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THE CORRELATION OF NUCLEATE BOILING BURNOUT DATA

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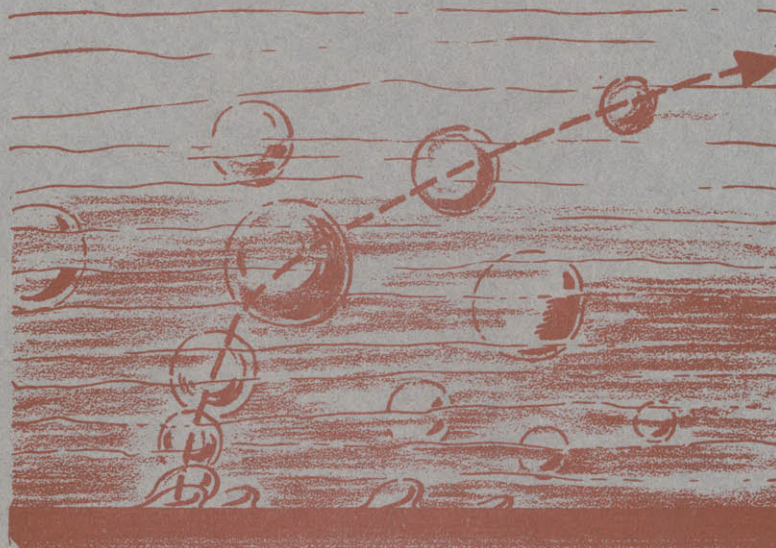
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THE CORRELATION OF NUCLEATE BOILING BURNOUT DATA

Peter Griffith *

ABSTRACT

A dimensionless correlation is developed for nucleate boiling burnout data including the following ranges of variables.

Fluids - Water

Benzene

n - Heptane

n - Pentane

Ethanol

Pressure - 0.0045 to 0.96 of critical pressure

Velocity - 0 to 110 ft/sec.

Subcooling - 0 to 280 °F

Quality - 0 to 70%

The data is drawn from a variety of sources and has been collected on widely varying types of systems. Over 300 points are correlated with 94% of the points included between the $\pm 33\%$ envelope drawn around the best line through the points. The correlation includes only fluid properties and quantities which can be calculated on the assumption of equilibrium conditions at the burnout point.

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LIST OF SYMBOLS

Capitals

- A - Area
 A_1, A_2, A_3 - Dimensionless constants in Equation (6).
 Values given in (8).
 C_q - Fraction of heat transferred to the bubbles. Defined by
 Equation (2).
 C_{vb} - Constant defined by Equation (1).
 D - Pipe diameter.
 D_b - Bubble diameter.
 F - Factor correcting pool boiling heat fluxes for changes in
 conditions. Value given in Equation (8).
 N_{Re} - Reynolds number = $VD\rho_1/\mu_1$.
 P - Pressure.
 P_c - Critical pressure.
 T - Temperature.
 V - Velocity.

Small Letters

- c - Specific heat.
 f - Frequency bubbles form at point on surface.
 g - Acceleration of gravity.
 h - Enthalpy.
 k - Thermal conductivity.
 m - Power on product term of Equation (6) equal to 0.5.
 n - Number of bubbles/unit area at burnout point.
 q - Heat flux.

Greek Letters

- ρ - Density
 μ - Viscosity

Unexplained Subscripts

- | | |
|----------------------|----------------|
| b - Bulk | l - Liquid |
| f - Liquid | m - Maximum |
| g - Vapor | s - Saturation |
| fg - Liquid to vapor | w - Wall |

INTRODUCTION

A large number of dimensional empirical correlations exist in the literature for nucleate boiling burnout data in which an experimenter satisfactorily correlates his data using his particular set of independent variables usually raised to various nonintegral powers with, perhaps, an arbitrary coefficient (1, 3, 4). Attempts to apply these correlations to other experimenter's data have usually met with failure because the coefficients and exponents include system characteristics which are usually different for the different systems. In addition, no general correlation scheme exists, so that it is impossible for a designer to take burnout data for water, for instance, and use it to calculate the burnout heat flux for some organic fluid under similar conditions. This work has been undertaken to fill the need for a general correlation.

Because the burnout process is so complicated, it is unlikely that every possible variable can be included in the correlation. Therefore, attention has been focused on the most important variables and the secondary variables have been ignored. This naturally limits the application to certain classes of problems. The primary variables included in this correlation are pressure, velocity, quality, and subcooling; the secondary variables include surface geometry, surface conditions, and heater strip orientation.

The burnout heat flux is visualized as a local phenomenon, i.e., it is determined by the conditions existing in the immediate vicinity of the heater strip. With the high heat fluxes often associated with burnout, there are large departures from equilibrium conditions in the vicinity of the strip. However, the magnitude and exact nature of these departures are also unknown and cannot be used to determine burnout heat fluxes. Likewise, quantities such as wall superheat, maximum bubble diameter, or the thickness of a superheated layer near the surface, which also are unmeasured or unknown, should not appear in the correlation even though they are well defined and, perhaps, significant in their effect on the burnout process. Therefore, a useful correlation must relate the burnout heat flux to some fictitious equilibrium properties which can be computed. This has been done here.

In any general correlation scheme, the central fact which must be recognized from an examination of the burnout data, is that when small changes in velocity, pressure, subcooling or quality are imposed on the system, there are no resulting abrupt changes in burnout heat flux. This implies that there is no sudden change in the burnout mechanism over most of the ranges of variables considered. This

fact has suggested the correlation scheme presented here.

A good correlation for saturated pool boiling burnout for a number of fluids under a wide variety of pressures already exists (7, 8). This correlation with a rational correction factor containing empirically determined coefficients and powers has been used as a starting point. This factor contains several dimensionless groups incorporating the effects of changes in velocity, quality, and subcooling. In the following sections this correlation scheme will be developed and the limitations on its application pointed out.

THE CORRELATION

When the boiling heat flux is increased, both the number of bubbles on the surface and the fraction of surface covered by bubbles increase. At some point the fraction covered exceeds a certain value, the heat flux is decreased and the heater burns out. Essentially then, burnout is viewed as occurring at a certain critical packing of bubbles on the surface. At the highest qualities this may not be the limiting process, but within very wide limits such a critical packing criterion seems to be valid. In the following section a saturated pool boiling burnout correlation will be developed which has this as its basis. It will then be altered and extended to include the various possible departures from saturated pool boiling conditions.

Let us start by imagining that at burnout the surface is packed with bubbles of uniform size. The number of bubbles per unit length will be equal to $1/D_b$ and the number per unit area will be equal to $1/D_b^2$. The bubbles may not be packed in this manner so let us assume that the number of bubbles per unit area is actually

$$n = \frac{C_{vb}}{D_b^2} \quad (1)$$

in which C_{vb} is a constant which is, at most, a function of pressure. It has been observed that the heat transferred to the bubbles is proportional to the net heat transfer or

$$\left(\frac{q}{A}\right)_{\text{boiling}} = C_q h_{fg} \rho_g n \left(\frac{\pi}{6}\right) D_b^3 f \quad (2)$$

where C_q is the fraction of heat transferred to the bubbles. When (1) is substituted into (2) and the C's evaluated at the maximum heat flux point the

result is

$$\frac{\left(\frac{q}{A}\right)_m}{h_{fg} \rho_g (fD_b)} = \left(\frac{\pi}{6}\right) C_q C_{vb} \quad (3)$$

The constants on the right of equation (6) are assumed to be, at most, functions of pressure. This analysis was presented in reference (8) and correlated data with an error of approximately $\pm 11\%$ by the following equation

$$\frac{(q/A)_m}{\rho_g h_{fg}} = 143 \left(\frac{\rho_l - \rho_g}{\rho_g} \right)^{0.6} \quad (4a)$$

The quantity (fD_b) in equation (3) is the product of the frequency and departure size of the bubbles and is primarily a function of their average growth rate. Equation (3) can then be rewritten as

$$\frac{(q/A)_m}{h_{fg} \rho_g V_{\text{growth}}} = f \left(\frac{P}{P_c} \right) \quad (4b)$$

When conditions are altered from those of saturated pool boiling by changes in velocity or by subcooling, the average growth velocity also changes. Let us continue by considering what the effect is of these changes on the average growth velocity. In reference (7) results of calculations are presented which show that the average growth velocity of a bubble growing on a wall in a temperature gradient is a function of three things: a thermal layer thickness "b", a bubble growth coefficient $\int_1 c_1 (T_w - T_s) / \rho_g h_{fg}$, and a temperature parameter $(T_w - T_s) / (T_w - T_b)$. In order to use these growth rate curves to correlate data, however, it is found necessary to make a number of assumptions about the actual processes occurring near the wall.

These assumptions and the development of the expression for the average growth velocity at saturation conditions are presented in Appendix C. For this application, however, this expression is perhaps best regarded as a group of terms with the correct dimensions and a plausible basis. Perhaps other groups are better, but this seems as good as any. The average growth velocity then, is

$$V_{\text{growth}} = \left[\frac{(\rho_l - \rho_g)g}{\mu_l} \left(\frac{k_1}{\rho_l c_1} \right)^2 \right]^{1/3} \times f \left(\frac{P}{P_c} \right) \quad (5)$$

If equation (5) is substituted in equation (4b) and the resulting burnout group is plotted versus P/P_c , the $f(P/P_c)$ in equation (5) need not be specified.

Next let us consider how equation (4b) should be altered to accommodate changes in flowing velocity and subcooling. The primary effect of both flowing velocity and subcooling is a reduction in the maximum size achieved by the bubble. This is accomplished through a reduction in the thickness of the superheated layer of liquid near the surface. The bubble growth rate calculations in (7) show that as the maximum bubble size is decreased the average growth velocity increases, all other conditions being equal. Therefore, the pool boiling bubble growth velocity should be altered by a factor which introduces this fact in an appropriate manner. This has been accomplished by defining a Reynolds number

$$N_{Re} = \frac{VD\rho_l}{\mu_l}$$

in which the V is the velocity of the material flowing past the burnout point. As the layer of interest is in the liquid, liquid properties have been used. The use of this velocity implies that the liquid and vapor are both moving at the same velocity. The "D" in this number is the hydraulic diameter for the flow passage. For large values of this Reynolds number, the burnout heat flux seemed to be proportional to the Reynolds number, so the first power on the number was chosen with the coefficient left to be determined. Let us now turn our attention to the appropriate subcooling compensation factor.

The bubble growth rates were found to be a function of two parameters;

$$\frac{\rho_l c_l (T_w - T_s)}{\rho_g h_{fg}} \quad \text{and} \quad \left(\frac{T_w - T_s}{T_w - T_b} \right).$$

This pair of parameters could as well have been replaced by the pair

$$\frac{\rho_l c_l (T_w - T_s)}{\rho_g h_{fg}} \quad \text{and} \quad \frac{\rho_l c_l (T_s - T_b)}{\rho_g h_{fg}}$$

in which the unknown " T_w " appears in only one of the parameters. Previously, however, the first parameter was found to be a function primarily of P/P_c which is treated as a variable by plotting the burnout group of equations (4) and (5)

versus P/P_c . The subcooling effect can now be handled by means of the dimensionless group $\rho_1 c_1 (T_s - T_b) / \rho_g h_{fg}$ alone. A cross plot of the computed bubble growth rate results (7) indicated the power of this group should be about 1.

From theoretical considerations it seems also that a product of the subcooling and velocity terms is also desirable so this has been included in the correction factor. This correction factor is then:

$$F = 1 + A_1 \left(\frac{VD \rho_1}{\mu_1} \right) + A_2 \left(\frac{\rho_1 c_1 (T_s - T_b)}{\rho_g h_{fg}} \right) + A_3 \left[\left(\frac{VD \rho_1}{\mu_1} \right) \left(\frac{\rho_1 c_1 (T_s - T_b)}{\rho_g h_{fg}} \right) \right]^m \quad (6)$$

We shall now briefly turn our attention to a change in equation (4b) which should be made to extend its range of application. After this we shall return to consider how the constants in equation (6) are determined.

When a bubble departs from the surface in saturated pool boiling, the liquid rushes in to replace has an enthalpy equal to h_f . Some of this liquid is converted to vapor, experiencing a change in enthalpy h_{fg} . If the liquid in the bulk is subcooled, the liquid has an enthalpy h_b and experiences a change in enthalpy $h_g - h_b$. Therefore, it is appropriate to replace h_{fg} by $(h_g - h_b)$. The same reasoning would apply in the quality region, except that what replaces a bubble would generally include both liquid and vapor. Burnout heat flux decreases when this enthalpy difference decreases and when $(q/A)_m$ goes to zero (at 100% quality) this enthalpy difference also is zero.

The various coefficients in equation (6) for the factor "F" were determined as follows. Equation (4b) is written below with the change suggested in the previous paragraph and the V_{growth} as given in equation (5).

$$\frac{(q/A)_m}{(h_g - h_b) \rho_g \left[\left(\frac{\rho_1 - \rho_g}{\mu_1} \right) g \left(\frac{k_1}{\rho_1 c_1} \right)^2 \right]^{1/3} F} = f \left(\frac{P}{P_c} \right) \quad (7)$$

where F is defined by equation (6). When the degree of subcooling is 0, both the product term and the subcooling term are 0. Thus "A₁" is determined by plotting the left side of (7), with F omitted, versus $VD \rho_1 / \mu_1$ for various pressures. All the forced convective data with $h_b > h_f$ has been used to determine A₁. The coefficient A₂ is determined by plotting, in the same manner

as above, the only pool boiling subcooled data available (2) and choosing the best value for A_2 . With the values determined earlier for A_1 and A_2 , with $A_3 = 0$, the plot of equation (7) versus the product $(VD \rho_1 / \mu_1) (\rho_1 c_1 (T_s - T_b) / \rho_g h_{fg})$ has been made for the various pressures and the best values for the exponent and the coefficient chosen. These data included points at both high and low pressures. When the factor "F" is written with the factors A_1 , A_2 , and A_3 , and the exponent "m" inserted, it becomes

$$F = 1 + 10^{-6} \left(\frac{VD \rho_1}{\mu_1} \right) + 0.014 \left(\frac{\rho_1 c_1 (T_s - T_b)}{\rho_g h_{fg}} \right) + 0.5 \times 10^{-3} \left[\left(\frac{VD \rho_1}{\mu_1} \right) \frac{\rho_1 c_1 (T_s - T_b)}{\rho_g h_{fg}} \right]^{0.5} \quad (8)$$

No more significant figures are presented for the coefficients than are felt to be justified by the scatter in the data. It is desired also to keep the functional relationships as simple as possible and the coefficients as few as possible. For the data correlated here, F has the range 1 to 9 with its value in the high pressure range varying from 1 to 2.5. Other correlation schemes are possible with these data and dimensionless groups, yet it does not seem possible to materially reduce the complication of the correlation scheme or the number of arbitrary constants.

DATA CORRELATED

All the burnout data available has been collected and examined. Not all that is available, however, appears in this correlation. The excluded data has been omitted for any one of the following reasons.

1. The burnout was due to flow instability so that the burnout heat fluxes were characteristic of the size of the flow fluctuation and the system dynamics, not only time average local conditions.
2. The burnout data was badly inconsistent internally or scattered so much as to cast serious doubt as to the care taken in its collection.
3. The quality at the burnout point was above 70%. These high quality points did not correlate and their exclusion is justified because it is felt the limiting process is not the same as for the other data. A correlation of these data based on a more appropriate limiting process is under consideration now.

Individual points which deviated, though they did not come under one of the three categories listed above, were not excluded as it was felt that the validity

of the correlation would be seriously open to question if such points were arbitrarily excluded.

It is appropriate at this time to comment on the burnout process and on how some of the deviations from this correlation might arise. Most nucleate boiling is at a sufficiently high heat flux so that if a pulsation of flow, heat flux, or quality occurs, for even a rather short period, it is possible that the transition to film boiling could occur at an average heat flux which is appreciably lower than the maximum attainable steady state heat flux. However, it is only the peak flux on the temperature difference-heat flux curve which is unique, so that a burnout heat flux below this peak value is characteristic of a disturbed system. A heat flux measurement taken on this kind of system without any information as to the nature and magnitude of the disturbance is difficult to interpret. Therefore, the highest burnout heat fluxes measured for a given set of operating conditions represent the reproducible values. In running the burnout apparatus at M.I.T. it has been found that a premature burnout occurs if the power on the test section is increased too rapidly or too unevenly. It is, therefore, only the reproducible points in the M.I.T. data which are reported.

RESULTS

The results are presented in Figure 1. The maximum range of variables covered on this curve are as follows:

$(q/A)_m$ - 0.075 to 11.4×10^6 BTU/hr-ft²

Velocity - 0 to 110 ft/sec

Fluids - Water

Benzene

n - Pentane

n - Heptane

Subcooling - 0 to 280 °F

Pressure - 0.0045 to 0.97 of critical pressure

Quality - 0 to 70%

94% of the points are included between the $\pm 33\%$ envelope drawn around the best line through the points.

In order to aid in the calculation of the burnout heat flux for water at a given condition, the property groups appearing in equations (5), (7), and (8) are plotted in Figure 2.

The range of validity of the correlation is limited by the nature of the

data that was used to determine the empirical coefficients. The types of systems included in this correlation are tabulated in Appendix A. Heater strips or tubes thinner than those mentioned in Appendix A are likely to decrease the burnout heat flux. This is so because the possibility of a local hot spot being able to dissipate its excess heat to the surrounding material is limited if the strip is very thin. Gunther (1) indicates that the strip used in his experiments was thin enough to affect his results. However, when the velocity, pressure or subcooling increases the dependence of the burnout heat flux on the strip thickness decreases. This is so because the maximum bubble size is decreased by any of these changes, and the possibility of a local hot spot under a single large bubble is decreased. An examination of his forced convection points indicates that only low velocity, pressure and subcooling points are outside the $\pm 33\%$ limits.

There is also a minimum channel height for which this correlation is valid. The data of References (3 & 9) is close to it. Aside from its effect on the Reynolds number in the factor "F", the channel height seems to be important at approximately the height at which an individual bubble is able to span the channel. Under these circumstances the nature of the boiling process is changed. A large number of the points outside the lower envelope at $P/P_c = 0.62$ are from the small tubes. Enough points correlate, however, so that it is felt that the small tube data should be included.

In general, the heater geometry does not seem to be important in its effect on the burnout heat flux since wires, tubes, plates, and strips all correlate here. Likewise, surface conditions, such as roughness or contact angle, do not appear to be important variables. Some experimenters apparently find that large contact angles led here to premature burnouts, but our experience has been that it is difficult to maintain a large contact angle. Therefore, the surface is not an important variable from system to system.

In the course of this correlation work, several gaps in the experimental programs became apparent. Very little data exists below 500 psia for water, in particular very little has been taken between 100 psia and 500 psia. Also, all the coefficients for "F" are calculated on water because, to the author's knowledge, no forced convective quality or subcooled data has been published for other fluids. It would be worthwhile to test some other substance to see if the dimensionless coefficients calculated on water are as general as is hoped.

ACKNOWLEDGMENTS

Thanks are due to John Reynolds for the information on the importance of the running procedure on the heat flux observed at burnout and for the many interesting discussions we have had on the nature of burnout.

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Source	Fluid	Pressure	Geometry	Length	Material	Heating	Comment
(1)	Water	14.7 to 164 psia	$\frac{1}{16}$ " by $\frac{1}{2}$ " channel with $\frac{1}{8}$ " x 0.004" strip.	6"		DC	Low subcooling and velocity points low because of heater strip thickness.
(2)	Water	14.7 psia	Strip in lucite .002" thick x .125" wide	4"	304 Stain- less steel	DC	Pool boiling saturated and subcooled.
(3)	Water	2000 psia	0.18" I Dia 0.306" I Dia	11.6" and 23.2	Nickel "A"	DC	
(4)	Water	500 1000 2000 psia	Tube 0.226 ID	24.6"	Stainless	DC	All subcooled forced con- vection points for UCLA data.
(5)	Ethanol n-Pentane n-Heptane Benzene	14.7 to 195 psia	Flat surface facing up		Chromium plated copper	Indirect	Only clean surface data used, pool boiling.
(6)	Water	14.7 to 2650 psia	0.024" OD wire		Platinum	DC	Pool boiling saturated. It is felt the highest pressure points are higher than could be attained on a plane surface.
(9)	Water	500 to 1500 psia	Tube .18" ID	9"	Nickel	DC	

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APPENDIX C

Development of the Expression of the Bubble Growth Velocity

The bubble/growth rate calculations of reference (7) give dimensionless bubble radius and growth velocity as a function of dimensionless time for various values of $(T_w - T_s)/(T_w - T_b)$ and $\rho_1 c_1 (T_w - T_b) / \rho_g h_{fg}$. The definitions of the various terms mentioned here are:

- b = thickness of heated layer near the surface, dimensions of length
 $Y = R/b$, dimensionless bubble radius C-1
 $\Upsilon = k_1 t / \rho_1 c_1 b^2$, dimensionless time C-2
 $\theta = (T - T_b) / (T_w - T_b)$, dimensionless temperature C-3
 $v = V_{\text{growth}} b (\rho_1 c_1 / k_1)$, dimensionless velocity C-4

When the bulk of the liquid is at saturation temperature the growth velocity becomes a function of $\rho_1 c_1 (T_w - T_b) / \rho_g h_{fg}$ alone. In the course of the calculations (7) it became apparent that this parameter is primarily a function of the critical pressure ratio (P/P_c) so that $\rho_1 c_1 (T_w - T_s) / h_{fg} \rho_g$ can be replaced by a function, $f(P/P_c)$, which is left unspecified.

Let us start the calculation for the average growth velocity by specifying the characteristic dimension "b", then see how "b" is fixed by the boiling process, and finally calculate the average growth velocity. Assume the true temperature distribution in the liquid is approximated by a constant temperature in the bulk and a straight line temperature distribution from the surface temperature to the bulk temperature. Call the distance from the wall to the point where there is a constant bulk temperature "b". The length "b" can now be determined by considering the unsteady heat transfer to the liquid after a bubble departs. It has been noticed that the delay between bubble departure and the initiation of the next bubble is approximately equal to the life of the bubble on the surface*. During this time the cool liquid rushing in after the bubble departs is being heated. If the liquid is treated as an infinite slab of stagnant liquid then the temperature distribution is represented by an error function (erf) curve. Approximating the "erf" curve by two straight lines leads to

$$b = 2 \sqrt{\frac{k_1 t_b}{\rho_1 c_1}} \quad \text{C-5}$$

* Heat Transfer, M. Jakob, John Wiley and Sons, Vol. 1, p. 632, 1955.

where t_b is time the bubble spends on the surface and is assumed equal to the delay between bubbles. Eliminate "b" between C-2 and C-5 and the result is

$$\tau = 0.25. \quad C-6$$

This is the dimensionless time at which the dimensionless radius and velocity are evaluated. The range of bubble sizes encountered is such that the rise velocity is adequately represented by Stokes Law

$$V_{\text{rise}} = \frac{g(\rho_1 - \rho_g)D_b^2}{18\mu_1} \quad C-7$$

The definition of dimensionless velocity at any time gives

$$V_{\text{growth}} = \frac{v}{b} \left(\frac{k_1}{\rho_1 c_1} \right) \quad C-8$$

The definition of Y gives

$$R = bY \quad C-9$$

As the bubble grows on the surface it is approximately hemispherical but when it departs it is approximately spherical. Therefore, the diameter of the spherical bubble equal in volume to that of a hemisphere of radius R is

$$D_b = 1.59 bY \quad C-10$$

in which R has been eliminated by use of C-9. When a bubble departs, its rise velocity must be at least equal to its growth velocity. Let us assume this is a sufficient condition and specify $V_{\text{rise}} = V_{\text{growth}}$ at departure and equate C-7 and C-8. When D_b is eliminated with C-10, then an expression for b is

$$b = \left[\frac{18}{(1.59)^2} \left(\frac{v}{Y^2} \right) \right]^{1/3} \left[\frac{\mu_1}{g(\rho_1 - \rho_g)} \left(\frac{k_1}{\rho_1 c_1} \right)^2 \right]^{1/3} \quad C-11$$

The average growth velocity is

$$V_{\text{growth average}} = \frac{D_b}{t_b} \quad C-12$$

When equations C-5, C-8, C-9, C-10, and C-11 are substituted into C-12 then

$$v_{\text{growth average}} = \left[\frac{1.59 Y}{(.25) \left[\frac{18}{(1.59)^2} \left(\frac{v}{Y^2} \right) \right]^{1/3}} \right] \left\{ \frac{\mu_1}{g(\rho_1 - \rho_g)} \left(\frac{\rho_1}{k_1 c_1} \right)^2 \right\}^{1/3} \quad \text{C-13}$$

The terms appearing in the first brackets are dimensionless and at $\tau = .25$ are constants or functions of (P/P_c) . Therefore, let us rewrite C-13 as

$$v_{\text{growth average}} = \left[\frac{\mu_1}{g(\rho_1 - \rho_g)} \left(\frac{\rho_1}{k_1 c_1} \right)^2 \right]^{1/3} f\left(\frac{P}{P_c}\right)$$

This is the average growth velocity which is to be substituted in equation (5) of the text.

CAPTIONS

Figure 1 Water property groups for substitution in equations 7 and 8 with the assistance of Figure 2.

Figure 2 Burnout correlation. The experimental conditions are tabulated in Appendix A. The strokes to the right of the vertical lines are points at the pressure corresponding to the vertical lines.

