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MECHANISMS OF ISOTHERMAL AND NON-ISOTHERMAL FLOW OF FLUIDS IN PIPES.

by

Eugene Chen Koo

S.B., Massachusetts Institute of Technology, 1929

Submitted in Partial Fulfillment of the
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DOCTOR OF SCIENCE
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Signature of Author. . . . .

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v. 1

38

Cambridge, Massachusetts
May 13, 1932

Professor A.L. Merrill
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dear Sir:

I submit herewith a thesis entitled "Mechanisms of Isothermal and Non-Isothermal Flow of Fluids in Pipes", in partial fulfillment of the requirements for the degree of Doctor of Science in Chemical Engineering.

Respectfully yours,

Eugene Chen Koo

186840

ACKNOWLEDGMENT

The author wishes to express his sincere appreciation of the ~~critical~~ criticisms and helpful suggestions of Professor W.H. McAdams, under whose direction this investigation was performed, and Mr. T.B. Drew, under whose supervision the experimental part was carried out. It is a pleasure to acknowledge the financial aid received from the Chemical Engineering Department and the Chinese Educational Mission at Washington. He wishes to thank Mr. H.C. Sung for his assistance with a part of the experimental work.

APPENDIX K

Table of Nomenclature

a = Velocity Distribution Exponent in Eq. $V/V_{max.} = (1-r/R)^a$.

A = Area in square feet.

b = Temperature Distribution Exponent in Eq.

$$\frac{t_w - t}{t_w - t_a} = (1-r/R)^b$$

C = Specific heat of fluid at constant pressure.

d = Differential Prefix.

D or I.D. Inside pipe diameter in any convenient unit.

D" = Inside pipe diameter in inches.

f = Fanning Friction Factor (no units)

F = Surface friction between Fluid and Wall

g = 32.2 ft. per sec. per sec.

h = film coefficient of heat transfer, pipe to water, B.t.u. per hour per foot^2 per ° F., based on arithmetic mean temperature drop across water film.

Δh = Differential head and pressure drop, expressed in feet of fluid.

k = thermal conductivity, B.t.u. per hour per square foot per ° F. per foot thickness.

L = Friction Length of straight pipe in feet.

L'= Prandtl's mixing Length.

m = velocity distribution exponent in Eq.

$$\frac{V}{V_{max.}} = \left[1 - (r/R)^2\right]^m.$$

m' = velocity distribution exponent in Eq.

$$\frac{V}{V_{max.}} = \left[1 - (r/R)^{1.25}\right]^{m'}$$

P = Absolute Pressure in lbs. per square feet.

ΔP = Differential Pressure and Pressure Drop.

Q = Quantity of heat transferred in B.t.u.

Q' = Rate of flow of Fluid in ft.^{3}/sec.

r = variable radius or distance from axis of Pipe.

R = Inside pipe radius.

r/R = Fraction of radius of Pipe.

S = Specific gravity of fluid at temperature in question.

t = Variable temperature at distance r from pipe axis in ° C.

t_a = Axial temperature in ° C.

t_e = Average downstream exit temperature in ° C.

t_f = Average film temperature or effective film temperature in ° C.

t_i = Average upstream inlet temperature in ° C.

t_m = Mixing cup temperature in ° C. (obtained through graphical integration).

t_w = Inside wall temperature of pipe in ° C. (Calculated).

$t_{o.w.}$ Outside wall temperature of pipe in ° C. (Measured).

$t_{ave.}$ Average cross-sectional temperature in ° C. (obtained through graphical integration.)

$\frac{\Delta t}{\Delta t_{max}} = \frac{t_w - t}{t_w - t_a}$ = Fraction of temperature drop, variable with r.

U = Average velocity in feet per second.

v or V = Variable velocity at distance r from pipe axis.

V_{av} = Average velocity in feet per second.

$V_{max.}$ = Maximum or axial velocity

$V_{ave.}/V_{max.}$ = Ratio of average to maximum velocity, or velocity ratio.

X = Calming Section Length in feet.

Z_m = viscosity, centipoises, taken at arithmetic mean of average cross-sectional temperature between two sections.

Z = viscosity, Centipoises.

μ (Mu)=Absolute viscosity of fluids.

ρ(Rho)=Fluid density as lbs. per ft.^{3}

ν(Nu)= Kinematic Viscosity of fluid = μ/ρ

λ(Lamba)=4f

θ (Theta)=Time in any convenient unit.

$Re = \frac{D\ V_{av}\ \rho}{\mu}$ = Reynolds number in Consistent Units (Dimensionless)

$Re_{max} = \frac{D\ V_{max}\ \rho}{\mu}$ Maximum Reynolds number.

$Pr, = \frac{c\mu}{k}$ = Prandlt's number in consistent units (Dimensionless)

Pe' = Peclet Number = (Re)(Pr)

ABSTRACT

The primary object of this investigation has been to determine experimentally the velocity and temperature distributions over a pipe cross-section obtained when water is heated or cooled as it is pumped through a vertical copper pipe. From the experiments, the mechanism of non-isothermal flow of fluids is explained and compared with the isothermal case. To make this comparison possible, a critical survey of literature on friction factor problem and velocity distribution measurements was necessary.

By collecting, calculating, and plotting the available experimental data of reliable nature on the friction factor it is determined that the "General Index Law Equation"

$$f = \alpha + \beta \, \mathrm{Re}^{c} \qquad \ldots\ldots(1)$$

should be used to express the relation between Fanning friction factor, f, and Reynolds number, Re. The following table shows the constants in this formula for the classes of pipes considered:

Kind of Pipe	α	β	c	Range of Re.	Eq.
1. Drawn Brass, etc.	0.00140	0.1252	-0.32	3000-3,000,000	(2)
2. Commercial Iron, etc.	0.00307	0.1886	-0.38	3000-2,500,000	(3)

The first class of pipe includes all clean "technically smooth" pipes, i.e., those of copper, brass, lead, and glass. The data for this class is fitted by the formula within ± 5% over the entire range of Re. The inside diameters of the pipes used in obtaining the data ranged from 0.107 inches to 4.97 inches. The fluids involved are air, water, steam, and oils. No trend with varying diameter is found. The new equation agrees well with that of Lees over the range of the data represented by Lee's equation.

The second class of pipe comprises ordinary clean commercial iron and steel pipe. The data is fitted within ± 10% over the entire range of Reynolds number. The pipe sizes ranged from 0.42 inches to 12 inches and now consistent trend with change of diameter is found. Air, steam, water, and brine have been used in the tests, thus this equation is recommended to use almost for any fluid. This equation checks very well with that proposed by Wilson, McAdams, and Seltzer in 1922, but it is in disagreement with that of McAdams and Sherwood for air and steam proposed in 1926.

The mechanism of the isothermal flow of fluids in pipes may be explained briefly as follows by considering the above proposed equations. At low Reynolds number in the turbulent flow region, the laminar film at the boundary controls or counts mostly for the friction which, of course, decreases proportionally as velocity increases since an increase of velocity will reduce the film thickness, while at very high Reynolds number, the pipe wall roughness controls or counts mostly for the friction; thus an increase of velocity will only reduce the friction slightly. It is obvious from Eq. (2) and (3) that as Reynolds number approaches infinity, the friction factor is equivalent to a constant which might be considered is purely due to internal pipe wall roughness, thus this factor for iron and steel pipes is found to be more than twice as big as that found for copper and brass. This consideration predicts that for non-isothermal case the film temperature, instead of the average temperature should be used in calculating the Reynolds number to obtain the friction factor value.

From a study of the isothermal velocity distribution problem in literature, it is recommended the following equation be used

$$\frac{V}{V_{max.}} = \left(1 - \frac{r}{R}\right)^{a} = \left(\frac{y}{R}\right)^{a} \qquad .....(4)$$

for the velocity distribution in pipes, where V = velocity at the point at the variable radius, r, in the pipe; $V_{max.}$ = axial velocity; R = Inside radius of pipe; y = R-r = distance from pipe wall. Using Levy's method, a semi-theoretical relation is derived between the exponent "a" in Eq. (4) and General Index Law equation for friction factor as follows:

$$a = -1.5 + 0.5 \sqrt{9-8 \left(\frac{Re.df}{f\ d\ Re}\right)} \qquad .....(5)$$

Therefore, Eq. (5) enables one to calculate the velocity distribution exponent "a" by substituting in the friction factor equation for any kind of pipe. Another useful relation has been found for the ratio of average to maximum velocity over a pipe cross-section is as follows:

$$\frac{V_{ave.}}{V_{max.}} = \frac{2}{(a+1)(a+2)} = \frac{1}{1 - \frac{Re.\ df}{f\ d\ Re.}} \qquad .....(6)$$

It is apparent from Equations (2), (3), and (5) that the velocity distribution exponent "a" will vary with Re., decreasing as Re. increases; this relation checks by all the experimental data found in literature and checks approximately by the writer's isothermal velocity distribution experiments. Therefore, it is necessary to modify Prandtl-von Karman's one-seventh potential law, proposed in 1921, on velocity distribution which states for turbulent flow the exponent "a" in Eq. (5) is a constant and equal to 1/7. From Eqs. (2), (3), and (6), one can also see that the velocity ratio should increase as Reynolds number increases.

The applicability of Eq. (6) is greatly inspired by the **ex**cellent correlation with Stanton and Pannell's data.

After a rather thorough review of literature, the experiments were then carried out. An apparatus was specially built in the course of investigation. The apparatus consists essentially of a centrifugal pump, a water reservoir, calming sections over 100 I.D., and two vertical heat transfer sections of 2" hard drawn copper pipe which are jacketed by 3-1/2" iron pipes and either heated by condensing steam or cooling water. Special pitot tube and thermocouple sets have been designed for velocity and temperature explorations. Fifty-six isothermal runs of water were first carried out, and their results check ~~very~~ well with what is required by Eq. (5) and (6). On the non-isothermal runs, mostly heating runs, one pipe was made to run parallel-currently and the other counter-currently. There is no appreciable difference in result of these two groups of runs. Temperature distribution calculations were made use of, the following relation, readily derived by assuming

$$\frac{dt}{dr} \propto \frac{dV}{dr} ,$$

$$\frac{\Delta t}{\Delta t_{max.}} = \frac{t_w - t}{t_w - t_a} = \left(1 - \frac{r}{R}\right)^b = \left(\frac{y}{R}\right)^b \qquad .....(7)$$

Thirty-**six** non-isothermal velocity distribution runs and fifty-eight temperature distribution runs, including only six runs during cooling, were obtained.

The experimental result reveals an important fact that the exponent "b" is far from being equal to the exponent "a", i.e., temperature distribution is not equal at all to the velocity distribution. The elementary form of Reynolds analogy requires that these two exponents should be equal, assuming that there is similarity between momentum transfer and heat transfer. This non-similarity of mechanism of momentum transfer and heat transfer clarifies the question of applicability of Reynolds analogy to liquids, although it has been found applicable to gases by J.R. Pannell's experiments. From the present experiments, it has been found that the temperature drop through the laminar film at the wall is estimated to be 80-90% of the temperature difference between temperatures at the wall and at the center of the pipe, as compared with a ratio of 40-50% in Pannell's experiments on air. This phenomenon is what to be expected, since in case of gases the rapid to and fro motion of their molecules causes the heat transfer very efficiently, but in case of liquids the transmission of heat, according to Caldwell, is believed and has been shown experimentally to take place not through diffusion of molecules but through the actual contact of the molecules themselves. The film temperature has been used in calculating Reynolds number, and the correlation of the non-isothermal velocity distribution exponents with the isothermal ones is carried out. ~~good~~. The temperature distribution exponent "b" has been found to be dependent on the product of Peclet number and a proposed temperature gradient ratio, $\frac{t_{wall}-t_{axis}}{t_{ave.}-t_{axis}}$.

TABLE OF CONTENTS

APPENDIXES

TABLE OF CONTENTS (Cont.)

MECHANISMS OF ISOTHERMAL AND
NON-ISOTHERMAL FLOW OF
FLUIDS IN PIPES.

I. INTRODUCTION

The primary object of the work herein described was to determine experimentally the velocity and temperature distributions obtained when a liquid is heated or cooled as it is pumped through a vertical pipe. Reynolds analogy suggests the similarity between the transfer of momentum and heat, but the results obtained by using this analogy in its elemental form are found applicable only in case of gases, not for liquids. The failure of his analogy in applying to liquids has often been explained from many different points of view. However, up to the present time, very little ~~no~~ actual experimental work on the simultaneous velocity and temperature distribution of a liquid has been available, so that it has been difficult to determine whether, or in what manner, the assumptions involved in the various derivations should be changed. It is thus hoped that a determination of the facts as to the velocity and temperature distributions may lead to a clarification of heat transfer theory. Recently, Eagle and Ferguson, in England, Keevil, in this Institute, worked independently on the non-isothermal friction factor during heat transfer. The first two worked with water; the second with oils. The question is still far from solved for fluids in general. This investigation might throw some light on the latter problem also.

The easiest way to study the above stated problems is to compare the non-isothermal velocity distribution and temperature distribution with the isothermal velocity distribution. In studying the isothermal flow of fluids in pipes, the two most obvious measurable factors are the pressure gradient and the velocity distribution. A theoretical friction factor equation generally has some concomitant velocity distribution equation. Considerable experimental work on turbulent flow friction factor has been found in literature and numerous graphical correlations and empirical equations have been proposed, but few of these have been compared with any extensive part of the now-available data. Until recently, comparatively little work on isothermal velocity distribution is found in literature. Consequently, a critical survey of literature on the friction factor problem, as given in Chapter II, and on isothermal velocity distribution, as given in Chapter III, was of primary importance, while the collection of additional experimental data on isothermal velocity distribution was thought necessary.

The importance of the present investigation is evident because of its vast applications in commercial flow of fluids and heat transfer work, besides the theoretical interest on the mechanism of flow isothermally and during heat transfer.

2

II

Isothermal Friction Factor of Fluids in Circular Pipes.

A. General Law of Surface Resistance.

B. Hagen-Poiseuille's Law for Laminar Flow.

C. General Index Law for Turbulent Flow.

D. New Equation for Friction Factor of Fluids in Smooth Pipes.

E. New Equation for Friction Factor of Fluids in Iron and Steel Pipe.

F. Application of von Karman's New Theory.

G. Effect of Entrance Conditions on Friction Factor.

H. Effect of Roughness of Pipe Walls on Friction Factor.

I. Literature References.

II. Isothermal Friction Factor of Fluids in Circular Pipes.

A. General Law of Surface Resistance.

It is a well known fact that as a fluid moves over a solid surface there is produced at the boundary frictional or shearing stress just as in the case of two solid surfaces sliding over each other. This is called the skin friction or surface resistance and this accounts for the pressure drop of fluids in pipe lines. The differential form of Bernoulli's theorem (1)+ may be written as

$$\frac{2fV^2\,dL}{gD} = -\frac{V\,dV}{g} - \frac{dP}{\rho} \qquad \ldots\ldots\ldots(1)$$

In case of liquids, the density may be taken as constant throughout the system; Thus, integrating Eq. (1),

$$f = \frac{gD}{2\,V^2\,L}\left(\frac{V_1^2 - V_2^2}{2g} + \frac{\Delta p}{\rho}\right) \qquad \ldots\ldots(1a)*$$

In case of isothermal flow of gases, substituting in the gas law and integrating,

$$f = \frac{gD}{2\,V^2\,L}\left(\frac{V_1^2 - V_2^2}{2g} + BT \ln \frac{p_1}{p_2}\right) \quad \ldots.(1b)$$

The kinetic energy term is often negligible in case of liquids, but it is not negligible in case of gases

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+Numerals in brackets refer to literature references at the end of each chapter.

*Refer to Nomenclature Table at the end of thesis.

especially at high Reynolds Number; thus, in the former case of Eq. (1a) may be simplifies to

$$\Delta p = \frac{4\ f\ L\rho\ V^2}{2gD} \qquad \text{.........(1c)}$$

which is commonly known as the Fanning Equation.

Rayleigh (2)(3) has shown from his principle of dimensional similarity that surface friction depends ~~only~~ on the diameter of the pipe, and on the velocity, density, and kinematic viscosity of the fluid. He further stated that this is a general law of the resistance of bodies of geometrically similar shape immersed in fluids moving relative to them, while flow of fluids in pipes is only a special case. If this is true, the expression for the surface resistance may be written as,

$$F/(\rho V^2) = \text{Function of Reynolds No.} = \varphi(\text{Re.}) \qquad (2)$$

The friction factor, f, is equivalent to $2F/\rho V^2$, Equation (2) then may be written as

$$f = 2F/\rho V^2 = \varphi'(\text{Re.}) \qquad \text{.......(2a)}$$

It has been pointed out by Lees (4) that on the theoretical side of this problem, whether established by an examination of the equations of motion of fluids as by Stokes and Helmholtz or by dimensional analysis as by Rayleigh, shows that cases having the same value of Reynolds number will give same value of friction factor.

Reynolds (5) has definitely shown that there are two entirely different types of fluid motion. One is called laminar or viscous flow which is characterized in a straight pipe, by the straight line motion of the fluid particles parallel to the axis. The other is called turbulent flow which is characterized by an erratic eddying motion of individual fluid particles, although the net flow is only in the direction of the axis. The Reynolds number at which one type of flow changes to the other is defined as the critical value. From an extensive study of this critical value by Schiller (6)(7)(8)(9), it is found that the critical value depends greatly upon the inlet length or calming section and the entrance disturbances. According to him, 2,320 may be taken as the lower limit of this criterion. A more detailed discussion of this point will be given later (p. 30).

Hsiao (10) recently has derived a general equation, based upon dimensional analysis, for friction factor in terms of Reynolds number and an exponent 'n' which is equal to 1 for laminar flow and varies from 1.7 to 2.0 for turbulent flow depending on the pipe wall roughness. His equation reads:

$$f = 16 \left[1918 \left(1.918 - \frac{1}{n-1} \right) \right]^{1-n} Re.^{n-2} \qquad ....(3)$$

Thus for viscous region, where n = 1

$$f = 16 \; Re.^{-1} = 16/Re. \qquad ....(4)$$

And for turbulent region, where n = 1.75 for smooth pipes,

$$f = 0.0826\ \mathrm{Re}.\boxed{-0.25} \qquad ....(5)$$

He used the general equation (3) to correlate his experiments on the effect of corrosion on the resistance to the flow of water in metallic pipes.

B. Hagen-Poiseuille's Law for Viscous Flow

In 1839 Hagen (11) first studied the flow of water through brass tubes at different temperatures, and in 1842 Poiseuille (12) used very small glass capillaries to study the flow of water. They practically reached the same equation as follows:

$$Q = K \frac{P\ D^4}{L} \qquad .....(6)$$

Later, Wiedemann and Hagenbach (11) deduced mathematically the following modified equation

$$Q = \frac{\pi\ P\ R^4}{8\ L\ \mu} \qquad ......(7)$$

which is better known as Poiseuille's Law, though it ought to be named as Hagen-Poiseuille's Law which name has been adopted by Prandlt and Tietjens (13).

Hagenbach deduced Eq. (7) from a theoretical derivation of a velocity distribution formula for laminar flow

$$V = \frac{P}{4\ L\ \mu}\left[(R^2 - (r)^2\right] \qquad ....(8)$$

The volume of liquid passed per unit time is equal to the volume of a paraboloid, since Eq. (8) is its profile and is in the form of a parabola. Thus the volume of the paraboloid will be

$$Q = 2\pi \int_0^R vr\,dr = 2w \int_0^R \frac{P}{4L\mu}(R^2 - r^2)r\,dr = \frac{\pi P R^4}{8 L \mu} \qquad \ldots.(9$$

which is identical with Eq. (7). Then the law of surface resistance for laminar flow can be easily obtained from Eq. (7) and (1a) and definition of friction factor 'f' in Eq. (2a). Then

$$f = 16\,\mathrm{Re.}^{-1} = 16/\mathrm{Re.} \qquad \ldots\ldots\ldots(10)$$

This theoretically derived equation has been found to be correct by the early investigators on the viscosity problem and more recently by the experiments of Blasius (14), Stanton and Pannell (15), Mills (16), Clapp and FitzSimons (17) and Keevil (18). Therefore, the resistance law for viscous flow is proved to be exact and it has been accepted by the scientific world.

C. General Index Law for Turbulent Flow.

Reynolds (5), from his experiments on lead pipe, made a log-log plot of his observed values of surface resistance against velocity. He found two straight branches; the lower one corresponding to observations below the critical value of Re., the upper one to those above the critical. He suggested for the turbulent region a simple Index Law

$$f = \beta\,\mathrm{Re}^{\alpha} = \beta\,\mathrm{Re}^{n-2} \qquad \ldots\ldots(11)$$

where n varies as the nature of the roughness of surface.

A general Index Law has been suggested by many authorities on this problem and has been used by them in correlating their experimental results on turbulent flow:

$$f = \alpha + \beta \, Re^{c} \qquad \ldots\ldots(12)$$

It is obvious that Eq. (12) reduces to the Reynolds simple form if a=0, so it may be said that the latter is only a special case of the former. A study of the available data on smooth pipes and commercial pipes shows that this equation can be used to represent the surface resistance law in turbulent region very satisfactorily. The coefficients α, β, and c are constants only for classes of pipes of the same degree of wall roughness. A rough pipe would be expected to have coefficients different from those for smooth pipe. However, all available experimental data reveal that for any particular type of pipe these coefficients are constants, - a fact which simplifies the study of the present problem and gives further support to the General Index Law. It is to be noticed that many sets of experimental results have been presented in other forms, some of which are very complicated. These will not be mentioned here, since a comprehensive review~~s~~ of these forms have been made by Eason (19) on air and gases, and by Gibson (20) and Forchheimer (21) ~~respectively~~ on water.

A brief summary of Index Law equations proposed by various authors for turbulent flow of fluids in smooth pipes, with their experimental particulars is shown in Table I. Glass, lead, copper and drawn brass are considered to belong to the class of "technically smooth" pipes. It is seen from Table 1 that experimenters who studied a comparatively small range of Reynolds number have generally recommended that the friction factor be expressed by the simple Index Law having $\alpha = 0$. As the range of Reynolds number increases, it is seen that the general Index Law equation is advocated. Schiller and Ombeck are exceptions, but it is seen from Schiller's original paper that beyond Re. = 28,000 his own data deviates from his proposed equation, which is identical with that of Blasius. Indeed, the simple Index Law requires that friction factor is a linear functions of Reynolds number on a log-log plot, which is found to be not true for a wide range of Re. The simple Index Law requires that friction factor approaches zero as Re. approaches infinity, while the general Index Law states that friction factor will approach α as a limit as Re. approaches infinity.

D. New Equation for Friction Factor of Fluids in Smooth Pipes (for Re. 3,000-3,000,000)

A wide range of data on friction factor in glass, lead, copper and especially drawn brass has been collected and recalculated with the object of obtaining a more accurate general Index Law equation which will cover the biggest range of Reynolds number possible.

TABLE 1

SUMMARY OF GENERAL INDEX LAW EQUATIONS FOR TURBULENT FLOW IN SMOOTH PIPES

Equation: $f = \alpha + \beta \, Re^{c}$

Authority and Reference	Fluid	Pipe Material	Range of Re	Coefficients in Eq. (12) α	 β	 $-c$
1. Saph and Schoder (22) (as calculated by Blasius) (14)	Water	drawn brass	3,000 to 100,000	0	.0791	.250
2. H. Blasius (14)	Water	glass, lead	3,000 to 76,300	0	.0791	.250
3. H. Ombeck (23)	Air	drawn brass	6,590 to 481,500	0	.0605	.224
4. M. Jakob (24)	air, Water	drawn brass	3,000 to 70,000	0	.0818	.254
5. L. Schiller (6)	Water	drawn brass	3,000 to 400,000	0	.0791	.250
6. H. Richter (25)	Water	drawn copper	4,100 to 72,000	0	.0873	.267
7. C. Y. Hsiao (10)	Water	copper, glass, lead-lined	2,690 to 86,200	0	.0826	.250
8. Jakob, Erk. (26)	Air, water	drawn brass	86,000 to 462,000	.00179	.153	.350
9. Stanton, Pannell (15) (as calculated by Lees) (4)	Air, water	drawn brass	3,000 to 430,000	.00180	.153	.350
10. Mises (27)		smooth pipe		.0024	.425	.500
11. R. Biel (28)		smooth pipe		.0024	.610	.500
12. R. Hermann (30)	Water	drawn brass	20,200 to 1,900,000	.00135	.099	.300
13 R. Hermann (30)	Water	drawn copper	37,700 to 1,330,000	.00132	.0998	.300
Author			3,000 to 3,000,000	.00140	.125	.320

The available data used for this purpose are all taken from the original tabulated data in the literature, except Nikuradse's data (32) which are presented only in the form of a log-log plot of 4f against Reynolds number. These were necessarily read from an enlarged photograph eight times as big as the original which enabled the writer to read the values accurately to three significant figures. Ombeck and Nusselt's data on compressed air in drawn brass pipes had already been corrected for change of kinetic energy which is only negligible below Re. = 10,000 in their work. Stanton and Pannell's experiments on air have not been corrected for kinetic energy changes but this is ~~not serious~~ negligible in their case since their runs on air are mostly below Re. = 10,000. In so far as is known, all data having an inlet length below 40D, or of an unreliable character for other reasons are excluded. As a result of these considerations, 17 groups of reliable data having calming length of more than 40D, consisting of 1339 tests, have been recalculated and are plotted in Figure I using 4f as ordinate and Re. as abscissa, the adoption of 4f instead of f as ordinate is merely for convenience since all data in German literature are expressed in this way. Table II summarizes the particulars of these 17 groups of experiments having range of diameter, kind of pipe, range of Reynolds number, and inlet length in terms of diameters of the pipe tested all indicated.

TABLE II

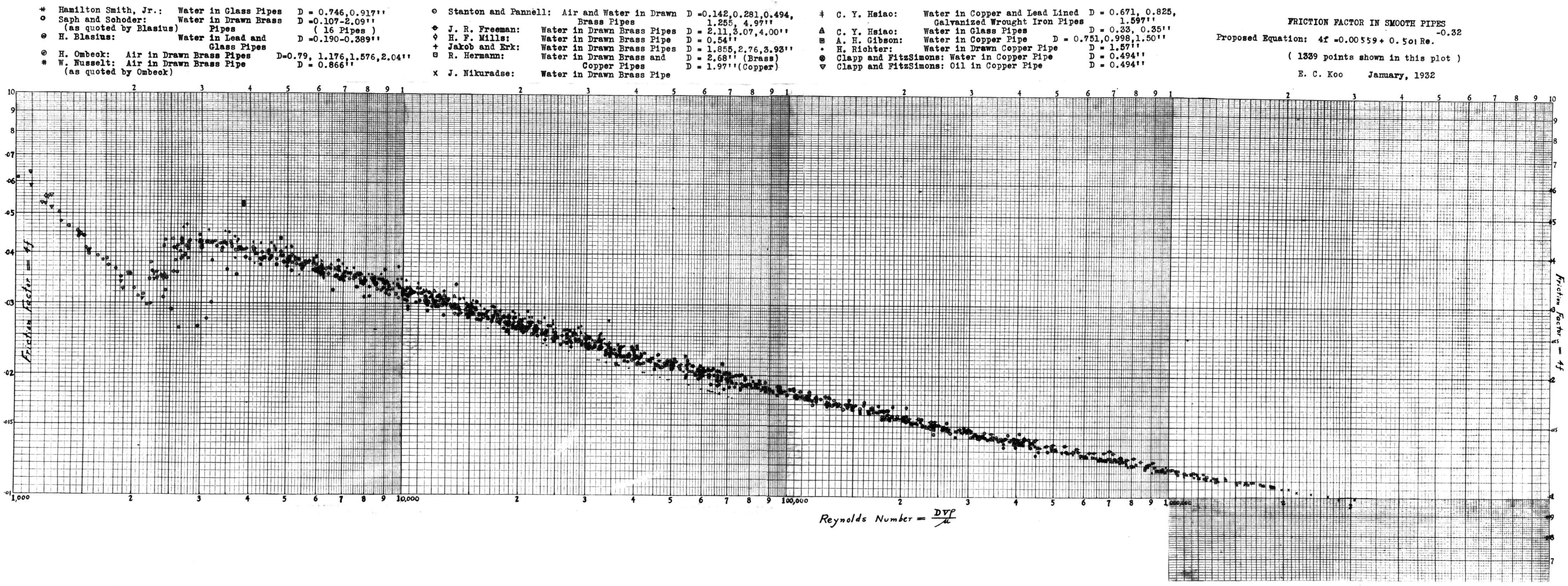
SUMMARY OF PREVIOUS EXPERIMENTS ON FRICTION FACTOR IN SMOOTH PIPES.

Authority and Reference	Fluid Used	Pipe Material	I.D. of Pipe in In.	Range of Re	No. of Tests	Inlet Length	Remarks
1. H.Smith,Jr. (33)	Water	Glass	0.746 0.917	7,450-34,200	9		Excluding 4 runs on 0.502" pipe, which is quoted as unreliable by author.
2. Saph and Schoder(22)(as calculated by Blasius)(14)	Water	Drawn Brass	16 Pipe varying from 0.107-2.09	3,000-100,000	39		These 39 tests are chosen as representative of about 300 actual run
3. H. Blasius (14)	Water	Lead	0.190	1,450-23,600	68	98.2D	45 runs on lead pipe wi inlet length of 20.7D are excluded.
H. Blasius (14)	Water	Glass	0.389	4,470-76,300	75	51.7D	Glass pipes not having uniform cross-sections.
4. H. Ombeck (23)	Air	Drawn Brass	0.79, 1.176, 1.576, 2.04	6,592-481,496	145		
5. W.Nusselt (as calculated by Ombeck) (23)	Air	Drawn Brass	0.866	6,208-151,700	10		W. Nusselt's data were also calculated by Blasius; results are no much different.
6. Stanton and Pannell (15)	Air and Water	Drawn Brass	0.142, 0.281, 0.494, 1.255, 4.97	1012-430,000	308	90-140D	Only 7 runs were done o 4.97" pipe.
7. J.R. Freeman (16)	Water	Drawn Brass	2.108, 3.067, 4.000	11,420-908,000	59		3" pipe is said to be slightly rougher than either 2" or 4" pipe.
8. H.F. Mills (16)	Water	Drawn Brass	0.54	2,580-12,680	20		

Authority and Reference	Fluid Used	Pipe Material	I.D. of Pipe in In.	Range of Re	No. of Tests	Inlet Length	Remarks
). Jakob and Erk (26)	Water	Drawn Brass	0.54	86,330-461,620	40	40-51D	
.0. R. Hermann (30)	Water	Drawn Brass	2.68	20,200-1,898,000	97	169D	
	Water	Drawn Copper	1.97	37,640-1,328,000	74	249D	
.1. J. Nikuradse (32)	Water	Drawn Brass		41,200-3,070,000	94	Greater than 55D	Diameter of pipe not stated in the reference
.2. C.Y. Hsiao (10)	Water	Copper, & lead lined galvanized wrought iron	0.671, 0.825 1.597	2,690- 86,200	120	75-145D	Only 80 representative runs are chosen from 359 runs on 0.825" copper pipe.
.3. C.Y. Hsiao (10)	Water	Glass	0.330, 0.350	4,800- 36,710	63	91-137D	Glass pipes none ~~all not~~ having uniform cross-section; excluding 30 I.D. inlet length runs.
.4. A.H. Gibson (20)	Water	Copper	0.751, 0.998 1.500	10,200-101,800	15		
.5. H. Richter (25)	Water	Drawn Copper	1.57	4,100- 72,000	38	43D	
6. Clapp and Fitzsimons (17)	Water	Copper	0.494	2,660- 36,600	45	100D	
7. Clapp and Fitzsimons (17)	Oil	Copper	0.494	1,087-6,740	20	100D	Range of Temp. of oil varied from 30-98°C.

Figure 1

Friction Factor
in
Smooth Pipes



FRICTION FACTOR IN SMOOTH PIPES

Proposed Equation: $4f = 0.00559 + 0.501\,Re.^{-0.32}$

(1339 points shown in this plot)

E. C. Koo January, 1932

A new equation in the form of general Index Law has been obtained by applying the method of averages to the 26 points so chosen as to represent the densest place in the band of points on Fig. 1 at these values of the abscissae indicated in Table III. The equation is

$$4f = 0.00559 + \frac{0.5009}{Re.^{0.32}} \qquad ....(13)$$

or

$$f = 0.00140 + 0.125\ Re.^{-0.32} \qquad ....(13a)$$

The accuracy of the proposed equation and its applicability to a very wide range of Reynolds number are shown in Figure 2 and Table III. It is important to note that this equation covers a range of Reynolds number from 3,000 to 3,000,000 and fits all available data within ± 5%. The new equation checks excellently with Lees Equation within the range of Re. of Stanton and Pannell. (see Fig. 2a) It may be further pointed out that it is justified to classify copper, brass, glass, and lead together as smooth surfaces, that is to say they are hydraulically of the same degree of roughness. Although except for 20 runs by Clapp and FitzSimons (17) on oil, only air and water have been used in determining the equation, it is believed, however, from the principle of dimensional similarity that the new equation should be applicable to any kind of fluid. It might be mentioned here that Gibbons (47) has made a series of tests on brine solutions in copper pipes

TABLE 3

NEW EQUATION FOR ISOTHERMAL FRICTION FACTOR IN SMOOTH PIPES

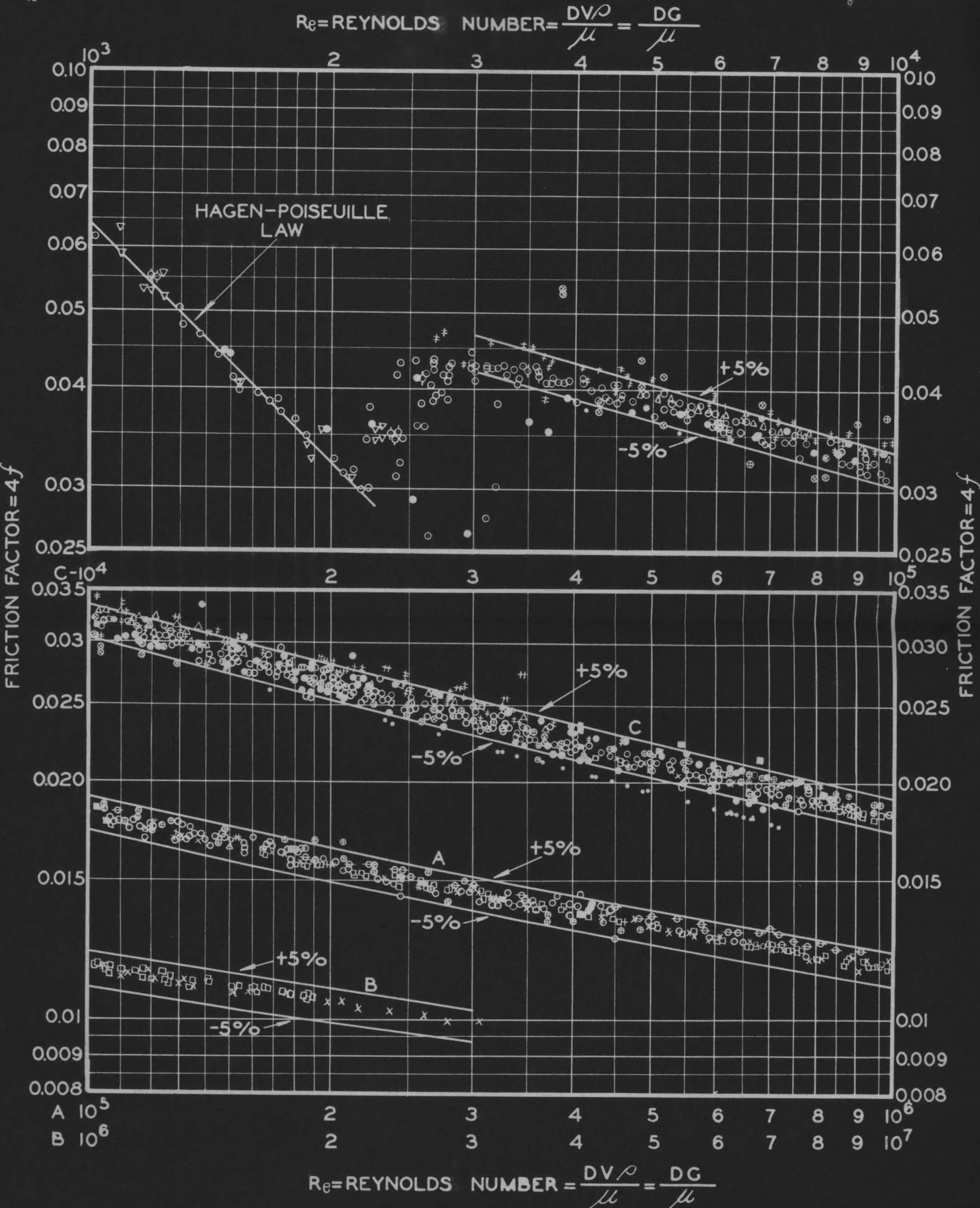
Re	4f (read from figure)	4f (calc. from Eq)	0.95(4f)	1.05(4f)
3,000	0.0435	0.04423	0.0420	0.0464
4,000	0.0409	0.04083	0.0388	0.0429
5,000	0.0386	0.03845	0.03655	0.0404
6,000	0.0368	0.03655	0.0347	0.0384
8,000	0.0341	0.03382	0.03215	0.0355
10,000	0.0322	0.03188	0.0303	0.0335
15,000	0.0289	0.02868	0.0272	0.0301
20,000	0.0268	0.02665	0.0253	0.0280
30,000	0.0241	0.02409	0.0229	0.0253
40,000	0.0223	0.02246	0.0214	0.0236
50,000	0.0212	0.02130	0.0202	0.0224
60,000	0.0203	0.02041	0.0194	0.0214
80,000	0.0190	0.01910	0.01814	0.02005
100,000	0.0181	0.01817	0.01727	0.01908
150,000	0.0166	0.01664	0.0158	0.01750
200,000	0.0157	0.01567	0.0149	0.01645
250,000	0.0149	0.01497	0.01422	0.0157
300,000	0.0144	0.01444	0.01372	0.01518
400,000	0.0137	0.01366	0.01298	0.01434
500,000	0.0132	0.01311	0.01247	0.01378
600,000	0.0127	0.01268	0.01205	0.01331
800,000	0.0121	0.01206	0.01146	0.01267
1,000,000	0.0117	0.01161	0.01103	0.01220
1,500,000	0.01095	0.01088	0.01033	0.01142
2,000,000	0.01045	0.01041	0.00989	0.01094
3,000,000	0.0995	0.00983	0.00934	0.01031

Proposed Equation: $4f = 0.00559 + \frac{0.5009}{Re.^{0.32}}$

or $f = 0.00140 + \frac{0.1252}{Re.^{0.32}}$

Figure 2

SMOOTH PIPES



EQUATION: $4f = 0.00559 + 0.501\, Re^{-0.32}$

Figure 2a

Comparison of New Equations

with Others

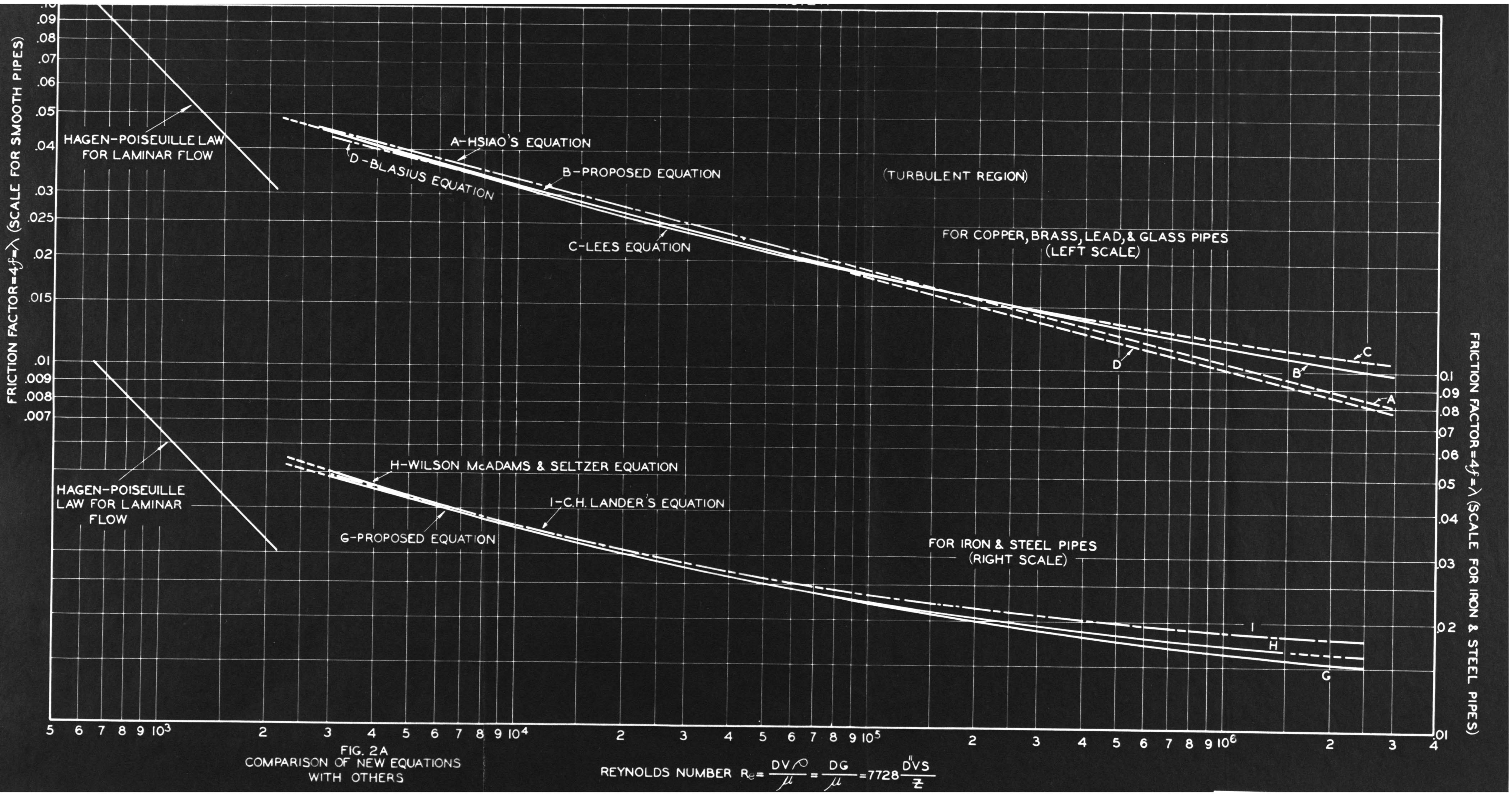


FIG. 2A
COMPARISON OF NEW EQUATIONS
WITH OTHERS

and found that the resistance law holds for water as well as for the brine solution if the difference of density of these solutions are taken care of in the calculation. His data on brine is not available in his original paper, hence they are not included in Figure I. Six groups of calculated results of friction data from previous workers are tabulated in Appendix A, because they have not been calculated in terms of f and Re. by previous workers themselves. The data of other groups may be easily found from their original references.

E. New Equation for Friction Factor of Fluids in Iron and Steel Pipes (for Re. = 3,000 to 2,500,000.)

The friction factor problem of fluids in iron and steel pipe is very much complicated by the corrosion effect. This effect is negligible when dry air, oil, or similar non-corrosive fluid is used but it is important when water, steam or brine solution is used as a fluid. The recent work of Fair, Whipple and Hsiao (61) (also see Hsiao's thesis (10)) has revealed the hydraulic service characteristics of small metallic pipes having Cambridge water. It is found that the effect of corrosion will cause an increase of pipe wall roughness as well as reduction of diameter for commercial wrought iron and steel pipes. For a 3/4" iron or steel pipe, there is a reduction of diameter to one third of the original after passing 3,000 cu.ft. of hot water. Of course, the effect of corrosion will be more pronounced at higher temperature, and also for smaller pipes due to the accompanying reduction of diameter. Besides the corrosion effect, temperature effect, calming length effect, and in cases of gases, kinetic energy change effect should be always considered. In hydraulics one often neglects the temperature effect. This is not justified, since the water temperature may often change from 35° F. in winter to 75° F. in summer, corresponding to a change of kinematic viscosity of 1.82×10^{-5} to 0.99×10^{-5}. (ft^2/sec)

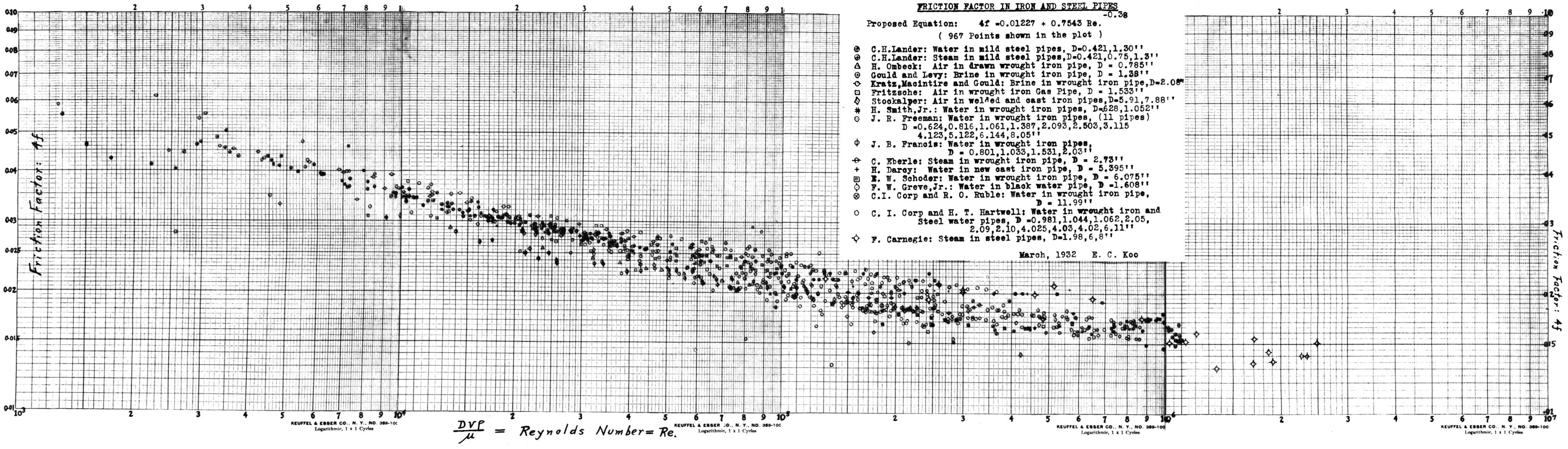
With all these variables affecting the friction factor, one might expect to find very divergent data in literature. However, by limiting oneself to new and clean iron and steel pipes including plain cast iron or wrought iron without any coating whatsoever, to tests in which the experimental temperature is definitely known, and to those conducted with a reasonable calming length it has been possible to assemble 16 groups of reliable data from literature consisting of 967 tests, using either air, steam, water, or brine as a fluid. A plot of friction factor versus Reynolds number on a log-log plot reveals a relation like that in the case of smooth pipes as shown in Figure 3. This relation can be expressed in the General Index Law form as

Figure 3

Friction Factor

in

Iron & Steel Pipes



FRICTION FACTOR IN IRON AND STEEL PIPES

Proposed Equation: $4f = 0.01227 + 0.7543\ Re.^{-0.38}$

(967 Points shown in the plot)

⊕ C.H.Lander: Water in mild steel pipes, D=0.421,1.30''
⊕ C.H.Lander: Steam in mild steel pipes,D=0.421,0.75,1.3''
△ H. Ombeck: Air in drawn wrought iron pipe, D = 0.785''
⊙ Gould and Levy: Brine in wrought iron pipe, D = 1.38''
-○- Kratz,Macintire and Gould: Brine in wrought iron pipe,D=2.08''
□ Fritzsche: Air in wrought iron Gas Pipe, D = 1.533''
◇ Stockalper: Air in welded and cast iron pipes,D=5.91,7.88''
H. Smith,Jr.: Water in wrought iron pipes, D=.628,1.052''
⊙ J. R. Freeman: Water in wrought iron pipes, (11 pipes)
D =0.624,0.816,1.061,1.387,2.093,2.503,3.115
4.123,5.122,6.144,8.05''
ɸ J. B. Francis: Water in wrought iron pipes,
D = 0.801,1.033,1.531,2.03''
-θ- C. Eberle: Steam in wrought iron pipe, D = 2.73''
\+ H. Darcy: Water in new cast iron pipe, D = 5.395''
▣ E. W. Schoder: Water in wrought iron pipe, D = 6.075''
◊ F. W. Greve,Jr.: Water in black water pipe, D =1.608''
⊗ C.I. Corp and R. O. Ruble: Water in wrought iron pipe,
D = 11.99''
○ C. I. Corp and H. T. Hartwell: Water in wrought iron and
Steel water pipes, D =0.981,1.044,1.062,2.05,
2.09,2.10,4.025,4.03,4.02,6.11''
✧ F. Carnegie: Steam in steel pipes, D=1.98,6,8''

March, 1932 E. C. Koo

Table 4 Summary of Previous Experiments on Friction Factor in Iron and Steel Pipes

Authority and Reference ~~Symbol in Fig.~~	Fluid Used	Pipe Material	I.D. in Inches	Range of Re.		No. of Tests	Calming Length and Remarks
1. C.H. Lander (50)	Water	Mild Steel	0.421, 1.30	4,560	58,100	15	171D, 55.5 D
" "	Steam	Mild Steel	0.421, 0.75 1.30	17,400	523,000	61	171D, 96D, 55.5D
2. H. Ombeck (23)	Air	Drawn Wrought Iron	0.785	13,510	106,000	16	Discarding 23 runs of much less friction.
3. Fritzsche (23)	Air	Wrought Iron	1.533	8,155	154,276	41	Discarding 42 runs in corroded pipe.
4. H. Darcy (16)	Water	New polished Cast Iron	5.395	17,900	565,000	10	This is the smoothest of his four test pipes.
5. H. Smith, Jr. (33)	Water	New Wrought Iron	0.628, 1.052	4,610	40,600	20	
6. J.B. Francis (16)	Water	New Wrought Iron	0.801, 1.033, 1.531, 2.03	9,110	117,000	57	As quoted by H.F. Mills.
7. J.R. Freeman (16)	Water	New Wrought	0.624, 0.816 1.061, 1.387 2.093, 2.503, 3.115, 4.123 5.122, 6.144, 8.050	1,270	863,000	162	Discarding 13 runs. in 0.36" pipe having higher friction as quoted by H.F. Mills.
8. E.W. Schoder (51)	Water	Wrought Iron	6.075	76,300	443,500	32	40 –76D
9. C. Eberle (52)	Steam	Wrought Iron	2.73	150,000	687,000	13	164D
10. Stockalper (23)	Air	Welded and Cast Iron	5.91, 7.88	235,530	557,560	5	As quoted by H.Ombeck.

Authority and Reference ~~Symbol in Fig.~~	Fluid Used	Pipe Material	I.D. in Inches	Range of Re.		No. of Tests	Calming Length and Remarks
11. Gould and Levy (53)	Brine Sol.	New Wrought Iron	1.38	1,300	50,900	111	About 130D.
12. Kratz, Macintire and Gould (54)	Brine Sol.	New Wrought Iron	2.08	2,280	93,600	52	About 87D.
13. F.W. Greve, Jr. (55)	Water	Black water, Pipe	1.608	4,840	227,000	42	151.5D
14. Corp and Ruble (56)	Water	Wrought Iron or Steel Gas and Water Pipe.	11.99	96,500	1,116,000	58	40D; Discarding expts. on 7 other pipes which temp. is not definitely known.
15. Corp and Hartwell (57)	Water	Wrought Iron or Steel Water Pipe	0.981, 1.044, 1.062, 2.05, 2.09, 2.10, 4.025, 4.03, 4.02, 6.11	15,180	861,000	251	40D; Temp. of water definitely known.
16. F. Carnegie (62)	Steam	Solid-drawn, hot rolled and ordinary steel pipe.	1.98, 6.0, 8.0	131,200	2,488,000	21	Superheated Steam.

Total 967 Tests

$$4f = 0.01227 + 0.7543\ \mathrm{Re.}^{-0.38} \qquad \text{......(14)}$$

or

$$f = 0.00307 + 0.1886\ \mathrm{Re.}^{-0.38} \qquad \text{......(14a)}$$

All the test data available to the author at present are found to fit the equation within a deviation of ± 10% as shown in Figure 4. This equation is recommended to be used for new commercial iron and steel pipes of 1/2 to 12 inches in diameter having Reynolds number varying from 3,000 to 2,500,000. This equation checks very well with that proposed by Wilson, McAdams and Seltzer (58), in 1922 (See Figure 2A). It is in disagreement with that of McAdams and Sherwood published in 1926 (59). A tabulation of test specifications, calculated values of friction factor for different Reynolds numbers and a comparison with other proposed empirical equations will be found in Tables 4, 5, 6, respectively. (also see Figure 2a). It must be pointed out here that the former workers did not succeed in collecting such a wide range of data which were available in literature but used Fanning's Tables or Smith's Tables instead, especially for the high Reynolds number range. Ten groups of calculated results of friction data from previous workers who have not given the results as friction factor and Reynolds number are tabulated in Appendix B.

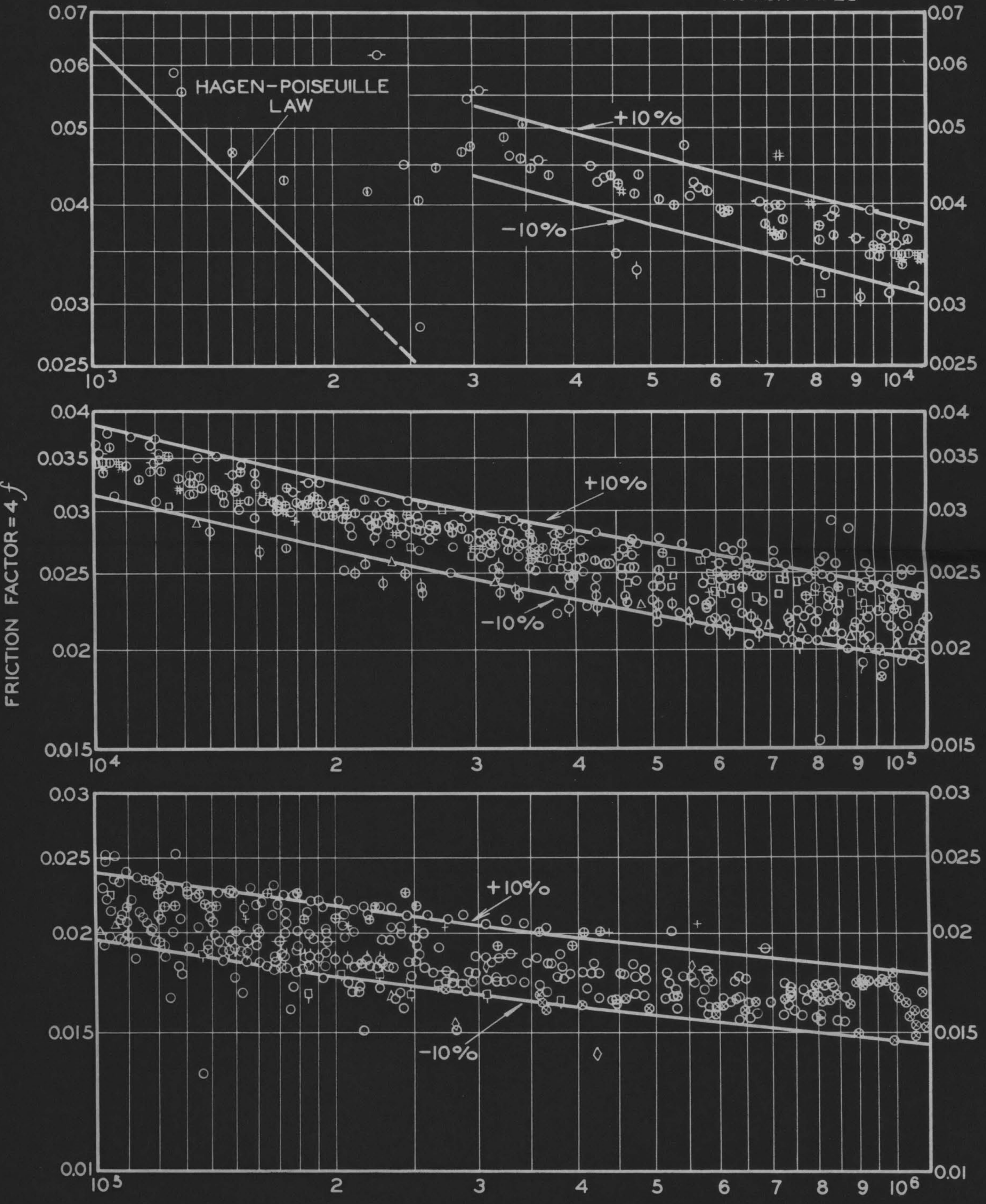
Among the adopted 16 groups of data on friction factor, Eberle's group was the only set of experiments which was tested including 2 to 3 90° elbows in the test section, others all had straight test sections. Eberle's experiment is purposely included here, because of its high range of Reynolds number using steam as a fluid medium. Stockalper's experiment was found to have not sufficiently accurate pressure measurement, however, it is included here because of it classical interest. Carpenter and Sickles' experiment on steam (66) has been excluded, for their test pipes are not new but have been in service for quite a long time. Hussey and Wattles (67) experimental data were taken at 3-1/2 pipe diameter after a 90° elbow; the insufficient calming length of their experiments is apparent.

The effect of pipe diameter on the friction factor is illustrated by several sets of experimental data in Figure 4A. There are indications that small size pipes, say below 1", have high values of friction factor while large size pipes have low values at same Reynolds number. However, for large size pipes, the effect of diameter on friction factor is less pronounced, for example, in Figure 4A, there is practically no difference in friction factor between Freeman's data on 4" and 8" wrought iron pipes, and Carnegie's experiment on steam shows that the friction factor for his 6" steel pipe is higher than that for his 2" steel pipe. Usually, drawn steel or iron tubes have less friction due to their polished surfaces. Further conclusion on this question cannot yet be drawn until more experimental data on different sizes of same kind of pipe are secured.

The decrease of the slope of the friction factor curve to a considerable degree at high Reynolds number for both smooth and iron and steel pipes may be explained as follows. Below Re. = 100,000 the film at the boundary of the pipe controls or counts for the friction which of course decreases proportionally as velocity increases since an increase of velocity will decrease the film thickness, while above that number the pipe wall roughness controls or counts for the friction mostly thus an increase of velocity will not decrease the friction as much as before. It is also obvious from the empirical equation that as Re. approaches infinity there is still a definite value of friction factor which might be said is due purely to wall roughness. In other words, one might visualize the mechanism better if one considers the constant α in the General Index Law as friction due to pipe wall roughness and the remainder or the entire second term as friction due to that laminar film at the wall which is a function of Reynolds number.

Figure 4

ROUGH PIPES



Re = REYNOLDS NUMBER $= \frac{DV\rho}{\mu} = \frac{DG}{\mu}$

EQUATION: $4f = 0.01227 + 0.7543\ Re^{-0.38}$

Table 5 Proposed Equation of Isothermal Friction Factor for New Iron&Steel Pipes.

(Range of Diameter 1/2" to 12")

Equation: $\lambda = 4f = 0.01227 + 0.7543\ Re^{-0.38}$

Re	4f (Calculated from Eq.) Use Log Table.	0.9 (4f) (= -10%)	1.1 (4f) (= +10%)	4f (Calculated from Wilson, McAdams and Seltzer's Eq.)
3,000	0.04827	0.0435	0.0531	0.0498
4,000	4454	401	490	
5,000	4192	377	461	.04255
6,000	3993	359	439	
8,000	3707	334	408	.0376
10,000	3505	315	386	.03525
15,000	3180	286	350	
20,000	2978	268	328	.02996
30,000	2728	246	300	
40,000	2573	231	283	
50,000	2463	222	271	.02479
60,000	2380	214	262	
80,000	2261	203	249	.0228
100,000	2177	196	239	.02202
150,000	2041	184	225	
200,000	1957	176	215	.02002
250,000	1898	171	209	
300,000	1853	167	204	
400,000	1788	161	197	
500,000	1743	157	192	.01807
650,000	1694	152	186	
850,000	1648	148	181	
1,000,000	1623	146	179	.01702
1,500,000	1566	141	172	.01654
2,500,000	1507	1355	166	
(3,500,000)*	(1473)	(1326)	(162)	
(4,500,000)	(1451)	(1305)	(160)	

* Parentheses indicate range of Reynolds number without enough experimental proof.

Table 6 Summary of General Index Law Equations for Turbulent Flow in Iron and Steel Pipes.

$$f = \alpha + \beta \, Re.^{+c}$$

Authority & Reference	Fluids	Pipe Material	Range of Test or Source of Data	Coefficients		
				α	β	$-c$
1. C.H. Lander (50)	Water and Steam	Mild Steel Steam Pipe	Re. = 4,560 to 648,000	0.0040	0.282	0.440
2. Wilson, McAdams and Seltzer (58)	Oil, Water, Steam	Commercial Iron and Steel	Re. = 3,000 to 1,500,000	0.0035	0.264	0.424
3. W.H. McAdams and T.K. Sherwood (59)	Air and Steam	Iron and Steel	Re. = 11,600 to 3,865,000	0.0054	46.4	1.000
4. M.D. Aisenstein (60)	Water	Commercial Smooth Pipes		0	0.167	0.170
5. Author	Air, Steam, Water, and Brine Solutions.	New clean Iron and Steel Pipes	Re. = 3,000 to 2,500,000	0.00307	0.1886	0.380

F. Application of von Karman's New Theory

Before stating von Karman's new theory of resistance law, it is necessary to consider Prandtl's theory of turbulence (34) (1925), based upon which von Karman's new theory (1930) is derived. In laminar flow, it is always assumed that the shearing stress F may be expressed as follows:

$$F = \mu\ (dv/dy) = \rho\nu\ (dv/dy) \qquad ....(14)$$

where v is the velocity component relative to rectangular axis X. In turbulent flow, a similar relation has been assumed possible by Prandtl, thus giving

$$F = \rho\ L^2\ (dv/dy)^2 \qquad ......(15)$$

If we let $\nu' = L^2\ (dv/dy)$, then

$$F = \rho\ \nu'\ (dv/dy) \qquad ......(15a)$$

Prandlt calls L', "Mischungsweg" or mixing length which is the distance through which a particle of fluid is displaced from one layer to another and is somewhat analogous to the idea of mean free path in the kinetic theory. Defining ν' as the 'apparent kinematic viscosity' or 'effective kinematic viscosity' in turbulent flow, we notice the similarity between Equations (14) and (15a). But ν', unlike the ordinary kinematic viscosity ν, which is a constant at a definite temperature, will vary from layer to ~~different~~ layer~~s~~ in the fluid and will also be a function of velocity.

In 1930, ~~Recently,~~ von Karman (35) based upon his revised equation for velocity distribution between parallel walls and his definition of Prandtl's mixing length, L', he suceeded in deriving a new resistance law which is quite different from the simple index law or the general index law. For a detailed discussion, one must refer to his original paper (35) or in English to J.W.Maccoll's review (37). For symmetrical flow in circular pipes, his revised velocity distribution equation may be written as follows:

$$V = V_{max.} + \frac{1}{k} \sqrt{F/\rho} \left[\text{Log} \left(1 - \sqrt{r/R}\right) + \sqrt{r/R} \right] \quad \ldots.(17)$$

where k is said to be a universal constant, and his definition of mixing length L' is,

$$L' = 2kR \sqrt{r/R} \left(1 - \sqrt{r/R}\right) \quad \ldots..(18)$$

Thus, the calculation yields,

$$V_{max.} = \frac{1}{k} \sqrt{F/\rho} \left(\text{Log} \frac{R\sqrt{F/\rho}}{\nu} + K \right) \quad \ldots..(19)$$

where K is another constant independent of k.

By the definition of f and Re., a new resistance law is derived

$$A/\sqrt{f} = \text{Log} \left(Re.\sqrt{f}\right) + B \quad \ldots..(20)$$

or expressing in term of 4f,

$$A'/\sqrt{4f} = \text{Log} \left(Re.\sqrt{4f}\right) + B' \quad \ldots..(21)$$

It must be mentioned here that in von Karman's original equation for resistance law the maximum velocity is used as a reference quantity instead of average velocity, thus in applying his original equation some accurate method must be used to obtain the maximum velocity from the average velocity . Unfortunately, the ratio V_{av}/V_{max} is known to a less order of precision than is the friction factor, hence it is impossible to apply his theoretical equation as it stands.

As a matter of interest, equation (20) or (20a) may be interpreted in terms of the usual Reynolds number and friction factor, though such interpretation is without theoretical basis. The representative values of friction factors read from Fig. I are again used to determine the constants in von Karman's new resistance law by the method of averages. The details of determining the constants are shown in Table 7. A plot of $1/\sqrt{4f}$ against $Re.\sqrt{4f}$ on a semi-log paper is shown in Figure 5 and gives a straight line relation which proves the applicability of von Karman's formula to actual experimental data if interpreted in terms of the average velocity. It is interesting to notice that this correlation is quite as good as that ~~much better than what~~ is shown in von Karman's original article - a fact which indicates a further merit to the new proposed equation based upon all available data. Substituting the constants found in Eq. (20a)

$$\frac{0.496}{\sqrt{4f}} = \text{Log}_{10} \; Re.\sqrt{4f} - 0.446 \qquad \text{....(20a)}$$

or we may express this equation in terms of f

$$0.248/\sqrt{f} \quad \text{Log}_{10} \; 2\,Re.\sqrt{f} - 0.446 \qquad \text{...(20b)}$$

Table 7 Verification of von Karman's New Theory

(for Smooth Pipes)

Re	4f (Read from Fig.)	$1/\sqrt{4f}$	$Re\sqrt{4f}$	$\log_{10} Re.\sqrt{4f}$
3,000	0.0435	4.79	626	2.796
4,000	402	4.98	803	2.904
5,000	379	5.13	975	2.989
6,000	361	5.26	1,140	3.057
8,000	333	5.48	1,460	3.164
10,000	314	5.65	1,772	3.249
15,000	284	5.93	2,530	3.403
20,000	263	6.17	3,244	3.512
30,000	237	6.49	4,620	3.665
40,000	222	6.71	5,960	3.776
50,000	211	6.89	7,270	3.862
60,000	202	7.04	8,530	3.931
80,000	189	7.27	11,000	4.042
100,000	180	7.46	13,420	4.128
150,000	166	7.75	19,320	4.286
200,000	157	8.00	25,060	4.400
250,000	150	8.16	30,600	4.486
300,000	145	8.30	36,200	4.559
400,000	137	8.55	46,800	4.670
500,000	132	8.70	57,450	4.759
600,000	127	8.86	67,700	4.830
800,000	121	9.09	88,000	4.944
1,000,000	116	9.27	107,700	5.032
1,500,000	109	9.56	156,700	5.195
2,000,000	1044	9.80	204,400	5.311
3,000,000	0992	10.04	298,800	5.476

von Karman Eq.: $\frac{A'}{\sqrt{4f}} = \log_{10} Re.\sqrt{4f} + B'$

Using method of Averages: $A' = 0.496$
$B' = -0.446$

Proposed Friction Factor: $\frac{0.496}{\sqrt{4f}} = \log_{10} Re\sqrt{4f} - 0.446$

Equation based on von Karman's New Theory.

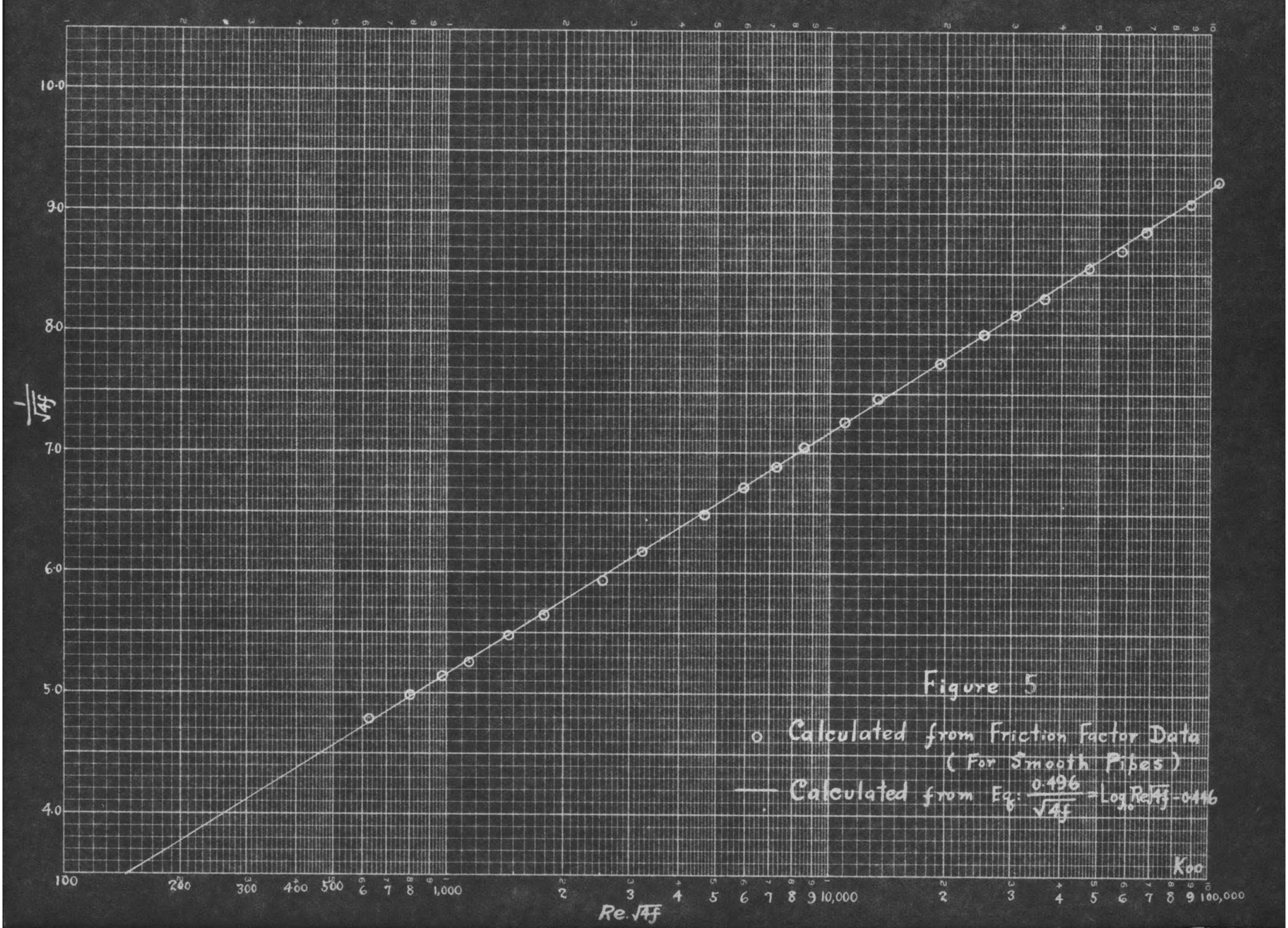


Figure 5

$\frac{1}{\sqrt{4f}}$
10.0
9.0
8.0
7.0
6.0
5.0
4.0
100
200
300
400
500
6
7
8
1,000
2
3
4
5
6
7
8
9
10,000
2
3
4
5
6
7
8
9
100,000
$Re.\sqrt{4f}$
○ Calculated from Friction Factor Data
(For Smooth Pipes)
— Calculated from Eq: $\frac{0.496}{\sqrt{4f}} = \log_{10} Re\sqrt{4f} - 0.446$
Koo

The biggest disadvantage of the above found equation is that for a given Re., friction factor has to be obtained through trial and error method. However, it has the particular advantage of calculating velocity of flow when pressure drop is known. (See Figure 5a). For iron and steel pipes, von Karman relation fails to be a straight line as shown in Figure 5a.

G. Effect of Entrahce Conditions on Friction Factor.

1. On Viscous Flow:

In viscous flow of fluids a parabolic velocity distribution curve has been theoreticallt derived as given in Eq. (8). An excellent review on the effect of entrance conditions on friction factor has recently been given by Prandlt and Tietjens (13). At the entrance of a round inlet, the velocity distribution is in the form of a trapezoid for laminar flow. As the fluid is flowing away from the inlet, the length of the shorter base of the trapezoid is diminished gradually, and finally the velocity distribution is in the form of a parabola. This inlet length which is required to build up the normal parabolic velocity distribution (better known as calming length) is found by Boussinesq (68) and Schiller (38) to be a function of pipe diameter and Reynolds number. As early as 1891, Boussinesq (68) proposed the following equation,

$$X = 0.065\ D\ Re \qquad (21)^*$$

while Schiller's equation proposed in 1922 has a smaller

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*Derived for rounded inlet, but on account of negligible contraction at square edge inlet in viscous region the formula should be a good approximation in most cases.

constant

$$X = 0.0288 \, D \, Re \qquad \ldots\ldots(21a)$$

From Nikuradse experiments as quoted by Prandlt and Tietjens (13), it is seen that Boussinesq's constant in Eq. (21) holds good only at high X/D Re, while Schiller's constant in Eq. (21a) holds good at only low X/D Re. However, Schiller's equation checks very well with friction factor measurements. Schiller has experimentally shown that due to insufficient calming length, the friction factor will be bigger than what is required by Hagen-Poiseuille's law. Therefore, the importance of sufficient calming length for friction factor determination should not be overlooked.

2. On Critical Value:

The critical value or the value of the critical Reynolds number which is the transition point from laminar to turbulent flow depends very much on entrance conditions & but slightly on the roughness of the pipe wall as has been found by Schiller (6)(9). He illustrated this by varying the distance of an adjustable plate placed near the mouth of a pipe which has a round inlet; the entrance disturbances will be the greatest as the distance is nearest to the mouth. He found that an increase in the inlet disturbance results in a lowering of the critical value, approaching 2,320 as a lower limit, while the upper limit can be controlled to 12,000. With a sharp-edged inlet, the critical value was found to be 2,800. On an artificially 1.6 cm. roughened pipe having a spiral thread of 0.4 mm. pitch and 0.3 m.m. depth, the critical value was found to be still the same. Upon fixing a rounded inlet to this roughened pipe, critical value was found to be as high as 20,000. It was emphatically pointed out by Schiller that some of the previous investigators who did find still lower critical value of Reynolds number than 2,320 was due to the use of too short an inlet length that led to erroneous results. For the upper limit, Ekman(39) found by careful elimination of disturbances that the critical value can reach as high as 40,000. It is thus seen how the critical value is dependent on the entrance conditions, and for ordinary sharp-edged inlet of sufficient inlet length (i.e., $X/DRe > 0.0288$), the critical value may be taken as 2,800. Substituting Re. = 2,800 in Eq. (21), inlet length ought to be greater than 80.7 D. Schiller & Kirsten (63)(64)(65) found that for a given inlet disturbance, the critical number continuely decreases with increasing tube length. They found that in one case where X/D = 1128, Re. = 3,500. Thus, the lower critical number, 2,320 has not been reached in their experiments. It might be explained that as the calming length is increased, inlet disturbances eliminate away proportionally, but wall roughness might come into picture.

3. On Turbulent Flow:

The effect of inlet length or calming section on the turbulent friction factor is well illustrated in Hermann's recent experiments (30) on water. It seems that the inlet length requirement is not so serious as compared with the viscous flow. About 25 D it was found that the turbulent friction factor will deviate ± 4% at Re. = 830,000, ~~while~~ and ± ~~at~~ 3% at Re. = 262,000. About 100 D, the friction factor will deviate ± 2.5% at Re. = 830,000 and ± 1.7% at Re. = 262,000. About 250D, it will deviate ± 0.9% at Re. = 200,000. The inlet length effect will be still less important at lower Reynolds numbers in turbulent flow. Nikurades (32) has shown recently that at Re. = 900,000, the velocity distribution curves show no difference at inlet length 100D, 65D, or 40D. However, it is recommended to use as long an inlet length as possible to eliminate any entrance disturbances, at least 40D.

The effect of form of inlet mouth of the pipe on the friction factor has also been worked out by Hermann(30) in the same paper. About 50D, for a rounded inlet the friction factor will be 3.5% higher than the ordinary sharp-edged inlet, and for a ring-shaped inlet the factor will be 2% higher. Above 100 D, this effect is less than 1%.

H. Effect of Roughness of Pipe Wall on Friction Factor

Stanton (40) in his 1911 paper on The Mechanical Viscosity of Fluids stated that by artificially roughening the pipe wall, he was able to make the surface friction or F of two pipes of different diameter vary as the square of the velocity; that means the friction factor f in his case will not be a function of Reynolds number. Recently, an exhaustive study of the roughness problem accompanied with their experimental work in rectangular channels has been undertaken by Hopf (41)(42), Fromm (43)(44), and Fritsch (45). Their study has covered a wide field of surfaces varying from smooth-drawn brass, wood, cement, sheet iron, and artificially roughened surfaces of wavy & of saw-like shape; ~~while~~ the sizes range from the smallest pipe tested in the laboratory to conduit of 18 feet in diameter. They found that there are two distinct families of curves on a log-log plot of friction factor against Reynolds number. In the first, all the curves approximate to horizontal straight lines, i.e., friction factor varies very little or does not vary at all with the Reynolds number; while in the second all the curves are almost parallel to the experimental curves of Blasius and other workers on smooth pipes, i.e., the friction factor decreases as Reynolds number increases. The first family includes very rough iron, rough cement, and saw-shaped pipe walls. The second family includes wood, sheet-iron,

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and wavy-shaped walls. It is obvious that the first family of curves indicate that the friction factor is independent of Reynolds number thus surface friction varies with the square of velocity as it has been found by Stanton before, while the second family of curves indicate that the friction factor is still dependent on Reynolds number, thus follows the general index law but with different coefficients as compared with in the case of "technically smooth" pipes. Attention must be called here to distinguish surface friction F from friction factor f, the relation between them is given in Eq. (2a). Engineers in interpreting Fanning equation often state that pressure drop or loss of head in a pipe is directly proportional to the square of velocity in turbulent flow. This is correct only for the first family of very rough pipes, while in other cases in making this statement they assume that f is a constant and ignore the effects of diameter, velocity and kinematic viscosity which constitute the Reynolds number.

In all cases, however, friction factor in rough pipes is always greater, sometimes eight times as big as shown by Fritsch (45), ~~than~~ as that ih smooth ones, therefore, it might be said that the new equation suggested by the writer serves as the lower limit of all the cases. Davies and White (46) in reviewing the laws of flow of fluids in pipes and channels have suggested that any ordinary surface can be defined completely by two factors, one is size factor and the other is shape factor. But the hard problem is how definitely to measure these factors.

von Mises (27) in generalizing pipes of all kinds of roughness to a single law by merely adding a roughness factor to the above stated general index law suggested the following equation:

$$f = \alpha + \sqrt{e/R} + \beta \, Re.^{C} \qquad ......(22)$$

substituting his recommended coefficients,

$$f = 0.0024 + \sqrt{e/R} + 0.424 \, Re.^{-0.50} \qquad .....(22a)$$

where e is a roughness coefficient having a linear dimension and is a constant for any particular pipe. The values of the roughness coefficient for different materials are given in Table 8 after von Mises.

Eq. (22a) suggested by Mises is consistent with ~~corresponding to~~ dimensional homogenity, and is recommended by Schiller (48) to be used for practical purposes. von Mises' equation may be called the "modified general index law", since it is different from the General Index Law only by an additional factor. Of course, this modified form can be only applied to the second family of pipes, while the friction factor in any class of pipes of first family will be a constant. Granting that Eq. (22) can be applied generally to technically smooth pipes and rough pipes of the second family, it is obvious to see the effect of diameter on the friction factor;

TABLE 8

ROUGHNESS COEFFICIENT e (AFTER von MISES)

Material	10^6 x e in cm.	10^3 x $\sqrt{e}$ in $\sqrt{cm}$.
Glass	0.2 to 0.8	0.45 to 0.9
Drawn brass, lead, copper	0.2 to 1.0	0.45 to 1.0
Polished cement	7.5 to 15	2.7 to 3.9
Rough cement	20 to 40	4.5 to 6.3
Rubber tubing, smooth	6 to 12	2.4 to 3.5
Rubber tubing, rough	15 to 30	2.7 to 5.5
Gas pipes	20 to 50	4.5 to 7
Asphalt lead or cast iron	30 to 60	5.5 to 7.7
Cast iron - New	100 to 200	10 to 14
Cast iron - used	250 to 500	16 to 22
Riveted tin pipe	200 to 500	14 to 22
Wood - smooth	25 to 50	5 to 7
Wood - polished	50 to 100	7 to 10
Wood - rough	200 to 400	14 to 20
Masonry pipes - smooth	200 to 400	14 to 20
Masonry pipes - rough	2,000 to 4,000	45 to 63
Earth wall	10,000 to 20,000	100 to 140

i.e., at the same Reynolds number the friction factor of a small pipe will be bigger than that of a large pipe; in other words, a very smooth tube will already be hydrodynamically rough in regard to Reynolds number. This effect has been well illustrated by Wildhagen (49) in his experiments on compressed air using glass capillaries. He found that only above 0.5665 m.m. in diameter, the friction factor in glass capillaries agrees with Blasius' experiments; while below that size the friction factor is greater, the smaller the size the greater is the friction factor. The smallest capillary he used is 0.286 mm. in diameter.

The complicated roughness problem has been gradually cleared up by the previous workers. It is believed that the problem might be further cleared by applying the modified general index law to all the available data on rough pipes at present and determining the coefficients for every class of roughness. The most difficult problem is that there is no sound way of measuring the roughness factor; so it has to be calculated from actual data for any individual class of pipe.

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III. REVIEW OF ISOTHERMAL VELOCITY DISTRIBUTION OF FLUIDS IN CIRCULAR PIPES

III. REVIEW OF ISOTHERMAL VELOCITY DISTRIBUTION OF FLUIDS IN CIRCULAR PIPES.

A. Velocity Distribution Formula for Laminar Flow*

In deriving Hagen-Poisseuille's Law, a parabolic velocity distribution formula has been used. A theoretical derivation of this formula was first given by Hagenbach(2) in 1860. Hatschek(3) has given Hagenbach's ~~his~~ derivation as follows. Let us consider a vertical length, L, of a circular pipe having its radius, R. A difference of pressure, p, is maintained between the two ends of this vertical pipe, which causes the fluid to flow through the pipe. We assume the flow to be such that every particle of fluid moves parallel to the axis of the pipe with a constant velocity, v. For reasons of symmetry, this velocity will be the same for all points lying on the same circle, so that we may consider the fluid composed of cylindrical laminae moving with velocities which are functions of their radii. The force exerted by pressure p on a cylinder of radius r will be,

$$F_p = \pi r^2 p \qquad .....(1)$$

while the resistance around the surface of the cylinder caused by the viscosity of fluid, will, according to

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* For a detailed discussion, refer to Lamb's book(1)

Newton's fundamental hypothesis on the motion of liquids, be given by the product: area x viscosity coefficient x velocity gradient, i.e.

$$F_V = 2\pi r \, L\mu dv/dr \qquad .....(2)$$

For steady flow, i.e., v is to remain constant, the forces acting on the cylinder of fluid in consideration must be equal and opposite, therefore

$$rp = - 2 \, L\mu dv/dr \qquad .....(3)$$

or

$$dv/dr = - \frac{rp}{2 \, L\mu} \qquad .....(3a)$$

By integration we get,

$$v = - \frac{r^2 p}{4 \, L\mu} + \text{constant} \qquad .....(4)$$

The usual assumption is that the velocity is zero at the wall, i.e., v = 0 for r = R. Then

$$v = - \frac{r^2 p}{4 \, L\mu} + \frac{R^2 p}{4 \, L\mu} = \frac{p}{4 \, L\mu} \, (R^2 - r^2) \qquad ...(5)$$

This is the equation of a parabola. For the maximum velocity, $V_{max.}$ i.e., r = 0, Eq. (5) becomes

$$V_{max.} = \frac{P}{4 \, L\mu} \, R^2 \qquad .....(5a)$$

Thus,

$$v/V_{max.} = \left(\frac{R^2 - r^2}{R^2}\right) = 1 - (r/R)^2 \qquad ...(6)$$

Eq. (6) is a dimensionless velocity distribution formula for laminar flow which has the advantage that it may ~~to~~ be used in comparing velocity distribution of fluids at different velocities as well as for different size of pipes.

In 1929, ~~Recently~~, Levy(4) proposed a general velocity distribution formula which may be written as follows:

$$v/V_{max.} = \left[1 - (r/R)^2\right]^m \qquad .....(7)$$

For laminar flow, he stated that m = 1, thus Eq. (7) reduces to Eq. (6).

Very little work has been done on the isothermal velocity distribution of fluids in laminar flow.

It seems Morrow's data(5) on water in glass pipes is the only one available at present. Unfortunately, his velocity distribution measurements were made at an inlet length of 30 diameters only, which will lead to erroneous results (see p. __).

L P.

B. Velocity Distribution Formulae for Turbulent Flow.

Many empirical formulae have been proposed by hydraulic engineers, but each formula seems to be only applicable to either one set of experiments or to one kind of pipe. An excellent review of these forms may be found in Gümbel's article(6) or Forchheimer's book on hydraulics(7). In 1904, Christen(8) found a new formula which can be applied to Freeman's experiment(9) on half inch brazed brass pipe as well as to Bazin's experiment(10, 11) on 31-inch cement pipe. His formula may be written as

$$v = C(R-r)^{1/8} \qquad .....(8)$$

where C is a constant.

In 1911, Stanton(12) on actually measuring the velocity distribution of air in both smooth and rough pipes proposed the following equation which is in the form of a parabola,

$$v = V_{max.} - A\, r^2 \qquad .....(9)$$

where A is a constant.

In 1917, Sasvari(13) based upon Biel's friction factor equations(14) derived an approximate form for velocity distribution in circular pipes. It reads

$$v/V_{max.} = 1.25 \left[1 - (r/R)^2\right]^{1/4} \qquad ....(10)$$

Prandtl(15) in 1920 and von Karman(15) in 1921 based upon Blasius' resistance equation and few assumptions reached the following equation

$$v/V_{max.} = (1 - r/R)^{1/7} \qquad .....(11)$$

The one-seventh exponent ~~potential~~ has been supported by the earlier experiments of Nikuradse(16), but it is shown ~~disproved~~ by Nikuradse himself in his recent experiments(17) that the exponent ~~potential~~ actually decreases as Reynolds number increases. In 1929, Levy(4) proposed a general formula which has been given already as Eq. (7),

$$v/V_{max.} = \left[1 - (r/R)^2\right]^{m} \qquad .....(7)$$

where m is a function of Reynolds number which decreases as Reynolds number increases, varying from 0.318 at Re. = 2,320 to 0.179 at Re. = 1,000,000. The merit of Levy's equation is that his equation may be applied both to turbulent as well as to laminar flow. The latest equation is proposed by von Karman(18) in 1930, reviewed recently by Maccoll(19) in English, based upon Prandtl's mixing length theory(20) in turbulent flow. For isothermal flow in pipes, his equation may be written as

$$v = V_{max.} + (1/k)\sqrt{F/\rho}\left[\text{Log.}(1-\sqrt{r/R}) + \sqrt{r/R}\right] \quad \ldots(12)$$

or,

$$v/\sqrt{F/P} \quad = \quad a + b\ \text{Log.}\left(\frac{r\sqrt{F/\rho}}{\nu}\right) \quad \ldots\ldots(13)$$

This equation is derived originally for flow between parallel walls.

From a brief review of all the important formulae for turbulent flow, it follows that Equations (8), (11), and (7) may be generalized into the following dimensionless equation

$$v/V_{max.} = \left[1 - (r/R)^n\right]^x \quad \ldots\ldots(14)$$

When n = 1, Eq. (14) reduces to Eq. (8) if x = 1/8, and to Eq. (11) if x = 1/7. When n = 2, Eq. (14) reduces directly to Eq. (7). Consequently,

$$\text{Log.}(v/V_{\text{max.}}) = x \text{ Log. } \left[1 - (r/R)^n\right] \quad \ldots\ldots(14a)$$

Thus, a log-log plot of $(v/V_{\text{max.}})$ against $1-(r/R)^n$, where n is assumed to be known, ought to give a straight line having its slope x. This is the method adopted to test the applicability of the above proposed equations to experimental data.

C. Proposed Velocity Distribution Formula for Turbulent Flow.

In order to find the correct formula to use, all available velocity distribution experimental data in literature have been calculated and tabulated in Appendix D. They are the work of Stanton(12), Freeman(9), Lawrence and Braunworth(22), Nikuradse(16, 17), and Marshall(21). ~~Graphical~~ Plots of these data have been made by assuming n = 1 and n = 2, respectively, (see Fig. 6-36). It is found that the exponent of Prandtl-Karman's formula (i.e., n = 1) does not equal to 1/7, but it varies, while Levy's formula (i.e., n = 2) fits the experimental data only near the center of the pipe not near the wall. By assuming that the exponent in Prandtl-Karman's formula is a variable instead of 1/7, it is found the modified formula will fit the data very well except near the center of the pipe. It is obvious then that Levy's formula has over-corrected the defect in Prandtl-Karman's modified form. It must be recalled that Blasius' resistance law is in the form of a simple index law (i.e., $f = b\ Re.^{c}$) which has been proved to be incorrect in the previous chapter for a wide range of Reynolds number but might be used as an approximation for a small range. Nevertheless, instead of the General Index Law which has been advocated by the writer, one may use the simple index law for a certain range of Reynolds number but keep on changing the exponent, which corresponds of changing the slope of the friction factor in a log-log plot against Reynolds number, as one changes the range of Reynolds number. Since Prandtl-Karman's original formula of one-seventh potential is based upon Blasius' simple index law, it is already seen from a study of friction factor problem that the velocity distribution exponent should be a variable instead of a constant. Thus, the modified form may be written as

$$v/V_{max.} = (1-r/R)^{a} \qquad .....(14b)$$

It is quite expected that if we let n = 1.25 or n = 1.50, Eq. (14) might be applicable to the experimental data still better than Eq. (14b). The result shows that if n = 1.25, i.e.,

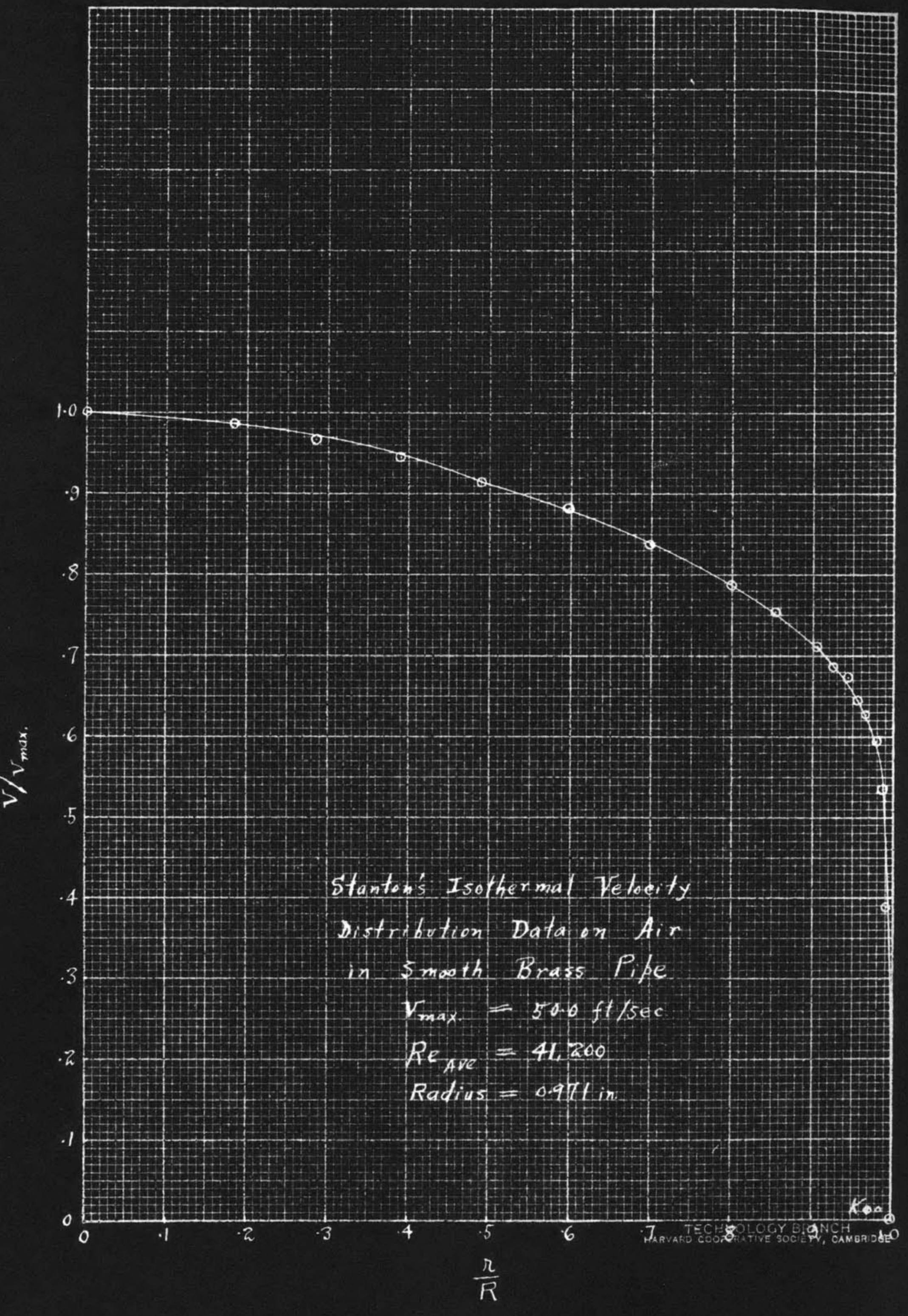
$$v/V_{max.} = \left[1 - (r/R)^{1.25}\right]^{x} \qquad .....(14c)$$

the formula is found to fit the data even better than Eq. (14b). However, for its simplicity and its further theoretical application in the following treatments, the modified form of Prandtl-Karman's formula, i.e., Eq. (14b) is advocated for ordinary use. The velocity distribution exponents "a" corresponding to all the experimental data in literature for smooth pipes have been graphically solved and their values will be found in Table 9. (See Figs. 6-36).

D. Relation Between Velocity Distribution and Friction Factor - Dependence of Velocity Distribution on Reynolds Number

Stanton(12) stated in his 1911 paper that from his experiments on air in smooth brass pipes the velocity distribution curves are only identical when the values of Reynolds number are equal. He further stated that for different Reynolds numbers the distribution curves are identical up to 0.8 R(R = radius of the pipe) but separated beyond this ratio, indicating a region of laminar flow near the wall. An examination of the available isothermal data by previous workers

reveals that the velocity distribution exponent actually decreases as Reynolds number increases. Recently, Nikuradse(17) has shown definitely the decrease of the exponent at high Reynolds numbers (Fig. 29). How can one explain this fact then?



Stanton's Isothermal Velocity
Distribution Data on Air
in Smooth Brass Pipe
$V_{max.} = 50·0$ ft/sec
$Re_{Ave} = 41,200$
Radius = 0·971 in.
V/Vmax.
1·0
·9
·8
·7
·6
·5
·4
·3
·2
·1
0
0
·1
·2
·3
·4
·5
·6
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1·0
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TECHNOLOGY BRANCH
HARVARD COOPERATIVE SOCIETY, CAMBRIDGE

Fig. 6

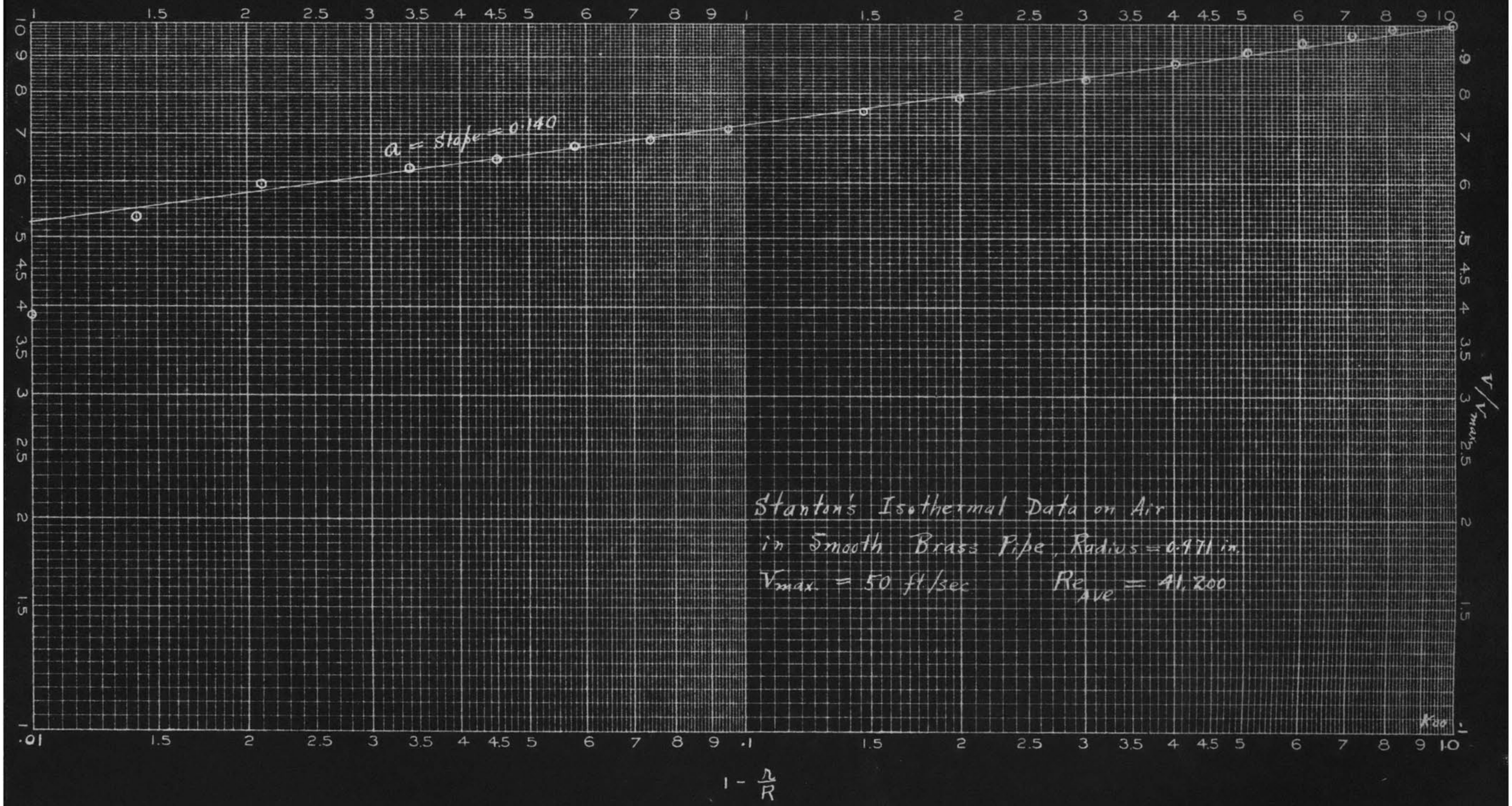
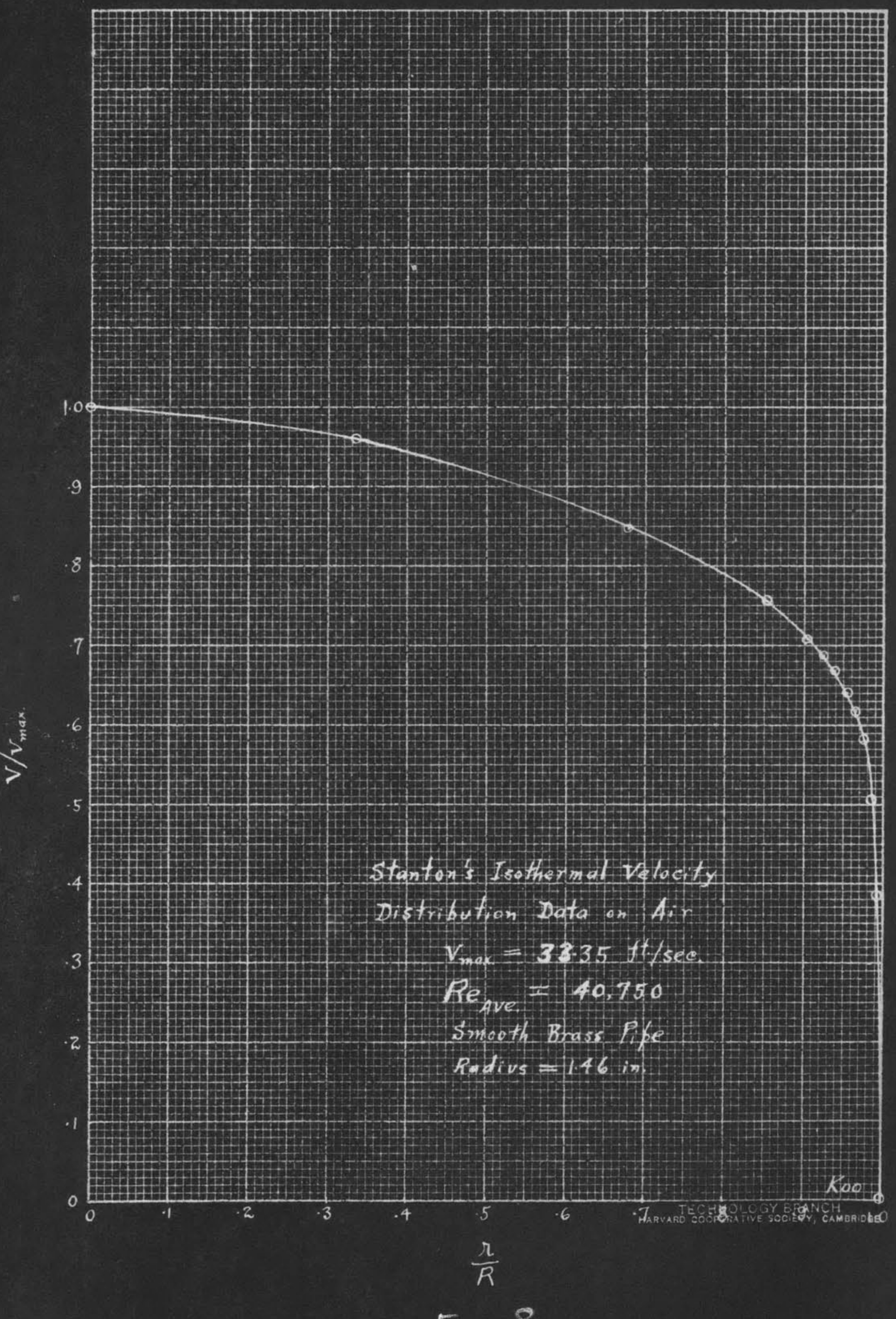

a = slope = 0.140
Stanton's Isothermal Data on Air
in Smooth Brass Pipe, Radius = 0.971 in.
$V_{max.}$ = 50 ft/sec
$Re_{Ave.}$ = 41,200
v/v_{max}
1 - $\frac{r}{R}$
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.9
.8
.7
.6
.5
.4
.1
Koo

Fig. 7



V/V_max.
1.0
·9
·8
·7
·6
·5
·4
·3
·2
·1
0
0
·1
·2
·3
·4
·5
·6
·7
1.0
r/R
Stanton's Isothermal Velocity
Distribution Data on Air
V_max = 33·35 ft/sec.
Re_Ave. = 40,750
Smooth Brass Pipe
Radius = 1·46 in.
Koo
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Fig. 8

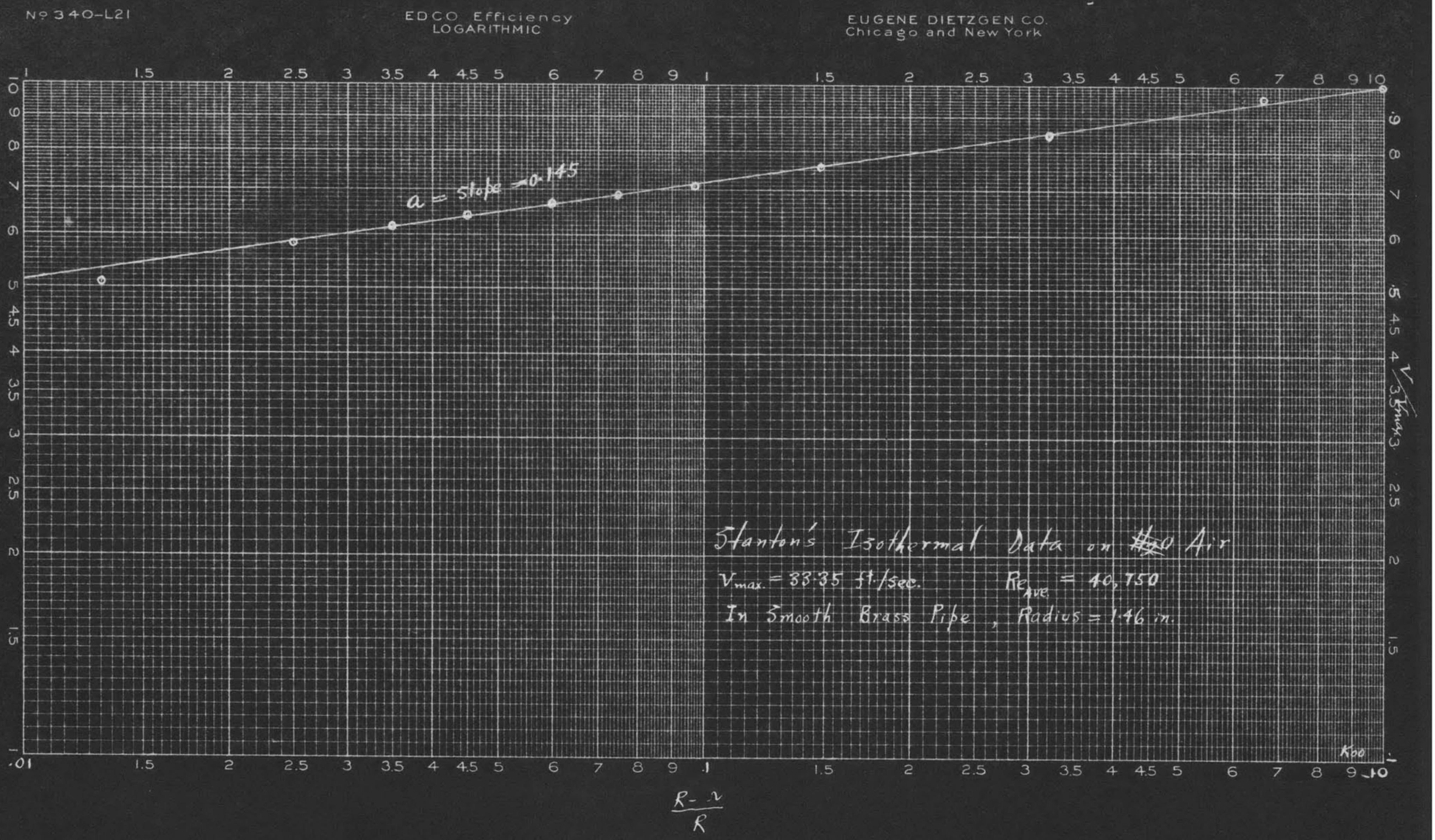
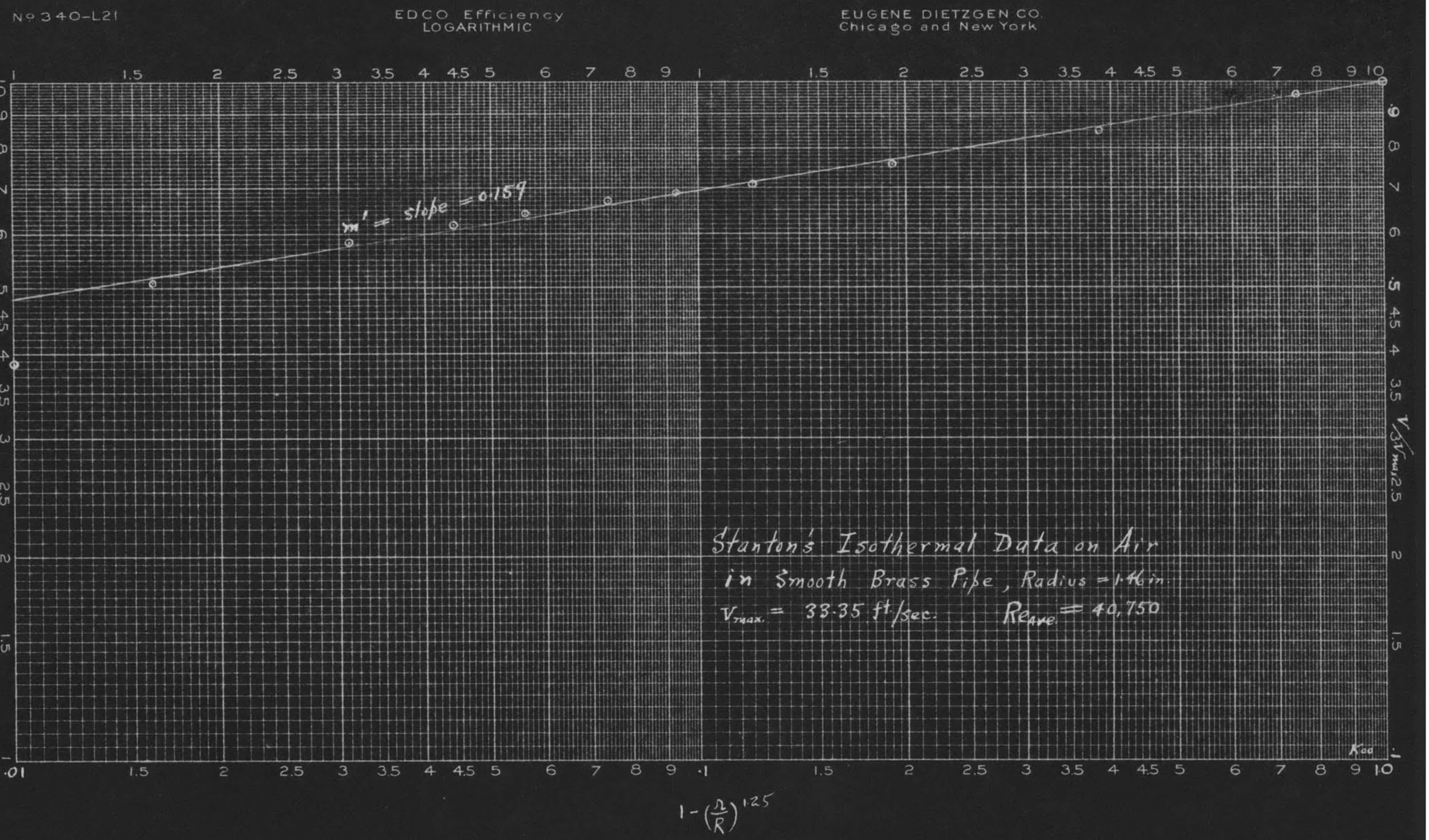

No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
a = slope = 0.145
Stanton's Isothermal Data on ~~H2O~~ Air
$V_{max.}$ = 33.35 ft./sec.
$Re_{AVE.}$ = 40,750
In Smooth Brass Pipe, Radius = 1.46 in.
$\frac{V}{V_{max}}$
$\frac{R-r}{R}$
.01
.1
1.0
Koo

Fig. 9



Stanton's Isothermal Data on Air in Smooth Brass Pipe, Radius = 1.46 in.

$V_{max.} = 33.35$ ft/sec. $Re_{ave} = 40{,}750$

Fig. 10

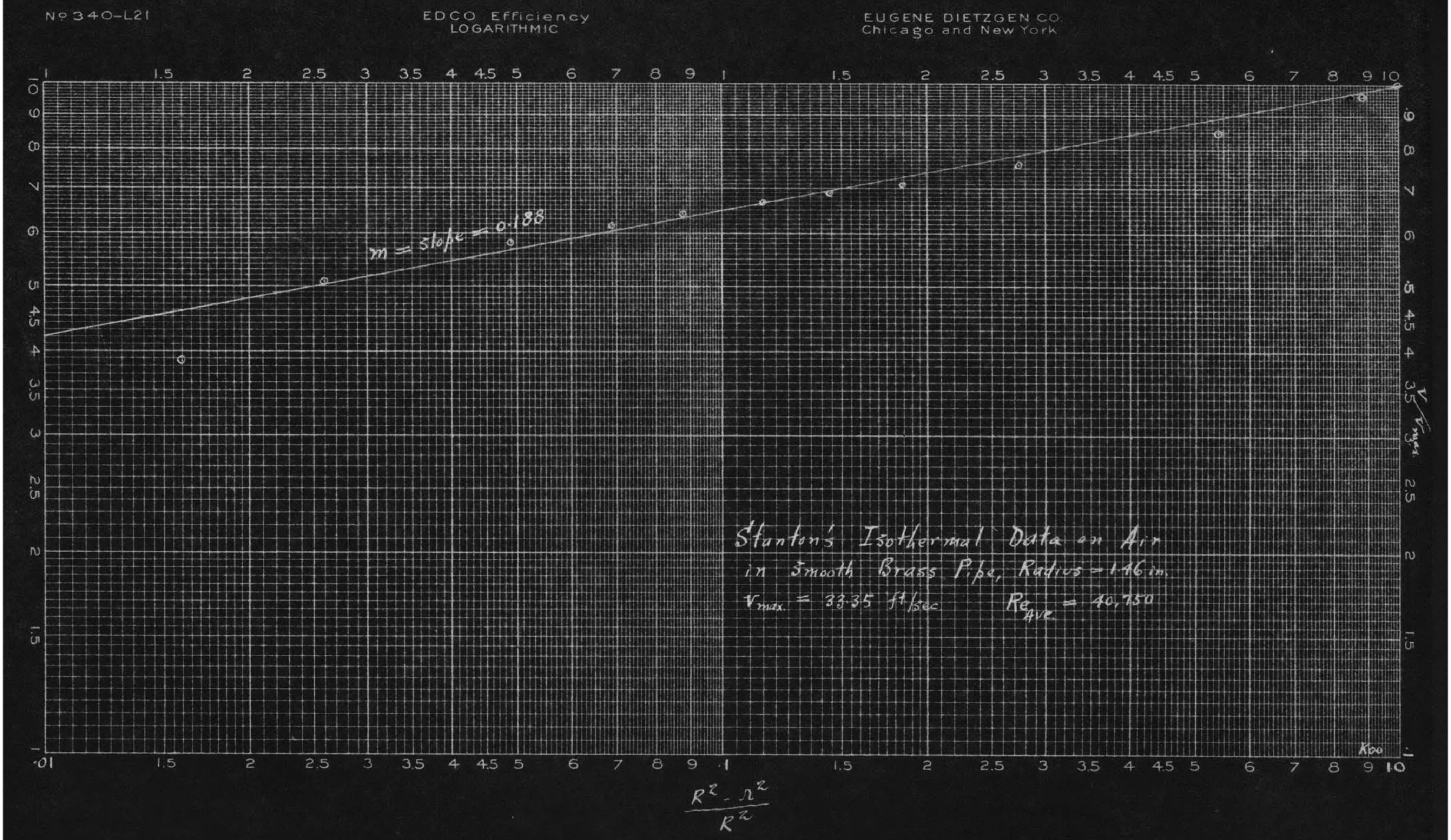
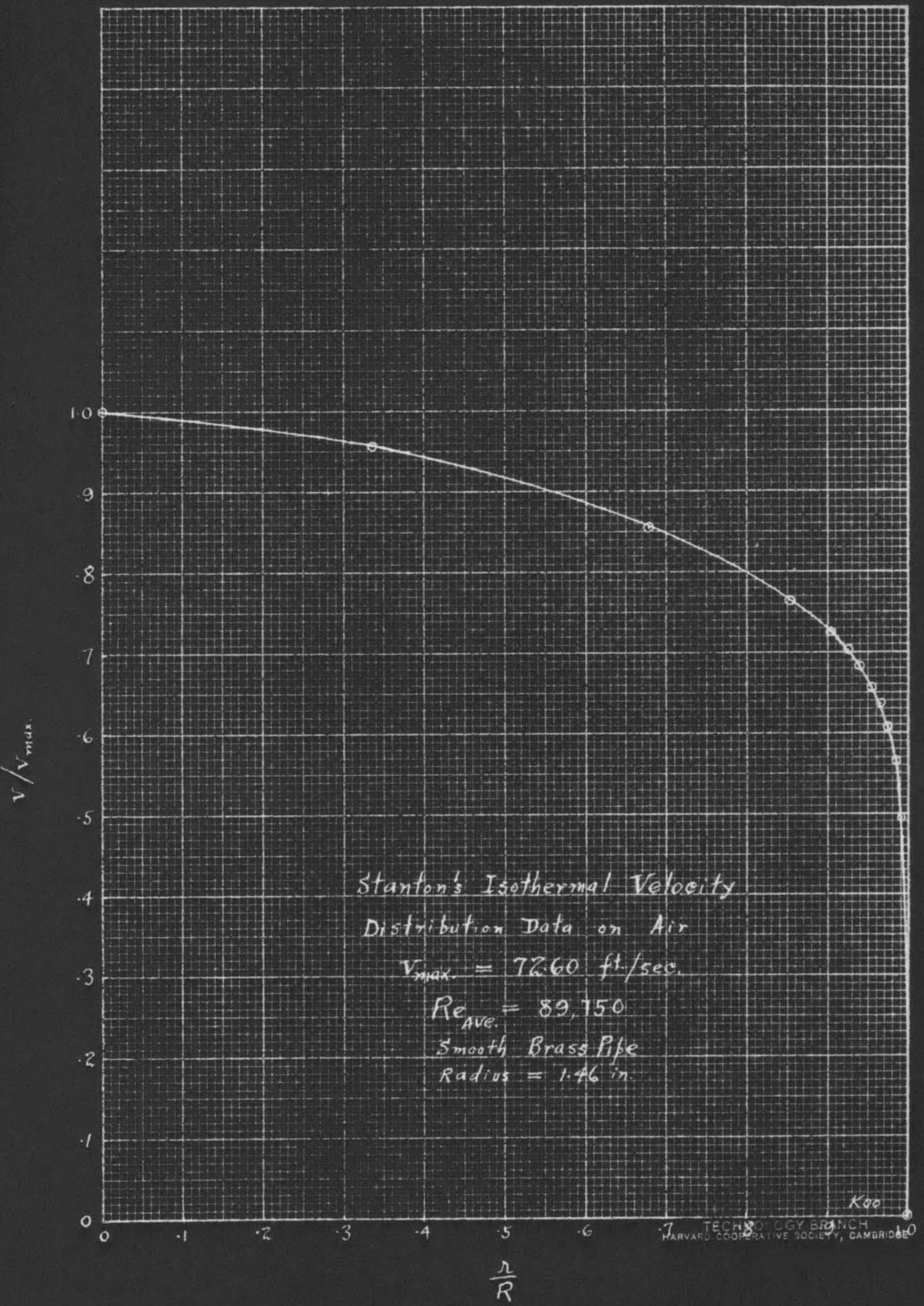

Nº 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
m = slope = 0.188
Stanton's Isothermal Data on Air
in Smooth Brass Pipe, Radius = 1.46 in.
$v_{max.}$ = 33.35 ft/sec
$Re_{Ave.}$ = 40,750
$v/v_{max.}$
$\frac{R^2 - r^2}{R^2}$
·01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
·1
1.0
·9
·8
·7
·6
·5
·4
·3
·2
K00

Fig. 11



Stanton's Isothermal Velocity
Distribution Data on Air
$V_{max.} = 72.60$ ft./sec.
$Re_{Ave.} = 89,150$
Smooth Brass Pipe
Radius = 1.46 in.
$V/V_{max.}$
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
$\frac{r}{R}$
Koo
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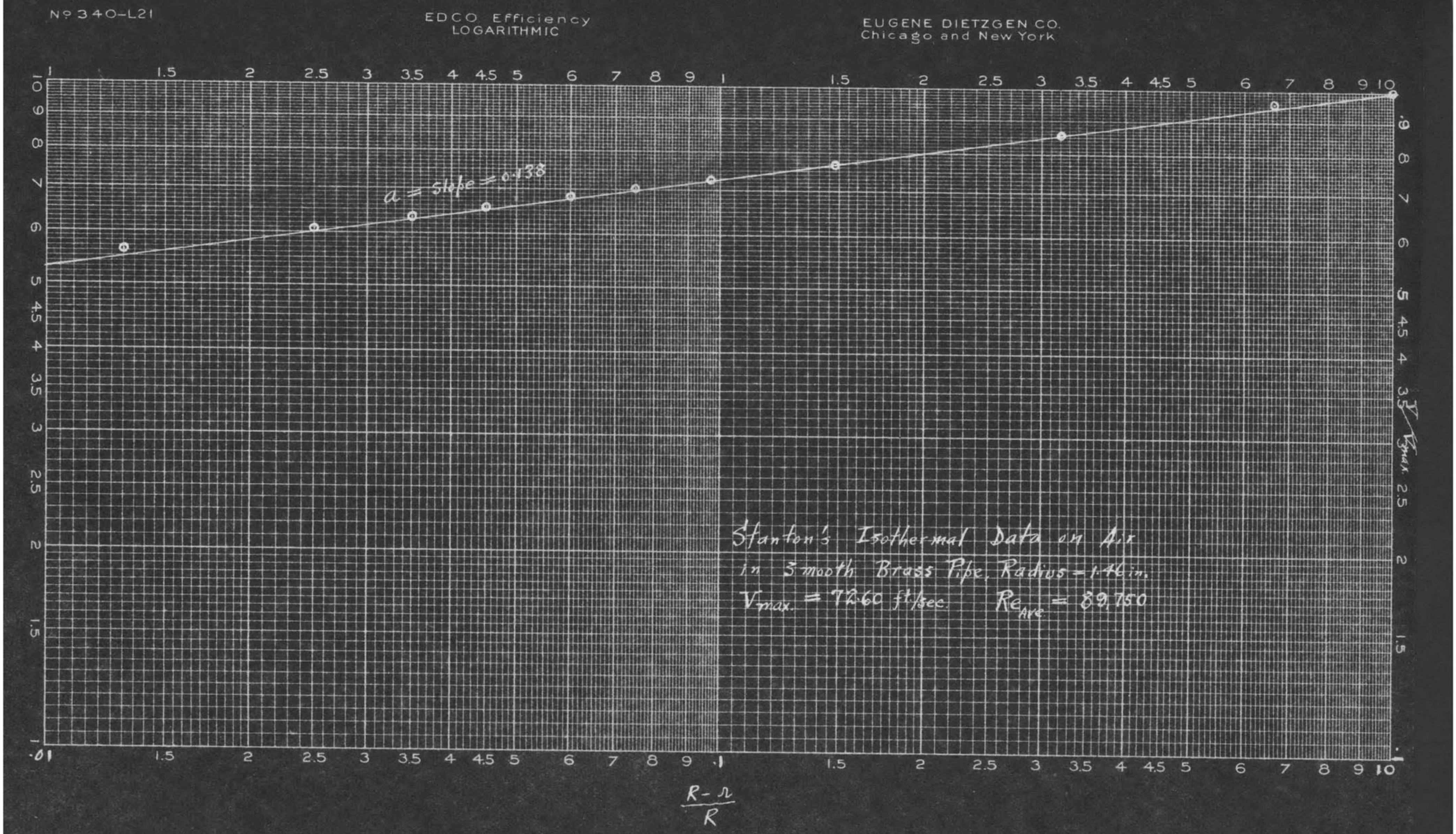
Fig. 12

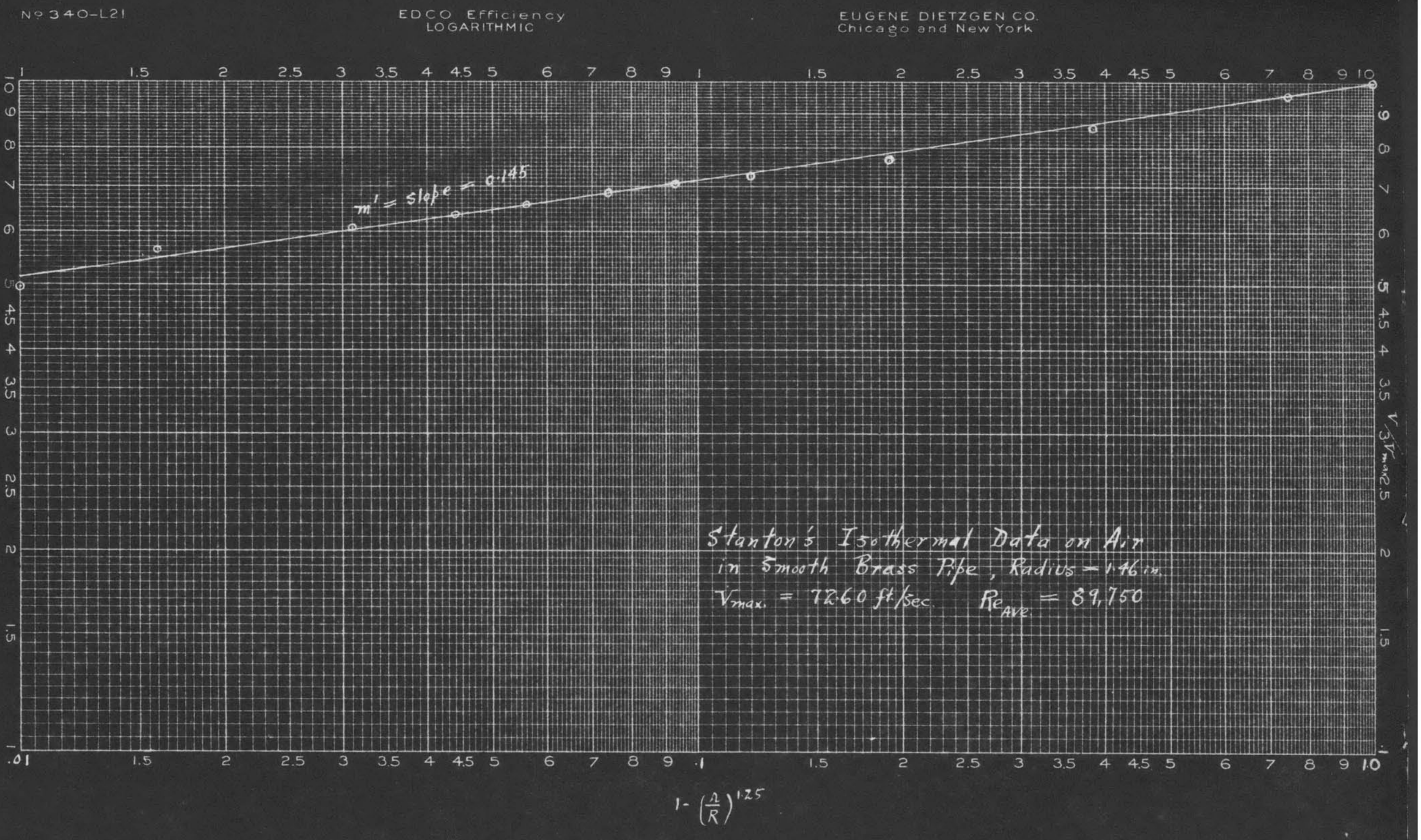


a = slope = 0.138

Stanton's Isothermal Data on Air in Smooth Brass Pipe, Radius = 1.46 in.

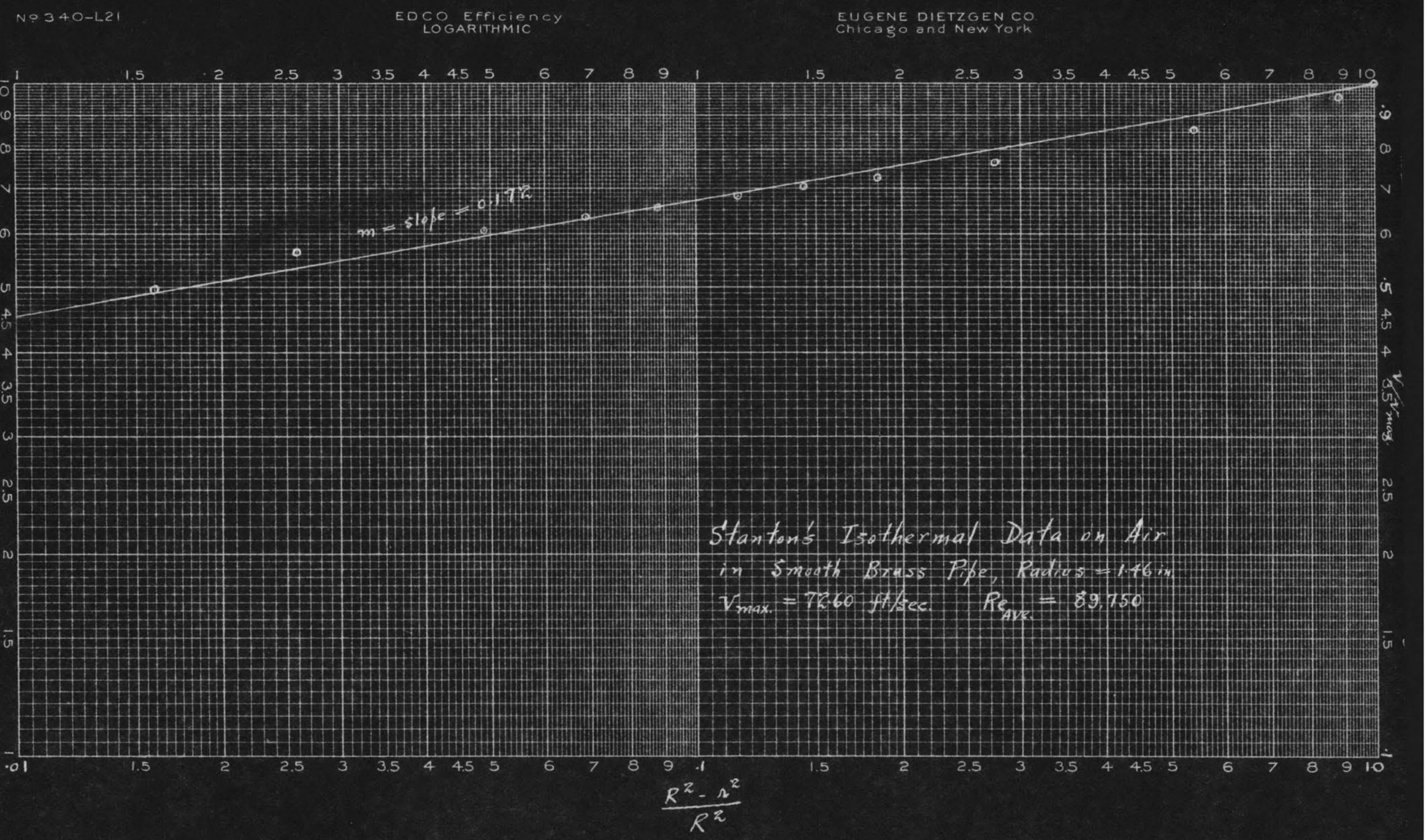
$V_{max.}$ = 72.60 ft/sec. $Re_{Ave.}$ = 89,750

Fig. 13



No 340-L21
EDCO Efficiency LOGARITHMIC
EUGENE DIETZGEN CO. Chicago and New York
m' = slope = 0.145
Stanton's Isothermal Data on Air
in Smooth Brass Pipe, Radius = 1.46 in.
$V_{max.}$ = 72.60 ft/sec. $Re_{AVE.}$ = 89,750
V/V_{max}
$1-\left(\frac{r}{R}\right)^{1.25}$
.01
.1
1.0
.1
.5
1.0

Fig. 14



Stanton's Isothermal Data on Air
in Smooth Brass Pipe, Radius = 1.46 in.
$V_{max.}$ = 72.60 ft/sec. $Re_{AVE.}$ = 89,750

Fig. 15

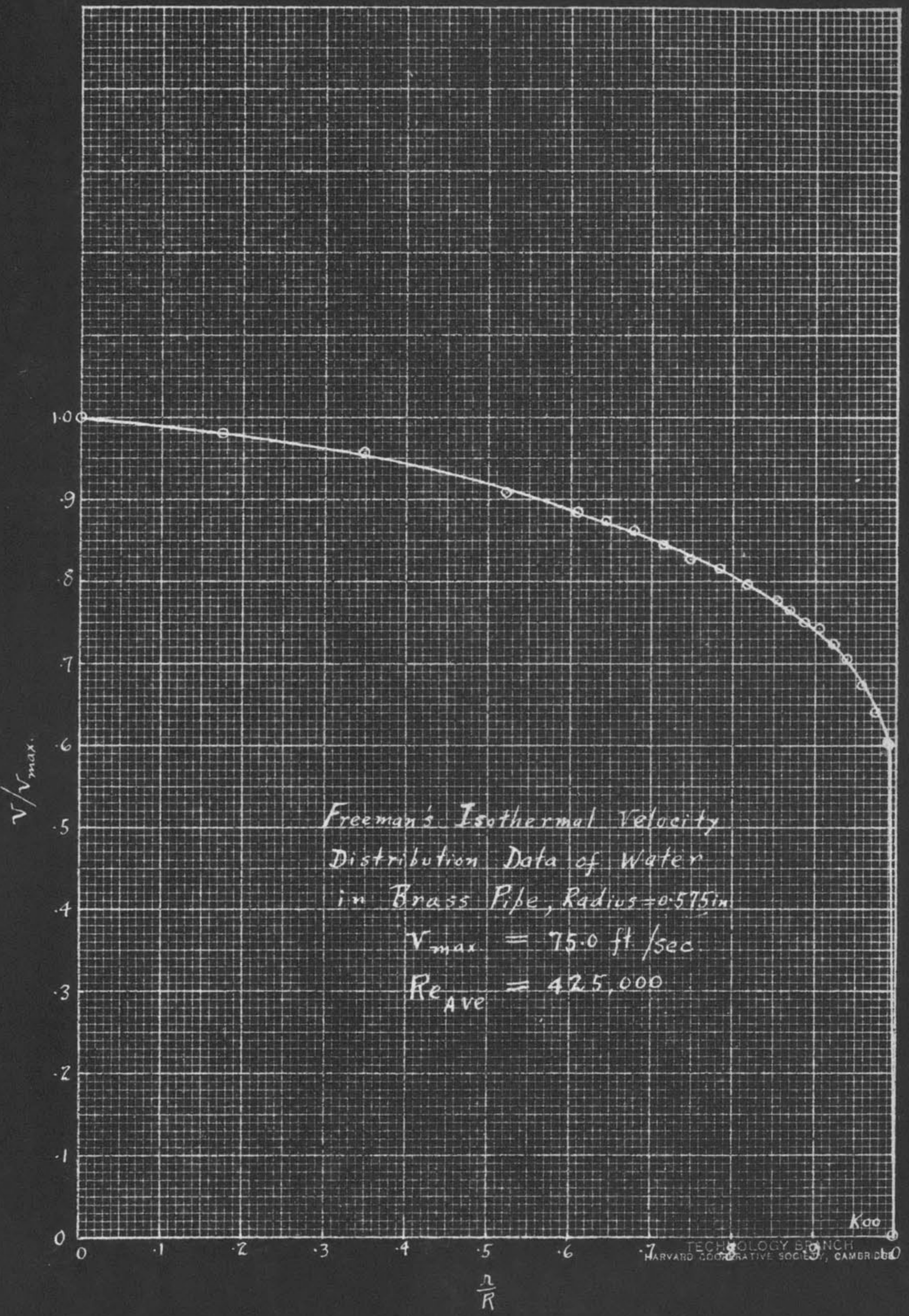

Freeman's Isothermal Velocity
Distribution Data of Water
in Brass Pipe, Radius = 0·575 in
V max = 75·0 ft /sec
Re Ave = 425,000
V/V max.
1·0
·9
·8
·7
·6
·5
·4
·3
·2
·1
0
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1·0
r/R
Kao
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Fig. 16

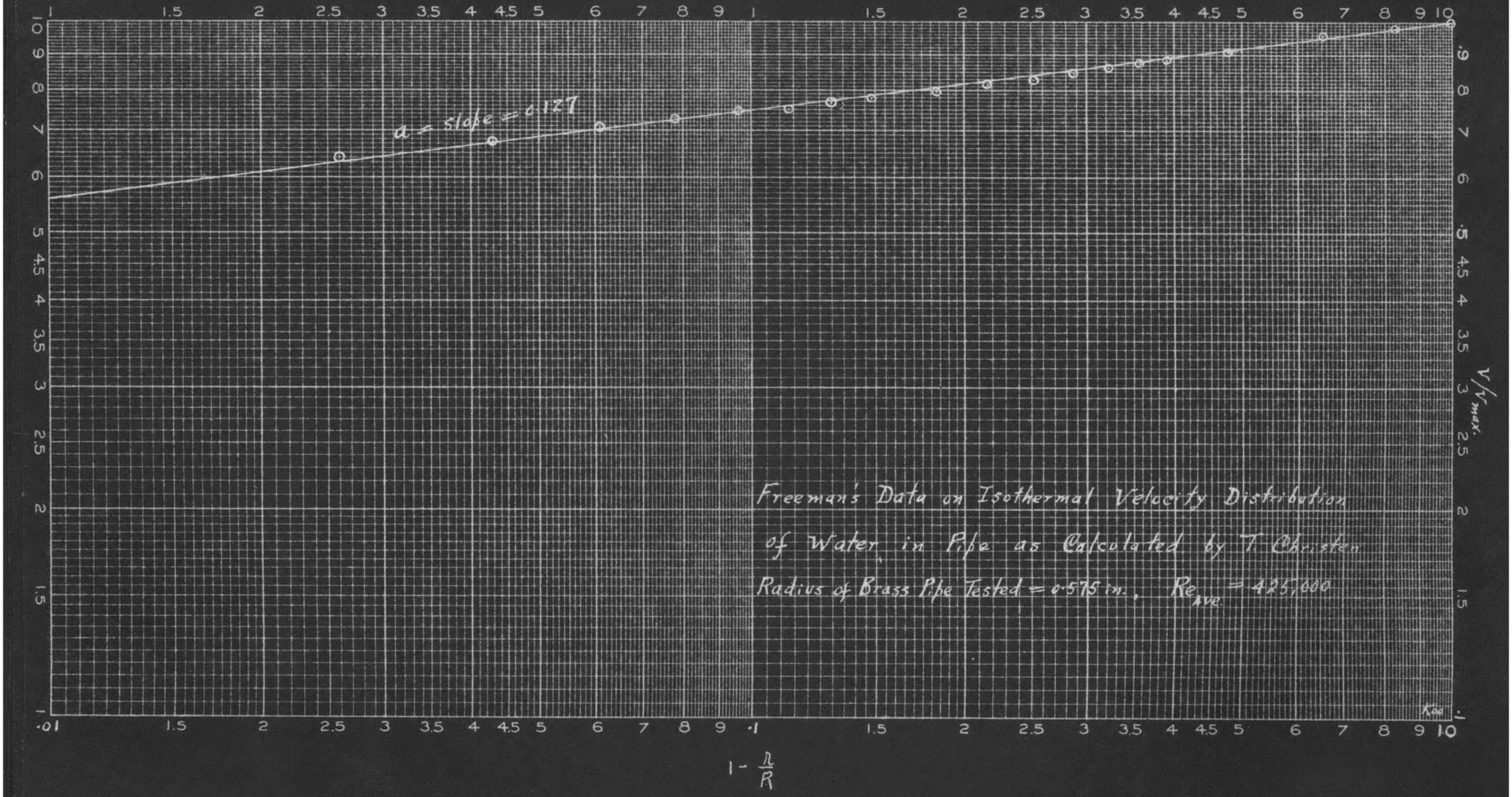

a = slope = 0.127
Freeman's Data on Isothermal Velocity Distribution
of Water in Pipe as Calculated by T. Christen
Radius of Brass Pipe Tested = 0.575 in., $Re_{ave.}$ = 425,000
1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 10
·01 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 ·1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 10
$1 - \frac{r}{R}$
10 9 8 7 6 5 4.5 4 3.5 3 2.5 2 1.5 1
·9 8 7 6 ·5 4.5 4 3.5 3 2.5 2 1.5 ·1
$V/V_{max.}$
Keo

Fig. 17

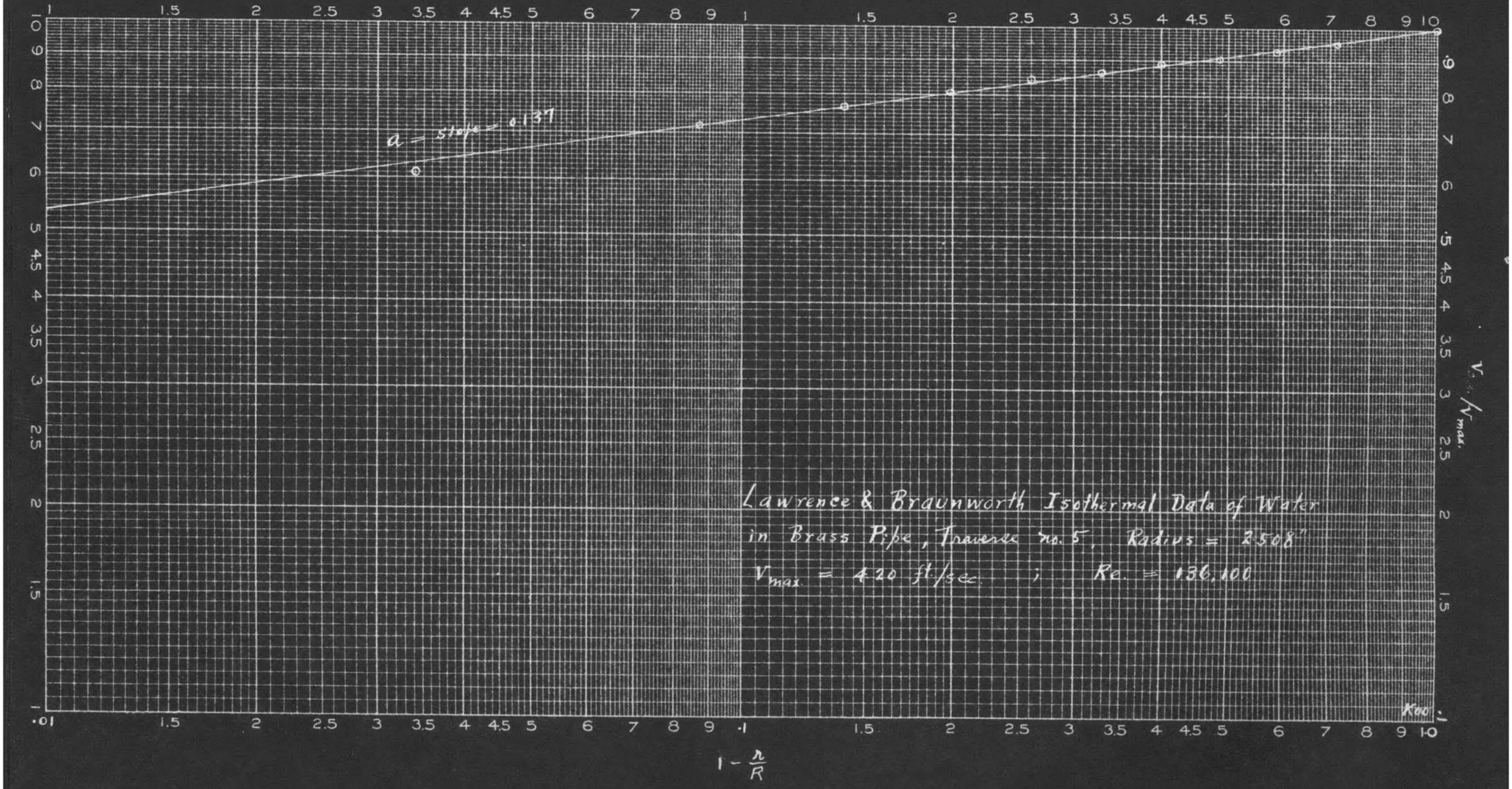

a = slope = 0.137
Lawrence & Braunworth Isothermal Data of Water
in Brass Pipe, Traverse no. 5, Radius = 2.508"
$V_{max.}$ = 4.20 ft/sec. ; Re. = 136,100
1 - r/R
V / $V_{max.}$
·01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
·1
1.0
·9
·8
·7
·6
·5
·1
K00

Fig. 18

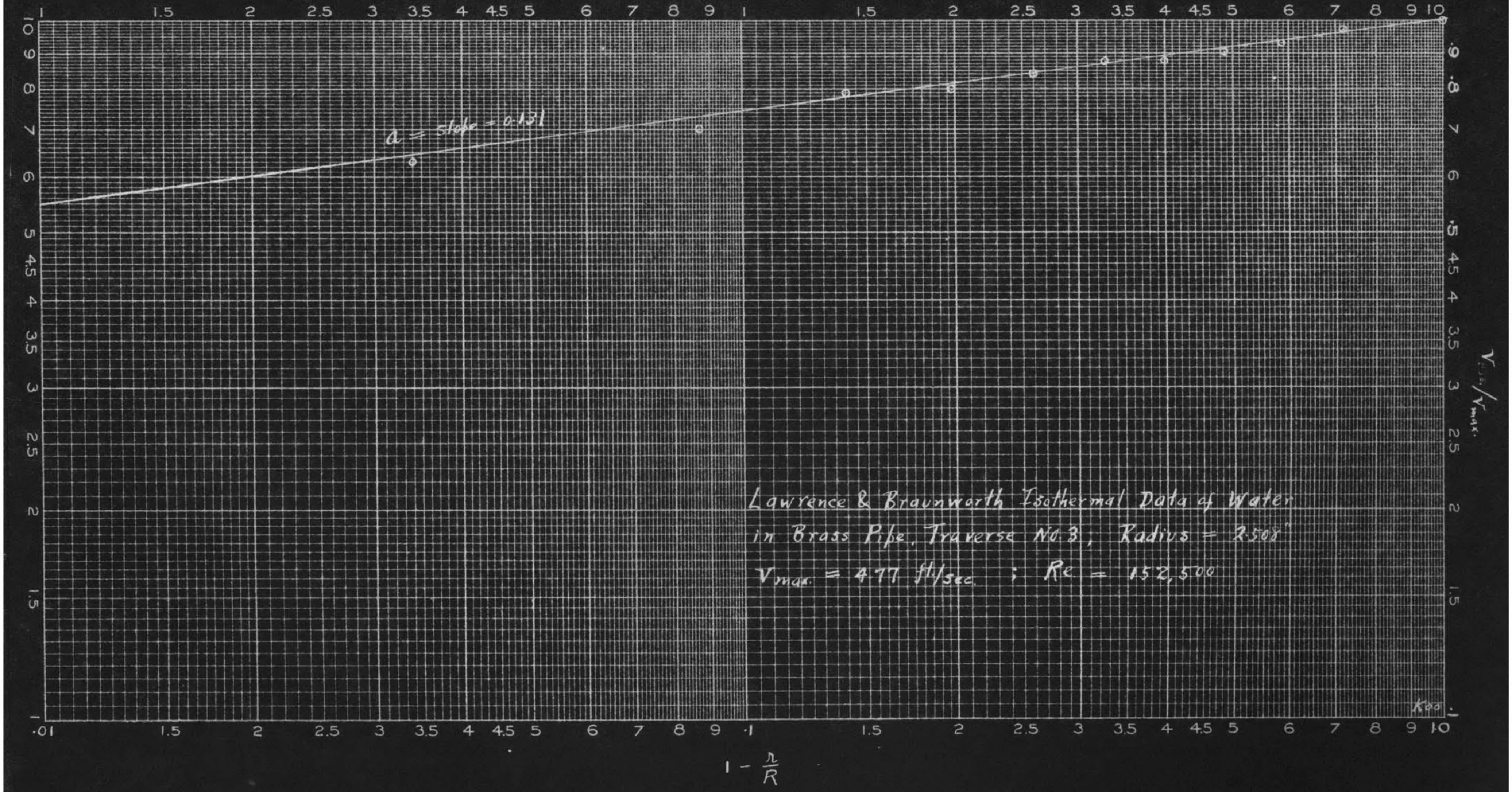

a = slope = 0.131
Lawrence & Braunworth Isothermal Data of Water
in Brass Pipe, Traverse No. 3, Radius = 2.508"
Vmax. = 4.77 ft/sec ; Re = 152,500
1 - r/R
V/Vmax.
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.9
.8
.7
.6
.5
.45
.4
.35
.3
.25
.2
.15
Koo

Fig. 19

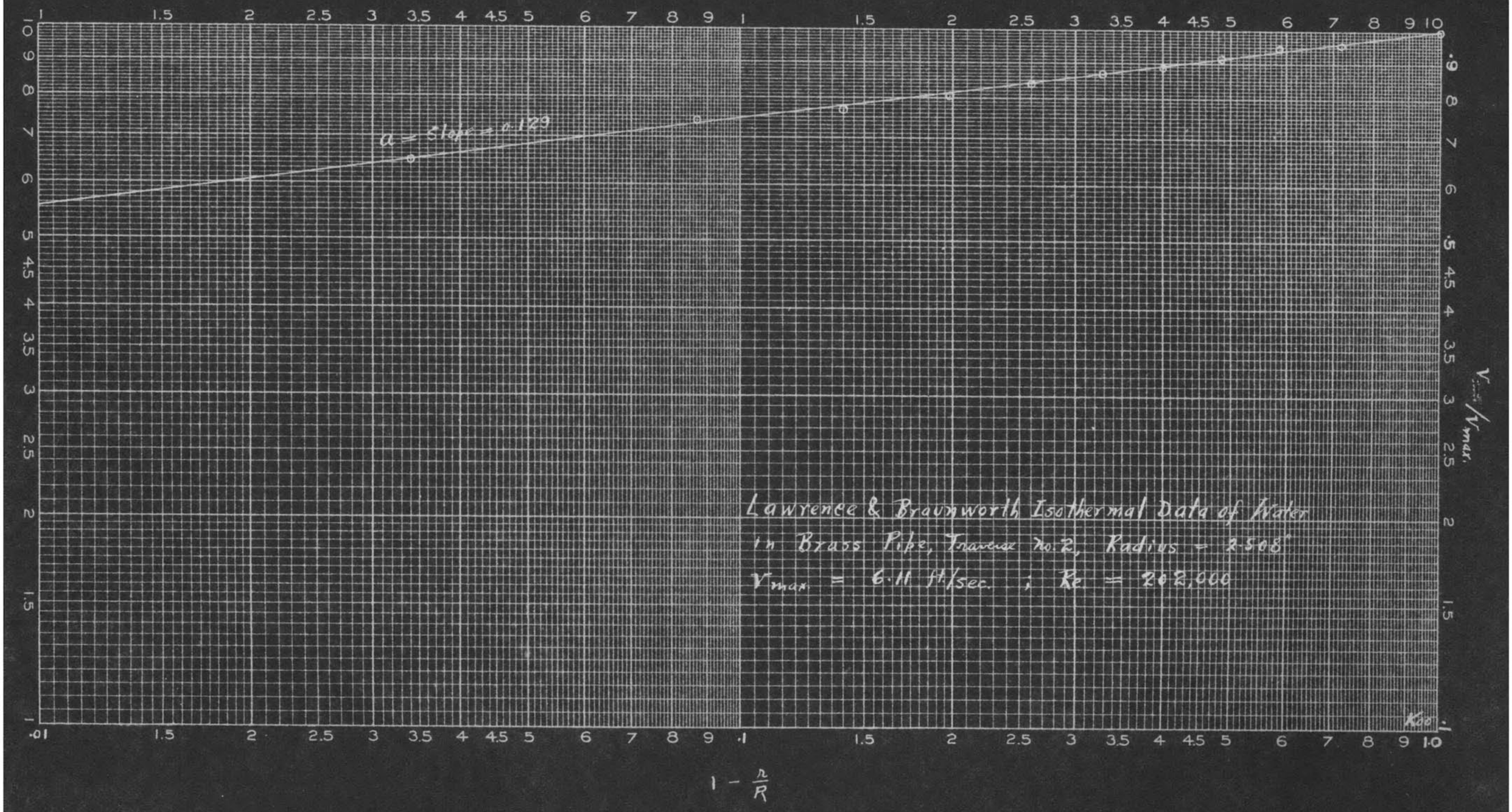

1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 10
10 9 8 7 6 5 4.5 4 3.5 3 2.5 2 1.5 1
·9 8 7 6 ·5 4.5 4 3.5 3 2.5 2 1.5 ·1
a = Slope = 0.129
Lawrence & Braunworth Isothermal Data of Water
in Brass Pipe, Traverse no. 2, Radius = 2.508"
$V_{max.}$ = 6.11 ft/sec. ; Re = 202,000
Koo
·01 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 ·1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 1.0
$1 - \frac{r}{R}$
$V/V_{max.}$

Fig. 20

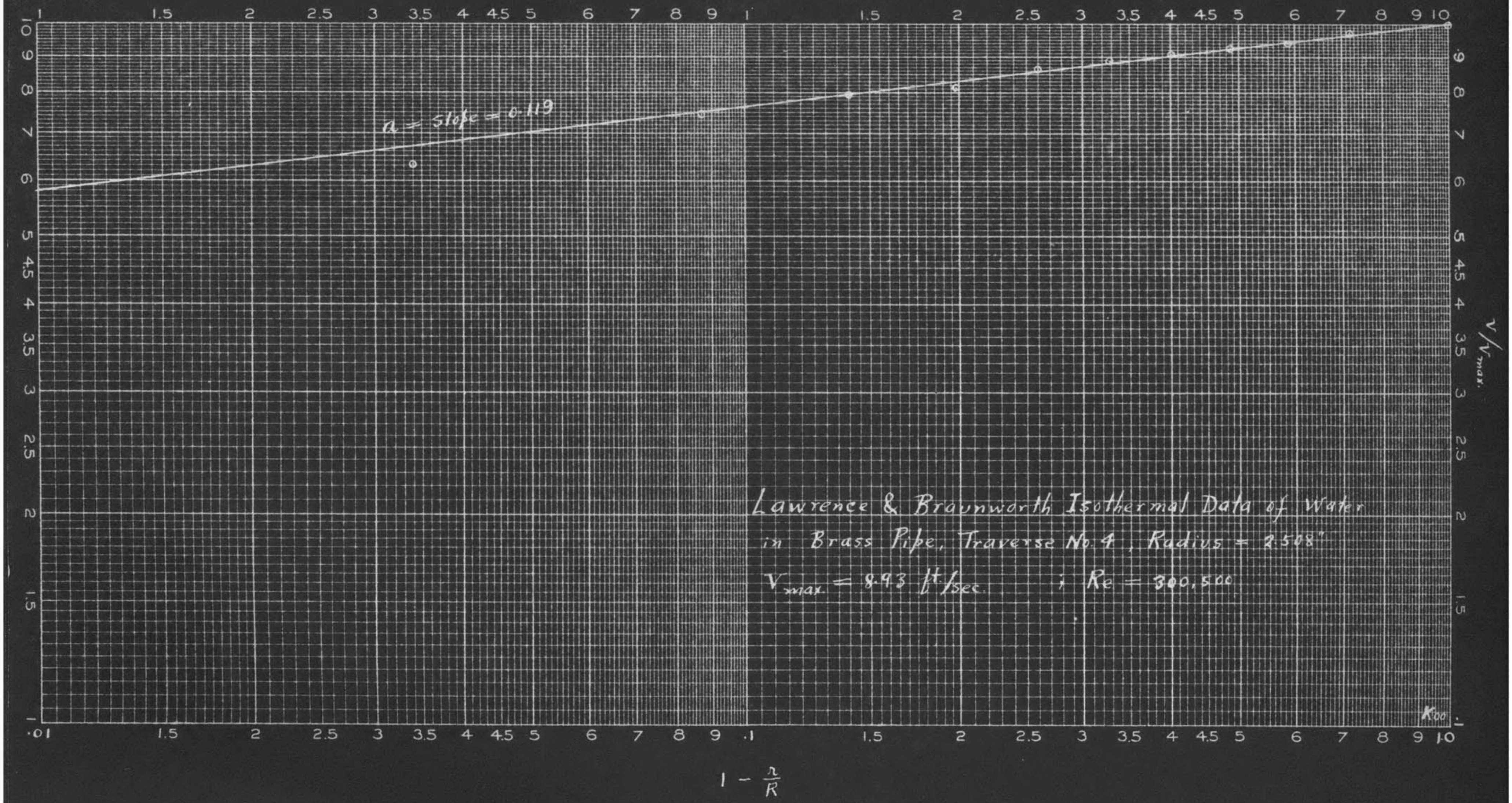
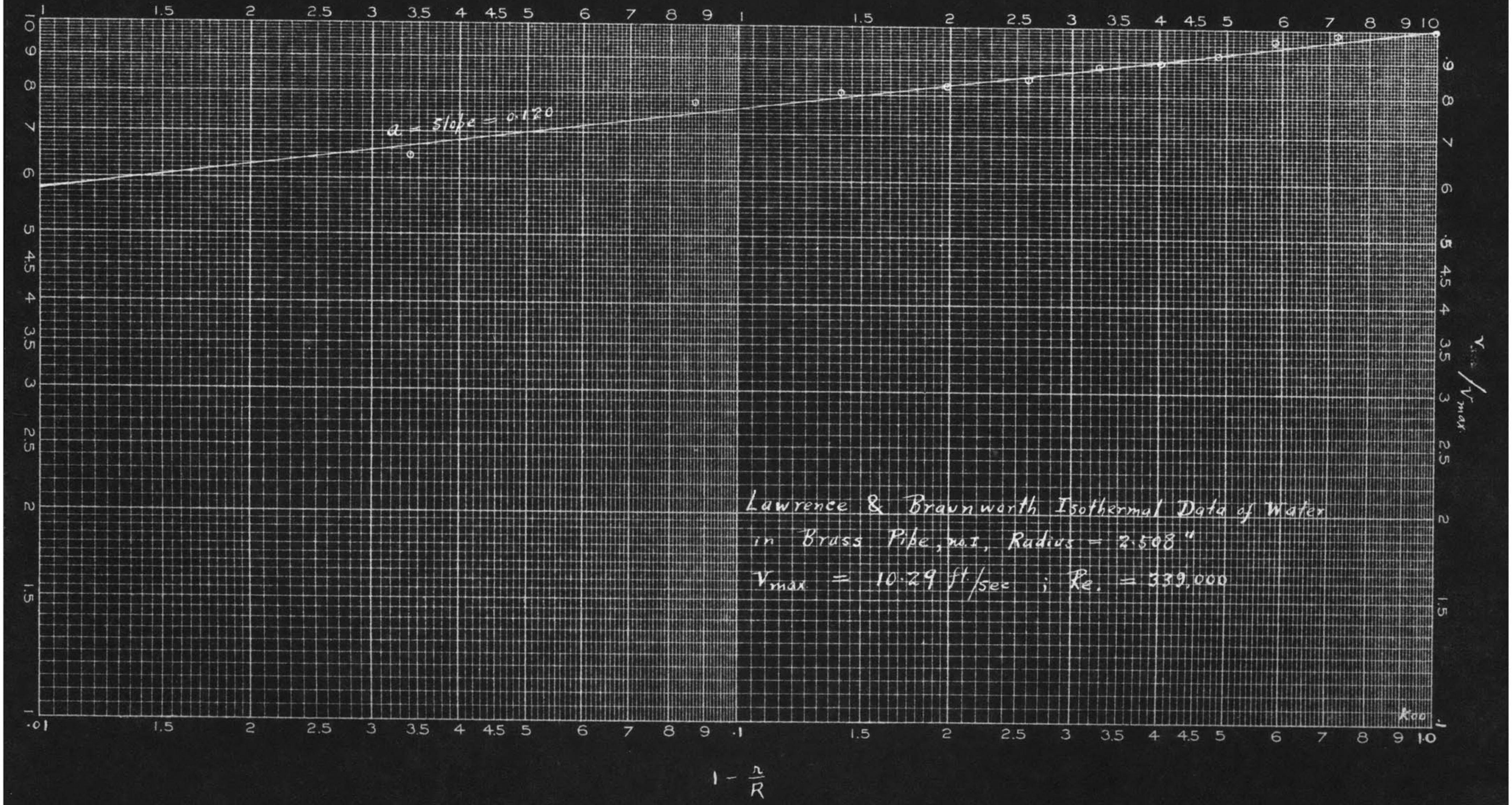

a = slope = 0.119
Lawrence & Braunworth Isothermal Data of Water
in Brass Pipe, Traverse No. 4, Radius = 2.508"
V max = 8.93 ft/sec ; Re = 300,500
1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 10
.01 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 .1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 1.0
10 9 8 7 6 5 4.5 4 3.5 3 2.5 2 1.5 1
.9 8 7 6 5 4.5 4 3.5 3 2.5 2 1.5 .1
V/V max.
1 − r/R
K00

Fig. 21

LOGARITHMIC

Chicago and New York 1922



Lawrence & Braunworth Isothermal Data of Water
in Brass Pipe, no. I, Radius = 2.508"
V_{max} = 10.29 ft/sec ; Re. = 339,000

Fig. 22

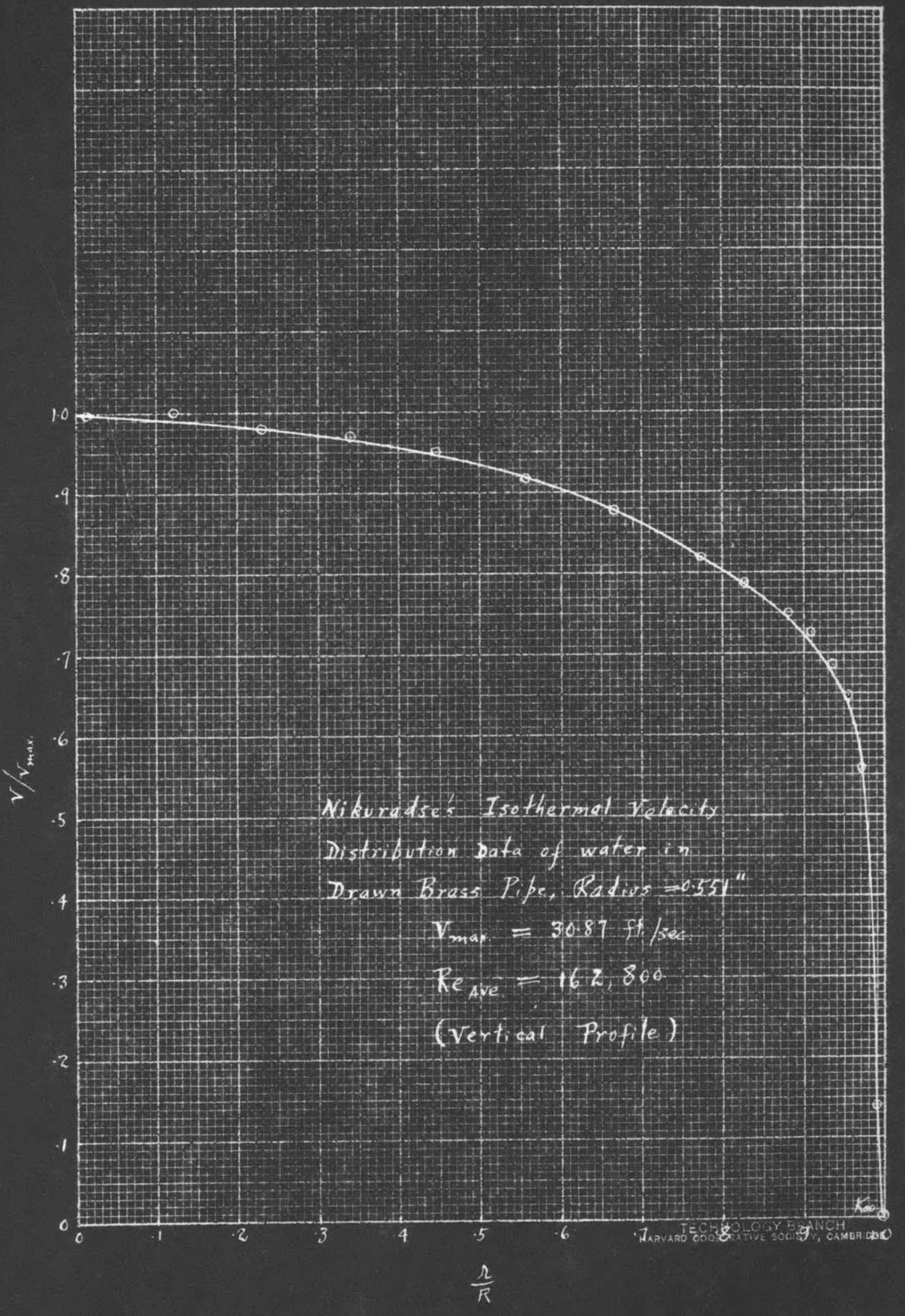

Nikuradse's Isothermal Velocity
Distribution Data of water in
Drawn Brass Pipe, Radius = 0.551"
V_max = 30.87 ft/sec
Re_Ave = 162,800
(Vertical Profile)
v/v_max.
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
r/R
Koo
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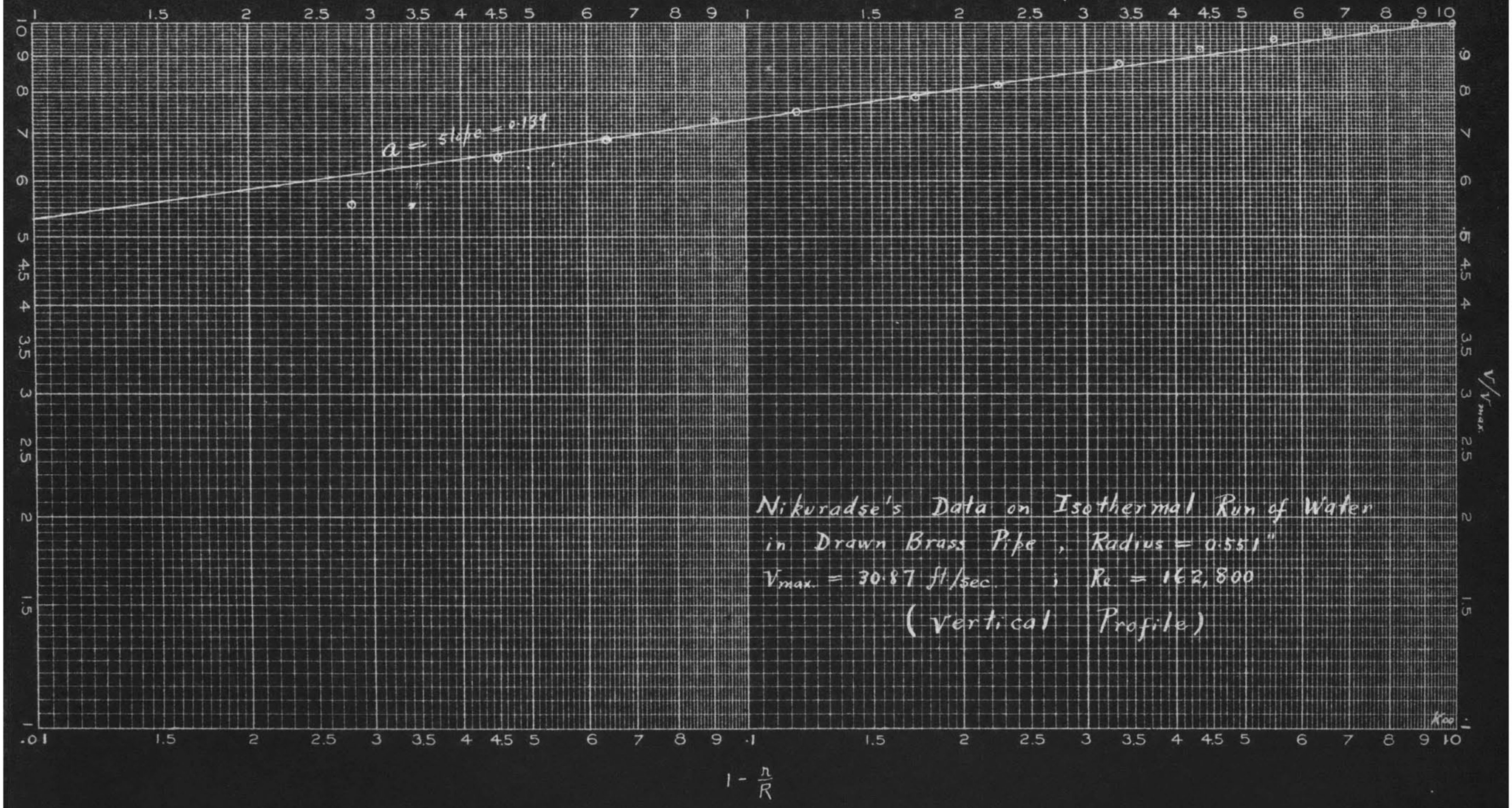
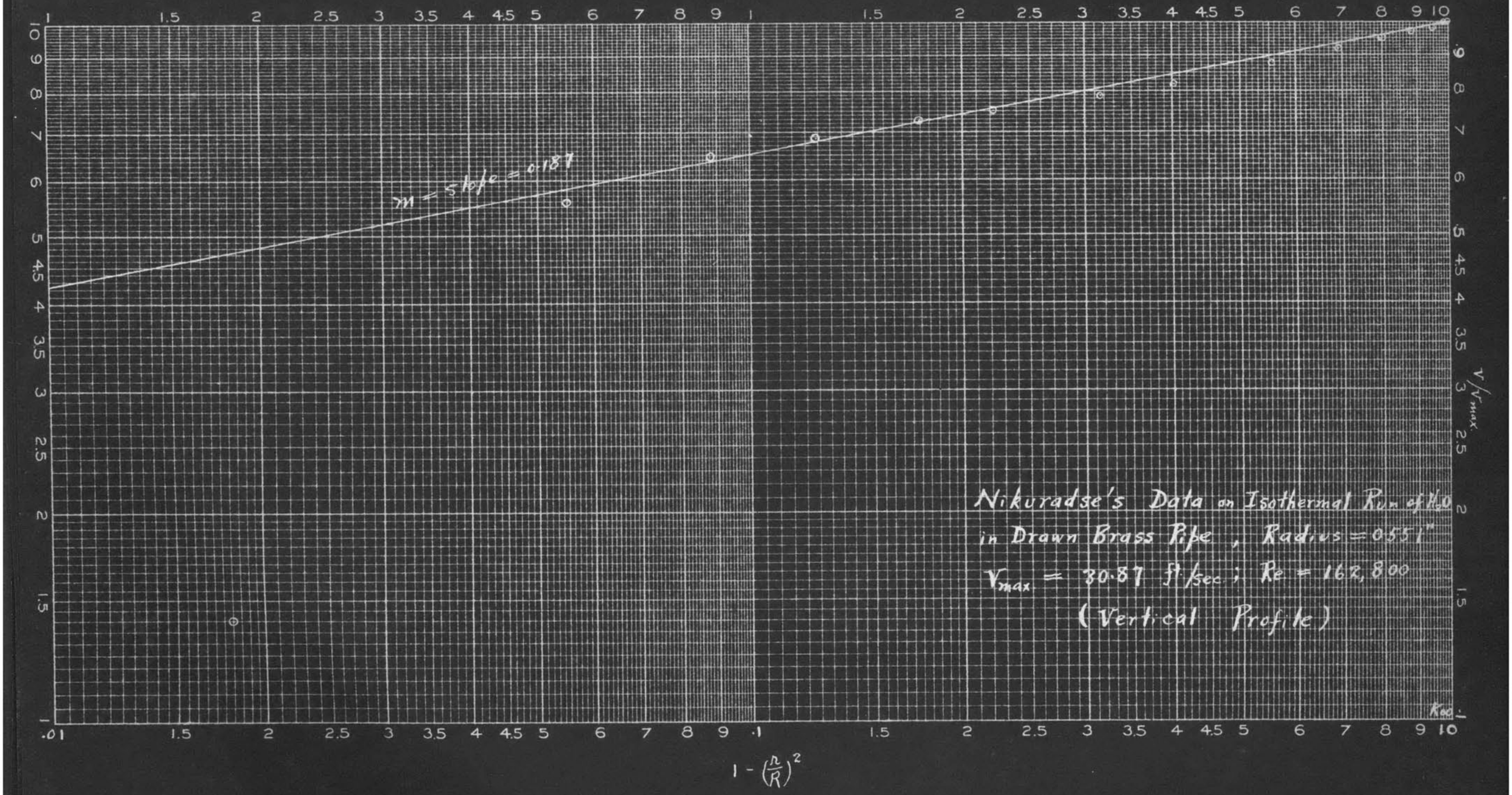
Fig. 23



Nikuradse's Data on Isothermal Run of Water in Drawn Brass Pipe, Radius = 0.551"
$V_{max.}$ = 30.87 ft./sec. ; R_e = 162,800
(Vertical Profile)

Fig. 24



m = slope = 0.187
Nikuradse's Data on Isothermal Run of H_2O
in Drawn Brass Pipe, Radius = 0.551"
V_{max} = 30.87 ft/sec ; Re = 162,800
(Vertical Profile)
v/v_{max}
$1-\left(\frac{r}{R}\right)^2$
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
K&E

Fig. 25

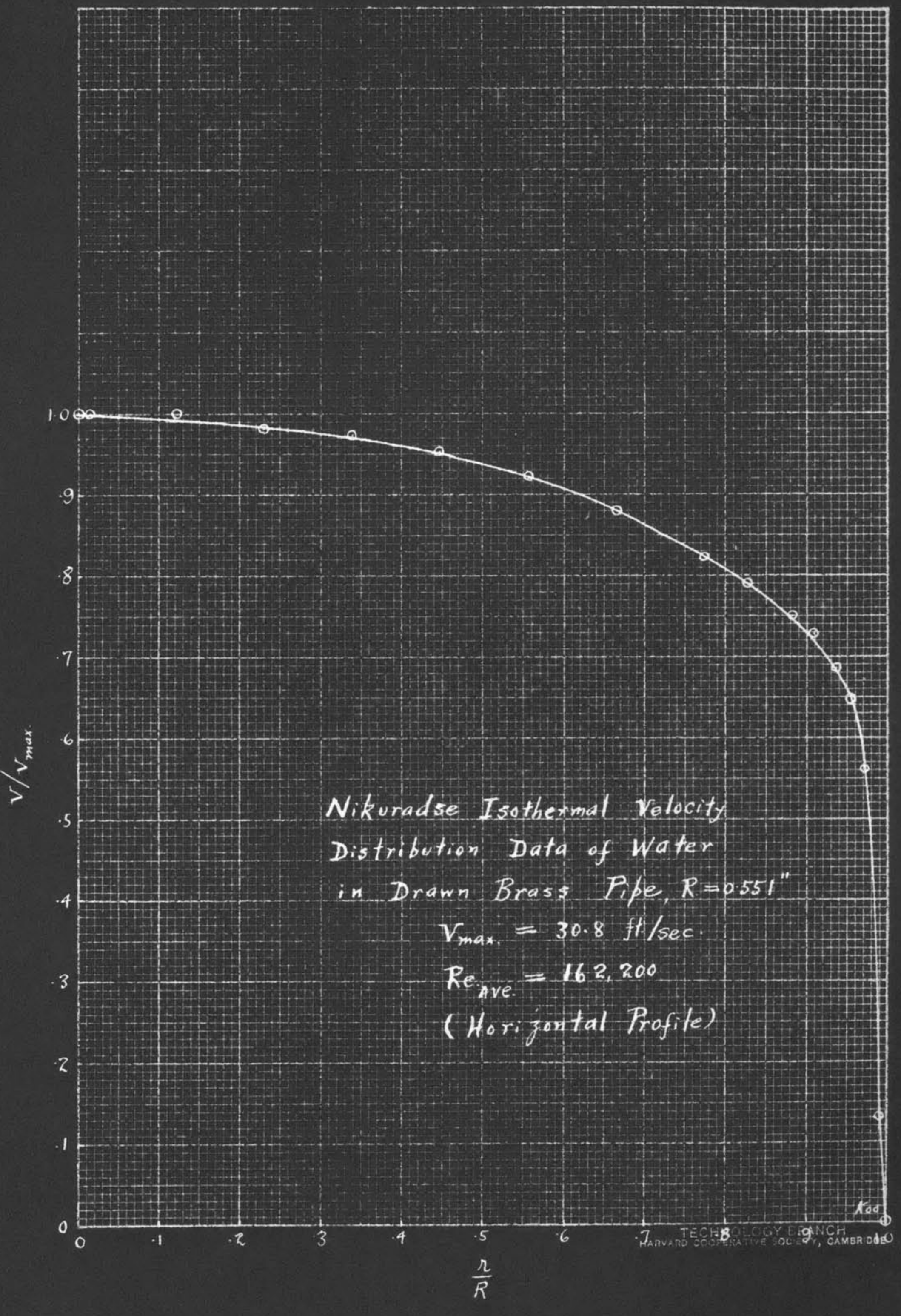

Nikuradse Isothermal Velocity
Distribution Data of Water
in Drawn Brass Pipe, R = 0.551"
$V_{max.}$ = 30.8 ft/sec
$Re_{Ave.}$ = 162,200
(Horizontal Profile)
$V/V_{max.}$
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
$\frac{r}{R}$
Koo
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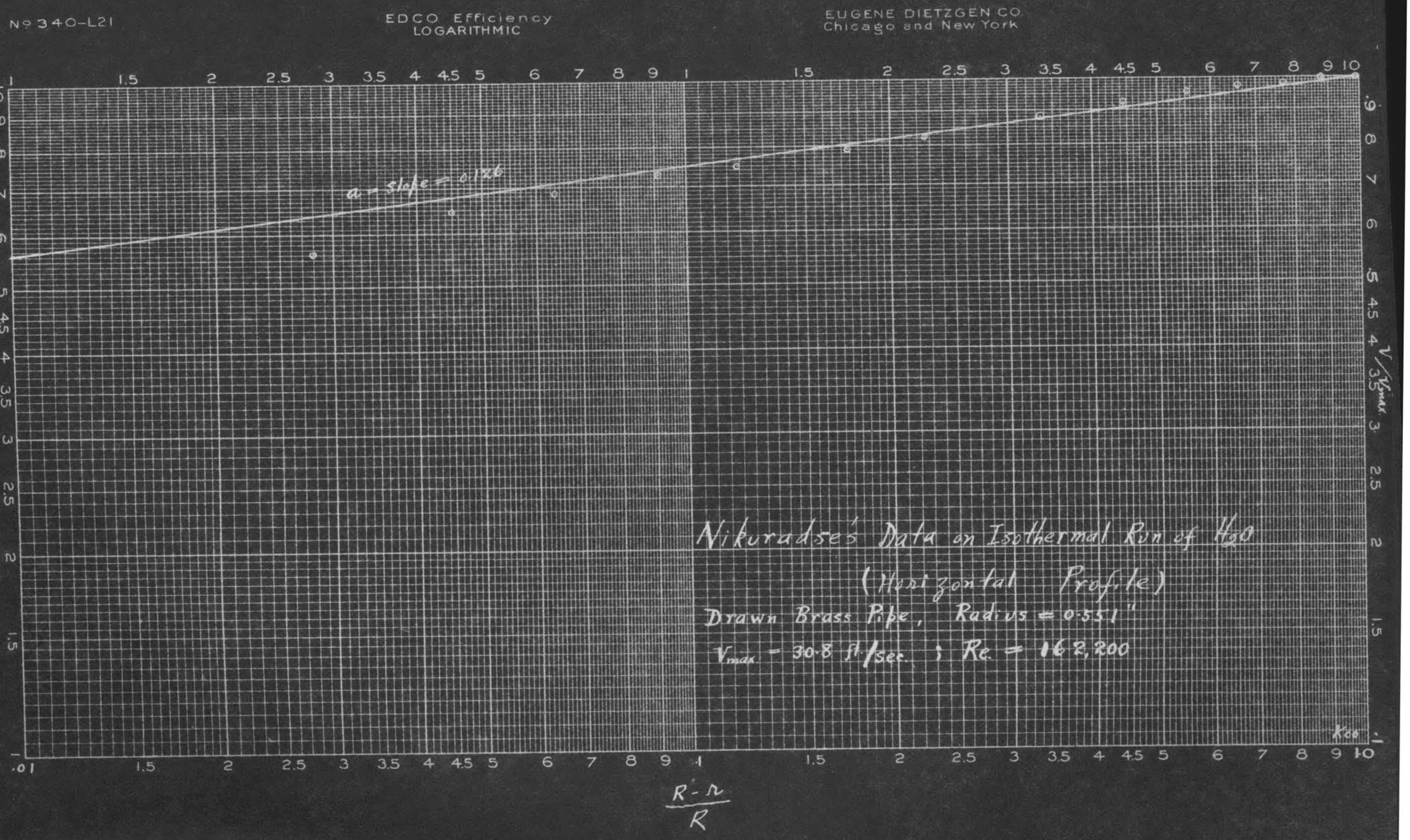
Fig. 26



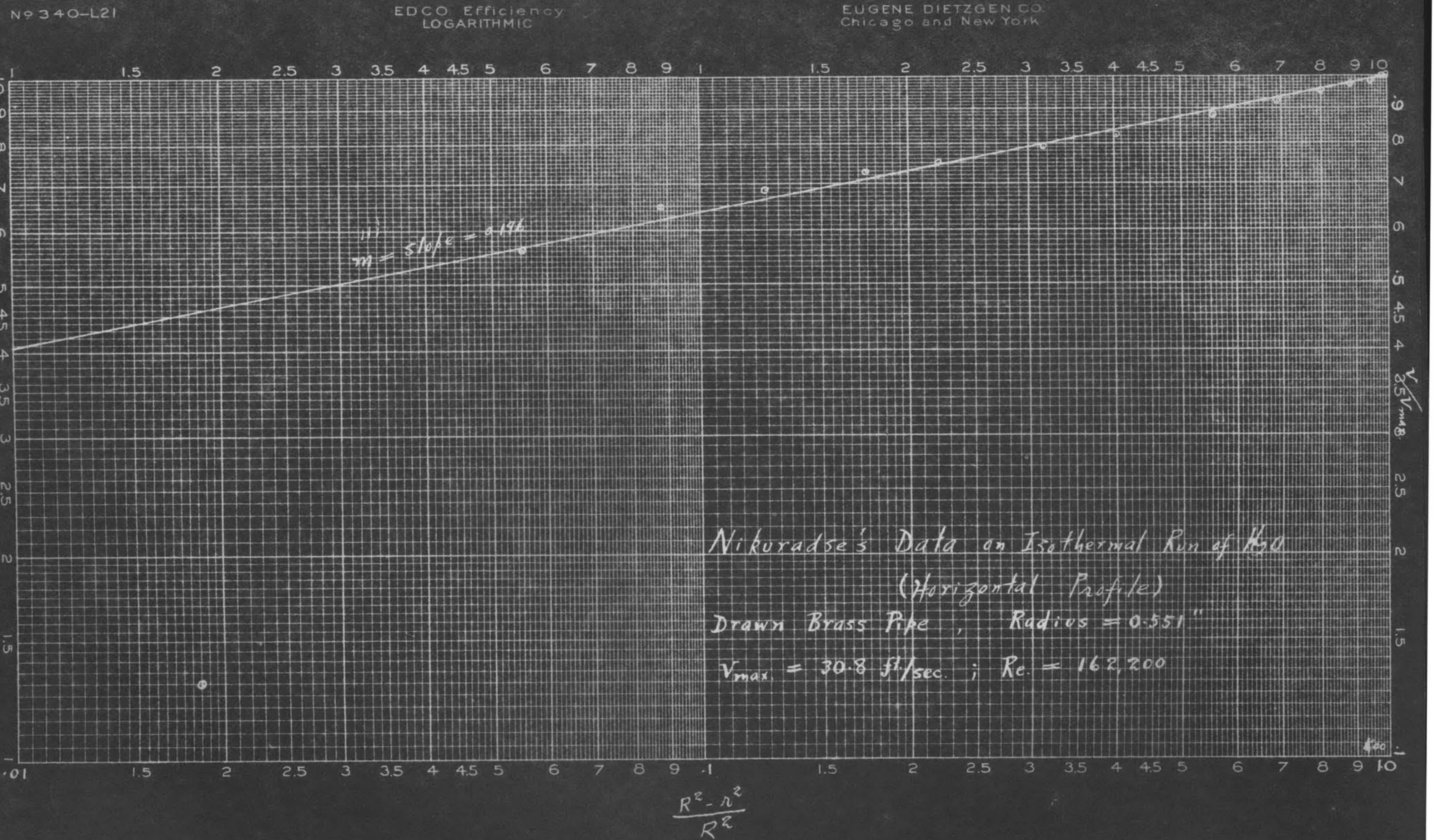
Nikuradse's Data on Isothermal Run of H_2O

(Horizontal Profile)

Drawn Brass Pipe, Radius = 0.551"

V_{max} = 30.8 ft/sec. ; Re. = 162,200

Fig. 27



Nikuradse's Data on Isothermal Run of H_2O

(Horizontal Profile)

Drawn Brass Pipe, Radius = 0.551"

$V_{max.}$ = 30.8 ft./sec. ; Re. = 162,200

Fig. 28

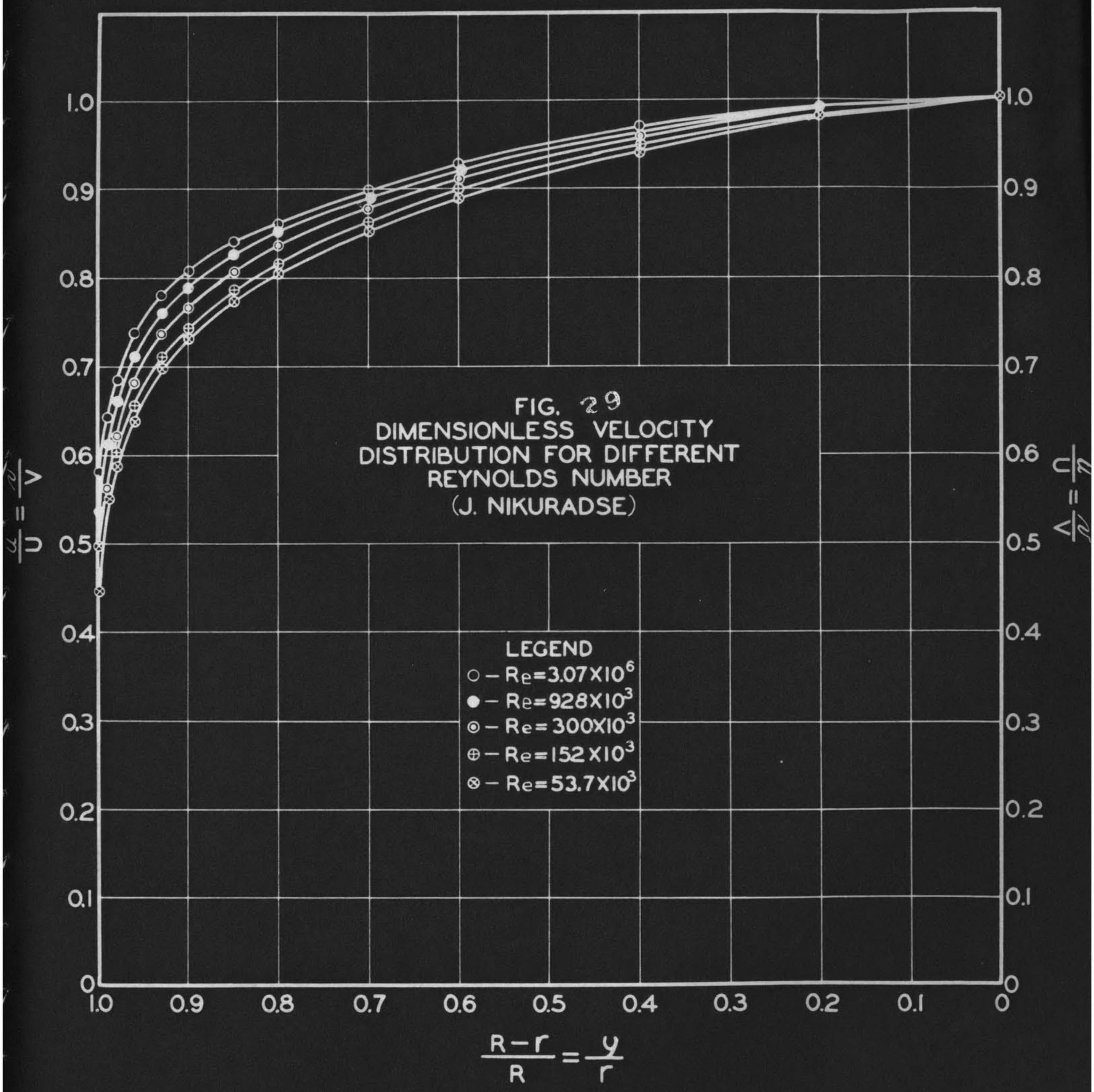


FIG. 29
DIMENSIONLESS VELOCITY DISTRIBUTION FOR DIFFERENT REYNOLDS NUMBER
(J. NIKURADSE)

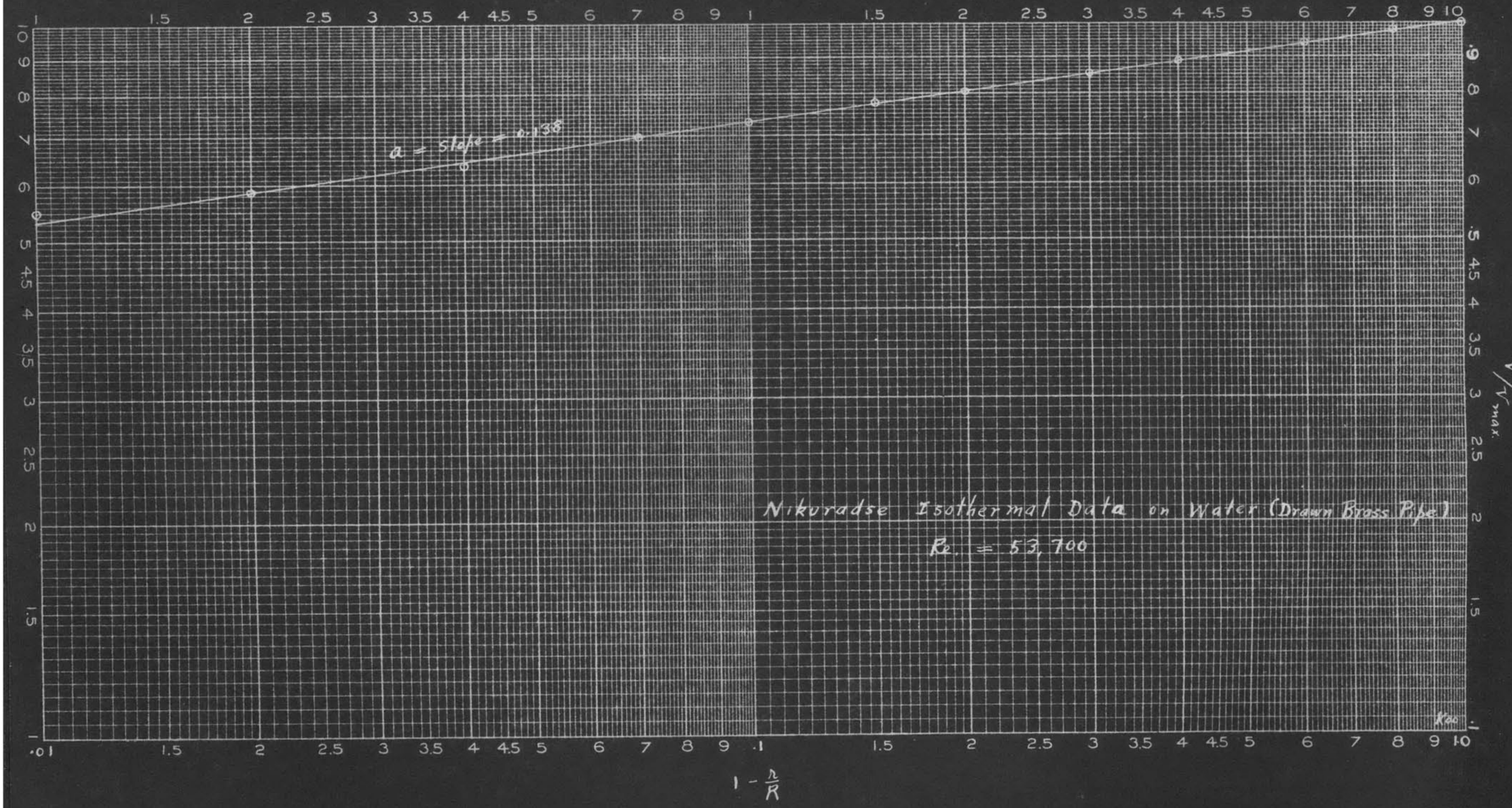
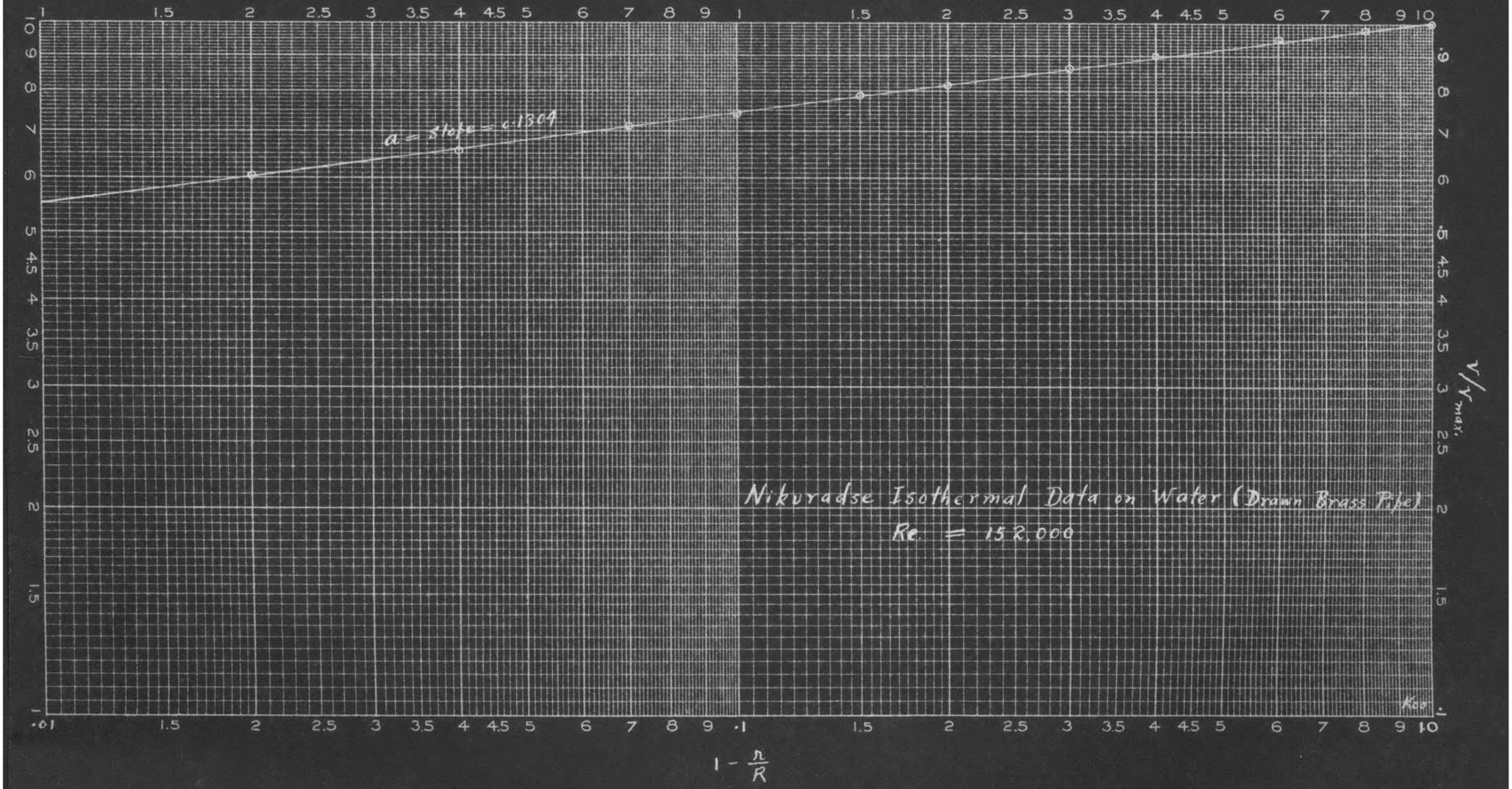
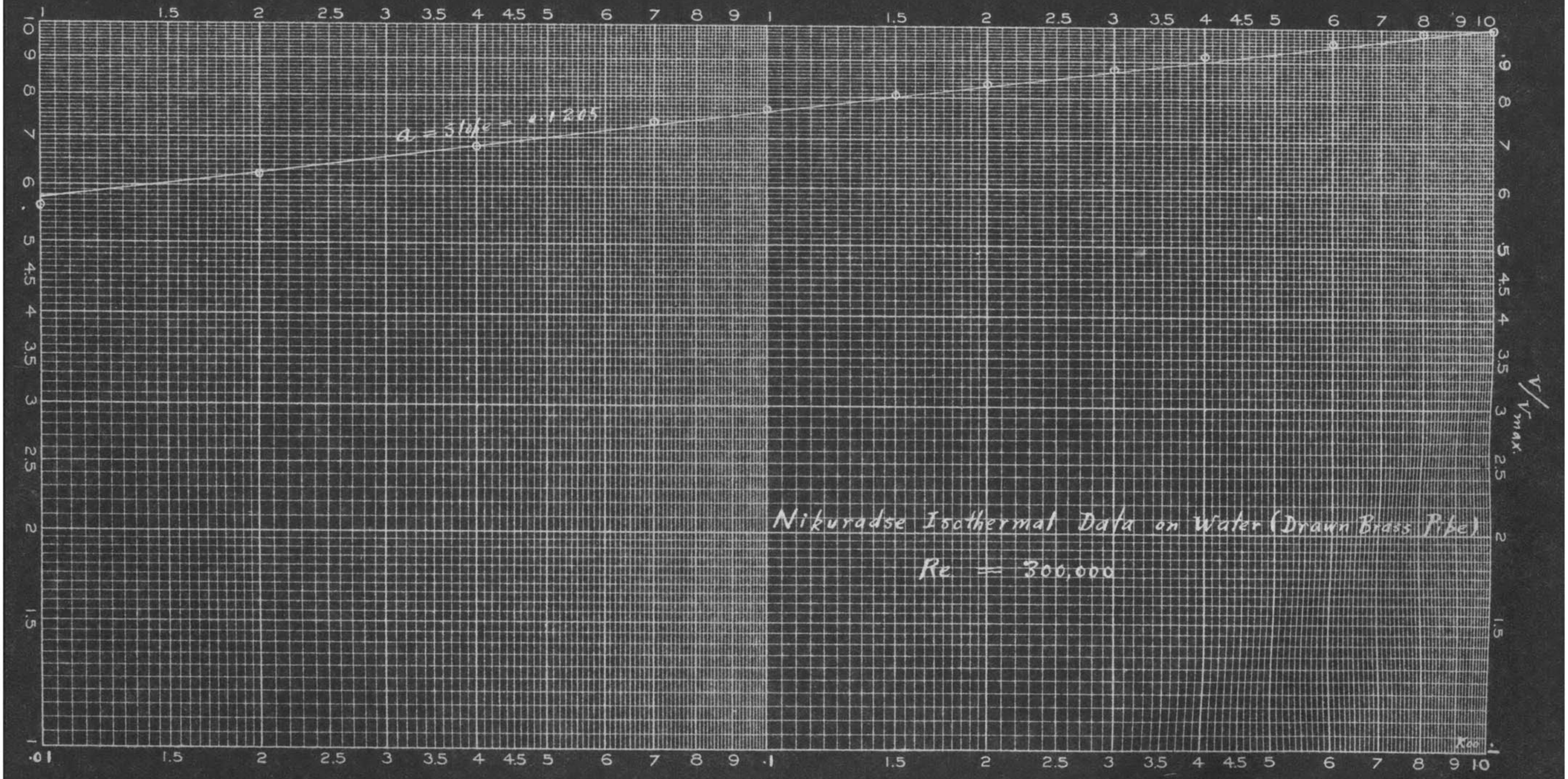
a = slope = 0.138
Nikuradse Isothermal Data on Water (Drawn Brass Pipe)
Re. = 53,700
v/v max.
1 - r/R
.01
.1
1.0
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
.5
.9

Fig. 30



a = slope = 0.1304
Nikuradse Isothermal Data on Water (Drawn Brass Pipe)
Re = 152,000
1 - r/R
v/v max.
·01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
·1
1.0
·5
·8
·9
·1
Koo

Fig. 31



Nikuradse Isothermal Data on Water (Drawn Brass Pipe)

Re = 300,000

Fig. 32

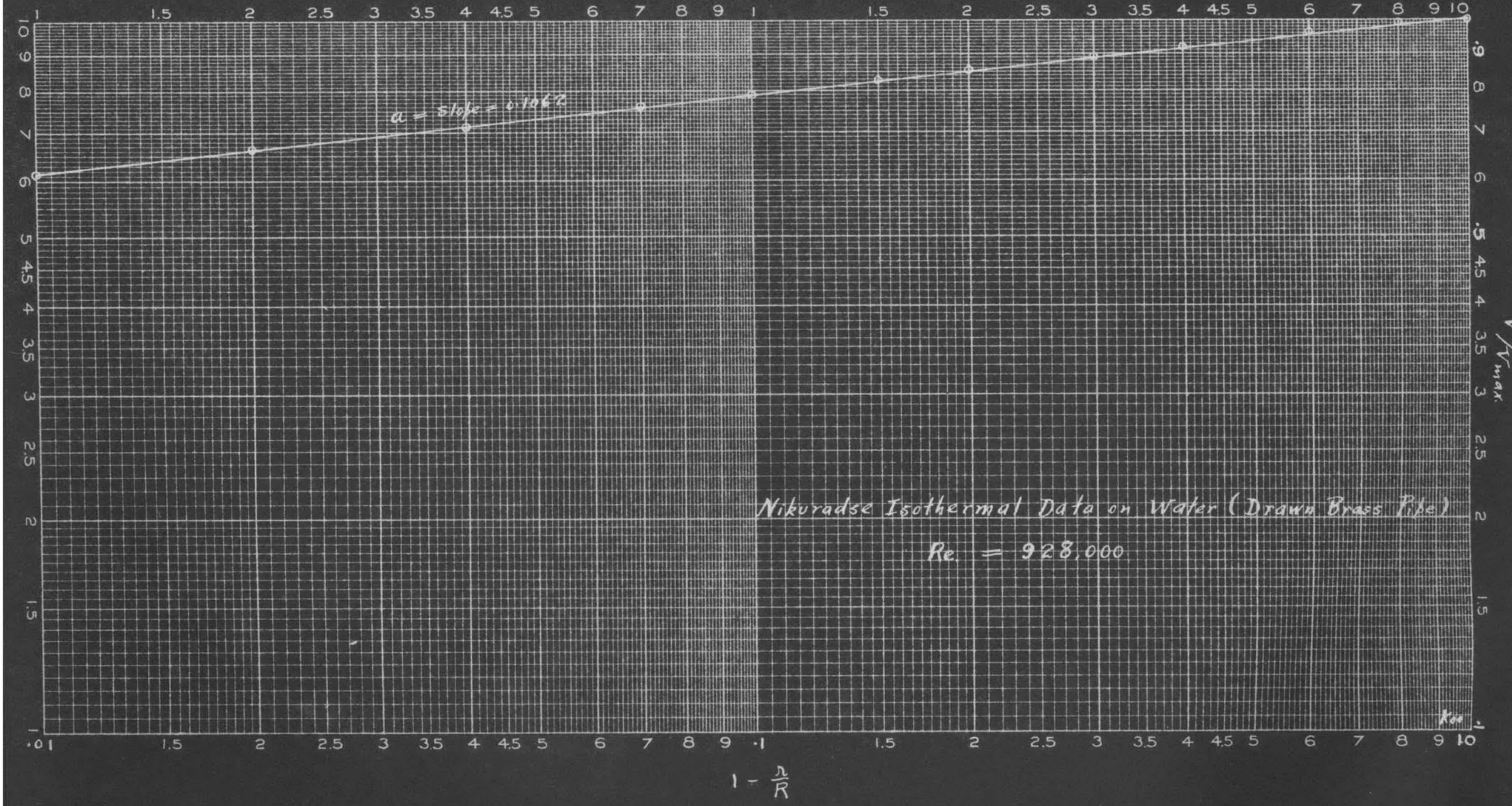

a = slope = 0.1062
Nikuradse Isothermal Data on Water (Drawn Brass Pipe)
Re = 928,000
$v/v_{max.}$
$1 - \frac{r}{R}$
·01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
·1
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
1·0
·9
8
7
6
·5
4.5
4
3.5
3
2.5
2
1.5
·1

Fig. 33

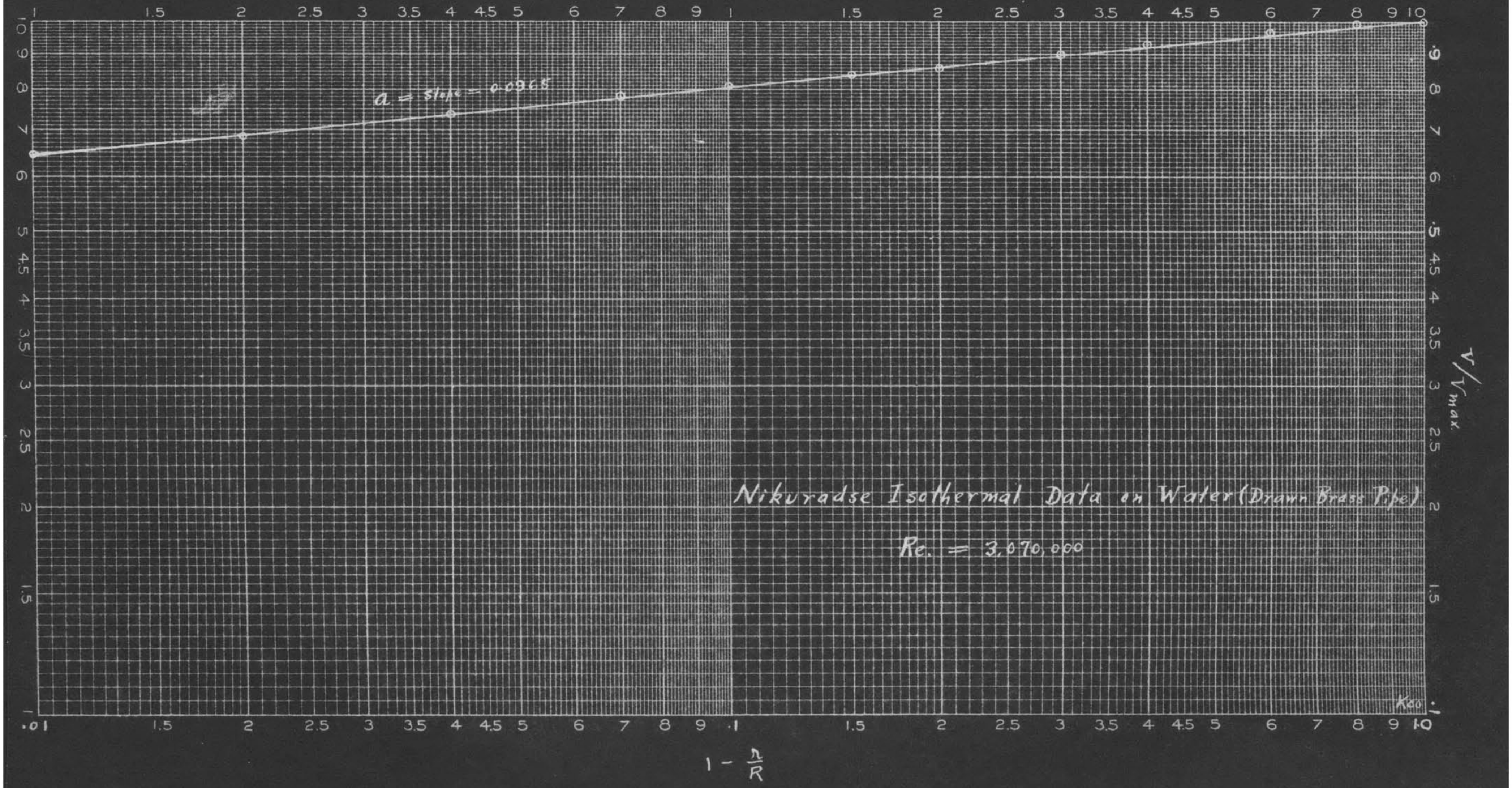

a = slope = 0.0965
Nikuradse Isothermal Data on Water (Drawn Brass Pipe)
Re. = 3,070,000
1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 10
.01 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 .1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 1.0
1 − r/R
10 9 8 7 6 5 4.5 4 3.5 3 2.5 2 1.5 1
1.0 .9 8 7 6 .5 4.5 4 3.5 3 2.5 2 1.5 .1
v/V_max
Koo

Fig. 34

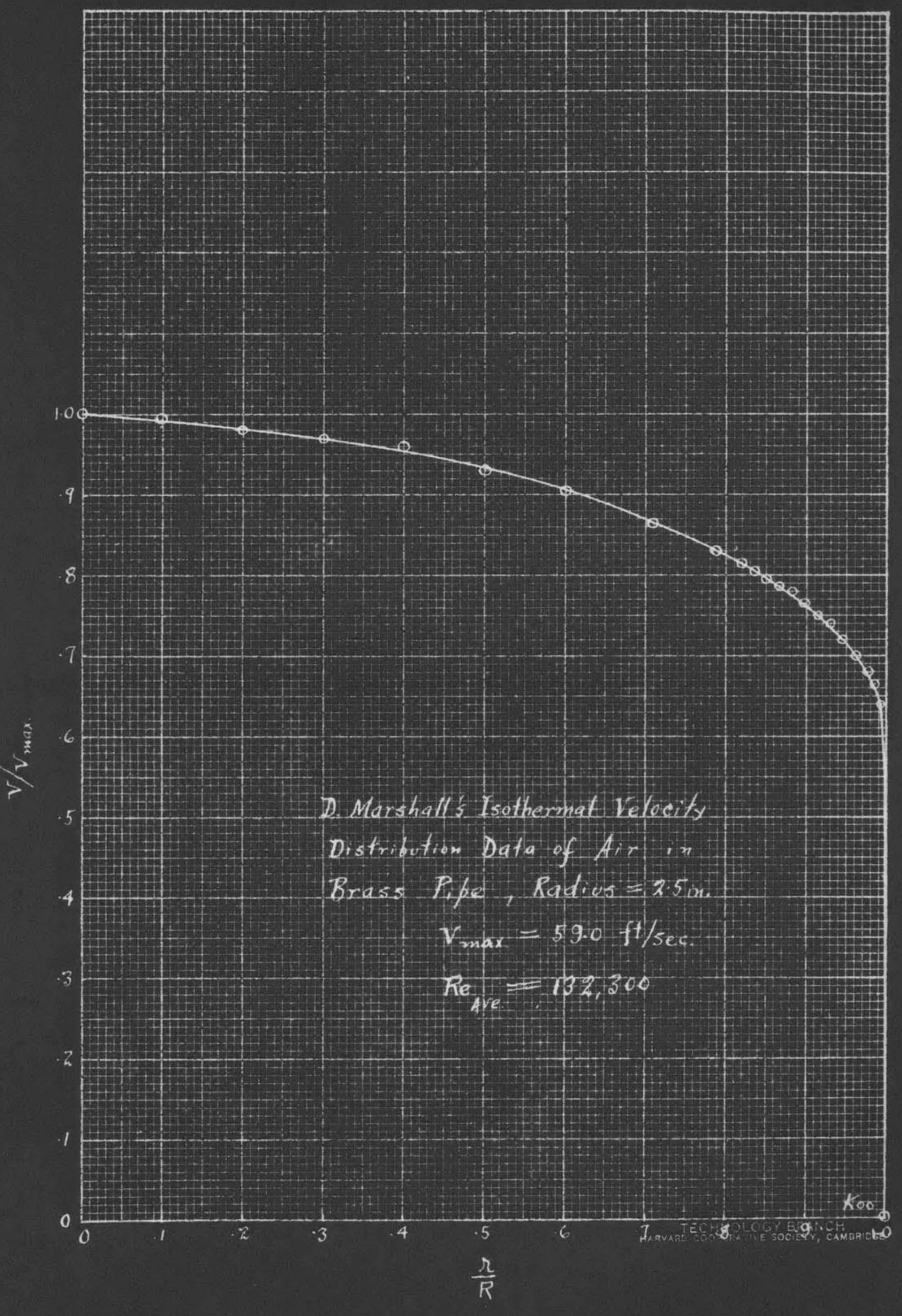

D. Marshall's Isothermal Velocity
Distribution Data of Air in
Brass Pipe, Radius = 2·5 in.
V_{max} = 59·0 ft/sec.
Re_{Ave} = 132,300
V/V_{max}
1·0
·9
·8
·7
·6
·5
·4
·3
·2
·1
0
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1·0
$\frac{r}{R}$
Koo
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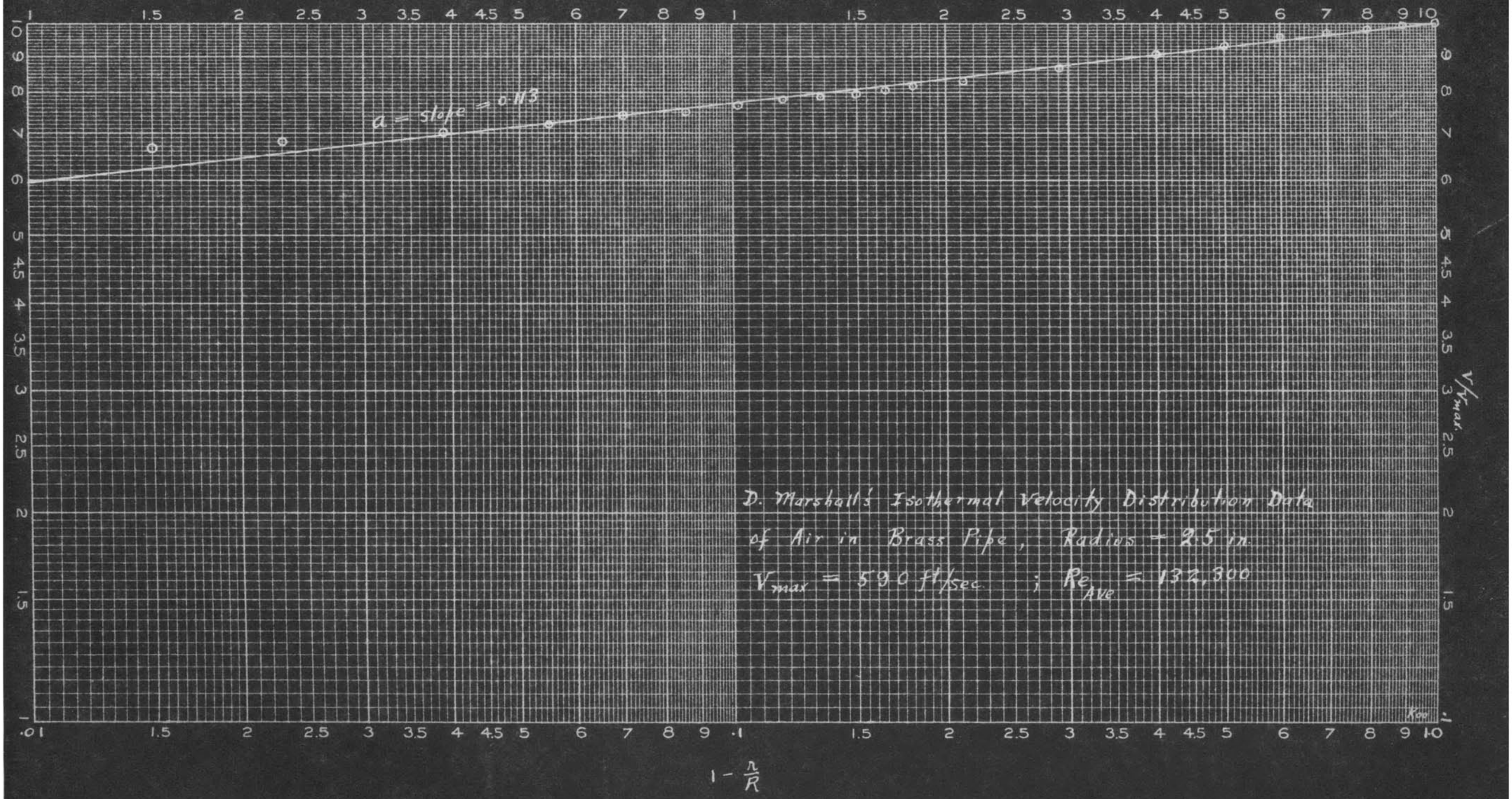
Fig. 35



D. Marshall's Isothermal Velocity Distribution Data of Air in Brass Pipe, Radius = 2.5 in.

V_{max} = 590 ft/sec ; Re_{Ave} = 132,300

Fig. 36

TABLE 9

EXPERIMENTAL VELOCITY DISTRIBUTION EXPONENTS FOR SMOOTH PIPES FROM LITERATURE

Authority	Fluid used	Pipe Material and set up	I.D. Inches	Re.	a	Remarks
T.E.Stanton	Air	Smooth brass (Vertical Position)	2.92	40,750	0.145	Inlet length = 67.6D
			1.94	41,200	0.140	" " = 101 D
			2.92	89,750	0.138	" " = 67.6D
J.R.Freeman	Water	Brazed brass (Inclined)	1.15	425,000	0.127	Inlet length = 104 D
Lawrence and Braunworth	Water	Seamless brass (Vertical Position)	5.02	136,100	0.137	Very big inlet length possibly greater than 250D
			5.02	152,500	0.131	
			5.02	202,000	0.129	
			5.02	300,500	0.119	
			5.02	339,000	0.120	
J. Nikuradse	Water	Drawn brass (Horizontal Position)	1.10	162,800	0.139	Vertical profile
			1.10	162,200	0.126	Horizontal profile
J. Nikuradse	Water	Drawn brass (Horizontal Position)		53,700	0.138	Inlet length greater than 55D
				152,000	0.130	
				300,000	0.121	
				928,000	0.106	
				3,070,000	0.0965	
D. Marshall	Air	Brass (Horizontal)	5.00	132,300	0.113	Inlet length = 192D

It has been stated before that Prandtl-Karman's equation is derived from the friction law, then a theoretical relation is to be expected to exist between velocity distribution and friction factor. From the General Index Law which the writer has just proposed for technically smooth pipes such as copper, lead, glass and drawn brass as

$$f = 0.00140 + 0.125 \text{ Re.}^{-0.32} \qquad .....(15)$$

it is seen that the friction factor decreases as Reynolds number increases and its slope on a log-log plot changes with Reynolds number. Therefore, a decrease of velocity distribution exponent "a" is corresponding to a decrease of friction factor slope on a log-log plot Reynolds number increases.

In 1929, Levy(4) derived a quasi-theoretical relation between "m" in his equation (Eq. (7)) and 4f based upon Jakob and Erk's friction factor equation which is very similar to Eq. (15). Unfortunately, instead of using that relation he derived, he proceeded to deduce some empirical equation in terms of π. It seems, therefore, his relation has attracted very little attention even among German writers. A similar attack with experimental verification on this relation seems to be necessary in order to explain the mechanism of turbulent flow.

Starting from Levy's equation also,

$$V/V_{max} = \left[1 - \left(\frac{r}{R}\right)^2\right]^m \quad \text{.........(7)}$$

the ratio of average to maximum velocity can be expressed in terms of m after integration as

$$\frac{V_{ave}}{V_{max}} = \frac{1}{1+m} \quad \text{..........(16)}$$

$$\text{Since, } V_{ave} = \frac{2\,\pi \int_0^R Vr\,dr}{\pi\,R^2} \quad \text{..........(17)}$$

It is noticed that when m = 1, Eq. (7) reduces to Eq. (6) which has been theoretically derived for laminar flow. Let us consider Hagen-Poisseuille's Law which may be written in terms of friction factor as

$$f = 16\ Re.^{-1} \quad \text{.......(18)}$$

On a log-log plot of f against Re., it is apparent that a straight line of 45° inclined to the right will be obtained. Mathematically, the relation states that

$$\text{Tan. } \alpha_{laminar} = \frac{d\ \ln f}{d\ \ln Re.} = \frac{Re.\ df}{f\ dRe.} = -\ 1 \quad \text{.....(19)}$$

Therefore,

$$\text{Tan. } \alpha_{laminar} = -\ m \quad \text{.....(20)}$$

Assuming that this relation between Tangent α and m can be extended to turbulent flow (note this is the only assumption made), then

$$\text{Tan.}\ \alpha_{\text{turbulent}} = -\ m = \frac{Re\ df}{f\ d\ Re} \qquad .....(21)$$

It has been shown already that Levy's equation is not applicable to turbulent flow, however, it is possible that Eq. (16) may still hold true for turbulent region. Fig. 36a gives a graph of $\frac{V_{max}}{V_{ave}}$ versus m, where $\frac{V_{max}}{V_{ave}}$ is the reciprocal of the observed value of $\frac{V_{ave}}{V_{max}}$ found in literature as given in Table II, while m is the negative slope read from Fig. 40a for corresponding Reynolds number. It is seen that Eq. (16) may be accepted as an approximate relation for turbulent flow if not near the critical region.

Eq. (14b) has been accepted by the writer as the velocity distribution equation for turbulent flow

$$\frac{V}{V_{max}} = (1 - \frac{r}{R})^{a} \qquad ......(14b)$$

from Eq. (14b) and Eq. (17) we obtained

$$\frac{V_{ave}}{V_{max}} = \frac{2}{(a+1)(a+2)} \qquad .....(22)$$

It follows:

$$m = \frac{a^2 + 3a}{a} = 0.5\ a^2 + 1.5\ a \qquad ......(23)$$

Equating Eq. (21) and Eq. (23),

$$m = -\ \frac{Re.\ df}{f\ dRe} = \frac{a^2 + 3a}{2} \qquad ....(23a)$$

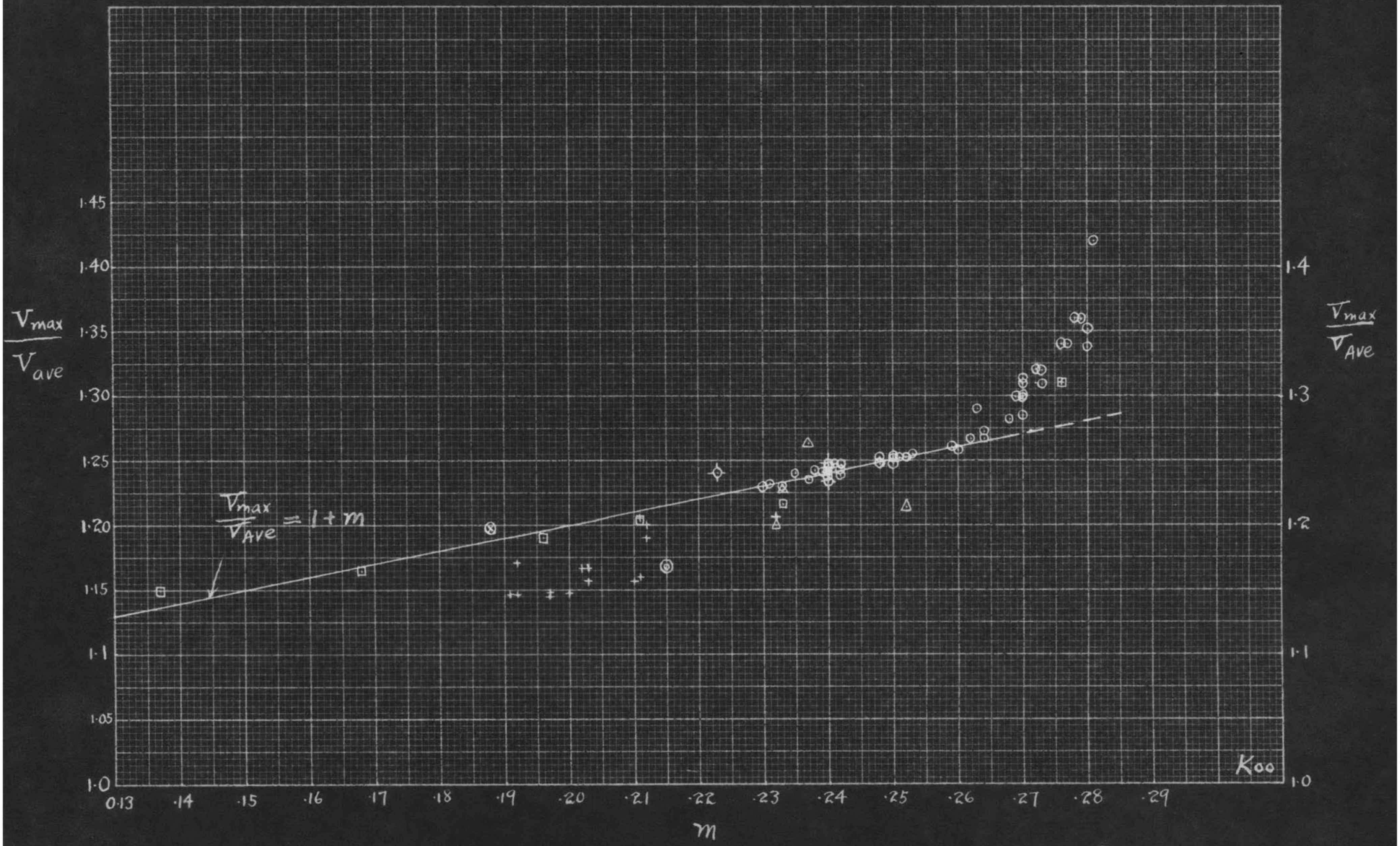


Figure 36 a   Legend same as Fig. 39
Except ⊡ Nikuradse Data

Therefore,

$$a = \frac{-3 + \sqrt{9 - 8\left(\dfrac{\mathrm{Re.}\ df}{f\ d\mathrm{Re}}\right)}}{2} \quad \ldots\ldots(24)$$

This is a relation theoretically found between velocity distribution exponent "a", friction factor "f" and Reynolds number "Re".

For turbulent flow of fluids in smooth pipes, velocity distribution exponent can then be calculated from the following equation, based upon the friction factor equation as given in Eq. (15)

$$a = -\ 1.5 + 0.5\sqrt{9 + \frac{2.56}{1 + 0.0112\ \mathrm{Re.}^{0.32}}} \quad \ldots\ldots\ldots(25)$$

Since Eq. (15) is derived from experimental data from Re. = 3,000 up to Re. = 3,000,000, the values of "a" calculated from Eq. (25) ought to be applicable to the said range of Reynolds number. The computed values of "a" corresponding to different Reynolds numbers are tabulated in Table 10 and shown as a smooth curve in Figure 37. The available experimental data all on brass pipes, consisting of 17 runs, have already been plotted on log-log paper, and their corresponding values of exponents are tabulated in Table 9, and plotted also in Fig. 37. A very good correlation of experimental exponents with calculated values is noticed. This verification proves the found relation between velocity distribution and friction factor or between velocity distribution and Reynolds number for smooth pipes. Furthermore, this verification gives another evidence of the applicability of General Index Law, not the Simple Index Law; for if the latter law holds, velocity distribution will be independent of Reynolds number in the same pipe. Figure 38 gives a picture of velocity distributions changing from laminar to turbulent flow, and also illustrates the gradual increase of flow near the boundary as Reynolds number increases.

The mechanism of production of turbulence has been explained by Prandtl(20), Tollmien(24), and many

Table 10 Relation of Velocity Distribution to Reynolds Number for Smooth Pipes.

Re = $\frac{DV_{ave}\rho}{\mu}$	Velocity Distribution Exponent "a" (Calc. from Eq. (25))	$V_{ave.}/V_{max.}$ (Calc.from Eq.(21c))	$Re_{max.} = \frac{DV_{max.}\rho}{\mu}$
3,000	0.1760	0.781	3,840
4,000	1739	783	5,110
5,000	1724	785	6,370
6,000	1710	787	7,630
8,000	1685	789	10,140
10,000	1665	791	12,630
15,000	1628	795	18,880
20,000	1591	799	25,050
30,000	1554	804	37,350
40,000	1524	8065	49,600
50,000	1497	810	61,750
60,000	1475	8115	74,000
80,000	1439	815	98,200
100,000	1409	819	122,100
150,000	1355	825	181,900
200,000	1295	830	241,000
250,000	1281	833	300,000
300,000	1253	836	359,000
400,000	1210	842	475,000
500,000	1177	845	592,000
600,000	1148	849	707,000
800,000	1103	854	938,000
1,000,000	1066	858	1,166,000
1,500,000	1002	866	1,732,000
2,000,000	0958	871	2,297,000
3,000,000	0890	879	3,415,000

Figure 37

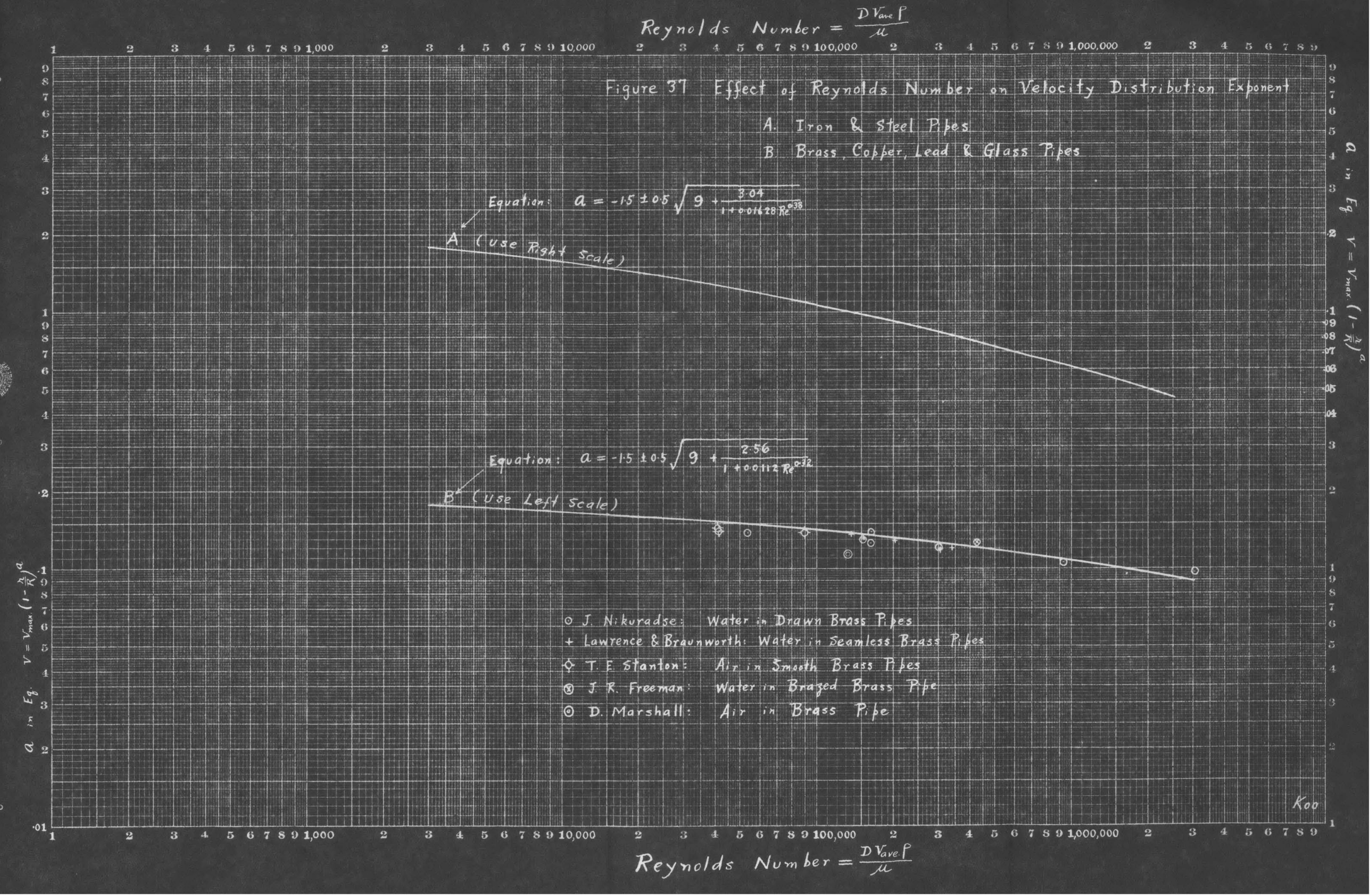


Figure 37 Effect of Reynolds Number on Velocity Distribution Exponent

A. Iron & Steel Pipes

B. Brass, Copper, Lead & Glass Pipes

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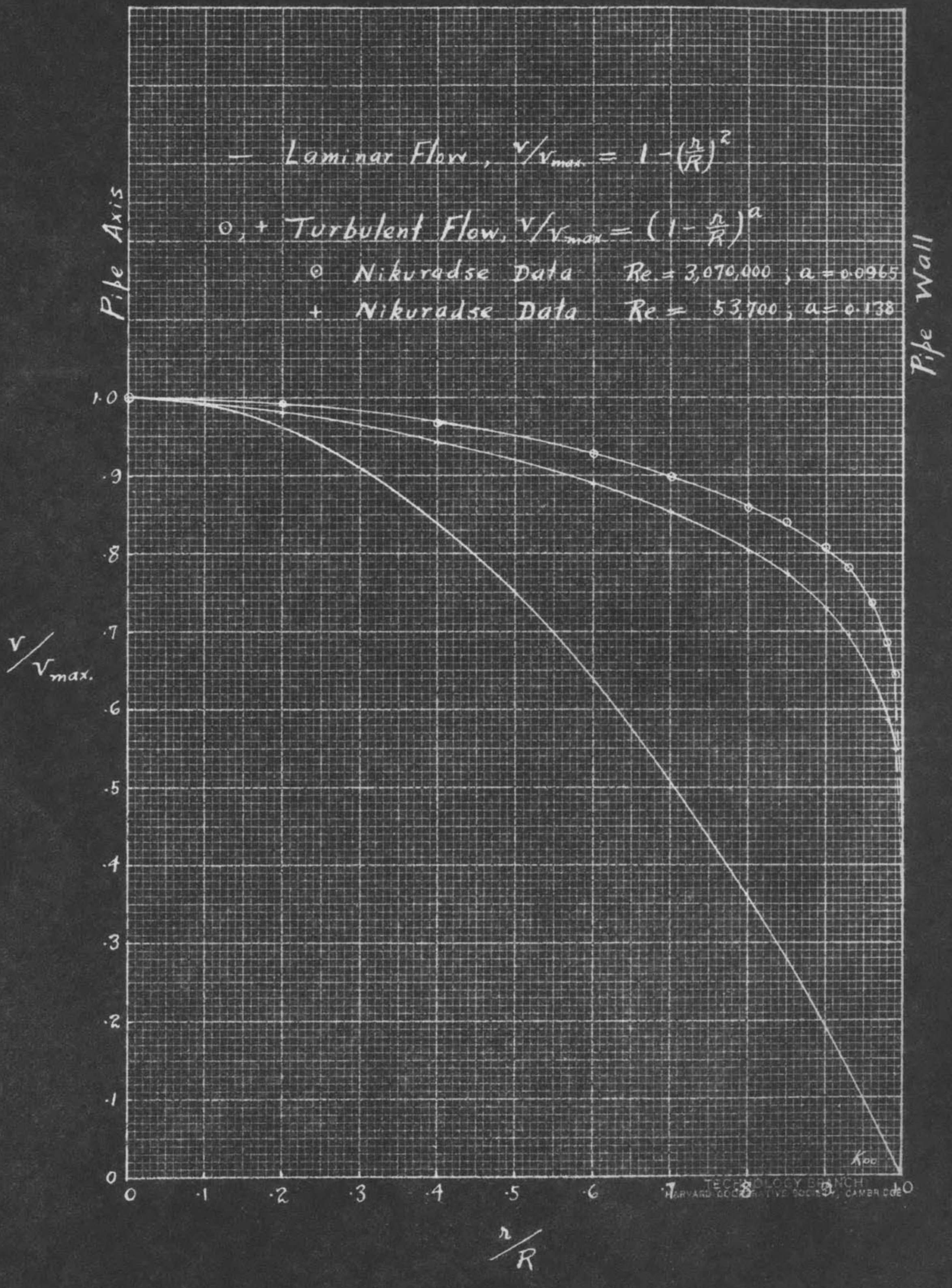

Figure 38
Effect of Re. on Velocity Distribution

others. Especially, Prandtl's boundary layer consideration will be helpful in explaining the velocity distribution phenomenon as Reynolds number increases. His theory has been recently reviewed and further explained by Glauert$^{(25)}$ in English, which may be briefly mentioned here. The narrow region along the surface of the pipe wall, in which the frictional forces are important, is defined as boundary layer or called ~~as~~ Prandtl's boundary layer. The conception of such a layer is merely an approximation to the actual case, and it is rather arbitrary to define its outer.limit. (Velocity distribution for isothermal flow near the wall has been recently reviewed also by Drew and Ryan$^{(26)}$, and it is shown the laminar flow near the wall by Hegge Zijnen's experiments.) From the drag experiments, it is believed that at ~~a low scale or~~ low Reynolds number, the flow is laminar and the fluid moves smoothly parallel to the axis, but at high Reynolds number the laminar motion becomes unstable and gradually changes to a turbulent type. From this consideration, it might be due to this gradual change of type of motion of this boundary layer that causes the change of velocity distribution as Reynolds number changes.

E. Relation Between Velocity Distribution Exponent and the Ratio of Average to Axial Velocity - Dependence of Velocity Ratio on Reynolds Number.

In deriving the relation between velocity distribution and friction factor, a very useful relation has been obtained as expressed by Eq. (22)

$$V_{ave}/V_{max} = \frac{2}{(a+1)(a+2)} \qquad \ldots\ldots(22)$$

also Levy's hypothesis that

$$V_{ave}/V_{max} = \frac{1}{1 - \dfrac{Re.\ df}{f\ dRe.}} = \frac{1}{(1+a)} \qquad \ldots\ldots(16)$$

was found to be a good approximation except near the critical region.

For laminar flow, m = 1, therefore,

$$V_{ave}/V_{max} = 1/2 \qquad \ldots\ldots\ldots(16a)$$

that is to say, the average velocity in laminar flow is just one-half of the axial velocity which is what has been found in Stanton and Pannell's experiments(27) as shown in Fig. 39.

For turbulent flow in smooth pipes, substituting the friction factor equation, Eq. (15) into Eq. (16), we obtain

$$V_{ave}/V_{max} = \frac{1}{1 + \dfrac{0.32}{1 + 0.0112\ Re^{0.32}}} = \frac{1 + 0.0112\ Re^{0.32}}{1.32+0.0112\ Re^{0.32}} \qquad \ldots.(16b)$$

The above equation expresses the relation between velocity ratio and Reynolds number in smooth pipes. The useful application of this equation is to calculate velocity ratio from a given Reynolds number, or to estimate the mean velocity after knowing the axial velocity in a pipe. It is believed that no previous worker has ever expressed such a relation algebraicly ~~quantitatively~~. Many hydraulic engineers believed that this velocity ratio is always a constant for all pipes, thus we can find in literature this constant varies from 0.753 to 0.950 as reviewed by Eason(28). Stanton and Pannell(27) first showed experimentally the variation of velocity ratio with Reynolds number from laminar to turbulent region, and the increase of the ratio in the turbulent region as Reynolds number increases. However, no explanation of this change is given by them.

The following important conclusions can be drawn based upon Eq. (21a) with an understanding of the friction resistance problem:

(1) Velocity ratio (i.e., $V_{ave.}/V_{max.}$) will be a constant only when friction factor obeys the simple index law (i.e., $f = b\ Re.^{c}$), which is only an approximation for a small range of Reynolds number.

(2) Velocity ratio will increase gradually as Reynolds number increases for almost any individual pipe, since friction factor generally obeys the General Index Law (i.e., $f = a + b\ Re.^{c}$).

(3) Velocity ratio will be only identical at identical Reynolds number in the same pipe, or in two pipes

of same degree of roughness.

(4) Velocity ratio will be usually different even at identical Reynolds number in two pipes of different degree of roughness (say copper and cast iron), since the resistance law coefficients will be different.

Based upon the above generalizations, we are able to explain some of the facts concerning velocity distribution problem found in literature. It is apparent that it is not reasonable to compare Darcy and Threllfall's experiments on cast iron pipe with Stanton and Pannell's experiments on drawn brass pipe as it was attempted by the latter experimenters.

From Eq. (16b) the velocity ratios corresponding to a range of Reynolds number from 3,000 to 3,000,000 have been calculated and tabulated in Table 10 also. They are plotted in Figure 39 as a smooth curve, while velocity ratios of available data have been tabulated in Table 11 and shown also in Fig. 39, Fig. 39A. It is observed that experimental data deviate systematically from the theoretical curve only at low or high Reynolds number, while Stanton and Pannell's data fit the curve remarkably good. It must be noticed that the velocity

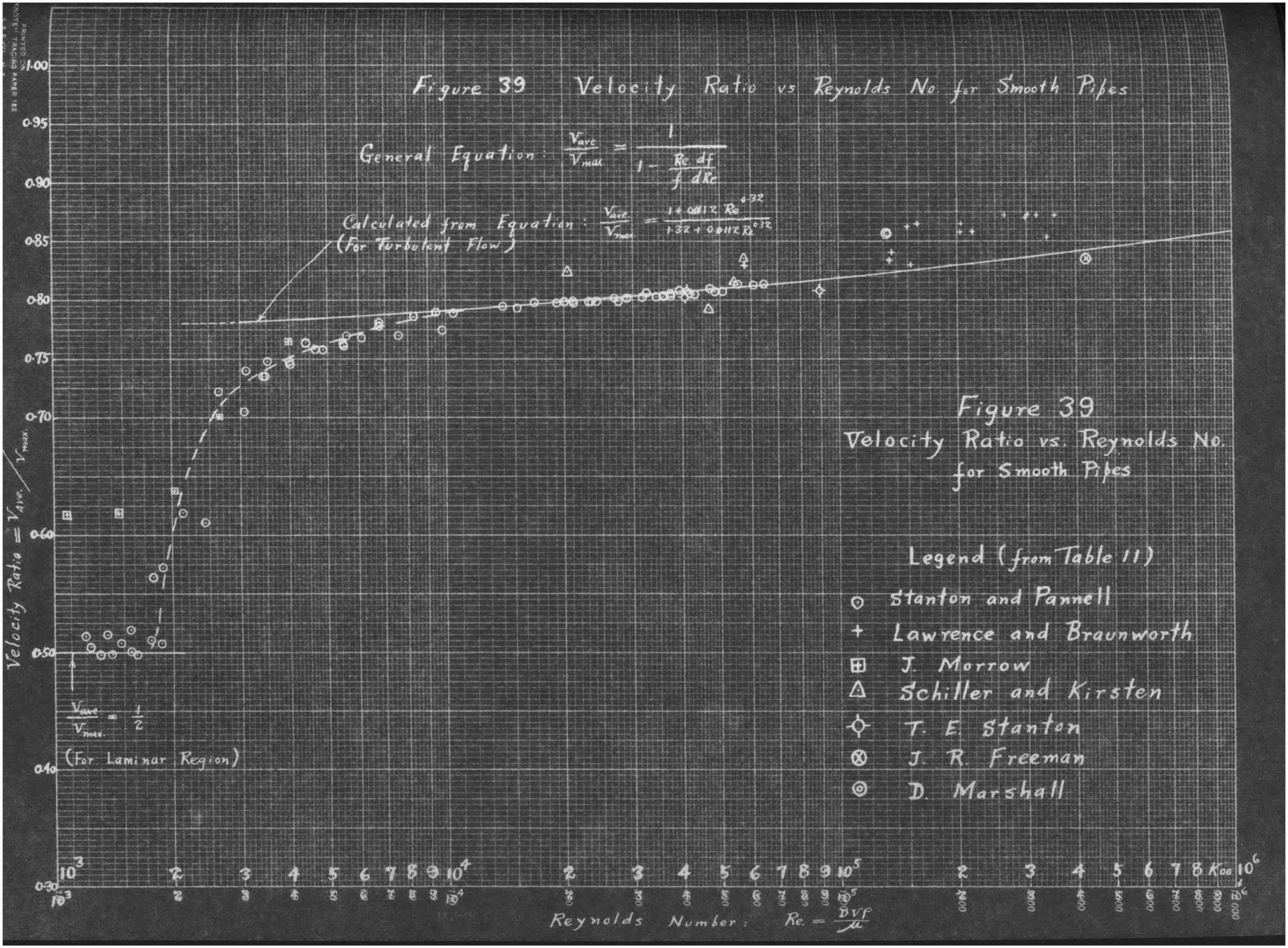

Figure 39
Velocity Ratio vs. Reynolds No. for Smooth Pipes

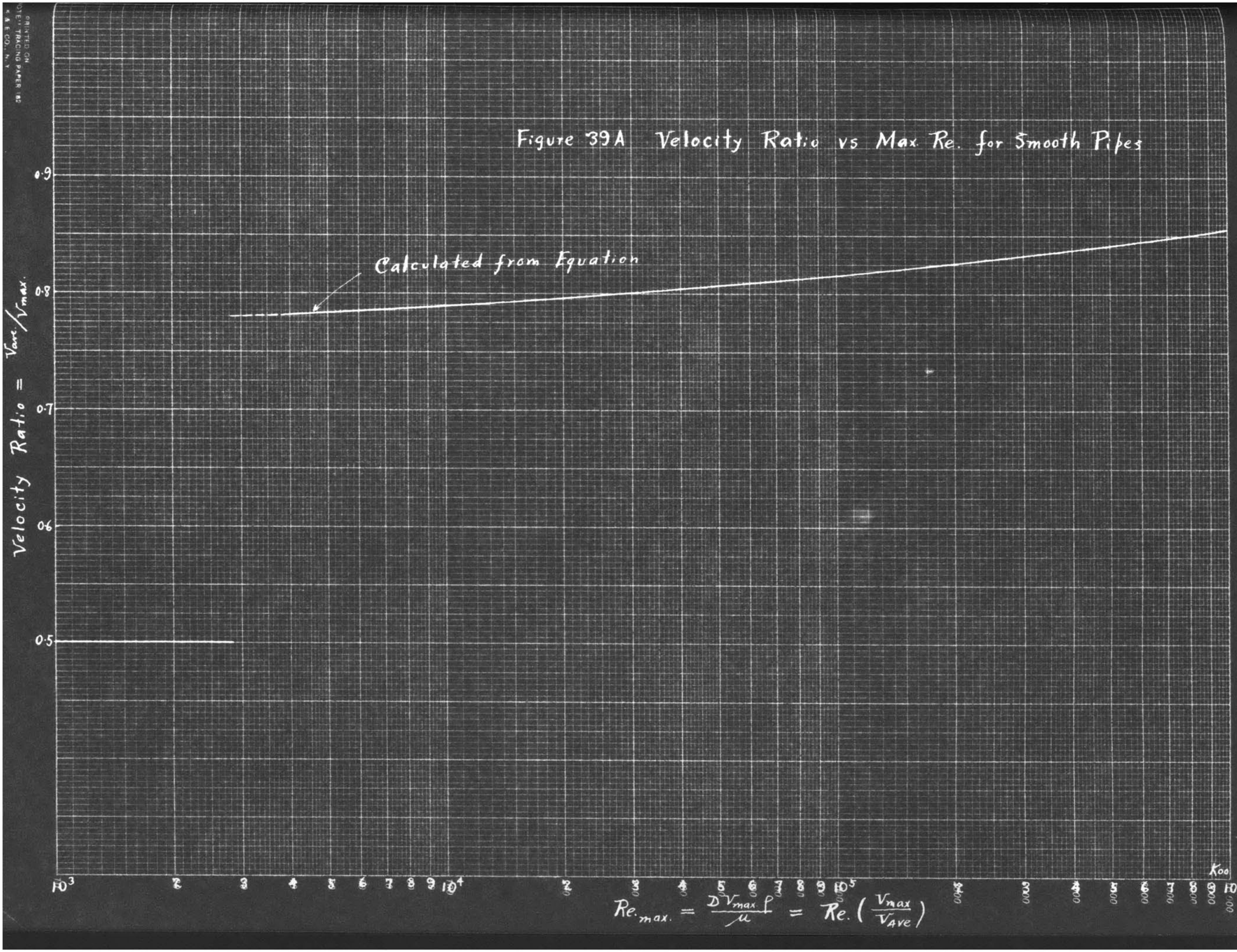

Figure 39A Velocity Ratio vs Max Re. for Smooth Pipes

Table 11. Mean to Axial Velocity Ratio vs. Reynolds Number for Smooth Pipes.

(Calculated from Literature)

Authority	Fluid Used	Pipe Material	I.D. (inches)	Re	$V_{ave.}/V_{max.}$
Stanton and Pannell	Water ($t_{ave.}$ = 10.76°C.)	Drawn Brass	1.124	37,400	0.803
				31,800	.802
				28,950	.802
				27,000	.802
				23,550	.798
				21,100	.799
				21,050	.798
				19,100	.798
				16,870	.797
				14,000	.795
				9,450	.789
				8,250	.786
				34,200	.802
				32,500	.805
				35,800	.803
				37,350	.806
				48,450	.807
				41,800	.806
Stanton and Pannell	Water (t_{ave}= 9.6°C.)	Drawn Brass	0.281	10,480	0.789
				7,850	.780
				6,780	.778
				5,620	.770
				4,400	.764
				3,510	.748
				3,100	.740
				2,640	.722
				2,125	.619
				1,890	.572
				1,783	.564
				1,556	.519
				1,470	.508
				1,360	.515

Authority	Fluid Used	Pipe Material	I.D. (inches)	Re.	V_{ave}/V_{max}
				1,306	0.498
				1,198	.514
				27,650	.799
				24,200	.799
				20,200	.799
				15,200	.793
				36,000	.805
				39,450	.808
				42,950	.805
				47,000	.810
				50,650	.807
				55,400	.813
				60,200	.812
				64,600	.814
Stanton and Pannell	Air ($T_{ave.}$=15°C.)	Drawn Brass	0.494	4,870	0.758
				6,110	.768
				9,800	.775
				5,500	.761
				4,000	.747
				3,085	.705
				3,430	.735
				3,480	.735
				7,590	.770
"	"	"	1.124	4,650	.758
				5,470	.762
				4,005	.745
"	"	"	0.281	2,430	.611
				1,400	.499
				1,608	.498
				1,752	.511
				1,873	.508
				1,233	.505
				1,560	.501

Authority	Fluid Used	Pipe Material	I.D. (inches)	Re.	$V_{ave.}/V_{max.}$
Lawrence and Braunworth	Water	Seamless-Brass	5.016	57,850	0.829
				134,800	.833
				136,100	.840
				150,300	.862
				152,500	.830
				159,000	.865
				202,000	.858
				204,000	.865
				219,000	.858
				263,000	.872
				299,000	.870
				300,500	.873
				318,000	.872
				339,000	.854
				352,000	.872
J. Morrow	Water	Glass	2.005	709	0.590
				1,082	.617
				1,460	.619
				2,025	.638
				2,655	.701
				3,990	.765
Schiller and Kirsten	Air	Brass	2.99	20,340	0.824
				46,600	.792
				54,260	.815
				57,600	.835
T.E. Stanton	Air	Drawn Brass	2.92	40,750	.802
			1.94	41,200	.8105
			2.92	89,750	.808
J.R. Freeman	Water	Brazed Brass	1.15	425,000	0.835
D. Marshall	Air	Brass	5.00	132,300	0.856

ratio as given by Eq. (16b), although it approaches the limit 1 at high Reynolds number, increases more rapidly than the usual Stanton and Pannell's plot indicates at Re. about 10^5.

A much simpler way of obtaining the velocity distribution exponent "a" in Eq. (14b)

$$\frac{V}{V_{max}} = \left(1 - \frac{r}{R}\right)^{a} \qquad \text{.....(14b)}$$

is to calculate from the ratio of average to maximum velocity as given by Eq. (22)

$$\frac{V_{ave}}{V_{max}} = \frac{2}{(a+1)(a+2)} \qquad \text{......(22)}$$

which is integrated from Eq. (14b).

It must be pointed out that Eq. (22) is not limited to any class of pipe, no matter whether smooth or rough. Thus, a careful measurement of average and axial velocity alone will enable one to predict the velocity distribution in a pipe from Eq. (22). An extended application of this relation will be particularly helpful in predicting the velocity distribution during heat transfer, so long as Eq. (14b) will still hold true for non-isothermal gases.

F. On the Radius of Mean Velocity

For laminar flow, the radius of mean velocity is easily obtained by equating Eq. (6) and Eq. (16a), thus

$$V_{ave}/V_{max} = 1 - (r_{ave}/R)^2 = 1/2 \qquad \text{.....(26)}$$

Let $(r_{ave}/R) = X$, then

$$X = 1/\ \ 2 = 0.707 \qquad \text{......(26a)}$$

For turbulent flow, a similar relation, but much more complicated, can be obtained from Eq. (14b) and Eq. (22) as

$$\text{Log.}(1-X) = \frac{\text{Log.}(V_{\text{ave.}}/V_{\text{max.}})}{a} = \frac{\text{Log.}\left[\frac{2}{(a+1)(a+2)}\right]}{a} \quad \ldots(27)$$

This relation states that X, the radius of mean velocity as per cent of radius of pipe, is a function of velocity exponent "a", and is also depending on Reynolds number, since "a" is a function of Re. for a certain pipe. Thus, X can be calculated from given or known values of "a" from Eq. (27). Fortunately, from actual calculation the variation of X is very small with respect to "a", varying only from 0.753 at a = 0.175 to 0.761 at a = 0.086. Consequently, in smooth pipes, one may take it as a constant by using its average,

$$X_{\text{ave.}} = 0.757 \quad \ldots\ldots(28)$$

for a wide range of Reynolds number from 3,000 to 3,000,000. This reveals a very simple method of measuring the average velocity of turbulent flow in pipes.

G. Factors Affecting Velocity Distribution

There are three important factors which will affect velocity distribution in pipes, i.e., inlet length, inlet shape and roughness of pipe wall. Concerning the first two factors, which may be grouped together as entrance conditions, considerable work has been carried out by Schiller and Kirsten(29) experimentally

They tested these effects by means of measuring velocity distribution of air in a three inch brass pipe which is attached to the suction side of a ventilating fan, using a special steel pitot tube for measuring the velocity. They used both a round mouth inlet, having its biggest diameter 10" at the open end and gradually beveled to connect with the 3" main pipe, and a sharp edged inlet which is the main pipe alone. Their velocity distributions were taken at different distances away from the inlet which is called inlet length. Their results are tabulated in Table 12, and may be briefly summarized as follows:

(1) For a round mouth inlet, velocity distribution is almost a flat shape near the inlet, but gradually approach the elliptical shape as the measuring station is moved away from the inlet, and a fully developed distribution is to be found at an inlet length equivalent to 50 diameters.

(2) For a sharp edged inlet, similar effect as a round inlet has been found, but, due to more disturbance caused by the sharp inlet, the inlet length effect is less significant at the start, as shown by graphs in their original paper, but it was found that in order to obtain a fully developed distribution, an inlet length of 50 D is also necessary.

TABLE 12.

EFFECT OF ENTRANCE CONDITIONS ON VELOCITY DISTRIBUTION*

X = Inlet length

D = diameter of pipe

A. Round Inlet

Re = 19,060		Re. = 22,940		Re. = 57,600		Re. = 87,440	
$\frac{X}{D}$	$\frac{V_{ave}}{V_{max}}$	$\frac{X}{D}$	$\frac{V_{ave}}{V_{max}}$	$\frac{X}{D}$	$\frac{V_{ave}}{V_{max}}$	$\frac{X}{D}$	$\frac{V_{ave}}{V_{max}}$
1.32	0.962	2.21	0.952	2.21	0.980	2.21	0.975
3.29	0.917	6.81	0.946	4.18	0.954	4.18	0.960
15.79	0.889	16.68	0.886	6.81	0.937	6.81	0.943
22.37	0.856	39.05	0.841	12.08	0.906	12.08	0.887
38.17	0.836	50.89	0.824	16.68	0.865	16.68	0.865
				19.31	0.835	25.89	0.820
				23.26	0.814	39.05	0.809
				25.89	0.806		
				39.05	0.806		
				50.89	0.804		

B. Sharp Edged Inlet

Re. = 20,340		Re. = 46,600		Re. = 54,260		Re. = 57,600		Re. = 100,980	
$\frac{X}{D}$	$\frac{V_{ave}}{V_{max}}$	$\frac{X}{D}$	$\frac{V_{ave}}{V_{max}}$	$\frac{X}{D}$	$\frac{V_{ave}}{V_{max}}$	$\frac{X}{D}$	$\frac{V_{ave}}{V_{max}}$	$\frac{X}{D}$	$\frac{V_{ave}}{V_{max}}$
38.17	0.848	100	0.792	25.00	0.835	25.00	0.880	25.00	0.864
50.00	0.824	135	0.792	38.17	0.839	38.17	0.835	38.17	0.865
				50.00	0.815	50.00	0.835		

*Experimental Results of Schiller and Kirsten

(3) For a round mouth inlet, the velocity ratio, i.e., $V_{ave.}/V_{max.}$, approaches 0.8 at their experimental range, while for a sharp edged inlet, this value is higher at the same inlet length.

The effect of roughness of pipe wall will increase the surface friction, decrease the quantity of flow, thus will naturally affect the velocity distribution. The velocity distribution exponent and velocity ratio at different Reynolds numbers for iron and steel pipes have been calculated from the proposed friction factor equation, and those values will be found in Table 13 and plotted on Figures 37 and 40 in the form of smooth curves. Their equations are as follows: (For iron and steel pipes)

$$\frac{V_{ave.}}{V_{max.}} = \frac{1 + 0.01628\ Re.^{0.38}}{1.38 + 0.01628\ Re.^{0.38}} \qquad .....(29)$$

and

$$a = -1.5 + 0.5\sqrt{9 + \frac{3.04}{1 + 0.01628\ Re.^{0.38}}} \qquad ..(30)$$

Based upon these equations, it is noticed that velocity will be quite different in a rough pipe as compared with a smooth pipe, at the same Reynolds number the velocity distribution exponent is smaller and velocity ratio is greater. This indicates more turbulence in a rough pipe than in a smooth pipe. Very little work has been found on the velocity distribution in iron and

Table 13. Relation of Velocity Distribution to Reynolds Number in Iron and Steel Pipes (Calculated).

$$f = 0.003068 + 0.1886\ Re^{-0.38} \qquad \frac{V_{ave}}{V_{max}} = \frac{1+0.01628\ Re^{0.38}}{1.38+0.01628\ Re^{0.38}} \qquad (29)$$

$$a = -1.5 \pm 0.5 \sqrt{9 + \frac{3.04}{1+0.01628\ Re^{0.38}}} \qquad (30)$$

Re.	Velocity Distribution Exponent "a"	V_{ave}/V_{max}
3,000	0.178	0.779
4,000	0.175	.784
5,000	.170	.788
6,000	.166	.792
8,000	.161	.797
10,000	.156	.803
15,000	.148	.811
20,000	.142	.817
30,000	.133	.828
40,000	.127	.834
50,000	.122	.841
60,000	.118	.844
80,000	.115	.851
100,000	.1068	.858
150,000	.0978	.869
200,000	.0917	.876
250,000	.0870	.881
300,000	.0832	.887
400,000	.0775	.894
500,000	.0731	.900
650,000	.0680	.906
850,000	.0633	.913
1,000,000	.0606	.915
1,500,000	.0539	.924
2,500,000	.04625	.934

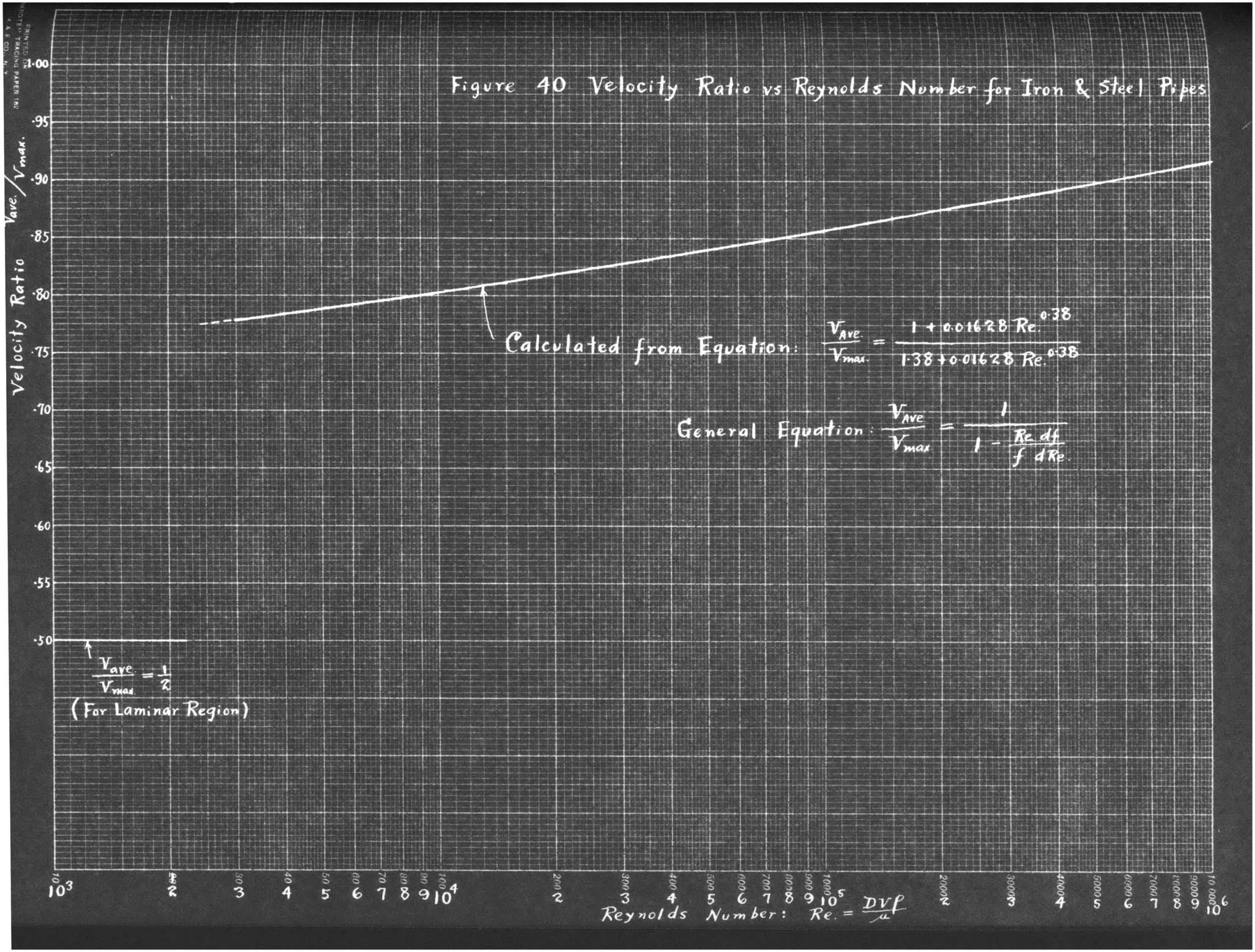
Figure 40 Velocity Ratio vs Reynolds Number for Iron & Steel Pipes

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Fig. 40a

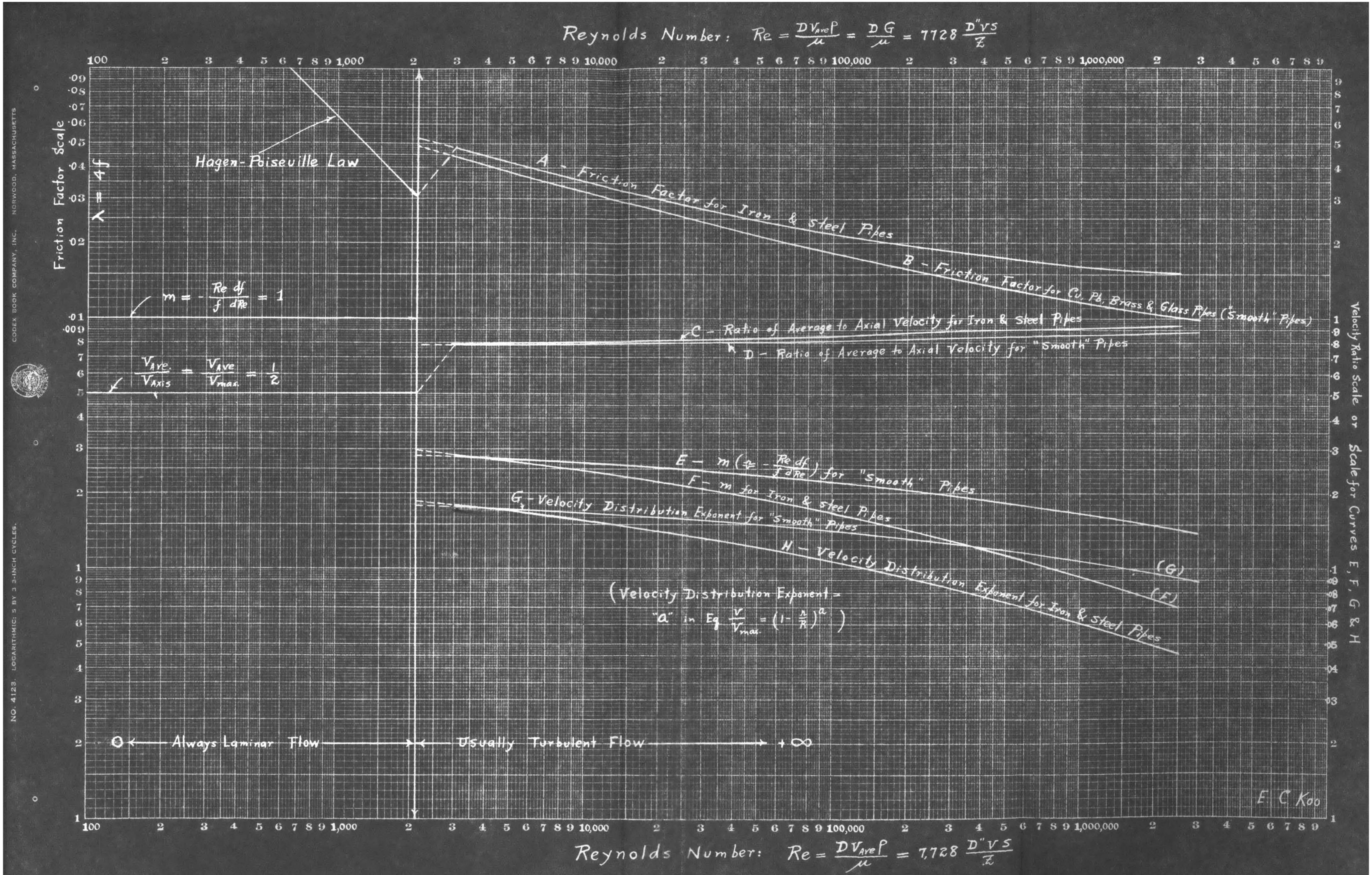
Reynolds Number: $Re = \frac{D V_{Ave} \rho}{\mu} = \frac{D G}{\mu} = 7728 \frac{D'' V S}{Z}$
100 2 3 4 5 6 7 8 9 1,000 2 3 4 5 6 7 8 9 10,000 2 3 4 5 6 7 8 9 100,000 2 3 4 5 6 7 8 9 1,000,000 2 3 4 5 6 7 8 9
Friction Factor Scale
$\lambda = 4f$
Hagen-Poiseuille Law
A - Friction Factor for Iron & Steel Pipes
B - Friction Factor for Cu, Pb, Brass & Glass Pipes ("Smooth" Pipes)
$m = -\frac{Re}{f}\frac{df}{dRe} = 1$
C - Ratio of Average to Axial Velocity for Iron & Steel Pipes
D - Ratio of Average to Axial Velocity for "Smooth" Pipes
$\frac{V_{Ave}}{V_{Axis}} = \frac{V_{Ave}}{V_{max}} = \frac{1}{2}$
E - $m\left(= -\frac{Re}{f}\frac{df}{dRe}\right)$ for "Smooth" Pipes
F - m for Iron & Steel Pipes
G - Velocity Distribution Exponent for "Smooth" Pipes
H - Velocity Distribution Exponent for Iron & Steel Pipes
(G)
(F)
(Velocity Distribution Exponent = "a" in Eq $\frac{V}{V_{max}} = (1 - \frac{r}{R})^a$)
0 ← Always Laminar Flow → ← Usually Turbulent Flow → ∞
Velocity Ratio Scale or Scale for Curves E, F, G & H
E C Koo
Reynolds Number: $Re = \frac{D V_{Ave} \rho}{\mu} = 7,728 \frac{D'' V S}{Z}$
CODEX BOOK COMPANY, INC. NORWOOD, MASSACHUSETTS
NO. 4123. LOGARITHMIC: 5 BY 3 3-INCH CYCLES.

Figure 40 a

steel pipes in literature except those of Threlfall(30) and Longridge(31). Their experiments on air and gases all prove a much higher velocity ratio than that calculated in an ordinary smooth pipe, the average ratio of Longridge's experiments is as high as 0.921. This at least indicates definitely that velocity distribution in iron and steel pipes is different from that in smooth pipes, thus Figures 37 and 40 may be suggested to be used at the present time for iron and steel pipes.

The experiments of Stanton(12) on artificially roughened pipe, Bazin(10, 11) and Krey(23) on cement pipes, however, indicate a shift in the value of "a" ~~the~~ opposite to the direction as given by Eq. (29) and (30).

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IV. REVIEW OF NON-ISOTHERMAL VELOCITY AND TEMPERATURE DISTRIBUTION OF FLUIDS IN CIRCULAR PIPES.

Pannell(1) in 1916, measured the velocity and temperature distributions of air in a vertically heated pipe. In 1930, Jakob, Erk, and Eck(2) made the similar measurements on steam in a vertical pipe while condensing. Recently, Kraussold(3) measured the temperature distributions of oil in vertical pipes mostly in viscous region. In this Institute, Woolfenden(4), in 1927, performed an experiment on velocity and temperature distributions of water in a horizontal pipe. Jakob, Erk and Eck's work is on a subject involving change of state. Woolfenden's experimental arrangement is rather unfortunate for comparison with the present work for he used a horizontal ~~vertical~~ pipe, instead of a vertical ~~horizontal~~ one, which position caused the non-symmetry of his velocity and temperature distributions. Hence, Pannell's work on air still remains as the only experiments of importance in turbulent region.

Therefore, it is believed to be very desirable to review Pannell's work more or less in detail. The calculated results of his four experiments are to be found in Table 14 with the accompanying Figures 41 to 46 . The temperature distribution equation used was due to

Schack(5). Its derivation is briefly outlined as follows. According to Schack, Prandtl's theory of similarity may be written as

$$\frac{dt}{dr} = \frac{kdV}{dr} \qquad .....(1)$$

That is to say, the temperature at any point ought to be directly proportional to the velocity at that point. The velocity distribution equation is

$$\frac{V}{V_{max.}} = (1 - \frac{r}{R})^{a} \qquad .....(2)$$

Differentiate Eq. (2) and substitute in (8), and integrate,

$$\int_{t_a}^{t_w} \frac{dt}{k} = \frac{a\ V_{max.}}{R} \cdot \int_{o}^{R} (1 - \frac{r}{R})^{a-1} dr \qquad .....(3)$$

$$\therefore \qquad t_w - t_a = -\ k\ V_{max.} \qquad .....(4)$$

and

$$\int_{t}^{t_w} \frac{dt}{k} = \frac{a\ V_{max.}}{R} \cdot \int_{r}^{R} (1 - \frac{r}{R})^{a-1} dr \qquad .....(5)$$

$$\therefore \qquad t_w - t = -k\ V_{max.}\ (1 - \frac{r}{R})^{a} \qquad .....(6)$$

Dividing Eq. (6) by Eq. (4), the temperature distribution equation is obtained, thus,

$$\frac{t_w - t}{t_w - t_a} = (1 - \frac{r}{R})^{a} \qquad .....(7)$$

This equation states that the temperature distribution exponent is identical with the corresponding velocity distribution exponent, if the similarity holds. In order to differentiate the actual temperature exponent from the velocity distribution exponent, "a" in Eq. (7) is changed to "b", thus

$$\frac{\Delta t}{\Delta t_{max.}} = \frac{t_w - t}{t_w - t_a} = \left(1 - \frac{r}{R}\right)^{b} \qquad \ldots\ldots(7a)$$

In Table 14 , it is seen that the graphically determined exponents "a" and "b" on Pannell's data are almost identical. The graphical plot of his Experiment IV is illustrated here. Thus, the similarity is found to be applicable to gases. However, its application to liquids is still unknown.

Table 14 Summarized Calculated Results of Pannell's Data on Heating of Air

(Column) (1) Experiment No.	(2) $\frac{V_{ave.}}{(Cm./Sec.)}$	(3) $\frac{t_{ave.}}{(°C.)}$	(4) $\frac{t_w - t_a}{(°C.)}$	(5) $\frac{t_w}{(°C.)}$	(6) $\frac{t_w - t_{ave.}}{t_w - t_a}$	(7) $\frac{t_{ave.} - t_a}{t_w - t_a}$	(8) $\frac{V_{ave.}}{V_{max.}}$	(9) $Re_{f\,ave.}$	(10) a	(11) b	(12) $Pe.\frac{(t_w - t_a)}{(t_{ave.} - t_a)}$
I	542	24.82	13.3	35.5	0.803	0.197	0.788	17,120	0.169	0.170	100,000
II	1,200	23.7	16.7	37.4	0.821	0.1796	0.809	38,200	0.154	0.150	245,000
III	1,482	23.7	17.4	38.0	0.822	0.1781	0.815	47,300	0.143	0.141	305,000
IV	2,180	27.3	18.6	43.0	0.844	0.1558	0.816	67,400	0.138	0.136	497,500

Calculated Results of Pannell's Data on Heating of Air

(Tech. Reports, Adv. Comm. for Aero. (Great Britain)
Vol. I, p. 22 (1916-7)).

Experiment I. $R = 2.44$ Cm.

$(\frac{r}{R})$	$1 - \frac{r}{R}$	$1 - (\frac{r}{R})^2$	$\frac{t_w - t}{t_w - t_a}$	$\frac{V}{V_{max.}}$
0.00	1.00	1.000	1.000	1.000
.15	.85	.978	1.000	.995
.30	.70	.910	.985	.963
.45	.55	.797	.940	.927
.60	.40	.640	.880	.873
.70	.30	.510	.842	.828
.80	.20	.360	.775	.773
.85	.15	.278	.737	.742
.90	.10	.190	.690	.701
.93	.07	.135	.638	.662
.95	.05	.098	.593	.618
.96	.04	.078	.563	.581
.97	.03	.059	.518	.517
.98	.02	.040	.465	.395
.99	.01	.020	.375	.190
1.00	.00	.000	.000	.000

$V_{max.} = 688$ Cms./sec.

$t_w = 35.5°C.$

$V_{ave.} = 542$ Cms./sec.

$t_{ave.} = 24.82°C.$

$t_w - t_a = 13.3°C.$

Calculated Results of Pannell's Data on Heating of Air

Experiment II $R = 2.44$ Cm.

$(\frac{r}{R})$	$(1 - \frac{r}{R})$	$(1 - (\frac{r}{R})^2$	$\frac{t_w - t}{t_w - t_a}$	$\frac{V}{V_{max.}}$
0.00	1.00	1.000	1.000	1.000
.15	.85	.978	.994	.988
.30	.70	.910	.975	.959
.45	.55	.797	.952	.929
.60	.40	.640	.898	.874
.70	.30	.510	.850	.837
.80	.20	.360	.802	.786
.85	.15	.278	.754	.755
.90	.10	.190	.700	.711
.93	.07	.135	.652	.674
.95	.05	.098	.610	.642
.96	.04	.078	.575	.623
.97	.03	.059	.545	.600
.98	.02	.040	.515	.545
.99	.01	.020	.467	.310
1.00	.00	.000	.000	.000

$V_{max.} = 1,483$ Cms./sec. $t_w = 37.4°C.$

$V_{ave.} = 1,200$ Cms./sec. $t_{ave.} = 23.7°C.$

$t_w - t_a = 16.7°C.$

Calculated Results of Pannell's Data on Heating of Air

Experiment III R = 2.44 Cm.

$(\frac{r}{R})$	$1 - \frac{r}{R}$	$1 - (\frac{r}{R})^2$	$\frac{t_w - t}{t_w - t_a}$	$\frac{V}{V_{max.}}$
0.00	1.00	1.000	1.000	1.000
.15	.85	.978	.994	.990
.30	.70	.910	.983	.967
.45	.55	.797	.948	.930
.60	.40	.640	.896	.882
.70	.30	.510	.850	.842
.80	.20	.360	.793	.794
.85	.15	.278	.752	.762
.90	.10	.190	.707	.720
.93	.07	.135	.661	.687
.95	.05	.098	.632	.656
.96	.04	.078	.615	.633
.97	.03	.059	.592	.598
.98	.02	.040	.563	.549
.99	.01	.020	.517	.478
1.00	.00	.000	.000	.000

$V_{max.}$ = 1820 Cms./sec.

$V_{ave.}$ = 1482 Cms./sec.

t_w = 38.0°C.

$t_{ave.}$ = 23.7°C.

$t_w - t_a$ = 17.4°C.

Calculated Results of Pannell's Data on Heating of Air

Experiment IV. R = 2.44 Cm.

$(\frac{r}{R})$	$1 - \frac{r}{R}$	$1 - (\frac{r}{R})^2$	$\frac{t_w - t}{t_w - t_a}$	$\frac{V}{V_{max.}}$
0.00	1.00	1.000	1.000	1.000
.15	.85	.978	1.000	.992
.30	.70	.910	.993	.969
.45	.55	.797	.966	.938
.60	.40	.640	.924	.884
.70	.30	.510	.870	.864
.80	.20	.360	.811	.796
.85	.15	.278	.762	.763
.90	.10	.190	.719	.727
.93	.07	.135	.676	.697
.95	.05	.098	.650	.670
.96	.04	.078	.627	.647
.97	.03	.059	.601	.621
.98	.02	.040	.574	.584
.99	.01	.020	.515	.475
1.00	.00	.000	.000	.000

$V_{max.}$ = 2,670 Cms./sec.

$V_{ave.}$ = 2,180 Cms./sec.

t_w = 43.0°C.

$t_{ave.}$ = 27.3°C.

$t_w - t_a$ = 18.6°C.

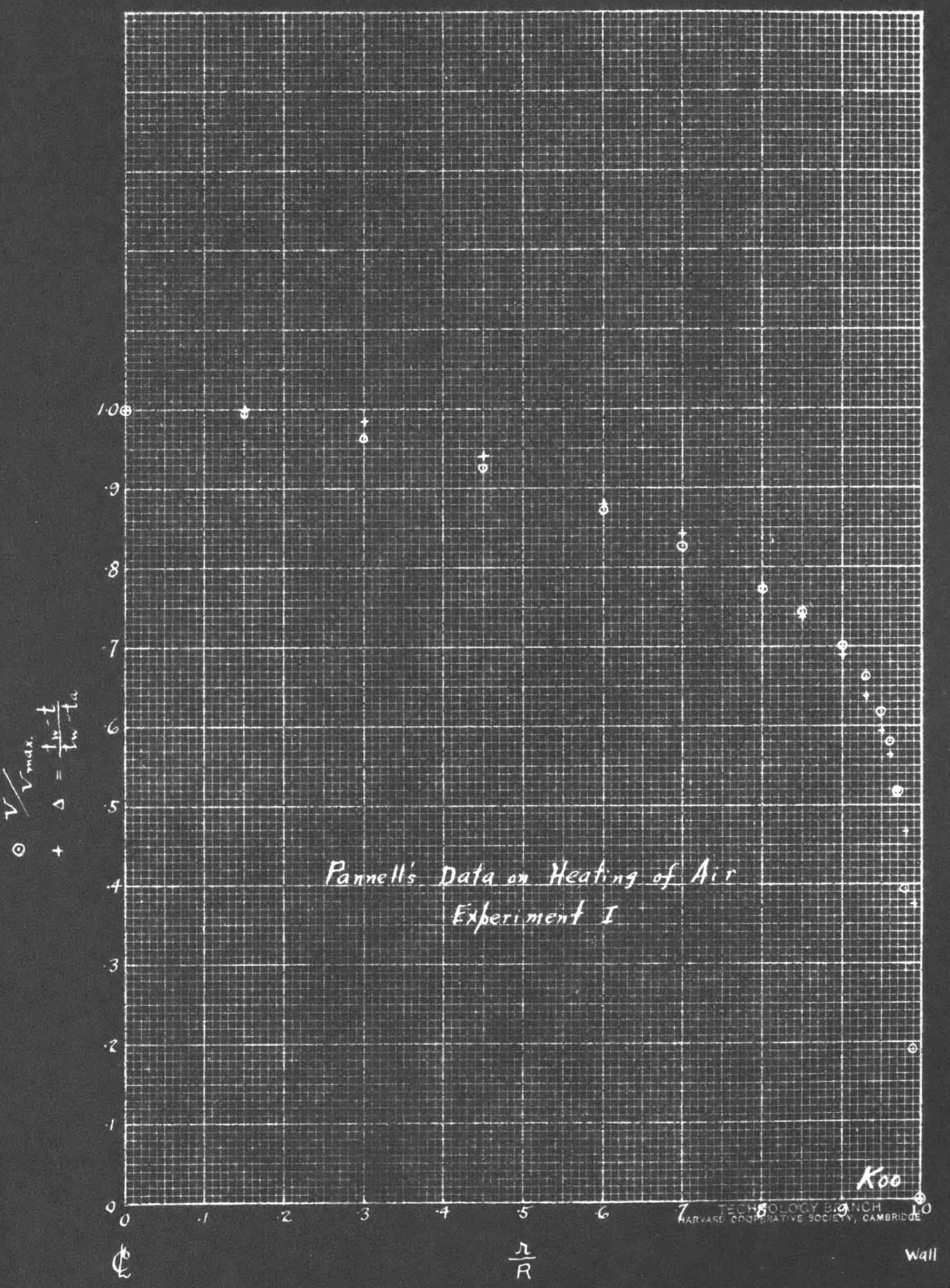
Pannell's Data on Heating of Air
Experiment I
⊙ V/V max.
+ Δ = (t_w - t)/(t_w - t_a)
1.0
·9
·8
·7
·6
·5
·4
·3
·2
·1
0
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1.0
℄
r/R
Wall
Koo
TECHNOLOGY BRANCH
HARVARD COOPERATIVE SOCIETY, CAMBRIDGE

Fig. 41

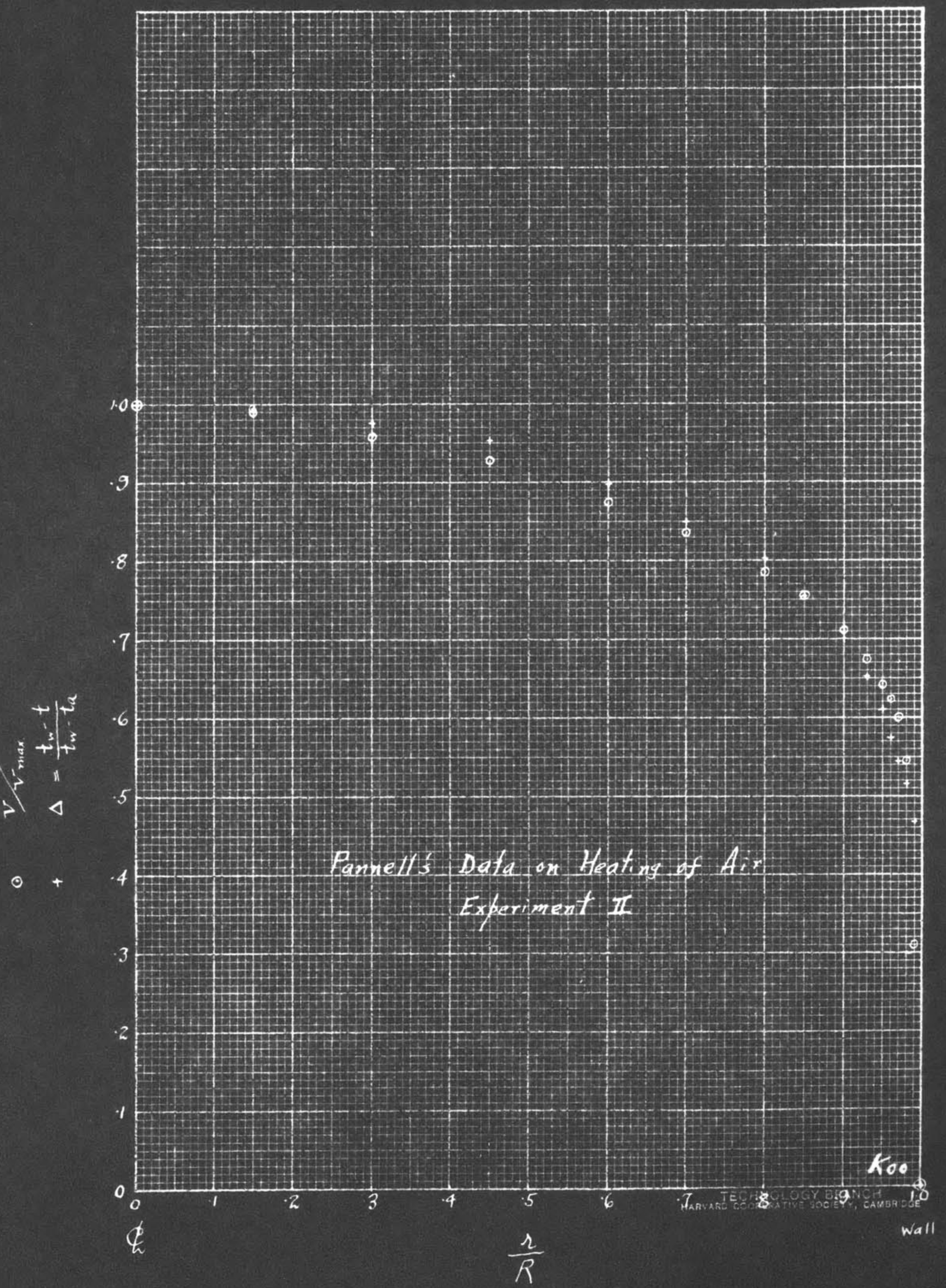
⊙ v/v_max
+ Δ = (t_w - t)/(t_w - t_a)
Pannell's Data on Heating of Air
Experiment II
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
℄
r/R
Wall
Koo
TECHNOLOGY BRANCH
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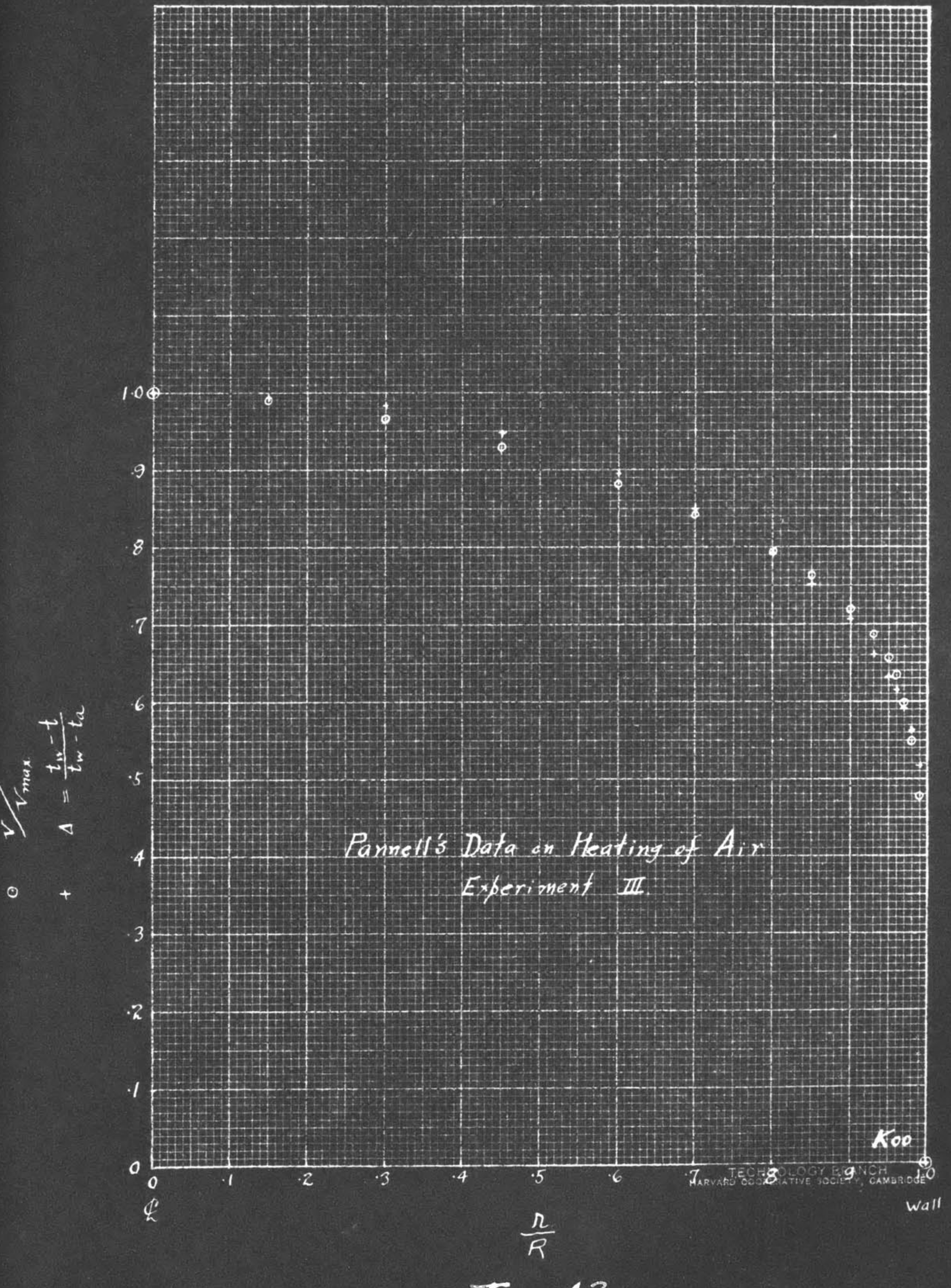
Fig. 42



Pannell's Data on Heating of Air
Experiment III.

Fig. 43

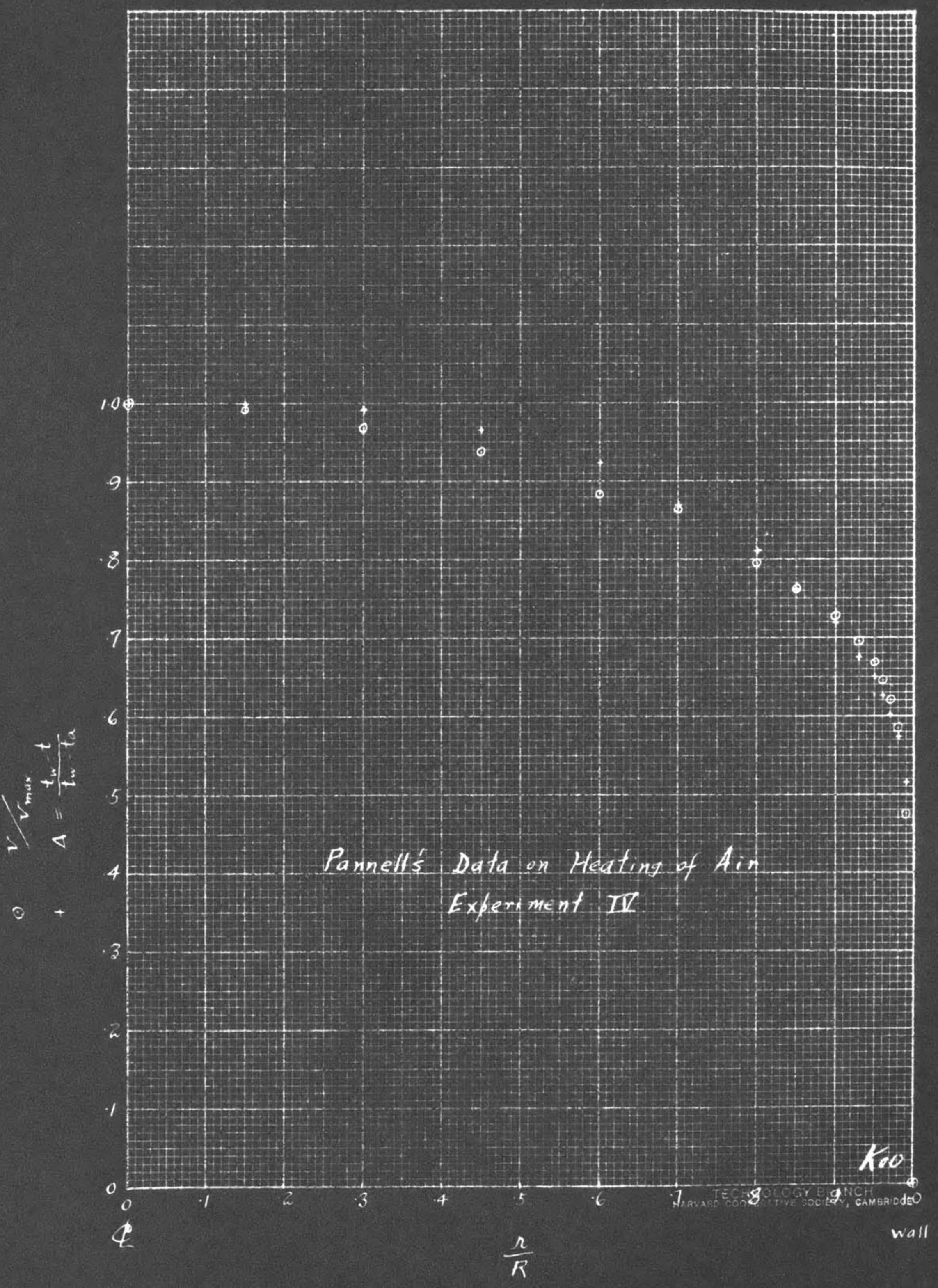
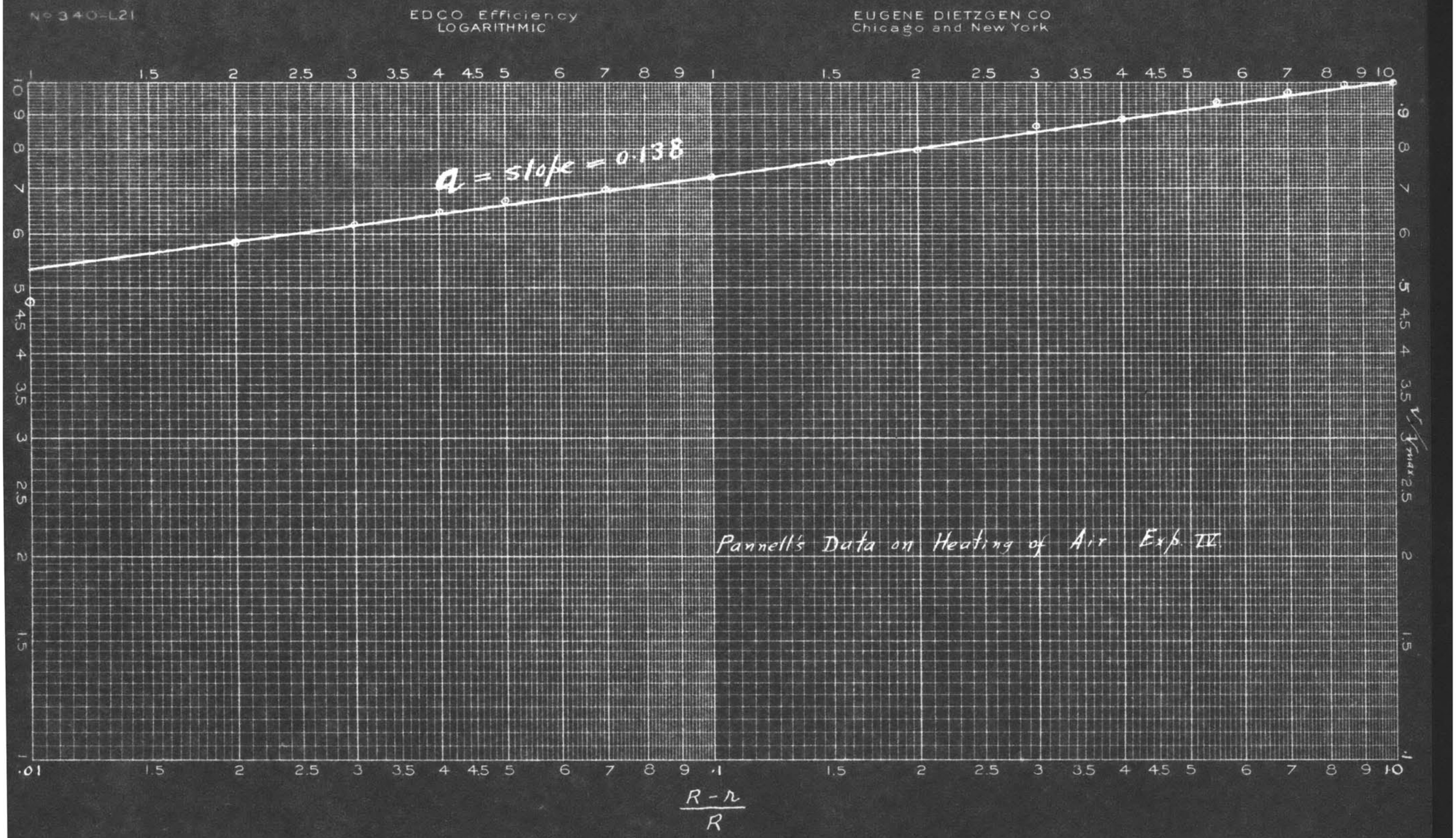
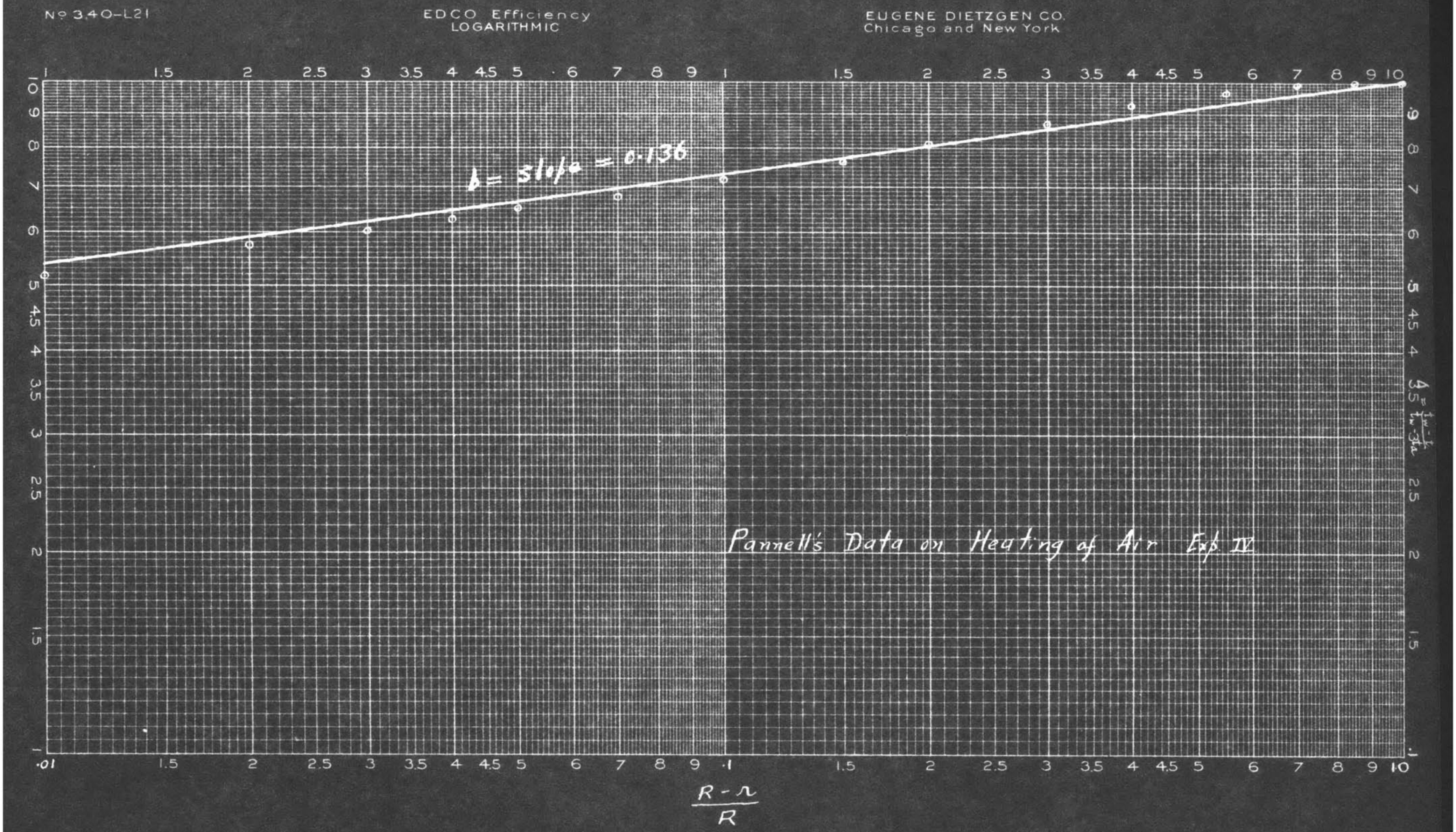

⊙ v/v_max
+ Δ = (t_w − t)/(t_w − t_a)
1.0
·9
·8
·7
·6
·5
·4
·3
·2
·1
0
Pannell's Data on Heating of Air
Experiment IV
Koo
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1.0
℄
wall
r/R
TECHNOLOGY BRANCH
HARVARD COOPERATIVE SOCIETY, CAMBRIDGE

Fig. 44



No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO
Chicago and New York
a = slope = 0.138
Pannell's Data on Heating of Air Exp. IV
V/V_{max}
$\frac{R-r}{R}$
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.1
.5
.9

Fig. 45



No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
b = slope = 0.136
Pannell's Data on Heating of Air Exp IV
$A = \frac{t_w - t}{t_w - 3t_a}$
$\frac{R - r}{R}$
.01
.1
10
1
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9

Fig. 46

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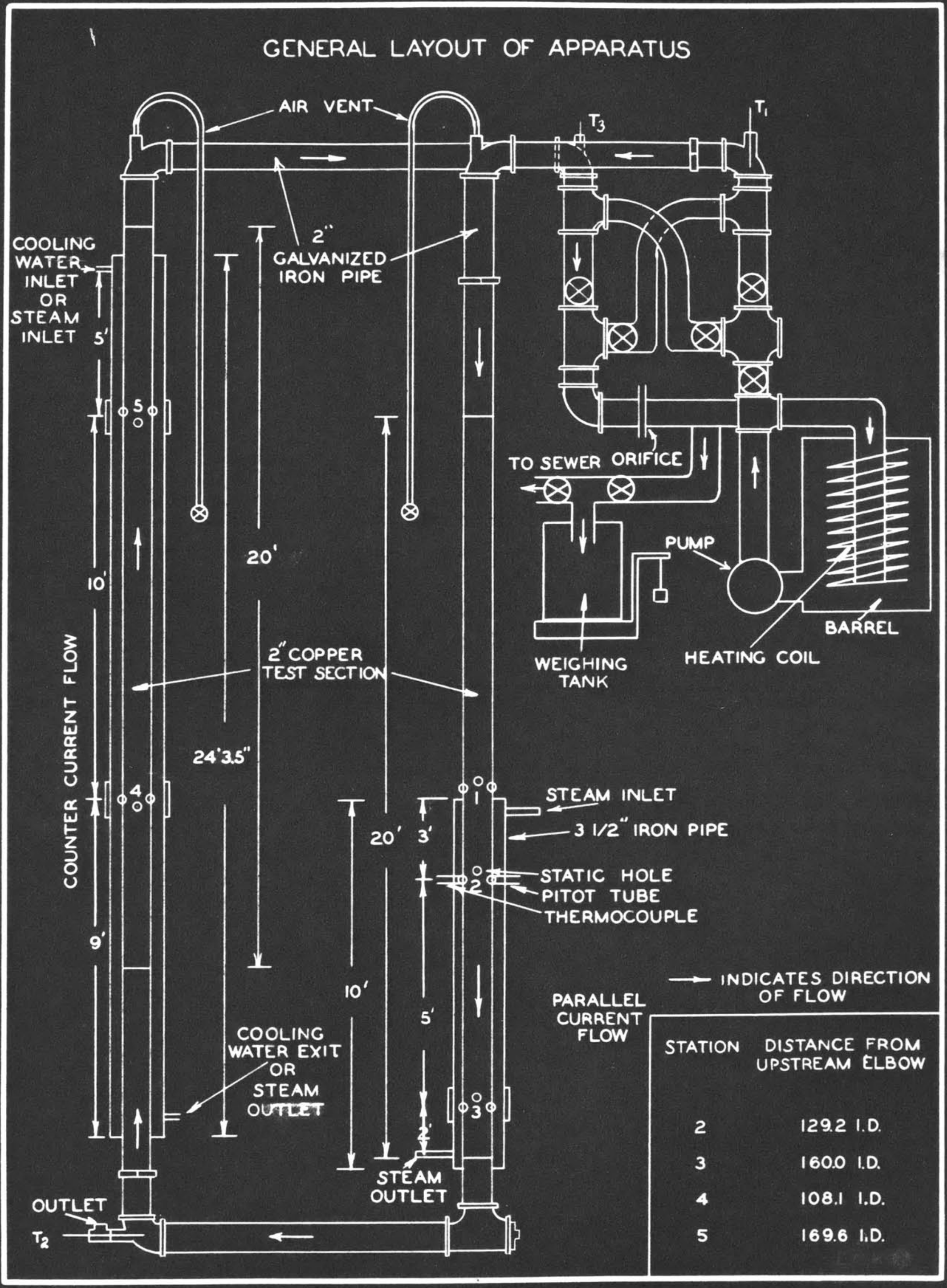
V. DESCRIPTION OF APPARATUS

In the course of investigation, an apparatus was built in the summer of 1930. Additional parts were added on from time to time. The general layout of the apparatus in the final form is shown diagrammatically in Figure 47, where the arrows in the pipe line indicate the regular direction of flow of water in the system. The apparatus consists mainly of pump and reservoir calming sections, vertical heat transfer sections which are either heated by condensing steam or cooled by cooling water in the outside jacket, and velocity and temperature measuring instruments which are specially designed. The two test sections are both made of No. 10 stubs gage seamless hard drawn copper and of 20 feet in length, having an inside diameter of 1.95 in. The details of the several important parts will be given in the following.

Pump and Reservoir

The water reservoir is a 55 gallon drum, to which are connected two feed lines of water of 1-inch pipe. Water was pumped by means of a centrifugal pump at a constant rate from the reservoir. The pump capacity is about 100 gallons per minute. The pump intake is always submerged in water, thus there is no chance of having air sucked into the system. The quantity of water pumped is regulated by means of a main by-pass valve, and another 1/4-inch needle valve



GENERAL LAYOUT OF APPARATUS

STATION	DISTANCE FROM UPSTREAM ELBOW
2	129.2 I.D.
3	160.0 I.D.
4	108.1 I.D.
5	169.6 I.D.

Figure 47

is also provided for minute adjustment. At the top of each valve is a Bourdon gauge to indicate the pressure. The direction of flow can be reversed in the whole system by means of cross pipes and valves. The water, after passing the test sections, may be either discarded to the sewer or return to the reservoir and recirculated. Due to the immense quantity of water needed in the system, it is impossible to get the maximum rate without recirculating through the system. Even running the apparatus at moderate rate without recirculating, it takes nearly all the water from the two supply mains of the laboratory.

Calming Sections

Except the test sections, all pipes in the apparatus are made of standard 2-inch galvanized iron pipes. The length of calming section in this investigation is defined as the length of straight pipe from the upstream elbow or tee to the test section. There are five available test cross-sections, but only test cross-sections 2, 3, 4, and 5 have been used. (These test cross-sections will be called "stations" from now on.) The calming lengths expressed in terms of the inside diameter of the test pipe for different stations are:

Station No.	Calming Length in I.D. of Test Pipe
2	129.2
3	160.0
4	108.1
5	169.6

It is believed that the present apparatus with such a long calming length for each station will eliminate any entrance disturbance (cf. Nikuradse (17) Chap. III). ✓

Test Sections or Heat Transfer Sections

There are two test sections: A 10 ft. length primarily for parallel current flow and a 20 ft. length primarily for countercurrent flow. They are set in a vertical position, so that by reasons of symmetry, the velocity and temperature distribution curves should be symmetrical with respect to the axis of the pipe. They are made of copper as mentioned before; copper being chosen because of its high conductivity and purity, so the standard value of its thermal conductivity can be used in the heat transfer calculations. The short section is jacketed with a 3.5-inch iron pipe 10 ft. long, while the long section is jacketed with a 24 ft. length of the same size. The jacket and the pipe are held together with two successively reducing steel bushings which are specially made to fit the present size and a 3.5-inch coupling. The exact way of fitting is shown in Fig. 48 . Between these bushings, steam valve packing glands have been used to make the joint steam tight. These joints are flexible so that the jacket may be slid to other parts of the test pipe, if necessary. Steam or water was fed into the long jacket at the top and exhausted at the bottom, but only steam was fed to the short jacket. One condensate trap was ✓

provided to remove the condensate, before the steam was fed to the heating jacket. Air vents were provided at the top of each vertical section of the apparatus in order to remove air in the system.

The outside diameter of the copper pipe was measured by means of a vernier caliper at different stations, its average value was found to be 2.25". By means of a depth gage, the value of the inside diameter plus one wall thickness was obtained, thus subtracting this value from the measured outside diameter will give the wall thickness of the pipe, which was found to be 0.15". Then the inside diameter of the pipe can be readily calculated; its average is 1.95 inch. The dimensions of different stations are given in Appendix J, and found to have very little difference from each other.

Orifices

There are three orifices provided in the system, one for the main water line, one for cooling water, and one for steam. Only the main line orifice is mostly used during the test to determine approximately the quantity of water going through the system. The main line orifice chamber consists of a thin plate orifice of 1-11/16 inch in diameter, the chamber being 3-inch standard iron pipe. During the calibration of the orifice, the water, after passing through it instead of being discharged to the sewer, was discharged to the

FIG. 49
IMPACT TUBE

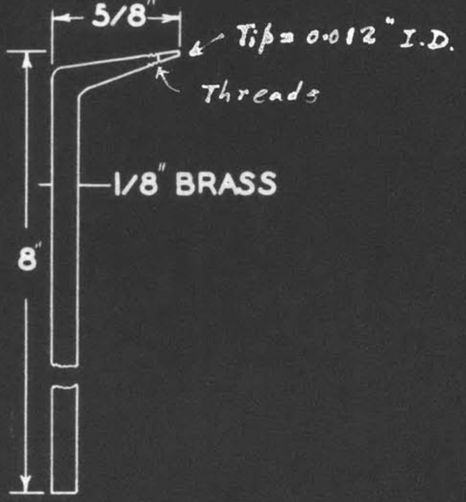


FIG. 50
THERMOCOUPLE

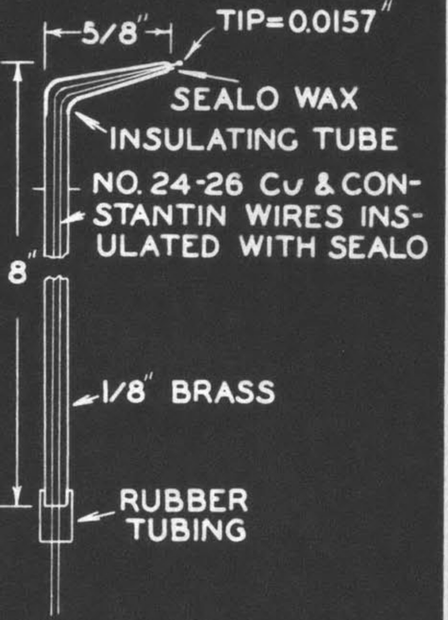
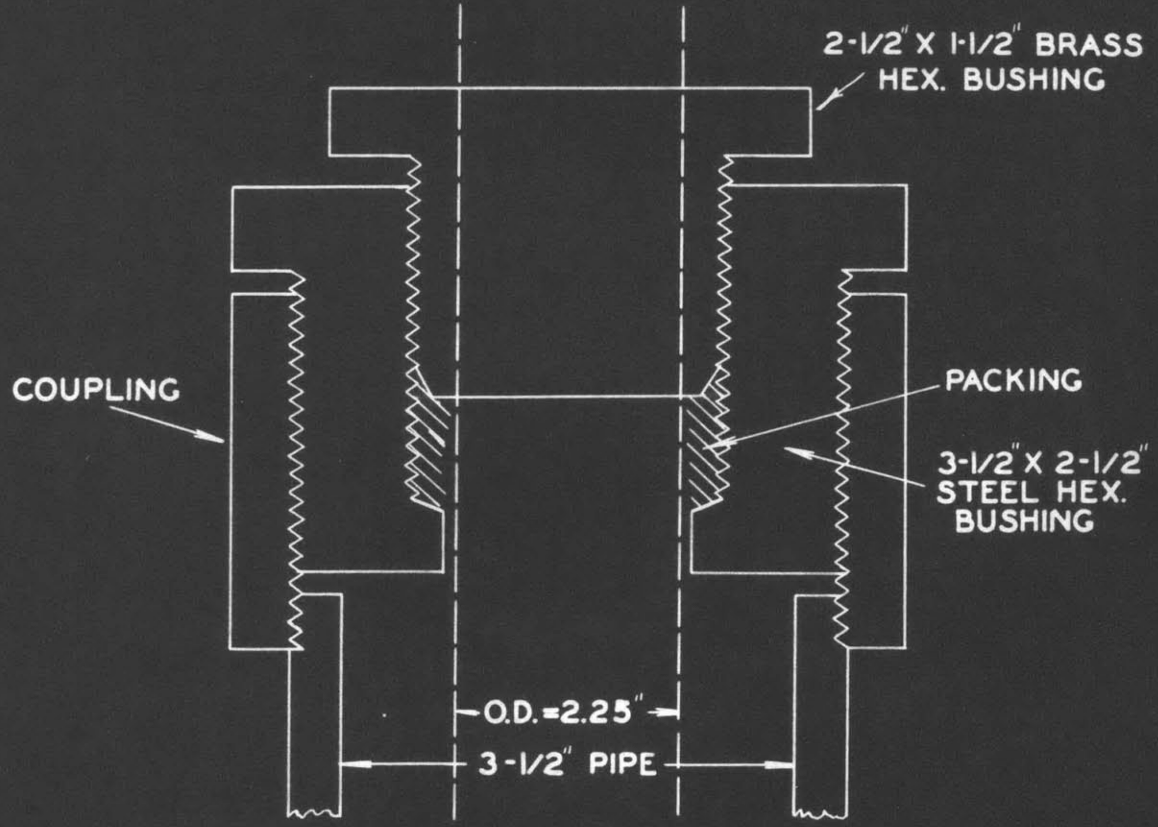
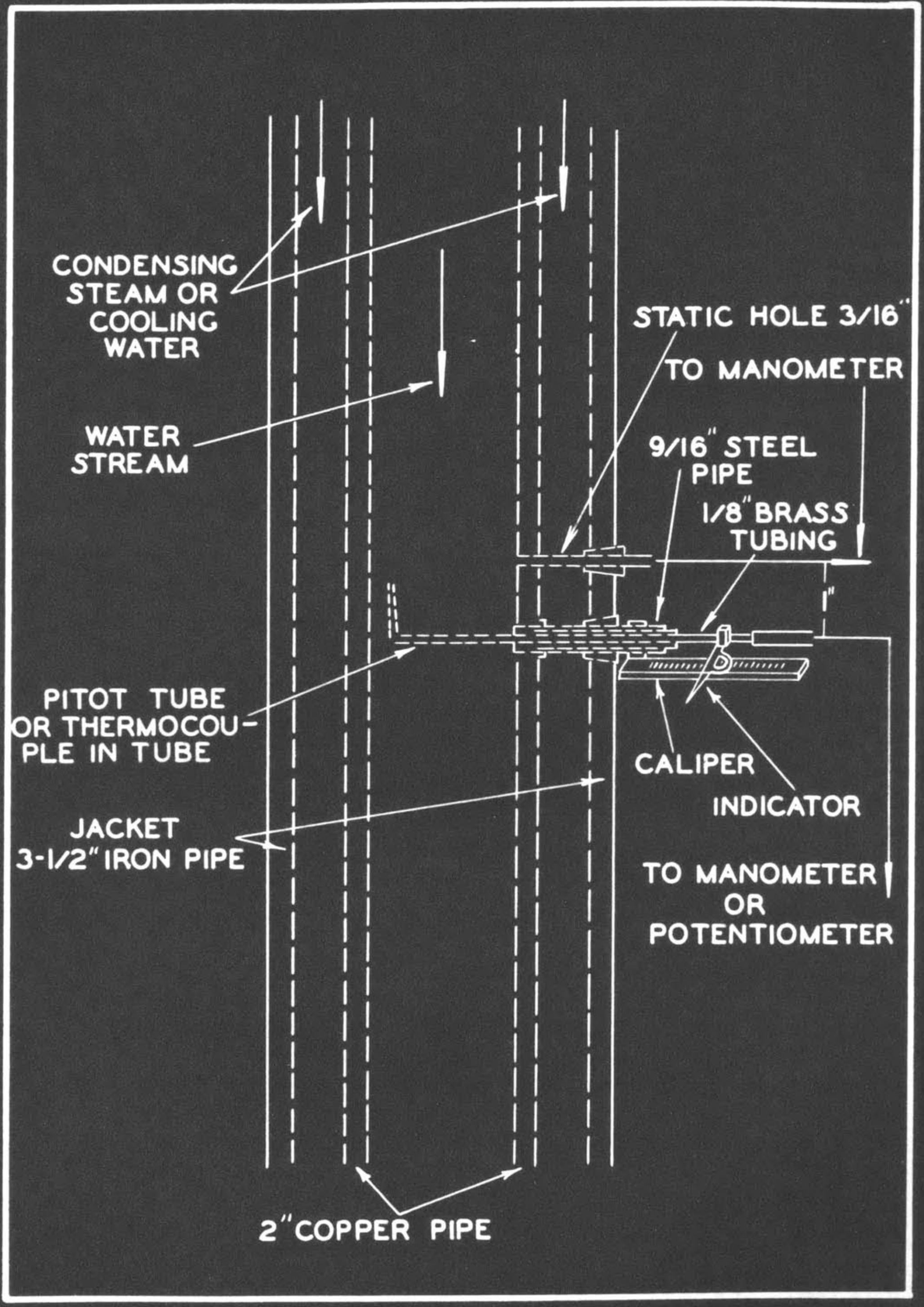


FIG. 48
FLEXIBLE JOINT





CONDENSING STEAM OR COOLING WATER
STATIC HOLE 3/16"
TO MANOMETER
WATER STREAM
9/16" STEEL PIPE
1/8" BRASS TUBING
1"
PITOT TUBE OR THERMOCOUPLE IN TUBE
CALIPER
INDICATOR
JACKET 3-1/2" IRON PIPE
TO MANOMETER OR POTENTIOMETER
2" COPPER PIPE

Figure 51

weighing tank on a platform scale. The calibration data are tabulated in Appendix J. The coefficient of this orifice was found to be 0.608.

Pitot Tube Set for Velocity Exploration (see Figure 51)

The pitot tube set consists of a static tube made of 3/16" copper tubing and a specially made impact tube made of 1/8" brass tubing. Considerable effort has been devoted in choosing the metal for the impact tube. Hypodermic-needle steel tubing cannot be used in this case, since it will rust easily in hot water and is liable to plug up the impact hole. Other tubings of alloy, such as monel, are not available to such a fine size. Copper tubing was once tried, but found to be too soft, besides its disadvantage of high thermal conductivity. The brass impact tube consists of a tip part and the stem. The tip part is a converging cone having its smaller end 0.4 mm. in inside diameter, while the bigger end, threaded, can be fitted to the main stem. (See Fig. 49). The tip part, together with a short length of brass tubing, is bent from the main stem to form an oblique angle of about 110°, thus the tip part is leaning outward. This arrangement was found to be necessary in order to get velocity exploration near to the pipe wall with the least possible disturbance. However, the plane of the tip was filed down to be parallel to the main stem, thus the tip is facing perpendicularly to the direction of flow of water in the pipe. The main stem of the impact tube was jacketed with a 5-inch length of 9/16" iron pipe which was threaded at one end to be fitted to the copper pipe; this end of the iron pipe was plugged with a bakelite stopper with a hole drilled at its center just big enough for the impact tube to slide to and fro. A hard rubber stopper, instead of bakelite, was tried at the beginning, but found to be not satisfactory because hard rubber shrinks when it is heated. The other end of the 9/16" iron pipe was threaded with an intermediate brass tubing which was used to tighten the packing in the annular space between the impact tube and its iron jacket. This packing was necessary in order to prevent any leakage. The iron jacket extended from the copper pipe, passed the annular space between the copper pipe and its 3-1/2" jacket and fitted tightly to the latter by means of a rubber stopper and litharge glycerine. A caliper which was readable to 0.01 cm. was fitted to the 9/16" steel pipe with its sliding indicator attached to the 1/8" impact tubing, thus by sliding the indicator, the position of the impact tube in the copper pipe was changed. As the impact tube was moving away from the fitting of its jacket on the copper pipe wall, it would touch the other side of the pipe wall diagonally.

Since the impact tube was electrically insulated by bakelite from its jacket and consequently from the copper pipe, an electric device by combining electric bell, dry cells and other connections was used to determine the positions of the impact tube when it touched the wall. The bell rang as soon as the impact tube touched the wall. The bell rang as soon as the impact tube touched the copper pipe, and readings on the caliper were taken. This device was found to be very desirable in order to determine the exact position as well as to protect the tubing from breakage or bending through "over pushing" against the wall, due to ignoring the exact position.

Thermocouple Set for Temperature Exploration

The other fittings of this set were exactly the same as those used in the pitot tube set, but the impact tube was replaced by a thermocouple tubing which contained either No. 24 or No. 26 copper constantan wires. The thermocouple tubing was just the main impact tube, as mentioned before, less its tip part. These wires were cotton-covered and coated with sealo wax, which was found to be a satisfactory insulator for the present purpose. The hot junction was soldered and filed down to a fine tip having its diameter from 0.4 to 0.6 mm. Just below the junction these wires were again insulated and protected by a spaghetti tubing (see Figure 50), which was obtained from one electric supply store, before they were fitted into the brass tubing. At the outlet of the brass tubing, a short length of rubber tubing was connected, the thermocouples were passed through the rubber tubing and connected to the potentiometer. By tying together the rubber tubing and the wires inside with a copper wire, leakage was eliminated entirely.

Other Thermocouples and Thermometers

No. 24 copper constantan thermocouples were used throughout to measure temperature before and after heating or cooling, and the outside wall temperatures of the copper pipe at the test cross-sections. These mixing cup temperatures were taken at the 2"-2"-1" tees, where water was assumed to be very turbulent and well mixed. These wires were enclosed in a 3/16" glass tubing held firmly near the tip with the aid of duPont cement, while the hot junction was left uncovered outside the glass tubing. The glass protecting tubing was passed through a rubber stopper which was inserted into the 1" side of the tee and held tight by means of a reducing coupling fitted on the 1" side led the way out from the fluid and made the attachment leakage proof.

Those thermocouples used to take outside wall temperatures were soldered into grooves on the wall while the excess solder was removed by a file and the surface was smoothed by sand paper. From the grooves, these wires were attached to, but electrically insulated from, the pipe wall for about three inches, by means of putting a piece of mica underneath the wires and electric tape above the wire and mica to hold them together at the wall, before they were led to the outside of the jacket.

All cold junctions of thermocouples were immersed in ice-filled thermo bottles. The potentiometer used was a Brown Precision Portable Type, readable to 0.01 of a millivolt (equivalent to 0.25°C.), and estimable to 0.005 of a millivolt.

One thermometer was inserted immediately after the steam orifice, and another in the water reservoir.

Manometers

The manometers were made from 3/16" pyrex glass tubing of well selected uniform cross-section. For taking main line orifice readings, a magnified mercury manometer inclined with a slope of 0.228 was used. For taking the static head of the test cross-sections, vertical mercury manometers were used, having one column connected to the static tube while the other open to the atmosphere. For taking velocity head at the test cross-sections, vertical manometers filled with carbon tetrachloride which was dyed red with azobenzene, were used, having one column connected to the impact tube, while the other connected to the static tube. The carbon tetrachloride used was left together with excess amount of water in a liter bottle for several weeks, thus it ought to be saturated with water. Its specific gravity at room temperature was found to be 1.5762, so the manometer reading for velocity head was magnified 1.74 times the velocity head expressed in water itself. Bromobenzene and chlorobenzene were found to be good, but their readings magnified too much that the manometers built approximately 2-1/2 ft. in height were not high enough, especially for readings near the center of the pipe. The leads of manometers were a combination of rubber and glass tubing, the reason of combining some glass tubings was to detect the air bubbles in the leads. In the manometer system, sufficient air vents were provided.

Lagging

The main part of the apparatus was lagged with 2-1/2" magnesia lagging from T_1 to T_2, as shown in Figure of the General Layout of Apparatus.

VI. EXPERIMENTAL PROCEDURE

Before any run, water was recirculated through the system, keeping a maximum rate of flow by shutting all the by-pass valves. During recirculating, air was removed from the system through the numerous air vents provided for this purpose and especially through the static and impact pressure leads to the manometers. Complete elimination of air in the system is regarded as a primary necessity in this investigation; because its presence in the system would effect the velocity and temperature distributions in the test cross-section, while its presence in the manometer leads would interfere with the pressure, measurements by giving erroneous readings, and sometimes even cause the manometer liquid to be sucked to the system. For most of the isothermal runs, water was recirculated, thus no air bubbles were detected after they had been excluded. For the heating runs, water was not recirculated but discharged to the sewer, the problem of getting rid of the air bubbles became more serious, because as the water was being heated in the pipe, its capacity ~~of~~ for dissolving air ~~in it~~ is gradually diminished, thus giving out air to the system. To overcome this difficulty, occasional opening of the air vents during the test worked out successfully except at very low velocity runs. At low velocity runs, when countercurrent heating runs were performed on the long section, there was considerable difficulty to stop the air bubbles from coming through the impact tube to the manometer leads, for the impact tube in these runs were facing downward, giving a favorable condition for air bubbles to go in. Apparently, in the parallel current runs, where the impact tube was facing ~~down~~upward, air bubbles were not liable to go into the tubing even if they were evolved. This explains why, in the present investigation, there are fewer simultaneous velocity and temperature distribution measurements in the ~~former~~ Counter-current case.

Because of the short distance to the bottom tee, stations 2 and 3 can be only run parallel currently. Stations 4 and 5 were used for counter-current runs only, whether during or cooling or isothermally. Preliminary runs of testing symmetry of velocity and temperature distributions at different stations were carried out at the very beginning of the experimentation. Table 15 and Figure 52 illustrate the symmetry of velocity distribution at Station 5, using bromobenzene as a manometer liquid which magnifies the reading more than the latter used carbon tetrachloride. The symmetry shown is what one expects; that little deviation might be due to the effect of fitting, while the velocity distribution on the other

Table 15.

Preliminary Run of Testing Symmetry of Velocity Distribution (Station No. 5) (Water Running Upward)

$\frac{r}{R}$	Δh (cm. Bromobenzene)	$V/V_{max.}$	Remarks
0.788	25.8	0.771	**Near fitting**
.748	27.4	.794	
.655	29.8	.828	
.534	33.4	.875	
.449	36.4	.915	
.348	38.6	.942	
.244	40.8	.970	
.147	42.4	.988	
.046	43.4	1.000	
.003	43.4	1.000	
.048	43.4	1.000	**Away from fitting**
.136	43.4	1.000	
.245	42.0	.983	
.342	40.4	.964	
.439	38.4	.940	
.540	36.2	.913	
.640	33.4	.875	
.741	29.6	.825	
.846	25.4	.764	
.894	23.6	.737	
.939	20.2	.682	
.967	17.0	.625	
.979	15.6	.599	
.987	14.2	.572	

Static Pressure = 32.5 cm. Hg. Gauge

t = 25°C.

$V_{ave.}$ (Manometer) = 6.02 ft./sec.

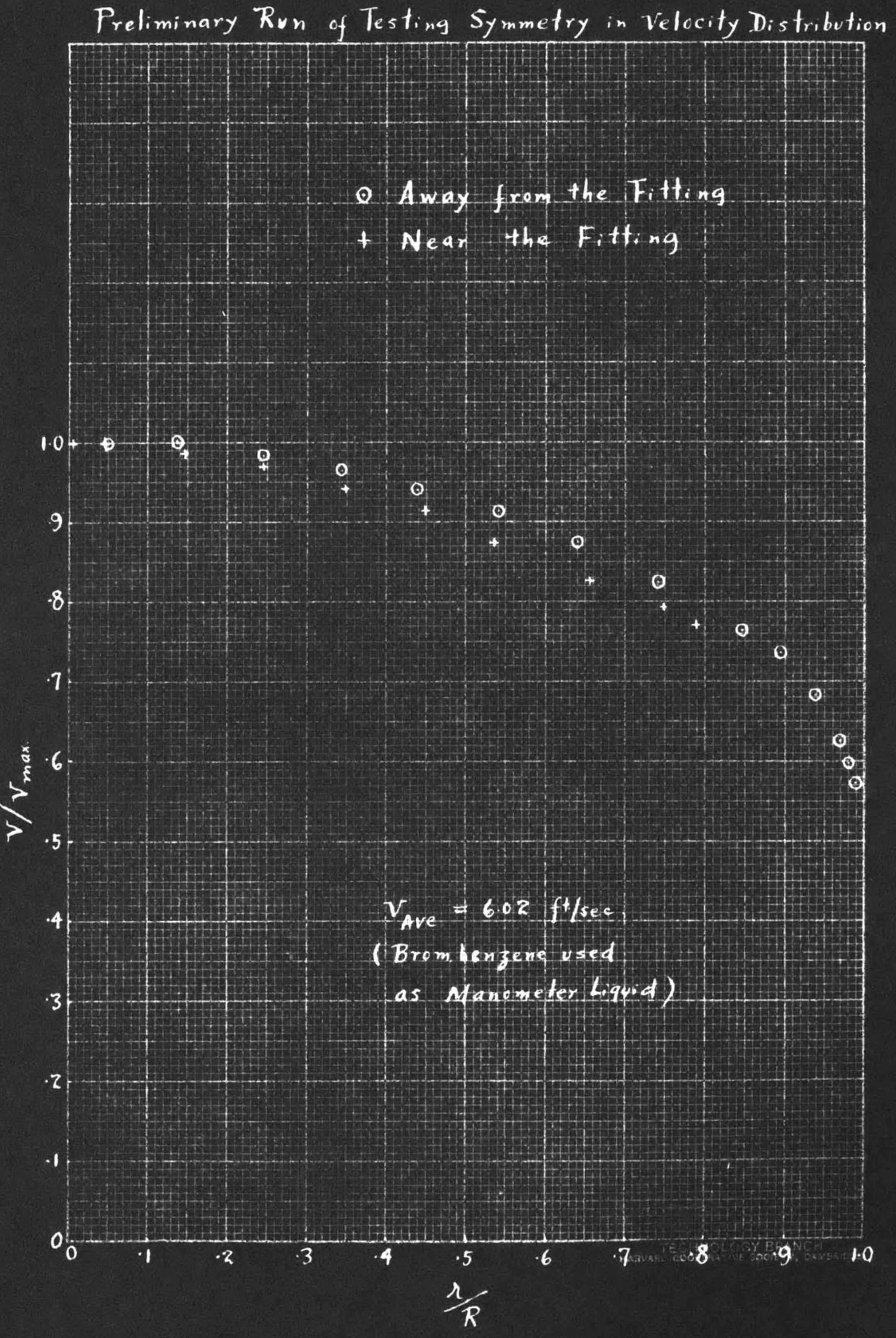

Preliminary Run of Testing Symmetry in Velocity Distribution
⊙ Away from the Fitting
+ Near the Fitting
V_{Ave} = 6.02 ft/sec
(Brombenzene used
as Manometer Liquid)
V/V max.
1.0
·9
·8
·7
·6
·5
·4
·3
·2
·1
0
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1.0
r/R

Fig. 52

half of the pipe away from the fitting indicates a remarkably good exploration. With these facts in mind, it was decided to explore that half of the radius away from the fitting for both velocity and temperature measurements, believing that the symmetry holds for all the time and for all cases in a vertical pipe.

For runs on isothermal velocity distribution, water was recirculated save in a few exceptional cases; while for non-isothermal runs, the water was discharged to the sewer. The discharging of water after its being heated or cooled is regarded as the only practical way of obtaining steady temperature distribution measurement irrespective of the time, since the inlet water from main was at constant temperature and wall temperature at a definite station was always kept constant. For isothermal runs, only velocity explorations have to be made, besides taking main line orifice manometer reading and water temperature reading. For each velocity exploration, the position of the impact tube in the pipe was changed with the aid of the sliding indicator and caliper attached to the pitot tube set. It took about 3 minutes for the CCl_4 manometer to give a steady reading of velocity head, this reading was taken together with its corresponding static pressure which was almost constant throughout a run. The exploration schedule was set for 15 readings, but, in the later runs, it was reduced to 10 readings. It took about 10 minutes to get the steady running condition at the start, and, combining together with the velocity explorations, the whole run took about 1-1/2 hour.

For non-isothermal runs, it took more than 2-1/2 hours to complete a run. At the beginning, steam in case of heating cooling water in case of cooling was turned on, the rate of the water flow inside the test pipes was also adjusted. The apparatus was allowed to run 20 minutes or more to obtain a constant and steady fluid flow and heat flow in the system. This condition was tested by measuring inlet and outlet water temperature and the pipe wall temperature at different stations. When the condition was steady, either velocity exploration or temperature exploration was performed besides the thermocouple readings of pipe wall, inlet and outlet water temperature, the thermometer reading at the steam orifice and main line orifice manometer readings. The steam used was found to be always superheated about 10°C.

An attempt was made to take readings at the four stations, two in the heating and two in the cooling section (which is the long pipe), simultaneously, having water in the system recirculated. It was found that even the cooling water was run at maximum rate while the steam at minimum, the temperature of water in the system was constantly increasing instead of keeping constant.

VII. ISOTHERMAL VELOCITY DISTRIBUTION RUNS

Fifty-six isothermal runs of water were made during the investigation, but only 32 runs which are considered to be more representative and reliable are presented here. Their data, with calculations and plots, are mostly included in Appendix D, while about half of the number of plots not included will be available in the files of the Department of Chemical Engineering. These runs cover a range of Reynolds number from 15,000 to 234,000. The average velocity varied from 1.43 to 7.66 feet per second. The temperature of water ranged ~~varied~~ from 7 to 57°C. The lowest temperature runs were performed on cold winter nights, while the highest temperature runs were made possible by means of preheating the water first, then circulating through the system, having a constant amount of steam passed through the long jacket to keep the water at almost constant temperature.

It might be mentioned here that the writer, during this investigation, observed through these experiments that Reynolds number has a marked effect on the velocity distribution, before the recent articles of Nikuradse(1) were available.

The summarized results of these isothermal runs are given in Table 16 . The velocity distribution exponent "a" in the following equation,

Table 16. Summarized Results of Isothermal Velocity Distribution Data

Run No.	$V_{ave.}$ (ft./sec.) (Graph. Integration)	Re= $\frac{DV_{ave}\rho}{\mu}$	a (from Plot)	V_{ave}/V_{max}	Station No.	Calming Length (Distance from upstream elbow)
{V I-10*	5.52	93,100	0.111	0.853	2	129.2D
{V I-11	5.54	93,500	.111	.855	3	160 D
V I-15	1.77	26,500	.151	.800	3	160 D
{V I-17	6.83	120,500	.109	.841	2	129.2D
{V I-18	6.70	118,200	.115	.825	3	160 D
V I-20	3.50	61,750	.119	.846	2	129.2D
{V I-24	5.30	103,400	.110	.843	2	129.2D
{V I-26	5.30	103,400	.118	.838	4	108.1D
V I-27	4.36	80,900	.118	.838	2	129.2D
{V I-28	7.56	125,000	.113	.850	3	160 D
{V I-29	7.45	123,000	.121	.840	4	108.1D
V I-31	6.57	118,700	.125	.830	4	108.1D
{V I-32	6.71	124,000	.109	.842	2	129.2D
{V I-34	6.64	123,200	.125	.830	4	108.1D
V I-36	2.88	49,900	.133	.818	3	160 D
{V I-37	5.85	106,200	.122	.838	3	160 D
{V I-38	5.81	105,600	.127	.830	4	108.1D
{V I-39	3.29	59,500	.138	.820	3	160 D
{V I-40	3.26	59,000	.145	.808	4	108.1D
V I-44	4.01	69,000	.137	.821	4	108.1D
V I-45	2.79	49,400	.133	.812	4	108.1D
V I-46	4.42	74,700	.139	.807	4	108.1D
{V I-47	1.43	15,030	.160	.803	2	129.2D
{V I-48	1.43	15,070	.149	.804	3	160 D
{V I-49	7.66	135,000	.111	.858	3	160 D
{V 1-50	7.55	133,000	.119	.845	4	108.1D
V 1-51	7.66	234,000	.101	.875	3	160 D
V 1-52	3.82	41,250	.141	.830	3	160 D
V I-53	1.44	16,040	.148	.815	3	160 D
V I-54	3.38	36,950	.130	.829	3	160 D
V I-55	1.52	17,000	.169	.797	3	160 D
V I-56	2.07	22,600	.160	.813	3	160 D

[*{ means Runs takeng together]

$$\frac{V}{V_{max.}} = (1 - \frac{r}{R})^{a} \qquad .....(1)$$

is determined graphically from a log-log plot for each individual run. For most of the isothermal runs another velocity distribution exponent "m'" in the equation

$$\frac{V}{V_{max.}} = (1 - (\frac{r}{R})^{1.25})^{m'} \qquad .....(2)$$

is also determined graphically. For example, for Run V I-38 (see the accompanied plots)(Figures 52A and 52B)

$a = 0.127 \quad ; m' = 0.122$

Confidence in the accuracy of the data is inspired by the smoothness of the curve plotted as velocity ratio against fraction of radius on an ordinary graph paper. (Fig.55)

In Figure 53 , velocity distribution exponent "a" is plotted against Reynolds number. Obviously, this exponent decreases as the Reynolds number increases, but the present experimental points all lie below the curve which is calculated from Eq. (25), Chapter III. It must be recalled that in the first place, the friction data on smooth pipes are mostly on brass pipes, and, secondly, the roughness of pipe wall has a remarkable effect on the shape of the curve,as illustrated in Chapter III by Eq. (30) and also Figure 40. Hard drawn copper has been used in the present case, so Eq. (25) might not be exact for copper pipe as the present pipe

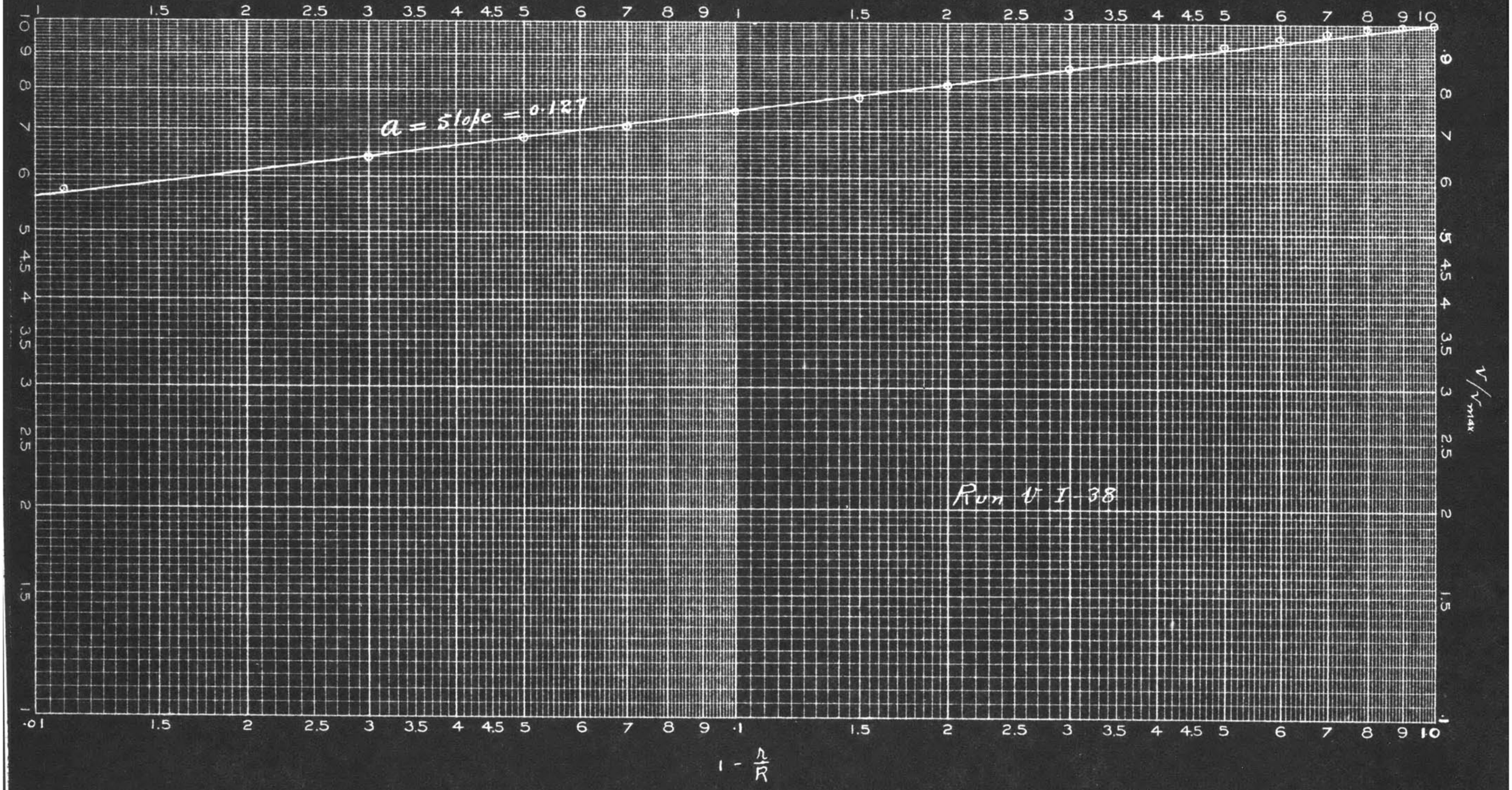

a = slope = 0.127
Run V I-38
v/v_max
1 - r/R
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1

Fig. 52 A

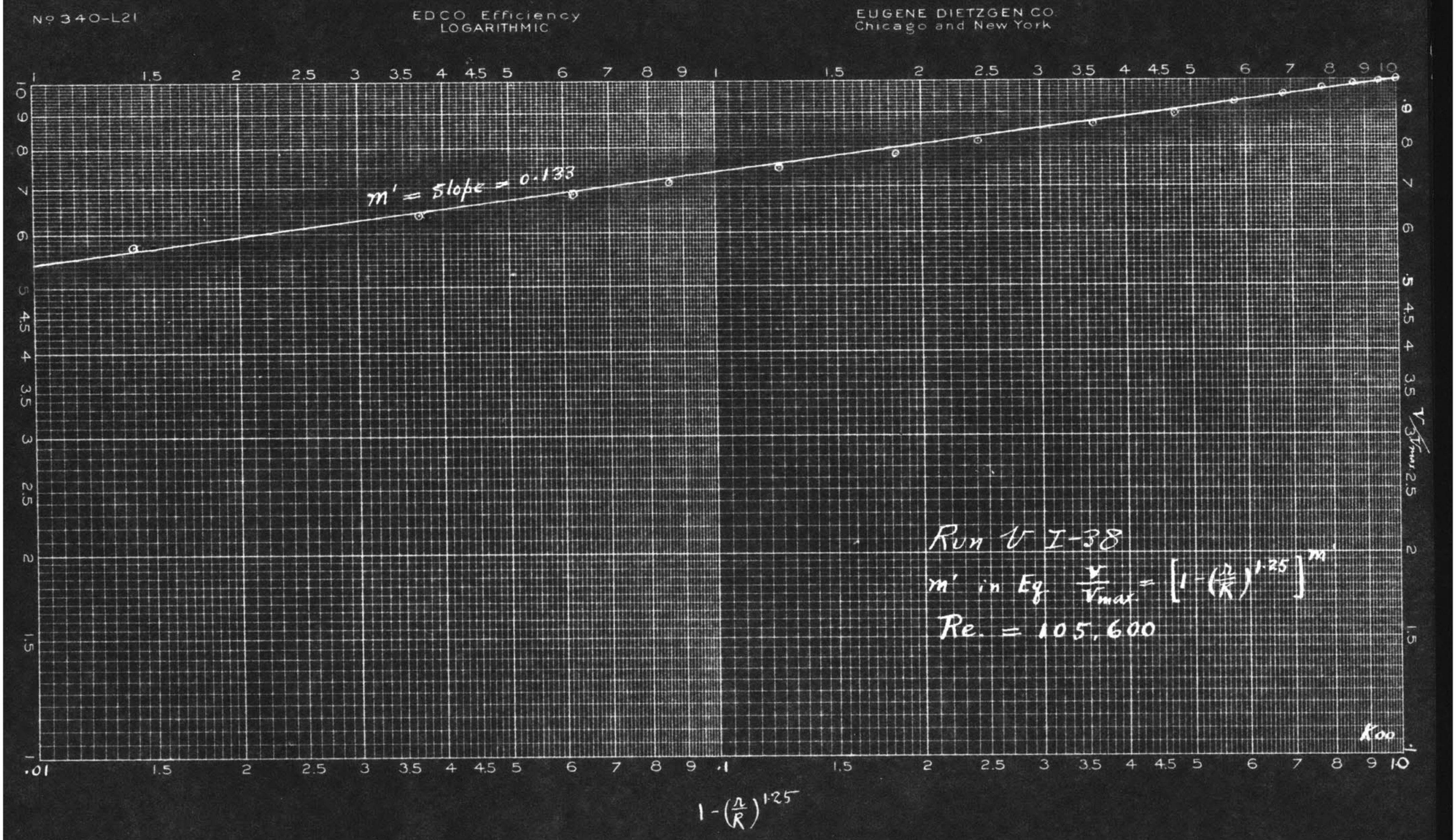
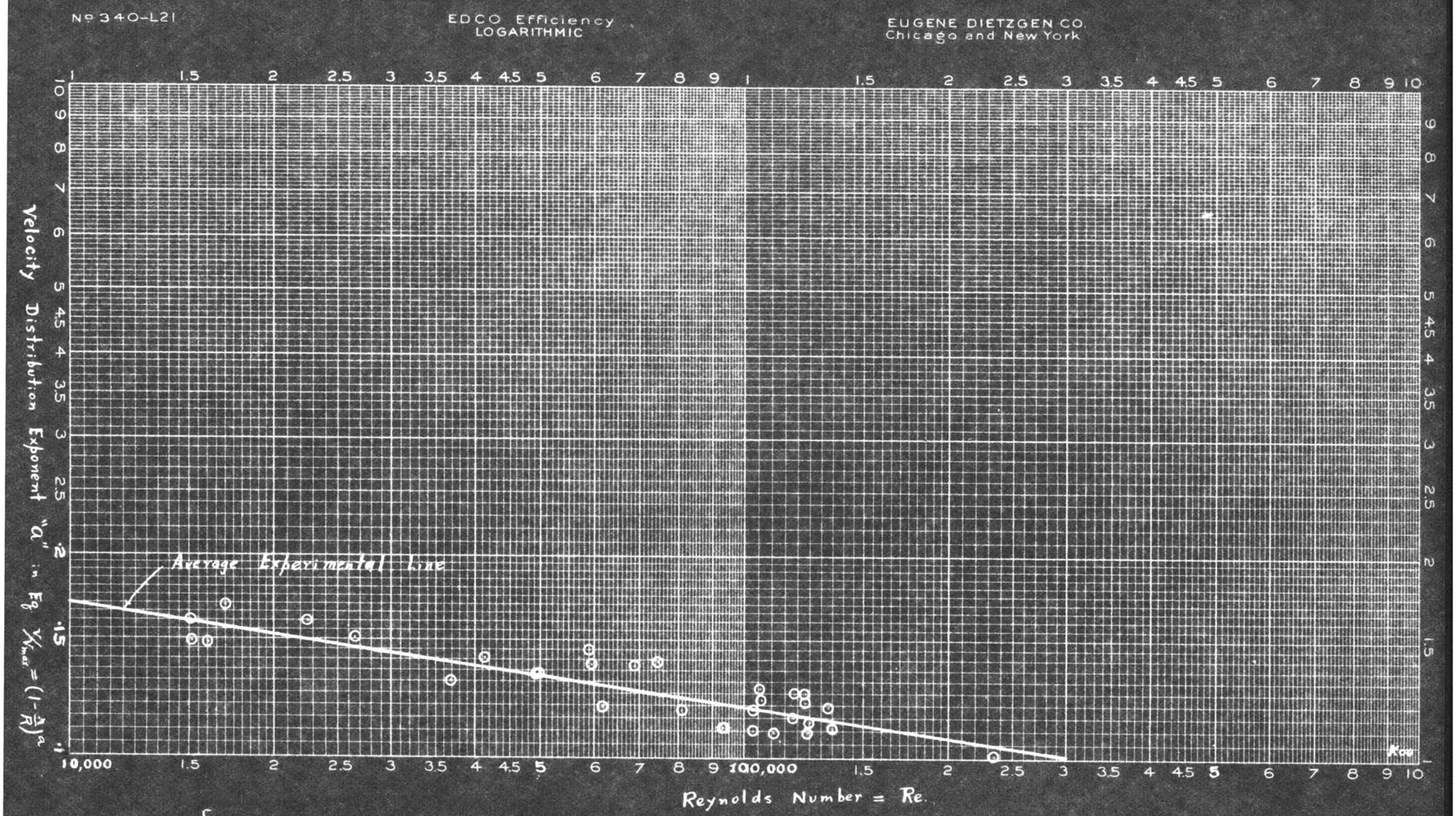
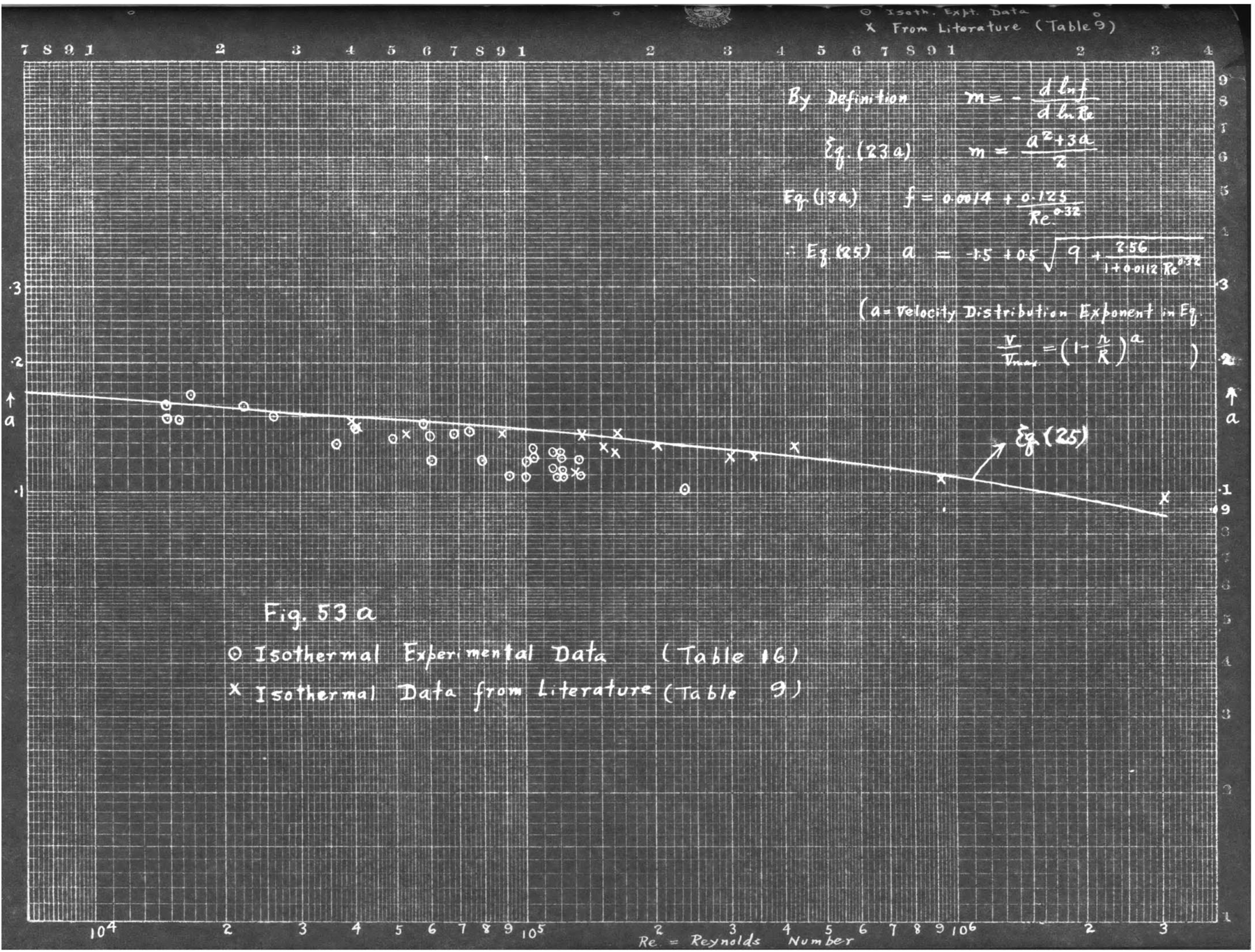

No 340-L21
EDCO Efficiency LOGARITHMIC
EUGENE DIETZGEN CO. Chicago and New York
m' = slope = 0.133
Run V I-38
m' in Eq. $\frac{V}{V_{max}} = \left[1-\left(\frac{r}{R}\right)^{1.25}\right]^{m'}$
Re. = 105,600
$\frac{V}{V_{max}}$
$1-\left(\frac{r}{R}\right)^{1.25}$
.01
.1
10
Koo

Fig. 52 B



Isothermal Velocity Distribution

Figure 53



By Definition $$m \equiv -\frac{d \ln f}{d \ln Re}$$

Eq. (23 a) $$m = \frac{a^2+3a}{2}$$

Eq. (13a) $$f = 0.0014 + \frac{0.125}{Re^{0.32}}$$

∴ Eq. (25) $$a = -1.5 + 0.5\sqrt{9 + \frac{2.56}{1+0.0112\,Re^{0.32}}}$$

(a = Velocity Distribution Exponent in Eq. $\frac{V}{V_{max}} = \left(1-\frac{r}{R}\right)^a$)

Fig. 53 a

⊙ Isothermal Experimental Data (Table 16)

× Isothermal Data from Literature (Table 9) is rougher hydraulicly than the average smooth pipes. Therefore, instead of following Eq. (25) in Chapter III, an average experimental line is drawn, as shown in Figure 53 . This line gives approximately the following values of "a" for different Reynolds numbers:

a	Re.
0.17	10,000
0.132	50,000
0.119	100,000
0.100	300,000

The decreasing of values of "a" as Reynolds number is increasing proves definitely the occurrence of more turbulence and the swelling up of the velocity distribution curve. This phenomenon might be caused by the decrease of the thickness of the viscous film near the wall, thus causing more flow near the boundary.

Values of distribution exponent m' in Eq. (2) are tabulated in Table 17 and plotted against Reynolds numbers in Figure 54 . The object of this plotting is to illustrate that Eq. (2) actually fits the data better than Eq. (1) for velocity distribution, although it is more complicated in application. It is noticed that the experimental points gather closer together, while the slope is also evident. This plot also illustrates that practically the velocity distribution at stations 2, 3, 4, of different calming lengths

Table 17 Isothermal Velocity Distribution Exponent in Eq.

$$\frac{V}{V_{max.}} = \left[1 - \left(\frac{r}{R}\right)^{1.25}\right]^{m'}$$

Run No.	Station No.	Re.	m' (From Plot)
(V I-10	2*	93,100	0.125
(V I-11	3	93,500	.119
V I-15	3	26,500	.166
(V I-17	2	120,500	.124
(V I-18	3	118,200	.131
V I-20	2	61,750	.136
(V I-24	2	103,400	.125
(V I-26	4	103,400	.132
V I-27	2	80,900	.133
(V I-28	3	125,000	.120
(V I-29	4	123,000	.131
V I-31	4	118,700	.136
(V I-32	2	124,000	.116
(V I-34	4	123,200	.135
V I-36	3	49,900	.145
(V I-37	3	106,200	.129
(V I-38	4	105,600	.133
(V I-39	3	59,500	.147
(V I-40	4	59,000	.153
V I-44	4	69,000	.151
V I-45	4	49,400	.144
V I-46	4	74,700	.154

* Station No.	Calming Length in I.D. of Copper Pipe
2	129.2
3	160.0
4	108.1

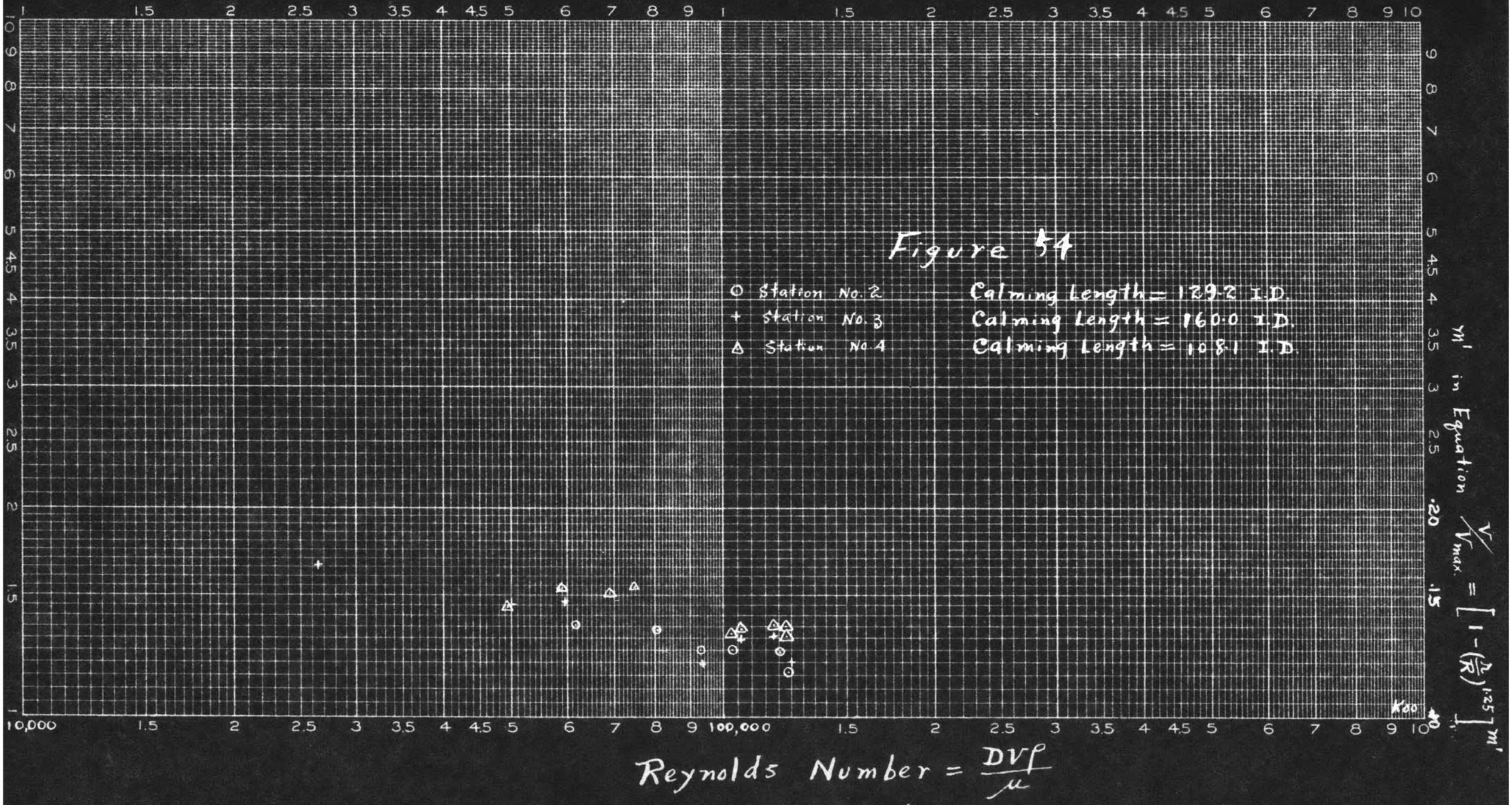


Figure 54

○ Station No. 2 Calming Length = 129.2 I.D.

+ Station No. 3 Calming Length = 160.0 I.D.

△ Station No. 4 Calming Length = 108.1 I.D.

may be considered constant at a certain Reynolds number. It is noticed that this long calming length has eliminated entrance disturbance effect on the velocity distribution, thus, during non-isothermal runs on Stations 2 and 3, corresponding isothermal runs may be taken on station 4 to compare the effect of heating on velocity distribution.

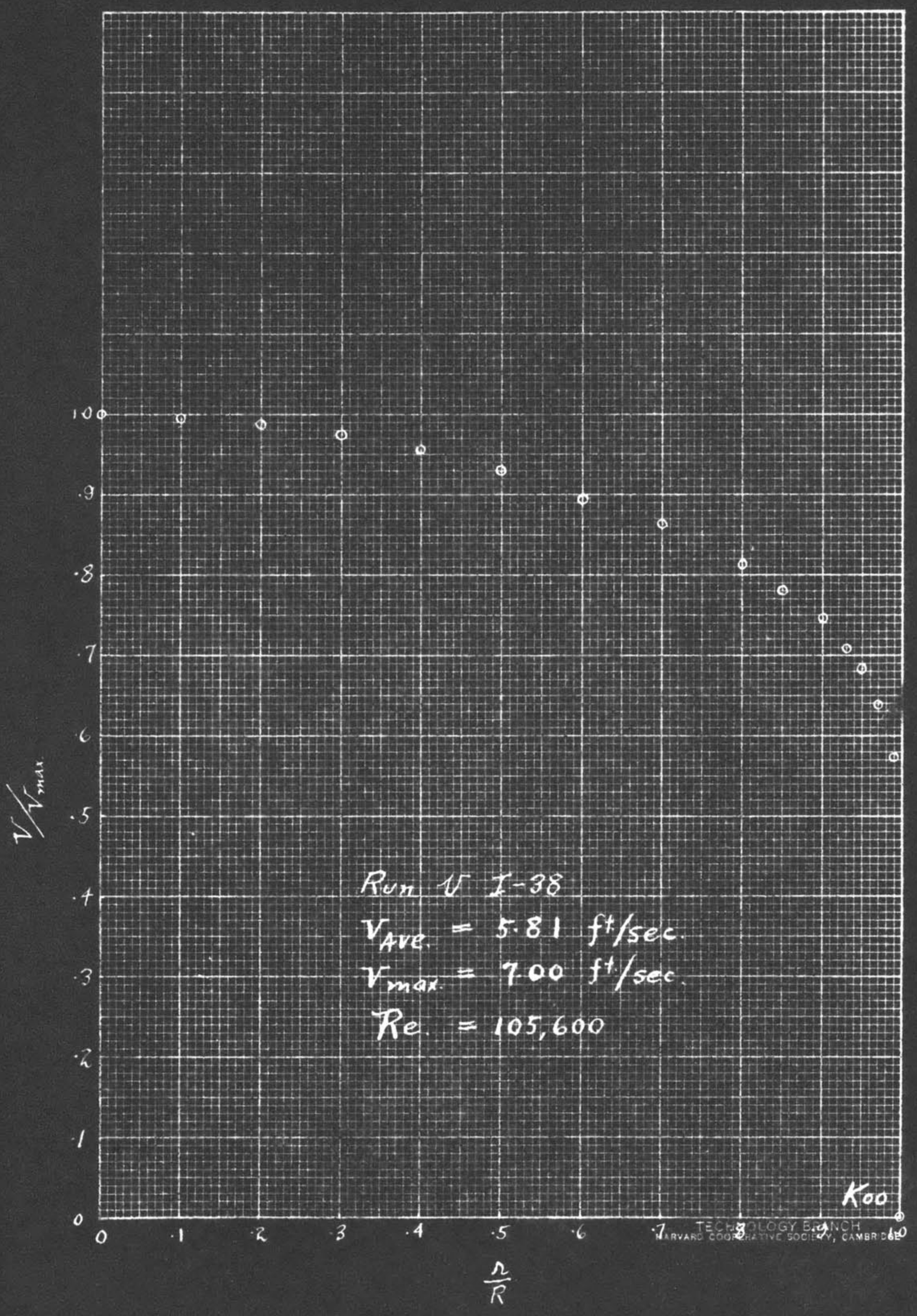
The velocity ratio, i.e., the ratio of average velocity to the maximum velocity which is at the axis of the pipe, for each run is calculated from the actually measured velocity at the axis and the graphically integrated values of $V_{ave.}$ which can be expressed as,

$$V_{ave.} = \frac{2\pi \int_0^R V\ r\ dr}{\pi R^2} = \frac{2}{R^2} \int_0^R V\ r\ dr \quad \ldots(3)$$

or

$$V_{ave.} = 2\int_0^R V(\tfrac{r}{R})\ d(\tfrac{r}{R}) \quad \ldots\ldots(3a)$$

The value in the integral is obtained through graphical integration - as illustrated in Figure 56 for Run V I-38. Thus, the velocity ratio for every run is similarly obtained. They are given in Table 16 and plotted in Figure 57 . The apparent rise of this ratio with increase of Reynolds number is similar to the calculated line from Eq. (21c) in Chapter III, except these values are higher than the calculated values which is what is expected since the corresponding velocity distribution



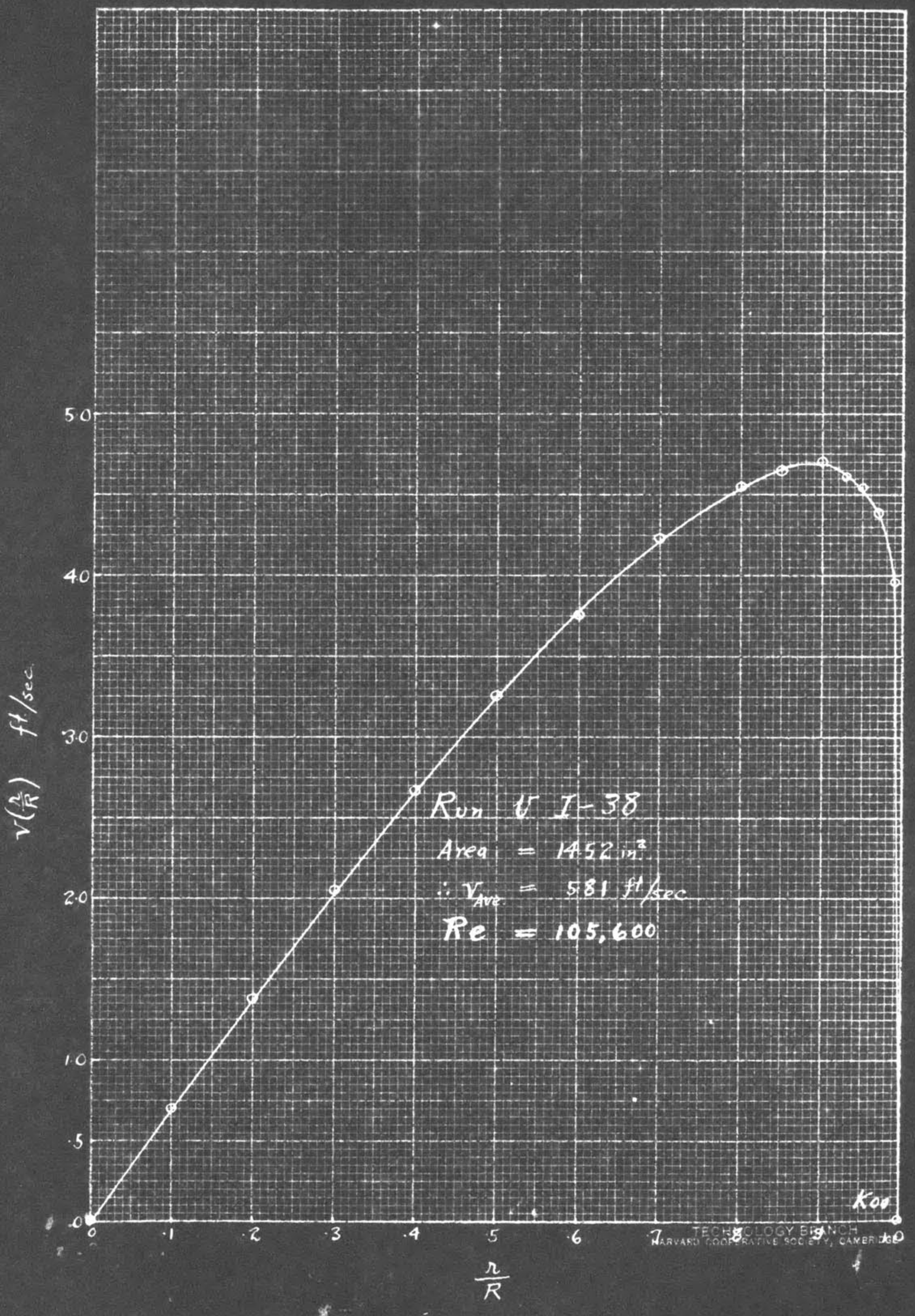
Run V I-38

$V_{Ave.} = 5.81$ ft/sec.

$V_{max.} = 7.00$ ft/sec.

$Re. = 105,600$

Fig. 55



Run V I-38
Area = 1452 in²
∴ V_Ave. = 581 ft/sec
Re = 105,600
v(r/R) ft/sec
5.0
4.0
3.0
2.0
1.0
.5
.0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
r/R
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Fig. 56

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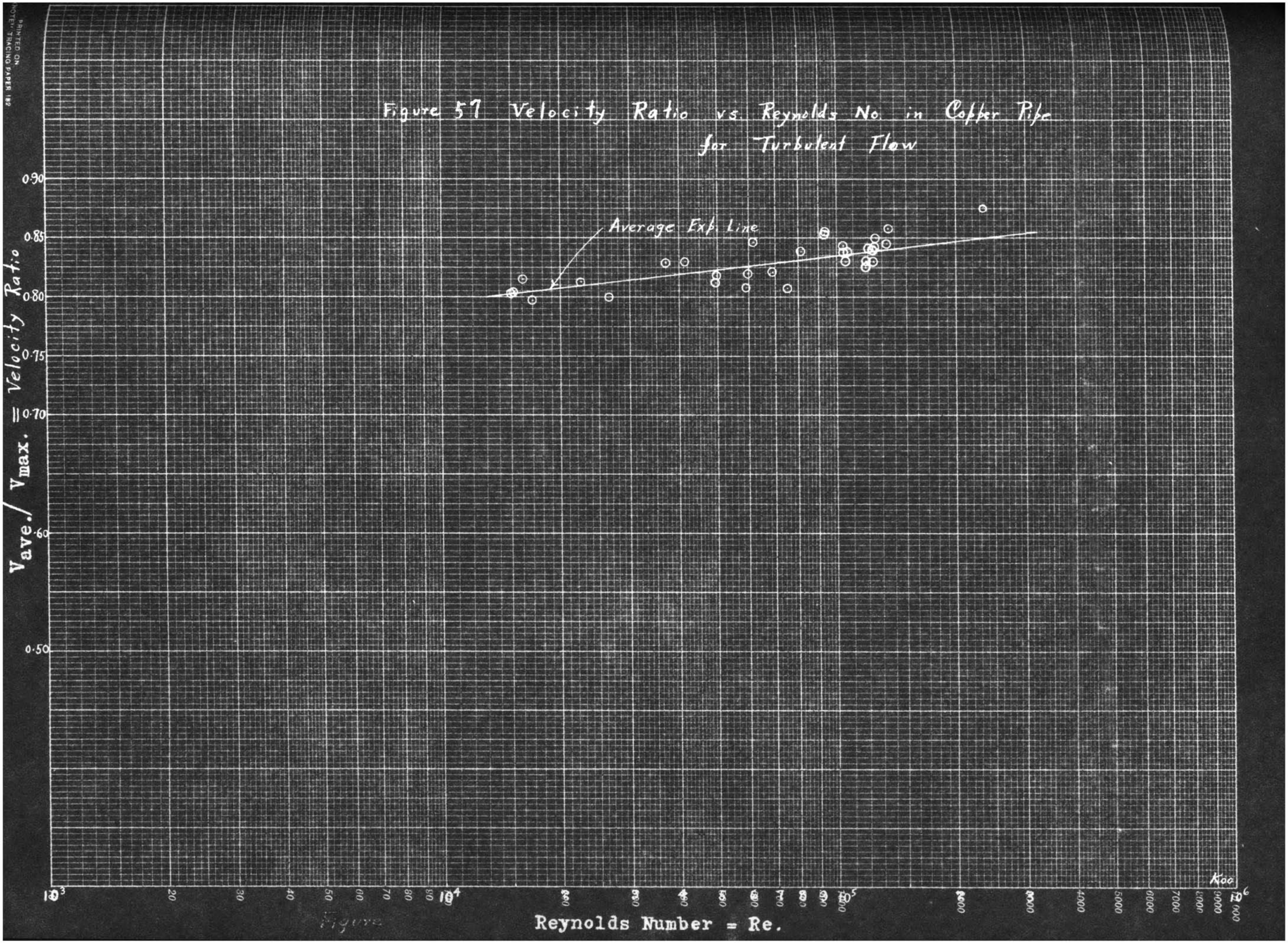

Figure 57 Velocity Ratio vs. Reynolds No. in Copper Pipe for Turbulent Flow

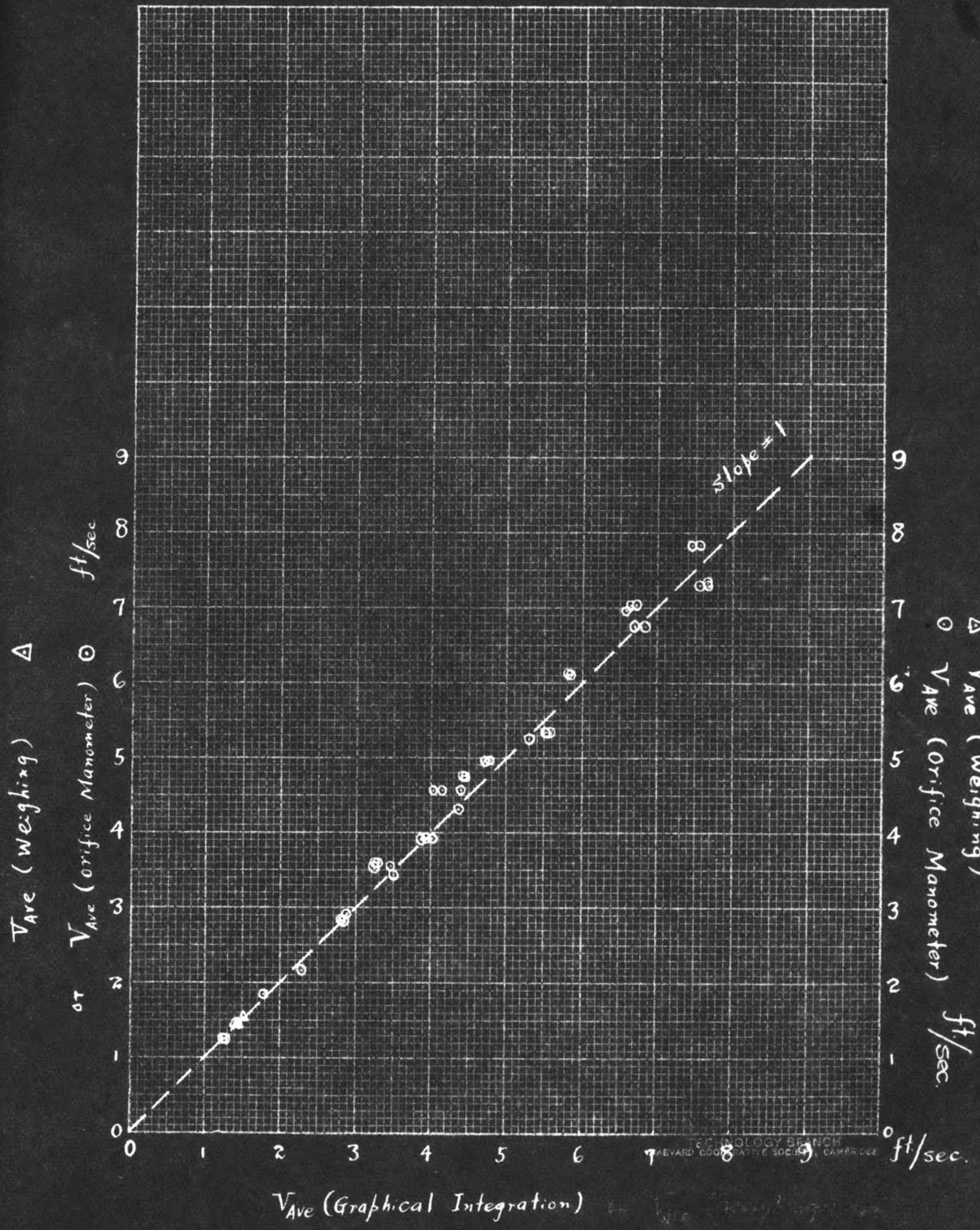

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Figure 57a Comparison of Average Velocities obtained from Different Methods exponents "a" are lower than those calculated.

It must be pointed out here that the average velocity obtained from graphical integration is satisfactory as compared with orifice meter readings and actual weighing. Its deviation from orifice meter reading is about ± 30%, while its deviation is still much less as compared with actual weighing (See Fig. 57a). For $\frac{V_{ave}}{V_{max}}$ calculations, values of graphically integrated V_{ave} are recommended to use, since any error that may introduce in the pitot tube coefficient will apparently be eliminated this way.

References

(1) J. Nikuradse: Article in A.Giles, L. Hopf and Th.von Karman: Aerodynamik und verwandter Gebiete, Julius Springer, Berlin (1930).

(2) J. Nikuradse: Article in Proceedings of the 3rd International Congress for Applied Mechanics, Vol. 1 (1931).

VIII. VELOCITY AND TEMPERATURE DISTRIBUTION RUNS DURING HEATING.

(Counter-Current and Parallel-Current)

A. General Discussion.

B. On Velocity Distribution During Heating.

C. On the Effective Film Temperature.

D. On Temperature Distribution During Heating.

E. Non-Similarity between Temperature and Velocity Distribution of Liquids.

F. Parallel-Current vs Counter-Current Heating.

G. On Temperature Rise Between Two Cross-Sections.

VIII. Velocity and Temperature Distribution Runs During Heating

(Counter-Current and Parallel Current)

A. General Discussion.

Parallel current runs were carried out at Stations 2 and 3, having water flowing downward; while counter current runs were carried out at Stations 4 and 5, having water flowing upward~~s~~. Twenty-one simultaneous velocity and temperature distribution runs with one extra temperature distribution run were obtained, operating with parallel current~~s~~ and their data with calculations and plots are included in Appendices E and G. Due to the difficulty in obtaining accurate velocity distribution measurements when the water is being heated and flowing upward as explained in Chapter VI, only ten simultaneous velocity and temperature distribution runs with six extra temperature distribution measurements were obtained on counter current runs. These data with calculations and plots are to be found in Appendices D and H. Graphical integration plots of average velocity, average temperature and mixing cup temperature are not included, but they are available in the Heat Transmission file in the Chemical Engineering Department.

The velocity distribition exponent "a" is graphically determined for each run, just like the isothermal runs, and the average velocity over the cross section is found by graphical integration. Based upon equation (7A) derived in Chapter IV, which is

$$\frac{\Delta t}{\Delta t_{max}} = \frac{t_w - t}{t_w - t_a} = \left(1 - \frac{r}{R}\right)^b \qquad \ldots\ldots\ldots(1)$$

the value of "b", which is defined as temperature distribution exponent, is similarly graphically determined for each run. The average "space" temperature over the cross-section can readily be seen to be equal to

$$t_{av} = \frac{2\pi \int_0^R tr\ dr}{\pi R^2} = 2\int_0^R t\left(\frac{r}{R}\right)d\left(\frac{r}{R}\right) \qquad \ldots\ldots(2)$$

However, this average cross-sectional temperature is different from the mean fluid temperature (better known as mixing cup temperature) which is equal to

$$t_m = \frac{2\pi \int_0^R t\rho\ vr\ dr}{2\pi \int_0^R \rho v\ r\ dr} = \frac{2\int_0^R t\rho v\ (r/R)d(r/R)}{(\rho V)_{av}} \qquad \ldots\ldots(3)$$

These operations are illustrated for Run V H-16, Run T H-26 in the parallel-current runs, and for Run V H-31, Run T H-45 in the counter-current runs by graphs herein inserted. (See Figure 58-63). All these graphically determined values are tabulated in Tables 18 and 19.

Table 18 Velocity and Temperature Distribution During Heating (Parallel Current)

(Column) (1) Velocity Run	(2) Temp. Run	(3) Station No.	(4) V_{ave} (ft./sec)	(5) $\frac{V_{ave}}{V_{max}}$	(6) t_{ave} (°C.)	(7) t_m (°C.)	(8) t_w (°C.)	(9) $t_w - t_a$ (°C.)	(10) a	(11) b	(12) a/b	(13) t_f (°C.)
V H-3	T H-14	2	2.28	0.869	25.52	23.77	78.0	56.4	0.100	0.039	2.56	52.3
V H-5	T H-16	2	4.13	0.857	22.92	22.69	70	48.85	0.103	.0223	4.62	45.2
(V H-8	T H-18)	2	1.275	0.768	28.56	27.17	86.7	63.45	0.169	.0495	3.41	67.5
(V H-9	T H-19)	3	1.244	0.778	37.60	36.11	80.3	47.55	0.153	.054	2.83	60.98
(V H-10	T H-20)	2	4.39	0.869	25.85	25.14	74.2	51.40	0.101	.030	3.37	56.35
(V H-11	T H-21)	3	4.02	0.778	28.80	-	65.0	38.0	0.157	.0278	5.65	49.2
(V H-12	T H-22)	2	3.45	0.847	25.92	25.00	76.41	54.56	0.109	.0417	2.62	58.8
(V H-13	T H-23)	3	3.22	0.791	30.80	30.33	69.0	41.0	0.155	.0397	3.90	52.05
(V H-14	T H-24)	2	3.928	0.825	18.72		73.21	57.81	0.124	.0384	3.23	54.63
(V H-15	T H-25)	3	3.870	0.809	24.28		66.36	45.76	0.140	.0415	3.37	48.13
(V H-16	T H-26)	2	2.806	0.838	19.36		77.11	61.38	0.123	.0364	3.38	57.0
(V H-17	T H-27)	3	2.806	0.826	25.52		71.51	49.36	0.131	.0422	3.10	53.52
(V H-18	T H-28)	2	4.434	0.832	17.60		76.01	61.51	0.123	.0288	4.27	55.75
(V H-19	T H-29)	3	4.432	0.828	21.60		66.91	48.48	0.127	.0324	3.92	44.81
(V H-20	T H-30)	2	1.557	0.814	10.48		78.41	73.01	0.141	.0434	3.25	55.80
(V H-21	T H-31)	3	1.560	0.767	18.40		76.01	61.51	0.177	.0348	5.08	49.7
(V H-22	T H-32)	2	2.120	0.859	9.20		75.03	70.53	0.103	.0414	2.49	53.24
(V H-23	T H-33)	3	2.05	0.830	17.12		73.91	60.75	0.123	.0348	3.53	47.84
	T H-34A)	2			8.40		79.01	75.01	-	.0342	-	55.32
V H-24	T H-34)	3	2.350	0.877	15.28		74.71	63.01	0.0905	.0279	3.24	47.06
(V H-25	T H-35)	2	4.770	0.892	7.40		73.21	69.61	0.0806	.0318	2.54	50.4
(V H-26	T H-36)	3	4.705	0.856	12.30		65.71	56.74	0.1054	.0275	3.83	40.6

Table (Cont.)

(Column) (1) Velocity Run	(2) Temp. Run	(14) $Re_{t_{ave.}}$	(15) Re_{t_f}	(16) Pr_{t_f}	(17) Pe_{t_f}	(18) $\frac{t_w - t_{ave}}{(°C.)}$	(19) $\frac{t_w - t_{ave}}{t_w - t_a}$	(20) $\frac{t_{ave} - t_a}{t_w - t_a}$	(21) $Pe_{t_f} \left(\frac{t_w - t_a}{t_{ave} - t_a}\right)$
V H-3	T H-14	38,900	64,200	3.47	223,000	52.48	0.931	0.0695	3,210,000
V H-5	T H-16	66,800	103,500	3.97	411,000	47.08	0.963	.0363	11,320,000
(V H-8	T H-18)	23,200	44,700	2.67	119,400	58.14	0.918	.0838	1,426,000
(V H-9	T H-19)	26,900	39,500	2.56	101,200	42.7	0.899	.102	993,000
(V H-10	T H-20)	74,700	131,800	3.21	423,000	48.35	0.941	.0594	7,120,000
(V H-11	T H-21)	73,800	108,000	3.68	398,000	36.2	0.953	.0474	8,400,000
(V H-12	T H-22)	59,600	107,800	3.10	334,000	50.48	0.925	.0747	4,470,000
(V H-13	T H-23)	61,600	90,500	3.48	315,000	38.2	0.932	.0684	4,605,000
(V H-14	T H-24)	57,200	114,800	3.32	348,000	54.49	0.943	.0575	6,050,000
(V H-15	T H-25)	64,100	102,000	3.74	381,000	42.08	0.920	.0804	4,740,000
(V H-16	T H-26)	41,500	85,200	3.19	272,000	57.75	0.942	.0592	4,600,000
(V H-17	T H-27)	47,850	80,600	3.38	272,000	45.99	0.931	.0683	3,980,000
(V H-18	T H-28)	62,700	132,000	3.25	429,000	58.41	0.950	.0504	8,520,000
(V H-19	T H-29)	69,000	110,400	4.00	442,000	45.31	0.935	.0655	6,750,000
(V H-20	T H-30)	18,200	46,400	3.24	150,200	67.93	0.931	.0696	2,160,000
(V H-21	T H-31)	22,500	42,100	3.63	153,000	57.61	0.938	.0634	2,415,000
(V H-22	T H-32)	23,900	60,600	3.40	206,000	65.83	0.934	.0666	3,090,000
(V H-23	T H-33)	28,600	53,800	3.76	202,000	56.79	0.935	.0652	3,100,000
-	T H-34A)	-	69,500	3.28	228,000	70.61	0.942	.0587	3,885,000
V H-24	T H-34)	31,300	60,850	3.82	232,000	59.43	0.943	.0568	4,085,000
(V H-25	T H-35)	51,000	130,400	3.58	467,000	65,81	0.945	.0546	8,560,000
(V H-26	T H-36)	57,900	108,800	4.36	475,000	53.41	0.941	.0587	8,100,000

Table 19. Velocity and Temperature Distribution During Heating
(Counter Current)

(Column) (1) Velocity Run	(2) Temp. Run	(3) Station No.	(4) $V_{ave.}$ (ft/sec)	(5) $\frac{V_{ave}}{V_{max}}$	(6) t_{ave} (°C.)	(7) t_m (°C.)	(8) t_w (°C.)	(9) $(t_w - t_a)$ (°C.)	(10) a	(11) b	(12) a/b	(13) t_f (°C.)
	(T H-37	4	1.302		22.80	-	87.24	69.42	-	0.0637	-	67.4
	(T H-38	5	(manometer)		38.56	-	86.64	55.44	-	.0895	-	62.9
V H-27	(T H-39	4	3.76	0.822	8.96	8.93	32.30	24.10	0.131	.0239	5.48	23.6
	(T H-40	5			18.60	-	73.20	57.15		.0308	-	42.83
(V H-28	(T H-41	4	2.98	0.829	12.12	11.33	65.93	56.93	0.119	.0330	3.61	46.80
(V H-29	(T H-42	5	3.03	0.851	24.48	23.9	79.63	58.37	0.111	.0330	3.36	48.06
V H-30	(T H-43	4	1.442	0.824	21.92	20.6	83.27	66.82	0.129	.0563	2.29	62.8
	(T H-44	5	-	-	36.56	-	85.57	53.87	-	.0553	-	59.53
(V H-31	(T H-45	4	3.33	0.837	10.88	10.13	62.95	54.75	0.121	.0283	4.27	45.0
(V H-32	(T H-46	5	3.30	0.830	22.08	21.68	78.95	59.75	0.123	.0322	3.82	49.67
	(T H-47	4	1.102		20.32	-	94.33	79.03	-	.0537	-	70.46
	(T H-48	5	(Weighing)		35.44	-	93.18	59.33	-	.0378	-	63.2
(V H-33	(T H-49	4	1.526	0.807	19.84	19.63	79.41	64.11	0.151	.0517	2.92	59.9
(V H-34	(T H-50	5	1.510	0.791	34.48	33.76	86.37	55.57	0.161	.0438	3.68	57.93
(V H-35	(T H-51	4	2.02	0.844	17.64	16.28	74.66	61.51	0.117	.0418	2.80	55.0
(V H-36	(T H-52	5	2.02	0.835	29.92	29.55	83.56	56.36	0.125	.0334	3.75	53.8

(Column) (1) Velocity Run	(2) Temp. Run	(14) $Re_{t_{ave.}}$	(15) Re_{t_f}	(16) Pr_{t_f}	(17) Pe_{t_f}	(18) $\frac{t_w-t_{ave}}{(°C.)}$	(19) $\frac{t_w-t_{ave}}{t_w-t_a}$	(20) $\frac{t_{ave}-t_a}{t_w-t_a}$	(21) $Pe_{t_f}\left(\frac{t_w-t_a}{t_{ave}-t_a}\right)$
	(T H-37	20,800	45,950	2.67	122,700	64.44	0.929	0.0718	1,710,000
	(T H-38	28,950	43,000	2.87	123,500	48.08	.868	0.133	929,000
V H-27	(T H-39	42,450	62,000	6.54	405,000	23.34	.969	.0319	12,700,000
	(T H-40	54,900	91,200	4.18	381,000	54.60	.955	.0446	8,540,000
(V H-28	(T H-41	36,800	77,300	3.84	296,500	53.81	.945	.0548	5,410,000
(V H-29	(T H-42	50,000	78,900	3.75	296,000	55.15	.945	.0551	5,380,000
(V H-30	(T H-43	22,600	47,500	2.87	136,300	61.35	.918	.0820	1,662,000
	(T H-44	30,850	45,350	3.03	137,200	49.01	.910	.0902	1,522,000
(V H-31	(T H-45	39,400	83,200	4.00	333,000	52.07	.951	.049	6,800,000
(V H-32	(T H-46	52,500	90,000	3.63	326,500	56.87	.952	.0482	6,770,000
	(T H-47	16,650	40,000	2.52	100,800	74.01	.936	.0635	1,588,000
	(T H-48	23,400	36,600	2.85	104,400	57.74	.974	.0268	3,900,000
(V H-33	(T H-49	22,700	48,000	3.00	144,000	59.57	.930	.0708	2,035,000
(V H-34	(T H-50	31,250	46,700	3.12	145,600	51.89	.933	.0662	2,200,000
(V H-35	(T H-51	28,700	59,350	3.30	195,800	57.02	.927	.0730	2,685,000
(V H-36	(T H-52	38,100	58,300	3.35	195,200	53.64	.953	.0483	4,040,000

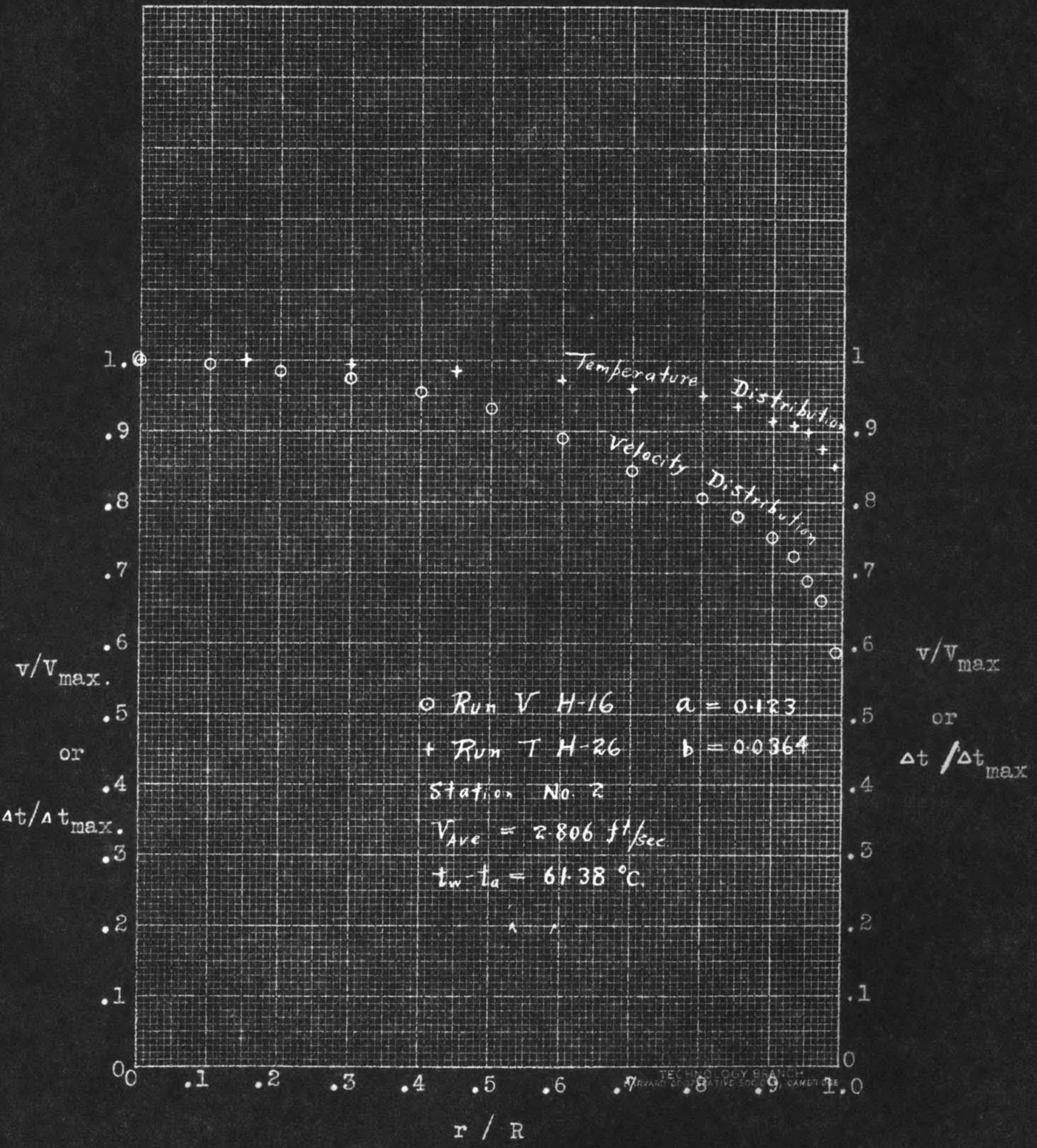
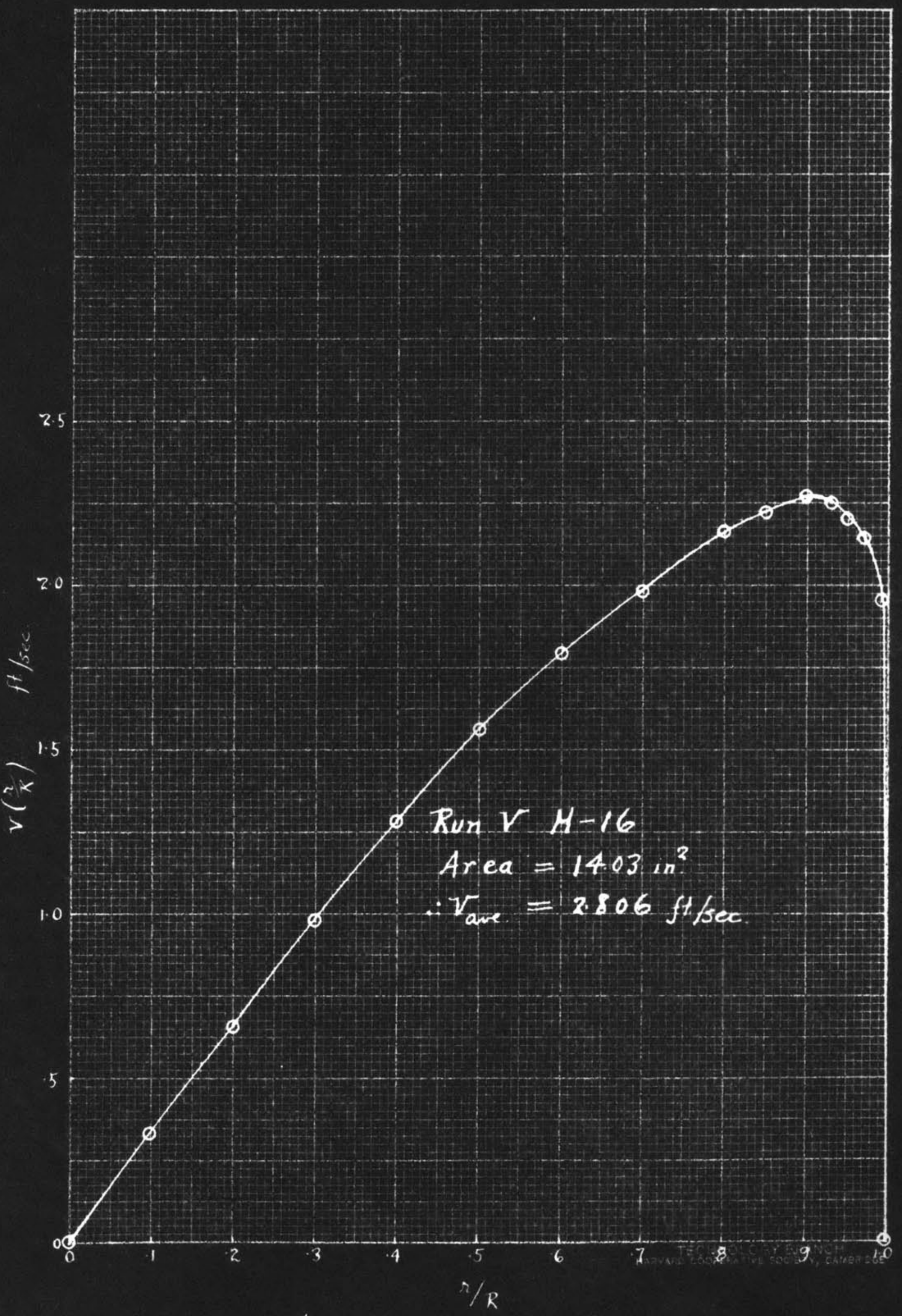
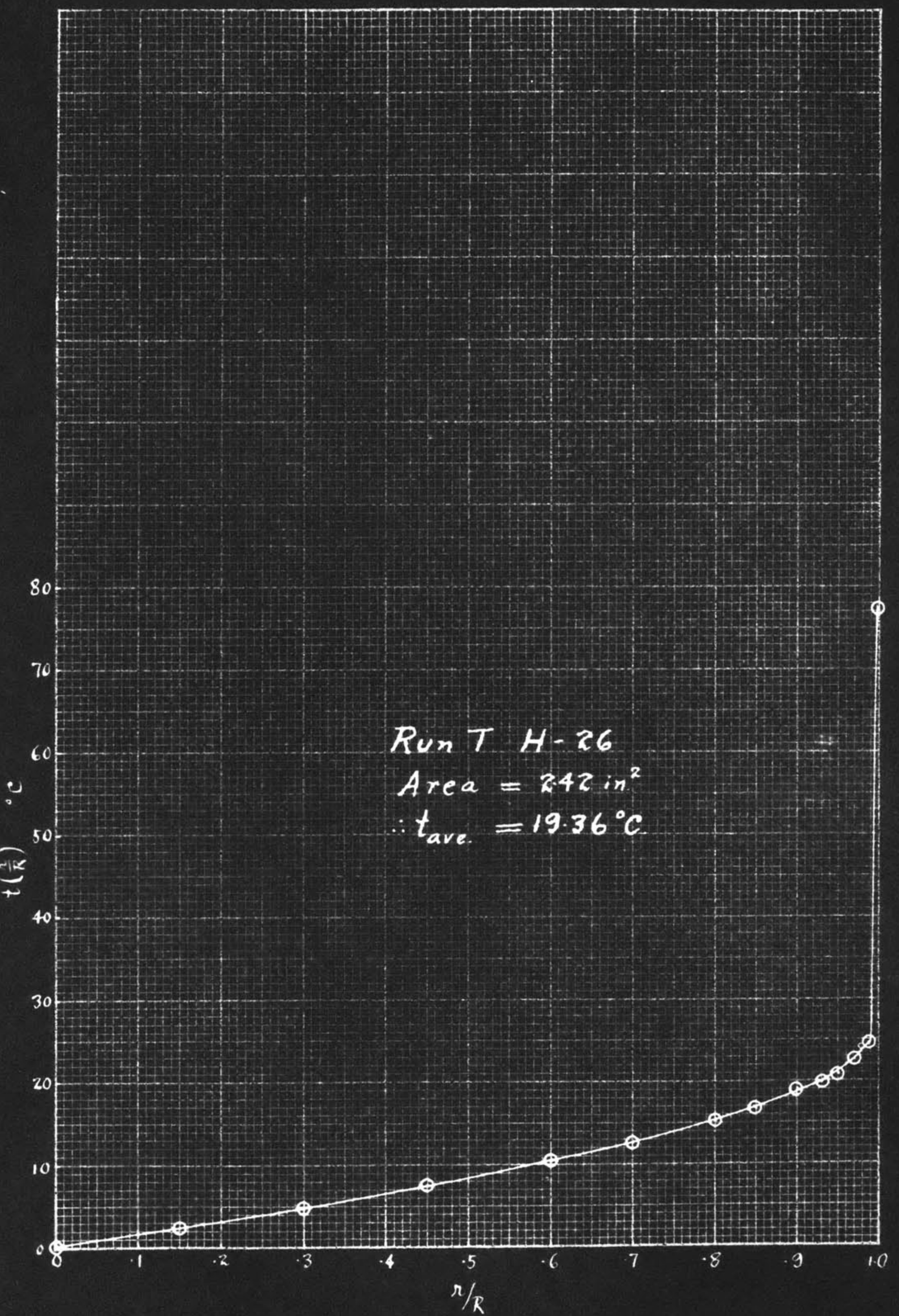


Fig. 58 Comparison of Simultaneous Velocity and Temperature Distribution During Heating



Graphical Integration
of V_{ave}

Fig. 59



Graphical Integration of $t_{ave.}$

Fig. 60

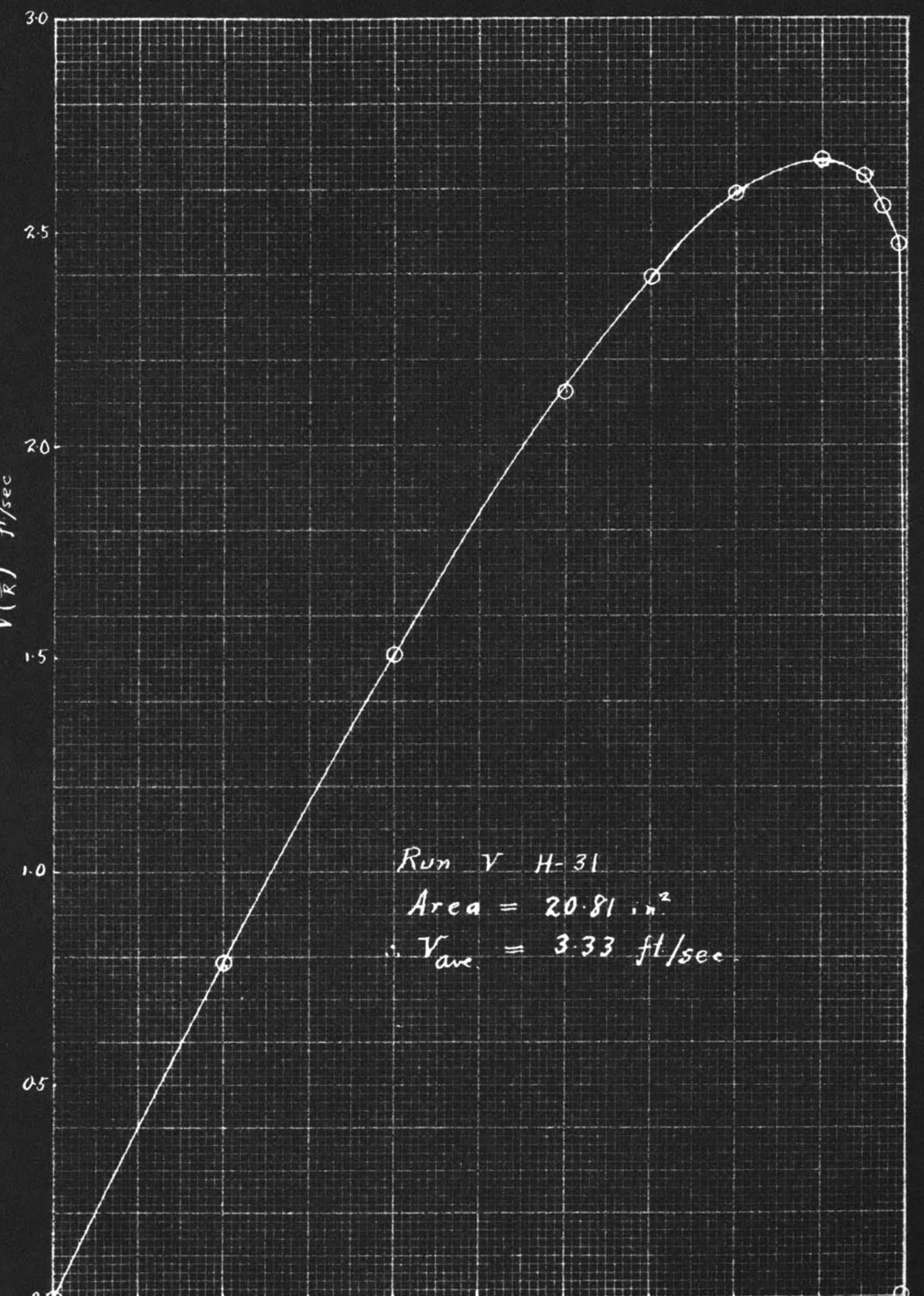
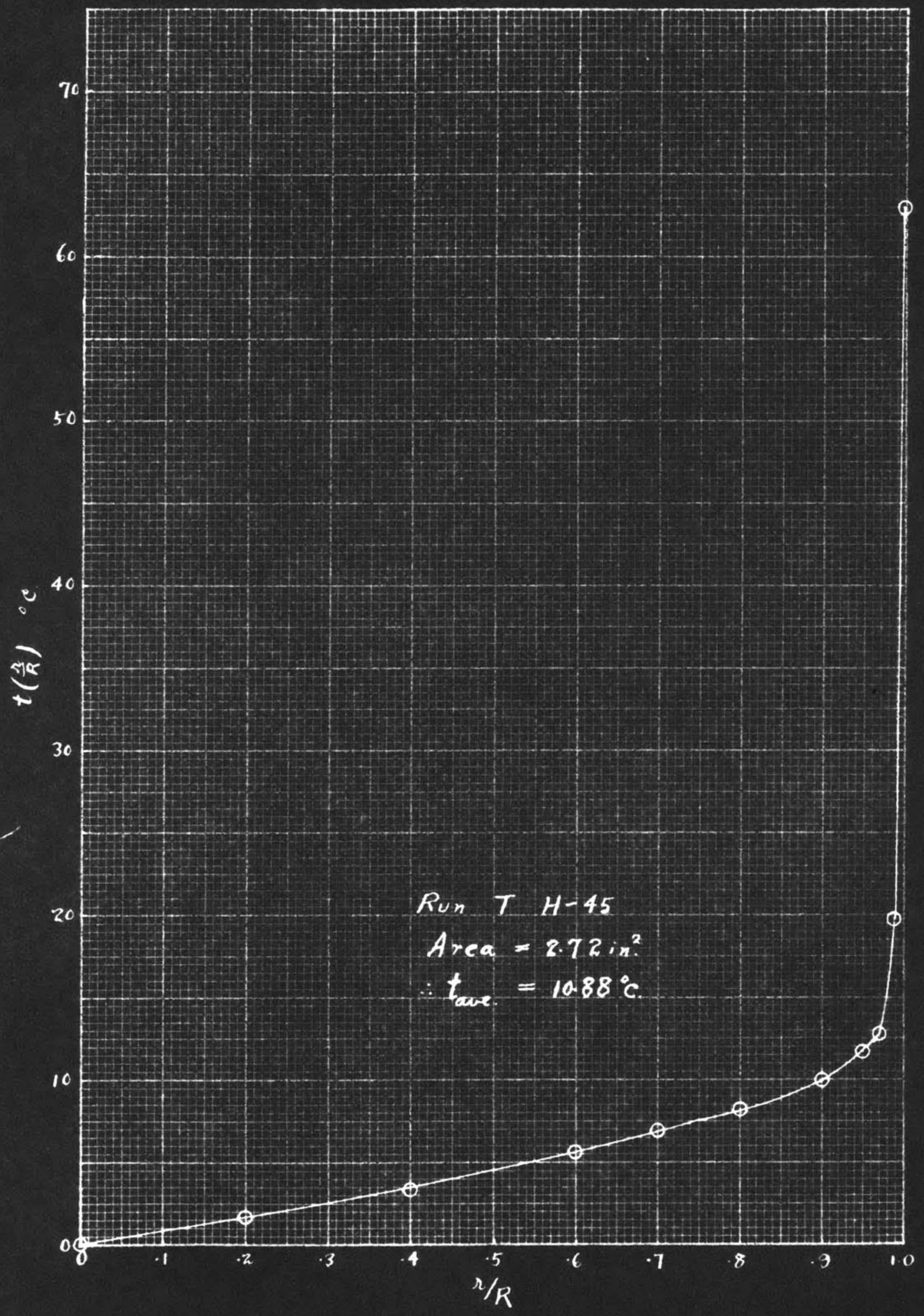

3.0
2.5
2.0
1.5
1.0
0.5
0
$v(\frac{r}{R})$ ft/sec
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
r/R
Run V H-31
Area = 20.81 in²
∴ V_{ave} = 3.33 ft/sec.

Fig. 61



t(r/R) °C
70
60
50
40
30
20
10
0
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1·0
r/R
Run T H-45
Area = 2.72 in²
∴ t_ave. = 10.88 °C.

Fig. 62

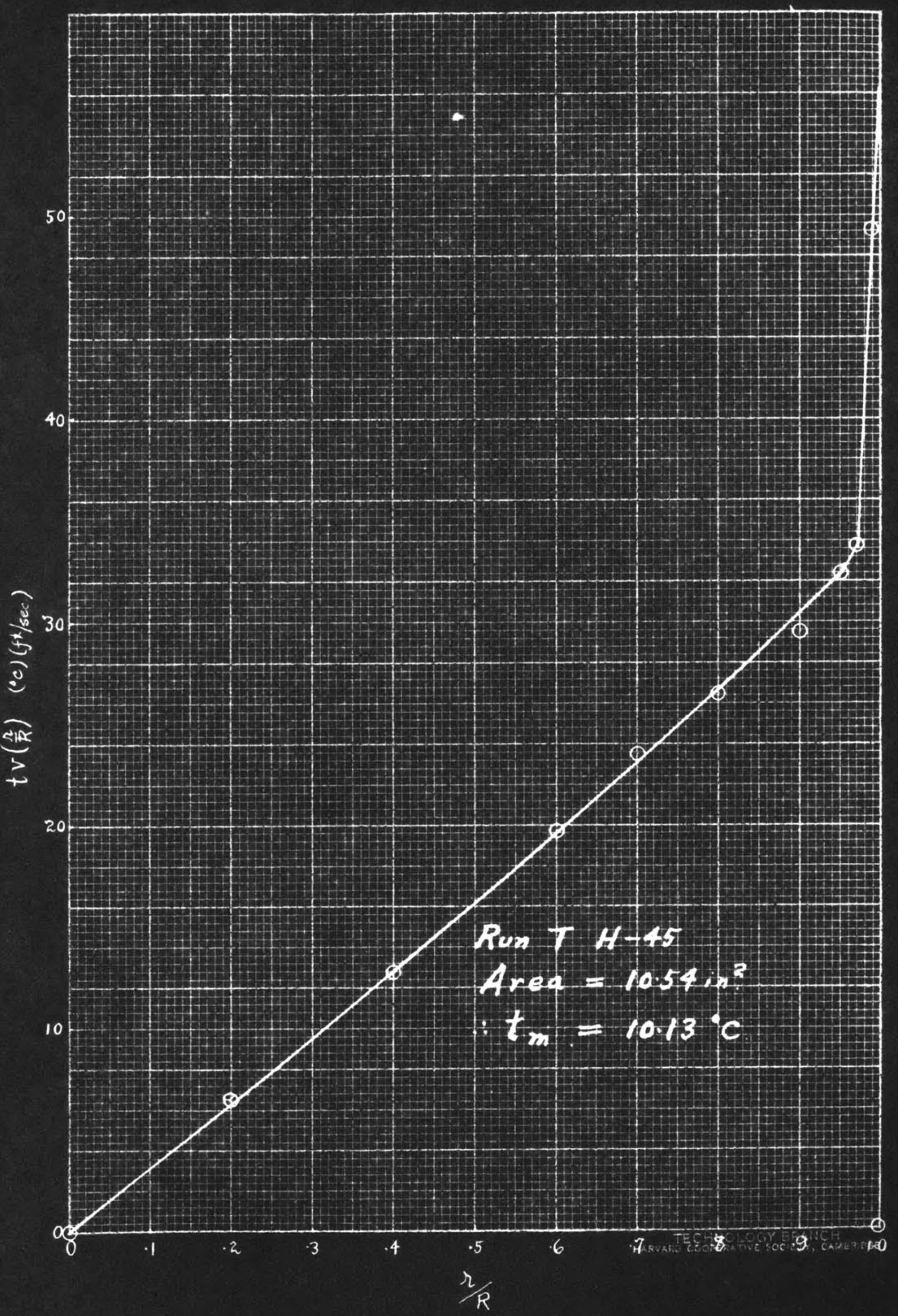


Fig. 63 Graphical Integration of Mixing Cup Temperature.

It is a very significant fact that these temperature distribution exponents "b" are found to be always much smaller than the corresponding velocity distribution exponents "a". The apparent difference of these two exponents is to be expected as discussed before and to be observed in Figure 58 which illustrates dimensionless plots of the sample runs V H-16 and T H-26. It is recalled that from Pannell's experiments on air (1) (See Figures 41-44, Chapter IV), the corresponding velocity and temperature distribution exponents are almost equal. However, in the present investigation, instead of being equal, their ratio (a/b) varies from 2 to 5; a fact which reveals definitely that the elementary form of Reynolds analogy as adopted by Prandtl (See Chapter IV) between momentum and heat transfer fails to apply to liquids.

In the same run, there is a noticeable difference in pipe wall temperature between two sections. In the parallel current runs, the pipe wall temperature of Station 3, i.e., the station farther away from the inlet water end, is always 2 to 10° C. lower than that of Station 2; consequently the maximum temperature gradient, $(t_w - t_a)$, at Station 3 will be always smaller than that at the other station. It must be remembered that excess amount of steam was used in each heating run so the decrease of pipe wall temperature cannot be due to insufficient steam.

However, in the counter-current runs, the pipe wall temperature of Station 5, which is the station farther away from the inlet water end but nearer to the steam inlet end, is usually greater than that at Station 4; thus, this equalizes the maximum temperature gradients between these sections to some extent, although they are seldom equal to each other. Of course, the difference of pipe wall temperatures will gradually decrease as the rate of water flowing inside the pipe being heated decreases or the velocity of the steam in the jacket increases. Uniform wall temperature seems to be almost impossible to maintain, if condensing steam is employed as a heating medium either in parallel-current or counter-current cases. In view of these facts it should be realized that while the results herein presented show the phenomena actually obtaining in real cases of heating water by steam, they may differ from the phenomena associated with a truly uniform wall temperature. Such differences as may exist cannot be determined by the present apparatus.

B. On Velocity Distribution During Heating.

In analyzing the difference of flow conditions between non-isothermal case and the isothermal one, the following factors must be considered:

1. The effective temperature based upon which Reynolds number ought to be calculated.

2. The density gradient due to the temperature gradient in the cross-section, thus producing natural convection.
3. The viscosity gradient also due to the temperature gradient in the cross-section.

The density effect will favor the flow near the boundary of the pipe for upward flow, while the reverse is true for the downward flow. The viscosity effect should apparently be to increase flow near the boundary, whether the water is flowing upward or downward during heating.

In plotting the velocity distribution exponents "a" in Fig. 64 against usual Reynolds number, the kinematic viscosity in which is based on the "space average" cross-sectional temperature, t_{ave}, it is observed that they generally lie below the experimental isothermal line. However, by substituting in calculating the Reynolds number, a mean film temperature, t_f, or effective film temperature taken at 0.995R from the corresponding temperature distribution for each velocity distribution run, it is shown in Fig. 65 that these exponents correlate better with the average isothermal line.

Keevil and McAdams (3) (4) have recently pointed out the effect of both density and viscosity gradients on the viscous flow during heat transfer; an understanding of their mental picture on the subject will be very helpful in explaining the present investigation. As the water is being heated, due to the high temperature near the pipe wall and the low temperature near the center of the pipe, the water near the wall will have a lower viscosity and lower density than that at the center portion. In comparison with the isothermal case, the liquid near the wall will flow at a greater velocity relative to that in the central portion for water flowing upward, and also for water flowing downward, provided that in the latter case, the density effect is negligible compared with the viscosity effect. Let us consider a long vertical pipe being heated at its middle position by a steam jacket (See Figure 65') the following phenomena are what are to be expected:

(1) Since the total flow at any pipe cross-section will be a constant, some radial flow will be produced as water is flowing from the isothermal to the heated section.

(2) Assume curve a in Fig. 65' represents the velocity distribution in the isothermal section A before heating, curve b will be the velocity distribution in the heating section B, since water will flow from central portion to the boundary due to radial flow as compared with the isothermal curve.

(3) After heating, the velocity distribution in the isothermal section C ought to go back to the isothermal case, curve a, thus some water has to flow backward from boundary to central portion as compared with the velocity distribution curve b during heating.

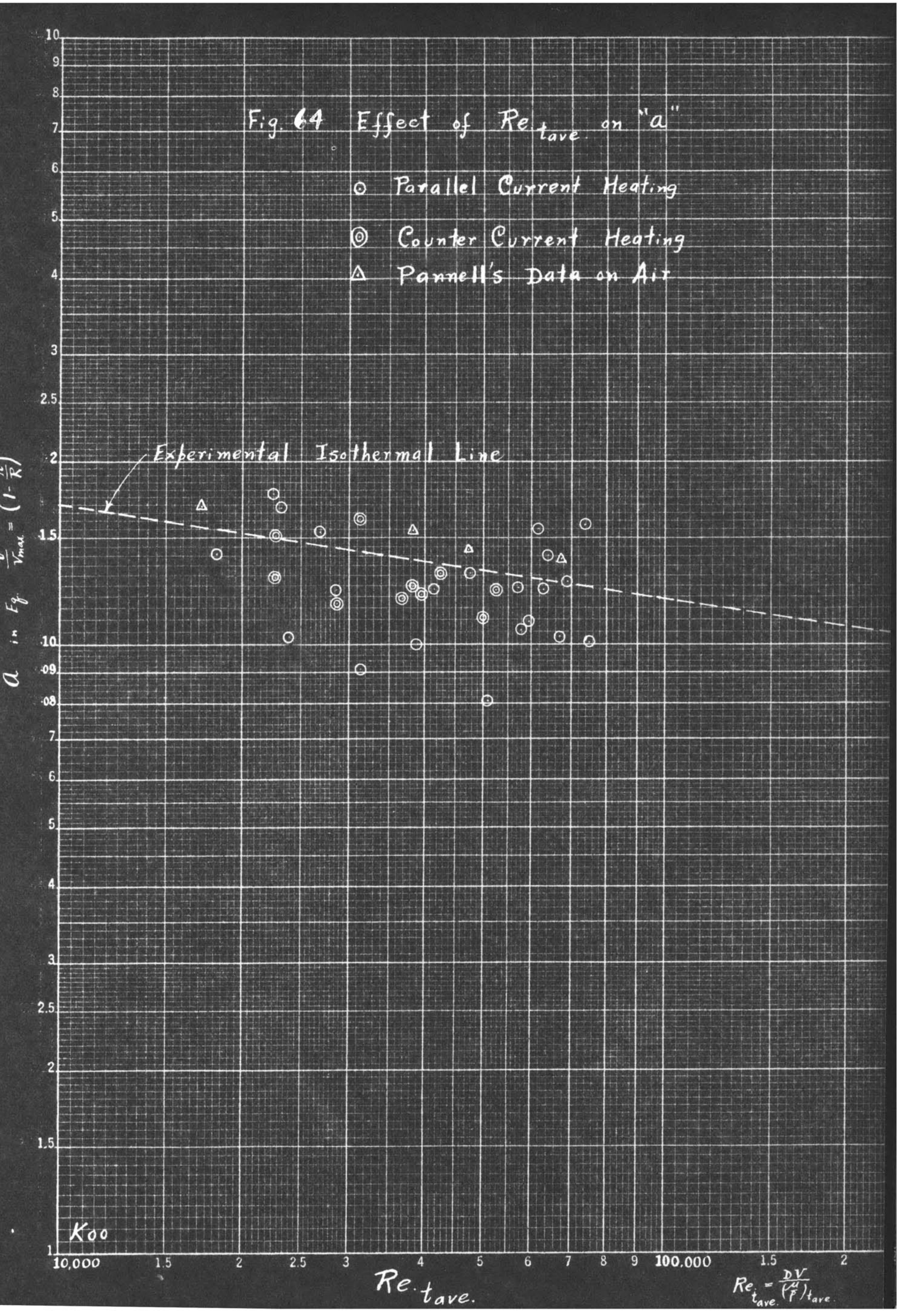
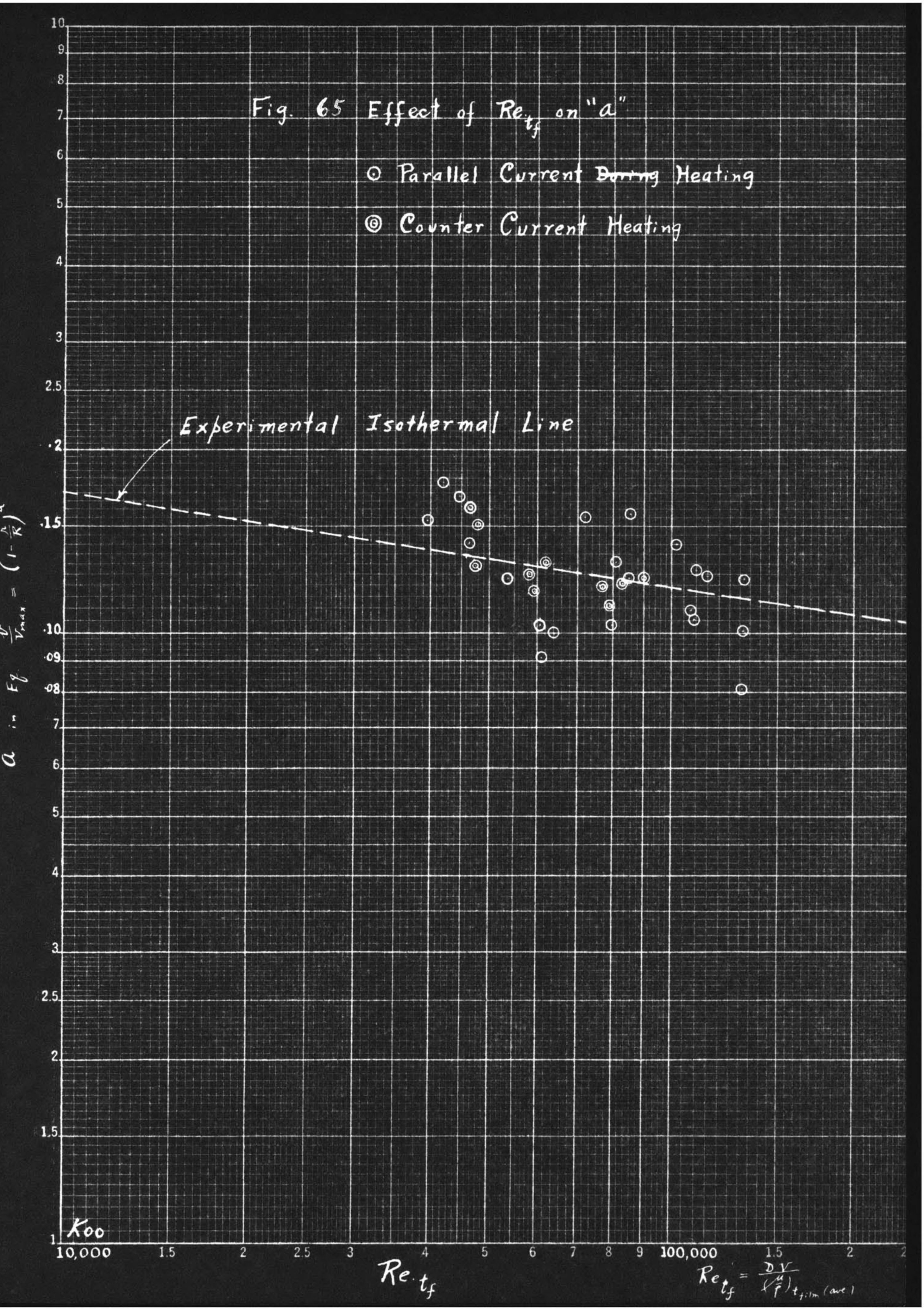


Fig. 64 Effect of $Re_{t_{ave}}$ on "a"



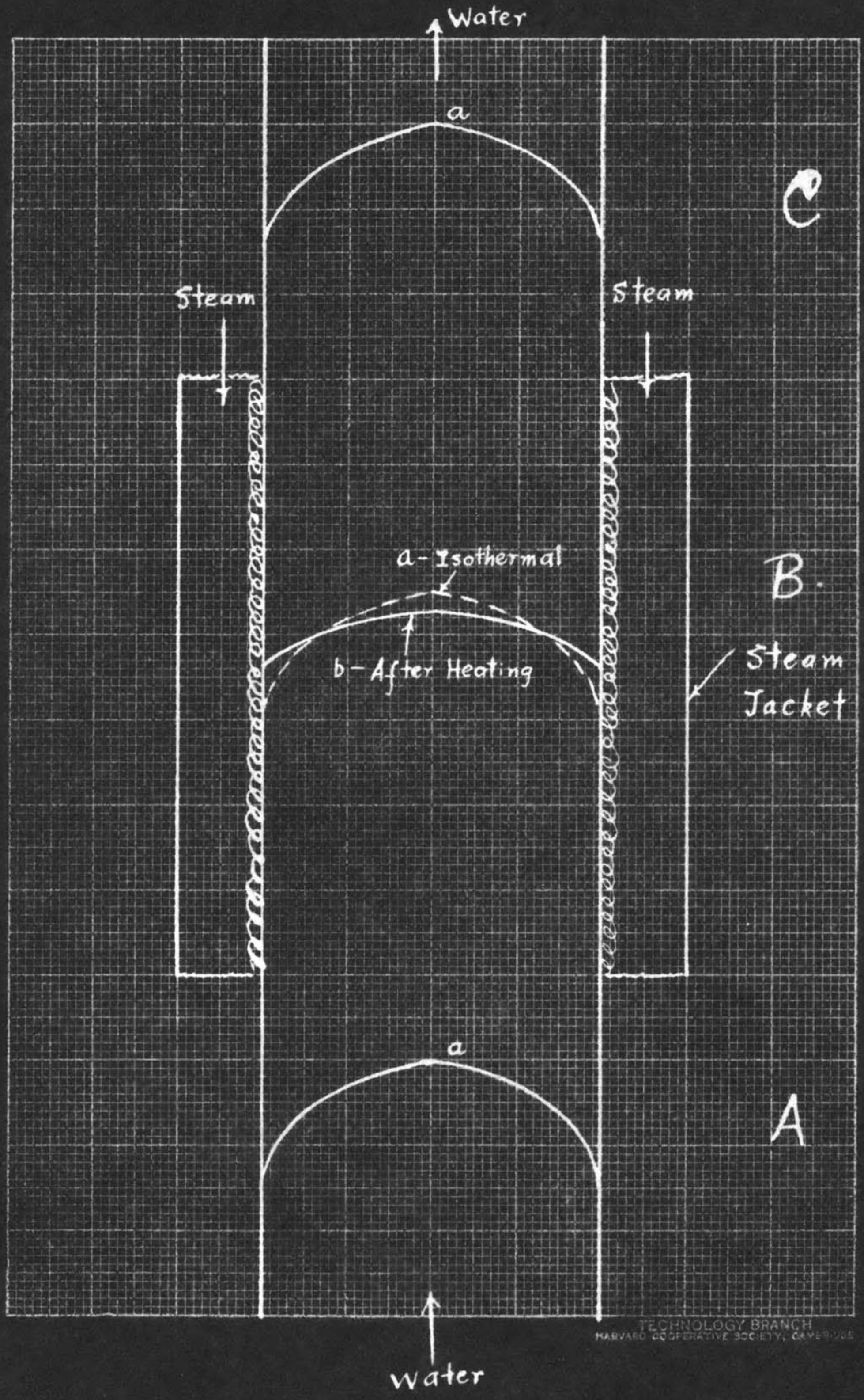


Figure 65'
Effect of Heating on
Velocity Distribution

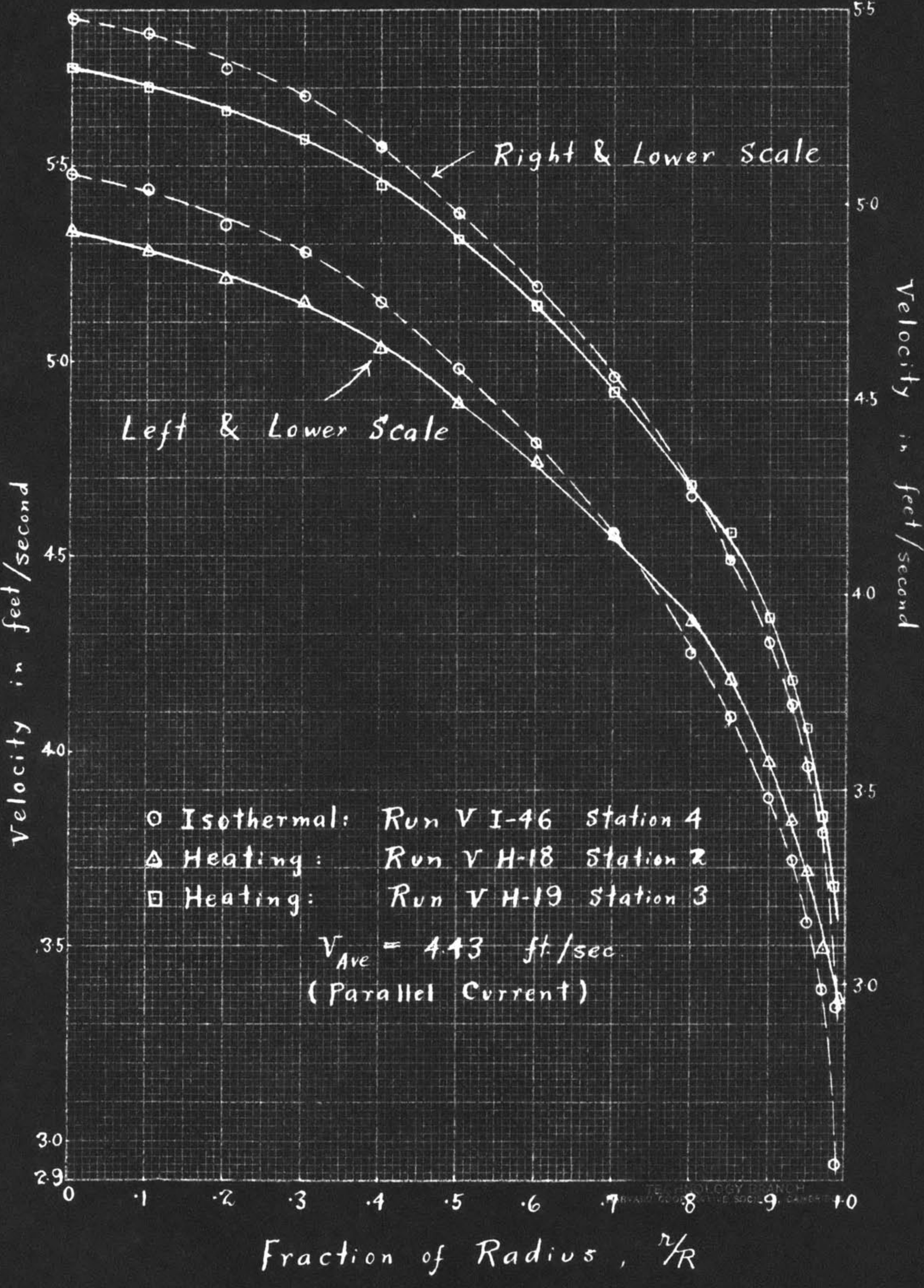

Right & Lower Scale
Left & Lower Scale
○ Isothermal: Run V I-46 Station 4
△ Heating: Run V H-18 Station 2
□ Heating: Run V H-19 Station 3
V_{Ave} = 4.43 ft./sec
(Parallel Current)
Velocity in feet/second
Velocity in feet/second
Fraction of Radius, r/R
5.5
5.0
4.5
4.0
3.5
3.0
2.9
5.5
5.0
4.5
4.0
3.5
3.0
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1.0

Figure 65 A

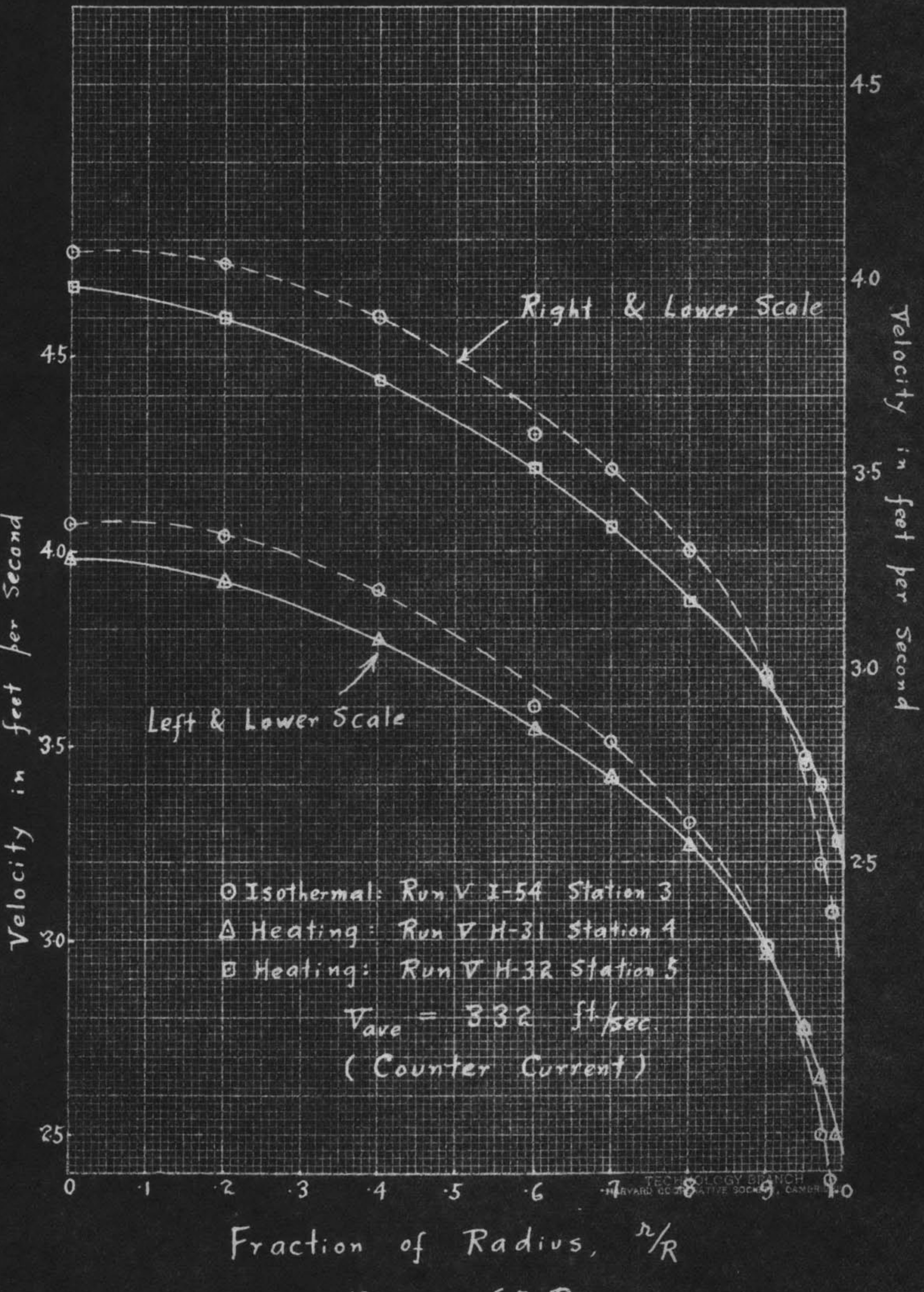

Right & Lower Scale
Left & Lower Scale
Velocity in feet per Second
Velocity in feet per Second
4.5
4.0
3.5
3.0
2.5
4.5
4.0
3.5
3.0
2.5
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1.0
Fraction of Radius, r/R
⊙ Isothermal: Run V I-54 Station 3
△ Heating: Run V H-31 Station 4
⊡ Heating: Run V H-32 Station 5
V_ave = 332 ft/sec.
(Counter Current)

Figure 65 B

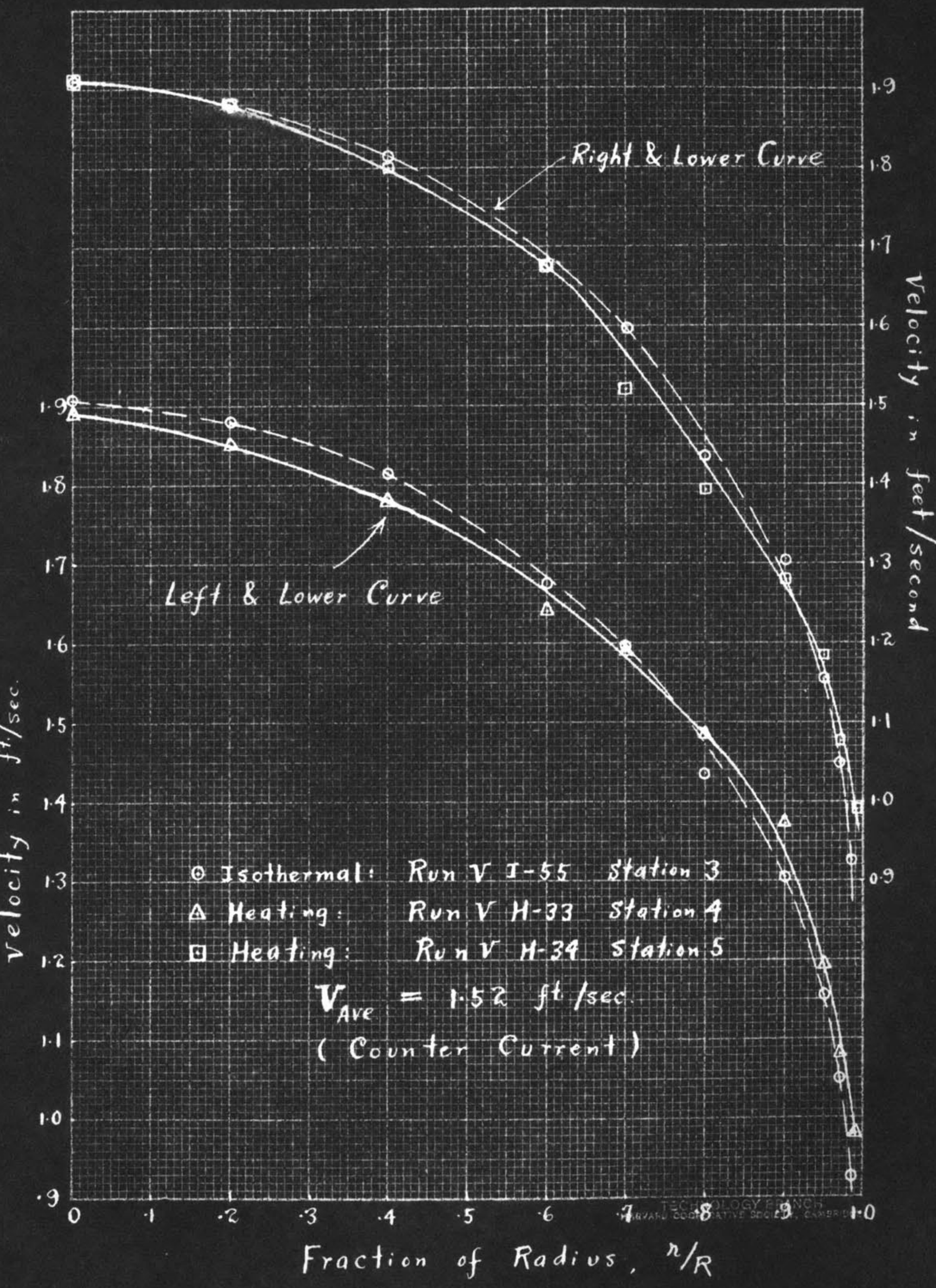

Right & Lower Curve
Left & Lower Curve
Isothermal: Run V I-55 Station 3
Heating: Run V H-33 Station 4
Heating: Run V H-34 Station 5
V_{Ave} = 1.52 ft/sec
(Counter Current)
Velocity in ft/sec.
Velocity in feet/second
Fraction of Radius, r/R
·9
1·0
1·1
1·2
1·3
1·4
1·5
1·6
1·7
1·8
1·9
0·9
1·0
1·1
1·2
1·3
1·4
1·5
1·6
1·7
1·8
1·9
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1·0

Figure 65 C

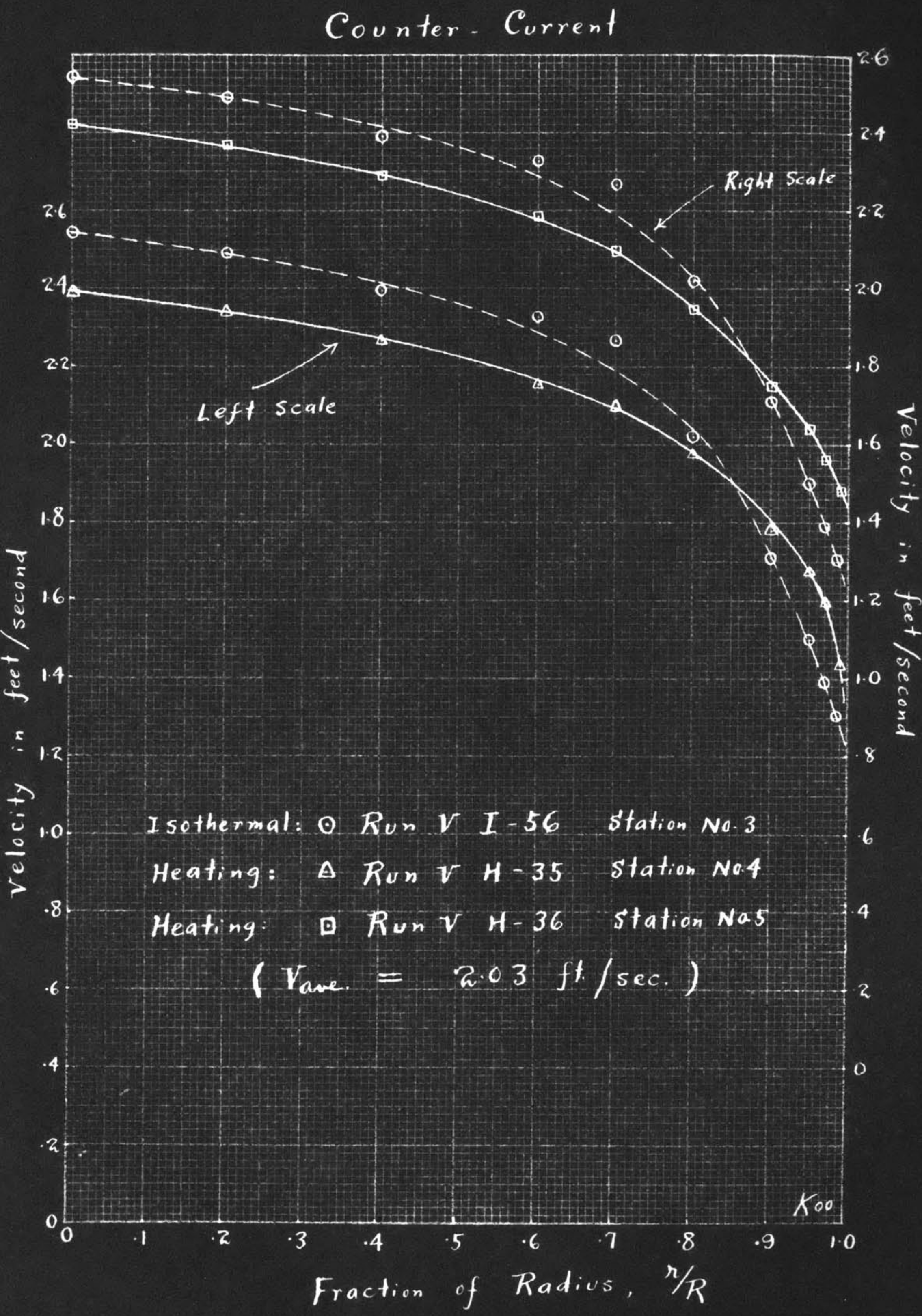
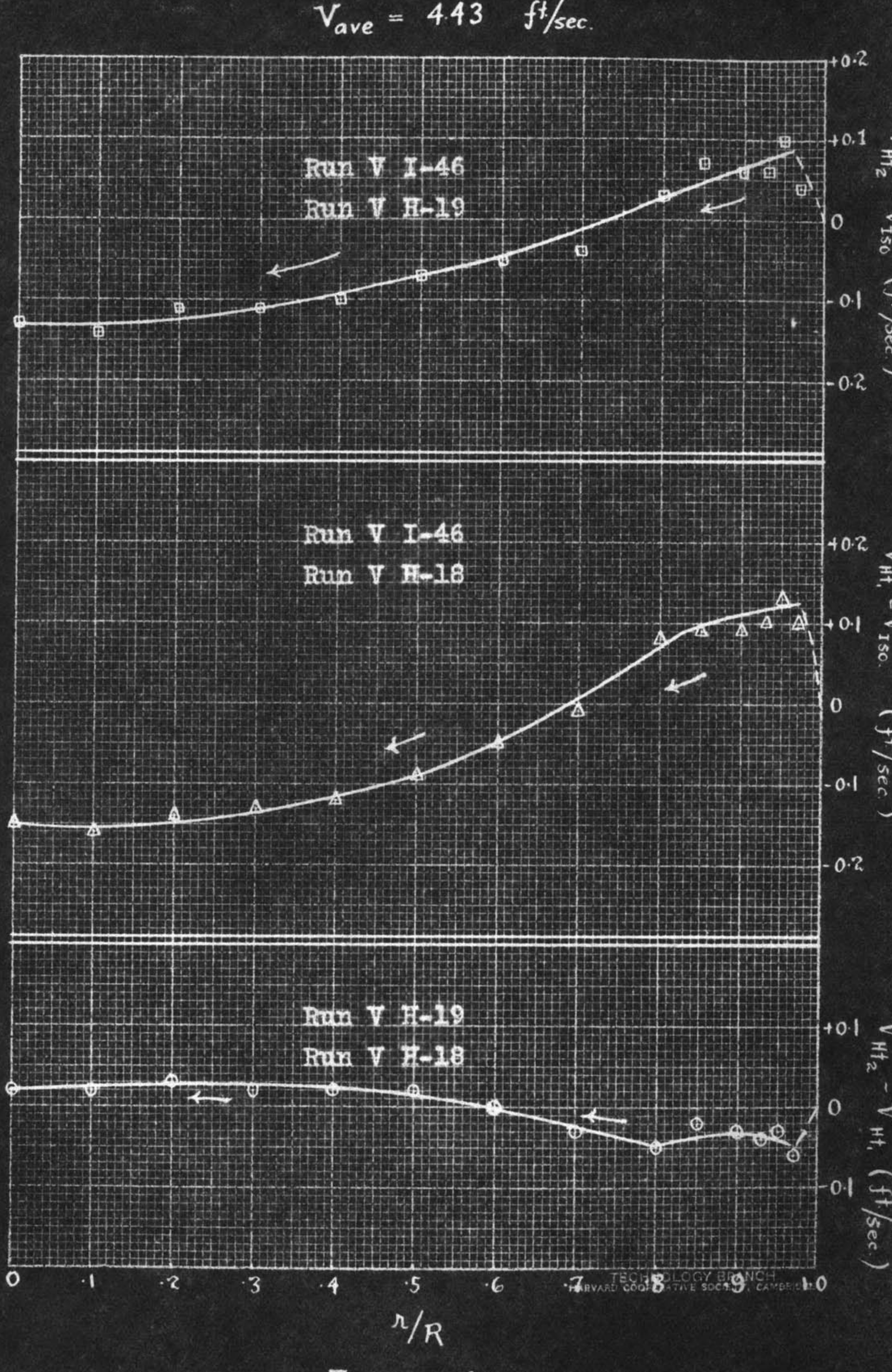


Figure 65D. Effect of Heating on Velocity Distribution Curve at Constant $V_{ave.}$


V_{ave} = 4.43 ft/sec.
Run V I-46
Run V H-19
$V_{Ht_2} - V_{ISO}$ (ft/sec.)
+0.2
+0.1
0
-0.1
-0.2
Run V I-46
Run V H-18
$V_{Ht_1} - V_{ISO}$ (ft/sec.)
+0.2
+0.1
0
-0.1
-0.2
Run V H-19
Run V H-18
$V_{Ht_2} - V_{Ht_1}$ (ft/sec.)
+0.1
0
-0.1
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1.0
r/R
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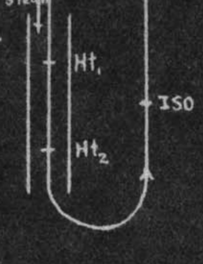

water
steam
Ht₁
Ht₂
ISO

Figure 65 E.

$V_{Ave} = 3.32$ ft/sec.

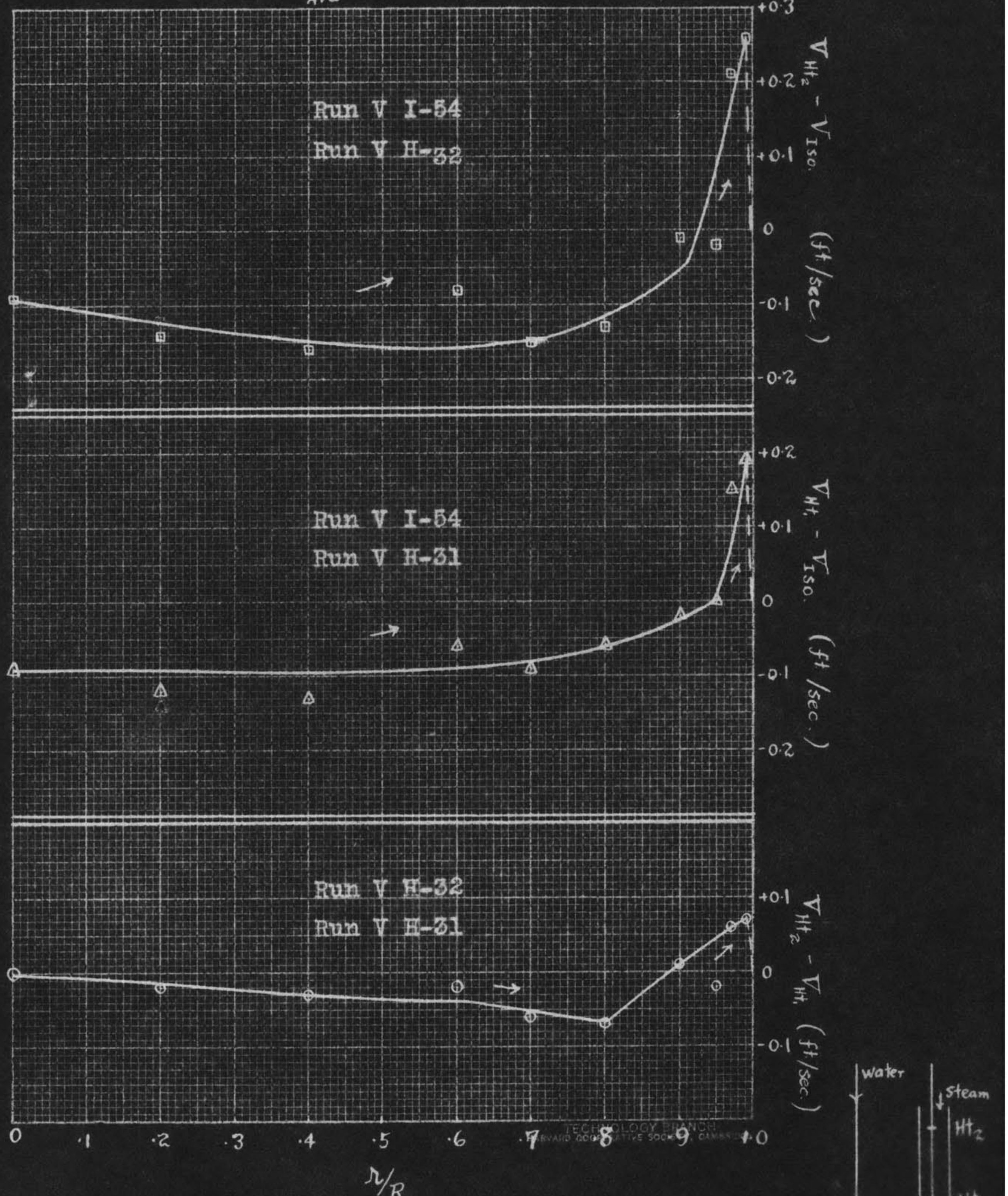
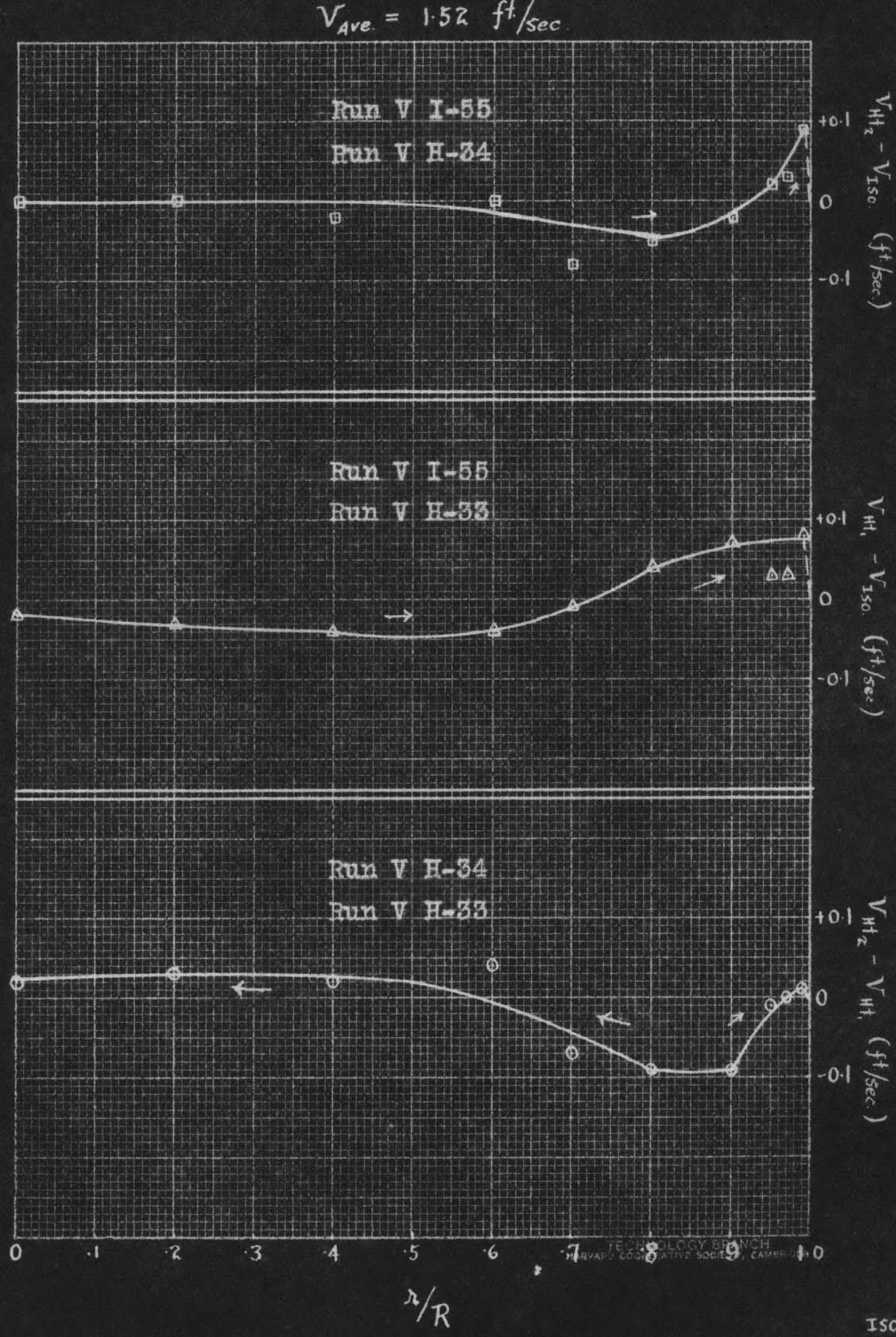


Figure 65 F



$V_{Ave.} = 1.52$ ft/sec
Run V I-55
Run V H-34
$V_{Ht_2} - V_{ISO.}$ (ft/sec.)
Run V I-55
Run V H-33
$V_{Ht_1} - V_{ISO.}$ (ft/sec.)
Run V H-34
Run V H-33
$V_{Ht_2} - V_{Ht_1}$ (ft/sec.)
+0.1
0
-0.1
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1·0
r/R
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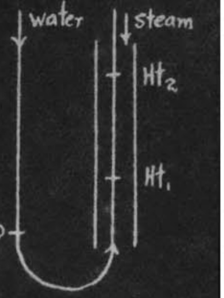

water
steam
Ht_2
Ht_1
ISO

Figure 65 G

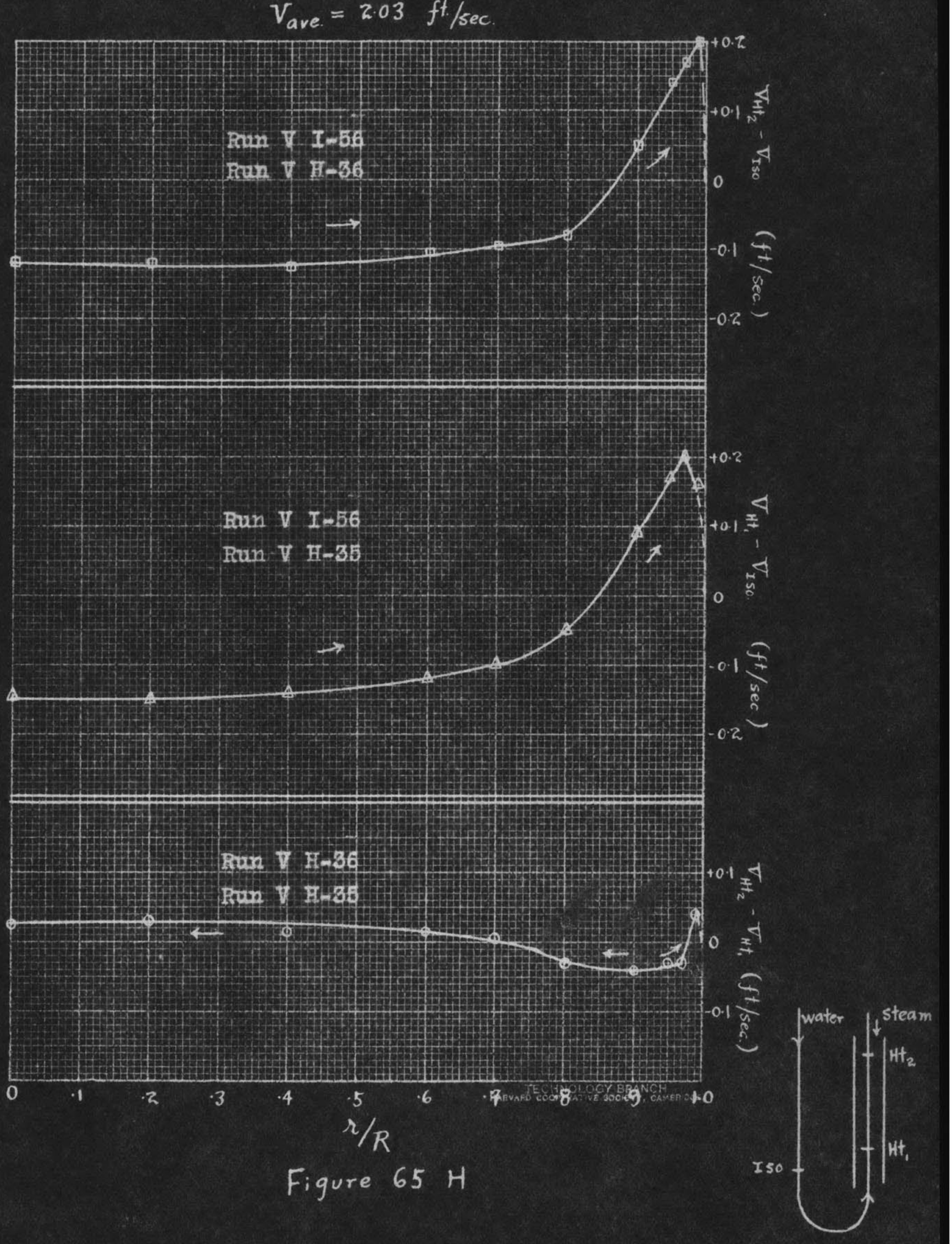
Vave. = 2.03 ft./sec.
Run V I-56
Run V H-36
VHt2 - VIso (ft/sec.)
+0.2
+0.1
0
-0.1
-0.2
Run V I-56
Run V H-35
VHt1 - VIso (ft/sec)
+0.2
+0.1
0
-0.1
-0.2
Run V H-36
Run V H-35
VHt2 - VHt1 (ft/sec.)
+0.1
0
-0.1
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1.0
r/R
water
steam
Ht2
Ht1
Iso
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Figure 65 H

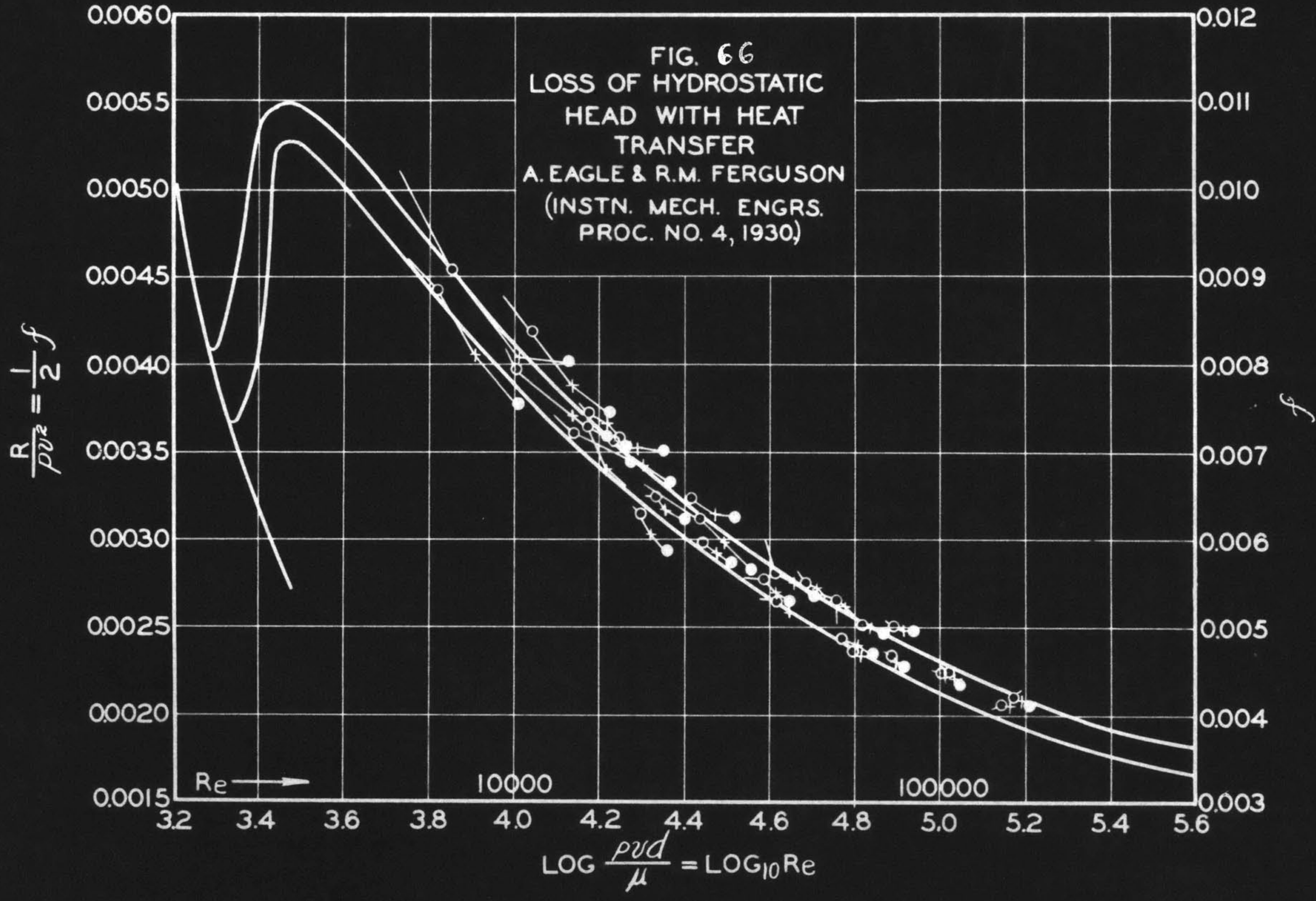

FIG. 66

LOSS OF HYDROSTATIC HEAD WITH HEAT TRANSFER

A. EAGLE & R.M. FERGUSON

(INSTN. MECH. ENGRS. PROC. NO. 4, 1930)

The above statements are the possible phenomenon which describe the effect of heating on velocity distribution. Experimental verification is still lacking in literature, and is to be attempted as follows from the present investigation.

In the present investigation, some sets of non-isothermal runs were taken simultaneously with isothermal velocity distribution runs which were taken either before or after the water was heated. (Tabulated in Table 19A). The apparent swelling up of the velocity distribution near the wall and its shrinkage at the central portion may be observed in Figures 65A to 65D.

Figures 65E to 65H illustrate more clearly the radial flow phenomenon by plotting the difference of velocity between corresponding isothermal and non-isothermal runs against fraction of radius. The arrows in these figures indicate the direction of radial flow as one passes from one pipe cross-section to another following the main stream. It is seen that in case the isothermal run is taken before the heating runs, the direction of radial flow is from central portion to the boundary, and in case that the isothermal run is taken after the heating runs, the direction of radial flow is reversed, i.e., some portion of water near the boundary flows backward toward the central portions as the case is changed from heating to isothermal. It must be remarked that the density variation over a cross section is only about 0.3%. Thus, these figures may be considered to show changes in the local mass velocity. ~~Data not accurate enough, etc., for giving amount of radial flow.~~

Table 19A. Comparison of Isothermal and Non-Isothermal Velocity Distributions at Constant Mean Velocity.

Isothermal Velocity Distribution.				Velocity Distribution During Heating				
Run No.	a	Re	Station No. (after heating)	Run No.	a	$Re_{t_{av}}$	Re_{t_f}	Stat. No.
V I-44	0.137	69,000	4	V H-14	0.124	57,200	114,800	2
				V H-15	0.140	64,100	102,000	3
V I-45	0.133	49,400	4	V H-16	0.123	41,500	85,200	2
				V H-17	0.131	47,850	80,600	3
V I-46	0.139	74,700	4	V H-18	0.123	62,700	132,000	2
				V H-19	0.127	69,000	110,400	3
				(Parallel-Current Runs above)				
				(Counter-Current Runs below)				
			(Before Htg.)					
V I-52	0.141	41,250	3	V H-27	0.131	42,450	62,000	4
V I-43	0.148	16,040	3	V H-30	0.129	22,600	47,500	4
V I-54	0.130	36,950	3	V H-31	0.121	39,400	83,200	4
				V H-32	0.123	52,500	90,000	5
V I-55	0.169	17,000	3	V H-33	0.151	22,700	48,000	4
				V H-34	0.161	31,250	46,700	5
V I-56	0.160	22,600	3	V H-35	0.117	28,700	59,350	4
				V H-36	0.125	38,100	58,300	5

The data of the present investigation are still insufficient to express quantitatively the effect of radial flow, which is due to the temperature gradient, on the non-isothermal velocity distribution exponent. As an approximation, the mechanism of non-isothermal flow may be regarded as chiefly due to the reduction of film resistance near the pipe wall; thus, the mean film temperature, which may be taken roughly as the arithmetical mean of the pipe wall and average water temperature, can be used in calculating the Reynolds number approximately. It is recommended that the isothermal velocity distribution line be used to non-isothermal cases when Re has been computed in this way.

C. On the Effective Film Temperature.

Eagle and Ferguson (5)(6) in correlating their non-isothermal friction data on water during heating used wall temperature in obtaining the Reynolds number. Their data (6) are reproduced here as Fig. 66. However, they did state that for high temperature difference between pipe wall and water, a temperature of the sum of the average water temperature plus 90 per cent of the temperature difference between the water and the pipe should be used, i.e., the effective temperature is

$$t_{\text{effective}} = t_{av} + 0.9(t_w - t_{av}) = 0.9\ t_w + 0.1\ t_{av} \qquad \ldots.(4)$$

It is interesting to note that Keevil (4) in correlating his non-isothermal friction data on oil for viscous region suggested the following formulae,

(For heating) $t_{eff.} = 0.21\ t_w + 0.79\ t_{av}$ (5)

and

(For cooling) $t_{eff.} = 1.29\ t_{ave} - 0.29\ t_w$ (6)

The above three equations can be generalized as the following one,

$$\frac{t_{eff} - t_{av}}{t_w - t_{av}} = K \qquad .....(7)$$

where K may be considered as a constant for a certain experimental range.

The adoption of 0.995R as the point to take the mean film temperature or effective film temperature by the writer is similar to ten Bosch (7) and Prandlt's (8) idea of using that in calculating the Prandlt's number, $C\mu/k$, in heat transfer equations. Prandlt (8) proposed an equation for this mean film temperature, which reads,

$$t_{eff} = t_w + (t_{ave} - t_w)\ \frac{(V_1/V_{av})\ Pr}{1+(V_1/V_{av})Pr-1)} \qquad .....(8)$$

where, V_1 = marginal velocity between the viscous film at the boundary and the turbulent core of fluid

and

$$\frac{V_1}{V_{av}} = \frac{V_{max}}{V_{av}} \quad \frac{1.363}{\left(\frac{V_{max}\ R}{v}\right)^{1/8}} \qquad .....(9)$$

The deviation of Eq. (9), however, is based upon his one-seventh potential velocity distribution, By the use of Equation (9) the writer calculated from several of his velocity distributions the values of V_1/V_{av}. These quantities when substituted in Equation 8 gave values for $t_{eff.}$ equal in most cases to the actual temperature of the fluid occurring, as shown by the temperature distributions, at about 0.995R. It must be remarked here that in actual velocity distribution the change from laminar film near the wall to the turbulent region must be gradual, so such assumption of a sharp margin is purely arbitrary and for the sake of convenience. Granting that 0.995R is the place to take the effective temperature for the present experimental range, the average value of k in Eq. (7) is found to be 0.73, thus

$$\frac{t_{eff} - t_{ave}}{t_w - t_{ave}} = 0.73 \qquad \text{.....(10)}$$

or

$$t_{eff.} = \cancel{t_f}\ \cancel{\times}\ 0.73\ t_w + 0.27\ t_{ave} \qquad \text{.....(10A)}$$

Undoubtedly, K in Eq. (7) is a variable for different running conditions and for different velocity or temperature. The exact formula of K cannot be deduced as yet, its solution can be obtained through a collection of more non-isothermal friction data taken together with velocity and temperature explorations.

D. On Temperature Distribution During Heating.

The factors which will affect the temperature distribution exponent of a liquid have not been enumerated yet. As found graphically, the rapid change of this exponent cannot be explained by Reynolds number alone, whether taking the average temperature or mean film temperature for the kinematic viscosity factor. The other factor which is to be considered will naturally be the Prandlt number, $c\mu/k$, since this number in the present experimental range has been found to vary from 2.50 to 6.50 at the mean film temperature. It is believed that the mean film temperature should be based upon in calculating this factor as pointed out by Prandlt (8). Plotting the temperature distribution exponent, "b", with the product of Reynolds number and Prandlt's number, which is usually called Peclet number, the decrease of this exponent with increase of Peclet number is noticed in Figure 67. The Peclet number adopted here is based upon mean film temperature, and it is obvious that this will almost be equal tp that based upon average temperature, since the group $)c\beta/k)$ with change very little with temperature. This relation means that in a given pipe and at the same time temperature, the temperature distribution exponent is dependent entirely on the average velocity in the pipe, decreasing as increase of velocity. Since the increase of velocity is equivalent to saying increase of turbulence in the pipe, as a result improving the mixing of liquid from the boundary layer to the central portion, thus less temperature gradient or the decrease of "b" is what is to be expected. As a limiting case, when the water in the pipe is infinitely turbulent, i.e., the velocity distribution exponent approaches zero, the different molecules of water over the pipe cross-section are so well mixed that there should not be any temperature gradient present, i.e., "b" = 0, also, which is equivalent to the isothermal flow.

However, the above method is found to be insufficient and not conclusive to explain all the experimental data secured, since it is very evident in examining the present data that at the same average velocity, the running conditions are not necessarily identical, therefore the temperature gradient across the pipe varies with different runs as the conditions may be. Consequently, the product of the Peclet number and a dimensionless temperature gradient ratio expressed as $\left(\frac{t_w - t_a}{t_{av} - t_a}\right)$ is recommended to be used in plotting with the temperature distribution exponent. Figure 68 illustrates a more prominent effect of this product on "b" than what has already been shown in Fig. 67. It is noticed that this correlation not only fits the present data on both heating and cooling runs but also fits fairly well on Pannell's data (1) on heating of air.

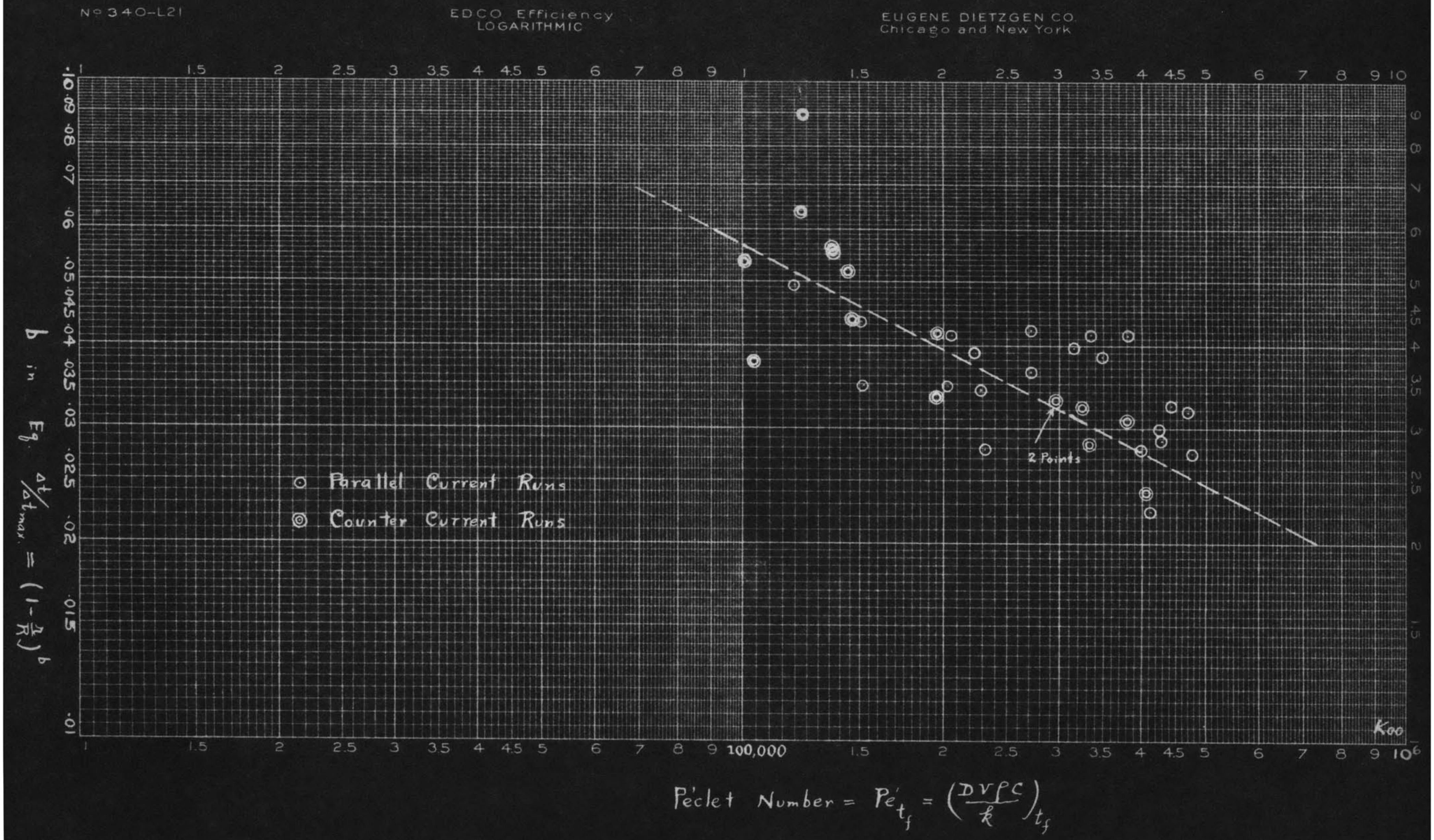

Fig. 67 Effect of Péclet Number on Temperature Distribution Exponent

Fig. 68

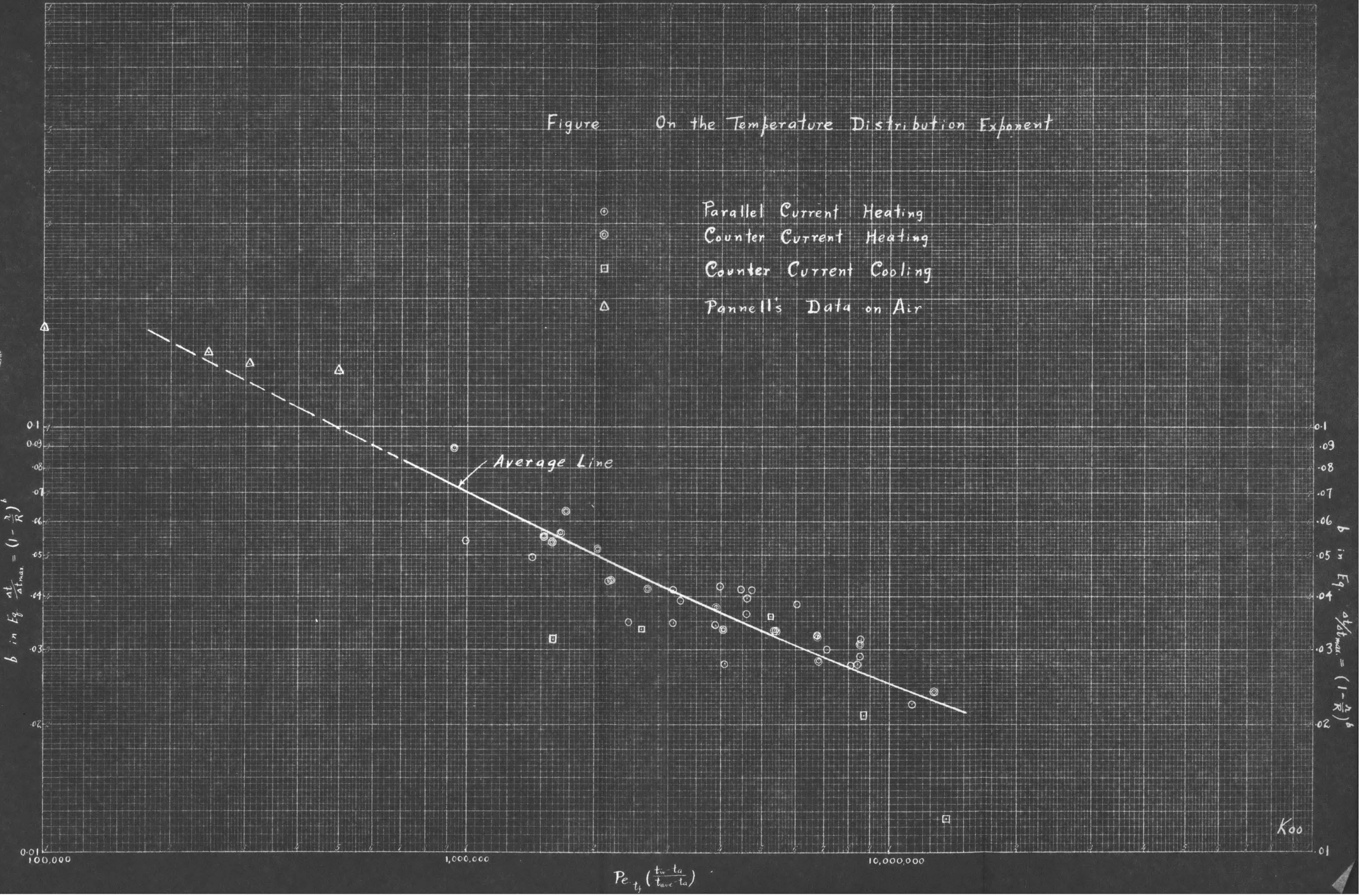

Figure On the Temperature Distribution Exponent

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Based upon Eq. (1) and (2), two useful relations of temperature gradient ratio can readily be derived as follows:

$$\frac{t_w - t_{av}}{t_w - t_a} = \frac{2}{(b+1)(b+2)} \qquad \ldots\ldots(11)$$

and,

$$\frac{t_{av} - t_a}{t_w - t_a} = 1 - \frac{2}{(b+1)(b+2)} \qquad \ldots\ldots(12)$$

It is noticed that Eq. (11) is exactly analogous to Eq. (22) in Chapter III, having the temperature gradient taking the place of velocity and its distribution exponent taking the place of velocity exponent. Sung (9) proposed in his recent thesis that a constant ratio may be used for $\frac{t_w - t_{ave}}{t_w - t_a}$, i.e.,

$$\frac{t_w - t_{ave}}{t_w - t_a} = 0.917 \qquad \ldots\ldots\ldots(13)$$

While in the present investigation, which covers a much wider range than Sung's the average of this ratio is found approximately to be 0.938 as shown in Fig. 69. It must be realized that this temperature gradient ratio is by no means constant as required by Eq. (11), that it can only be considered as a constant for a very limited experimental range as an approximation. The error made in assuming it as a constant is just as bad as of saying the mean to axial velocity ratio is a constant for different Reynolds numbers. The values

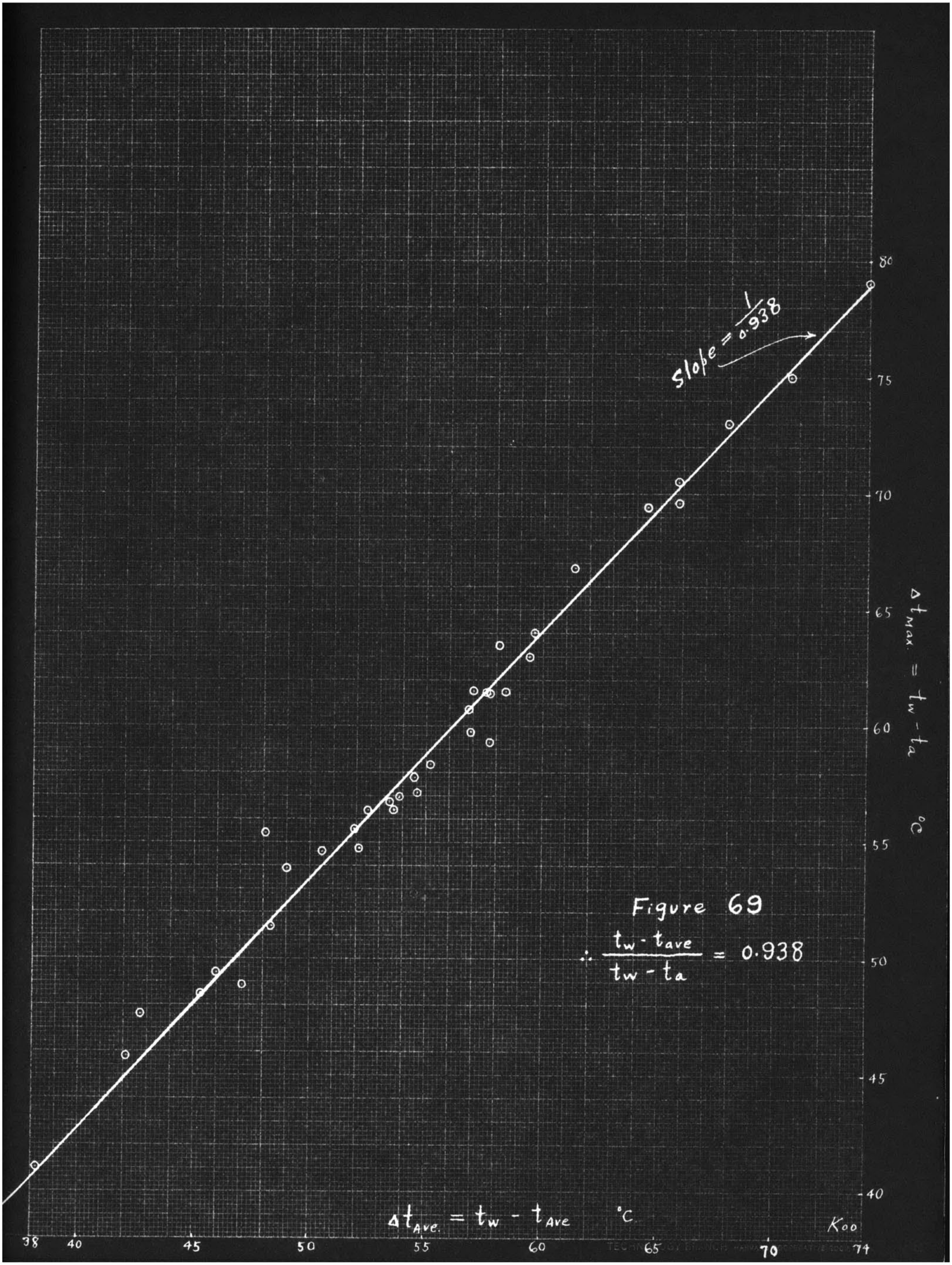


Figure 69

$$\therefore \frac{t_w - t_{ave}}{t_w - t_a} = 0.938$$

of these temperature gradient ratios are calculated from Eq. (11) and (12) and tabulated in Table 19B for the range of "b" from 0 to 0.15. The verification of this relation by the present data is to be found in Figure 69A. This verification states indirectly the justification and applicability of the relation expressed by

$$\frac{t_w - t}{t_w - t_a} = \left(1 - \frac{r}{R}\right)^{b} \qquad \ldots\ldots(1)$$

If a straight line may be drawn through the experimental points shown in Fig. 68, the following relation can readily be found for the temperature distribution exponent by simply applying Eq. (12),

$$b = \text{constant} \left(\frac{Pe}{1 - \frac{2}{(b+1)(b+2)}} \right)^{\text{slope}} \qquad \ldots.(14)$$

Table 19B. Dependance of Temperature Gradient Ratios on Temperature Distribution Exponent.*

b	$\frac{t_w-t_{av}}{t_w-t_a}$ (Calculated from Eq 11)	$\frac{t_{av}-t_a}{t_w-t_a}$ (Calculated from Eq. 12)
0	1.000	0
0.01	0.985	0.015
.02	.971	.029
.03	.957	.043
.04	.943	.057
.05	.929	.071
.06	.915	.085
.07	.902	.098
.08	.890	.110
.09	.878	.122
.10	.866	.134
.11	.854	.146
.12	.843	.157
.13	.831	.169
.14	.820	.180
.15	.808	.192

*The present experimental range is from b = 0.02 to b = 0.09.

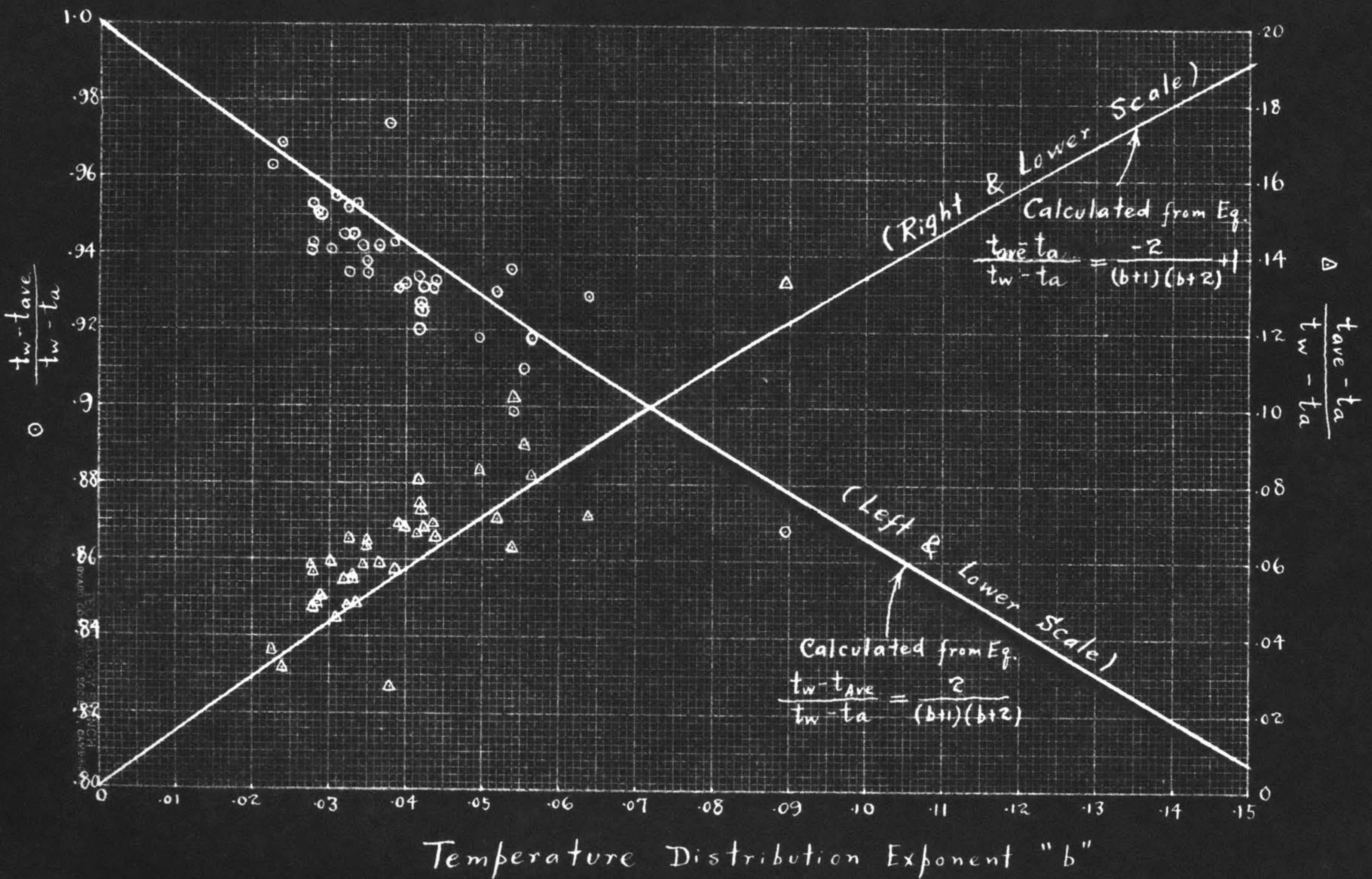


Figure 69A Dependence of Temperature Gradient Ratios on Temperature Distribution Exponent, "b".

Where, Slope = the slope on the log-log plot in Fig. 68

Or

$$b = Pe^{Slope} \left(\frac{t_w - t_a}{t_{ave} - t_a} \right)^{Slope} \qquad \text{......(14a)}$$

A simple way of explaining the above complicated relation is to regard that the temperature gradient ratio signifies the ineffectiveness of heat transmission from pipe wall to the main body of fluid inside the pipe, thus at constant Peclet number the bigger is the ratio, the bigger will the temperature distribution exponent, that means the less efficient is the way of heat transmission. One notices that in the case of gases this temperature gradient ratio is much smaller than that in the case of liquids. For example, comparing the present data on water with Pannell's data on air (1) at about the same Reynolds number in the following table, it proves the statement at least qualtitatively.

Table 19C. Comparison of Temperature Gradient Ratio for Gases and Liquids.

Fluid used	$Re_{t_{av}}$	$\frac{t_w - t_a}{t_{av} - t_a}$	Remarks
Air	38,200	5.56	Expt.II(Pannell)
Water	38,900	14.4	Run V H-3
Air	67,400	6.42	Expt. IV
Water	66,800	27.5	Run V H-5

The explanation of this fact will be given in the section following.

E. Non-Similarity between Temperature and Velocity Distribution of Liquids.

The derivation of the temperature distribution equation given in Chapter IV assumes the validity of the similarity between temperature and velocity distributions over a pipe cross-section, that is to say, the temperature at any point ought to be directly proportional to the velocity at that point. If this assumption holds true for liquids, the experimental velocity distribution exponent and temperature distribution exponent should be equal to each other, as it has been found from Pannell's data on air. The far from being equal to these two exponents as shown in Tables 18 and 19 of the present investigation is sufficient to believe the non-similarity between temperature and velocity distribution in case of liquids. This statement proves indirectly but conclusively that the mechanism of heat transfer is different from momentum transfer.

The significant facts which are believed to account for the difference in the mechanisms of transfer for gases and liquids are these:

(1) In gases where the rapid to and fro motion of the molecules occurs, which leads to the momentum and viscosity conduction, is very similar to the molar difference of particles in a turbulent fluid, as it is recently stated

by Caldwell (10). Based upon Iyer's latest experiment (11), he further stated that in liquids, the transmission of momentum and heat takes place mainly, not by diffusion of molecules, but by transmission through the molecules themselves which are more or less in actual contact all the time.

(2) It is found from the present temperature distribution data on water that the temperature drop through the laminar film attached to the pipe wall is about 80-90% of the total drop from pipe wall to axis; while in Pannell's data on air, this ratio is found to be 40-50%. (The calculation is based upon the assumption that 0.90R will be the outer margin of the laminar film, for both cases).

(3) The viscosity gradient effect will be different for gases and liquids, since viscosity increases with temperature for gases but decreases with temperature for liquids.

From the above established facts, it is obvious that the mechanisms of transfer of heat and momentum for gases will be different from those for liquids. It might be remarked here that if one excludes the laminar film and considers the turbulent core in the pipe alone, one will get the almost same velocity distribution exponent but a uite different temperature distribution exponent for a definite run since the temperature drop through the film is very great.

By this modified way, these two exponents may expect to approach each other. However, the method of calculating film thickness is still doubtful, so such application to the experimental data is not possible at present.

In spite of the fact that there is no similarity between velocity and temperature distribution, Eq. (1) is still recommended for the temperature distribution equation to be used in comparing non-isothermal data, and Eq. (13) is recommended to be used to calculate the average temperature from known axial and pipe wall temperatures.

F. Parallel-Current vs Counter-Current Heating

In all these heating runs, whether the water is flowing upward or downward, i.e., counter-current or parallel current with the condensing steam in the outside jacket, there is practically no difference observed in velocity or temperature distribution at a certain Reynolds number. At very low velocity, about or below 1 ft./sed., velocity distribution measurements were very difficult and sometimes inaccurate due to the air bubbles evolved from water during heating.

As the only difference in upward and downward flow will be due to the effect of density gradient which causes the natural convection, it might be concluded here that in case of forced convection and turbulent flow the natural convection is comparatively negligible as compared with the effect of forced convection.

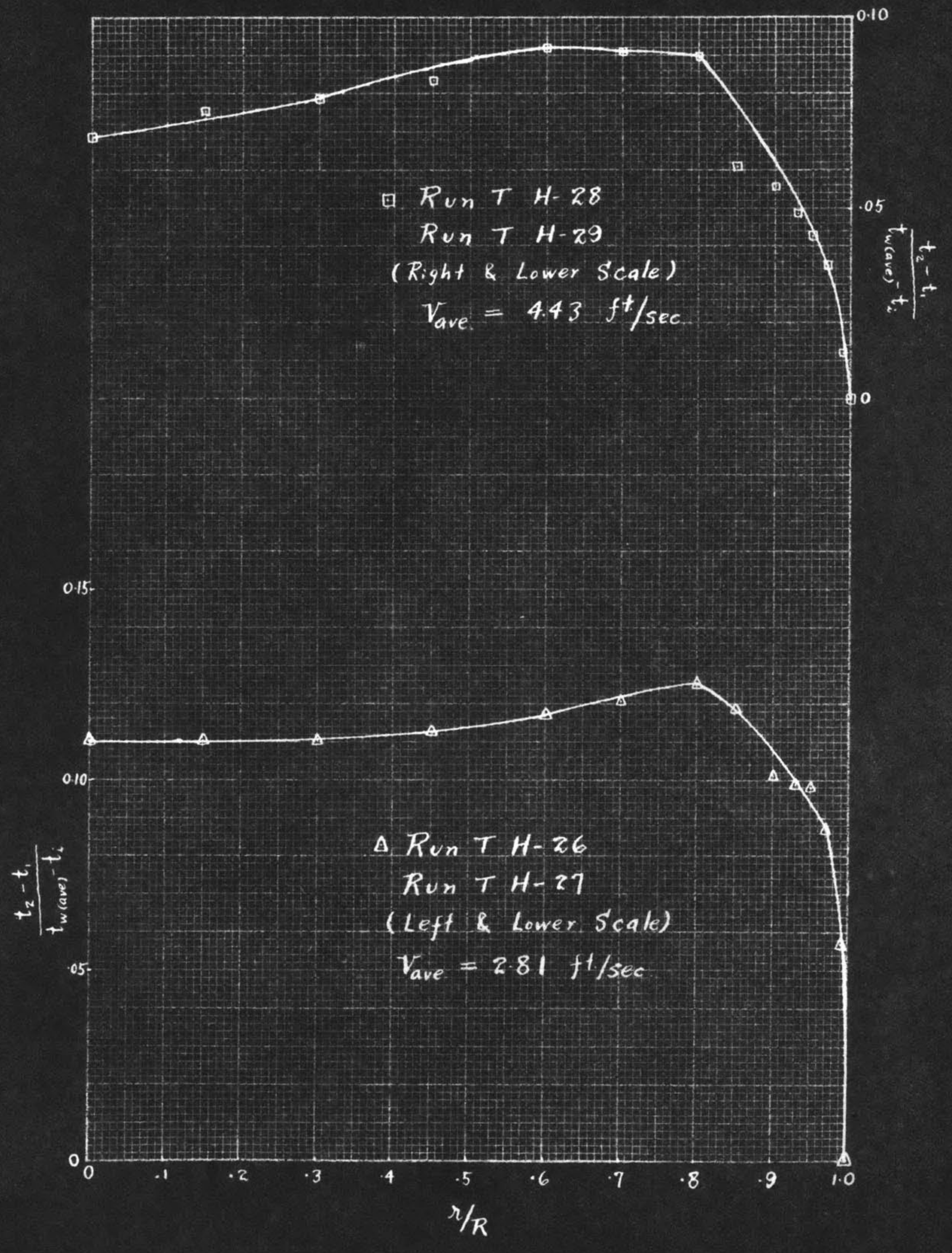
In examining the data on parallel-current and counter-current heating, the following significant facts which account for the more efficient heating during counter-current runs are noticed.

Parallel-Current Flow	Counter-Current Flow
(1) Pipe wall temperature decreases with direction of flow.	(1) Pipe wall temperature increases with direction of flow.
(2) Maximum temperature gradient, $(t_w - t_a)$, always decreases with direction of flow.	$(t_w - t_a)$ decreases slightly with direction of flow but increases at high velocity of water.
(3) Mean film temperature decreases with direction of flow.	(3) Mean film temperature increases with direction of flow.
(4) Re_{t_f} decreases with direction of flow.	(4) Re_{t_f} increases with direction of flow.
(5) "a" increases with direction of flow.	(5) "a" decreases with direction of flow.
(6) "b" usually decreases with direction of flow.	(6) "b" usually about constant with direction of flow.

From the discussion on velocity and temperature distribution exponents in above sections, the favorable conditions for efficient heat transfer have been found to be at maximum temperature exponent and minimum velocity distribution exponent possible. Consequently, counter-current heating should be more efficient from the above listed facts. The heat transfer coefficients for counter-current run is found to be higher than that for parallel-current run at same Peclet number as it will be found, in Chapter X.

G. On Temperature Rise between Two Cross-Sections.

The temperature rise between two test sections for each run has been calculated. In their results as to be found in Appendices G and H and are also illustrated in Figures 70-79. The maximum temperature rise is observed to be at 0.6 to 0.9 of the pipe radius. This phenomena may be accounted as the way of travelling of heat wave with the direction of flow. It is recalled that a similar phenomenon has been observed by Pannell (1) in his experiments on heating of air; the maximum temperature rise on his experiments varies at 0.85 to 0.95 R.



0·10
·05
0
$\frac{t_2 - t_1}{t_{w(ave)} - t_1}$
0·15
0·10
·05
0
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1·0
r/R
□ Run T H-28
Run T H-29
(Right & Lower Scale)
$V_{ave} = 4.43$ ft/sec
△ Run T H-26
Run T H-27
(Left & Lower Scale)
$V_{ave} = 2.81$ ft/sec

Fig. 70

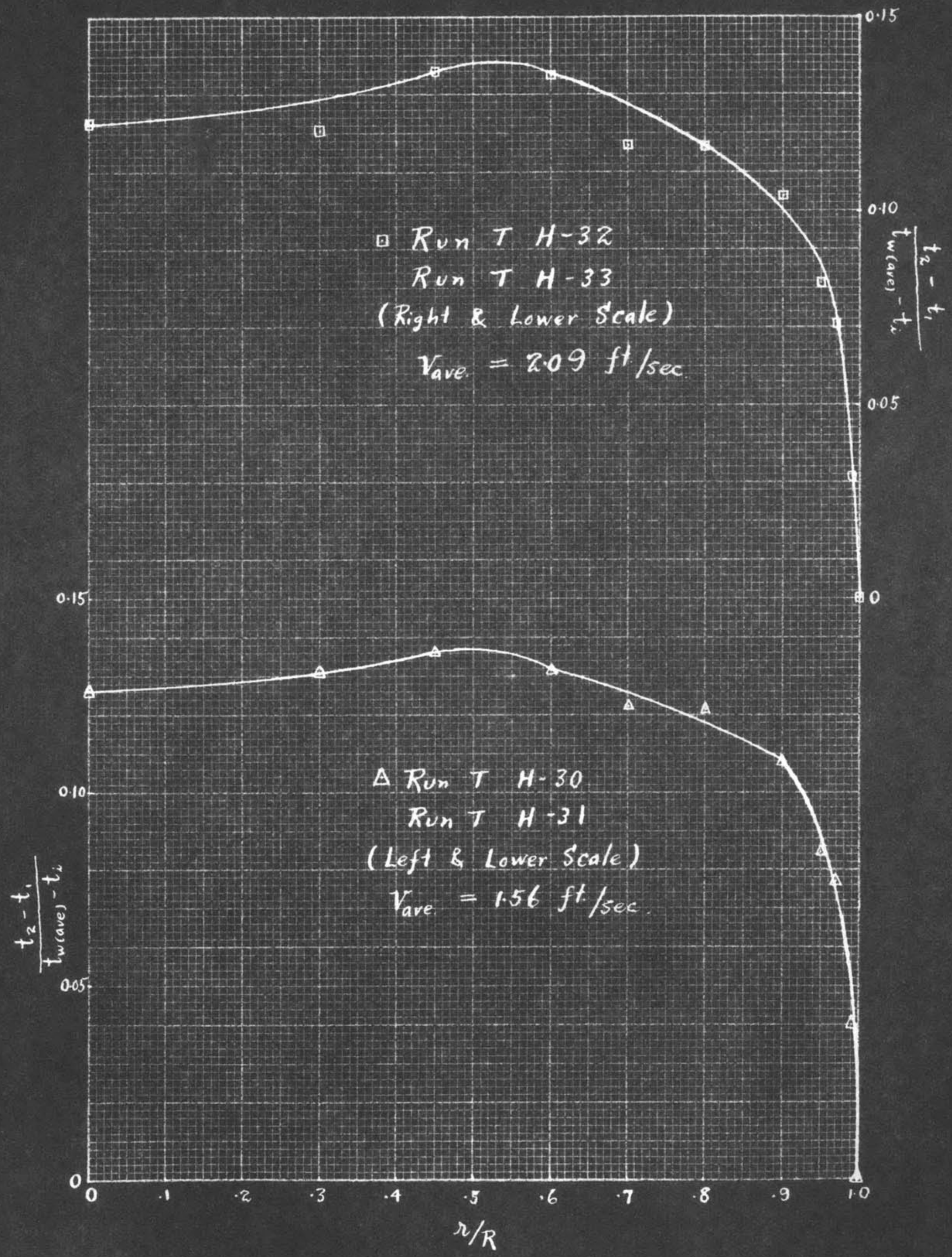
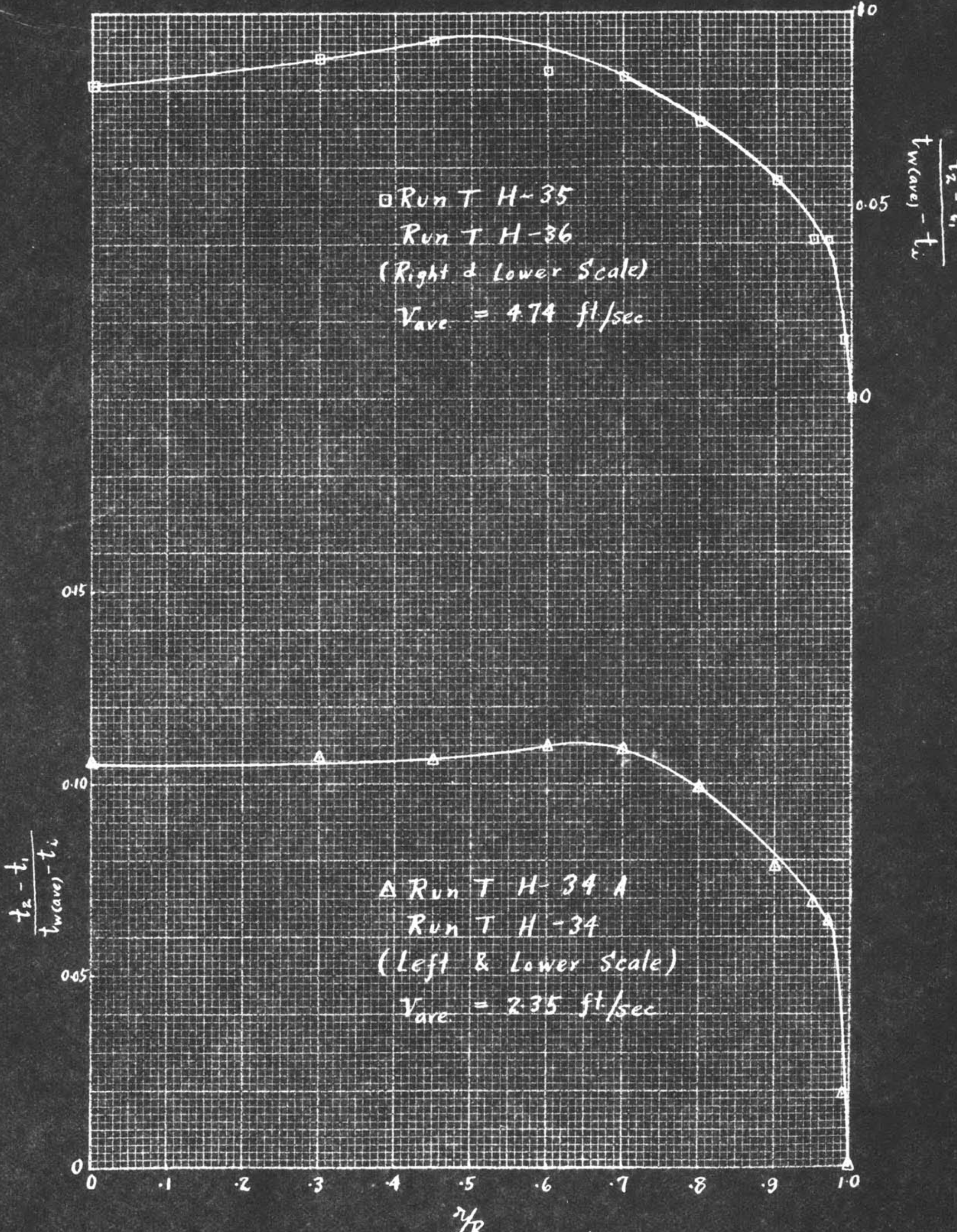

0.15
0.10
0.05
0
$\frac{t_2 - t_1}{t_{w(ave)} - t_1}$
□ Run T H-32
Run T H-33
(Right & Lower Scale)
$V_{ave.}$ = 2.09 ft/sec
0.15
0.10
0.05
0
$\frac{t_2 - t_1}{t_{w(ave)} - t_1}$
△ Run T H-30
Run T H-31
(Left & Lower Scale)
$V_{ave.}$ = 1.56 ft/sec
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1.0
r/R

Fig. 71



□ Run T H-35
Run T H-36
(Right & Lower Scale)
V_{ave} = 4.74 ft/sec
△ Run T H-34 A
Run T H-34
(Left & Lower Scale)
V_{ave} = 2.35 ft/sec
$\frac{t_2 - t_1}{t_{w(ave)} - t_i}$
$\frac{t_2 - t_1}{t_{w(ave)} - t_i}$
0.10
0.05
0
0.15
0.10
0.05
0
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1·0
r/R

Fig. 72

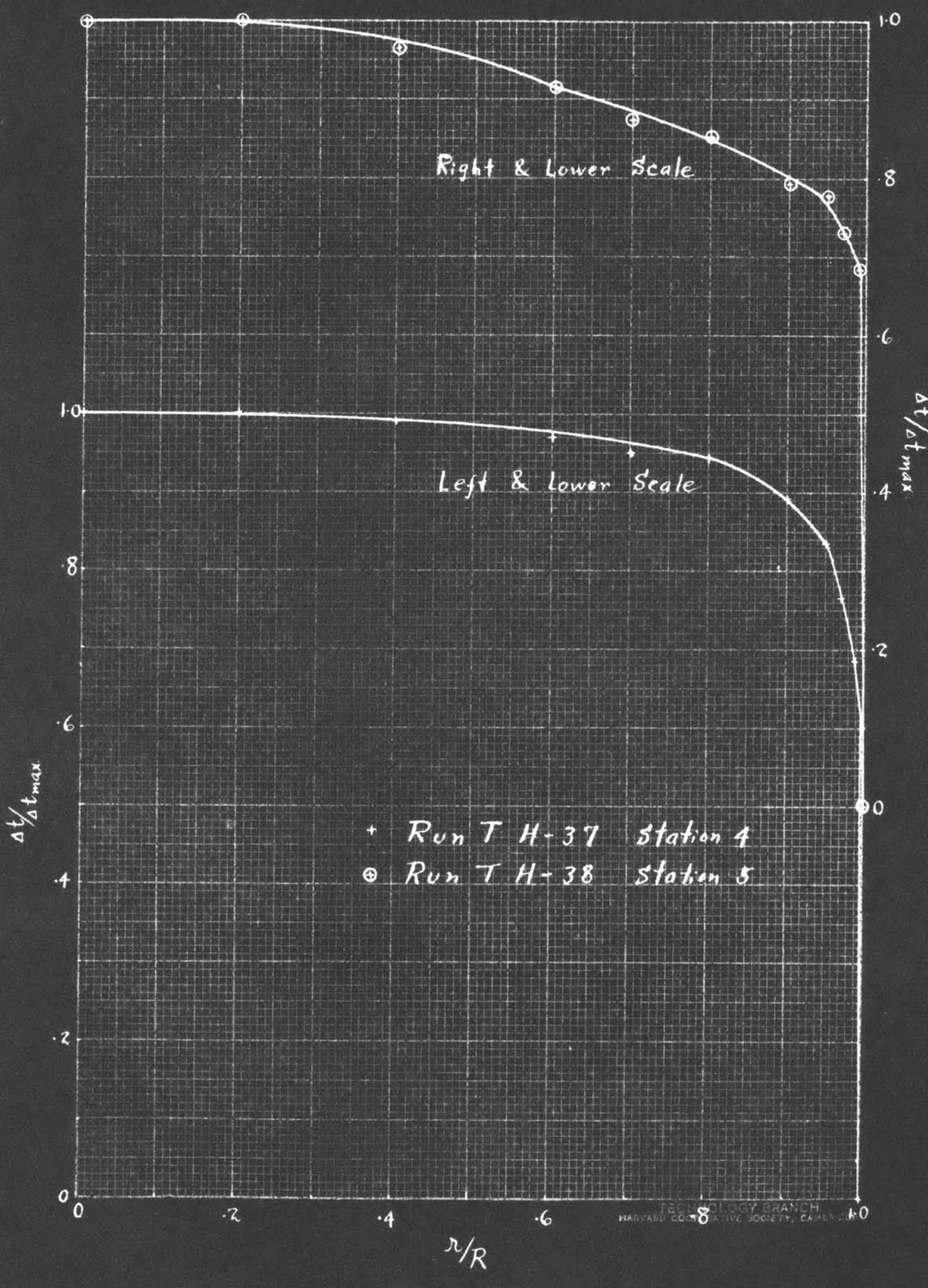

Right & Lower Scale
Left & lower Scale
+ Run T H-37 Station 4
⊕ Run T H-38 Station 5
Δt/Δt max
Δt/Δt max
r/R
0
·2
·4
·6
·8
1·0
0
·2
·4
·6
·8
1·0
0
·2
·4
·6
·8
1·0
TECHNOLOGY BRANCH
HARVARD COOPERATIVE SOCIETY, CAMBRIDGE

Fig. 73

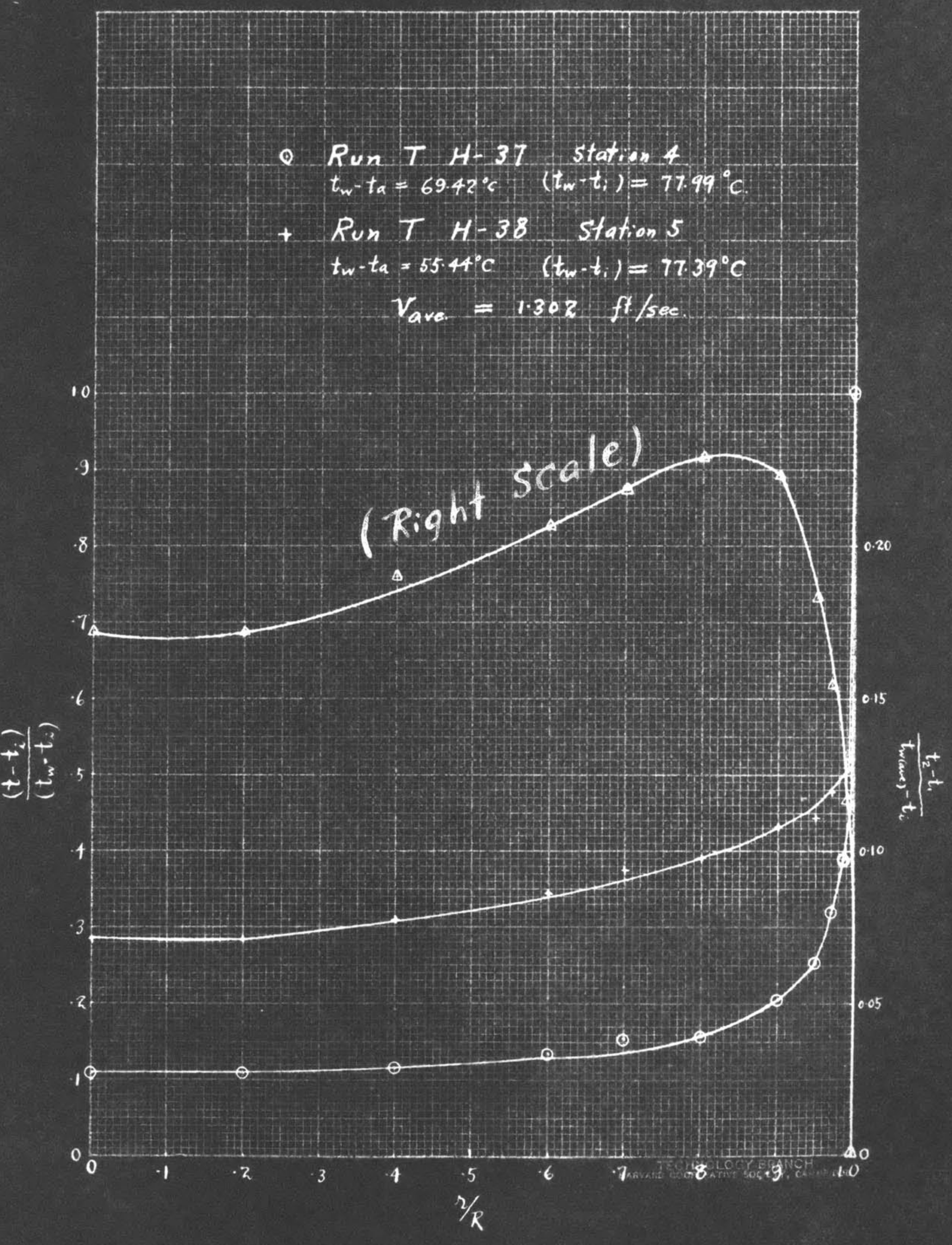

○ Run T H-37 Station 4
$t_w - t_a = 69.42°c$ $(t_w - t_i) = 77.99°C$.
+ Run T H-38 Station 5
$t_w - t_a = 55.44°C$ $(t_w - t_i) = 77.39°C$
$V_{ave.} = 1.302$ ft/sec.
(Right Scale)
$\frac{(t - t_i)}{(t_w - t_i)}$
$\frac{t_2 - t_i}{t_{w(ave)} - t_i}$
1.0
·9
·8
·7
·6
·5
·4
·3
·2
·1
0
0.20
0.15
0.10
0.05
0
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1.0
r/R
TECHNOLOGY BRANCH
HARVARD COOPERATIVE SOCIETY, CAMBRIDGE

Fig. 74

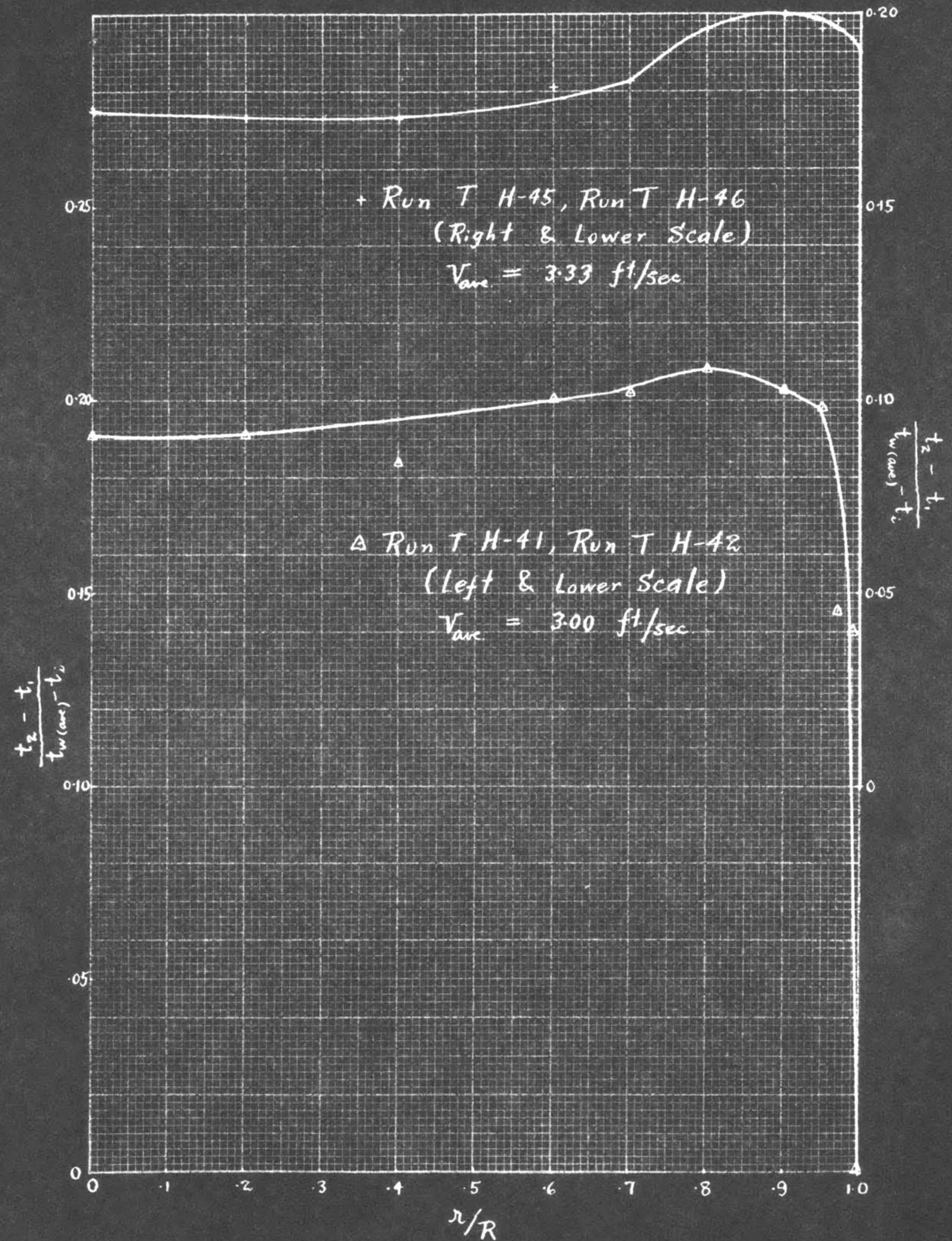

+ Run T H-45, Run T H-46
(Right & Lower Scale)
$V_{ave} = 3.33$ ft/sec
△ Run T H-41, Run T H-42
(Left & Lower Scale)
$V_{ave} = 3.00$ ft/sec
$\frac{t_2 - t_1}{t_{w(ave)} - t_1}$
0.25
0.20
0.15
0.10
.05
0
0.20
0.15
0.10
0.05
0
$\frac{t_2 - t_1}{t_{w(ave)} - t_1}$
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1·0
r/R

Fig. 75

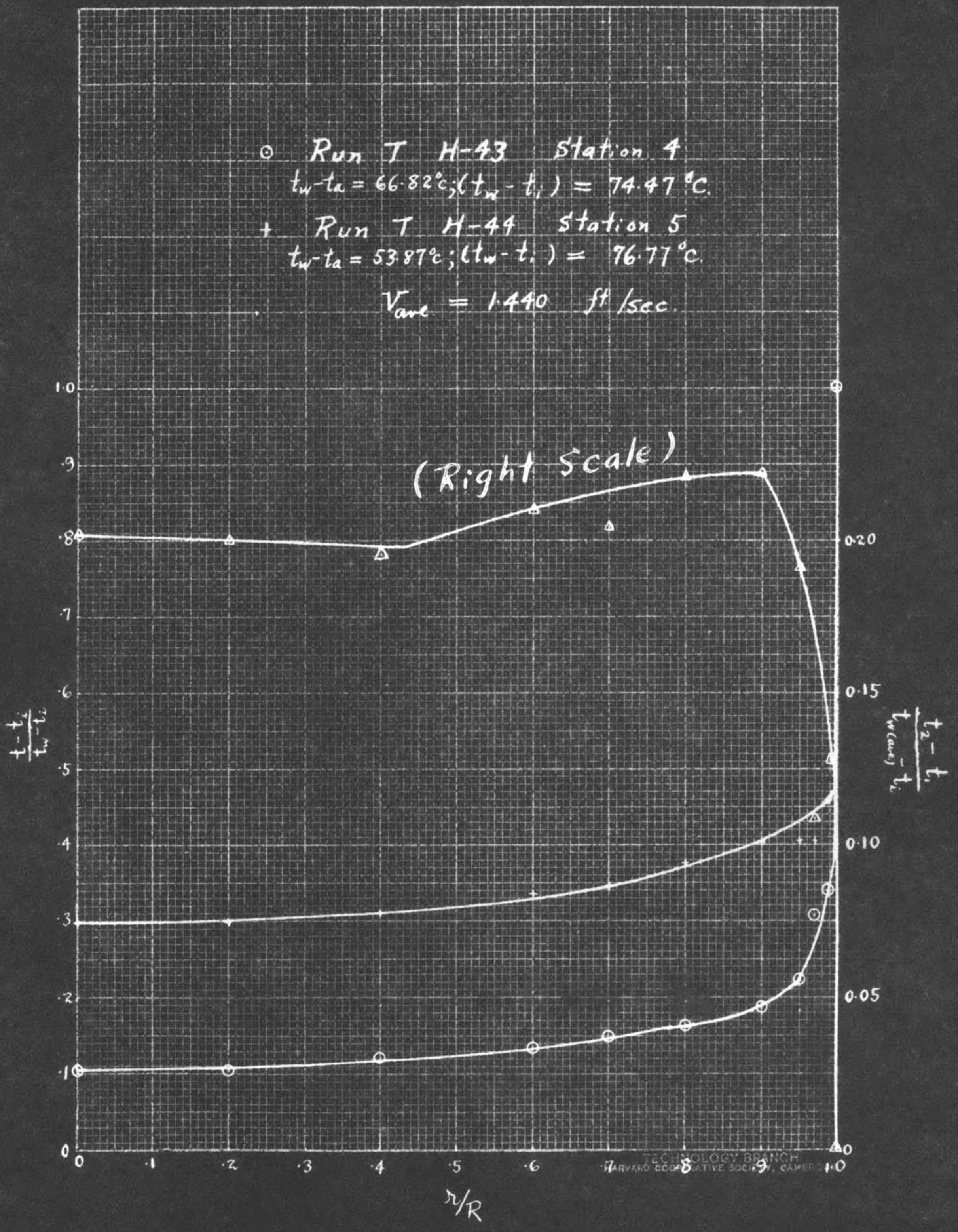

⊙ Run T H-43 Station 4
$t_w - t_a = 66.82°c; (t_w - t_i) = 74.47°C.$
+ Run T H-44 Station 5
$t_w - t_a = 53.87°c; (t_w - t_i) = 76.77°C.$
$V_{ave} = 1.440$ ft/sec.
(Right Scale)
$\frac{t - t_i}{t_w - t_i}$
$\frac{t_2 - t_i}{t_{w(ave)} - t_i}$
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1·0
0.05
0.10
0.15
0.20
r/R
TECHNOLOGY BRANCH
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Fig. 76

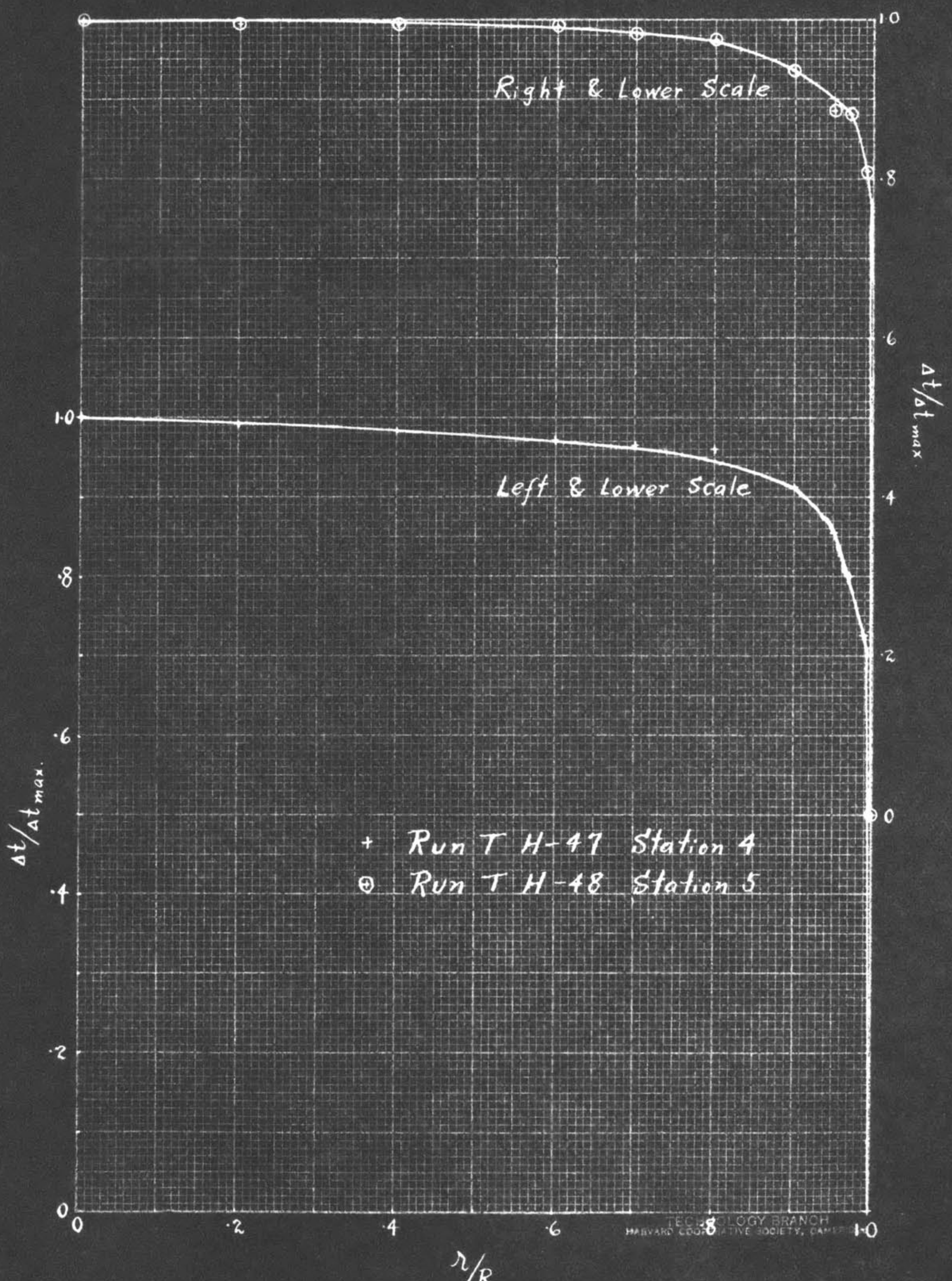
Right & Lower Scale
Left & Lower Scale
+ Run T H-47 Station 4
⊕ Run T H-48 Station 5
Δt/Δtmax.
Δt/Δtmax
1·0
·8
·6
·4
·2
0
0
·2
·4
·6
·8
1·0
r/R
TECHNOLOGY BRANCH
HARVARD COOPERATIVE SOCIETY, CAMBR

Fig. 17

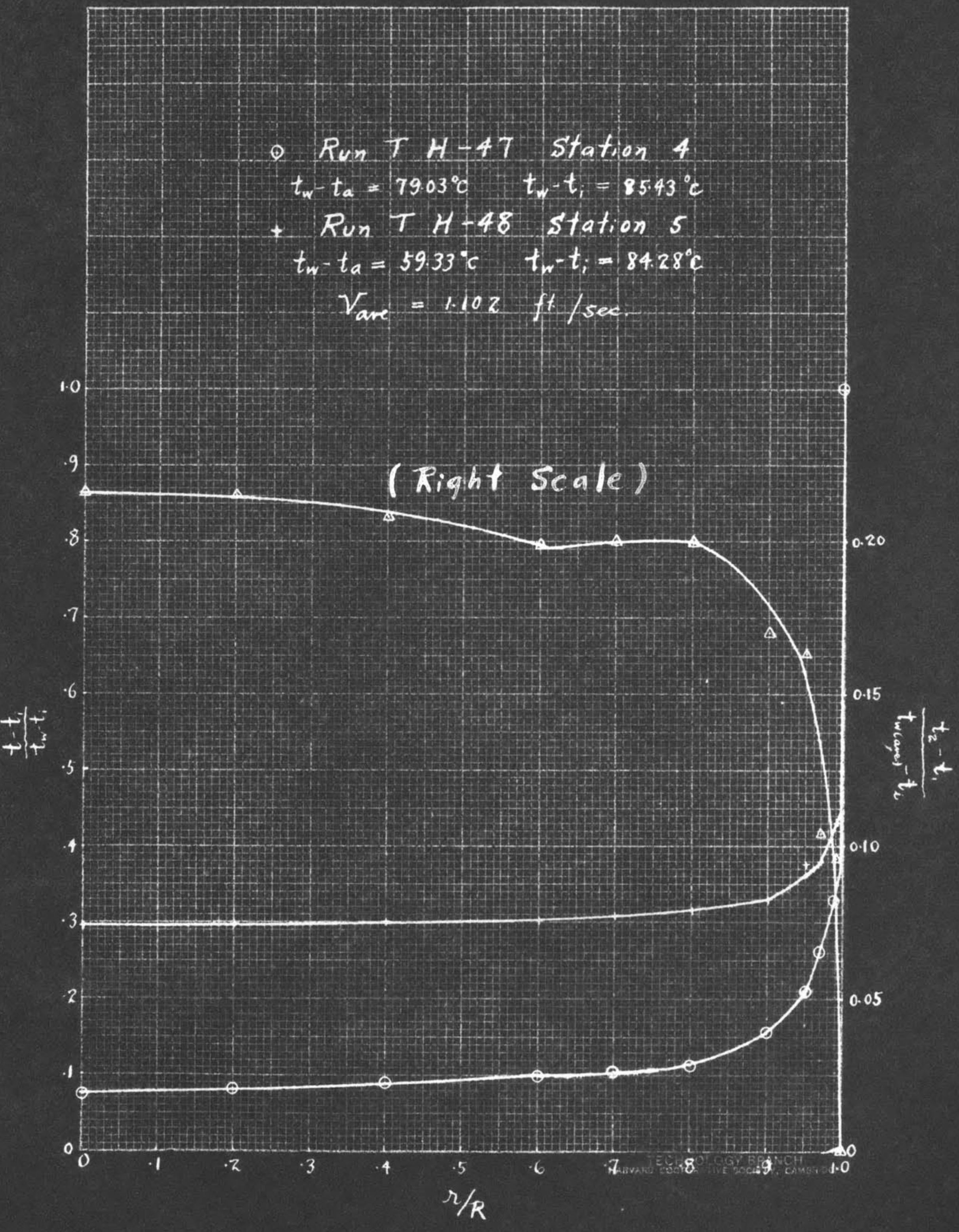
⊙ Run T H-47 Station 4
$t_w - t_a = 79.03°C$ $t_w - t_i = 85.43°C$
+ Run T H-48 Station 5
$t_w - t_a = 59.33°C$ $t_w - t_i = 84.28°C$
$V_{ave} = 1.102$ ft/sec.
(Right Scale)
$\frac{t - t_i}{t_w - t_i}$
$\frac{t_2 - t_i}{t_{w(ave)} - t_i}$
r/R
1.0
·9
·8
·7
·6
·5
·4
·3
·2
·1
0
0.20
0.15
0.10
0.05
0
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1.0

Fig. 78

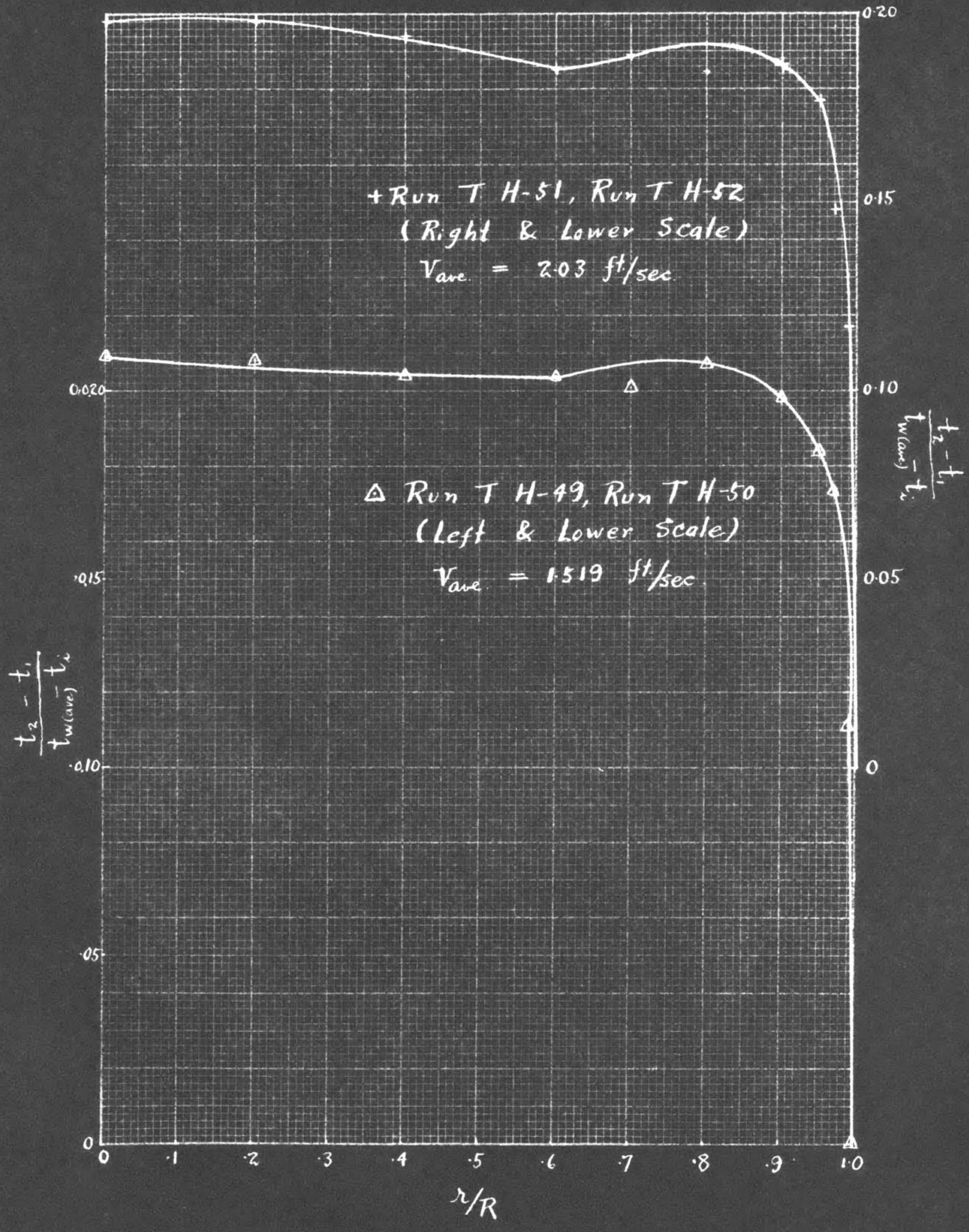
+ Run T H-51, Run T H-52
(Right & Lower Scale)
$V_{ave} = 2.03$ ft/sec
△ Run T H-49, Run T H-50
(Left & Lower Scale)
$V_{ave} = 1.519$ ft/sec
$\frac{t_2 - t_1}{t_{w(ave)} - t_1}$
$\frac{t_2 - t_1}{t_{w(ave)} - t_1}$
0.20
0.15
0.10
0.05
0
0.020
.015
.010
.05
0
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1·0
r/R

Fig. 79

Literature References

(1) J.R.Pannell: Tech.Reports Adv.Comm. for Aero. (Great Britain) Vol. I p. 22 (1916-17)

(2) Chapter II, this thesis.

(3) C.S.Keevil and W.H.McAdams, Chem. and Met. Vol. 36, p. 8 (1929)

(4) C.S.Keevil: Sc.D.Thesis, M.I.T. (1930).

(5) A. Eagle and R.M.Ferguson: Proc.Roy.Soc. Vol. 127A p. 806 (1930)

(6) A.Eagle and R.M.Ferguson: Proc.Institution Mech.Engrs.,p.985 (1930)

(7) ten Bosch: Z.V.D.I., Vol. 70, p. 911 (1926)

(8) L.Prandtl: Physik. Zeitschr. Vol. 29, p. 487 (1928)

(9) H.C.Sung: M.S.Thesis, M.I.T. (1932)

(10) J.Caldwell: Jour.Roy.Tech.College, Glasgow, p. 409 (1931)

(11) M.P.V. Iyer: Indian Journal of Physics, Vol. V, Part III (1930)

IX. Temperature Distribution Runs During Counter Current Cooling

Only six temperature distribution runs during counter current cooling were obtained. Their data and calculated results are shown in Appendix I. As it is mentioned in Chapter VI on Experimental Procedure, it is believed that the present apparatus is not sufficiently good to take cooling runs due to the practical impossibility of recirculating of the main line water without an addition of a series of auxiliary coolers to the present layout. The low temperature gradient between the pipe wall and the preheated water will not, it is believed, change the shape of isothermal velocity distribution to any appreciable extent, thus the lack of the simultaneous velocity distribution runs are not considered to be series.

From the summarized results of these runs in Table **20** , it is noticed that the temperature gradient from the pipe wall to the axis is very little, varying from 7.00 to 18.5° C., as compared with that in heating runs which have a gradient of as big as 79° C. Due to the small gradient and majority of the temperature drop occurs through the film, the temperature gradient in the runs TC-3 and TC-5 is almost negligible, thus the accuracy of the graphical method used in determining their temperature distribution exponents is greatly reduced. Therefore, the value determined graphically by one can hardly be checked by another; so in Sung's

Table 20.Summarized Results of Temperature Distribution Data During Countercurrent Cooling

(Column) (1) Run No.	(2) Station No.	(3) $V_{ave.}$ (ft./sec.)	(4) $t_{ave.}$ (°C.)	(5) $Re_{t_{ave.}}$	(6) t_a-t_w (°C.)	(7) $\frac{t_w-t_{ave}}{t_w-t_a}$	(8) $\frac{t_{ave}-t_a}{t_w-t_a}$	(9) b	(10) $Pe_{t_{ave.}}$	(11) $Pe_{t_f}(\frac{t_w-t_a}{t_{ave.}-t_a})$
T C-1	4	3.69	40.06	85,700	18.46	0.934	0.0719	0.0358	377,000	5,250,000
(T C-2	4	3.67	30.22	63,000	8.33	0.966	.0397	.021	346,000	8,720,000
(T C-3	5	3.67	28.33	67,650	9.60	0.972	.0289	.012	392,000	13,560,000
(T C-4	4	0.855	34.0	17,680	8.14	0.963	.0340	.0318	88,400	2,600,000
(T C-5	5	0.855	25.76	14,830	6.96	0.987	.0191	.00995	92,000	4,830,000
T C-6	4	0.769	35.73	16,310	12.41	0.954	.0491	.0318	78,300	1,600,000

thesis (1) these exponential values, graphically determined, are quite different from the values here adopted.

The correlation of the present data with the already secured heating data is shown in Figure 68 , plotting "b" against $P_e' \, t_f \left(\frac{t_w-t_a}{t_{av}-t_a}\right)$. The tendency of the decrease of "b" with the increase of latter group is also apparent. It is realized that some other efficient cooling agents such as brine or ammonia might be used, instead of cooling water, in order to build up a big temperature gradient over the pipe cross-section; the data thus obtained should be more comparable with the present heating data.

Literature Reference:

(1) H.C.Sung, M.S. Thesis, M.I.T., Course X (1932)

X. HEAT TRANSFER CALCULATIONS

From the results of velocity and temperature distribution measurements, heat transfer calculations are made possible. These calculations applied to sections which have been preceded by a reasonable length of heated pipe so that the temperature distribution has been fairly uniformly built up are considered to be more reliable. No great precision can, of course, be attributed to the heat transfer coefficients found in this way because the apparatus in use is not really well suited for the measurements of such quantities.

The inside heating area between Stations 2 and 3 is 2.55 sq.ft., while between Stations 4 and 5 is 5.103 sq.ft. In calculating temperature distributions, the inside wall temperature for each run has been estimated through overall heat balances, thus these temperatures can be readily used. The heat transfer equation reduces down to

$$\frac{Q}{\theta} = h\ A\ (\Delta t)_{ave.} = h\ A\ (t_w - t_{ave.})_{ave.} \qquad \ldots\ldots(1)$$

and

$$\frac{Q}{\theta} = 3{,}600 \times 1.8 \times 0.02075 \times 6.24\ V_{ave.}(\text{Temp.Rise}) \qquad \ldots(2)$$

$$\therefore \quad \text{For short section:} \quad h = \frac{8{,}370\ V_{ave.}\ \Delta\ T}{4.59(\Delta t)_{ave.}} \qquad \ldots(3)$$

$$\text{and} \quad \text{For long section:} \quad h = \frac{8{,}370\ V_{ave.}\ \Delta\ T}{9.18\ (\Delta t)_{ave.}} \qquad \ldots(4)$$

The average cross-sectional temperature, $t_{ave.}$, instead of mixing cup temperature, t_m, is used; this seems justifiable within the possible accuracy since the difference of these readings are employed in the calculation and $t_{ave.}$ and t_m differ from each other but little and always in the same direction. The calculated results are given in Table 21 and 22 .

In plotting the heat transfer coefficient against the modulus, $\frac{DUS}{Z}$, which is equivalent to $\frac{Re.}{7,728}$, $\frac{(\frac{hD''}{k})}{(\frac{CZ_m}{k})^{0.5}}$ and $\frac{(\frac{hD''}{k})}{(\frac{CZ_m}{k})^{0.4}}$ have both been tried. (See Figure 80). It is seen that according to the first group, the slope is found to be 0.82; while, according to the second group, the slope is found to be 0.84. Lawrence and Sherwood(1) proposed the heat transfer equation for water in copper pipes as follows,

$$\frac{hD}{k} = 550 \left(\frac{DUS}{Z}\right)^{0.7} \left(\frac{CZ_m}{k}\right)^{0.5} \qquad .....(5)$$

Thus, the present investigation taking the middle portion of the heated pipe for the calculation checks well with other workers.

Schiller(2) and Burbach(3) proposed a very simple equation as follows:

$$\frac{hd}{k_w} = 0.0395 \, (Pe'_w)^{0.75} \qquad .....(6)$$

Table 21. Results of Heat Transfer Calculations
(Stations No. 2 and 3)(Parallel Current)

Run No.	Temp. Rise °C.	Q/Θ	$\frac{\Delta t_{ave}}{°C.}$	h	$\frac{hD''}{k}$	$\frac{CZ}{k}$	$(\frac{CZ}{k})^{0.4}$	$\frac{\frac{hD''}{k}}{(\frac{CZ}{k})^{0.5}}$	$\frac{\frac{hD''}{k}}{(\frac{CZ}{k})^{0.4}}$	$\frac{DUS}{Z}$	$(Pe' t_{ave.})$
(T H-18 (T H-19	9.04	95,250	50.42	412	2295	1.454	1.35	1578	1700	3.24	128,200
(T H-20 (T H-21	2.95	100,200	42.28	515.5	2880	1.567	1.43	1838	2013	9.63	442,000
(T H-22 (T H-23	4.88	136,000	44.34	669	3730	1.542	1.414	2420	2640	7.82	348,000
(T H-24 (T H-25	5.56	181,300	48.29	818.5	4560	1.69	1.52	2700	3000	7.85	418,500
(T H-26 (T H-27	6.16	144,800	51.87	608.5	3390	1.668	1.505	2030	2250	5.78	300,800
(T H-28 (T H-29	4.00	148,300	51.86	624	3480	1.73	1.552	2010	2240	8.52	477,000
(T H-30 (T H-31	7.92	103,300	62.77	358	1995	1.854	1.64	1076	1216	2.635	169,400
(T H-32 (T H-33	7.92	138,200	61.31	491.5	2740	1.888	1.66	1451	1650	3.40	226,500
(T H-34A (T H-34	6.88	135,300	65.02	453	2525	1.932	1.694	1307	1490	4.05	283,000
(T H-35 (T H-36	4.90	194,200	59.61	710	3960	2.00	1.742	1980	2275	7.05	528,000

Table 22. Results of Heat Transfer Calculations

(Station No. 4&5) (Counter Current)

Run No.	Temp. Rise (°C.)	Q/θ	$\Delta t_{ave.}$ (°C.)	h	$\frac{hD''}{k}$	$\frac{CZ}{k}$	$(\frac{CZ}{k})^{0.4}$	$\frac{hD''/k}{(\frac{CZ}{k})^{0.5}}$	$\frac{hD''/k}{(\frac{CZ}{k})^{0.4}}$	$\frac{DVS}{Z}$	$(Pe't_{ave.})$
(T H-37 (T H-38	15.76	171,600	56.26	332.5	1853	1.496	1.380	1240	1343	3.21	134,600
(T H-39 (T H-40	9.7	307,500	38.97	861	4795	1.873	1.653	2560	2900	6.28	412,000
(T H-41 (T H-42	12.36	310,000	54.48	620	3455	1.761	1.573	1961	2195	5.59	324,000
(T H-43 (T H-44	14.64	176,400	55.18	348.5	1942	1.528	1.403	1271	1384	3.44	150,400
(T H-45 (T H-46	11.20	312,000	54.47	625	3483	1.807	1.603	1929	2170	5.93	361,500
(T H-47 (T H-48	15.12	139,400	65.88	231	1288	1.558	1.424	826	904	2.56	115,800
(T H-49 (T H-50	14.64	186,000	55.73	363	2023	1.570	1.434	1289	1410	3.475	159,800
(T H-51 (T H-52	12.28	208,800	55.33	412	2295	1.638	1.484	1401	1546	4.300	216,000

Figure 80

Heat Transfer Plot

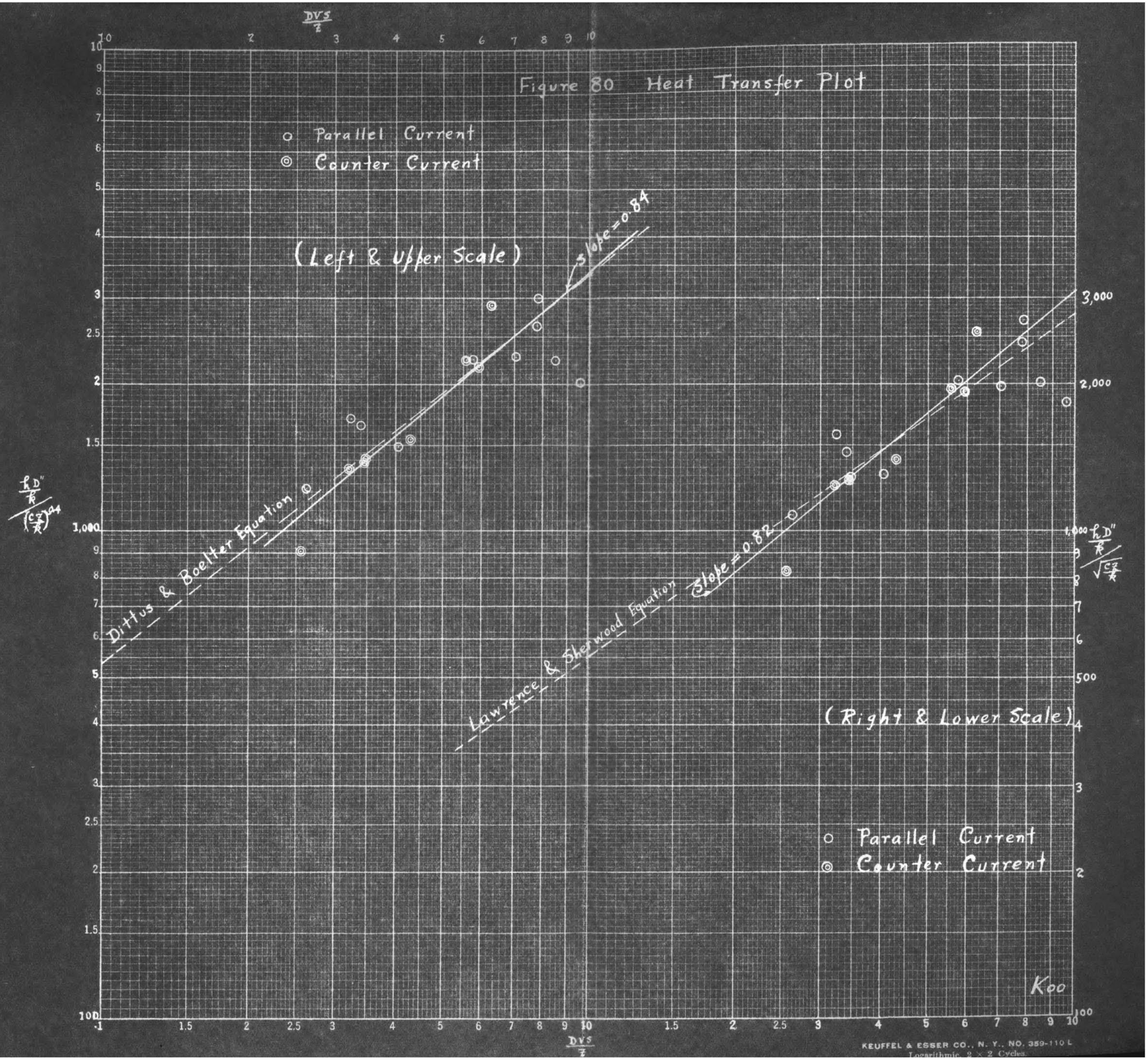

Figure 80 Heat Transfer Plot

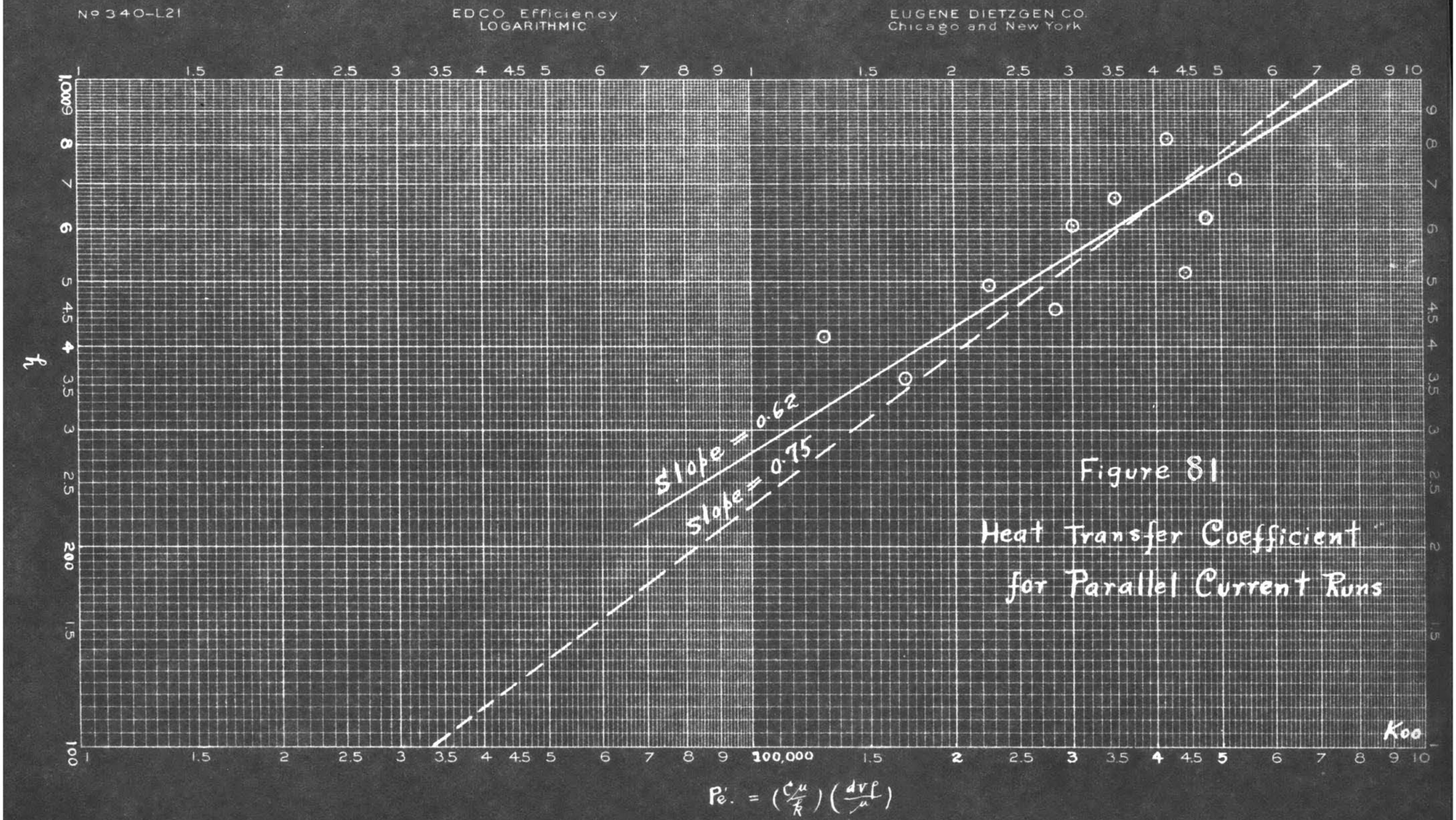

Figure 81

Heat Transfer Coefficient for Parallel Current Runs

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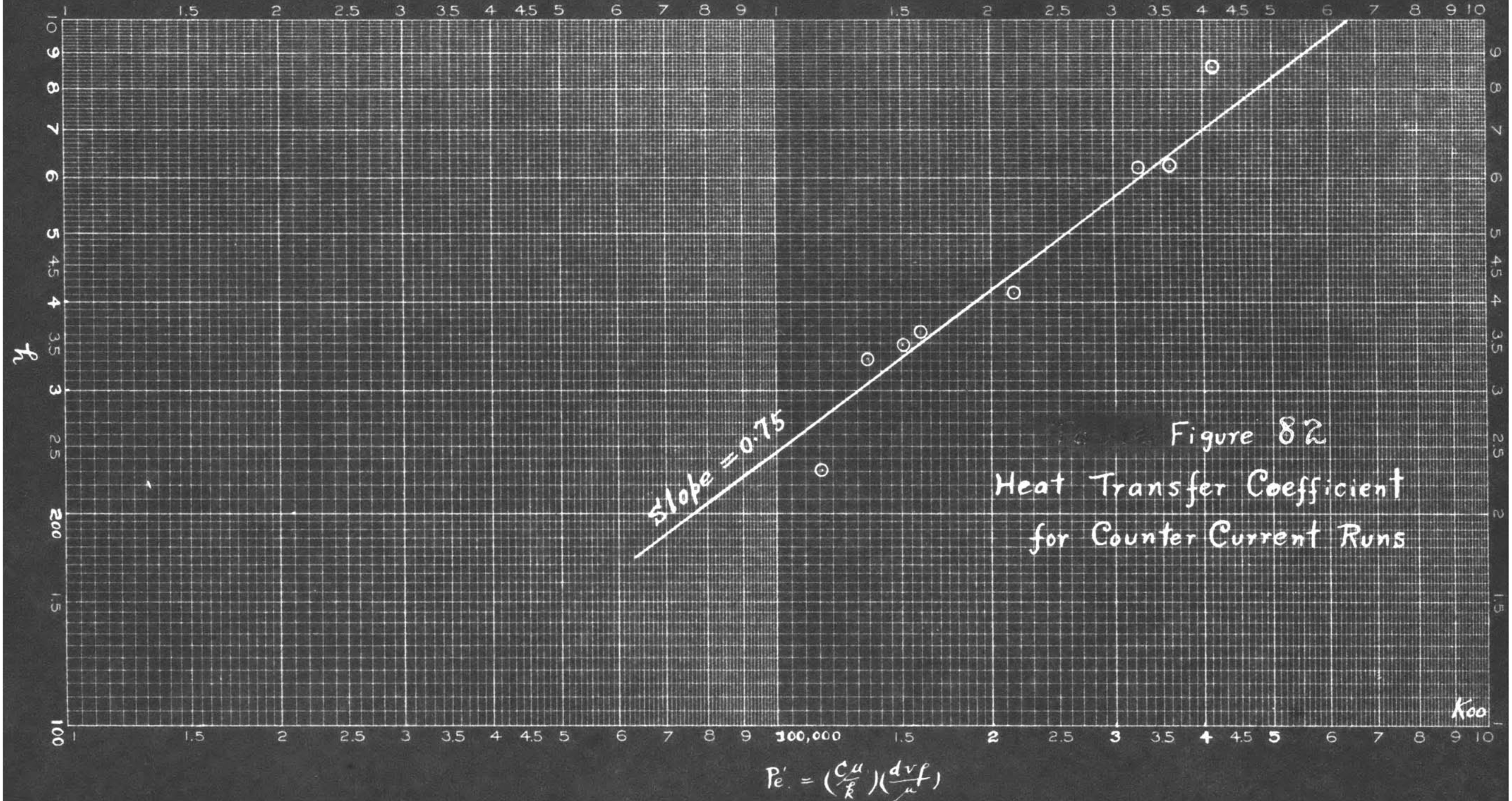

Figure 82
Heat Transfer Coefficient for Counter Current Runs

According to their equation, the plotting of heat transfer coefficient against the Peclet number ought to give a slope of 0.75. This method is applied to the present data, except Peclet number at average temperature is used instead of taking wall temperature. The 0.75 slope fits the present data very well, especially on the counter current runs. (Fig. 81-82).

Dittus and Boelter's equation on heat transfer (4) can be written as

$$\frac{hD''}{k} = 534 \left(\frac{DVS}{Z}\right)^{0.8} \left(\frac{CZ}{k}\right)^{0.4} \qquad \ldots\ldots(7)$$

The present investigation checks well with their equation as shown in Fig. 80.

The present data indicates that countercurrent runs have higher heat transfer coefficients at the same Reynolds number.

Literature References

(1) A.E.Lawrence and T.K.Sherwood: Ind. and Eng.Chem., Vol. 23, No. 3, p. 301 (1931).

(2) L.Schiller and Th. Burbach: Physik. Zeit., Vol. 29, p. 340 (1928).

(3) Th.Burbach: "Stromungsweiderstand und Wärmeübergang in Rohren", Akad. Verlag, Leipzig (1930).

(4) Dittus and Boelter: Univ.Calif.Publication in Engineering, Vol. 2, p. 443 (1930).

XI. CONCLUSION

From the experimental results on the simultaneous velocity and temperature distribution measurements of water when it is being either heated or cooled, as it is pumped through a pipe, it is definitely found that the velocity distribution exponent "a" in Eq.

$$\frac{V}{V_{max.}} = (1 - \frac{r}{R})^{a} \qquad \ldots\ldots(1)$$

and its corresponding temperature distribution exponent "b" in Eq.

$$\frac{\Delta t}{\Delta t_{max.}} = \frac{t_w - t}{t_w - t_a} = (1 - \frac{r}{R})^{b} \qquad \ldots\ldots(2)$$

are far from being equal; the latter exponent is always much smaller than the former one. Therefore, it is obvious that the form of the Reynolds analogy which states the similarity between the transfer of momentum and heat should not be applicable to liquids, although it is applicable to gases. From this non-similarity, it can be concluded indirectly that the mechanism of momentum transfer and heat transfer in case of liquids are very different. Consequently, any heat transfer theory which presupposes the validity of Reynolds analogy to liquids is erroneous.

From a critical survey of literature on friction factor problem, it is evident that the General Index Law form for the Fanning friction factor should be adopted,

$$f = a + b\ \mathrm{Re.}^{c} \qquad \ldots..(3)$$

It is recommended that the following equations should be used for smooth pipes,for commercial iron, and for steel pipes, respectively.

For Smooth Pipes, $f = 0.00140 + 0.125\ \mathrm{Re.}^{-0.32}$ ...(4)

and for Iron and Steel Pipes, $f = 0.00307 + 0.189\ \mathrm{Re.}^{-0.38}$ ...(5)

One might visualize the mechanism of flow better if one considers the constant a in Eq. (3) is due to pipe wall roughness, while the second term in that equation is due to the effect of laminar film at the wall.

From a critical survey of literature on the isothermal velocity distribution, a general empirical equation is recommended to calculate the velocity distribution exponent "a"

$$a = -1.5 \pm 0.5 \sqrt{9 - 8\left(\frac{\mathrm{Re.}df}{fd\ \mathrm{Re}}\right)} \qquad \ldots..(6)$$

It follows that for the velocity ratio,

$$\frac{V_{\mathrm{ave.}}}{V_{\mathrm{max.}}} = \frac{1}{1 - \frac{\mathrm{Re.}\ df}{f\ d\ \mathrm{Re}}} \qquad \ldots..(7)$$

The change of the distribution exponent and the velocity ratio with the Reynolds number is apparent from the above stated equations; this is supported by the experimental facts found in literature as well as by the results of the present investigation.

For non-isothermal flow, the velocity distribution exponent equation, Eq. (1), still holds true. It is found that if the film temperature of the water is taken in calculating the Reynolds number for non-isothermal flow, the velocity distribution exponential values correlate with the isothermal values found experimentally. From this fact, it is again indirectly concluded that the film temperature should be used in calculating Reynolds number for the non-isothermal friction factor.

Mechanisms of Isothermal and Non-Isothermal Flow of Fluids in Pipes.

Volume 2 - Appendix sections A - K (volume 2 contains separate page numbering from volume 1)

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(Volume 2)

MECHANISM OF ISOTHERMAL AND NON-ISOTHERMAL FLOW OF FLUIDS IN PIPES

APPENDIXES

APPENDIX A

TABULATION OF CALCULATED RESULTS OF ISOTHERMAL FRICTION DATA IN SMOOTH PIPES FROM PREVIOUS WORKERS.

1. J.R.Freeman
2. H.F.Mills
3. J. Nikuradse
4. H.Smith,Jr.
5. A.H.Gibson
6. C.Y.Hsiao

186841

John R. Freeman

Quoted by H.F.Mills

"The Flow of Water in Pipes" (1923)

First Series 17 Runs <u>Water in Smooth Drawn Brass Pipe</u>

$D = 2.108'' = 0.1755$ ft.

$t = 70°$ F.

Re	4f
11,420	0.0313
12,640	0.0309
16,700	289
30,200	2395
46,300	217
62,700	2025 →
98,600	183
137,000	171
171,400	163
209,000	1565 →
244,000	152
250,000	151
297,500	147
345,000	143
391,500	140
401,000	139
442,000	138

John R. Freeman

Quoted by H.F.Mills

Second Series 19 Runs Water in Drawn Brass Pipe

D = 3.067 in. = 0.2555 ft. t = 70° F.

$R_e = \frac{DV}{\nu}$	x = 4f
17,130	.0296
22,750	270
55,200	2135
108,600	182
146,000	1705
179,500	164
221,000	157
244,000	154
299,000	148
343,000	144
417,000	139
448,000	139
499,000	136
535,000	134
575,000	132
639,000	1295
678,000	129
715,000	128
748,000	126

John R. Freeman

Quoted by H.F.Mills

Third Series 23 Runs <u>Water in Drawn Brass Pipe</u>

$D = 4.00$ in. $= 0.333$ ft. $t = 70°$ F.

$Re = \frac{DV}{\nu}$	$x = 4f$
36,900	0.0235
50,200	214
52,900	212
113,000	178
184,500	161
239,000	154
239,300	151
284,000	148
302,000	147
371,000	139
374,000	.01405
435,000	137
480,000	134
537,000	130
580,000	130
622,000	129
700,000	131
722,000	125
778,000	124
808,000	.01255
845,000	123
888,000	121
908,000	.01225

H.F.Mills

"Flow of Water in Pipes" (1923)

Water in Smooth Drawn Brass Pipe D = 0.54 in. = .045 ft.

t = 70° F. 25 Runs

Re	4f
274	0.225
278	0.228
295	0.248
308	0.228
313	0.226
1188	0.0552
1210	0.0550
2360	0.0351
2390	346
2410	352
2580	411
3465	420
3585	414
4120	402
4530	391
5440	376
6160	368
7900	341
7920	343
8000	342
9190	330
10500	318
10430	319
11480	310
12680	302

J. Nikuradse: Water in Drawn Brass Pipe

Given as a plot in A. Giles, L. Hopf & Theo. v. Karman: Aerodynamik und verwandter Gebeite, Julius Springer, Berlin (1930) (Values in the following table read from an eight times enlarged plot)

4f	Re	4f	Re	4f	Re
.0219	41,200	.0147	285,000	.0113	1,100,000
214	45,100	145	303,000	114	1,120,000
212	48,800	145	286,000	114	1,150,000
206	52,800	143→	314,000	115	1,140,000
204	53,700	142	325,000	115	1,180,000
200	58,900	139	349,000	114	1,190,000
199	63,100	139	365,000	112	1,210,000
195	67,600	138	394,000	112	1,250,000
192	70,000	137	398,000	112	1,310,000
191	75,900	136	410,000	112	1,330,000
190	78,000	135	431,000	111	1,290,000
188	78,700	134	446,000	110	1,350,000
186	84,100	134	475,000	108	1,510,000
182	90,200	132	494,000	108	1,580,000
179	100,000	131	495,000	108	1,760,000
177	105,000	130	525,000	105	1,980,000
178	108,000	129	527,000	105	2,060,000
175	114,000	129	555,000	103	2,160,000
174	117,000	128	562,000	102	2,360,000
170	127,000	127	605,000	101	2,610,000
170	130,000	126	588,000	099	2,790,000
168	136,000	126	637,000	099	3,070,000
167	148,000	126	661,000		
165	148,000	125	668,000		
163	165,000	124→	689,000		
162	168,000	123	703,000		
162	173,000	123	728,000		
160	183,000	122	745,000		
158	182,000	122	776,000		
157	200,000	119	783,000		
156	206,000	120	813,000		
151	232,000	119	887,000		
151	242,000	119	925,000		
150	243,000	116	904,000		
149	263,000	119	986,000		
147	266,000	117	1,040,000		

Total 94 runs

Inlet Distance greater than 40D

Hamilton Smith, Jr.

"Hydraulics" (1886)

9 Runs Water in Smooth Glass Pipes t = 57-68° F.

Re	4f
32,800	0.0254
28,700	0.0262
34,200	0.0273
19,300	0.0288
12,800	0.0322
23,400	0.0277
19,550	0.0285
14,140	0.0309
7,450	0.0368

A.H.Gibson

Proc. Inst. Mech. Engr., p. 201 (1914)

15 Runs Water in Copper Pipes At 15° C. = 59° F.

Re	4f	Diameter (inches)
10,200	0.0316	0.751
20,400	273	"
30,600	2485	"
40,750	233	"
50,900	2195	"
13,560	298	0.998
27,100	258	"
40,650	236	"
54,200	223	"
67,800	214	"
20,400	257	1.500
40,750	221	"
61,100	205	"
81,500	193	"
101,800	185	"

C.Y.Hsiao: Sc.D. Thesis, Harvard Sanitary Eng.Dept. (1930)

Water in Glass Pipe

Glass Pipe No. 1 Entrance Length = 109D

Test no. G 1-3 1-16 D = 0.02747' = 0.33"

Re	4f
14,650	0.0300
13,570	312
12,840	314
12,100	322
11,520	322
10,900	328
10,400	328
9,800	330
9,170	335
8,430	348
7,800	354
7,200	357
6,660	365
6,130	373
5,650	379
5,060	390

Test No. G 1.10 1-17

36,710	0.0233
34,420	240
32,350	245
30,190	249
28,280	251
26,270	258
24,210	264
22,230	265
20,260	269
17,650	282
15,500	292
13,700	299
11,980	308
10,240	327
8,894	342
7,736	356
6,500	374

C.Y. Hsiao

Entrance Length = 137D

Glass Pipe No. 2 D = 0.02915' = 0.35" Test No. G 2-3 1-16

Re	4f
17,460	0.0284
16,490	285
15,500	290
14,720	294
13,990	296
13,310	298
12,450	312
11,890	314
11,320	320
10,470	325
9,765	332
9,078	340
8,405	342
7,593	354
6,822	362
6,074	368

Test No. G 2-1 1-14

13,200	308
12,550	311
11,820	312
11,200	318
10,520	324
9,840	329
9,000	339
8,380	344
7,700	351
7,040	359
6,400	368
5,840	379
5,240	390
4,800	394

C.Y.Hsiao

Copper Pipe No. 1

Test No. 1-1 1-22 D = 0.1331' 1.597"

Entrance Length = 75 D

Re	4f
10,660	0.0319
32,360	247
30,590	249
10,190	326
10,890	318
14,780	297
18,880	276
32,810	239
32,830	241
3,659	388
4,457	398
5,468	370
7,087	350
8,380	337
8,418	342
11,910	310
12,950	310
15,490	293
29,110	254
32,410	248
3,821	408
12,140	303

C.Y.Hsiao <u>Copper Pipe No. 16</u>

Test No. 16-1 1-16 D = 0.0688' 0.825"

Entrance Length = 145D

Re	4f
32,530	0.0251
33,460	239
24,570	268
24,580	267
15,320	294
15,360	293
12,490	316
12,470	315
9,444	337
8,860	341
5,853	388
5,838	383
4,660	410
4,622	417
3,716	431
3,696	427

Test No. 16-3 17-27

11,110	0.0328
9,900	346
8,775	354
6,981	377
5,937	394
4,970	412
4,543	424
4,082	425
3,532	450
3,132	455
2,749	470

C.Y.Hsiao

Copper Pipe No. 16

Test No. 16-11 1-21 D = 0.0688' = 0.825"

Re	4f
50,400	0.0216
45,910	224
41,450	227
37,660	236
33,920	238
30,510	240
27,700	250
24,610	259
22,500	272
21,310	278
20,200	274
17,790	289
16,300	291
15,050	300
13,150	304
11,750	311
10,090	337
8,305	354
7,140	368
66,970	204
80,850	198

Test No. 16-12 1-14

59,800	205
54,530	211
48,380	214
38,480	222
31,130	244
24,710	255
20,040	282
18,290	280
16,330	282
13,760	298
12,210	312
10,560	321
8,603	346
86,180	1845→

C.Y.Hsiao

Copper Pipe No. 16

Test No. 16-22 1-10 D = 0.0688' = 0.825"

Re	4f
29,100	.0256
25,040	261
20,480	275
15,910	298
11,320	324
8,133	354
6,134	382
4,660	405
3,650	438
2,686	463

Test No. 16-17 1-8

48,070	.0224
33,340	246
27,110	258
20,760	276
15,190	306
9,694	344
5,943	390
4,270	423

C.Y.Hsiao

Lead-Lined Galvanized Wrought Iron Pipe

Test No. 12-1 1-18 D = 0.0559' 0.671"

Entrance Length = 145 D

Re	4f
29,890	0.0253
29,130	264
27,970	259
24,560	272
21,700	278
17,350	294
14,440	309
10,190	342
12,410	312
10,500	324
9,265	324
7,940	348
7,105	360
6,174	366
5,327	386
4,122	410
3,436	452
2,897	428

APPENDIX B

TABULATION OF CALCULATED RESULTS OF ISOTHERMAL FRICTION DATA IN COMMERCIAL IRON AND STEEL PIPES FROM PREVIOUS WORKERS

1. H. Darcy
2. H.Smith,Jr.
3. J.B.Francis
4. J.R.Freeman
5. E.W.Schoder
6. C.Eberle
7. F.W.Greve,Jr.
8. C.I. Corp and R.O.Ruble
9. C.I.Corp and H.T.Hartwell
10. **F. Carnegie**

H. Darcy

Quoted by H.F.Milles "Flow of Water in Pipes" (1923)

Water in New Smooth Cast Iron Pipe 10 Runs

D = 0.450 ft. = 5.395" t = 59° F.

Re	4f
17,900	0.0291
35,900	0.02634
58,800	236
91,800	220
154,000	208
206,000	204
252,000	203
274,500	202
439,000	200
565,000	205

H. Smith, Jr.

"Hydraulics" John Wiley, N.Y. (1886)

Water in New Wrought Iron Pipe

I D = 0.0878 ft. = 1.052" t = 57 to 68° F.
(No funnel)

4f	Re
0.0253	40,100
260	35,200
269	29,800
281	23,900
314	16,200
344	10,720
461	7,220

II D = 0.0878 ft. = 1.052" (with funnel shaped mouth piece)

0.0253	40,600
260	35,400
269	30,100
280	24,300
309	16,560
342	10,810
400	7,940

VI D = 0.0523 ft. ~~ft.~~ = 0.628" (no funnel)

0.0299	17,400
309	15,180
320	12,830
338	10,300
370	7,070
417	4,610

J.B.Francis

(As quoted by H.F.Mills)

Water in New Wrought Iron Pipes

I. D = 0.801 in. = 0.06675 ft. t = 65.5° F.

Re	4f
9,110	0.0305
9,120	304
16,100	266
16,150	264
21,250	250
21,300	249
25,700	2395
25,800	2365

II. D = 1.033" = 0.0861 ft. t = 66.5° F.

Re	4f
13,960	0.0282
14,030	282
24,500	250
24,700	248
32,200	235
32,600	234
39,200	225
39,250	224

J.B.Francis

(As quoted by H.F.Mills)

Water in New Wrought Iron pipes

III. D = 1.531" = 0.1278 ft. t = 71° F.

Re	4f
21,850	0.0256
23,000	241
33,800	2375
33,950	2355
42,550	234
42,600	226
50,300	221
50,500	222
50,700	221
55,600	217
55,700	216
61,700	213
61,850	2115
67,700	209
68,000	209
68,200	209
72,850	206
73,300	207
73,700	204
76,900	204
77,100	2035

J.B.Francis

(As quoted by Mills) Water in New Wrought Iron Pipes

IV. D = 2.03" = 0.169 ft. t = 70.5° F.

Re	4f
50,000	0.0230
50,450	2275
66,300	220
66,500	220
73,750	2185
74,000	216
74,100	217
84,200	214
84,400	2135
91,000	212
91,100	211
101,000	208
101,300	2075
109,200	2065
109,500	2065
114,500	2085
115,500	205
116,000	203
116,100	203
117,000	200

J.R.Freeman 1.(12 Runs) D = 0.624" = 0.052 ft.

Water in New Wrought Iron Pipes

(Quoted by H.F.Mills) t = 70° F.

Re	4f
457	0.1488
749	.0892
1 ,502	462
2,465	448
4,300	425
5,670	425
7,150	3985
8,500	392
10,400	376
11,150	371
12,000	370
13,520	350

J.R.Freeman

(As quoted by H.F.Mills)

Water in New Wrought Iron Pipe

2. D = 0.816" = 0.068 ft. t = 70° F.

Re	4f
1,270	0.0587
3,345	460
7,260	399
15,330	341
24,750	308
33,600	292
42,500	282
51,000	276
61,400	269
71,000	266
82,100	2605
92,500	256

3. D = 1.061" = 0.0885 ft.

976	0.0676
2,565	280
5,590	408
11,790	362
19,000	310
25,800	304
32,600	281
39,200	284
47,150	276
54,500	271
63,000	266
71,000	2565

John R. Freeman

Quoted by H.F. Mills

"Water in New Wrought Iron Pipe"

t = 70°F.

4. D = 1.387" = 0.1155 ft.

Re	4f
5500	0.0472
9700	366
24800	290
34000	271
40500	260
60600	246
83000	2355 →
100000	234
121000	2273 →
147800	224
176500	222
190200	219

5. D = 2.093" = 0.1745 ft.

Re	4f
4520	0.0347
13200	3145 →
35700	251
47600	2445 →
82800	227
110600	220
138700	2155 →
166000	2105 →
203000	2035 →
240500	203
257000	199

John R. Freeman

Quoted by H.F. Mills

Water in New Wrought Iron Pipe

t = 70°F.

6. D = 2.503" = 0.2085 ft.

Re	4f
12,000	0.03085
39,500	246
56,500	231
78,900	2245
89,500	2155
120,400	2085
159,800	201
187,300	1965
251,500	1922
298,000	186
346,000	1823
380,000	1875
414,500	1782

7. D = 3.115" = 0.2595 ft.

Re	4f
8,260	0.0326
10,640	3135
13,270	316
17,680	299
35,200	254
58,800	229
80,400	2225
113,800	2115
127,400	208
179,100	201
227,500	195
269,000	191
356,000	185
419,500	182
469,500	1815
518,500	177
555,000	175

John R. Freeman

Quoted by H.F. Mills

Water in New Wrought Iron Pipe

t = 70°F.

8. D = 4.123" = 0.344 ft.

Re	4f
2,950	0.0541
4,360	4305
16,270	309
21,100	2795
25,650	2675
42,400	2395
53,900	222
60,150	2495
60,400	249
64,700	2215
86,100	2086
101,800	228
117,500	201
141,000	199
154,500	195
177,000	1785
178,000	1926
254,000	1875
306,500	186
381,000	181
455,000	1765
559,000	1734
625,000	173
634,500	175
735,000	170
764,000	169

John R. Freeman

Quoted by H.F. Mills

Water in New Wrought Iron Pipe

t = 70°F.

9. D = 5122" = 0.427 ft.

Re	4f
20,600	0.02515
28,000	250
39,500	241
78,000	2055
83,800	210
159,100	193
312,500	1825
405,000	1783
471,000	176
540,500	176
589,000	174
647,500	1745
724,000	172
830,000	1715
832,500	170

John R. Freeman

Quoted by H.F. Mills

Water in New Wrought Iron Pipe

t = 74°F.

10. D = 6.144" = 0.512 ft.

Re	4f
51,000	0.0241
99,700	213
125,000	200
147,500	196
245,500	1834
290,000	1688
377,000	175
486,000	1748
574,500	1684
609,000	165
797,000	1657
863,000	1655

11. D = 8.05" = 0.670 ft. t = 69.5°F.

Re	4f
31,700	0.0252
61,250	218
93,100	2065
125,000	199
194,500	189
252,000	184
342,500	172
366,000	182
370,000	182
391,000	177
426,500	1776
451,000	177
471,000	176
484,500	179
514,500	165
520,000	172
574,000	169
585,000	175
654,000	1556
739,000	167
776,000	1644
815,000	171
820,000	170

E.W. Schoder

Trans. Amer. Soc. Civil Engrs. Vol. 62, p. 67 (1909)

Water in 6" Wrought Iron Pipe

D = 6.075" = 0.506 ft.

<u>A</u>. I. <u>Pipe Length = 99.33 ft.</u>
Calming Length = 40.1 D
II. <u>Pipe Length = 46.10 ft.</u>
Calming Length= 76 D

t = 69°F.

Re	4f	4f
286,000	0.01716	0.01713
269,000	172	1718
247,500	1743	1742
227,500	177	1765
203,300	1783	1782
184,000	181	181
168,200	1832	1832
136,000	1885	1885
107,700	1955	1960
76,300	2025	202
<u>B</u>. t = 33°F.		
443,500	1638	1636
435,000	1640	1640
382,000	1642	1640
309,500	1674	1675
248,000	1683	1682
185,300	1672	1672

C. Eberle

(Forschungsarbeiten,Heft 78 (1909)).

Steam in Wrought Iron Pipe

D = 7.0 cm. = 2.73"

Corrected for Elbows, Equivalent length of 90°Elbow = 30 D.

Re	4f	
580,000	0.0179	3 elbows
687,000	191	
162,600	184	2 elbows
194,000	195	
150,000	201	
169,000	195	
212,000	182	Calming Length = 164 D
215,000	183	
220,500	181	
319,000	185	
330,000	188	
274,500	179	
250,500	184	

F.W. Greve, Jr.

Purdue Univ. Eng. Experim. Station, Bull. No. 1 (1918)

Water in Black Commercial Water Pipe

13. D = 1.6076" = 0.134 ft. Calming Length = 151.5 D
t = 59°F. Length = 40.513'

Re	4f
4,800	0.0330
9,940	309
20,950	283
22,600	278
32,200	249
32,200	254
39,750	248
39,750	2465
46,200	2335
46,500	229
52,950	2255
52,950	2275
58,300	2253
58,300	227
67,400	216
67,400	2165
91,800	2055
91,700	2065
109,000	198
109,700	1985
120,800	194
122,000	1932
138,000	1918
138,100	1920
148,000	190
148,400	1905
153,400	214
154,400	191
170,200	1878
171,300	1868
183,000	1862
184,200	1863
190,000	1864
191,000	1862
197,200	1868
199,000	1848
208,000	1843
208,000	1848
217,300	184
217,400	184
226,000	1845
227,000	184

C.I. Corp. and R.O. Ruble

Univ. Wisconsin Bull., Vol. 9, No. 1 (1922)

I. 12" Pipe Water in Wrought Iron or Steel Gas and Water Pipe

(New Clean)

14. D = 0.999 ft. = 11.99 in. t = 70°F.

(Calming Length > 40 D)

Re	4f	Re	4f
283,000	0.01503	844,000	0.0167
635,000	166	879,000	1719
460,000	1639	1,079,000	1676
736,000	168	951,000	1724
594,000	161	96,500	1839
796,000	171	366,000	1585
890,000	1725	250,000	1665
1,042,000	156	1,028,000	1633
1,098,000	152	1,059,000	153
738,000	161	1,069,000	1527
1,116,000	1525	1,010,000	1653
904,000	174	1,017,000	1623
787,000	1643	960,000	1741
644,500	1594	895,000	1718
491,000	160	835,500	1678
361,500	1624	788,000	1672
250,000	1667	726,000	1646
828,000	1645	667,000	163
909,000	1719	610,500	161
960,000	174	532,000	1669
904,000	1718	358,000	1661
1,086,000	1572	273,500	170
970,000	1737	1,098,000	1534
994,000	146	1,056,000	1592
897,000	149	994,000	1703
732,000	1614		
594,000	1581		
405,000	1615		
1,059,000	148		
990,000	1779		
920,000	1732		
909,000	1733		
810,000	164		

C.I. Corp. and H.T. Hartwell

Univ. Wisconsin Bulletin Engineering Series No. 66 (1927)
Water in New Wrought Iron or Steel Pipe (Temp. definitely known)

I. 1" Pipe D = 0.981" to 1.062" (8 Pipes)

Re	4f
15,180	0.0301
30,400	262
38,400	2526
45,700	2485
51,000	245
57,700	242
15,900	294
20,800	284
28,200	269
34,100	262
39,300	254
43,700	257
32,800	274
47,200	253
59,000	249
68,500	2475
119,700	234
105,400	234
98,300	237
79,900	239
91,500	239
112,300	234
25,450	294
38,200	280
47,900	272
57,700	262
65,500	2605
80,100	251
89,000	2475
73,300	256
81,100	258
93,800	249
98,800	245
102,700	250
23,550	272
46,800	2525

C.I. Corp. and H.T. Hartwell

Water in New Wrought Iron or Steel Water Pipe (Temp. definitely known)

II. 2" Pipe D = 2.05, 2.09 - 2.10"

Re	4f	Re	4f
182,000	0.0179	66,100	2025
177,000	1796	91,300	1915
171,000	180	127,000	←181
166,000	181	153,000	1674
160,400	183	213,000	1685
154,000	1843	236,000	165
145,800	1865	112,000	185
138,000	189	149,000	175
130,800	191	176,500	1705
119,200	195	209,500	1683
112,200	194		
104,000	197		
96,100	200		
86,300	2035		
80,200	2045		
72,900	2075		
62,300	211		
50,500	217		
38,000	2215		
114,200	215		
107,800	211		
102,900	2205		
98,000	218		
93,000	2185		
85,200	223		
77,100	2195		
68,600	229		
63,600	228		
56,200	230		
50,000	240		
42,600	246		
37,600	252		
31,300	265		
25,000	288		
19,200	324		
240,000	1677		
226,500	1694		
214,000	1712		
207,000	1723		
192,400	1754		
178,500	1782		
160,600	183		
144,000	184		
121,600	186		
97,300	191		
58,800	211		

C.I. Corp. and H.T. Hartwell

Water in New Wrought Iron or Steel Water Pipe (Temp. definitely known)

III. 4" Pipe   D = 4.025, 4.03, 4.02" (4 Pipes)

Re	4f	Re	4f
41,200	0.02265	298,500	0.0178
59,800	219	308,000	174
74,900	206	331,000	174
87,600	200	316,000	173
106,700	196	322,000	173
108,000	194	47,000	272
121,300	193	63,500	258
143,700	1896	81,400	248
142,200	195	96,500	2405
172,700	1797	109,300	238
168,000	1862	120,800	229
177,000	1933	129,800	228
189,000	182	142,200	224
186,200	1815	147,300	2265
197,300	1825	156,000	227
196,800	183	171,600	223
210,500	180	185,600	219
224,000	1803	200,800	219
59,000	218	216,000	2155
128,000	1765	226,000	216
152,000	189	245,000	210
153,000	187	259,500	210
178,000	190	274,500	2075
194,000	181	288,000	210
216,000	1828	308,500	206
206,700	1836	325,000	207
222,000	180	343,500	205
232,000	181	364,000	202
249,000	1762	80,500	153
264,000	178	102,400	192
280,000	172	124,300	165
288,000	172	136,100	132
273,000	173	154,000	194
280,500	176	172,500	200
292,000	1732	185,000	1995

C.I. Corp. and H.T. Hartwell (Cont.)

Re	4f
204,000	0.0198
216,500	150
233,000	196
250,500	197
194,000	1995
139,000	209
104,500	2135
79,900	2165
241,000	199
233,500	2025
204,500	211
176,300	2205
209,000	213
167,000	229
209,000	182
194,700	202
172,800	206
120,000	210
140,300	2075
89,300	2125
107,400	208
134,800	203
158,200	198
169,300	218
183,800	212
172,400	212
159,000	217
161,000	219
147,000	224
123,400	227
133,500	222
102,500	248
83,750	292
201,000	206
218,500	211
209,000	213
192,200	216
235,000	214
105,800	250
88,400	284
64,700	272
126,000	2605
230,000	226
229,500	226

C.I. Corp. and H.T. Hartwell

Water in New Standard Wrought Iron or Steel Water Pipe

IV. 6" Pipe I.D. = 0.509 ft. = 6.11 in.

Temp. of H_2O given.

Re	4f
806,000	0.0157
861,000	154
795,000	1558
745,000	1588
694,500	1632
653,000	1632
619,000	163
571,000	1633
524,500	1658
447,000	1632
385,000	1745
316,000	175
257,500	1808
167,000	1788
845,000	1547
805,000	1550
762,500	1575
724,000	158
671,000	158
630,500	164
583,000	1612
531,000	1691
472,000	1675
411,000	1695
359,500	1715
294,000	1781
234,000	1801
175,400	2595
635,000	1546
579,000	1580
533,000	1602
483,500	166
425,000	1652
365,000	1684
295,000	1720
242,500	1604
164,000	1810
75,900	223

F. Carnegie   Institution of Mechanical Engineers -
Proceedings   p. 473 (1930).

Steam in Steel Pipes
(H. Speyerer's Data on Viscosity of Steam (Z.V.D.I. Vol. 69, p. 747) was used.

A. D = 8" = 0.75 ft.   Test Length = 221.6 ft.
Pressure = 189 to 220 lbs. per sq.in. abs.
Temperature = 246-251°C.

Re	4f	Pipe Material	Remarks
483,300	0.01728	Solid-drawn steel pipes with welded flanges.	Quantity of steam measured by steam-metering system.
871,800	.01732		
1,022,000	.01512		
1,128,000	.01520		
1,357,000	.01300		
1,695,000	.01344		
2,351,000	.01396		
2,269,000	.01396		
2,488,000	.01512		
1,913,000	.01356		
1,708,000	.01548		
1,858,000	.01432		
1,209,000	.01588		

B. D = 6" = 0.50 ft.   Test Length = 130.0 ft.
Pressure = 82.5-88.5 lb./sq.in. abs.
Temperature = 196-199°C.

Re	4f	Pipe Material	Remarks
258,800	0.02112	Hot-rolled steel pipes.	Quantity of steam measured by steam-metering system.
459,500	.01988		
515,200	.02092		
649,000	.01940		

C. D = 1.98" = 0.165 ft. (actually measured); Test Length= 100 ft.
Pressure = 66.5-78.5 lb./sq.in. abs.
Temperature = 170-191°C.

Re	4f	Pipe Material	Remarks
131,200	0.02168	Ordinary welded steam-barrel pipe.	Quantity of steam measured by actual condensation.
244,500	.01932		
299,400	.02036		
249,400	.02020		

APPENDIX C

Calculated Results of Isothermal Velocity Distribution Data from Previous Investigators

I. T.E,Stanton
II. J.R.Freeman
III. F.E.Lawrence and P.L.Braunworth
IV. J. Nikuradse
V. D.Marshall

I. T.E.Stanton

Calculation of Stanton's Data on Isothermal velocity distribution of Air.

Proc.Royal Society, London, vol.85A, Table II, 1911

Radius = 2.465 cm.
0.971 in.

$V_{max.}$ = 1525 cm./sec.
50 ft./sec.

$\frac{r}{R}$	V (ft./sec.)	$\frac{V}{V_{max.}}$	$V(r/R)$ (ft./sec.)	1-r/R
.000	50.0	1.000	.00	1.000
.183	49.35	.987	9.04	.817
.284	48.4	.968	13.74	.716
.390	47.2	.945	18.40	.610
.491	45.75	.915	22.45	.509
.597	44.1	.882	26.30	.403
.698	41.9	.838	29.20	.302
.800	39.4	.788	31.5	.200
.853	37.65	.754	32.1	.147
.905	35.55	.711	32.15	.095
.926	34.4	.688	31.85	.074
.942	33.6	.672	31.65	.058
.955	32.25	.645	30.8	.045
.966	31.4	.628	30.3	.034
.979	29.75	.595	29.1	.021
.986	26.70	.534	26.3	.014
.990	19.40	.388	19.2	.010

Calming length = 101D

Assume t_{av} = 59° F.

Re = 41,200 V_{av} (from graph) = 40.5 ft./sec.

V_{av}/V_{max} = 40.5/50 = 0.8105

I. T.E.Stanton

Calculation of Stanton's Data on Isothermal Velocity Distribution of Air

Proc.Roy.Soc., London, vol. 85A, Table II, 1911

Radius = 3.7 cm.
= 1.46 in.

V_{max} = 1.017 cm.~~ft~~./sec.
33.35 ft./sec.

(r/R)	$\frac{v}{(ft./sec.)}$	$V/V_{max.}$	$v(\frac{r}{R})$	$1-\frac{r}{R}$	$1-(\frac{r}{R})^{1.25}$	$1-(\frac{r}{R})^{2}$
.000	33.35	1.000	000.0	1.000	1.000	1.000
.336	32.00	.960	10.75	.664	.744	.887
.678	28.25	.848	19.13	.322	.385	.540
.852	25.25	.757	21.47	.148	.192	.274
.903	23.60	.708	21.32	.097	.120	.185
.925	22.95	.688	21.22	.075	.093	.144
.939	22.28	.669	20.92	.061	.074	.116
.954	21.37	.641	20.40	.046	.056	.088
.965	20.60	.618	19.88	.035	.044	.069
.975	19.40	.582	18.93	.025	.031	.049
.986	16.92	.507	16.70	.014	.016	.026
.992	12.86	.385	12.73	.008	.010	.016

Calming Length = 67.6 D

Assume t_{air} = 59° F.

Re. = 40,750

$V_{ave.}$ (graphical integration) = 26.75 ft./sec.

$V_{ave.}/V_{max.}$ = 26.75/33.35 = 0.802

I.T.E. Stanton

Calculation of Stanton's Data on Isothermal Velocity Distribution of Air.

Proc.Royal Society, London, vol. 85A, 366, 1911

Radius = 3.7 cm. = 1.46 in. V_{max} = 2.215 cm./sec.
= 72.60 ft./sec.

$\left(\frac{r}{R}\right)$	$\frac{v}{\text{(ft./sec.)}}$	$\frac{v}{v_{max}}$	$v\left(\frac{r}{R}\right)$	$\left(1-\frac{r}{R}\right)$	$1-\left(\frac{r}{R}\right)^{1.25}$	$1-\left(\frac{r}{R}\right)^{2}$
.000	72.6	1.000	00.00	1.000	1.000	1.000
.336	69.5	.957	23.35	.664	.744	.887
.678	62.1	.855	42.1	.322	.385	.540
.852	55.52	.765	47.3	.148	.192	.274
.903	52.6	.725	47.5	.097	.120	.185
.925	51.0	.7025	47.2	.075	.093	.144
.939	49.5	.682	46.45	.061	.074	.116
.954	47.65	.657	45.45	.046	.056	.088
.965	46.15	.636	44.5	.035	.044	.069
.975	44.08	.607	43.0	.025	.031	.049
.986	40.95	.564	40.35	.014	.016	.026
.992	36.00	.496	35.65	.008	.010	.016

Calming Length = 67.6 D

Assuming t_{air} = 59° F.

Re. = 89,750

$V_{ave.}$(graphical integration) = 58.60 ft./sec.

$V_{ave.}/V_{max.}$ = 58.6/72.6 = 0.808

II. J.R. Freeman

Isothermal Velocity Distribution
Data of Water in Brazed Brass Pipe (As Calculated by T. Christen)
(Zeitschr. für Gewasserkunde, Vol. 6, p. 175 (1914)).

$R = 1.46$ cm. = 0.575 in. $V_{max.} = 75.0$ ft./sec.

$\frac{r}{R}$	$(1 - \frac{r}{R})$	$V/V_{ave.}$	$V/V_{max.}$
0	1.000	1.197	1.000
0.174	0.826	1.173	0.980
.348	.652	1.146	.957
.522	.478	1.088	.909
.609	.391	1.058	.884
.643	.357	1.046	.873
.678	.322	1.032	.862
.713	.287	1.013	.846
.748	.252	0.992	.828
.783	.217	.976	.815
.817	.183	.955	.797
.852	.148	.931	.777
.870	.130	.917	.766
.887	.113	.899	.750
.904	.096	.890	.743
.922	.078	.867	.724
.939	.061	.845	.705
.957	.043	.806	.673
.974	.026	.768	.641
.991	.009	.720	.601

$$\frac{V_{ave}}{V_{max}} = \frac{1}{1.197} = 0.835$$

Assume $t_{H_2O} = 10°C.$

Calming Length = 104 D

Re = 425,000

III. Lawrence, F.E. and Braunworth, P.L.

Isothermal Vel. Distribution of H_2O in Brass Pipes

(Am.Soc.Civil Engr. Trans., Vol. 57, p. 265 (1906)).#

Seamless Brass Pipe D = 5.016"

Traverse No. 1 $V_{max.}$ = 10.29 ft./sec.

$\frac{r}{R}$	V_1 (ft./sec.)	$V/V_{max.}$	$(1 - \frac{r}{R})$
0.000	10.29(ave.)	1.000	1.000
0.282	10.12	.984	.718
0.413	9.91	.962	.587
0.514	9.45	.918	.486
0.598	9.24	.897	.402
0.672	9.08	.882	.328
0.740	8.69	.844	.260
0.802	8.47	.823	.198
0.860	8.29	.805	.140
0.913	7.98	.775	.087
0.966	6.67	.648	.034

V_{ave} (Calculated from Vel. Dist.) = 8.79 ft./sec.

$$\frac{V_{ave}}{V_{max}} = \frac{8.79}{10.29} = 0.854$$

V_{ave}' (from actual measurement) = 9.072 ft./sec.

$$\frac{V_{ave}}{V_{max}} = \frac{9.072}{10.29} = 0.882$$

t_{H_2O} = 68°F. (assumed) Re = 339,000

~~Actual~~ data obtained from Cornell University through the kindness of Mr. C. H. Chang

III. Lawrence, F.E., and Braunworth, P.L.

Isothermal Velocity Distribution of Water in Brass Pipe

Am.Soc. Civil Engrs., Trams., Vol. 57 (1906)

Seamless Brass Pipe D = 5.016"

Traverse No.2 $V_{max} = 6.11$ ft./sec.

$\frac{r}{R}$	V_1 (ft./sec.)	$\frac{V_1}{V_{max}}$	$1 - \frac{r}{R}$
.000	6.11	1.000	1.000
.282	5.83	.955	.718
.413	5.76	.944	.587
.514	5.57	.913	.486
.598	5.42	.888	.402
.672	5.30	.868	.328
.740	5.12	.839	.260
.802	4.90	.802	.198
.860	4.68	.766	.140
.913	4.51	.739	.087
.966	3.95	.647	.034

V_{av} (Calculated from Vel.Dist.) = 5.293 ft./sec.

$$\therefore \frac{V_{av}}{V_{max}} = \frac{5.293}{6.110} = 0.865$$

V_{av}' (from actual meas.) = 5.244 ft./sec.

$$\frac{V_{av}'}{V_{max}} = \frac{5.244}{6.11} = 0.859$$

$t_{H_2O} = 68°$ F. Re = 204,000

III. Lawrence, F.E. and Braunworth, P.L.

Iso. Vel. Dist. of H_2O in Brass Pipe

(Am. Soc. Civil Engr. Trans. vol. 57, p. 265 (1906)).

Seamless Brass Pipe D = 5.016"

$V_{max.}$ = 4.77 ft./sec.

Traverse No. 3

$\frac{r}{R}$	$(1 - \frac{r}{R})$	V (ft./sec.)	$V/V_{max.}$
.000	1.000	4.77	1.000
.282	.718	4.64	.973
.413	.587	4.44	.930
.514	.486	4.30	.902
.598	.402	4.19	.878
.672	.328	4.17	.875
.740	.260	4.01	.840
.802	.198	3.81	.799
.860	.140	3.76	.788
.913	.087	3.34	.700
.966	.034	3.01	.630

$V_{ave.}$ (Calculated) = 3.956 ft./sec. $\frac{V_{ave}}{V_{max}}$ (calc.) = $\frac{3.956}{4.77}$ = 0.83

$V_{ave.}$ (Meas.) = 4.128

Re = 152,500

III. Lawrence, F.E., and Braunworth, P.L.

Isothermal Velocity Distribution of H_2O in Brass Pipe

Am.Soc.Civil Engrs. Trans., vol. 57, p. 265 (1906)

Seamless Brass Pipe D = 5.016"

Traverse No. 4 $V_{max.}$ = 8.93 ft./sec.

$\frac{r}{R}$	$(1 - \frac{r}{R})$	V_1 (ft./sec.)	$V_1/V_{max.}$
.000	1.000	8.93	1.000
.282	.718	8.68	0.972
.413	.587	8.39	0.940
.514	.486	8.25	0.925
.598	.402	8.09	0.907
.672	.328	7.92	0.888
.740	.260	7.72	0.865
.802	.198	7.25	0.812
.860	.140	7.09	0.795
.913	.087	6.64	0.745
.966	.034	5.64	0.632

V_{av} (calculated) = 7.80 ft./sec.

$$\frac{V_{av}}{V_{max}} = \frac{7.80}{8.93} = 0.873$$

V_{av} (meas.) = 7.76 ft./sec.

Re_{ave} = 38,550 x 7.80 = 300,500

III. Lawrence, F.E., and Braunworth, P.L.

Isothermal Velocity Distribution of H_2O in Brass Pipe

Am.Soc. Civil Engrs. Trans. Vol. 57, p. 265 (1906)

Seamless Brass Pipe D = 5.016"

Traverse No. 5 $V_{max.}$ = 4.20 ft./sec.

$\frac{r}{R}$	$(1 - \frac{r}{R})$	V_1 (ft./sec.)	$\frac{V_1}{V_{max.}}$
.000	1.000	4.20	1.000
.282	.718	4.00	.953
.413	.587	3.89	.927
.514	.486	3.79	.903
.598	.402	3.73	.889
.672	.328	3.62	.863
.740	.260	3.53	.840
.802	.198	3.38	.805
.860	.140	3.21	.765
.913	.087	3.00	.715
.966	.034	2.56	.610

$V_{av.}$ (calculated) = 3.53 ft./sec.; **$V_{ave.}/V_{max.}$=3.53/4.20=0.840**

$V_{av.}$ (meas.) = 3.495 ft./sec.

Re = 136,100

IV. J. Nikuradse

Calculation of Nikuradse's Isothermal Velocity Distribution data of water in drawn brass Pipe.

Forschungsarbeiten, V.D.I., Heft 281, 1926

Radius = 0.551" (Vertical Profile) $V_{max.}$ = 30.87 ft./sec.

$\frac{r}{R}$	$1 - \frac{r}{R}$	$1 - (\frac{r}{R})^2$	$v/v_{max.}$
.991	.009	.018	.140
.972	.028	.055	.557
.955	.045	.088	.647
.936	.064	.124	.685
.909	.091	.174	.726
.882	.118	.222	.750
.827	.173	.316	.787
.773	.227	.403	.819
.665	.335	.558	.878
.556	.444	.691	.918
.447	.553	.800	.950
.339	.661	.885	.970
.230	.770	.947	.980
.121	.879	.985	1.000
.014	.986	.998	.997

t_{H_2O} = 50° F.

Re. = 162,000

IV. J. Nikuradse

Calculation on Nikuradse's Isothermal Velocity Distribution of water in drawn brass pipe.

Forschungsarbeiten, V.D.I., Heft 281, 1926

Radius = 0.551" (Horizontal Profile) V_{max} = 30.8 ft./sec.

$\frac{r}{R}$	$1 - \frac{r}{R}$	$\frac{V}{V_{max}}$	$1 - (\frac{r}{R})^2$
.991	.009	.130	.018
.972	.028	.561	.055
.955	.045	.648	.088
.936	.064	.686	.124
.909	.091	.729	.174
.882	.118	.750	.222
.827	.173	.790	.316
.773	.227	.822	.403
.665	.335	.880	.558
.556	.444	.922	.691
.447	.553	.952	.800
.339	.661	.973	.885
.230	.770	.982	.947
.121	.879	1.000	.985
.014	.986	1.000	.998

t_{H_2O} = 10° C. = 50° F.

Re = 162,200

IV. J. Nikuradse

Isothermal Velocity Distribution of Water in Drawn Brass Pipes.

A. Gilles, L.Hoff and Theo. v. Karman = Aerodynamik und Verwandte Gebeite, Julius Springer, Berlin, 1930

Calming Length greater than 55D

Read from an enlarged plot of eight times as big as the original.

$\frac{r}{R}$	$(1 - \frac{r}{R})$	Re=53,700 $V/V_{max.}$	Re=152,000 $V/V_{max.}$	Re=300,000 $V/V_{max.}$	Re=928,000 $V/V_{max.}$	Re=3,070,00 $V/V_{max.}$
00	1.00	1.000	1.000	1.000	1.000	1.000
.20	.80	.980	.981	.990	.991	.992
.40	.60	.941	.950	.957	.962	.968
.60	.40	.889	.900	.912	.918	.928
.70	.30	.852	.862	.877	.890	.898
.80	.20	.803	.814	.836	.852	.859
.85	.15	.774	.786	.805	.826	.840
.90	.10	.730	.742	.766	.788	.808
.93	.07	.697	.710	.737	.760	.781
.96	.04	.637	.656	.681	.711	.737
.98	.02	.587	.603	.622	.663	.685
.99	.01	.549	----	.561	.612	.643

V. D. Marshal

Calculation of D. Marshall's Isothermal Velocity Distribution of Air in Pipes. (Brass Pipe)

(Gt. Brit. Aero. Research Comm., Repts. and Memo. no. 1004, 1925-6

Radius = 2.5"

$\frac{V_{ave.}}{V_{max.}} = \frac{50.5}{59.0} = 0.856$

$V_{max.}$ = 59.0 ft./sec.

$V_{ave.}$ = 50.5 ft./sec.

$t_{ave.}$ = 16°C. = 60.8°F.

Calming Length = 192 D.

$\frac{r}{R}$	$(1 - \frac{r}{R})$	$v/v_{max.}$	V (cm./sec.)
.000	1.000	1.000*	1810
.100	.900	.995*	1800
.200	.800	.980*	1773
.300	.700	.970*	1755
.400	.600	.960*	1738
.500	.500	.930*	1682
.600	.400	.905*	1638
.709	.291	.865*	1565
.788	.212	.830	1500
.819	.181	.815	1475
.835	.165	.805	1455
.850	.150	.795	1440
.867	.133	.788	1425
.882	.118	.78	1410
.898	.102	.765	1385
.914	.086	.75	1355
.930	.070	.74	1340
.945	.055	.72	1305
.961	.039	.70	1265
.977	.023	.68	1230
.985	.015	.665	1205
.992	.008	.64	1160

Re = 132,300

* Estimated from the graph given in the original paper.

Appendix D

ISOTHERMAL VELOCITY DISTRIBUTION DATA WITH CALCULATIONS AND PLOTS

Run V   I-10   Station No. 2

$(\frac{r}{R})$	Δh (cm. Ccl$_4$)	V (ft./sec.)	$\frac{V}{V_{max.}}$	$V(\frac{r}{R})$ (ft./sec)	$1-(\frac{r}{R})^2$	$(1-\frac{r}{R})$
0.985	12.80	3.95	0.611	3.89	.0298	.015
.970	14.90	4.26	.659	4.13	.0591	.030
.950	17.50	4.62	.714	4.39	.0975	.05
.930	18.64	4.76	.736	4.43	.1351	.07
.900	20.50	5.00	.774	4.50	.1900	.10
.850	22.34	5.21	.807	4.43	.2775	.15
.800	24.12	5.42	.838	4.34	.3600	.20
.700	26.60	5.69	.880	3.98	.51	.30
.600	28.70	5.92	.915	3.56	.64	.40
.500	30.10	6.06	.937	3.03	.75	.50
.400	32.10	6.25	.966	2.50	.84	.60
.300	33.10	6.35	.981	1.91	.91	.70
.200	33.84	6.42	.992	1.28	.96	.80
.100	34.30	6.46	.998	0.65	.99	.90
.000	34.40	6.47	1.000	0.00	1.00	1.00

Static Pressure = 17.0 Cm. Hg. Gauge   t = 25°C.

$V_{ave.}$ (manometer) = 5.35 ft./sec.

$V_{ave.}$ (graphical integration) = 5.52 ft./sec.

$V_{ave.}/V_{max.} = 5.52/6.47 = 0.853$

Re = 93,100

Run V I-11 Station No. 3

$\frac{r}{R}$	Δh (cm. CCl_4)	V (ft./sec.)	$\frac{V}{V_{max.}}$	$V(\frac{r}{R})$ (ft./sec.)	$1-(\frac{r}{R})^2$	$1-\frac{r}{R}$
0.988	12.90	3.96	0.612	3.92	.0239	.012
.970	15.11	4.29	.664	4.16	.0591	.03
.950	17.67	4.64	.718	4.41	.0975	.05
.930	19.10	4.82	.746	4.48	.1351	.07
.900	20.50	4.99	.772	4.49	.19	.10
.850	22.80	5.27	.815	4.48	.2775	.15
.800	24.30	5.44	.842	4.35	.36	.20
.700	26.50	5.68	.878	3.98	.51	.30
.600	28.65	5.91	.914	3.55	.64	.40
.500	30.15	6.06	.938	3.03	.75	.50
.400	32.60	6.30	.975	2.52	.84	.60
.300	33.25	6.36	.984	1.91	.91	.70
.200	33.80	6.42	.992	1.28	.96	.80
.100	34.30	6.46	.998	0.65	.99	.90
.000	34.40	6.47	1.000	0.00	1.00	1.00

Static Pressure = 17.6 Cm. Hg. Gauge t = 25°C.

$V_{ave.}$ (manometer) = 5.35 ft./sec.

$V_{ave.}$ (graphical integration) = 5.54 ft./sec.

$V_{ave.}/V_{max.}$ = 5.54/6.47 = 0.855

Re = 93,500

Run V   I-15   Station No. 3

$\frac{r}{R}$	Δh (Cm. CCl_4)	V (ft./sec.)	$V/V_{max.}$	$V(\frac{r}{R})$ (ft./sec.)	$1-(\frac{r}{R})^2$	$1-(\frac{r}{R})$
0.988	1.10	1.16	0.525	1.15	.0239	.012
.970	1.30	1.26	.570	1.22	.0591	.03
.950	1.70	1.44	.652	1.37	.0975	.05
.930	1.80	1.48	.670	1.38	.1351	.07
.900	1.96	1.55	.701	1.39	.1900	.10
.850	2.14	1.61	.729	1.37	.2775	.15
.800	2.30	1.67	.758	1.34	.3600	.20
.700	2.64	1.79	.811	1.25	.5100	.30
.600	2.90	1.88	.851	1.13	.6400	.40
.500	3.20	1.98	.896	0.99	.7500	.50
.400	3.56	2.08	.942	0.83	.8400	.60
.300	3.80	2.15	.974	0.65	.9100	.70
.200	4.00	2.21	1.000	0.44	.9600	.80
.100	4.00	2.21	1.000	0.22	.9900	.90
.000	4.00	2.21	1.000	0.00	1.0000	1.00

Static Pressure = 21.3 Cm. Hg. Gauge   t = 19.8°C.

$V_{ave.}$ (Manometer) = 1.84 ft./sec.

$V_{ave.}$ (graphical integration) = 1.77 ft./sec.

$V_{ave.}/V_{max.}$ = 1.77/2.21 = 0.800

Re = 26,500

Run V   I-17   Station No. 2

$\frac{r}{R}$	Δh (Cm. CCl_4)	V (ft./sec.)	$V/V_{max.}$	$V(\frac{r}{R})$ (ft./sec.)	$1-(\frac{r}{R})^2$	$1-\frac{r}{R}$
0.985	24.30	5.44	0.671	5.35	.0298	.015
.970	25.30	5.55	.684	5.38	.0591	.03
.950	28.16	5.85	.722	5.56	.0975	.05
.930	29.50	5.99	.738	5.57	.1351	.07
.900	31.44	6.19	.763	5.57	.1900	.10
.850	34.60	6.49	.800	5.51	.2775	.15
.800	36.20	6.64	.818	5.31	.3600	.20
.700	41.30	7.10	.874	4.97	.5100	.30
.600	44.50	7.36	.907	4.42	.6400	.40
.500	47.52	7.60	.937	3.80	.7500	.50
.400	49.50	7.77	.956	3.11	.8400	.60
.300	52.36	7.98	.983	2.40	.9100	.70
.200	53.10	8.04	.990	1.61	.9600	.80
.100	54.10	8.11	1.000	.81	.9900	.90
.000	54.10	8.12	1.000	.000	1.0000	1.00

Static Pressure = 26.5 Cm. Hg. Gauge   t = 27°C.

$V_{ave.}$ (manometer) = 6.75 ft./sec.

$V_{ave.}$ (graphical integration) = 6.83 ft./sec.

$V_{ave.}/V_{max.} = 6.83/8.12 = 0.841$

Re. = 120,500

Run V I-18 Station No. 3

$\frac{r}{R}$	Δh (Cm. CCl_4)	V (ft./sec.)	$V/V_{max.}$	$V(\frac{r}{R})$ (ft./sec.)	$1-(\frac{r}{R})^2$	$1-(\frac{r}{R})$
0.988	23.70	5.37	0.662	5.31	.0239	.012
.970	25.06	5.54	.682	5.37	.0591	.03
.950	27.10	5.75	.709	5.46	.0975	.05
.930	27.50	5.79	.714	5.39	.1351	.07
.900	29.84	6.03	.743	5.43	.1900	.10
.850	31.88	6.23	.768	5.29	.2775	.15
.800	34.36	6.47	.797	5.17	.3600	.20
.700	38.56	6.85	.844	4.79	.5100	.30
.600	42.60	7.21	.888	4.32	.6400	.40
.500	46.00	7.49	.922	3.74	.7500	.50
.400	49.70	7.78	.958	3.11	.8400	.60
.300	51.52	7.93	.976	2.38	.9100	.70
.200	53.44	8.07	.994	1.61	.9600	.80
.100	53.70	8.09	.996	0.81	.9900	.90
.000	54.10	8.12	1.000	0.00	1.0000	1.00

Static Pressure = 51.4 Cm. Hg. Gauge t = 27°C.

$V_{ave.}$ (manometer)= 6.75 ft./sec.

$V_{ave.}$ (graphical integration) = 6.70 ft./sec.

$V_{ave.}/V_{max.}$ = 6.70/8.12 = 0.825

Re. = 118,200

RUN V I-20 Station No. 2

Fraction of Radius $(\frac{r}{R})$	Δh (cm. CCl_4)	V (ft./sec.)	$\frac{V}{V_{max}}$	$(1-(\frac{r}{R})^2)$	$(1-(\frac{r}{R}))$	$V(\frac{r}{R})$ ft./sec.
0.985	4.79	2.42	0.583	0.0298	.015	2.38
0.970	6.20	2.75	0.663	0.0591	.03	2.66
0.950	6.80	2.88	0.694	0.0975	.05	2.74
0.930	7.40	3.00	0.724	0.1351	.07	2.79
0.900	7.90	3.10	0.748	0.1906	.10	2.79
0.850	8.80	3.27	0.791	0.2775	.15	2.78
0.800	9.42	3.39	0.817	0.3600	.20	2.71
0.700	10.60	3.59	0.867	0.51	.30	2.52
0.600	11.60	3.76	0.907	0.64	.40	2.26
0.500	12.30	3.87	0.935	0.75	.50	1.94
0.400	12.90	3.96	0.957	0.84	.60	1.58
0.300	13.54	4.06	0.979	0.91	.70	1.22
0.200	13.86	4.10	0.990	0.96	.80	0.82
0.100	14.04	4.13	0.997	0.99	.90	0.41
0.000	14.10	4.14	1.000	1.00	1.00	0.00

Static pressure = 7.54 cm. Hg. gauge

t = 27°C.

V_{ave} (Manometer) = 3.43 ft./sec.

V_{ave} (Graphical integration) = 3.50 ft./sec.

$\frac{V_{ave}}{V_{max}}$ = 3.50/4.14 = 0.846

Re. = 61,750

RUN V I-24 Station No. 2

Fraction of Radius $(\frac{r}{R})$	Δh (cm. CCl_4)	V (ft./sec.)	$\frac{v}{V_{max}}$	$V(\frac{r}{R})$ (ft./sec.)	$1-(\frac{r}{R})^2$	$1-\frac{r}{R}$
0.991	14.54	4.21	0.669	4.17	0.0179	0.009
0.970	15.74	4.38	0.696	4.25	0.0591	0.03
0.950	16.94	4.54	0.723	4.31	0.0975	0.05
0.930	17.94	4.68	0.743	4.35	0.1351	0.07
0.900	19.54	4.88	0.776	4.39	0.1900	0.10
0.850	20.54	5.00	0.794	4.25	0.2775	0.15
0.800	22.34	5.22	0.828	4.18	0.3600	0.20
0.700	24.34	5.44	0.865	3.80	0.5100	0.30
0.600	26.34	5.66	0.900	3.40	0.6400	0.40
0.500	27.74	5.81	0.924	2.90	0.7500	0.50
0.400	28.94	5.94	0.943	2.38	0.8400	0.60
0.300	30.34	6.08	0.965	1.82	0.9100	0.70
0.200	31.54	6.19	0.985	1.24	0.9600	0.80
0.100	32.14	6.26	0.994	0.63	0.9900	0.90
0.000	32.54	6.29	1.000	0.00	1.0000	1.00

Static Pressure = 16.74 cm. Hg. Gauge

t = 31.8°C.

V_{ave} (manometer) = 5.25 ft./sec.

V_{ave} (Graphical integration) = 5.30 ft./sec.

$$\frac{V_{ave}}{V_{max}} = \frac{5.30}{6.29} = 0.843$$

Re. = 103,400

RUN V I-26 Station No. 4

Fraction of Radius $(\frac{r}{R})$	Δh (cm. CCl_4)	V (ft./sec.)	$\frac{V}{V_{max}}$	$V(\frac{r}{R})$ (ft./sec)	$1-(\frac{r}{R})$	$1-\frac{r}{R}$
0.989	11.36	3.72	0.587	3.68	0.0219	0.011
0.970	13.76	4.09	0.646	3.96	0.0591	0.03
0.950	15.86	4.39	0.694	4.17	0.0975	0.05
0.930	17.16	4.57	0.722	4.25	0.1351	0.07
0.900	18.76	4.78	0.755	4.30	0.1900	0.10
0.850	19.96	4.93	0.779	4.19	0.2775	0.15
0.800	21.96	5.17	0.817	4.14	0.3600	0.20
0.700	24.86	5.50	0.869	3.85	0.5100	0.30
0.600	26.66	5.70	0.900	3.42	0.6400	0.40
0.500	28.66	5.91	0.933	2.96	0.7500	0.50
0.400	30.16	6.06	0.957	2.42	0.8400	0.60
0.300	30.96	6.14	0.969	1.84	0.9100	0.70
0.200	32.26	6.27	0.989	1.25	0.9600	0.80
0.100	32.76	6.31	0.996	0.63	0.9900	0.90
0.000	32.96	6.33	1.000	0.00	1.0000	1.00

Static Pressure = 13.2 cm. Hg. Gauge

t = 31.8°C.

V_{ave} (Manometer) = 5.25 ft./sec.

V_{ave} (Graphical integration) = 5.30 ft./sec.

$$\frac{V_{ave}}{V_{max}} = \frac{5.30}{6.33} = 0.838$$

Re = 103,400

Run V I-27 Station No. 2

$(\frac{r}{R})$	$^{2}\frac{\Delta h}{(\text{Cm. Ccl}_4)}$	$\frac{V}{(\text{ft./sec.})}$	$V/V_{max.}$	$\frac{V(\frac{r}{R})}{(\text{ft./sec.})}$	$1-(\frac{r}{R})^2$	$1-\frac{r}{R}$
0.991	7.90	3.10	0.596	3.07	.0179	.009
.970	10.30	3.54	.681	3.44	.0591	.03
.950	10.90	3.65	.700	3.46	.0975	.05
.930	11.90	3.81	.731	3.54	.1351	.07
.900	12.80	3.95	.759	3.56	.1900	.10
.850	13.80	4.10	.788	3.48	.2775	.15
.800	14.90	4.26	.819	3.41	.3600	.20
.700	16.60	4.50	.865	3.15	.5100	.30
.600	17.90	4.67	.897	2.80	.6400	.40
.500	19.14	4.83	.928	2.42	.7500	.50
.400	20.20	4.96	.953	1.98	.8400	.60
.300	21.10	5.07	.975	1.52	.9100	.70
.200	21.70	5.14	.988	1.03	.9600	.80
.100	21.96	5.17	.994	.52	.9900	.90
.000	22.20	5.20	1.000	.00	1.0000	1.00

Static Pressure = 12.10 cm. Hg. Gauge t = 29.3°C.

$V_{ave.}$ (Manometer)= 4.31 ft./sec.

$V_{ave.}$ (graphical integration) = 4.36 ft./sec.

$V_{ave.}/V_{max.}$ = 4.36/5.20 = 0.838

Re. = 80,900

Run V I-28 Station No. 3

$(\frac{r}{R})$	$\frac{\Delta h}{(Cm.\ CCl_4)}$	$\frac{V}{(ft./sec.)}$	$V/V_{max.}$	$\frac{V(\frac{r}{R})}{(ft./sec)}$	$(1-\frac{r}{R})$	$1-(\frac{r}{R})^{1.25}$
0.985	24.8	5.50	0.618	5.41	.015	.019
.970	28.6	5.90	.663	5.72	.030	.037
.950	32.8	6.32	.710	6.00	.050	.062
.930	35.8	6.60	.742	6.14	.070	.086
.900	38.6	6.85	.770	6.16	.100	.124
.850	42.8	7.22	.812	6.14	.150	.184
.800	46.6	7.53	.846	6.02	.200	.243
.700	51.8	7.95	.894	5.56	.300	.360
.600	56.0	8.26	.928	4.96	.400	.472
.500	59.2	8.50	.955	4.25	.500	.579
.400	61.8	8.68	.975	3.48	.600	.682
.300	63.8	8.81	.990	2.64	.700	.778
.200	64.4	8.85	.994	1.77	.800	.866
.100	64.8	8.89	.998	0.89	.900	.944
.000	65.0	8.90	1.000	0.00	1.000	1.000

Static Pressure = 35.3 cm. Hg. Gauge t = 24.0°C.

$V_{ave.}$ (manometer) = 7.83 ft./sec.

$V_{ave.}$ (graphical integration) = 7.56 ft./sec.

$V_{ave.}/V_{max.} = 7.56/8.90 = 0.850$

Re. = 125,000

Run v I-29 Station No. 4

$\frac{r}{R}$ (-)	Δh (cm. CCL_4)	$1.218\Delta h$ ($= v^2$)	v (ft./sec.)	$\frac{v}{v_{max.}}$	$V(r/R)$ (ft./sec.)	$\frac{1-r}{R}$	1.25 1-(r/r)
.989	22.6	27.6	5.25	0.592	5.19	0.011	0.014
.970	25.6	31.2	5.58	.630	5.51	.030	.037
.950	29.8	36.3	6.02	.679	5.72	.050	.062
.930	34.2	41.7	6.45	.727	6.00	.070	.086
.900	36.8	44.8	6.69	.755	6.02	.100	.124
.850	40.4	44.2	7.01	.791	5.96	.150	.184
.800	44.2	53.8	7.33	.827	5.86	.200	.243
.700	49.4	60.2	7.76	.876	5.43	.300	.360
.600	53.2	64.8	8.05	.908	4.83	.400	.472
.500	56.4	68.7	8.28	.934	4.14	.500	.579
.400	59.0	71.9	8.47	.955	3.39	.600	.682
.300	61.6	75.0	8.65	.975	2.60	.700	.778
.200	63.6	77.5	8.80	.992	1.76	.800	.866
.100	64.2	78.2	8.85	.998	.89	.900	.944
.000	64.6	78.7	8.87	1.000	.00	1.000	1.000

Static Pressure = 26.2 cm. Hg. Gauge t = 24° C.

$V_{av.}$ (manometer) = 7.83 ft./sec.

$V_{av.}$ (graphical integration) = 7.45 ft./sec.

$V_{av.}/V_{max.} = 7.45/8.87 = 0.840$

Re. = 123,000

Run v I-31 Station No. 4

$(\frac{r}{R})$	Δh (cm.CCL_4)	v (ft./sec.)	$\frac{v}{V_{max.}}$	$V(r/R)$ (ft./sec.)	$(1-\frac{r}{R})$	$1-(\frac{r}{R})^{1.25}$
.989	16.8	4.52	0.570	4.47	.011	.014
.970	20.4	4.98	.629	4.85	.030	.037
.950	23.6	5.36	.677	5.09	.050	.062
.930	25.8	5.60	.707	5.21	.070	.086
.900	28.4	5.88	.742	5.29	.100	.124
.850	32.2	6.26	.790	5.32	.150	.184
.800	34.6	6.49	.820	5.19	.200	.243
.700	38.8	6.87	.867	4.81	.300	.360
.600	42.0	7.15	.902	4.29	.400	.472
.500	44.2	7.33	.925	3.67	.500	.579
.400	47.0	7.56	.955	3.02	.600	.682
.300	49.2	7.73	.976	2.32	.700	.778
.200	50.4	7.83	.989	1.57	.800	.866
.100	51.0	7.88	.995	0.79	.900	.944
.000	51.6	7.925	1.000	0.00	1.000	1.000

Static Pressure = 21.2 cm. Hg. Gauge t = 28° C.

$V_{ave.}$ (manometer) = 6.97 ft./sec.

$V_{ave.}$ (graphical integration) = 6.57 ft./sec.

$V_{ave.}/V_{max.}$ = 6.57/7.925 = 0.830

Re. = 118,700

Run v I-32 Station No. 2

$\frac{r}{R}$	Δh (Cm. CCl_4)	v (ft./sec.)	$\frac{v}{v_{max.}}$	V(r/R) (ft./sec.)	$1-\frac{r}{R}$	$1-(\frac{r}{R})^{1.25}$
.991	20.8	5.04	0.632	4.99	0.009	0.011
.970	24.6	5.48	.687	5.31	.030	.037
.950	27.0	5.74	.720	5.45	.050	.062
.930	29.0	5.94	.745	5.52	.070	.086
.900	31.2	6.16	.773	5.55	.100	.124
.850	34.2	6.45	.810	5.48	.150	.184
.800	36.2	6.64	.833	5.31	.200	.243
.700	40.0	6.97	.875	4.88	.300	.360
.600	42.8	7.22	.905	4.335	.400	.472
.500	45.0	7.40	.928	3.70	.500	.579
.400	47.6	7.62	.955	3.05	.600	.682
.300	49.2	7.74	.970	2.32	.700	.778
.200	51.0	7.88	.987	1.58	.800	.866
.100	51.6	7.92	.992	0.79	.900	.944
.000	52.2	7.97	1.000	0.00	1.000	1.000

Static Pressure = 26.6 cm. Hg. Gauge t = 29.1° C.

$V_{ave.}$ (manometer) = 7.02 ft./sec.

$V_{ave.}$ (graphical integration) = 6.71 ft./sec.

$V_{ave.}/V_{max.} = 6.71/7.97 = 0.842$

Re. = 124,000

Run v I-34 Station No. 4

$\frac{r}{R}$	Δh (Cm.CCL_4)	v (ft./sec.)	$\frac{v}{V_{max.}}$	V(r/R) (ft./sec.)	$(1-\frac{r}{R})$	$1-(\frac{r}{R})^{1.25}$
.989	17.2	4.58	0.572	4.53	0.011	0.014
.970	21.4	5.11	.639	4.96	.030	0.037
.950	23.8	5.38	.673	5.11	.050	.062
.930	26.4	5.67	.709	5.28	.070	.086
.900	29.0	5.94	.743	5.35	.100	.124
.850	32.2	6.25	.781	5.31	.150	.184
.800	34.8	6.51	.814	5.21	.200	.243
.700	39.4	6.92	.865	4.85	.300	.360
.600	42.4	7.19	.899	4.31	.400	.472
.500	45.2	7.42	.928	3.71	.500	.579
.400	48.0	7.64	.955	3.06	.600	.682
.300	50.2	7.82	.978	2.34	.700	.778
.200	51.4	7.90	.988	1.58	.800	.866
.100	52.2	7.97	.996	0.80	.900	.944
.000	52.6	8.00	1.000	0.00	1.000	1.000

Static Pressure = 21.2 cm. Hg. Gauge t = 29.1° C.

$V_{ave.}$ (manometer) = 7.02 ft./sec.

$V_{ave.}$ (graphical integration) = 6.64 ft./sec.

$V_{ave.}/V_{max.} = 6.64/8.00 = 0.830$

Re. = 123,200

Run v I-36 Station No. 3

$(\frac{r}{R})$	$\frac{\Delta h}{(\text{Ccm.}CCL_4)}$	$\frac{v}{(\text{ft./sec.})}$	$\frac{v}{v_{max.}}$	$\frac{V(r/R)}{(\text{ft./sec.})}$	$(1-\frac{r}{R})$	$1-(\frac{r}{R})^{1.25}$
0.985	3.8	2.15	0.610	2.12	0.015	0.019
.970	4.4	2.31	.656	2.24	.030	.037
.950	4.8	2.42	.688	2.30	.050	.062
.930	5.0	2.46	.699	2.29	.070	.086
.900	5.4	2.56	.727	2.30	.100	.124
.850	5.8	2.66	.756	2.26	.150	.184
.800	6.2	2.74	.779	2.19	.200	.243
.700	7.2	2.96	.841	2.07	.300	.360
.600	8.0	3.12	.887	1.87	.400	.472
.500	8.6	3.23	.918	1.62	.500	.579
.400	9.0	3.30	.938	1.32	.600	.682
.300	9.4	3.38	.960	1.01	.700	.778
.200	9.8	3.45	.980	0.69	.800	.866
.100	10.0	3.48	.989	0.35	.900	.944
.000	10.2	3.52	1.000	0.00	1.000	1.000

Static Pressure = 29.0 cm. Hg. Gauge t = 26.1° C.

$V_{ave.}$ (manometer) = 2.92 ft./sec.

$V_{ave.}$ (graphical integration) = 2.88 ft./sec.

$V_{ave.}/V_{max.} = 2.88/3.52 = 0.818$

Re. = 49,900

Run v I-37 Station No. 3

$\frac{r}{R}$	Δh (CCm.CCL_4)	v (ft./sec.)	$\frac{v}{v_{max.}}$	v(r/R) (ft./sec.)	$(1-\frac{r}{R})$	$(1-(r/R))^{1.25}$
0.985	75.0	4.27	0.612	4.20	0.015	0.019
.970	16.4	4.47	.640	4.34	.030	.037
.950	19.2	4.83	.692	4.59	.050	.062
.930	21.2	5.07	.727	4.72	.070	.086
.900	23.2	5.31	.761	4.78	.100	.124
.850	25.2	5.54	.794	4.70	.150	.184
.800	27.2	5.75	.824	4.60	.200	.243
.700	30.4	6.08	.871	4.25	.300	.360
.600	32.4	6.27	.899	3.76	.400	.472
.500	34.6	6.49	.930	3.24	.500	.579
.400	36.2	6.63	.950	2.65	.600	.682
.300	37.8	6.78	.972	2.04	.700	.778
.200	39.0	6.89	.987	1.38	.800	.866
.100	39.6	6.94	.995	0.69	.900	.944
.000	40.0	6.98	1.000	0.00	1.000	1.000

Static Pressure = 21.0 cm.Hg. Gauge t = 28.35° C.

$V_{ave.}$ (,anometer) = 6.11 ft./sec.

$V_{ave.}$ (graphical integration) = 5.85 ft./sec.

$V_{ave.}/V_{max.}$ = 5.85/6.98 = 0.838

Re. = 106,200

Run v I-38 Station No. 4

$\frac{r}{R}$	$\frac{\Delta h}{(\text{ccm.}CCL_4)}$	$\frac{V}{(\text{ft./sec.})}$	$\frac{v}{v_{max.}}$	$\frac{V(r/R)}{(\text{ft./sec.})}$	$(1-r/R)$	$1-(r/R)^{1.25}$
.989	13.2	4.01	0.573	3.96	0.011	0.014
.970	16.4	4.47	.639	4.34	.030	.037
.950	18.8	4.78	.683	4.54	.050	.062
.930	20.2	4.96	.709	4.61	.070	.086
.900	22.4	5.22	.746	4.70	.100	.124
.850	24.6	5.47	.781	4.65	.150	.184
.800	26.6	5.69	.813	4.55	.200	.243
.700	30.0	6.04	.863	4.23	.300	.360
.600	32.2	6.26	.896	3.76	.400	.472
.500	34.8	6.51	.930	3.26	.500	.579
.400	36.6	6.68	.955	2.67	.600	.682
.300	38.2	6.81	.973	2.04	.700	.778
.200	39.2	6.91	.988	1.38	.800	.866
.100	39.8	6.96	.995	.70	.900	.944
.000	40.2	7.00	1.000	.00	1.000	1.000

Static Pressure = 16.0 cm. Hg. Gauge t = 28.35° C.

$V_{ave.}$ (manometer) = 6.11 ft./sec.

$V_{ave.}$ (graphical integration) = 5.81 ft./sec.

$V_{ave.}/V_{max.}$ = 5.81/7.00 = 0.830

Re. = 105,600

RUN V I-39 Station No. 3

$(\frac{r}{R})$	Δh (cm. CCl_4)	V (ft./sec)	$\frac{V}{V_{max}}$	$V(\frac{r}{R})$ (ft/sec)	$1-\frac{r}{R}$	$1-(\frac{r}{R})^{1.25}$
0.985	4.0	2.21	0.552	2.18	0.015	0.019
0.970	5.0	2.47	0.616	2.40	0.030	0.037
0.950	5.8	2.66	0.664	2.52	0.050	0.062
0.930	6.2	2.74	0.684	2.55	0.070	0.086
0.900	7.0	2.92	0.729	2.63	0.100	0.124
0.850	7.8	3.08	0.769	2.62	0.150	0.184
0.800	8.4	3.20	0.798	2.56	0.200	0.243
0.700	9.6	3.42	0.854	2.40	0.300	0.360
0.600	10.4	3.56	0.888	2.14	0.400	0.472
0.500	11.0	3.66	0.913	1.83	0.500	0.579
0.400	12.0	3.82	0.953	1.53	0.600	0.682
0.300	12.4	3.88	0.968	1.16	0.700	0.778
0.200	12.8	3.95	0.985	0.79	0.800	0.866
0.100	13.0	3.98	0.993	0.40	0.900	0.944
0.000	13.2	4.01	1.000	0.00	1.000	1.000

Static Pressure = 26.6 cm. Hg. Gauge

t = 28.2°C.

V_{ave} (Manometer) = 3.60 ft./sec.

V_{ave} (Graphical integration) = 3.29 ft./sec.

$$\frac{V_{ave}}{V_{max}} = \frac{3.29}{4.01} = 0.820$$

Re. = 59,500

RUN V I-40

Station No. 4

$(\frac{r}{R})$	Δh (cm. CCl_4)	V (ft./sec)	$\frac{V}{V_{max}}$	$V(\frac{r}{R})$ (ft./sec)	$(1-\frac{r}{R})$	$1-(\frac{r}{R})^{1.25}$
0.989	3.6	2.09	0.518	2.07	0.011	0.014
0.970	4.8	2.42	0.599	2.35	0.030	0.037
0.950	5.6	2.61	0.646	2.48	0.050	0.062
0.930	6.2	2.74	0.679	2.55	0.070	0.086
0.900	7.0	2.92	0.723	2.63	0.100	0.124
0.850	7.8	3.08	0.762	2.62	0.150	0.184
0.800	8.4	3.20	0.792	2.56	0.200	0.243
0.700	9.3	3.36	0.832	2.35	0.300	0.360
0.600	10.2	3.52	0.872	2.11	0.400	0.472
0.500	11.0	3.66	0.906	1.83	0.500	0.579
0.400	12.0	3.82	0.946	1.53	0.600	0.682
0.300	12.6	3.92	0.970	1.18	0.700	0.778
0.200	13.0	3.98	0.985	0.80	0.800	0.866
0.100	13.2	4.01	0.994	0.40	0.900	0.944
0.000	13.4	4.04	1.000	0.00	1.000	1.000

Static Pressure = 25.0 cm. Hg. Gauge

t = 28.2°C.

V_{ave} (Manometer) = 3.60 ft./sec.

V_{ave} (Graphical integration) = 3.26 ft./sec.

$$\frac{V_{ave}}{V_{max}} = \frac{3.26}{4.04} = 0.808$$

Re = 59,000

RUN V I-44 Station No. 4

(Taken simultaneously with Run V H-14 and V H-15)

$\frac{r}{R}$	Δh (cm. CCl_4)	V (ft./sec.)	$\frac{V}{V_{max}}$	$V(\frac{r}{R})$ (ft./sec.)	$1-(\frac{r}{R})$	$1-(\frac{r}{R})^{1.25}$
0.989	5.8	2.67	0.547	2.64	0.011	0.014
0.970	7.0	2.94	0.603	2.85	0.030	0.037
0.950	8.6	3.24	0.664	3.08	0.050	0.062
0.930	9.4	3.39	0.695	3.16	0.070	0.086
0.900	10.0	3.49	0.715	3.14	0.100	0.124
0.850	11.4	3.72	0.763	3.16	0.150	0.184
0.800	12.4	3.88	0.795	3.10	0.200	0.243
0.700	14.2	4.16	0.852	2.92	0.300	0.360
0.600	15.4	4.33	0.888	2.60	0.400	0.472
0.500	16.4	4.47	0.916	2.24	0.500	0.579
0.400	17.2	4.58	0.939	1.83	0.600	0.682
0.300	18.2	4.71	0.965	1.41	0.700	0.778
0.200	19.0	4.81	0.986	0.96	0.800	0.866
0.100	19.4	4.86	0.995	0.49	0.900	0.944
0.000	19.6	4.88	1.000	0.00	1.000	1.000

Static pressure = 32.2 cm. Hg. Gauge

t = 25.78°C.

V_{ave} (manometer) = 3.92 ft./sec.

V_{ave} (Graphical integration) = 4.01 ft./sec.

$$\frac{V_{ave}}{V_{max}} = \frac{4.01}{4.88} = 0.821$$

Re. = 69,000

RUN V I-45 Station No. 4

(Taken simultaneously with Run V H-16 and V H-17)

$\frac{r}{R}$	Δh (cm. CCl_4)	V (ft/sec.)	$\frac{V}{V_{max}}$	$V(\frac{r}{R})$ (ft/sec)	$(1-\frac{r}{R})$	$1-(\frac{r}{R})^{1.25}$
0.989	3.0	1.91	0.557	1.89	0.011	0.014
0.970	3.8	2.15	0.627	2.08	0.030	0.037
0.950	4.4	2.31	0.674	2.20	0.050	0.062
0.930	4.8	2.42	0.706	2.25	0.070	0.086
0.900	5.2	2.52	0.735	2.27	0.100	0.124
0.850	5.5	2.60	0.759	2.21	0.150	0.184
0.800	5.9	2.70	0.788	2.16	0.200	0.243
0.700	6.6	2.86	0.834	2.00	0.300	0.360
0.600	7.3	3.00	0.875	1.80	0.400	0.472
0.500	8.0	3.14	0.916	1.52	0.500	0.579
0.400	8.6	3.23	0.943	1.29	0.600	0.682
0.300	9.0	3.32	0.969	1.00	0.700	0.778
0.200	9.3	3.37	0.983	0.67	0.800	0.866
0.100	9.5	3.41	0.995	0.34	0.900	0.944
0.000	9.6	3.43	1.000	0.00	1.000	1.000

Static pressure = 30.8 cm. Hg. Gauge

t = 27.2°C.

V_{ave} (Manometer) = 2.84 ft./sec.

V_{ave} (Graphical integration) = 2.79 ft./sec.

$$\frac{V_{ave}}{V_{max}} = \frac{2.79}{3.43} = 0.812$$

Re. = 49,400

RUN V I-46 Station No. 4

(Taken simultaneously with Run V H-18 and V H-19)

$\frac{r}{R}$	Δh (cm. CCl_4)	V (ft./sec)	$\frac{V}{V_{max}}$	V (ft/sec.)	$(1-(\frac{r}{R})$	$1-(\frac{r}{R})^{1.25}$
0.989	7.0	2.94	0.536	2.91	0.011	0.014
0.970	9.4	3.39	0.618	3.29	0.030	0.037
0.950	10.4	3.56	0.649	3.38	0.050	0.062
0.930	11.4	3.72	0.679	3.46	0.070	0.086
0.900	12.4	3.88	0.708	3.49	0.100	0.124
0.850	13.8	4.09	0.746	3.48	0.150	0.184
0.800	14.8	4.25	0.776	3.40	0.200	0.243
0.700	17.0	4.56	0.832	3.19	0.300	0.360
0.600	18.8	4.79	0.874	2.87	0.400	0.472
0.500	20.4	4.98	0.909	2.49	0.500	0.579
0.400	21.8	5.15	0.940	2.06	0.600	0.682
0.300	23.0	5.28	0.964	1.58	0.700	0.778
0.200	23.6	5.35	0.976	1.10	0.800	0.866
0.100	24.4	5.44	0.992	0.54	0.900	0.944
0.000	24.8	5.48	1.000	0.00	1.000	1.000

Static Pressure = 39.2 cm. Hg. Gauge

t = 23.8°C.

V_{ave} (Manometer) = 4.76 ft./sec.

V_{ave} (Graphical integration) = 4.42 ft./sec.

$$\frac{V_{ave}}{V_{max}} = \frac{4.42}{5.48} = 0.807$$

Re. = 74,700

RUN V I-47 Station No. 2

$\frac{r}{R}$	Δh (cm. CCl_4)	V (ft./sec.)	$V (\frac{r}{R})$ (ft./sec.)	$(1 - \frac{r}{R})$
0.991	0.60	0.86	0.483	0.009
0.970	0.85	1.02	0.573	0.030
0.950	1.00	1.10	0.618	0.050
0.90	1.25	1.233	0.694	0.10
0.80	1.60	1.393	0.784	0.20
0.70	1.85	1.500	0.843	0.30
0.60	2.00	1.557	0.875	0.40
0.40	2.20	1.639	0.921	0.60
0.20	2.40	1.710	0.960	0.80
0.00	2.60	1.780	1.000	1.00

Static Pressure = 10.20 cm. Hg Gauge

t_{H_2O} = 6.85°C.

$V_{ave.}$ (Graphical Integration) = 1.429 ft./sec.

$$\frac{V_{ave}}{V_{max}} = \frac{1.429}{1.780} = 0.803$$

$$Re = \frac{0.1626 \times 1.429 \times 10^5}{1.544} = 15,030$$

RUN V I-48 Station No. 3

$\frac{r}{R}$	Δh (cm. CCl_4)	V (ft./sec.)	$\frac{v}{V_{max}}$	$V(\frac{r}{R})$ (ft./sec.)	$(1-\frac{r}{R})$
0.985	0.7	0.93	0.522	0.916	0.015
0.970	0.9	1.077	0.605	1.044	0.030
0.950	1.1	1.157	0.650	1.100	0.050
0.900	1.3	1.258	0.706	1.132	0.100
0.800	1.6	1.393	0.784	1.116	0.200
0.700	1.86	1.500	0.843	1.050	0.300
0.600	2.0	1.557	0.875	0.935	0.400
0.400	2.2	1.639	0.920	0.655	0.600
0.200	2.4	1.71	0.960	0.342	0.800
0.000	2.5	1.780	1.000	0.00	1.000

Static pressure = 10.8 Cm. Hg. Gauge

t_{H_2O} = 6.85°C.

V_{ave} (Graphical Integration) = 1.432 ft./sec.

$$\frac{V_{ave}}{V_{max}} = \frac{1.432}{1.780} = 0.804$$

$$Re = \frac{.1626 \times 1.432 \times 10^5}{1.544} = 15{,}070$$

RUN V I-49 Station No. 3

$\frac{r}{R}$	Δh (cm. CCl_4)	V (ft./sec.)	$\frac{V}{V_{max}}$	$V(\frac{r}{R})$ (ft./sec.)	$(1-\frac{r}{R})$
0.985	25.8	5.605	0.628	5.52	0.015
0.970	29.4	5.983	0.670	5.80	0.030
0.950	33.0	6.34	0.710	6.02	0.050
0.900	39.2	6.91	0.774	6.22	0.100
0.800	46.6	7.535	0.8435	6.02	0.200
0.700	51.4	7.91	0.885	5.54	0.300
0.600	55.6	8.24	0.923	4.94	0.400
0.400	61.4	8.66	0.970	3.465	0.600
0.200	63.4	8.81	0.986	1.762	0.800
0.000	65.5	8.935	1.000	0	1.000

Static pressure = 34.70 Cm. Hg. Gauge

t_{H_2O} = 27.0°C.

V_{ave} (manometer) = 7.30 ft./sec.

V_{ave} (Graphical Integration) = 7.660 ft./sec.

$$\frac{V_{ave}}{V_{max}} = \frac{7.660}{8.935} = 0.858$$

$$Re = \frac{0.1626 \times 7.66 \times 10^5}{0.923} = 135,000$$

RUN V I-50 Station No. 4

$\frac{r}{R}$	Δh (cm. CCl_4)	V (ft./sec.)	$\frac{v}{V_{max}}$	$V(\frac{r}{R})$ (ft./sec.)	$(1-\frac{r}{R})$
0.989	22.7	5.258	0.589	5.20	0.011
0.970	27.5	5.788	0.648	5.61	0.030
0.950	31.7	6.21	0.695	5.90	0.050
0.900	38.1	6.81	0.762	6.13	0.100
0.800	44.3	7.35	0.823	5.88	0.200
0.700	48.9	7.72	0.864	5.40	0.300
0.600	53.5	8.08	0.905	4.85	0.400
0.400	59.7	8.53	0.955	3.41	0.600
0.200	64.1	8.84	0.989	1.77	0.800
0.000	65.6	8.94	1.000	0	1.000

Static Pressure = 26.8 Cm. Hg. Gauge

t_{H_2O} = 27.0°C.

V_{ave} (Manometer) = 7.30 ft./sec.

V_{ave} (Graphical Integration) = 7.548 ft./sec.

$$\frac{V_{ave}}{V_{max}} = \frac{7.548}{8.94} = 0.845$$

$$Re = \frac{0.1626 \times 7.548 \times 10^5}{0.923} = 133,000$$

RUN V I-51 Station No. 3

$\frac{r}{R}$	Δh (cm. CCl_4)	V (ft./sec.)	$\frac{V}{V_{max}}$	$V(\frac{r}{R})$ (ft./sec.)	$(1-\frac{r}{R})$
0.985	27.0	5.735	0.655	5.65	0.015
0.970	30.8	6.12	0.699	5.94	0.030
0.950	34.4	6.47	0.739	6.15	0.050
0.900	40.2	7.00	0.799	6.30	0.100
0.800	46.6	7.53	0.860	6.02	0.200
0.700	51.0	7.88	0.900	5.52	0.300
0.600	54.4	8.15	0.930	4.89	0.400
0.400	59.2	8.49	0.970	3.395	0.600
0.200	61.4	8.65	0.987	1.73	0.800
0.000	62.9	8.76	1.000	0	1.000

t_{H_2O} = 57.4°C. (Constant temp. maintained by means of heating the water at first, then circulating through the pipe by adding a little steam to the cooling section).

V_{ave} (Manometer) = 7.34 ft./sec.

V_{ave} (Graphical Integration) = 7.664 ft./sec.

$$\frac{V_{ave}}{V_{max}} = \frac{7.664}{8.76} = 0.875$$

$$Re = \frac{.1626 \times 7.664 \times 10^5}{0.533} = 234,000$$

RUN V I-52 Station No. 3

(Taken simultaneously with Run V H-27)

$\frac{r}{R}$	Δh (cm. CCl_4)	V (ft./sec.)	$\frac{v}{V_{max}}$	$V(\frac{r}{R})$ (ft./sec.)	$(1-\frac{r}{R})$
0.985	5.30	2.542	0.552	2.50	0.015
0.970	6.60	2.832	0.615	2.745	0.030
0.950	7.40	3.00	0.652	2.85	0.050
0.90	9.00	3.31	0.719	2.98	0.100
0.80	11.00	3.66	0.795	2.925	0.200
0.70	13.00	3.98	0.865	2.785	0.300
0.60	14.40	4.19	0.910	2.515	0.400
0.40	16.10	4.43	0.962	1.772	0.600
0.20	17.20	4.577	0.995	0.915	0.800
0.00	17.40	4.603	1.000	0.00	1.000

Static Pressure = 24.2 Cm. Hg. Gauge

V_{ave} (Graphical Integration = 3.82 ft./sec.

$$\frac{V_{ave}}{V_{max}} = \frac{3.82}{4.603} = 0.830$$

t = 7.7°C.

$$Re = \frac{0.1626 \times 3.82 \times 10^5}{1.505} = 41,250$$

RUN V I-53 Station No. 3

(Taken Simultaneously with Run V H-30)

$\frac{r}{R}$	Δh (cm. CCl_4)	V (ft./sec.)	$\frac{V}{V_{max}}$	$V(\frac{r}{R})$ 2 (ft./sec.)	$(1-\frac{r}{R})$
0.985	0.73	0.95	0.538	0.936	0.015
0.970	0.92	1.06	0.601	1.028	0.030
0.950	1.07	1.14	0.646	1.083	0.050
0.90	1.35	1.28	0.725	1.152	0.10
0.80	1.60	1.393	0.790	1.115	0.20
0.70	1.81	1.48	0.839	1.036	0.30
0.60	1.99	1.556	0.882	0.935	0.40
0.40	2.25	1.658	0.940	0.664	0.60
0.20	2.43	1.72	0.975	0.344	0.80
0.00	2.55	1.764	1.000	0.00	1.00

Static Pressure = 39.6 cm. Hg. Gauge

V_{ave} (Weighing) = 1.434 ft./sec.

V_{ave} (Graphical Integration) = 1.438 ft./sec.

$$\frac{V_{ave}}{V_{max}} = \frac{1.438}{1.764} = 0.815$$

t_{H_2O} = 8.8°C.

$$Re = \frac{0.1626 \times 1.438 \times 10^5}{1.457} = 16,040$$

RUN V I-54 Station No. 3

(Taken simultaneously with Run V H-31, and Run V H-32)

$\frac{r}{R}$	Δh (cm. CCl_4)	V (ft./sec.)	$\frac{v}{V_{max}}$	$V(\frac{r}{R})$ (ft./sec.)	$(1-\frac{r}{R})$
0.985	4.60	2.368	0.582	2.33	0.015
0.970	5.10	2.494	0.613	2.42	0.030
0.950	6.30	2.77	0.681	2.63	0.050
0.900	7.30	2.98	0.732	2.68	0.100
0.80	8.90	3.295	0.810	2.635	0.200
0.70	10.10	3.51	0.863	2.455	0.300
0.60	11.0	3.60	0.885	2.16	0.400
0.40	12.5	3.90	0.959	1.56	0.600
0.20	13.4	4.04	0.992	0.808	0.800
0.00	13.6	4.07	1.000	0.00	1.000

Static Pressure = 28.40 cm. Hg. Gauge

V_{ave} (Graphical Integration) = 3.375 ft./sec.

$$\frac{V_{ave}}{V_{max}} = \frac{3.375}{4.07} = 0.829$$

t_{H_2O} = 8.16°C.

$$Re = \frac{0.1626 \times 3.375 \times 10^5}{1.485} = 36,950$$

RUN V I-55 Station No. 3

(Taken simultaneously with Run V H-33 and Run V H-34)

$\frac{r}{R}$	Δh (c. CCl_4)	V (ft./sec.)	$\frac{V}{V_{max}}$	$V(\frac{r}{R})$ (ft./sec.)	$(1-\frac{r}{R})$
0.985	0.70	0.925	0.484	0.911	0.015
0.970	0.90	1.05	0.550	1.018	0.030
0.950	1.10	1.156	0.605	1.098	0.050
0.90	1.40	1.304	0.683	1.174	0.10
0.80	1.70	1.436	0.752	1.148	0.20
0.70	2.10	1.597	0.836	1.118	0.30
0.60	2.30	1.676	0.878	1.006	0.40
0.40	2.70	1.815	0.950	0.726	0.60
0.20	2.90	1.880	0.984	0.376	0.80
0.00	3.00	1.910	1.000	0.0	1.00

Static Pressure = 38.0 Cm. Hg. Gauge

V_{ave} (Weighing) = 1.515 ft./sec.

V_{ave} (Graphical Integration) = 1.522 ft./sec.

$$\frac{V_{ave}}{V_{max}} = \frac{1.522}{1.910} = 0.797$$

t_{H_2O} = 8.88°C.

$$Re = \frac{0.1626 \times 1.522 \times 10^5}{1.454} = 17,000$$

RUN V I-56 Station No. 3

(Taken simultaneously with Run V H-35 and Run V H-36.)

$\frac{r}{R}$	Δh (Cm. CCl_4)	V (ft./sec.)	$\frac{v}{V_{max}}$	$V(\frac{r}{R})$ (ft./sec.)	$(1-\frac{r}{R})$
0.985	1.40	1.303	0.513	1.283	0.015
0.970	1.60	1.39	0.547	1.348	0.030
0.950	1.86	1.50	0.590	1.425	0.050
0.90	2.40	1.71	0.673	1.539	0.10
0.80	3.34	2.02	0.795	1.615	0.20
0.70	4.20	2.264	0.891	1.584	0.30
0.60	4.44	2.325	0.915	1.394	0.40
0.40	4.70	2.393	0.941	0.957	0.60
0.20	5.10	2.493	0.981	0.499	0.80
0.00	5.30	2.542	1.000	0.00	1.00

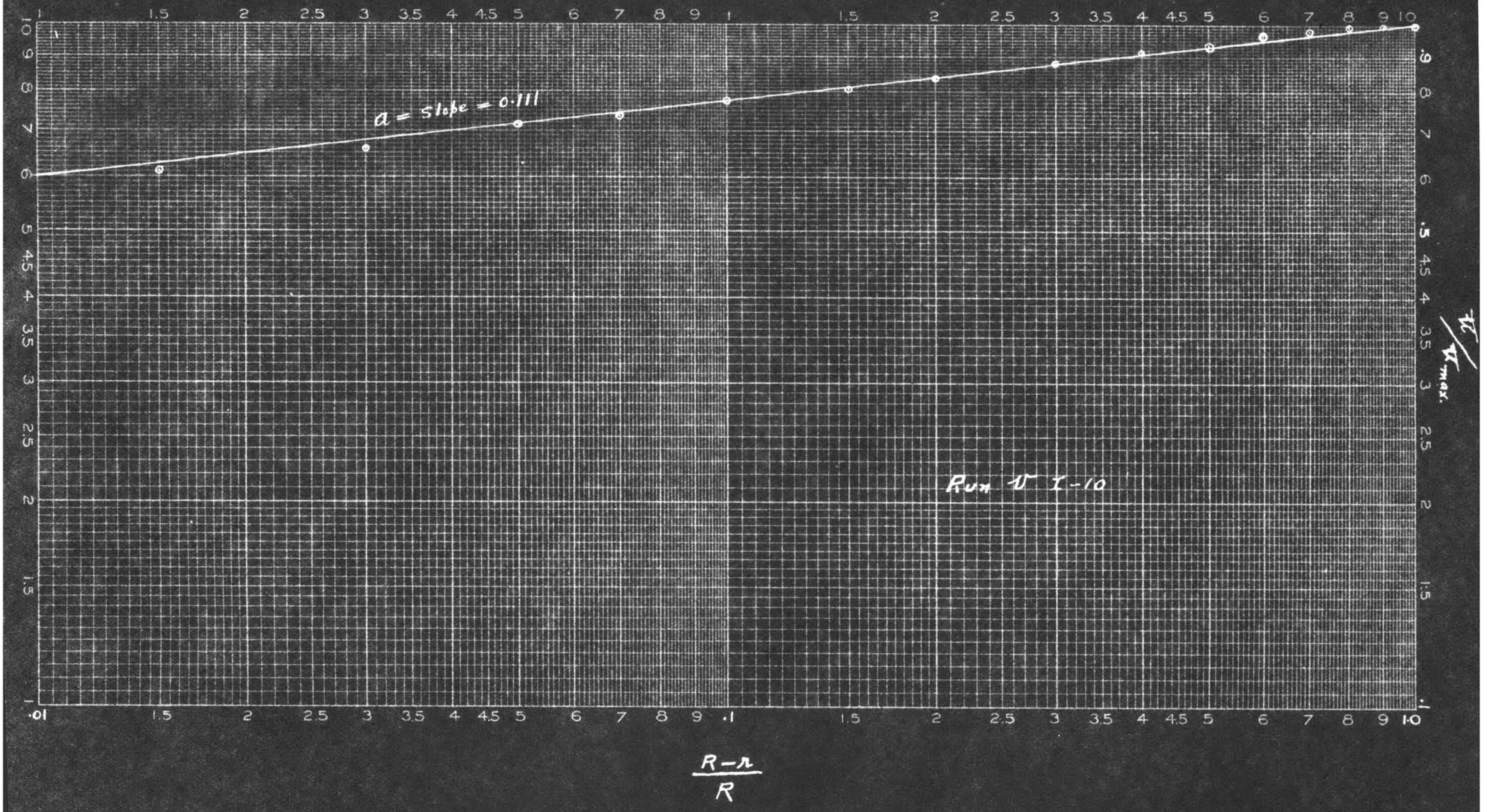
Static pressure = 35.10 Cm. Hg. Gauge

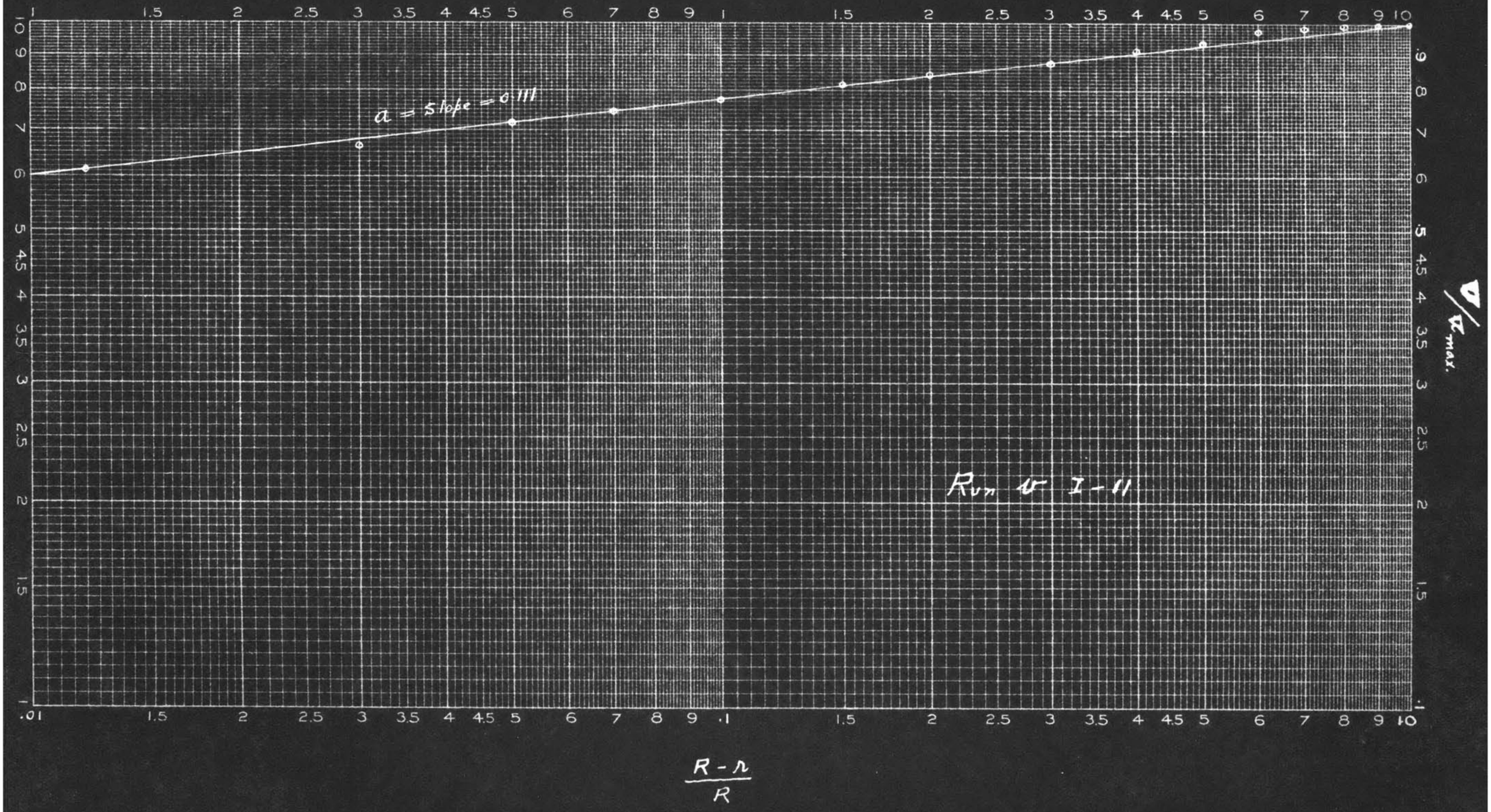
V_{ave} (Graphical Integration) = 2.065 ft./sec.

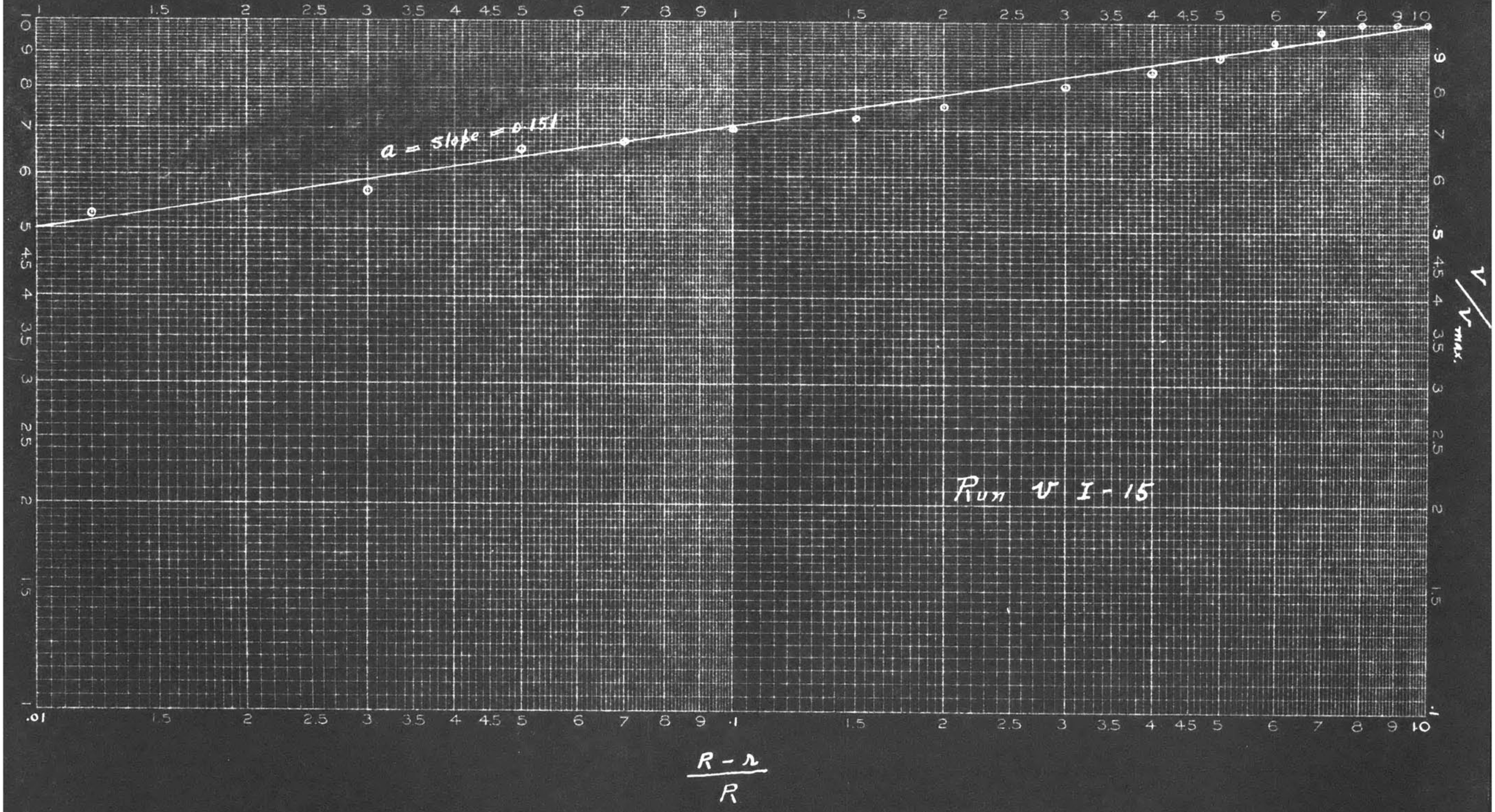
$$\frac{V_{ave}}{V_{max}} = \frac{2.065}{2.542} = 0.813$$

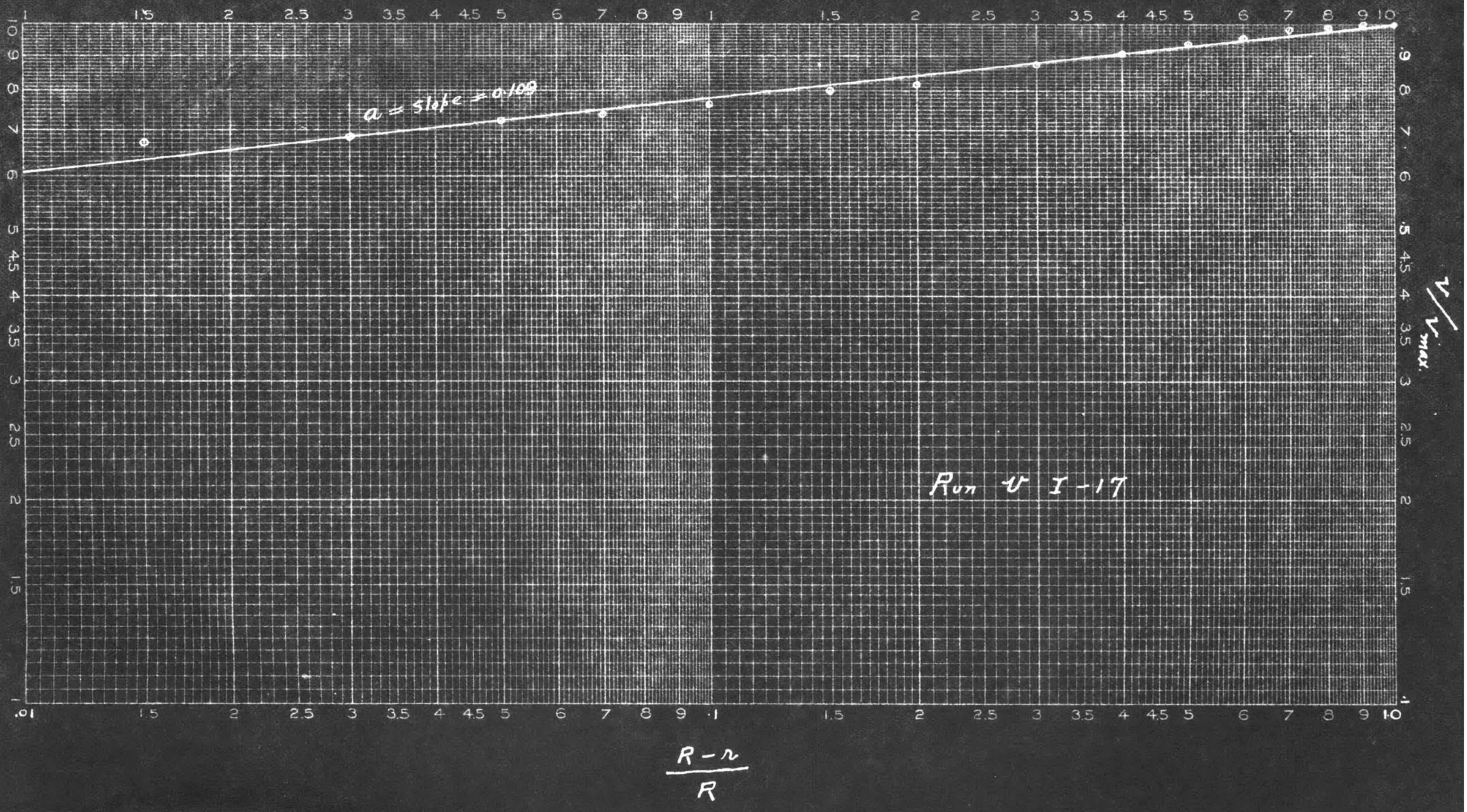
t_{H_2O} = 8.1°C.

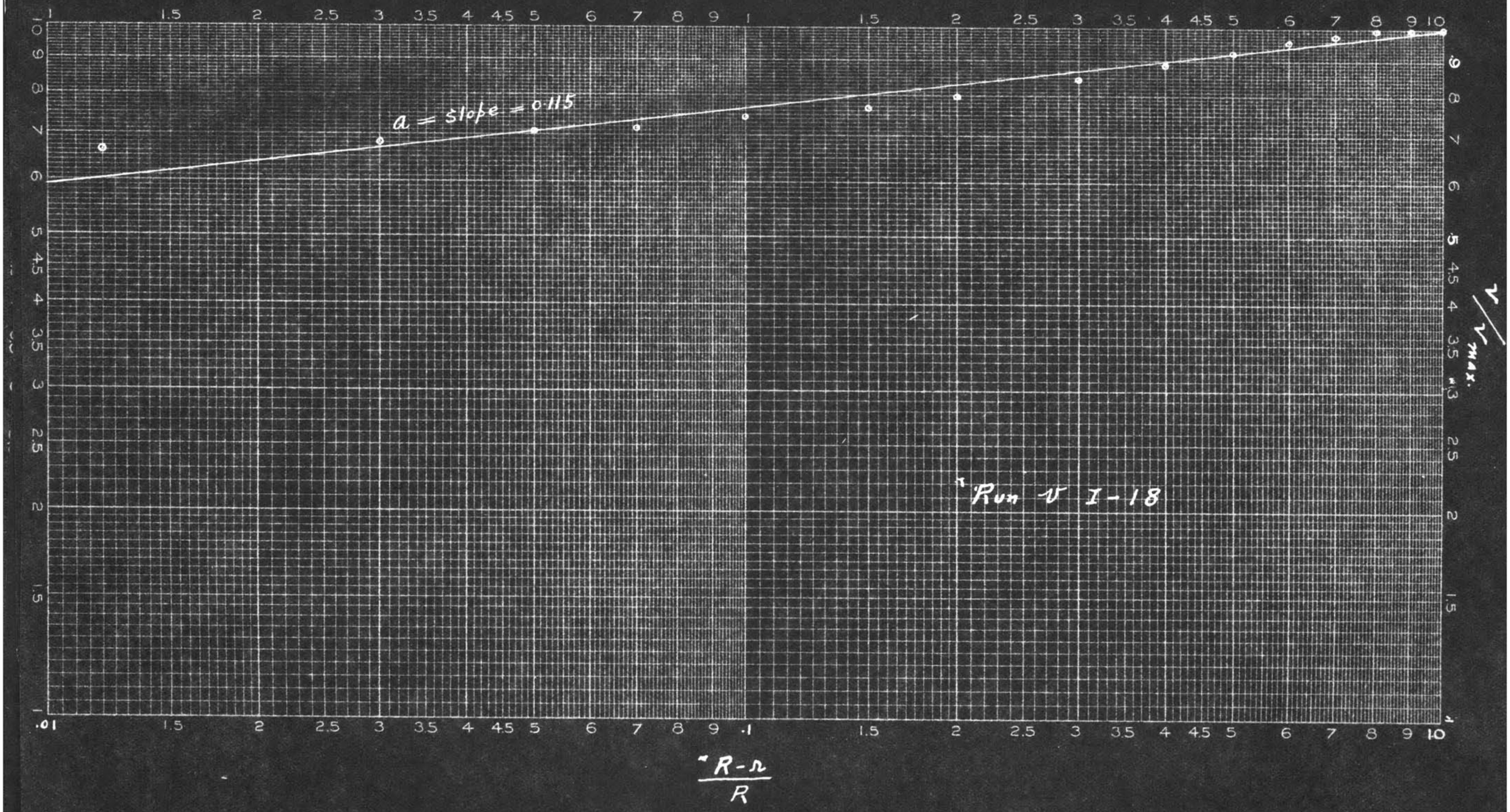
$$Re = \frac{0.1626 \times 2.065 \times 10^5}{1.487} = 22{,}600$$

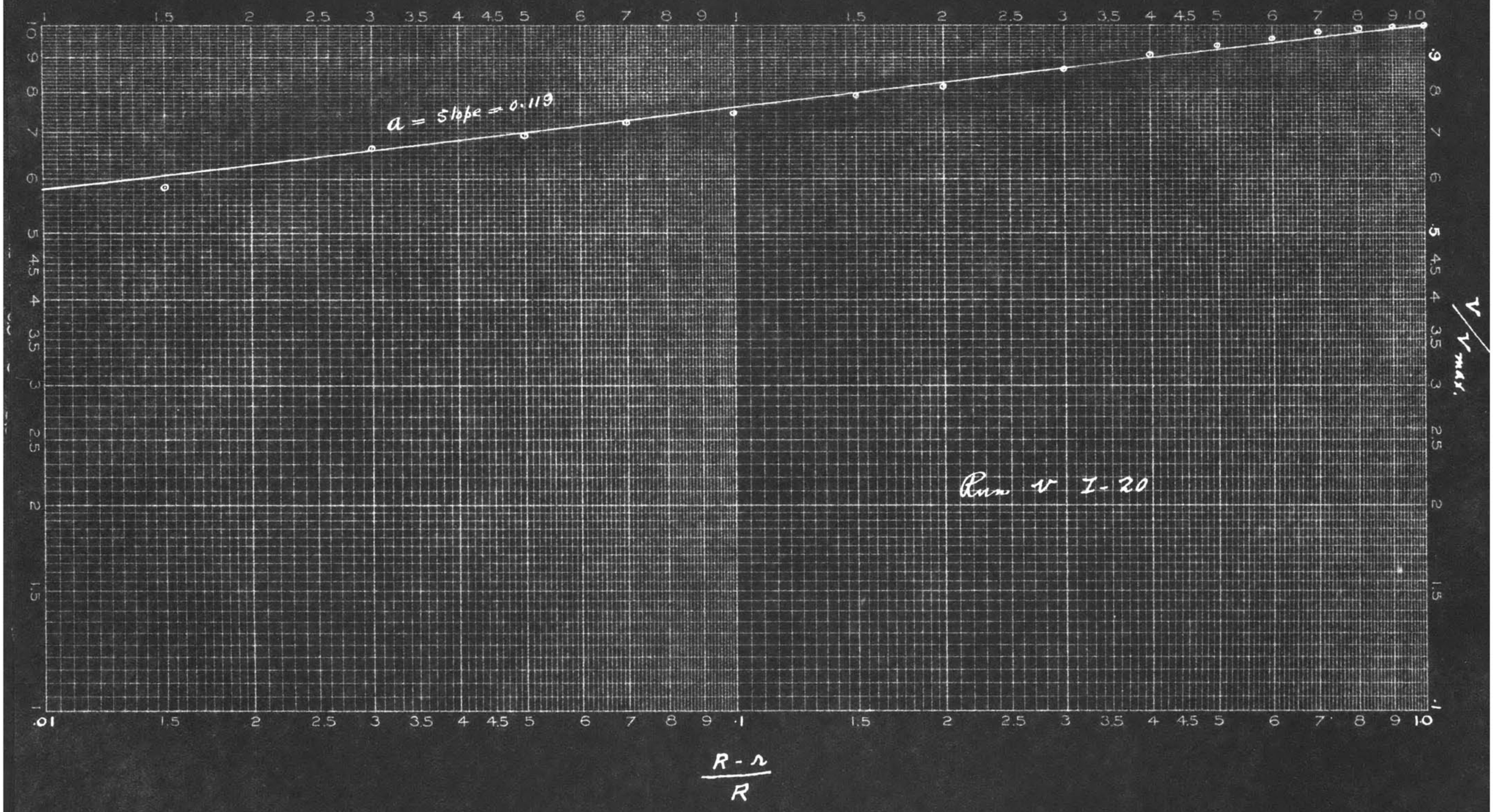
a = Slope = 0.111
Run V I-10
u/u max.
(R-r)/R
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.5
.9


a = slope = 0.111
Run v I-11
$\frac{R-r}{R}$
v/v max.
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
1.0
.1
1.5
2
2.5
3
3.5
4
4.5
.5
.6
.7
.8
.9
1.0

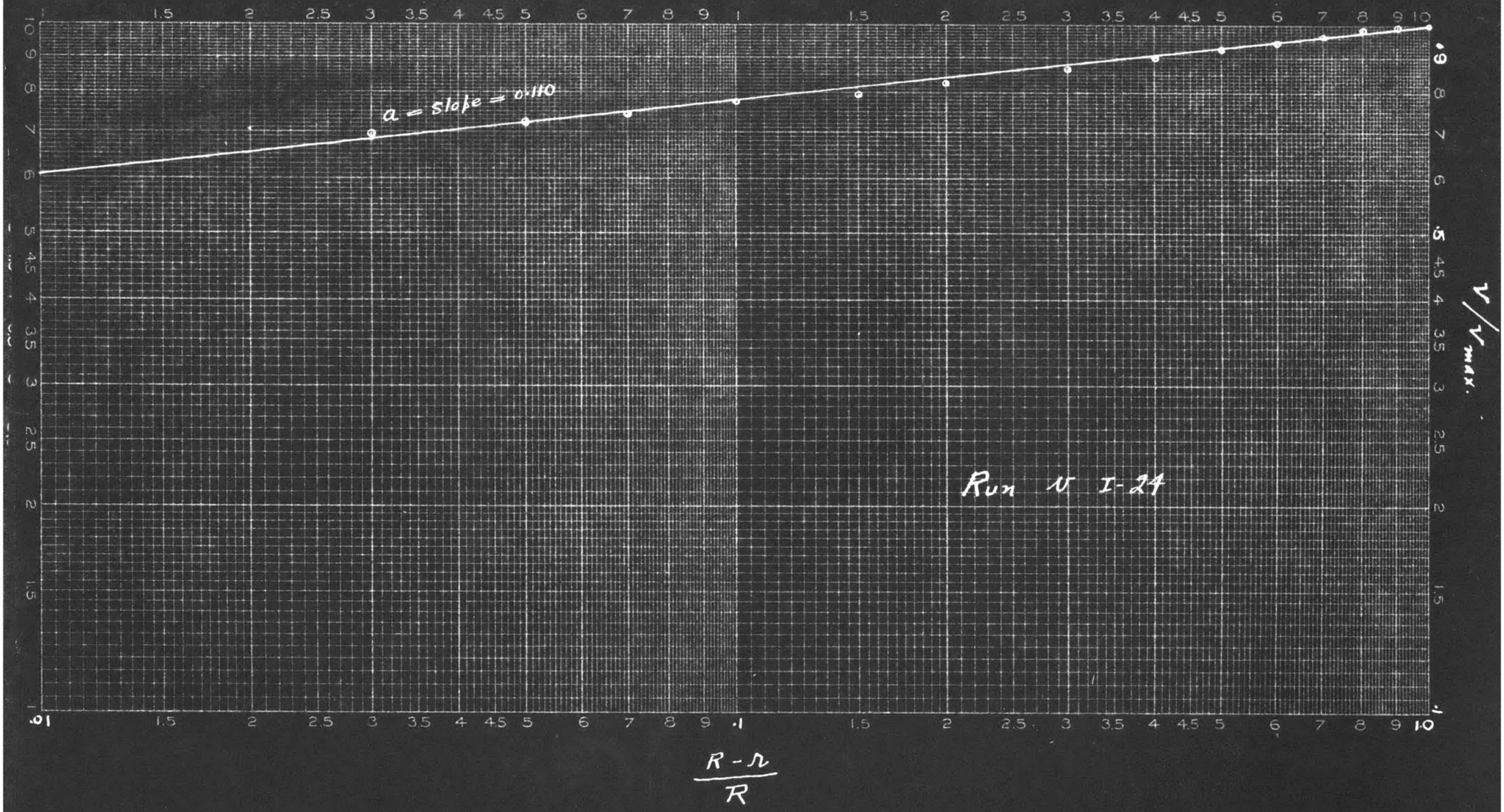

a = slope = 0.151
Run V I-15
R - r / R
V / V max.
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
1.0
.9
.8
.7
.6
.5
4.5
4
3.5
3
2.5
2
1.5
.1


a = slope = 0.109
Run V I-17
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
·1
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
1.0
$\frac{R-r}{R}$
.1
1.5
2
2.5
3
3.5
4
4.5
.5
6
7
8
.9
1.0
$v/V_{max.}$

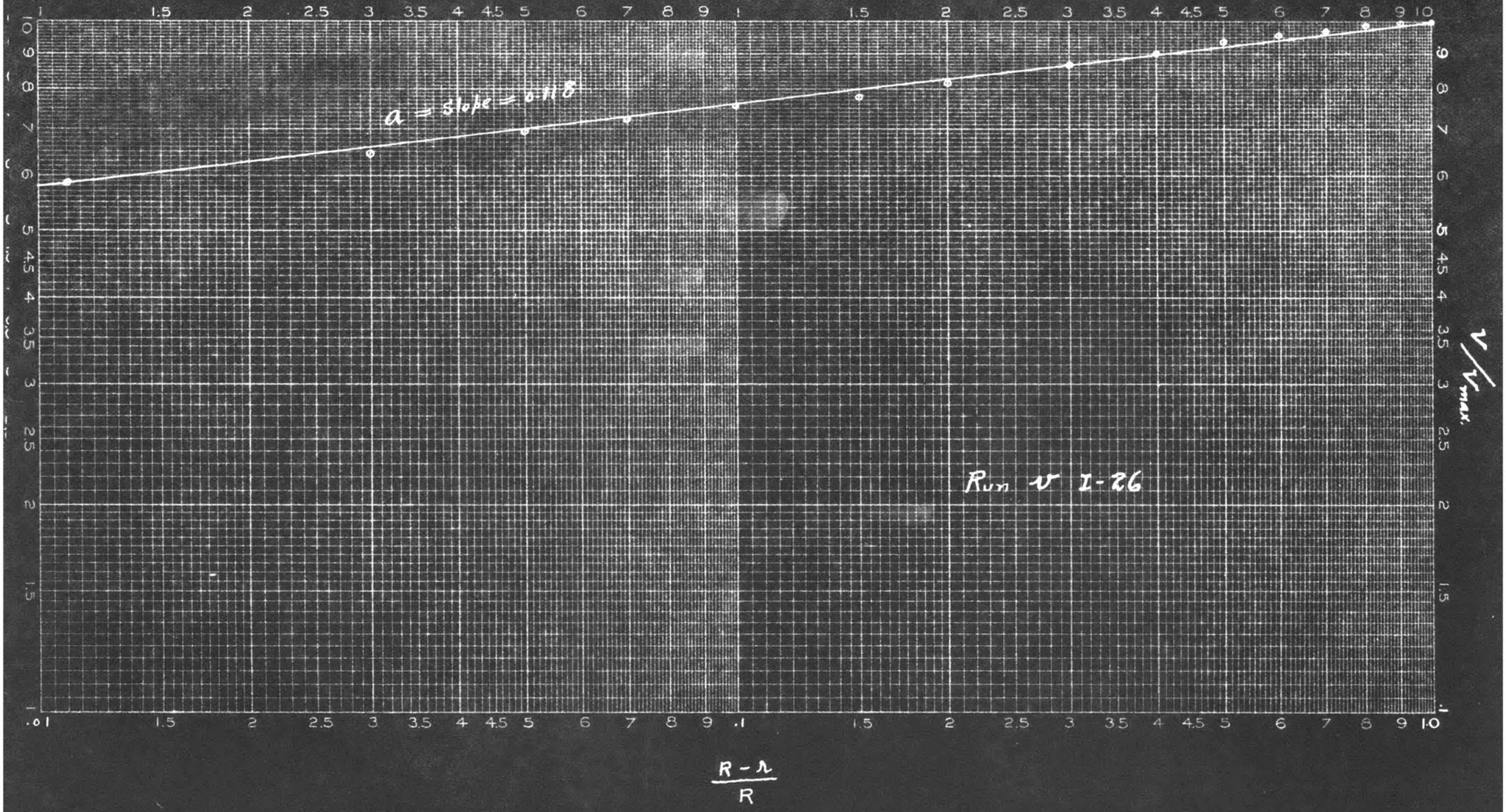

a = slope = 0·115
Run V I-18
R-r / R
v / V max.
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
10
1
.9
.8
.7
.6
.5
.4
.3
.2

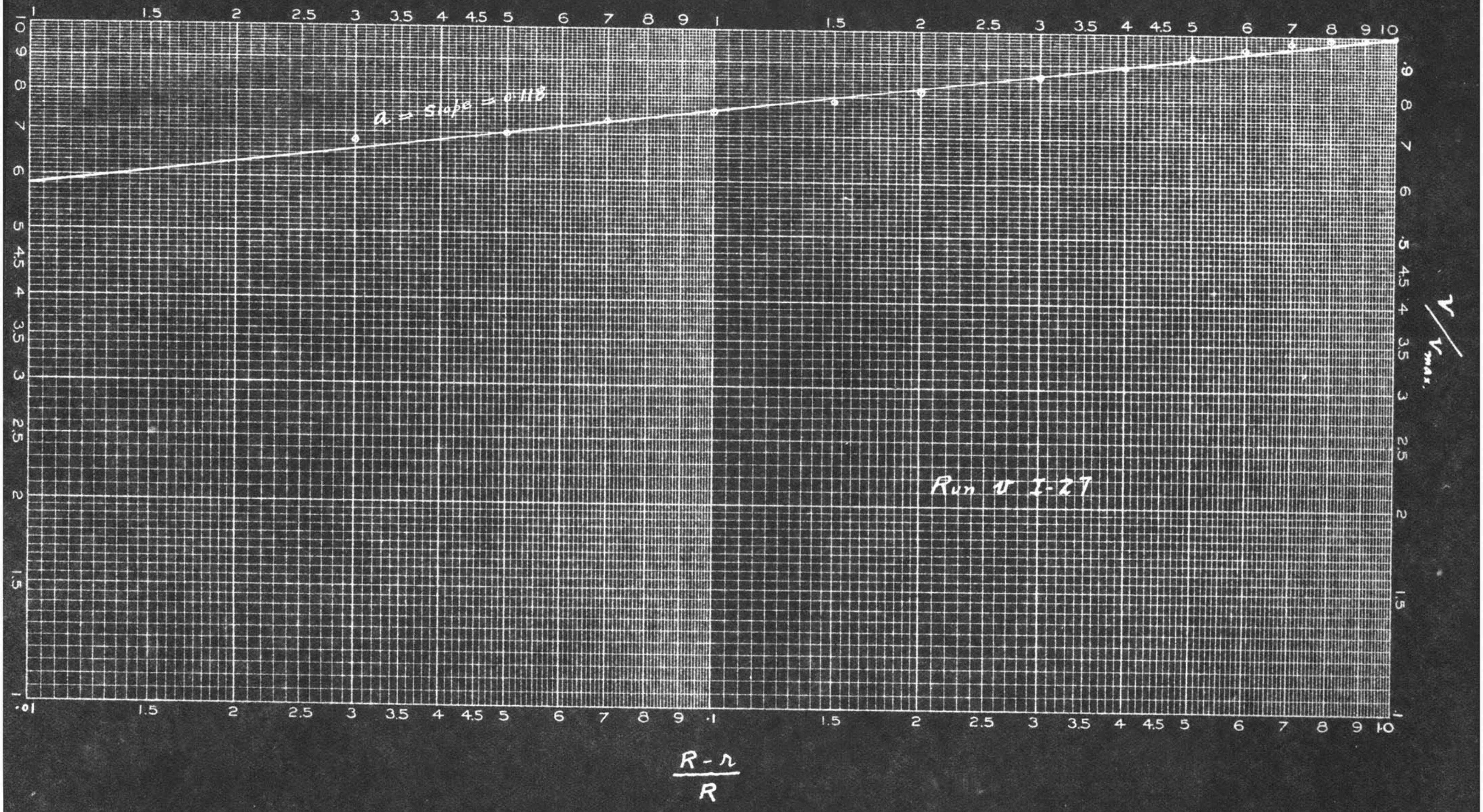


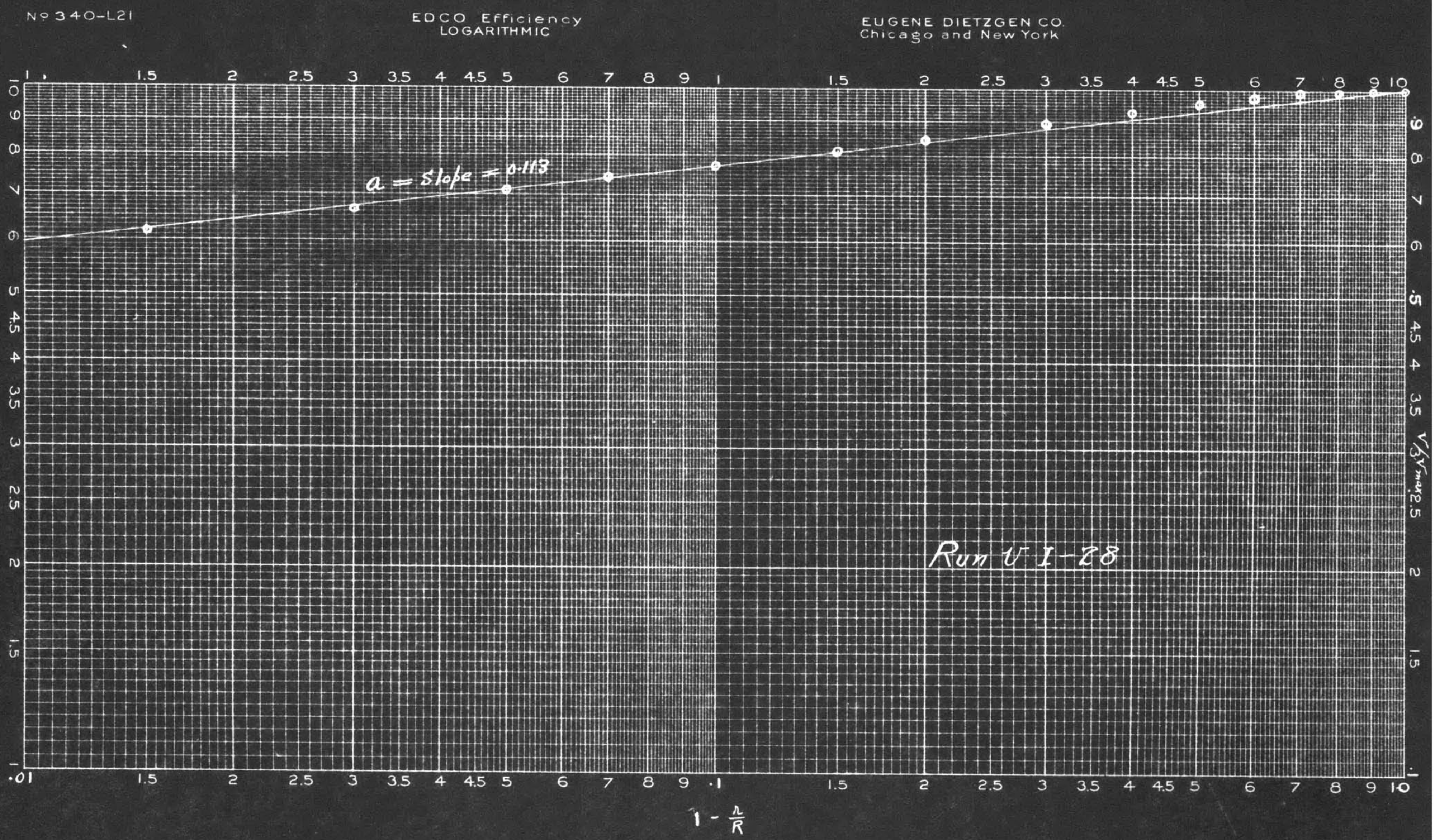
a = Slope = 0.119
Run V I-20
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.1
.5
.9
$\frac{R-r}{R}$
$V/V_{max.}$


a = slope = 0.110
Run V I-24
$\frac{R-r}{R}$
$V/V_{max.}$
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
·9
8
7
6
·5
4.5
4
3.5
3
2.5
2
1.5
·1

7


a = slope = 0.118
Run V I-26
v/V max.
(R - r)/R
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
1
10

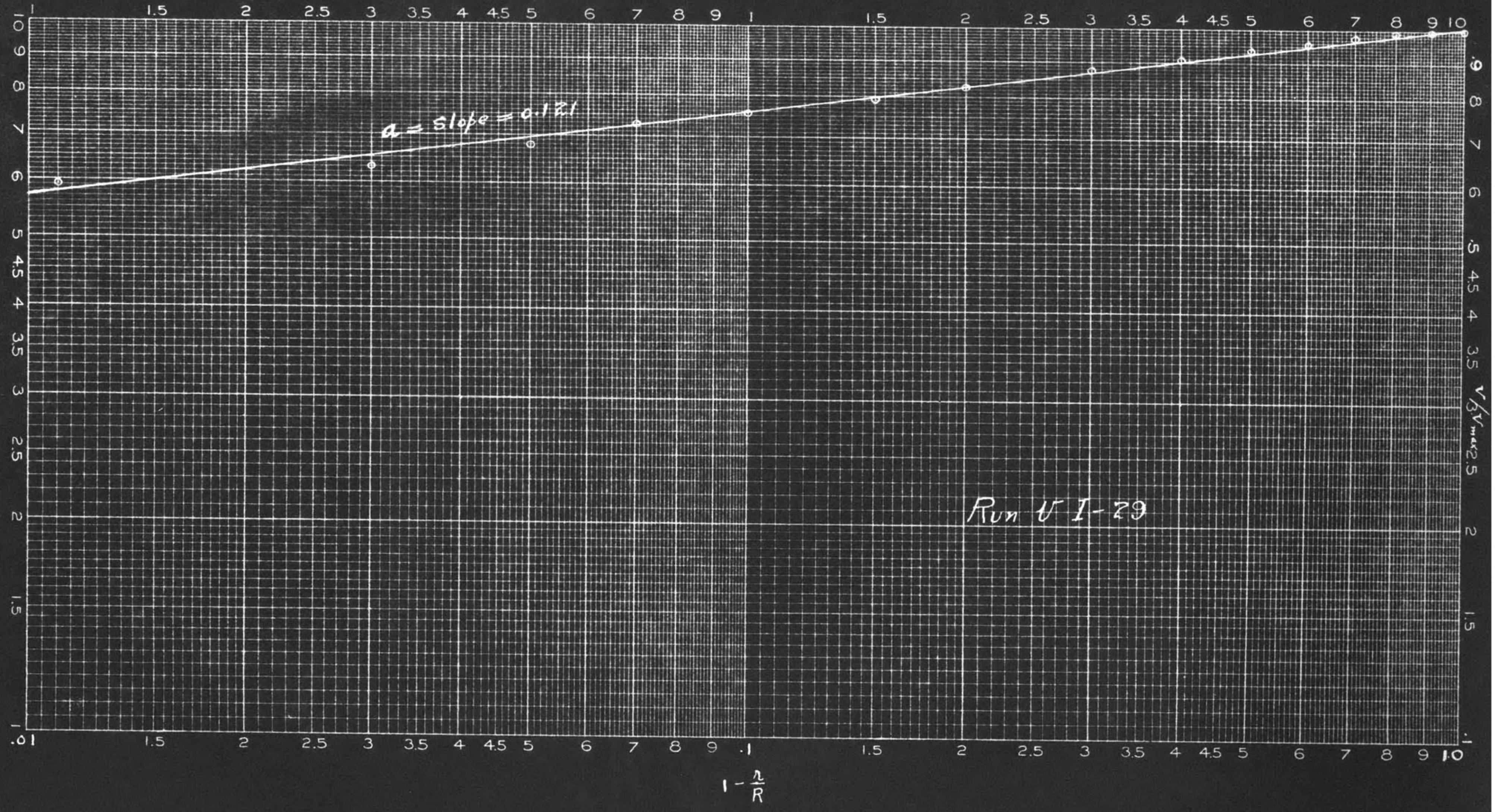

a = Slope = 0.118
Run V I-27
v/v_max.
(R - r)/R
·01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
·1
1·0
·9
·1

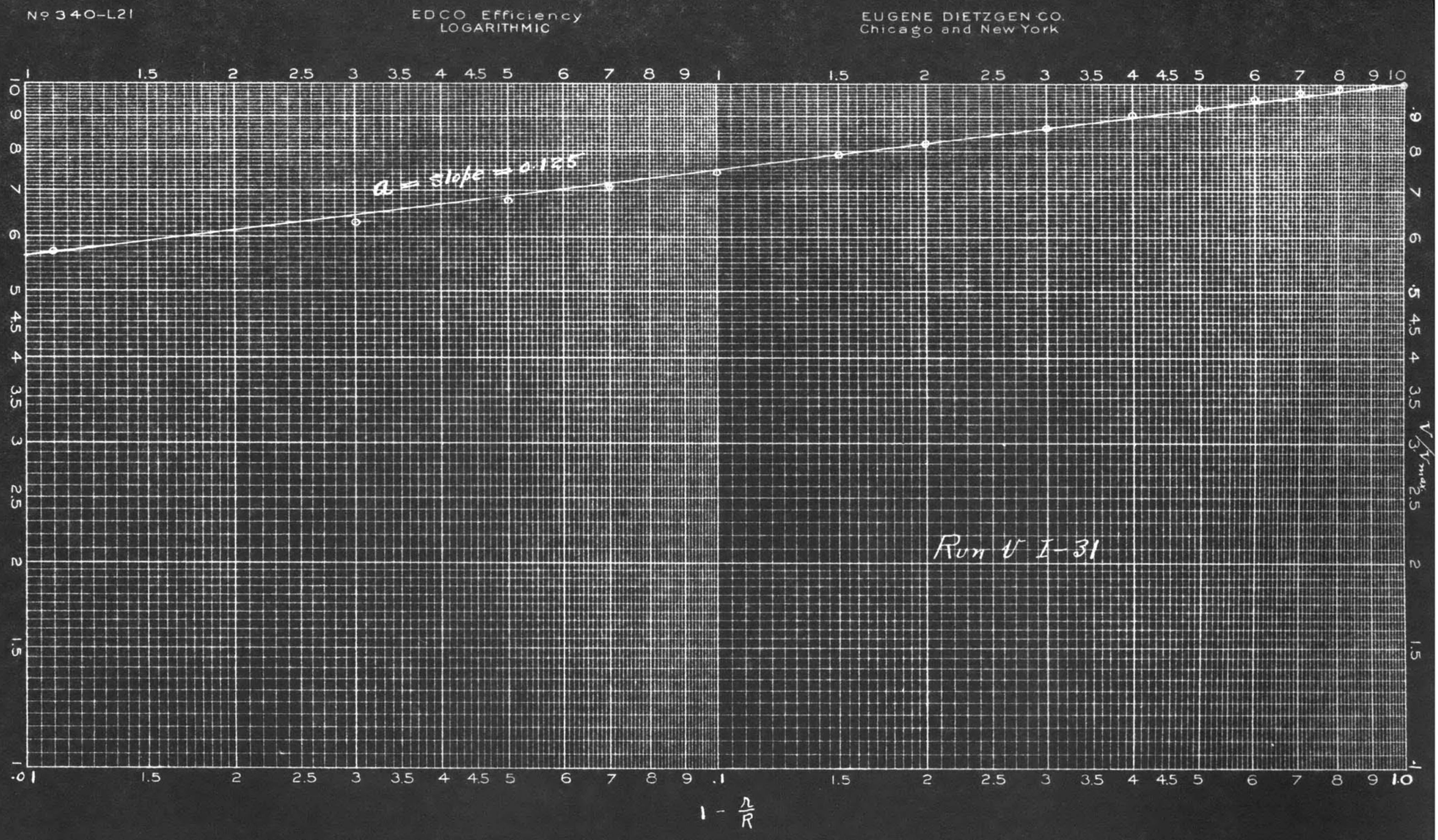

Nº 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
a = Slope = 0.113
Run V I-28
1 - r/R
V/V max
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1

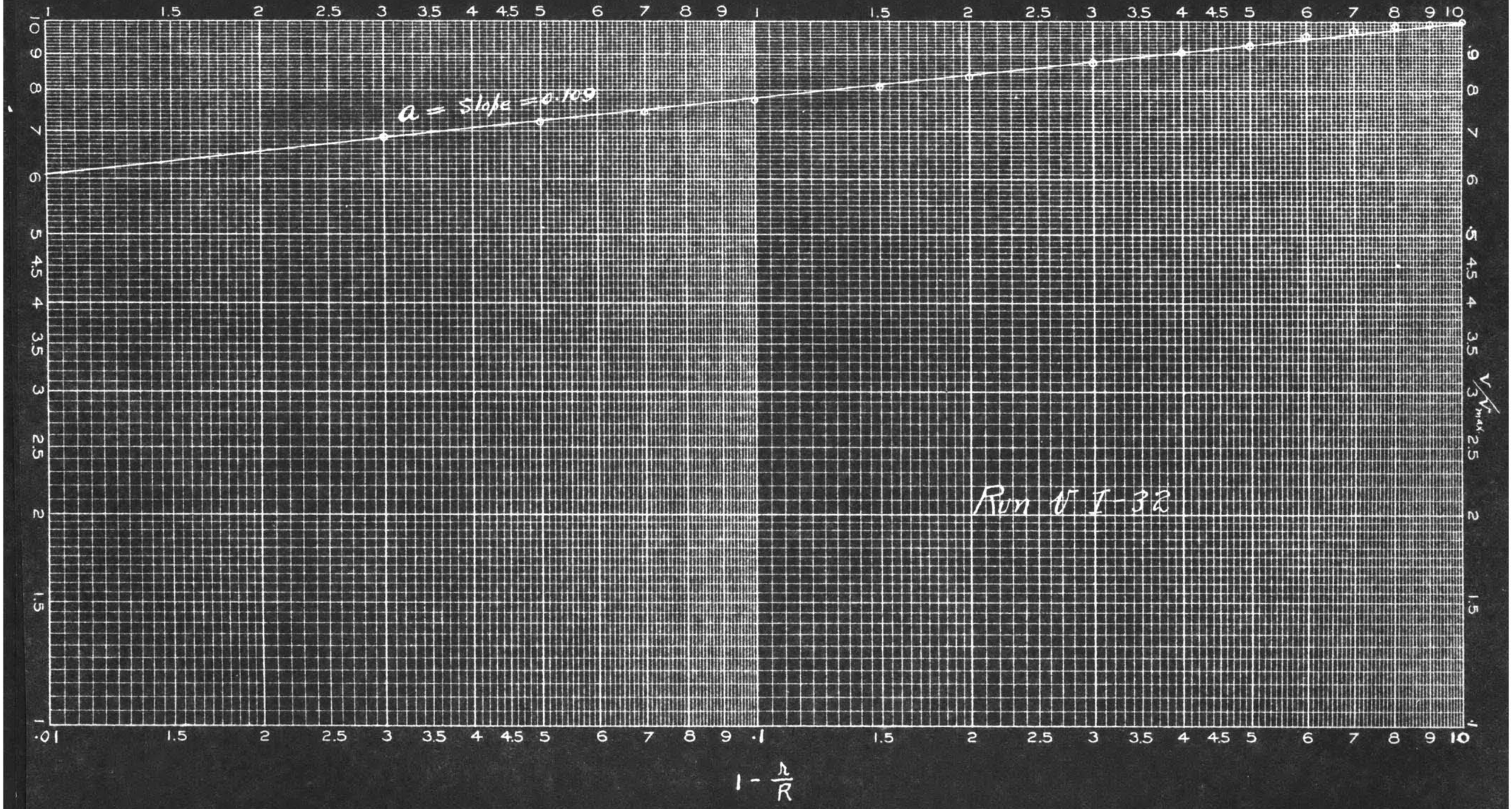
No 340-L21

EDCO Efficiency
LOGARITHMIC

EUGENE DIETZGEN CO
Chicago and New York




No. 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 10
10 9 8 7 6 5 4.5 4 3.5 3 2.5 2 1.5 1
.9 .8 .7 .6 .5 4.5 4 3.5 3 2.5 2 1.5 .1
.01 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 .1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 1.0
a = slope = 0.125
Run V I-31
V/V_{max}
$1 - \frac{r}{R}$



a = Slope = 0.109
Run V I-32
$1 - \frac{r}{R}$
v/v_{max}
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
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1
1.5
2
2.5
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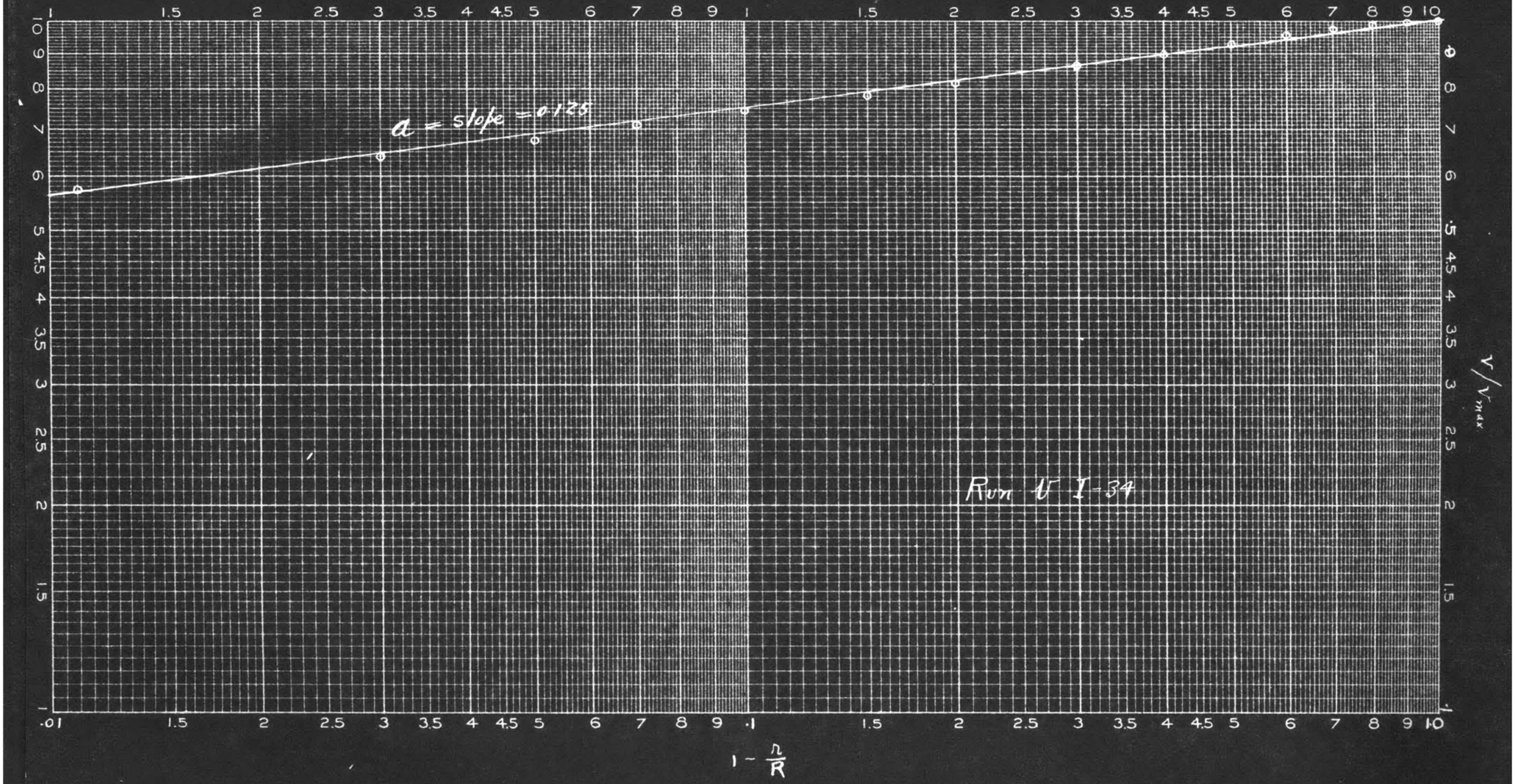
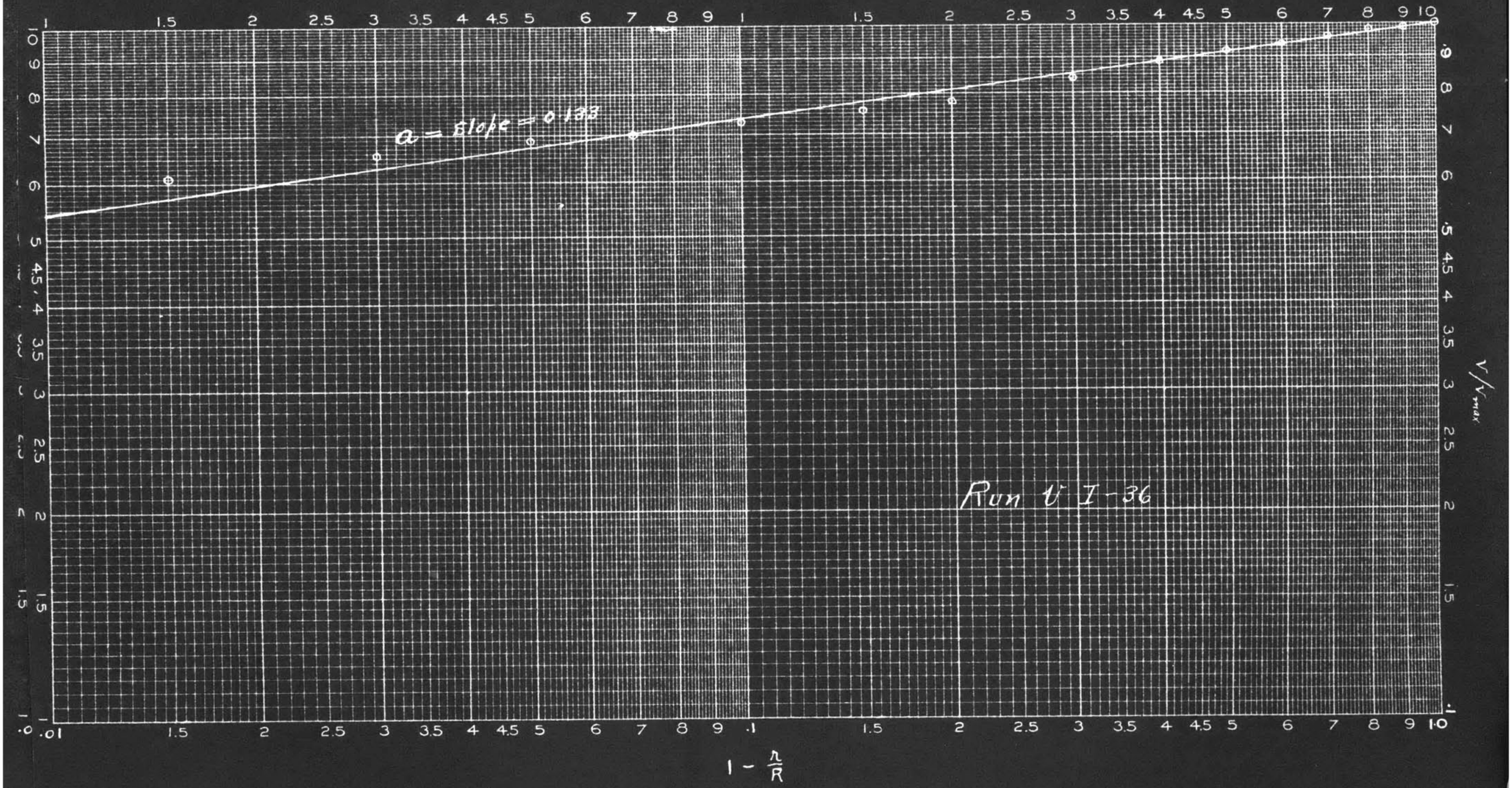
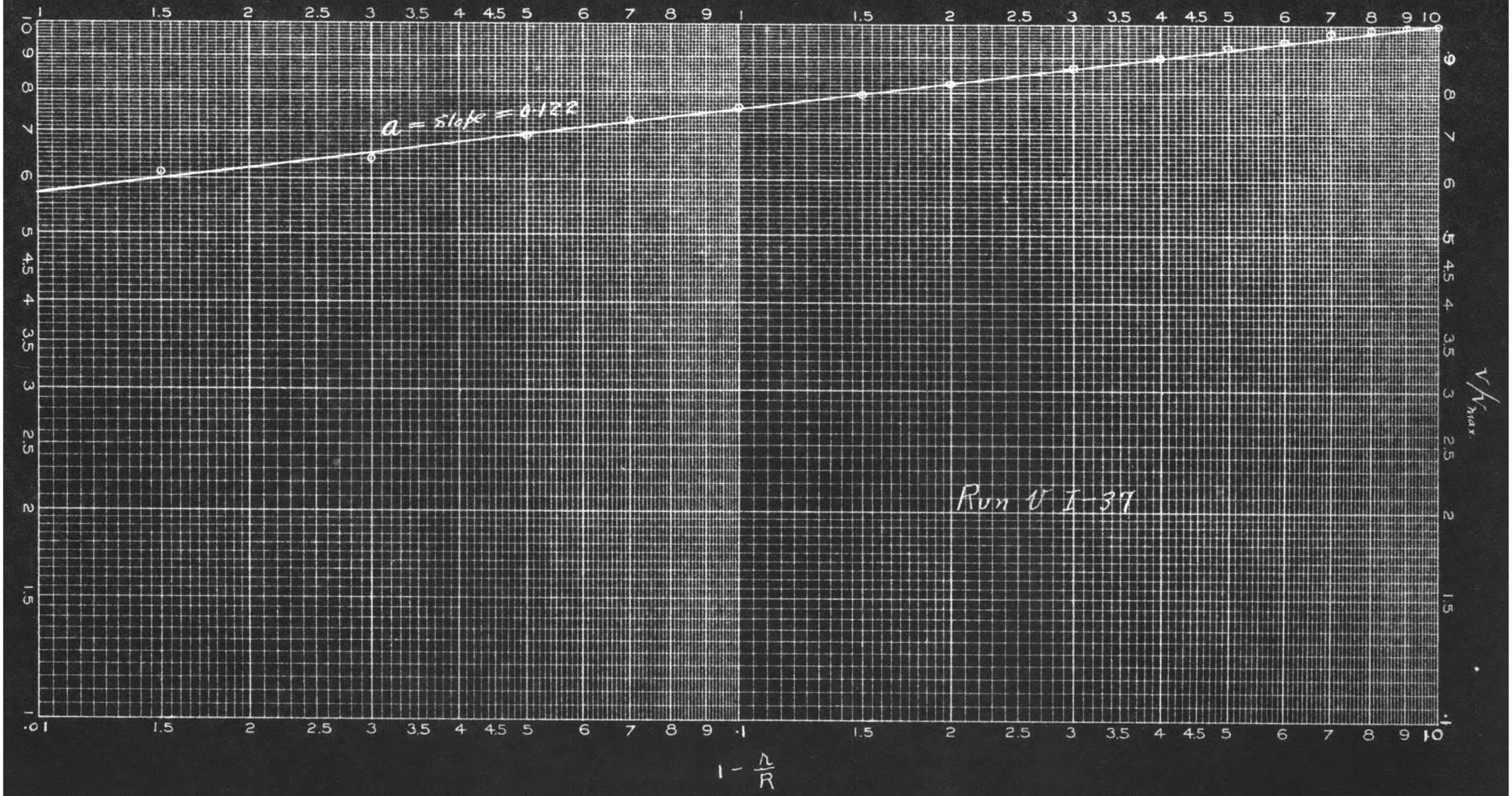
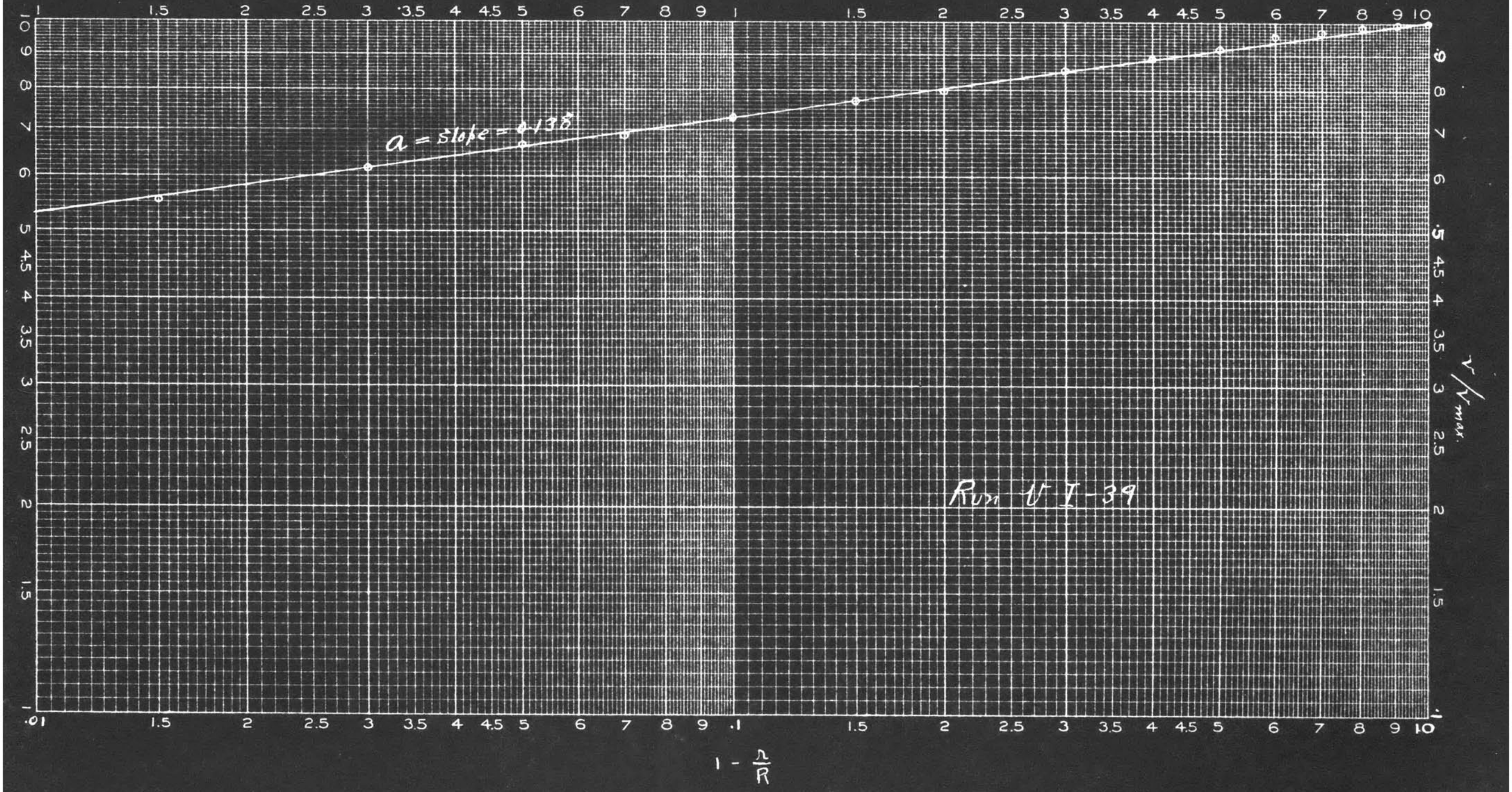
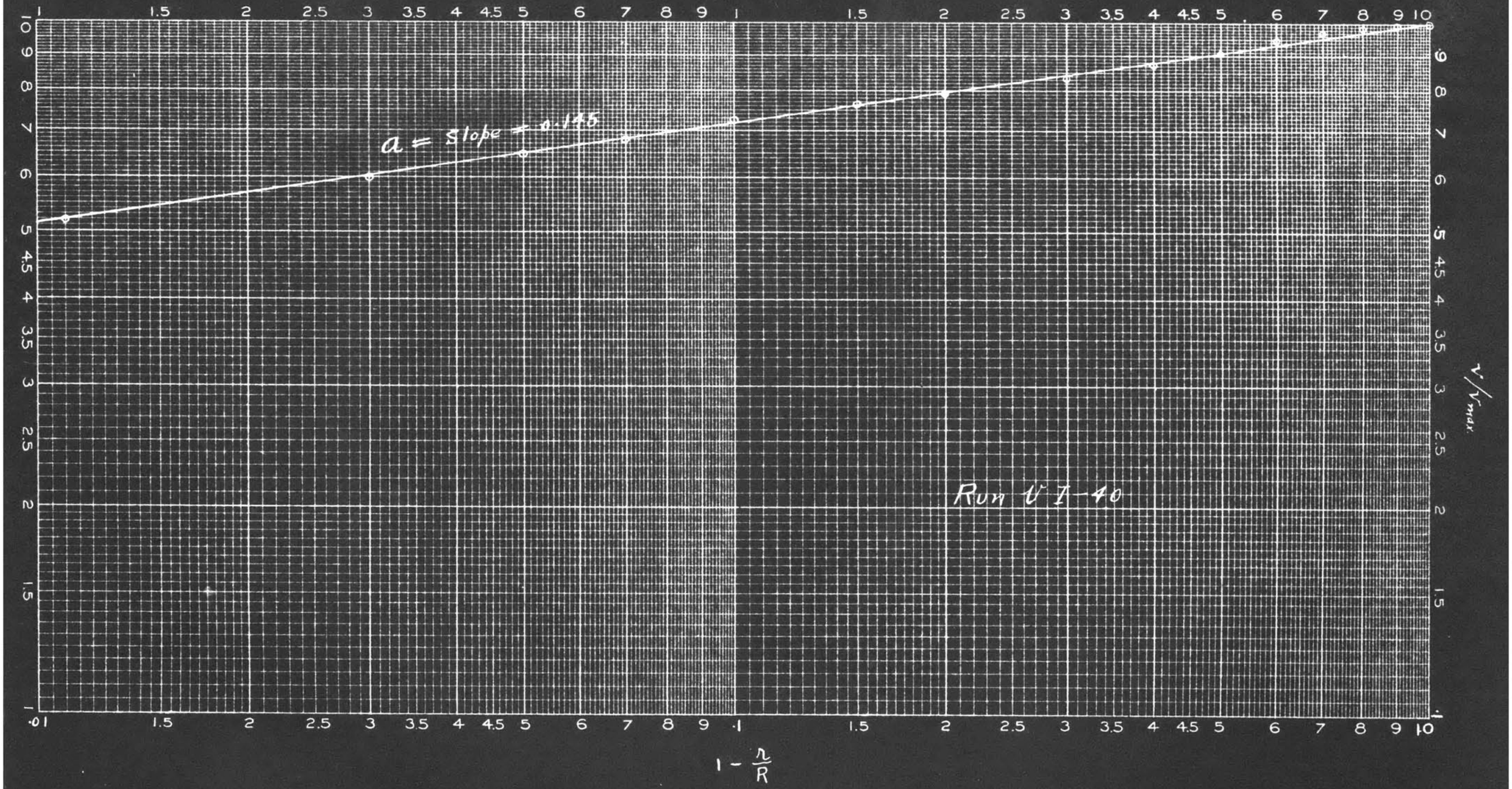
3.5
4
4.5
5
6
7
8
9
10

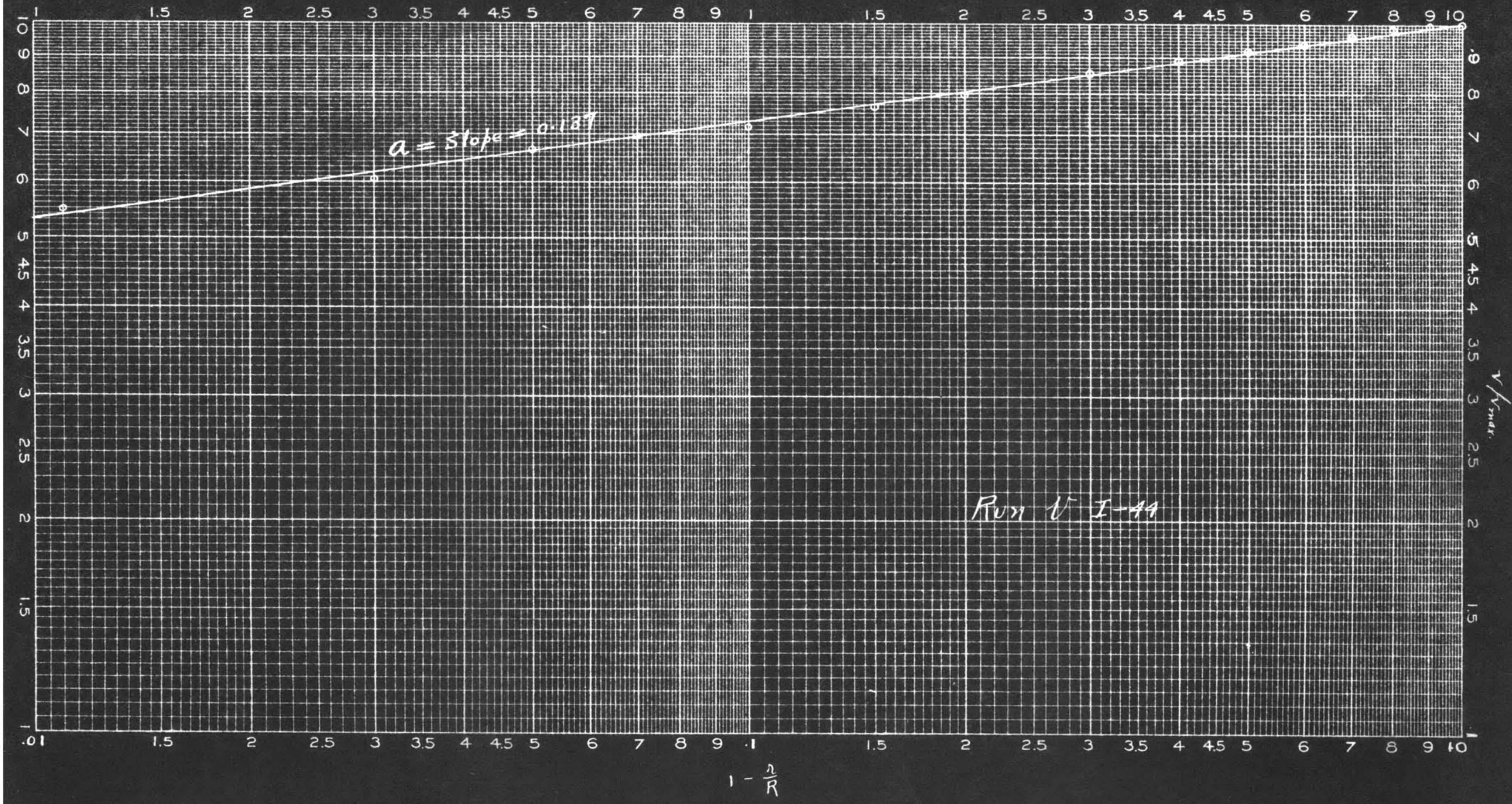

a = slope = 0.125
Run V I-34
V/Vmax
1 - r/R
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0

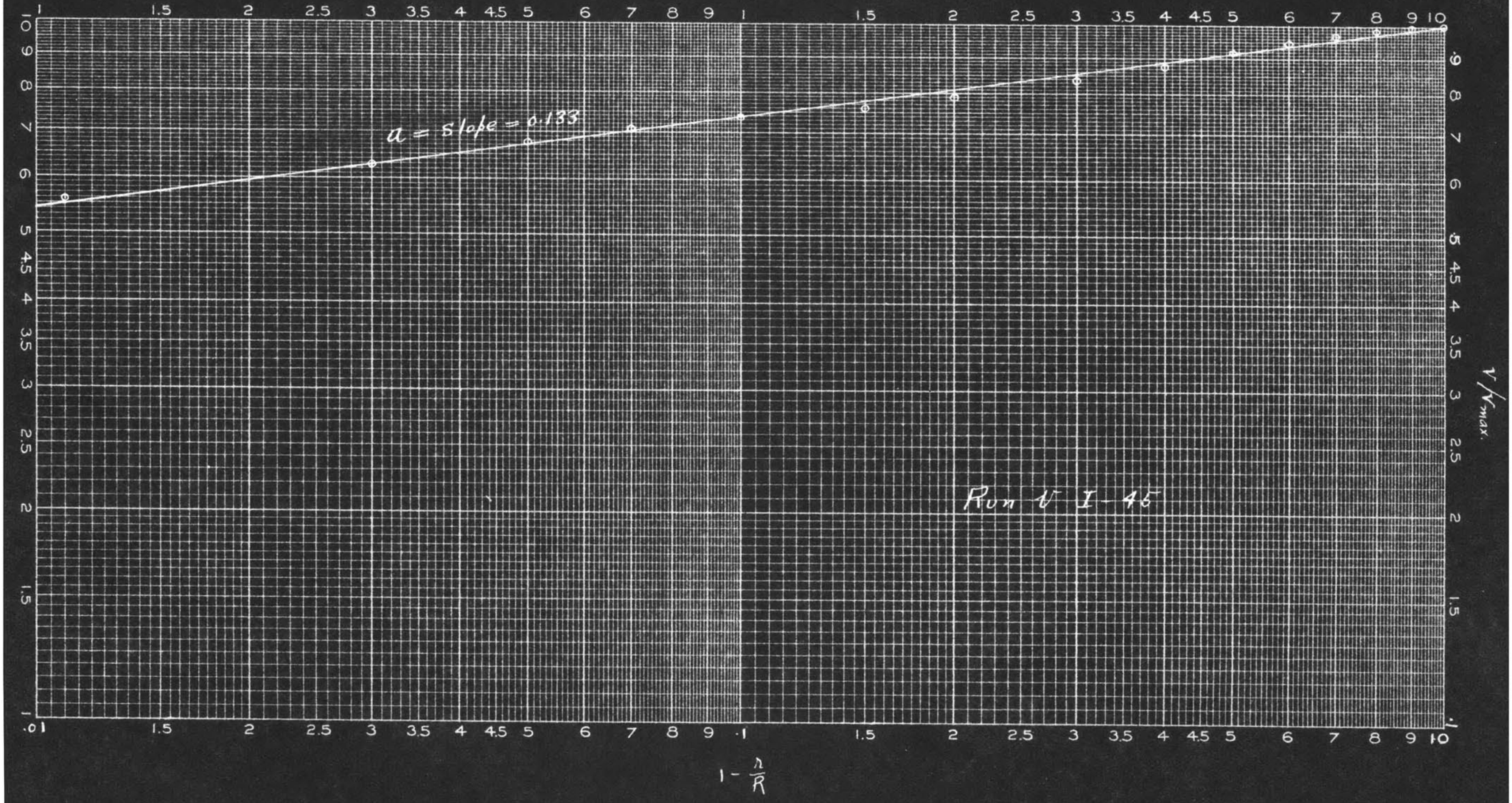

a = Slope = 0·133
Run V I-36
1 - $\frac{r}{R}$
V/Vmax
.01
.1
1.0
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.5
.9
.1

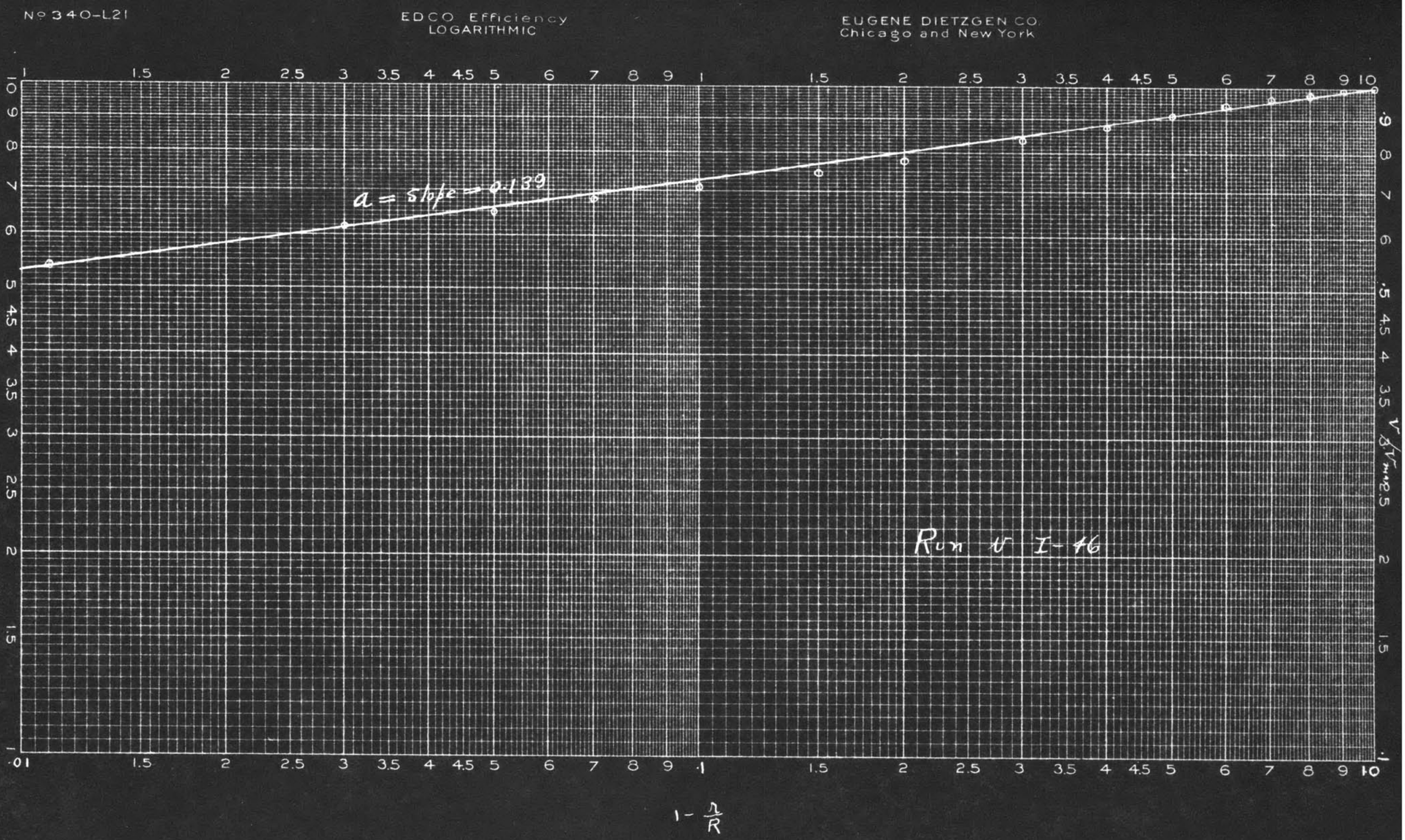

a = slope = 0·122
Run V I-37
1 - r/R
v/v max.
·01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
·1
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
1·0

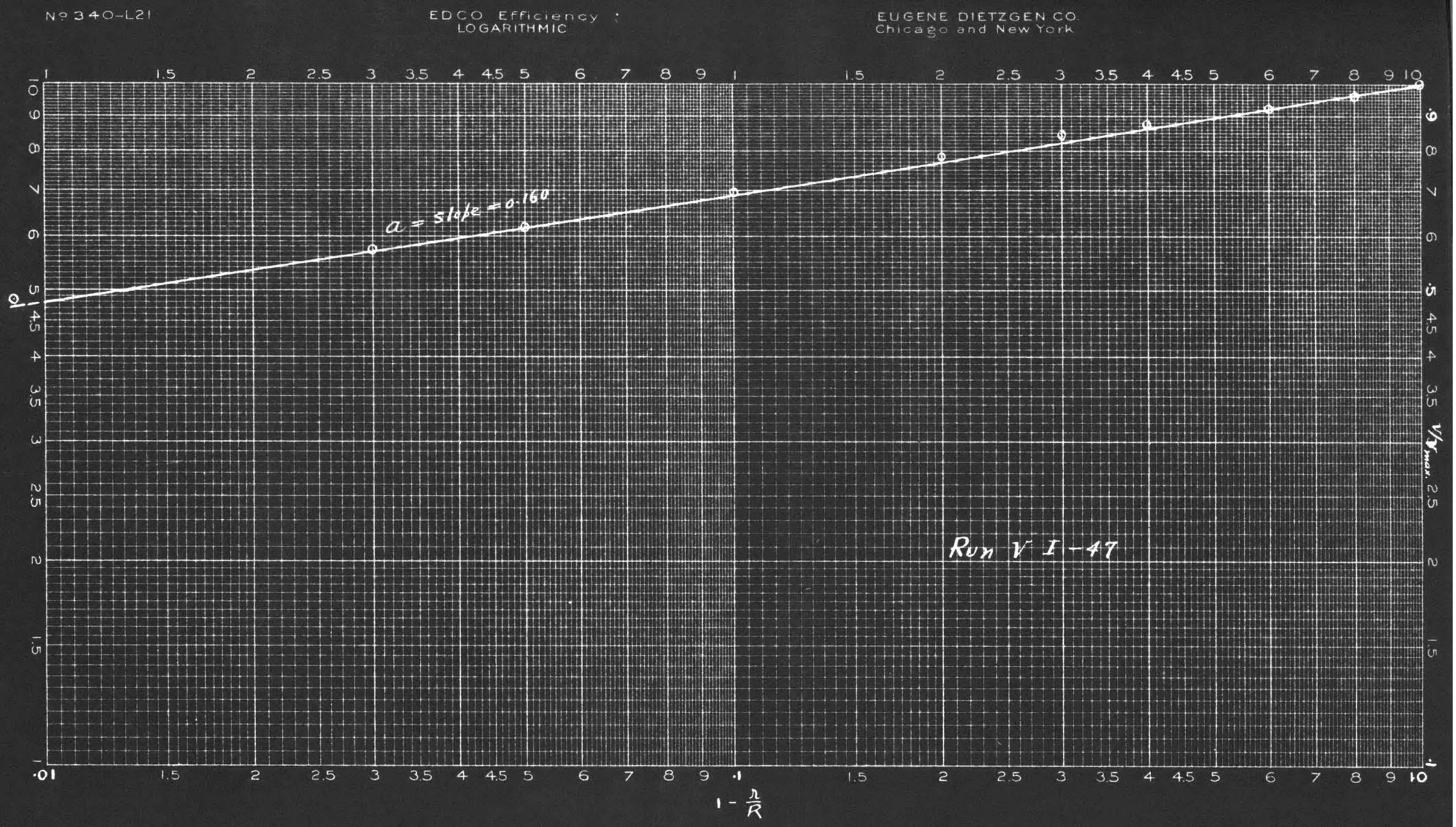

a = slope = 0.138
Run V I-39
$1 - \frac{r}{R}$
$v/V_{max.}$
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
1.0
.1
1.5
2
2.5
3
3.5
4
4.5
.5
6
7
8
.9
1.0

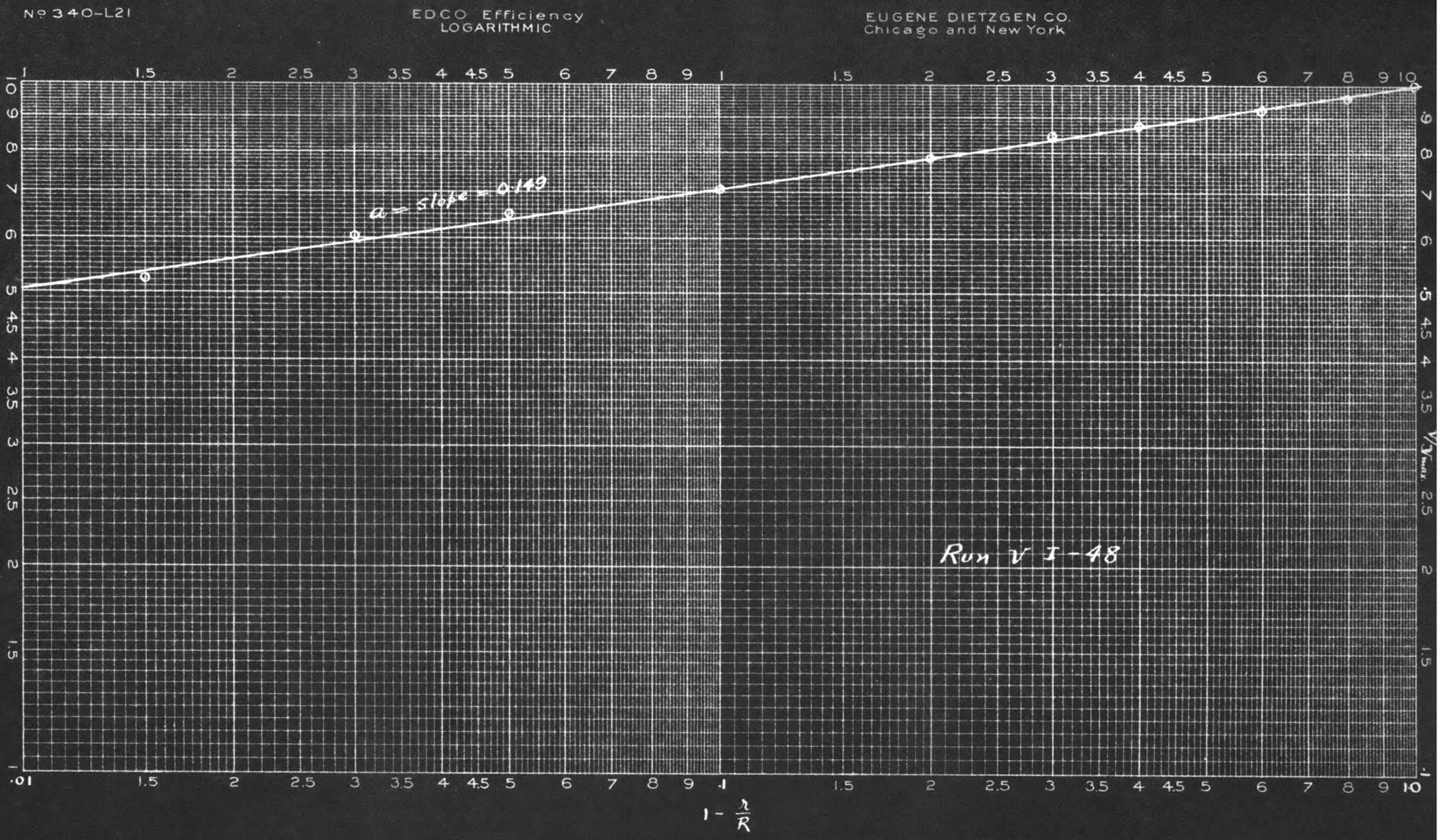

a = slope = 0.145
Run V I-40
1 - r/R
v/V_max
·01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
·1
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
1·0
·1
1.5
2
2.5
3
3.5
4
4.5
·5
6
7
8
·9
1·0


a = slope = 0·137
Run V I-44
$v/v_{max.}$
$1 - \frac{r}{R}$
.01
.1
1.0
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
10

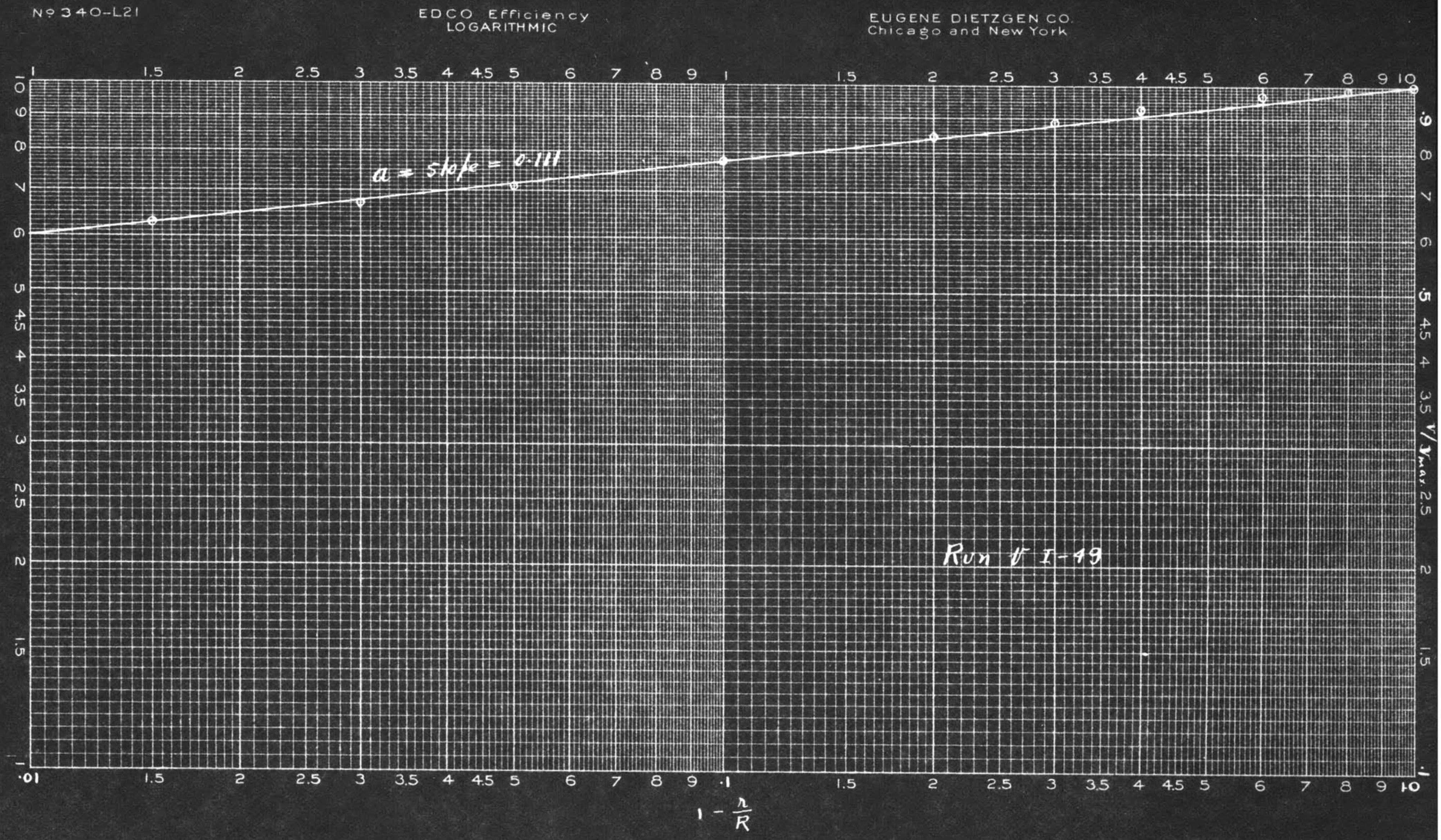

a = slope = 0.133
Run V I-45
1 - r/R
v/v_max
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1


No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
a = slope = 0.139
Run V I-46
1 - r/R
v/v_max
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.15
.1


Nº 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
a = slope = 0.160
Run V I-47
d
V/V max.
1 - r/R
·01
·1
1·0
·1
·5
·9
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9



Nº 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
a = slope = 0.149
Run V I-48
1 - r/R
V/V max.
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.1
.5
.9

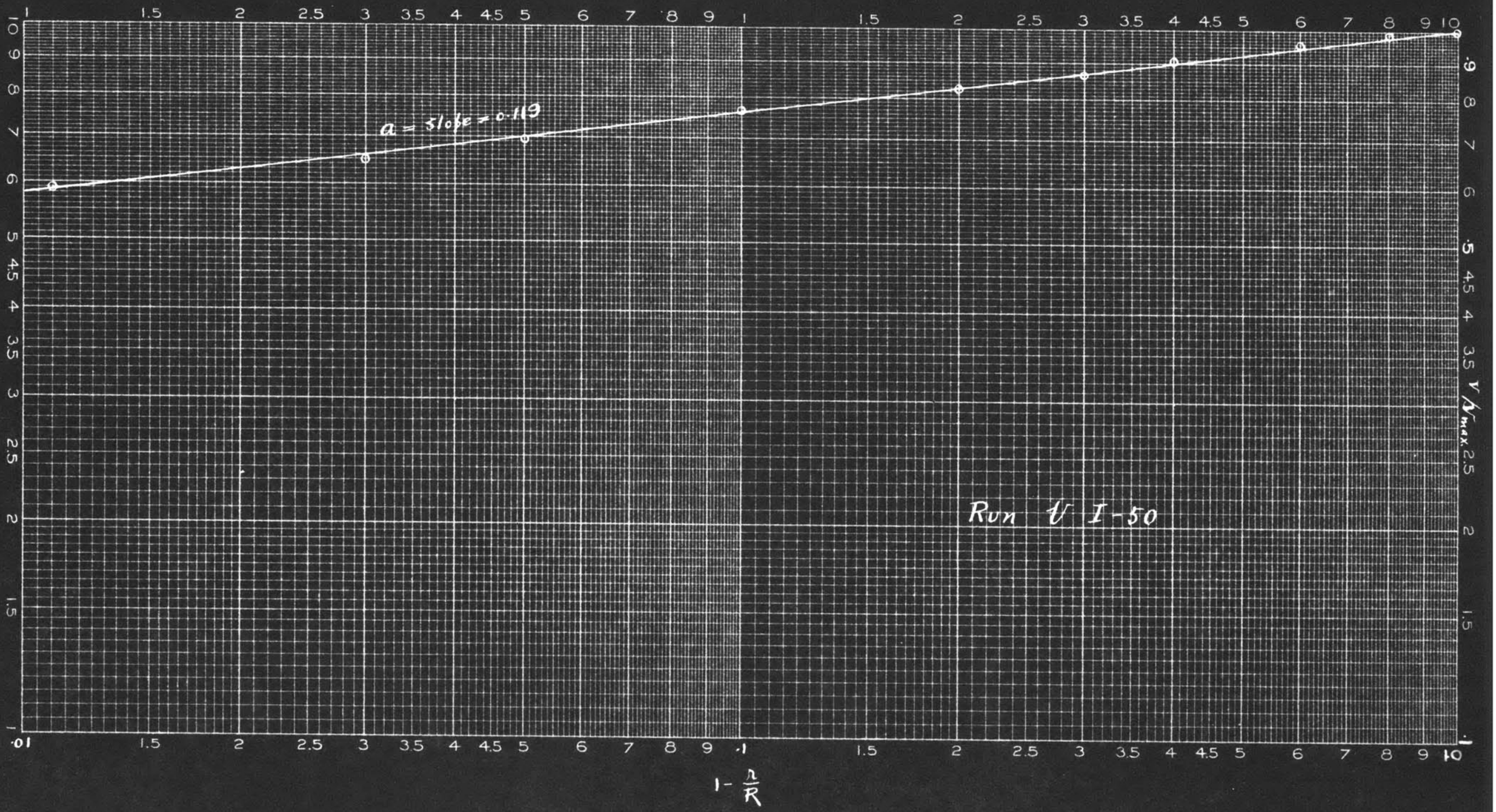


Nº 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
a = slope = 0.111
Run # I-49
$1 - \frac{r}{R}$
v/v_{max}
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.9
.8
.7
.6
.5

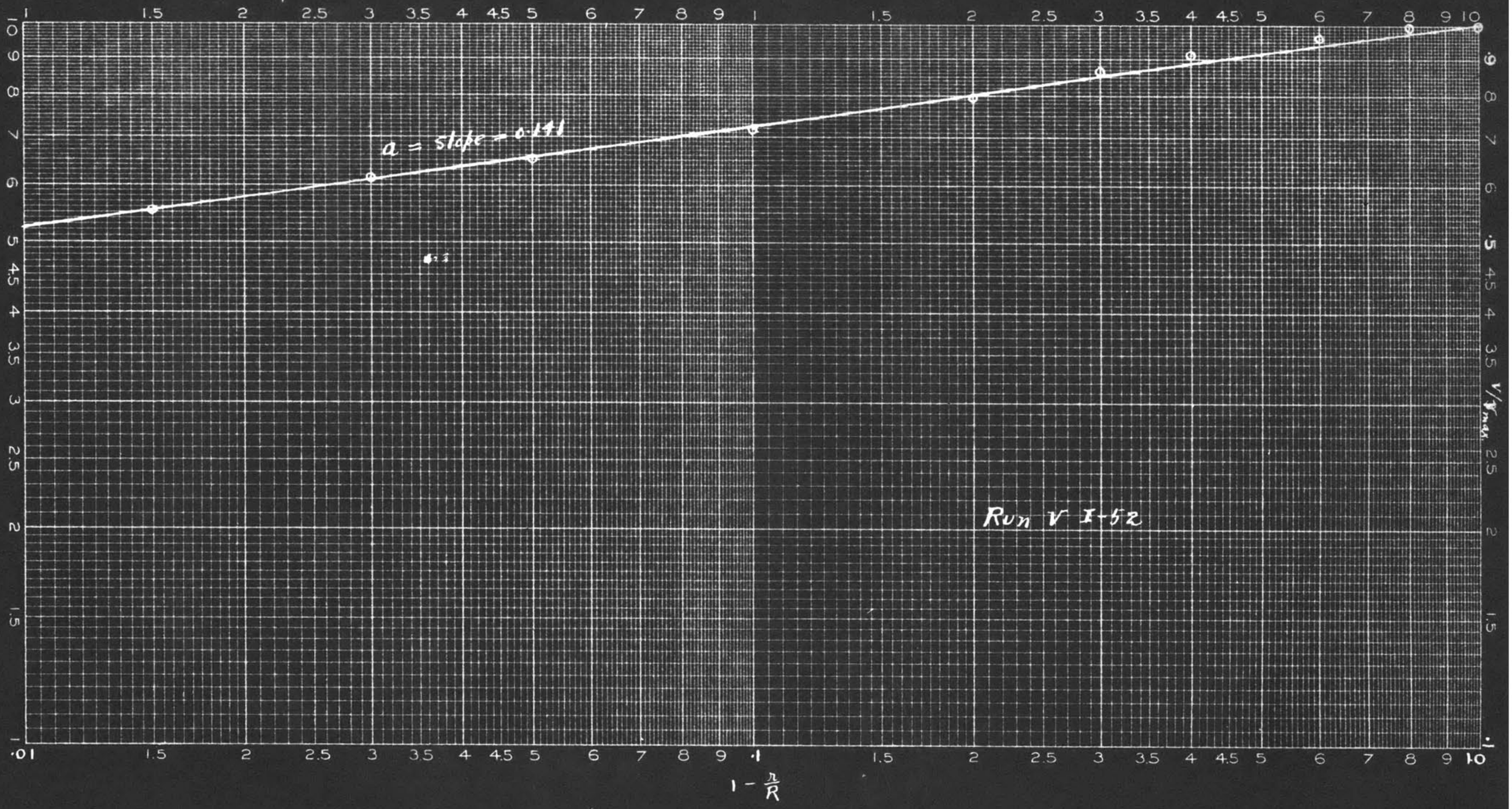
Nº 340-L21

EDCO Efficiency
LOGARITHMIC

EUGENE DIETZGEN CO
Chicago and New York



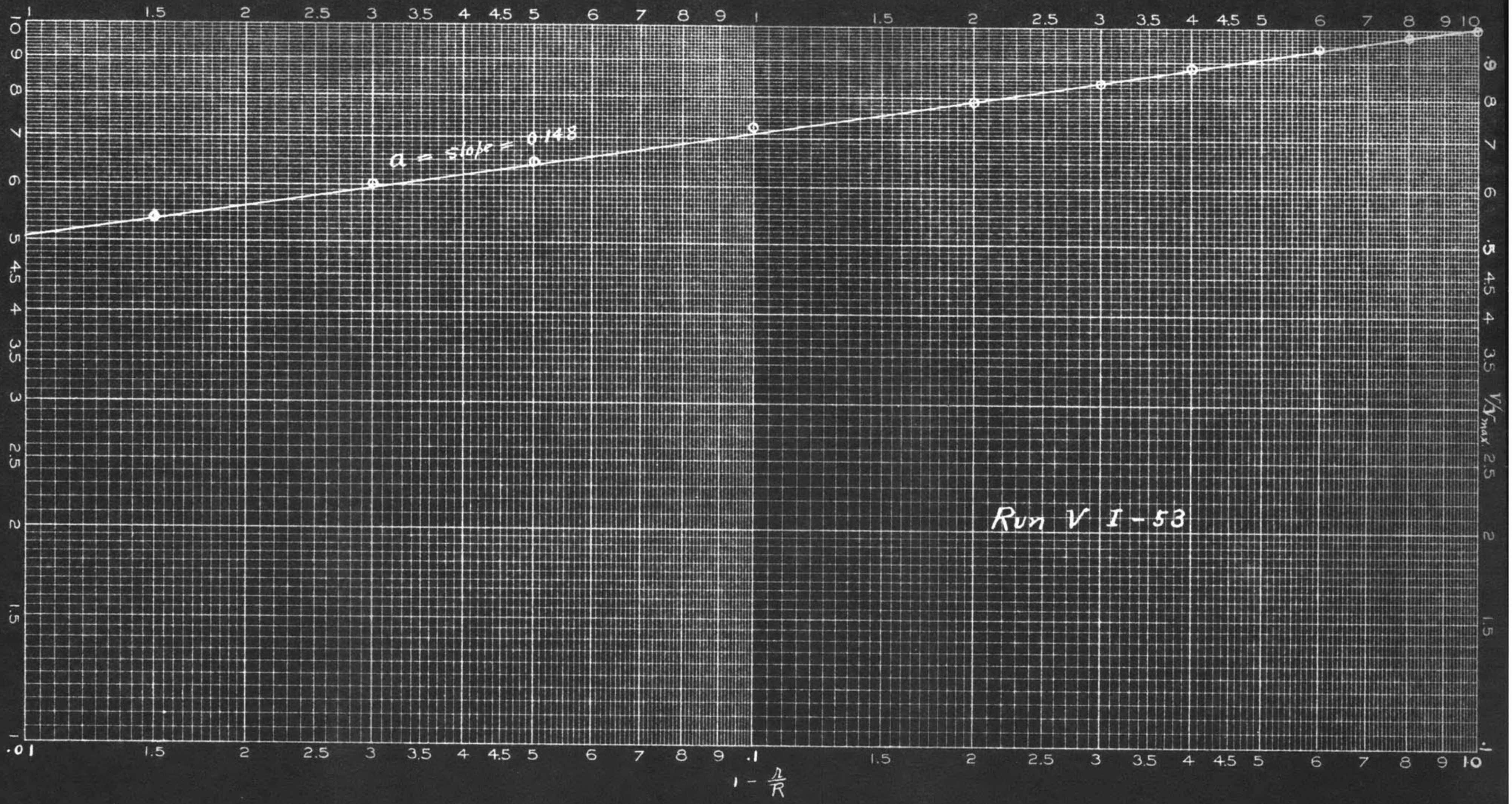
No 340-L21 EDCO Efficiency LOGARITHMIC EUGENE DIETZGEN CO. Chicago and New York



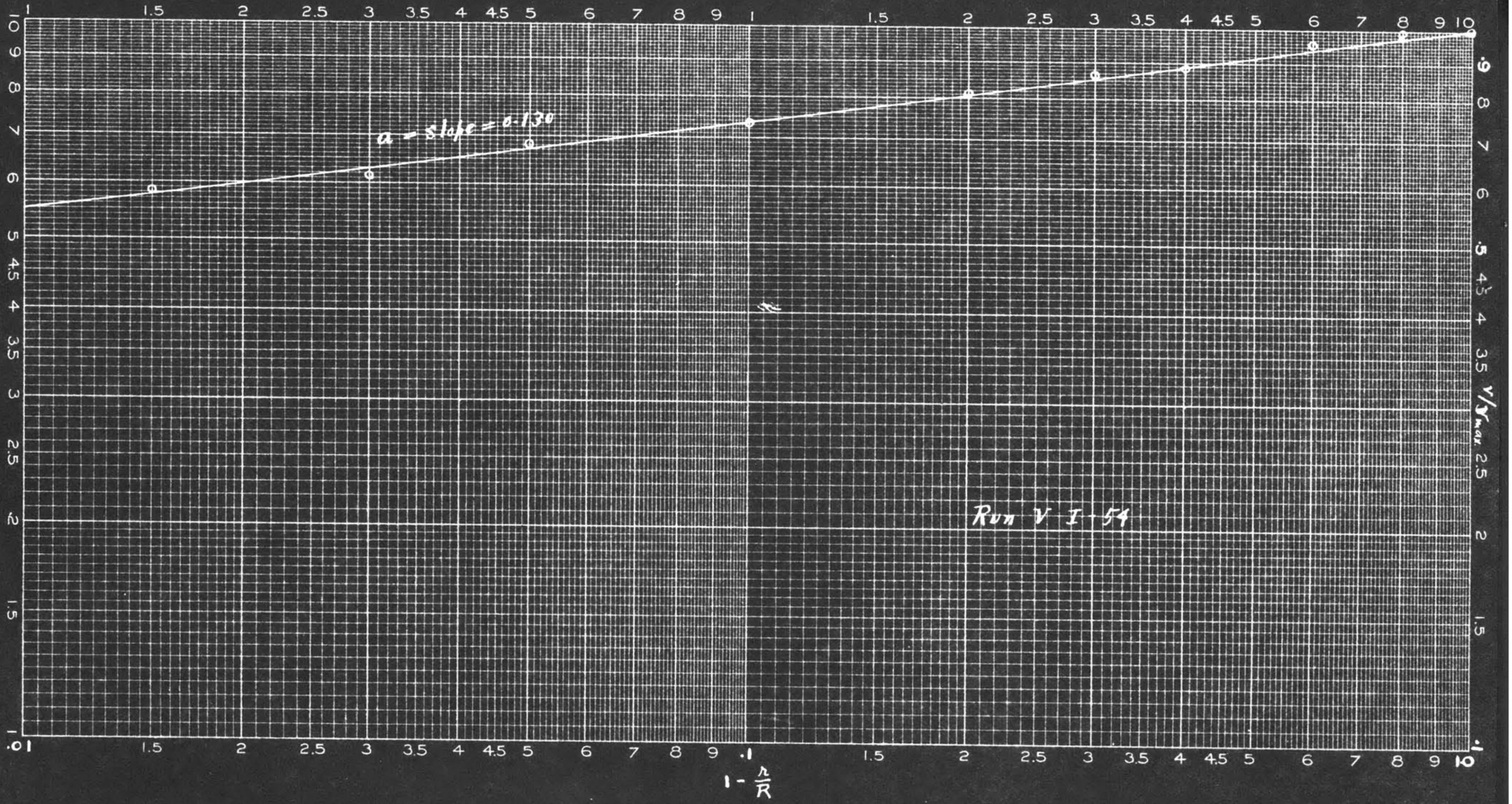
Nº 340-L21

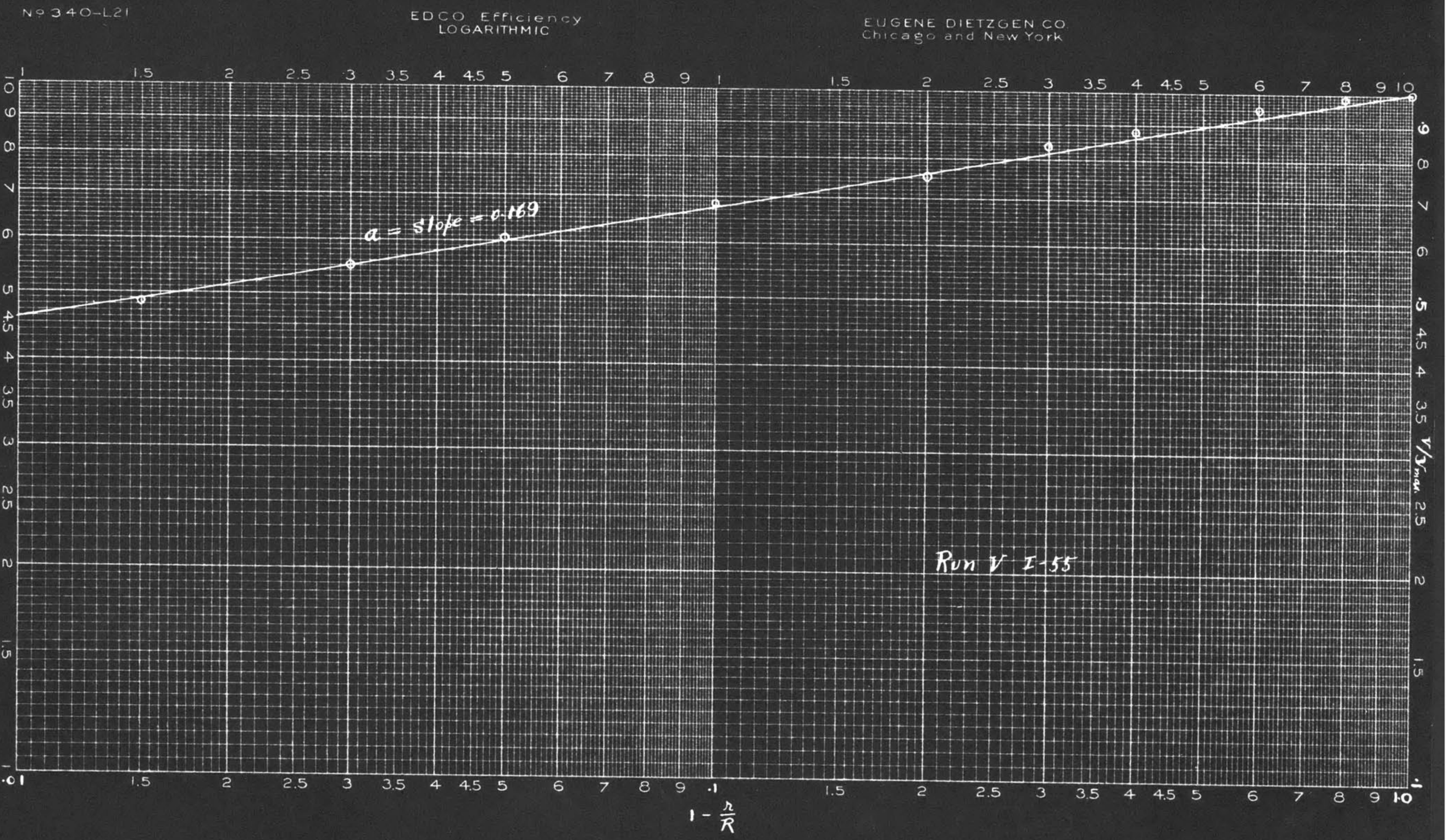
EDCO Efficiency
LOGARITHMIC

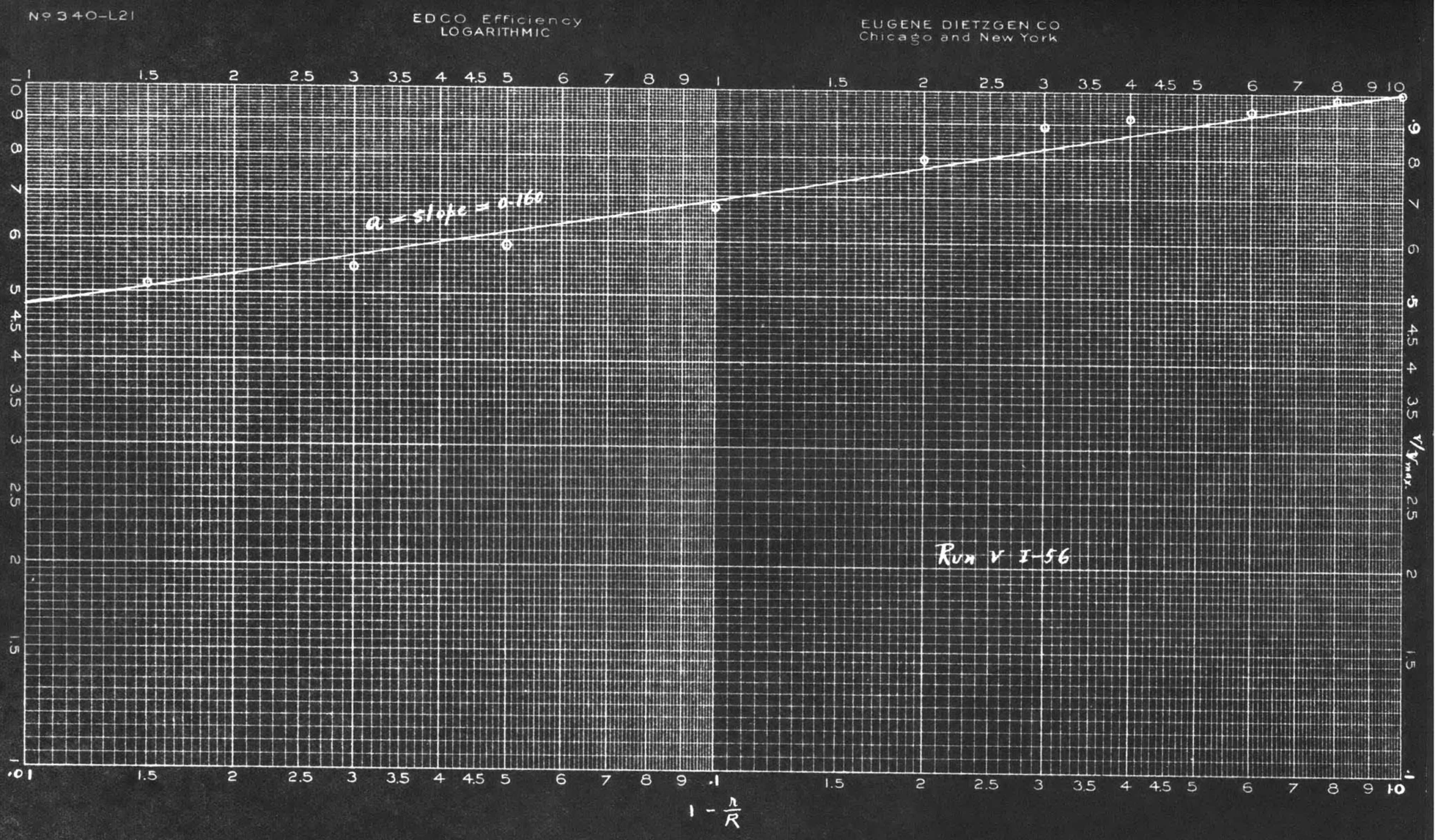
EUGENE DIETZGEN CO.
Chicago and New York



No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York




No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
a = slope = 0.169
Run V I-55
·01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
·1
1.0
1 - r/R
·1
·5
·9
V/V_max


Nº 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
a = slope = 0.160
Run V I-56
1 - r/R
v/v max.
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.9
.8
.7
.6
.5
.4
.35
.3
.25
.2
.15
.1

APPENDIX E

VELOCITY DISTRIBUTION DATA DURING PARALLEL CURRENT HEATING WITH CALCULATIONS AND PLOTS

(WATER FLOWING DOWNWARD)

Data of Koo and Sung

Run v H-3 Station No. 2

(See Run T H-14 for Corresponding Temperature Distribution)

r/R	Δh (cm.CCL_4)	V (ft./sec.)	$V/V_{max.}$	$V(r/R)$ (ft./sec.)	$1-(\frac{r}{R})^2$	$1-\frac{r}{R}$
.985	2.44	1.725	.658	1.700	0.0298	0.015
.970	2.64	1.793	.684	1.740	.0591	.03
.950	3.04	1.925	.734	1.83	.0975	.05
.930	3.46	2.055	.784	1.91	.1351	.07
.900	3.74	2.135	.814	1.92	.1900	.10
.850	3.94	2.190	.835	1.86	.2775	.15
.800	4.18	2.260	.862	1.81	.3600	.20
.700	4.68	2.390	.911	1.67	.5100	.30
.600	4.90	2.446	.932	1.47	.6400	.40
.500	4.94	2.454	.935	1.23	.7500	.50
.400	5.24	2.530	.963	1.01	.8400	.60
.300	5.34	2.552	.972	.77	.9100	.70
.200	5.64	2.623	1.000	.53	.9600	.80
.100	5.64	2.623	1.000	.26	.9900	.90
.000	5.64	2.623	1.000	.00	1.0000	1.00

Static Pressure = 33.48 Cm.Hg.Gauge

$V_{ave.}$ (manometer) = 2.175 ft./sec.

$V_{ave.}$ (Graphical Integration) = 2.28 ft./sec.

$V_{ave.}/V_{max.}$ = 2.28/2.623 = 0.869

$t_{ave.}$ (Graphical Integration) = 25.52° C.

Re. = 38,900

Data of Koo and Sung

Run v H-5 Station No. 2

(See Run T H-16 for Corresponding Temperature Distribution)

$\frac{r}{R}$	$\frac{\Delta h}{(\text{cm.}CCL_4)}$	$\frac{V}{(\text{ft./sec.})}$	$\frac{V}{V_{max.}}$	$1-(\frac{r}{R})^2$	$(1-\frac{r}{R})$	$\frac{V(r/R)}{(\text{ft./sec.})}$
0.985	8.30	3.184	0.661	0.0298	0.015	3.14
.970	8.80	3.280	.681	.0591	.03	3.18
.950	9.84	3.466	.720	.0975	.05	3.30
.930	10.70	3.610	,750	.1351	.07	3.36
.900	11.50	3.741	.777	.1900	.10	3.36
.850	12.60	3.917	.813	.2175	.15	3.32
.800	13.80	4.100	.851	.3600	.20	3.28
.700	14.90	4.260	.885	.5100	.30	2.98
.600	15.70	4.374	.908	.6400	.40	2.62
.500	16.70	4.510	.936	.7500	.50	2.26
.400	17.40	4.600	.954	.8400	.60	1.84
.300	17.80	4.660	.966	.9100	.70	1.40
.200	18.40	4.730	.981	.9600	.80	0.95
.100	18.90	4.800	.995	.9900	.90	.48
.000	19.10	4.820	1.000	1.0000	1.00	.00

Static Pressure = 35.0 cm.Hg.Gauge

$V_{ave.}$ (manometer) = 4.58 ft./sec.

$V_{ave.}$ (Graphical Integration) = 4.13 ft./sec.

$V_{ave.}/V_{max.}$ = 4.13/4.82 = 0.857

$t_{ave.}$ (graphical integration) = 22.92° C.

Re. = 66,800

Data of Koo and Sung

Run v H-8 Station No. 2

(See Run T H-18 for corresponding temperature distribution)

$\frac{r}{R}$	Δh ($Cm.CCL_4$)	V (ft./sec.)	$\frac{V}{V_{max.}}$	$V(r/R)$ (ft./sec.)	$1-(\frac{r}{R})^2$	$(1-\frac{r}{R})$
0.991	0.50	0.782	0.470	0.775	0.0179	0.009
.970	.70	.925	.557	.897	.0591	.03
.950	.80	.988	.595	.939	.0975	.05
.930	.86	1.025	.617	.953	.1351	.07
.900	1.10	1.160	.699	1.044	.1900	.10
.850	1.20	1.212	.730	1.03	.2775	.15
.800	1.30	1.260	.759	1.008	.3600	.20
.700	1.40	1.307	.787	.915	.5100	.30
.600	1.50	1.354	.816	.813	.6400	.40
.500	1.60	1.398	.842	.700	.7500	.50
.400	1.74	1.458	.878	.584	.8400	.60
.300	2.10	1.600	.964	.480	.9100	.70
.200	2.20	1.638	.986	.328	.9600	.80
.100	2.24	1.652	.995	.165	.9900	.90
0	2.26	1.660	1.000	0	1.0000	1.00

Static Pressure = 16.78 Cm.Hg.Gauge

$V_{ave.}$ (Manometer) = 1.25 ft./sec.

$V_{ave.}$ (Graphical Integration) = 1.275 ft./sec.

$V_{ave.}/V_{max.}$ = 1.275/1.66 = 0.768

$t_{ave.}$ (Graphical Integration) = 28.56° C.

$\therefore$ Re = 23,200

Data of Koo and Sung

Run v H-9 Station No. 3

(See Run T H-19 for corresponding temperature distribution)

$\frac{r}{R}$	$\frac{\Delta h}{(\text{Cm.CCL}_4)}$	$\frac{V}{(\text{ft./sec.})}$	$\frac{V}{V_{\text{max.}}}$	$\frac{V(r/R)}{(\text{ft./sec.})}$	$1-(r/R)^2$	$1-r/R$
0.985	0.60	0.855	0.534	0.842	0.0298	0.015
.970	.80	.987	.617	.957	.0591	.03
.950	.90	1.047	.655	.995	.0975	.05
.930	.94	1.072	.670	.997	.1351	.07
.90	1.04	1.128	.705	1.015	.1900	.10
.85	1.12	1.170	.732	.995	.2775	.15
.80	1.16	1.190	.744	.952	.36	.20
.70	1.24	1.230	.769	.862	.51	.30
.60	1.40	1.308	.817	.785	.64	.40
.50	1.60	1.400	.875	.700	.75	.50
.40	1.80	1.485	.928	.594	.84	.60
.30	1.90	1.553	.970	.466	.91	.70
.20	2.00	1.565	.978	.314	.96	.80
.10	2.10	1.600	1.000	.160	.99	.90
0	2.10	1.600	1.000	0	1.00	1.00

Static Pressure = 18.04 Cm.Hg.Gauge

$V_{\text{ave.}}$ (Manometer) = 1.25 ft./sec.

$V_{\text{ave.}}$ (Graphical Integration) = 1~~.23~~ 1.249 ft./sec.

$V_{\text{ave.}}/V_{\text{max.}}$ = 1.23/1.60 = ~~0.769~~ 0.778

$t_{\text{ave.}}$ (Graphical Integration) = 37.6° C.

Re. = 26,900

Data of Koo and Sung

Run v H-10 Station No. 2

(See Run T H-20 for Corresponding Temperature Distribution).

$\frac{r}{R}$	Δh (Cm.CCL_4)	V (ft./sec.)	$\frac{V}{V_{max.}}$	v(r/R) (ft./sec.)	$1-(\frac{r}{R})^2$	$1-\frac{r}{R}$
.991	9.04	3.30	.653	3.27	.0179	.009
.970	10.6	3.57	.707	3.46	.0591	.03
.950	11.1	3.66	.725	3.48	.0975	.05
.930	12.4	3.86	.764	3.59	.1351	.07
.900	13.16	3.98	.788	3.58	.1900	.10
.850	14.6	4.19	.830	3.56	.2775	.15
.800	15.50	4.32	.856	3.46	.3600	.20
.700	16.4	4.43	.878	3.10	.5100	.30
.600	17.7	4.61	.913	2.76	.6400	.40
.500	18.84	4.76	.943	2.38	.7500	.50
.400	19.84	4.89	.968	1.96	.8400	.60
.300	20.4	4.95	.980	14.9	.9100	.70
.200	20.7	4.99	.988	1.00	.9600	.80
.100	21.04	5.03	.997	.50	.9900	.90
.000	21.14	5.05	1.000	.00	1.0000	1.00

Static Pressure = 39.64 cm.Hg.Gauge

$V_{ave.}$ (manometer) = 4.57 ft./sec.

$V_{ave.}$ (graphical integration) = 4.39 ft./sec.

$V_{ave.}/V_{max.}$ = 4.39/5.06 = 0.869

$t_{ave.}$ (graphical integration) = 25.35° C.

Re. = 74,700

Data of Koo and Sung

Run v H-11 Station No. 3

(See Run T H-21 for Corresponding Temperature Distribution)

$\frac{r}{R}$	$\frac{\Delta h}{(\text{Cm.}CCL_4)}$	$\frac{V}{(\text{ft./sec.})}$	$\frac{V}{V_{max.}}$	$\frac{V(r/R)}{(\text{ft./s.})}$	$1-(r/R)^2$	$1-\frac{r}{R}$
.985	6.66	2.83	.547	2.78	.0298	.015
.970	7.56	3.01	.582	2.92	.0591	.03
.950	8.66	3.23	.625	3.07	.0975	.05
.930	9.66	3.42	.662	3.18	.1351	.07
.900	10.36	3.53	.683	3.18	.1900	.10
.850	11.56	3.73	.721	3.17	.2775	.15
.800	12.56	3.89	.753	3.12	.3600	.20
.700	13.88	4.08	.790	2.86	.5100	.30
.600	15.42	4.31	.834	2.58	.6400	.40
.500	17.66	4.61	.892	2.31	.7500	.50
.400	18.8	4.76	.921	1.90	.8400	.60
.300	20.56	4.86	.942	1.43	.9100	.70
.200	21.36	5.07	.980	1.01	.9600	.80
.100	21.76	5.11	.990	.51	.9900	.90
.000	22.16	5.17	1.000	.00	1.0000	1.00

Static Pressure = 42.14 cm.Hg.Gauge

$V_{ave.}$ (manometer) = 4.57 ft./sec.

$V_{ave.}$ (graphical integration) = 4.02 ft./sec.

$V_{ave.}/V_{max.} = 4.02/5.17 = 0.778$

$t_{ave.}$ (graphical integration) = 28.8° C.

Re. = 73,800

Data of Koo and Sung

Run v H-12 Station No. 2

(See Run T H-22 for Corresponding Temperature Distribution)

r/R	$\frac{\Delta h}{(cm.CCL_4)}$	$\frac{V}{(ft./sec.)}$	$\frac{V}{V_{max.}}$	$1-(\frac{r}{R})^2$	$(1-\frac{r}{R})$	$\frac{V(r/R)}{(ft.\ per\ sec.)}$
.991	5.30	2.52	0.620	.0179	0.009	2.50
.970	6.30	2.75	.675	.0591	.03	2.67
.950	7.32	2.96	.727	.0975	.05	2.82
.930	7.8	3.06	.752	.1351	.07	2.84
.900	8.4	3.18	.782	.1900	.10	2.86
.850	8.96	3.28	.806	.2775	.15	2.79
.800	9.50	3.38	.830	.36	.20	2.70
.700	10.24	3.51	.863	.51	.30	2.46
.600	11.10	3.66	.900	.64	.40	2.20
.500	12.2	3.83	.941	.75	.50	1.92
.400	12.64	3.90	.958	.84	.60	1.56
.300	13.14	3.98	.978	.91	.70	1.19
.200	13.30	4.00	.983	.96	.80	0.80
.100	13.80	4.07	1.000	.99	.90	0.41
.000	13.80	4.07	1.000	1.00	1.00	0.00

Static Pressure - 22.86 cm.Hg.Gauge

$V_{ave.}$ (manometer) = 3.56 ft./sec.

$V_{ave.}$ (graphical integration) = 3.4$\overset{5}{6}$ ft./sec.

$V_{ave.}/V_{max.}$ = 3.4$\overset{\checkmark}{6}$/4.07 = 0.850

$t_{ave.}$ (graphical integration) = 25.92° C.

Re. = 59,600

Data of Koo and Sung

Run v H-13 Station No. 3

(See Run T H-23 for Corresponding Temperature Distribution)

r/R	Δh (cm. CCl_4)	V (ft./sec.)	$V/V_{max.}$	$V(r/R)$ (ft. per sec.)	$1-(r/R)^2$	$1-r/R$
0.985	4.50	2.33	0.573	2.30	0.0298	0.015
.970	4.96	2.44	.600	2.36	.0591	.03
.950	5.6	2.59	.637	2.46	.0975	.05
.930	6.16	2.72	.668	2.53	.1351	.07
.900	6.76	2.85	.700	2.56	.1900	.10
.850	7.56	3.01	.740	2.56	.2775	.15
.800	8.16	3.14	.772	2.51	.3600	.20
.700	9.26	3.34	.821	2.34	.5100	.30
.600	10.24	3.51	.863	2.10	.6400	.40
.500	10.96	3.63	.892	1.82	.7500	.50
.400	11.76	3.76	.925	1.50	.8400	.60
.300	12.96	3.95	.970	1.19	.9100	.70
.200	13.36	4.01	.987	.80	.9600	.80
.100	13.66	4.05	.995	.41	.9900	.90
.000	13.80	4.07	1.000	.00	1.0000	1.00

Static Pressure = 24.28 cm.Hg.Gauge

$V_{ave.}$ (manometer) = 3.56 ft./sec.

$V_{ave.}$ (graphical integration) = 3.22 ft./sec.

$V_{ave.}/V_{max.} = 3.22/4.07 = 0.791$

$t_{ave.}$ (graphical integrayion) = 30.8 ° C.

Re. = 61,600

Run v H-14 Station No. 2

(For corresponding Temperature Distribution, see Run T H-24)

$\frac{r}{R}$	$\frac{\Delta h}{(Cm.CCL_4)}$	$\frac{v}{(ft./sec.)}$	$\frac{V}{V_{max.}}$	$\frac{v(r/R)}{(ft.\ per\ sec.)}$	$(1-\frac{r}{R})$
.991	7.4	3.02	0.634	2.99	0.009
.970	8.0	3.14	.660	3.045	.030
.950	8.8	3.29	.691	3.125	.050
.930	9.6	3.43	.720	3.19	.070
.900	10.4	3.56	.748	3.20	.100
.850	11.4	3.72	.782	3.16	.150
.800	12.0	3.82	.802	3.06	.200
.700	13.2	4.00	.840	2.80	.300
.600	14.2	4.16	.874	2.50	.400
.500	15.2	4.30	.903	2.15	.500
.400	16.0	4.42	.929	1.77	.600
.300	17.0	4.56	.959	1.37	.700
.200	17.8	4.66	.980	0.932	.800
.100	18.2	4.71	.990	0.471	.900
.000	18.6	4.76	1.000	0.00	1.000

Static Pressure = 32.6 cm.Hg.Gauge

$V_{ave.}$ (manometer) = 3.92 ft./sec.

$V_{ave.}$ (graphical integration) = 3.928 ft./sec.

$V_{ave.}/V_{max.}$ = 3.928/4.76 = 0.825

$t_{ave.}$ (graphical integration) = 18.72° C.

Re. = 57,200

Run v H-15 Station No. 3

(For Corresponding Temperature Distribution, see Run T H-25)

$\frac{r}{R}$	Δh (cm.CCL$_4$)	V (ft./sec.)	$\frac{V}{V_{max.}}$	V(r/R) (ft./sec.)	(1-r/R)
.985	6.2	2.77	0.578	2.73	0.015
.970	7.0	2.94	.614	2.85	.030
.950	8/0	3.14	.656	2.98	.050
.930	8.8	3.29	.687	3.06	.070
.900	9.6	3.43	.717	3.085	.100
.850	10.4	3.58	.747	3.04	.150
.800	11.2	3.69	.770	2.95	.200
.700	12.6	3.91	.816	2.74	.300
.600	14.0	4.12	.860	2.47	.400
.500	15.4	4.33	.905	2.17	.500
.400	16.4	4.47	.934	1.79	.600
.300	17.4	4.61	.963	1.38	.700
.200	18.2	4.71	.985	0.942	.800
.100	18.6	4.76	.995	0.476	.900
.000	18.8	4.79	1.000	0.000	1.000

Static Pressure = 35.0 cm.Hg.Gauge

$V_{ave.}$ (manometer) = 3.92 ft./sec.

$V_{ave.}$ (graphical integration) = 3.87 ft./sec.

$V_{ave.}/V_{max.}$ = 3.87/4.76 = 0.809

$t_{ave.}$ (graphical integration) = 24.28° C.

Re. = 64,100

Run v H-16 Station No. 2

(For corresponding Temperature Distribution, see Run T H-26)

$\frac{r}{R}$	$\frac{\Delta h}{(\text{cm.}CCL_4)}$	$\frac{V}{(\text{ft./sec.})}$	$\frac{V}{V_{\text{max.}}}$	$\frac{V(r/R)}{(\text{ft./sec.})}$	$(1-\frac{r}{R})$
.991	3.2	1.97	0.588	1.95	0.009
.970	4.0	2.21	.660	2.14	.030
.950	4.4	2.31	.690	2.195	.050
.930	4.8	2.42	.723	2.25	.070
.900	5.2	2.52	.752	2.27	.100
.850	5.6	2.61	.780	2.22	.150
.800	6.0	2.70	.806	2.16	.200
.700	6.6	2.83	.845	1.98	.300
.600	7.3	2.98	.890	1.79	.400
.500	8.0	3.12	.931	1.56	.500
.400	8.4	3.20	.955	1.28	.600
.300	8.7	3.26	.973	0.978	.700
.200	8.9	3.295	.984	0.659	.800
.100	9.1	3.33	.994	0.333	.900
.000	9.2	3.35	1.000	0.00	1.000

Static Pressure = 30.20 Cm.Hg.Gauge

$V_{av.}$ (for manometer) = 2.84 ft./sec.

$V_{ave.}$ (from Graphical Integration) = 2.806 ft./sec.

$$\therefore V_{ave.}/V_{max.} = \frac{2.806}{3.35} = 0.838$$

$t_{av} = 19.36° C.$

$$\therefore Re. = \frac{1.952 \times 2.806 \times 10^5}{12 \times 1.101} = 41,500$$

Run v H-17 Station No. 3

(For corresponding temperature distribution, see Run T H-27)

$\frac{r}{R}$	Δh (cm.CCL_4^o)	V (ft./sec.)	$\frac{V}{V_{max.}}$	V(r/R) (ft./sec.)	(1-r/R)
.985	3.2	1.97	0.580	1.94	0.015
.970	3.8	2.154	0.634	2.09	.030
.950	4.4	2.31	0.680	2.195	.050
.930	4.8	2.42	.712	2.25	.070
.900	5.2	2.52	.741	2.27	.100
.850	5.6	2.61	.768	2.22	.150
.800	6.0	2.70	.795	2.16	.200
.700	6.7	2.78	.818	1.946	.230
.600	7.4	3.00	.883	1.80	.300
.500	8.0	3.12	.918	1.56	.400
.400	8.5	3.22	.947	1.29	.500
.300	8.9	3.295	.970	.989	.600
.200	9.2	3.35	.985	.670	.700
.100	9.4	3.38	.994	.338	.800
.000	9.5	3.40	1.000	.000	.900

Static Pressure = 32.6 Cm.Hg.Gauge

$V_{ave.}$ (from manometer) = 2.84 ft./sec.

$V_{ave.}$ (from graphical integration) = 2.806 ft./sec.

$$\therefore V_{ave.}/V_{max.} = 2.806/3.40 = 0.826$$

t_{av} = 25.52° C.

$$\therefore Re = \frac{1.952 \times 2.806 \times 10^5}{12 \times 0.955} = 47,850$$

Run v H-18 Station No. 2

(for Corresponding Temperature Distribution, see Run T H-28)

$\frac{r}{R}$	Δh (Cm.CCL$_4$)	V (ft./sec.)	$\frac{V}{V_{max.}}$	v(r/R) (ft./sec.)	(1-r/R)
.991	9.2	3.36	0.630	3.33	0.009
.970	10.0	3.49	.655	3.385	.030
.950	11.2	3.69	.692	3.505	.050
.930	12.0	3.82	.717	3.55	.070
.900	13.0	3.97	.745	3.575	.100
.850	14.4	4.18	.785	3.555	.150
.800	15.3	4.33	.812	3.46	.200
.700	17.0	4.55	.853	3.18	.300
.600	18.4	4.74	.889	2.84	.400
.500	19.6	4.89	.918	2.445	.500
.400	20.8	5.03	.944	2.01	.600
.300	21.8	5.15	.966	1.545	.700
.200	22.4	5.21	.979	1.04	.800
.100	23.0	5.28	.991	0.528	.900
.000	23.4	5.33	1.000	0.000	1.000

Static Pressure = 39.8 Cm.Hg.Gauge

$V_{ave.}$ (for manometer) = 4.76 ft./sec.

$V_{ave.}$ (for graphical integration) = 4.434 ft./sec.

$\therefore V_{ave.}/V_{max.} = 4.434/5.33 = 0.832$

$t_{ave.} = 17.6°$ C.

$$Re = \frac{1.952 \times 4.434 \times 10^5}{12 \times 1.15} = 62{,}700$$

Run v H-19 Station No. 3

(For corresponding temperature distribution, see Run T H-29)

$\frac{r}{R}$	$\frac{\Delta h}{(Cm.CCL_4)}$	$\frac{v}{(ft./sec.)}$	$\frac{v}{V_{max.}}$	$\frac{v(r/R)}{(ft./sec.)}$	(1-r/R)
.985	8.6	3.25	0.607	3.20	0.015
.970	9.6	3.43	.641	3.33	.030
.950	11.0	3.66	.684	3.48	.050
.930	11.8	3.78	.707	3.515	.070
.900	12.8	3.94	.736	3.545	.100
.850	14.2	4.16	.778	3.535	.150
.800	15.0	4.28	.800	3.42	.200
.700	16.8	4.52	.845	3.165	.300
.600	18.4	4.74	.885	2.84	.400
.500	19.8	4.91	.918	2.455	.500
.400	21.0	5.05	.944	2.02	.600
.300	22.0	5.17	.966	1.55	.700
.200	22.6	5.24	.979	1.048	.800
.100	23.2	5.30	.991	.530	.900
.000	23.6	5.35	1.000	.000	1.000

Static Pressure = 42.2 Cm.Hg.Gauge

$V_{ave.}$ (Manometer) = 4.76 ft./sec.

$V_{ave.}$ (for Graphical Integration) = 4.432 ft./sec.

$$V_{ave.}/V_{max.} = \frac{4.432}{5.35} = 0.828$$

$t_{ave.}$ = 21.60° C.

$$Re. = \frac{1.952 \times 4.432 \times 10^5}{12 \times 1.043} = 69,000$$

Run v H-20 Station No. 2

(For Corresponding Temperature Distribution, see Run T H-30)

$\frac{r}{R}$	Δh (Cm.CCL_4)	V (ft./sec.)	$\frac{V}{V_{max.}}$	V(r/R) (ft./sec.)	(1-r/R)
.991	0.80	0.993	0.519	0.984	0.009
.970	1.00	1.103	0.577	1.070	0.030
.950	1.06	1.25	0.654	1.188	0.050
.900	1.60	1.394	0.730	1.255	0.100
.800	2.00	1.56	0.815	1.248	0.200
.700	2.20	1.64	0.858	1.149	0.300
.600	2.30	1.675	0.875	1.005	0.400
.400	2.60	1.78	0.931	0.684	0.600
.200	2.80	1.85	0.968	0.370	0.800
0.000	3.00	1.913	1.000	0.000	1.000

Static Pressure = 24.0 Cm.Hg.Gauge

(Manometer out of order)

$V_{ave.}$ (Graphical Integration) = 1.557 ft./sec.

$$\therefore V_{ave.}/V_{max.} = \frac{1.557}{1.913} = 0.814$$

$t_{ave.}$ = 10.48° C.

$$\therefore Re = \frac{1.952 \times 1.557 \times 10^{5}}{12 \times 1.39} = 18,200$$

Run v H-21 Station No. 3

(For corresponding temperature distribution, See Run TH-31)

$\frac{r}{R}$	Δh (Cm.CCL_4)	V (ft./sec.)	$\frac{V}{V_{max}}$	V(r/R) (ft./sec.)	(1-r/R)
.985	0.60	0.863	0.424	0.850	0.015
.970	0.80	0.993	0.488	0.963	.030
.950	1.20	1.208	0.593	1.148	.050
.900	1.40	1.33	0.654	1.197	.100
.800	1.80	1.48	0.727	1.184	.200
.700	2.20	1.64	0.806	1.148	.300
.600	2.40	1.71	0.840	1.027	.400
.400	2.90	1.88	0.924	0.752	.600
.200	3.20	1.977	0.971	0.396	.800
.000	3.40	2.036	1.000	0.000	1.000

Static Pressure = 25.2 Cm.Hg.Gauge

(Main Line Orifice out of Order)

$V_{ave.}$ (Graphical Integration) = 1.560 ft./sec.

$$\therefore V_{ave}/V_{max} = \frac{1.56}{2.036} = 0.767$$

t_{ave} = 18.4° C.

$$Re = \frac{1.952 \times 1.560 \times 10^5}{12 \times 1.127} = 22,500$$

Run v H-22 Station No. 2

(For corresponding temperature distribution, see Run T H-32)

$\frac{r}{R}$	Δh (Cm.CCL_4)	V (ft./sec.)	$\frac{V}{V_{max.}}$	V(r/R) ft./sec.	(1-r/R)
.991	2.3	1.673	0.678	1.66	0.009
.970	2.5	1.745	.707	1.693	0.030
.950	2.7	1.81	.733	1.72	0.050
.900	3.2	1.97	.798	1.773	0.100
.800	3.5	2.066	.838	1.655	0.200
.700	3.8	2.153	.872	1.507	0.300
.600	4.1	2.238	.906	1.342	0.400
.400	4.4	2.314	.936	0.926	0.600
.200	4.6	2.364	.956	0.473	0.800
.000	5.0	2.470	1.000	0.000	1.000

Static Pressure = 24.2 Cm.Hg.Gauge

(Main Line Orifice Out of Order)

$V_{ave.}$ (Graphical Integration) = 2.12 ft./sec.

$$\therefore V_{ave.}/V_{max.} = \frac{2.12}{2.47} = 0.859$$

$t_{ave.}$ = 9.2° C.

$$\therefore Re = \frac{1.952 \times 2.12 \times 10^5}{12 \times 1.44} = 23{,}900$$

Run V H-23 Station No. 3

(For corresponding temperature distribution, See Run T H-33)

$\frac{r}{R}$	Δh (Cm.CCL_4)	V (ft./sec.)	$\frac{V}{V_{max.}}$	V(r/R) (ft./sec.)	(1-r/R)
.985	2.0	1.56	0.632	1.536	0.015
.970	2.2	1.636	.662	1.586	.030
.950	2.4	1.71	.693	1.624	.050
.900	2.9	1.88	.762	1.693	.100
.800	3.2	1.976	.800	1.58	.200
.700	3.6	2.094	.848	1.467	.300
.600	3.9	2.184	.885	1.31	.400
.400	4.2	2.263	.917	0.905	.600
.200	4.6	2.368	.959	0.474	.800
.000	5.0	2.47	1.000	0.000	1.000

Static Pressure = 25.2 Cm.Hg.Gauge

(Main Line Orifice Manometer out of Order)

$V_{ave.}$ (Graphical Integration) = 2.050 ft./sec.

$\therefore V_{ave.}/V_{max.} = 2.05/2.47 = 0.830$

$t_{ave.} = 17.12° C.$

$$\therefore Re = \frac{1.952 \times 2.05 \times 10^5}{12 \times 1.164} = 28,600$$

Run v H-24 Station No. 3

(For Corresponding Temperature Distribution, see Run T H-34)

$\frac{r}{R}$	Δh (Cm.CCL_4)	V (ft./sec.)	$\frac{V}{V_{max.}}$	V(r/R) (ft./sec.)	(1-r/R)
.985	2.70	1.816	0.677	1.788	0.015
.970	3.20	1.977	0.738	1.917	.030
.950	3.50	2.064	0.770	1.960	.050
.900	3.90	2.184	0.815	1.968	.100
.800	4.30	2.290	0.855	1.832	.200
.700	4.60	2.369	0.884	1.658	.300
.600	5.10	2.494	0.930	1.497	.400
.400	5.50	2.59	0.966	1.038	.600
.200	5.70	2.636	0.982	0.527	.800
.000	5.90	2.680	1.000	0.000	1.000

Static Pressure = 44.0 Cm.Hg.Gauge

(No manometer reading, since H_2O was discarded at bottom)

$V_{ave.}$ (Graphical Integration) = 2.350 ft./sec.

$\therefore V_{ave.}/V_{max.} = 2.35/2.68 = 0.877$

$t_{ave.} = 15.28°$ C.

$$\therefore Re = \frac{1.952 \times 2.350 \times 10^5}{12 \times 1.22} = 31,300$$

Run v H-25 Station No. 2

(For corresponding Temperature Distribution, see Run T H-35)

$\frac{r}{R}$	Δh (Cm.CCL_4)	V (ft./sec.)	$\frac{V}{V_{max.}}$	V(r/R) (ft.sec.)	(1-r/R)
.991	11.6	3.758	0.702	3.72	0.009
.970	13.4	4.042	0.755	3.92	.030
.950	14.5	4.204	.786	3.99	.050
.900	15.9	4.402	.823	3.96	.100
.800	17.9	4.67	.874	3.74	.200
.700	19.6	4.887	.914	3.42	.300
.600	21.0	5.06	.945	3.04	.400
.400	22.2	5.202	.972	2.08	.600
.200	23.2	5.314	.994	1.063	.800
.000	23.5	5.35	1.000	0.00	1.000

Static Pressure = 33.3 Cm.Hg.Gauge

V_{ave} (manometer) = 4.97 ft.sec.

V_{ave} (Graphical Integration) = 4.77 ft./sec.

V_{ave}/V_{max} = 4.77/5.35 = 0.892

t_{ave} = 7.40° C.

$$\therefore Re = \frac{1.952 \times 4.77 \times 10^{5}}{12 \times 1.52} = 51,000$$

Run v H-26 Station No. 3

(For corresponding temperature distribution, See Run T H-36)

r/R	Δh (Cm.CCl_4)	V (ft./sec.)	$V/V_{max.}$	$V(r/R)$ (ft./sec.)	(1-r/R)
0.985	10.4	3.56	0.648	3.505	0.015
.970	11.8	3.79	0.690	3.68	.030
.950	13.0	3.98	.724	3.78	.050
.900	14.6	4.218	.767	3.80	.100
.800	16.9	4.539	.825	3.63	.200
.700	18.9	4.80	.873	3.36	.300
.600	20.9	5.048	.918	3.03	.400
.400	23.0	5.29	.962	2.115	.600
.200	24.4	5.452	.991	1.09	.800
.000	24.8	5.498	1.000	0.00	1.000

Static Pressure = 33.8 Cm.Hg.Gauge

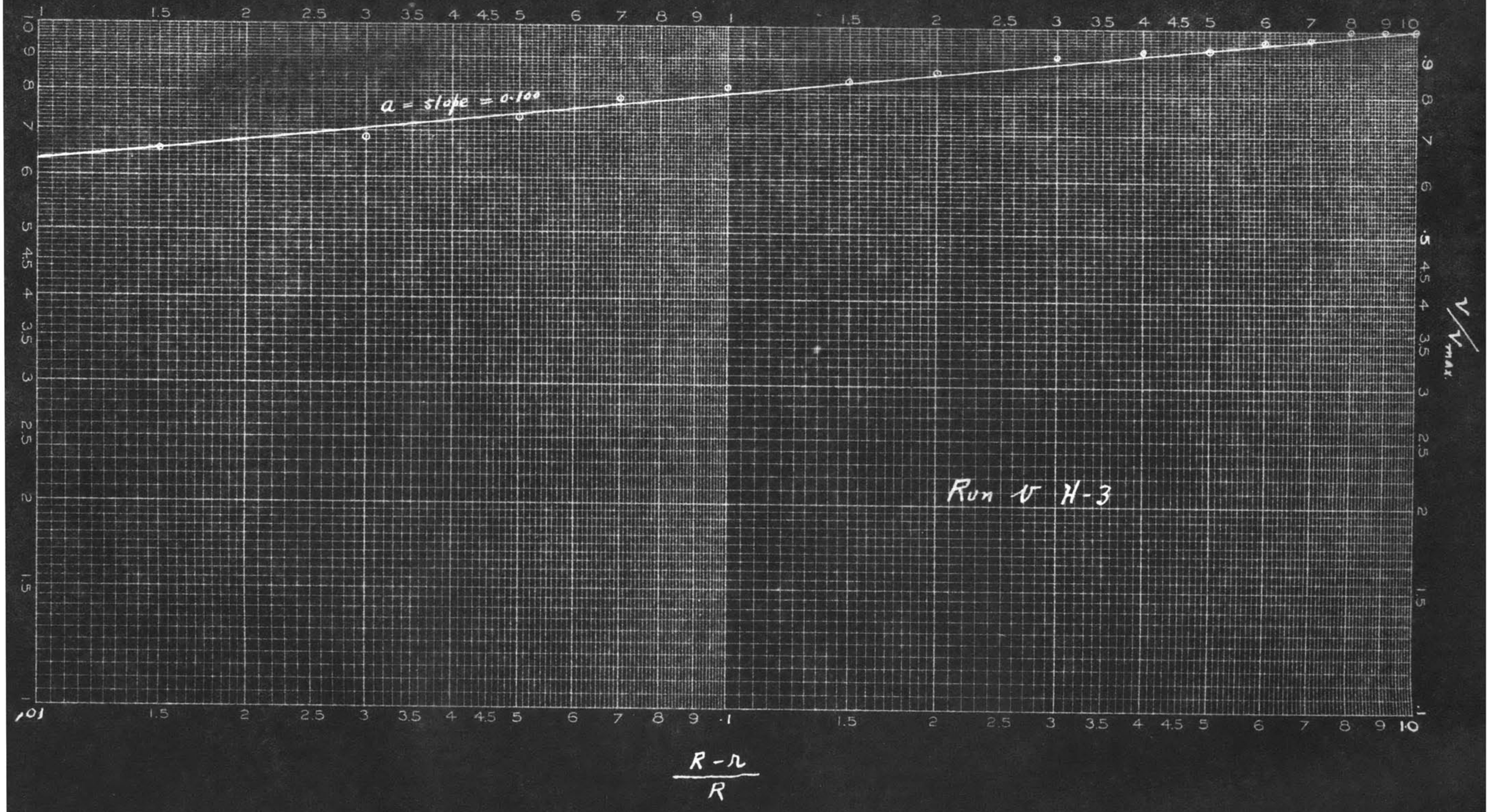
$V_{ave.}$ (Manometer) = 4.97 ft./sec.

$V_{ave.}$ (Graphical Integration) = 4.705 ft./sec.

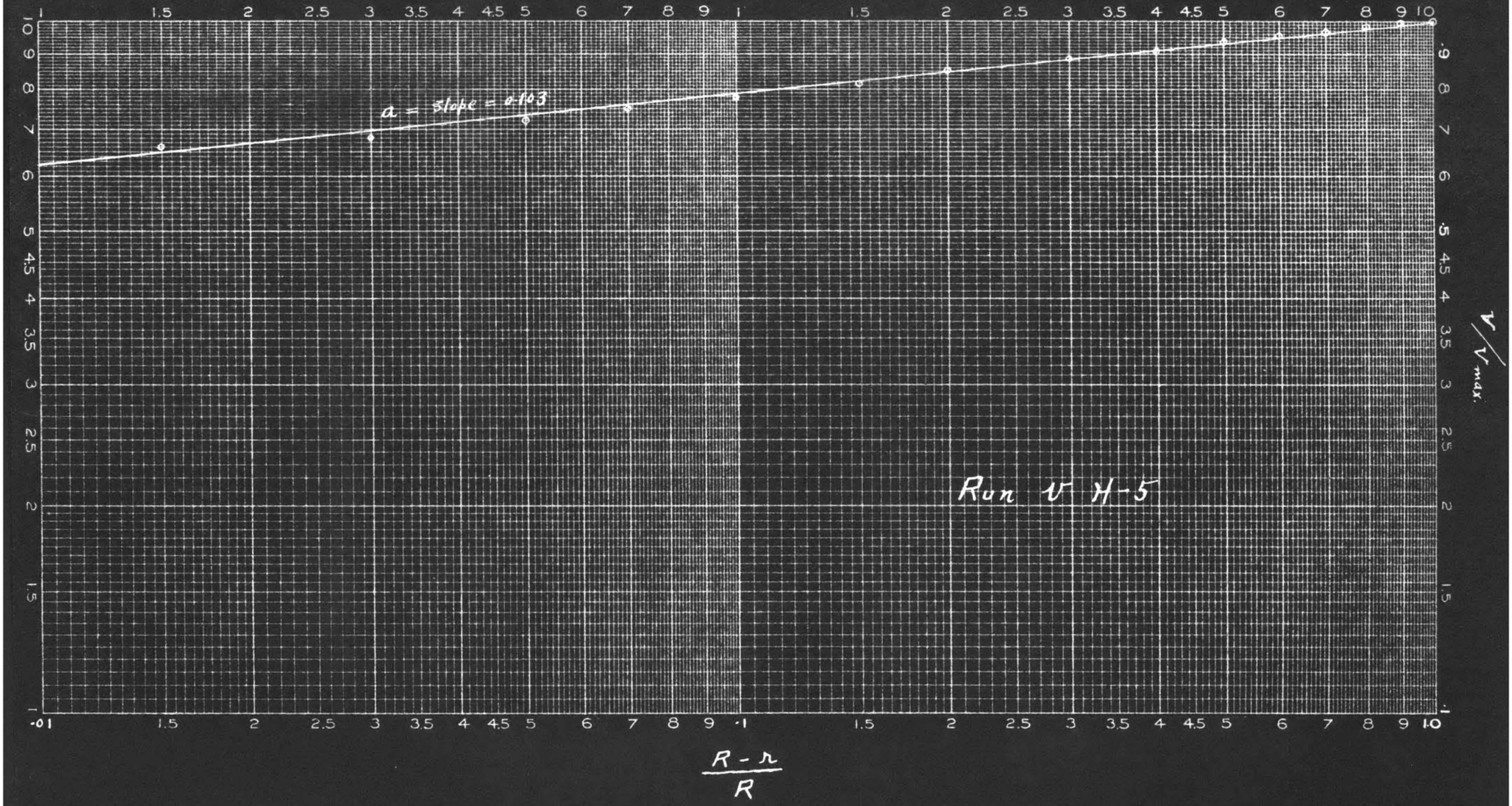
$V_{ave.}/V_{max.}$ = 4.705/5.498 = 0.856

$t_{ave.}$ = 12.3° C.

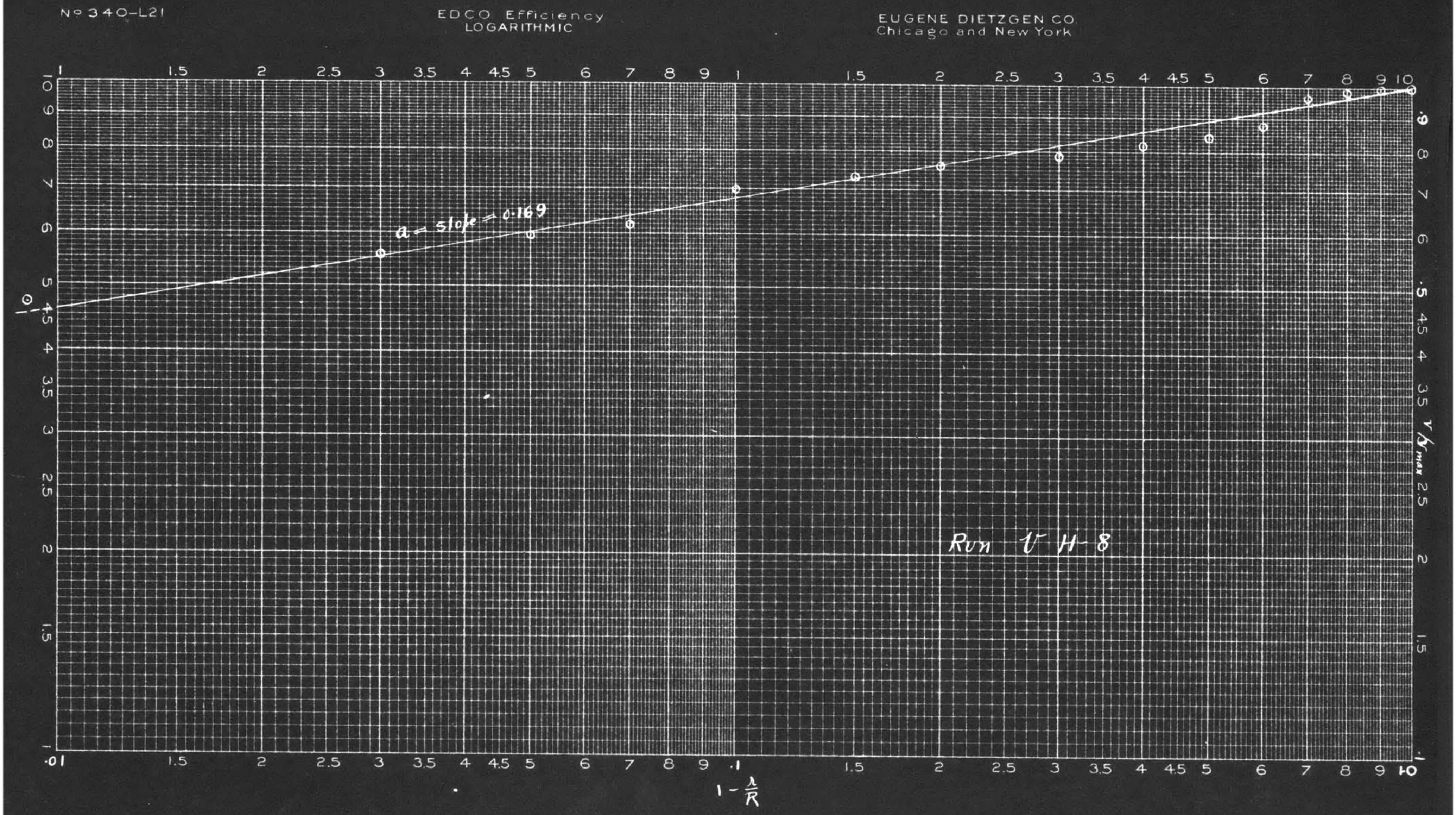
$$\therefore Re = \frac{1.952 \times 4.705 \times 10^5}{12 \times 1.321} = 57{,}900$$

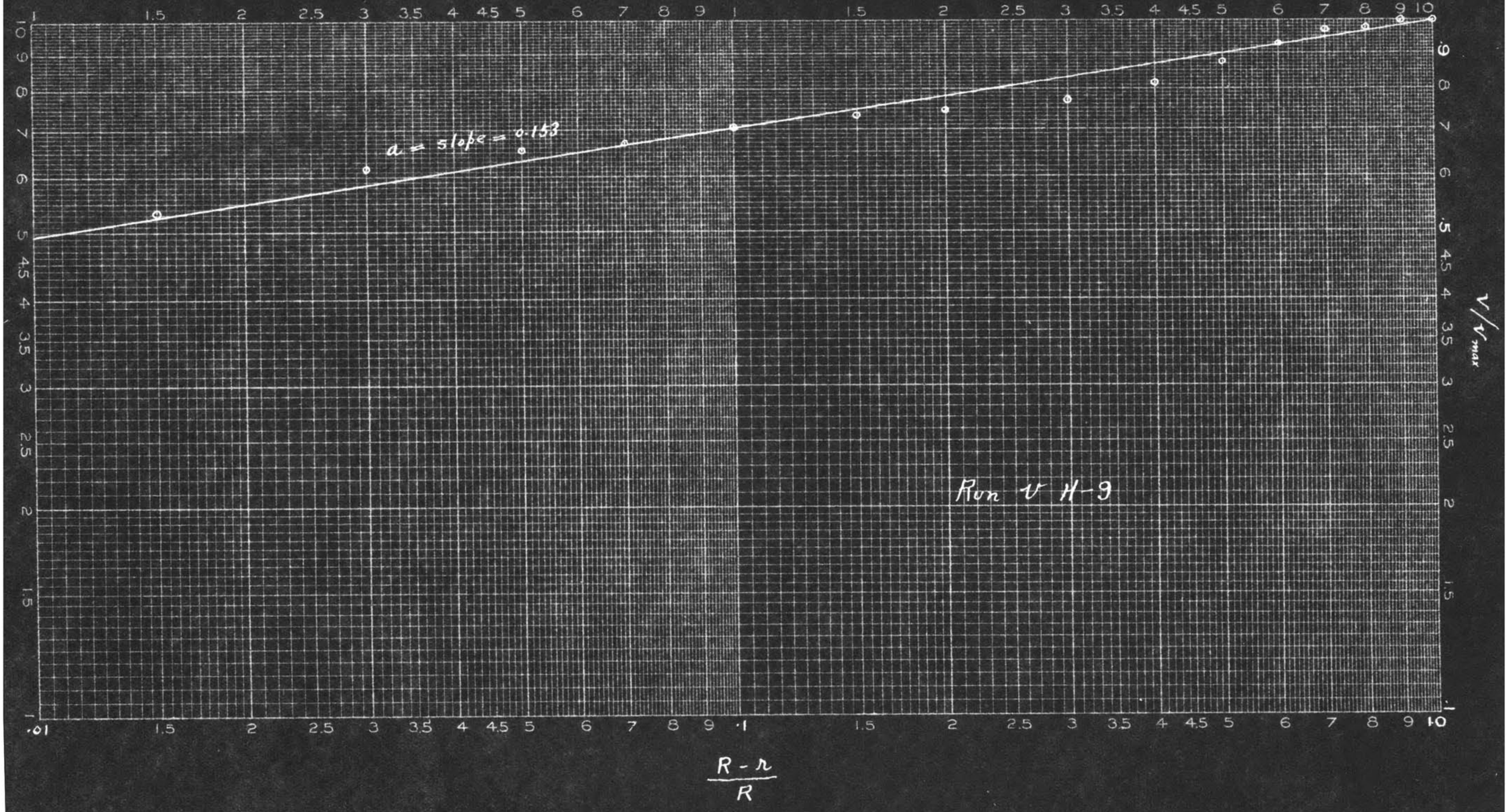


a = slope = 0.100
Run V H-3
V/Vmax.
R - r / R
.01
·1
1.0
·9
·5

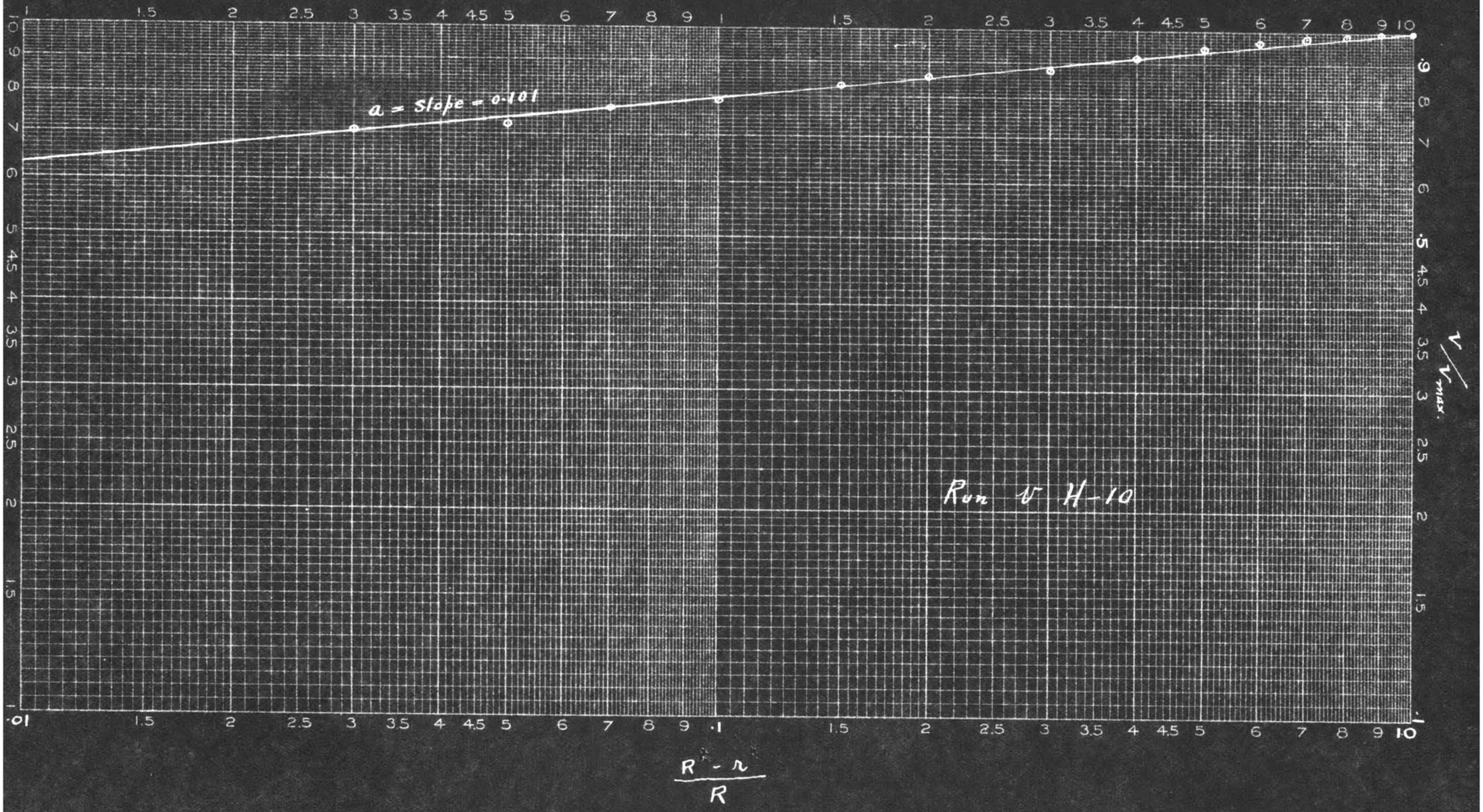


a = slope = 0.103
Run V H-5
R - r / R
v/V max.
·01
·1
1.0
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9

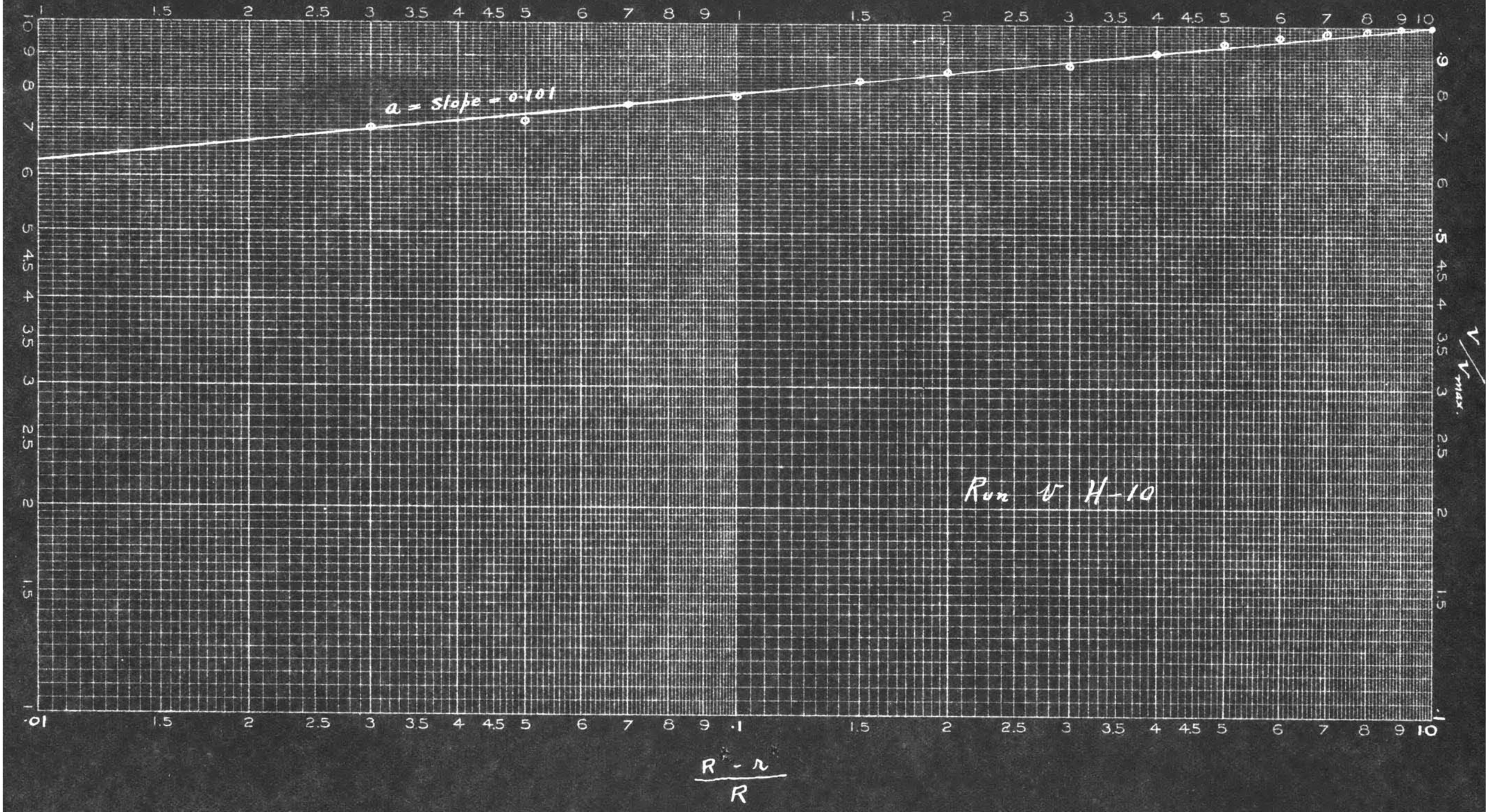

No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO
Chicago and New York
a = slope = 0.169
Run V H-8
1 - λ/R
v/v max
·01
·1
1·0
·9
·5
0
1
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
10



4



a = Slope = 0·101
Run V H-10
V/Vmax.
$\frac{R^2 - r^2}{R}$
·01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
·1
1.0
·9
·8
·7
·6
·5


a = Slope = 0.101
Run V H-10
$\frac{R^* - r^*}{R}$
$V/V_{max.}$
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.9
.5
.1

Nº 340-L21

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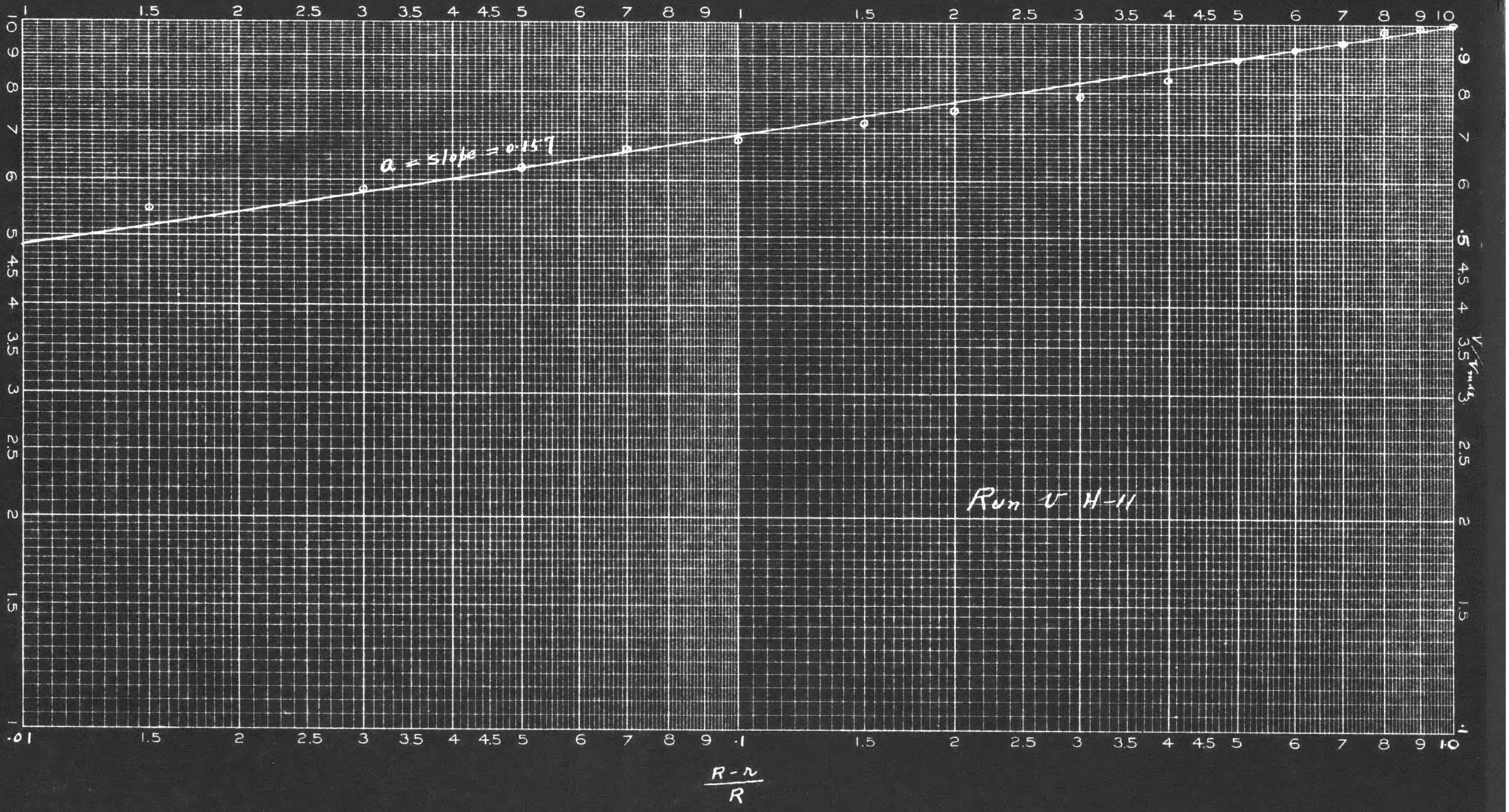
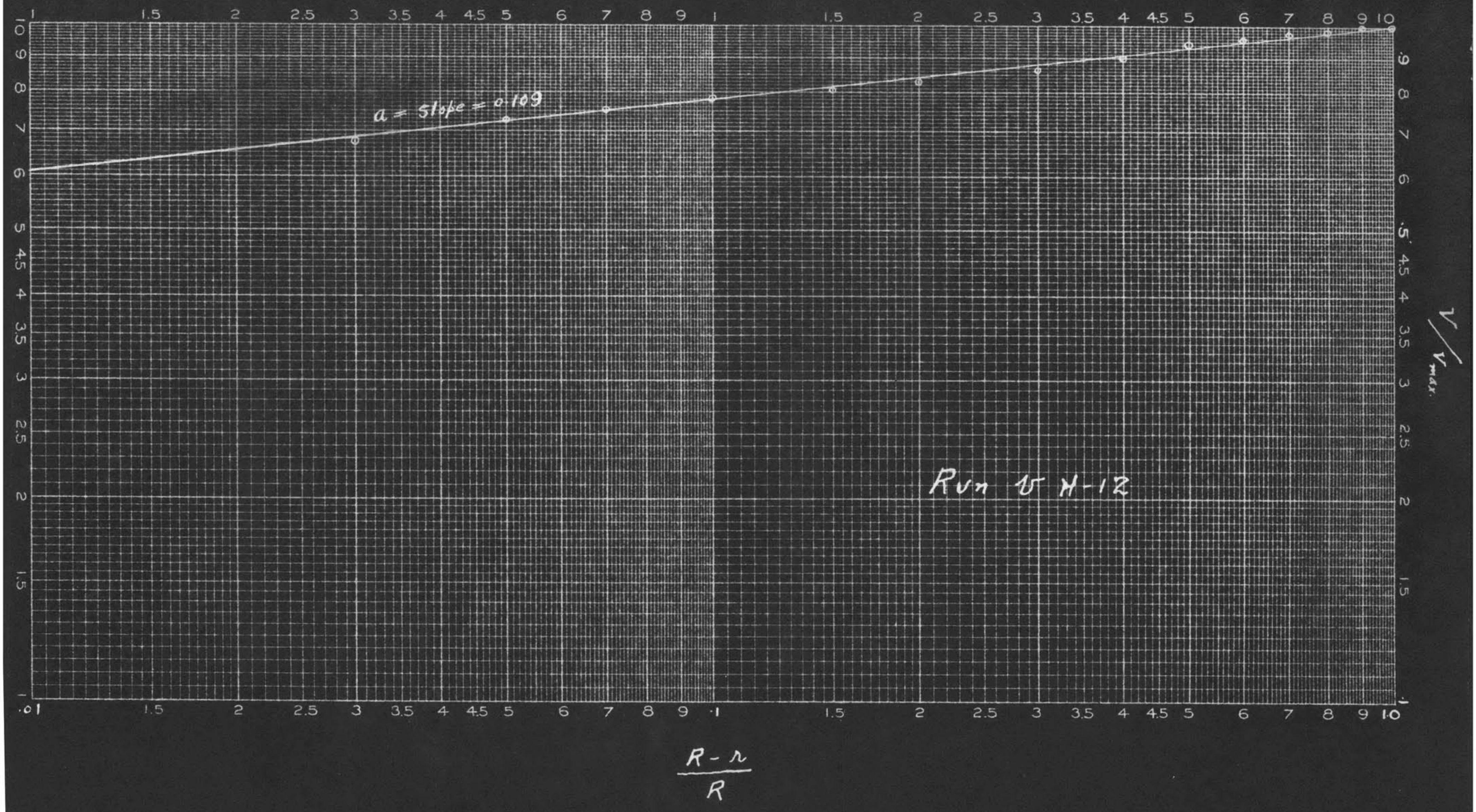
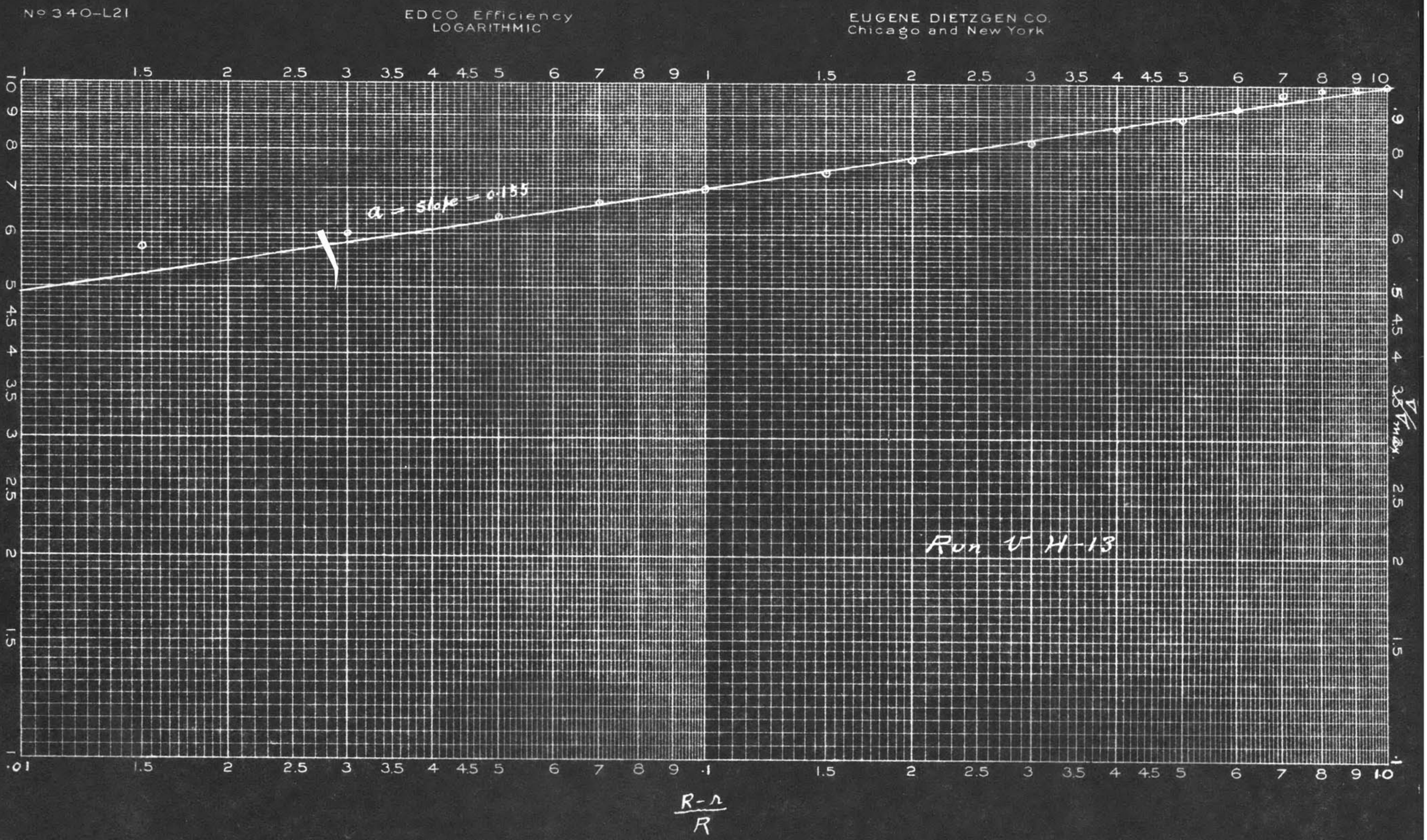



a = slope = 0.109
Run IV H-12
1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 10
.01 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 .1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 1.0
10 9 8 7 6 5 4.5 4 3.5 3 2.5 2 1.5 1
.9 .8 .7 .6 .5 4.5 4 3.5 3 2.5 2 1.5 .1
V/V_max.
R - r / R

V



Nº 340-L21
EDCO Efficiency
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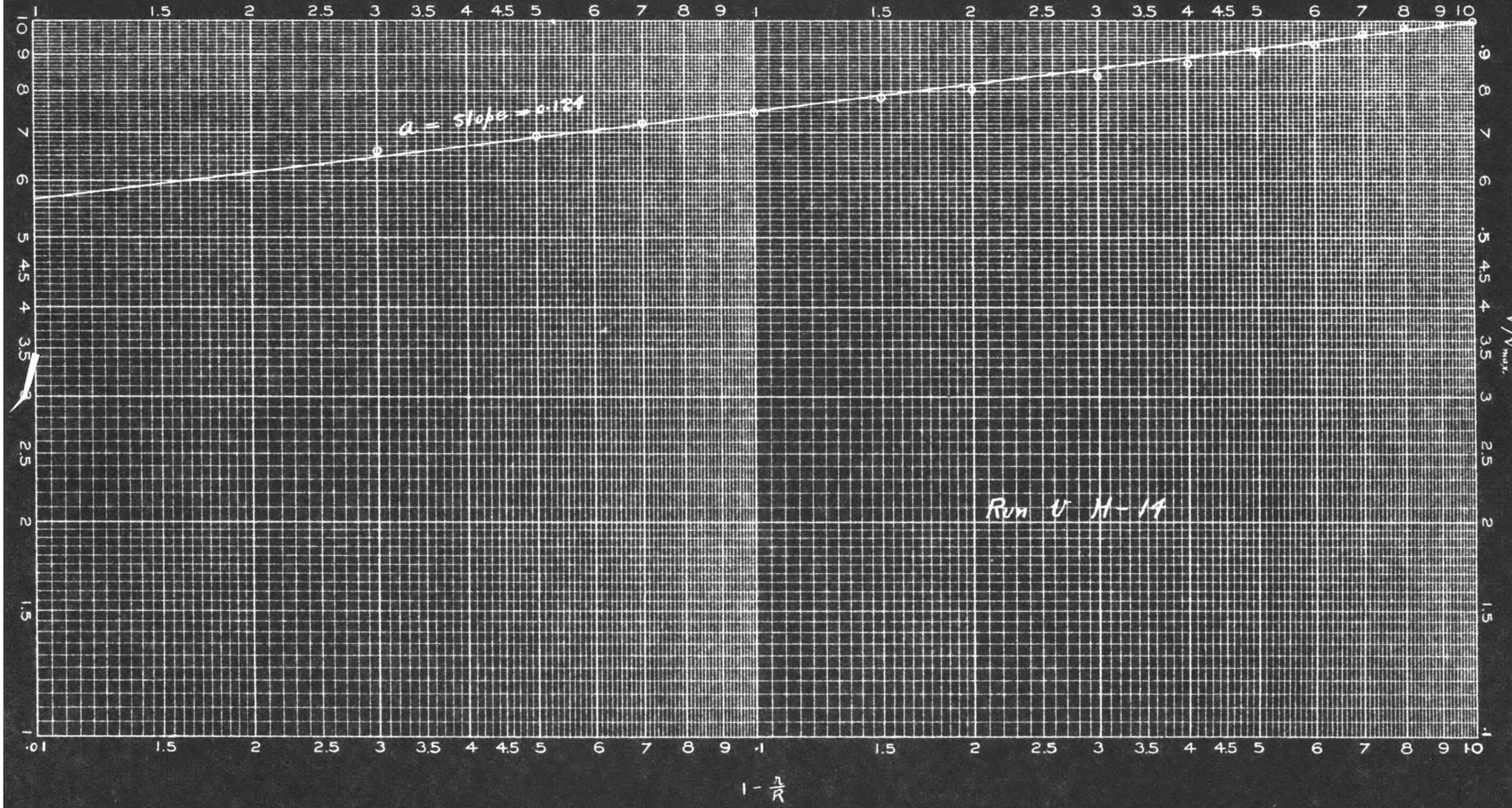
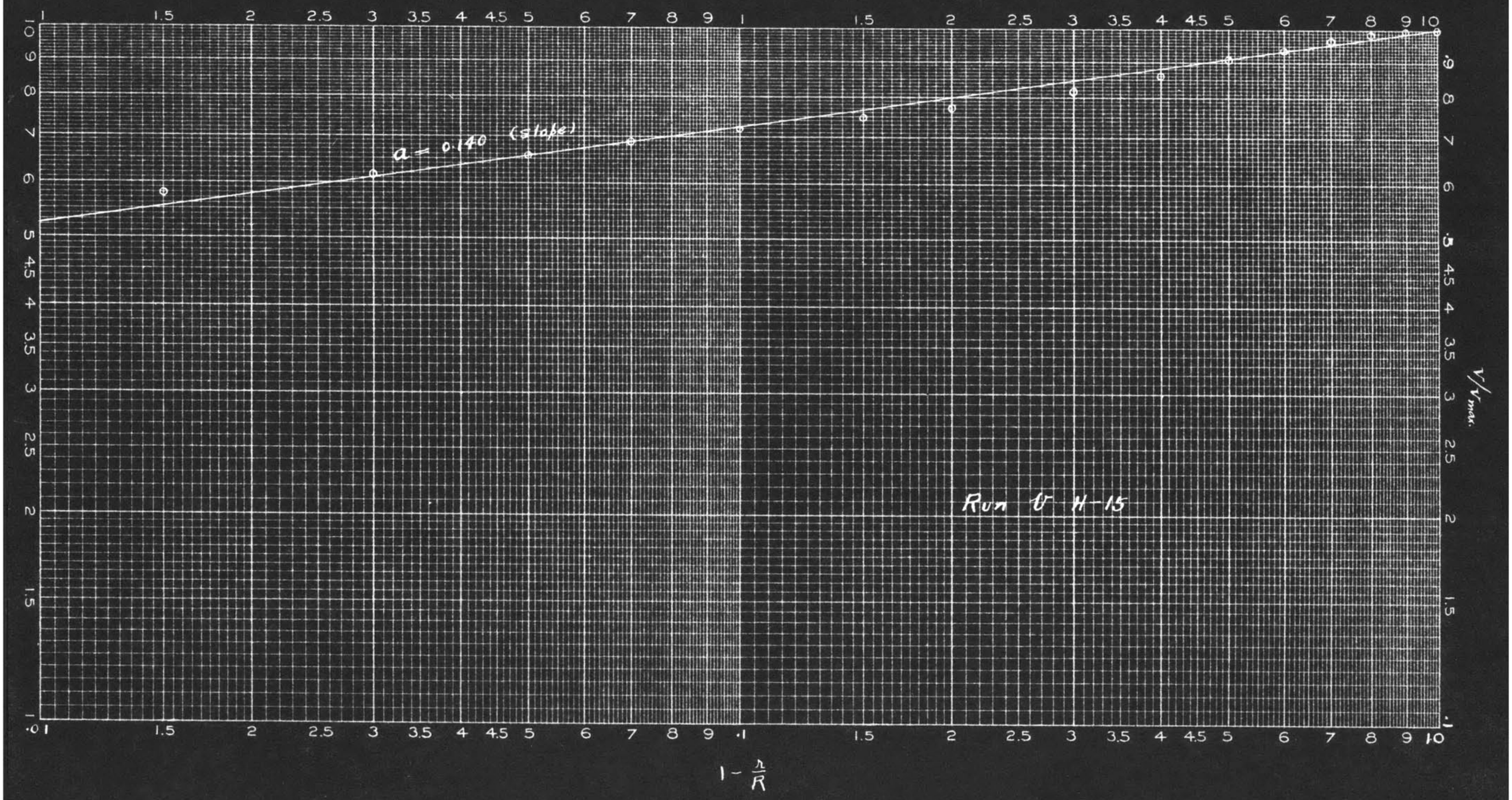
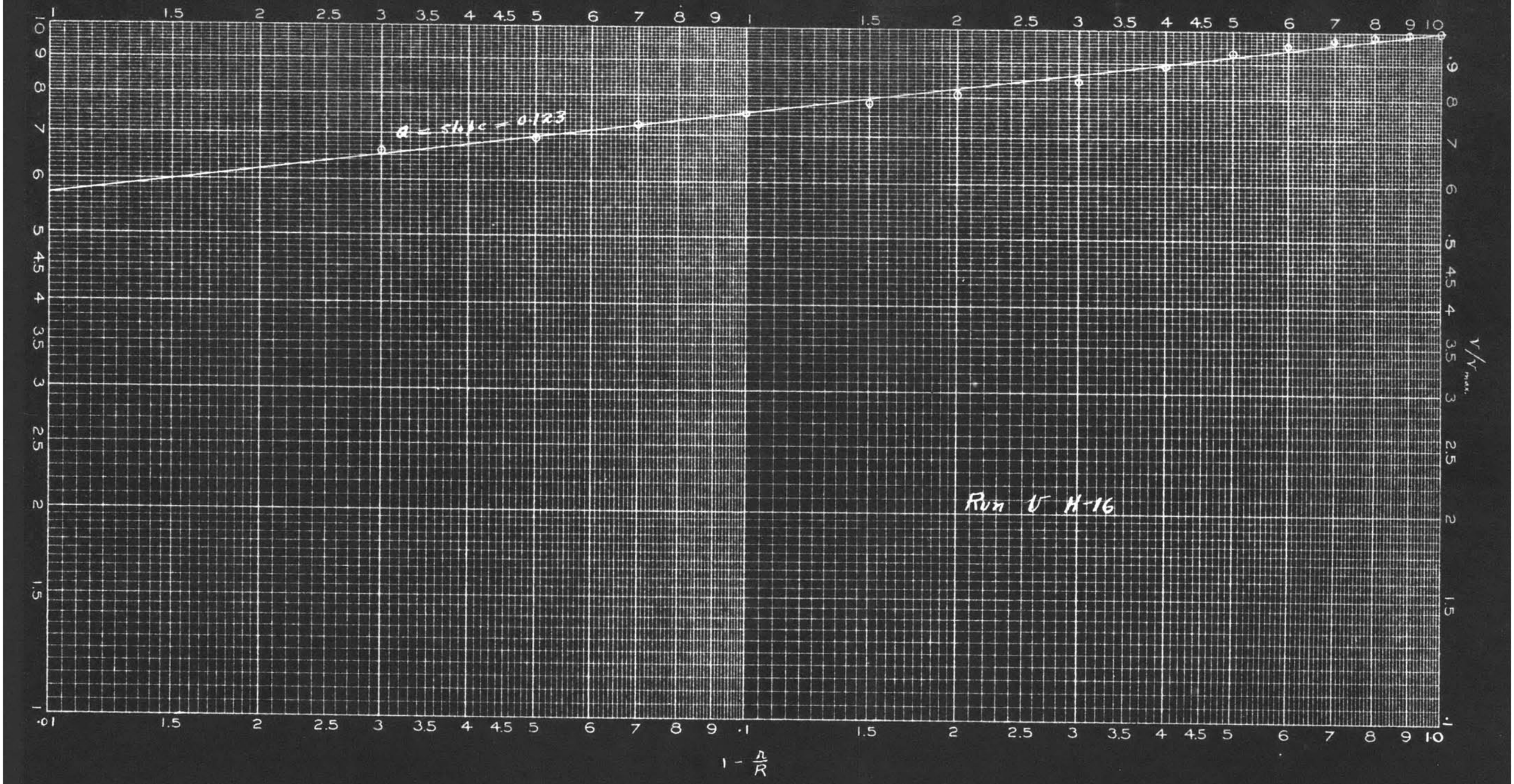
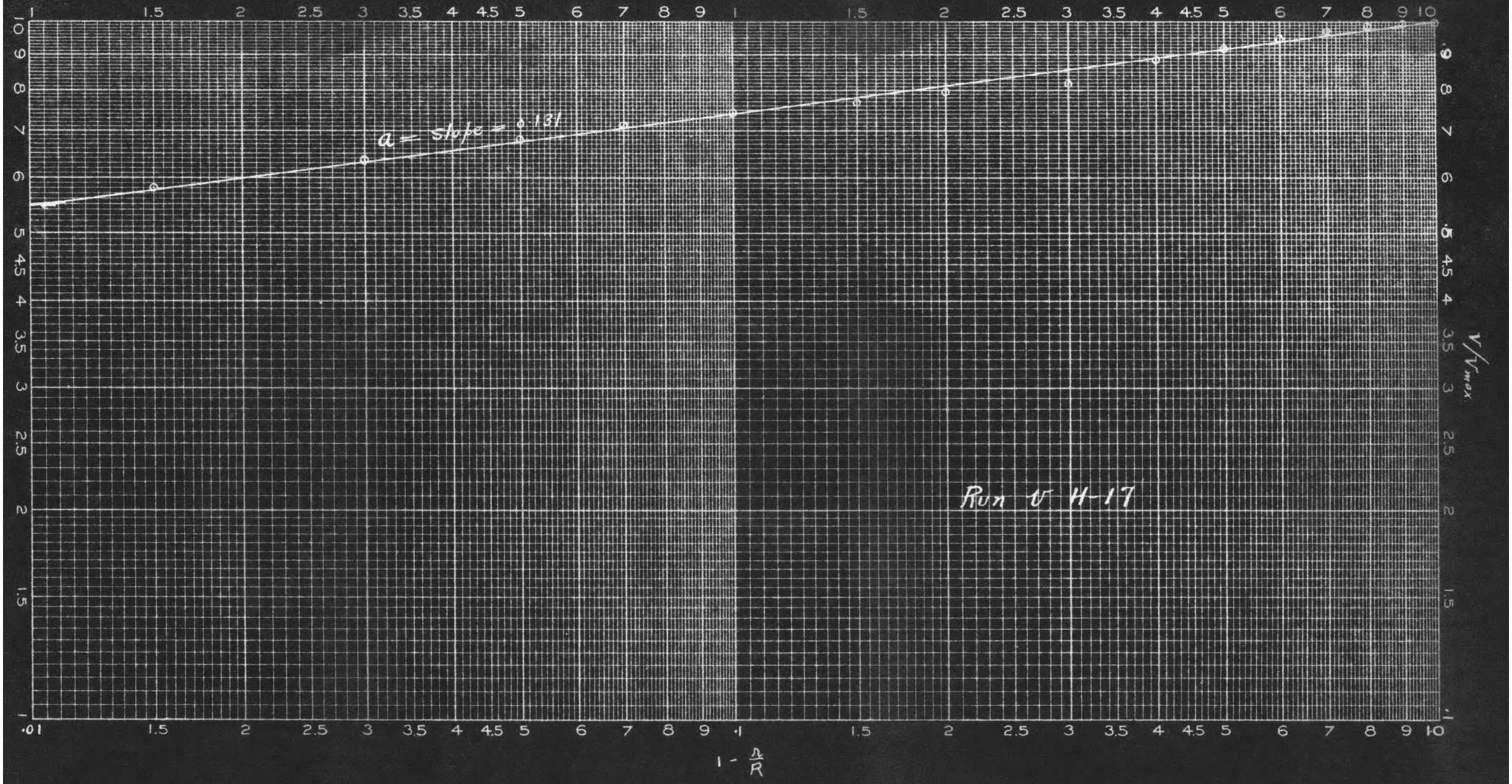
a = slope = 0.155
Run V H-13
$\frac{v}{v_{max}}$
$\frac{R-r}{R}$
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.1
.5
.9

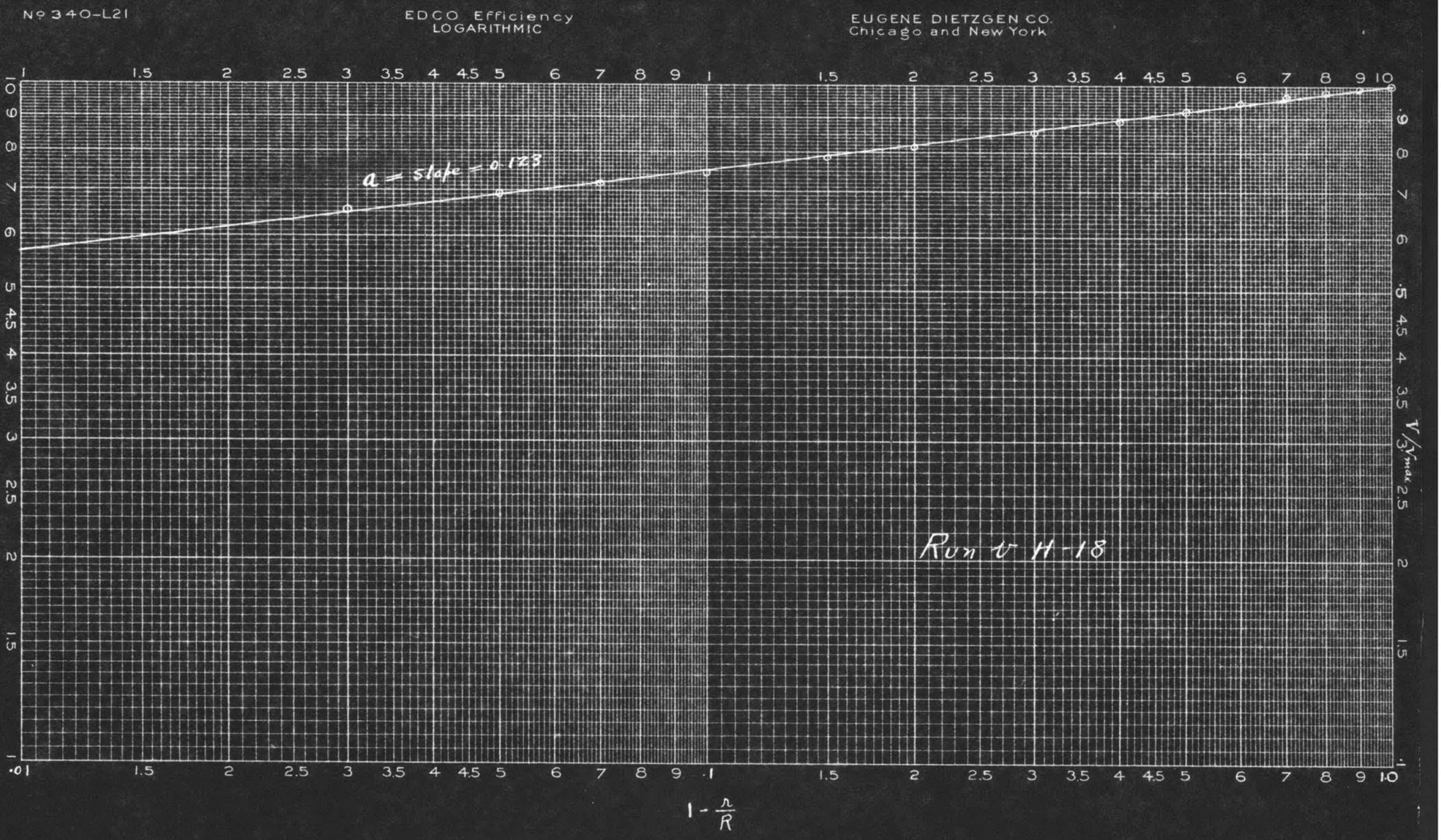


a = slope = 0.124
Run V H-14
1 - r/R
v/v_max
·01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
·1
1·0
·5
·9

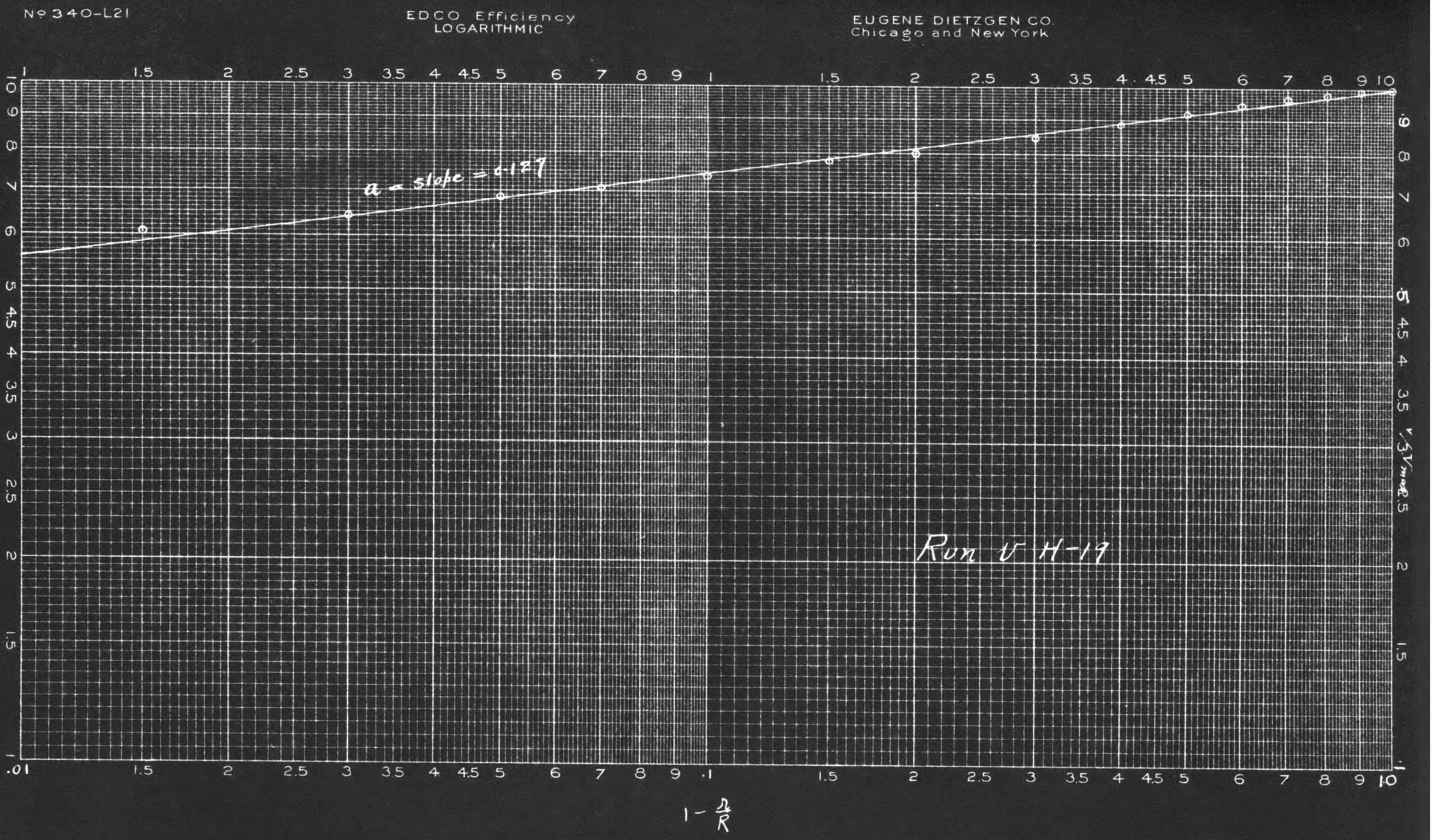

a = 0.140 (slope)
Run V H-15
V/Vmax.
1 - r/R
.01 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 .1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 1.0
1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 10
.1 1.5 2 2.5 3 3.5 4 4.5 .5 6 .7 .8 .9 1.0

a = slope = 0.123
Run V H-16
1 - r/R
v/v max.
·01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
·1
1·0
1
.9
.8
.7
.6
.5

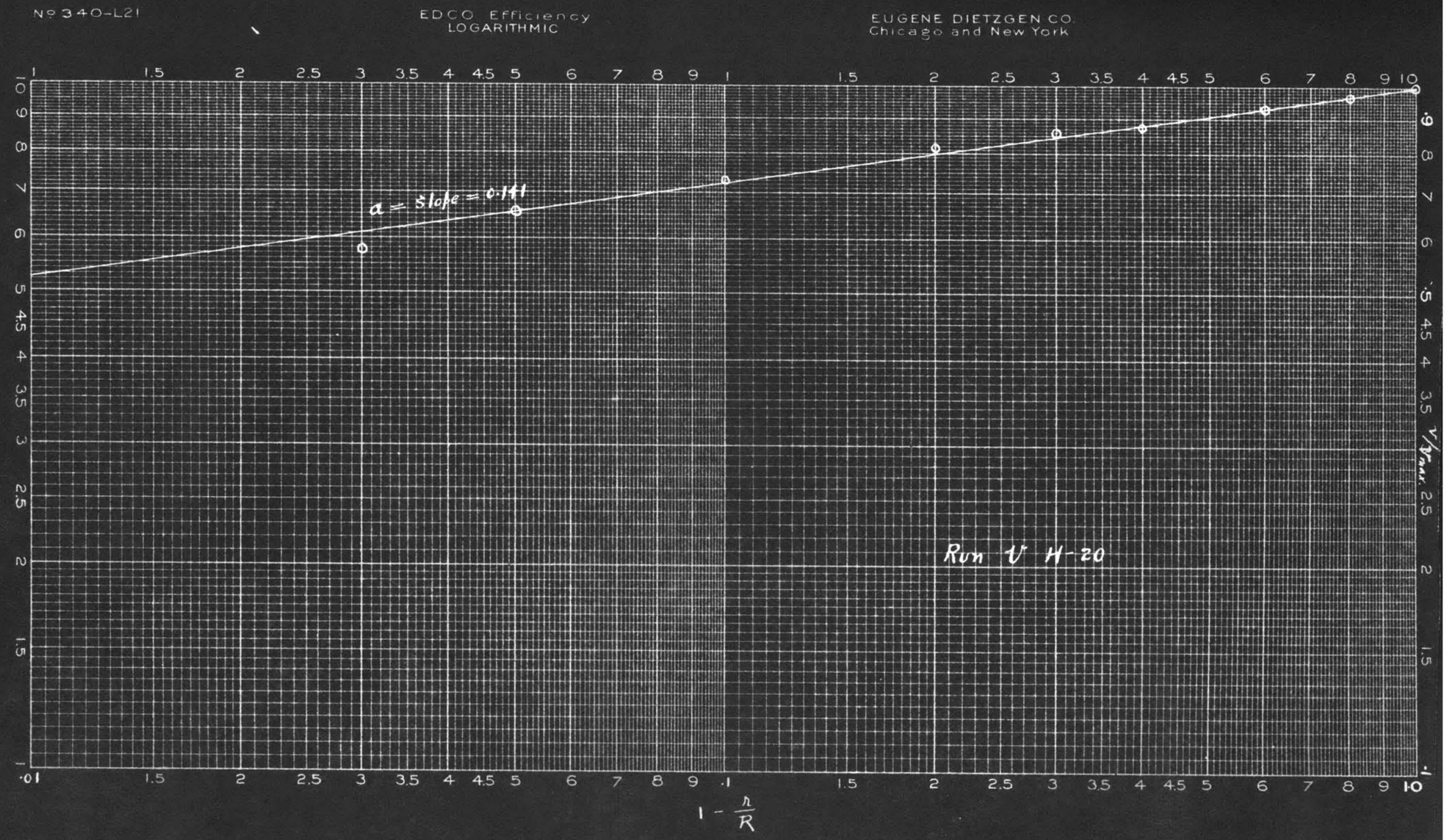

a = slope = .131
Run V H-17
1 - r/R
V/Vmax
·01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
·1
1.0
·9
·8
·7
·6
·5
·4
·3
·2
·1



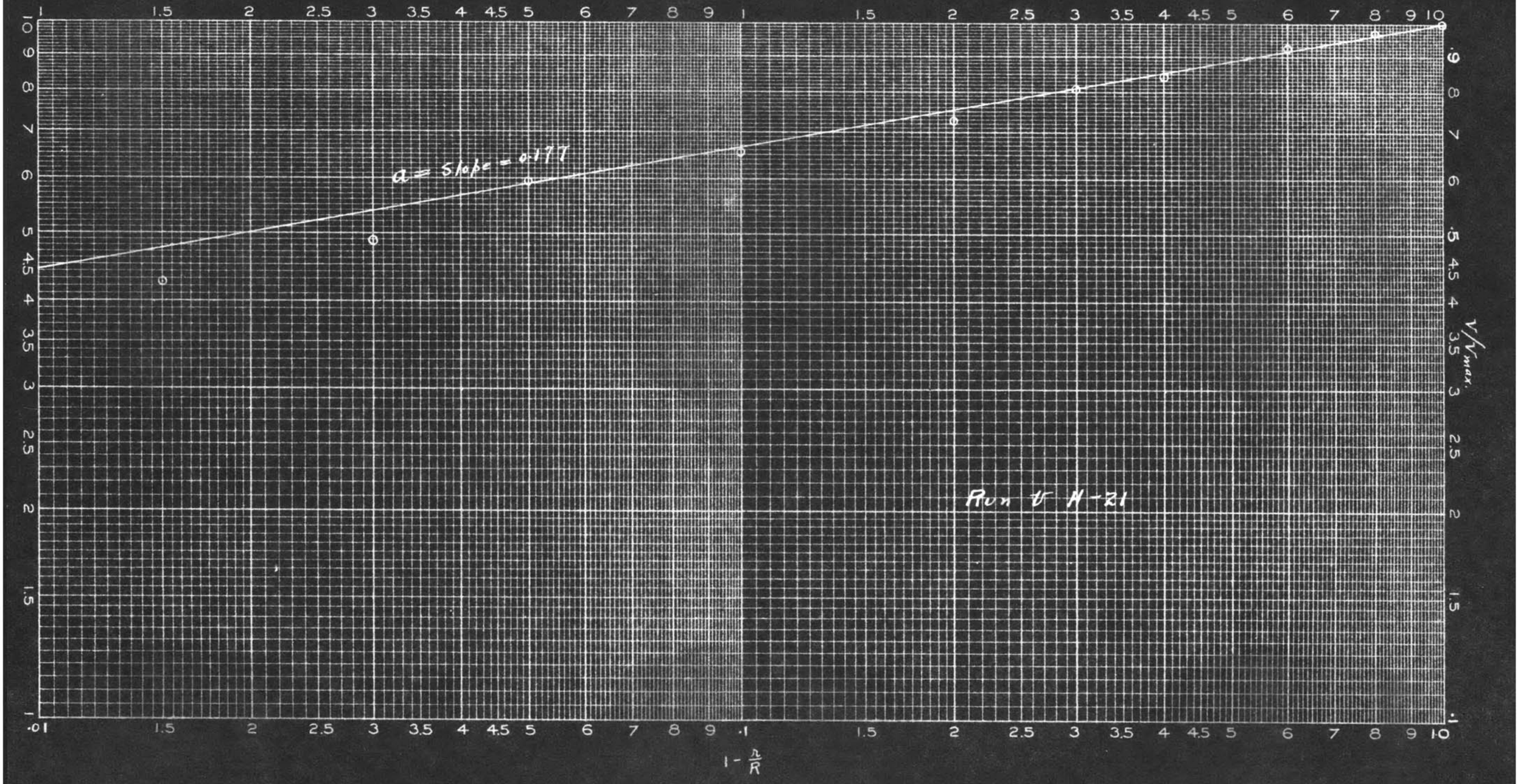
No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
a = slope = 0.128
Run v H-18
$1 - \frac{r}{R}$
V/V_{max}
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.1
.5
.9



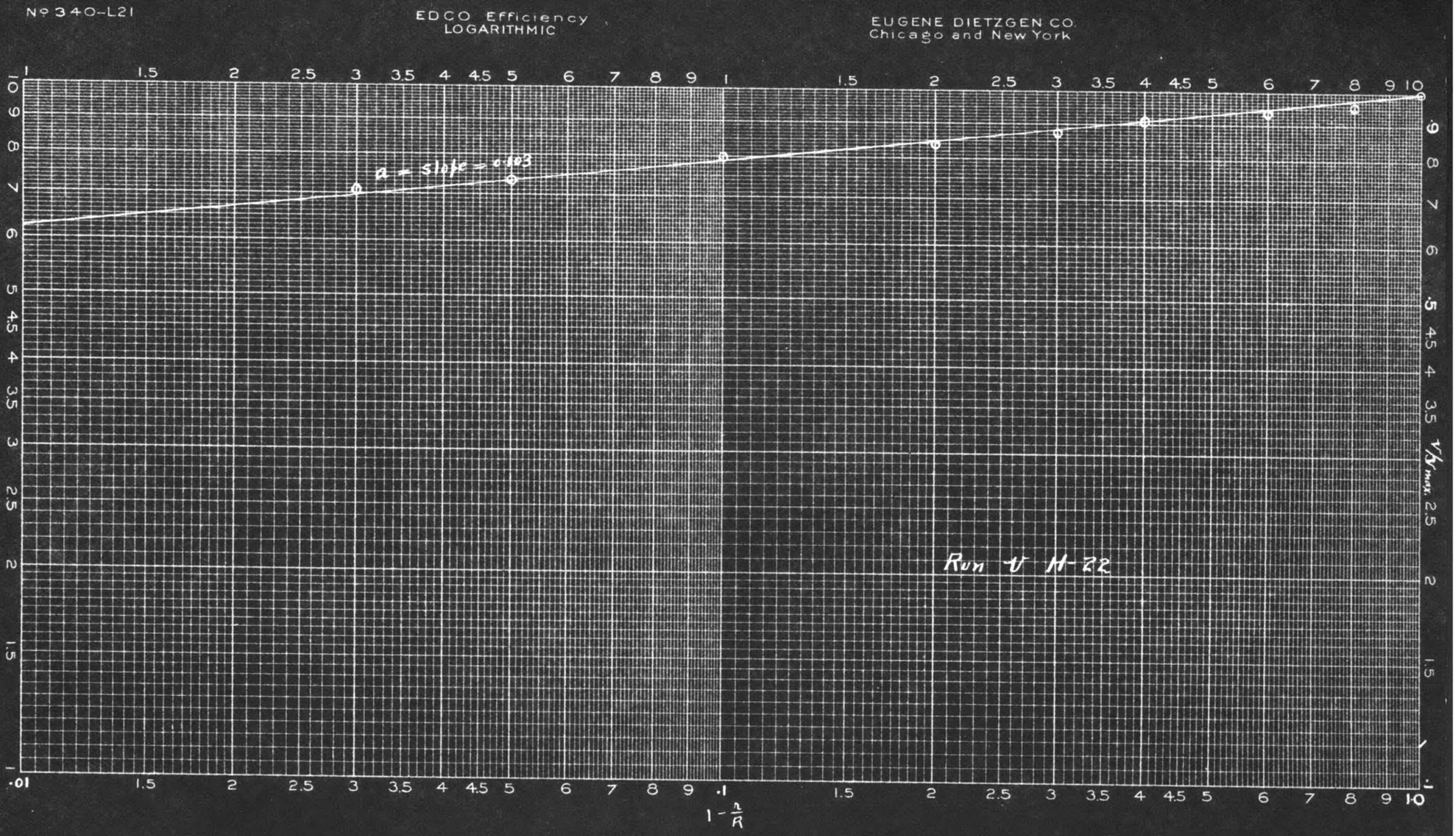
No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
a = slope = 0.127
Run V H-19
$1 - \frac{r}{R}$
v/V_{max}
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.1
.5
.9

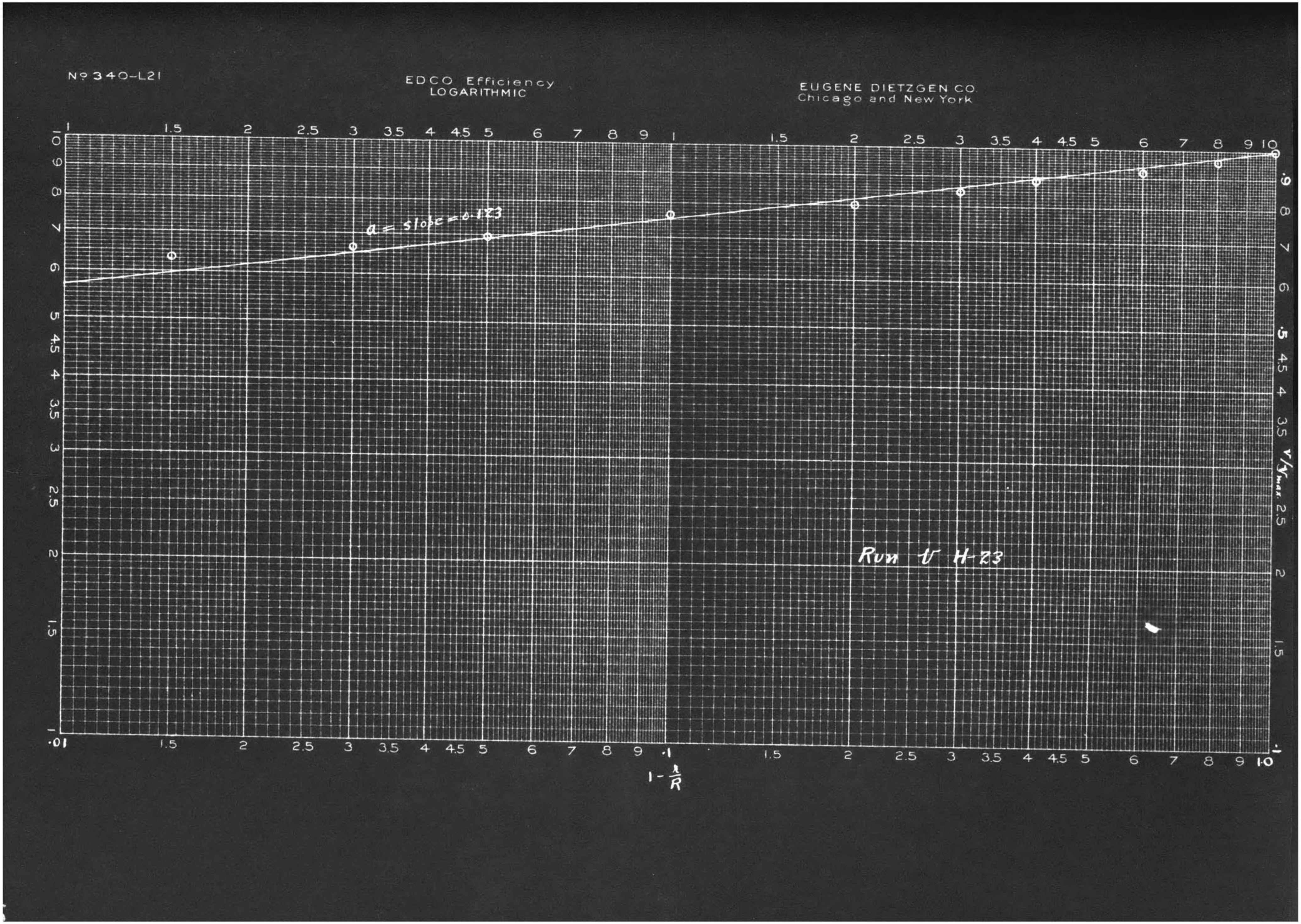


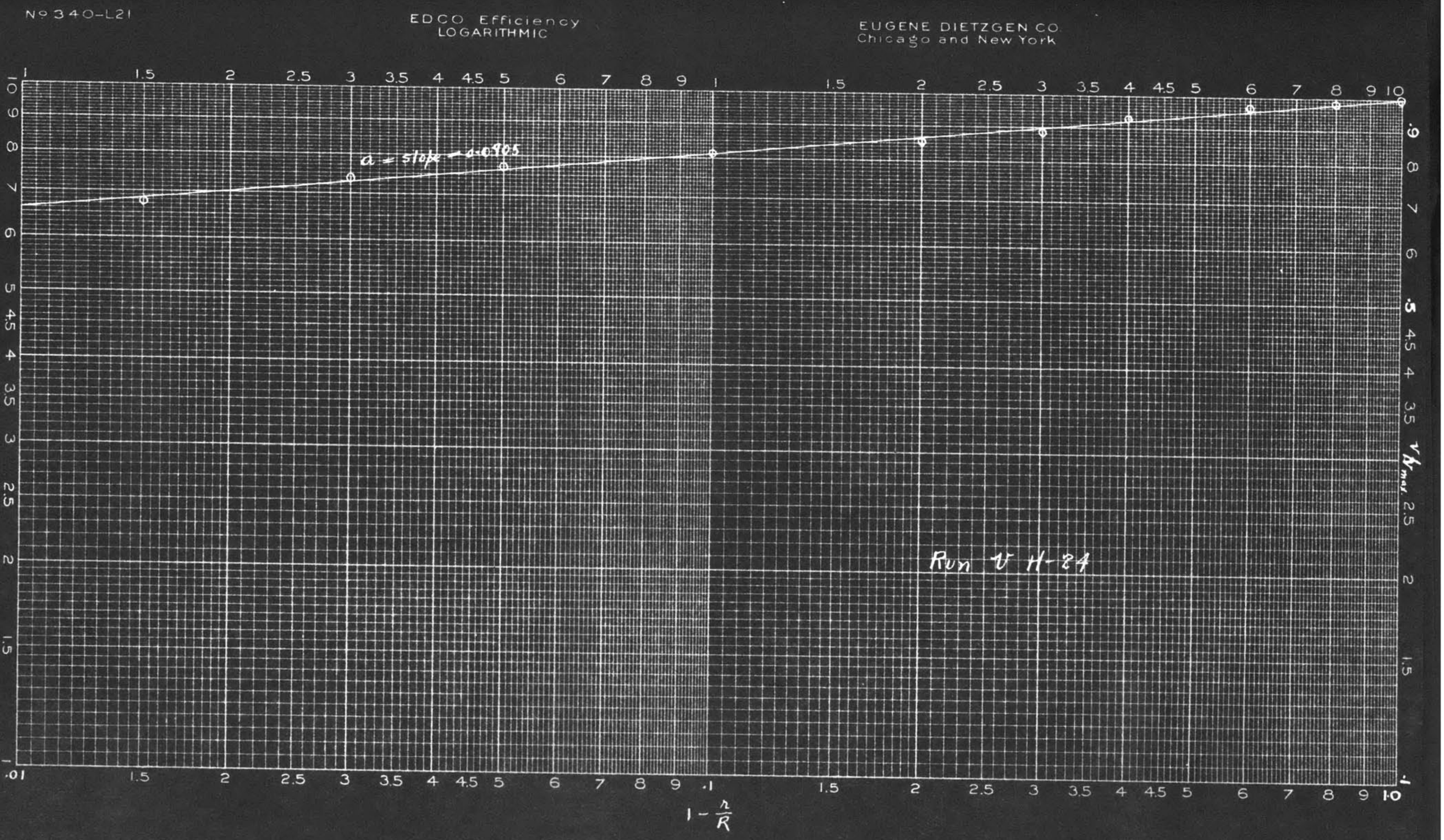
No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
a = slope = 0.141
Run V H-20
1 - r/R
v/v max.
·01
·1
1·0
·5
·9
1
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
10



a = slope = 0.177
Run V H-21
1 - r/R
V/Vmax.
.01
.1
1.0
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
10
.9
.8
.7
.6
.5
.4


Nº 340-L21
EDCO Efficiency LOGARITHMIC
EUGENE DIETZGEN CO. Chicago and New York
a = slope = 0.103
Run V M-22
$V/V_{max.}$
$1-\frac{r}{R}$
.01
.1
1.0
.9
.8
.7
.6
.5
.1

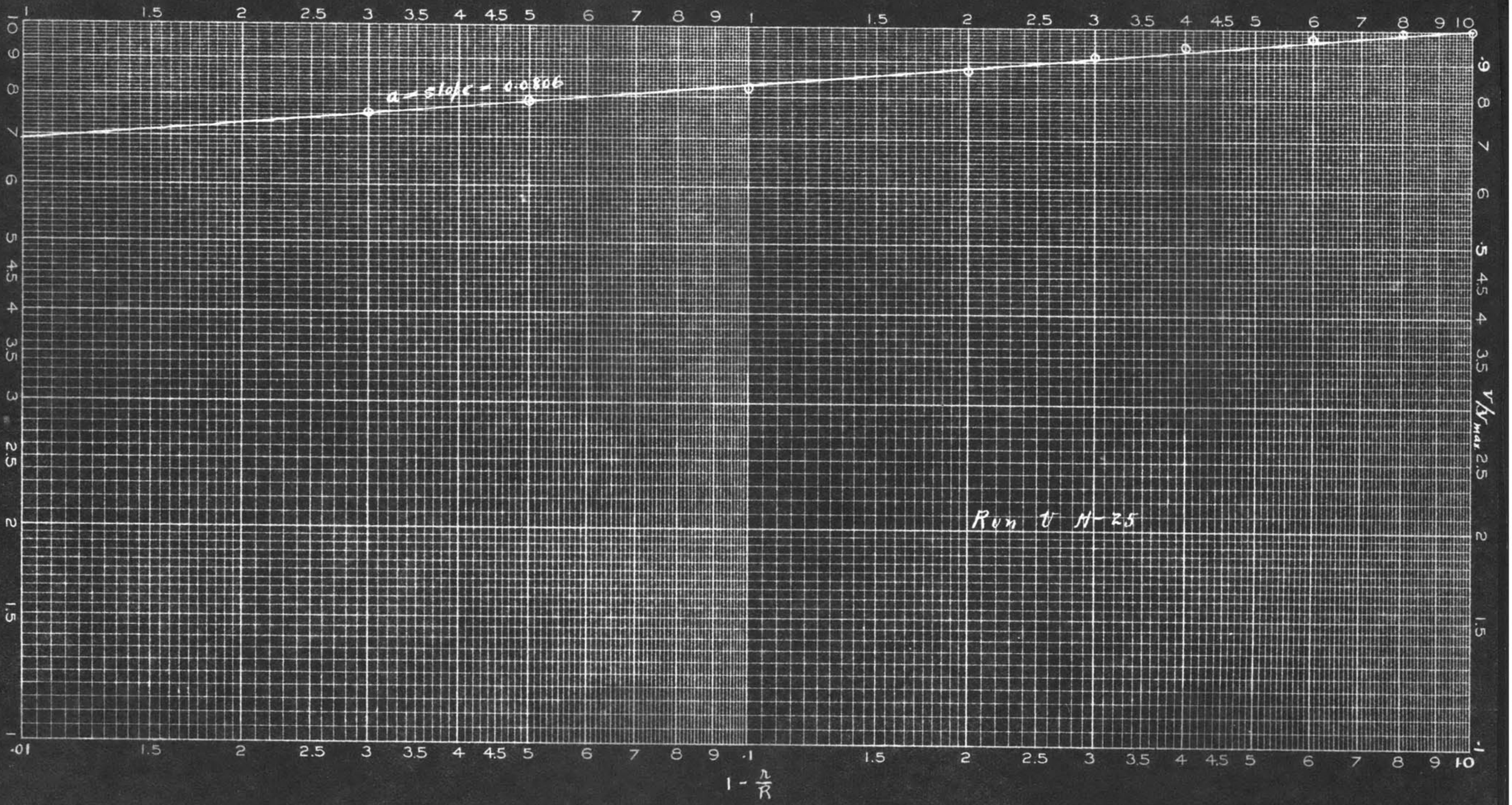

No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
a = slope = 0.123
Run V H-23
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
1 - r/R
v/v max.
.1
1.5
2
2.5
3
3.5
4
4.5
.5
6
7
8
.9

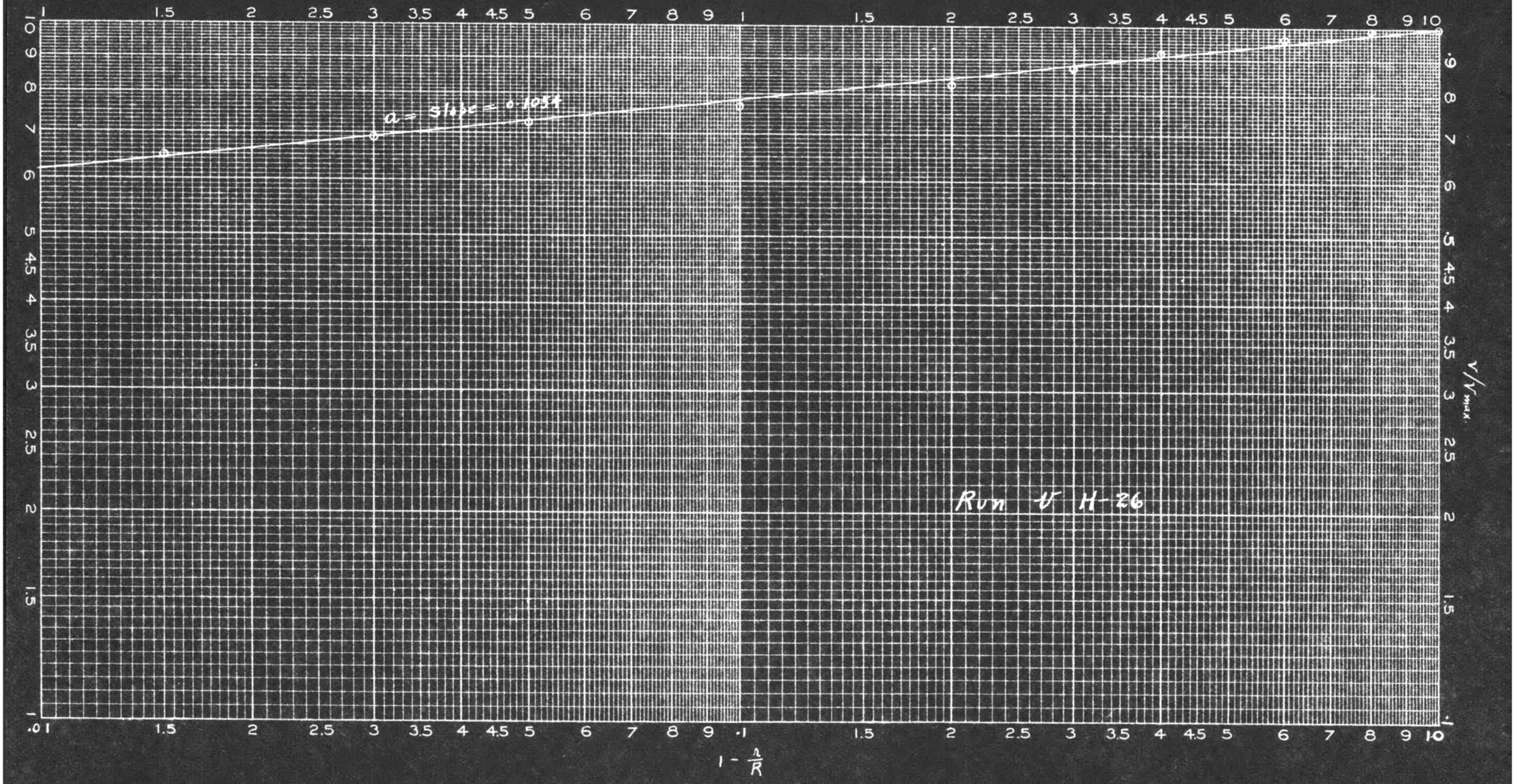

No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
a = slope = 0.0905
Run V H-24
$1 - \frac{r}{R}$
v/v_{max}
.01
.1
1.0
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
10
.9
.8
.7
.6
.5

No 340-L21

EDCO Efficiency
LOGARITHMIC

EUGENE DIETZGEN CO.
Chicago and New York




a = slope = 0.1054
Run V H-26
1 - r/R
v/v_max
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.9
.5

APPENDIX F

VELOCITY DISTRIBUTION DATA DURING COUNTER CURRENT HEATING WITH CALCULATIONS AND PLOTS.

(Water Flowing Upward)

Run V H-27(Counter current) Station No. 4

(For corresponding temperature distribution, see Run T H-39)

$\frac{r}{R}$	$\frac{\Delta h}{(\text{Cm.}CCL_4)}$	$\frac{v}{(\text{ft./sec.})}$	$\frac{v}{v_{max.}}$	$\frac{v(r/R)}{(\text{ft./sec.})}$	(1-r/R)
0.989	5.70	2.636	0.576	2.605	0.011
.970	7.00	2.92	.638	2.83	.030
.950	7.80	3.082	.674	2.925	.050
.90	9.00	3.31	.723	2.98	.10
.80	10.6	3.594	.786	2.875	.20
.70	12.4	3.887	.850	2.72	.30
.60	13.7	4.087	.894	2.455	.40
.40	15.4	4.332	.946	1.732	.60
.20	16.8	4.525	.989	.905	.80
.00	17.2	4.577	1.000	.000	1.00

Static Pressure = 19.2 Cm.Hg.Gauge

$V_{ave.}$ (Graphical Integration) = 3.76 ft./sec.

$$\therefore V_{ave.}/V_{max.} = \frac{3.76}{4.577} = 0.822$$

Run v H-28 (Counter Current) Station No. 4

(For corresponding temperature distribution, see Run T H-41)

$\frac{r}{R}$	Δh (Cm.CCL_4)	V (ft./sec.)	$\frac{V}{V_{max.}}$	V (r/R) (ft./sec.)	(1-r/R)
0.989	3.80	2.154	0.600	2.13	0.011
.970	4.60	2.368	.659	2.295	.050
.950	5.20	2.52	.701	2.395	.050
.90	5.80	2.658	.740	2.390	.10
.80	7.10	2.938	.818	2.35	.20
.70	7.90	3.103	.864	2.17	.30
.60	8.40	3.20	.891	1.92	.40
.40	9.40	3.386	.943	1.355	.60
.20	10.30	3.54	.985	.708	.80
.00	10.60	3.593	1.000	.000	1.00

Static Pressure = 24.8 Cm.Hg.Gauge

$V_{ave.}$ (Graphical Integration) = 2.98 ft./sec.

$$V_{ave.}/V_{max.} = \frac{2.98}{3.593} = 0.829$$

Run v H-29 (Counter Current) Station No. 5

(For corresponding temperature distribution, See Run TH-42)

$\frac{r}{R}$	Δh (Cm.CCL_4)	v (cu.ft./sec.)	$\frac{V}{V_{max.}}$	V(r/R) (ft./sec.)	1-r/R
0.991	4.20	2.264	0.636	2.243	0.009
.970	4.80	2.419	.680	2.345	.030
.950	5.40	2.567	.721	2.44	.050
.90	6.13	2.73	.767	2.455	.10
.80	7.30	2.98	.837	2.385	.20
.70	7.90	3.102	.872	2.17	.30
.60	8.52	3.203	.905	1.935	.40
.40	9.38	3.380	.950	1.354	.60
.20	10.10	3.508	.986	0.702	.80
.00	10.40	3.560	1.000	0.00	1.00

Static Pressure = 20.6 cm. Hg. Gauge

$V_{ave.}$ (Graphical Integration) = 3.03 ft./sec.

$\therefore V_{ave.}/V_{max.} = 3.03/3.56 = 0.851$

Run V H-30 (Counter Current) Station No. 4

(For corresponding temperature distribution, see Run TH-43)

$\frac{r}{R}$	Δh (Cm.CCL_4)	V (.ft./sec.)	$\frac{V}{V_{max.}}$	V(r/R) (ft./sec.)	(1-r/R)
0.989	0.84	1.015	0.580	1.004	0.011
.970	1.06	1.132	.647	1.098	.030
.950	1.17	1.192	.681	1.132	.050
.90	1.39	1.30	.743	1.170	.10
.80	1.69	1.43	.817	1.144	.20
.70	1.79	1.475	.843	1.033	.30
.60	1.94	1.533	.876	0.920	.40
.40	2.19	1.633	.934	0.654	.60
.20	2.36	1.697	.970	0.340	.80
.00	2.51	1.750	1.000	0	1.00

Static Pressure = 34.40 Cm.Hg.Gauge

$V_{ave.}$ (weighing) = 1.434 ft./sec.

$V_{ave.}$ (Graphical Integration) = 1.442 ft./sec.

$$\therefore V_{ave.}/V_{max.} = \frac{1.442}{1.750} = 0.824$$

Run V H-31 (Counter Current) Station No. 4

(For corresponding temperature distribution, See Run T H-45)

$\frac{r}{R}$	Δh (Cm.CCL$_4$)	V (ft./sec.)	$\frac{V}{V_{max.}}$	V(r/R) (ft./sec.)	(1-r/R)
0.989	5.10	2.494	0.627	2.467	0.011
.970	5.70	2.637	.662	2.557	.030
.950	6.30	2.77	.696	2.63	.050
.90	7.20	2.96	.744	2.663	.100
.80	8.60	3.24	.814	2.59	.20
.70	9.60	3.42	.860	2.393	.30
.60	10.30	3.54	.890	2.125	.40
.40	11.70	3.77	.947	1.51	.60
.20	12.60	3.92	.985	0.784	.80
.00	13.00	3.98	1.000	0.0	1.00

Static Pressure = 23.70 Cm.Hg.Gauge

$V_{ave.}$ (Graphical Integration) = 3.33 ft./sec.

$$\therefore V_{ave.}/V_{max.} = \frac{3.33}{3.98} = 0.837$$

Run V   H-32 (Counter Current)   Station No. 5

(For corresponding temperature distribution, see Run T H-46)

$\frac{r}{R}$	Δh (CCm.CCL$_4$)	V (ft./sec.)	$\frac{V}{V_{max.}}$	v(r/R) (ft./sec.)	(1-r/R)
0.991	5.35	2.554	0.642	2.530	0.009
.970	5.98	2.700	.679	2.62	.030
.950	6.23	2.75	.691	2.61	.050
.90	7.23	2.966	.745	2.67	.10
.80	8.23	3.166	.795	2.535	.20
.70	9.23	3.355	.843	2.35	.30
.60	10.15	3.515	.884	2.11	.40
.40	11.48	3.74	.940	1.496	.60
.20	12.47	3.90	.980	0.78	.80
.00	12.98	3.98	1.000	0.00	1.00

Static Pressure = 24.0 cm. Hg. Gauge

$V_{ave.}$ (Graphical Integration) = 3.30 ft./sec.

$$V_{ave.}/V_{max.} = \frac{3.30}{3.98} = 0.830$$

Run V H-33 (Counter Current) Station No. 4

(For corresponding temperature distribution, see Run T H-49)

$\frac{r}{R}$	Δh (CCm.CCL_4)	V (ft./sec.)	$\frac{V}{V_{max.}}$	V(r/R) (ft./sec.)	(1-r/R)
0.990	0.78	0.980	0.518	0.970	0.01
.970	0.962	1.08	.571	1.048	.03
.950	1.17	1.193	.631	1.133	.05
.90	1.56	1.372	.726	1.234	.10
.80	1.82	1.483	.785	1.186	.20
.70	2.08	1.59	.842	1.113	.30
.60	2.21	1.64	.868	0.985	.40
.40	2.60	1.78	.942	0.712	.60
.20	2.80	1.85	.979	0.370	.80
.00	2.93	1.89	1.000	0.00	1.00

Static Pressure = 32.2 Cm.Hg.Gauge

$V_{ave.}$ (weighing) = 1.515 ft.sec.

$V_{ave.}$ (Graphival Integration) = 1.526 ft./sec.

$$\therefore V_{ave.}/V_{max.} = \frac{1.526}{1.89} = 0.807$$

Run V H-34 (Counter Current) Station No. 5

(For corresponding temperature distribution, see Run T H-50)

$\frac{r}{R}$	$\frac{\Delta h}{(\text{CCm.CCL}_4)}$	$\frac{V}{(\text{ft./sec.})}$	$\frac{V}{V_{max.}}$	$\frac{v(r/R)}{(\text{ft./sec.})}$	(1-r/R)
0.991	0.80	0.990	0.518	0.981	0.009
.970	0.95	1.077	.564	1.045	.030
.950	1.15	1.184	.620	1.126	.050
.90	1.35	1.28	.670	1.152	.10
.80	1.60	1.394	.730	1.116	.20
.70	1.90	1.52	.796	1.064	.30
.60	2.30	1.676	.877	1.006	.40
.40	2.65	1.80	.943	0.720	.60
.20	2.90	1.880	.984	0.376	.80
.00	3.00	1.91	1.000	o.00	1.00

Static Pressure = 36.2 Cm.Hg.Gauge

$V_{ave.}$ (Weighing) = 1.515 ft./sec.

$V_{ave.}$ (Graphical Integration) = 1.510 ft./sec.

$\therefore V_{ave.}/V_{max.} = 1.51/1.91 = 0.791$

Run V H-35 (Counter Current) Station No. 4

(For corresponding temperature distribution, see Run T H-51)

$\frac{r}{R}$	$\frac{\Delta h}{(\text{CCm.CCL}_4)}$	$\frac{V}{(\text{ft./sec.})}$	$\frac{V}{V_{max.}}$	$\frac{v(r/R)}{(\text{ft./sec.})}$	(1-r/R)
0.990	1.70	1.436	0.600	1.421	0.010
.970	2.10	1.596	.667	1.547	.030
.950	2.30	1.676	.701	1.592	.050
.90	2.60	1.780	.745	1.602	.10
.80	3.20	1.976	.826	1.580	.20
.70	3.60	2.095	.876	1.468	.30
.60	3.80	2.155	.901	1.293	.40
.40	4.20	2.264	.947	0.906	.60
.20	4.50	2.340	.979	0.468	.80
.00	4.70	2.393	1.000	0.00	1.00

Static Pressure = 30.2 Cm.Hg.Gauge

$V_{ave.}$ (Graphical Integration) = 2.02 ft./sec.

$$\therefore V_{ave.}/V_{max.} = \frac{2.02}{2.393} = 0.844$$

Run V H-36 (Counter Current) Station No. 5

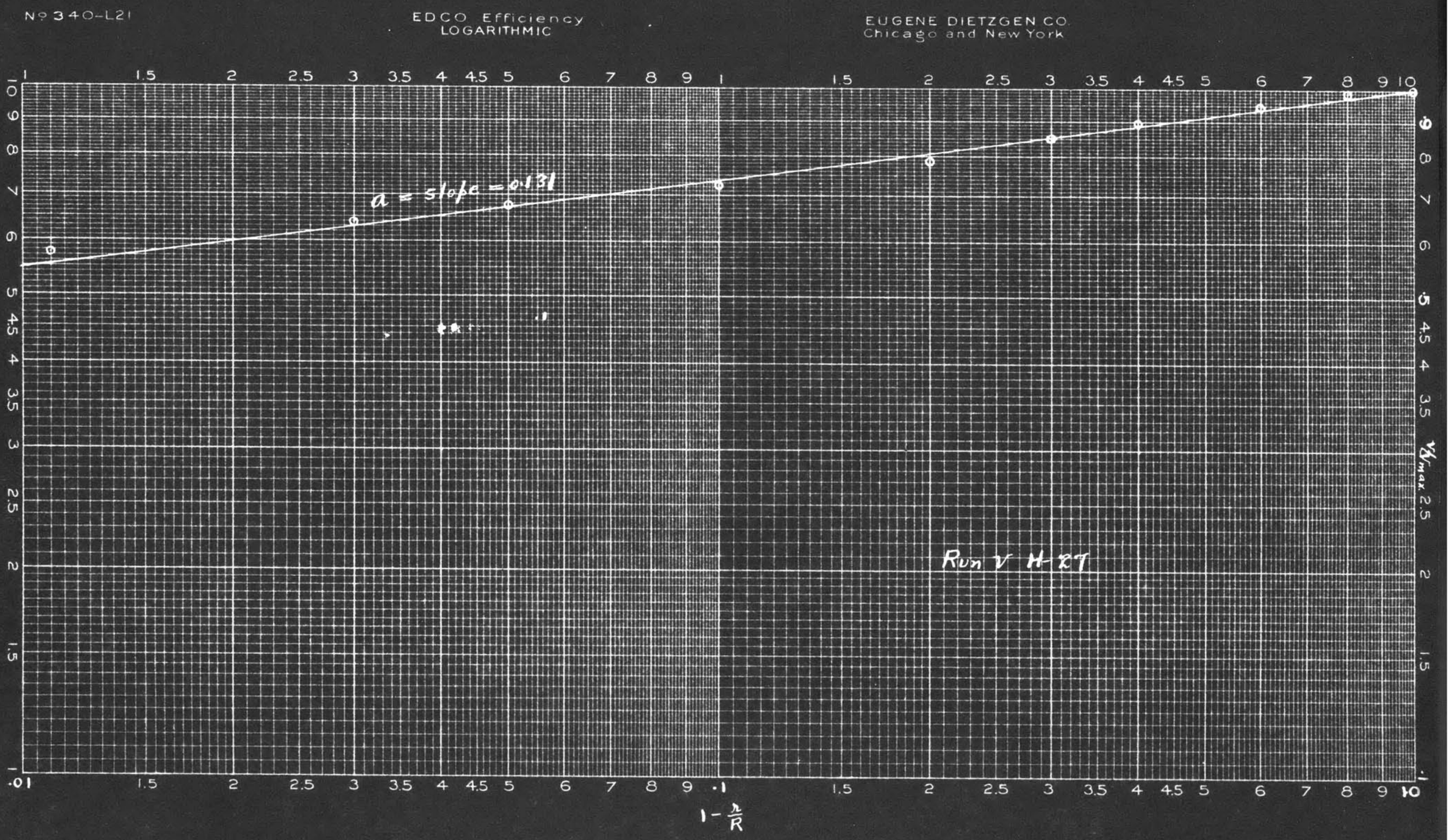
(For corresponding temperature distribution, see Run T H-52)

$\frac{r}{R}$	Δh (CCm.CCL_4)	V (ft./sec.)	$\frac{V}{V_{max.}}$	V(r/R) (ft./sec.)	(1-r/R)
0.990	1.80	1.48	0.612	1.467	.010
.970	2.00	1.56	.645	1.513	.030
.950	2.20	1.64	.678	1.558	.050
.90	2.50	1.746	.722	1.572	.10
.80	3.10	1.945	.804	1.557	.20
.70	3.60	2.095	.866	1.468	.30
.60	3.90	2.183	.904	1.310	.40
.40	4.30	2.29	.946	0.916	.60
.20	4.60	2.368	.979	0.473	.80
.00	4.80	2.42	1.000	0.00	1.00

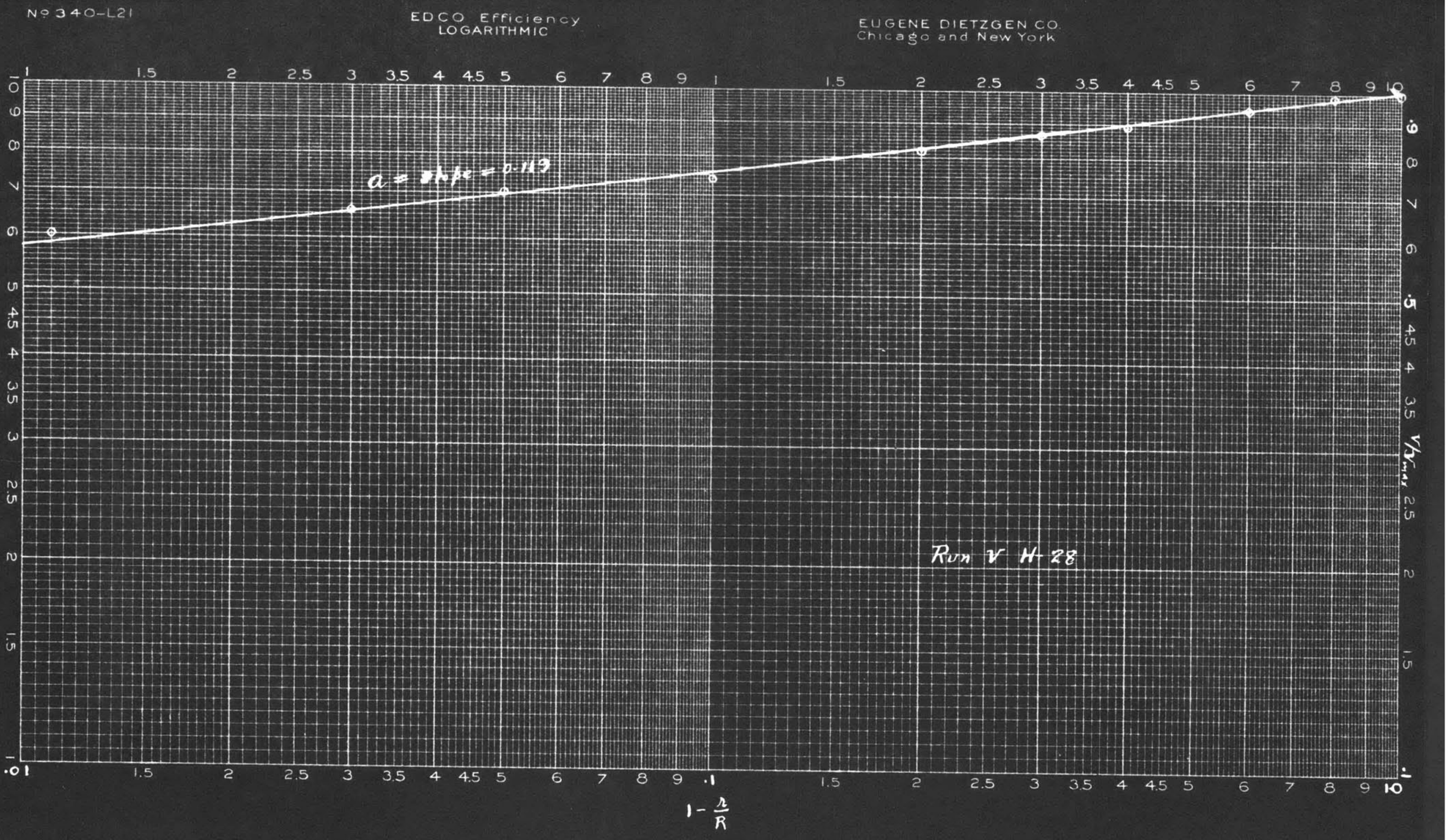
Static Pressure = 24.0 Cm.Hg.Gauge

$V_{ave.}$ (Graphical Integration) = 2.02 ft./sec.

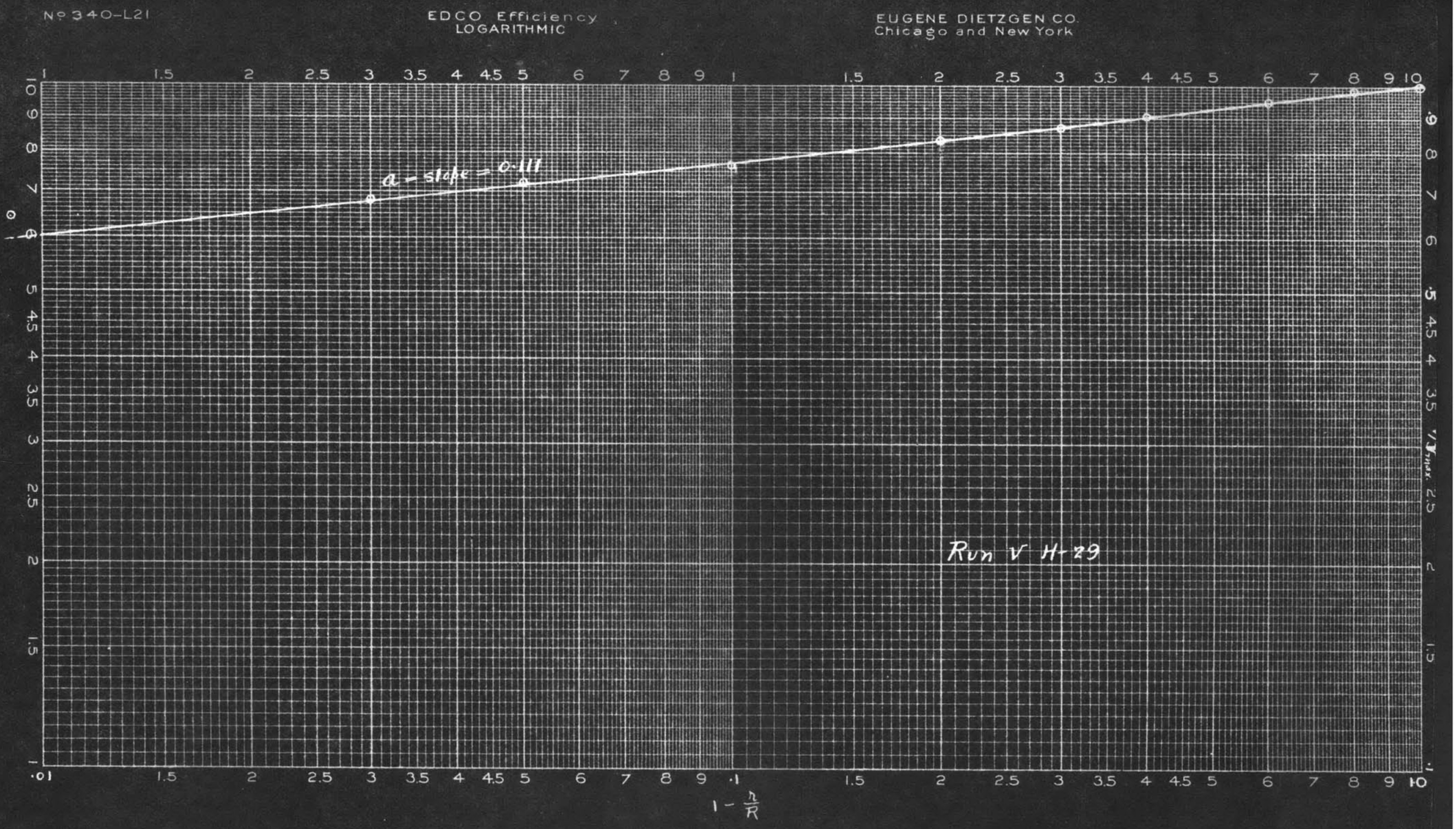
$\therefore V_{ave.}/V_{max.} = 2.02/2.42 = 0.835$



Nº 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
a = slope = 0.131
Run V H-27
1 - r/R
V/V_max
.01
.1
1.0

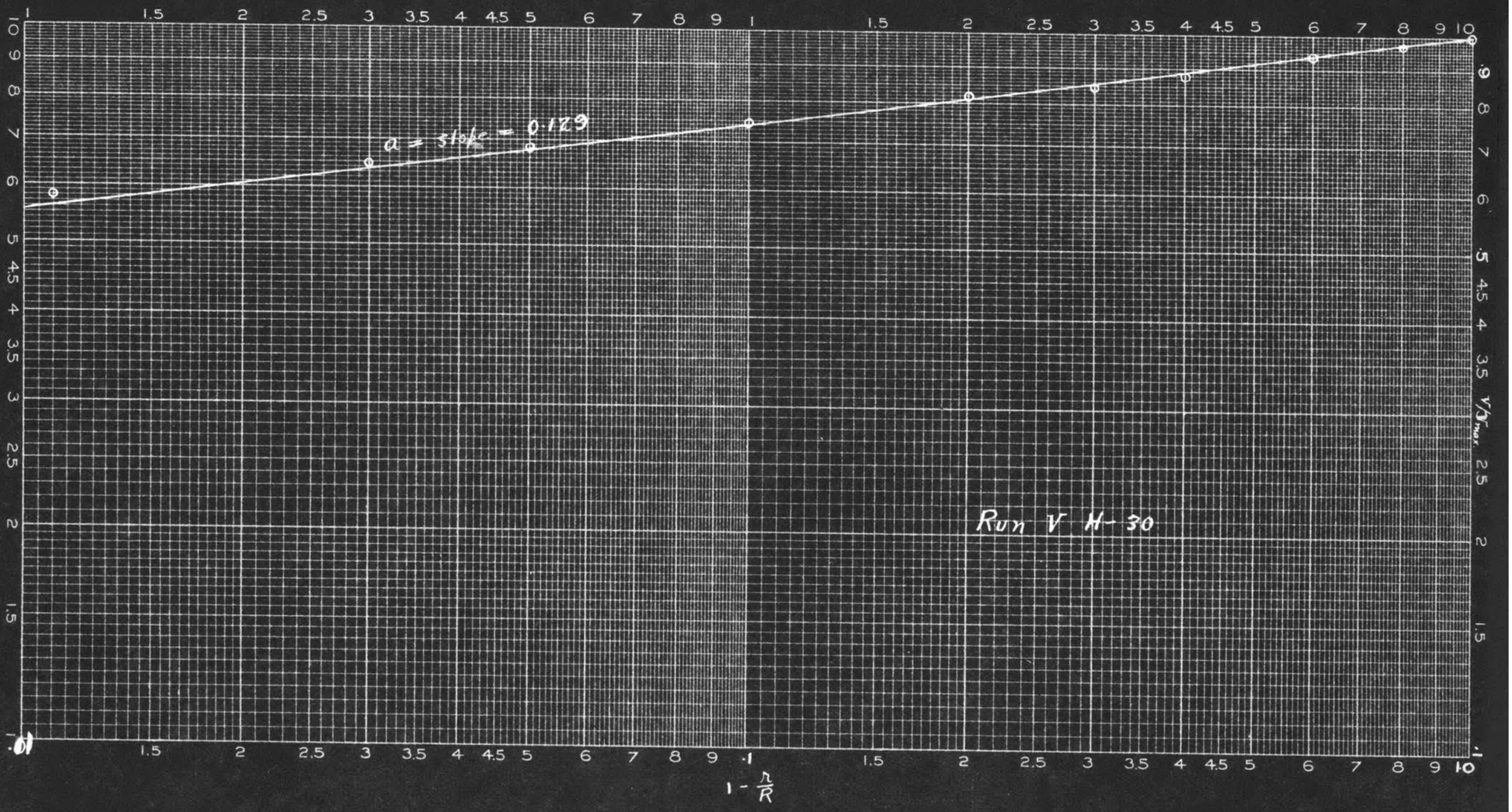


No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
a = slope = 0.119
Run V H-28
$1 - \frac{r}{R}$
V/V_{max}
.01
.1
1.0
.1
.5
.9
1
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
10



Nº 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
a - slope = 0·111
Run V H-29
1 - r/R
v/V max
·01
·1
1·0

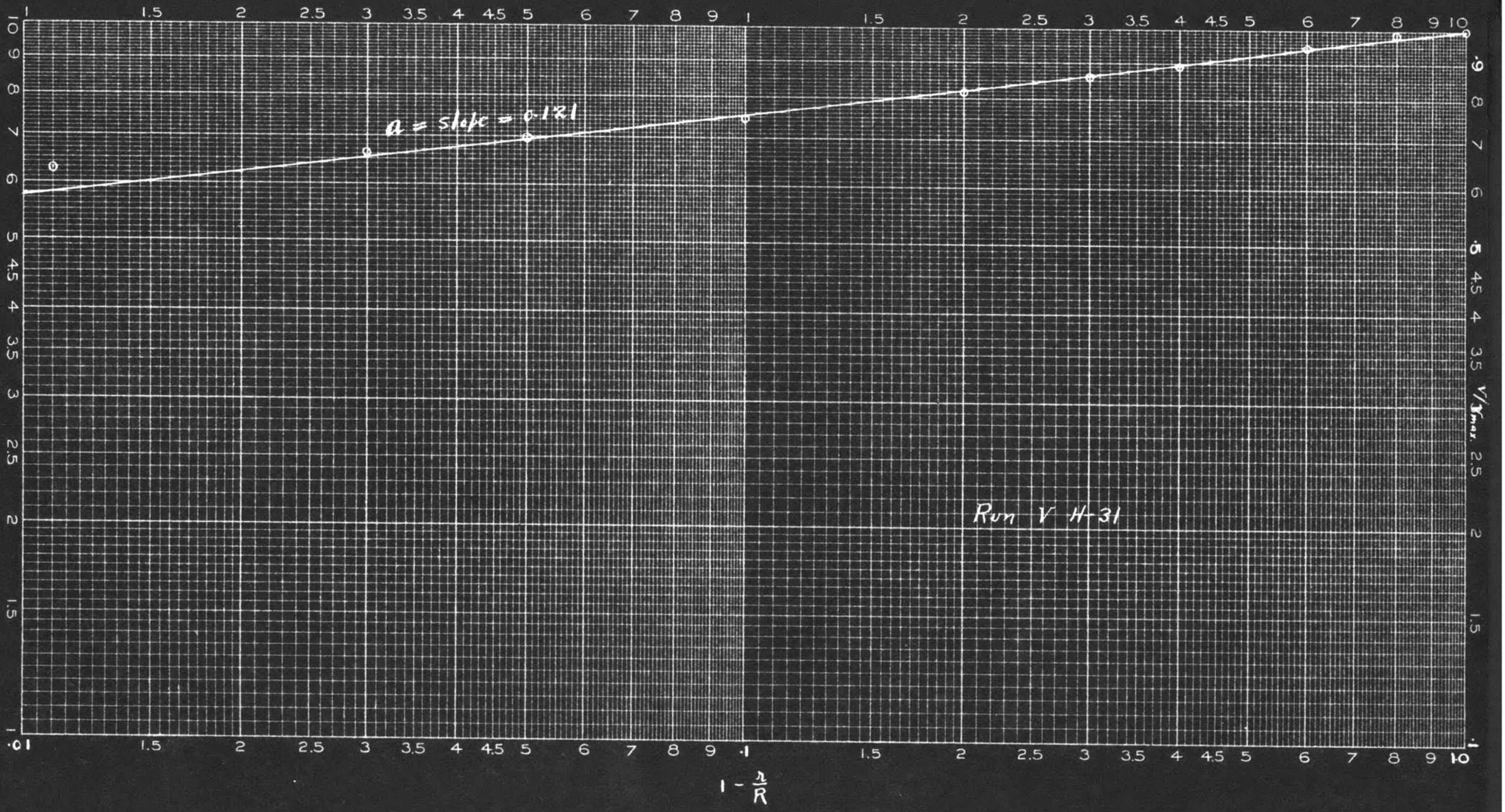
No 340-L21 EDCO Efficiency LOGARITHMIC EUGENE DIETZGEN CO. Chicago and New York

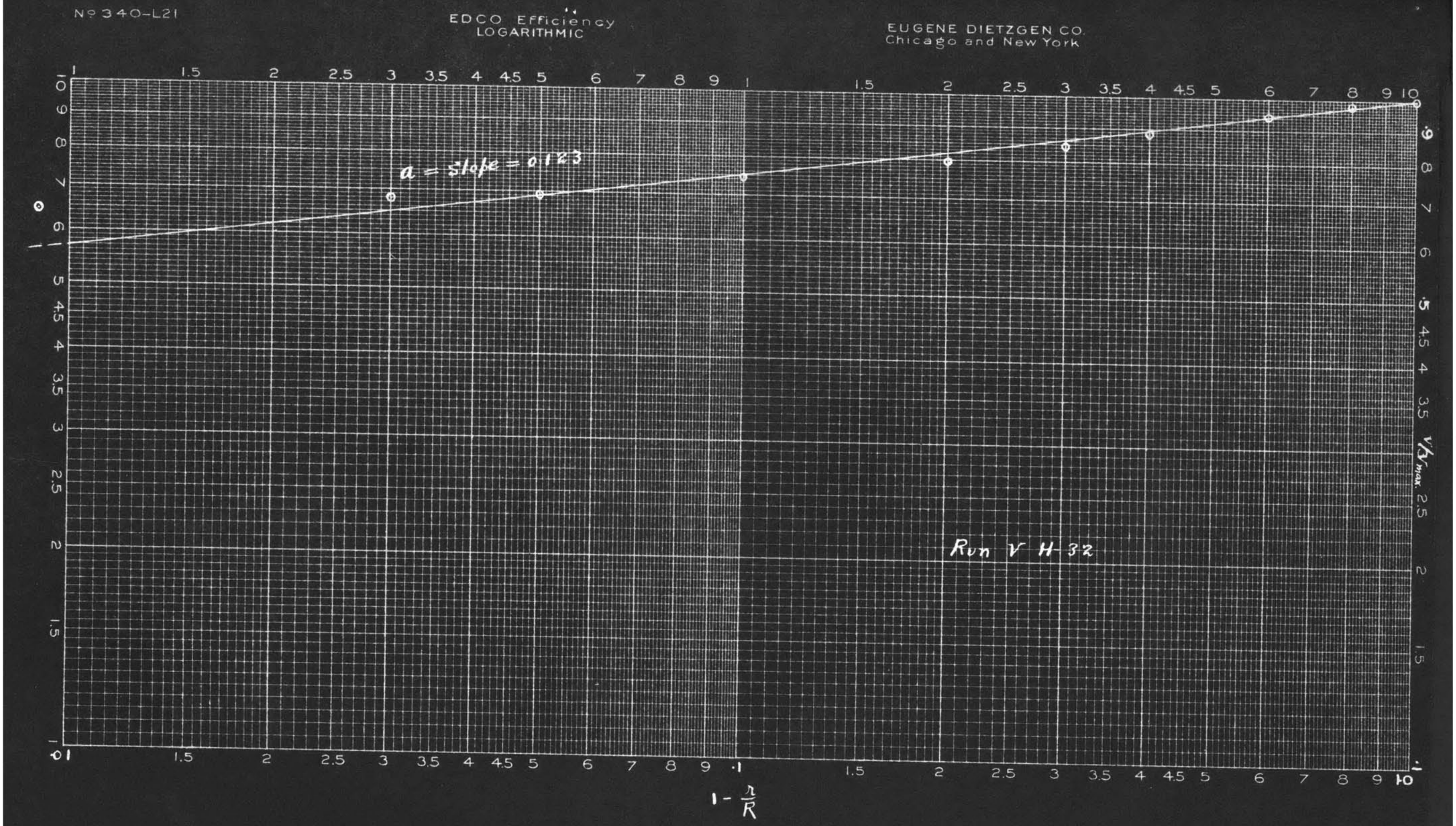


No 340-L21

EDCO Efficiency
LOGARITHMIC

EUGENE DIETZGEN CO.
Chicago and New York

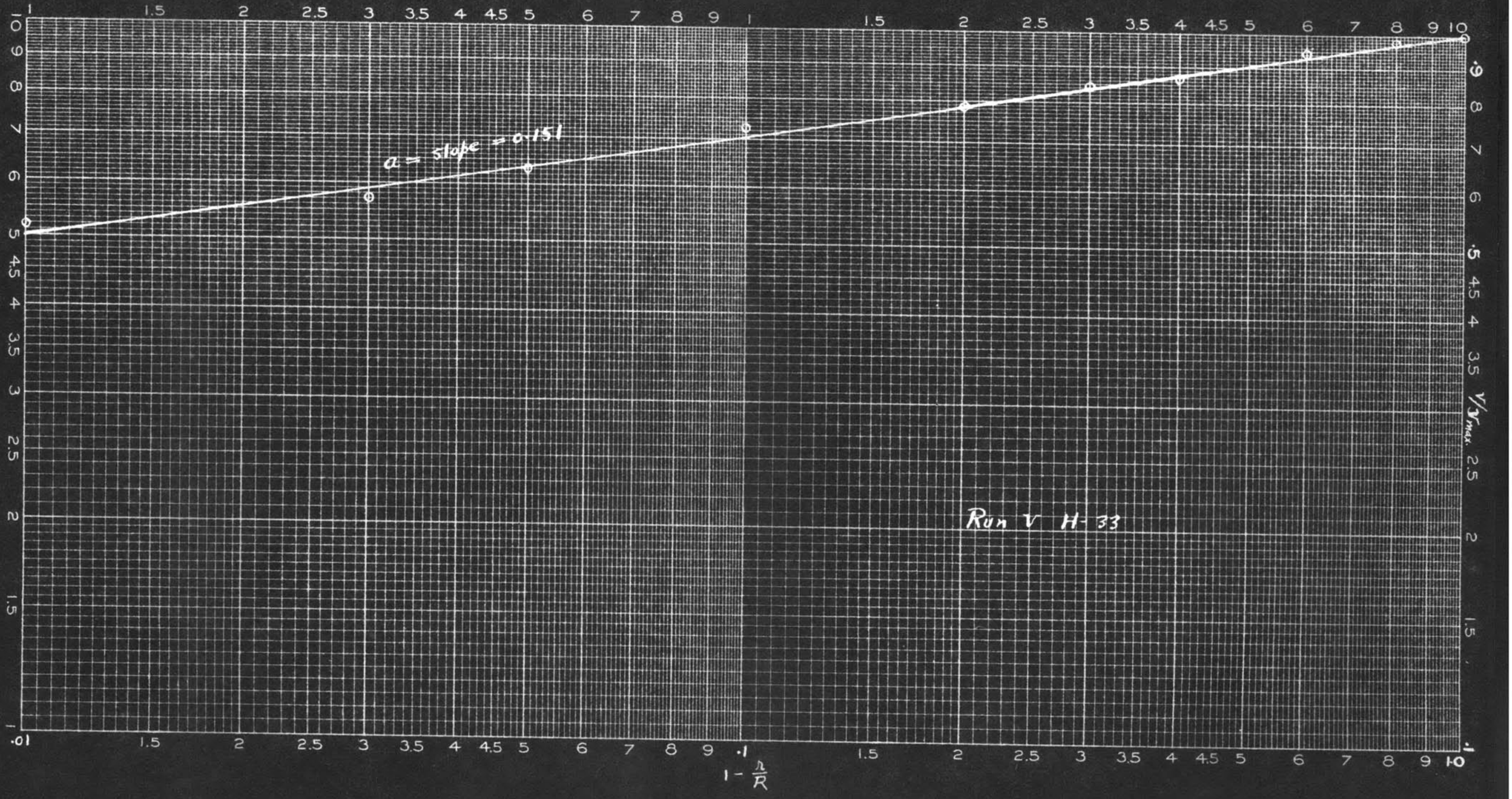



No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
a = slope = 0.123
Run V H-32
1 - r/R
V/V max.
·01
·1
1·0
·1
·5
·9

Nº 340-L21

EDCO Efficiency
LOGARITHMIC

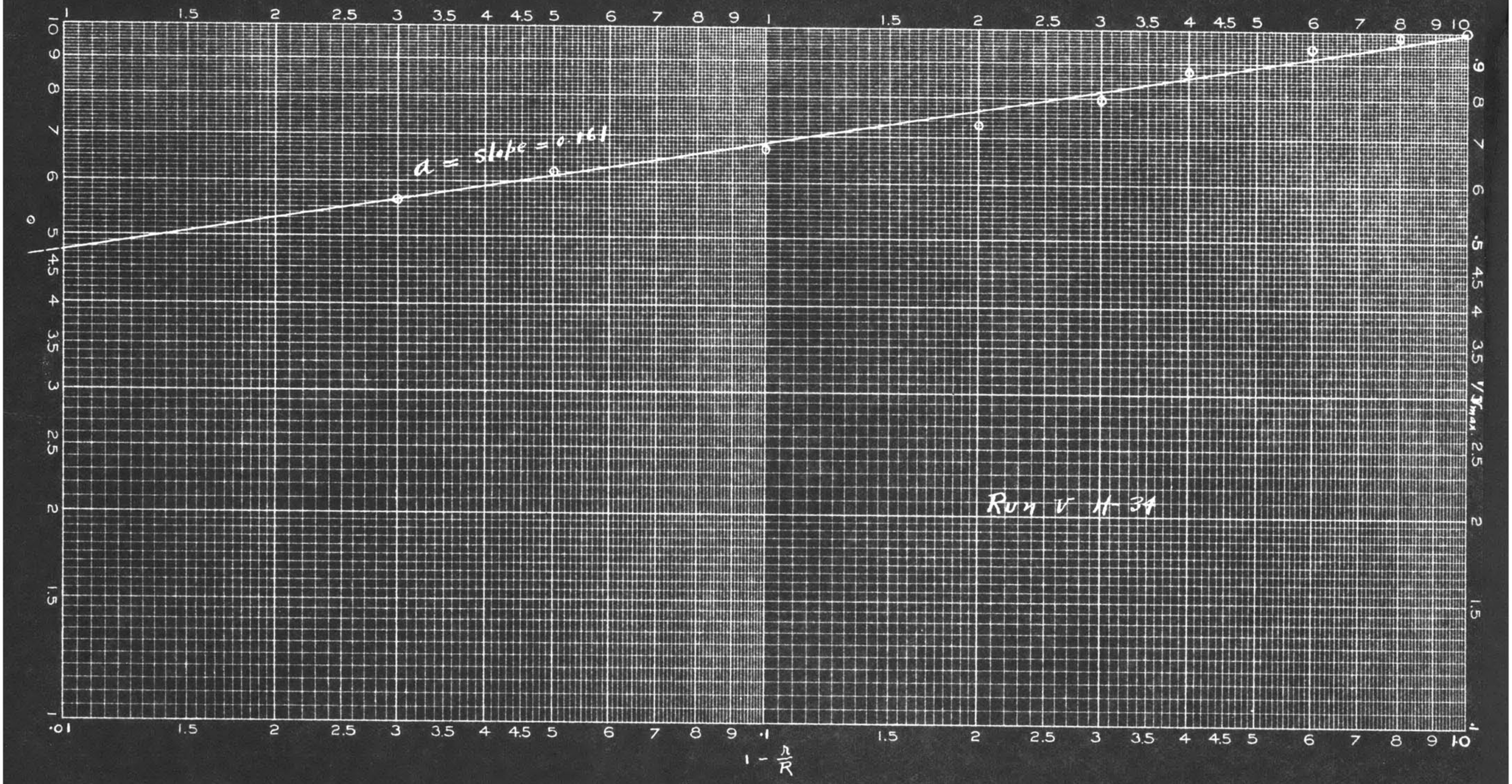
EUGENE DIETZGEN CO.
Chicago and New York



No 340-L21

EDCO Efficiency
LOGARITHMIC

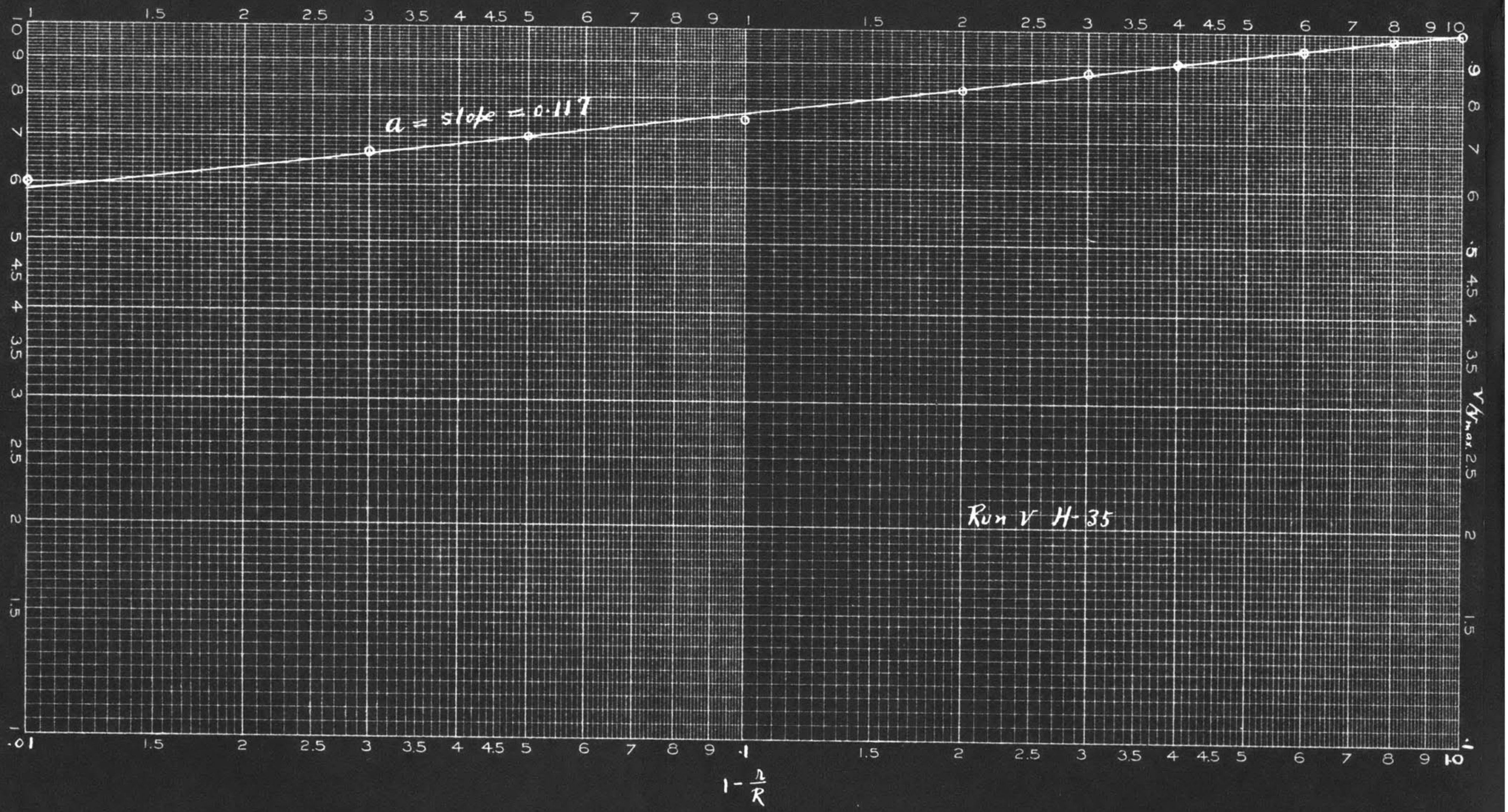
EUGENE DIETZGEN CO.
Chicago and New York

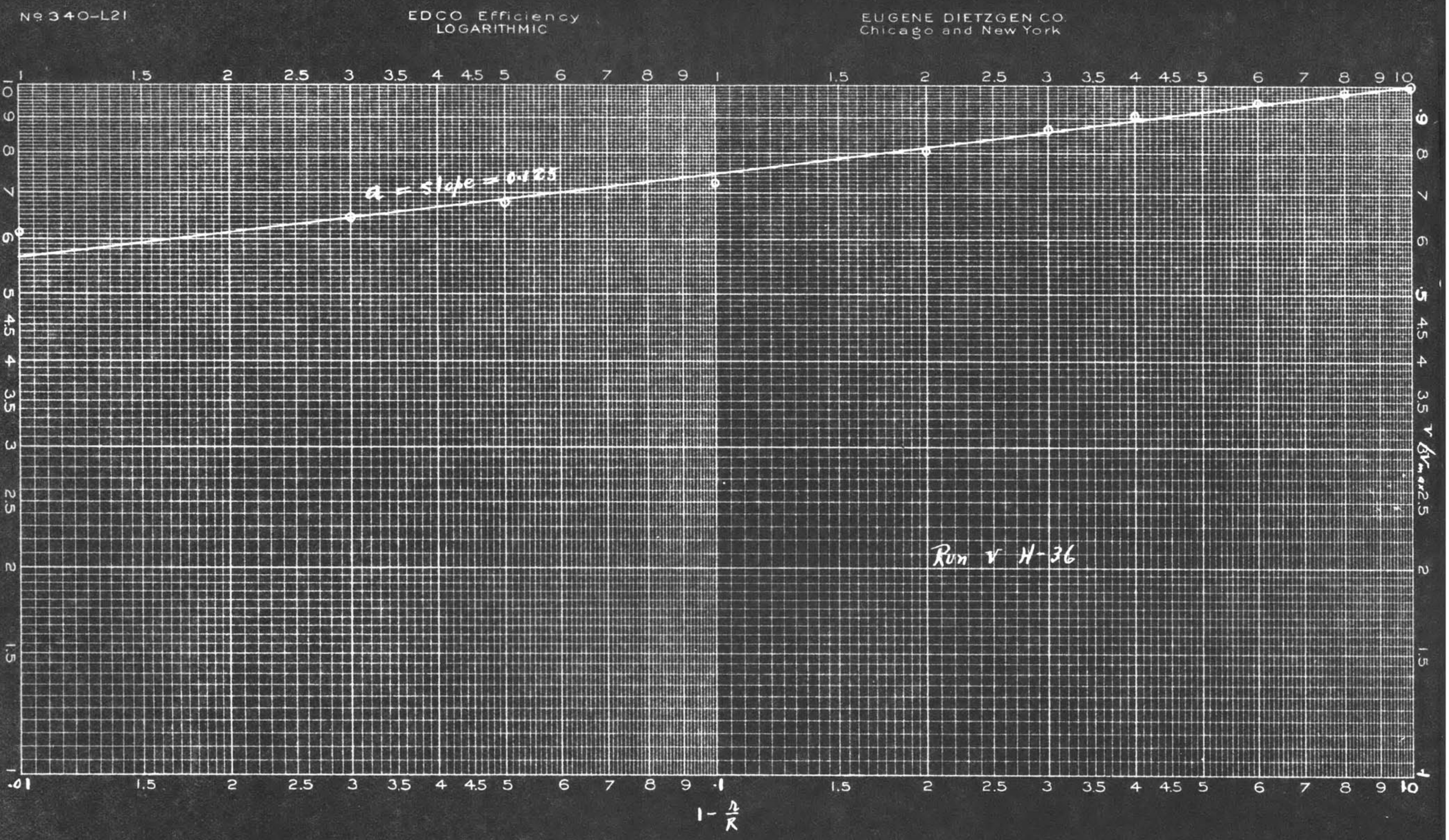


Nº 340-L21

EDCO Efficiency
LOGARITHMIC

EUGENE DIETZGEN CO
Chicago and New York





No 340-L21
EDCO Efficiency LOGARITHMIC
EUGENE DIETZGEN CO. Chicago and New York
a = slope = 0.125
Run V H-36
$1 - \frac{r}{R}$
v/v_{max}
.01
.1
10
.9
.5
1

APPENDIX G

TEMPERATURE DISTRIBUTION DATA DURING PARALLEL CURRENT HEATING WITH CALCULATIONS AND PLOTS

(Water Flowing Downward)

RUN T H-14 Station No. 2

(See V H-3 for Corresponding Vel. Distribution)

$\frac{r}{R}$	rdg. in mo.	t °C.	$\frac{t_w - t}{t_w - t_a}$	$t(\frac{r}{R})$ (°C)	$1-(\frac{r}{R})^2$	$1 - \frac{r}{R}$
1.000				(78.0)		
0.991	1.295	31.8	.819	31.5	0.0179	0.009
0.970	1.165	28.7	.874	27.8	0.0591	0.03
0.950	1.130	27.9	.888	26.5	0.0975	0.05
0.930	1.105	27.3	.899	25.4	0.1351	0.07
0.900	1.080	26.7	.910	24.0	0.1900	0.10
0.850	1.053	26.0	.922	22.1	0.2775	0.15
0.800	1.015	25.1	.938	20.1	0.3600	0.20
0.700	0.98	24.3	.952	17.0	0.5100	0.30
0.600	0.945	23.7	.962	14.2	0.6400	0.40
0.450	0.890	22.1	.991	10.0	0.798	0.55
0.300	0.870	21.6	1.000	6.5	0.910	0.70
0.150	0.870	21.6	1.000	3.2	0.978	0.85
0.000	0.870	21.6	1.000	0.0	1.000	1.00

t (outside wall) = 78.9°C.

V_{ave} (from graph) = 2.28 ft./sec.

$t_w - t_a$ = 56.4°C.

t_{ave} (from Graph) = 25.52°C.

t_m (Graphical integration) = 23.77°C.

RUN T H-16 Station No. 2

(For corresponding velocity distribution, See Run V H-5)

$\frac{r}{R}$	rdg. in mo.	t °C.	$\frac{t_w - t}{°C.}$	$\frac{t_w - t}{t_w - t_a}$	$t(\frac{r}{R})$ °C.	$(1 - \frac{r}{R})$
1.000					(70.0)	
0.991	1.020	25.3	44.7	0.915	25.0	0.009
0.970	0.995	24.7	45.3	0.926	24.0	0.030
0.950	0.985	24.4	45.6	0.933	23.2	0.050
0.930	0.970	24.1	45.9	0.940	22.4	0.070
0.90	0.960	23.9	46.1	0.943	21.5	0.10
0.85	0.945	23.5	46.5	0.951	20.0	0.15
0.80	0.930	23.1	46.9	0.960	18.5	0.20
0.70	0.895	22.25	47.75	0.977	15.6	0.30
0.60	0.885	22.0	48.0	0.981	13.2	0.40
0.45	0.865	21.5	48.5	0.992	9.7	0.55
0.30	0.860	21.4	48.6	0.994	6.4	0.70
0.15	0.855	21.25	48.75	0.997	3.2	0.85
0.00	0.850	21.15	48.85	1.000	0	1.00

$t_{o.w.}$ = 70.9 (assumed)

t_i = Inlet temperature = 20.4°C.

t_e = Exit temperature = 24.6°C.

t_w = 70.0°C.

$t_w - t_a$ = 48.85°C.

t_{ave} (Graphical integration) = 22.92°C.

t_m (Graphical integration) = 22.69°C.

RUN T H-18 Station No. 2

(See Run V H-8 for corresponding Vel. Distribution)

$\frac{r}{R}$	Rdg. in mo.	t °C.	$\frac{t_w-t}{t_w-t_a}$	$t(\frac{r}{R})$ (°C)	$1-(\frac{r}{R})^2$	$1-\frac{r}{R}$
1.000				(86.70)		
0.987	1.505	36.80	.787	36.30	0.0258	0.013
0.970	1.340	32.90	.848	31.90	0.0591	0.03
0.950	1.265	31.10	.876	29.60	0.0975	0.05
0.930	1.250	30.70	.883	28.60	0.1351	0.07
0.900	1.200	29.50	.902	26.60	0.1900	0.10
0.850	1.170	28.80	.913	24.50	0.2775	0.15
0.800	1.145	28.20	.922	22.60	0.3600	0.20
0.700	1.085	26.80	.944	18.80	0.5100	0.30
0.600	1.040	25.75	.960	15.45	0.6400	0.40
0.450	1.000	24.80	.975	11.17	0.7980	0.55
0.300	0.960	23.90	.989	7.17	0.9100	0.70
0.150	0.940	23.40	.997	3.51	0.9780	0.85
0.000	0.935	23.25	1.000	0.00	1.0000	1.00

t_w = 86.7°C.

t (outside wall) = 87.6°C.

$t_w - t_a$ = 63.45°C.

V_{ave} (from Graph) = 1.27 ft./sec.

t_{ave} (from Graph) = 28.56°C.

t_m (Graphical Integration) = 27.17°C.

t_i = 22.8°C.

t_e = 40.0°C.

RUN T H-19 Station No. 3

(See Run V H-9 for Corresponding Vel. Distribution)

$\frac{r}{R}$	Rdg. in mo.	t °C.	$\frac{t_w - t}{t_w - t_a}$	$t(\frac{r}{R})$ (°C)	$1-(\frac{r}{R})^2$	$1 - \frac{r}{R}$
1.000				(80.30)		
0.990	1.715	41.65	.813	41.30	0.0199	0.01
0.970	1.640	39.95	.849	38.80	0.0591	0.03
0.950	1.630	39.70	.854	37.70	0.0975	0.05
0.930	1.600	39.00	.869	36.30	0.1351	0.07
0.900	1.560	38.10	.888	34.30	0.1900	0.10
0.850	1.520	37.20	.907	31.60	0.2775	0.15
0.800	1.505	36.80	.915	29.40	0.3600	0.20
0.700	1.475	36.05	.930	25.20	0.5100	0.30
0.600	1.435	35.10	.951	21.05	0.6400	0.40
0.450	1.415	34.65	.960	15.60	0.7975	0.55
0.300	1.365	33.50	.985	10.06	0.9100	0.70
0.150	1.340	32.90	.997	4.94	0.9780	0.85
0.000	1.335	32.75	1.000	0.00	1.0000	1.00

t_w = 80.3°C.

t (outside wall) = 81.2°C.

$t_w - t_a$ = 47.55°C.

V_{ave} (from Graph) = 1.23 ft./sec.

t_{ave} (from Graph) = 37.6°C.

t_m (Graphical integration) = 36.11°C.

t_i = 22.8°C.

t_e = 40.0°C.

RUN T H-20 Station No. 2

(See Run V H-10 for corresponding Vel. Distribution)

$\frac{r}{R}$	Rdg. in mo.	t °C.	$\frac{t_w - t}{t_w - t_a}$	$t(\frac{r}{R})$ (°C)	$1 - (\frac{r}{R})^2$	$1 - \frac{r}{R}$
1.000				(74.2)		
0.987		27.75	.904	27.4	.0258	0.13
0.970		27.60	.908	26.75	.0591	0.03
0.950		27.50	.910	26.13	.0975	0.05
0.930		27.20	.915	25.30	.1351	0.07
0.900		27.00	.919	24.3	.1900	0.10
0.850		25.80	.942	21.9	.2775	0.15
0.800		25.35	.950	20.27	.3600	0.20
0.700		25.10	.956	17.58	.5100	0.30
0.600		24.35	.970	14.6	.6400	0.40
0.450		23.85	.979	10.74	.7975	0.55
0.300		22.90	.997	6.87	.9100	0.70
0.150		22.80	1.000	3.42	.9780	0.85
0.000		22.80	1.000	0	1.0000	1.00

t (outside wall) = 75.1°C.

t_w = 74.2°C.

t_i = 22.8°C.

t_e = 29.4°C.

$t_w - t_a$ = 51.4°C.

t_{ave} (Graphical Integration) = 25.85°C.

t_m (Graphical Integration) = 25.14°C.

RUN T H-21 Station No. 3

(For corresponding velocity distribution, See Run V H-11)

$\frac{r}{R}$	rdg. in mo.	t °C.	$t_w - t$ °C.	$\frac{t_w - t}{t_w - t_a}$	$t(\frac{r}{R})$ (°C)	$(1 - \frac{r}{R})$	$\frac{Vt(\frac{r}{R})}{(°C)(ft/sec.)}$
1.000					(65.0)		
0.990	1.360	33.3	31.7	0.834	33.0	0.01	
0.970	1.220	30.0	35.0	0.921	29.1	0.03	
0.950	1.215	29.9	35.1	0.924	28.4	0.05	
0.930	1.205	29.6	35.4	0.932	27.6	0.07	
0.90	1.200	29.5	35.5	0.934	26.6	0.10	
0.85	1.190	29.3	35.7	0.940	24.9	0.15	
0.80	1.175	28.9	36.1	0.950	23.1	0.20	
0.70	1.170	28.8	36.2	0.952	20.2	0.30	
0.60	1.155	28.4	36.6	0.963	17.0	0.40	
0.45	1.135	28.0	37.0	0.973	12.6	0.55	
0.30	1.115	27.5	37.5	0.986	8.3	0.70	
0.15	1.100	27.2	37.8	0.994	4.1	0.85	
0.00	1.095	27.0	38.0	1.000	0	1.00	

$t_{o.w.}$ = 65.89°C. (assumed)

t_w = 65.0°C.

$t_w - t_a$ = 38.0°C.

t_{ave} = 28.8°C. (Graphical Integration)

t_i = 22.8°C.

t_e = 29.4°C.

RUN T H-22 Station No. 2

(See Run V H-12 for Corresponding Velocity Distribution)

$\frac{r}{R}$	Rdg. in mo.	t °C.	$\frac{t_w-t}{t_w-t_a}$	$1-(\frac{r}{R})^2$	$(1-\frac{r}{R})$	$t(\frac{r}{R})$(°C)
1.000						(76.41)
0.987	1.245	30.6	0.839	0.0259	0.013	30.2
0.970	1.160	28.6	0.876	0.0591	0.03	27.7
0.950	1.130	27.8	0.891	0.0975	0.05	26.4
0.93	1.110	27.4	0.898	0.1351	0.07	25.5
0.90	1.100	27.15	0.902	0.1900	0.10	24.4
0.85	1.055	26.1	0.921	0.2775	0.15	22.2
0.80	1.030	25.75	0.928	0.36	0.20	20.6
0.70	1.000	24.8	0.945	0.51	0.30	17.4
0.60	0.970	24.1	0.958	0.64	0.40	14.5
0.45	0.925	23.0	0.977	0.798	0.55	10.4
0.30	0.895	22.0	0.995	0.910	0.70	6.6
0.15	0.890	21.85	1.000	0.978	0.85	3.3
0.00	0.890	21.85	1.000	1.000	1.00	0

$t_{o.w.}$ = 77.30°C.

t_w = 76.41°C. (temp. drop through pipe wall allowed = 0.89°C.)

$t_w - t_a$ = 54.56°C.

t_{ave} (Graphical Integration) = 25.92°C.

t_i = 23.04°C.

t_e = 31.9°C.

t_m = 25.00°C. (Graphical Integration)

RUN T H-23 Station No. 3

(For corresponding velocity distribution, see Run V H-13)

$\frac{r}{R}$	rdg. in mo.	t °C.	$t_w - t$ °C.	$\frac{t_w - t}{t_w - t_a}$	$t(\frac{r}{R})$ (°C)	$(1 - \frac{r}{R})$
1.000					(69.0)	
0.990	1.435	35.1	33.9	0.827	34.8	0.01
0.970	1.345	33.0	36.0	0.878	32.0	0.03
0.950	1.320	32.4	36.6	0.892	30.8	0.05
0.930	1.300	31.9	37.1	0.905	29.6	0.07
0.900	1.290	31.7	37.3	0.910	28.6	0.10
0.850	1.275	31.3	37.7	0.920	26.6	0.15
0.800	1.260	31.0	38.0	0.927	24.8	0.20
0.700	1.245	30.6	38.4	0.936	21.4	0.30
0.600	1.205	29.7	39.3	0.958	17.8	0.40
0.45	1.180	29.1	39.9	0.973	13.1	0.55
0.30	1.170	28.8	40.2	0.980	8.7	0.70
0.15	1.155	28.5	40.5	0.987	4.3	0.85
0.00	1.135	28.0	41.0	1.000	0	1.00

$t_{o.w.}$ = 69.89°C. (assumed)

t_w = 69°C.

$t_w - t_a$ = 41°C.

t_{ave} (Graphical integration) = 30.8°C.

t_m (Graphical integration) = 30.32°C.

t_i = 23.04°C.

t_e = 31.9°C.

RUN T H-24 Station No. 2

(For corresponding Velocity Distribution, See Run V H-14)

$\frac{r}{R}$	Rdg. in mo.	t °C.	$(t_w - t)$ (°C)	$\frac{t_w - t}{t_w - t_a}$	$t(\frac{r}{R})$ (°C)	$(1 - \frac{r}{R})$
1.000		(73.21)			(73.21)	
0.987	1.005	24.93	48.28	0.835	24.6	0.013
0.970	0.892	24.45	48.76	0.844	23.7	0.030
0.950	0.865	21.50	51.71	0.895	20.4	0.050
0.930	0.837	20.8	52.41	0.907	19.35	0.070
0.900	0.818	20.36	52.85	0.915	18.32	0.100
0.850	0.778	19.38	53.83	0.932	16.48	0.150
0.800	0.748	18.64	54.57	0.944	14.92	0.200
0.700	0.710	17.70	55.51	0.960	12.40	0.300
0.600	0.685	17.10	56.11	0.970	10.27	0.400
0.450	0.650	16.24	56.97	0.985	7.31	0.500
0.300	0.630	15.74	57.47	0.994	4.73	0.700
0.150	0.620	15.50	57.71	0.998	2.325	0.850
0.000	0.610	15.40	57.81	1.000	0.00	1.000

$t_{o.w.}$ = 74.10°C.

t_w = 74.10 - 0.89 = 73.21°C.

$t_w - t_a$ = 57.81°C.

t_{ave} (Graphical Integration) = 18.72°C.

t_i = 15.93°C.

t_e = 25.78°C.

RUN T H-25 Station No. 3

(For Corresponding Velocity Distribution, See Run V H-15)

$\frac{r}{R}$	Rdg. in mo.	t °C.	t_w-t °C	$\frac{t_w-t}{t_w-t_a}$	$t(\frac{r}{R})$ °C.	$(1-\frac{r}{R})$
1.000		(66.36)			(66.36)	
0.990	1.215	29.4	36.46	0.797	29.6	0.010
0.970	1.090	26.0	40.36	0.882	25.2	0.030
0.950	1.048	25.7	40.66	0.889	24.4	0.050
0.930	1.038	25.5	40.86	0.893	23.7	0.070
0.900	1.028	25.2	41.16	0.900	22.7	0.100
0.850	1.017	24.45	41.91	0.916	20.8	0.150
0.800	0.985	24.3	42.06	0.919	19.45	0.200
0.700	0.960	23.9	42.46	0.927	16.73	0.300
0.600	0.930	23.15	43.21	0.944	13.9	0.400
0.450	0.890	22.12	44.24	0.966	9.96	0.550
0.300	0.850	21.14	45.22	0.988	6.35	0.700
0.150	0.835	20.78	45.58	0.995	3.12	0.850
0.000	0.828	20.60	45.76	1.000	0.00	1.000

$t_{o.w.}$ = 67.25°C.

t_w = 67.25 - 0.89 = 66.36°C.

$t_w - t_a$ = 66.36 - 20.60 = 45.76°C.

t_{ave} (Graphical Integration) = 24.28°C.

t_i = 15.93°C.

t_e = 25.78°C.

RUN T H-26 Station No. 2

(For Corresponding Velocity Distribution, See Run V H-16)

$\frac{r}{R}$	Rdg. in mo.	t °C.	$t_w - t$ °C.	$\frac{t_w - t}{t_w - t_a}$	$t(\frac{r}{R})$	$(1 - \frac{r}{R})$
1.000		(77.11)			(77.11)	
0.987	1.005	24.9	52.21	0.851	24.6	0.013
0.970	0.940	23.4	53.71	0.876	22.7	0.030
0.950	0.886	22.0	55.11	0.899	20.9	0.050
0.930	0.860	21.4	55.71	0.909	19.9	0.070
0.900	0.848	21.1	56.01	0.914	19.0	0.100
0.850	0.790	19.65	57.46	0.935	16.7	0.150
0.800	0.760	18.90	58.21	0.950	15.1	0.200
0.700	0.730	18.18	58.93	0.960	12.7	0.300
0.600	0.700	17.47	59.64	0.972	10.5	0.400
0.450	0.670	16.74	60.37	0.985	7.54	0.550
0.300	0.640	15.98	61.13	0.996	4.80	0.700
0.150	0.630	15.73	61.38	1.000	2.36	0.850
0.000	0.630	15.73	61.38	1.000	0.00	1.000

Outside wall temperature = 78.0°C.

t_w = 78.0 - 0.89 = 77.11°C.

$t_w - t_a$ = 61.38°C.

t_{ave} (Graphical Integration) = 19.36°C.

t_i = 16.06°C.

t_o = 27.20°C.

RUN T H-27 Station No. 3

(For Corresponding Velocity Distribution, See Run V H-17)

$\frac{r}{R}$	Rdg. in mo.	t °C.	t_w-t (°C)	$\frac{t_w-t}{t_w-t_a}$	$t(\frac{r}{R})$ (°C)	$(1-\frac{r}{R})$
1.000		(71.51)			(71.51)	
0.990	1.453	35.6	35.91	0.728	35.2	0.010
0.970	1.155	28.45	43.06	0.872	27.6	0.030
0.950	1.122	27.70	43.81	0.888	26.3	0.050
0.930	1.100	27.15	44.36	0.900	25.25	0.070
0.900	1.095	27.00	44.51	0.902	24.3	0.100
0.850	1.075	26.55	44.96	0.910	22.55	0.150
0.800	1.058	26.20	45.31	0.918	20.95	0.200
0.700	1.018	25.20	46.31	0.938	17.63	0.300
0.600	0.978	24.30	47.21	0.957	14.60	0.400
0.450	0.936	23.30	48.21	0.976	10.50	0.550
0.300	0.901	22.40	49.11	0.995	6.72	0.700
0.150	0.891	22.15	49.36	1.000	3.32	0.850
0.000	0.891	22.15	49.36	1.000	0.00	1.000

Outside pipe wall temperature = 72.4°C.

$t_w = 72.4 - 0.89 = 71.51$°C.

$t_w - t_a = 49.36$°C.

$t_{ave} = 25.52$°C. (Graphical Integration)

$t_i = 16.06$°C.

$t_e = 27.20$°C.

RUN T H-28 Station No. 2

(For corresponding velocity distribution, See Run V H-18)

$\frac{r}{R}$	Rdg. in mo.	t °C.	$t_w - t$ °C.	$\frac{t_w - t}{t_w - t_a}$	$t(\frac{r}{R})$ (°C)	$(1 - \frac{r}{R})$
1.000		(76.01)			(76.01)	
0.987	0.938	23.35	52.66	0.856	23.05	0.013
0.970	0.820	20.40	55.61	0.905	19.80	0.030
0.950	0.795	19.8	56.21	0.914	18.8	0.050
0.930	0.780	19.4	56.61	0.920	18.04	0.070
0.900	0.760	18.9	57.11	0.929	17.0	0.100
0.850	0.743	18.5	57.51	0.935	15.7	0.150
0.800	0.695	17.3	58.71	0.955	13.85	0.200
0.700	0.663	16.56	59.45	0.966	11.6	0.300
0.600	0.640	16.0	60.01	0.975	0.96	0.400
0.450	0.610	15.2	60.81	0.988	0.684	0.550
0.300	0.590	14.8	61.21	0.995	0.444	0.700
0.150	0.585	14.6	61.41	0.998	0.219	0.850
0.000	0.580	14.5	61.51	1.000	0.000	1.000

Outside pipe wall Temperature = $t_{o.w.}$ = 76.9 °C.

t_w = 76.9 - 0.89 = 76.01°C.

$t_w - t_a$ = 61.51°C.

t_{ave} (Graphical integration) = 17.6°C.

t_i = 14.12°C.

t_e = 24.3°C.

RUN T H-29

Station No. 3

(For corresponding velocity distribution, See Run V H-19)

$\frac{r}{R}$	Rdg. in mo.	t °C.	$t_w - t$ (°C)	$\frac{t_w - t}{t_w - t_a}$	$t(\frac{r}{R})$	$(1 - \frac{r}{R})$
1.000		(66.91)			(66.91)	
0.990	1.040	25.7	41.21	0.851	25.40	0.010
0.970	0.900	22.4	44.51	0.918	21.73	0.030
0.950	0.895	22.25	44.66	0.922	21.1	0.050
0.930	0.893	22.18	44.73	0.923	20.6	0.070
0.900	0.890	22.1	44.81	0.925	19.9	0.100
0.850	0.885	22.0	44.91	0.9265	18.7	0.150
0.800	0.880	21.86	45.05	0.929	17.5	0.200
0.700	0.875	21.75	45.16	0.932	15.22	0.300
0.600	0.855	21.27	45.64	0.941	12.77	0.400
0.450	0.805	20.0	46.91	0.967	9.00	0.550
0.300	0.775	19.3	47.61	0.983	5.79	0.700
0.150	0.760	18.92	47.99	0.990	2.84	0.850
0.000	0.740	18.43	48.48	1.000	0.00	1.000

$t_{o.w.}$ = 67.8°C.

t_w = 66.91°C.

$t_w - t_a$ = 48.48°C.

t_{ave} (Graphical Integration) = 21.60°C.

t_i = 14.12°C.

t_e = 24.3°C.

RUN T H-30 Station No. 2

(For corresponding velocity distribution, See Run V H-20)

$\frac{r}{R}$	Rdg. in mo.	t °C	t_w-t	$\frac{t_w-t}{t_w-t_a}$	$t(\frac{r}{R})$ (°C)	$(1-\frac{r}{R})$
1.000		(78.41)			(78.41)	
0.987	0.787	19.6	58.81	0.806	19.33	0.013
0.970	0.615	15.4	63.01	0.864	14.93	0.030
0.950	0.555	13.9	64.51	0.884	13.2	0.050
0.900	0.485	12.21	66.21	0.907	10.98	0.100
0.800	0.430	10.85	67.56	0.926	8.68	0.200
0.700	0.380	9.60	68.81	0.942	6.72	0.300
0.600	0.320	8.06	70.35	0.964	4.84	0.400
0.450	0.265	6.67	71.74	0.983	3.00	0.550
0.300	0.240	6.05	72.36	0.992	1.815	0.700
0.000	0.215	5.40	73.01	1.000	0.00	1.000

$t_{o.w.}$ = 79.3°C.

t_w = 78.41

t_w-t_a = 73.01°C.

t_{ave} (Graphical Integration) = 10.48°C.

t_i = 5.0°C.

t_e = 21.77°C.

RUN T H-31 Station No. 3

(For corresponding velocity distribution, See Run V H-21)

$\frac{r}{R}$	Rdg. in mo.	t °C.	$t_w - t$ °C.	$\frac{t_w - t}{t_w - t_a}$	$t(\frac{r}{R})$ (°C)	$(1 - \frac{r}{R})$
1.000		(76.01)			(76.01)	
0.990	0.942	23.4	52.61	0.855	23.2	0.010
0.970	0.843	20.96	55.05	0.895	20.3	0.030
0.950	0.805	20.0	56.01	0.910	19.0	0.050
0.900	0.805	20.0	56.01	0.910	19.0	0.100
0.800	0.790	19.64	56.37	0.916	15.72	0.200
0.700	0.740	18.43	57.58	0.930	12.9	0.300
0.600	0.705	17.55	58.46	0.950	10.54	0.400
0.450	0.660	16.50	59.51	0.968	7.43	0.550
0.300	0.620	15.5	60.51	0.984	4.65	0.700
0.000	0.580	14.5	61.51	1.000	0.00	1.000

$t_{o.w.} = 76.9$°C.

$t_w = 76.01$°C.

$t_w - t_a = 61.51$

t_{ave} (Graphical integration) = 18.4°C.

$t_i = 5.0$°C.

$t_e = 21.77$°C.

RUN T H-32 Station No. 2

(For corresponding velocity distribution, See Run V H-22)

$\frac{r}{R}$	Rdg. in mo.	t °C.	$t_w - t$ °C.	$\frac{t_w - t}{t_w - t_a}$	$t(\frac{r}{R})$ (°C)	$(1 - \frac{r}{R})$
1.000		(75.03)			(75.03)	
0.987	0.745	18.54	56.49	0.801	18.3	0.013
0.970	0.575	14.38	60.65	0.860	13.95	0.030
0.950	0.535	13.40	61.63	0.874	12.73	0.050
0.900	0.445	11.20	63.83	0.905	10.08	0.100
0.800	0.365	9.20	65.83	0.933	7.36	0.200
0.700	0.320	8.07	66.96	0.950	5.65	0.300
0.600	0.260	6.54	68.49	0.970	3.92	0.400
0.450	0.210	5.28	69.75	0.989	2.375	0.550
0.300	0.200	5.00	70.03	0.992	1.500	0.700
0.000	0.180	4.50	70.53	1.000	0.00	1.000

$t_{o.w.}$ = 75.92°C.

t_w = 75.03

$t_w - t_a$ = 70.53°C.

t_{ave} (Graphical Integration) = 9.2°C.

t_i = 3.75°C.

t_e = 21.13°C.

RUN T H -33 Station No. 3

(For corresponding velocity distribution, See Run V H-23)

$\frac{r}{R}$	Rdg. in mo.	t °C.	$t_w - t$ °C.	$\frac{t_w - t}{t_w - t_a}$	$t(\frac{r}{R})$ (°C)	$(1 - \frac{r}{R})$
1.000		(73.91)			(73.91)	
0.990	0.875	21.76	52.15	0.859	21.53	0.010
0.970	0.780	19.4	54.51	0.898	18.8	0.030
0.950	0.770	19.18	54.73	0.901	18.2	0.050
0.900	0.745	18.55	55.36	0.912	16.7	0.100
0.800	0.700	17.45	56.46	0.930	13.97	0.200
0.700	0.655	16.36	57.55	0.948	11.46	0.300
0.600	0.645	16.10	57.81	0.952	9.66	0.400
0.450	0.595	14.88	59.03	0.972	6.70	0.550
0.300	0.540	13.52	60.39	0.994	4.05	0.700
0.000	0.525	13.16	60.75	1.000	0.00	1.000

$t_{o.w.}$ = 74.80°C.

t_w = 73.91°C.

$t_w - t_a$ = 60.75°C.

t_{ave} (Graphical Integration) = 17.12°C.

t_i = 3.75°C.

t_e = 21.13°C.

RUN T H-34A Station No. 2

(No corresponding Velocity Distribution Run)

$\frac{r}{R}$	Rdg. in mo.	t °C.	$t_w - t$ °C.	$\frac{t_w - t}{t_w - t_a}$	$t(\frac{r}{R})$ (°C)	$(1 - \frac{r}{R})$
1.000		(79.01)			(79.01)	
0.987	0.695	17.32	61.69	0.823	17.1	0.013
0.970	0.5035	12.64	66.37	0.885	12.27	0.030
0.950	0.470	11.80	67.21	0.896	11.2	0.050
0.900	0.410	10.32	68.69	0.916	9.29	0.100
0.800	0.330	8.30	70.71	0.943	6.64	0.200
0.700	0.260	6.54	72.47	0.965	4.58	0.300
0.600	0.230	5.78	73.23	0.976	3.47	0.400
0.450	0.210	5.30	73.71	0.983	2.385	0.550
0.300	0.180	4.50	74.51	0.993	1.35	0.700
0.000	0.160	4.00	75.01	1.000	0.00	1.000

$t_{o.w.}$ = 79.9°C.

t_w = 79.01°C.

$t_w - t_a$ = 75.01°C.

t_{ave} (Graphical Integration) = 8.40°C.

t_i = 3.88°C.

t_e = 19.47°C.

RUN T H-34 Station No. 3

(For Corresponding Velocity Distribution, See Run V H-24)

$\frac{r}{R}$	Rdg. in mo.	t °C.	$t_w - t$ °C.	$\frac{t_w - t}{t_w - t_a}$	$t(\frac{r}{R})$ (°C)	$(1 - \frac{r}{R})$
1.000		(74.71)			(74.71)	
0.990	0.780	19.40	55.31	0.878	19.20	0.010
0.970	0.695	17.32	57.39	0.911	16.8	0.030
0.950	0.675	16.85	57.86	0.920	16.0	0.050
0.900	0.640	16.00	58.71	0.932	14.4	0.100
0.800	0.620	15.50	59.21	0.940	12.4	0.200
0.700	0.580	14.50	60.21	0.955	10.15	0.300
0.600	0.550	13.78	60.93	0.967	8.28	0.400
0.450	0.520	13.04	61.67	0.980	5.87	0.550
0.300	0.490	12.30	62.41	0.991	3.69	0.700
0.000	0.465	11.70	63.01	1.000	0.00	1.000

$t_{o.w.}$ = 75.6°C.

t_w = 74.71°C.

$t_w - t_a$ = 63.01°C.

t_{ave} (Graphical Integration) = 15.28°C

t_i = 3.88°C.

t_e = 19.47°C.

RUN T H-35 Station No. 2

(For corresponding Velocity Distribution, See Run V H-25)

$\frac{r}{R}$	Rdg. in mo.	t °C.	$t_w - t$ °C.	$\frac{t_w - t}{t_w - t_a}$	$t(\frac{r}{R})$ (°C)	$(1 - \frac{r}{R})$
1.000		(73.21)			(73.21)	
0.987	0.555	13.9	59.31	0.852	13.72	0.013
0.970	0.445	11.2	62.01	0.891	10.87	0.030
0.950	0.410	10.35	62.86	0.904	9.84	0.050
0.900	0.350	8.82	64.39	0.925	7.94	0.100
0.800	0.280	7.05	66.16	0.951	5.64	0.200
0.700	0.225	5.67	67.54	0.970	3.97	0.300
0.600	0.200	5.00	68.21	0.980	3.00	0.400
0.450	0.165	4.12	69.09	0.992	1.855	0.550
0.300	0.150	3.74	69.47	0.998	1.122	0.700
0.000	0.145	3.60	69.61	1.000	0.000	1.000

$t_{o.w.}$ = 74.1°C.

t_w = 73.21°C.

$t_w - t_a$ = 69.61°C.

t_{ave} (Graphical Integration) = 7.40°C.

t_i = 3.36°C.

t_g = 15.2°C.

RUN T H-36 Station No. 3

(For Corresponding Velocity Distribution, See Run V H-26)

$\frac{r}{R}$	Rdg. in mo.	t °C.	$t_w - t$ (°C)	$\frac{t_w - t}{t_w - t_a}$	$t(\frac{r}{R})$ (°C)	$(1 - \frac{r}{R})$
1.000		(65.71)			(65.71)	
0.990	0.620	15.5	50.21	0.885	15.35	0.010
0.970	0.555	13.9	51.81	0.913	13.48	0.030
0.950	0.520	13.04	52.67	0.930	12.40	0.050
0.900	0.500	12.54	53.17	0.938	11.30	0.100
0.800	0.470	11.8	53.91	0.950	9.44	0.200
0.700	0.445	11.2	54.51	0.961	7.84	0.300
0.600	0.420	10.6	55.11	0.971	6.36	0.400
0.450	0.405	10.25	55.46	0.977	4.61	0.550
0.300	0.380	9.57	56.14	0.990	2.87	0.700
0.000	0.355	8.97	56.74	1.000	0.00	1.000

$t_{o.w.} = 66.6$°C.

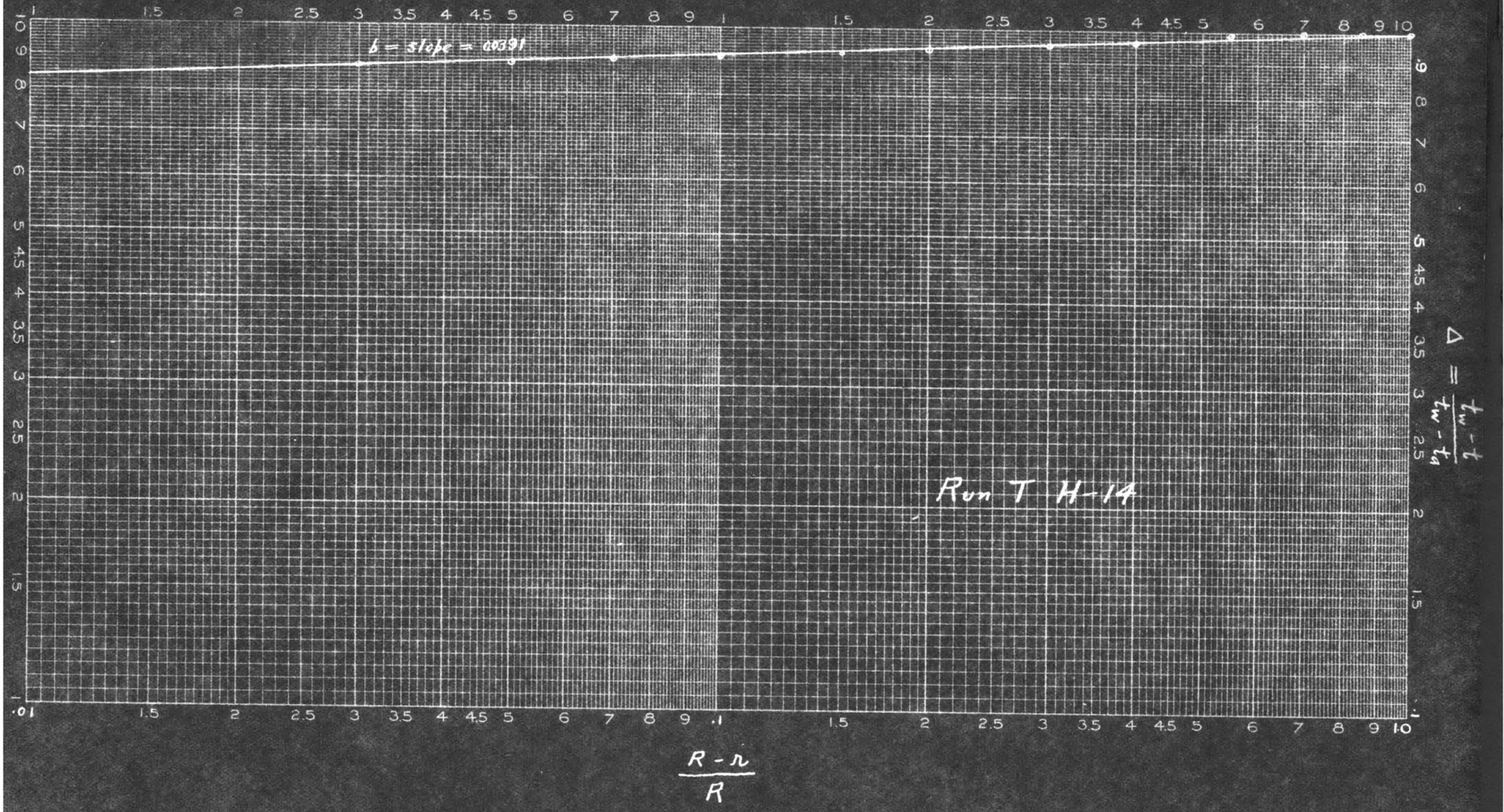
$t_w = 65.71$°C.

$t_w - t_a = \Delta t_{max} = 56.74$°C.

t_{ave} (Graphical Integration) = 12.3°C.

$t_i = 3.36$°C.

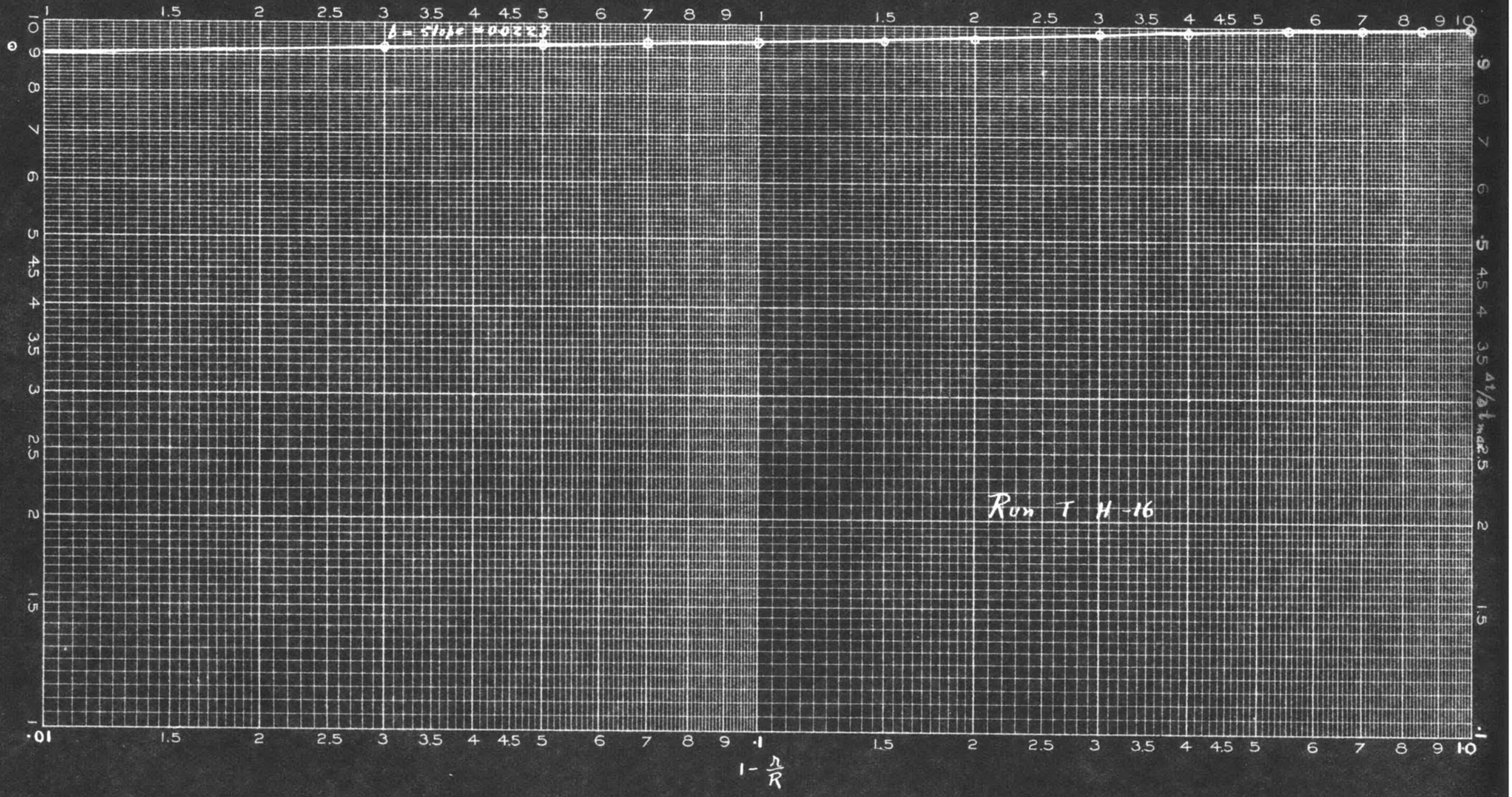
$t_e = 15.2$°C.

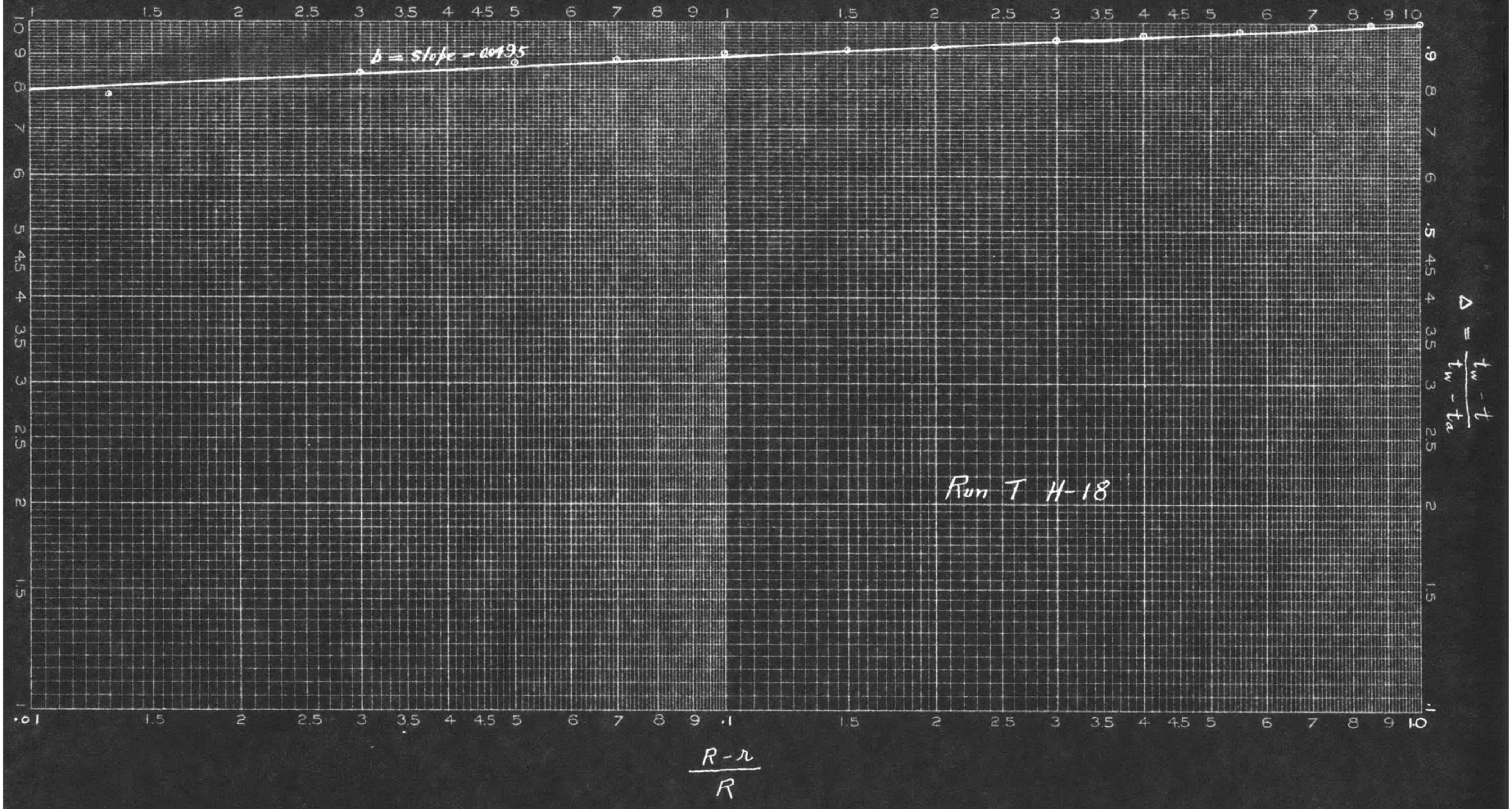

b = slope = 0.0391
Run T H-14
Δ = $\frac{t_w - t}{t_w - t_a}$
$\frac{R - r}{R}$
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.9

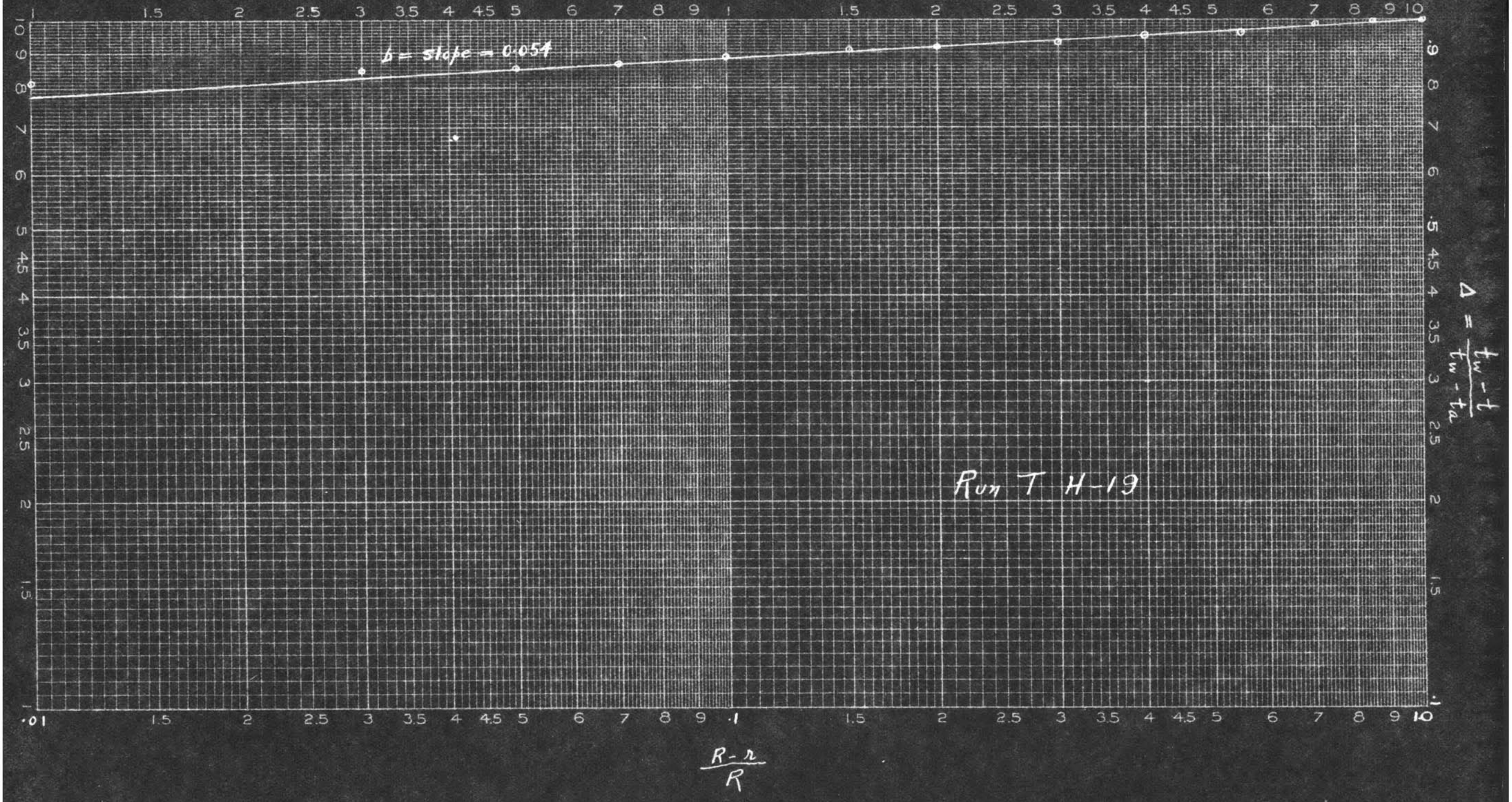
Nº 340-L21

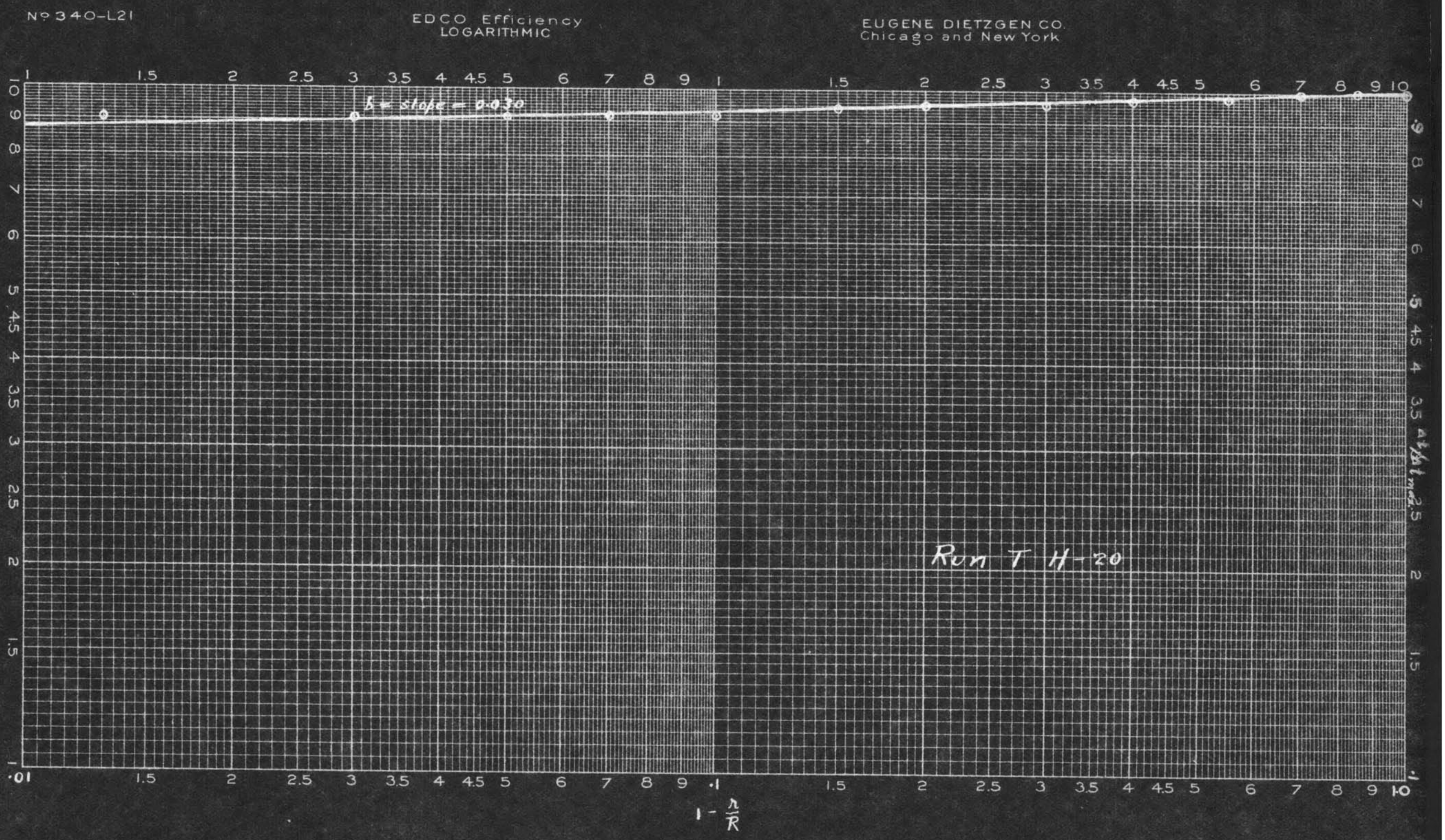
EDCO Efficiency
LOGARITHMIC

EUGENE DIETZGEN CO.
Chicago and New York

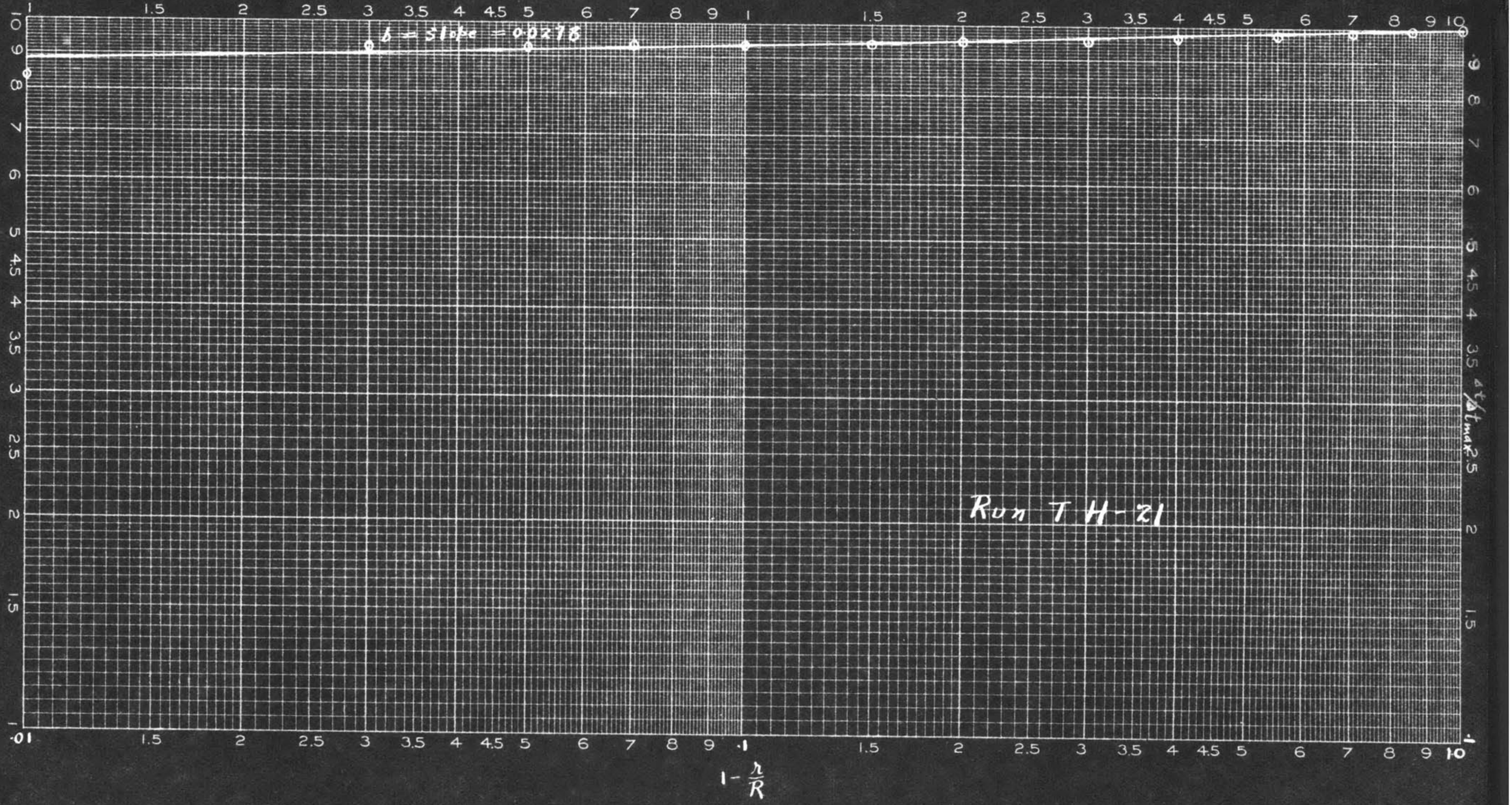


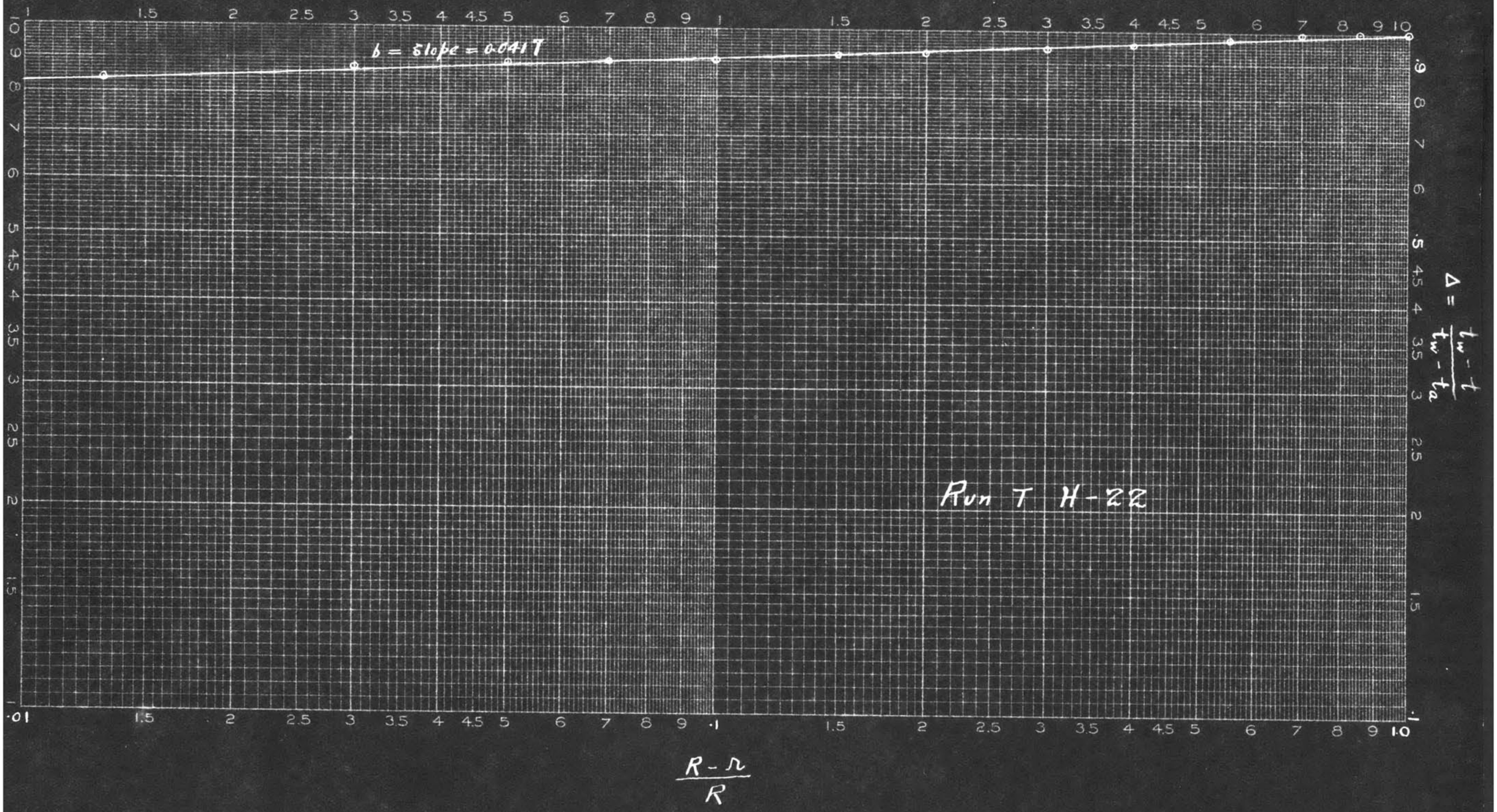

b = slope = 0.0495
Run T H-18
$\Delta = \frac{t_w - t}{t_w - t_a}$
$\frac{R - r}{R}$
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1

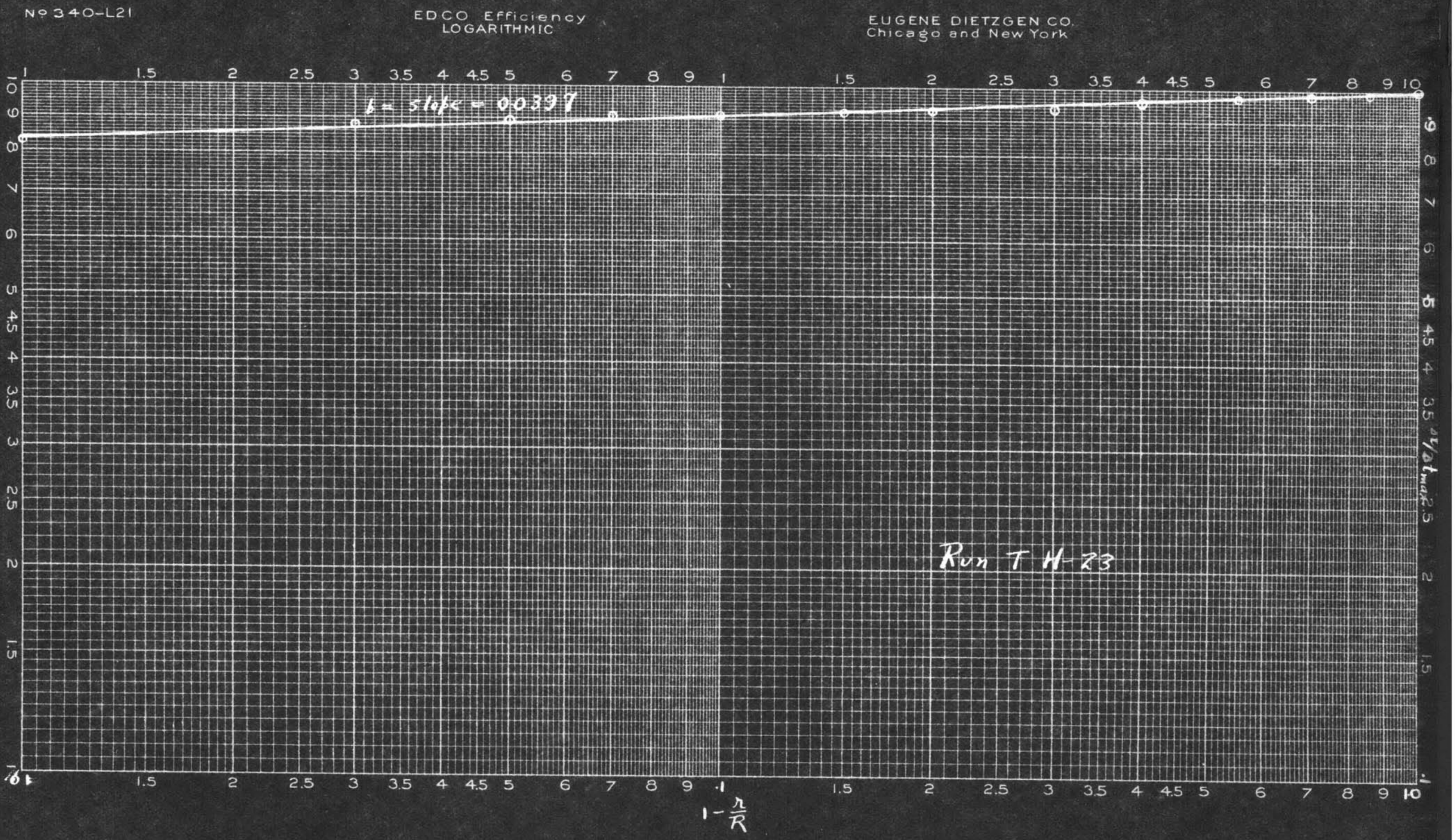
b = slope = 0.054
Run T H-19
Δ = (tw - t)/(tw - ta)
(R - r)/R
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
10
.9
.8
.7
.6
.5
.45
.4
.35
.3
.25
.2
.15
.1

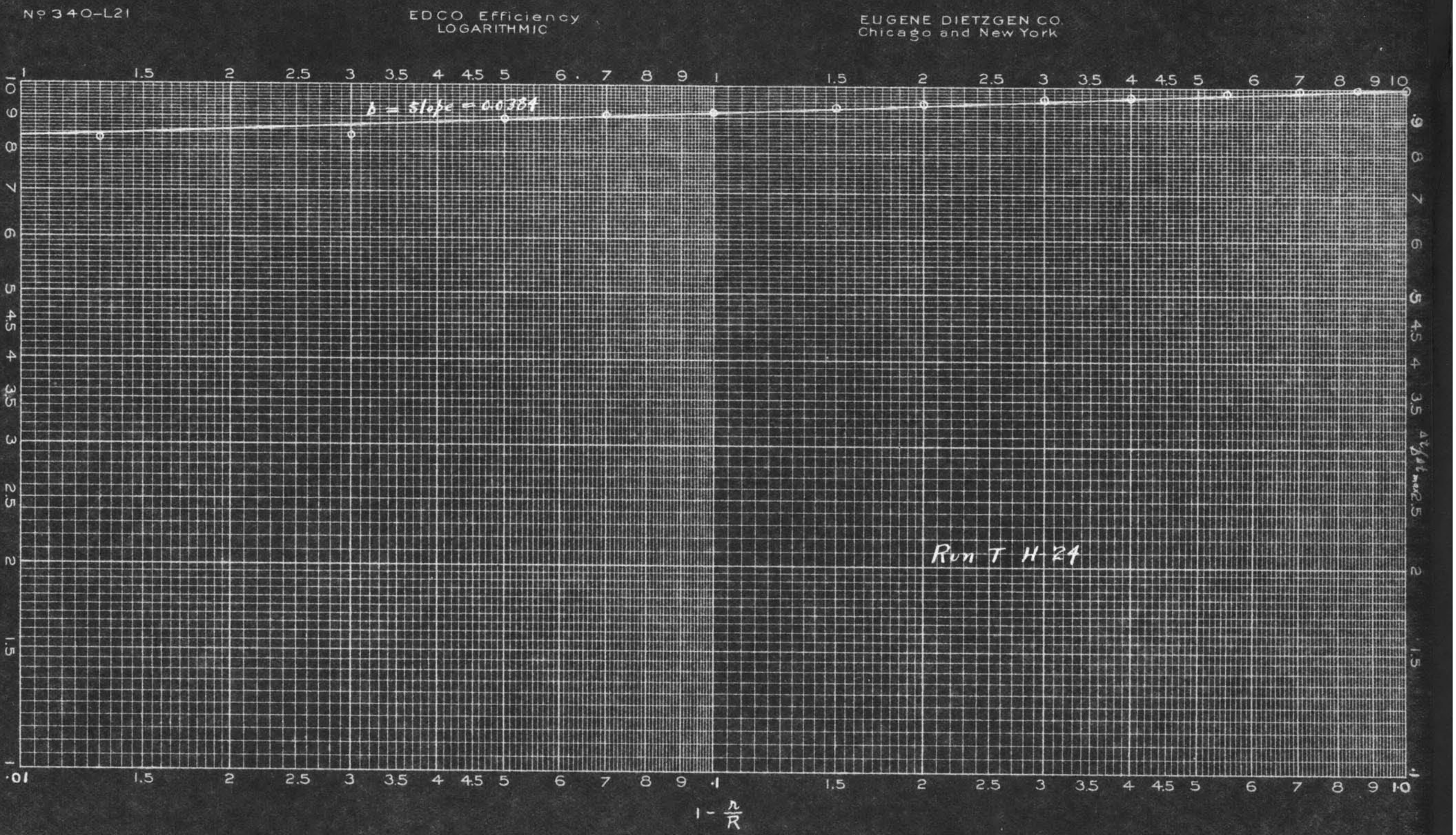
No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
b = slope = 0.030
Run T H-20
$1 - \frac{r}{R}$
a_t/a_{tm}
·01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
·1
1·0
·9
·8
·7
·6
·5
·4
·3
·2

No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York




b = slope = 0.0417
Run T H-22
$\Delta = \frac{t_w - t}{t_w - t_a}$
$\frac{R - r}{R}$
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.9
.5

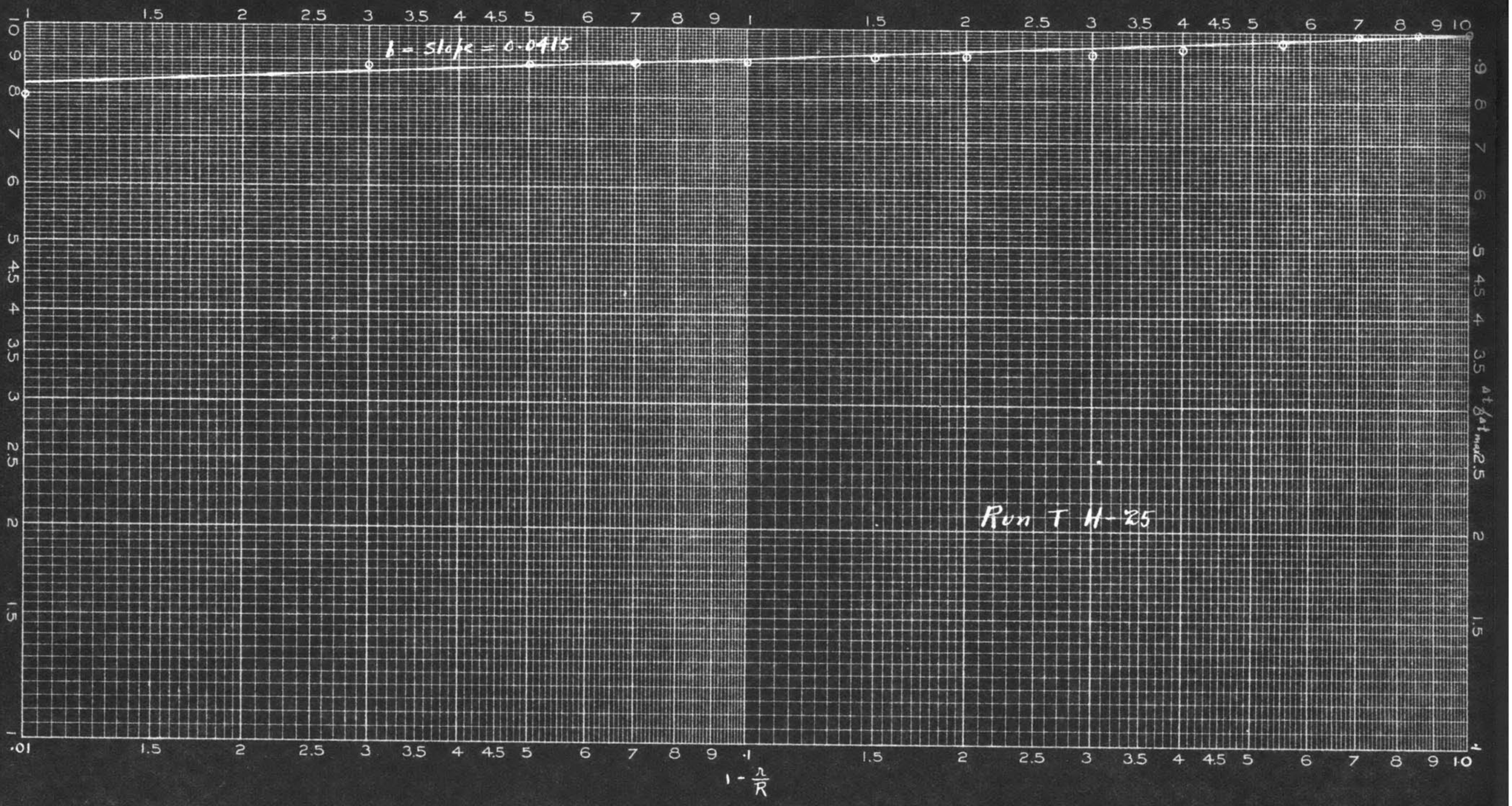

No 340-L21
EDCO Efficiency LOGARITHMIC
EUGENE DIETZGEN CO. Chicago and New York
b = slope = 0.0397
Run T H-23
1 - r/R
Δv/Δt max
.01
.1
1.0
.9
.8
.7
.6
.5
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9


Nº 340-L21
EDCO Efficiency LOGARITHMIC
EUGENE DIETZGEN CO. Chicago and New York
b = slope = 0.0384
Run T H-24
$\Delta t / \Delta t_{max}$
$1 - \frac{r}{R}$
·01
·1
1·0
·9
·5
·1

Nº 340-L21

EDCO Efficiency
LOGARITHMIC

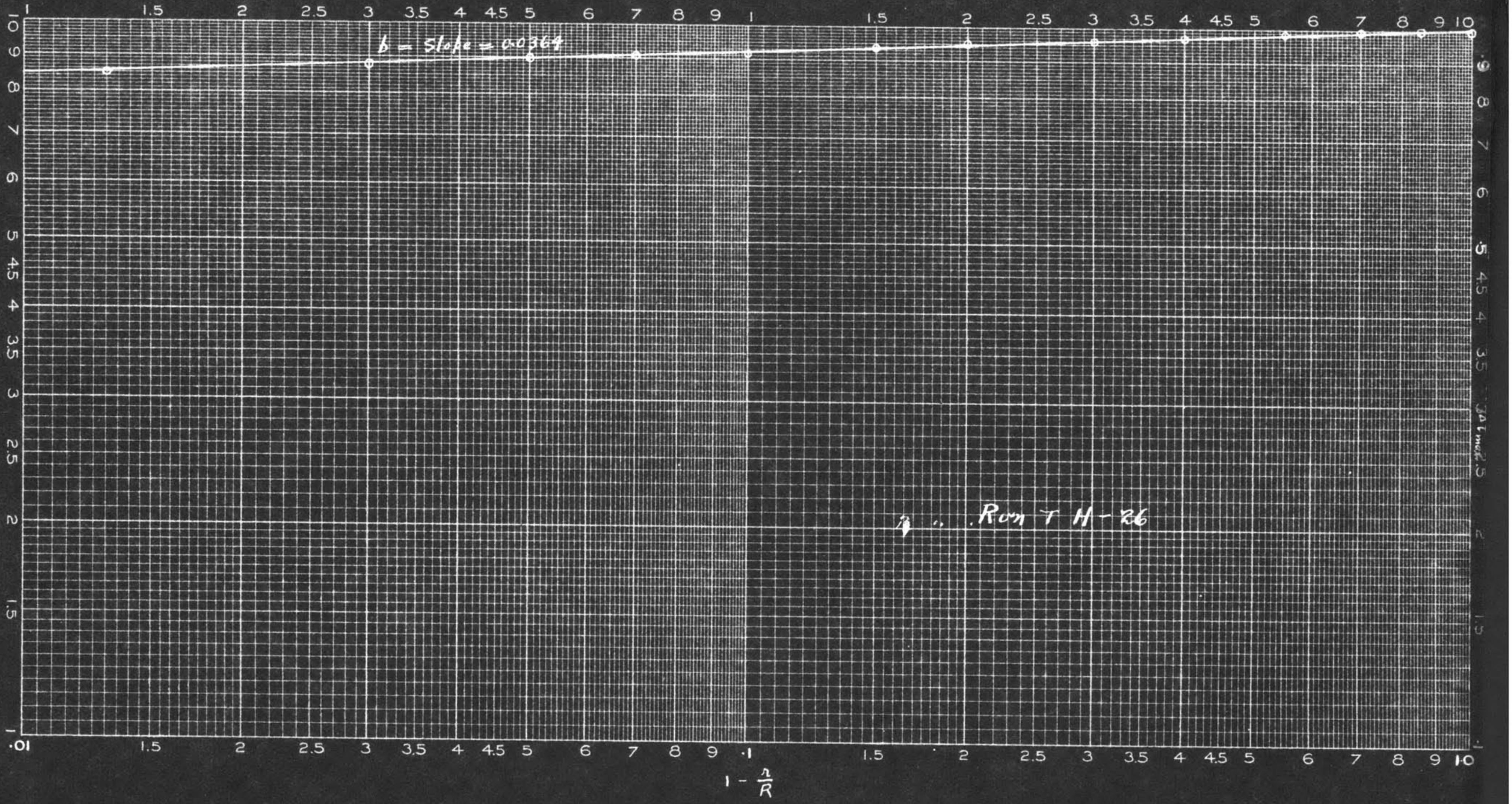
EUGENE DIETZGEN CO.
Chicago and New York



No 340-L21

EDCO Efficiency
LOGARITHMIC

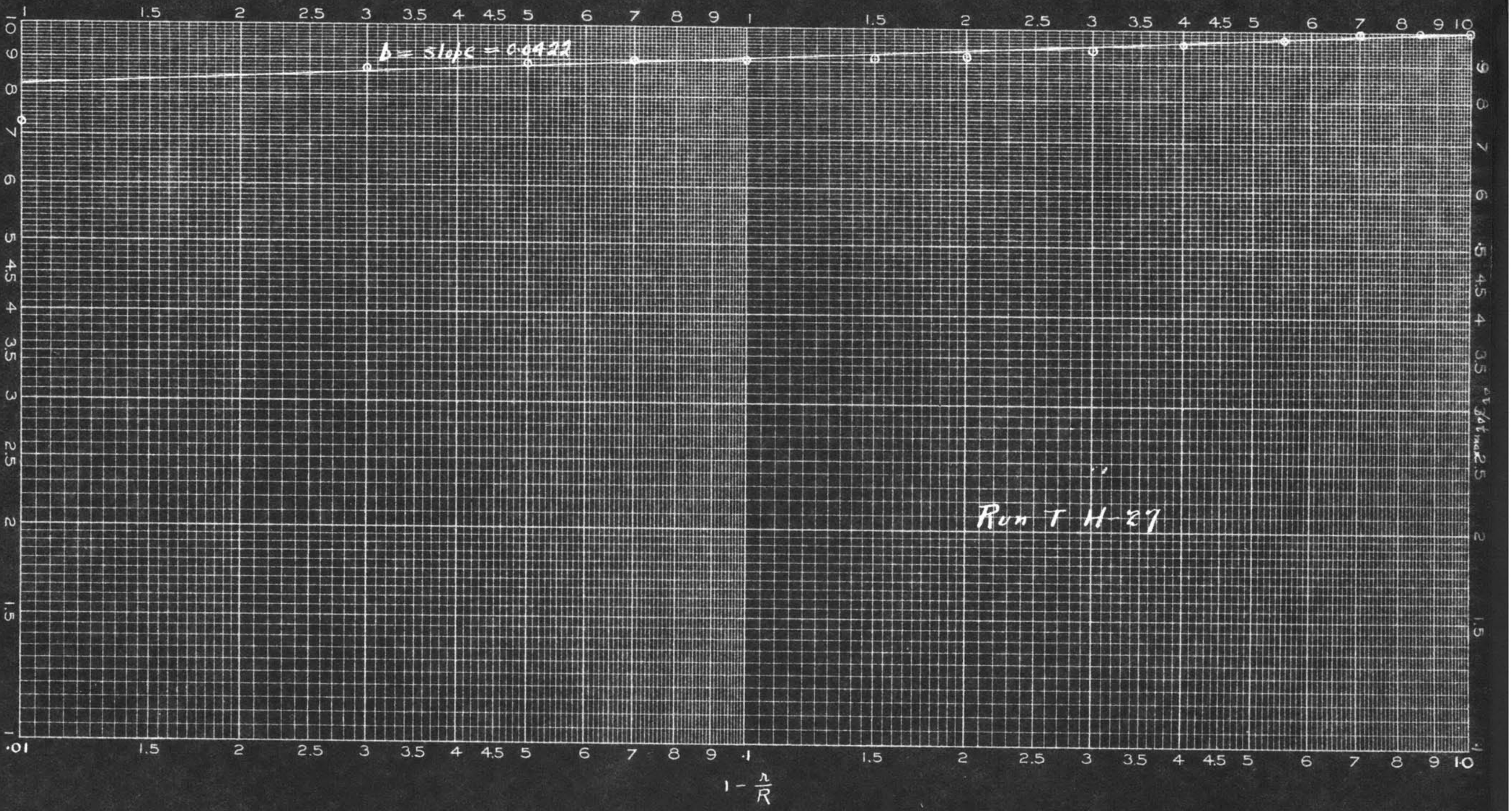
EUGENE DIETZGEN CO.
Chicago and New York



Nº 340-L21

EDCO Efficiency
LOGARITHMIC

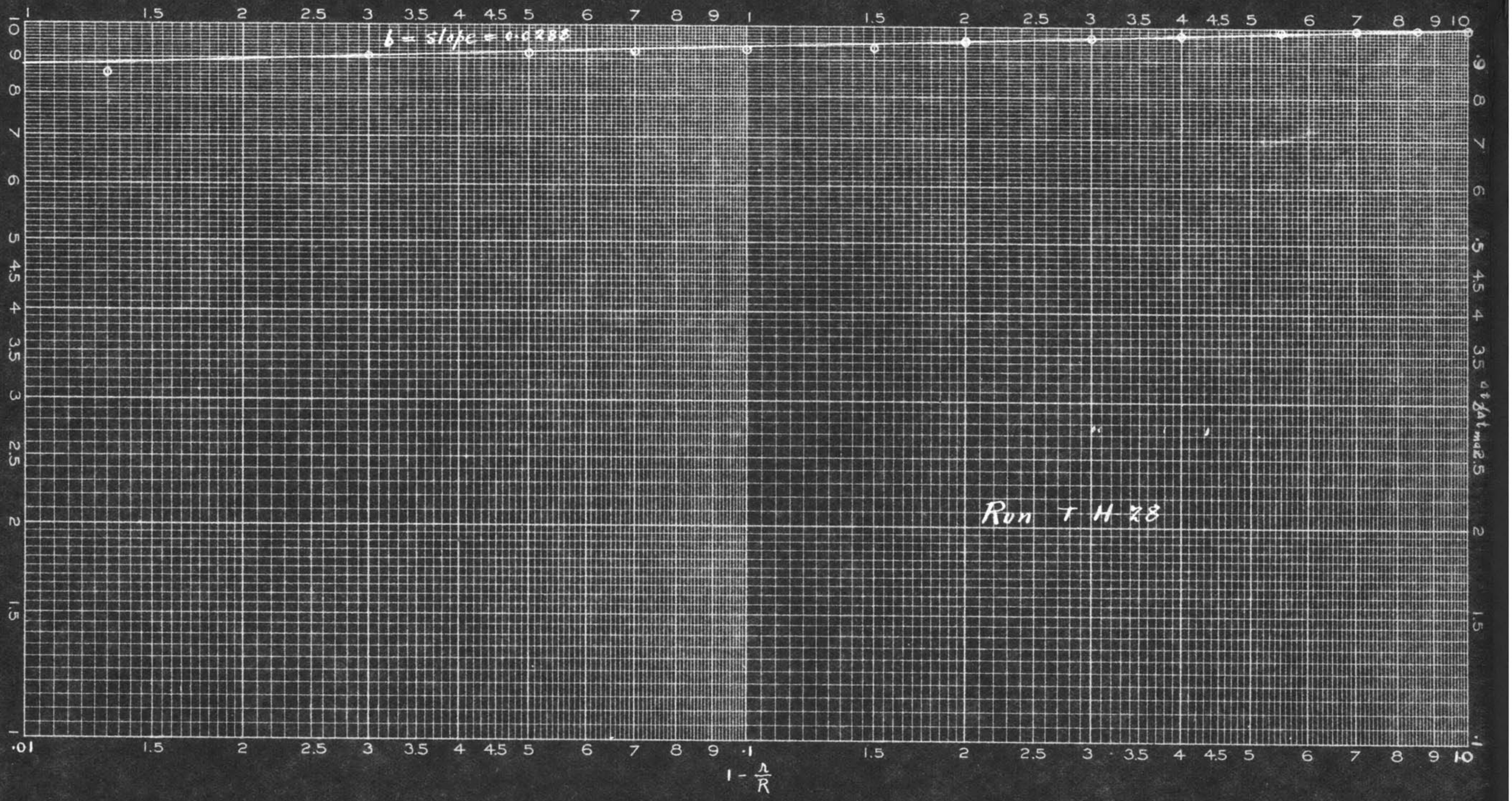
EUGENE DIETZGEN CO.
Chicago and New York



No 340-L21

EDCO Efficiency
LOGARITHMIC

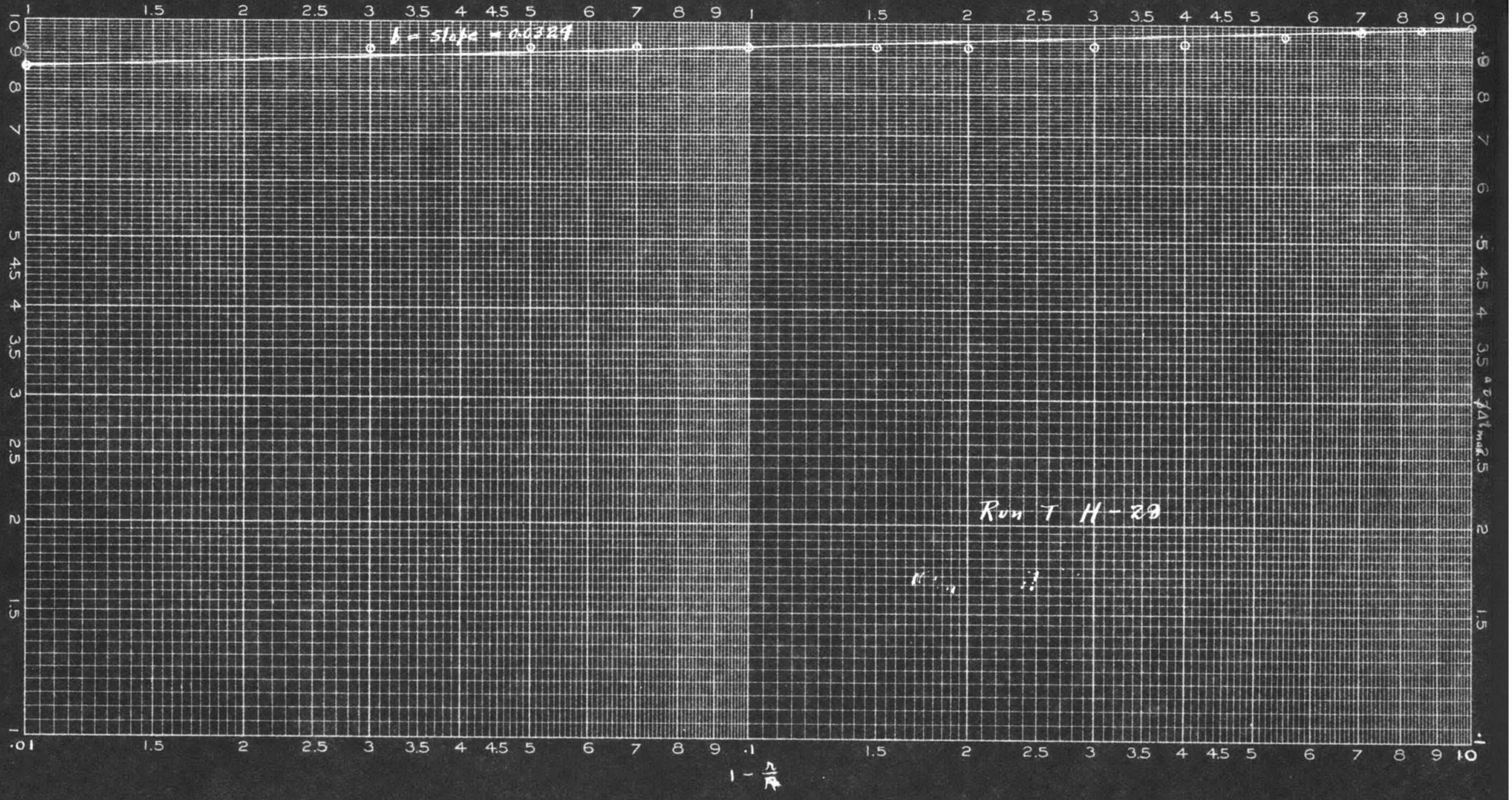
EUGENE DIETZGEN CO.
Chicago and New York

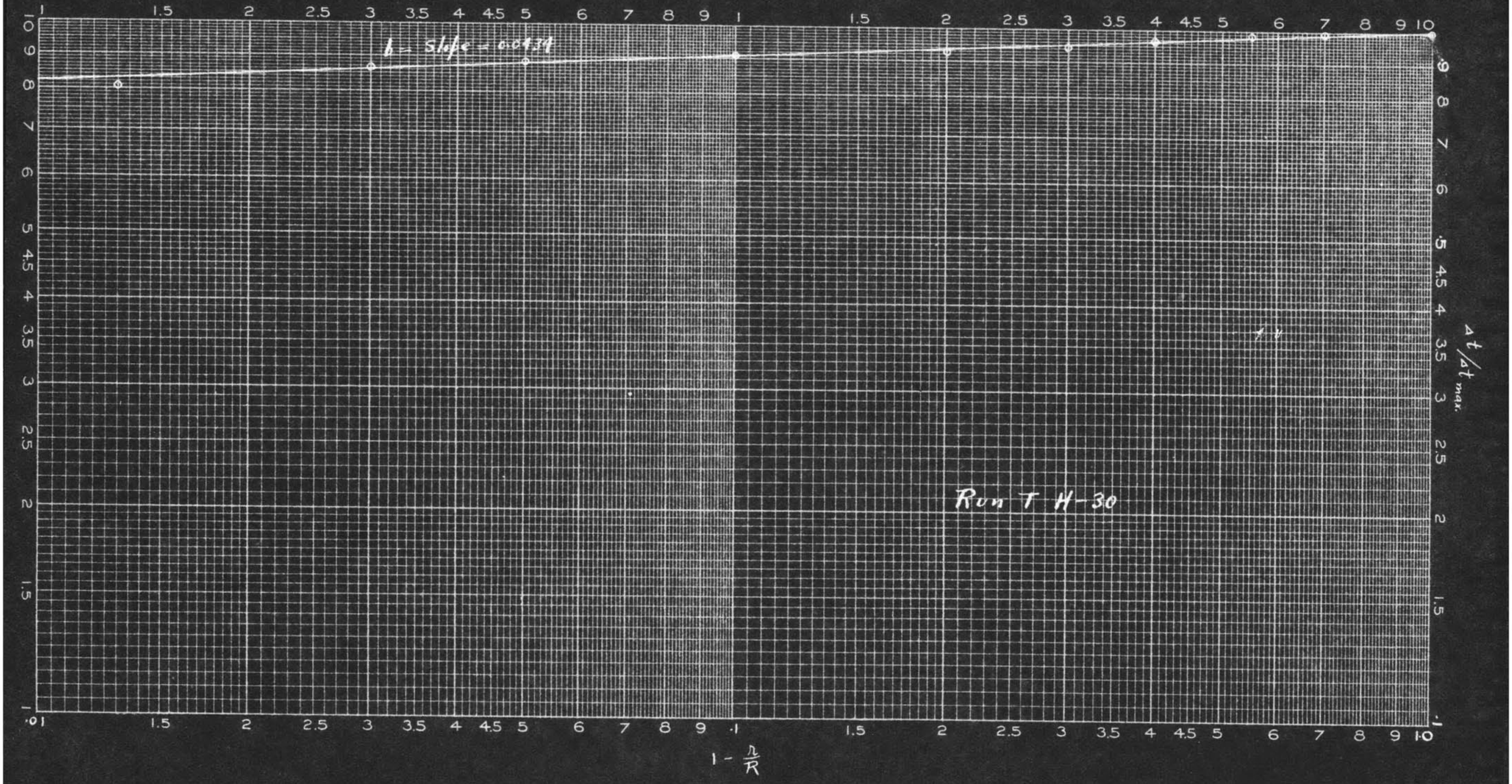


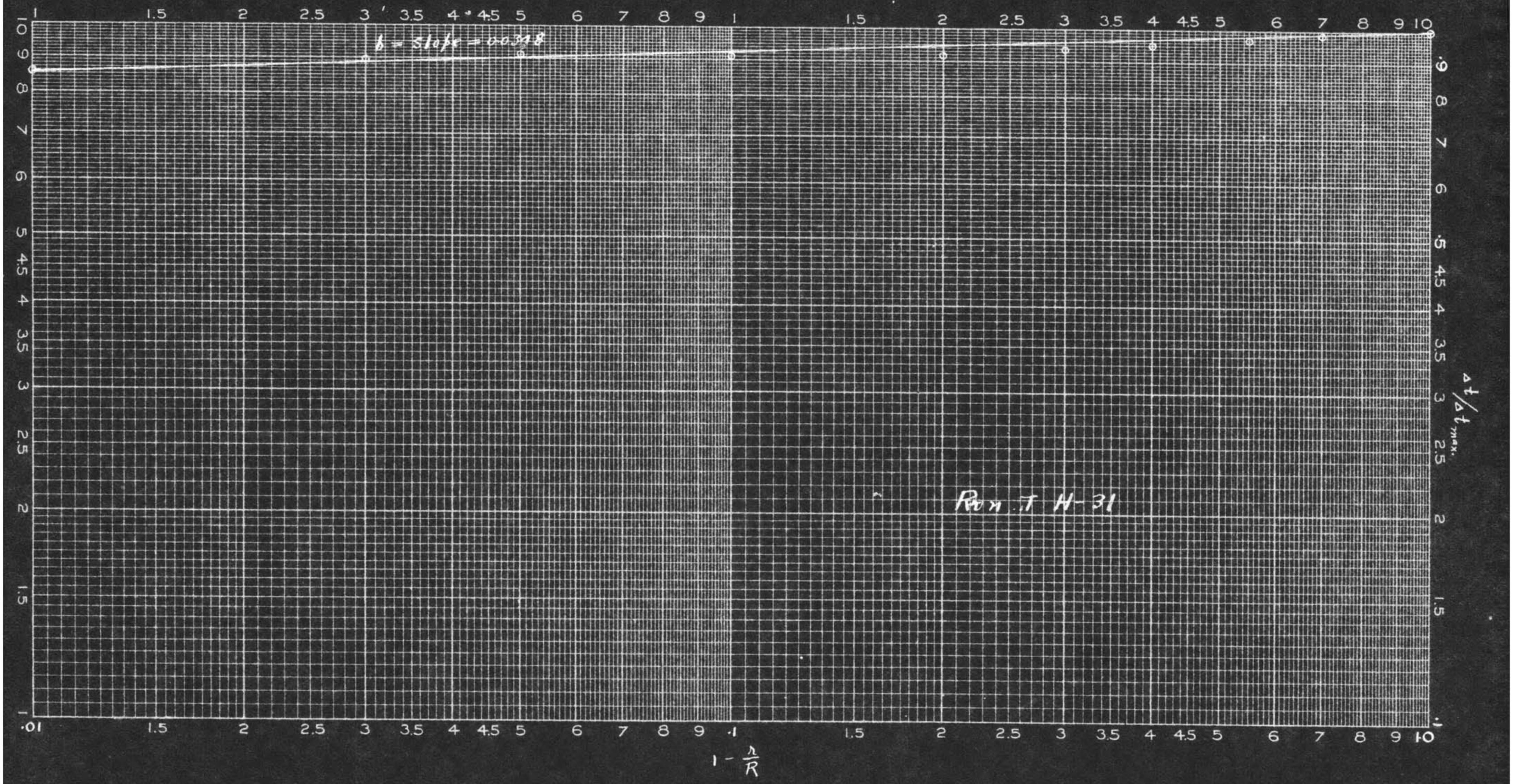
No. 340-L21

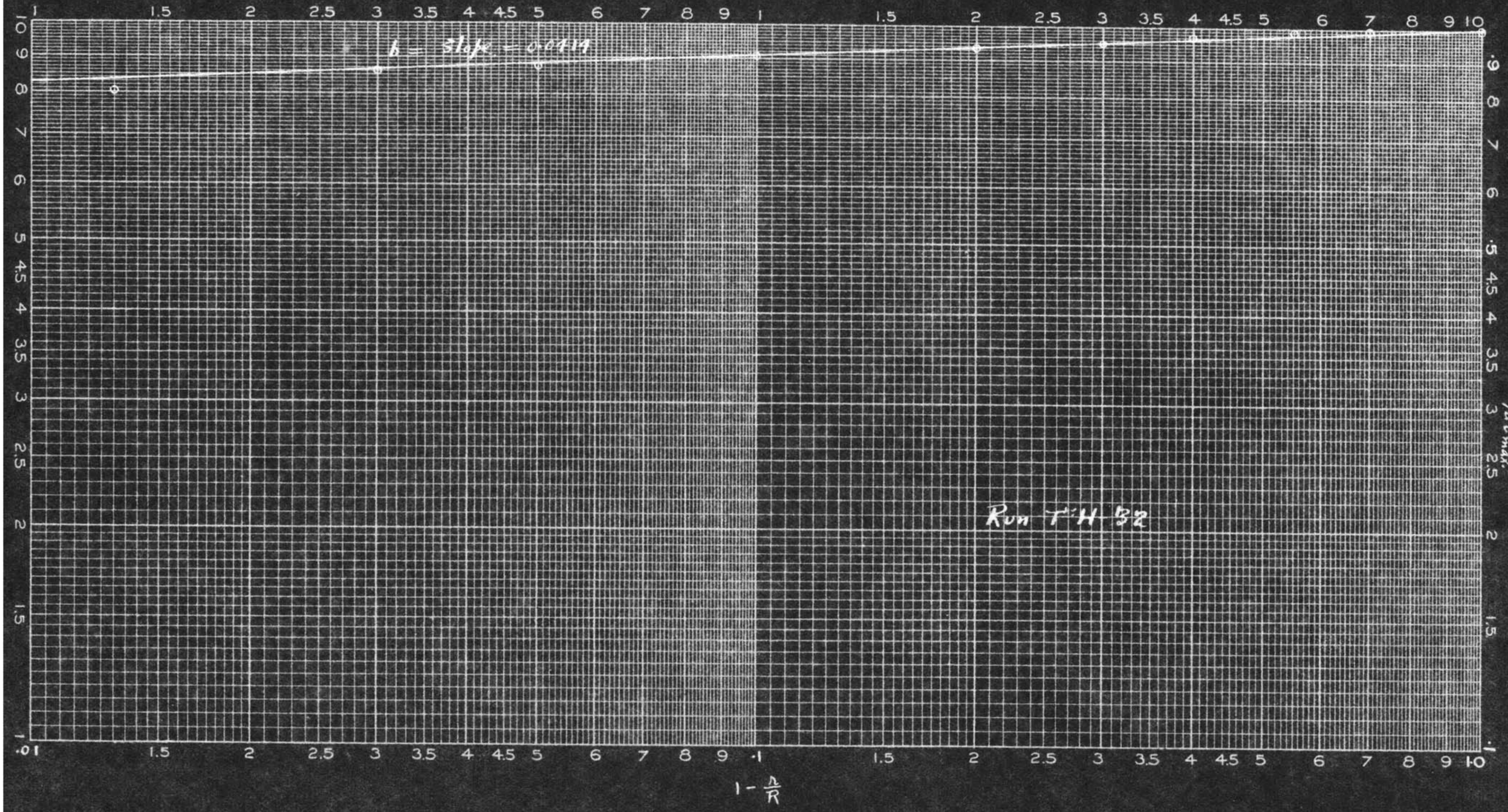
EDCO Efficiency
LOGARITHMIC

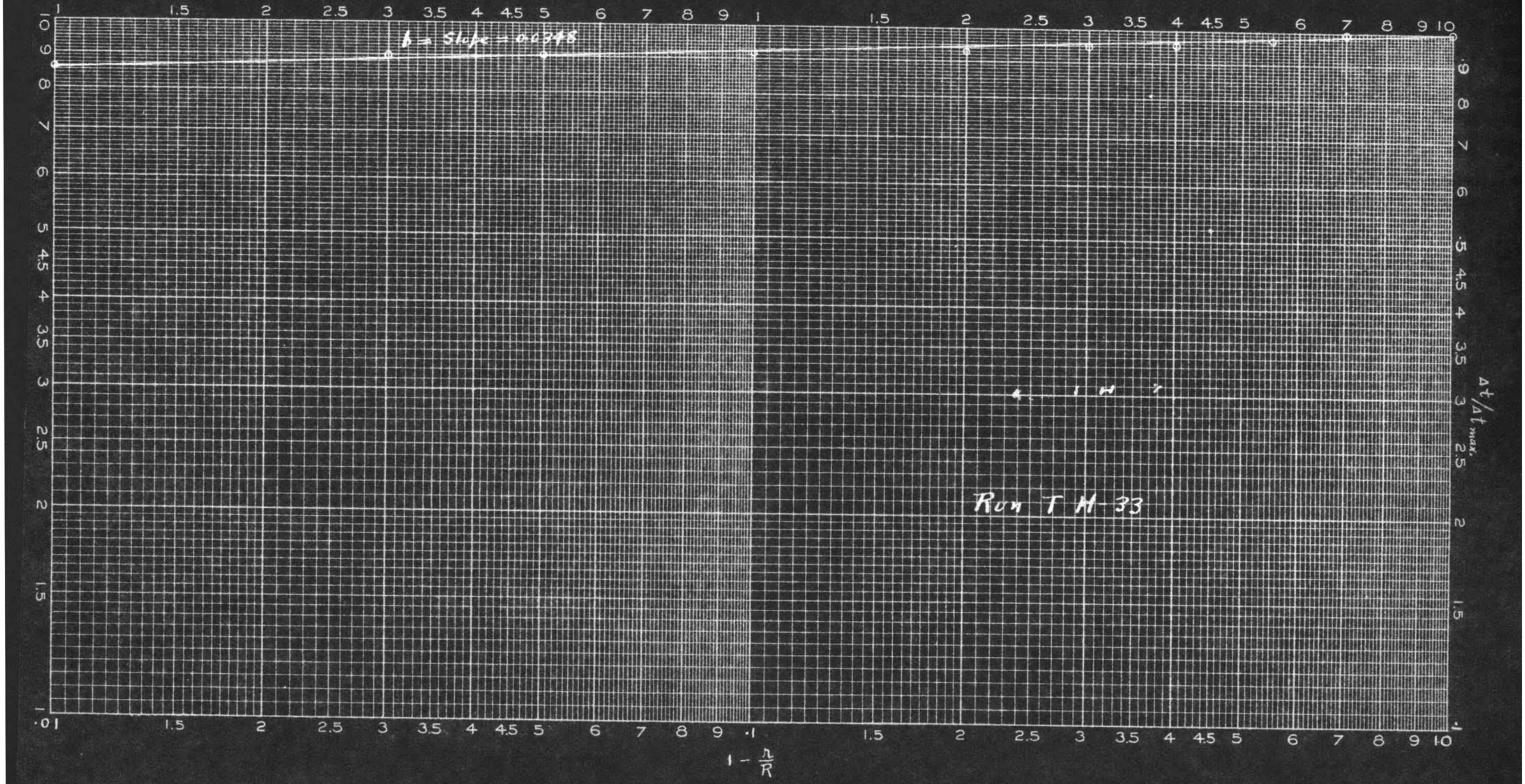
EUGENE DIETZGEN CO.
Chicago and New York

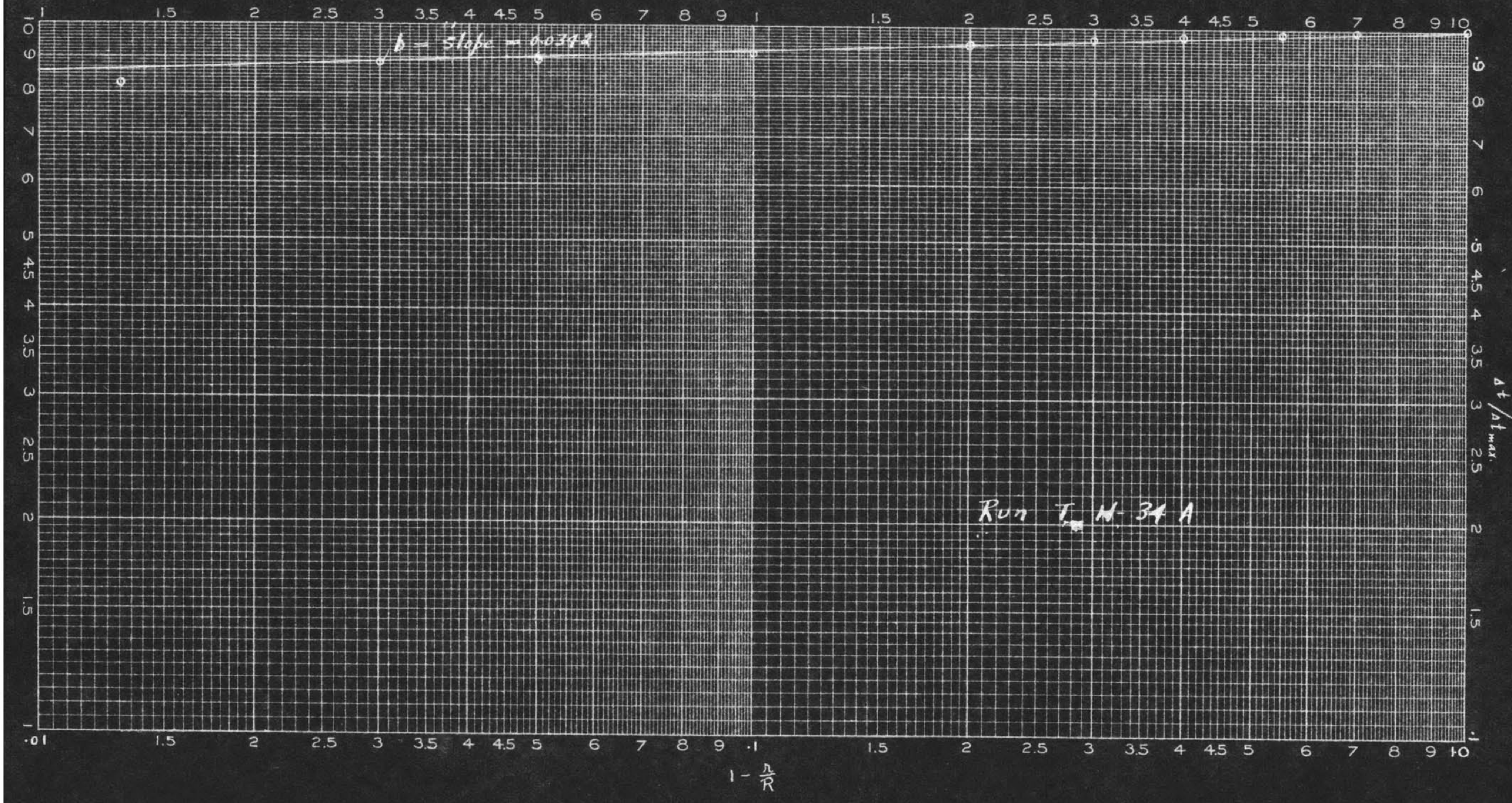


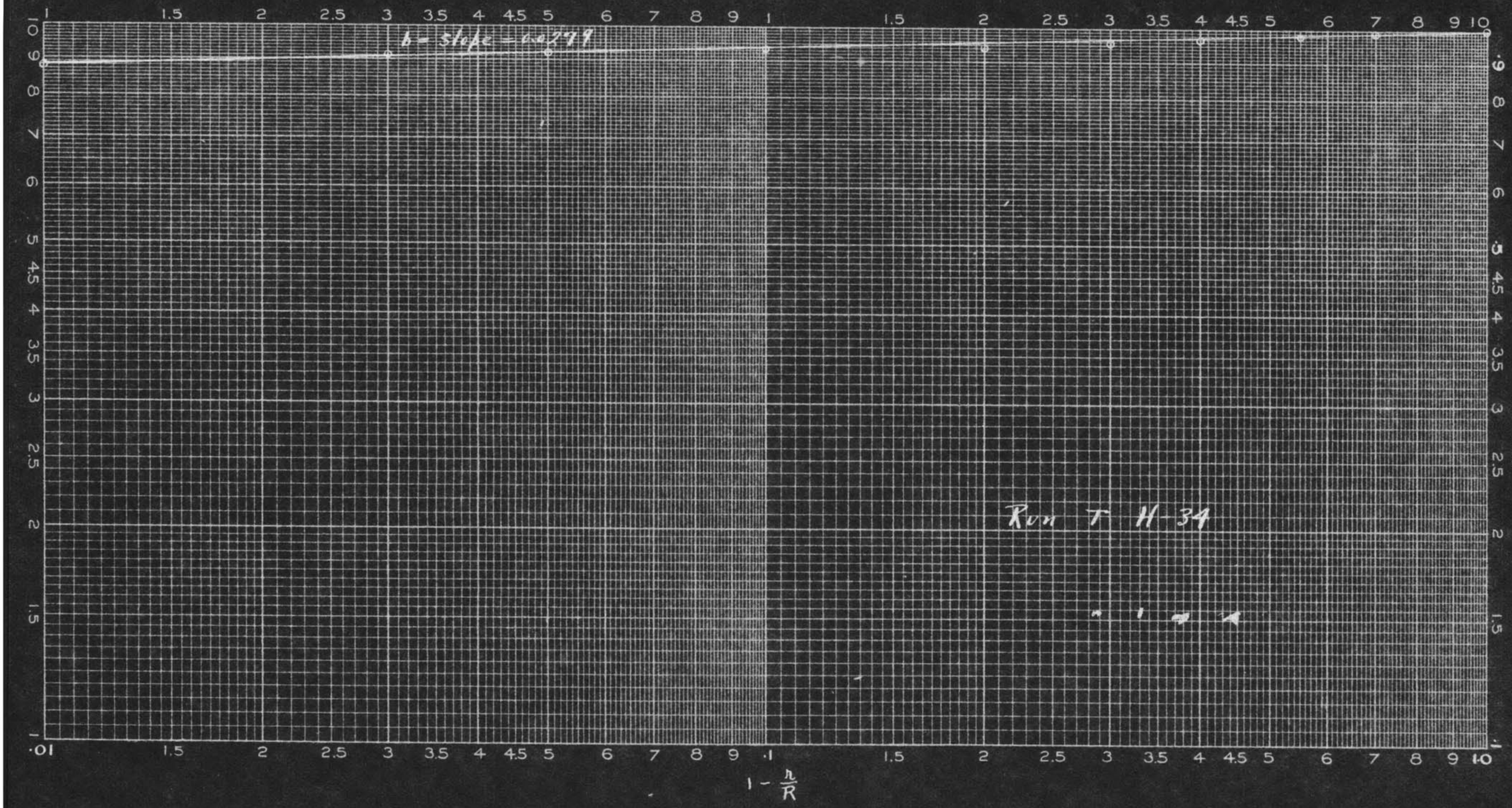

b = Slope = 0.0434
Run T H-30
1 - r/R
Δt/Δt max.
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
1

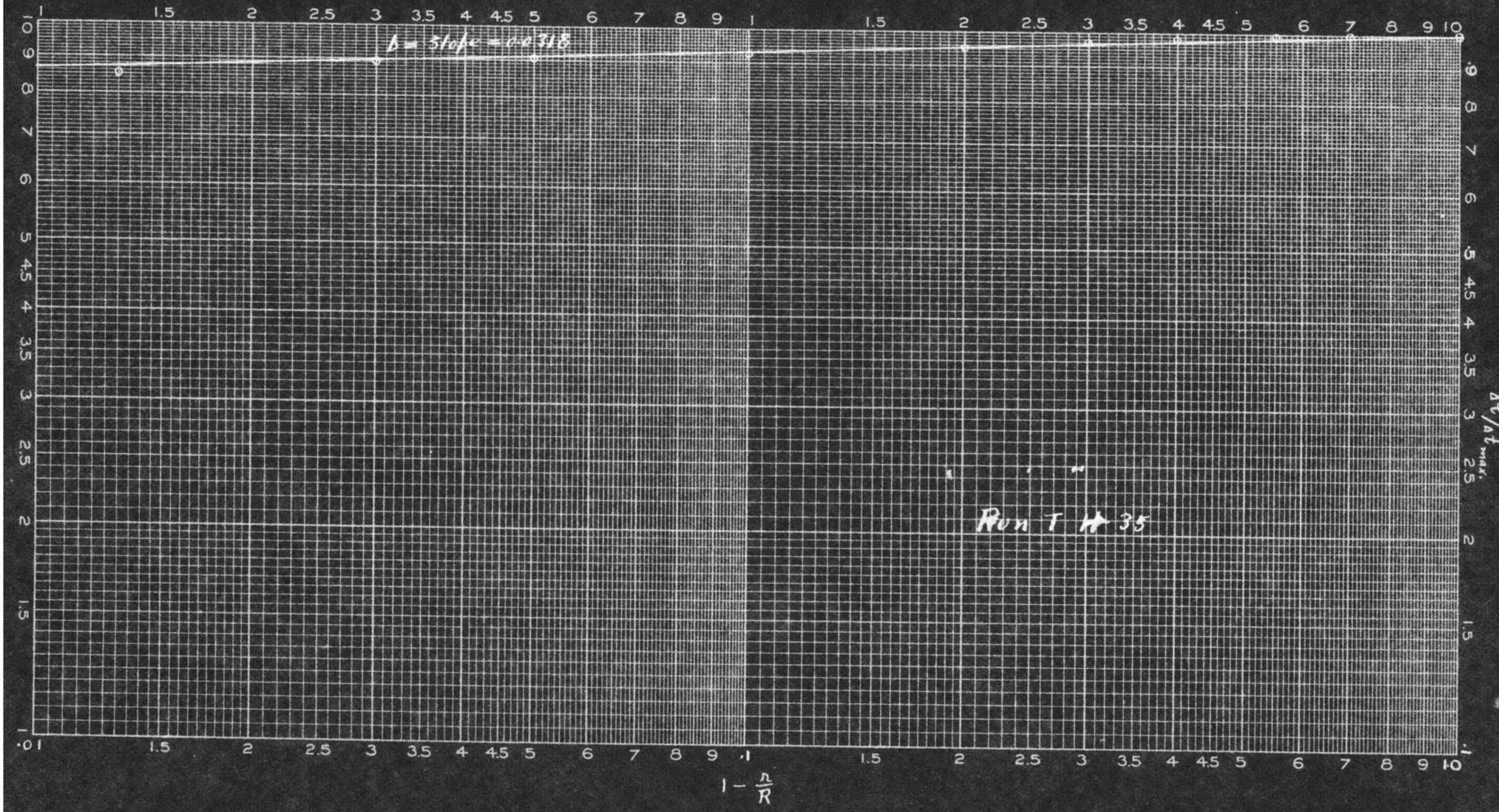

b = Slope = 0.0348
Run T H-31
1 - r/R
Δt/Δt max.
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.9
.5

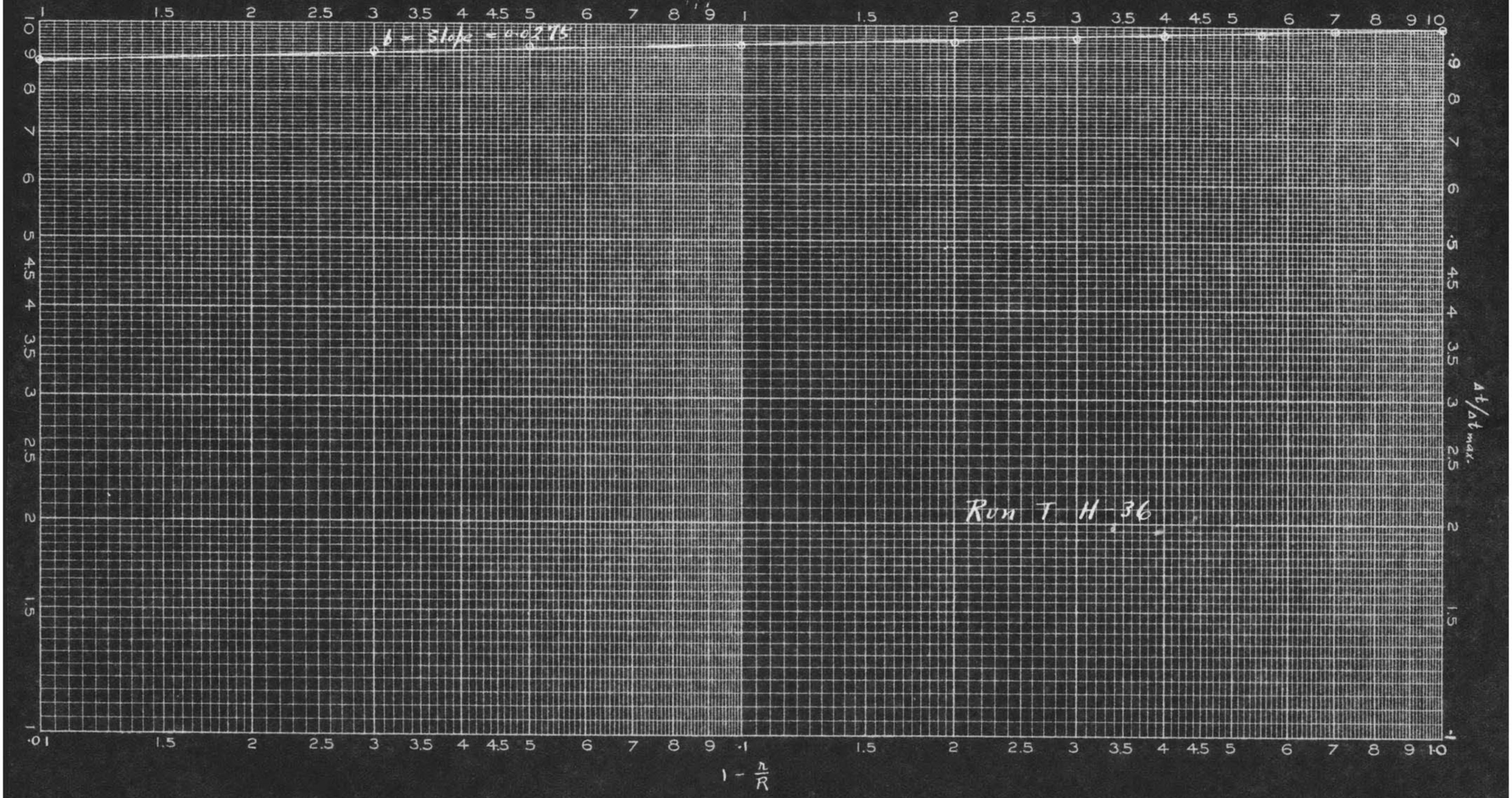

b = slope = 0.0414
Run T'H 32
1 - r/R
Δt/Δt max.
·01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
·1
1·0
·1
·9

b = Slope = 0.0348
Run T H-33
Δt/Δt max.
1 - r/R
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.9
.8
.7
.6
.5
.4
.35
.3
.25
.2
.15
.1


b = slope = 0.0342
Run T-H-34 A
1 - r/R
Δt/Δt max.
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.9
.5
.1


b = slope = 0.0279
Run T H-34
1 - $\frac{r}{R}$
$\Delta t / \Delta t_{max.}$
·01
·1
1·0
1
10
·9
·5
·1


b = Slope = 0.0318
Run T # 35
Δt/Δt$_{max.}$
1 − r/R
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0


b = Slope = 0·0275
Run T. H. 36
1 - $\frac{r}{R}$
$\Delta t / \Delta t_{max.}$
·01
·1
1·0
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
·9
·5
·1

Temperature Distribution in the Direction of Flow

r/R	0.99	0.97	0.95	0.93	0.90	0.85	0.80	0.70	0.60	0.45	0.30	0.15	0

Run T H-18 (t_1) Run T H-19 (t_2) $t_{w(ave.)}=83.5°$ C. $t_{w(ave.)} - t_i = 60.7°$ C.

r/R	0.99	0.97	0.95	0.93	0.90	0.85	0.80	0.70	0.60	0.45	0.30	0.15	0
t_2(°C.)	41.65	39.95	39.70	39.00	38.1	37.2	36.8	36.05	35.10	34.65	33.5	32.9	32.75
t_1(°C.)	40.00	32.9	31.1	30.7	29.5	28.8	28.2	26.8	25.75	24.8	23.9	23.4	23.25
t_2-t_1	1.65	7.05	8.60	8.3	8.6	8.4	8.6	9.25	9.35	9.85	9.6	9.5	9.5
$\left(\frac{t_2-t_1}{t_{w(av)}-t_i}\right)$	0.027	0.116	0.142	0.137	0.142	0.138	0.142	0.152	0.154	0.162	0.158	0.156	0156

Run T H-20 (t_1) Run T H-21 (t_2) $t_{w(av)}=69.6°$ C. $t_{w(av)} - t_i = 46.8°$ C.

t_2(°C.)	33.3	30.0	29.9	29.6	29.5	29.3	28.9	28.8	28.4	28.0	27.5
t_1(°C.)	32.0	27.6	27.5	27.2	27.0	25.8	25.35	25.1	24.35	23.85	22.9
t_2-t_1	1.30	2.4	2.4	2.4	2.5	3.5	3.55	3.7	4.05	4.15	4.6
$\left(\frac{t_2-t_1}{t_{w(av)}-t_i}\right)$	0.0278	0.0513	0.0513	0.0513	0.0534	0.0748	0.0759	0.0791	0.0865	.0887	0.0983

27.2	27.0
22.8	22.8
4.4.	4.2
0.094	0.0897

TEMPERATURE DISTRIBUTION IN THE DIRECTION OF FLOW

$\frac{r}{R}$	0.99	0.97	0.95	0.93	0.90	0.85	0.80	0.70	0.60	0.45	0.30	0.15	0
Run T H-22 (t_1) Run T H-23 (t_2)				$t_{w(ave)} = 72.71°C.$				$t_{w(ave)} - t_i = 49.67°C.$					
t_2(°C)	35.1	33.0	32.4	31.9	31.7	31.3	31.0	30.6	29.7	29.1	28.8	28.50	28.0
t_1(°C)	33.5	28.6	27.8	27.4	27.15	26.1	25.75	24.8	24.1	23.0	22.0	21.85	21.85
t_2-t_1	1.6	4.4	4.6	4.5	4.55	5.2	5.25	5.8	5.6	6.1	6.8	6.65	6.15
$\frac{t_2-t_1}{t_{w(ave)}-t_i}$	0.0322	0.0886	0.0926	0.0906	0.0916	0.1048	0.1058	0.1168	0.1128	0.123	0.137	0.134	0.124
Run T H-24 (t_1) Run T H-25 (t_2)				$t_{w(ave)} = 69.69°C.$				$t_{w(ave)} - t_i = 53.86°C.$					
t_2(°C)	29.9	26.0	25.7	25.5	25.20	24.45	24.30	23.9	23.15	22.12	21.14	20.78	20.60
t_1(°C)	29.0	24.45	21.5	20.8	20.36	19.38	18.64	17.70	17.10	16.24	15.74	15.50	15.40
t_2-t_1	0.9	1.55	4.2	4.7	4.84	5.07	5.66	6.2	6.05	5.88	5.40	5.28	5.20
$\frac{t_2-t_1}{t_{w(ave)}-t_i}$	0.0167	0.0288	0.078	0.0872	0.0899	0.0942	0.105	0.115	0.112	0.109	0.1003	0.098	0.0965

TEMPERATURE DISTRIBUTION IN THE DIRECTION OF FLOW

$\frac{r}{R}$	0.99	0.97	0.95	0.93	0.90	0.85	0.80	0.70	0.60	0.45	0.30	0.15	0
Run T H-26 Run T H-27	(t_1) (t_2)		$t_{w(ave)} = 74.31$°C.				$t_{w(ave)} - t_i = 58.25$°C.						
t_2(°C)	35.60	28.45	27.70	27.15	27.0	26.55	26.20	25.20	24.3	23.30	22.40	22.15	22.15
t_1(°C)	32.3	23.4	22.0	21.4	21.1	19.65	18.90	18.18	17.47	16.74	15.98	15.73	15.73
$t_2 - t_1$	3.3	5.05	5.7	5.75	5.9	6.90	7.3	7.02	6.83	6.56	6.42	6.42	6.42
$\frac{t_2 - t_1}{t_{w(ave)} - t_i}$	0.0566	0.0867	0.0979	0.0987	0.1013	0.1184	0.1253	0.1204	0.1172	0.1127	0.1102	0.1102	0.1102
Run T H-28 Run T H-29	(t_1) (t_2)		$t_{w(ave)} = 71.46$°C.				$t_{w(ave)} - t_i = 57.34$°C.						
t_2(°C)	25.7	22.4	22.25	22.18	22.1	22.0	21.86	21.75	21.27	20.0	19.3	18.92	18.43
t_1(°C)	25.0	20.4	19.8	19.4	18.9	18.5	17.3	16.56	16.0	15.2	14.8	14.6	14.5
$t_2 - t_1$	0.7	2.0	2.45	2.78	3.2	3.5	4.56	5.19	5.27	4.8	4.5	4.32	3.93
$\frac{t_2 - t_1}{t_{w(ave)} - t_i}$	0.0122	0.0349	0.0427	0.0485	0.0558	0.061	0.0795	0.0906	0.0920	0.0837	0.0785	0.0754	0.0685

Temperature Distribution in the Direction of Flow

r/R	0.99	0.97	0.95	0.90	0.80	0.70	0.60	0.45	0.30	0
Run T H-30 (t_1) Run T H-31 (t_2) $t_{w(av)}$=77.21° C. $t_{w(av)}-t_i$ = 72.21° C.										
t_2(°C.)	23.4	20.96	20.0	20.0	19.64	18.43	17.55	16.50	15.5	14.5
t_1(°C.)	20.5	15.4	13.9	12.2	10.85	9.60	8.06	6.67	6.05	5.4
t_2-t_1	2.9	5.56	6.1	7.8	8.79	8.83	9.49	9.83	9.45	9.1
$\frac{t_2-t_1}{t_{w(av)}-t_i}$	0.0402	0.0771	0.0845	0.108	0.1218	0.1223	0.1315	0.1362	0.131	0.126
Run T H=32 (t_1) Run T H-33 (t_2) $t_{w(av)}$ = 74.47° C. $t_{w(av)}$ - t_i = 70.72° C.										
t_2(°C.)	21.76	19.4	19.18	18.55	17.45	16.36	16.10	14.88	13.52	13.16
t_1 (°C.)	19.54	14.38	13.40	11.20	9.20	8.07	6.54	5.28	5.00	4.50
t_2-t_1	2.22	5.02	5.78	7.35	8.25	8.29	9.56	9.60	8.52	8.66
$\frac{t_2-t_1}{t_{w(av)}-t_i}$	0.0314	0.0071	0.0818	0.104	0.1166	0.1172	0.1352	0.1358	0.1204	0.1224

Temperature Distribution in the Direction of Flow

r/R	0.99	0.97	0.95	0.90	0.80	0.70	0.60	0.45	0.30	0
Run T H-34A (t_1) Run T H-34 (t_2) $t_{w(av)}=76.86°$ C. $t_{w(av)}-t_i = 72.98°$ C.										
t_2(°C.)	19.40	17.32	16.85	16.00	15.50	14.50	13.78	13.04	12.30	11.70
t_1(°C.)	18.00	12.64	11.80	10.32	8.30	6.54	5.78	5.30	4.50	4.00
t_2-t_1	1.40	4.68	5.05	5.68	7.2	7.96	8.00	7.74	7.8	7.7
$\frac{t_2-t_1}{t_{w(av)}-t_i}$	0.0192	0.0642	0.0692	0.0779	0.0986	0.109	0.1097	0.106	0.1068	0.1055
Run T H-35 (t_1) Run T H-36 (t_2) $t_{w(av)} = 69.46°$ C. $t_{w(av)}-t_i = 66.10°$ C.										
t_2(°C.)	15.5	13.9	13.04	12.54	11.8	11.2	10.6	10.25	9.57	8.97
t_1(°C.)	14.5	11.2	10.35	8.82	7.05	5.67	5.0	4.12	3.74	3.60
t_2-t_1	1.0	2.7	2.69	3.72	4.75	5.53	5.6	6.13	5.83	5.37
$\frac{t_2-t_1}{t_{w(av)}-t_i}$	0.0151	0.0409	0.0407	0.0563	0.0719	0.0837	0.0847	0.0928	0.0882	0.0813

APPENDIX H

TEMPERATURE DISTRIBUTION DATA DURING COUNTER CURRENT HEATING WITH CALCULATIONS AND PLOTS (WATER FLOWING UPWARD)

Run T H-37 (Counter Current) Station No. 4

(no corresponding Velocity Distribution Run)

$\frac{r}{R}$	Rdg. in mv. (no.24 wire)	t (°C.)	(t_w-t) (°C.)	$\frac{t_w-t}{t_w-t_a}$	$t(\frac{r}{R})$ (°C.)	$(1-\frac{r}{R})$	$\frac{t-t_i}{t_w-t_i}$
1.000					(87.24)		1.000
.988	1.555	39.6	47.64	0.687	39.10	0.012	0.390
.97	1.335	34.2	53.04	.765	33.15	.030	.320
.95	1.140	29.3	57.94	.835	27.85	.050	.257
.90	.985	25.4	61.84	.892	22.85	.10	.207
.80	.835	21.63	65.61	.946	17.3	.20	.159
.70	.820	21.26	65.98	.951	14.9	.30	.154
.60	.765	19.85	67.39	.971	11.92	.40	.136
.40	.710	18.46	68.78	.991	7.39	.60	.118
.20	.685	17.82	69.42	1.000	3.56	.80	.110
.00	.685	17.82	69.42	1.000	0.00	1.00	.110

$t_{o.w.}$ = 88.30°C.

t_w = 87.24°C. (Temp. drop through Pipe wall allowed = 1.06°C.)

t_w-t_a = 69.42°C.

$t_{ave.}$(Graphical Integration) = 22.80°C.

t_i = 9.25°C.

t_w-t_i = 77.99°C.

$V_{ave.}$(Manometer) = 1.302 ft./sec.

Run T H-38 (Counter Current) Station No. 5

(No Corresponding Velocity Distribution Run)

$\frac{r}{R}$	Rdg. in mv. (No.24 wire)	t (°C.)	(t_w-t) (°C.)	$\frac{t_w-t}{t_w-t_a}$	$t\ (\frac{r}{R})$ (°C.)	$(1-\frac{r}{R})$	$\frac{t-t_i}{t_w-t_i}$
1.000					(86.64)		1.000
0.992	1.935	48.65	37.99	0.685	48.4	0.008	.509
.970	1.830	46.15	40.49	.731	44.8	.030	.477
.95	1.720	43.50	43.14	.779	41.3	.050	.443
.90	1.685	42.7	43.94	.793	38.45	.10	.432
.80	1.550	39.45	47.19	.852	31.55	.20	.390
.70	1.500	38.2	48.44	.874	26.75	.30	.375
.60	1.405	35.9	50.74	.915	21.55	.40	.344
.40	1.295	33.2	53.44	.964	13.28	.60	.310
.20	1.215	31.2	55.44	1.000	6.24	.80	.284
.00	1.215	31.2	55.44	1.000	0.00	1.00	.284

$t_{o.w.}$ = 87.70°C.

t_w = 86.64°C. (Temp. drop through pipe wall allowed=1.06°C.)

t_w-t_a = 55.44°C.

$t_{ave.}$ (Graphical Integration) = 38.56°C.

t_i = 9.25°C.

t_w-t_i = 77.39

$V_{ave.}$ (Manometer) = 1.302 ft./sec.

Run T H-39 (Counter Current) Station No. 4

(For Corresponding Velocity Distribution, see Run V H-27)

$\frac{r}{R}$	Rdg. in mv. (No.24 Wire)	t (°C.)	(t_w-t) (°C.)	$\frac{t_w-t}{t_w-t_a}$	$t(\frac{r}{R})$ (°C.)	$(1-\frac{r}{R})$	$tv(\frac{r}{R})$ $\frac{(°C.)(ft.)}{(sec.)}$
1.000					(32.3)		
0.988	0.435	11.4	20.9	0.868	11.26	0.012	29.65
.970	.380	10.05	22.25	0.923	9.75	.030	28.45
.950	.365	9.65	22.65	.940	9.17	.050	28.27
.90	.360	9.50	22.8	.946	8.55	.10	28.26
.80	.330	8.73	23.57	.977	6.98	.20	25.08
.70	.325	8.60	23.7	.983	6.02	.30	23.40
.60	.320	8.48	23.82	.988	5.09	.40	20.80
.40	.320	8.48	23.82	.988	3.39	.60	14.67
.20	.310	8.20	24.1	1.000	1.64	.80	7.43
.00	.310	8.20	24.1	1.000	.00	1.00	0

$t_{o.w.}$ = 33.3°C.

t_w = 32.3°C. (Temp. drop through pipe wall allowed = 1.0°C.)

t_w-t_a = 24.1°C.

$t_{ave.}$ (Graphical Integration) = 8.96°C.

$V_{ave.}$ (Graphical Integration) = 3.76 ft./sec.

t_m (Graphical Integration) = 8.93°C.

Run T H-40 (Counter Current) Station No. 5

(No Corresponding Velocity Distribution Run)

$\frac{r}{R}$	Rdg. in mv. (No.24 Wire)	t (°C.)	(t_w-t) (°C.)	$\frac{t_w-t}{t_w-t_a}$	$t(\frac{r}{R})$ (°C.)	$(1-\frac{r}{R})$
1.000					(73.2)	
0.992	0.950	24.6	48.6	0.850	24.45	0.008
.970	0.865	22.4	50.8	.890	21.72	.030
.950	.805	20.9	52.3	.915	19.9	.050
.90	.765	19.85	53.35	.934	17.87	.10
.80	.755	19.6	53.6	.938	15.68	.20
.70	.705	18.3	54.9	.961	12.81	.30
.60	.680	17.7	55.5	.971	10.63	.40
.40	.635	16.56	56.64	.990	6.63	.60
.20	.615	16.05	57.15	1.000	3.21	.80
.00	.615	16.05	57.15	1.000	0.00	1.00

$t_{o.w.}$ = 75.1°C.

t_w = 73.2°C.(Temp. drop through pipe wall allowed = 1.9°C.)

t_w-t_a = 57.15°C.

$t_{ave.}$ (Graphical Integration) = 18.60°C.

Run T H-41 (Counter Current) Station No. 4

(For corresponding velocity distribution, See Run V H-28)

$\frac{r}{R}$	Rdg. in mv. (No.24 wire)	t (°C.)	(t_w-t) (°C.)	$\frac{t_w-t}{t_w-t_a}$	t(r/R) (°C.)	(1-r/R)	tv(r/R) (°C.)(ft./sec.
1.000					(65.93)		
0.988	0.775	20.1	45.83	0.805	19.87	0.012	42.8
.970	.675	17.56	48.37	.850	17.03	.030	40.4
.950	.535	14.0	51.93	.912	13.3	.050	33.5
.900	.5025	13.2	52.73	.927	11.88	.100	31.6
.80	.440	11.56	54.37	.955	9.25	.20	27.2
.70	.405	10.66	55.27	.971	7.46	.30	23.15
.60	.380	10.04	55.89	.983	6.02	.40	19.26
.40	.365	9.60	56.33	.990	3.84	.60	13.0
.20	.340	9.00	56.93	1.000	1.80	.80	6.38
.00	.340	9.00	56.93	1.000	0.00	1.00	0

$t_{o.w.}$ = 67.30° C.

t_w = 65.93° C. (Temp. drop, thru pipe wall allowed = 1.37° C.)

t_w-t_a = 56.93° C.

$t_{ave.}$ (Graphical Integration) = 12.12° C.

$v_{ave.}$ (Graphical Integration) = 2.98 ft./sec.

t_m (Graphical Integration) = 11.33° C.

Run T H-42 (Counter Current) Station No. 5

(For corresponding velocity distribution, See Run V H-29)

$\frac{r}{R}$	Rdg. in mv. (no.24 wire)	$\frac{t}{(°C.)}$	$\frac{(t_w-t)}{(°C.)}$	$\frac{t_w-t}{t_w-t_a}$	$\frac{t(r/R)}{(°C.)}$	(1-r/R)	$\frac{tv(r/R)}{(°C.)(ft./sec.)}$
1.000					(79.63)		
0.992	1.130	29.1	50.53	0.866	28.95	0.008	65.6
.970	1.045	26.9	52.73	.904	26.1	.030	63.2
.950	1.035	26.7	52.93	.907	25.4	.050	65.2
.90	1.015	26.2	53.43	.915	23.55	.10	64.3
.80	0.965	24.9	54.73	.937	19.9	.20	59.35
.70	0.950	23.6	56.03	.960	16.52	.30	51.2
.60	.885	22.9	56.73	.972	13.74	.40	44.3
.40	.825	21.4	58.23	.997	8.56	.60	28.95
.20	.820	21.26	58.37	1.000	4.25	.80	14.91
.00	.820	21.26	58.37	1.000	0	1.00	0

$t_{o.w.}$ = 81.0° C.

t_w = 79.63° C. (Temp. drop thru pipe wall allowed = 1.37° C.)

t_w-t_a = 58.37° C.

$t_{ave.}$ (Graphical Integration) = 24.48° C.

$V_{ave.}$ (Graphical Integration) = 3.03 ft./sec.

t_m (Graphical Integration) = 23.9° C.

Run T H-43 (Counter Current) Station No. 4

(For corresponding velocity distribution, See Run V H-30)

$\frac{r}{R}$	Redg. in mv. (No.24 wire)	$\frac{t}{(°C.)}$	$\frac{(t_w-t)}{(°C.)}$	$\frac{t_w-t}{t_w-t_a}$	$\frac{t(r/R)}{(°C.)}$	(1-r/R)	$\frac{tv(r/R)}{(°C.)}$ (ft./sec.)	$\frac{t-t_i}{t_w-t_i}$
1.000					(83.27)			0.341
.988	1.335	34.2	49.07	0.735	33.8	0.012	34.3	1.000
.970	1.235	31.7	51.57	.773	30.75	.030	34.8	0.308
.950	0.982	25.4	57.87	.866	24.12	.050	28.77	.223
.900	.885	22.9	60.37	.904	20.6	.10	26.8	.189
.80	.808	20.97	62.30	.933	16.76	.20	23.95	.163
.70	.770	20.0	63.27	.947	14.0	.30	20.65	.150
.60	.720	18.7	64.57	.967	11.22	.40	17.20	.133
.40	.685	17.82	65.45	.980	7.13	.60	11.64	.121
.20	.635	16.56	66.71	.998	3.31	.80	5.61	.104
.00	.630	16.45	66.82	1.000	0.0	1.00	0	.103

$t_{o.w.}$ = 84.3° C. t_w = 83.27 (Temp. drop thru film wall allowed = 1.03° C.)

t_w-t_a = 66.82° C. t_{ave} (Graphical Integration) = 21.92° C.

v_{av} (Graphical Integration) = 1.442 ft./sec. t_i = 8.8° C,

t_w-t_i = 74.47° C. t_m (Graphical Integratinn) = 20.6° C.

Run T H-44 (Counter Current) Station No. 5

(No corresponding velocity distribution run)

$\frac{r}{R}$	Rdg. in mv. (no.24 wire)	t (°C.)	(t_w-t) (°C.)	$\frac{t_w-t}{t_w-t_a}$	$t(r/R)$ (°C.)	$(1-\frac{r}{R})$	$\frac{t-t_i}{t_w-t_i}$
1.000					(85.57)		1.000
.992	1.735	43.9	41.67	0.775	43.6	0.008	.4575
.970	1.568	39.9	45.67	.849	38.7	.030	.405
.950	1.565	39.8	45.77	.851	37.8	.050	.404
.90	1.560	39.7	45.87	.853	35.7	.10	.403
.80	1.475	37.7	47.87	.890	30.15	.20	.377
.70	1.385	35.4	50.17	.932	24.75	.30	.347
.60	1.350	34.6	50.97	.946	20.75	.40	.336
.40	1.270	32.6	52.97	.983	13.04	.60	.310
.20	1.235	31.7	53.87	1.000	6.34	.80	.298
.00	1.235	31.7	53.87	1.000	0.0	1.00	.298

$t_{o.w.}$ = 86.6° C.

t_w = 85.57° C. (Temp. drop thru pipe wall = 1.03° C.)

t_w-t_a = 53.87° C.

$t_{av.}$ (Graphical Integration) = 36.56° C.

t_i = 8.8° C.

t_w-t_i = 76.77° C.

Run T H-45 (Counter Current) Station No. 4

(For corresponding velocity distribution, see Run V H-31)

$\frac{r}{R}$	Rdg. in mv. (no.24 wire)	$\frac{t}{(°C.)}$	$\frac{(t_w-t)}{(°C.)}$	$\frac{t_w-t}{t_w-t_a}$	$\frac{t(r/R)}{(°C.)}$	$(1-\frac{r}{R})$	$\frac{tv(r/R)}{(°C.)(ft./sec.)}$
1.000					(62.95)		
0.988	0.770	20.0	42.95	0.785	19.76	0.012	49.3
.970	.503	13.2	49.75	.909	12.80	.030	33.75
.950	.470	12.3	50.65	.925	11.68	.050	32.4
.900	.420	11.06	51.89	.448	9.95	.10	29.45
.80	.385	10.2	52.75	.964	8.16	.20	26.45
.70	.370	9.8	53.15	.972	6.86	.30	23.45
.60	.350	9.27	53.68	.980	5.56	.40	19.68
.40	.320	8.50	54.45	.994	3.40	.60	12.82
.20	.315	8.35	54.60	.997	1.67	.80	6.55
.00	.310	8.20	54.75	1.000	0.00	1.00	0

$t_{o.w.}$ = 64.40° C.

t_w = 62.95° C. (Temp. drop thru pipe wall allowed = 1.45° C.)

t_w-t_a = 54.75° C.

t_{av} (Graphical Integration) = 10.88° C.

v_{av} (Graphical Integration) = 3.33 ft./sec.

t_m (Graphical Integration) = 10.13° C.

Run T H-46 (Counter Current) Station No. 5

(For corresponding velocity distribution, see Run v H-32)

$\frac{r}{R}$	Rdg. in mv. (#24 wire)	$\frac{t}{(°C.)}$	$\frac{(t_w-t)}{(°C.)}$	$\frac{t_w-t}{t_w-t_a}$	$\frac{t(r/R)}{(°C.)}$	(1-r/R)	$\frac{tV(r/R)}{(°C.)}$ (ft./sec.
1.000	1.250				(78.95)		
.992		32.1	46.85	0.784	31.9	0.008	81.5
.970	.990	25.6	53.35	.893	24.83	.030	67.1
.950	.950	24.6	54.35	.910	23.35	.050	64.25
.90	.910	23.6	55.35	.926	21.25	.10	63.1
.80	.870	22.5	56.45	.945	18.0	.20	57.0
.70	.820	21.26	57.69	.965	14.9	.30	50.0
.60	.795	20.62	58.33	.976	12.37	.40	48.5
.40	.745	19.34	59.61	.997	7.74	.60	28.94
.20	.740	19.20	59.75	1.000	3.84	.80	14.98
.00	.740	19.20	59.75	1.000	0.00	1.00	0

$t_{o.w.}$ = 80.40

t_w = 78.95 (Temp. drop thru pipe wall allowed = 1.45° C.)

$t_w - t_a$ = 59.75° C.

t_{av} (Graphical Integration) = 22.08° C.

v_{av} (Graphical Integration) = 3.30 ft./sec.

t_m (Graphical Integration) = 21.68° C.

Run T H-47 (Counter Current) Station No. 4

(No corresponding velocity distribution Run)

$\frac{r}{R}$	Rdg. in mv. (No. 24 Wire)	$\frac{t}{(°C.)}$	$\frac{(t_w-t)}{(°C.)}$	$\frac{t_w-t}{t_w-t_a}$	$\frac{t(r/R)}{(°C.)}$	(1-r/R)	$\frac{t-t_i}{t_w-t_i}$
1.000					(94.33)	0.012	1.000
0.988	1.45	37.06	57.27	0.725	36.65		0.330
.970	1.215	31.20	63.13	0.799	30.25	.030	.261
.950	1.04	26.8	67.53	.855	25.45	.050	.210
.90	0.860	22.28	72.05	.912	20.05	.10	.157
.80	0.710	18.47	75.86	.960	14.78	.20	.112
.70	.690	17.96	76.37	.966	12.58	.30	.106
.60	.670	17.46	76.87	.972	10.48	.40	.100
.40	.635	16.57	77.76	.983	6.63	.60	.090
.20	.605	15.80	78.53	.992	3.16	.80	.081
.00	.585	15.30	79.03	1.000	0.0	1.00	.075

$t_{o.w.}$ = 95.5° C.

t_w = 94.33° C. (Temp. drop thru pipe wall allowed = 1.17° C.)

$t_w - t_a$ = 79.03° C.

$t_{ave.}$ (Graphical Integration) = 20.32° C.

t_i = 8.90° C.

$t_w - t_i$ = 85.43° C.

$V_{ave.}$ (Weighing) = 1.102 ft./sec.

Run T H-48 (Counter Current) Station No. 5

(No corresponding velocity distribution run)

$\frac{r}{R}$	Rdg. in mv. (No.24 Wire)	$\frac{t}{(°C.)}$	$\frac{(t_w-t)}{(°C.)}$	$\frac{t_w-t}{t_w-t_a}$	$\frac{t(r/R)}{(°C.)}$	(1-r/R)	$\frac{t-t_i}{t_w-t_i}$
1.000					(93.18)		1.000
0.992	1.79	45.2	47.98	0.809	44.90	0.008	1.431
.970	1.61	40.9	52.28	.881	39.65	.030	.380
.95	1.60	40.65	52.53	.886	38.60	.050	.377
.90	1.435	36.7	56.48	.935	33.0	.10	.330
.80	1.385	35.4	57.78	.975	28.3	.20	.315
.70	1.365	34.95	58.23	.982	24.45	.30	.309
.60	1.340	34.35	58.83	.991	20.6	.40	.302
.40	1.335	34.2	58/98	.994	13.68	.60	.300
.20	1.330	34.05	59.13	.996	6.81	.80	.298
.00	1.32	33.85	59.33	1.000	0.0	1.00	.296

$t_{o.w.}$ = 93.80° C.

t_w = 93.18° C. (Temp. drop through pipe wall = 0.62° C.)

t_w-t_a = 59.33° C.

$t_{ave.}$ = 35.44° C. (Graphical Integration)

t_i = 8.90° C.

t_w-t_i = 84.28° C.

V_{ave} = 1.102 ft./sec. (Weighing)

Run T H-49 (Counter Current) Station No. 4

(For corresponding velocity distribution, see Run V H-33)

$\frac{r}{R}$	Rdg. in mv. (No.24 wire)	t (°C.)	$t_w - t$ (°C.)	$\frac{t_w - t}{t_w - t_a}$	$t(r/R)$ (°C.)	$(1-r/R)$	$tv(r/R)$ (°C.) (ft./sec.)
1.000					(79.41)		
0.988	1.275	32.7	46.71	0.729	32.3	0.012	31.65
.970	1.005	25.9	53.51	.835	25.1	.030	27.1
.950	0.925	23.9	55.51	.866	22.7	.050	27.1
.90	.840	21.76	57.65	.900	19.58	.10	26.9
.80	.770	19.35	60.06	.937	15.48	.20	22.95
.70	.720	18.70	60.71	.947	13.1	.30	20.85
.60	.665	17.3	62.11	.969	10.38	.40	17.02
.40	.620	16.2	63.21	.986	6.48	.60	11.52
.20	.590	15.4	64.01	.998	3.08	.80	5.70
.00	.585	15.3	64.11	1.000	0.0	1.00	0

$t_{o.w.}$ = 80.80° C.

t_w = 79.41° C. (Temp. drop through pipe wall allowed =1.39° C.)

$t_w - t_a$ = 64.11° C.

$t_{ave.}$ (Graphical Integration) = 19.84° C.

$v_{ave.}$ (Graphical Integration) = 1.526 ft./sec.

t_m (Graphical Integration) = 19.63° C.

Run T H-50 (Counter Current) Station No. 5

(For corresponding velocity distribution, see Run V H-34)

$\frac{r}{R}$	Rdg. in mv. (No.24 wire)	$\frac{t}{(°C.)}$	$\frac{(t_w-t)}{(°C.)}$	$\frac{t_w-t}{t_w-t_a}$	$\frac{t(r/R)}{(°C.)}$	(1-r/R)	$\frac{tv(r/R)}{(°C.)(ft. per sec.)}$
1.000					(86.37)		
0.992	1.610	40.9	45.47	0.818	40.7	0.008	40.3
.970	1.520	38.75	47.62	.857	37.55	.030	40.4
.950	1.470	37.5	48.87	.880	35.6	.050	42.2
.90	1.425	36.4	49.97	.899	32.75	.10	41.95
.80	1.355	34.7	51.67	.930	27.75	.20	38.7
.70	1.310	33.6	52.77	.949	23.5	.30	35.75
.60	1.265	32.4	53.97	.970	19.44	.40	32.6
.40	1.220	31.3	55.07	.991	12.52	.60	22.53
.20	1.200	30.8	55.57	1.000	6.16	.80	11.58
.00	1.200	30.8	55.57	1.000	0.00	1.00	0

$t_{o.w.}$ = 87.20° C.

t_w = 86.37° C. (Temp. drop thru pipe wall allowed = 0.83° C.)

$t_w - t_a$ = 55.57° C.

$t_{ave.}$ = 34.48° C. (Graphical Integration)

$V_{ave.}$ (Graphical Integration) = 1.510 ft./sec.

t_m (Graphical Integration) = 33.76° C.

Run T H-51 (Counter Current) Station No. 4

(For corresponding velocity distribution, see Run V H-35)

$\frac{r}{R}$	Rdg. in mv. (No.24 wire)	$\frac{t}{(°C.)}$	$\frac{(t_w-t)}{(°C.)}$	$\frac{t_w-t}{t_w-t_a}$	$\frac{t(r/R)}{(°C.)}$	(1-r/R)	$\frac{tv(r/R)}{(°C.)(ft./sec.)}$
1.000					(74.66)		
0.988	1.07	27.6	97.06	0.765	27.3	0.012	39.2
.970	0.885	22.9	51.76	.842	22.2	.030	35.45
.950	0.770	20.0	54.66	.890	19.0	.050	31.85
.90	.705	18.34	56.32	.915	16.5	.10	29.4
.80	.640	16.70	57.96	.941	13.36	.20	26.4
.70	.600	15.68	58.98	.959	10.98	.30	23.0
.60	.580	15.17	59.49	.966	9.11	.40	19.63
.40	.535	14.00	60.66	.986	5.60	.60	12.68
.20	.502	13.15	61.51	1.000	2.63	.80	6.15
.00	.502	13.15	61.51	1.000	0.00	1.00	0

$t_{o.w.}$ = 75.90° C.

t_w = 74.66° C. (Temperature drop thru pipe wall allowed = 1.24° C.)

$t_w - t_a$ = 61.51° C.

t_{ave} (Graphical Integration) = 17.64° C.

v_{ave} (Graphical Integration) = 2.02 ft./sec.

t_m (Graphical Integration) = 16.28° C.

Run T H-52 (Counter Current) Station No: 5

(For corresponding velocity distribution, see Run V H-36)

$\frac{r}{R}$	Rdg. in mv. (No.24 wire)	$\frac{t}{(°C.)}$	$\frac{(t_w-t)}{(°C.)}$	$\frac{t_w-t}{t_w-t_a}$	$\frac{t(r/R)}{(°C.)}$	$(1-r/R)$	$\frac{tv(r/R)}{(°C.)}$ (ft./sec.)
1.000					(83.56)		
0.992	1.405	35.9	47.66	0.846	35.70	0.008	52.85
.970	1.305	33.4	50.16	.890	32.4	.030	50.6
.950	1.270	32.6	50.96	.904	30.95	.050	50.75
.90	1.230	31.6	51.96	.921	28.45	.10	49.7
.80	1.160	29.8	53.76	.955	23.85	.20	46.4
.70	1.130	29.1	54.46	.965	20.40	.30	42.75
.60	1.100	28.3	55.26	.980	17.0	.40	37.1
.40	1.080	27.8	55.76	.990	11.12	.60	25.46
.20	1.055	27.2	56.36	1.000	5.44	.80	12.90
.00	1.055	27.2	56.36	1.000	0.00	1.00	0

$t_{o.w.}$ = 84.80° C.

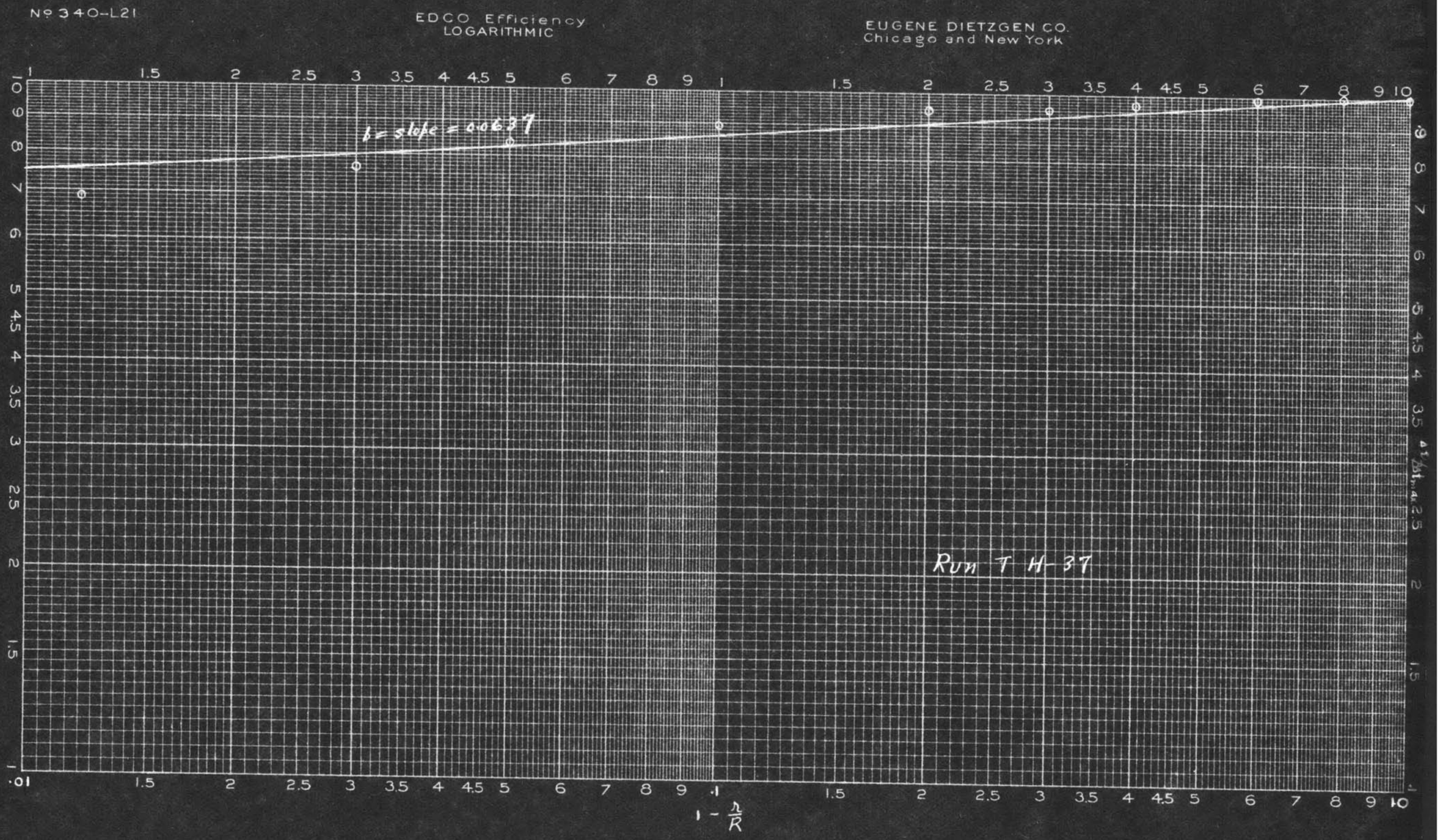
t_w = 83.56° C. (Temperature difference thru pipe wall = 1.24° C.)

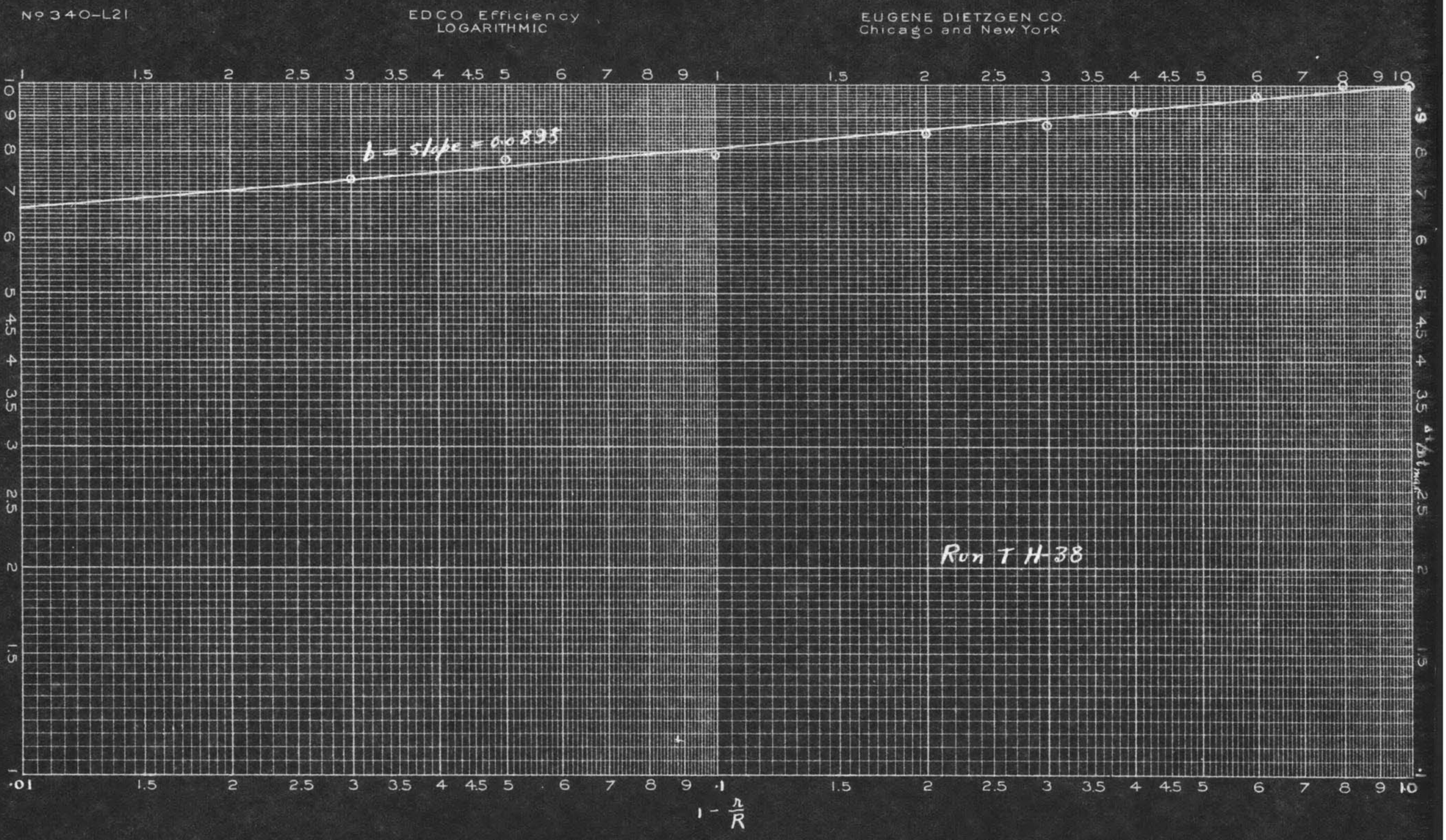
t_w-t_a = 56.36° C.

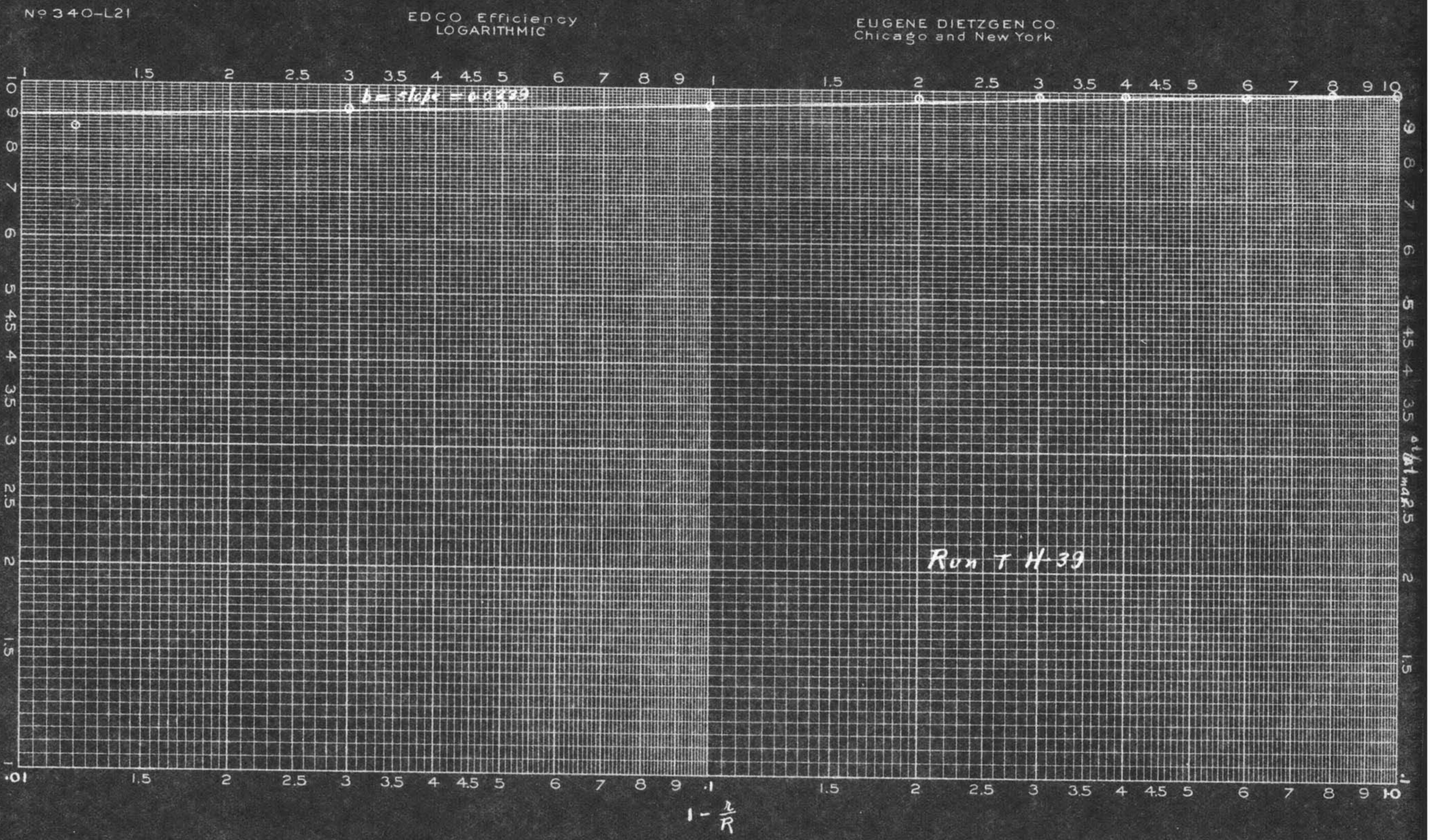
t_{ave} (Graphical Integration) = 29.92° C.

V_{ave} (Graphical Integration) = 2.02 ft./sec.

t_m (Graphical Integration) = 29.55° C.

Nº 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
b = slope = 0.0637
Run T H-37
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
1.0
$1 - \frac{r}{R}$
.9
.8
.7
.6
.5
.4
.3
.2
1.5
.1

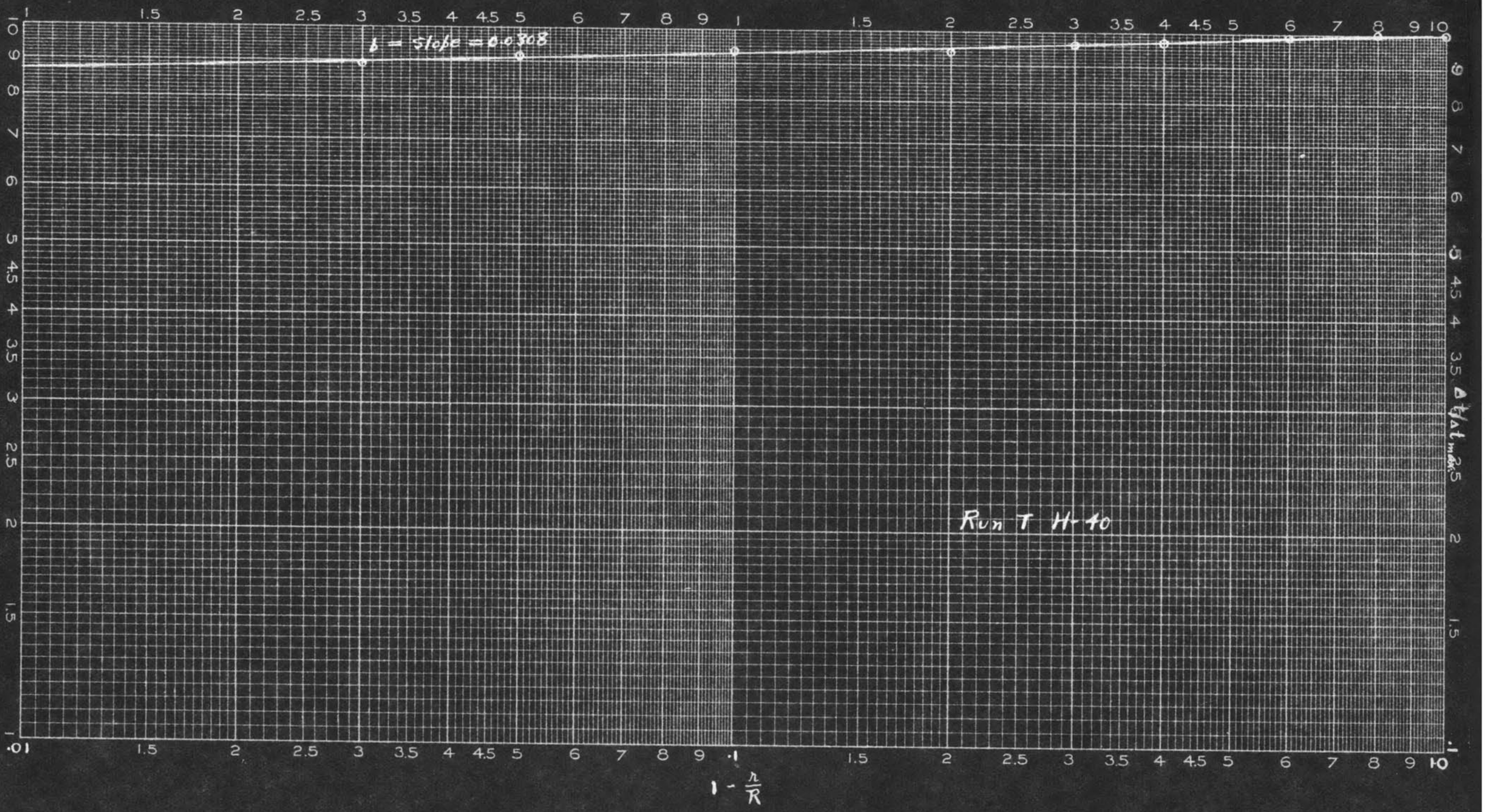
No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
b = slope = 0.0895
Run T H-38
$\Delta t / \Delta t_{max}$
$1 - \frac{r}{R}$
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
1.0
.1
1.5
2
2.5
3
3.5
4
4.5
.5
6
7
8
.9
1.0


No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
b = slope = 0.0239
Run T H-39
1 - $\frac{r}{R}$
Δt/Δt max
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.0
.9
.8
.7
.6
.5
4.5
4
3.5
3
2.5
2
1.5
.1

No 340-L21

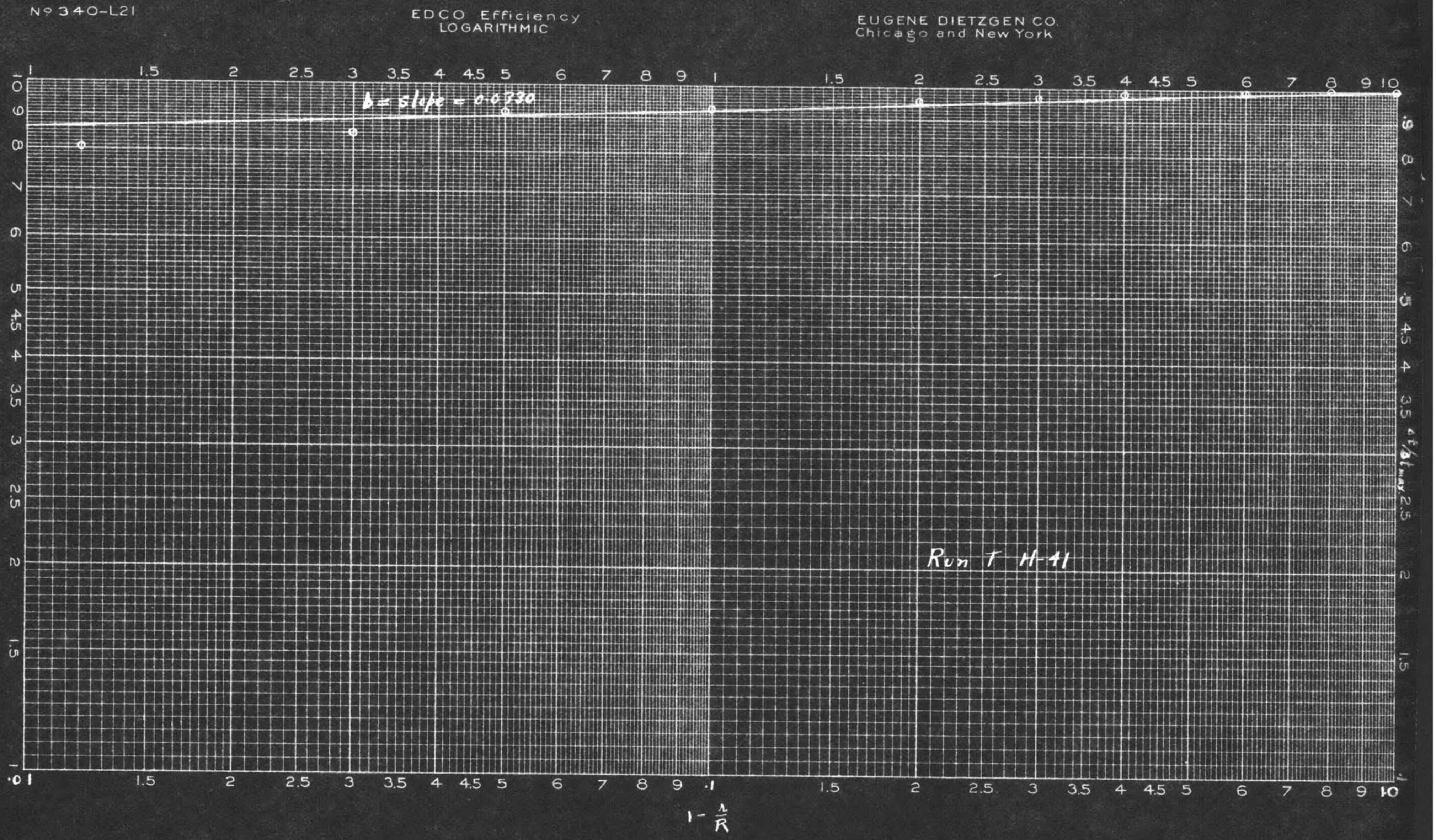
EDCO Efficiency
LOGARITHMIC

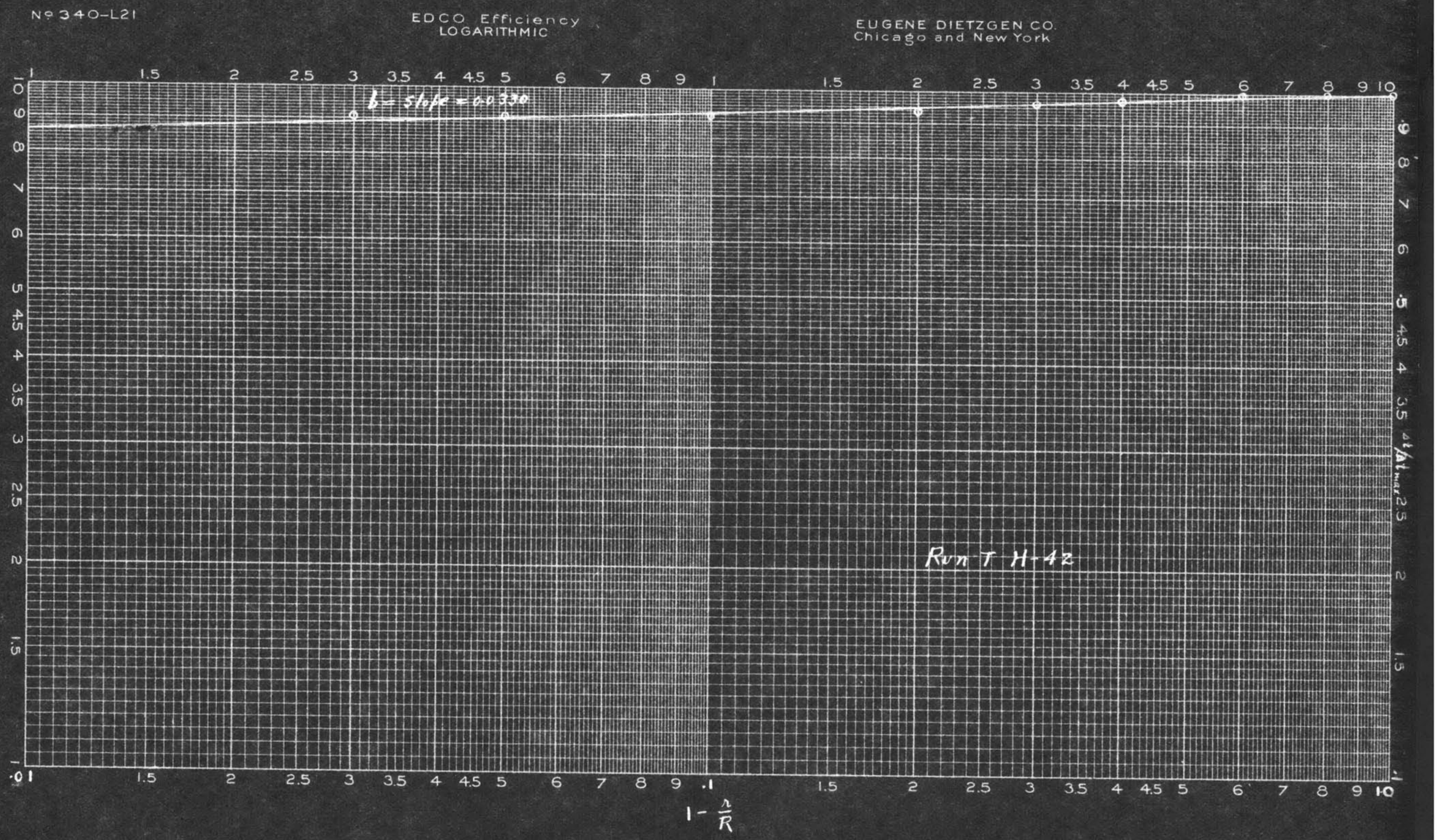
EUGENE DIETZGEN CO.
Chicago and New York

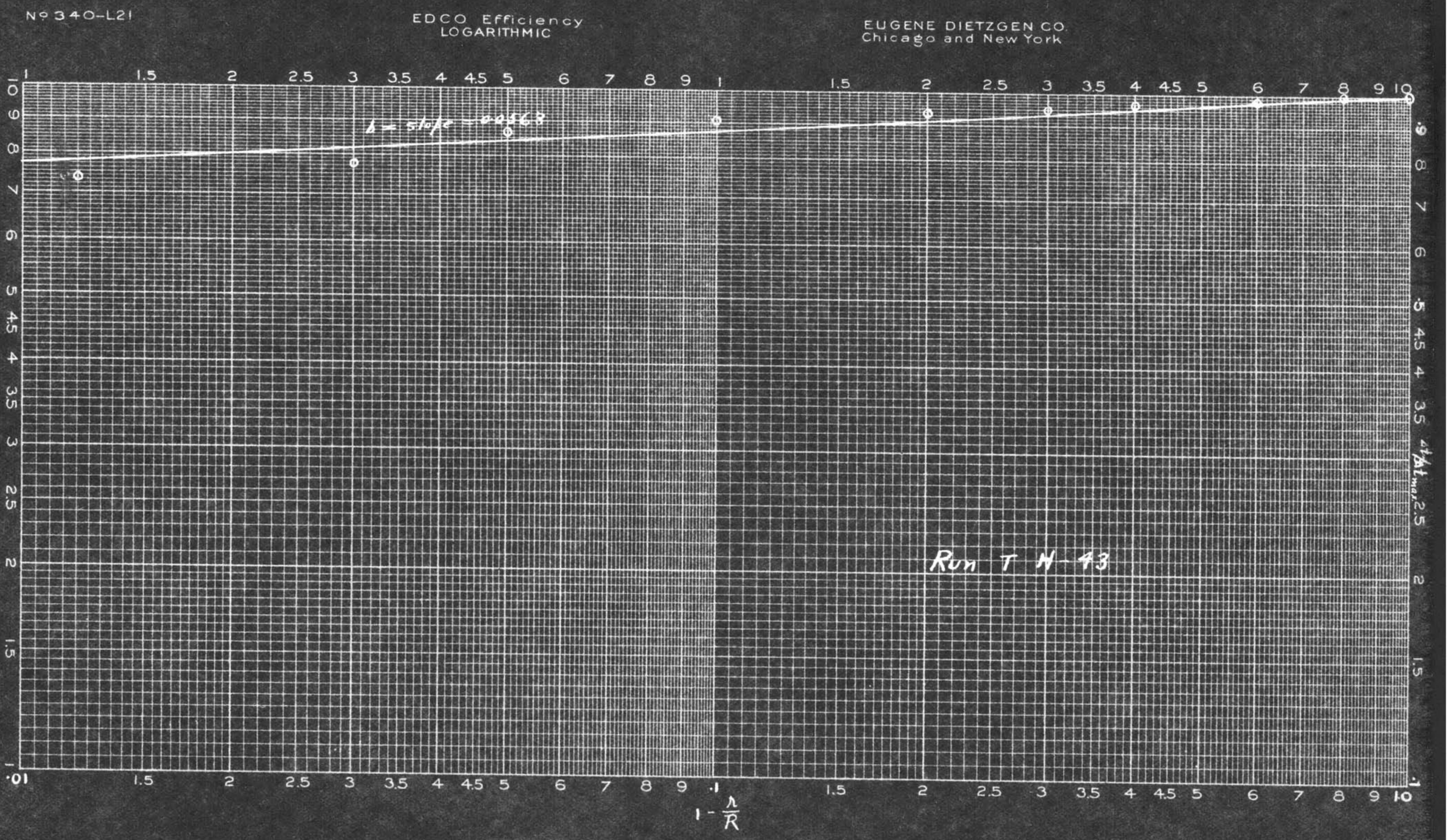


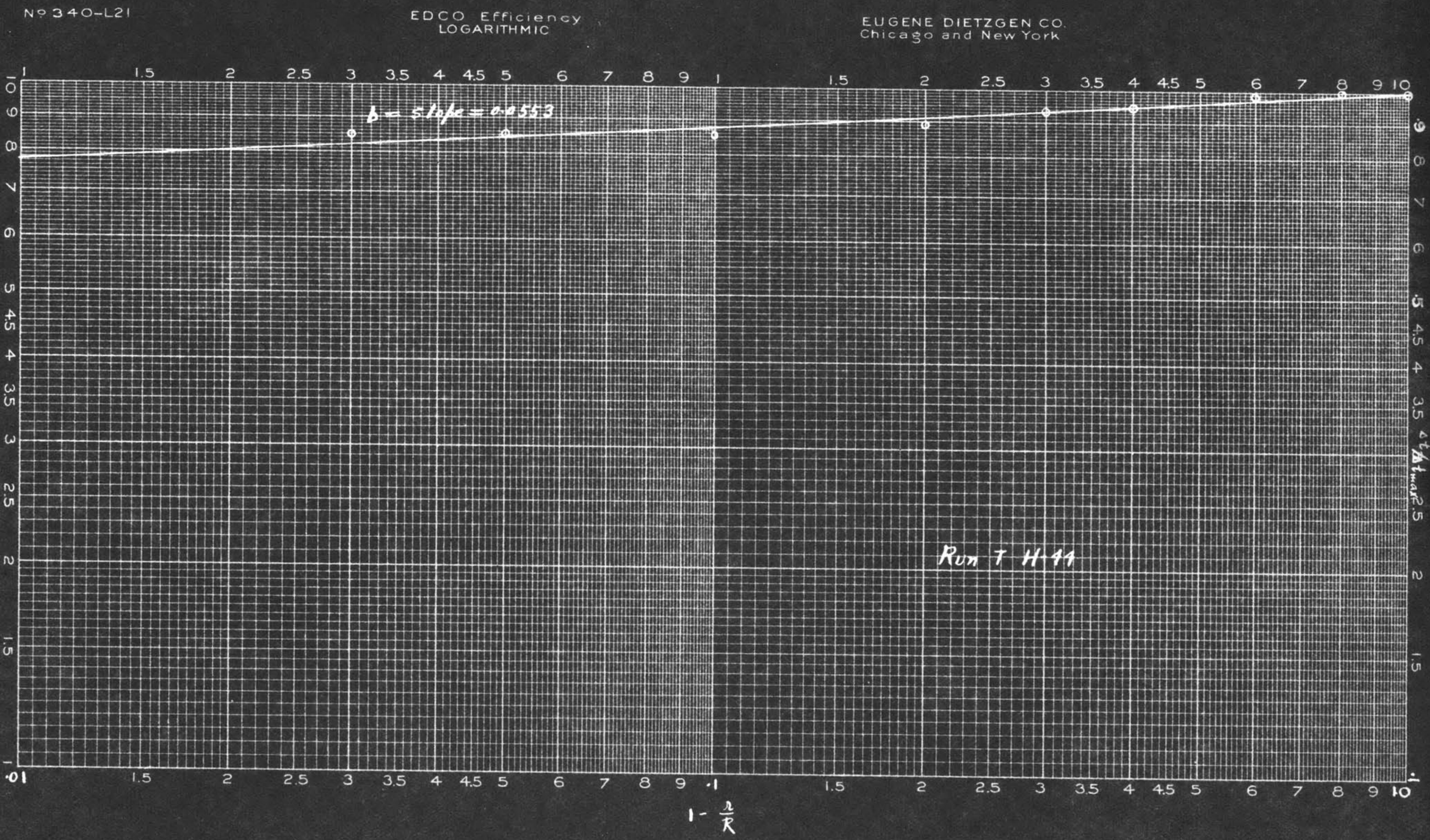
δ = slope = 0.0308

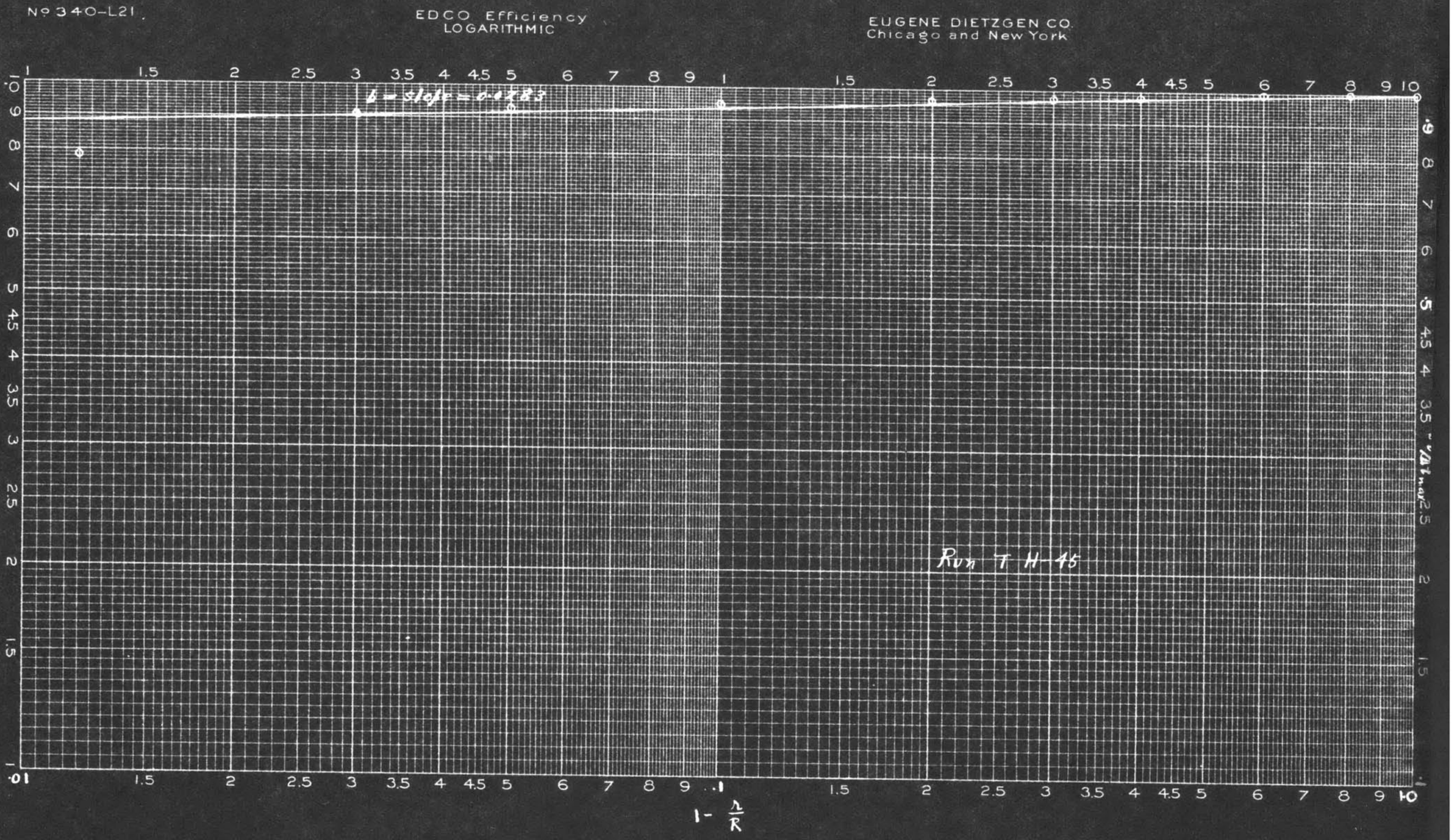
Run T H-40

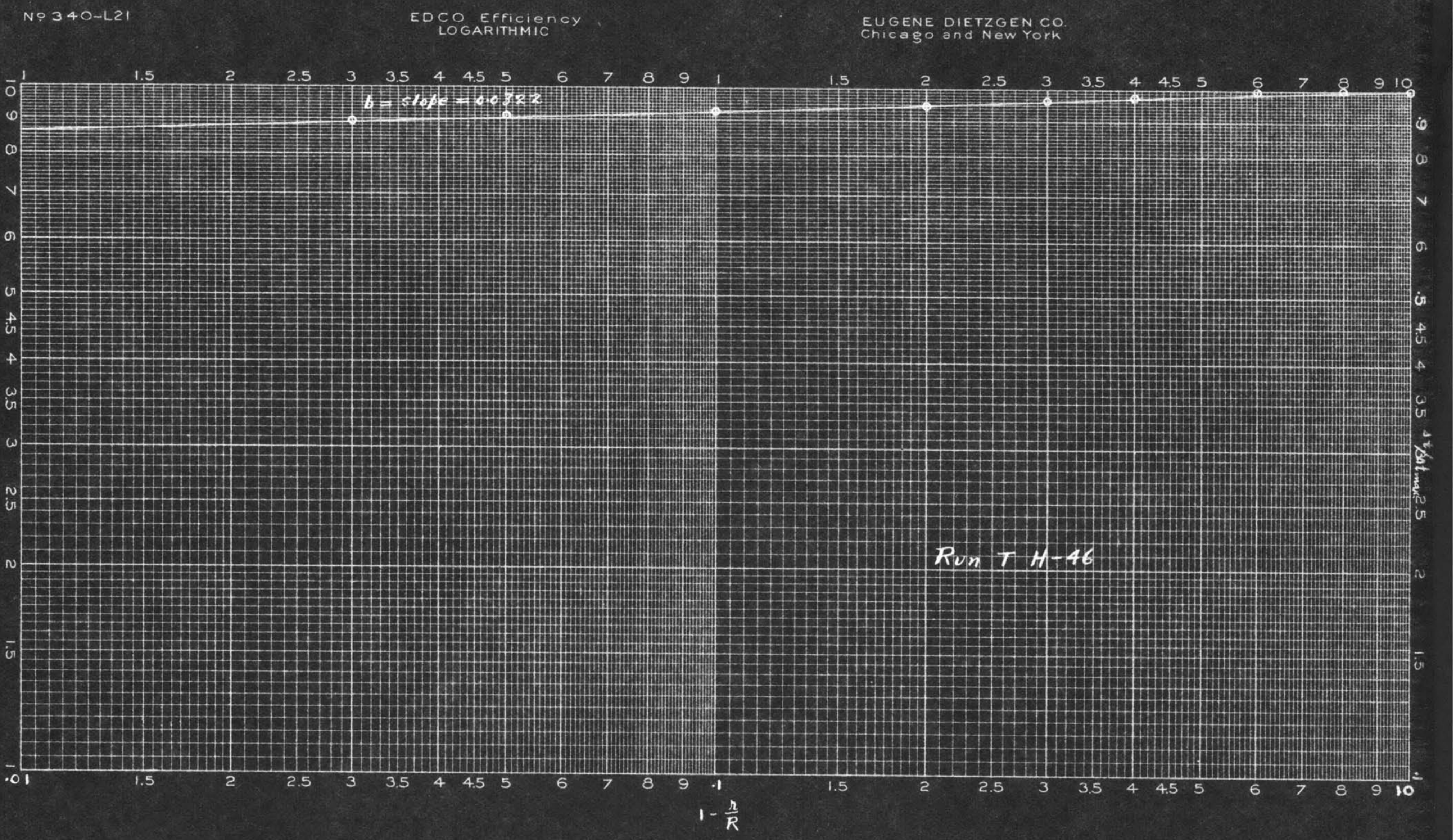
No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
b = slope = 0.0330
Run T H-41
1 - λ/R
Δt/Δt max
.01
.1
1.0
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9

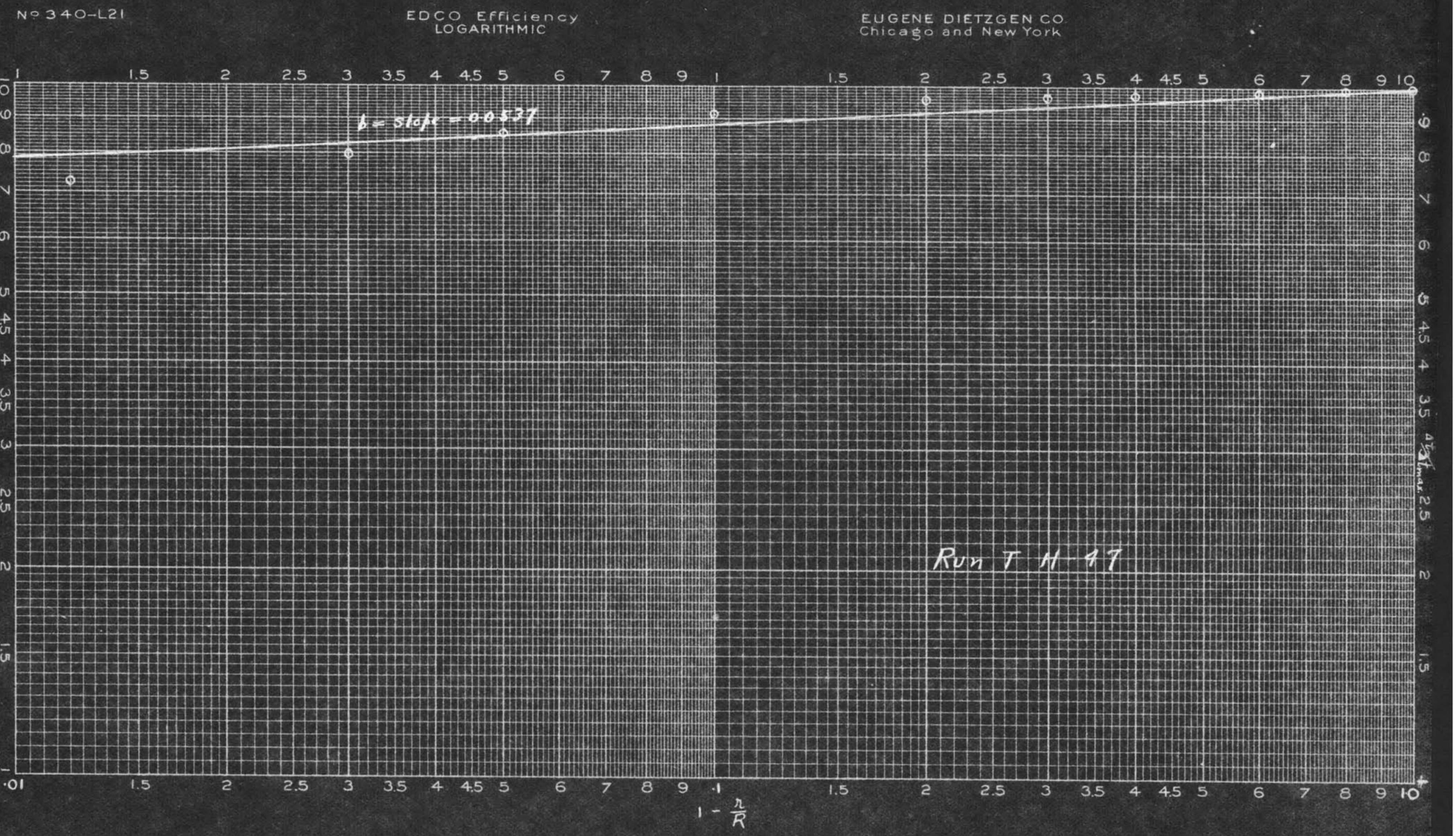
Nº 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
b = slope = 0.0330
Run T H-42
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
1.0
1 - r/R
Δt/Δt max
.9
.8
.7
.6
.5
.4
.3
.2
.1

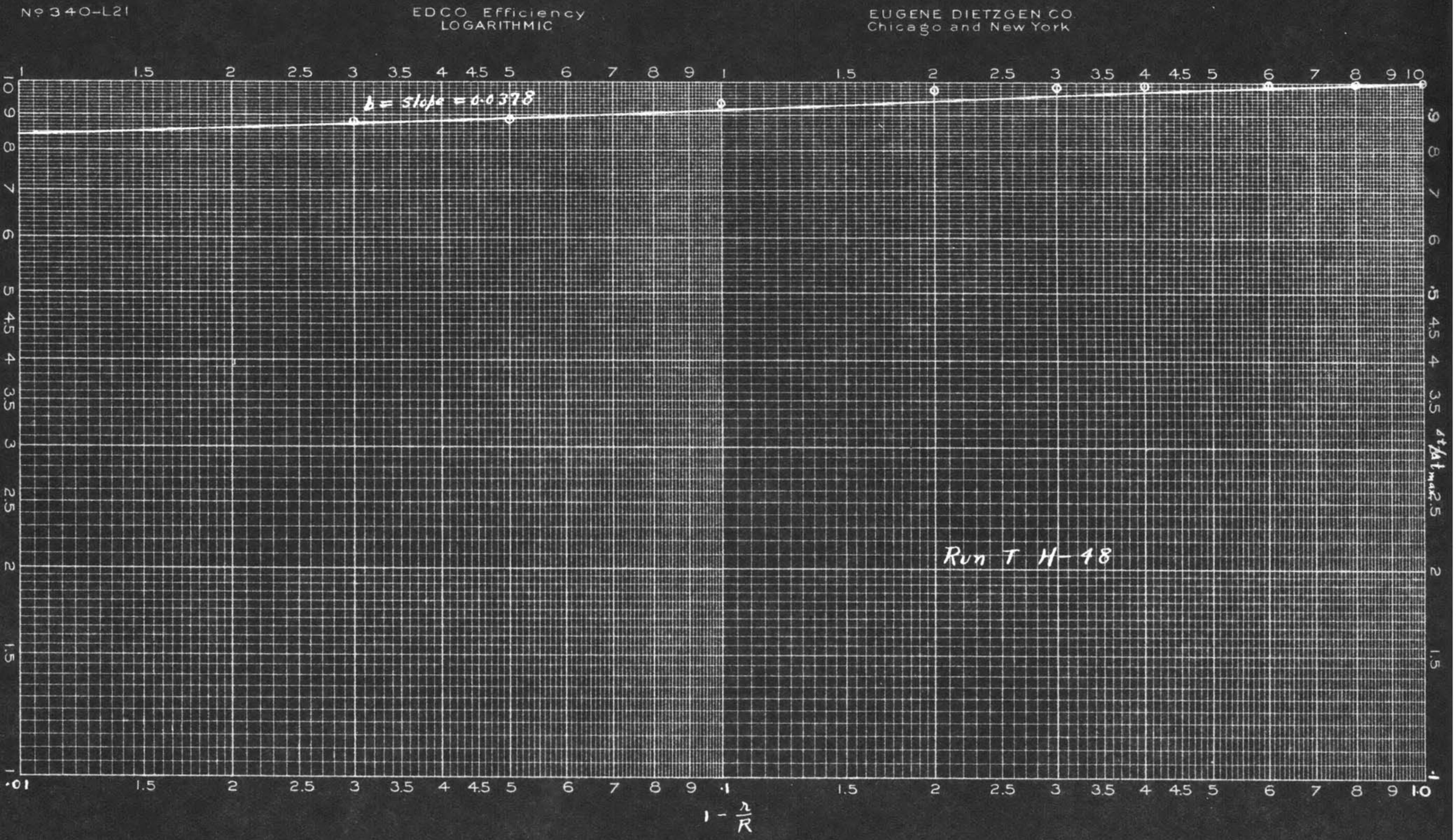

No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
b = slope = .00568
Run T H-43
$1 - \frac{r}{R}$
$\Delta t / \Delta t_{max.}$
.01
.1
1.0
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.9
.8
.7
.6
.5
.4
.3
.2


No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
b = slope = 0.0553
Run T H-44
1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 10
10 9 8 7 6 5 4.5 4 3.5 3 2.5 2 1.5 1
.9 8 7 6 .5 4.5 4 3.5 2.5 2 1.5 .1
.01 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 .1 1.5 2 2.5 3 3.5 4 4.5 5 6 7 8 9 10
$1 - \frac{r}{R}$
$\Delta t / \Delta t_{max}$

No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
b = slope = 0.0783
Run T H-45
$1 - \frac{r}{R}$
v/v max
·01
·1
1·0
·9
·5
·1
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9

No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
b = slope = 0.0322
Run T H-46
$1 - \frac{r}{R}$
$\Delta t / \Delta t_{max}$
.01
.1
10
.1
.5
.9
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9

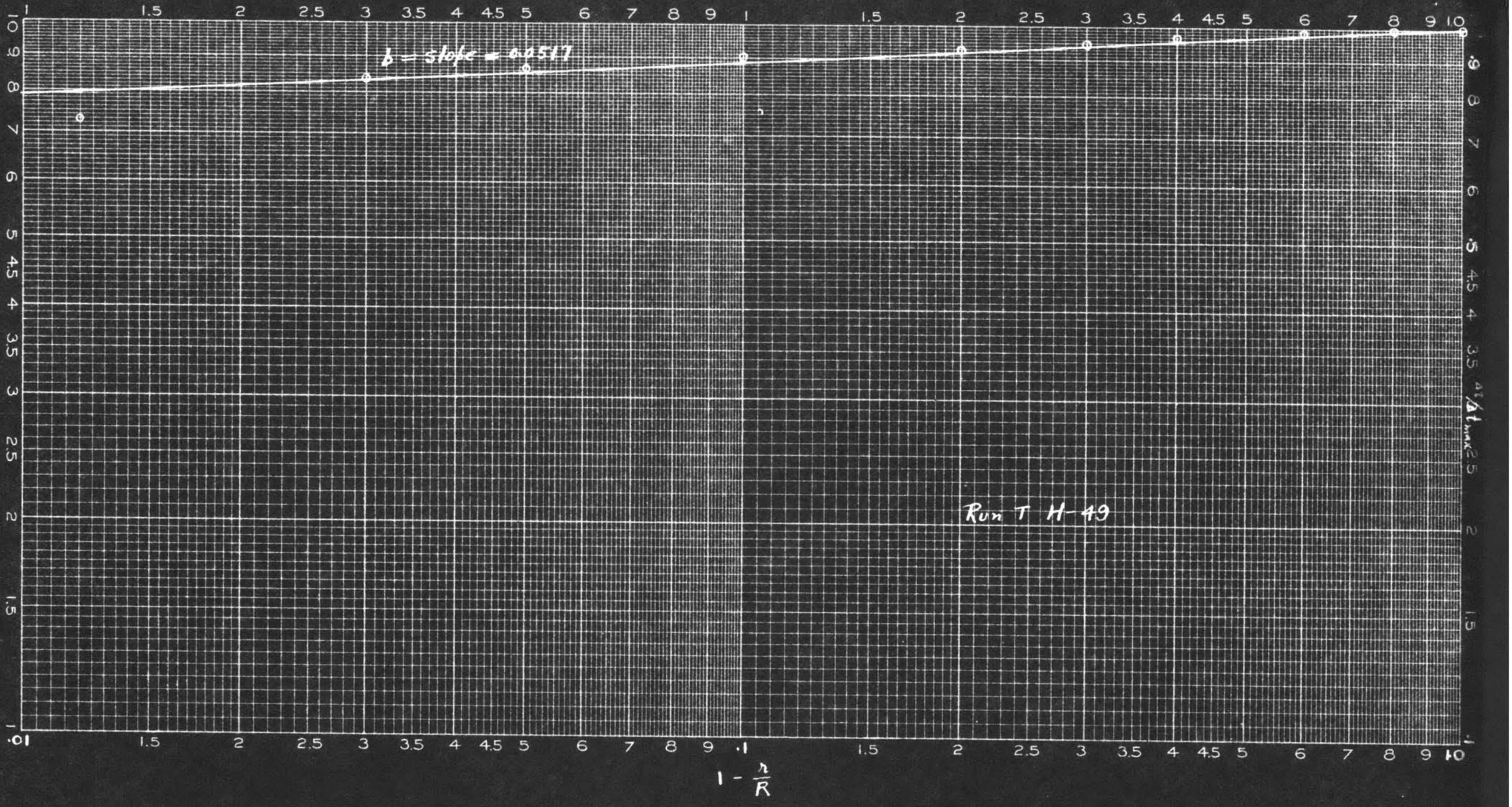

No 340-L21
EDCO Efficiency LOGARITHMIC
EUGENE DIETZGEN CO. Chicago and New York
b = slope = 0.0537
Run T H 47
$1 - \frac{r}{R}$
$\Delta t / \Delta t_{max}$
.01
.1
1
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
10


No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO.
Chicago and New York
b = slope = 0.0378
Run T H-48
Δt/Δt max
1 - r/R
·01
·1
1·0
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
10
·9
·5
·1

No 340-L21

EDCO Efficiency
LOGARITHMIC

EUGENE DIETZGEN CO.
Chicago and New York



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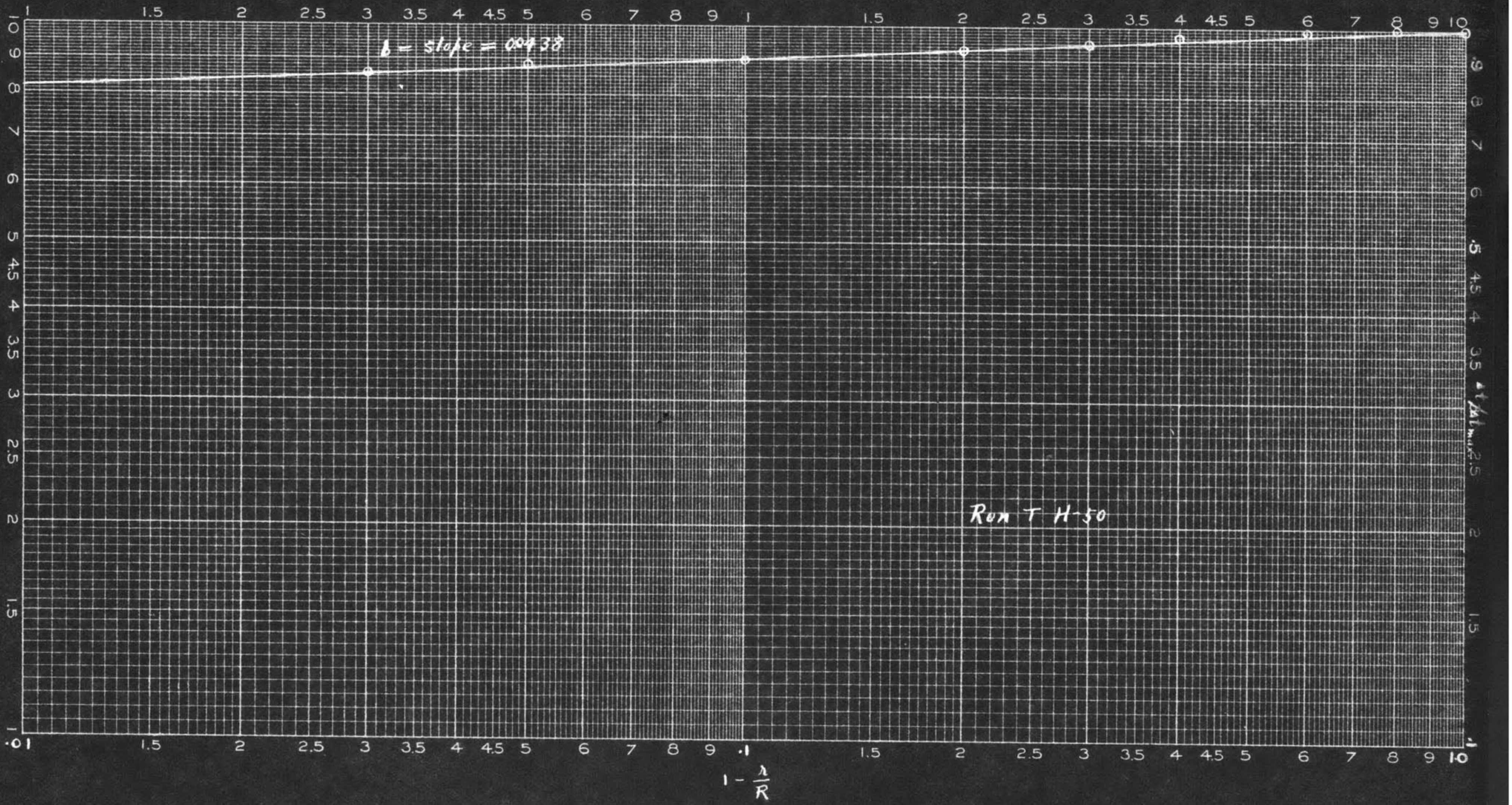
EDCO Efficiency
LOGARITHMIC

EUGENE DIETZGEN CO.
Chicago and New York



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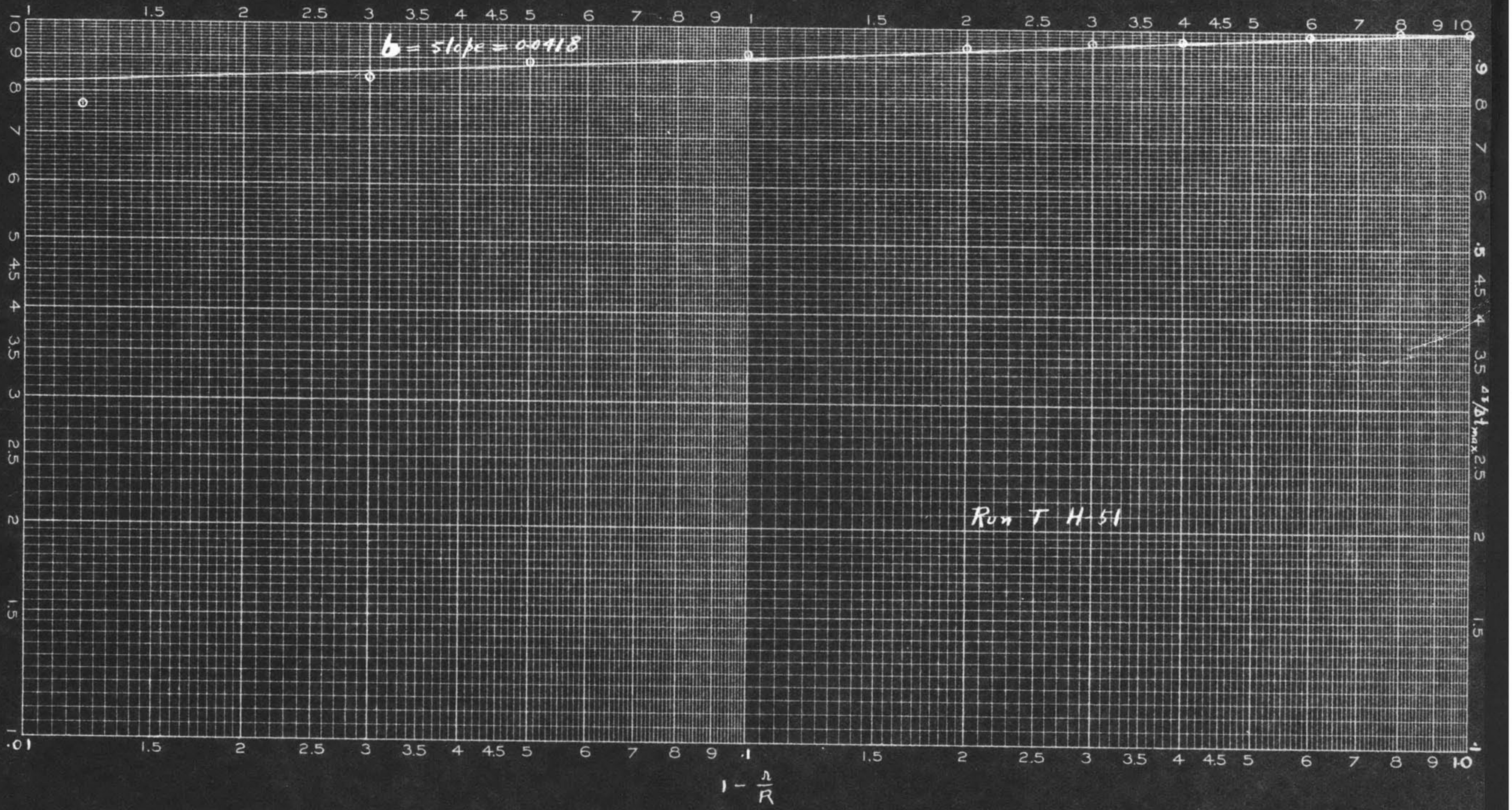
EDCO Efficiency
LOGARITHMIC

EUGENE DIETZGEN CO.
Chicago and New York



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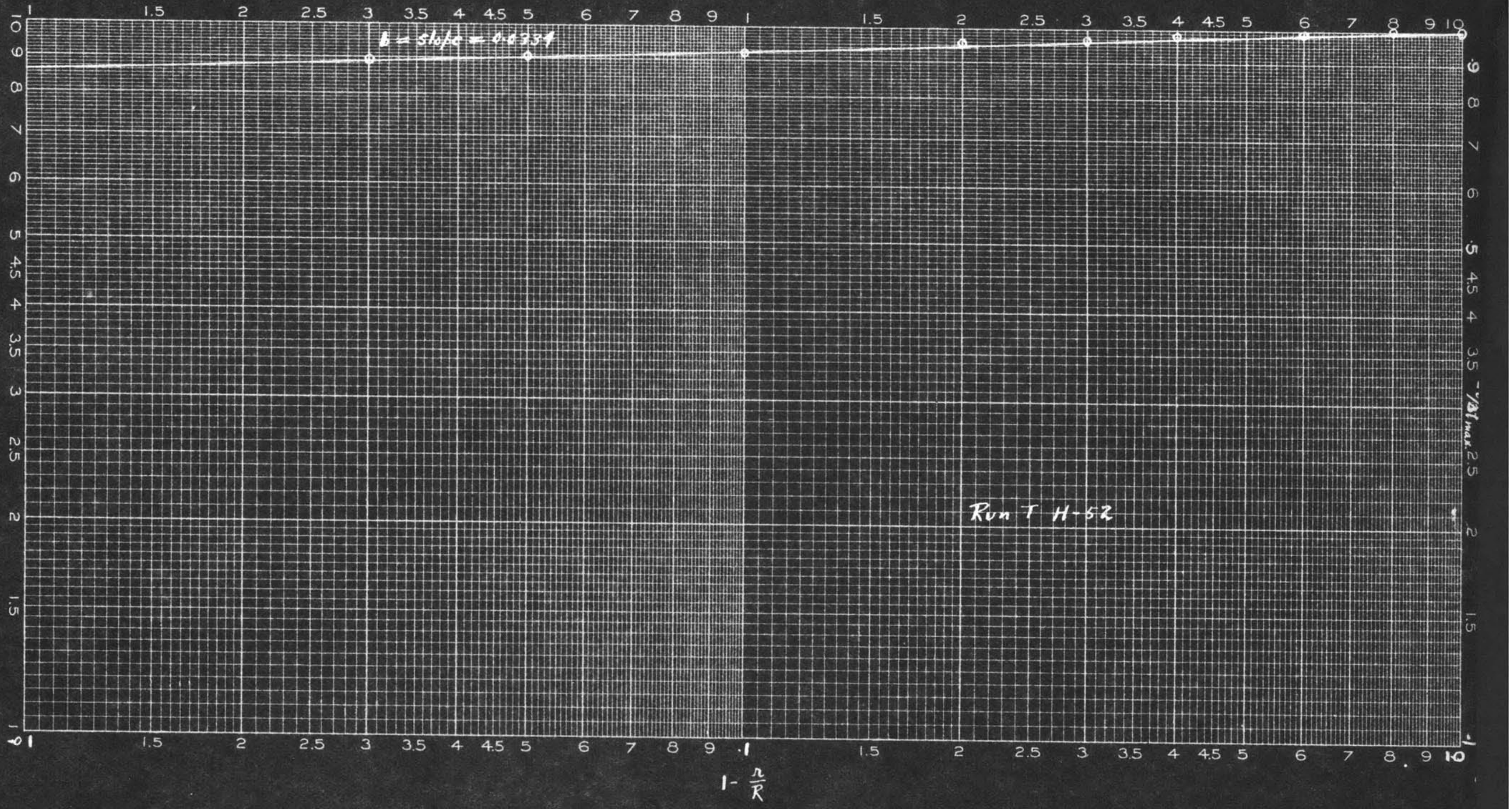
EDCO Efficiency
LOGARITHMIC

EUGENE DIETZGEN CO.
Chicago and New York



Temperature Distribution in the Direction of Flow

Run T H-37 (t_1) t_w (ave.) = 86.94° C. $t_{w(ave.)}-t_1$ = 77.69° C.
Run T H-38 (t_2)

r/R	0.99	.97	.95	.90	.80	.70	.60	.40	.20	0
t_2	48.65	46.15	43.50	42.7	39.45	38.2	35.9	33.2	31.2	31.2
t_1	39.6	34.2	29.3	25.4	21.63	21.26	18.85	18.46	17.82	17.82
(t_2-t_1)	9.05	11.95	14.2	17.3	17.82	16.94	16.05	14.74	13.38	13.38
$\left(\frac{t_2-t_1}{t_{w(av)}-t_1}\right)$	0.117	0.154	0.183	0.223	0.229	0.218	0.2065	0.190	0.172	0.172

Run T H-41 (t_1) $t_{w(ave.)}$ = 72.78° C. $t_{w(ave.)}-t_1$ = 64.18° C.
Run T H-42 (t_2)

r/R	0.99	0.97	0.95	0.90	0.80	0.70	0.60	0.40	0.20	0
t_2(°C.)	29.1	26.9	26.7	26.2	24.9	23.6	22.9	21.4	21.26	21.26
t_1(°C.)	20.1	17.56	14.0	13.2	11.56	10.66	10.04	9.6	9.0	9.0
(t_2-t_1)	9.0	9.34	12.7	13.0	13.34	12.94	12.86	11.8	12.26	12.26
$\frac{t_2-t_1}{t_{w(av)}-t_1}$	0.140	0.1455	0.198	0.2027	0.208	0.202	0.2005	0.184	0.191	0.191

Temperature Distribution in the Direction of Flow

Run T H-43 (t_1)
Run T H-44 (t_2)

$t_{w(ave.)} = 84.42°$ C. $t_{w(ave.)} - t_i = 75.62°$ C.

r/R	0.99	0.97	0.95	0.90	0.80	0.70	0.60	0.40	0.20	0
t_2 (°C.)	43.9	39.9	39.8	39.7	37.7	35.4	34.6	32.6	31.7	31.7
t_1 (°C.)	34.2	31.7	25.4	22.9	20.97	20.0	18.7	17.82	16.56	16.45
$(t_2 - t_1)$	9.7	8.2	14.4	16.8	16.73	15.4	15.9	14.78	15.14	15.25
$\frac{t_2 - t_1}{t_{w(ave.)} - t_i}$	0.128	0.1085	0.191	0.222	0.221	0.204	0.210	0.1955	0.200	0.202

Run T H-45 (t_1)
Run T H-46 (t_2)

$t_{w(ave.)} = 70.95°$ C. $t_{w(ave,)} - t_i = 62.79°$ C,

r/R	0.99	0.97	0.95	0.90	0.80	0.70	0.60	0.40	0.20	0
t_2 (°C.)	32.1	25.6	24.6	23.6	22.5	21.26	20.62	19.34	19.20	19.20
t_1 (°C.)	20.0	13.2	12.3	11.06	10.2	9.87	9.27	8.50	8.35	8.20
$(t_2 - t_1)$	12.1	12.4	12.3	12.54	12.3	11.46	11.35	10.84	10.85	11.00
$\left(\frac{t_2 - t_1}{t_{w(av)} - t_i}\right)$	0.193	0.198	0.196	0.200	0.196	0.1825	0.181	0.173	0.173	0.175

Temperature Distribution in the Direction of Flow

Run T H-47 (t_1) Run T H-48 (t_2) $t_{w(ave.)}$=93.76° C. $t_{w(ave.)}-t_i$ = 84.86° C.

r/R	0.99	0.97	0.95	0.90	0.80	0.70	0.60	0.40	0.20	0
t_2(°C.)	45.2	40.9	40.65	36.7	35.4	34.95	34.35	34.2	34.05	33.85
t_1(°C.)	37.06	31.2	26.8	22.28	18.47	17.96	17.46	16.57	15.80	15.30
t_2-t_1	8.14	9.7	13.85	14.42	16.93	16.99	16.89	17.63	18.25	18.35
$\left(\frac{t_2-t_1}{t_{w(av)}-t_1}\right)$	0.096	0.1043	0.163	0.170	0.1994	0.200	0.199	0.208	0.215	0.216

Run T H-49 (t_1) Run T H-50 (t_2) $t_{w(ave.)}$ = 82.89° C. $t_{w(ave.)}-t_i$ = 74.01° C.

r/R	0.99	0.97	0.95	0.90	0.80	0.70	0.60	0.40	0.20	0
t_2(°C.)	40.9	38.75	37.5	36.4	34.7	33.6	32.4	31.3	30.8	30.8
t_1(°C.)	32.7	25.9	23.9	21.76	19.35	18.70	17.3	16.2	15.4	15.3
t_2-t_1	8.2	12.85	13.6	14.64	15.35	14.9	15.1	15.1	15.4	15.5
$\left(\frac{t_2-t_1}{t_{w(av)}-t_i}\right)$	0.111	0.1736	0.184	0.198	0.207	0.201	0.204	0.204	0.208	0.209

Run T H-51 (t_1) Run T H-52(t_2) $t_{w(ave.)}$=79.11° C, $t_{w(ave.)}-t_i$ = 71.01° C.

r/R	0.99	0.97	0.95	0.90	0.80	0.70	0.60	0.40	0.20	0
t_2(°C.)	35.9	33.4	32.6	31.6	29.8	29.1	28.3	27.8	27.2	27.2
t_1(°C.)	27.6	22.9	20.0	18.34	16.7	15.68	15.17	14.0	13.15	13.15
(t_2-t_1)	8.3	10.5	12.6	13.26	13.1	13.42	13.13	13.8	14.05	14.05
$\left(\frac{t_2-t_1}{t_{w(av)}-t_1}\right)$	0.117	0.148	0.177	0.1865	0.1844	0.189	0.185	0.194	0.198	0.198

APPENDIX I

TEMPERATURE DISTRIBUTION DATA DURING COUNTER CURRENT COOLING

(With Calculations and Plots)

Run T C-1 —— Run T C-6

Temperature Distribution Data During Cooling

Data of Koo and Sung

Run T C-1 (Counter Current) Station No. 4

$\frac{r}{R}$	Rdg. in mv.	$\frac{t}{(°C.)}$	$\frac{t_w-t}{t_w-t_a}$	$(1-\frac{r}{R})$
1.000	1.48	(22.84)	0	0
0.980	1.55	36.2	0.722	0.02
.975	1.625	37.85	.811	.025
.95	1.630	39.55	.901	.05
.94	1.630	39.65	.910	.06
.92	1.665	39.65	.910	.08
.80	1.685	40.50	.955	.20
.70	1.700	40.95	.976	.30
.60	1.700	41.3	1.000	.40
.45	1.700	"	"	.55
.30	1.700	"	"	.70
.15	1.700	"	"	.85
0	1.700	"	"	1.00

$t_{o.w.}$ = 22.16°C.

t_w = 22.84°C. (Temp. rise through pipe wall allowed = 0.68°C.)

$V_{ave.}$(Manometer) = 3.69 ft./sec.

t_a-t_w = 18.46°C.

$t_{ave.}$ (Graphical Integration) = 40.06°C.

Data of Koo and Sung

Run T C-2 (Counter Current) Station No. 4

(Taken Simultaneously with Run T C-3)

$\frac{r}{R}$	Rdg. in mv.	t (°C.)	$\frac{t_w-t}{t_w-t_a}$	$(1 - \frac{r}{R})$
0.99	1.16	28.6	0.768	0.01
.98	1.21	29.8	.914	.02
.97	1.22	30.0	.934	.03
.96	1.23	30.2	.954	.04
.94	1.23	30.2	.954	.06
.92	1.23	30.2	.954	.08
.90	1.23	30.2	.954	.10
.85	1.235	30.3	.980	.15
.80	1.237	30.4	.993	.20
.70	1.240	30.5	1.000	.30
.60	"	"	"	.40
.45	"	"	"	.55
.30	"	"	"	.70
.15	"	"	"	.85
0	"	"	"	1.00

$V_{ave.}$ = 3.67 ft./sec. (Manometer)

$t_{o.w.}$ = 21.9°C.

t_w = 22.17°C. (Temp. rise through pipe wall allowed = 0.27°C.)

$t_a - t_w$ = 8.33°C.

$t_{ave.}$ (Graphical Integration) = 30.22°C.

Data of Koo and Sung

Run T C-3 (Counter Current) Station No. 5

(Taken Simultaneously with Run T C-2)

$\frac{r}{R}$	Rdg. in mv.	$\frac{t}{(°C.)}$	$\frac{t_w-t}{t_w-t_a}$	$(1-\frac{r}{R})$
0.980	1.120	27.65	0.901	0.02
.972	1.135	27.95	.930	.028
.962	1.150	28.3	.971	.038
.94	1.150	28.3	.971	.06
.92	1.150	28.3	.971	.08
.90	1.150	28.3	.971	.10
.85	1.155	28.45	.982	.15
.80	1.155	28.45	.982	.20
.70	1.160	28.60	1.000	.30
.60	1.160	28.60	1.000	.40
.45	"	"	"	.55
.30	"	"	"	.70
.15	"	"	"	.85
0	"	"	"	1.00

$V_{ave.}$ = 3.67 ft./sec. (Manometer)

$t_{o.w.}$ = 18.7°C.

t_w = 19.0°C. (Temp. rise through pipe wall allowed = 0.3°C.)

$t_a - t_w$ = 9.6°C.

$t_{ave.}$ (Graphical Integration) = 28.33°C.

Data of Koo and Sung

Run T C-4 (Counter Current) Station No. 4

(Taken Simultaneously with Run T C-5)

$\frac{r}{R}$	Rdg. in mv.	$\frac{t}{(°C.)}$	$\frac{t_w - t}{t_w - t_a}$	$(1 - \frac{r}{R})$
0.990	1.312	32.2	0.748	0.01
.980	1.348	33.05	.850	.02
.970	1.365	33.45	.898	.03
.96	1.383	33.85	.952	.04
.94	1.385	33.95	.960	.06
.90	1.385	33.95	.960	.10
.85	1.390	34.05	.973	.15
.80	1.390	34.05	.973	.20
.45	1.390	34.05	.973	.55
.30	1.400	34.30	1.000	.70
.15	1.400	"	1.000	.85
0	1.400	"	1.000	1.00

$V_{ave.}$ = 0.855 ft./sec. (Manometer)

$t_{o.w.}$ = 26.0°C.

t_w = 26.16°C. (Temp. rise through pipe wall allowed = 0.16°C.)

$t_a - t_w$ = 8.14°C.

$t_{ave.}$ (Graphical integration) = 34.0°C.

Data of Koo and Sung

Run T C-5 (Counter Current) Station No. 5

(Taken Simultaneously with Run T C-4)

$\frac{r}{R}$	Rdg. in mv.	$\frac{t}{(°C.)}$	$\frac{t_w-t}{t_w-t_a}$	$(1 - \frac{r}{R})$
0.980	1.025	25.4	0.928	0.02
.972	1.038	25.7	.976	.028
.962	1.040	25.75	.984	.038
.940	1.042	25.8	.992	.06
.92	"	"	.992	.08
.90	"	"	.992	.10
.85	"	"	.992	.15
.80	"	"	.992	.20
.70	1.045	25.85	1.000	.30
.60	"	"	"	.40
.45	"	"	"	.55
.30	"	"	"	.70
.15	"	"	"	.85
0	"	"	"	1.00

$V_{ave.}$ (Manometer) = 0.855 ft./sec.

$t_{o.w.}$ = 18.84°C.

t_w = 18.89°C. (Temp. rise through pipe wall allowed = 0.05°C.)

$t_a - t_w$ = 6.96°C.

$t_{ave.}$ (Graphical Integration) = 25.76°C.

Data of Koo and Sung

Run T C-6 (Counter Current) Station No. 4

$\frac{r}{R}$	Rdg. in mv.	$\frac{t}{(°C.)}$	$\frac{t_w-t}{t_w-t_a}$	$(1 - \frac{r}{R})$
0.990	1.315	32.26	0.673	0.01
.982	1.410	34.55	.857	.018
.97	1.422	34.8	.880	.03
.96	1.432	35.05	.898	.04
.94	1.440	35.25	.915	.06
.90	1.448	35.45	.928	.10
.85	1.460	35.7	.951	.15
.80	1.468	35.9	.970	.20
.70	1.478	36.15	.987	.30
.60	1.482	36.25	.9955	.40
.45	1.482	36.25	"	.55
.30	1.485	36.3	1.000	.70
.15	"	"	"	.85
0	"	"	"	1.00

$V_{ave.}$ (Manometer) = 0.769 ft./sec.

$t_{o.w.}$ = 23.8°C.

t_w = 23.89 (Temp. rise through pipe wall allowed = 0.09°C.)

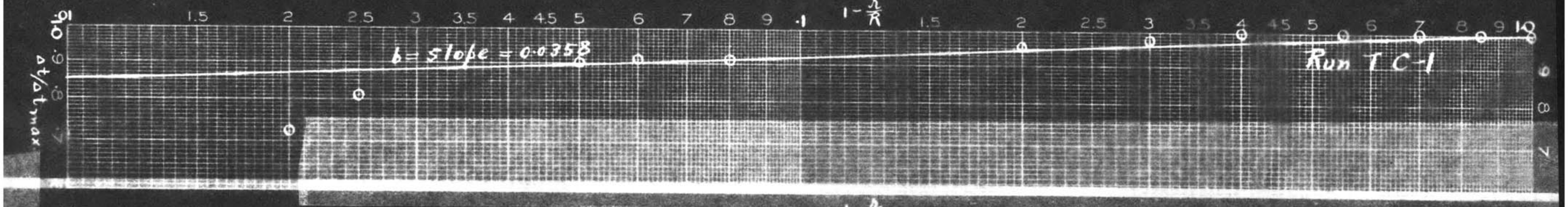
$t_a - t_w$ = 12.41°C.

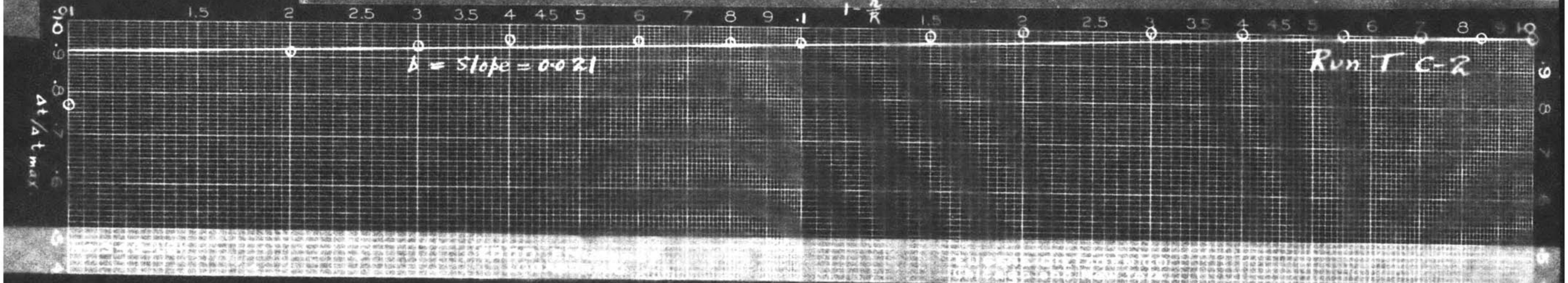
$t_{ave.}$ (Graphical Integration) = 35.73°C.

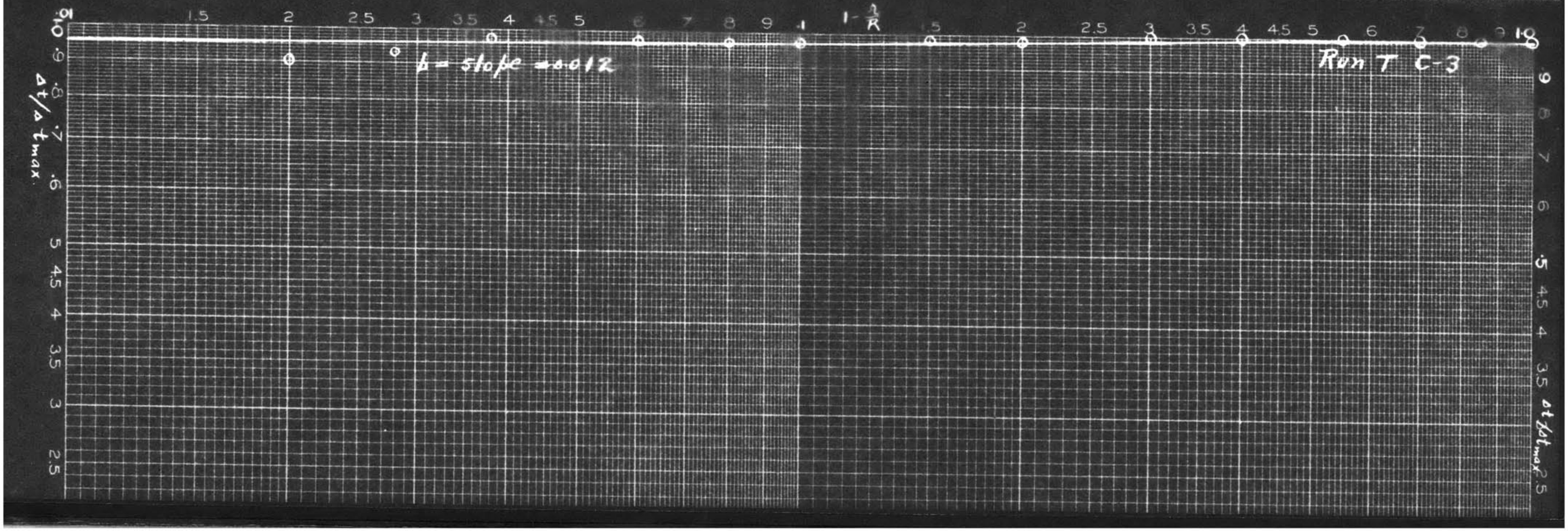
No 340-L21

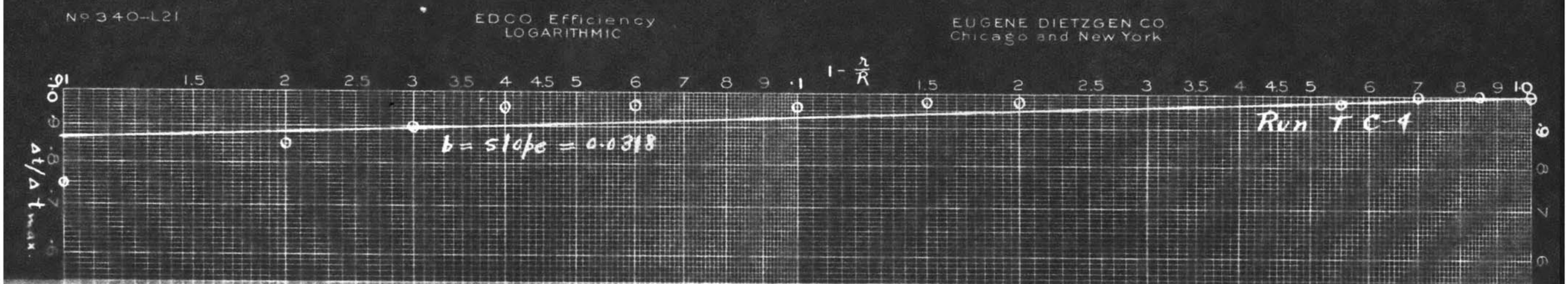
EDCO Efficiency
LOGARITHMIC

EUGENE DIETZGEN CO
Chicago and New York

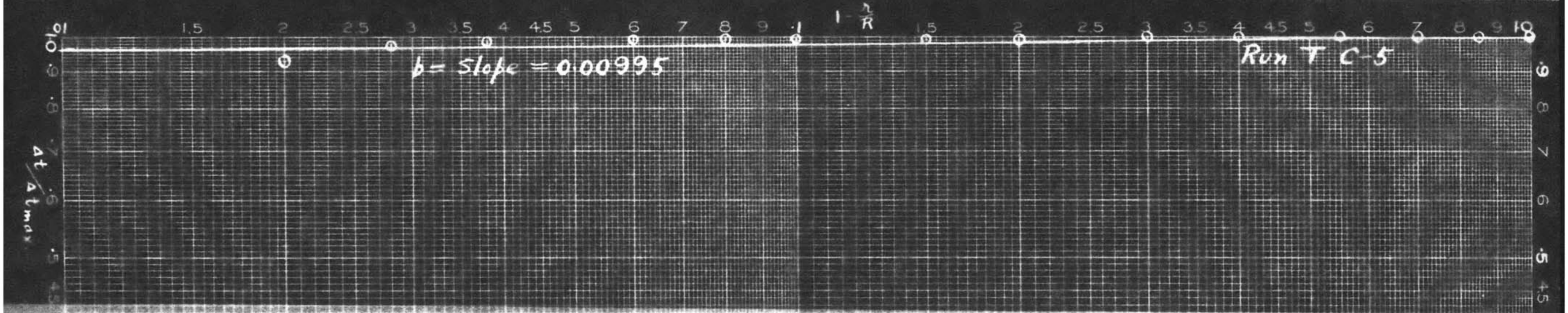




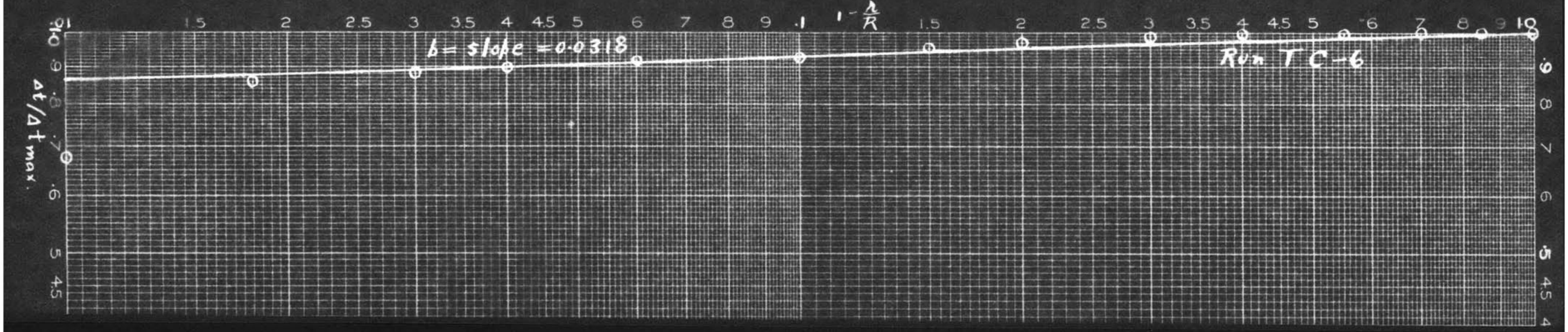




No 340-L21
EDCO Efficiency
LOGARITHMIC
EUGENE DIETZGEN CO
Chicago and New York
$1-\frac{r}{R}$
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
1.0
$\Delta t/\Delta t_{max}$
1.0
.9
.8
.7
.6
b = slope = 0.0318
Run TC-4



$1-\frac{r}{R}$
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
1.0
$\Delta t/\Delta t_{max}$
1.0
.9
.8
.7
.6
.5
4.5
b = slope = 0.00995
Run TC-5



$1-\frac{r}{R}$
.01
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
.1
1.5
2
2.5
3
3.5
4
4.5
5
6
7
8
9
1.0
$\Delta t/\Delta t_{max}$
1.0
.9
.8
.7
.6
.5
4.5
b = slope = 0.0318
Run TC-6

APPENDIX J

A. Calibration of Water Orifice.

B. Calibration of Thermocouples (Charts).

C. Measurements on Thermocouples and Pitot Tubes.

D. Determination of Specific Gravity of CCl_4.

E. Measurements on Copper Pipe.

F. Kinematic Viscosity of Water (Table and Chart).

G. Prandtl's Number of Water (Table and Chart)

H. Conversion of Cm. CCl_4 to Velocity in Feet per Second (Table and Chart).

APPENDIX J

A. Calibration of Water Orifice

The manometer used was a mercury manometer inclined with a slope of 0.228 so that the reading was magnified $\frac{1}{0.228}$ or 4.385 times. Inside diameter of nominal 3" pipe = 3.07"

Diameter of orifice = $1\frac{11}{16}$" = 1.6875"

Area of orifice opening = 0.01554 sq.ft.

$$\therefore \quad V_2^2 - V_1^2 = C^2\,(2g\Delta h)$$

where Δh in ft. of water

$$\text{or} \quad V_2^2 \quad 1 - \left(\frac{1.6875}{3.07}\right)^4 = C^2\,(2g\Delta h)$$

$$\therefore \quad V_2 = 8.41\, C \sqrt{\Delta h} \quad \text{ft./sec.}$$

$$\text{and } Q = 0.01554\, V_2 = 0.1308\, C \sqrt{\Delta h} \quad \text{cu.ft./sec.}$$

$$\text{or} \quad C = \frac{7.65Q}{\sqrt{\Delta h}}$$

Let h' be the change of mercury levels before and when running in cm. of one column of the inclined manometer, then

$$\Delta h\ (\text{ft.of water}) = \frac{h' \times 2 \times 13.6}{4.385 \times 2.54 \times 12} = 0.203\, h'$$

$$\therefore \quad C = \frac{7.65\, Q}{0.203\sqrt{h'}} = \frac{17.28\, Q}{\sqrt{h'}}$$

h'	$\sqrt{h'}$	Q_1 (ft^3/sec.) measured	C	Ave
7.00	2.646	0.0952	0.621	
5.95	2.440	0.0949	0.672	
8.55	2.925	0.0978	0.578	
6.88	2.623	0.0948	0.623	
			(2.494)	0.6235
12.75	3.570	0.1210	0.586	
12.35	3.513	0.1213	0.598	
11.75	3.425	0.1224	0.617	
12.90	3.590	0.1217	0.585	
			(2.386)	0.5965
15.00	3.875	0.1366	0.609	
14.60	3.820	0.1368	0.619	
16.60	4.070	0.1358	0.576	
16.40	4.050	0.1352	0.577	
			(2.381)	0.5952
17.20	4.150	0.1472	0.614	
18.90	4.345	0.1550	0.616	
16.70	4.080	0.1508	0.638	
19.00	4.350	0.1493	0.593	
			(2.461)	0.6152

$$\therefore \quad C_{ave} = 0.6076$$

$$= 0.608$$

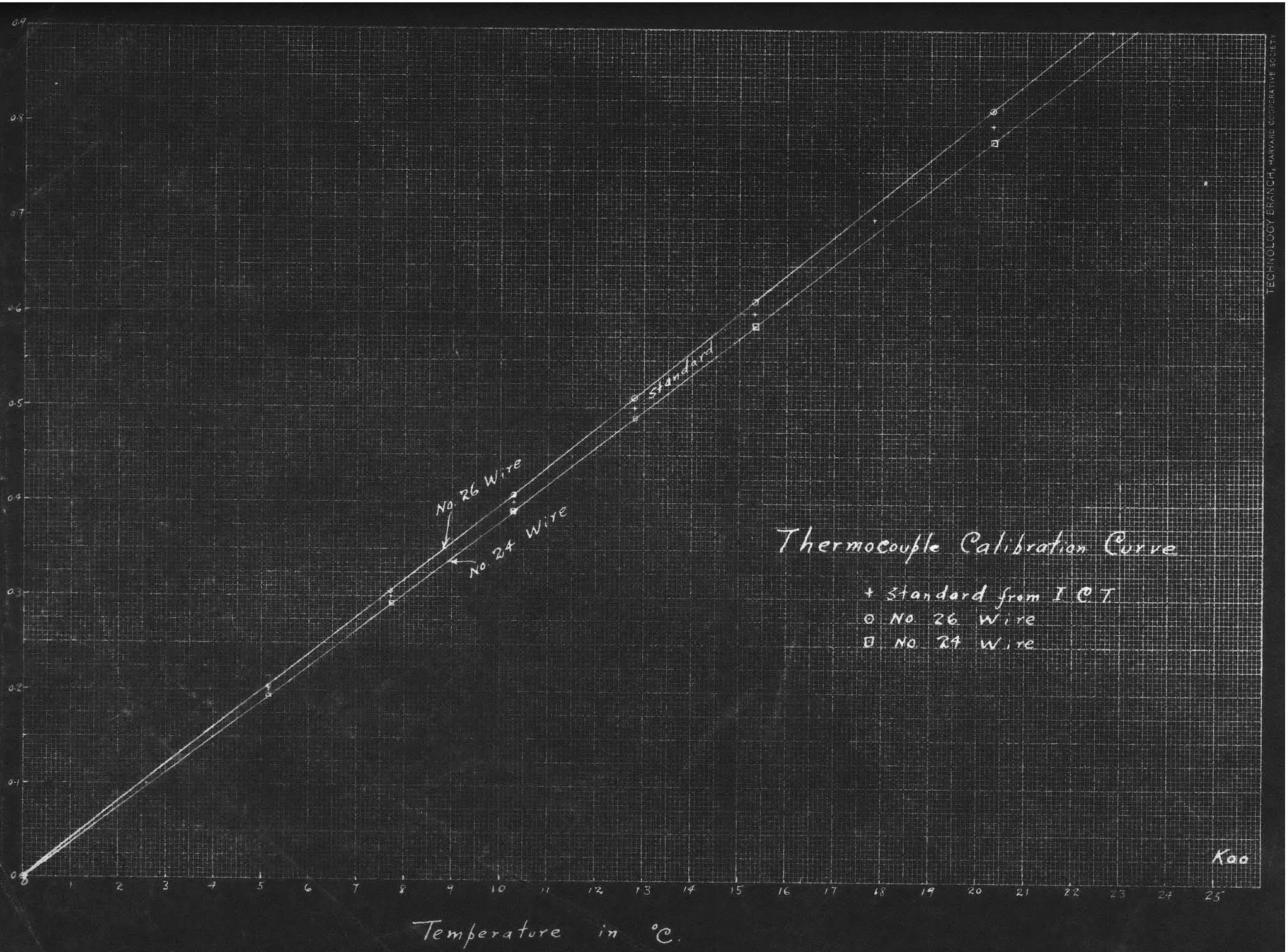
$$\therefore \quad Q = \frac{0.608 \quad h'}{17.28} = 0.0352 \sqrt{h'} \text{ cu.ft./sec.}$$

and

$$V_{ave} = 2.263 \sqrt{h'} \quad \text{ft./sec.}$$

B. <u>Calibration of Thermocouples</u>

Both No. 24 and No. 26 constantan-copper thermocouples were calibrated against ice and boiling water. The intermediate points were interpretated from the standard values given in International Critical Tables. The results are shown in Figure



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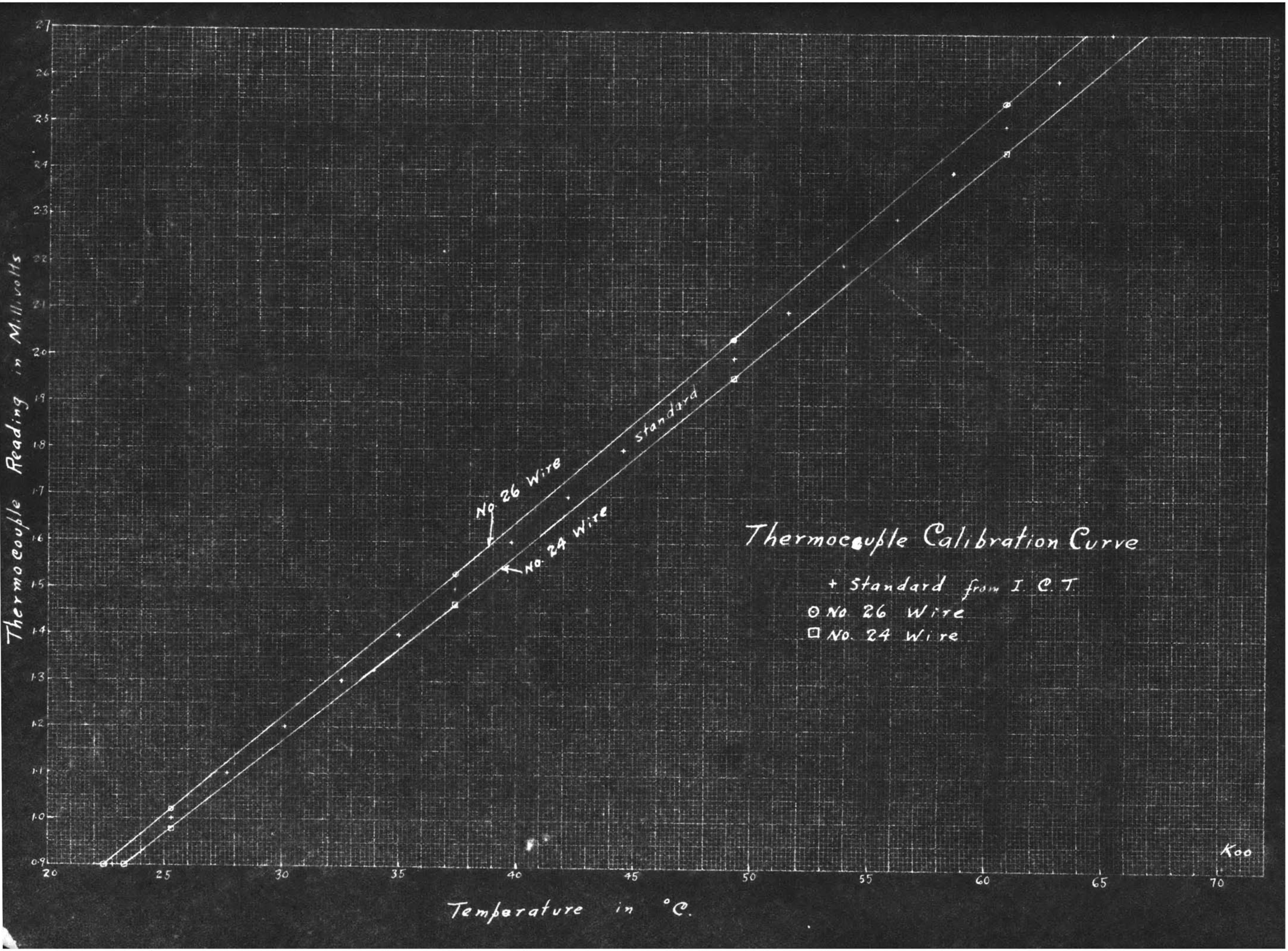
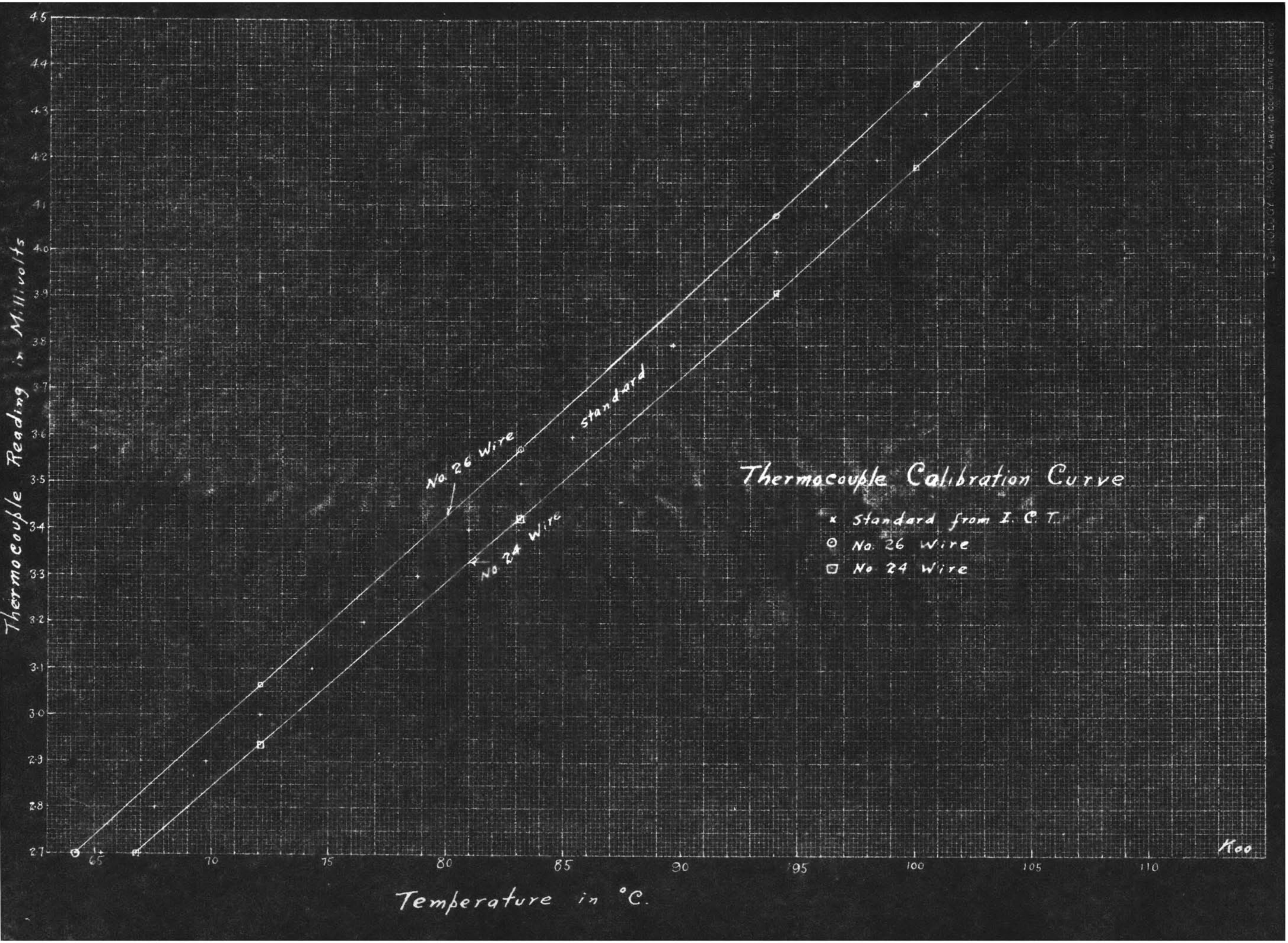

Thermocouple Calibration Curve
+ Standard from I. C. T.
⊙ No. 26 Wire
☐ No. 24 Wire
No. 26 Wire
No. 24 Wire
Standard
Thermocouple Reading in Millivolts
Temperature in °C.
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
2.4
2.5
2.6
2.7
20
25
30
35
40
45
50
55
60
65
70
Koo


Thermocouple Calibration Curve
× Standard from I. C. T.
⊙ No. 26 Wire
⊡ No. 24 Wire
No. 26 Wire
Standard
No. 24 Wire
Thermocouple Reading in Millivolts
Temperature in °C.
4.5
4.4
4.3
4.2
4.1
4.0
3.9
3.8
3.7
3.6
3.5
3.4
3.3
3.2
3.1
3.0
2.9
2.8
2.7
65
70
75
80
85
90
95
100
105
110
Koo

C. Measurements on Thermocouples and Pitot Tubes

Thermocouples:

No.	Diameter of Pipe in mm.	Outer edge to center line of tip in mm.
1	0.595	1.185
2	0.627	0.485
3	0.595	0.870
4	0.735	0.579

Pitot Tubes:

	Inside diameter in mm.	Outer edge to center line of tip in mm.
1	0.280	0.361
2	0.396	0.278
3	0.414	0.320
4	0.424	0.305
5	0.380	0.312
6	0.388	0.300
7	0.369	0.309
8	0.417	0.307

D. Determination of Specific Gravity of CCl_4

No. Determinations	Wt. of CCl_4
1	3.1643
2	3.1685
3	3.1510
4	3.1542
5	3.1545
Average	3.1585

Calibrated capacity of 2 cc. pipette = 2.0038 cc.

$$\therefore \text{ Specific gravity of } CCl_4 = \frac{3.1585}{2.0038} = 1.5762$$

E. Measurements on Copper Pipe

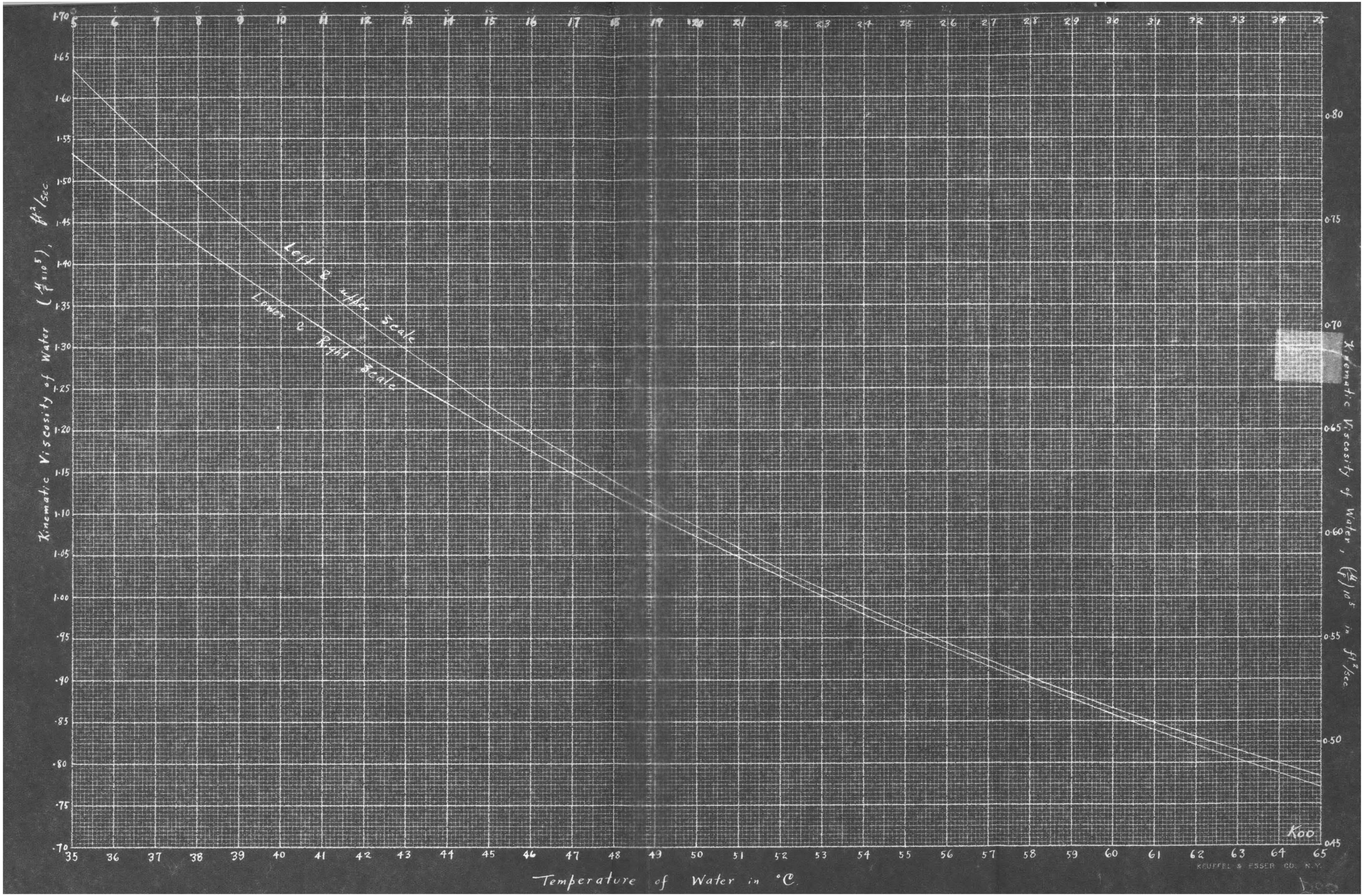
Test cross section No. (or Station No.)	Inside Diameter cm.	Wall Thickness cm.
1	4.9585	0.3784
2	4.9597	0.3778
3	4.9535	0.3846
4	4.9570	0.3830
5	4.9549	0.3818

Table **Kinematic Viscosity of Water**

(Calculated from Smithsonian
Tables by C.Y. Hsiao)

Temperature °C.	Temperature °F.	Viscosity (centipoise)	Density (gm./cc.)	Kinematic Viscosity (cm.^{2}/sec.) X 10^2	Kinematic Viscosity (ft.^{2}/sec.) X 10^5
0	32.0	1.7921	0.99987	1.79233	1.92924
1	33.8	1.7313	.99993	1.73142	1.86368
2	35.6	1.6728	.99997	1.67285	1.80063
3	37.4	1.6191	.99999	1.61912	1.74280
4	39.2	1.5674	1.00000	1.56740	1.68713
5	41.0	1.5188	.99999	1.51882	1.63484
6	42.8	1.4728	.99997	1.47284	1.58535
7	44.6	1.4284	.99993	1.42850	1.53762
8	46.4	1.3860	.99988	1.38617	1.49205
9	48.2	1.3462	.99981	1.34646	1.44931
10	50.0	1.3077	.99973	1.30805	1.40797
11	51.8	1.2713	.99963	1.27177	1.36892
12	53.6	1.2363	.99952	1.23689	1.33137
13	55.4	1.2028	.99940	1.20352	1.29545
14	57.2	1.1709	.99927	1.17176	1.26127
15	59.0	1.1404	.99913	1.14139	1.22858
16	60.8	1.1111	.99897	1.11225	1.19721
17	62.6	1.0828	.99880	1.08410	1.16691
18	64.4	1.0559	.99862	1.05736	1.13813
19	66.2	1.0299	.99843	1.03152	1.11031
20	68.0	1.0050	.99823	1.00678	1.08368
21	69.8	.9810	.99802	0.98295	1.05803
22	71.6	.9579	.99780	.96001	1.03334
23	73.4	.9358	.99757	.93808	1.00974
24	75.2	.9142	.99733	.91665	.98667
25	77.0	.8937	.99708	.89632	.96479
26	78.8	.8737	.99682	.87649	.94344
27	80.6	.8545	.99655	.85746	.92296

Temperature		Viscosity (centipose)	Density (gm./cc.)	Kinematic Viscosity	
°C.	°F.			(cm.^{2}/sec.) X 10^2	(ft.^{2}/sec.) X 10^5
28	82.4	.8360	.99627	.83913	.90323
29	84.2	.8180	.99598	.82130	.88404
30	86.0	.8007	.99568	.80417	.86560
31	87.8	.7840	.99537	.78765	.84782
32	89.6	.7679	.99506	.77171	.83066
33	91.4	.7523	.99473	.75629	.81406
34	93.2	.7371	.99440	.74125	.79787
35		.7225	.99406	.72682	.78234
36		.7085	.99371	.71294	.76744
37		.6947	.99336	.69934	.75276
38		.6814	.99300	.68620	.73862
39		.6685	.99263	.67346	.72490
40		.6560	.99225	.66112	.71162
41		.6439	.99187	.64918	.69877
42		.6321	.99147	.63754	.68624
43		.6207	.99107	.62629	.67413
44		.6097	.99066	.61545	.66246
45		.5988	.99025	.60470	.65089
46		.5883	.98982	.59435	.63975
47		.5782	.98940	.58439	.62903
48		.5683	.98896	.57464	.61853
49		.5588	.98852	.56529	.60847
50		.5494	.98807	.55603	.59850
55		.5064	.98573	.51373	.55297
60		.4688	.98324	.47679	.51321
65		.4355	.98059	.44412	.47804
70		.4061	.97781	.41532	.44704
75		.3799	.97489	.38968	.41945
80		.3565	.97183	.36683	.39485
85		.3355	.96865	.34636	.37282
90		.3165	.96534	.32786	.35290
95		.2994	.96192	.31125	.33503
100		.2838	.95838	.29612	.31874



Kinematic Viscosity of Water $(\frac{\mu}{\rho} \times 10^5)$, ft²/sec
1.70
1.65
1.60
1.55
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
.95
.90
.85
.80
.75
.70
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47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65
Left & upper Scale
Lower & Right Scale
0.80
0.75
0.70
0.65
0.60
0.55
0.50
0.45
Kinematic Viscosity of Water, $(\frac{\mu}{\rho})10^5$ in ft²/sec
Koo
Temperature of Water in °C.
KEUFFEL & ESSER CO. N.Y.

Table Prandtl's Number of Water

(I.C.T. Vol. 5, p. 10)

t(°C.)	t(°F.)	(Millipoises)	C_p (Joules per Gram)	k_t $(10^{-5} \frac{\text{watt}}{(\text{cm})(°\text{C})})$	$P_r = \frac{C\mu}{k}$
0	32	17.938	4.220	554.0	13.68
10	50	13.097	4.199	570.0	9.65
20	68	10.087	4.184	587.0	7.19
30	86	8.004	4.1805	603.5	5.55
40	104	6.536	4.180	620	4.41
50	122	5.492	4.1825	636	3.61
60	140	4.699	4.186	653.5	3.01
70	158	4.071	4.1887	670	2.55
80	176	3.570	4.192	685	2.18
90	194	3.166	4.1948	702	1.891
100	212	2.839	4.198	719	1.657
110		2.56	4.202	736	1.462
120		2.32	4.2065	752	1.298
130		2.12	4.2115	769	1.162
140		1.96	4.2166	785	1.054
150		1.84	4.222	801	0.972

$$\text{Prandtl's Number} = \text{Pr.} = (\text{millipoises})\left(\frac{\text{Joules}}{\text{Gram}}\right)\left(\frac{1}{k_t}\right)100$$

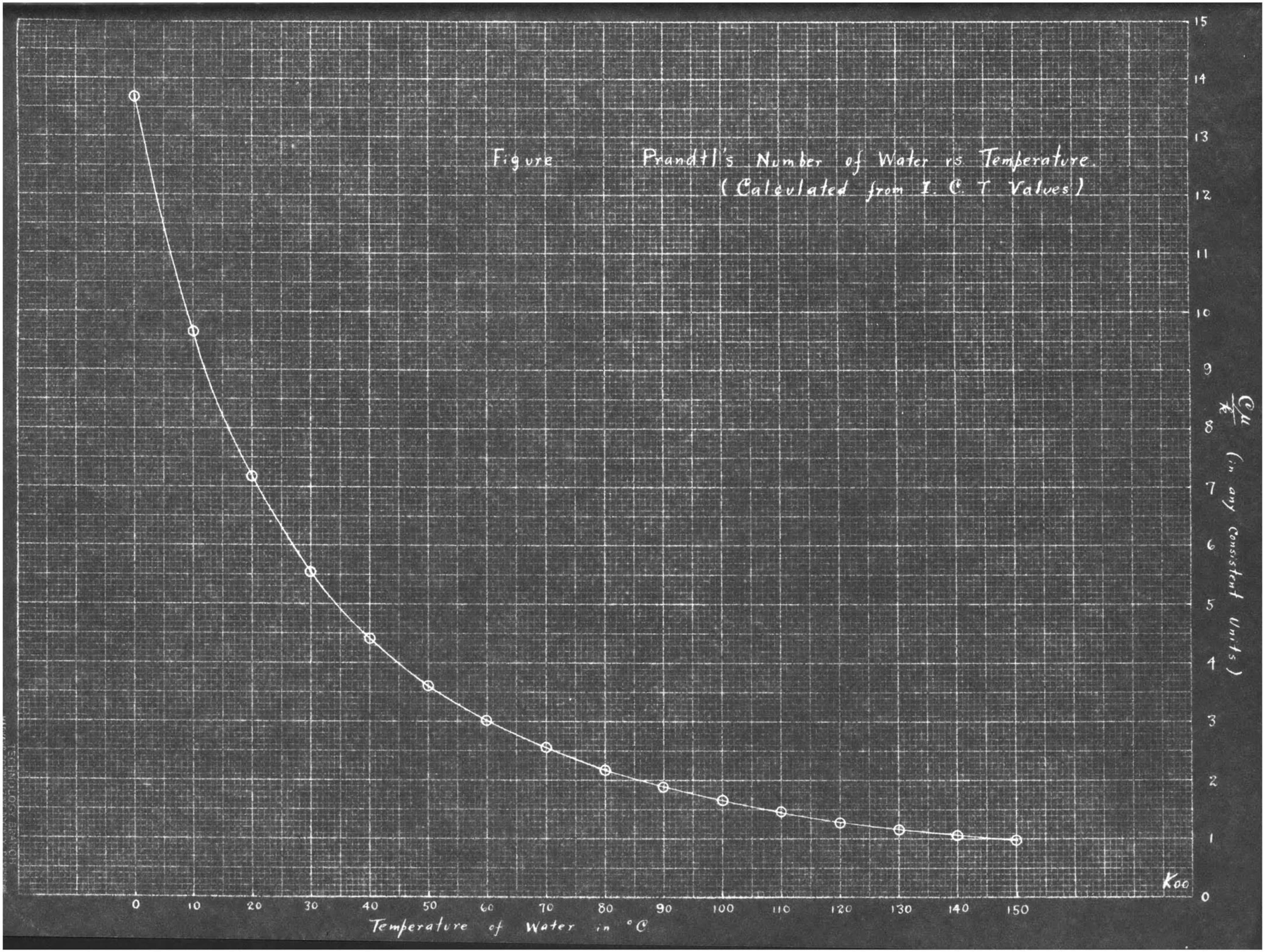


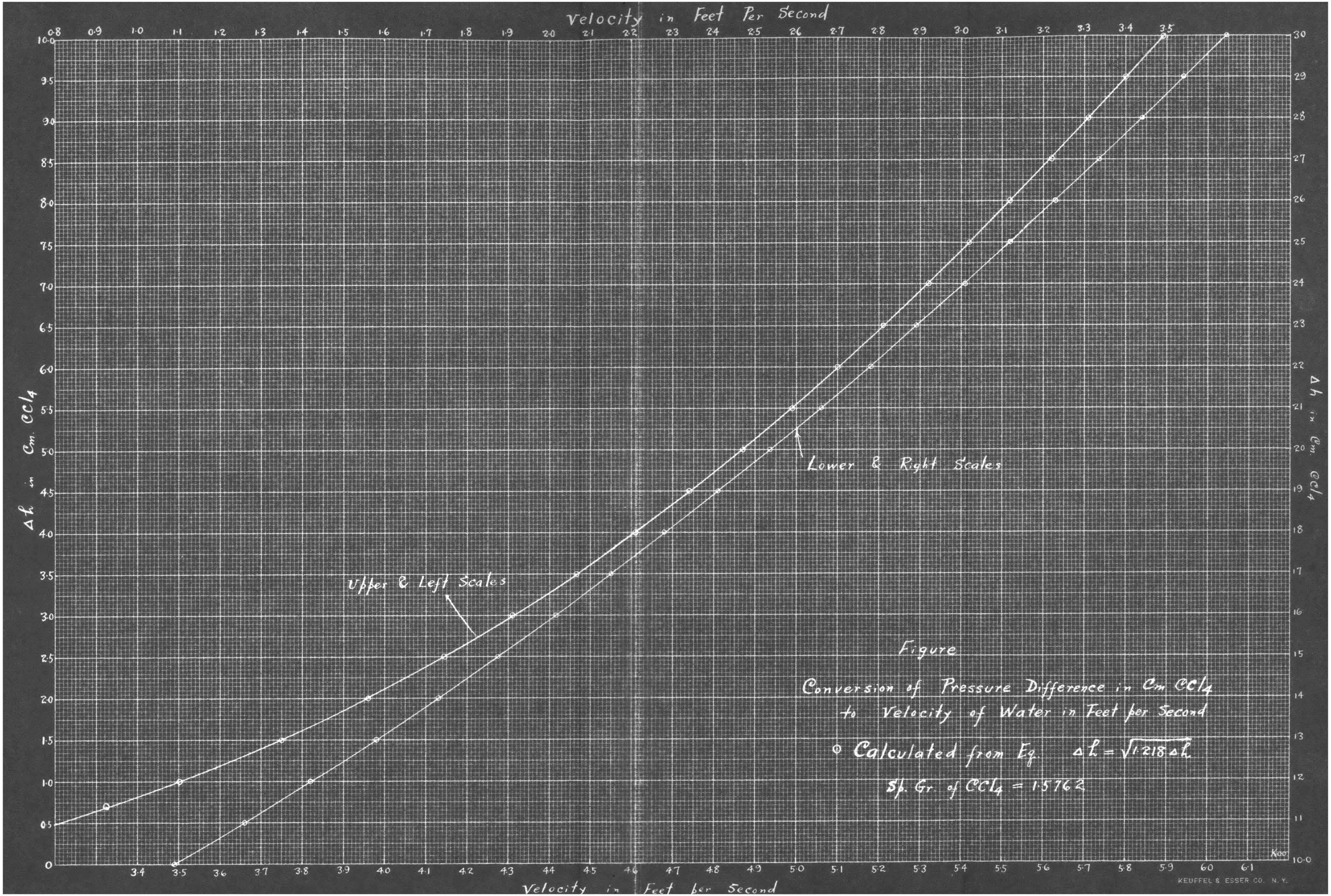
Figure Prandtl's Number of Water vs Temperature.
(Calculated from I. C. T Values)

Table Conversion of Pressure Drop in Cm. CCl_4 to Velocity of Water in feet per Second.

(Values plotted in Fig.)

Equation: $\Delta h = 1.218 \Delta h$

Δh (Cm. CCl_4)	V (ft./sec.)
0.05	0.247
.10	.349
.20	.494
.3	.605
.4	.698
.5	.780
.7	.923
1.0	1.101
1.5	1.35
2.0	1.56
2.5	1.745
3.0	1.91
3.5	2.065
4.0	2.21
4.5	2.34
5.0	2.47
5.5	2.59
6.0	2.70
6.5	2.81
7.0	2.92
7.5	3.02
8.0	3.12
8.5	3.22
9.0	3.31
9.5	3.40
10.0	3.49
11.0	3.66
12.0	3.82
13.0	3.98
14.0	4.13
15.0	4.275
16.0	4.415
17.0	4.55
18.0	4.68
19.0	4.81
20.0	4.936
21.0	5.06
22.0	5.18
23.0	5.29
24.0	5.41
25.0	5.52
26.0	5.63
27.0	5.735
28.0	5.84
29.0	5.94
30.0	6.045



Figure

Conversion of Pressure Difference in Cm CCl_4 to Velocity of Water in Feet per Second

○ Calculated from Eq. $\Delta h = \sqrt{1.218 \Delta h}$

Sp. Gr. of CCl_4 = 1.5762

APPENDIX K

Table of Nomenclature

a = Velocity Distribution Exponent in Eq. $V/V_{max.} = (1-r/R)^a$.

A = Area in square feet.

b = Temperature Distribution Exponent in Eq.
$$\frac{t_w - t}{t_w - t_a} = (1-r/R)^b$$

C = Specific heat of fluid at constant pressure.

d = Differential Prefix.

D or I.D. = Inside pipe diameter in any convenient unit.

D" = Inside pipe diameter in inches.

f = Fanning Friction Factor (no units)

F = Surface friction between Fluid and Wall.

g = 32.2 ft. per sec. per sec.

h = film coefficient of heat transfer, pipe to water, B.t.u. per hour per foot^2 per ° F., based on arithmetic mean temperature drop across water film.

Δh = Differential head and pressure drop, expressed in feet of fluid.

k = thermal conductivity, B.t.u. per hour per square foot per ° F. per foot thickness.

L = Friction Length of straight pipe in feet.

L' = Prandlt's mixing Length.

m = velocity distribution exponent in Eq.
$$\frac{V}{V_{max.}} = \left[1 - (r/R)^2\right]^m.$$

m' = velocity distribution exponent in Eq.
$$\frac{V}{V_{max.}} = \left[1 - (r/R)^{1.25}\right]^{m'}.$$

P = Absolute Pressure in lbs. per square feet.

ΔP = Differential Pressure and Pressure Drop.

Q = Quantity of heat transferred in B.t.u.

Q = Rate of flow of Fluid in ft.^{3}/sec.

r = variable radius or distance from axis of Pipe.

R = Inside pipe radius.

r/R = Fraction of radius of Pipe.

S = Specific gravity of fluid at temperature in question.

t = Variable temperature at distance r from pipe axis in ° C.

t_a = Axial temperature in ° C.

t_e = Average downstream exit temperature in ° C.

t_f = Average film temperature or effective film temperature in ° C.

t_i = Average upstream inlet temperature in ° C.

t_m = Mixing cup temperature in ° C. (obtained through graphical integration).

t_w = Inside wall temperature of pipe in ° C. (Calculated).

$t_{o.w.}$ = Outside wall temperature of pipe in ° C, (Measured).

$t_{ave.}$ = Average cross-sectional temperature in ° C. (obtained through graphical integration.)

$$\frac{\Delta t}{\Delta t_{max.}} = \frac{t_w - t}{t_w - t_a}$$ = Fraction of temperature drop, variable with r.

U = Average velocity in feet per second.

v or V Variable velocity at distance r from pipe axis.

V_{av} = Average velocity in feet per second.

V_{max} = Maximum or axial velocity.

$V_{ave.}/V_{max.}$ = Ratio of average to maximum velocity, or velocity ratio.

X = Calming Section Length in Feet.

Z_m = viscosity, centipoises, taken at arithmetic mean of average cross-sectional temperature between two sections.

Z = viscosity, Centipoises.

μ(Mu) = Absolute viscosity of fluid~~s~~.

ρ(Rho) = Fluid density as lbs. per ft.^{3}

ν(Nu) = Kinematic Viscosity of fluid = μ/ρ

λ(Lambda) = 4f

θ(Theta) = Time in any convenient unit.

$$Re = \frac{D\ V_{av}\ \rho}{\mu} =$$ Reynolds Number in Consistent Units (Dimensionless)

$$Re_{max.} = \frac{D\ V_{max.}\rho}{\mu}$$ = Maximum Reynolds number.

$$Pr, = \frac{c\mu}{k} =$$ Prandtl's number in consistent units (Dimensionless)

Pe', = Peclet Number = (Re)(Pr)