LECTURE 24

Last time:

- Using random binning in broadcast channels
- Marton's region

Lecture outline

- Fading Channels
  - Model and Problems
  - Outage vs Ergodic capacity
  - CSI

Review

- Marton's region

$$
\begin{aligned}
R_1 &\leq I(U;Y_1) \\
R_2 &\leq I(V;Y_2) \\
R_1 + R_2 &\leq I(U;Y_1) + I(V;Y_2) - I(U;V)
\end{aligned}
$$

A few important points

- Random binning, even useful in deterministic channels
- Controlling the output, range of channel function vs. typical set.
- Deterministic channel and random channel. By choosing $2^{nI(U;Y_1)} < 2^{nI(X;Y_1)}$ $\underline{U}$ sequences, I ensure that for each $\underline{X}$, there is at most one typical $\underline{U}$ that is jointly typical.

Wireless Channels

Key difference from AWGN channel Multi-path Fading

Flat fading model

$$Y_i = H_i X_i + W_i$$

where W_i is the AWGN with variance σ_W^2.

- $\{H_i\}$ is a random process, for which the marginal distribution is modelled as **unit variance** Rayleigh or Ricean
- The rate at which $\{H_i\}$ changes over time depends on
  - the speed that receiver moves,
  - the environment
  - the carrier frequency
  - the symbol period

The wireless challenge

- How do we define a capacity for this random channel
  - the assumptions on the availability of CSI
  - the assumptions on the channel time-variation
- How well the different assumptions apply to practical channels
- How do the optimal signaling change with the channel assumptions
- How do these apply when we have a network

Example TCP

First Simple Example

Assume H is fixed

$$C(H) = \max_{P_X} I(X;Y) = \frac{1}{2}\log\left(1 + |H|^2 \frac{\sigma_X^2}{\sigma_W^2}\right)$$

Optimal input $X \sim N(0, \sigma_X^2)$.

Outage Capacity and Ergodic Capacity

- Let's assume the input to be $X \sim N(0, \sigma_X^2)$.
- What if H is randomly chosen fixed value?
  - Rayleigh
  - Ricean
- The Shannon capacity is 0.
- **Outage capacity** with a small exception, say 1%, what is the rate that can be supported?

$$\max\{R : P(I(H) < R) \leq 1\%\}$$

- Assume now $\{H_i\}$ is an ergodic random process. Coherence time measures the speed $\{H_i\}$ changes over time.
- **Ergodic capacity**

$$C = E_H[I(H)]$$

- To achieve the ergodic capacity, we need to code over a time range that is much larger then the coherence time.

Closer Look at the Ergodic Capacity

- Interleaving, delay, and burstiness.
- w.o.l.g. assume H_i's are i.i.d.
- Evaluating the ergodic capacity is hard.
- as SNR $\to 0$,

$$\begin{aligned} C &= E_H[\frac{1}{2}\log(1+|H|^2 SNR)] \\ &\approx \frac{1}{2}E[|H|^2] + \frac{1}{2}SNR \end{aligned}$$

  Same as the AWGN channel

- as SNR $\to \infty$,

$$\begin{aligned} C &= E_H[\frac{1}{2}\log(1+|H|^2 SNR)] \\ &= \frac{1}{2}\log SNR + \frac{1}{2}E[\log|H|^2] \end{aligned}$$

  Differ from AWGN channel by a constant factor.

Wait a Minute

Why should we fix the input to be i.i.d. Gaussian ?

- If we allow the input distribution to depend on H_i, then water-filling is the optimal input.
- Extreme case, we have very litter power, then we only transmit when H is extremely high.
- Requires transmitter side channel state information, CSI
- Power efficiency, feedback, burstiness again.

Without Tx CSI

Assume the transmitter does not have CSI, but the receiver does.

$$C = \max_{P_X} I(X;Y)$$

where

- the input P_X must not depend on H, distinguish from the notation $I(X;Y|H)$.
- the channel can be written as $P_{Y|X} \sim N(hX, \sigma_W^2)$.
- the optimal input must be Gaussian, so that Y is Gaussian
- the input must be i.i.d. by concavity argument.
- Can generalize to any stationary random process $\{H_i\}$.

How do We obtain CSI at the Receiver

- send training sequences to help the transmitter estimate the channel

$$\underline{y}_t = H\underline{x}_t + \underline{w}_t$$

- Tradeoff: more time and energy to estimate the channel, less time to communicate.
- Modelling the time variation of the channel states
  - block fading model
  - Markov process

Example block fading with block length $l > 1$.

$$\underline{y} = \hat{H}\underline{x} + \tilde{H}\underline{x} + \underline{w}$$

- the error in channel measurement becomes the extra noise
- lose $3dB$ (double the noise power) at high SNR

- resulting channel capacity per block is

$$\frac{l - l_t}{l} \log SNR - c$$

- Use one symbol time to train.
- very inefficient when SNR is low.

Non-coherent Communication

- Can we communicate without the receiver knowing the channel?

Example Communication with the direction of vectors.

- Do we really know the channel or not?

Communication is feasible when the corresponding randomness is resolved

Example Low SNR peaky signaling.