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STATISTICS OF THE ELASTIC PARAMETERS AND THEIR POSSIBLE CORRELATION WITH THE SEDIMENTARY ROCKS

by

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TABLE OF CONTENTS

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Submitted to the Department of Geology and Geophysics on August 20, 1956, in partial fulfillment of the requirements for the Degree of Master of Science.

Having in mind the problem of obtaining more information from the seismogram and the possibility of making an analysis of noise, the theory of the seismic scattering model, as studied by R. Bowman(2) in the case of Lame's constants varying and density constant, is reviewed and summarized. One illustrative example using a velocity log as model is worked out to determine to what degree the scattered energy contributes to the recording. The result indicates that the scattered energy is small but significant.

As a complement to the problem of deducing statistics about the distribution of the elastic parameters from the observed statistics (auto- and crosscorrelation of seismic traces) a first attempt is carried out to find connections between several types of sedimentary rocks and the distribution of elastic parameters. Work is done in 14 different sections of continuous velocity logs, which from the general appearance seems to satisfy the scattering model, corresponding to several classes of sedimentary rocks belonging to different areas and geologic formations. It is found that $\overline{\Delta\lambda^2}$ (variance of fluctuations of Lame's constant) and $I_2 = \int_0 R\,\varphi(R)\,dR$ (where $\varphi(R)$ is the normalized space autocorrelation of the above fluctuations) are probably the best parameters to differentiate between types of sedimentary rocks. Due to lack of enough samples in some of the rock types, these conclusions are somewhat tentative.

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I. INTRODUCTION

During the past years the seismic reflexion method of geophysical exploration has been used with ever increasing success to determine the geologic structure of buried formations, mapping conditions which are favorable for the accumulation of oil and gas. In spite of the effectiveness of the technique, however, a vast number of problems that have arisen since seismic prospecting has been used as an exploratory tool remain yet to be solved. Progress has proceeded simultaneously along various lines but perhaps the greatest advance has been made in the design and construction of recording equipment. The development of interpretation methods of the information recorded on the seismograms has not progressed so much. The main problem in this interpretation is the picking up of reflexions, separating the signal from the noise and to this aim has been drawn most of the efforts of the geophysicists. This is not a very difficult task when we are working with an ideal seismogram in which the reflexions are separated by quiet, almost noise-free intervals. In poorer seismograms the reflexions can be located by virtue of the "line-ups" but the intervals between the reflexions are filled with oscillations above amplitudes are comparable with those of the reflexions. In the limit the line-ups disappear and one has an energetic record which appears to be full of reflexions, but none of these can be identified or used as a basis for interpretation. In this particular case the analysis for signal is quite difficult and therefore the analysis of the "noise" itself has been suggested. This therefore represents an attempt to obtain a new type of information from the seismogram.

The first step in figuring out this problem will be to find a mechanism which produces the amplitude changes on the seismogram. Several models have been tried. In some cases it seems that the most suitable model is the scattering one. The noise on the seismogram could be produced by a large number of scatterers (small inhomogeneities) distributed throughout the earth. These scatterers may be caused by localized variations in the physical properties of the earth, by bedding planes of limited horizontal extent, or by any other type of departure from homogeneity. Each scatterer produces a small reflected pulse which reaches the geophone, and the seismogram trace represents the sum of a large number of these pulses.

Lifshits and Parkhomovski[(1)] were the first to study the scattering model with small inhomogeneity mainly on anisotropic media since they were interested in propagation through randomly oriented crystals.

Later on R. Bowman[(2)] applied this model to conditions existing in the earth, and to the seismological problem to find the wave propagation in statistically inhomogeneous medium, with continuous and stationary distribution of inhomogeneity: knowing the autocorrelation of the inhomogeneities - that is, the only statistical property known since the exact distribution of the inhomogeneities in space is unknown - he studied the elastic wave motion in the medium, finding the plane wave solution for the average motion - for the case of low frequencies - the two models used by him are isotropic. In one, Lame's constants vary and density is constant; in the other only the density varies. It has been thought that a continuous velocity log could be a reasonable representation of that model.

A first step in the analysis of noise - in this particular model - is an investigation of the degree to which the scattered energy contributes to noise.

In the first part of this thesis we are going to make a first approach to that problem, finding out if there is enough energy in the wave produced by scattering to account for the seismic noise using the first model represented by a velocity log, in which we make also the assumption that the inhomogeneities in λ are a constant multiple of those in μ.

This assumption fits pretty well for our particular velocity log.

Another important problem is the inverse of the one treated by R. Bowman: given the observed statistics in a medium, find out the statistics of the generating mechanism.

This problem has not been treated yet either theoretically or experimentally. It would be very important to figure out this problem at least experimentally in order to be able to deduct statistics about the distribution of the elastic parameters from the observed statistics, that is, the auto- and crosscorrelations of seismic traces. A very important complement to that problem is to try to find out connections between the different types of sedimentary rocks and the already mentioned distribution of the elastic parameters. A first attempt to find out those connections is worked out in the second part of this thesis. We have taken several continuous velocity logs which from their general appearance seem to satisfy the requirements of the scattering model.

We have studied the different classes of statistics of the elastic parameters, trying to find out their connections with the different earth materials.

If it is possible to find actual connections between the rock type and the elastic parameters distribution, and if also we can go back, at least experimentally, in the R. Bowman's work finding relations between the observed statistics of the seismogram - that is the auto- and crosscorrelation of seismic traces and the statistics of the generating mechanism - it will be possible to have some information about the kind of lithology inside the earth as a function of the statistical properties of the seismogram.

II. THEORY

As a background to the actual work of this thesis we are going to review and summarize the theory of the small inhomogeneities scattering model with constant density and variables λ and μ, as well as the problem of scattering of elastic waves by small inhomogeneities of the elastic parameters as studied by R. Bowman[2], since the velocity logs which we are going to work with approximate this physical situation.

a) Small inhomogeneities scattering model

This model is that of an infinite isotropic, perfectly elastic solid body in which the density is constant but the elastic parameters fluctuate randomly in space with no trends or preferred directions in the fluctuations. The exact distribution of the inhomogeneities is unknown but we know that the value of each elastic parameter along any direction is equal to the average value (ensemble average or space average) plus a deviation from the average.

b) Wave propagation in small inhomogeneities scattering model

We do not know the exact distribution of the inhomogeneities in space - we only know their autocorrelation, therefore we cannot obtain the exact solution, but we can find an average behavior of the medium. To do this, the concept of ensemble is introduced, which represents a collection of specific distribution of inhomogeneities; the members of the ensemble have common statistical properties (mean and autocorrelation), but differ from the other in exact form. The ensemble, then, include all the possible forms that the distribution of inhomogeneities may take in a given situation and gives the statistical properties of every distribution. The average behavior is the average of all the exact solutions of all the media within the ensemble.

Therefore to find the equation for the average solution in the medium with constant density we write the equation for the displacement in one of the media of the ensemble. In tensor form, this is:*

$$\rho \omega^2 u_i + \frac{\partial}{\partial x_k}\left[A_{iklm}(\vec{r}) \frac{\partial u_l}{\partial x_m} \right] = 0 \quad ; \quad i, k, l, m = 1, 2, 3 \tag{1}$$

$A_{iklm}(r)$ = elastic modulus tensor i = medium of the ensemble

then we write:

$$u_i(\vec{r}) = \bar{u}_i + \Delta u_i(\vec{r}) \tag{2}$$

$u_i(\vec{r})$ = displacement in the medium i

$\bar{u}_i$ = average displacement

and the same with the elastic modulus tensor then, we

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*The remainder of Section II is a summary of Bowman[(2)], pages 29 to 45.

substitute those values into the equation of motion and we take the average over the whole ensemble, obtaining the equation

$$\rho \omega^2 \bar{u}_i + \bar{A}_{iklm} \frac{\partial^2 \bar{u}_l}{\partial x_k \partial x_m} = - \overline{\frac{\partial}{\partial x_k} \Delta_{iklm} \frac{\partial \Delta \bar{u}_l}{\partial x_m}} \qquad (3)$$

$\frac{\partial \Delta \bar{u}_l}{\partial x_n}$ = derivative of the deviation terms.

Now, to figure out the equation of motion in the specific medium, we write the integral solution for , as a sum of a volume integral and two surface integrals, using the elastic Green's function and then substituting this into the equation (3) we get:

$$\rho \omega^2 \bar{u}_i + A_{iklm} \frac{\partial^2 \bar{u}}{\partial x_k \partial x_m} = \frac{\Delta A_{ikpq} \Delta A_{rstlm}}{4\pi\rho} \times \qquad (4)$$

$$\times \frac{\partial}{\partial x_k} \int_{V'} G_{ps} \frac{\partial^2 \cdot}{\partial x'_q \partial x'_t} \varphi(R) \frac{\partial \bar{u}_l}{\partial x'_m} dV'$$

This is the equation for the average displacement. The normal solution for this equation, for $\bar{u}_i(\vec{r})$ in the form of plane wave, is

$$\bar{u}_i(\vec{r}) = B_i e^{i\vec{q}\cdot\vec{r}} \qquad (5)$$

and in the resulting equation we solve for q.

We find the characteristic values of the propagation factor q_l and q_t for longitudinal and transversal waves

$$q_\ell^2 = \frac{\rho\omega^2}{(\bar{\lambda}+2\mu)(\alpha+2\beta)} \qquad q_t^2 = \frac{\rho\omega^2}{\bar{\mu}+\beta} \tag{6}$$

These expressions for the propagation factors can be simplified in the particular case when we have low frequencies compared with the size of the inhomogeneities, obtaining the following values:

$$\begin{aligned} q_\ell &= \frac{\omega}{C_\ell}\left[1 + a_1 + a_2\omega^2 I_2\right] + i\,a_3\,\omega^4 I_3 \\ q_t &= \frac{\omega}{C_t}\left[1 + b_1 + b_2\omega^2 I_2\right] + i\,b_3\,\omega^4 I_3 \end{aligned} \tag{7}$$

where C_ℓ and C_t are the longitudinal and transversal velocity.
$I_2 = \int_0^\infty R\,\varphi(R)\,dR$ and $I_3 = \int_0^\infty R^2\,\varphi(R)\,dR$.
a_1, a_2, a_3, b_1, b_2, b_3 are parameter functions of the velocities and $\bar{\lambda}$, $\bar{\mu}$, $\overline{\Delta\lambda}$, $\overline{\Delta\mu}$.

Therefore we see that for low frequencies comparing with the scale of the inhomogeneities the damping factor of the average plane wave motion, in this particular medium, is proportional to ω^4 and to $I_3 = \int_0^\infty R^2\varphi(R)\,dR$ where $\varphi(R)$ is the autocorrelation function of the inhomogeneities.

III. EXPERIMENT 1

We are going to make a first approach to the problem of determining if there is enough energy involved in the scattered wave to account for seismic noise as recorded on surface.

The only suitable data we were able to locate was a set of continuous velocity (v_p and v_s) logs taken in Leer County, New Mexico, by the Shell Company(3): Fig. 1 shows a portion of this log taken from Figure 4(b) of Vogel's paper.

The region from 3000' to 4000' was chosen for study because of its apparently stationary behavior. In this region the section consists of Permian dolomites and anhydrites for which we have taken a density of 3.0. Variations in shear velocity tend to follow those in compressional velocity and the assumption $\lambda = \mu$ is a satisfactory one for our present purposes. A plot of λ vs. depth (with readings every 2.262 feet) is shown in Figure 2 as derived from these assumptions.

The propagation factor q_l for longitudinal waves (Eqn. 7) reduces for $\lambda = \mu$, to

$$q_\ell = \frac{\omega}{c_\ell}\left\{ 1 + \frac{\varepsilon}{30}\left[47 + 8\left(\frac{c_\ell}{c_t}\right)^2 + \frac{\varepsilon}{30 c_\ell^4}\left[31 + 4\left(\frac{c_\ell}{c_t}\right)^4\right]\omega^2 \int_0^\infty R\,\varphi(R)\,dR \right\}$$

$$+ i\left\{ \frac{\varepsilon}{30 c_\ell^4}\left[47 + 8\left(\frac{c_\ell}{c_t}\right)\right]^5 \omega^4 \int_0^\infty R^2 \varphi(R)\,dR \right\} \qquad (8)$$

where

$$\varepsilon = \frac{\overline{\lambda'^2}}{9\bar{\lambda}^2}$$

φ = normalized autocorrelation function of $\lambda' = \lambda - \bar{\lambda}$

$$= \lim_{n \to \infty} \frac{\sum_{-n}^{n} \lambda'_i \lambda'_{i+p}}{\sum_{-n}^{n} \lambda'^2} \qquad (9)$$

The solid line in Figure 3 shows the sample auto-correlation function (and power spectrum) computed to a lag of 226 feet over the 1000 foot section. This wave is not suitable for integration on several counts and must be modified. First of all a low frequency component is apparent (wavelength about 450 feet). It must be removed since we are interested in the scattering from small scale variations. Secondly, values of the function for large lags are both inaccurate and too large (in absolute value). These errors follow from sampling difficulties and would be greatly magnified by the integrations necessary. What we have done is first to remove the low frequency component (by subtraction) and then to fit an analytic function to the early part of the remainder. The function fitted was of the form :

$$C_1 e^{-aR} + C_2 e^{-bR} \cos(cR+d) \tag{10}$$

and this curve is shown in Fig. 3. The integrals of (1) may then be performed analytically and we find

$$q_\ell = 55.10^{-6}\omega + 97.10^{-20}\omega^2 + 470.10^{-19}\omega^4 i \tag{11}$$

Figure 4 shows the wave amplitude loss due to scattering by this medium for each 1000 feet of propagation. We also show in this Figure the group velocity curve

$$\bar{C}_\ell(\omega) = \left\{ 1 - \frac{\varepsilon}{30}\left[47 + 8\left(\frac{\bar{C}_\ell}{\bar{C}_t}\right)^2\right] - \frac{\varepsilon}{5\bar{C}_\ell^{\,4}}\left[31 + 4\left(\frac{\bar{C}_\ell}{\bar{C}_t}\right)^4\right]\omega^2 \int_0^\infty R\,\varphi(R)\,dR \right\} \tag{12}$$

A study of true ground motion amplitude on seismograms connection with scattering results of this type should reveal if the scattered energy contributes materially to the recording. The numbers indicate the scattered energy is small but it may not be out of line with the wide dynamic ranges actually observed.

It would furthermore be of interest to examine the experimental damping measurements for possible phase shifts which might be explainable in terms of the above predicted dispersion.

IV. EXPERIMENT 2

In this experiment we try to find connections between the rock type and the elastic parameter distribution.

Prior to this, we review the general classification of sedimentary rocks considering the nature of the corresponding sediments according to the classification of Professor R. R. Shrock(4) in the following diagram.

Next, we consider the factors affecting the elastic properties of rocks - as far as is known at the present time from empirical results - in order to find out later any relationship existing between these properties and our results.

The experimental data from all the material which we have found show considerable dependence of velocity upon porosity.

Apart from porosity velocity may be considered to depend also on matrix material or materials, grain size distribution and shape, cementation, liquid in pores, pressure and temperature(5).

It has been observed that, on the average, internal velocities in sand-shale sections increase with depth and that this increase appears to be proportional to the one-sixth root of depth(6). Sedimentary rocks show marked differences in elasticity depending on petrologic composition

A CLASSIFICATION OF SEDIMENTARY ROCKS

NATURE OF SEDIMENTS			SEDIMENTARY ROCKS	
DOMINANTLY FRAGMENTAL	Angular particles more than 2 mm. in greatest dimension	Rubble composed of sharpstones	SHARPSTONE	CONGLOMERATE
	Rounded particles more than 2 mm. in greatest dimension	Gravel composed of roundstones	ROUNDSTONE	
	Angular and rounded particles of rocks and minerals ranging in greatest dimension from 2 mm. to 0.1 mm.	Volcanic fragments = Tuff	TUFFSTONE	SANDSTONE
		Mixture of rock and mineral fragments	GRAYWACKE	
		Quartz + Feldspar	ARKOSE	
		Quartz + other minerals in large amount		
		Quartz + other minerals in small amount		
	Rock and mineral particles ranging in greatest dimension from 0.1 mm. to 0.001 mm. and colloidal particles less than 0.001 mm. in greatest dimension	Volcanic ash	ASHSTONE	SHALE
		Silt particles — 0.1 to 0.01 mm.	SILTSTONE	
		Clay minerals less than 0.01 mm.	CLAYSTONE	
		Silt + Clay + Water = Mud	MUDSTONE	
PARTLY FRAGMENTAL / PARTLY PRECIPITATED	Fe^{II} and Fe^{III} compounds precipitated inorganically and organically as concretions, nodules and layers. Impurities commonly present in the layers	Iron concretions	Concretionary	IRONSTONE
		Iron compounds + mud, silica, etc.	Precipitated	
	Siliceous inorganic fragments less than 2 mm. in greatest dimension	Inorganic fragments	Fragmental	SILICASTONE
	Siliceous organic hard parts and their fragments	Diatom frustules, radiolarian skeletons and sponge spicules		
	Silica precipitated as oölites, pisolites, etc.	Siliceous concretions	Concretionary	
	Silica precipitated from suspensions and solutions	Chert, flint, sinter, etc.	Precipitated	
	Plant structures — spores, fronds, leaves, wood, etc. Inorganic sediment	Plant debris; inorganic impurities		COAL
	Waxes, resins, etc. from decomposition of plants	Plant fluids		
	Calcite and Aragonite fragments. Calcareous organic hard parts — shells, exoskeletons, plates, spines, and fragments		Fragmental	LIMESTONE
	Organically and inorganically precipitated concretions		Concretionary	
	Inorganically precipitated $CaCO_3$ — Evaporation, etc.		Precipitated	
	Organically precipitated $CaCO_3$ — (1) by NH_3 from decomposition; (2) loss of CO_2 to plants; etc.			
			Recrystallized	
	Dolomite fragments. Dolomitized organic hard parts		Fragmental	DOLOSTONE
	Dolomitic concretions		Concretionary	
	Inorganically precipitated dolomite. Organically precipitated dolomite		Precipitated	
POSSIBLY FRAGMENTAL / DOMINANTLY PRECIPITATED	Fragments of anhydrite, gypsum, halite, alkali, nitrate caliche, etc.		Fragmental	SALINASTONE
	Evaporites — minerals precipitated during evaporation of saline waters	Anhydrite, Gypsum, Chlorides, Nitrates, Other rare salts — Precipitated	ANHYDROCK, GYPROCK	

Robert R. Shrock

December 7, 1946

Elastic sediments such as shale are less elastic than the sediments composed partially or wholly of crystalline matter such as limestone, dolomite, and the like. Elastic properties of sedimentary rocks depend much more on texture and geologic history than on mineral composition(7). The effect of porosity and decomposition is to decrease the modulus of elasticity and the wave velocity of a sediment.

In areas of great thickness of sedimentary rocks the porosity decreases with depth. Therefore the modulus of elasticity increases and with it the wave velocity(8). Related to change in porosity is the variation of Young's modulus with pressure. For small pressures rock appears to be more compressible since any cavities present have to be closed before the pressure can begin to act on the rock matter itself(9).

Excessive compressibilities resulting from porosity are accompanied by high values of Poisson's ratio. It must be expected that the ratio of longitudinal to transverse wave speed changes considerably with depth in consolidated sediments.

The effect of "moisture" or water content on the velocity in sedimentary beds is rather involved. In consolidated beds (sandstone, limestone, slates and the like) moisture appears to decrease the velocity; in unconsolidated beds moisture increases appreciably in velocity[10]. In reflexion work practical use is made of the increase of velocity and improvement of the transmission characteristic by placing the shots in or below the ground water table.

Many observations of elastic wave speed appear to indicate a direct relation between geologic <u>age</u> and <u>elasticity</u>. However, the controlling factor is the amount of "<u>diastrophism</u>" to which a formation has been subjected in its geologic history[illegible]. An increase in age merely increased the probability that it has undergone a greater degree of dynamometamorphism. Consequently, the greater the geologic age of the formation, the less is the velocity change with depth of burial.

So far, we have considered the general factors affecting the elastic properties of sedimentary rocks. However, we are interested mainly in finding out a class of statistics of the elastic parameter which characterize, within a range of variation, a particular type of rock independent of the physical or chemical property affecting different rocks corresponding to the same type. However, it will always be very interesting to try to explain some aspects of our results with some of the above properties, especially with respect to different samples of rocks belonging to the same family but differing in physical and chemical properties.

Experimental procedure and data

We have been able to gather 14 sections of continuous velocity logs. Three logs have been taken in different parts of the United States of America and Venezuela but always with the same type of apparatus and procedure. The transmitter and receiver are placed vertically 5 feet apart. All the sections we have chosen seem to satisfy the requirements of the scattering model, showing stationary behavior so that the oscillations about the mean value had an amplitude less than a third of the mean. We have removed the low frequency components from all the sections since we are interested in the scattering from small scale variations.

The characteristics of the velocity logs are summarized in the following page:

Continuous Velocity Logs

Group	Rock Type	Geological Age	Depth	Location
I	(1) Sandstone		5835'-6250'	McKenzie County, N.D.
II	(2) Freites Shale		1310'-2000'	Anzoategui, Venezuela
	(3) Pierre Shale		1805'-2400'	McKenzie County, N.D.
	(4) Pierre Shale		2405'-3250'	McKenzie County, N.D.
	(5) Silty Shale		9200'-9590'	Buttonwillow Basin, Wy.
III	(6) Grn-grn Clay		1000'-1500'	Buttonwillow Basin, Wy
	(7) Sandy Clay		2900'-3300'	Buttonwillow Basin, Wy
IV	(8) Limestone	Devonian	8960'-9850'	Crane County, Texas
	(9) Limestone	Devonian	2280'-3025'	Crane County, Texas
V	(10) Dolomite	Permian	3155'-3645'	Crane County, Texas
	(11) Dolomite	Permian	3650'-4050'	Crane County, Texas
	(12) Dolomite	Permian	4450'-5050'	Crane County, Texas
	(13) Dolomite and Anhydrite	Permian	3000'-4000'	Leer County, New Mexico
VI	(14) Gypsum and Anhydrite	Rustler	705'-1500'	Crane County, Texas

The velocity logs do not show the variations in shear wave velocity. We have been obliged to assume the Poisson condition $\lambda = \mu$ and therefore we have $\lambda = \mu = \rho \frac{V_P}{3}$. We have assumed ρ = constant for each different formation (11)

Dakota Sandstone	= 2.1
Freites Shale	= 2
Pierre Shale	= 2.1
Clay	= 2
Limestone	= 2.7
Dolomite	= 3.0
Gypsum and Anhydrite	= 2.6

With those values we have calculated $\lambda(=\mu)$. A plot of λ vs. depth for the different sections of the corresponding velocity logs is shown in Fig. 6.

From the variation of $\lambda(=\mu)$ we have calculated the variance $\overline{\Delta\lambda^2}$, also $\frac{\overline{\Delta\lambda^2}}{\bar{\lambda}^2}$ and the normalized autocorrelation function of $\lambda' = \lambda - \bar{\lambda}$ as well as the spectrum - as we can see in Fig. 7.

Mere inspection of the autocorrelations and spectra does not yield enough information to make broad qualitative distinctions between the different kinds of rocks. Therefore to be able to make some comparisons quantitatively we have taken the first part of each autocorrelation (ten first lags), as more accurate, and we have calculated $I_2 = \int_0^{10} R\,\varphi(R)\,dR$ and

$I_2 = \int_0^{10} R^2 \varphi(R)\, dR$ (as we know the integral of the products of the autocorrelation of the inhomogeneity multiplied by the lag and the square of the lag respectively) after smoothing the shape of this part of the autocorrelation as we can see in Fig. 8.

V. RESULTS

All the results are summarized in the table that we present in the following page.

A graphical representation of all the results are presented in Fig. 9.

VALUES OF THE PARAMETERS FOR THE CORRESPONDING TYPES OF ROCKS STUDIED

Rock Type	V_p 1000'/second	$\overline{\Delta\lambda^2}$	$\bar{\lambda}$	$\overline{\Delta\lambda^2}/\bar{\lambda}^2$	Wave Length (feet)	I_2	I_3
(1) Sandstone	10.90	140.3	86	.019	14.1'	12.4	11.3
(2) Freites Shale	6.30	4.3	28	.0055	17.6'	11.8	383
(3) Pierre Shale	7.25	4	37	.003	11.3'	-14.8	-495
(4) Pierre Shale	7.45	4.9	38	.0034	13.5'	3.3	46.2
(5) Silty Shale	8.30	2.7	49	.0012	15.3'	9.7	43.7
(6) Gry-grn Clay	7.05	15.2	34	.013	17.5'	2	1330
(7) Sandy Clay	6.5	2.5	29	.003	15.1'	10.5	191
(8) Limestone	19.4	11.2	34	.0097	13.0'	9.68	-72.21
(9) Limestone	22.5	48.8	46	.023	18.0'	-55.4	-850
(10) Dolomite	19.5	11.1	36	.0086	18.4'	32.2	270
(11) Dolomite	18.8	21.6	35	.0176	20.8'	36.2	572
(12) Dolomite	18.7	17.0	37	.0124	16.2'	9.5	174.0
(13) Dolomite and Anhydrite	18.2	11.2	85	.0015	15.7'	28	535
(14) Gypsum and Anhydrite	18.0	95	33	.0086	15.2'	90.6	2140

VI. EVALUATION, LIMITATIONS AND CONCLUSIONS

We see that probably the most sensitive parameter of those studied is variance = $\overline{\Delta \lambda^2}$ which, in the case of the four samples of shale (2 Pierre Shale, 1 Freites Shale and 1 Silty Shale) has almost an equal value, and in its magnitude is quite different from those of all other rocks. In the case of the dolomites also we have a fairly constant value for variance = $\overline{\Delta \lambda^2}$ in the four samples and their average value is also quite different in magnitude from those of all other rocks. In the case of the parameter $\frac{\overline{\Delta \lambda^2}}{\overline{\lambda}^2}$, variations between different elements of the same family are greater. However, they are not so large as to prevent broad separation among different families of rocks. Therefore this parameter could be used to point out the difference among samples of the same family, if this behavior is confirmed. As far as the difference between families this parameter does not give us, in this particular study, any more information than the variance itself.

As far as the average velocity $\overline{V_p}$ is concerned, we have, in general, a fairly common value for all elements belonging to the same type of rock. However, as we have discussed before the factors which characterize the velocity depend mostly on the physical and chemical properties such as the porosity, matrix characteristics, etc., and therefore with this parameter it is impossible to try to distinguish different types of rocks. However, it would be interesting to try to find the connection between average velocity and the parameters which are the object of our study.

The average wave length presents pretty much the same values for the different samples of the same type of rock and if we take the average of the corresponding families we find a good distinction of values between the different rocks. We do not however see any correlation between the average wave length and the corresponding average velocities. We see that wave length increases with $\overline{\Delta\lambda^2}$ except for the case of limestone. As far as the autocorrelation of the inhomogeneities is concerned we look to the integrals I_2 and I_3. It seems that I_2 gives us a truer picture.than I_3, since for large lags, the inaccuracies brought about by the presence of R raised to the second power are emphasized and cause greater deviation from the actual values.

I_3 is one of the damping coefficients in the propagation factor.

As we see, there is a weak correlation between the values of I_2 and I_3. There is of course a good correlation between the values of $\overline{\Delta\lambda^2}$ and I_2. The increase in I_2 follows fairly regularly the increase in variance = $\overline{\Delta\lambda^2}$. The average values of I_2 for the different families of rocks, as in the case of $\overline{\Delta\lambda^2}$, are pretty sensitive to the distinction between two different rocks.

Summarizing we can say that $\overline{\Delta\lambda}$ and I_2 are probably the best parameters (i.e. most sensitive) to differentiate among types of sedimentary rocks. As a whole the results do not show good enough correlations to provide a clear distinction among different types of rocks. However, we have to take into account that we did not have enough samples for some of the rock types. We have also to consider that the rocks belong to far distant areas. It is possible that much better results might be obtained in the case of rocks belonging to the same area.

Our main limitation in this work is that we have used only such portions of the velocity logs which from the general appearance seem to satisfy the requirements of the scattering model.

VI. FUTURE WORK

Since the results are not discouraging it seems that it will be worthwhile to do further work along the following lines:

a) From the theoretical point of view it will be interesting to try to see how well the velocity logs we have used represent the physical situation which satisfy the scattering model such as that studied in R. Bowman's thesis[(2)].

b) Try to apply our kind of work to a single area.

c) Study logs with both v_p and v_s velocities because it might not be possible to apply the Poisson condition in many of cour cases. As we know there are quantitative differences between the propagation characteristics of transverse and longitudinal waves in the scattering model.

d) Work, if possible, with greater number of samples and longer sections showing the same behavior.

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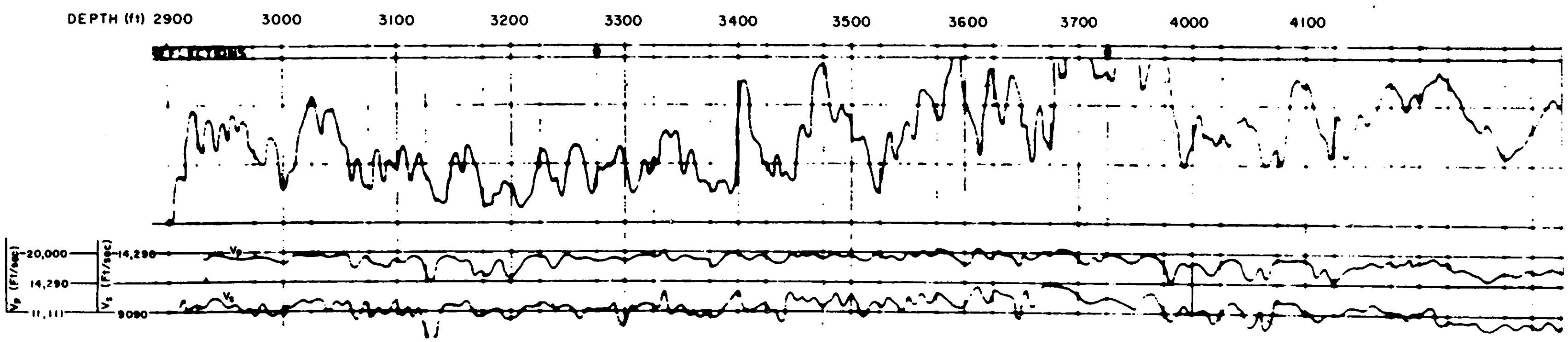

Fig. 1 A portion of a continuous velocity log from Fig. 4(b) of Vogel's article (1952, pg. 592).

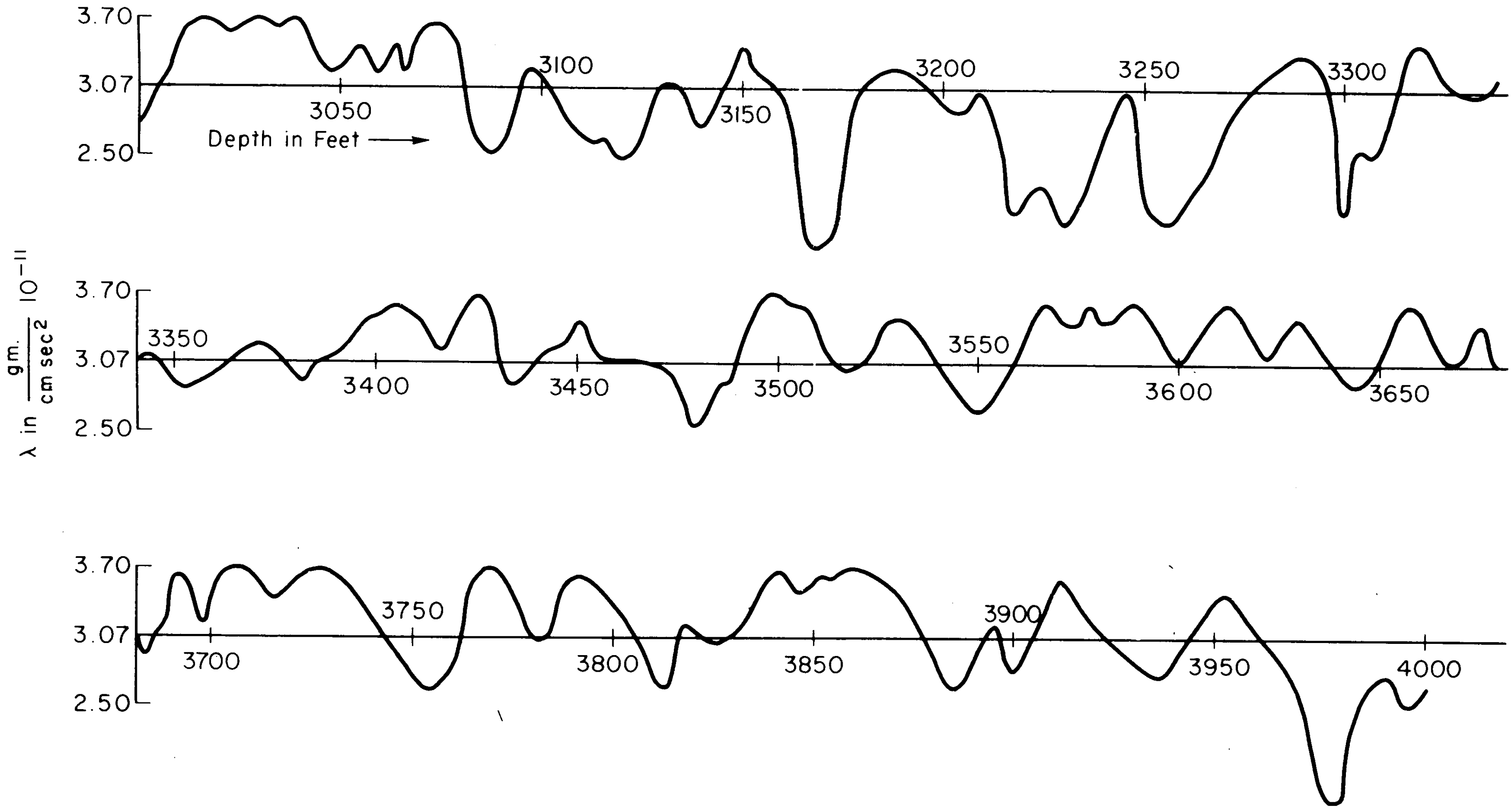

FIG. 2 – VARIATION OF $\lambda (= \mu)$ WITH DEPTH FROM FIG. 5.1

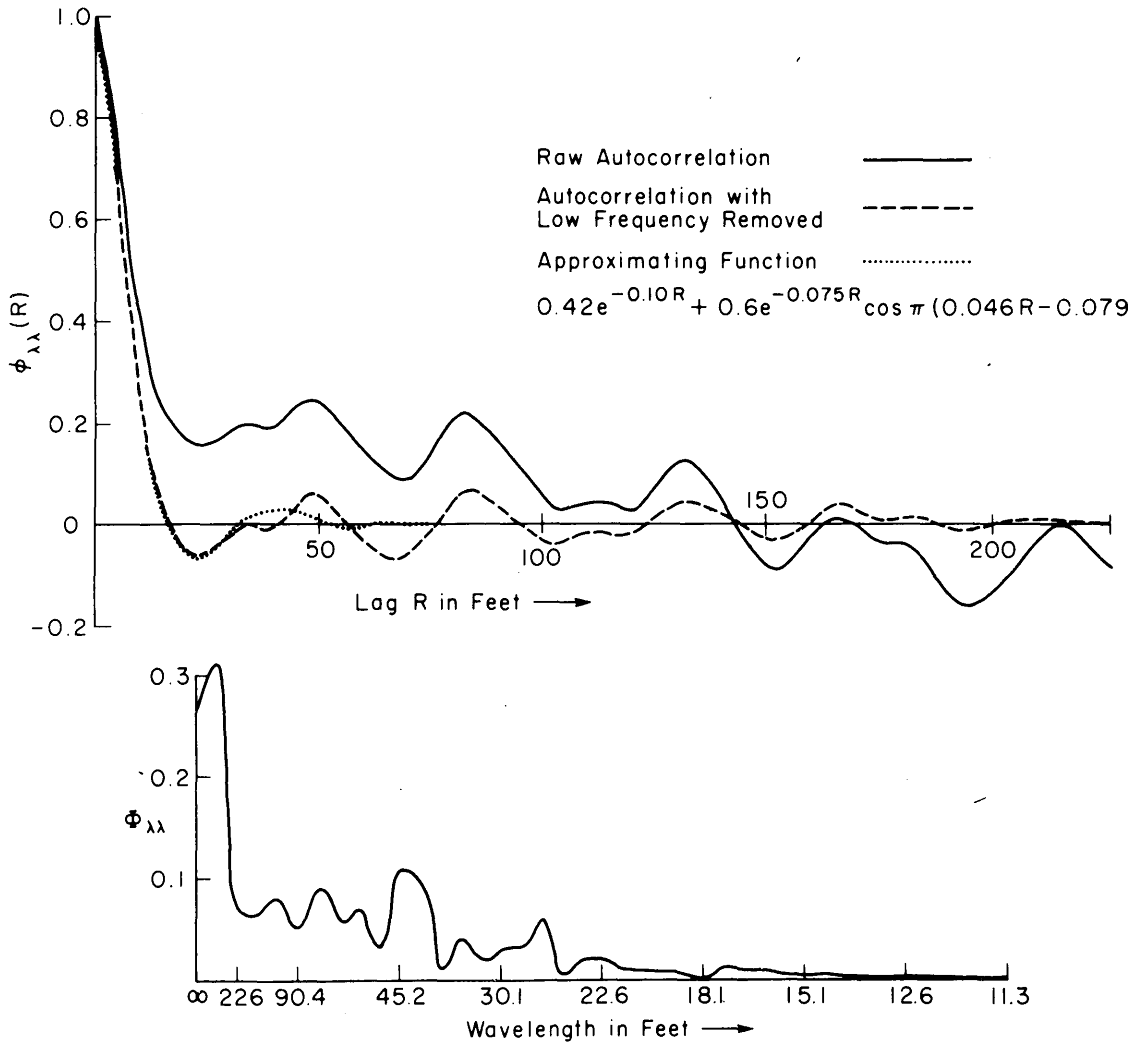

FIG. 3 - AUTOCORRELATION AND SPECTRUM OF λ VARIATIONS

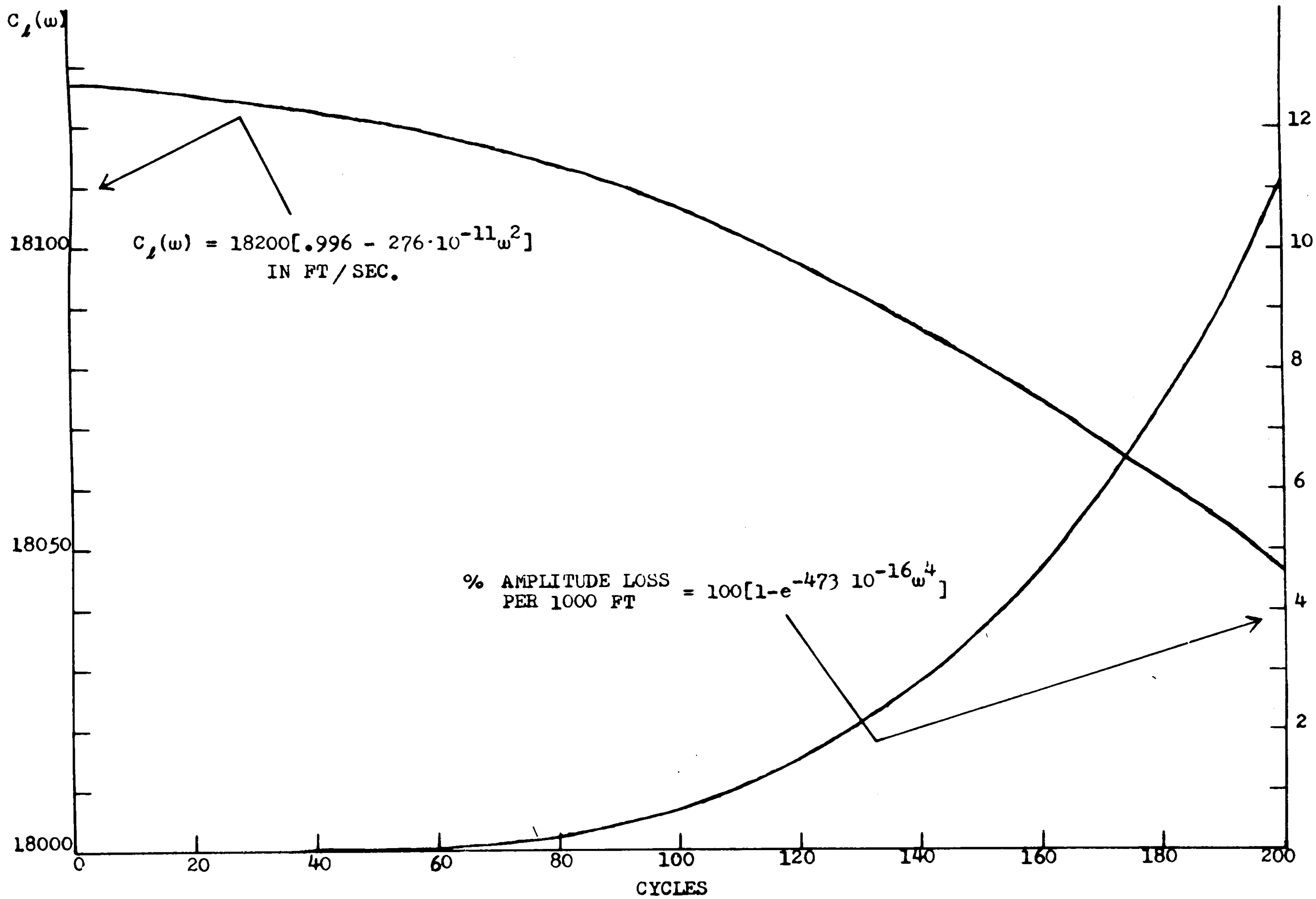

FIG .4 ATTENUATION AND GROUP VELOCITIES AS FUNCTIONS OF FREQUENCY

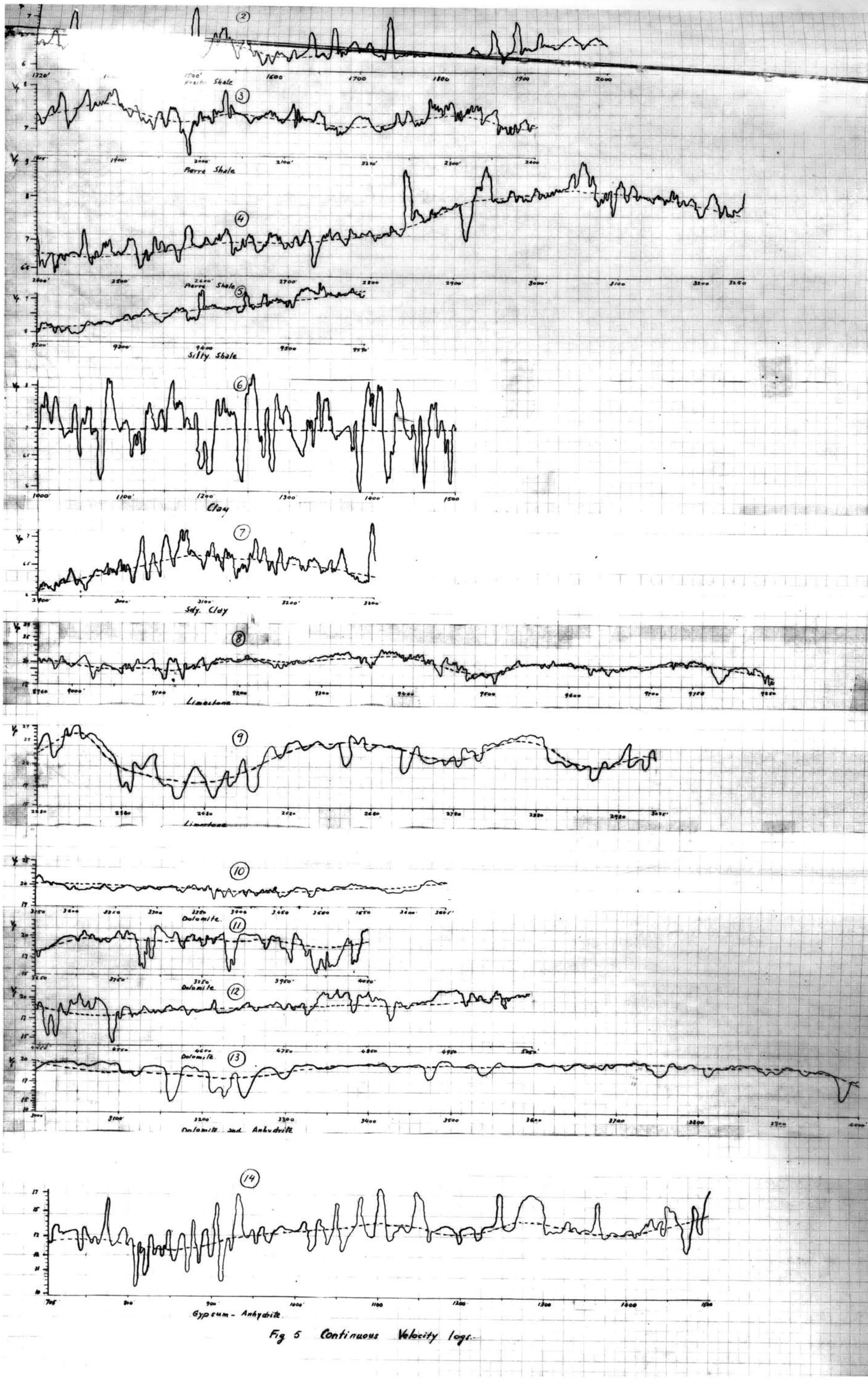

Fig 5 Continuous Velocity logs.

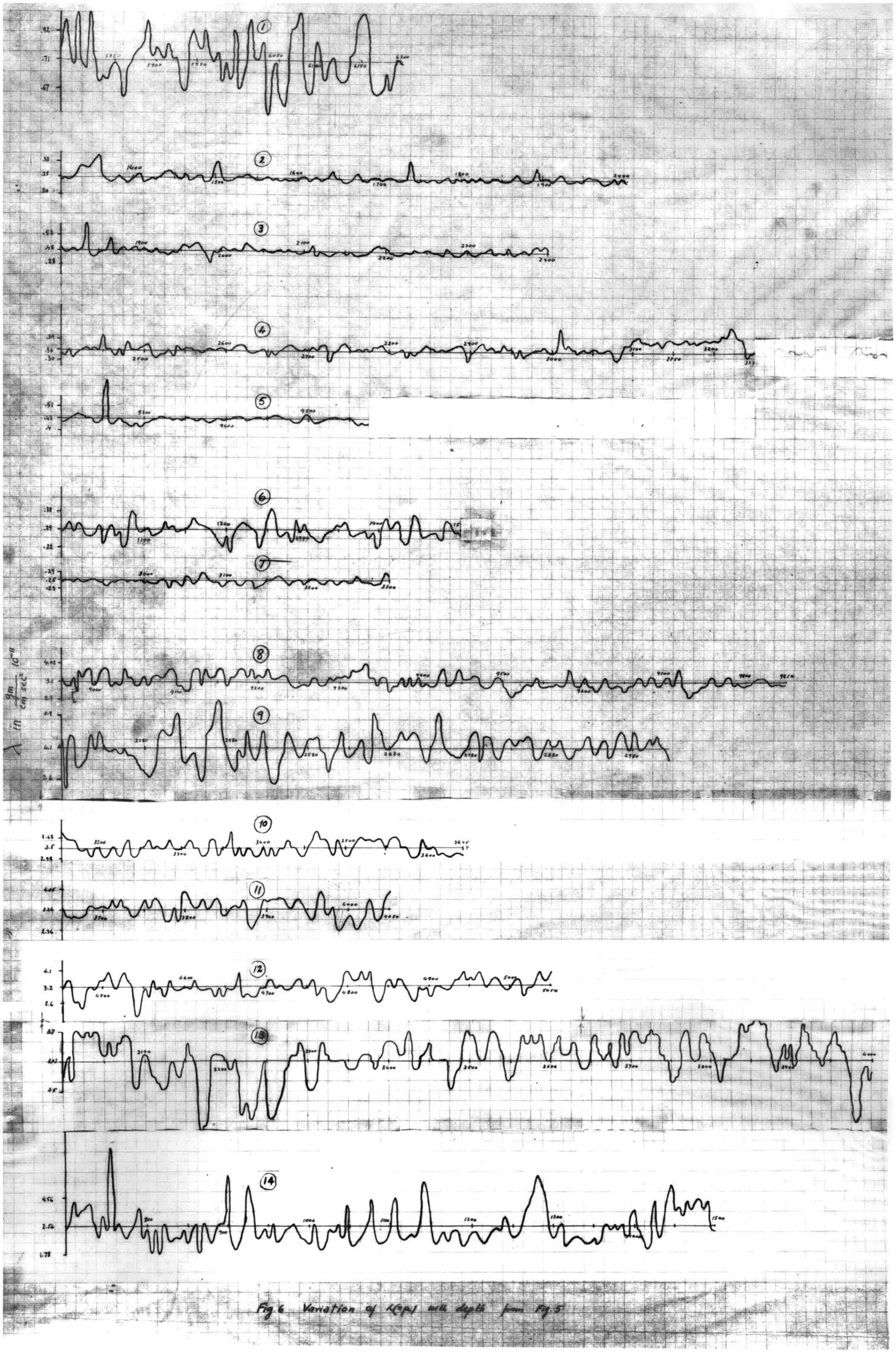

Fig. 6 Variation of K(ρ) with depth from Fig. 5

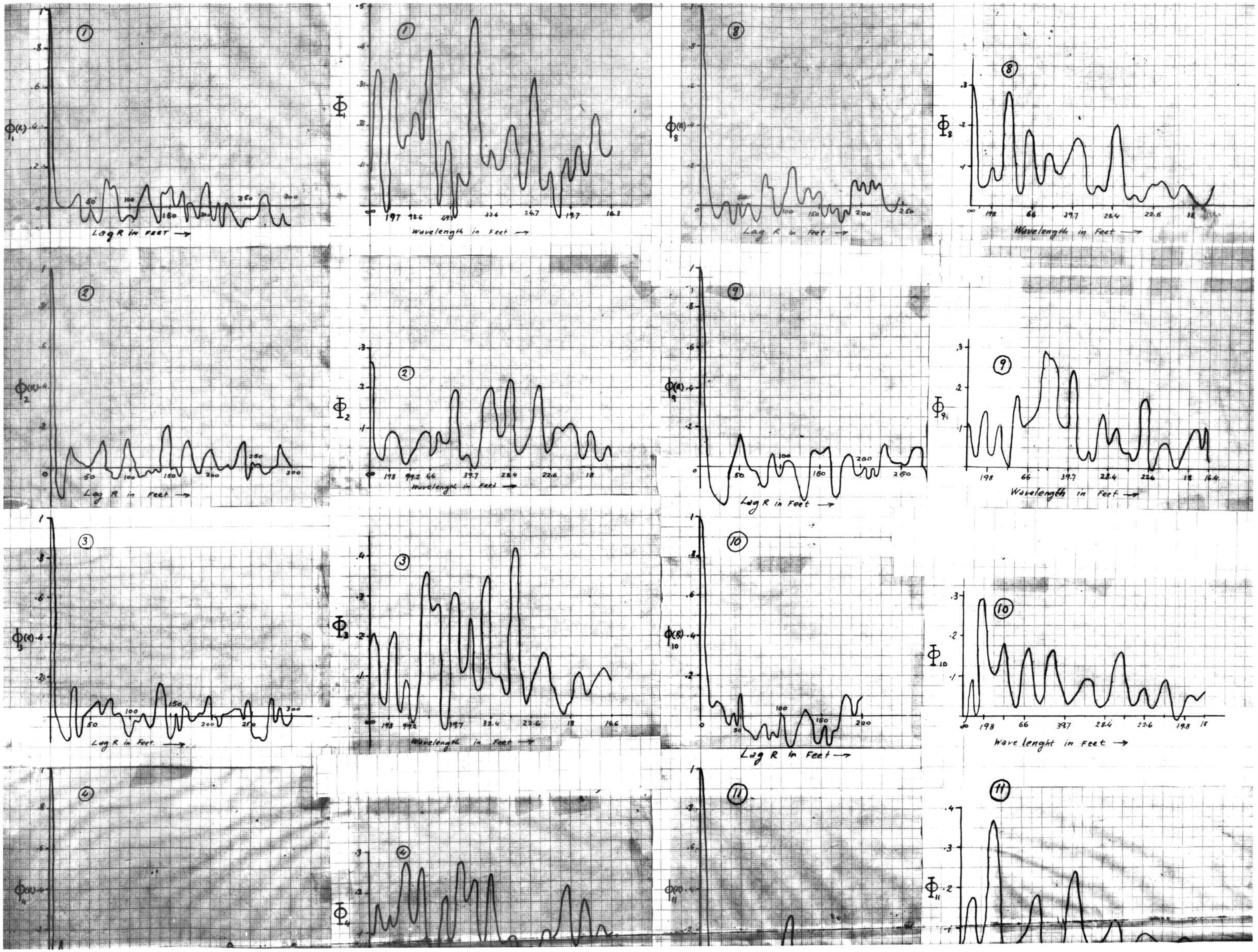

Fig 7. Autocorrelations and Spectrums of λ variations (continued next page)

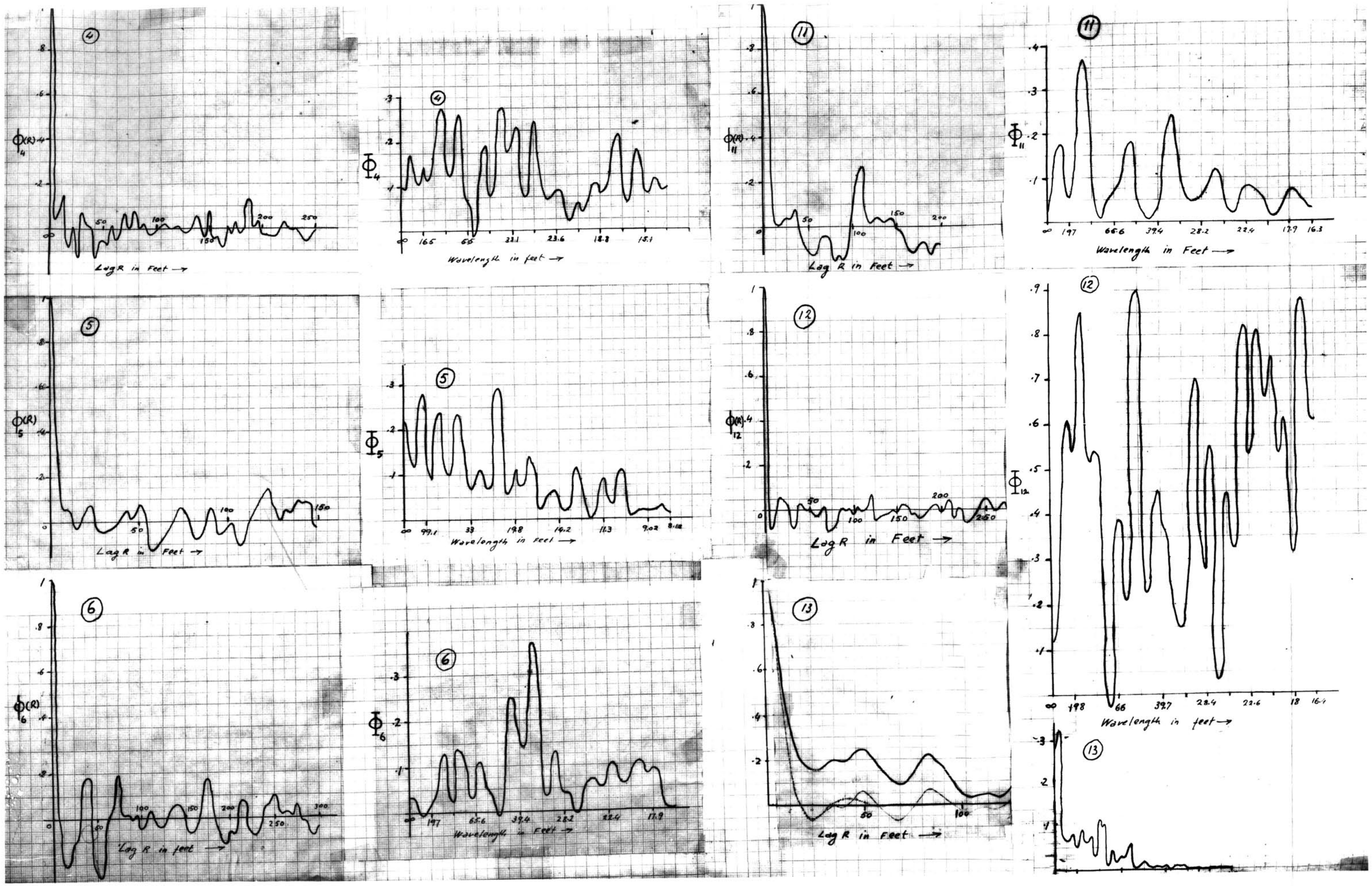

Fig 7. Autocorrelations and Spectrums of λ variations (continued next page)

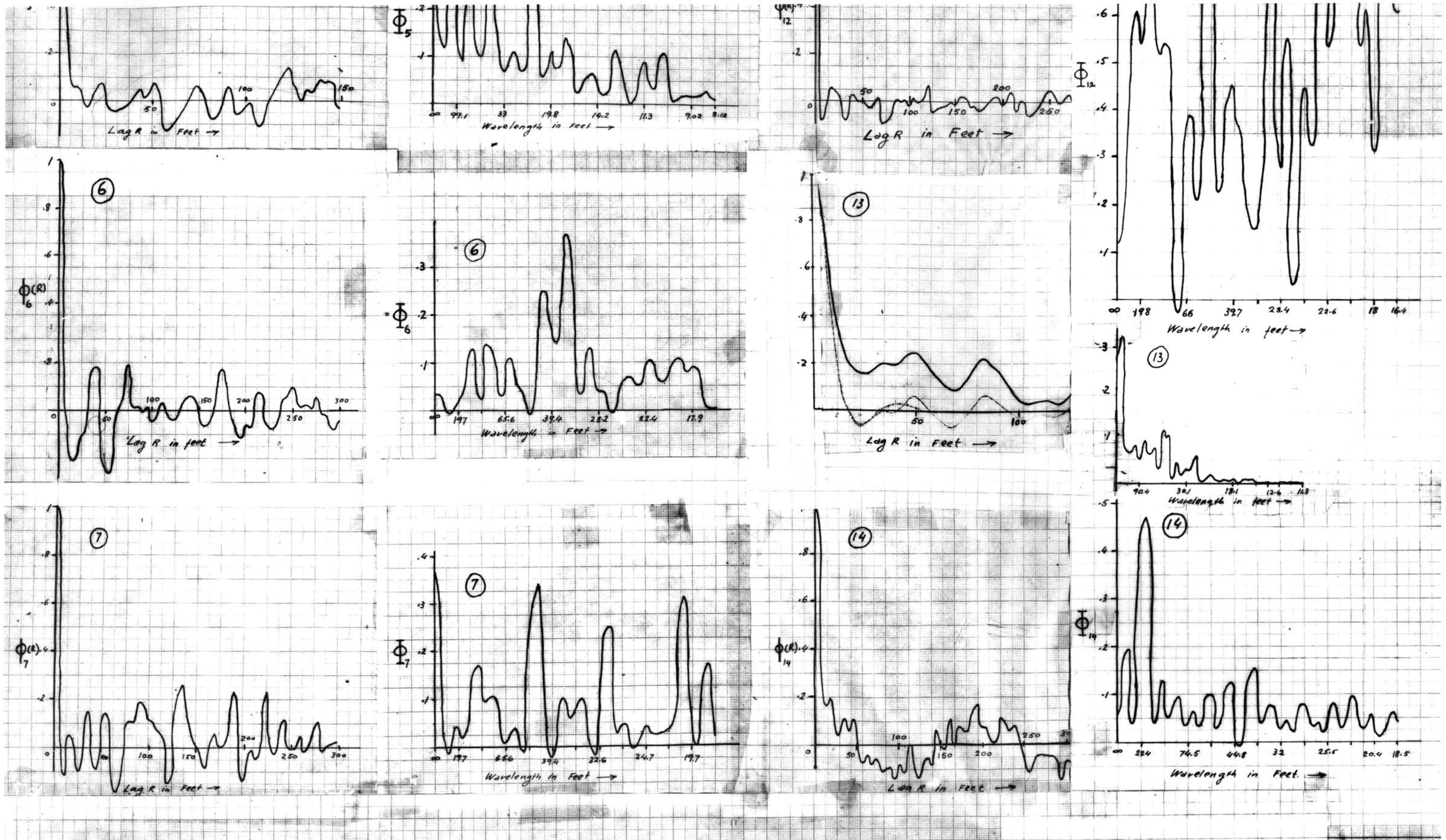

Fig 7 Autocorrelations and Spectrums of λ variations.-

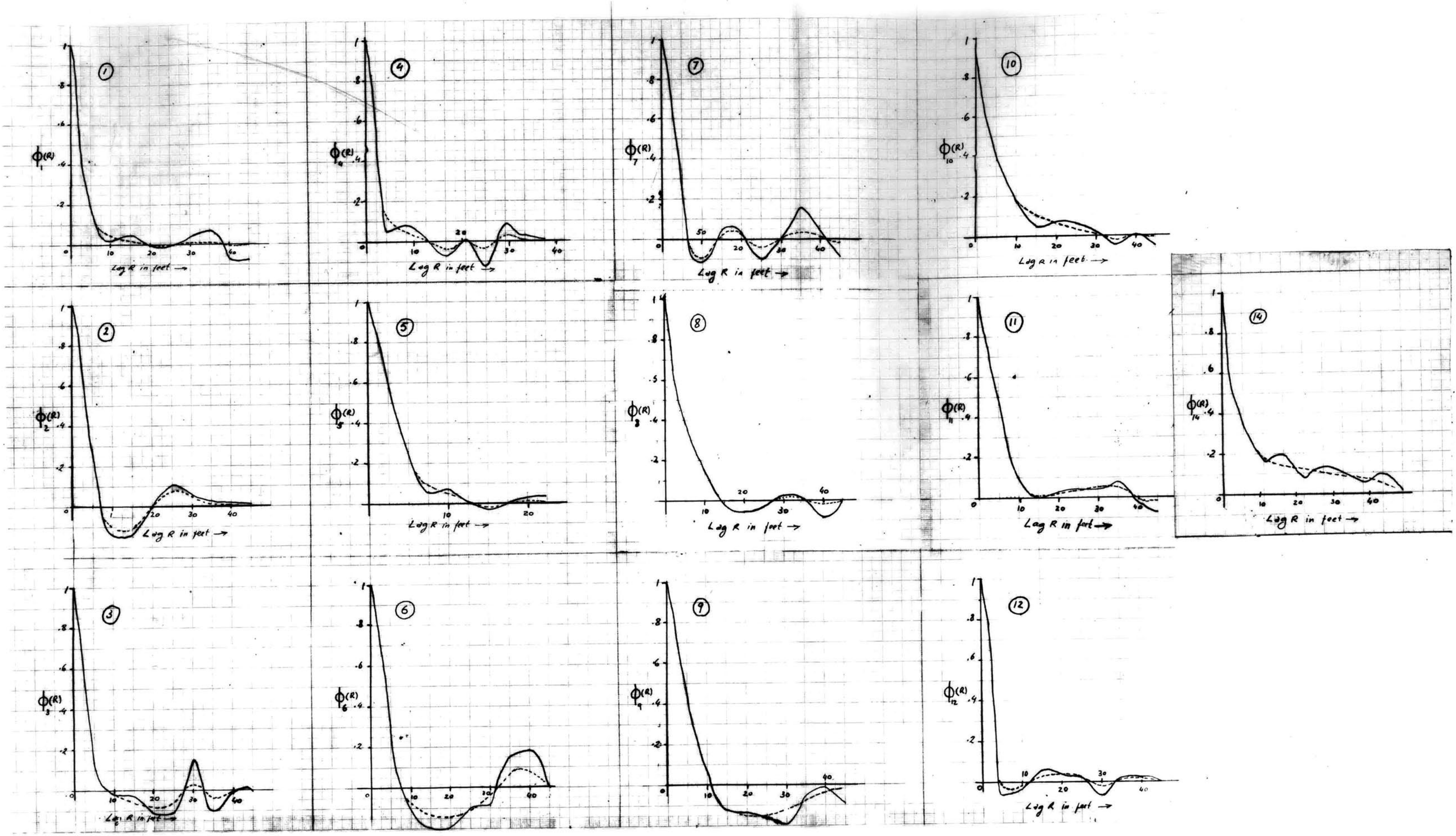

Fig. 8. Smoothened Autocorrelation Functions of λ Variations for the First Ten Lags.

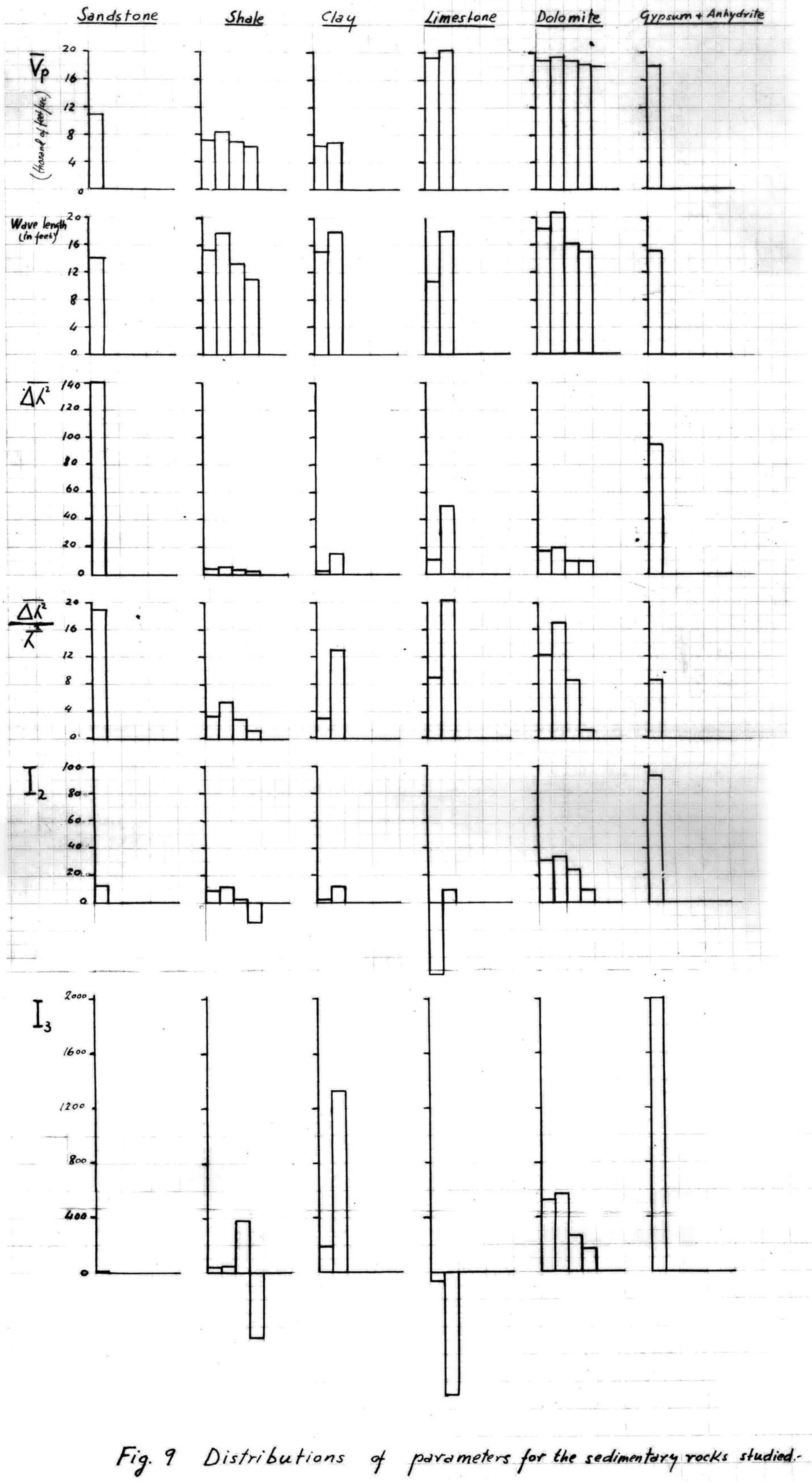

Fig. 9 Distributions of parameters for the sedimentary rocks studied.