematic. The analysis at the beginning of this chapter suggests that both the debt and the equity would have to be taken into consideration in any duration formula.

These problems can be handled to a large extent by γ. The γ on an asset attributes some of that asset's returns to factors relating to the return on the long term debt, i.e., $\gamma(R_L)$. The returns on long term default free debt are primarily a function of interest rates. Because of this, γ is a good proxy for the amount of interest rate exposure for a given asset. We would expect, then, that to a very close approximation two securities with the same γ's would behave identically with respect to interest rate fluctuations. Therefore, if we have an asset and a liability of the same value and γ, the position would be approximately immunized against interest rate fluctuations. The formulation of the problem in this manner avoids the computational and theoretical problems inherent in the duration analysis, yet still provides the same benefits. Moreover, γ has the very desirable property that the weighted average of the γ's of the assets in a portfolio gives the γ for a portfolio. This suggests that diversification across maturities does not necessarily make sense, since the interest rate exposure will be, to a close approximation, the same as a bond with γ equal to the γ of the portfolio.

Using the γ analysis, immunization against interest rate changes is fairly easy. The rule would be to set the weighted average of the γ's--assets minus liabilities--to be equal to zero. This process could be made even easier by the publication of β's and γ's for different securities or assets. This would enable intelligent management with respect to market risk and interest rate exposure. Since γ's are definitely not constant over time, it would probably make sense to use daily data and revise estimates monthly. Nonetheless, this would provide a valuable tool for managing interest rate risks.

Chapter VI

SUMMARY

The purpose of this paper was not to measure the performance of bond portfolios per se, but rather to develop the methodology. It was shown that if assets are restricted to bonds, it makes sense to evaluate the returns using a naive portfolio constructed from the risk free asset, the market portfolio, and long term default free debt. When this measure was tested against nine mutual bond funds there was no significant indication of superior performance. There were methodological problems due to changing risk levels, but this is basically unavoidable.

Although the primary focus of this paper was on the development of a benchmark portfolio for the measurement of bond fund performance, perhaps the most significant result is the use of the regression coefficient γ as a proxy for interest rate exposure. It represents a simpler approach to the problem of interest rate immunization than the measure of duration, since it is easily computed from readily available data. Furthermore, it is applicable to any type of asset or liability if returns are available, regardless of riskiness, call provisions, or other features. Finally, it provides a useful tool in the management of portfolios with exposure to interest rate risks.

Appendix A

SUMMARY INFORMATION FOR MUTUAL FUNDS USED IN STUDY [21]

<u>TABLE A-1</u>

At Year-Ends								Annual Data		
					% of Assets in					
Year	Total Net Assets ($)	Net Asset Value Per Share ($)	Offering Price ($)	Yield ($)	Cash & Equiv-alent	Bonds & Pre-ferreds	Common Stocks	Income Divi-dends	Capital Gains Dis-tribution	Expense Ratio (%)
1976	114,833,026	9.01	9.85	7.4	2	98	--	0.73	--	0.73
1975	79,352,108	8.00	8.74	8.35	9	92	--	0.73	--	0.72
1974	52,121,621	7.27	7.95	9.18	6	94	--	0.73	--	0.81
1973	47,177,044	8.80	9.62	7.48	13	87	--	0.72	--	0.79

<u>Channing</u>:

Investment objective is to seek interest income while conserving capital. It invests in a diversified portfolio of marketable debt securities, including convertable debentures. The rate of portfolio turnover during 1976 was 34 percent of average assets.

<u>TABLE A-2</u>

	At Year-Ends							Annual Data		
					% of Assets in					
Year	Total Net Assets ($)	Net Asset Value Per Share ($)	Offering Price ($)	Yield ($)	Cash & Equiv-alent	Bonds & Pre-ferreds	Common Stocks	Income Divi-dends	Capital Gains Dis-tribution	Expense Ratio (%)
1976	27,417,399	9.57	10.02	7.3	9	91	--	0.73	--	0.91
1975	17,043,530	8.47	8.87	8.5	7	93	--	0.75	--	0.98
1974	13,489,361	7.99	8.73	8.1	10	90	--	0.71	--	0.98
1973	11,931,245	9.45	10.33	7.0	14	86	--	0.72	--	1.06

<u>Delchester</u>:

Investment objective is to earn as leberal a current income as is consistent with providing reasonable safety. At least 80 percent of the fund's assets must be invested in bonds. Rate of portfolio turnover during 1976 was 72 percent of average assets.

TABLE A-3

Keystone Bond Funds

	Total Net Assets	Per Share Net Asset Value	Dividends	Expense Ratio
B1	56,691,664	18.29	1.41	0.31%
B2	39,830,982	19.86	1.63	0.63
B4	449,333,777	8.41	0.72	0.64

B1: Assets are invested in highly marketable bonds limited to Government agencies or other bond issues of high or good grade. Objective is price stability with liberal yield.

B2: Assets are invested in medium grade bonds. Investment objective is to obtain maximum income possible without undue risk of principal.

B4: Assets are invested primarily in discount bonds. Domestic, foreign and restricted securities are allowable. The objective is generous income return yet is characterized by relatively wide ranges of price and income payment fluctuation.

TABLE A-4

	At Year-Ends				% of Assets in			Annual Data		
Year	Total Net Assets ($)	Net Asset Value Per Share ($)	Offering Price ($)	Yield (%)	Cash & Equiv-alent	Bonds & Pre-ferreds	Common Stocks	Income Divi-dends	Capital Gains Dis-tribution	Expense Ratio (%)
1976	174,779,324	11.54	12.60	6.6	9	89*	2	0.83	--	0.83
1975	137,284,578	9.52	10.40	7.7	7	93	--	0.80	--	0.83
1974	101,754,596	8.03	8.78	9.5	15	85	--	0.83	--	0.87
1973	98,708,575	9.30	10.16	7.2	14	86	--	0.81	0.16	0.86

* Includes a substantial proportion in convertible issues.

Lord Abbett Bond-Debenture Fund:

Investment objective is to provide a high current yield and the opportunity for capital appreciation through a managed portfolio consisting primarily of lower-rated securities. The rate of portfolio turnover during 1976 was 42.2 percent of average assets.

TABLE A-5

	At Year-Ends			% of Assets in			Annual Data		
Year	Total Net Assets ($)	Net Asset Value Per Share ($)	Yield (%)	Cash & Equiv- alent	Bonds & Pre- ferreds	Common Stocks	Income Divi- dends	Capital Gains Dis- tribution	Expense Ratio (%)
1976	134,025,041	15.01	8.4	(7)	104	3	1.265	--	0.69
1975	86,960,338	13.34	9.1	-	95	5	1.22	--	0.69
1974	61,670,487	12.51	9.4	-	87	5	1.17	--	0.65
1973	68,105,561	14.79	7.7	5	93	2	1.14	--	0.66

Northeast Investors Trust:

Continuous income is the trust's primary objective, with capital appreciation an important but secondary goal. Investment policy is flexible, although principle holdings are in fixed income securities. Loans are employed to increase earning power and capital leverage. Portfolio turnover for 1976 was 23.4 percent of average assets.

TABLE A-6

At Year-Ends								Annual Data		
					% of Assets in					
Year	Total Net Assets ($)	Net Asset Value Per Share ($)	Offering Price ($)	Yield (%)	Cash & Equiv-alent	Bonds & Pre-ferreds	Common Stocks	Income Divi-dends	Capital Gains Dis-tribution	Expense Ratio (%)
1976	120,742,878	4.58	4.94	7.9	4	96	--	0.39	--	0.80
1975	86,440,133	4.03	4.40	8.6	4	96	--	0.38	--	0.93
1974	55,134,230	3.70	4.04	9.7	13	87	--	0.39	--	0.88
1973	40,260,611	4.54	4.96	7.7	16	84	--	0.38	--	0.99

National Bond Fund:

The objective of National Bond Fund is to provide an investment in a diversified group of bonds, including convertible bonds, which are selected for income. Portfolio turnover during 1976 was 63 percent of average assets.

TABLE A-7

At Year-Ends								Annual Data		
					% of Assets in					
Year	Total Net Assets ($)	Net Asset Value Per Share ($)	Offering Price ($)	Yield (%)	Cash & Equiv-alent	Bonds & Pre-ferreds	Common Stocks	Income Divi-dends	Capital Gains Dis-tribution	Expense Ratio(%)
1976	228,793,861	7.57	8.27	7.3	3	97	--	0.60	--	0.50
1975	147,403,924	6.75	7.40	8.1	6	94	--	0.60	--	0.43
1974	115,754,224	6.49	7.11	8.4	10	90	--	0.60	--	0.35
1973	112,402,923	7.75	8.49	7.1	17	83	--	0.60	--	0.33

United Bond Fund:

United Bond Fund was initially offered in March 1964 as a medium for investors primarily interested in a portfolio of fixed-dollar securities offering a reasonable return with more emphasis on preservation of capital invested. Only debt securities may be purchased for the portfolio. Portfolio turnover in 1976 was 43.6 percent of average assets.

FOOTNOTES

1. There have been numerous studies of this nature. The primary ones are Jensen (1968), McDonald (1974), and Sharpe (1966).
2. Sharpe (1964), Litner (1965), and Mossin (1966). For an excellent review, see Jensen (1972).
3. Jensen (1969), Sharpe (1966), Treynor (1965), and Fama (1972), have developed measures of portfolio performance.
4. Jensen (1968, 1969).
5. Alternative derivations of asset pricing models can be found in Black (1972), Fama and MacBeth (1973), Hakansson (1971), and Merton (1973b). See Jensen (1972) or Sharpe (1964) for standard assumptions.
6. Black, Jensen and Scholes (1972).
7. Black (1972), Fama and MacBeth (1973), and Merton (1973b).
8. Merton (1973b).
9. Merton (1973a).
10. Merton (1974).
11. Black and Scholes (1973).
12. Jensen (1968, 1969), Sharpe (1966), Treynor (1965), and Fama (1972).

Footnotes, continued

13. The model might be biased against high betas and biased for low betas as evidenced by the results of Black, Jensen and Scholes (1972).
14. Wiesenberger (1973-1976a).
15. Ibbotson-Sinquefield tapes have the monthly returns 1926-1976 for stocks, Government bonds, Treasury Bills, and Corporate Bond Index. They are provided by the Center for Research in Security Prices.
16. Ibbotson and Sinquefield (1976).
17. Modigliani and Miller (1958).
18. Macaulay (1938).
19. Macaulay (1938) and Hicks (1939) independently suggest this. See Fisher and Weil (1971) for a complete treatment.
20. See Cox, Ingersoll, and Ross (1977b) for a more complete treatment of the issues.
21. These figures were taken from Wiesenberger (1976n), as were the summary descriptions of the funds' objectives.

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