

# A STUDY OF THE DYNAMICS OF SHOTCRETE FORMWORK

by MICHAEL DAVID

Bachelor of Engineering in Civil Engineering

The Cooper Union, 2009

SUBMITTED TO THE DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING IN  
PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF ENGINEERING IN CIVIL AND ENVIRONMENTAL ENGINEERING

AT THE

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2010

© 2010 MICHAEL STEVEN DAVID. All rights reserved.

The author hereby grants to MIT permission to reproduce and to distribute publicly  
paper and electronic copies of this thesis document in whole or in part in any medium  
now known or hereafter created.

Signature of Author: \_\_\_\_\_

Department of Civil and Environmental Engineering  
May 7, 2010

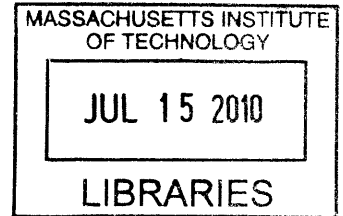
Certified by: \_\_\_\_\_

Jerome J. Connor  
Professor of Civil and Environmental Engineering  
Thesis Supervisor

Accepted by: \_\_\_\_\_

Daniele Veneziano  
Chairman, Departmental Committee for Graduate Students

ARCHIVES





# A STUDY OF THE DYNAMICS OF SHOTCRETE FORMWORK

by MICHAEL DAVID

Submitted to the Department of Civil and Environmental Engineering on May 7, 2010 in Partial  
Fulfillment of the Requirements for the Degree of

Master of Engineering  
in Civil and Environmental Engineering  
at the Massachusetts Institute of Technology

## ABSTRACT

This study models and analyzes the dynamic behavior of shotcrete formwork during standard application procedure. Based on standard shotcrete application, a program was developed to simulate shotcrete application and the dynamic behavior of shotcrete formwork.

This study shows that the random behavior standard shotcrete application have minimal impact on the maximum values of displacement and acceleration of a shotcrete formwork system, which justifies the significance of the simulations based on the precision of the results obtained. Standard design parameters were varied in order to determine their impact on the behavior of a formwork system, and determine which parameters had the greatest ability to control the displacement and acceleration of formwork during shotcrete application.

Thesis Supervisor: Jerome J. Connor

Title: Professor of Civil and Environmental Engineering



# ACKNOWLEDGEMENTS

I would like to thank Professor Jerry Connor for his advice and support. His knowledge has helped me develop the topic of this paper and has helped provide me with the skills to perform this study. His encouragement and inspiration has helped me not only in this paper, but in my studies and my life.

I would also like to thank Simon Laflamme for his guidance and for providing me with the tools and knowledge required for the work done in this study. His guidance and help has played a significant role in the development of this paper.

I would like to thank my family for encouraging me in my studies and supporting me throughout my education. My parents and siblings are the reason I have accomplished what I have in my studies and my life. I will always be grateful for everything they have done for me.



# TABLE OF CONTENTS

1	Introduction .....	11
1.1	Properties of Shotcrete .....	11
1.2	Popular Uses of Shotcrete .....	11
1.3	Current Formwork Systems .....	13
2	Purpose .....	15
2.1	Analysis of Formwork .....	15
2.2	Designing Formwork .....	15
3	Simulation & Programming .....	17
3.1	Overview .....	17
3.2	Analysis Parameters .....	18
3.3	Stiffness Matrix of Formwork .....	20
3.4	Application of Shotcrete .....	24
3.5	State-Space Analysis .....	26
3.6	Master File.....	28
3.7	Simulations .....	29
4	Results .....	31
4.1	Simulation .....	31
4.2	Maximum Displacement.....	32
4.3	Maximum Acceleration.....	34
5	Conclusion.....	37
5.1	Accuracy of Simulation .....	37
5.2	Maximum Deflection/Acceleration.....	37
5.3	Suggestions for Future Research .....	38
6	References.....	41

7	Appendix I ( <i>MATLAB Program</i> ) .....	43
7.1	Parameters for Analysis .....	44
7.2	Developing Stiffness Matrix For Plate on Springs.....	45
7.3	Apply Shotcrete to Formwork .....	49
7.4	State-Space File .....	52
7.5	Master File.....	54
7.6	Manipulation of Solutions.....	60
8	Appendix II ( <i>Simulations</i> ).....	63
8.1	Simulation A-1 .....	65
8.2	Simulation A-2.....	67
8.3	Simulation A-3 .....	69
8.4	Simulation B-1 .....	71
8.5	Simulation B-2.....	73
8.6	Simulation B-3 .....	75
8.7	Simulation C-1 .....	77
8.8	Simulation C-2 .....	79
8.9	Simulation C-3 .....	81
8.10	Simulation D-1.....	83
8.11	Simulation D-2.....	85
8.12	Simulation D-3.....	87



# LIST OF TABLES

Table 1 - Rebound for Conventional Shotcrete ..... 12  
Table 2 - Maximum Production Rates..... 12  
Table 3 - Set Parameters..... 18  
Table 4 - Varied Parameters ..... 18  
Table 5 - Varying Parameters..... 29  
Table 6 - Results..... 31

# LIST OF FIGURES

Figure 1 - Finite Labels..... 20  
Figure 2 - DOF of Finite Points..... 21  
Figure 3 - Structure of a Compiled Matrix..... 21  
Figure 4 - Stiffness Matrix of Single Finite Element ..... 22  
Figure 5 - Compiled Stiffness Matrix ..... 23  
Figure 6 - Proper Nozzle Motion for Shotcrete Application ..... 24  
Figure 7 - Shotcrete Application..... 25  
Figure 8 - State Space Deflection of Formwork..... 27  
Figure 9 - Master File Flow Chart ..... 28  
Figure 10 - Maximum Displacement..... 32  
Figure 11 - Maximum Acceleration..... 34



# 1. INTRODUCTION

## *1.1 PROPERTIES OF SHOTCRETE*

Shotcrete is a common method for applying concrete to surfaces. It is a process where concrete is sprayed through a nozzle and adheres to the surface it is applied to. Similar to concrete, it is often reinforced. Because of its ability to adhere it is often used as a repair system for damaged or deficient concrete structures.

The most common uses of shotcrete are for tunnel linings (*Underground Support*, Vol. VII & X). It is able to comply with the uneven geometry that often results from tunneling. Cast-in-place concrete would require formwork which would have to be able to change geometry. Another benefit of shotcrete is that in tunneling there is no formwork required. This reduces the amount of space taken up by temporary formwork and supports. Even when formwork is required only a single face of formwork is needed, unlike in typical cast-in-place construction. This helps to simplify forming problems that arise during construction (Hurd, 15.27-15.28).

Shotcrete is naturally compacting because of its method of application. By applying layer after layer the impact force generated compresses each layer. This prevents the common problem of honeycombing and makes the shotcrete stronger than traditional cast-in-place. Common shotcrete has a compressive strength of 5ksi, while standard concrete has a compressive strength of 4ksi.

## *1.2 POPULAR USES OF SHOTCRETE*

Shotcrete is feasible up to a certain scale of construction. For large-scale projects the cost of labor for applying shotcrete can exceed the cost of constructing formwork. Because of the method of application, skilled nozzleman need to be hired who know the proper methods for applying shotcrete to surfaces. If shotcrete is not applied in the proper way excessive rebound can occur which will result in sandy porous material being embedded in the shotcrete. Rebound is the sand and concrete which does not fully adhere to the surface the shotcrete is being applied to. On average 5% of the shotcrete sprayed ends up as rebound which is blown free from the surface and cleared off the site. Table 1 shows the percent rebound by mass for the different mix systems.

**Table 1 - Rebound for Conventional Shotcrete**

Work surfaces (1)	Percent of Rebound, by Mass	
	Dry-mix (2)	Wet-mix (3)
Floors or slabs	5-15	0-5
Sloping and vertical walls	5-25	5-10
Overhead work	25-50	10-20

*Courtesy of Standard Practice for Shotcrete*

In addition to nozzleman, contractors need to be experienced with shotcrete construction. The methods of mixing and the equipment used vary from those for tradition concrete construction. There are two standard methods of shotcrete mixing: dry and wet. Both methods include water, but dry mixing combines a dry mixture of aggregates, cement, and admixtures with water within the nozzle, while wet mixing combines these together in a storage container before entering the nozzle (Lamond, 619-620). Both methods have different nozzles which have different properties, but both methods have similar application techniques. Table 2 lists the different production rates for dry mix guns.

**Table 2 - Maximum Production Rates**

Compressor capacity at 100 psi (ft <sup>3</sup> /min) (1)	Inside diameter of delivery hose (in.) (2)	Maximum production rate (yd <sup>3</sup> /hr) (3)
365	1	4
425	1.25	6
500	1.5	9
700	1.75	10
900	2	12
1,000	2.5	15

*Courtesy of Standard Practice for Shotcrete*

One of the reasons why shotcrete is popular in tunneling and is less common in other forms of construction is that, in tunneling, the bedrock or soil remaining after excavation acts as the formwork. Both of these surfaces have such high inertial masses that there are almost zero vibrations during application. This results in a formwork which does not need to be constructed or removed and is infinitely stiff. In other types of construction a formwork surface would need to be constructed and there would inevitably be some deflections and vibrations.

### *1.3 CURRENT FORMWORK SYSTEMS*

There are two main types of formwork used during shotcrete construction: temporary and permanent formwork. Examples of temporary formwork are standard ply-wood, which is often used for in-ground pools and inflatable formwork, which has been researched as an option for affordable housing construction. Common examples of permanent formwork are ply-wood forms left for cost, or architectural purposes and corrugated steel, which is the common formwork used for flooring systems. Permanent formwork can be located on the exterior, or embedded within the interior of the shotcrete.

By having permanent formwork, the properties of the form become part of the system. Therefore, thermal panels and acoustic panels can also be used as the formwork for shotcrete applications (Renner-Smith, 67). This is beneficial, because materials that would be introduced into the structural system, without any structural or construction benefit, could replace the need for temporary formwork. At the same time acoustic and thermal panels can be embedded within shotcrete, and therefore will be moved away from the surface, being protected.



## 2. PURPOSE

### *2.1 ANALYSIS OF FORMWORK*

There are several difficulties with trying to model shotcrete formwork. The most important factor which affects how formwork would behave under the loading of shotcrete application is the randomness of application. There are set guidelines for how shotcrete should be applied, however, these guidelines allow for variation. These variations in application of shotcrete provided uncertainties that may be required for proper analysis. This raises the question of whether or not the dynamic effects that shotcrete application has on formwork can be accurately replicated and analyzed.

One of the purposes of this study is to determine whether or not the randomness of shotcrete application allows for precise analysis of formwork systems for shotcrete. A computer program will be constructed in MATLAB in order to simulate the state-space properties of shotcrete formwork in order to determine how significant the variation of behavior is between simulations of shotcrete application following the *Standard Practice for Shotcrete* specifications.

### *2.2 DESIGNING FORMWORK*

The dynamic properties of formwork are what influence the behavior during shotcrete application. These properties are dependent on the stiffness and mass of the system, which are affected by different parameters which vary during common construction processes. Many shotcrete application studies state that formwork needs to have sufficient stiffness in order to properly receive the shotcrete and allow for acceptable quality of production (Austin). It is not discussed how this stiffness is obtained.

The different parameters which affect the stiffness and mass of the formwork system will be varied independently in order to determine the effect of each parameter on the dynamic response of the system. It will be determined within a practical range of values for each parameter which has the greatest positive impact on controlling the displacement and acceleration of the formwork.





## 3. SIMULATION & PROGRAMMING

### 3.1 OVERVIEW

In order to analyze Shotcrete formwork several MATLAB programs were developed. The MATLAB code for each program can be found in Appendix I. The first program determines the stiffness matrix for a plate supported by spring elements. The program was written so that the dimensions of the plate, the spacing of the supports and the size of the finite elements could be chosen and the stiffness matrix would be compiled based on these constraints.

The second program developed simulates the application of shotcrete to a surface. In order to *apply* the shotcrete an algorithm was written so that the shotcrete is applied as per *Standard Practice for Shotcrete*. This program was used for the dynamic analysis of the system, by varying the mass and force matrix over time.

Before the shotcrete was *applied* a program was written to perform the dynamic analysis of the system under a uniform load. The stiffness program was used and the state-space equation was incorporated in order to determine deflection over a time interval specified. The state-space equation is a dynamic representation of the equation of motion for the system and will be discussed later in this chapter.

Once these programs were fully developed a *Master* program was constructed combining all three programs together. This final program was used to vary different parameters in order to determine which had the most significant effect on deflection and acceleration.

### 3.2 ANALYSIS PARAMETERS

Parameters which are related to the shotcrete application were held constant and parameters which are dependent on the shotcrete formwork were varied. The varying parameters were changed independently for each simulation in order to determine the impact each had on the dynamic response of the system. The set and varying parameters used for simulation are shown in Tables 3 & 4.

**Table 3 - Set Parameters**

<b>Set Parameters</b>	
<b>h</b>	<b>Formwork Height (ft)</b>
<b>w</b>	<b>Formwork Width (ft)</b>
<b>c</b>	<b>Damping Ratio</b>
<b>E</b>	<b>Formwork Surface Modulus of Elasticity (psi)</b>
<b>rof</b>	<b>Formwork Density (lbs/ft<sup>3</sup>)</b>
<b>v</b>	<b>Poisson's Ratio</b>
<b>Qs</b>	<b>Shotcrete Spray Speed (ft<sup>3</sup>/min)</b>
<b>vs</b>	<b>Shotcrete Application Speed on Contact (in/s)</b>
<b>ds</b>	<b>Shotcrete Spray Diameter on Contact (in)</b>
<b>ros</b>	<b>Shotcrete Density (lbs/ft<sup>3</sup>)</b>
<b>ts</b>	<b>Desired Shotcrete Thickness (in)</b>
<b>s</b>	<b>Maximum slope</b>

**Table 4 - Varied Parameters**

<b>Varied Parameters</b>	
<b>s1</b>	<b>Vertical Spacing of Supports (ft)</b>
<b>s2</b>	<b>Horizontal Spacing of Supports (ft)</b>
<b>ks</b>	<b>Support Stiffness (lbs/ft)</b>
<b>tf</b>	<b>Formwork Surface Thickness (in)</b>

\*values for all parameters used in the simulations can be found in Appendix II

The parameters were determined based on *Standard Practice for Shotcrete* specifications and common practices for shotcrete formwork. For the analysis of the varied parameters the height and width of the formwork were chosen to be 2ft and 3ft, respectively. These dimensions could be varied, however, for this analysis and the computing time required, these values were chosen and used for all of the simulations that were run.

The damping ratio was chosen to be 2% of the stiffness for the formwork. This value was chosen to account for material damping and to allow the vibrations in the system to dissipate as they naturally would.

The modulus of elasticity, density and Poisson's ratio for the formwork were chosen to be 1001ksi, 43.7lbs/ft<sup>3</sup> and 0.22 respectively. These values were chosen for standard ply-wood, which is a common formwork surface for shotcrete application.

The shotcrete production rate was chosen as 10yd<sup>3</sup>/h, a standard application rate used in common practice. This value, along with the desired thickness of shotcrete, chosen to be 5in, affects the amount of time the simulation needs to run, in order for the proper thickness of shotcrete to be applied. The application speed on contact is the speed at which the point of application moves across the surface of the formwork, which was chosen to be 20in/s. This number affects the time step of the state space equation. The diameter of the shotcrete spray on contact determines the size of the finite elements the formwork is broken down into, and was chosen to be 3in. The diameter, application speed and density of the shotcrete applied (150lbs/ft<sup>3</sup>) are used to determine the change in the mass and force matrix of the system per time step.

The maximum slope of the shotcrete was incorporated into the program in order for the shotcrete to be applied somewhat uniformly over the surface. Using this number, the program will not apply shotcrete at any point where the slope will become too great. This is to account for proper application technique, where the nozzleman applies shotcrete in areas where there is minimal thickness.

All parameters can be varied in the program, but only four parameters were varied for analysis. Standard values were determined for the varying parameters. The standard spacing of the supports in both the vertical and horizontal directions was chosen to be 1ft. The standard support stiffness of the formwork was chosen to be 250 kip/ft. This value is equivalent to having a standard 4"x4" post 3ft long, angled at 30° to the formwork. The standard thickness of the formwork was determined to be 0.5in, which is as per standard construction practice. These four values were then varied independently in order to determine how each parameter affected the deflection and acceleration of the system.

### 3.3 STIFFNESS MATRIX OF FORMWORK

The stiffness matrix for the formwork was constructed in order to determine the deflections and accelerations over the formwork surface. The formwork was modeled as a continuous plate subdivided into finite elements. The finite points and areas along the formwork are labeled as shown in Figure 1. The finite element areas and points were labeled in this order, following standard matrix notation for easy referencing, but most importantly because of the ease in which the stiffness matrix for the plate could be compiled.

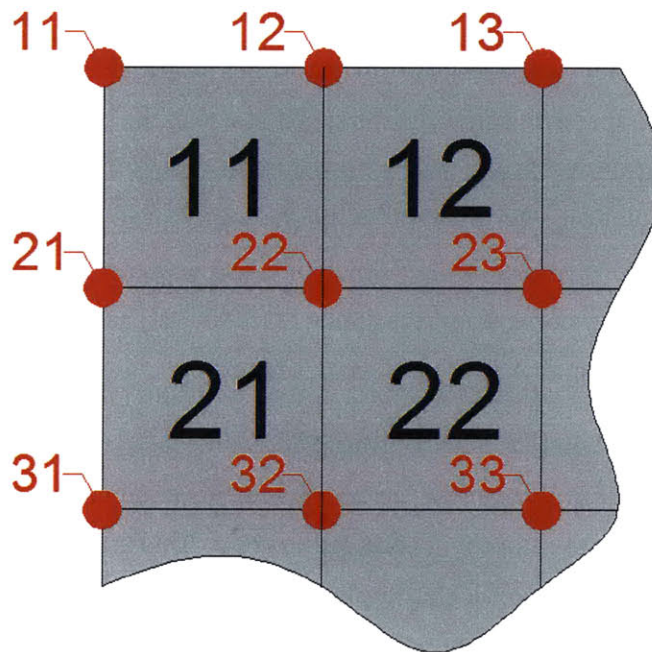


Figure 1 - Finite Labels

Each finite element has 4 corresponding nodes. Each node then has three degrees of freedom: translation in the z-axis and rotation about both the x and y axes, as shown in Figure 2. Using these three degrees of freedom the stiffness matrix for a single finite element was established. Each finite element was viewed as a four point plate element whose stiffness matrix was developed using *Theory of Matrix Structural Analysis*.<sup>1</sup>

<sup>1</sup> It is important to note that Przemieniecki labels the finite element points on a four point plate differently, which changes the arrangement of the stiffness matrix.

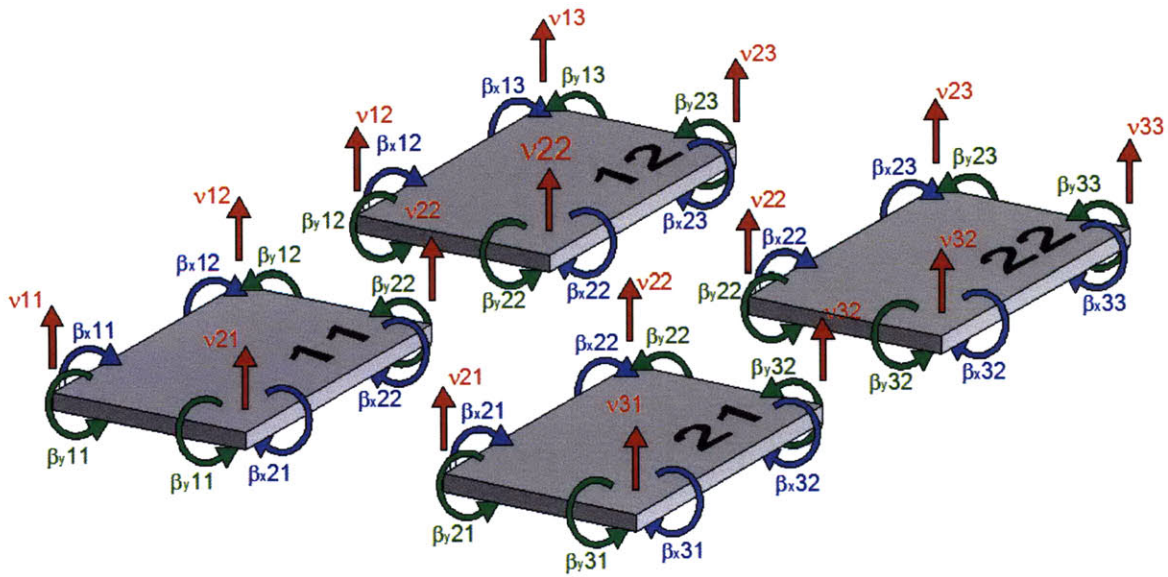


Figure 2 - DOF of Finite Points

In order for the rows and columns of the compiled stiffness matrix of the system to follow the progression of points as shown in Figure 3, the stiffness matrix for a single finite element would vary based on the number of points along the width of the formwork as a whole. This figure represents the structure of a plate consisting of 9 finite elements; 3 high by 3 wide, with 16 finite points, labeled as shown in Figure 1. Each box will consist of a 3x3 matrix corresponding to the 3 degrees of freedom for each finite point.

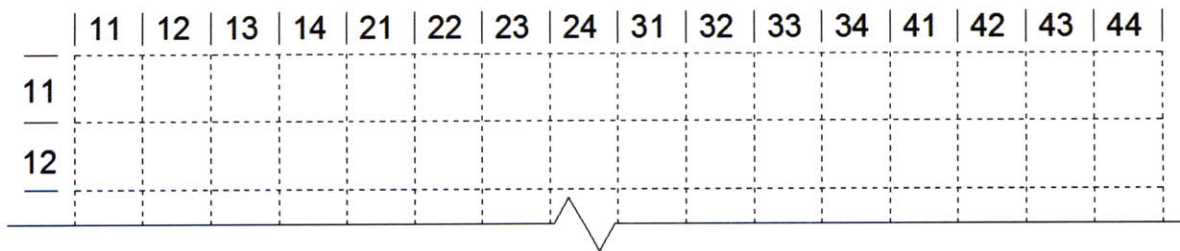
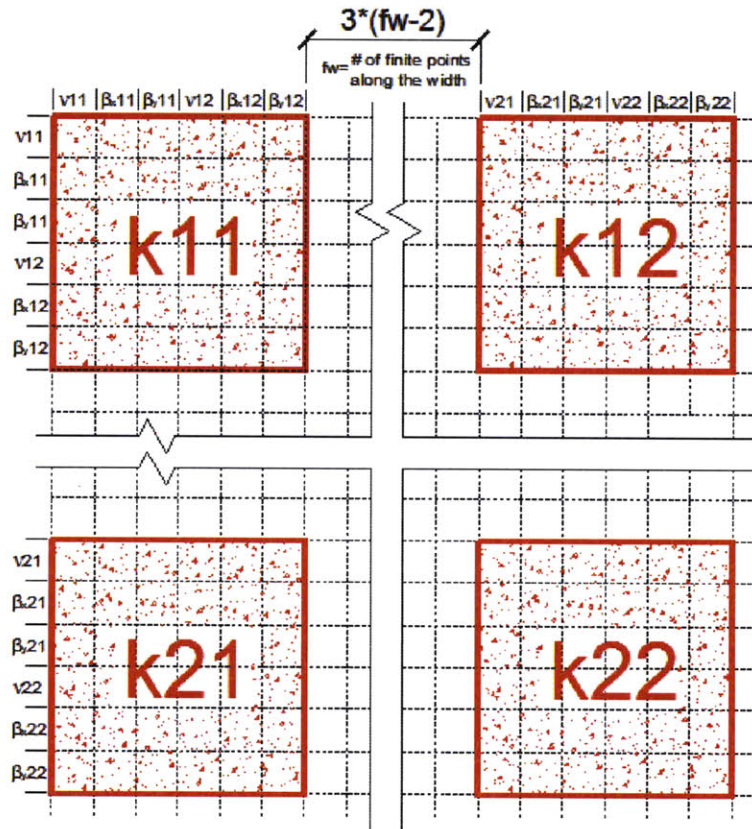


Figure 3 - Structure of a Compiled Matrix

The size of the stiffness matrix for a single finite element would need to vary to account for the number of rows and columns within the compiled stiffness matrix which separates corresponding points on a single finite element. This is shown in Figure 4, along with the relationship between this space and the number of finite element points along the entire width of the formwork.



**Figure 4 - Stiffness Matrix of Single Finite Element**

One of the advantages of developing a single stiffness matrix in this way is the ease in which the matrices compile to form the stiffness matrix of the formwork as a whole. Figure 5 shows the way in which stiffness matrices for single finite elements combine for a formwork divided into 4 finite elements; 2 high, by 2 wide. By looking at this figure it can be seen that the number for finite points along the width of the entire formwork is what determines the way in which the individual matrices are overlapped. An algorithm was developed in order to easily compile single stiffness matrices.

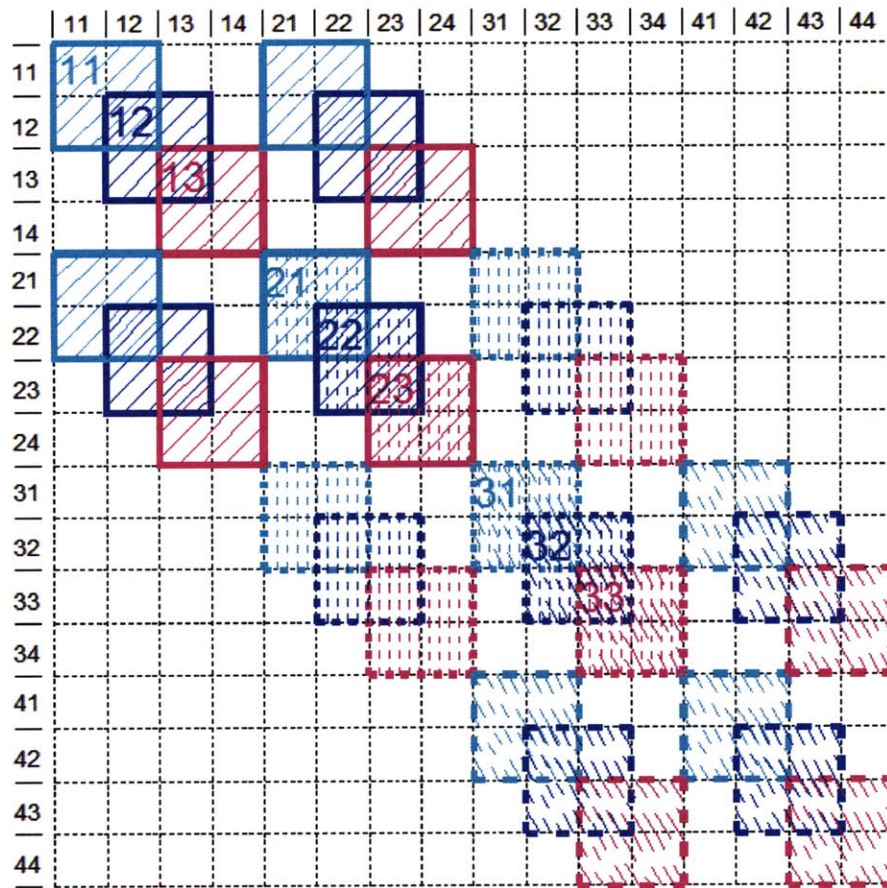


Figure 5 - Compiled Stiffness Matrix

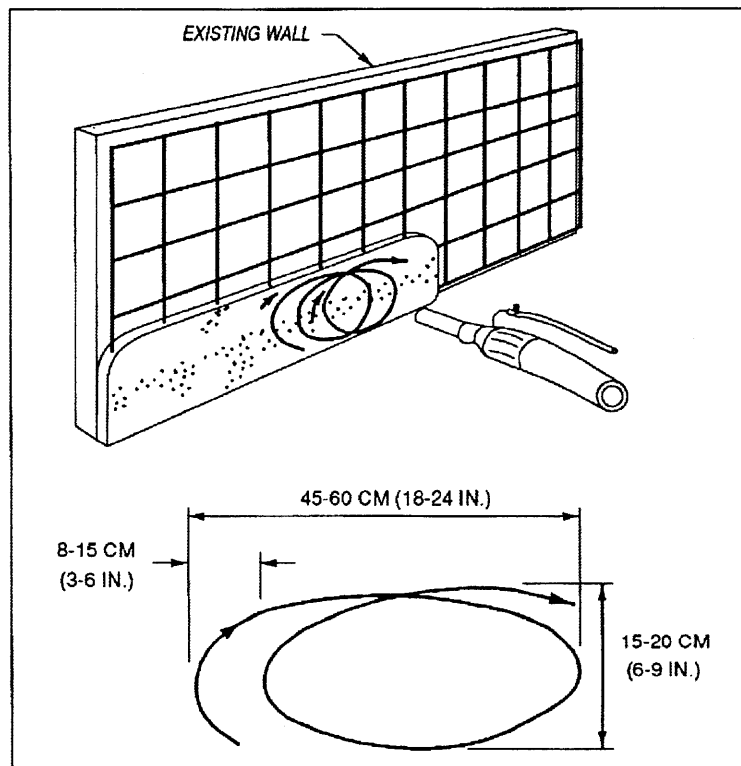
The compiled stiffness matrix was the stiffness matrix for the plate alone. The plate was modeled as supported by springs. This was done in order to replicate the way in which the formwork is supported in common construction practice. Therefore, at all locations where supports were specified to be located the stiffness of the supports ( $k_s$ ) was added to the compiled stiffness matrix. This allowed for the elasticity of the supports to be taken into account and also kept the stiffness matrix from needing to be reduced, which would have been the case if the supports were modeled as fixed.

Once the stiffness matrix was completed and the boundary conditions were specified a distributed load, determined based on the shotcrete application parameters, was applied to the formwork in order to display the deflection. The dynamic analysis, later preformed, was linear based on these small deflections.

### 3.4 APPLICATION OF SHOTCRETE

The application of shotcrete was simulated as per *Standard Practice for Shotcrete* specifications. An algorithm was developed so that shotcrete would be applied in an amount determined by the previously discussed parameters. The Shotcrete Application program can be found in Appendix I.

The shotcrete is applied at a random starting point and then follows an elliptical pattern which moves across the surface in random directions and with the dimensions of the ellipse varying, while remaining within the applications restraints shown in Figure 6.



**Figure 6 - Proper Nozzle Motion for Shotcrete Application**

*Courtesy of Standard Practice for Shotcrete*

Based on the parameters for how the shotcrete is applied to the surface two different time steps are established: time step for the change in location of the point of application ( $dta$ ) and the time step used for the state-space formulation ( $dt$ ). The simulations ran  $dt$  is one fourth of  $dta$ . Therefore, every time the location of the point of application varied four time steps were analyzed for the dynamic analysis.



For every time step (dta) the load matrix would change, while for every time step (dt) the mass matrix and thickness would change. The load on the formwork accounted for the force the formwork experienced based on the shotcrete application parameters. The mass and thickness matrices were also determined based on these parameters, but also accounted for 5% rebound of shotcrete, which lowered the amount of shotcrete that would adhere to the surface. The application of shotcrete to the surface of the formwork over time is shown in Figure 7.

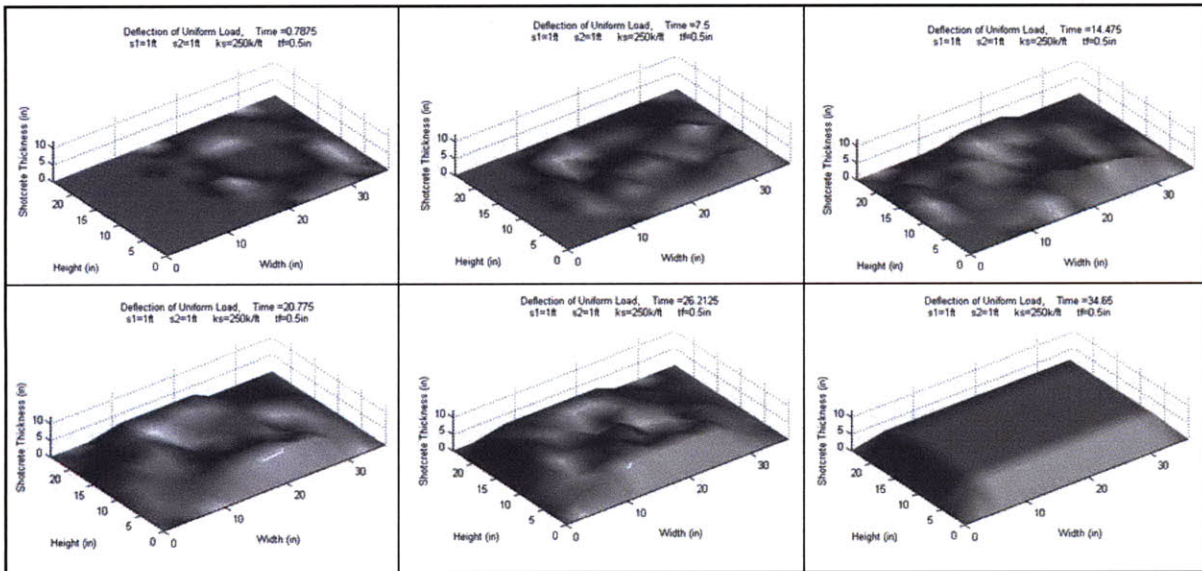


Figure 7 - Shotcrete Application

The random components of the application of the shotcrete are to account for the human component during application. Each time the Shotcrete Application file is run there is a different pattern to how the shotcrete is applied. Therefore, several simulations were run for each analysis case. While the distribution of the maximum displacement for each finite point varied for each simulation, the maximum displacement of the entire surface varied by only 2%. Therefore, in Appendix I, only a single simulation for each analysis case is displayed.

### 3.5 STATE-SPACE ANALYSIS

In order to determine the maximum displacement and accelerations of each finite point at each time step the state-space equation was used, (Connor). The state space formulation consists of 4 equations, which were discretized over the time step  $dt$ . Because there is no control of the system matrices  $C_{ss}$  and  $D_{ss}$  are constant over time.  $A_{ss}$  and  $B_{ss}$  varied over time based on the changing mass and load location matrix.

$$A_{ss} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \quad \text{size} = (2 \cdot \text{dof}) \times (2 \cdot \text{dof})$$

$$B_{ss} = \begin{bmatrix} 0 \\ M^{-1}E \end{bmatrix} \quad \text{size} = (2 \cdot \text{dof}) \times 1$$

$$C_{ss} = [I] \quad \text{size} = (2 \cdot \text{dof}) \times (2 \cdot \text{dof})$$

$$D_{ss} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix} \quad \text{size} = (2 \cdot \text{dof}) \times 1$$

These matrices were developed for each time step and used in the state space equation. The stiffness matrix (K) and the damping matrix (C) did not change with time. This followed the assumption that the wet shotcrete did not add stiffness to the system. The mass matrix (M) vary with every time step  $dt$ , while the load location matrix (E) varied with every time step  $dt$ . It is important to note that matrix  $E \cdot F$  is equal to the load matrix (P). The MATLAB program written for this analysis contained a loop varying time over time step  $dt$ , which output the displacement and acceleration of the formwork at every time step. This program can be found in Appendix I.

#### Equation 1 - State-Space Equation

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}E \end{bmatrix} F$$

The MATLAB program written for this analysis contained a loop varying time over time step  $dt$ , which output the displacement and acceleration of the formwork at every time step. This program can be found in Appendix I.

A uniform load was applied to the formwork in order to verify that the state space analysis properly determined the deflection. Figure 8 shows the deflection of the formwork at different time steps under a uniform distributed load. As discussed earlier, because of the magnitude of the deflections the system was linearly analyzed. These small deflections also resulted in the rotational inertia of the formwork to be neglected. Therefore, because of the zero values in the mass matrix, the pseudo inverse was used for the state-equation.

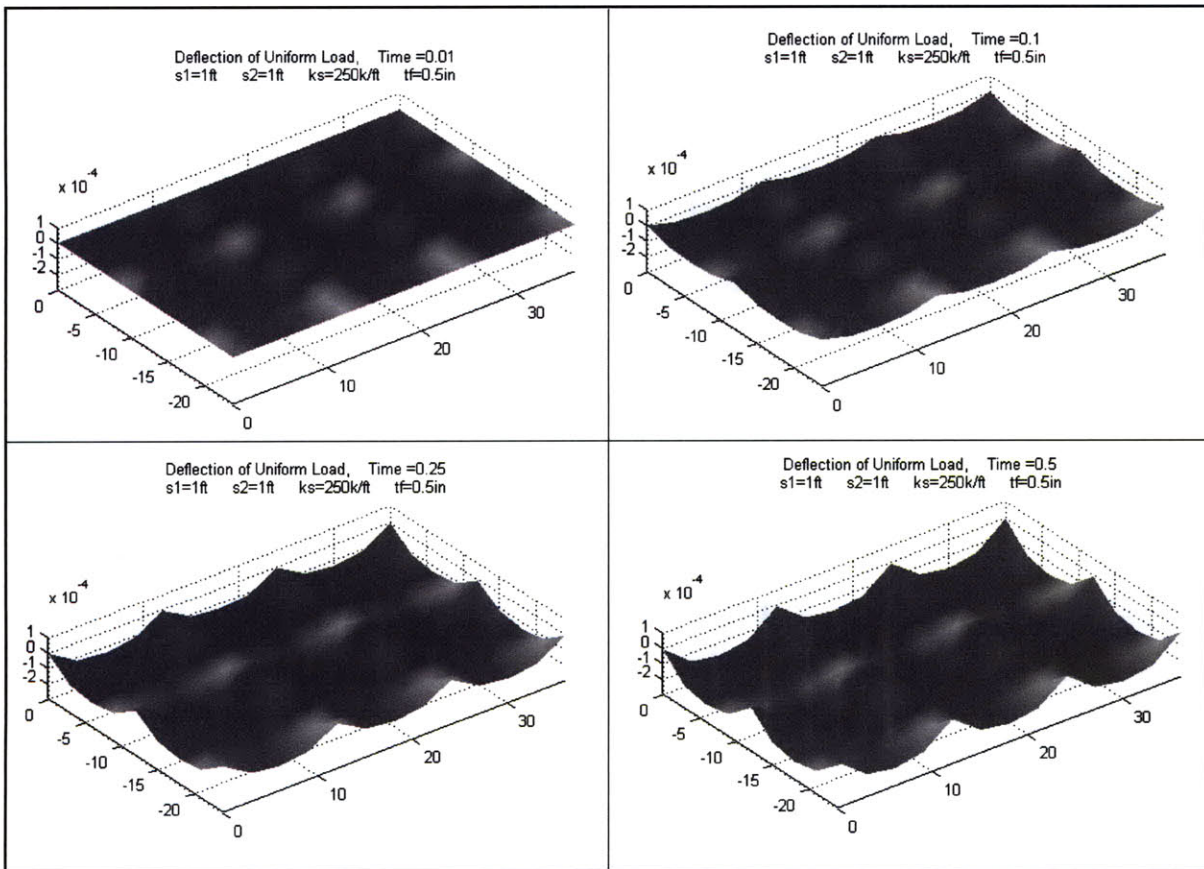


Figure 8 - State Space Deflection of Formwork

### 3.6 MASTER FILE

Once these three files were completed they were compiled in order to form a single Master File, which can be found in Appendix I. The program consists of a loop stepping time  $dt$  for a time of  $dta$  which varies the mass matrix, within a loop which varies the point of application of the force. In other words, the state space program is embedded within the Shotcrete Application program, both of which follow the Stiffness Matrix program. Figure 9 shows a flow chart of the Master File utilizing the three different programs.

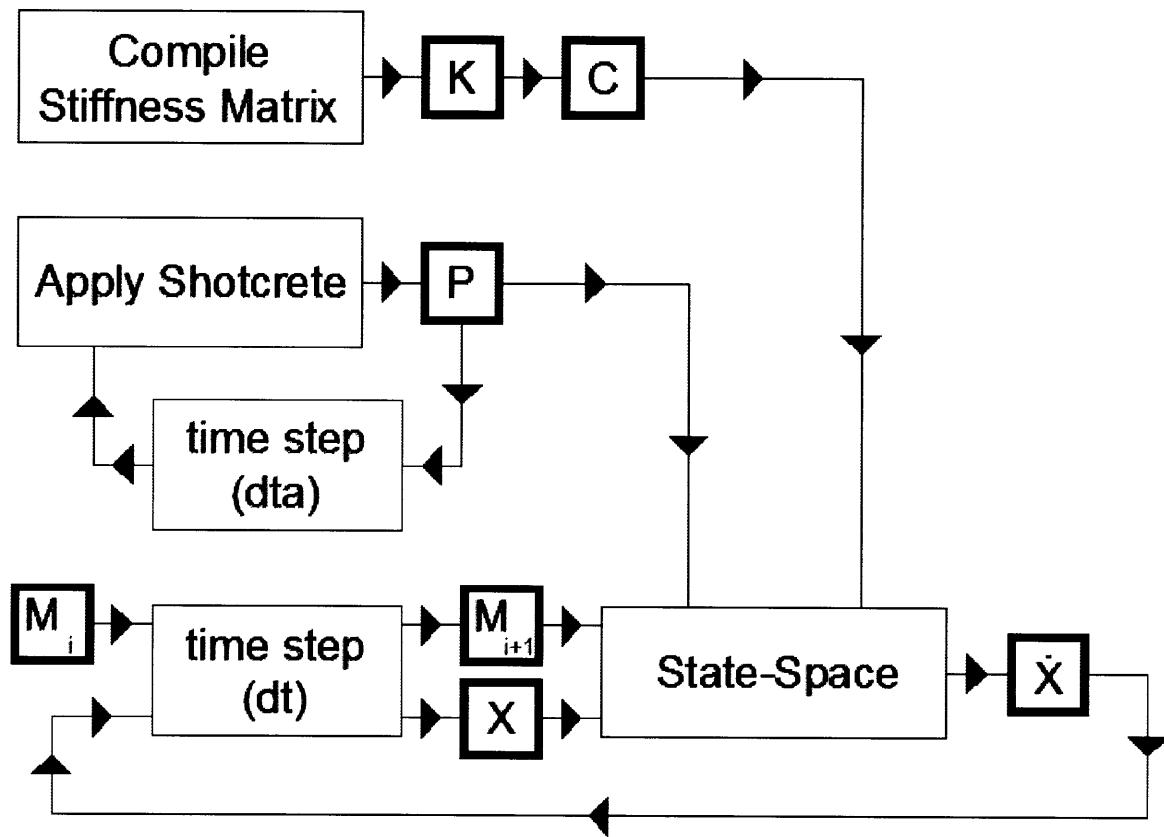


Figure 9 - Master File Flow Chart

### 3.7 SIMULATIONS

The master file produces matrices which represent the displacement, velocity and acceleration for each degree of freedom for the entire system and every time step (dt). Because the formwork is a three dimensional object the Master File has the option to display the application of the shotcrete or the displacement of the formwork as a video, where the image varies over time.

In order to sort out all of the different values a Solution Manipulation program was created which sorts through every displacement and acceleration matrix. The maximum deflection of each point is stored and an image is produced. This was also done with the accelerations. These images for each simulation run can be found in Appendix II.

Each image represents the maximum displacement or acceleration for each finite point of the system. Each plot was then studied in order to determine the maximum value of displacement and acceleration for the entire formwork surface. These values are shown in the Results section of the report.

Maintaining the set parameters each varying parameter was changed independently in order to determine which parameter had the greatest affect on the maximum displacement and acceleration of the formwork. The range over which each parameter was varied is shown in Table 5.

**Table 5 - Varying Parameters**

Simulation		Varying Parameter	
<b>A</b>	<b>1</b>	<i>s1 (ft)</i>	0.5
	<b>2</b>		1
	<b>3</b>		2
<b>B</b>	<b>1</b>	<i>Avg(s1,s2) (ft)</i>	0.5
	<b>2</b>		1
	<b>3</b>		1.75
<b>C</b>	<b>1</b>	<i>ks (kip/ft)</i>	150
	<b>2</b>		250
	<b>3</b>		350
<b>D</b>	<b>1</b>	<i>tf (in)</i>	0.25
	<b>2</b>		0.5
	<b>3</b>		1



# 4. RESULTS

## 4.1 SIMULATION

The maximum displacement and acceleration for each simulation can be found in Appendix II. Table 6 shows the varying parameters for each analysis case and the resulting maximum displacement and acceleration for the entire formwork surface. As discussed earlier, because of the random components of the shotcrete application program the simulation was run several times, with the maximum deflection over the entire formwork varying by 2%.

**Table 6 - Results**

Simulation		Varying Parameter		Maximum Displacement $\times 10^{-5}$ (in)	Maximum Acceleration $\times 10^{-3}$ (in/s <sup>2</sup> )
<b>A</b>	<b>1</b>	<i>s1</i> (ft)	0.5	1.6	6
	<b>2</b>		1	2.5	6.5
	<b>3</b>		2	6	6.5
<b>B</b>	<b>1</b>	<i>Avg(s1,s2)</i> (ft)	0.5	1.5	6
	<b>2</b>		1	2.5	6.5
	<b>3</b>		1.75	4	7
<b>C</b>	<b>1</b>	<i>ks</i> (kip/ft)	150	2.5	6
	<b>2</b>		250	2.5	6.5
	<b>3</b>		350	2.5	6
<b>D</b>	<b>1</b>	<i>tf</i> (in)	0.25	3	11
	<b>2</b>		0.5	2.5	6.5
	<b>3</b>		1	0.14	1.1

\*unless specified - s1 = 1 ft, s2 = 1 ft, ks = 250 kip/ft, tf = 0.5 in

## 4.2 MAXIMUM DISPLACEMENT

The resulting displacements for the varying parameters are displayed in Figure 10. By decreasing the spacing of the vertical, or both the vertical and horizontal supports the maximum deflection decreased. However, the displacement having both spacing at 6in was the same as only having the vertical spacing at 6in with the horizontal at 1ft. This shows that below a certain point it is not advantageous to reduce both the vertical and horizontal spacing.

As the stiffness of the supports increased there was almost zero noticeable difference in the maximum displacement. The stiffness of the supports has very little impact on the maximum displacement of the formwork.

The thickness of the formwork significantly impacted the maximum deflection. As the thickness of the formwork increased the maximum displacement decreased. The change of the vertical spacing of the supports had a more drastic impact of the maximum deflection. However, within reasonable values, the thickness of the formwork has the greatest potential to lower the maximum displacement.

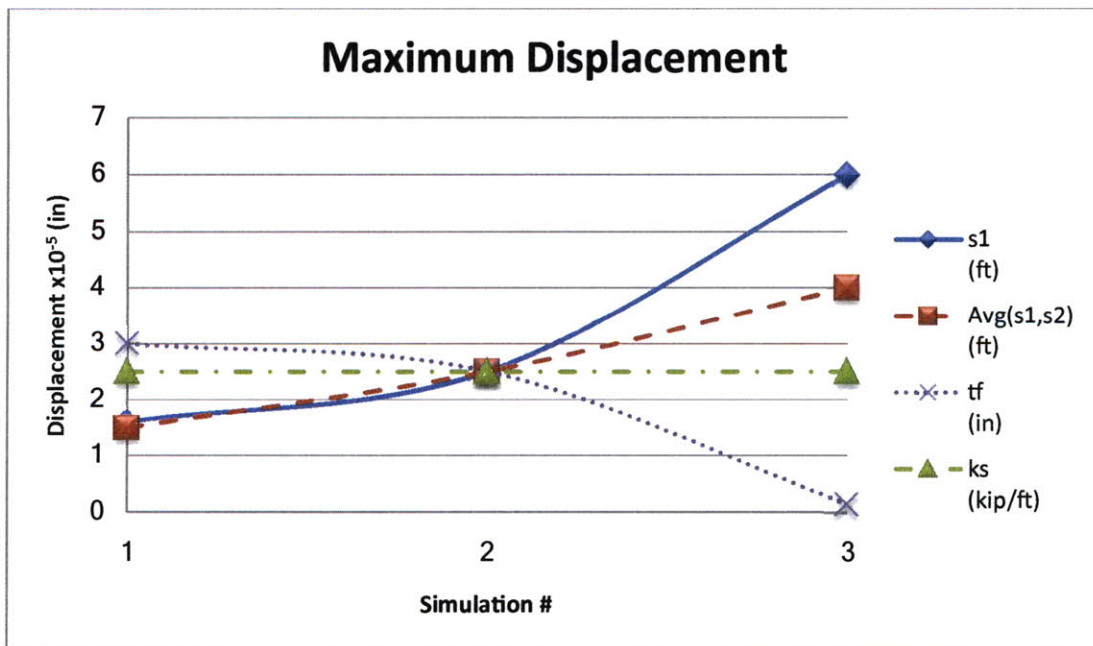


Figure 10 - Maximum Displacement



It is also important to look at, the maximum displacements of each finite point, which is what is shown in Appendix II.

When the vertical spacing of the supports was small there was a small area which had significant maximum displacements. As the vertical spacing of the supports increased these areas increased. At the same time the areas with almost zero displacement increased. As the supports were moved closer together the max displacement had the most drastic changes over the surface of the formwork.

As both the vertical and horizontal spacing changed the area with displacement close to the maximum varied almost linearly. This was also the case with the thickness of the formwork, while the support stiffness had almost zero affect of the displacement distributions.

### 4.3 MAXIMUM ACCELERATION

The resulting accelerations for the varying parameters are displayed in Figure 11. The only varied parameter that had a significant effect on the maximum acceleration of the formwork was the thickness. As the thickness of the formwork increased the maximum acceleration decreased. Varying the spacing of the support and the stiffness of the supports had almost no effect on the maximum acceleration.

By varying the thickness of the formwork the distribution of maximum acceleration was greatly affected. By decreasing the thickness of the formwork, the area which experienced maximum accelerations greatly increased. Increasing the thickness of the formwork resulted in very few points on the formwork experiences values similar to the maximum acceleration of the entire area. This could be caused by the increased stiffness, the increased mass, or a combination of the two.

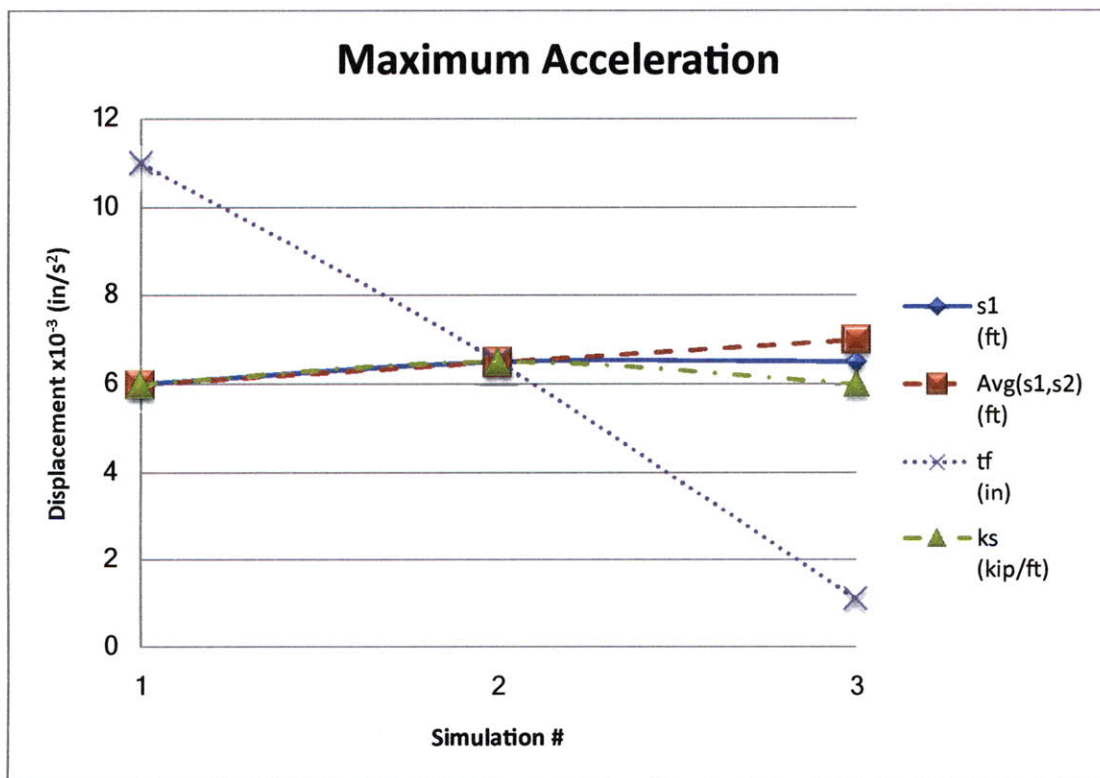


Figure 11 - Maximum Acceleration

The purpose of these simulations was not to find the value of the parameters required for a target displacement or acceleration, but to find what parameters had the greatest impact on displacement and acceleration within a reasonable range for standard practice. The thickness of formwork had the greatest impact on both deflection and acceleration and was able to reduce these values much more significantly than the other parameters tested.



## 5. CONCLUSION

### *5.1 ACCURACY OF SIMULATION*

After accounting for the random components of the shotcrete application, the results from several simulations, varied by only 2%, while the same parameters were maintained.

This proves that while application is random the maximum values of displacement and acceleration that the formwork would experience varies minimally. Only the distribution of the displacements and accelerations that individual finite points experience will vary. Because the formwork will be uniform and the exact pattern of application will not be predetermined only the maximum values of the entire surface should be considered in the design of the formwork system.

This shows that the pattern in which the shotcrete is applied can vary and the program will still provide precise results, if the application follows the guidelines set out by *Standard Practice for Shotcrete*. Therefore, the program reasonably predicts how the formwork would behave during standard shotcrete application.

### *5.2 MAXIMUM DEFLECTION/ACCELERATION*

Based on the parameters which were analyzed and the range over which each parameter was varied it was found that the thickness of the formwork had the greatest impact on the maximum displacement and acceleration of the formwork.

The spacing of the supports impacts the maximum displacement of the formwork, but has little effect on the acceleration. Within reasonable values, the spacing of the supports does not control the acceleration and displacement of the system, as well as the thickness of the formwork.

By reducing the displacement and acceleration the amount of rebound will be reduced, resulting in a more efficient application process (Bindiganavile). In addition, the reduced deflections and accelerations will help to prevent delamination of the formwork and the shotcrete before the shotcrete has cured (Zynda, 34). By reducing the vibration of the system the shotcrete will also be more compact, resulting in a higher and more uniform strength. This will allow for shotcrete formwork to be used on a much larger scale than in common practice.

### *5.3 SUGGESTIONS FOR FUTURE RESEARCH*

The most important future research for this topic would be physical testing of the results of this program. A model of the formwork should be constructed and the shotcrete should be applied in a pattern specified by the program in order to determine whether or not the results are accurate. It was found by repeated analysis that the program provides precision, based on percent difference of the result from repeated simulations. However, the accuracy of the program cannot be determined without physical testing.

In order to obtain a better understanding of the behavior of shotcrete the varied parameters should be studied over a larger range of values. This will more accurately determine the relationship between each parameter and the maximum displacements and accelerations. In addition, the simulation should be run on a larger scale of formwork. This would more accurately replicate common shotcrete practice and could allow for an optimal support spacing to be determined.

The varied parameters should also be varied throughout the formwork. In other words, the stiffness of the supports should decrease with elevation, while the spacing of the supports also decreases. This would provide an understanding of how shotcrete formwork could be used for large scale applications.

In the simulations run, the supports were analyzed as being pin connected to points on the surface of the formwork. In reality the supports would provide some moment connection and would most likely be attached at more than one point. This should be accounted for in further research in order to more accurately model the system. Other stiffening devices should also be modeled, such as beams spanning between supports. This would change the stiffness and therefore, the response of the system.

For the simulations carried out, the application of the shotcrete was determined by *Standard Practice for Shotcrete*. In further research the pattern in which the shotcrete is applied should be varied in order to determine if there is an optimal pattern for application. In addition, application to both sides of the formwork should be studied in order to account for imbedded formwork.

Increasing the thickness of the formwork increases the stiffness of the formwork and also increases the initial mass. Therefore, future research should test both the stiffness and mass of the formwork in order to determine the effect that each individual parameter has on the deflection and acceleration. This will help to determine what material would work best for the formwork. If the

stiffness is the leading factor affecting deflections, different materials with higher elasticity values could be more beneficial. On the other hand if the initial mass of the system is the leading factor, denser material should be researched, or a base layer of shotcrete should be applied to increase the mass of the formwork receiving the load. The study of additional materials can also open up the possibility of using thermal or acoustic panels as the formwork for shotcrete.





## 6. REFERENCES

1. Hurd, M. *Formwork for Concrete 7th edition*. 7th ed. Annapolis: Amer Concrete Institute, 2005. *Google Books*. Web. 24 Nov. 2009.  
<<http://books.google.com/books?id=0G9n7dfvNEC&printsec=frontcover#v=onepage&q=&f=false>>.
2. Lamond, Joseph F., and James H. Pielert. *Significance of Tests and Properties of Concrete*. West Conshohocken: ASTM International, 2006. *Google Books*. Web. 24 Nov. 2009.  
<<http://books.google.com/books?id=isTMHD6yIy8C&pg=PT613&dq=shotcrete#v=onepage&q=shotcrete&f=false>>.
3. *Shotcrete for Underground Support*. Vol. VII. New York: American Society of Civil Engineers, 1995. Print.
4. *Shotcrete for Underground Support*. Vol. X. New York: American Society of Civil Engineers, 2006. Print.
5. Austin, S. A., and P. J. Robins. *Sprayed Concrete: Properties, Design, and Application*. Latheronwheel, Caithness, Scotland: Whittles Pub., 1995. Print.
6. *Standard Practice for Shotcrete*. New York, N.Y.: ASCE, 1995. Print.
7. Renner-Smith, Susan. "Sprayed-Concrete Sandwich." *Popular Science* May (1961): 67. Web.
8. Zynda, Chris. "Safety Shooter." *Shotcrete* Summer (2009): 34. Web.
9. Bindiganavile, Vivek, and Nemkumar Banthia. "Effect of Particle Density on Its Rebound in Dry-Mix Shotcrete." *JOURNAL OF MATERIALS IN CIVIL ENGINEERING* February (2009): 58-64. Web.
10. Przemieniecki, J. S. "5.12." *Theory of Matrix Structural Analysis*. New York: McGraw-Hill, 1968. 115-22. Print.
11. Connor, Jerome J. *Introduction to Structural Motion Control*. New Jersey: Prentice Hall, 2002. Print.



## 7. APPENDIX I

### *MATLAB PROGRAM*

## 7.1 PARAMETERS FOR ANALYSIS

```
%Set Parameters
```

```
h = 2;           %Formwork Height (ft)
w = 3;           %Formwork Width (ft)
c = 0.01;        %Damping Ratio
E = 1001000;     %Formwork Surface Modulus of Elasticity (psi)
rof = 43.7;      %Formwork Density (lbs/ft3)
Qs = 4.5;        %Shotcrete Spray Speed (ft3/min) ==> (10yd3/h)
vs = 20;         %Shotcrete Application Speed on Contact (in/s)
ds = 3;          %Shotcrete Spray Diameter on Contact (in)
ros = 150;       %Shotcrete Density (lbs/ft3)
ts = 6;          %Desired Shotcrete Thickness (in)
s = 10;          %Maximum slope
v = 0.22;        %Poisson's Ratio
```

```
%Varied Parameters
```

```
s1 = 1;          %Vertical Spacing of Supports (ft)
s2 = 1;          %Horizontal Spacing of Supports (ft)
ks = 250000;     %Support Stiffness (lbs/ft)
tf = 0.5;        %Formwork Surface Thickness (in)
```

```
sim_time = 600;
```

```
%Calculated Parameters
```

```
wf = rof*tf/12;           %Weight of Formwork (lbs/ft2)
a = ds;                   %Finite Element Height & Width (in)
fh = h*12/a+1;           %Finite Element Points Over Total Height
fw = w*12/a+1;           %Finite Element Points Over Total Width
dta = 1/vs*a;            %Application Time Step (s)
dt = dta/4;              %Time Step (s)
steps = sim_time/dt;     %Number of Time Steps
dts = 0.95*Qs*dt/(a^2)*(12^3)/60; %Change In Thickness of Concrete per Time Step(in)
```

## 7.2 DEVELOPING STIFFNESS MATRIX FOR PLATE ON SPRINGS

```

clear all
clc

%%
%Parameters

h = 2;           %Formwork Height (ft)
w = 3;           %Formwork Width (ft)
s1 = 1;          %Vertical Spacing of Supports (ft)
s2 = 1;          %Horizontal Spacing of Supports (ft)
ks = 250000;     %Support Stiffness (lbs/ft)
c = 0.01;        %Damping Ratio
E = 1001000;    %Formwork Surface Modulus of Elasticity (psi)
tf = 0.5;        %Formwork Surface Thickness (in)
rof = 43.7;     %Formwork Density (lbs/ft3)
Qs = 4.5;        %Shotcrete Spray Speed (ft3/min) ==> (10yd3/h)
ds = 3;          %Shotcrete Spray Diameter on Contact (in)
ros = 150;      %Shotcrete Density (lbs/ft3)
vs = 20;        %Shotcrete Application Speed on Contact (in/s)
ts = 6;         %Desired Shotcrete Thickness (in)
s = 10;         %Maximum slope
v = 0.22;       %Poisson's Ratio

sim_time = 600;

wf = rof*tf/12;  %Weight of Formwork (lbs/ft2)
a = ds;          %Finite Element Height & Width (in)
fh = h*12/a+1;  %Finite Element Points Over Total Height
fw = w*12/a+1;  %Finite Element Points Over Total Width
dta = 1/vs*a;   %Application Time Step (s)
dt = dta/4;     %Time Step (s)
steps = sim_time/dt; %Number of Time Steps
dts = Qs*dt/(a^2)*(12^3)/60; %Change In Thickness of Concrete per Time Step(in)

j = 0;
for i=0:1:w*12/a
    j = j+1;
    x(j) = i*a;
end
j = 0;
for i=0:1:h*12/a
    j = j+1;
    y(j) = i*a;
end

```

$$\frac{EA}{L}$$

Stiffness Matrix for Single Finite Element

```

k11 = [8+(14-4*v)/5      (2+(1+4*v)/5)*a      -(2+(1+4*v)/5)*a      -2-(14-4*v)/5      (2+(1-v)/5)*a      (-1+(1+4*v)/5)*a;
      (2+(1+4*v)/5)*a    (4/3+4/15*(1-v))*a^2    -v*a^2              -(2+(1-v)/5)*a    (2/3-(1-v)/15)*a^2    0;
      -(2+(1+4*v)/5)*a    -v*a^2              (4/3+4/15*(1-v))*a^2  (-1+(1+4*v)/5)*a    0                    (2/3-4/15*(1-v))*a^2;
      -2-(14-4*v)/5      -(2+(1-v)/5)*a      (-1+(1+4*v)/5)*a      8+(14-4*v)/5      -(2+(1+4*v)/5)*a      -(2+(1+4*v)/5)*a;
      (2+(1-v)/5)*a      (2/3-(1-v)/15)*a^2  0                    -(2+(1+4*v)/5)*a  (4/3+4/15*(1-v))*a^2  v*a^2;
      (-1+(1+4*v)/5)*a    0                    (2/3-4/15*(1-v))*a^2  -(2+(1+4*v)/5)*a  v*a^2              (4/3+4/15*(1-v))*a^2];

k21 = [-2-(14-4*v)/5      (1-(1+4*v)/5)*a      (2+(1-v)/5)*a      -4+(14-4*v)/5      (1-(1-v)/5)*a      (1-(1-v)/5)*a;
      (1-(1+4*v)/5)*a      (2/3-4/15*(1-v))*a^2  0                    (-1+(1-v)/5)*a      (1/3+(1-v)/15)*a^2    0;
      -(2+(1-v)/5)*a      0                    (2/3-(1-v)/15)*a^2  (-1+(1-v)/5)*a      0                    (1/3+(1-v)/15)*a^2;
      -4+(14-4*v)/5      (-1+(1-v)/5)*a      (1-(1-v)/5)*a      -2-(14-4*v)/5      (-1+(1+4*v)/5)*a      (2+(1-v)/5)*a;
      (1-(1-v)/5)*a      (1/3+(1-v)/15)*a^2  0                    (-1+(1+4*v)/5)*a      (2/3-4/15*(1-v))*a^2  0;
      (-1+(1-v)/5)*a      0                    (1/3+(1-v)/15)*a^2  -(2+(1-v)/5)*a      0                    (2/3-(1-v)/15)*a^2];

k12 = k21';

k22 = [8+(14-4*v)/5      (2+(1+4*v)/5)*a      (2+(1+4*v)/5)*a      -2-(14-4*v)/5      (2+(1-v)/5)*a      (1-(1+4*v)/5)*a;
      (2+(1+4*v)/5)*a      (4/3+4/15*(1-v))*a^2  v*a^2              -(2+(1-v)/5)*a      (2/3-(1-v)/15)*a^2    0;
      (2+(1+4*v)/5)*a      v*a^2              (4/3+4/15*(1-v))*a^2  (1-(1+4*v)/5)*a      0                    (2/3-4/15*(1-v))*a^2;
      -2-(14-4*v)/5      -(2+(1-v)/5)*a      (1-(1+4*v)/5)*a      8+(14-4*v)/5      -(2+(1+4*v)/5)*a      (2+(1+4*v)/5)*a;
      (2+(1-v)/5)*a      (2/3-(1-v)/15)*a^2  0                    -(2+(1+4*v)/5)*a      (4/3+4/15*(1-v))*a^2  -v*a^2;
      (1-(1+4*v)/5)*a      0                    (2/3-4/15*(1-v))*a^2  (2+(1+4*v)/5)*a      -v*a^2              (4/3+4/15*(1-v))*a^2];

```

```

%%

%Compile k Matrix
ksize = 12+3*(fw-2);
k = zeros(ksize,ksize);
k(1:6,1:6) = k11;
k(7+3*(fw-2):12+3*(fw-2),1:6) = k21;
k(1:6,7+3*(fw-2):12+3*(fw-2)) = k12;
k(7+3*(fw-2):12+3*(fw-2),7+3*(fw-2):12+3*(fw-2)) = k22;
k = E*tf^3/(12*(1-v^2)*a^2)*k;

%%
%Compile Stiffness (K) Matrix

Ksize = ((fw-1)*(fh-1)+(fh-1)-1)*3+ksize-3;
K = zeros(Ksize);

j = 0;
for(i=0.5:1+1/(fw-1):(fw-1)*(fh-1)+(fh-1))
    j = round(i)*3-3+1;
    K(j:j+ksize-1,j:j+ksize-1) = K(j:j+ksize-1,j:j+ksize-1) + k;
end

%%
%Uniformly Distributed Load
P = zeros(Ksize,1);
for i=1:3:Ksize
    P(i)=- (ros*Qs) * (Qs*12^2/a^2)/32.2/3600;
end

%%
%Set Constraints
for i=1:s1*12/a:h*12/a+1
    for j=1:s2*12/a:w*12/a+1
        ij = (3*((i-1)*(fw)+j)-2);
        K(ij,ij)=K(ij,ij)+ks;           %Apply Springs to Formwork
    end
end

%%
%Solve U Matrix
U = K\P;

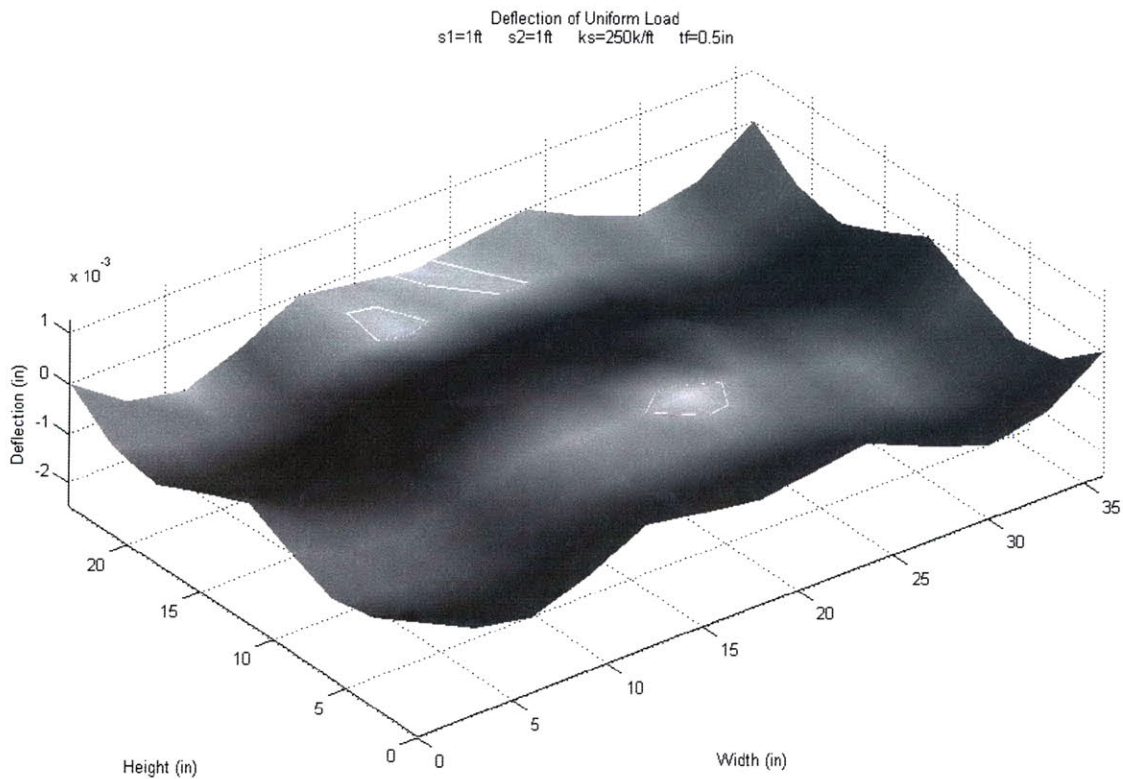
%Create Displacement Vector
j=1;
for i=1:3:Ksize
    u(j,1) = U(i);
    j=j+1;
end

```

## MATLAB Program

```
%Create Displacement Matrix
uM = zeros(fh,fw);
l = 1;
for i=1:1:fh
    for j=1:1:fw
        uM(i,j) = u(l);
        l=l+1;
    end
end

zmin = min(transpose(min(uM)));
surf1(x,y,uM);
shading interp;
colormap(gray);
axis([0 w*12 0 h*12 2*zmin -zmin])
daspect([max(w,h) max(w,h) -zmin])
title({'Deflection of Uniform Load', ['s1=',num2str(s1),'ft      s2=',
    num2str(s2),'ft      ks=',num2str(ks/1000),'k/ft      tf=',
    num2str(tf),'in']});
```





## 7.3 APPLY SHOTCRETE TO FORMWORK

```

%%
%Develop Dimension Vectors

j = 0;
for i=0:1:w*12/a
    j = j+1;
    x(j) = i*a;
end
j = 0;
for i=0:1:h*12/a
    j = j+1;
    y(j) = i*a;
end

%%
%Apply Shotcrete

z = zeros(h*12/a+1,w*12/a+1); %Initial Shotcrete Thickness (Zero)
m = wf*1*ones(h*12/a+1,w*12/a+1); %Initial Mass (Mass of Formwork)

r1 = int8((w*11/a)*rand(1)+1); %Random Start Point
r2 = int8((h*11/a)*rand(1)+1); %Random Start Point
t(1) = 0; %Start Time (s)
k = 2;

hw = waitbar(0,'Progress');
l=1;
l1=1;
for i=0:1:sim_time/dta
    waitbar(i/(sim_time/dta),hw);
    if(z(r2,r1)<ts)
        j=0;
        %Edge Conditions
        if(r1==w*12/a+1),r1=w*12/a;end
        if(r1==1),r1=2;end
        if(r2==h*12/a+1),r2=h*12/a;end
        if(r2==1),r2=2;end
        %Slope Conditions
        if((z(r2,r1)<z(r2+1,r1+1)+s)&&(z(r2,r1)>z(r2+1,r1+1)-s)),j=j+1;end
        if((z(r2,r1)<z(r2+1,r1)+s)&&(z(r2,r1)>z(r2+1,r1)-s)),j=j+1;end
        if((z(r2,r1)<z(r2+1,r1-1)+s)&&(z(r2,r1)>z(r2+1,r1-1)-s)),j=j+1;end
        if((z(r2,r1)<z(r2,r1+1)+s)&&(z(r2,r1)>z(r2,r1+1)-s)),j=j+1;end
        if((z(r2,r1)<z(r2,r1)+s)&&(z(r2,r1)>z(r2,r1)-s)),j=j+1;end
        if((z(r2,r1)<z(r2,r1-1)+s)&&(z(r2,r1)>z(r2,r1-1)-s)),j=j+1;end
        if((z(r2,r1)<z(r2-1,r1+1)+s)&&(z(r2,r1)>z(r2-1,r1+1)-s)),j=j+1;end
        if((z(r2,r1)<z(r2-1,r1)+s)&&(z(r2,r1)>z(r2-1,r1)-s)),j=j+1;end
        if((z(r2,r1)<z(r2-1,r1-1)+s)&&(z(r2,r1)>z(r2-1,r1-1)-s)),j=j+1;end
    end
end

```

```

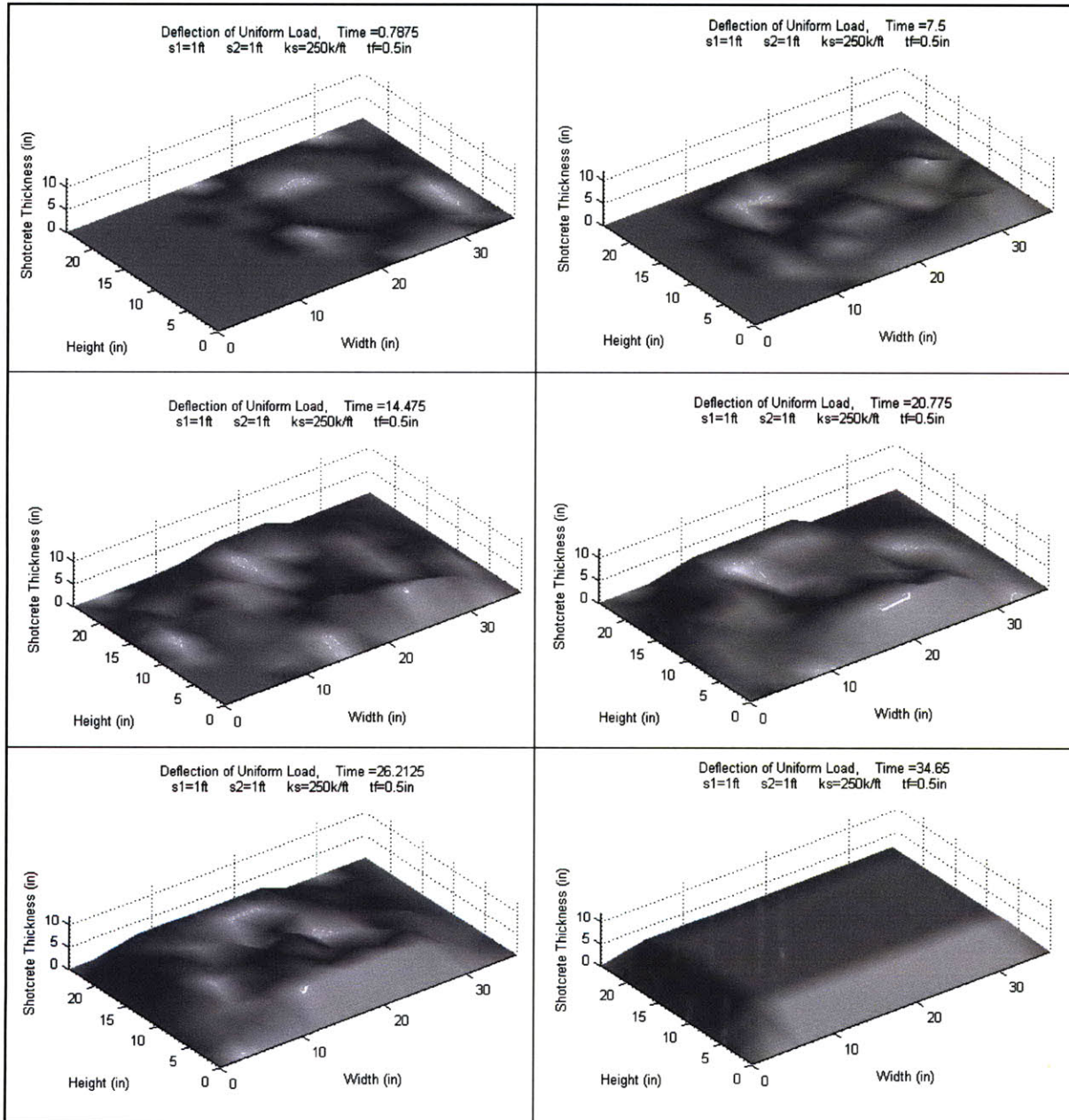
    if(j>=5&&(z(r2,r1)<ts))
        z(r2,r1)=z(r2,r1)+dts;           %Thickness (in)
        P = zeros(h*12/a+1,w*12/a+1);
        P(r2,r1) = (ros*Qs)*(Qs*12^2/a^2)/32.2; %Load (lbs)
        m(r2,r1) = m(r2,r1)+ros*Qs*dt; %Mass (lbs)
        t(k) = t(k-1)+dt; %Time (s)
        k = k+1;

        %Display Application of Shotcrete
        surf(x,y,z);
        shading interp;
        colormap(gray);
        axis([0 w*12 0 h*12 0 ts*2])
        daspect([max(w,h) max(w,h) ts])
        F = getframe;
    end
end

%Choose next point
if (l1>6/a)
    r3 = int8(8*rand(1));
    if(r3==0),r1=r1-1;r2=r2-1;end
    if(r3==1),r1=r1-1;end
    if(r3==2),r1=r1-1;r2=r2+1;end
    if(r3==3),r2=r2-1;end
    if(r3==5),r2=r2+1;end
    if(r3==6),r1=r1+1;r2=r2-1;end
    if(r3==7),r1=r1+1;end
    if(r3==8),r1=r1+1;r2=r2+1;end
    if (l<8)
        l=l+1;
    else
        l=(1/8);
    end
    end
    l1=1;
else
    l1=l1+1;
    if(l==1),r1=r1-1;end
    if(l==2),r1=r1-1;r2=r2+1;end
    if(l==3),r2=r2+1;end
    if(l==4),r1=r1+1;r2=r2+1;end
    if(l==5),r1=r1+1;end
    if(l==6),r1=r1+1;r2=r2-1;end
    if(l==7),r2=r2-1;end
    if(l==8),r1=r1-1;r2=r2-1;end
end

end
close(hw)

```



## 7.4 STATE-SPACE FILE

```

%%
%Set Constraints

for i=1:s1*12/a:fh
    for j=1:s2*12/a:w*12/a+1
        ij = (3*((i-1)*(fw)+j)-2);
        K(ij,ij)=K(ij,ij)+ks;           %Apply Springs to Formwork
    end
end

M = zeros(Ksize,Ksize);
for i=1:3:Ksize
    M(i,i) = wf*a^2;                   %Initial Mass Matrix (Mass of Formwork)
end

C = 0.2*K;

%%
%Uniformly Distributed Load

P = zeros(Ksize,1);
for i=1:3:Ksize
    P(i)=- (ros*Qs) * (Qs*12^2/a^2)/32.2/3600;
end

pinvM = pinv(M);
A_ss = [zeros(dof) eye(dof); -pinvM*K -pinvM*C];
B_ss = [zeros(dof,1); pinvM*P];
C_ss = eye(dof*2);
D_ss = zeros(dof*2,1);

%%
%Time Definitions
dt = 0.01;
sim_time = 1;
steps = sim_time/dt;
t = 0:dt:sim_time;

%Conversion to Discrete
[A_ss,B_ss,C_ss,D_ss]=c2dm(A_ss,B_ss,C_ss,D_ss,dt);

%Initial Condition
X=zeros(dof*2,steps);
Xd=zeros(dof*2,steps);

```

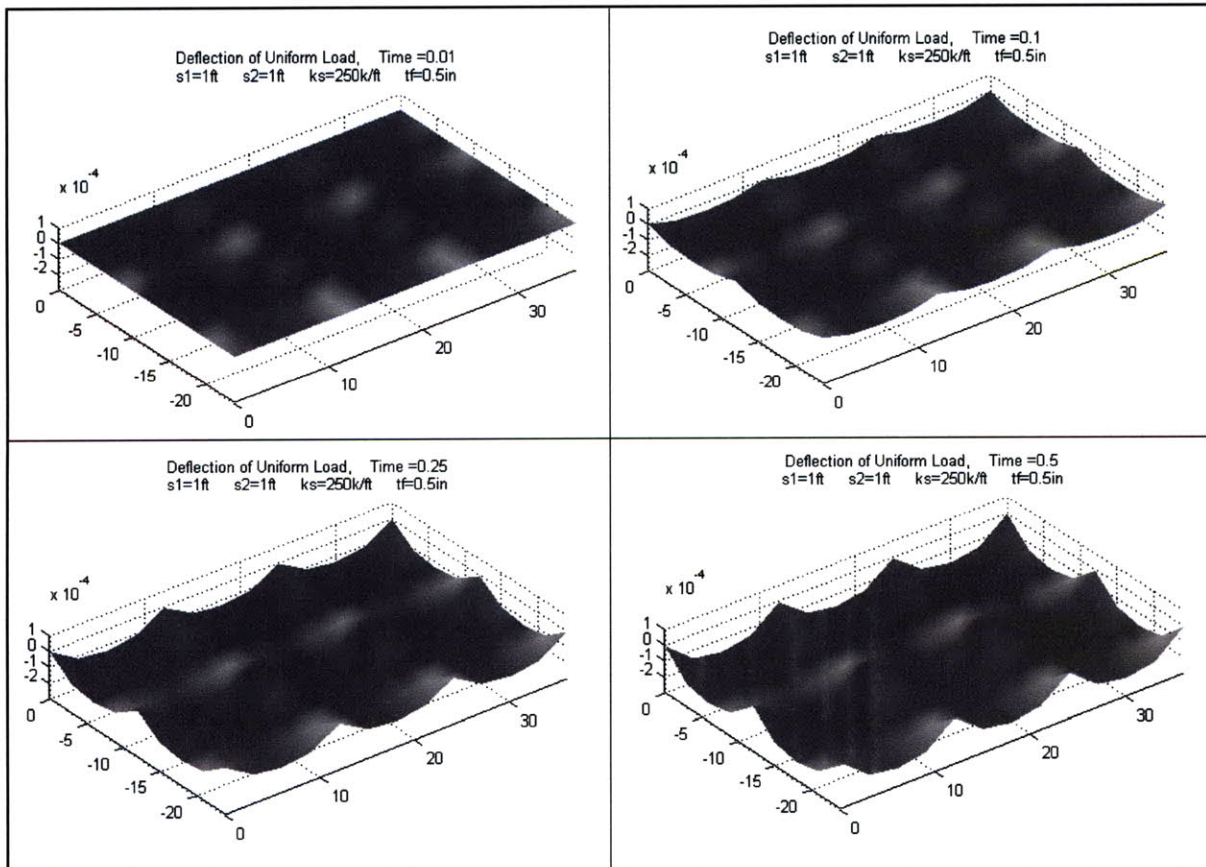
```

hw = waitbar(0,'Progress');
for i=1:1:steps;
    waitbar(i/steps,hw);
    X(:,i+1)=A_ss*X(:,i)+B_ss;
    Xd(:,i+1)=(X(:,i+1)-X(:,i))/dt;

    ijk = 1;
    for i2=1:1:fh
        for j2=1:1:fw
            XM(i2,j2) = X(ijk*3-2,i+1);
            ijk=ijk+1;
        end
    end

    surfc(x,-y,XM);
    shading interp;
    colormap(jet);
    title(['Deflection of Uniform Load',',', Time =',
          num2str(i*dt)],['s1=',num2str(s1),'ft', s2=',num2str(s2),
          'ft', ks=',num2str(ks/1000),'k/ft', tf=',num2str(tf),'in']});
    axis([0 w*12 -h*12 0 -0.0003 0.0001])
    daspect([max(w,h) max(w,h) 0.0002])
    F = getframe;
end
close(hw)

```



## 7.5 MASTER FILE

```

clc
clear all

%Set Parameters
h = 2;          %Formwork Height (ft)
w = 3;          %Formwork Width (ft)
c = 0.01;       %Damping Ratio
E = 1001000;    %Formwork Surface Modulus of Elasticity (psi)
rof = 43.7;     %Formwork Density (lbs/ft3)
Qs = 4.5;       %Shotcrete Spray Speed (ft3/min) ==> (10yd3/h)
vs = 20;        %Shotcrete Application Speed on Contact (in/s)
ds = 3;         %Shotcrete Spray Diameter on Contact (in)
ros = 150;      %Shotcrete Density (lbs/ft3)
ts = 6;         %Desired Shotcrete Thickness (in)
s = 10;         %Maximum slope
v = 0.22;       %Poisson's Ratio

%Varied Parameters
s1 = 1;         %Vertical Spacing of Supports (ft)
s2 = 1;         %Horizontal Spacing of Supports (ft)
ks = 250000;    %Support Stiffness (lbs/ft)
tf = 0.5;       %Formwork Surface Thickness (in)

sim_time = 600;

%Calculated Parameters
wf = rof*tf/12;          %Weight of Formwork (lbs/ft2)
a = ds;                 %Finite Element Height & Width (in)
fh = h*12/a+1;          %Finite Element Points Over Total Height
fw = w*12/a+1;          %Finite Element Points Over Total Width
dta = 1/vs*a;           %Application Time Step (s)
dt = dta/4;             %Time Step (s)
steps = sim_time/dt;    %Number of Time Steps
dts = 0.95*Qs*dt/(a^2)*(12^3)/60; %Change In Thickness of Concrete per Time Step(in)

%%
%Develop Dimension Vectors

j = 0;
for i=0:1:w*12/a
    j = j+1;
    x(j) = i*a;
end
j = 0;
for i=0:1:h*12/a
    j = j+1;
    y(j) = i*a;
end

```

%%

%Stiffness Matrix for Single Finite Element

```

k11 = [8+(14-4*v)/5      (2+(1+4*v)/5)*a      -(2+(1+4*v)/5)*a      -2-(14-4*v)/5      (2+(1-v)/5)*a      (-1+(1+4*v)/5)*a;
      (2+(1+4*v)/5)*a    (4/3+4/15*(1-v))*a^2    -v*a^2                -(2+(1-v)/5)*a    (2/3-(1-v)/15)*a^2    0;
      -(2+(1+4*v)/5)*a    -v*a^2                (4/3+4/15*(1-v))*a^2  (-1+(1+4*v)/5)*a  0                      (2/3-4/15*(1-v))*a^2;
      -2-(14-4*v)/5      -(2+(1-v)/5)*a        (-1+(1+4*v)/5)*a      8+(14-4*v)/5      -(2+(1+4*v)/5)*a      -(2+(1+4*v)/5)*a;
      (2+(1-v)/5)*a      (2/3-(1-v)/15)*a^2    0                      -(2+(1+4*v)/5)*a  (4/3+4/15*(1-v))*a^2  v*a^2;
      (-1+(1+4*v)/5)*a    0                      (2/3-4/15*(1-v))*a^2  -(2+(1+4*v)/5)*a  v*a^2                (4/3+4/15*(1-v))*a^2];

k21 = [-2-(14-4*v)/5    (1-(1+4*v)/5)*a      (2+(1-v)/5)*a        -4+(14-4*v)/5      (1-(1-v)/5)*a      (1-(1-v)/5)*a;
      (1-(1+4*v)/5)*a    (2/3-4/15*(1-v))*a^2  0                      (-1+(1-v)/5)*a    (1/3+(1-v)/15)*a^2  0;
      -(2+(1-v)/5)*a      0                      (2/3-(1-v)/15)*a^2  (-1+(1-v)/5)*a    0                      (1/3+(1-v)/15)*a^2;
      -4+(14-4*v)/5      (-1+(1-v)/5)*a        (1-(1-v)/5)*a        -2-(14-4*v)/5      (-1+(1+4*v)/5)*a    (2+(1-v)/5)*a;
      (1-(1-v)/5)*a      (1/3+(1-v)/15)*a^2    0                      (-1+(1+4*v)/5)*a  (2/3-4/15*(1-v))*a^2  0;
      (-1+(1-v)/5)*a      0                      (1/3+(1-v)/15)*a^2  -(2+(1-v)/5)*a    0                      (2/3-(1-v)/15)*a^2];

k12 = k21';

k22 = [8+(14-4*v)/5      (2+(1+4*v)/5)*a      (2+(1+4*v)/5)*a      -2-(14-4*v)/5      (2+(1-v)/5)*a      (1-(1+4*v)/5)*a;
      (2+(1+4*v)/5)*a    (4/3+4/15*(1-v))*a^2  v*a^2                -(2+(1-v)/5)*a    (2/3-(1-v)/15)*a^2  0;
      (2+(1+4*v)/5)*a    v*a^2                (4/3+4/15*(1-v))*a^2  (1-(1+4*v)/5)*a  0                      (2/3-4/15*(1-v))*a^2;
      -2-(14-4*v)/5      -(2+(1-v)/5)*a        (1-(1+4*v)/5)*a      8+(14-4*v)/5      -(2+(1+4*v)/5)*a    (2+(1+4*v)/5)*a;
      (2+(1-v)/5)*a      (2/3-(1-v)/15)*a^2    0                      -(2+(1+4*v)/5)*a  (4/3+4/15*(1-v))*a^2  -v*a^2;
      (1-(1+4*v)/5)*a    0                      (2/3-4/15*(1-v))*a^2  (2+(1+4*v)/5)*a  -v*a^2                (4/3+4/15*(1-v))*a^2];

```

55

```

%%

%Compile k Matrix
ksize = 12+3*(fw-2);
k = zeros(ksize,ksize);
k(1:6,1:6) = k11;
k(7+3*(fw-2):12+3*(fw-2),1:6) = k21;
k(1:6,7+3*(fw-2):12+3*(fw-2)) = k12;
k(7+3*(fw-2):12+3*(fw-2),7+3*(fw-2):12+3*(fw-2)) = k22;
k = E*tf^3/(12*(1-v^2)*a^2)*k;

%%
%Compile Stiffness (K) Matrix

Ksize = ((fw-1)*(fh-1)+(fh-1)-1)*3+ksize-3;
dof = Ksize;
K = zeros(Ksize);

j = 0;
for i=0.5:1+1/(fw-1):(fw-1)*(fh-1)+(fh-1)
    j = round(i)*3-3+1;
    K(j:j+ksize-1,j:j+ksize-1) = K(j:j+ksize-1,j:j+ksize-1) + k;
end

C = c*K;    %Proportional Damping Matrix

%%
%Set Constraints

for i=1:s1*12/a:fh
    for j=1:s2*12/a:w*12/a+1
        ij = (3*((i-1)*(fw)+j)-2);
        K(ij,ij)=K(ij,ij)+ks;    %Apply Springs to Formwork
    end
end

%%
%Initial Conditions
X = zeros(dof*2,steps);
Xd = zeros(dof*2,steps);
XM = zeros(fh,fw);

t(1) = 0;    %Start Time (s)

z = zeros(fh,fw);    %Initial Shotcrete Thickness (Zero)
m = wf*a^2*ones(fh,fw);    %Initial Mass (Mass of Formwork)

```



```

%%
%Apply Shotcrete

M = zeros(Ksize,Ksize);
for i=1:3:Ksize
    M(i,i) = wf*a^2;      %Initial Mass Matrix (Mass of Formwork)
end

r1 = int8((w*l1/a)*rand(1)+1);      %Random Start Point
r2 = int8((h*l1/a)*rand(1)+1);      %Random Start Point

ij = 1;
l = 1;
l1 = 1;
app_steps = ceil(ts/dts)*fw*fh;
hw = waitbar(0,'Progress');
for i=0:l:sim_time/dta
    if(z(r2,r1)<ts)
        j=0;

        %Edge Conditions
        if(r1==w*12/a+1),r1=w*12/a;end
        if(r1==1),r1=2;end
        if(r2==h*12/a+1),r2=h*12/a;end
        if(r2==1),r2=2;end

        %Slope Conditions
        if((z(r2,r1)<z(r2+1,r1+1)+s)&&(z(r2,r1)>z(r2+1,r1+1)-s)),j=j+1;end
        if((z(r2,r1)<z(r2+1,r1)+s)&&(z(r2,r1)>z(r2+1,r1)-s)),j=j+1;end
        if((z(r2,r1)<z(r2+1,r1-1)+s)&&(z(r2,r1)>z(r2+1,r1-1)-s)),j=j+1;end
        if((z(r2,r1)<z(r2,r1+1)+s)&&(z(r2,r1)>z(r2,r1+1)-s)),j=j+1;end
        if((z(r2,r1)<z(r2,r1)+s)&&(z(r2,r1)>z(r2,r1)-s)),j=j+1;end
        if((z(r2,r1)<z(r2,r1-1)+s)&&(z(r2,r1)>z(r2,r1-1)-s)),j=j+1;end
        if((z(r2,r1)<z(r2-1,r1+1)+s)&&(z(r2,r1)>z(r2-1,r1+1)-s)),j=j+1;end
        if((z(r2,r1)<z(r2-1,r1)+s)&&(z(r2,r1)>z(r2-1,r1)-s)),j=j+1;end
        if((z(r2,r1)<z(r2-1,r1-1)+s)&&(z(r2,r1)>z(r2-1,r1-1)-s)),j=j+1;end
        if(j>=5&&(z(r2,r1)<ts))
            p = zeros(h*12/a+1,w*12/a+1);
            p(r2,r1) = (ros*Qs)*(Qs*12^2/a^2)/32.2/3600;      %Load (lbs)
            P = zeros(Ksize,1);
            P((3*((r2-1)*(fw)+r1))-2)=p(r2,r1);      %Load Matrix (lbs)

            for il=0:dt:dta
                waitbar(ij/(app_steps),hw);
                z(r2,r1)=z(r2,r1)+dts;      %Thickness (in)
                m(r2,r1) = 0.95*m(r2,r1)+ros*Qs*dt;      %Mass (lbs)
                M((3*((r2-1)*(fw)+r1))-2,(3*((r2-1)*(fw)+r1))-
                    2)=m(r2,r1);      %Mass Matrix (lbs)
                t(ij+1) = t(ij)+dt;      %Time (s)

                pinvM = pinv(M);
                A_ss = [zeros(dof) eye(dof); -pinvM*K -pinvM*C];
                B_ss = [zeros(dof,1); pinvM*P];
                C_ss = eye(dof*2);
                D_ss = zeros(dof*2,1);
            end
        end
    end
end

```

```

[A_ss,B_ss,C_ss,D_ss]=c2dm(A_ss,B_ss,C_ss,D_ss,dt);
    %Conversion to Discrete

X(:,ij+1)=A_ss*X(:,ij)+B_ss;
Xd(:,ij+1)=(X(:,ij+1)-X(:,ij))/dt;

    %Compile XM for Display of Formwork Deflections
    %ijk = 1;
    %for i2=1:1:fh
    %    for j2=1:1:fw
    %        XM(i2,j2) = X(ijk*3-2,ij+1);
    %        ijk=ijk+1;
    %    end
    %end
    ij = ij+1;
end

%Display Application of Shotcrete
    %surf(x,y,z);
    %shading interp;
    %colormap(gray);
    %axis([0 w*12 0 h*12 0 ts*2])
    %daspect([max(w,h) max(w,h) ts])
    %Fa = getframe;

%Display Formwork Deflections
    %surf(x,-y,XM);
    %shading interp;
    %colormap(gray);
    %axis([0 w*12 -h*12 0 -0.0003 0.0001])
    %daspect([max(w,h) max(w,h) 0.0002])
    %F = getframe;
end
end

```

```

%Choose next point
    if (l1>6/a)
        r3 = int8(8*rand(1));
        if(r3==0),r1=r1-1;r2=r2-1;end
        if(r3==1),r1=r1-1;end
        if(r3==2),r1=r1-1;r2=r2+1;end
        if(r3==3),r2=r2-1;end
        if(r3==5),r2=r2+1;end
        if(r3==6),r1=r1+1;r2=r2-1;end
        if(r3==7),r1=r1+1;end
        if(r3==8),r1=r1+1;r2=r2+1;end
        if (l<8)
            l=l+1;
        else
            l=(l/8);
        end
        l1=1;
    else
        l1=l1+1;
        if(l==1),r1=r1-1;end
        if(l==2),r1=r1-1;r2=r2+1;end
        if(l==3),r2=r2+1;end
        if(l==4),r1=r1+1;r2=r2+1;end
        if(l==5),r1=r1+1;end
        if(l==6),r1=r1+1;r2=r2-1;end
        if(l==7),r2=r2-1;end
        if(l==8),r1=r1-1;r2=r2-1;end
    end
end

end
close(hw)

%Check that Simulation Time Allowed for Entire Application of Shotcrete
for i=2:1:fh-1
    for j=2:1:fw-1
        if(z(i,j)<ts)
            display('Increase Simulation Time (or Vary Application Area or
Slope Conditions)');
        end
    end
end

end

%%

```

## 7.6 MANIPULATION OF SOLUTIONS

```

clc
%Max Displacement & Acceleration
Xmax = zeros(dof/3,1);
Amax = zeros(dof/3,1);
for i=1:1:dof/3
    Xmax(i) = min(X(i*3-2,:));
    Amax(i) = max(abs(Xd(i*3-2+dof,:)));
end

%Create Displacement Matrix
XmaxM = zeros(fh,fw);
l = 1;
for i=1:1:fh
    for j=1:1:fw
        XmaxM(i,j) = Xmax(l);
        l=l+1;
    end
end

%Create Acceleration Matrix
AmaxM = zeros(fh,fw);
l = 1;
for i=1:1:fh
    for j=1:1:fw
        AmaxM(i,j) = Amax(l);
        l=l+1;
    end
end

surfc(x,-y,XmaxM);
shading interp;
colormap(jet);
title({'Maximum Displacement', ['s1=', num2str(s1), 'ft      s2=', num2str(s2),
    'ft      ks=', num2str(ks/1000), 'k/ft      tf=', num2str(tf), 'in']});

figure
[loc1,hgt1] = contourf(x,-y,XmaxM);
clabel(loc1,hgt1)
title({'Maximum Displacement Contours', ['s1=', num2str(s1), 'ft      s2=',
    num2str(s2), 'ft      ks=', num2str(ks/1000), 'k/ft      tf=',
    num2str(tf), 'in']});

figure
surfc(x,-y,AmaxM);
shading interp;
colormap(jet);
title({'Maximum Acceleration', ['s1=', num2str(s1), 'ft      s2=', num2str(s2),
    'ft      ks=', num2str(ks/1000), 'k/ft      tf=', num2str(tf), 'in']});

```

```
figure
[loc2,hgt2] = contourf(x,-y,AmaxM);
clabel(loc2,hgt2)
title({'Maximum Acceleration Contours', ['s1=', num2str(s1), 'ft      s2=',
num2str(s2), 'ft      ks=', num2str(ks/1000), 'k/ft      tf=',
num2str(tf), 'in']});
```



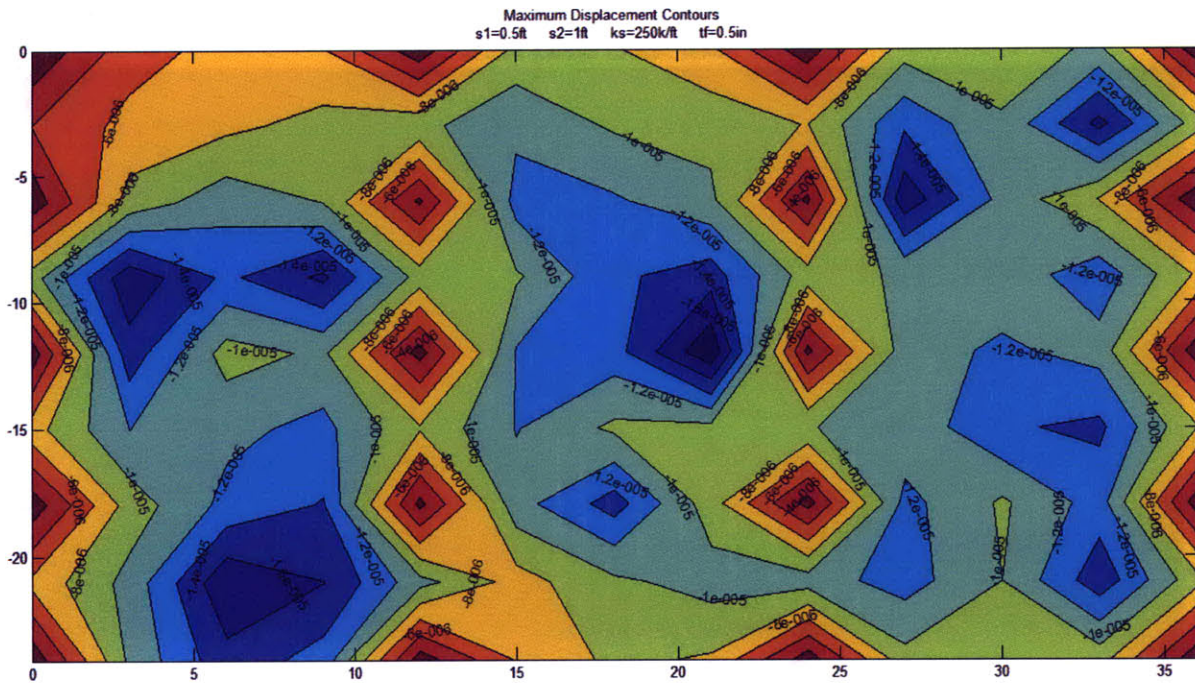
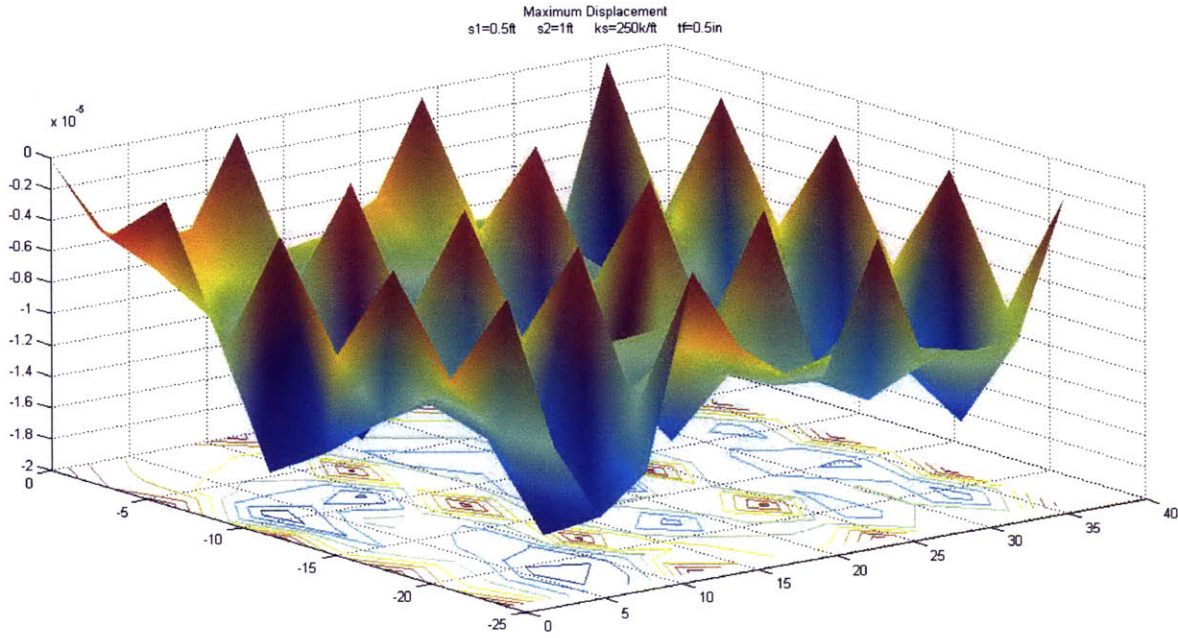
## 8. APPENDIX II

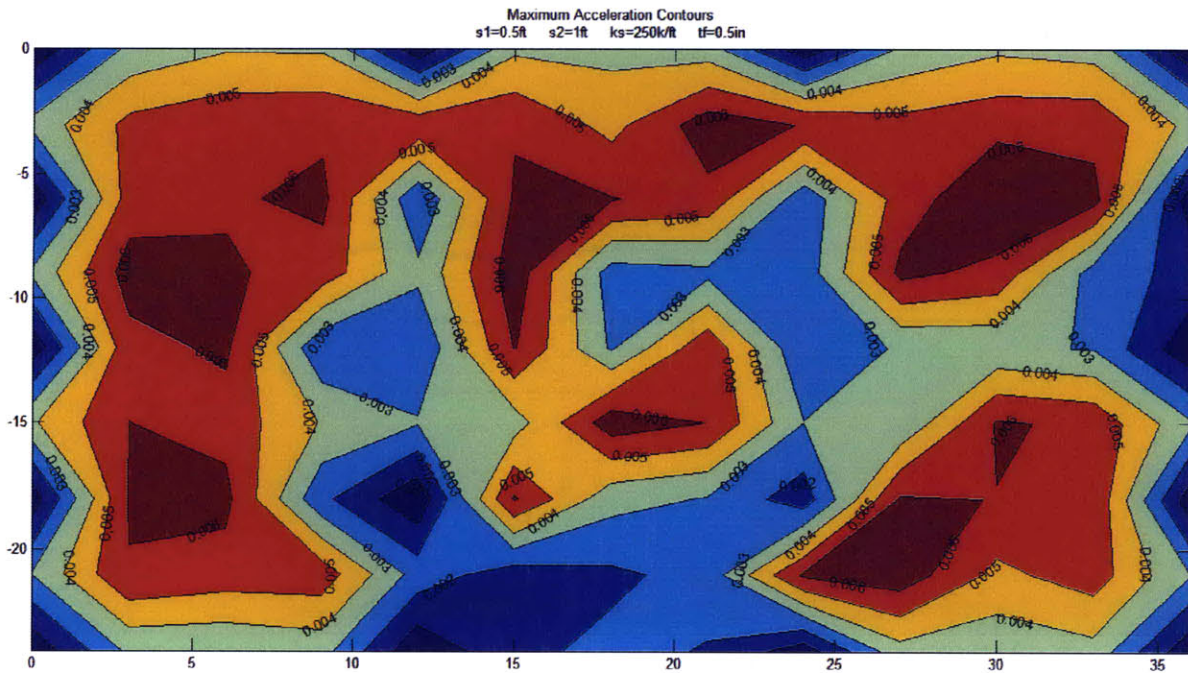
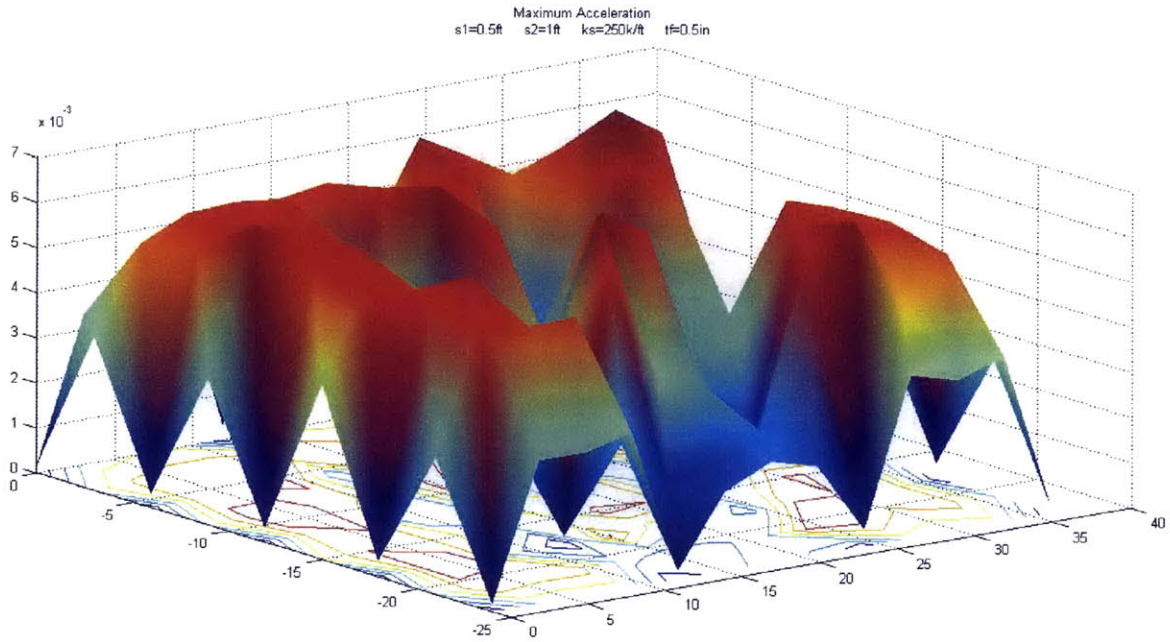
### *SIMULATIONS*

Simulation		Varying Parameter	$s1$ (ft)	$s2$ (ft)	$ks$ (kip/ft)	$tf$ (in)
<b>A</b>	1	$s1$	0.5	1	250	0.5
	2		1	1	250	0.5
	3		2	1	250	0.5
<b>B</b>	1	$s1$ & $s2$	0.5	0.5	250	0.5
	2		1	1	250	0.5
	3		2	1.5	250	0.5
<b>C</b>	1	$ks$	1	1	150	0.5
	2		1	1	250	0.5
	3		1	1	350	0.5
<b>D</b>	1	$tf$	1	1	250	0.25
	2		1	1	250	0.5
	3		1	1	250	1

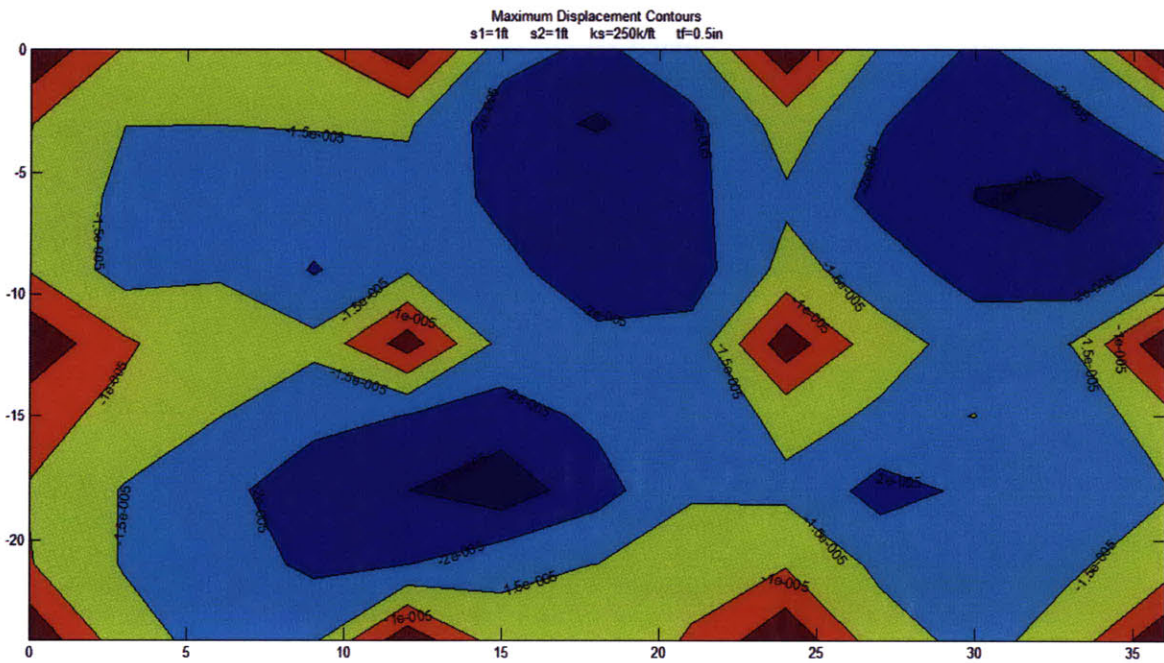
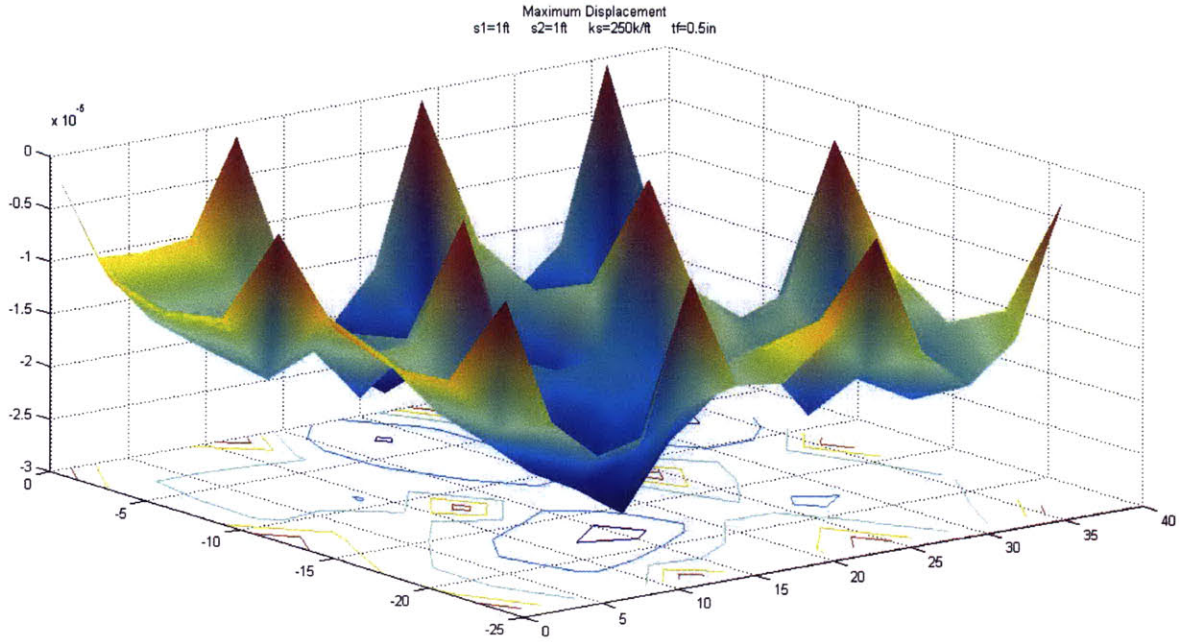


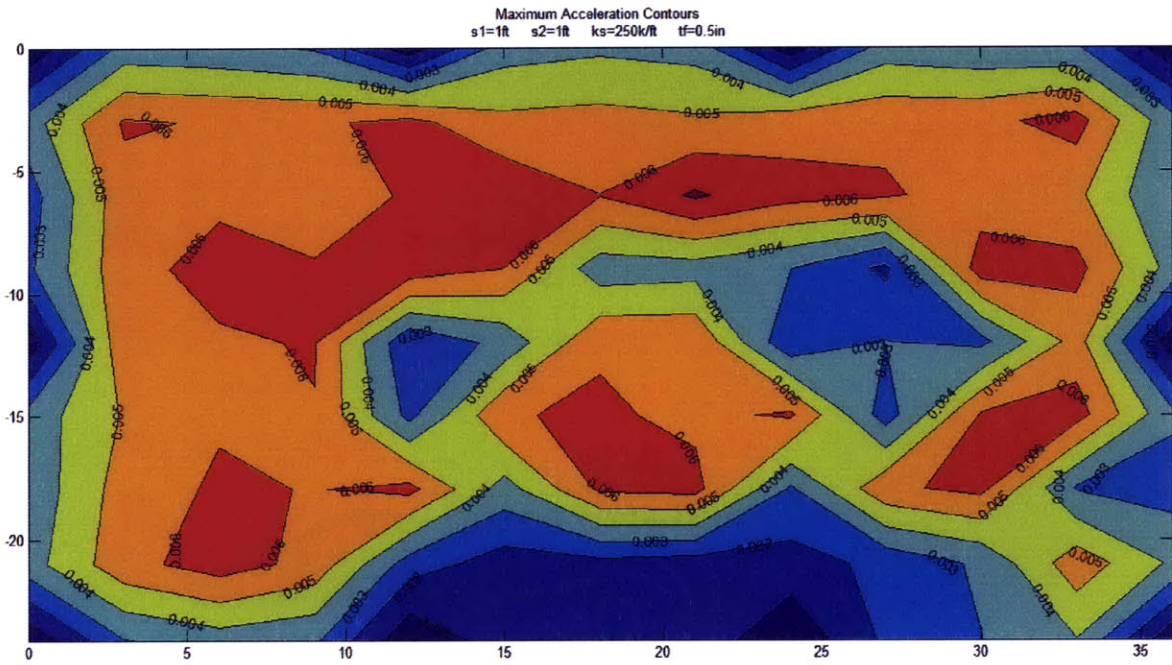
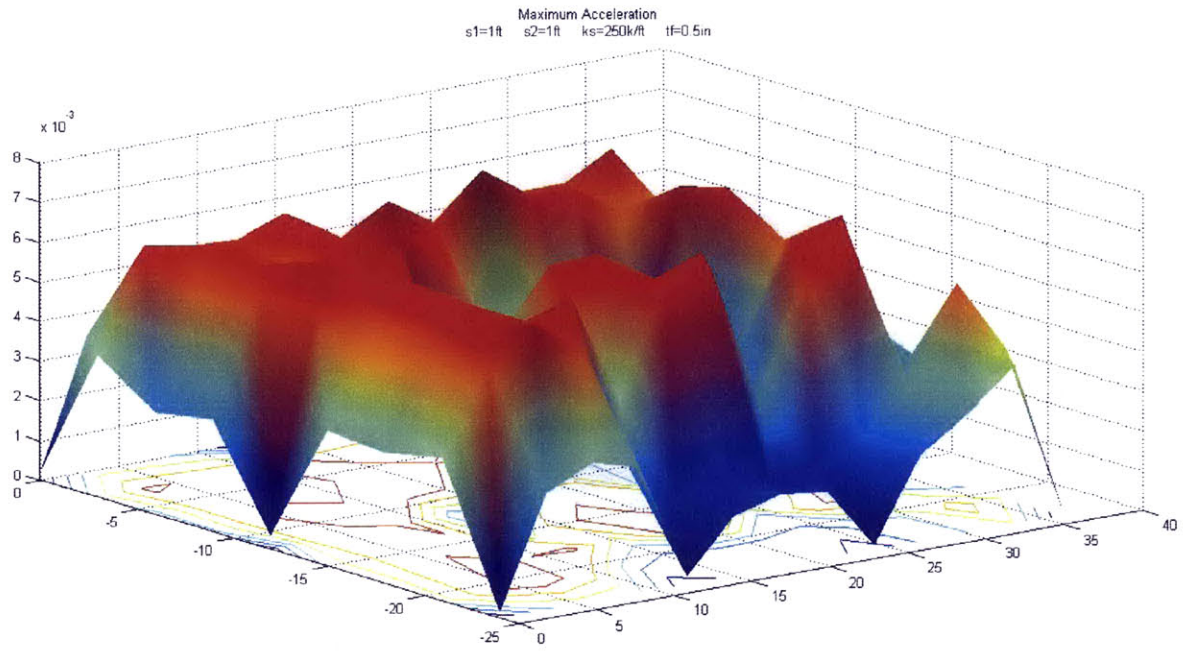
### 8.1 SIMULATION A-1



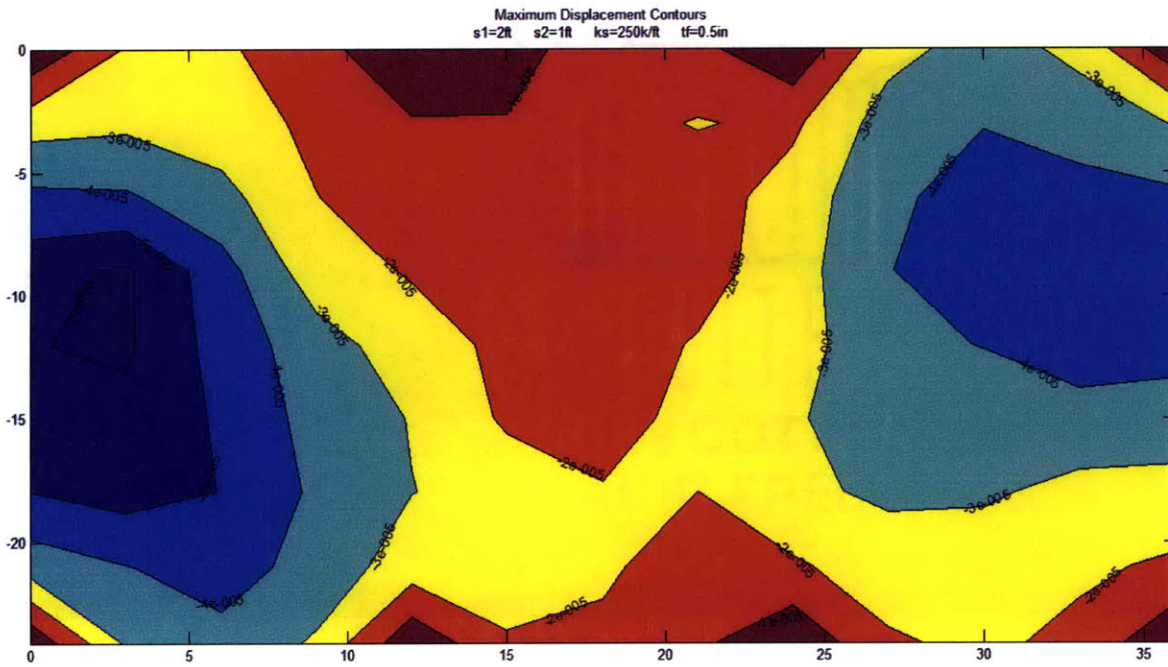
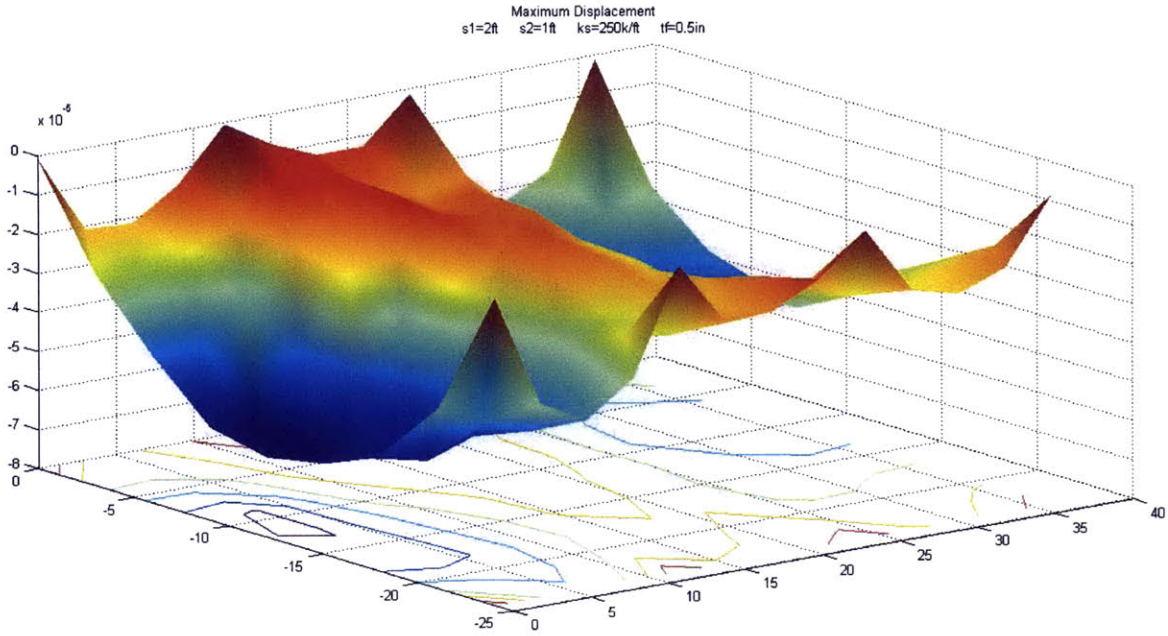


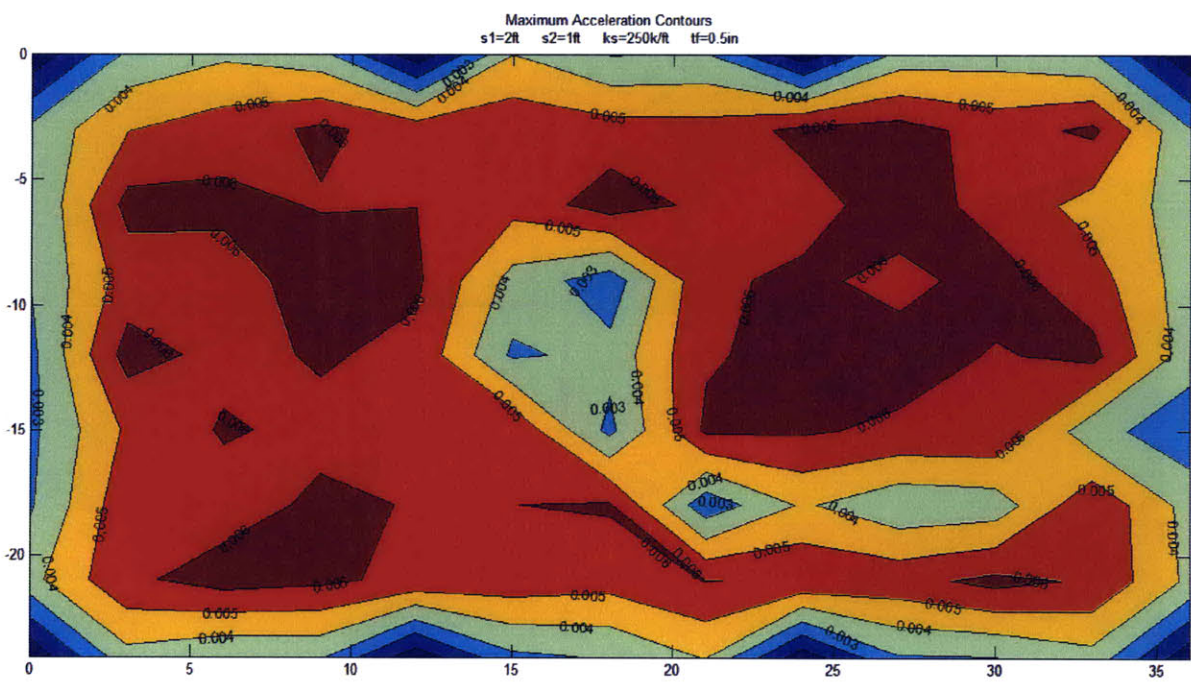
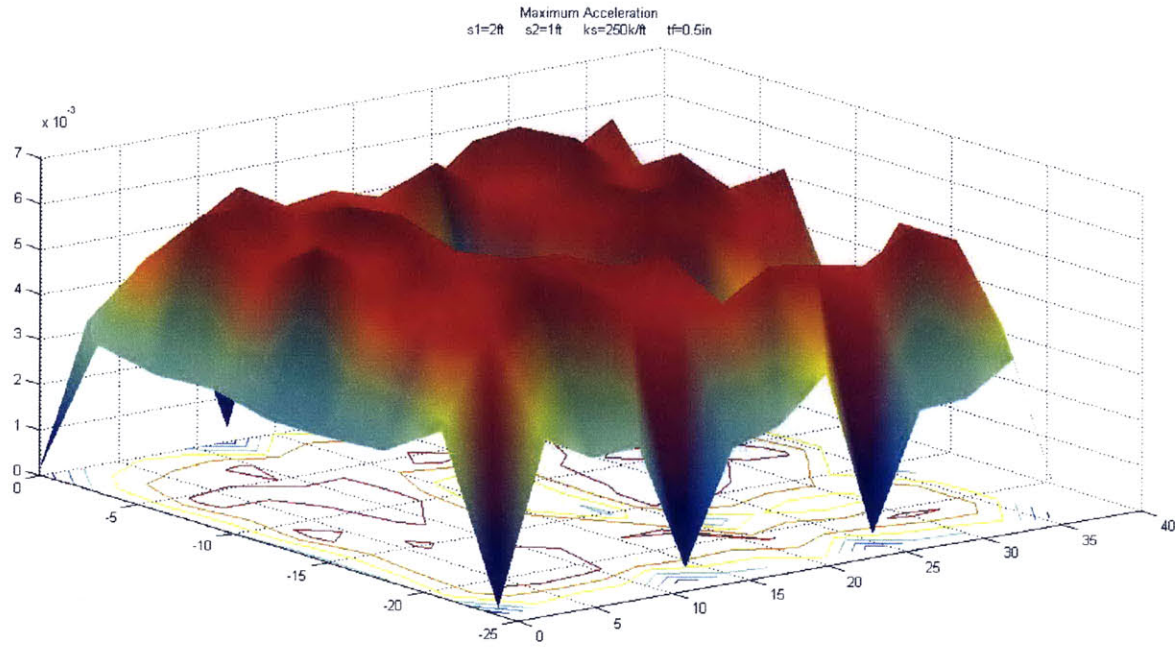
## 8.2 SIMULATION A-2



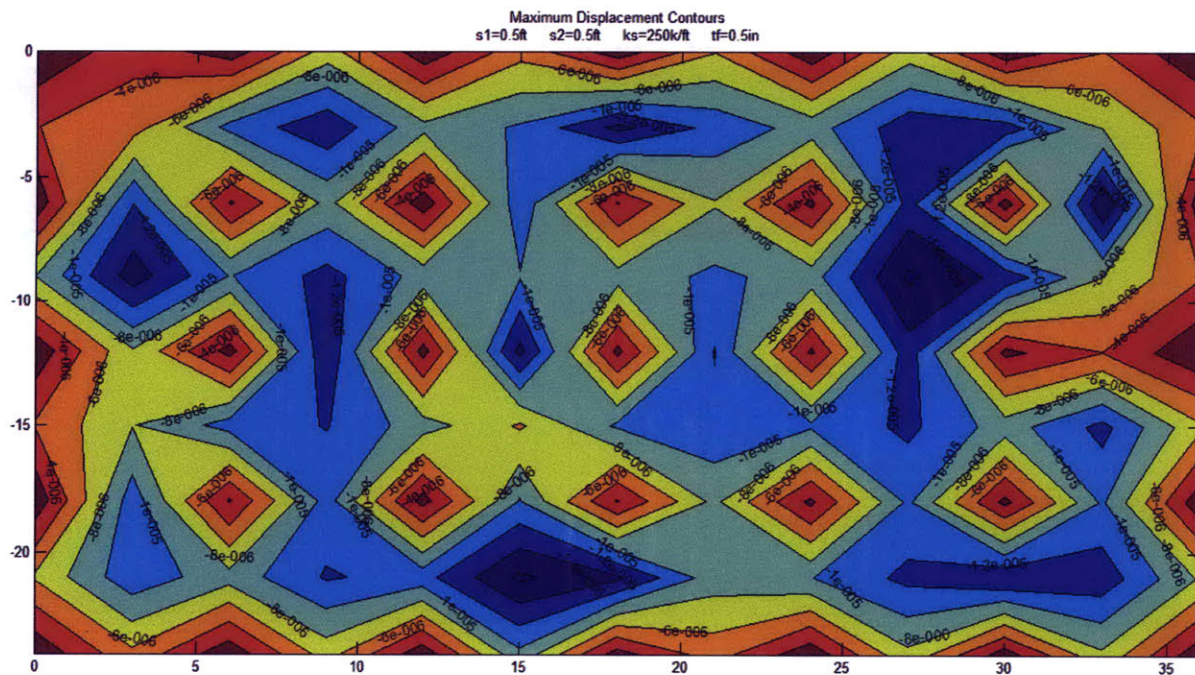
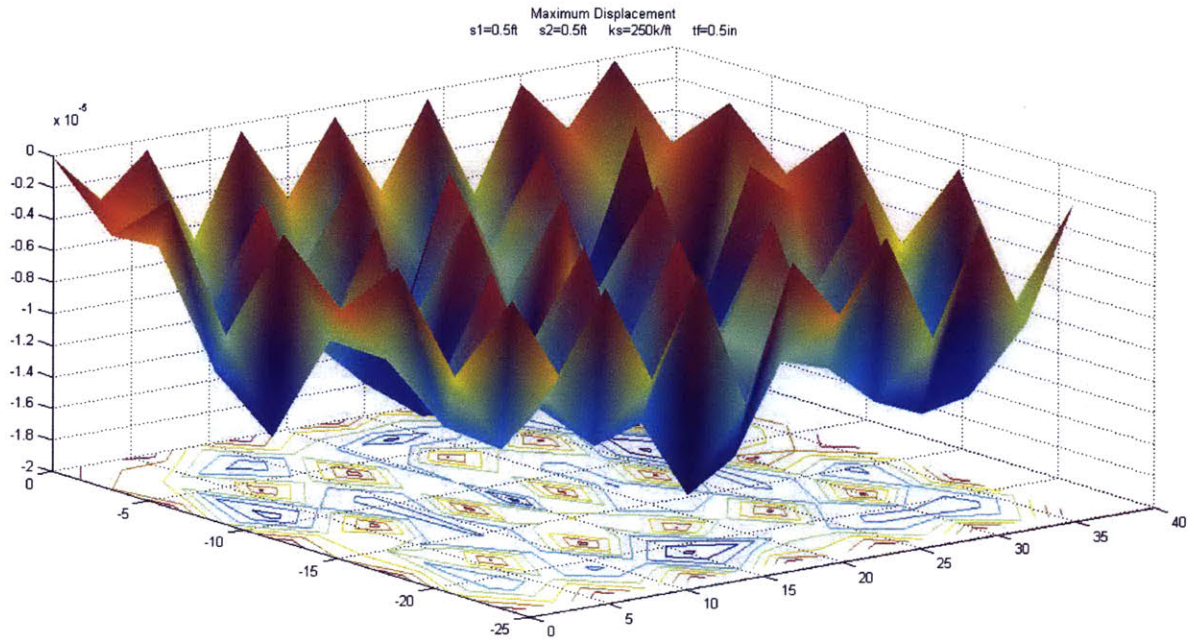


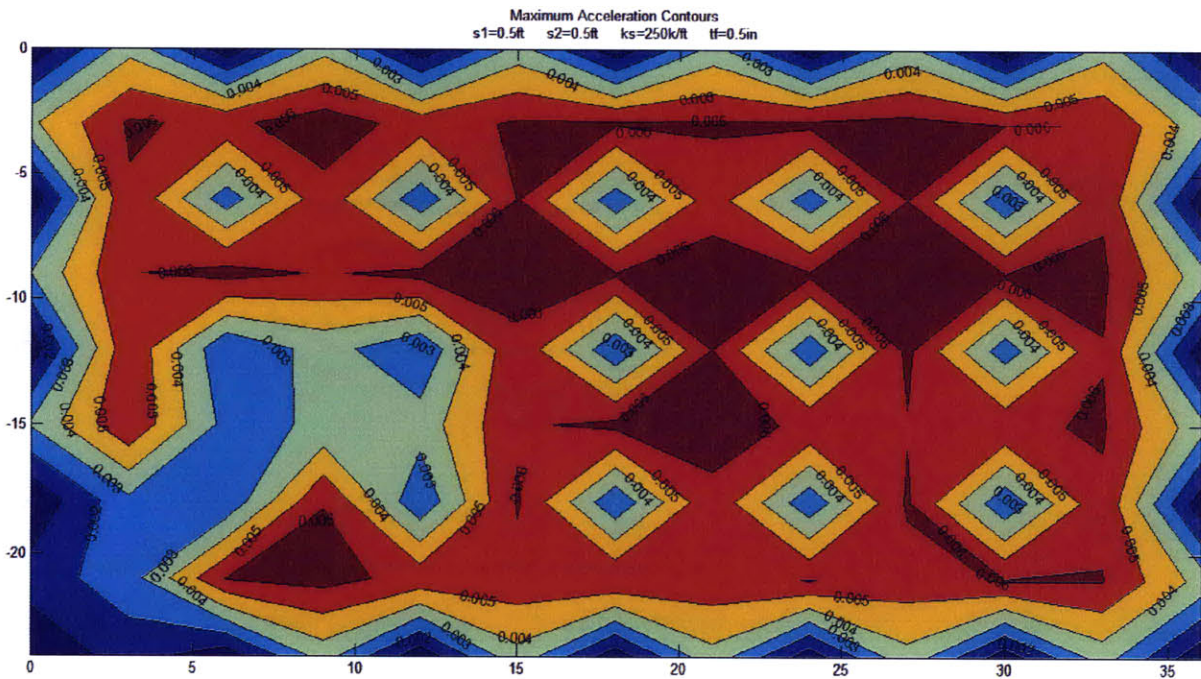
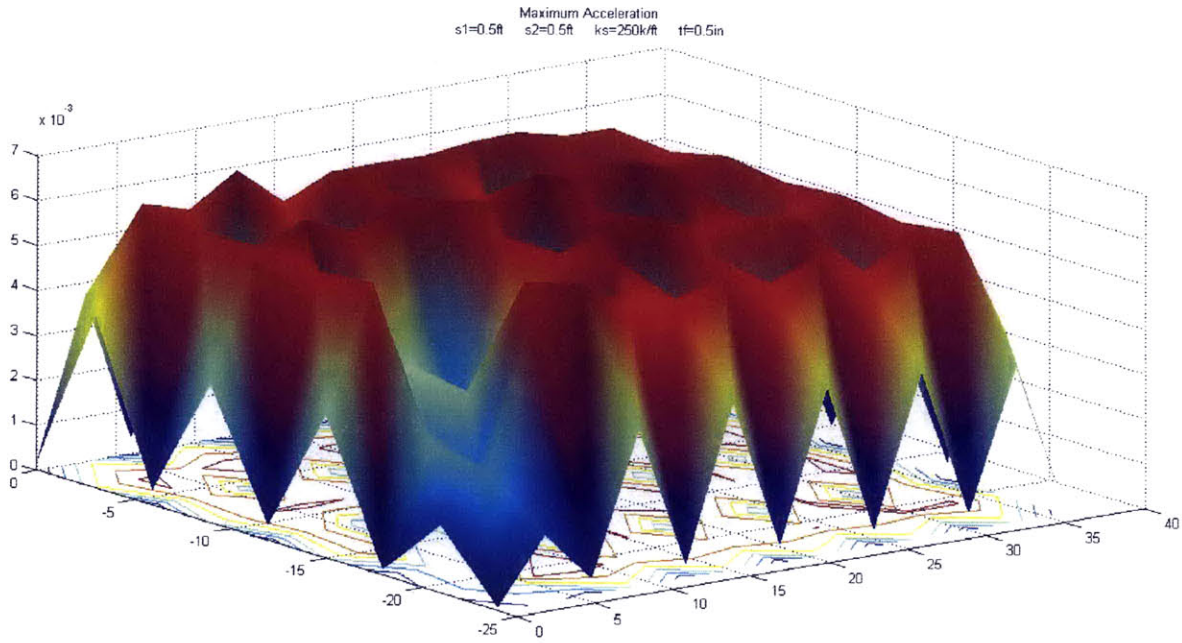
### 8.3 SIMULATION A-3





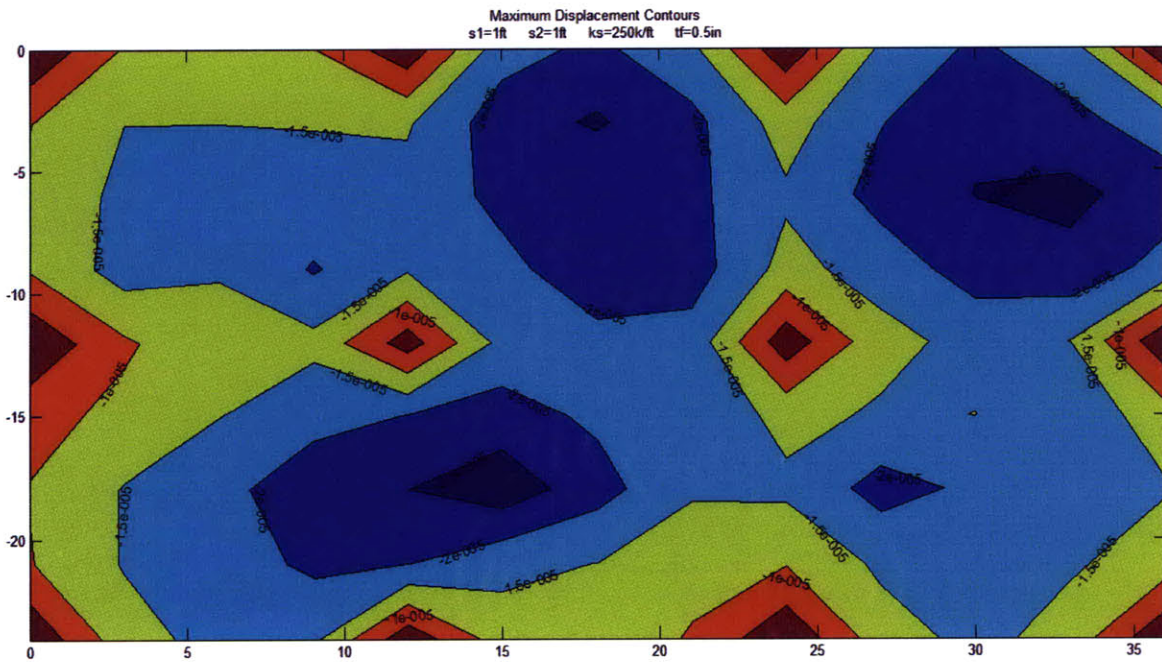
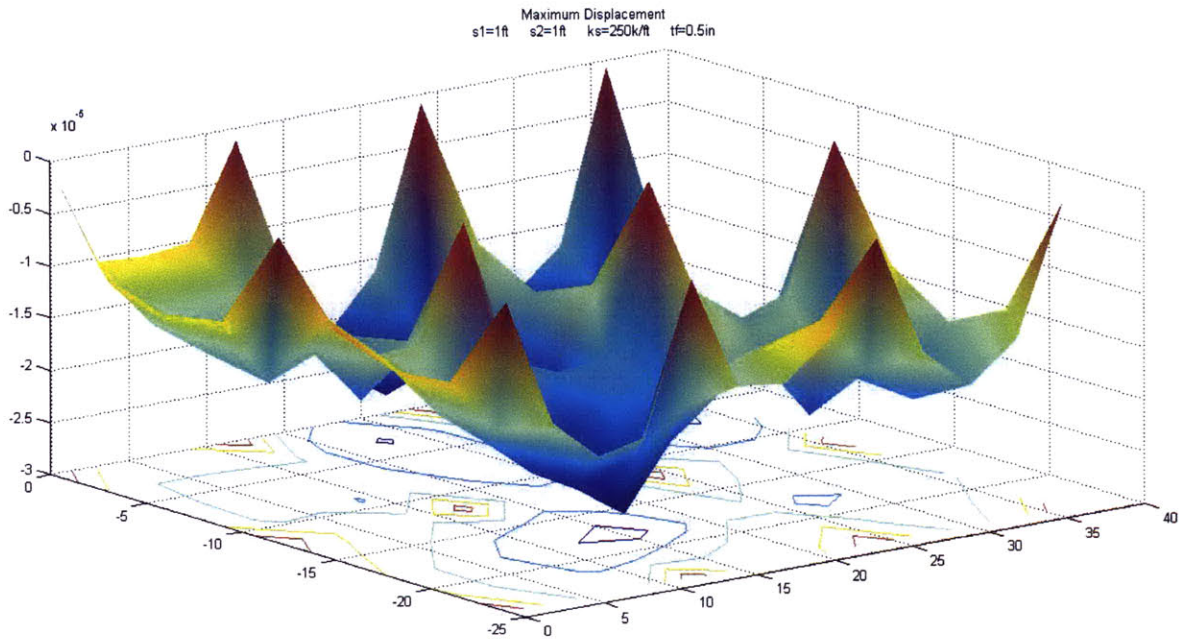
## 8.4 SIMULATION B-1

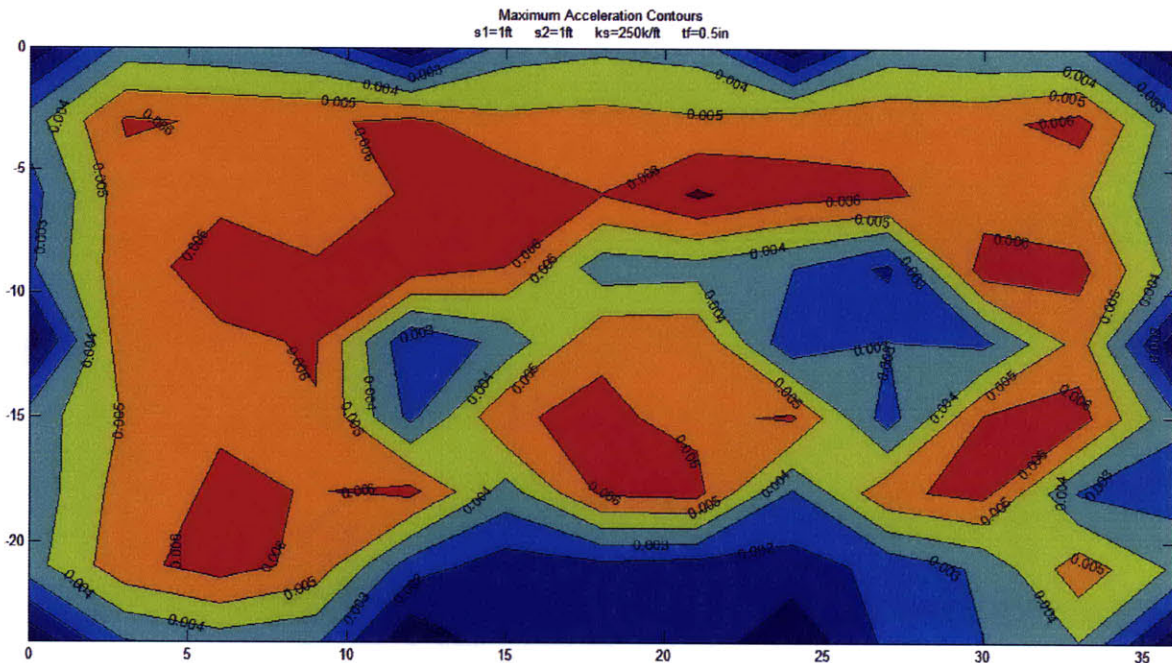
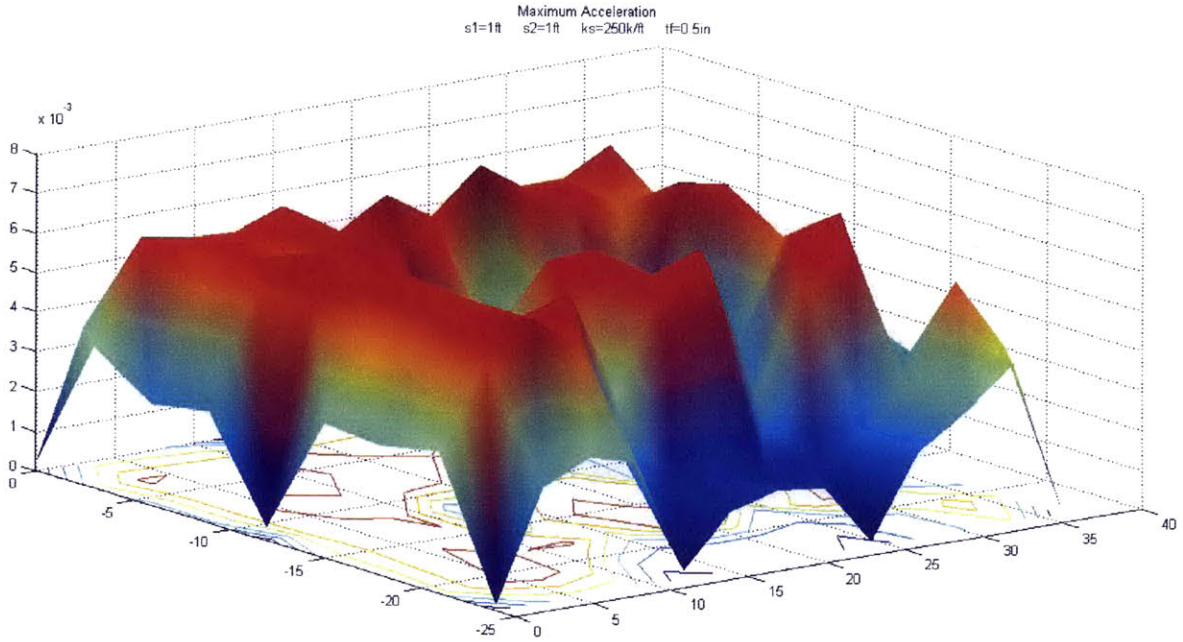




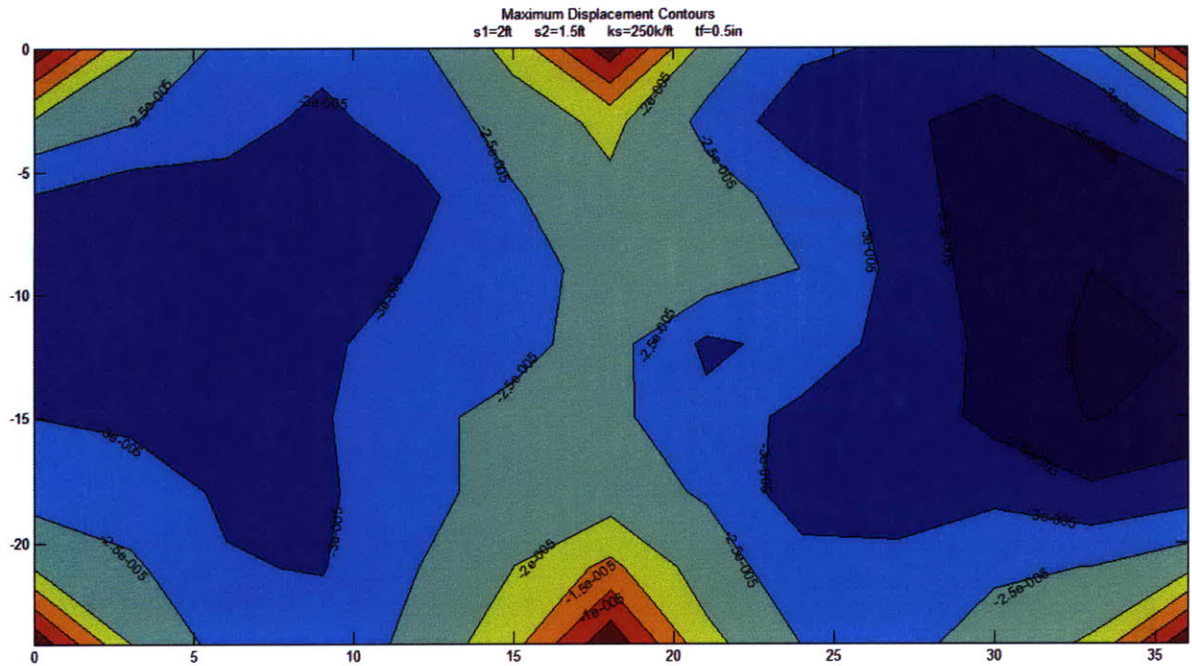
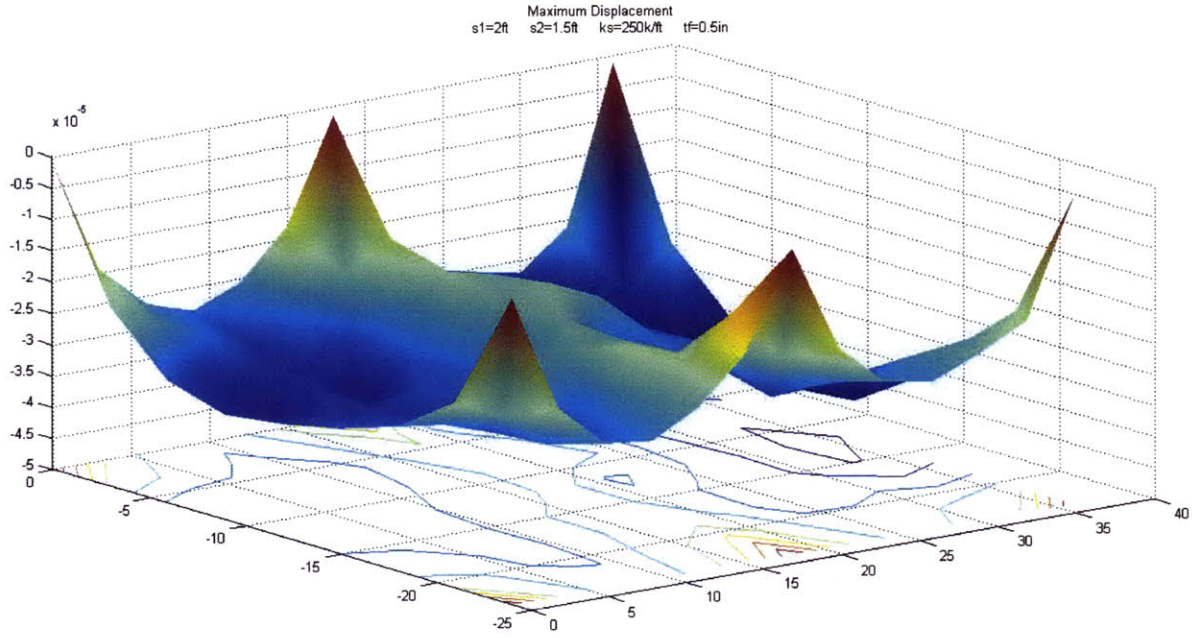


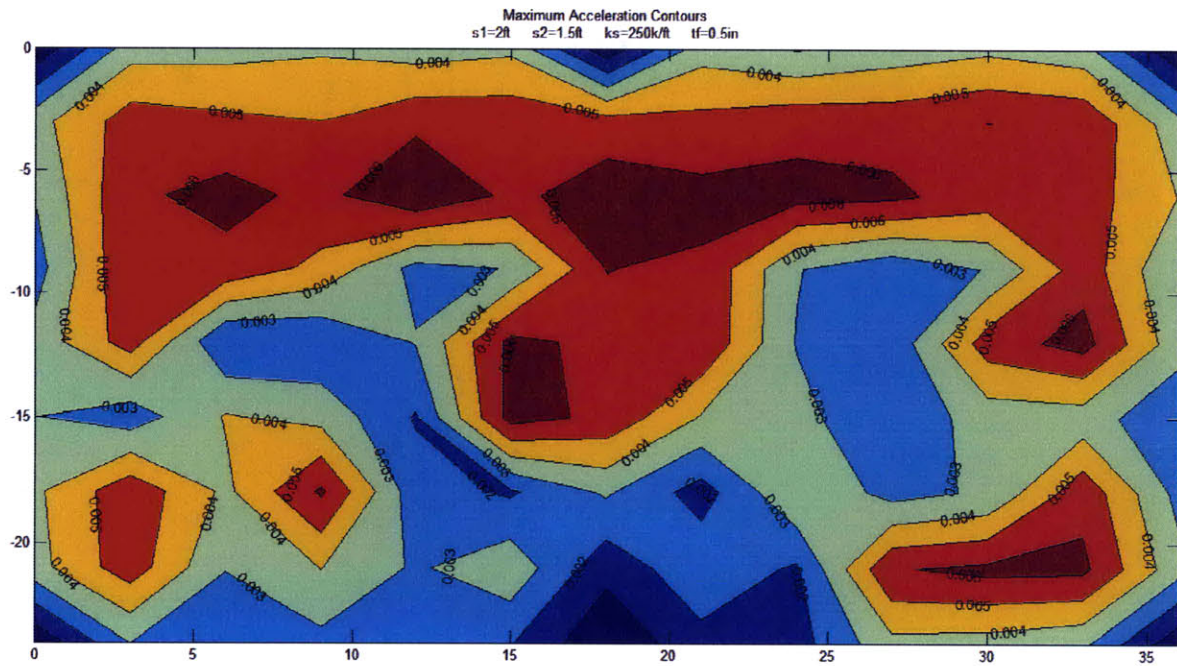
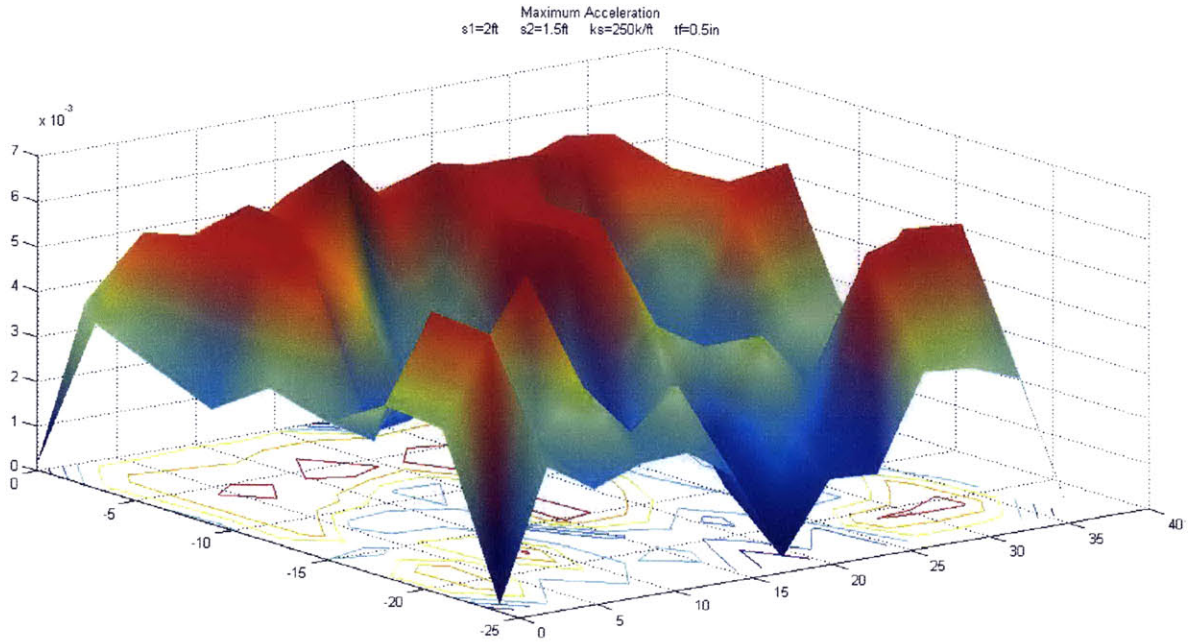
### 8.5 SIMULATION B-2



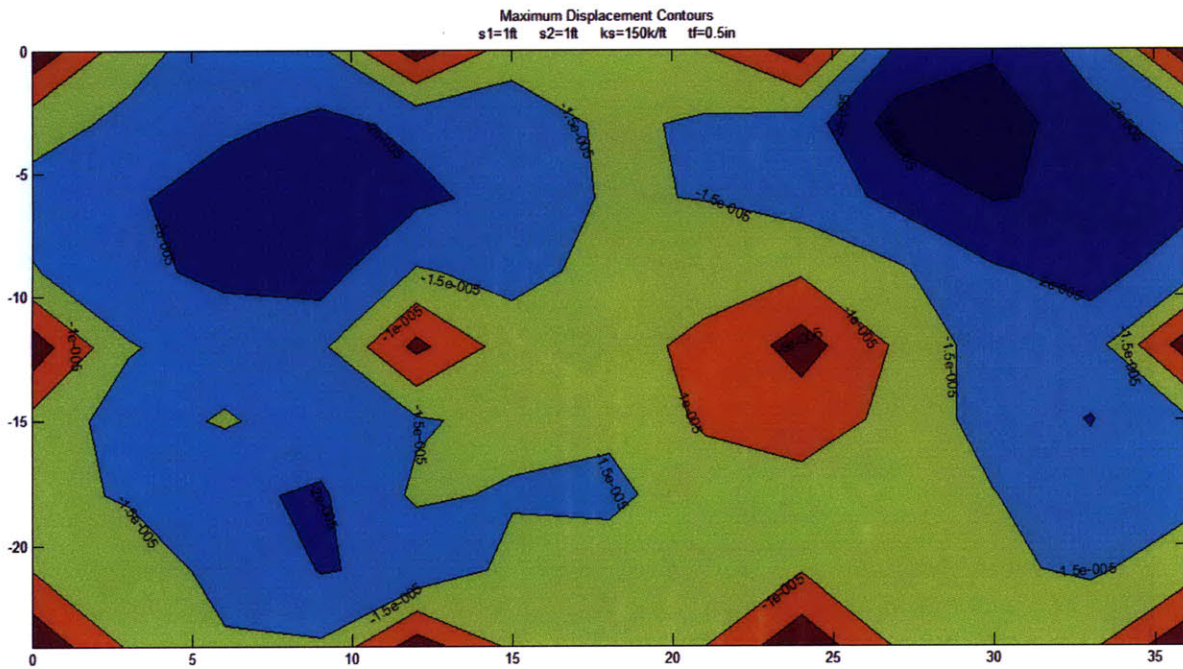
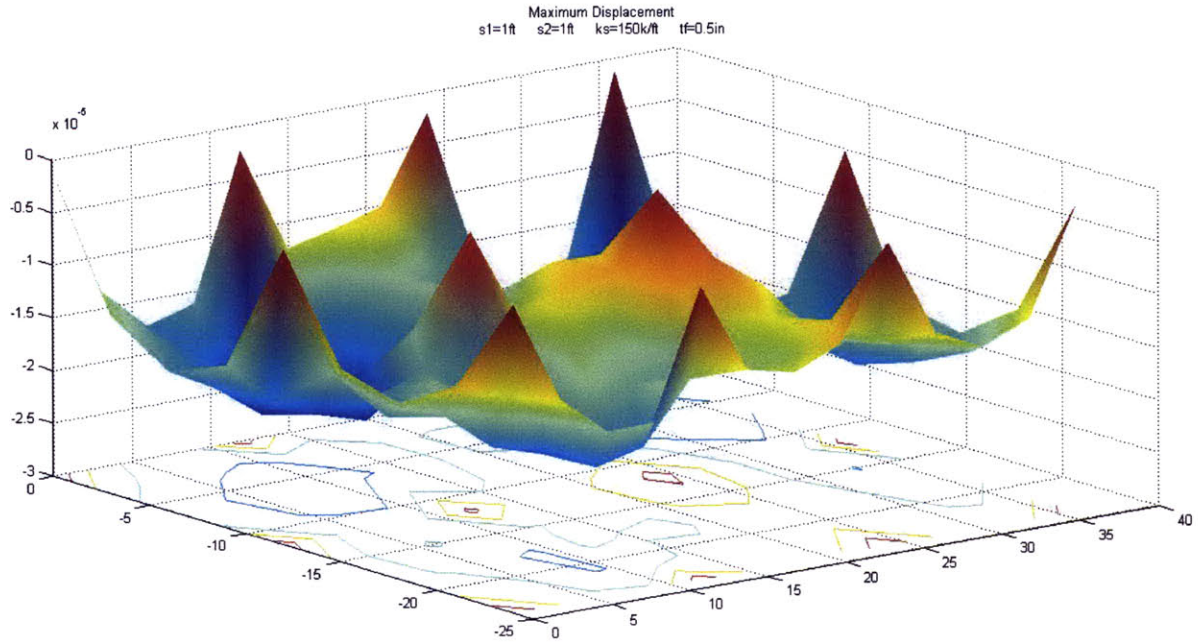


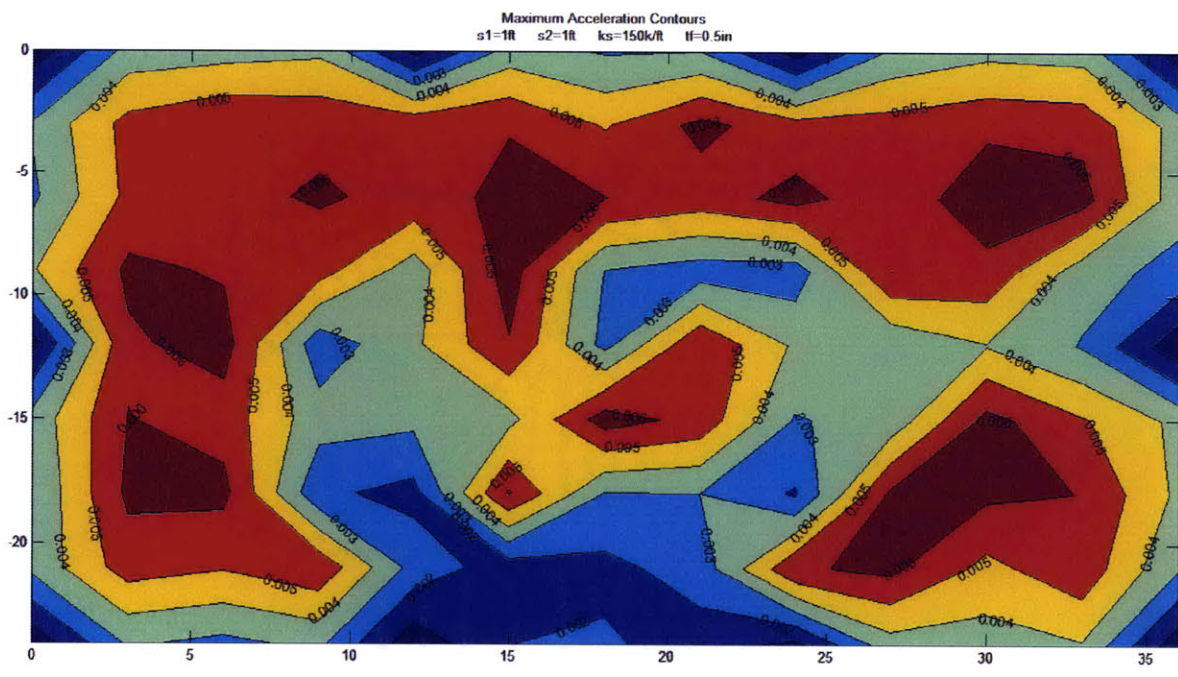
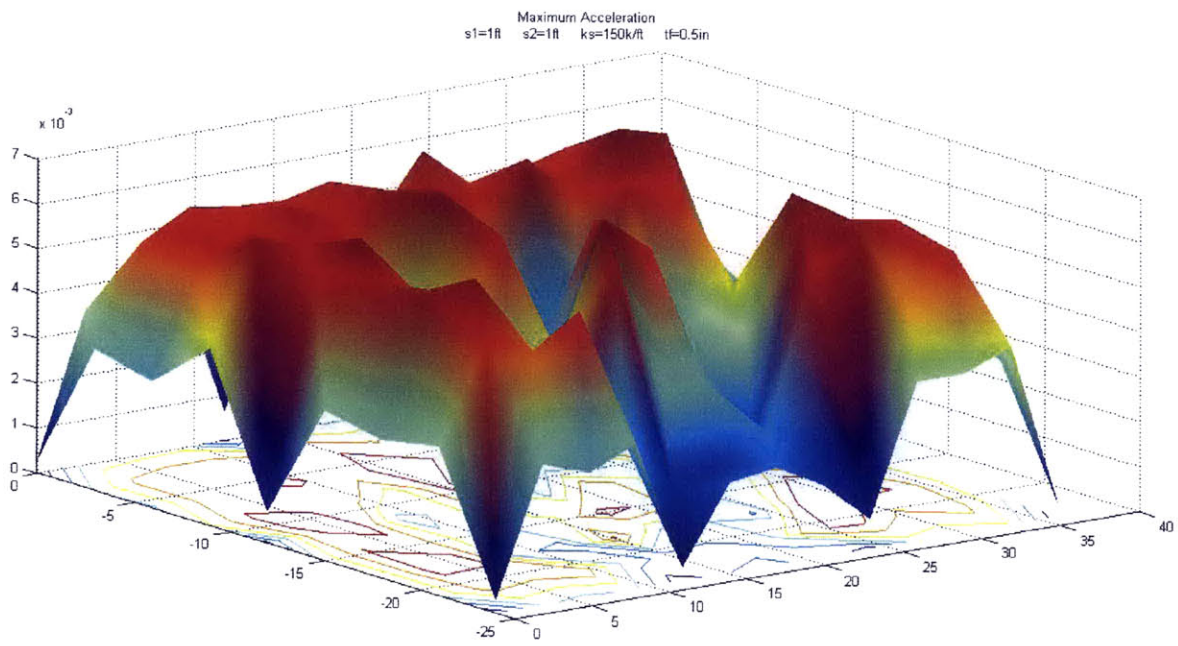
### 8.6 SIMULATION B-3



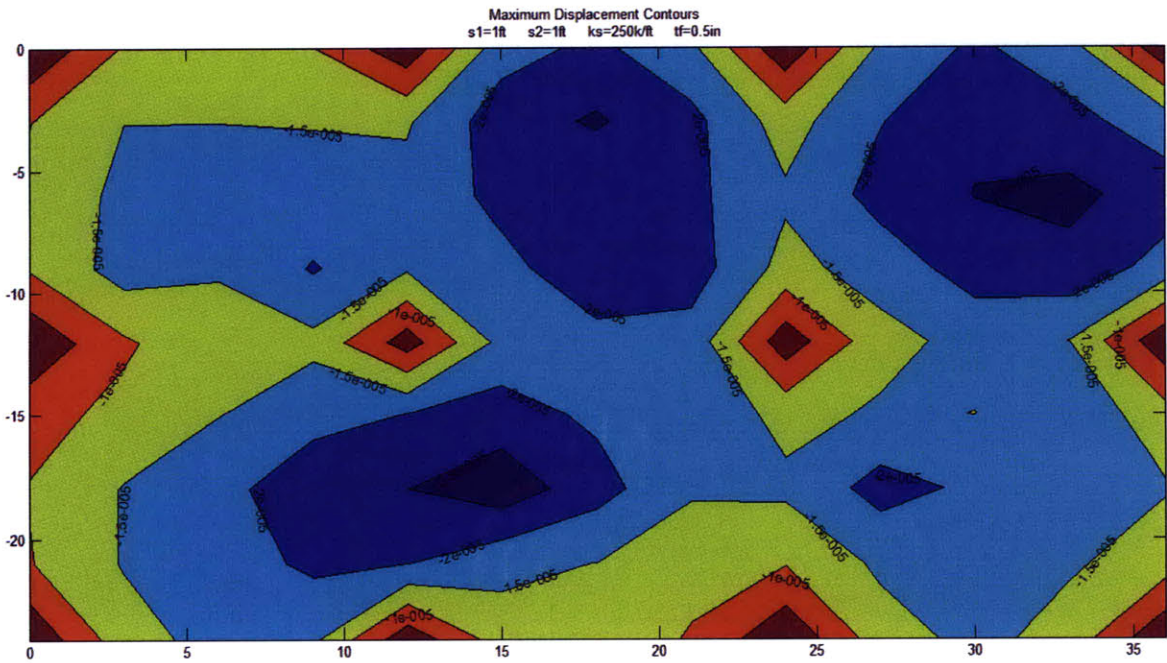
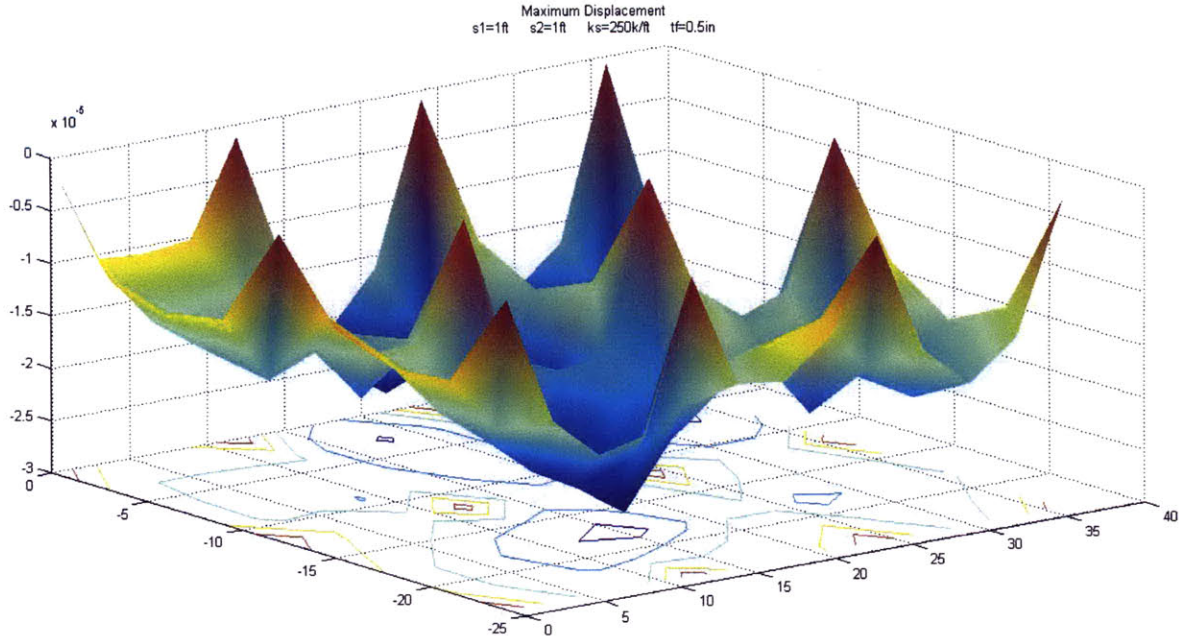


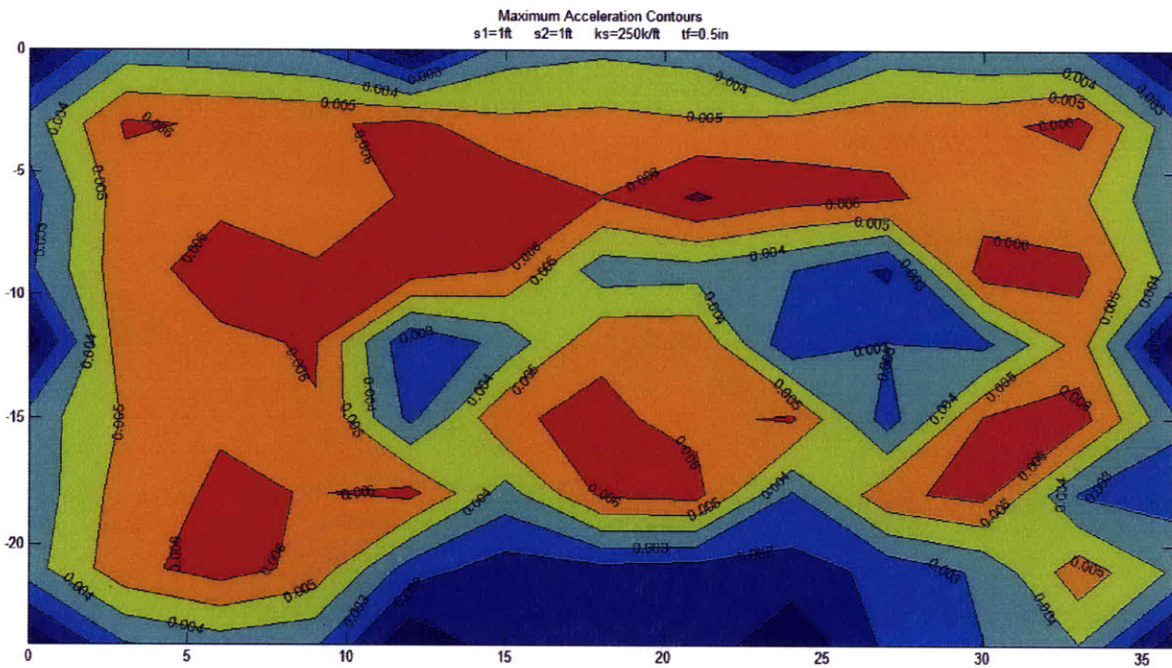
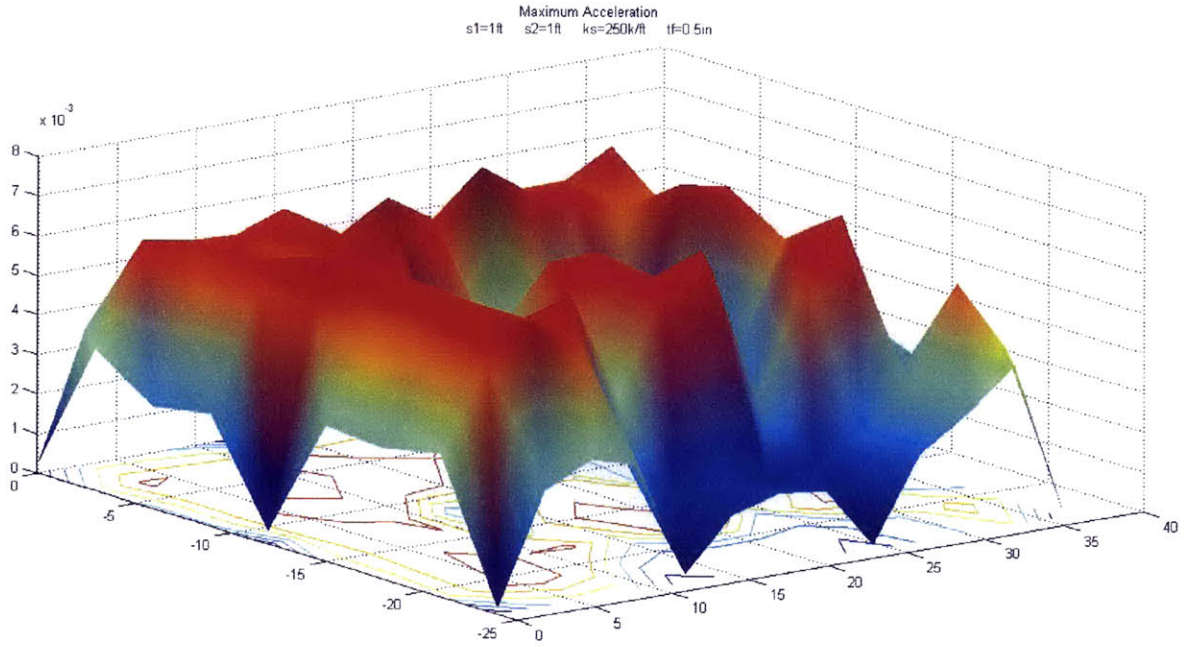
### 8.7 SIMULATION C-1





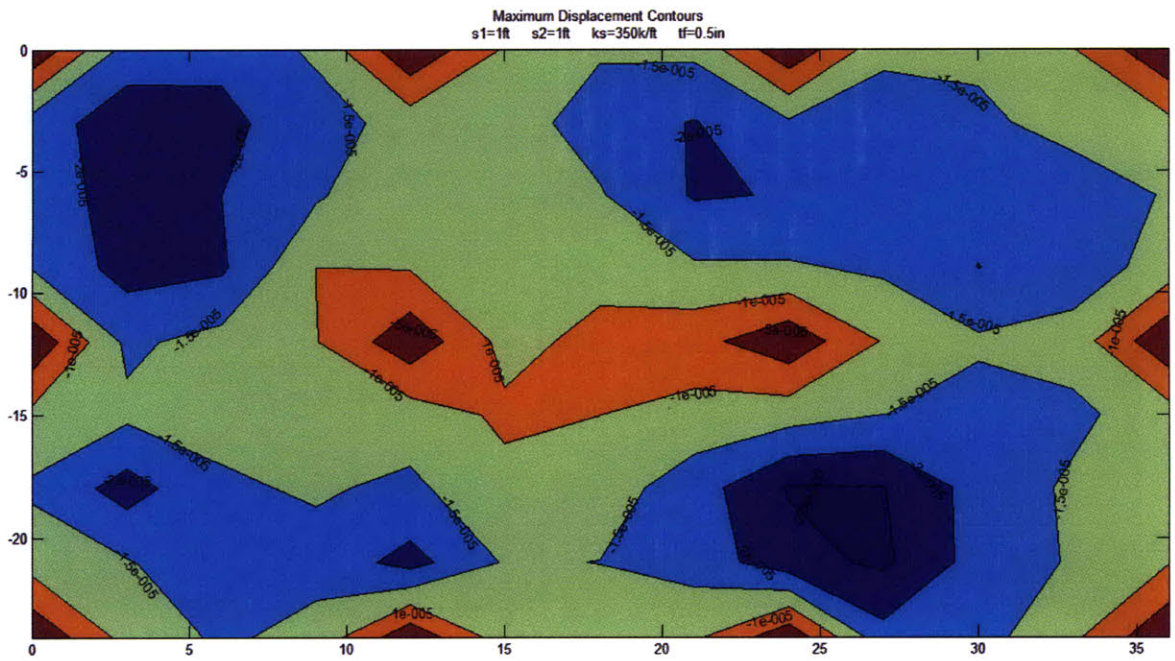
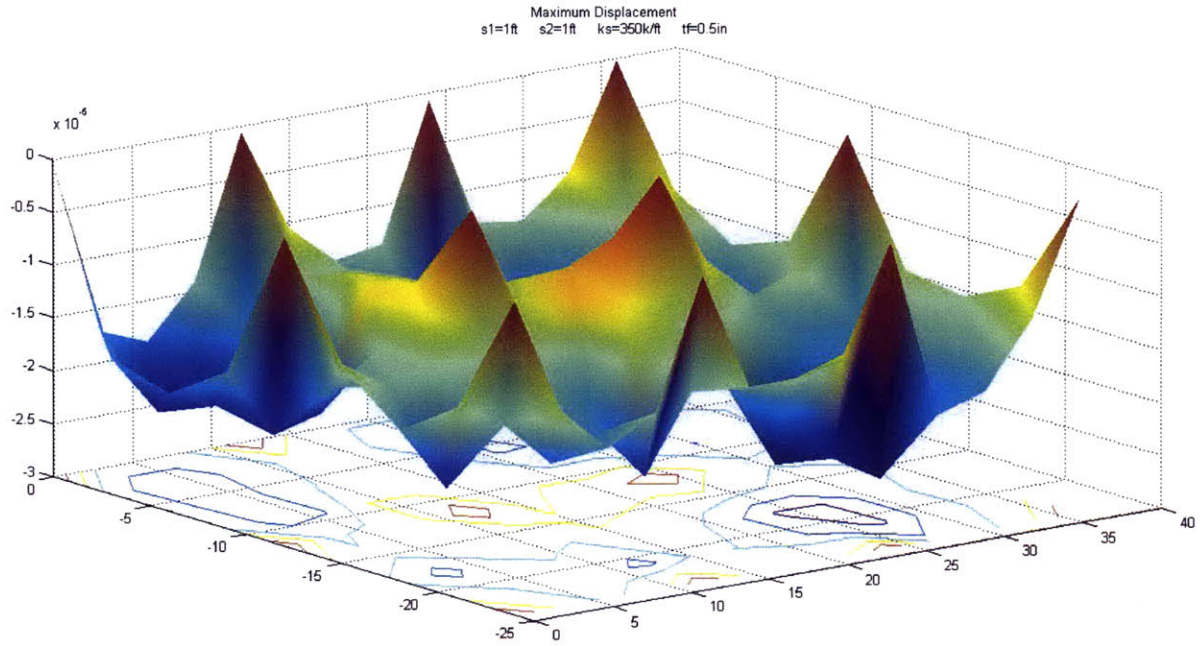
### 8.8 SIMULATION C-2

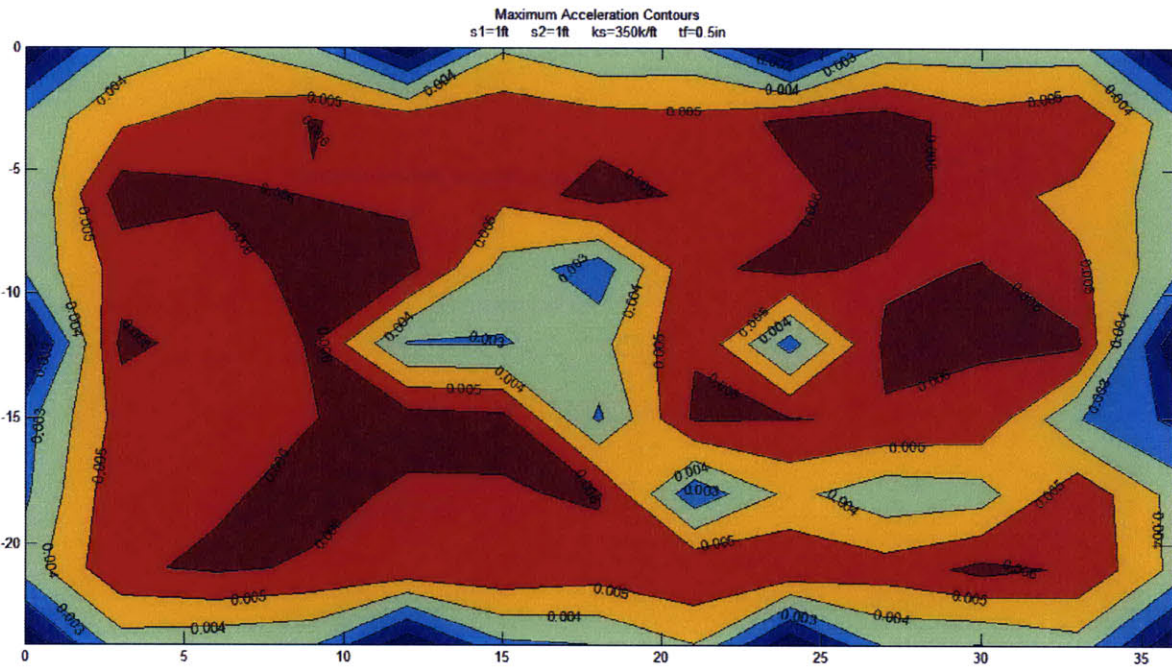
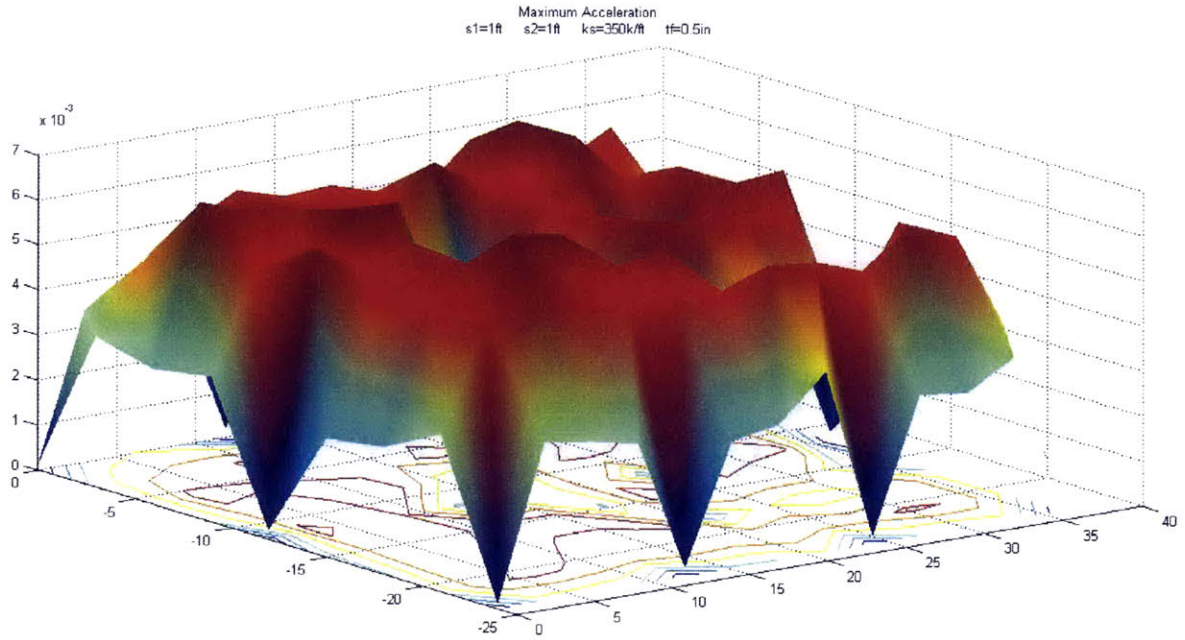




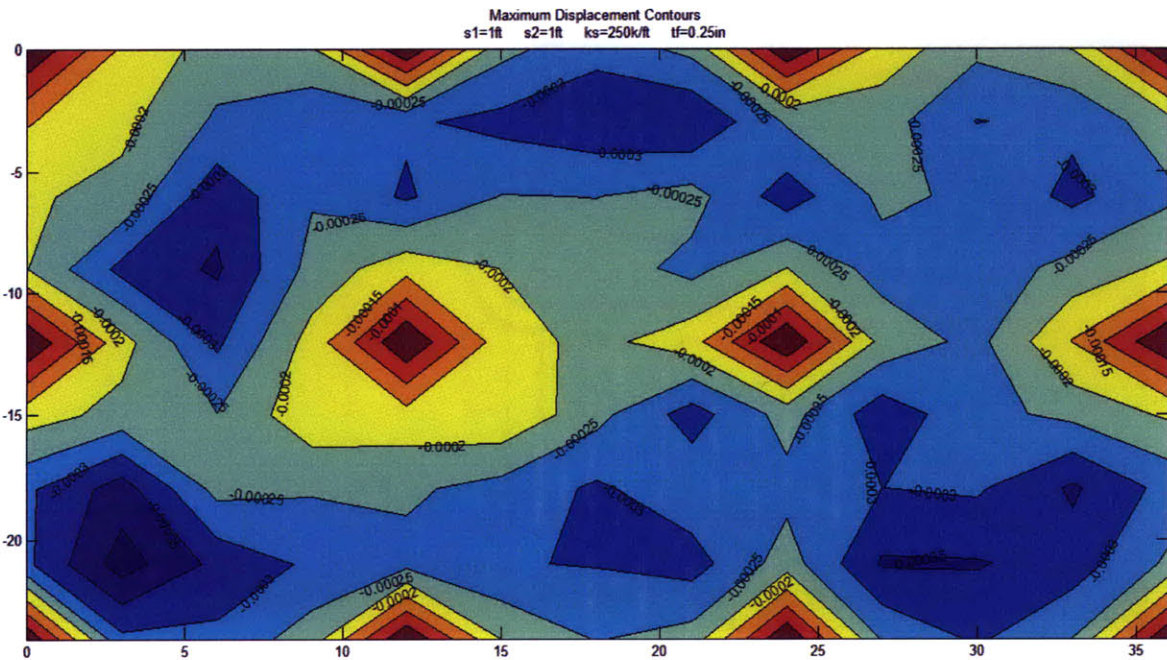
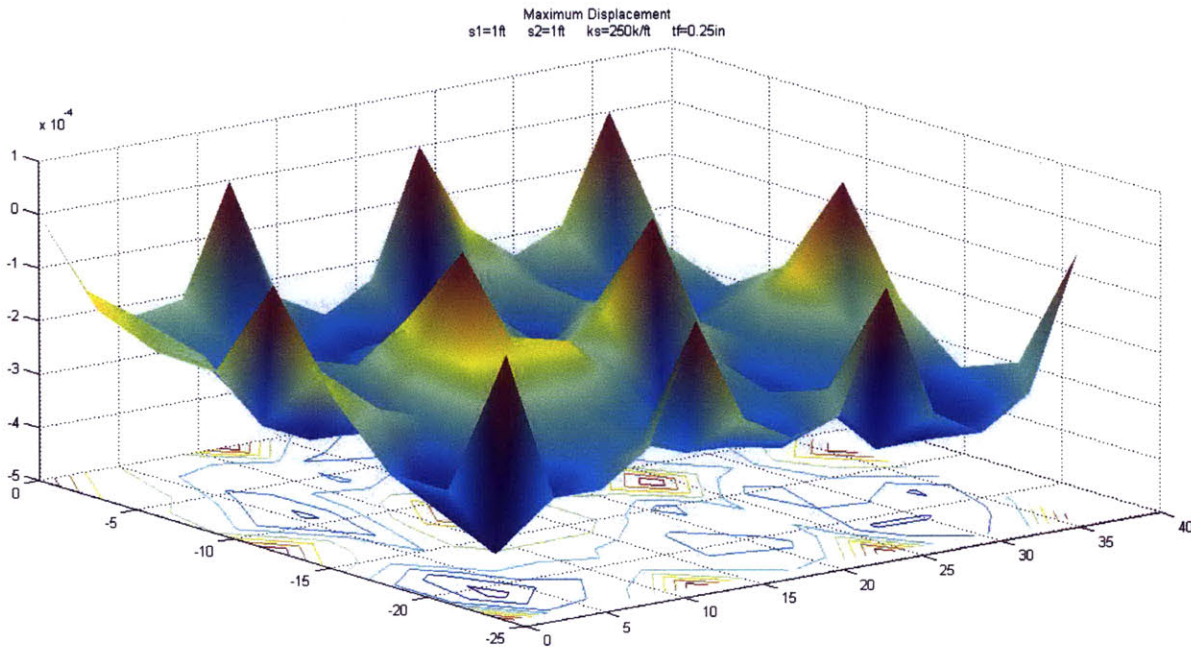


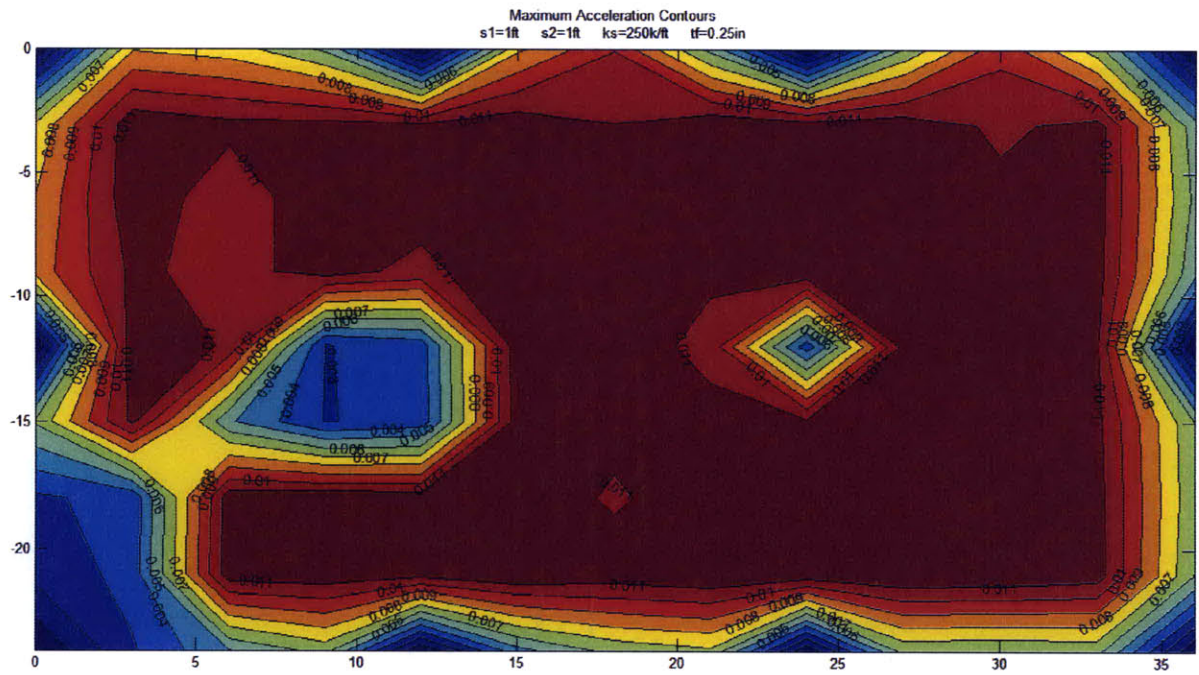
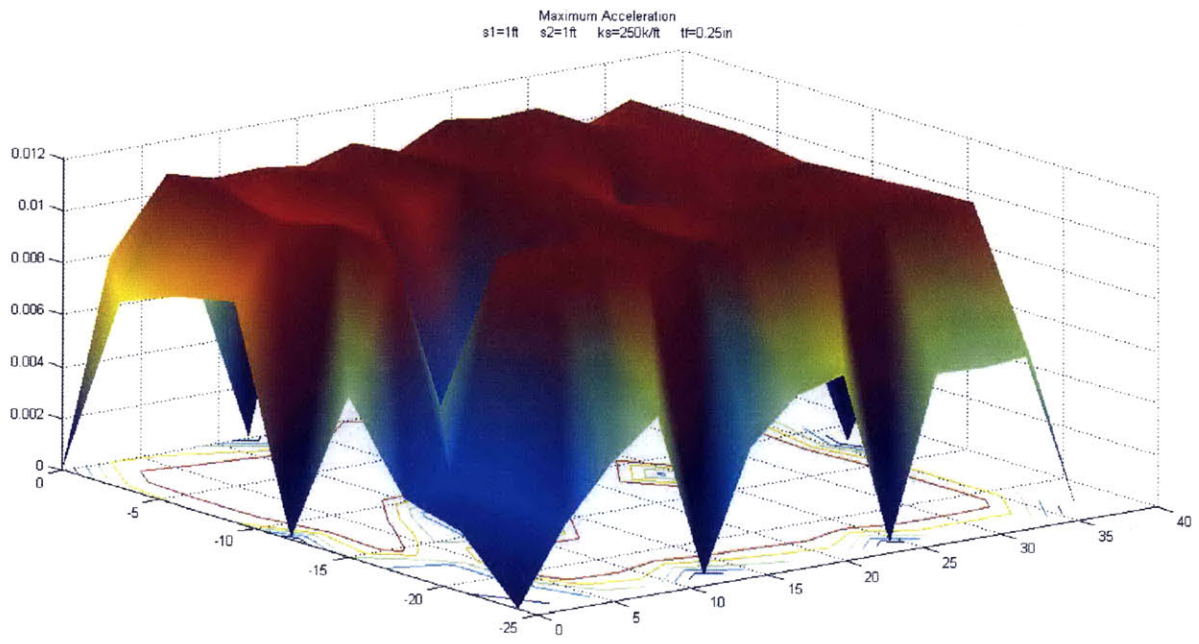
### 8.9 SIMULATION C-3



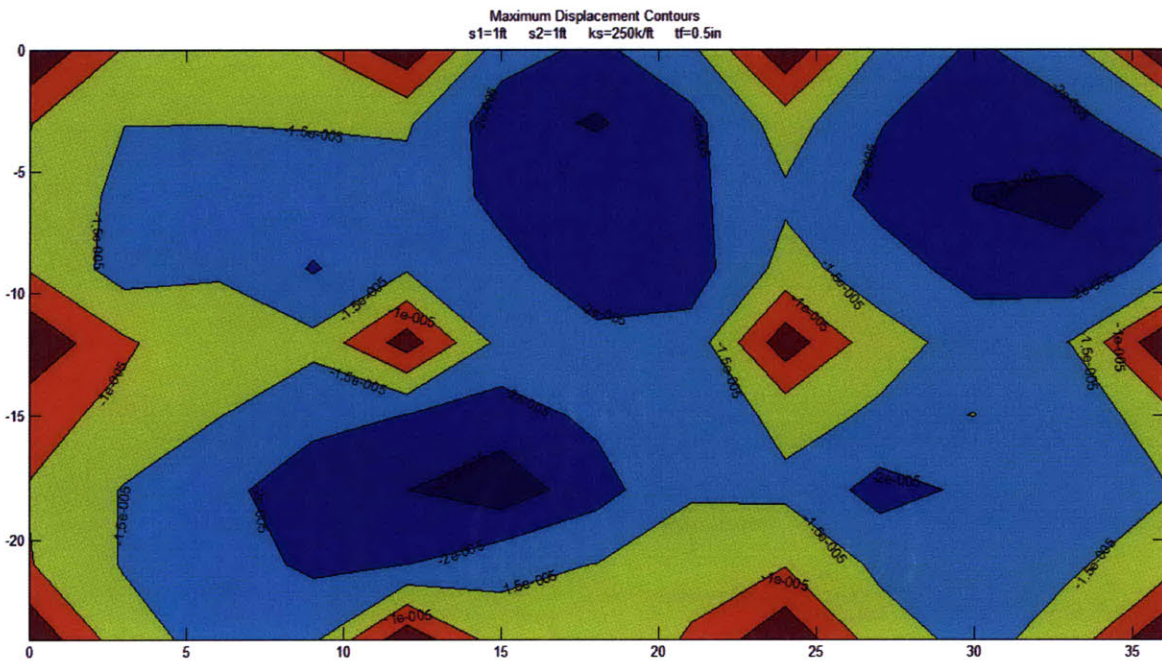
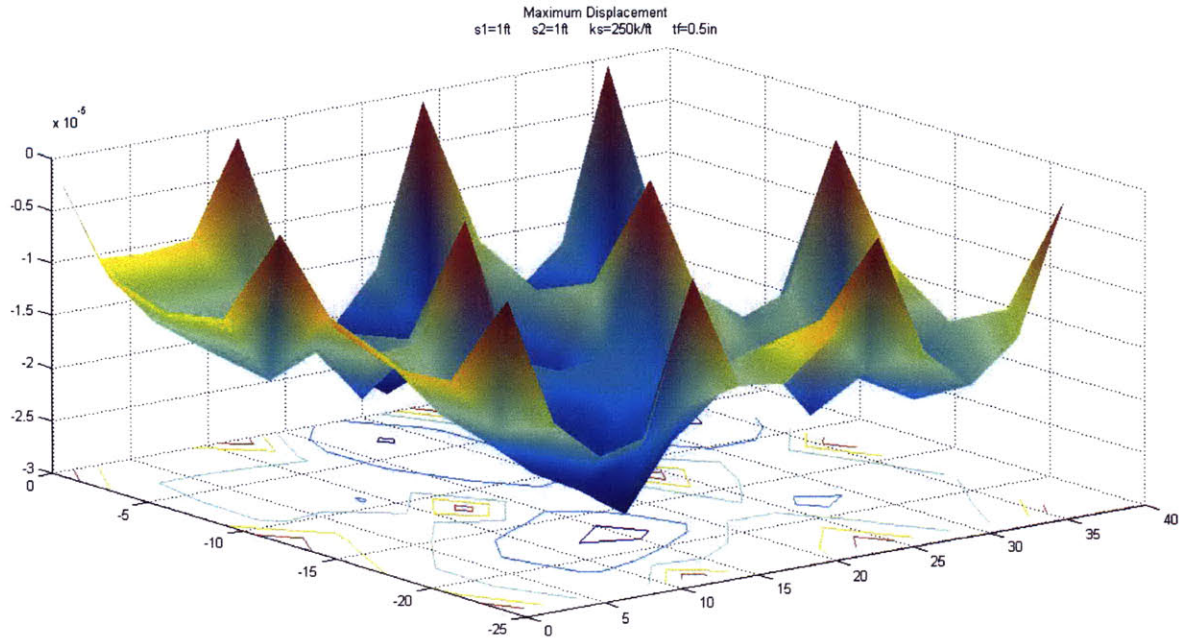


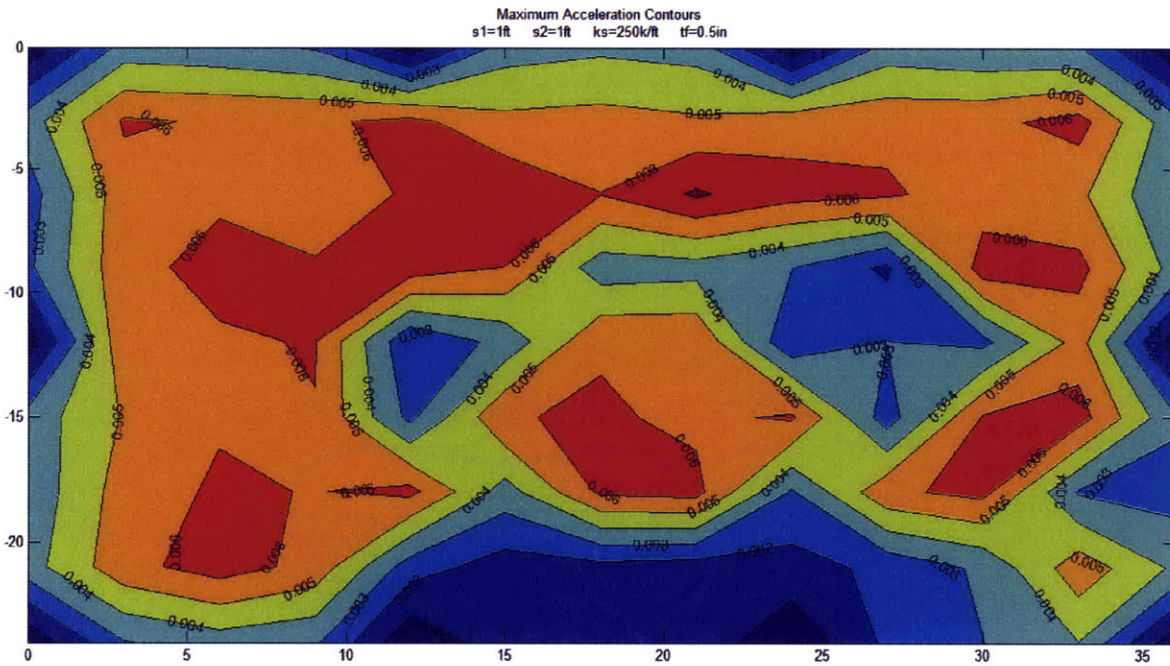
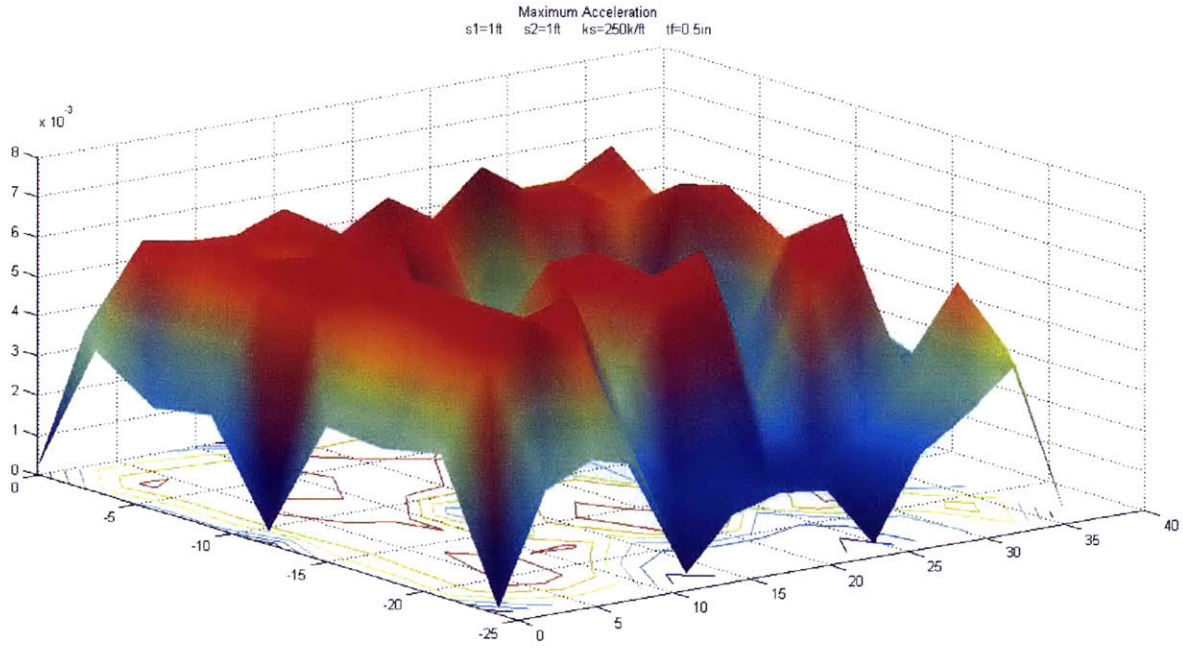
### 8.10 SIMULATION D-1





### 8.11 SIMULATION D-2





### 8.12 SIMULATION D-3

