

A Frequency Domain Strip Theory

Applied to the Seakeeping of the Zumwalt-Class Destroyer

by

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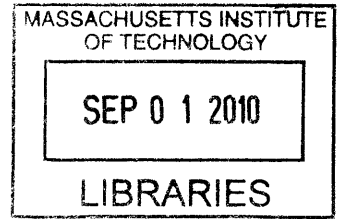
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ABSTRACT

Seakeeping analysis of the Zumwalt-Class destroyer was carried out in the framework of linear strip theory and potential flow. First, the problem was formulated and solved analytically. Second, a program called Ship Motions Analyzer (SMA) was written in MATLAB™ to carry out the seakeeping analysis for regular waves in a discretized frequency range. SMA calculates sectional added mass and damping coefficients first. Then, it calculates excitation forces and moments acting on a ship advancing at constant forward speed with arbitrary heading for sway, heave, roll, pitch and yaw modes of motion. Finally, SMA evaluates Response Amplitude Operators (RAO's) in the same modes of motion. In addition, it also includes a subroutine which evaluates steady drift forces acting on a ship in the plane of undisturbed free surface.

The added mass and damping coefficients of a fully submerged heaving circle and a semi-circle in heave and sway were calculated to validate the results of SMA. The results were compared to the results of Vugst [1] and Frank [2]. They match each other exactly. In addition, the magnitudes of heave and pitch excitation force and moment, and RAO's in the same modes of motions were calculated. The results agree with the theory. Finally, added resistance of Mariner type ship was calculated by SMA to compare the results to the ones given by Salvase [3] and to validate the calculations. These results are also in very good agreement with the available computational and experimental results.

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Biographical Note and Acknowledgements

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Table 1 Nomenclature

Chapter 2	
\vec{V}	Velocity Vector
∇	Nabla Operator
a	Sectional added mass coefficient
b	Sectional damping coefficient
c	Position of a pulsating source defined in the complex plane
f	Complex force coefficient
g	Acceleration of gravity
i	Space complex variable
I_{km}	Influence matrix which contains velocities induced by the panels at the collocation points
j	Time complex variable
J_{km}	Influence matrix which contains potentials induced by the panels at the collocation points
N	Number of panels
n	Unit normal vector pointing into the body in x - y plane
n_x	x component of unit normal vector in x - y plane
n_y	y component of unit normal vector in x - y plane
p	Hydrodynamic pressure
p_a	Atmospheric pressure
Q	Complex source strength
r	Position vector with respect to the origin of designated coordinate system
u	Horizontal velocity component x - y plane
U_n	Fluid velocity in normal direction
v	Vertical velocity component x - y plane
x	Horizontal axis of two dimensional Cartesian coordinate system Real axis of complex plane
y	Vertical axis of two dimensional Cartesian coordinate system Imaginary axis of complex plane
z	Complex Variable Position of any point in fluid domain defined in complex plane
η	Free surface elevation
ν	Kinematic Viscosity Wave Number
ρ	Density of fluid
ϕ	Velocity Potential
ϕ_b	Velocity potential of a two dimensional body
ϕ_p	Velocity potential of a panel
ϕ_s	Velocity potential of a point source
Ψ	Stream Function
ω	Vorticity Wave frequency (rd/s)
Φ	Complex potential
Chapter 3	
A_{jk}	Added mass of the ship
B	Ship's beam
B_{jk}	Damping of the ship

C_{jk}	Hydrostatic restoring coefficients
F_j	Amplitude of excitation force/moment
F_j	Excitation force/moment
$f_j(x)$	Sectional Froude-Krylov force
G_j	Amplitude of force/moment due to body motions
H_j	Amplitude of total force/moment
$h_j(x)$	Sectional diffraction force
I_4	Moment of inertia due to roll motion
I_{46}	Product of inertia
I_5	Moment of inertia due to pitch motion
I_6	Moment of inertia due to yaw motion
k	Wave number
L_s	Ship's length(LBP)
M_{jk}	Generalized mass matrix of the ship
N_j	2D generalized normal
n_j	3D generalized normal. $j=1, 2, \dots, 6$
T_{jk}	Hydrodynamic force/moment in the j^{th} direction due to unit amplitude motion in k^{th} direction
t_{jk}	Sectional hydrodynamic force/moment in the j^{th} direction due to unit amplitude motion in k^{th} direction
U	Ship's speed
β	Incident wave angle
ζ_j	Complex amplitudes of oscillatory body motions
λ	Wave length
ϕ_j	3D oscillatory potential
ϕ_j^0	Speed independent part of oscillatory potential
ϕ_j^U	Speed dependent part of oscillatory potential
ϕ_D	Diffraction potential
ϕ_I	Incident wave potential
ϕ_T	Complex amplitude of unsteady potential
ψ_k	2D oscillatory body potential. $k=2,3,4$
ω	Wave frequency (rd/s)
ω_e	Frequency of encounter (rd/s)
Φ	Total velocity potential
Chapter 4	
β	Incident wave angle
$\phi_T^{(1)}$	First order amplitude of time dependent potential
$\phi_T^{(2)}$	Second order amplitude of time dependent potential
$\phi_{DC}^{(2)}$	Second order DC potential
Φ_B	Body potential due to body motions including diffraction effects
\mathcal{F}	Second order steady state force
ϕ_I^*	Complex conjugate of incident wave potential
\mathcal{F}_j^I	Steady state force component due to incident waves
\mathcal{F}_j^D	Steady state force component due to body motions
\mathcal{F}_D	Steady state force component due to diffraction potential
\mathcal{M}	Steady state moment

CHAPTER 1

INTRODUCTION

1 Introduction

Seakeeping analysis is one of the major steps of ship design. The major interests in seakeeping analysis can be summarized as the following:

- Calculation of the magnitudes of forces exerted by waves coming toward the ship from different directions,
- Calculation of the amount of displacements in different modes of motion,
- Calculation of added resistance due to waves,
- Calculation of the number of occurrences of green water on the deck,
- Calculation of the possibility of slamming of the ship,
- Calculation of the magnitude of bending moment and shear force acting on any section of the ship.

Understanding the motions of a ship as a response to the forces exerted by the waves is important to naval architects. The forces and motions must be well understood because they are closely related to the safety of both personnel and cargo, and to the strength of the ship's structure.

1.1 Historical Background

Calculating the forces exerted by waves on floating bodies and the motions of the bodies as a response to the wave forces has been of interest to many researchers for many years. The hydrodynamic properties of a ship must be known to calculate the responses of a ship in any sea state. Calculation of the hydrodynamic coefficients of arbitrary ship sections and the excitation

forces and moments are the primary objectives of this thesis. Frank's [2] close-fit source distribution method constitutes the main frame of the calculation of hydrodynamic coefficients of arbitrary ship sections. Many people contributed to the calculation of hydrodynamic coefficients of cylinders in a free surface for different modes of motion. Frank [2] summarizes their works as follows.

“Ursell formulated and solved the boundary-value problem for the semi-immersed cylinder using the linearized free-surface theory. He represented the velocity potential as the sum of an infinite set of multipoles, each satisfying the linear free-surface condition and each being multiplied by a coefficient determined by requiring the series to satisfy the kinematic boundary condition at a number of points on the cylinder. Grim used a variation of the Ursell method to solve the problem for two-parameter, Lewis-form cylinders by conformal mapping onto a circle. Tasai and Porter, using the Ursell approach, obtained the added mass and damping for oscillating contours mappable onto a circle by the more general Theodorsen transformation. Ogilvie calculated the hydrodynamic forces on completely submerged heaving circular cylinders.” [2]

The boundary of any arbitrary two dimensional body can be represented easily using Frank's [2] method. Frank [2] represented the velocity potential of an arbitrary two dimensional body by a distribution of pulsating sources over the submerged portion of the body. First, he approximated body contour as a sum of two dimensional panels and distributed constant – strength sources over each panel. Then, he calculated unknown source strengths by satisfying the kinematic boundary condition on the body boundary. Finally, he obtained added mass and damping coefficients of different cylinders oscillating in or below the free surface in different

modes of motion. The method described in his research will be used to calculate added mass and damping coefficients of arbitrary ship sections.

Vugst [1] conducted experiments on different cylinders in different modes of motion and compared the experimental results to theoretical results. His research gives added mass and damping coefficients of semi-circular, rectangular, triangular cylinders and Lewis-form sections in heave; sway; roll motions and coupling coefficients of roll into sway and sway into roll. The added mass and damping coefficients presented in Vugst's [1] research are used to compare the added mass and damping coefficients obtained by Frank's [2] method.

Wave induced forces and moments exerted on a ship advancing at constant forward speed must also be well understood since they are other key elements in determining the ship motions. Many studies have been done to predict the wave induced forces, moments and motions of the ships since the late 1950's and various researchers contributed to this subject in different ways. The theory of Korvin-Kroukovsky and Jacobs[4] to predict heave and pitch motions in head waves is considered, "the first motion theory suitable for numerical computations" [5]. St.Denis and Pierson showed that the responses of ships in irregular waves can be computed by summing the responses of the ship in regular waves. "Gerritsma and Beukelman validated a strip theory with experimental results." [6] Salvasen, *et al.* [5] discussed the calculation of wave induced forces and moments including forward speed effects. The study of Salvasen, *et al.* [5] constitutes the mainframe of the calculations regarding excitation forces and moments exerted on a ship advancing at constant forward speed in regular waves with arbitrary heading.

1.2 Goal of This Thesis

Hydrodynamic coefficients of a ship and the excitation forces and moments should be calculated to determine the responses of a ship advancing at constant forward speed in regular waves. The goal of this thesis is to develop a computational tool to calculate added mass and damping coefficients of arbitrary ship sections and to evaluate excitation forces and moments induced by the waves. First, the boundary integral equations given by Frank [2] will be evaluated in a step-by-step manner. Second, a program in MATLAB will be written to calculate sectional added mass and damping coefficients. Results obtained by the MATLAB program will be compared to the results presented by Vugst [1]. Third, the excitation forces and moments will be calculated using the equations given by Salvasen, *et al.*[5]. In addition, sectional Froude-Krylov and diffraction forces will be calculated by integrating the pressure field around the body due to incident wave system and diffracting waves, respectively. After that, responses of a ship in regular head waves will be calculated. Finally, the calculation of added resistance in waves, steady drift forces and slowly varying forces will be covered briefly.

CHAPTER 2

FORMULATION AND SOLUTION OF RADIATION PROBLEM

2 Panel Method

Panel or boundary element methods were developed by Hess and Smith [7] in the late 1950's to early 1960's. The technique simply consists of representing the boundary of a body in a fluid domain by a number of panels. Each panel is considered to generate a simple flow field. The strength of each panel is determined by satisfying appropriate boundary conditions on each panel. The body boundary is represented by the sum of all panels.

Panel methods have some limitations. They require the flow field to be described by potential flow. They are incapable of modeling the viscous effects that are evident in real world applications including boundary layers and flow separation. This thesis requires the investigation of only small amplitude harmonic motions of arbitrary ship sections, so nonlinear effects will be ignored throughout the evaluation of added mass and damping coefficients.

2.1 Potential Flow

A flow must satisfy certain conditions to be considered as potential. The flow must be

- Incompressible
- Irrotational ($\omega=0$)
- Inviscid ($\nu=0$)

In addition, the flow must satisfy Laplace's equation everywhere in the fluid domain. For a 2D flow, Laplace's equation is given as the following:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad (1)$$

If the flow satisfies all of these conditions, the velocity vector can be represented as the gradient of a scalar function. This scalar function is called the potential function. Potential flow theory can also be used to model irrotational, compressible flows, but the fluid velocities considered in this work has low Mach numbers.

As described in Newman [8], “the velocity field can be represented by analytic functions of a complex variable. Assuming that the flow depends only on the coordinates x and y , which are taken to be the real and imaginary parts of the complex variable $z = x + iy$, where i is the imaginary unit. The complex potential $F(z)$ is defined to be

$$F(z) = \phi + i\psi \quad (2)$$

where ϕ is the velocity potential and ψ the stream function. The two velocity components can be determined by differentiating either of these real functions:”

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} \quad (3)$$

The real and imaginary parts of F satisfy *Cauchy-Riemann equations*. Therefore, the complex potential F is an analytic function of the complex variable z , and its derivative is

$$\frac{dF}{dz} = u - iv \quad (4)$$

If the velocity vector at any point in the fluid domain for any time is known, pressure can also be calculated at that point, which will lead to the calculation of the total force on a body.

The pressure at any point in the flow field is given by Bernoulli’s Equation:

$$p = -\rho \left(\frac{\partial \phi}{\partial t} + \frac{1}{2} \nabla \phi \cdot \nabla \phi + gy \right), \quad (5)$$

where ρ is the fluid density, g the acceleration of gravity and y the depth of the point in question. The second order term in Bernoulli's Equation will be neglected and the linearized form of this equation will be used to calculate pressure on the surface of the body while calculating added mass and damping coefficients.

2.2 Boundary Value Problem

Certain boundary conditions must be satisfied by the velocity potential to solve the radiation problem. These are

- Linearized Free Surface Boundary Condition
- Bottom Boundary Condition
- Infinite Boundary Condition
- Kinematic Body Boundary Condition.

2.2.1 Linearized Free Surface Boundary Condition

As described by Newman [8], "the physical nature of a free surface requires both a kinematic and a dynamic boundary condition. The kinematic boundary condition states that the normal velocities of the fluid and of the boundary surface must be equal and the pressure on the free surface must be atmospheric according to the dynamic boundary condition."

The motions of the body and the disturbances in the free surface are assumed small compared to body length and the wave length. The linearized form of the free surface boundary condition is of interest and higher-order terms in the wave amplitude and associated fluid motions will be neglected throughout this thesis.

First, in order to formulate the linearized free surface boundary condition, a Cartesian coordinate system will be adapted, with $y = 0$ the plane of the undisturbed free surface and the y -

axis positive upwards. The vertical elevation of any point on the free surface may be defined by a function $y = \eta(x, z, t)$. In the case of a two dimensional fluid motion, parallel to the x - y plane, the dependence of z will be deleted. The linearized kinematic boundary condition is given by

$$\frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial y} \quad (6)$$

This approximate boundary condition simply states that the vertical velocities of the free surface and the fluid particles are equal.

The dynamic boundary condition is obtained from Bernoulli's equation. The exact condition to be satisfied on the free surface is

$$-\frac{1}{\rho}(p - p_a) = \frac{\partial \phi}{\partial t} + \frac{1}{2} \nabla \phi \cdot \nabla \phi + gy = 0 \quad (7)$$

Substituting the free-surface elevation for y and neglecting the second order term in the fluid velocity, the linearized equation for the free-surface elevation is

$$\eta = -\frac{1}{g} \frac{\partial \phi}{\partial t} \quad (8)$$

On the plane of $y = 0$, the dynamic boundary condition can be differentiated with respect to time and combined with the kinematic boundary condition. This combination gives a single boundary condition for the velocity potential:

$$\frac{\partial^2 \phi}{\partial t^2} + g \frac{\partial \phi}{\partial y} = 0 \quad (9)$$

2.2.2 Other Boundary Conditions

The body in question is bounded horizontally in a horizontally infinite fluid domain and it is stationary but steadily oscillates in the free surface. It does not extend to infinity horizontally and the fluid is infinitely deep. In this case, it is reasonable to impose that the fluid motion will vanish far below the body and outgoing waves at infinity on all sides will be generated. The bottom boundary condition can be expressed as the following:

$$\frac{\partial \phi}{\partial y} = 0 \quad y \rightarrow -\infty \quad (10)$$

The last boundary condition to be satisfied is the kinematic body boundary condition. This condition states that the fluid velocity on the body boundary in normal direction must be equal to the body velocity in normal direction. The fluid particles can slip on the body surface since the viscosity is assumed to be 0, but the fluid cannot pass the body boundary.

2.3 Evaluation of Added Mass and Damping Coefficients of a 2D Ship Section

The seakeeping analysis will be carried out in the framework of linear strip theory, so the sectional added mass and damping coefficients of ship sections should be calculated first. The added mass and damping coefficients of a ship will be obtained by integrating the sectional added mass and damping coefficients along the ship's length. Added mass and damping coefficients of two dimensional ship sections will be calculated using 2D panel method and the principles of potential flow.

First, the boundary of a ship section will be approximated by 2D panels. The contour of the boundary will be divided into N equal segments. Then, pulsating point sources will be distributed over each panel. The potential of a pulsating source is given by Wehausen and Laitone [9] as

$$\begin{aligned} \Phi(z, t) = & \left[\frac{1}{2\pi} (\log(z - c) - \log(z - \bar{c})) - \frac{1}{\pi} PV \int_0^\infty \frac{e^{-ik(z-\bar{c})}}{k - \nu} dk \right] Q \cos(\omega t) \\ & - [e^{-i\nu(z-\bar{c})}] Q \sin(\omega t) \end{aligned} \quad (11)$$

where $\nu = \omega^2/g$. In solving oscillatory problems it is convenient to eliminate the time variable by the use of complex notation. In this thesis, i will represent the imaginary part of space complex variables and j will represent the imaginary part of time complex variables. The potential function can now be written in a frequency dependent form and converted to the form given in (10) by multiplying by $e^{j\omega t}$ and taking the time real part. Thus, the potential function in frequency domain is

$$\Phi(z) = \left[\frac{1}{2\pi} (\log(z - c) - \log(z - \bar{c})) - \frac{1}{\pi} PV \int_0^\infty \frac{e^{-ik(z-\bar{c})}}{k - \nu} dk \right] Q + j[e^{-i\nu(z-\bar{c})}] Q \quad (12)$$

Next, the boundary value problem will be solved to calculate the unknown source strengths (Q). The strength of each point source on a panel is assumed to be constant, but the source strengths of other panels may be different. Finally, the pressure distribution around the body will be calculated and this will help to determine the added mass and damping coefficients of a ship section.

2.3.1 Calculation of Source Strengths

Equation (11) satisfies all boundary conditions and Laplace's Equation except body boundary condition. This problem should be solved to determine the unknown source strengths. The kinematic boundary condition on the body boundary states that the fluid velocity in normal direction is equal to body velocity in normal direction:

$$\nabla\Phi \cdot n = U_n \quad (13)$$

where n is the unit normal vector pointing into the body. First, the influences of all panels on the collocation points will be calculated. The collocation points are the mid-points of all panels. The influence matrix includes the velocities induced on the collocation points by all panels with unit source strength:

$$[I_{km}][Q] = [U_n] \quad (14)$$

The motions of interest in this thesis are harmonic motions of a body in the free surface of a deep fluid. Particularly, source strengths will be calculated for heave, sway and roll motions. The normal velocities on the collocation points are

$$U_{n_{Heave}} = j\omega n_y \quad (15)$$

$$U_{n_{Sway}} = j\omega n_x \quad (16)$$

$$U_{n_{Roll}} = j\omega(r \times n) \quad (17)$$

where n_y is the y component of unit normal vector pointing into the body, n_x the x component of unit normal vector and r is the position vector.

For N panels, the influence matrix will be a $N \times N$ matrix. U_n matrix includes normal velocities at N collocation points. Finally, the unknown source strengths of panels are obtained by

$$[Q] = [I_{km}] \setminus [U_n] \quad (18)$$

After source strengths of all panels are found, the hydrodynamic pressure can be evaluated on the collocation panels, and eventually it will lead to the calculation of complex force coefficient.

2.3.2 Calculation of Added Mass and Damping Coefficients

Final task is the calculation of pressures on collocation points and the calculation of total force exerted on the body. This total force is also known as the “complex force coefficient” [8], which will be used to obtain the added mass and damping coefficients. Total force can be calculated by integrating pressures on collocation points over the body boundary. The pressures will be obtained using the linearized Bernoulli’s Equation:

$$p = -\rho \frac{\partial \Phi}{\partial t} \quad (19)$$

The total force is given by integrating dynamic pressure over the body boundary:

$$f = \oint p n ds \quad (20)$$

Complex force coefficient has strong frequency dependence and it can be written as

$$f = \omega^2 a - i\omega b \quad (21)$$

where ω is the wave frequency, a the added mass coefficient and b the damping coefficient.

Finally, added mass and damping coefficients are obtained by separating real and imaginary parts of the complex force coefficient and normalizing them by ω^2 and $-\omega$, respectively:

$$a = \frac{\text{Re}\{f\}}{\omega^2} \quad (22)$$

$$b = \frac{-\text{Im}\{f\}}{\omega} \quad (23)$$

2.3.3 Evaluation of the Components of Influence Matrix

The potential of a pulsating source is given by Wehausen and Laitone [9] as following:

$$\Phi(z) = \left[\frac{1}{2\pi} (\log(z - c) - \log(z - \bar{c})) - \frac{1}{\pi} PV \int_0^{\infty} \frac{e^{-ik(z-\bar{c})}}{k - v} dk \right] Q + j[e^{-iv(z-\bar{c})}]Q \quad (24)$$

Here, z is the position of any point in fluid domain, and c is the position of a pulsating source. z and c are defined in the complex plane. \bar{c} is the complex conjugate of c and Q is the unknown complex source strength.

$$z = x + y * i \quad (25)$$

$$c = a + b * i \quad (26)$$

where $-\infty < x < \infty$, $-\infty < y < 0$, $-\infty < a < \infty$, $-\infty < b \leq 0$, $v = \omega^2/g$.

The contour of the body is divided into N straight line elements or panels, and the velocities induced by each panel at the midpoints of all other panels will be calculated. It is convenient to solve this problem for each term separately.

Velocity Components Induced by the First Term: $\Phi_1 = \frac{1}{2\pi} \log(z - c)$

Complex potential function is constituted of two parts. The real part of complex potential is called the velocity potential and the imaginary part is called the stream function. Φ_1 can be written as

$$\Phi_1 = \frac{1}{2\pi} \left(\log[(x - a)^2 + (y - b)^2]^{0.5} + i * \tan^{-1} \frac{(y - b)}{(x - a)} \right) \quad (27)$$

and the velocity potential can be written as

$$\phi_1 = \frac{1}{4\pi} \log[(x - a)^2 + (y - b)^2] \quad (28)$$

The velocity components in x and y directions can be calculated by differentiating the first term with respect to x and y and then integrating along the panel. The velocity components can be written in the following forms:

$$u_1 = \frac{1}{2\pi} \int \frac{(x - a)}{[(x - a)^2 + (y - b)^2]} ds \quad (29)$$

$$v_1 = \frac{1}{2\pi} \int \frac{(y - b)}{[(x - a)^2 + (y - b)^2]} ds \quad (30)$$

These integral equations are evaluated between the end points of the panels. The body is defined in third and fourth quadrants as described in Figure 1. The direction from c_1 to c_2 is assumed to be counter clockwise for each panel, e.g.: c_1 is in the undisturbed free surface and c_2 is in the third quadrant of the complex plain, in other words in the fluid domain for the panel, which is drawn in red in Figure 1.

Evaluating these integrals in this form is quite hard. Adapting a local coordinate system on the panel whose horizontal axis is aligned with the panel will simplify the solution.

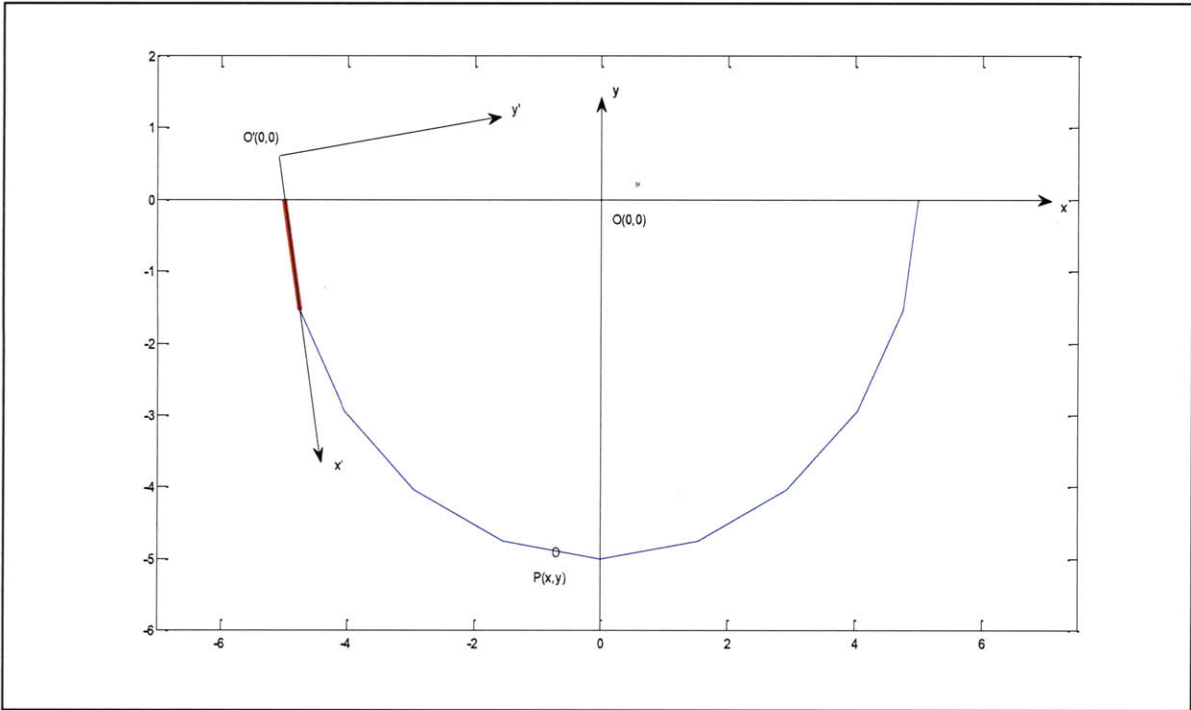


Figure 1: Representation of body surface by panels, body coordinate system (x, y) and adaptation of local coordinate system (x', y') on a panel.

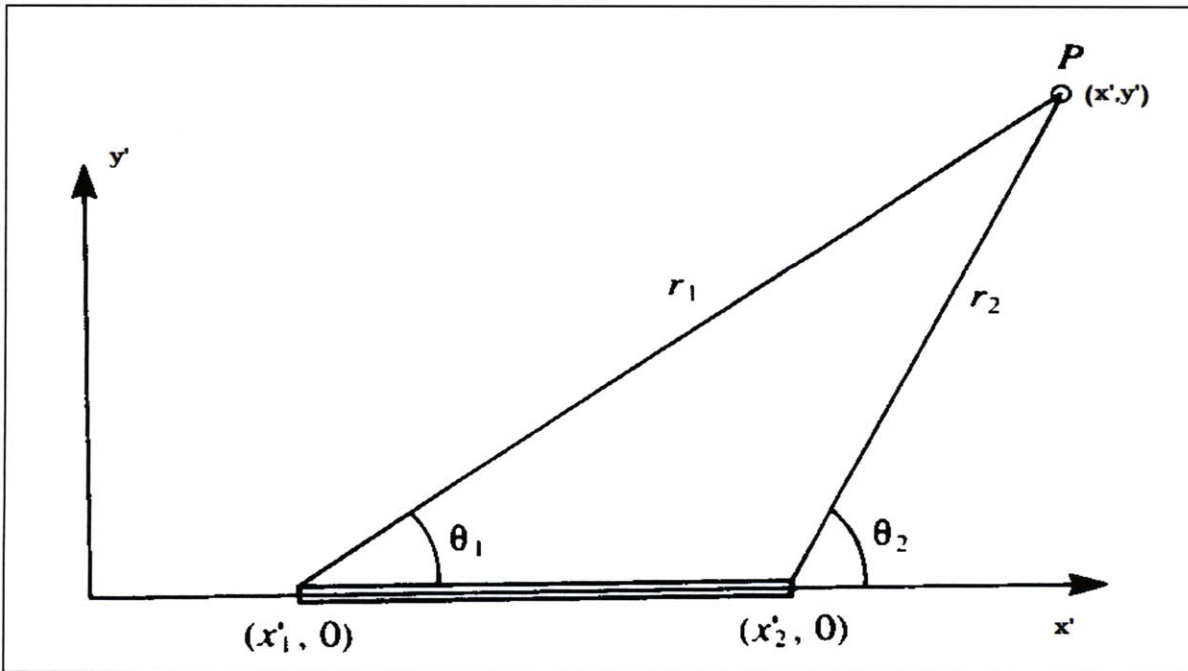


Figure 2: Local coordinate system adapted on the panel. (Adapted from Katz and Plotkin [10])

Since a local coordinate system is adapted and the integral equations will be solved in the local coordinates, the coordinates of the midpoints of panels, which are defined in body coordinate system (x, y) , should be defined in the local coordinate system.

$$x_m = (x_i + x_{i+1})/2 \quad (31)$$

$$y_m = (y_i + y_{i+1})/2 \quad (32)$$

where $i = 1, 2, 3, \dots, N$.

$$x_m^* = x_m - x_0 \quad (33)$$

$$y_m^* = y_m - y_0 \quad (34)$$

$$x'_m = x^* \sin \alpha + y^* \cos \alpha \quad (35)$$

$$y'_m = -x^* \sin \alpha + y^* \cos \alpha \quad (36)$$

Here, x_m and y_m are the coordinates of the midpoint of a panel, x_0 and y_0 are the coordinates of the origin of the local coordinate system in body coordinate system. α is the angle between the positive horizontal axes of both coordinate systems. The coordinates of the midpoint of a panel in the local coordinate system can be found using equations from (31) to (36).

The integral equation for u_1 and v_1 can now be written in the following forms:

$$u'_1 = \frac{Q}{2\pi} \int_{x'_1}^{x'_2} \frac{(x'_m - x_0)}{(x'_m - x_0)^2 + y'_m{}^2} dx_0 \quad (37)$$

$$v'_1 = \frac{Q}{2\pi} \int_{x'_1}^{x'_2} \frac{y'_m}{(x'_m - x_0)^2 + y'_m{}^2} dx_0 \quad (38)$$

The evaluation of these integral equations can also be found in Katz and Plotkin [10]. They can be solved using appropriate variable transformations:

$$X = (x'_m - x_0)^2 + y'_m{}^2 \quad (39)$$

$$dX = -2(x'_m - x_0)dx_0 \quad (40)$$

$$u'_1 = \frac{-Q}{4\pi} \int_{X_1}^{X_2} \frac{dX}{X} \quad (41)$$

$$X_2 = (x'_m - x'_2)^2 + y'_m{}^2 \text{ and } X_1 = (x'_m - x'_1)^2 + y'_m{}^2 \quad (42)$$

$$u'_1 = \frac{Q}{4\pi} \log \frac{(x'_m - x'_1)^2 + y'_m{}^2}{(x'_m - x'_2)^2 + y'_m{}^2} \quad (43)$$

$$Y = \frac{y'_m}{(x'_m - x_0)} \quad (44)$$

$$dY = \frac{y'_m}{(x'_m - x_0)^2} dx_0 \quad (45)$$

$$v'_1 = \frac{Q}{2\pi} \int_{x'_1}^{x'_2} \frac{\frac{y'_m}{(x'_m - x_0)^2}}{1 + \left[\frac{y'_m}{(x'_m - x_0)}\right]^2} dx_0 \quad (46)$$

$$v'_1 = \frac{Q}{2\pi} \int_{Y_1}^{Y_2} \frac{dY}{1 + Y^2} \quad (47)$$

$$Y_2 = \frac{y'_m}{(x'_m - x'_2)} \text{ and } Y_1 = \frac{y'_m}{(x'_m - x'_1)} \quad (48)$$

$$v'_1 = \frac{Q}{2\pi} (\tan^{-1} Y_2 - \tan^{-1} Y_1) \quad (49)$$

$$v'_1 = \frac{Q}{2\pi} \left(\tan^{-1} \frac{y'_m}{(x'_m - x'_2)} - \tan^{-1} \frac{y'_m}{(x'_m - x'_1)} \right) \quad (50)$$

Recalling r_1 , r_2 , θ_1 and θ_2 from Figure 2, (43) and (50) can be written as the following:

$$u'_1 = \frac{Q}{4\pi} \log \frac{r_1^2}{r_2^2} \quad (51)$$

$$v'_1 = \frac{Q}{2\pi} (\theta_2 - \theta_1) \quad (52)$$

where

$$r_1 = \sqrt{(x'_m - x'_1)^2 + y_m'^2} \quad (53)$$

$$r_2 = \sqrt{(x'_m - x'_2)^2 + y_m'^2} \quad (54)$$

$$\theta_1 = \tan^{-1} \frac{y'_m}{x'_m - x_1} \quad (55)$$

$$\theta_2 = \tan^{-1} \frac{y'_m}{x'_m - x_2} \quad (56)$$

θ_1 and θ_2 are in radians.

In the special case when the midpoint is on the panel itself, where $z = \mp 0$, the velocity components become

$$u'_1 = 0 \quad (57)$$

$$v'_1 = \mp \frac{Q}{2} \quad (58)$$

at the panel center and

$$u'_1 = \infty \quad (59)$$

at the panel edges.

Equations (43) and (50) give velocity components in x' and y' directions respectively. Finally, the velocity components in x and y directions in the body coordinate system can be found using following equations:

$$u_1 = u'_1 \cos \alpha - v'_1 \sin \alpha \quad (60)$$

$$v_1 = u'_1 \sin \alpha + v'_1 \cos \alpha \quad (61)$$

Velocity Components Induced by the Second Term: $\Phi_2 = \frac{1}{2\pi} \log(z - \bar{c})$

The evaluation of velocity components induced by the second term is identical to the first term except that this time the path of the line integral is reversed. \bar{c} represents the positions of corresponding imaginary sources. The path of the integration to calculate velocity components u'_2 and v'_2 is between the end points of the corresponding imaginary panel. Recalling the example used to demonstrate the path of integration for the first term, the beginning point of the first imaginary panel \bar{c}_1 is going to be in the undisturbed free surface and the end point \bar{c}_2 is going to be in the second quadrant of the complex plane. This time, a right handed local coordinate system is required to be adapted on the imaginary panel such that the positive x' axis will be aligned with the imaginary panel and it will point $\bar{c}_1 - \bar{c}_2$ direction.

Velocity Components Induced by the Third Term: $\Phi_3 = \frac{1}{\pi} PV \int_0^{\infty} \frac{e^{-ik(z-\bar{c})}}{k-\nu} dk$

Solving this integral equation is easier by using complex variables than using real variables. PV stands for Cauchy Principal Value of the integral. One can easily see that the integrand becomes indefinite when $k = \nu$. This integral equation can be solved as the following

$$PV \int_0^{\infty} \frac{e^{-ik(z-\bar{c})}}{k-\nu} dk = \int_0^{\infty} \frac{e^{-ik(z-\bar{c})}}{k-\nu} dk + \delta i\pi e^{-i\nu(z-\bar{c})} \quad (62)$$

where $\delta i\pi e^{-i\nu(z-\bar{c})}$ is the residue of PV integral at $k = \nu$, and δ is defined by Holloway [11] as the following:

$$\delta = 1 \quad \text{if} \quad \text{Re}(z) \leq \text{Re}(\bar{c}) \quad (63)$$

$$\delta = -1 \quad \text{if} \quad \text{Re}(z) > \text{Re}(\bar{c}) \quad (64)$$

Applying a variable transformation, $\int_0^\infty \frac{e^{-ik(z-\bar{c})}}{k-\nu} dk$ is given by Frank [2] as the following:

$$t = i(k - \nu)(z - \bar{c}) \quad (65)$$

$$dt = i(z - \bar{c})dk \quad (66)$$

$$\int_0^\infty \frac{e^{-ik(z-\bar{c})}}{k-\nu} dk = e^{-i\nu(z-\bar{c})} \int_{-i\nu(z-\bar{c})}^\infty \frac{e^{-t}}{t} dt \quad (67)$$

The integral equation on the right hand side of (67) is given by Abramowitz and Stegun [12] as the following:

$$E_1(Z) = \int_Z^\infty \frac{e^{-t}}{t} dt = -\gamma - \log Z - \sum_{n=1}^\infty \frac{(-1)^n Z^n}{n \cdot n!} \quad (|\arg(Z)| < \pi) \quad (68)$$

Here, $Z = -i\nu(z - \bar{c})$, \log is the natural logarithm and γ is the Euler-Mascheroni constant.

$\gamma = 0.577215665\dots$ Finally, (62) can be written in the following form:

$$PV \int_0^\infty \frac{e^{-ik(z-\bar{c})}}{k-\nu} dk = e^{-i\nu(z-\bar{c})} \left\{ \left[-\gamma - \log(-i\nu(z-\bar{c})) - \sum_{n=1}^\infty \frac{(-1)^n [-i\nu(z-\bar{c})]^n}{n \cdot n!} \right] + \delta i\pi \right\} \quad (69)$$

$$PV \int_0^\infty \frac{e^{-ik(z-\bar{c})}}{k-\nu} dk = e^{-i\nu(z-\bar{c})} \{E_1[-i\nu(z-\bar{c})] + \delta i\pi\} \quad (70)$$

Velocity components u_3 and v_3 induced by each panel on the collocation points can be calculated as the following:

$$u_3 = \text{Re} \left\{ \frac{1}{\pi} \int \frac{\partial}{\partial x} [e^{-i\nu(z-\bar{c})} \{E_1[-i\nu(z-\bar{c})] + \delta i\pi\}] ds \right\} \quad (71)$$

$$v_3 = \text{Re} \left\{ \frac{1}{\pi} \int \frac{\partial}{\partial y} [e^{-i\nu(z-\bar{c})} \{E_1[-i\nu(z-\bar{c})] + \delta i\pi\}] ds \right\} \quad (72)$$

Applying a variable transformation and chain rule, the derivatives under the integral sign can be taken as the following:

$$r = -iv(z - \bar{c}) \quad (73)$$

$$\frac{\partial}{\partial x} [e^r \{E_1[r] + \delta i\pi\}] = \frac{\partial e^r}{\partial x} E_1[r] + e^r \frac{\partial E_1[r]}{\partial x} + \delta i\pi \frac{\partial e^r}{\partial x} \quad (74)$$

$$\frac{\partial e^r}{\partial x} = \frac{\partial e^r}{\partial r} \frac{\partial r}{\partial x} = e^r (-iv) = -ive^{-iv(z-\bar{c})} \quad (75)$$

$$\frac{\partial E_1[r]}{\partial x} = \frac{\partial E_1[r]}{\partial r} \frac{\partial r}{\partial x} = -\frac{e^{-r}}{r} (-iv) = iv \frac{e^{-r}}{r} = iv \frac{e^{iv(z-\bar{c})}}{(-iv(z-\bar{c}))} \quad (76)$$

Equation (74) can be written as the following by combining (75) and (76).

$$\begin{aligned} & \frac{\partial}{\partial x} [e^r \{E_1[r] + \delta i\pi\}] \\ &= -ive^{-iv(z-\bar{c})} E_1[-iv(z-\bar{c})] + e^{-iv(z-\bar{c})} iv \frac{e^{iv(z-\bar{c})}}{(-iv(z-\bar{c}))} \\ &+ \delta i\pi (-ive^{-iv(z-\bar{c})}) \end{aligned} \quad (77)$$

$$\frac{\partial}{\partial x} [e^r \{E_1[r] + \delta i\pi\}] = -ive^{-iv(z-\bar{c})} \{E_1[-iv(z-\bar{c})] + \delta i\pi\} - \frac{1}{z-\bar{c}} \quad (78)$$

$$u_3 = \text{Re} \left\{ \frac{1}{\pi} \int \left\{ -ive^{-iv(z-\bar{c})} \{E_1[-iv(z-\bar{c})] + \delta i\pi\} - \frac{1}{z-\bar{c}} \right\} ds \right\} \quad (79)$$

Following the same steps in (74), (75) and (76) the integral in (72) can be calculated as the following:

$$\frac{\partial}{\partial y} [e^r \{E_1[r] + \delta i\pi\}] = \frac{\partial e^r}{\partial y} E_1[r] + e^r \frac{\partial E_1[r]}{\partial y} + \delta i\pi \frac{\partial e^r}{\partial y} \quad (80)$$

$$\frac{\partial e^r}{\partial y} = \frac{\partial e^r}{\partial r} \frac{\partial r}{\partial y} = ve^r = ve^{-iv(z-\bar{c})} \quad (81)$$

$$\frac{\partial E_1[r]}{\partial y} = \frac{\partial E_1[r]}{\partial r} \frac{\partial r}{\partial y} = -v \frac{e^{-r}}{r} = -i \frac{e^{iv(z-\bar{c})}}{(z-\bar{c})} \quad (82)$$

$$\frac{\partial}{\partial y} [e^r \{E_1[r] + \delta i\pi\}] = v e^{-iv(z-\bar{c})} E_1[-iv(z-\bar{c})] - i e^{-iv(z-\bar{c})} \frac{e^{iv(z-\bar{c})}}{z-\bar{c}} + \delta i\pi v e^{-iv(z-\bar{c})} \quad (83)$$

$$\frac{\partial}{\partial y} [e^r \{E_1[r] + \delta i\pi\}] = v e^{-iv(z-\bar{c})} \{E_1[-iv(z-\bar{c})] + \delta i\pi\} - \frac{i}{z-\bar{c}} \quad (84)$$

$$v_3 = \text{Re} \left\{ \frac{1}{\pi} \int \left\{ v e^{-iv(z-\bar{c})} \{E_1[-iv(z-\bar{c})] + \delta i\pi\} - \frac{i}{z-\bar{c}} \right\} ds \right\} \quad (85)$$

Here, ds is the infinitesimal length variable over the boundary of the body. It can also be expressed using the positions of sources along the boundary of the body:

$$c = a + ib \quad (86)$$

$$dc = da + idb \quad (87)$$

$$dc = ds \cos \alpha + ids \sin \alpha \quad (88)$$

$$ds = dce^{-i\alpha} \quad (89)$$

Since the integrals in (79) and (85) are over the imaginary panels, (89) can be written in the following form.

$$ds = d\bar{c} e^{-i\beta} \quad (90)$$

$$\beta = \tan^{-1} \left\{ \frac{\text{Im}(\bar{c}_2 - \bar{c}_1)}{\text{Re}(\bar{c}_2 - \bar{c}_1)} \right\} \quad (91)$$

where β is the angle between the positive x axis of the body coordinate system and the imaginary panel. Equations (79) and (85) can now be written as

$$u_3 = \text{Re} \left\{ \frac{1}{\pi} \int_{\bar{c}_1}^{\bar{c}_2} \left\{ -i v e^{-iv(z-\bar{c})} \{E_1[-iv(z-\bar{c})] + \delta i\pi\} - \frac{1}{z-\bar{c}} \right\} d\bar{c} e^{-i\beta} \right\} \quad (92)$$

$$v_3 = \operatorname{Re} \left\{ \frac{1}{\pi} \int_{\bar{c}_1}^{\bar{c}_2} \left\{ \nu e^{-i\nu(z-\bar{c})} \{E_1[-i\nu(z-\bar{c})] + \delta i\pi\} - \frac{i}{z-\bar{c}} \right\} d\bar{c} e^{-i\beta} \right\} \quad (93)$$

These integral equations can be solved applying Integration by Parts (IBP).

$$m = E_1[-i\nu(z-\bar{c})] \quad (94)$$

$$dm = \frac{e^{i\nu(z-\bar{c})}}{z-\bar{c}} d\bar{c} \quad (95)$$

$$dn = e^{-i\nu(z-\bar{c})} d\bar{c} \quad (96)$$

$$n = \frac{e^{-i\nu(z-\bar{c})}}{i\nu} \quad (97)$$

$$u_3 = \operatorname{Re} \left\{ \frac{1}{\pi} \left\{ -i\nu \left(E_1[-i\nu(z-\bar{c})] \frac{e^{-i\nu(z-\bar{c})}}{i\nu} - \int_{\bar{c}_1}^{\bar{c}_2} \frac{e^{-i\nu(z-\bar{c})}}{i\nu} \frac{e^{i\nu(z-\bar{c})}}{z-\bar{c}} d\bar{c} \right) \right. \right. \\ \left. \left. - i\nu \int_{\bar{c}_1}^{\bar{c}_2} \delta i\pi e^{-i\nu(z-\bar{c})} d\bar{c} - \int_{\bar{c}_1}^{\bar{c}_2} \frac{d\bar{c}}{z-\bar{c}} \right\} e^{-i\beta} \right\} \quad (98)$$

$$u_3 = \operatorname{Re} \left\{ \frac{1}{\pi} [-e^{-i\nu(z-\bar{c})} E_1[-i\nu(z-\bar{c})] + \delta i\pi e^{-i\nu(z-\bar{c})}]_{\bar{c}_1}^{\bar{c}_2} e^{-i\beta} \right\} \quad (99)$$

Following the same steps taken to calculate u_3 , v_3 can be calculated as the following:

$$v_3 = \operatorname{Re} \left\{ \frac{1}{\pi} \left\{ \nu \left(E_1[-i\nu(z-\bar{c})] \frac{e^{-i\nu(z-\bar{c})}}{i\nu} - \int_{\bar{c}_1}^{\bar{c}_2} \frac{e^{-i\nu(z-\bar{c})}}{i\nu} \frac{e^{i\nu(z-\bar{c})}}{z-\bar{c}} d\bar{c} \right) \right. \right. \\ \left. \left. + \nu \int_{\bar{c}_1}^{\bar{c}_2} \delta i\pi e^{-i\nu(z-\bar{c})} d\bar{c} - \int_{\bar{c}_1}^{\bar{c}_2} \frac{id\bar{c}}{z-\bar{c}} \right\} e^{-i\beta} \right\} \quad (100)$$

$$v_3 = \operatorname{Re} \left\{ \frac{1}{\pi} [-ie^{-i\nu(z-\bar{c})} E_1[-i\nu(z-\bar{c})] - \delta \pi e^{-i\nu(z-\bar{c})}]_{\bar{c}_1}^{\bar{c}_2} e^{-i\beta} \right\} \quad (101)$$

Velocity Components Induced by the Fourth Term: $\Phi_4 = e^{-iv(z-\bar{c})}$

$$u_4 = Re \left\{ \int \frac{\partial e^{-iv(z-\bar{c})}}{\partial x} ds \right\} \quad (102)$$

$$u_4 = Re \left\{ \int_{\bar{c}_1}^{\bar{c}_2} (-ive^{-iv(z-\bar{c})}) d\bar{c} e^{-i\beta} \right\} \quad (103)$$

$$\mathbf{u}_4 = Re \left\{ [-e^{-iv(z-\bar{c})}]_{\bar{c}_1}^{\bar{c}_2} e^{-i\beta} \right\} = Re \left\{ [e^{-iv(z-\bar{c}_1)} - e^{-iv(z-\bar{c}_2)}] e^{-i\beta} \right\} \quad (104)$$

$$v_4 = Re \left\{ \int \frac{\partial e^{-iv(z-\bar{c})}}{\partial y} ds \right\} \quad (105)$$

$$v_4 = Re \left\{ \int_{\bar{c}_1}^{\bar{c}_2} (ve^{-iv(z-\bar{c})}) d\bar{c} e^{-i\beta} \right\} \quad (106)$$

$$\mathbf{v}_4 = Re \left\{ [-ie^{-iv(z-\bar{c})}]_{\bar{c}_1}^{\bar{c}_2} e^{-i\beta} \right\} = Re \left\{ i[e^{-iv(z-\bar{c}_1)} - e^{-iv(z-\bar{c}_2)}] e^{-i\beta} \right\} \quad (107)$$

2.3.4 Evaluation of the Potentials of Panels

The potential of a two dimensional body can be calculated by integrating the potentials of point sources along the body boundary.

$$\phi_b = \int_C \phi_s(z) ds \quad (108)$$

Since the body boundary is divided into N panels, this integral equation can be written as the sum of the potentials of all panels.

$$\phi_b = \sum_{i=1}^N \phi_{p_i} \quad (109)$$

Following similar steps in the calculation of the influences of panels, the potentials of all panels will be calculated term by term.

Potential Induced by the First Term: $\Phi_1 = \frac{1}{2\pi} \log(z - c)$

Recalling (27) and writing it by using variables in local coordinate system, the potential of a panel due to first term can be calculated as the following:

$$\phi_{p_1} = \frac{1}{4\pi} \int_{x_1}^{x_2} \log((x'_m - x_0)^2 + y'_m{}^2) dx_0 \quad (110)$$

$$X = (x'_m - x_0) \quad (111)$$

$$dX = -dx_0 \quad (112)$$

$$\phi_{p_1} = -\frac{1}{4\pi} \int_{x'_m - x_1}^{x'_m - x_2} \log(X^2 + y'_m{}^2) dX \quad (113)$$

Equation (113) can be solved by applying Integration by Parts:

$$m = \log(X^2 + y'_m{}^2) \quad (114)$$

$$dm = \frac{2X}{X^2 + y'_m{}^2} dX \quad (115)$$

$$dn = dX \quad (116)$$

$$n = X \quad (117)$$

$$\phi_{p_1} = -\frac{1}{4\pi} \left\{ X \log(X^2 + y'_m{}^2) - 2 \int \frac{X^2}{X^2 + y'_m{}^2} dX \right\} \quad (118)$$

$$\phi_{p_1} = -\frac{1}{4\pi} \left\{ X \log(X^2 + y'_m{}^2) - 2 \int \left(1 - \frac{dX}{1 + \left(\frac{X}{y'_m}\right)^2} \right) dX \right\} \quad (119)$$

$$\phi_{p_1} = -\frac{1}{4\pi} \left\{ X \log(X^2 + y'_m{}^2) - 2X + 2y'_m \tan^{-1} \frac{X}{y'_m} \right\} \quad (120)$$

$$\begin{aligned} \phi_{p_1} = \frac{1}{4\pi} & \left[-(x'_m - x_0) \log((x'_m - x_0)^2 + y'_m{}^2) + 2(x'_m - x_0) \right. \\ & \left. - 2y'_m \tan^{-1} \frac{x'_m - x_0}{y'_m} \right]_{x'_1}^{x'_2} \end{aligned} \quad (121)$$

Equation (121) is given in a different form by Katz and Plotkin [10] using the variables defined in equations (53) to (56). Finally, (121) becomes

$$\phi_{p_1} = \frac{1}{4\pi} [(x'_m - x'_1) \log r_1^2 - (x'_m - x'_2) \log r_2^2 + 2y'_m(\theta_2 - \theta_1)] \quad (122)$$

Returning to original variables, (122) can be written as the following:

$$\begin{aligned} \phi_{p_1} = \frac{1}{4\pi} & [(x'_m - x'_1) \log((x'_m - x'_1)^2 + y'_m{}^2) - (x'_m - x'_2) \log((x'_m - x'_2)^2 + y'_m{}^2)] \\ & + 2y'_m \left[\tan^{-1} \left(\frac{y'_m}{x'_m - x'_2} \right) - \tan^{-1} \left(\frac{y'_m}{x'_m - x'_1} \right) \right] \end{aligned} \quad (123)$$

Potential Induced by the Second Term: $\Phi_{p_2} = \frac{1}{2\pi} \log(z - \bar{c})$

Calculation of the potential due to the second term is identical to the calculation of potential due to the first term. One must pay attention while calculating the potentials due to the second term that the integration is over the corresponding imaginary panel. So, the variables should be calculated accordingly.

Potential Induced by the Third Term: $\Phi_3 = \frac{1}{\pi} PV \int_0^\infty \frac{e^{-ik(z-\bar{c})}}{k-v} dk$

$$\phi_{p_3} = Re \left(\frac{1}{\pi} \int_{\bar{c}_1}^{\bar{c}_2} \{ e^{-iv(z-\bar{c})} \{ E_1[-iv(z-\bar{c})] + \delta i\pi \} d\bar{c} e^{-i\beta} \} \right) \quad (124)$$

$$r = -iv(z - \bar{c}) \quad (125)$$

$$dr = ivd\bar{c} \quad (126)$$

$$m = E_1[r] = E_1[-iv(z - \bar{c})] \quad (127)$$

$$\frac{dm}{d\bar{c}} = \frac{dm}{dr} \frac{dr}{d\bar{c}} = -\frac{e^{-r}}{r} iv \quad (128)$$

$$dm = \frac{e^{iv(z-\bar{c})}}{z-\bar{c}} d\bar{c} \quad (129)$$

$$dn = e^{-iv(z-\bar{c})} d\bar{c} \quad (130)$$

$$n = \frac{e^{-iv(z-\bar{c})}}{iv} \quad (131)$$

$$\begin{aligned} \phi_{p_3} = \text{Re} \left\{ \left(\frac{1}{\pi} \left(\frac{e^{-iv(z-\bar{c})}}{iv} E_1[-iv(z-\bar{c})] - \int_{\bar{c}_1}^{\bar{c}_2} \frac{e^{-iv(z-\bar{c})}}{iv} \frac{e^{iv(z-\bar{c})}}{z-\bar{c}} d\bar{c} \right) \right. \right. \\ \left. \left. + \delta i\pi \int_{\bar{c}_1}^{\bar{c}_2} e^{-iv(z-\bar{c})} d\bar{c} \right) e^{-i\beta} \right\} \end{aligned} \quad (132)$$

$$\phi_{p_3} = \text{Re} \left[\frac{1}{\pi} \left(\left(\frac{e^{-iv(z-\bar{c})}}{iv} E_1[-iv(z-\bar{c})] + \frac{1}{iv} \log(z-\bar{c}) \right) + \frac{\delta\pi}{v} e^{-iv(z-\bar{c})} \right) e^{-i\beta} \right]_{\bar{c}_1}^{\bar{c}_2} \quad (133)$$

Potential Induced by the Fourth Term: $\Phi_4 = e^{-iv(z-\bar{c})}$

$$\phi_{p_4} = \text{Re} \left\{ \int_{\bar{c}_1}^{\bar{c}_2} e^{-iv(z-\bar{c})} d\bar{c} e^{-i\beta} \right\} \quad (134)$$

$$\phi_{p_4} = \text{Re} \left\{ \frac{1}{iv} [e^{-iv(z-\bar{c})}]_{\bar{c}_1}^{\bar{c}_2} e^{-i\beta} \right\} \quad (135)$$

2.3.5 Influence Matrix and Complex Force Coefficient

The final task to obtain added mass and damping coefficients is to calculate the unknown source strengths of panels. As mentioned before, this problem is solved by satisfying body boundary condition on the body surface. The $N \times N$ influence matrix I can be formed as the following:

$$I_{km} = \begin{bmatrix} I_{11} & \cdots & I_{1N} \\ \vdots & \ddots & \vdots \\ I_{N1} & \cdots & I_{NN} \end{bmatrix} \quad (136)$$

I_{km} means the influence of m^{th} panel at the midpoint of k^{th} panel, where $k, m = 1, 2, 3, \dots, N$. Recalling the velocity components evaluated in 2.3.3, I_{km} can be calculated using appropriate panel midpoint coordinates and panel end points coordinates.

$$I_{km} = (u_1 - u_2 - u_3 + ju_4)n_x + (v_1 - v_2 - v_3 + jv_4)n_y \quad (137)$$

If the normal velocity matrix for sway, heave and roll modes of motion and unknown source strength matrix are written as the following respectively:

$$U_n = \begin{bmatrix} U_1 \\ \vdots \\ U_N \end{bmatrix}, \quad (138)$$

$$Q = \begin{bmatrix} Q_1 \\ \vdots \\ Q_N \end{bmatrix}, \quad (139)$$

Q can be obtained as the following:

$$Q = [I_{km}] \setminus U_n \quad (140)$$

Once the source strengths of all panels are determined, added mass and damping coefficients of ship sections can be calculated easily. The complex force coefficient can be calculated following the same steps taken to calculate influence matrix:

$$J_{km} = \begin{bmatrix} J_{11} & \cdots & J_{1N} \\ \vdots & \ddots & \vdots \\ J_{N1} & \cdots & J_{NN} \end{bmatrix} \quad (141)$$

where J_{km} represents the potential induced by the m^{th} panel at the midpoint of k^{th} panel.

$$J_{km} = \phi_{p1} - \phi_{p2} - \phi_{p3} + j\phi_{p4} \quad (142)$$

$$f = -\rho j\omega \sum J_{km} Q dl \quad (143)$$

j represents time complex unit in (142) and (143) and dl is the length of each panel. Multiplication of J and Q matrices produces a $N \times 1$ matrix. Multiplying the elements of this matrix with the corresponding panel lengths and summing all of them gives the complex force coefficient. Finally, added mass and damping coefficients can be found by decomposing (143) as shown in (21), (22) and (23).

2.4 Results and Discussions

The added mass and damping coefficients of a semicircle in heave and sway motions calculated by SMA can be seen in Figure 3 and Figure 4, respectively. It can be seen in both figures that the results of panel method and Vugst [1] are in very good agreement. The added mass and damping coefficients of a fully submerged circle is also investigated. The results can be seen in Figure 5. The results of panel method and Frank [2] are in good agreement. One also can check the accuracy of results for a fully submerged body by integrating the source strengths of panels along the body boundary. The integral of source strengths must be 0 for a fully submerged body.

$$\int_C Q ds = 0 \quad (144)$$

This integral equation can also be written in a summation form.

$$\sum_{i=1}^N Q_i ds = 0 \quad (145)$$

One of the problems with the panel method solution is the presence of irregular frequencies which can be seen easily in Figures 3 and 4. The presence of irregular frequencies is not related to the physics of the problem but is related to the solution of the boundary integral equations. The solutions of boundary integral equations suffer from natural frequencies of the enclosed fluid inside the body that manifest themselves as irregular frequencies and the panel method estimates added mass and damping coefficients with great inaccuracy. Note that if the body is not surface piercing and if it is fully submerged, the effects of irregular frequencies will not be seen in the solutions. The irregular frequencies can be suppressed by paneling the free surface inside the body, also known as the “lid method” or by modifying the potential function. The studies of Ursell [13] , Lee and Sclavounos [14] and Liapis [15] can be mentioned here regarding irregular frequencies and methods used to suppress them, however suppression of irregular frequencies is not within the scope of this thesis.

The second problem with the panel method is that it cannot provide accurate results for bulbous sections. According to Frank [2], “if any portion of a surface piercing body remains outside of a cylinder drawn downward from the intersection of the body and the free surface, the panel method will fail to give accurate added mass and damping coefficients.” The added mass and damping coefficients of one of the bulbous sections of the Zumwalt-Class destroyer, which are shown in Figure 6, seem to support this argument, however without any supporting experimental results it is hard to comment on the accuracy of these results.

2.5 Conclusions

The results obtained by using panel method are satisfactory except the presence of the irregular frequencies. SMA can be improved by solving the problem regarding irregular frequencies. The effects of irregular frequencies were observed during the calculation of the sectional added mass and damping coefficients of the Zumwalt-Class destroyer. The results were almost useless for some sections, especially at high frequency range.

One also must pay attention while using panel method to calculate added mass and damping coefficients of bulbous sections. The results may sometimes be inaccurate at very low frequencies as shown in Figure 6, but this method can also give inaccurate results in medium frequencies as well, as it was observed during the calculation of sectional added mass and damping coefficients of the Zumwalt-Class destroyer.

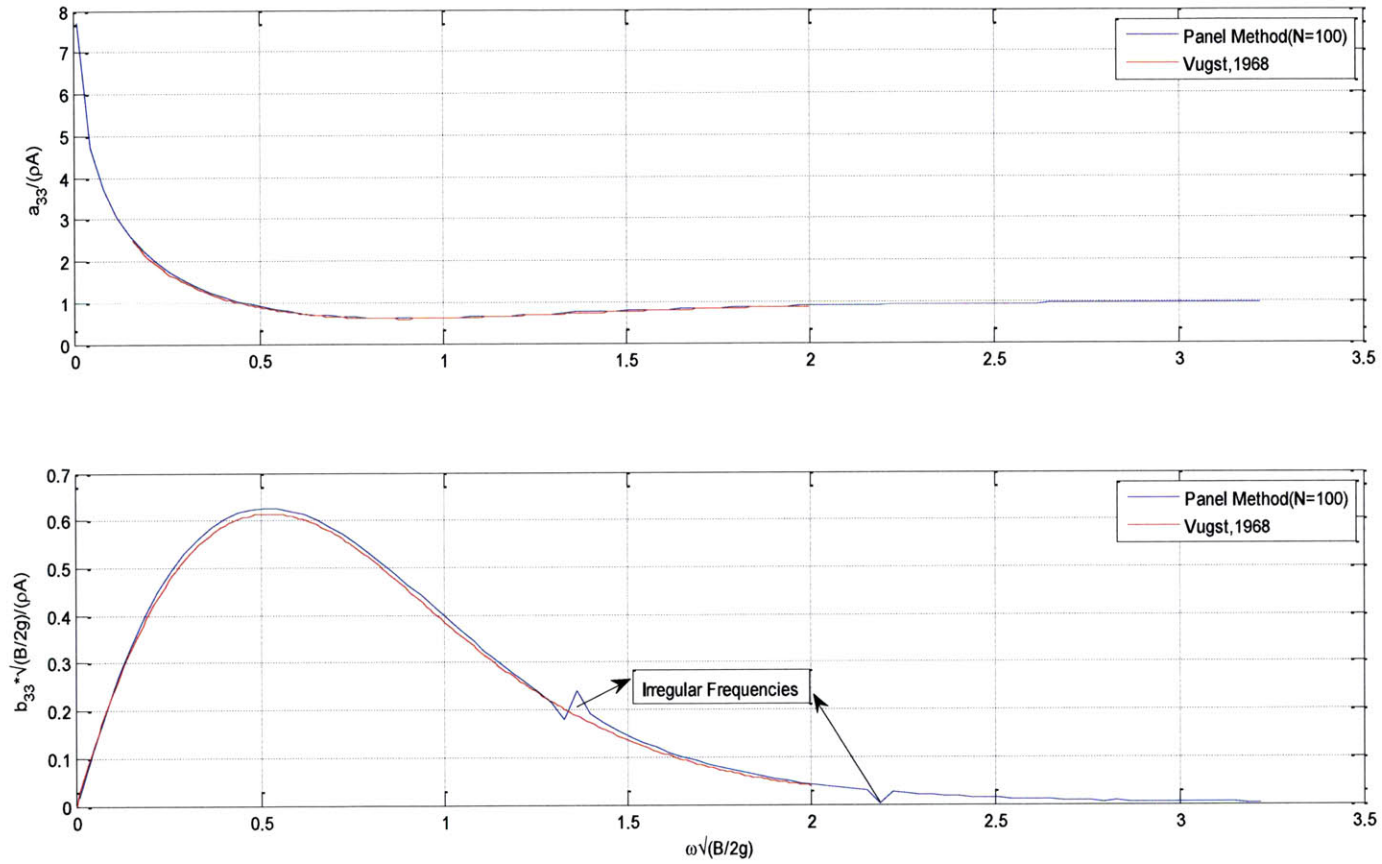


Figure 3: Heave added mass and damping coefficients of a semicircle with $R=5\text{m}$.calculated by SMA

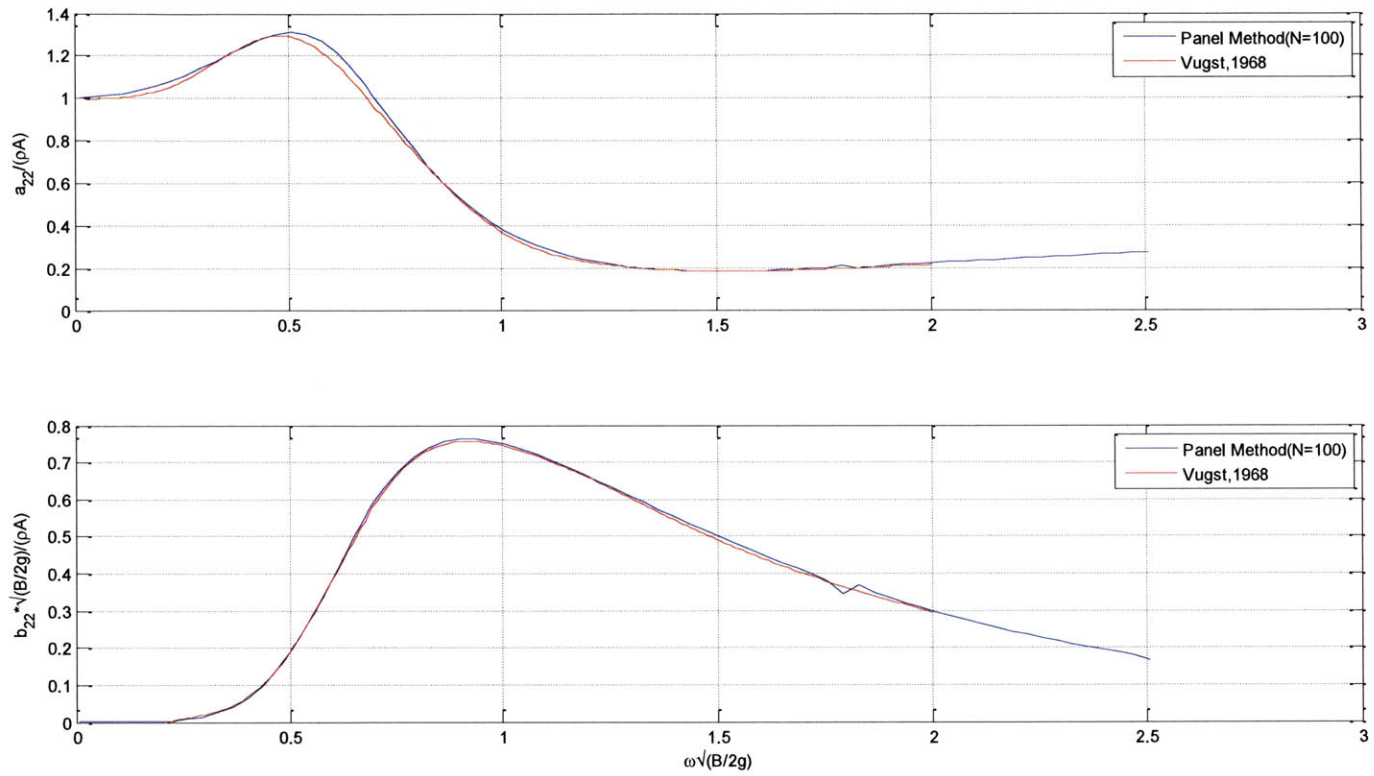


Figure 4: Sway added mass and damping coefficients of a semicircle with $R=5m$.calculated by SMA

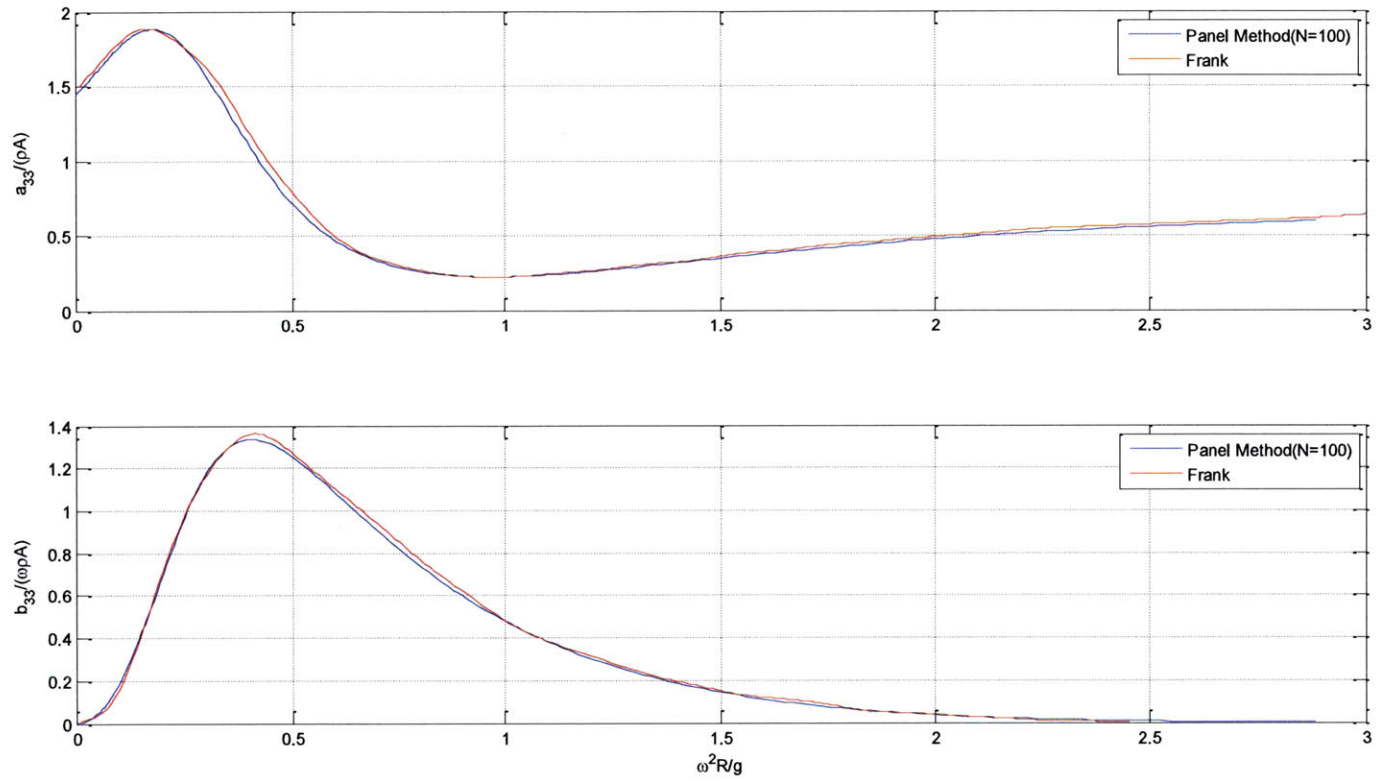


Figure 5: Added mass and damping coefficients of a circle whose center is $1.25R$ below the free surface ($R=7$ m.)

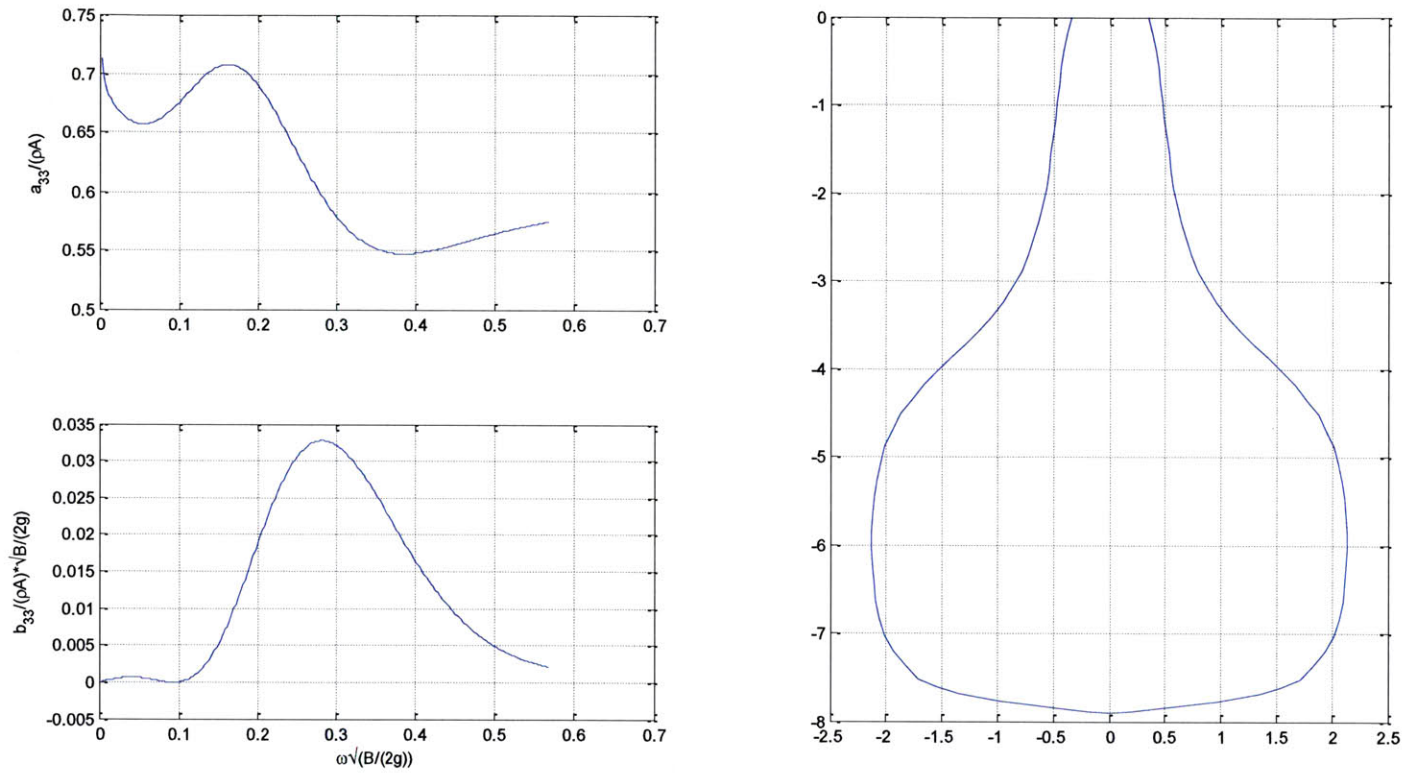


Figure 6: Added mass and damping coefficients of a bulbous section (One of the bulbous sections of the Zumwalt-Class destroyer)

CHAPTER 3

SHIP MOTIONS IN REGULAR WAVES

3 Introduction

The motions of a ship in six degrees of freedom are of great interest for naval architects since the performance of a ship in a seaway is the ultimate criterion for a ship's design. Hydrodynamic coefficients of the ship's hull and excitation forces/moments in the equation of motion must be known to calculate the motions of the ship advancing at a constant forward speed in waves.

In the previous chapter, a method was developed to calculate sectional added mass and damping coefficients. The added mass and damping coefficients of a ship are evaluated using these sectional hydrodynamic coefficients in the framework of linear strip theory. In addition, the excitation forces and moments are derived both using sectional Froude-Krylov and diffraction forces and taking forward speed effects into account. The theory of Salvasen *et al.* [5] constitutes the mainframe of above mentioned calculations.

3.1 Evaluation of Added Mass and Damping Coefficients of a Ship

The oscillatory motions in six degrees of freedom are assumed to be linear and harmonic for a ship moving at constant forward speed with arbitrary heading in regular sinusoidal waves. A right-handed Cartesian coordinate system (x , y , and z) is fixed with respect to the mean hull position of the ship, with z vertically upward through the center of gravity of the ship, x in the

direction of forward motion and the origin in the plane of undisturbed free surface. The ship is assumed to oscillate as a rigid body in six degrees of freedom with amplitudes ζ_j ($j=1, 2, \dots, 6$). $j = 1, 2, 3, 4, 5, 6$ refer to surge, sway, heave, roll, pitch and yaw respectively. The incident wave angles, β , with respect to the direction of forward motion are defined as in Figure 8 such that $\beta = 0$ degrees for following seas and $\beta = 180$ degrees for head seas.

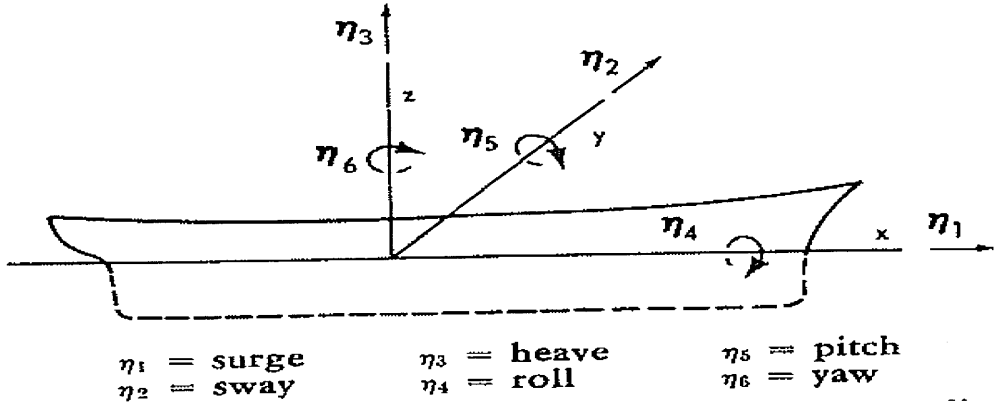


Figure 7: Cartesian coordinate system fixed to the ship and modes of motions in six degrees of freedom (Adapted from Salvasen et al. [4])

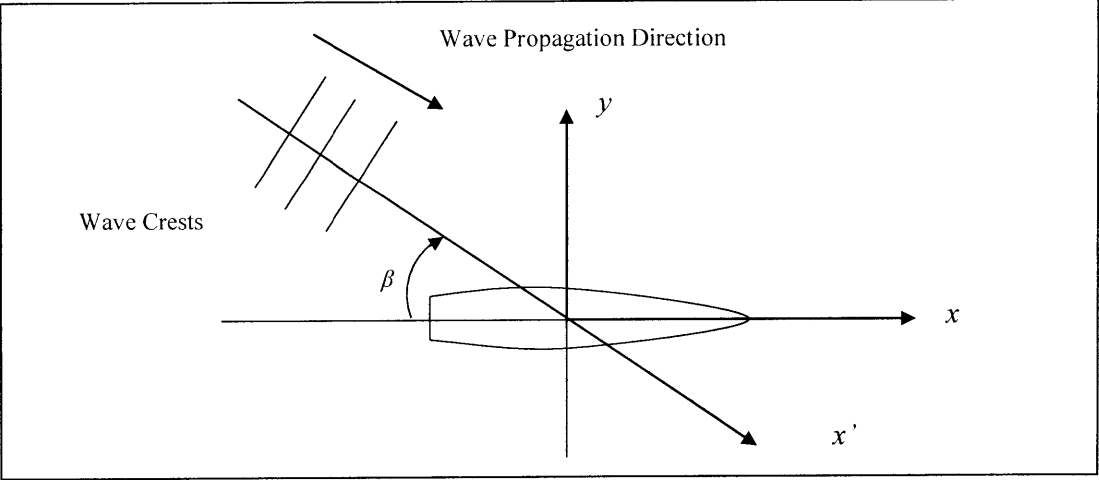


Figure 8: Definition of incident wave angle

In addition, the ship oscillates with the frequency of encounter, ω_e , which is related to wave frequency, ω , with

$$\omega_e = \omega - kU \cos \beta \quad (146)$$

where k is wave number, U is forward speed of the ship and β is incident wave angle.

If viscous effects are ignored, the fluid motions can be assumed irrotational and the problem can be solved in the framework of potential flow theory.

The total velocity potential can be written in the following form:

$$\Phi(x, y, z, t) = [-Ux + \phi_S(x, y, z)] + \phi_T(x, y, z)e^{i\omega_e t} \quad (147)$$

where $-Ux + \phi_S(x, y, z)$ is the steady contribution with U the forward speed of the ship, ϕ_T is the complex amplitude of the unsteady potential. As discussed in the previous chapter, this potential function must also satisfy Laplace's Equation in the fluid domain, linearized free surface condition, bottom boundary condition, radiation condition in the far field and body boundary condition on the surface of the body.

In order to linearize body boundary condition and free surface condition, it is assumed that the hull geometry is such that the steady perturbation potential, ϕ_S , and its derivatives are small. Since small oscillatory motions are of interest, potential ϕ_T and its derivatives are also assumed small. The problem can be linearized by disregarding higher order terms in ϕ_S and ϕ_T , and taking first order terms into account. The amplitude of the time dependent part of the potential, ϕ_T , can be written in the following form.

$$\phi_T = \phi_I + \phi_D + \sum_{j=1}^6 \zeta_j \phi_j \quad (148)$$

Here ϕ_I is the incident wave potential, ϕ_D is the diffraction potential and ϕ_j is the contribution to the velocity potential from the j^{th} mode of motion. All of these potentials must satisfy following boundary conditions individually.

Using the notations of Salvasen *et al.* [5], the problem is formulated as follows. The steady perturbation potential, ϕ_S , must satisfy the body condition on the mean hull position.

$$\frac{\partial}{\partial n} [-Ux + \phi_S] = 0 \quad (149)$$

It must also satisfy free surface boundary condition on the undisturbed free surface.

$$U^2 \frac{\partial^2 \phi_S}{\partial x^2} + g \frac{\partial \phi_S}{\partial z} = 0 \quad \text{on } z = 0 \quad (150)$$

The incident wave potential and diffraction potential must satisfy the following boundary condition on the mean position

$$\frac{\partial \phi_I}{\partial n} + \frac{\partial \phi_D}{\partial n} = 0 \quad (151)$$

and following boundary condition on the undisturbed free surface.

$$\left[\left(i\omega_e - U \frac{\partial}{\partial n} \right)^2 + g \frac{\partial}{\partial z} \right] (\phi_I, \phi_D) = 0 \quad \text{on } z = 0 \quad (152)$$

The oscillatory potentials, ϕ_j , ($j=1, 2, \dots, 6$) must satisfy the following body boundary condition

$$\frac{\partial \phi_j}{\partial n} = i\omega_e n_j + U m_j \quad (153)$$

on the hull at mean position and

$$\left(i\omega_e - U \frac{\partial}{\partial x} \right)^2 \phi_j + g \frac{\partial \phi_j}{\partial z} = 0 \quad \text{on } z = 0 \quad (154)$$

at the undisturbed free surface.

Here n_j is the generalized normal defined by $(n_1, n_2, n_3) = n$ and $(n_4, n_5, n_6) = r \times n$ where n is the unit normal vector and r is the position vector with respect to the origin of the coordinate system. $m_j=0$ for $j=1, 2, 3, 4$ while

$$m_5 = n_3 \text{ and } m_6 = -n_2 \quad (155)$$

The hull condition (153) for oscillatory potential components can be simplified by dividing the oscillatory potential into speed independent and speed dependent parts.

$$\phi_j = \phi_j^0 + \frac{U}{i\omega_e} \phi_j^U \quad (156)$$

Here ϕ_j^0 is the speed independent part and ϕ_j^U is the speed dependent part of oscillatory potential ϕ_j .

So (153) can be expressed as the following:

$$\frac{\partial \phi_j^0}{\partial n} = i\omega_e n_j \text{ and } \frac{\partial \phi_j^U}{\partial n} = i\omega_e m_j \quad (157)$$

ϕ_j^0 and ϕ_j^U must also satisfy Laplace's Equation in the fluid domain, free surface condition, radiation condition, bottom boundary condition and body boundary condition. It follows from the relationships in (155) that $\phi_j^U=0$ for $j=1, 2, 3, 4$ and

$$\phi_5^U = \phi_3^0 \text{ and } \phi_6^U = -\phi_2^0 \quad (158)$$

It can be seen from the last equation that the oscillatory potential components can be expressed in terms of speed independent part of the potential, ϕ_j^0 .

$$\phi_j = \phi_j^0 \text{ for } j = 1, 2, 3, 4 \quad (159)$$

$$\phi_5 = \phi_5^0 + \frac{U}{i\omega_e} \phi_3^0 \quad (160)$$

$$\phi_6 = \phi_6^0 - \frac{U}{i\omega_e} \phi_2^0 \quad (161)$$

The pressure in the fluid can be found by applying Bernoulli's Equation.

$$p = -\rho \left(\frac{\partial \Phi}{\partial t} + \frac{1}{2} |\nabla \Phi|^2 + gz \right) \quad (162)$$

If the pressure is linearized by including only first order terms in ϕ_S and ϕ_T , and the steady pressure term is ignored, then the linearized time-dependent pressure on the hull can be written as the following:

$$p = -\rho \left(i\omega_e - U \frac{\partial}{\partial x} \right) \phi_T e^{i\omega_e t} - \rho g (\zeta_3 + \zeta_4 y - \zeta_5 x) e^{i\omega_e t} \quad (163)$$

“The last term in (163) gives the ordinary buoyancy restoring force and moment, which is ignored in the derivation of hydrodynamic coefficients and excitation forces and moments.”[5]

The amplitudes of forces and moments acting on the hull can be calculated by integrating (163) on the mean hull position. The amplitudes of forces and moments can be evaluated as the following:

$$H_j = -\rho \iint_S n_j \left(i\omega_e - U \frac{\partial}{\partial x} \right) \phi_T ds \quad j = 1,2,3,4,5,6 \quad (164)$$

Here H_1, H_2, H_3 are the force components in x, y and z directions, and H_4, H_5, H_6 are the moments about x, y and z axes. The force and moment components can be decomposed as following using (148).

$$H_j = F_j + G_j \quad (165)$$

Here F_j represents excitation forces and moments, and G_j represents force and moment due to the motions of the body in six degrees of freedom.

$$F_j = -\rho \iint_S n_j \left(i\omega_e - U \frac{\partial}{\partial x} \right) (\phi_I + \phi_D) ds \quad (166)$$

$$G_j = -\rho \iint_S n_j \left(i\omega_e - U \frac{\partial}{\partial x} \right) \sum_{k=1}^6 \zeta_k \phi_k ds = \sum_{k=1}^6 T_{jk} \zeta_k \quad (167)$$

Here T_{jk} refers to hydrodynamic force and moment in the j^{th} direction due to the motion with unit amplitude in k^{th} direction.

$$T_{jk} = -\rho \iint_S n_j \left(i\omega_e - U \frac{\partial}{\partial x} \right) \phi_k ds \quad (168)$$

T_{jk} can be separated into its real and imaginary parts as the following:

$$T_{jk} = \omega_e^2 A_{jk} - i\omega_e B_{jk} \quad (169)$$

Here A_{jk} is the added mass and B_{jk} is the damping coefficient of the ship. These hydrodynamic coefficients are expressed in terms of three dimensional oscillatory potentials, ϕ_k ($k = 1, 2, 3, 4, 5, 6$). A_{jk} and B_{jk} will now be expressed as the integral of sectional added mass and damping coefficients along the length of the ship. Following a variant of Stokes' theorem given as following by Salvasen *et al.* [5],

$$\iint_S n_j U \frac{\partial}{\partial x} \phi_k ds = U \iint_S m_j \phi_k ds - U \int_{C_x} n_j \phi_k dl \quad (170)$$

(168) can be written in a new form:

$$T_{jk} = -\rho i\omega_e \iint_S n_j \phi_k ds + U\rho \iint_S m_j \phi_k ds - U\rho \int_{C_A} n_j \phi_k dl \quad (171)$$

Here C_A refers to the aftermost section of the ship. Using the definition of oscillatory potential given in (156), speed independent part of T_{jk} and speed independent part of the line integral at any cross section can be written as following respectively:

$$T_{jk}^0 = -\rho i \omega_e \iint_S n_j \phi_k^0 ds \quad (172)$$

$$t_{jk} = -\rho i \omega_e \int_{C_x} n_j \phi_k^0 dl \quad (173)$$

Finally, the added mass and damping coefficients (171) can be expressed in terms of speed independent terms (172) and (173) by applying the potential functions given in (159), (160) and (161).

For $j, k = 1, 2, 3, 4$

$$T_{jk} = T_{jk}^0 + \frac{U}{i\omega_e} t_{jk}^A \quad (174)$$

where t_{jk}^A represents the hydrodynamic force/moment evaluated at the aftermost section.

For $j=5, 6$ and $k=1, 2, 3, 4$

$$T_{5k} = T_{5k}^0 - \frac{U}{i\omega_e} T_{3k}^0 + \frac{U}{i\omega_e} t_{5k}^A \quad (175)$$

$$T_{6k} = T_{6k}^0 + \frac{U}{i\omega_e} T_{2k}^0 + \frac{U}{i\omega_e} t_{6k}^A \quad (176)$$

For $j=1, 2, 3, 4$ and $k=5, 6$

$$T_{j5} = T_{j5}^0 + \frac{U}{i\omega_e} T_{j3}^0 + \frac{U}{i\omega_e} t_{j5}^A - \frac{U^2}{\omega_e^2} t_{j3}^A \quad (177)$$

$$T_{j6} = T_{j6}^0 - \frac{U}{i\omega_e} T_{j2}^0 + \frac{U}{i\omega_e} t_{j6}^A - \frac{U^2}{\omega_e^2} t_{j2}^A \quad (178)$$

For $j=k=5, 6$

$$T_{55} = T_{55}^0 + \frac{U^2}{\omega_e^2} T_{33}^0 + \frac{U}{i\omega_e} t_{55}^A - \frac{U^2}{\omega_e^2} t_{53}^A \quad (179)$$

$$T_{66} = T_{66}^0 + \frac{U^2}{\omega_e^2} T_{22}^0 + \frac{U}{i\omega_e} t_{66}^A + \frac{U^2}{\omega_e^2} t_{62}^A \quad (180)$$

In order to reduce the surface integrals in speed independent terms, T_{jk}^0 , to integrals along the length of the ship, following assumptions are made by Salvasen *et al.*[5].

- The beam and the draft of the ship are much smaller than her length ($L \gg B, T$). So, the surface integration variable ds and T_{jk}^0 can be written as following the respectively:

$$ds = dx dl \quad (181)$$

Here dx is the integration variable in the x direction.

$$T_{jk}^0 = -\rho i \omega_e \int_L \int_{C_x} n_j \phi_k^0 dl dx = \int_L t_{jk} dx \quad (182)$$

- Since the hull is assumed to be long and slender, the derivatives in the longitudinal direction must be much smaller than those in lateral directions.

$$\frac{\partial}{\partial x} \ll \frac{\partial}{\partial y} \text{ or } \frac{\partial}{\partial z} \quad (183)$$

- It follows from the slender hull assumption that the x component of the hull normal should be much smaller than y and z components.

$$n_1 \ll n_2, n_3 \quad (184)$$

Following this assumption, three components of the three dimensional generalized normal, $n_j (j=2, 3, 4)$ can be replaced by the two dimensional generalized normal, N_j , in the y - z plane. ($j=2, 3, 4$)

$$n_5 = -xN_3 \text{ and } n_6 = xN_2 \quad (185)$$

- Finally, in order to linearize the free surface condition, it is assumed that the frequency of encounter is high ($\omega_e \gg U \frac{\partial}{\partial x}$). This assumption requires that the wave length is of the order of ship's beam. Although this assumption seems to be inappropriate for the low frequency range,

calculations show that the heave and pitch motions are predicted very accurately by the theory since these motions are dominated by hydrostatic terms in the low frequency range.

The three dimensional Laplace's Equation and boundary conditions reduce to two dimensional Laplace's Equation and boundary conditions under these assumptions. The three dimensional problem can now be considered as a two dimensional problem of a cylinder with cross section C_x oscillating in the free surface. The speed independent three dimensional oscillatory potential ϕ_k^0 can be replaced with a two dimensional potential:

$$\phi_k^0 = \psi_k \text{ for } k = 2, 3, 4 \quad (186)$$

$$\phi_5^0 = -x\psi_3 \text{ and } \phi_6^0 = x\psi_2 \quad (187)$$

Here ψ_k represents the potential of any cross section in sway, heave and roll for $k = 2, 3, 4$ respectively, and it can be calculated using the panel method described in the previous chapter.

The hydrodynamic force or moment can now be written using the two dimensional sectional potential as the following:

$$t_{jj} = -\rho i \omega_e \int_{C_x} N_j \psi_j dl = \omega_e^2 a_{jj} - i \omega_e b_{jj} \quad (188)$$

Here a_{jj} and b_{jj} are the sectional added mass and damping coefficients for sway, heave and roll.

($j = 2, 3, 4$) The cross-coupling coefficient can also be written in the following form:

$$t_{24} = -\rho i \omega_e \int_{C_x} N_2 \psi_4 dl = \omega_e^2 a_{24} - i \omega_e b_{24} \quad (189)$$

Considering only ships with lateral symmetry, the non-zero added mass and damping coefficients are as follows.

$$A_{33}^0 = \int_L a_{33} dx \quad (190)$$

$$A_{33} = \int_L a_{33} dx - \frac{U}{\omega_e^2} b_{33}^A \quad (191)$$

$$B_{33}^0 = \int_L b_{33} dx \quad (192)$$

$$B_{33} = \int_L b_{33} dx + U a_{33}^A \quad (193)$$

$$A_{35} = - \int_L x a_{33} dx - \frac{U}{\omega_e^2} B_{33}^0 + \frac{U}{\omega_e^2} x_A b_{33}^A - \frac{U^2}{\omega_e^2} a_{33}^A \quad (194)$$

$$B_{35} = - \int_L x b_{33} dx + U A_{33}^0 - U x_A a_{33}^A - \frac{U^2}{\omega_e^2} b_{33}^A \quad (195)$$

$$A_{53} = - \int_L x a_{33} dx + \frac{U}{\omega_e^2} B_{33}^0 + \frac{U}{\omega_e^2} x_A b_{33}^A \quad (196)$$

$$B_{53} = - \int_L x b_{33} dx - U A_{33}^0 - U x_A a_{33}^A \quad (197)$$

$$A_{55} = \int_L x^2 a_{33} dx + \frac{U^2}{\omega_e^2} A_{33}^0 - \frac{U}{\omega_e^2} x_A^2 b_{33}^A + \frac{U^2}{\omega_e^2} x_A a_{33}^A \quad (198)$$

$$B_{55} = \int_L x^2 b_{33} dx + \frac{U^2}{\omega_e^2} B_{33}^0 + U x_A^2 a_{33}^A + \frac{U^2}{\omega_e^2} x_A b_{33}^A \quad (199)$$

$$A_{22}^0 = \int_L a_{22} dx \quad (200)$$

$$A_{22} = \int_L a_{22} dx - \frac{U}{\omega_e^2} b_{22}^A \quad (201)$$

$$B_{22}^0 = \int_L b_{22} dx \quad (202)$$

$$B_{22} = \int_L b_{22} dx + U a_{22}^A \quad (203)$$

$$A_{24}^0 = A_{42}^0 = \int_L a_{24} dx \quad (204)$$

$$A_{24} = A_{42} = \int_L a_{24} dx - \frac{U}{\omega_e^2} b_{24}^A \quad (205)$$

$$B_{24}^0 = B_{42}^0 = \int_L b_{24} dx \quad (206)$$

$$B_{24} = B_{42} = \int_L b_{24} dx + Ua_{24}^A \quad (207)$$

$$A_{26} = \int_L xa_{22} dx + \frac{U}{\omega_e^2} B_{22}^0 - \frac{U}{\omega_e^2} x_A b_{22}^A + \frac{U^2}{\omega_e^2} a_{22}^A \quad (208)$$

$$B_{26} = \int_L xb_{22} dx - UA_{22}^0 + Ux_A a_{22}^A + \frac{U^2}{\omega_e^2} b_{22}^A \quad (209)$$

$$A_{44} = \int_L a_{44} dx - \frac{U}{\omega_e^2} b_{44}^A \quad (210)$$

$$B_{44} = \int_L b_{44} dx + Ua_{44}^A + B_{44}^* \quad (211)$$

$$A_{46} = \int_L xa_{24} dx + \frac{U}{\omega_e^2} B_{24}^0 - \frac{U}{\omega_e^2} x_A b_{24}^A + \frac{U^2}{\omega_e^2} a_{24}^A \quad (212)$$

$$B_{46} = \int_L xb_{24} dx - UA_{24}^0 + Ux_A a_{24}^A + \frac{U^2}{\omega_e^2} b_{24}^A \quad (213)$$

$$A_{62} = \int_L xa_{22} dx - \frac{U}{\omega_e^2} B_{22}^0 - \frac{U}{\omega_e^2} x_A b_{22}^A \quad (214)$$

$$B_{62} = \int_L xb_{22} dx + UA_{22}^0 + Ux_A a_{22}^A \quad (215)$$

$$A_{64} = \int_L xa_{24} dx - \frac{U}{\omega_e^2} B_{24}^0 - \frac{U}{\omega_e^2} x_A b_{24}^A \quad (216)$$

$$B_{64} = \int_L xb_{24} dx + UA_{24}^0 + Ux_A a_{24}^A \quad (217)$$

$$A_{66} = \int_L x^2 a_{22} dx + \frac{U^2}{\omega_e^2} A_{22}^0 - \frac{U}{\omega_e^2} x_A^2 b_{22}^A + \frac{U^2}{\omega_e^2} x_A a_{22}^A \quad (218)$$

$$B_{66} = \int_L x^2 b_{22} dx + \frac{U^2}{\omega_e^2} B_{22}^0 + Ux_A^2 a_{22}^A + \frac{U^2}{\omega_e^2} x_A b_{22}^A \quad (219)$$

In equations (190) to (219), x_A is the x -coordinate of the aftermost section of the ship, a_{jj}^A and b_{jj}^A are the added mass and damping coefficients of the aftermost section of the ship; U is the forward speed of the ship.

In equation (211), B_{44}^* represents viscous roll damping. Although the viscous effects are ignored throughout the derivations of all hydrodynamic forces and moments, they should be included in the calculation of roll damping coefficient. The roll motion is strongly affected by viscous damping. In Salvasen *et al.*[5] B_{44}^* is given by $K\dot{\eta}_{4_{max}}$ where K is a function of frequency, viscosity, the bilge keel dimensions and the hull geometry. $\dot{\eta}_{4_{max}}$ represents maximum roll velocity and it must be estimated before the motions are calculated. If the difference between estimated and calculated values of maximum roll velocities is too high, a new value must be estimated and the motions must be calculated using the new value. Here, the studies of Ikeda *et al.*[16], Ikeda *et al.*[17] and Ikeda [18] can be cited as references regarding the calculation of viscous roll damping.

3.2 Evaluation of Excitation Forces and Moments

The excitation forces and moments were given by (166) as following in the previous section.

$$F_j = -\rho \iint_S n_j \left(i\omega_e - U \frac{\partial}{\partial x} \right) (\phi_I + \phi_D) ds \quad (220)$$

This expression can be separated into its incident wave part and diffraction part respectively as the following:

$$F_j^I = -\rho \iint_S n_j \left(i\omega_e - U \frac{\partial}{\partial x} \right) \phi_I ds \quad (221)$$

$$F_j^D = -\rho \iint_S n_j \left(i\omega_e - U \frac{\partial}{\partial x} \right) \phi_D ds \quad (222)$$

The incident wave potential, which satisfies linear free surface condition, Laplace's Equation in the fluid domain, infinite radiation condition and consistent with the coordinate system and wave angle defined in the beginning of this chapter, is

$$\phi_I = \frac{igA}{\omega} e^{-ik(x \cos \beta - y \sin \beta)} e^{kz} \quad (223)$$

where A is the wave amplitude, g the acceleration of gravity, ω the wave frequency and k the wave number which is ω^2/g for deep water. This form of the incident wave potential is consistent with the coordinate system defined in the beginning of this chapter and the incident wave angle defined in Figure 8. The incident wave part of the excitation forces and moments can be computing by introducing (223) to (221).

$$F_j^I = -\rho i \iint_S n_j (\omega_e + kU \cos \beta) \frac{igA}{\omega} e^{-ik(x \cos \beta - y \sin \beta)} e^{kz} ds \quad (224)$$

This equation can be written in a simpler form using the relationship given in (146),

$$F_j^I = -\rho i \omega \iint_S n_j \phi_I ds \quad (225)$$

and known as Froude-Krylov force and moment.

The diffraction part of excitation force and moment can be written as following by applying Stokes Theorem[5]:

$$F_j^D = -\rho \iint_S (i\omega_e n_j - U m_j) \phi_D ds - \rho U \int_{C_A} n_j \phi_D dl \quad (226)$$

Recalling the hull conditions $\frac{\partial \phi_j^0}{\partial n} = i\omega_e n_j$ and $\frac{\partial \phi_j^U}{\partial n} = i\omega_e m_j$ from (157), (226) is written as

$$F_j^D = -\rho \iint_S \frac{\partial}{\partial n} \left(\phi_j^0 - \frac{U}{i\omega_e} \phi_j^U \right) \phi_D ds - \frac{\rho U}{i\omega_e} \int \frac{\partial \phi_j^0}{\partial n} \phi_D dl \quad (227)$$

“For any two functions ϕ and ψ satisfying the same Laplace’s Equation, the free surface condition, the radiation condition at infinity and the bottom condition, it is found by using Green’s second identity [5], that”

$$\iint_S \phi \frac{\partial \psi}{\partial n} ds = \iint_S \psi \frac{\partial \phi}{\partial n} ds \quad (228)$$

This relationship is also valid for two dimensional cases and it can be applied to both surface integrals and line integrals.[5] Now, (227) can be written in a new form using the relationship in (228).

$$F_j^D = -\rho \iint_S \left(\phi_j^0 - \frac{U}{i\omega_e} \phi_j^U \right) \frac{\partial \phi_D}{\partial n} ds - \frac{\rho U}{i\omega_e} \int_{C_A} \phi_j^0 \frac{\partial \phi_D}{\partial n} dl \quad (229)$$

This equation can be expressed as following by introducing hull condition (151).

$$F_j^D = \rho \iint_S \left(\phi_j^0 - \frac{U}{i\omega_e} \phi_j^U \right) \frac{\partial \phi_I}{\partial n} ds + \frac{\rho U}{i\omega_e} \int_{C_A} \phi_j^0 \frac{\partial \phi_I}{\partial n} dl \quad (230)$$

The excitation forces and moments can be written in their final forms using the relationships in (158) and combining (230) with (225) as the following:

$$F_j = -\rho i\omega \iint_S n_j \phi_I ds + \rho \iint_S \left(\phi_j^0 - \frac{U}{i\omega_e} \phi_j^U \right) \frac{\partial \phi_I}{\partial n} ds + \frac{\rho U}{i\omega_e} \int_{C_A} \phi_j^0 \frac{\partial \phi_I}{\partial n} dl \quad (231)$$

$$F_1 = -\rho \iint_S \left(i\omega n_1 \phi_I - \phi_1^0 \frac{\partial \phi_I}{\partial n} \right) ds + \frac{\rho U}{i\omega_e} \int_{C_A} \phi_1^0 \frac{\partial \phi_I}{\partial n} dl \quad (232)$$

$$F_2 = -\rho \iint_S \left(i\omega n_2 \phi_I - \phi_2^0 \frac{\partial \phi_I}{\partial n} \right) ds + \frac{\rho U}{i\omega_e} \int_{C_A} \phi_2^0 \frac{\partial \phi_I}{\partial n} dl \quad (233)$$

$$F_3 = -\rho \iint_S \left(i\omega n_3 \phi_I - \phi_3^0 \frac{\partial \phi_I}{\partial n} \right) ds + \frac{\rho U}{i\omega_e} \int_{C_A} \phi_3^0 \frac{\partial \phi_I}{\partial n} dl \quad (234)$$

$$F_4 = -\rho \iint_S \left(i\omega n_4 \phi_I - \phi_4^0 \frac{\partial \phi_I}{\partial n} \right) ds + \frac{\rho U}{i\omega_e} \int_{C_A} \phi_4^0 \frac{\partial \phi_I}{\partial n} dl \quad (235)$$

$$F_5 = -\rho \iint_S \left(i\omega n_5 \phi_I - \phi_5^0 \frac{\partial \phi_I}{\partial n} \right) ds + \frac{\rho U}{i\omega_e} \int_{C_A} \phi_5^0 \frac{\partial \phi_I}{\partial n} dl \quad (236)$$

$$F_6 = -\rho \iint_S \left(i\omega n_6 \phi_I - \phi_6^0 \frac{\partial \phi_I}{\partial n} \right) ds + \frac{\rho U}{i\omega_e} \iint_S \phi_2^0 \frac{\partial \phi_I}{\partial n} ds + \frac{\rho U}{i\omega_e} \int_{C_A} \phi_6^0 \frac{\partial \phi_I}{\partial n} dl \quad (237)$$

The surface integrals can be transformed into integrals along the length of the ship following the assumptions in (181) and (184). In addition, excitation forces and moments can be written as the following by introducing the normal derivative of incident wave potential.

$$\frac{\partial \phi_I}{\partial n} = (in_2 \sin \beta + n_3)k\phi_I \quad (238)$$

$$F_1 = -\rho \iint_S [i\omega n_1 - \phi_1^0 (in_2 \sin \beta + n_3)k] \phi_I ds + \frac{\rho U}{i\omega_e} \int_{C_A} \phi_1^0 (in_2 \sin \beta + n_3)k\phi_I dl \quad (239)$$

$$F_2 = -\rho \iint_S [i\omega n_2 - \phi_2^0 (in_2 \sin \beta + n_3)k] \phi_I ds + \frac{\rho U}{i\omega_e} \int_{C_A} \phi_2^0 (in_2 \sin \beta + n_3)k\phi_I dl \quad (240)$$

$$F_3 = -\rho \iint_S [i\omega n_3 - \phi_3^0 (in_2 \sin \beta + n_3)k] \phi_I ds + \frac{\rho U}{i\omega_e} \int_{C_A} \phi_3^0 (in_2 \sin \beta + n_3)k\phi_I dl \quad (241)$$

$$F_4 = -\rho \iint_S [i\omega n_4 - \phi_4^0 (in_2 \sin \beta + n_3)k] \phi_I ds + \frac{\rho U}{i\omega_e} \int_{C_A} \phi_4^0 (in_2 \sin \beta + n_3)k\phi_I dl \quad (242)$$

$$F_5 = -\rho \iint_S [i\omega n_5 - \phi_5^0 (in_2 \sin \beta + n_3)k] \phi_I ds - \frac{\rho U}{i\omega_e} \iint_S \phi_3^0 (in_2 \sin \beta + n_3)k\phi_I ds + \frac{\rho U}{i\omega_e} \int_{C_A} \phi_5^0 (in_2 \sin \beta + n_3)k\phi_I dl \quad (243)$$

$$F_6 = -\rho \iint_S [i\omega n_6 - \phi_6^0 (in_2 \sin \beta + n_3)k] \phi_I ds + \frac{\rho U}{i\omega_e} \iint_S \phi_2^0 (in_2 \sin \beta + n_3)k\phi_I ds + \frac{\rho U}{i\omega_e} \int_{C_A} \phi_6^0 (in_2 \sin \beta + n_3)k\phi_I dl \quad (244)$$

The amplitudes of both motion and force in surge direction are assumed to be small compared to other modes of motion while analyzing ship motions in the framework of strip theory, so surge motion and corresponding force are neglected. If the generalized three dimensional normal and

speed independent oscillatory potentials are replaced by generalized two dimensional normal and two dimensional potentials respectively, the remaining forces can be expressed as the integral of sectional Froude-Krylov, $f_j(x)$, and diffraction forces, $h_j(x)$, along the length of the ship:

$$f_j(x) = \rho g A e^{-ikx \cos \beta} \int_{C_x} N_j e^{iky \sin \beta} e^{kz} dl; \quad j = 2, 3, 4 \quad (245)$$

$$h_j(x) = \rho \omega A e^{-ikx \cos \beta} \int_{C_x} (iN_3 - N_2 \sin \beta) e^{iky \sin \beta} e^{kz} \psi_j dl; \quad j = 2, 3, 4 \quad (246)$$

$$F_j = \int_L (f_j + h_j) dx + \frac{U}{i\omega_e} h_j^A; \quad j = 2, 3, 4 \quad (247)$$

$$F_5 = - \int_L \left[x(f_3 + h_3) + \frac{U}{i\omega_e} h_3 \right] dx - \frac{U}{i\omega_e} x_A h_3^A \quad (248)$$

$$F_6 = \int_L \left[x(f_2 + h_2) + \frac{U}{i\omega_e} h_2 \right] dx - \frac{U}{i\omega_e} x_A h_2^A \quad (249)$$

In these equations, sectional diffraction forces with superscript A refer to the sectional diffraction force calculated for the aftermost section of the ship.

3.3 Calculation of Ship Motions by Ship Motions Analyzer

The ultimate goal of this thesis is to develop a computational tool to calculate ship motions. A program called Ship Motions Analyzer (SMA) is written in MATLAB™ using the equations derived in Chapter 2 and Chapter 3 to calculate added mass and damping coefficients of arbitrary ship sections and excitation forces and moments respectively. In order to calculate ship motions in six degrees of freedom, the six linear coupled differential equations of ship motions must be solved. The equation of ship motions can be written in the following form:

$$\sum_{k=1}^6 [(M_{jk} + A_{jk}) \ddot{\eta}_k + B_{jk} \dot{\eta}_k + C_{jk} \eta_k] = F_j e^{i\omega_e t}; \quad j = 1, 2, 3, 4, 5, 6 \quad (250)$$

Here, M_{jk} is the generalized mass matrix, A_{jk} and B_{jk} are the added mass and the damping coefficients of the ship, C_{jk} are the hydrostatic restoring coefficients, and F_j is the complex magnitude of excitation force and moment. The dots refer to time derivatives, so $\ddot{\eta}_k$ and $\dot{\eta}_k$ represent acceleration and velocity terms. For a ship with lateral symmetry, generalized mass matrix, added mass and damping coefficient matrices can be formed as the following:

$$M_{jk} = \begin{bmatrix} M & 0 & 0 & 0 & Mz_c & 0 \\ 0 & M & 0 & -Mz_c & 0 & 0 \\ 0 & 0 & M & 0 & 0 & 0 \\ 0 & -Mz_c & 0 & I_4 & 0 & -I_{46} \\ Mz_c & 0 & 0 & 0 & I_5 & 0 \\ 0 & 0 & 0 & -I_{46} & 0 & I_6 \end{bmatrix} \quad (251)$$

$$A_{jk}(B_{jk}) = \begin{bmatrix} A_{11} & 0 & A_{13} & 0 & A_{15} & 0 \\ 0 & A_{22} & 0 & A_{24} & 0 & A_{26} \\ A_{31} & 0 & A_{33} & 0 & A_{35} & 0 \\ 0 & A_{42} & 0 & A_{44} & 0 & A_{46} \\ A_{51} & 0 & A_{53} & 0 & A_{55} & 0 \\ 0 & A_{62} & 0 & A_{64} & 0 & A_{66} \end{bmatrix} \quad (252)$$

In (251), M is the mass of the ship, z_c is the coordinate of the vertical center of gravity of the ship measured from the origin of the coordinate system defined in the beginning of this chapter, I_4 , I_5 and I_6 are the moments of inertia in roll, pitch and yaw modes of motion respectively, and I_{46} is the product of inertia. These terms can be calculated using following equations.

$$I_4 = \iiint_V \rho_S (y^2 + z^2) dv \quad (253)$$

$$I_5 = \iiint_V \rho_S (x^2 + z^2) dv \quad (254)$$

$$I_6 = \iiint_V \rho_S (x^2 + y^2) dv \quad (255)$$

$$I_{46} = \iiint_V \rho_S (-xz) dv \quad (256)$$

ρ_s represents the mass density of the ship, dv the integration variable over the volume of the ship in these equations. Although the weight distribution is different all over a ship, the weight distribution is assumed homogeneous and ρ_s is set equal to the density of sea water in the calculations ($\rho_s = 1025 \text{ kg/m}^3$).

The only non-zero hydrostatic restoring coefficients for a ship with lateral symmetry are C_{33} , C_{44} , C_{55} and $C_{35}=C_{53}$.

$$C_{33} = \rho g \int_L b dx \quad (257)$$

$$C_{35} = C_{53} = -\rho g \int_L x b dx \quad (258)$$

$$C_{55} = \rho g \int_L x^2 b dx \quad (259)$$

$$C_{44} = \rho g \nabla \overline{GM} \quad (260)$$

Here ρ is the density of fluid, g the acceleration of gravity, b the sectional beam, ∇ the displacement volume of the ship, and \overline{GM} the well known metacentric height.

For a ship with lateral symmetry, surge, heave and pitch motions are decoupled from sway, roll and yaw motions. Therefore, six linear coupled differential equations are reduced to two sets of three linear coupled differential equations. As mentioned previously, since surge motion is small compared to other modes of motion and ignored, one set of equation remains for coupled heave and pitch motions and another set of equation remains for coupled sway, roll and yaw motions. If linear and harmonic motions of the ship are expressed as the following

$$\zeta_j = \hat{\zeta}_j e^{i\omega_e t} \quad ; \quad j = 2,3,4,5,6 \quad (261)$$

where $\hat{\zeta}_j$ is the complex magnitude of motion in j_{th} mode, equations of motions for coupled heave-pitch and coupled sway-roll-yaw motions can be written in the matrix form as following respectively.

$$\begin{bmatrix} \hat{\zeta}_3 \\ \hat{\zeta}_5 \end{bmatrix} = \left\{ -\omega_e^2 \begin{bmatrix} (M + A_{33}) & A_{35} \\ A_{53} & (I_5 + A_{55}) \end{bmatrix} + i\omega_e \begin{bmatrix} B_{33} & B_{35} \\ B_{53} & B_{55} \end{bmatrix} + \begin{bmatrix} C_{33} & C_{35} \\ C_{53} & C_{55} \end{bmatrix} \right\} \setminus \begin{bmatrix} F_3 \\ F_5 \end{bmatrix} \quad (262)$$

$$\begin{bmatrix} \hat{\zeta}_2 \\ \hat{\zeta}_4 \\ \hat{\zeta}_6 \end{bmatrix} = \left\{ -\omega_e^2 \begin{bmatrix} (A_{22} + M) & (A_{24} - Mz_c) & A_{26} \\ (A_{42} - Mz_c) & (A_{44} + I_4) & (A_{46} - I_{46}) \\ A_{62} & (A_{64} - I_{46}) & (A_{66} + I_6) \end{bmatrix} + i\omega_e \begin{bmatrix} B_{22} & B_{24} & B_{26} \\ B_{42} & B_{44} & B_{46} \\ B_{62} & B_{64} & B_{66} \end{bmatrix} \right. \\ \left. + \begin{bmatrix} 0 & 0 & 0 \\ 0 & C_{44} & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\} \setminus \begin{bmatrix} F_2 \\ F_4 \\ F_6 \end{bmatrix} \quad (263)$$

In the last two equations, “\” means matrix division and the results of these operations are matrices which contain the complex magnitudes of motions for a given frequency of encounter.

3.4 Results and Discussions

The ship motions were calculated by SMA for the bare hull of DDG-1000 (Zumwalt-Class destroyer) for regular head waves in a frequency range between 0.01 rd/s and 1.5 rd/s, and the following results were obtained for different forward speeds. Figures 9 to 13 show heave response amplitude operator (RAO), heave excitation force normalized by wave amplitude, A , pitch RAO, pitch moment normalized by wave number, k , and wave amplitude, A , and magnitude of pitch moment.

The results for heave motion seem to be consistent with the linear strip theory. The magnitude of heave force normalized by the wave amplitude converges to hydrostatic restoring coefficient C_{33} and the amplitude of heave motion normalized by the wave amplitude converges to 1, while the frequency of encounter goes to 0. Since the excitation forces and moments are

dominated by the hydrostatic forces in the very low frequency range, and the motions of the ships are assumed to be linear with fluid motions, the present results support these assumptions.

For the case of pitch motion, the magnitude of pitch moment and the magnitude of pitch motion normalized by wave number and the wave amplitude should converge to hydrostatic restoring coefficient C_{55} and 1 respectively, while the frequency of encounter goes to 0. Figures 11 and 12 do not seem to support this argument. Although these results tend to converge above mentioned values, the theory breaks down in very low frequencies.

According to Lewis [19], the maximum value of pitch moment generally occurs around $L_S/\lambda = 0.75$. The results shown in Figure 13 support this argument. It can be seen that the maximum values of pitch moment at different speeds occur around frequencies of encounter which correspond to wave frequencies where $L_S/\lambda = 0.75$

For both modes of motion, the magnitudes of excitation forces (moments) and the amplitudes of motions decay in the high frequency range since the waves cancel out each other's effects along the ship. It is seen in Figure 10 that heave excitation force is not predicted very well in high frequency range. This result may be because of the choice of the number of panels used to describe the boundaries of ship sections. The number of panels was 100 in these calculations and the diffraction part of the excitation force may not have been predicted accurately. Increasing the number of panels will definitely increase the sensitivity of the calculations.

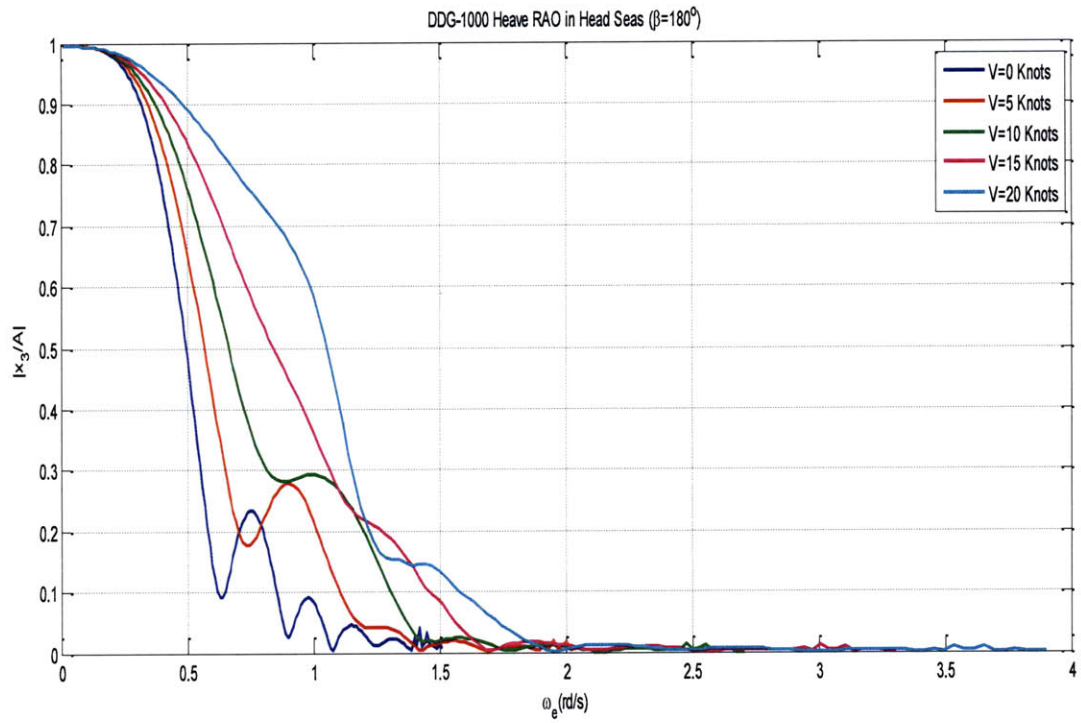


Figure 9: DDG 1000 Heave RAO in Head Seas

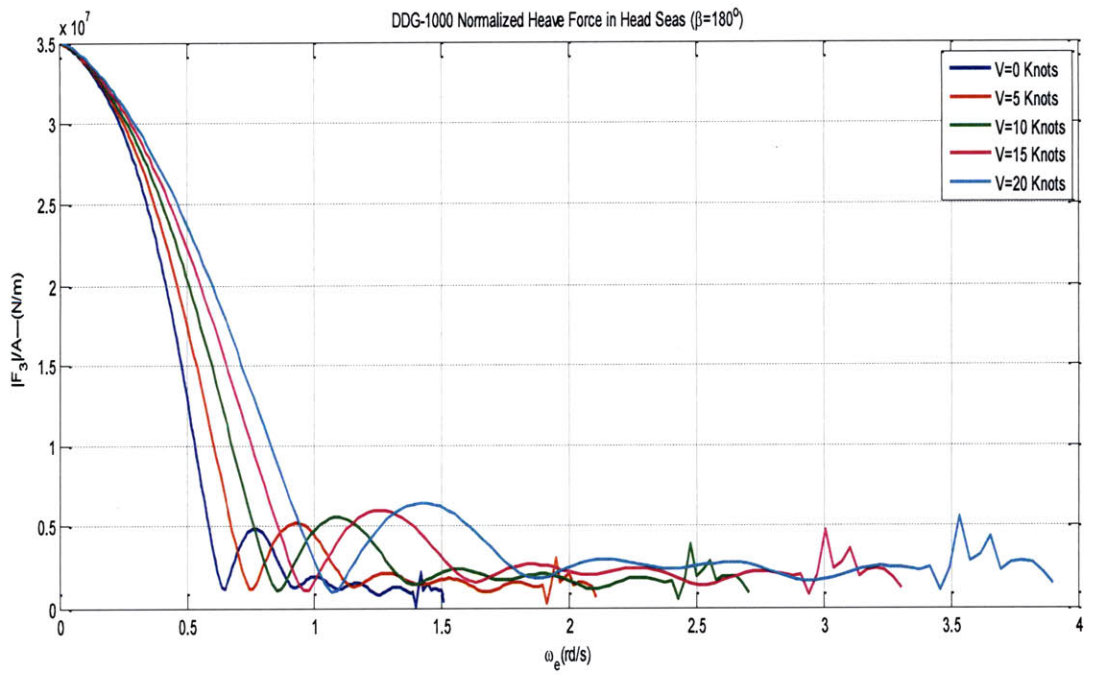


Figure 10: DDG 1000 Normalized Heave Excitation Force in Head Seas

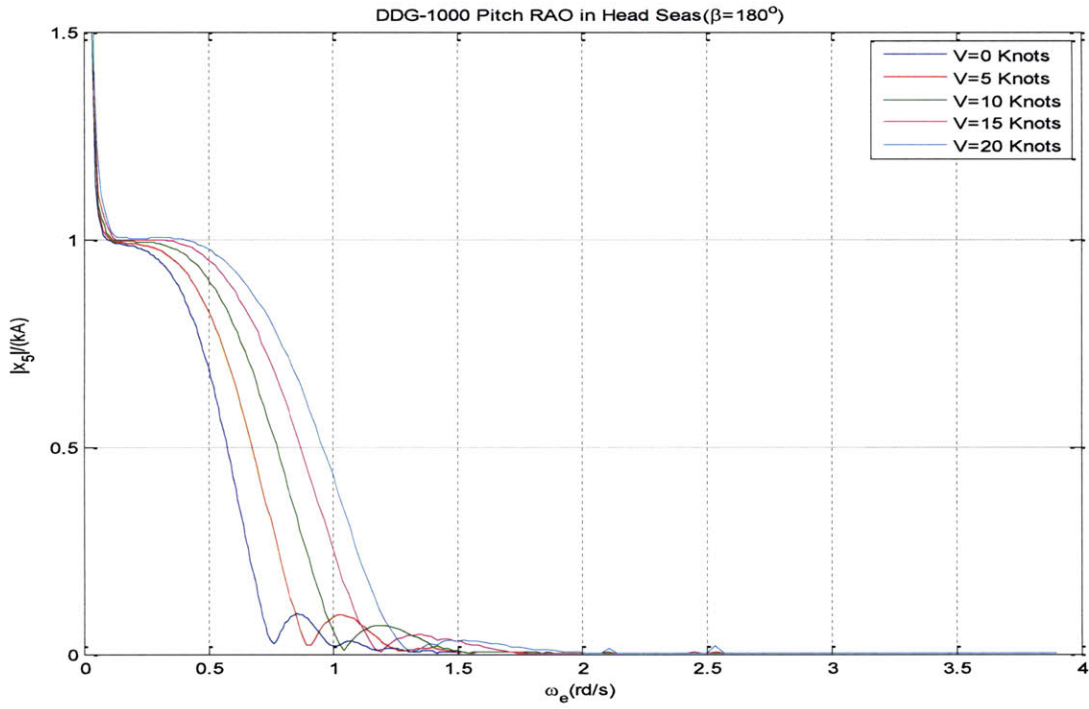


Figure 11: DDG 1000 Pitch RAO in Head Seas

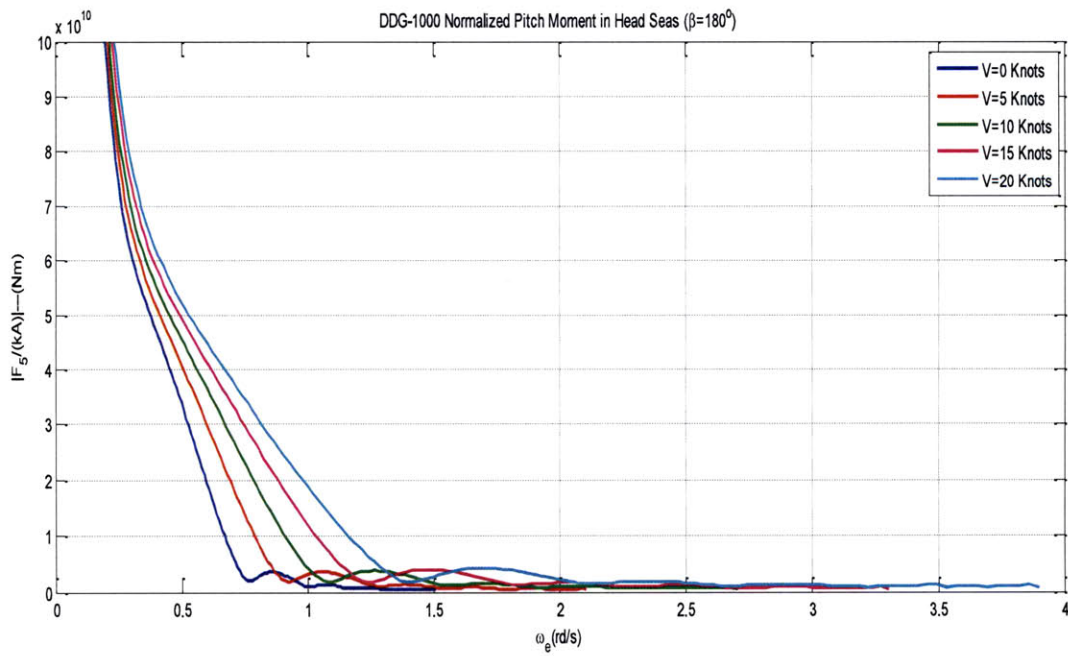


Figure 12: DDG 1000 Normalized Pitch Moment in Head Seas

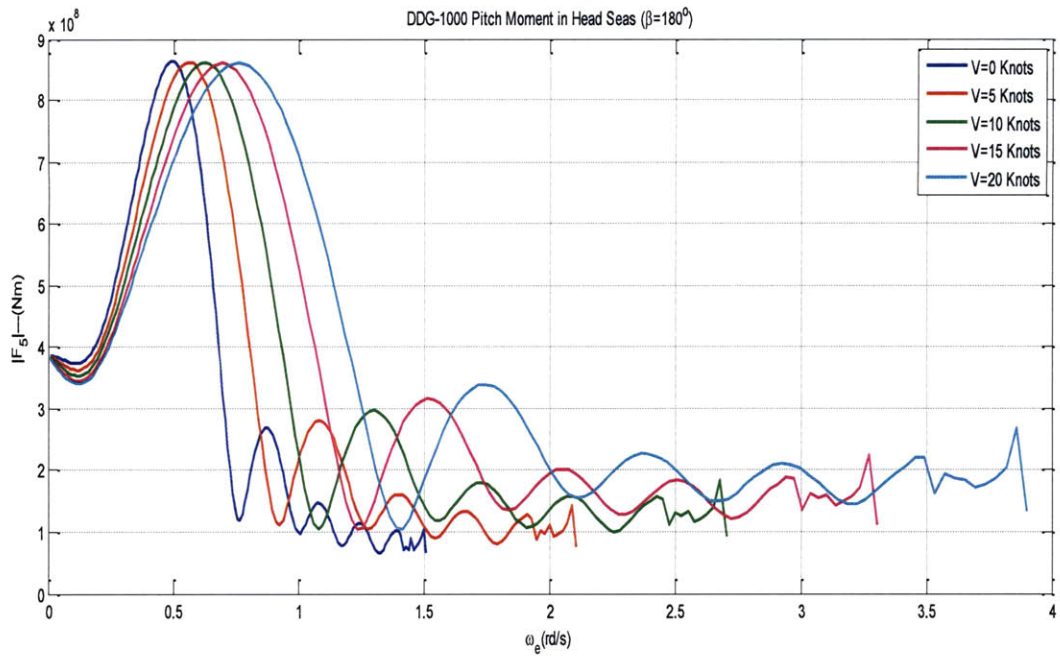


Figure 13: DDG 1000 Magnitude of Pitch Moment in Head Seas

3.5 Conclusions

It can be seen that the heave motion in head waves is predicted well by the theory. The pitch motion also seems to agree with the theory except for very low frequencies. The key assumption of this theory is that the hull is long and slender. DDG 1000 has a length to beam ratio of $L_s/B=182/24=7.58$. The same calculations were repeated with a semi cylinder whose length is 180 m. and beam is 14 m. This semi cylinder has the same mass with DDG 1000 and its length to beam ratio is $L_s/B =180/14=12.86$. It was observed that the pitch RAO converged to 1 for both 0 speed and different forward speed cases. This result suggests that the slenderness of the ship may affect the results significantly. The same convergence was not observed for pitch moment.

In order to determine the effects of forward speed and body geometry, more calculations should be carried out with different types of hulls. In addition, the results shown here must be supported by experimental results which were not available during the theoretical calculations.

CHAPTER 4

SECOND ORDER FORCES AND MOMENTS

4 Introduction

The responses of a ship to regular waves were assumed to be linear and harmonic in the previous chapter. However, the ship experiences steady drift motions in the horizontal plane due to the lack of hydrostatic restoring forces and moments. “Similarly, in irregular seas, a ship will experience slowly varying surge, sway and yaw motions with non zero means in addition to motions with frequency components equal to frequency of encounter of the individual wave components.”[3]

The steady state force component in the direction of the ship’s longitudinal axis is known as the added resistance and the component in the lateral direction in horizontal plane is known as the sideways drift force. These forces are of particular interest in this study since the added resistance is an important factor affecting the ship’s additional power requirement to maintain a particular speed in a seaway and the drift force is closely related to the course keeping of the ship. In determining the total power requirement of a ship, an allowance is added to the calm water resistance of the ship. Added resistance due to waves has a significant contribution to this allowance. This quantity is known as “Sea Margin” or “Weather Margin.”[20]

In this study, a method developed by Salvasen [3] is chosen to evaluate added resistance and drift force acting on a ship in oblique regular waves since this method is based on the results obtained by Frank’s [2] close-fit source distribution method and the linear strip theory, which were covered in chapters 2 and 3 respectively.

4.1 Evaluation of Second Order Steady State Force

The steady state second order force component in the horizontal plane (x - y plane) is of interest in this study. It is assumed that the ship is advancing at constant forward speed with arbitrary heading in regular sinusoidal waves. The same coordinate system which was used in Chapter 3 will be used in the derivation of second order forces and moments. The incident wave angle should be measured as shown in the following figure with respect to the direction of forward motion and wave propagation direction. This definition is consistent with the way the steady state force is derived in Salvasen [3] and the direction of the steady state force. Incident wave angle becomes 0 for following waves ($\beta=0$) and 180 for head waves ($\beta=180$).

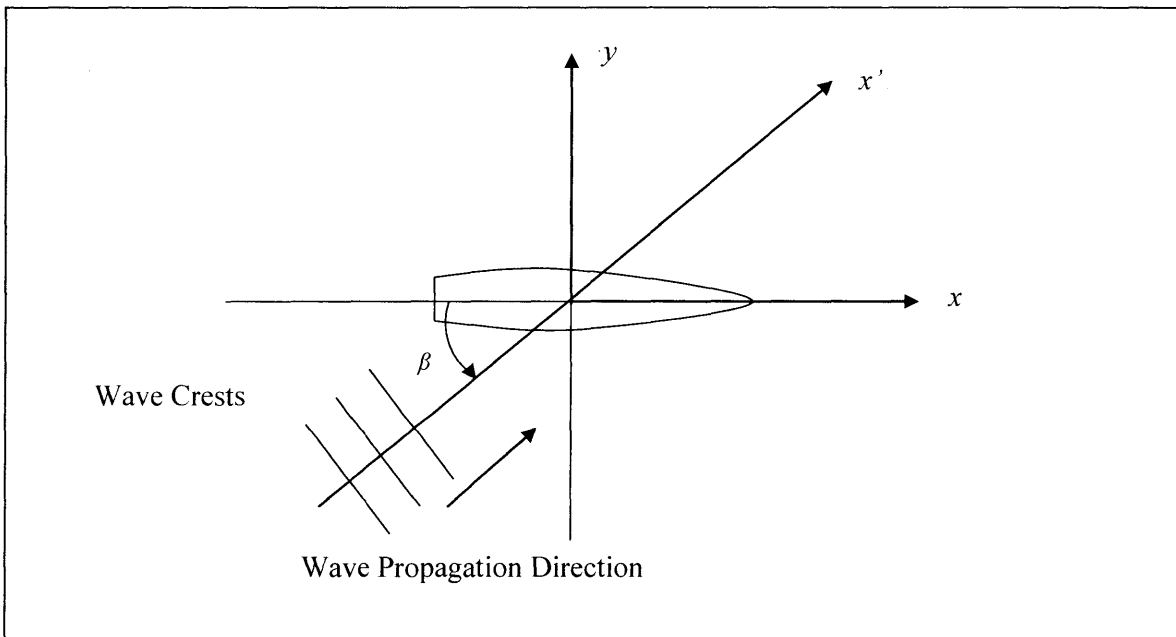


Figure 14: Definition of incident wave angle.

The hydrodynamic force acting on a body in the free surface can be expressed in the following form:

$$\vec{F} = -\rho \frac{d}{dt} \iint_{S_B+S_F} \Phi n ds - \rho \iint_{S_\infty} \left[\frac{\partial \Phi}{\partial n} \nabla \Phi - \frac{1}{2} |\nabla \Phi|^2 n \right] ds \quad (264)$$

Here, Φ is the total velocity potential, n the generalized normal pointing out of the fluid domain, S_B the body surface, S_∞ a control surface in the far field and S_F the portion of the free surface inside the far field control surface. The total velocity potential can be written in the following form

$$\left(\phi_S^{(1)} + \phi_S^{(2)} + \dots \right) + \left(\phi_T^{(1)} e^{i\omega_e t} + \phi_T^{(2)} e^{i2\omega_e t} + \phi_{DC}^{(2)} + \dots \right) \quad (265)$$

neglecting third order terms. Here, $\phi_T^{(1)}$ and $\phi_T^{(2)}$ are the first and second order amplitudes of the time dependent potential, and $\phi_{DC}^{(2)}$ is the second order DC potential. Considering only the first integral in (264), and substituting (265) into this integral results in an expression as the following since the time derivative of the steady part of the potential does not have any contribution.

$$\iint_{S_B+S_F} \Phi n ds = A + B e^{i\omega_e t} + C e^{i2\omega_e t} + \dots \quad (266)$$

The total potential can now be written as the following:

$$\Phi = \Phi_I + \Phi_B \quad (267)$$

Here, Φ_I is the incident wave potential given as the following

$$\phi_I = \frac{igA}{\omega} e^{-ik(x \cos \beta + y \sin \beta)} e^{kz} \quad (268)$$

and Φ_B the potential due to the body motions in six degrees of freedom and diffraction effects. Substituting (267) into (264) and “performing some manipulations, Newman has shown that the second integral in equation (264) can be expressed as the following”[3]:

$$F = \rho \iint_{S_\infty} \left(\Phi_B \frac{\partial}{\partial n} - \frac{\partial \Phi}{\partial n} \right) \left(\nabla \Phi_I + \frac{1}{2} \nabla \Phi_B \right) ds \quad (269)$$

Following Newman's "weak scatterer" assumption, the body potential Φ_B can be assumed small compared to incident wave potential. This is a reasonable assumption if the ship's beam and draft are much smaller than its length. By assuming $\Phi_B \ll \Phi_I$ and neglecting second order terms in body potential Φ_B , the force can be written in the following form:

$$\vec{F} = \rho \iint_{S_\infty} \left(\Phi_B \frac{\partial}{\partial n} - \frac{\partial \Phi_B}{\partial n} \right) \nabla \Phi_I ds \quad (270)$$

If the body potential is expressed as $\Phi_B = \phi_S + \phi_B e^{i\omega_e t}$ and the incident wave potential as $\Phi_I = \phi_I e^{i\omega_e t}$, and by taking the mean value of (270), the steady state second order force can be written as

$$\vec{\mathcal{F}} = \frac{1}{2} \rho \iint_{S_\infty} \left(\phi_B \frac{\partial}{\partial n} - \frac{\partial \phi_B}{\partial n} \right) \nabla \phi_I^* ds \quad (271)$$

Here, \mathcal{F} refers to second order steady state force and ϕ_I^* is the complex conjugate of incident wave potential. The integration over S_∞ can be converted to integration over the surface of the body by applying Green's Theorem[3]:

$$\vec{\mathcal{F}} = -\frac{1}{2} \rho \iint_{S_B} \left(\phi_B \frac{\partial}{\partial n} - \frac{\partial \phi_B}{\partial n} \right) \nabla \phi_I^* ds \quad (272)$$

The horizontal component of steady state force is of interest in this study, so the two dimensional gradient of the conjugate of incident wave potential is

$$\nabla \phi_I^* = ik(\cos \beta \vec{i} + \sin \beta \vec{j}) \phi_I^* \quad (273)$$

Here, \vec{i} represents the x component, \vec{j} the y component of the second order steady state force in x - y plane, and ϕ_I^* can be written as the following:

$$\phi_i^* = \frac{-igA}{\omega} e^{kz} e^{ik(x \cos \beta + y \sin \beta)} \quad (274)$$

The second order force can be expressed as the following by substituting (273) into (272),

$$\vec{\mathcal{F}} = -\frac{i}{2} \rho k (\cos \beta \vec{i} + \sin \beta \vec{j}) \iint_{S_B} \left(\phi_B \frac{\partial}{\partial n} - \frac{\partial \phi_B}{\partial n} \right) \phi_i^* ds \quad (275)$$

and its magnitude can be written as

$$\mathcal{F} = -\frac{i}{2} \rho k \iint_{S_B} \left(\phi_B \frac{\partial}{\partial n} - \frac{\partial \phi_B}{\partial n} \right) \phi_i^* ds \quad (276)$$

The x and y components of \mathcal{F} are given by the following equations. The negative of the x component of \mathcal{F} is usually known as the added resistance and y component as the sideways drift force.

$$\mathcal{F}_x = \mathcal{F} \cos \beta \quad (277)$$

$$\mathcal{F}_y = \mathcal{F} \sin \beta \quad (278)$$

“The integral in (276) is known as the Kochin function.”[3] If the body potential ϕ_B is expressed in terms of the diffraction potential and the oscillatory body potentials

$$\phi_B = \sum_{j=1}^6 \zeta_j \phi_j + \phi_D \quad (279)$$

(276) can be written as the following:

$$\mathcal{F} = -\frac{i}{2} \rho k \sum_{j=1}^6 \left\{ \zeta_j \iint_{S_B} \left(\phi_j \frac{\partial}{\partial n} - \frac{\partial \phi_j}{\partial n} \right) \phi_i^* ds \right\} - \frac{i}{2} \rho k \iint_{S_B} \left(\phi_D \frac{\partial}{\partial n} - \frac{\partial \phi_D}{\partial n} \right) \phi_i^* ds \quad (280)$$

Equation (280) can be separated into three components.

$$\mathcal{F} = \sum_{j=1}^6 (\mathcal{F}_j^I + \mathcal{F}_j^D) + \mathcal{F}_D \quad (281)$$

Here, \mathcal{F}_j^I represents the contribution to second order steady state force from j^{th} mode of motion due to incident waves, \mathcal{F}_j^D the contribution from oscillatory body potentials in j^{th} mode of motion and \mathcal{F}_D the contribution from the diffraction potential. These components can be expressed as the following:

$$\mathcal{F}_j^I = \frac{i}{2} \rho k \zeta_j \iint_{S_B} \frac{\partial \phi_j}{\partial n} \phi_I^* ds \quad (282)$$

$$\mathcal{F}_j^D = -\frac{i}{2} \rho k \zeta_j \iint_{S_B} \phi_j \frac{\partial \phi_I^*}{\partial n} ds \quad (283)$$

$$\mathcal{F}_D = \frac{i}{2} \rho k \iint_{S_B} \frac{\partial \phi_D}{\partial n} \phi_I^* ds - \frac{i}{2} \rho k \iint_{S_B} \phi_D \frac{\partial \phi_I^*}{\partial n} ds \quad (284)$$

It can be shown that the first integral in (284) is zero by applying the hull boundary condition (151), so \mathcal{F}_D becomes

$$\mathcal{F}_D = -\frac{i}{2} \rho k \iint_{S_B} \phi_D \frac{\partial \phi_I^*}{\partial n} ds \quad (285)$$

Equations (282), (283) and (285) can now be evaluated as follows.

The normal derivative of oscillatory body potential in j^{th} mode of motion can be expressed using (156) and (157), so (282) can be written as the following:

$$\mathcal{F}_j^I = \frac{i}{2} \rho k \zeta_j \iint_{S_B} (i\omega_e n_j + U m_j) \phi_I^* ds \quad (286)$$

In this equation, n_j is the three dimensional generalized normal and m_j is as given in (155).

Applying a variant of Stoke's theorem as given in Salvase *et al.* [5],

$$\iint_S U m_j \phi ds = \iint_S n_j U \frac{\partial}{\partial x} \phi ds \quad (287)$$

(286) now takes the following form:

$$\mathcal{F}_j^I = \frac{i}{2} \rho k \zeta_j \iint_{S_B} i n_j (\omega_e + Uk \cos \beta) \phi_j^* ds \quad (288)$$

Recalling the expression for frequency of encounter given in (146), (288) can be written as the following:

$$\mathcal{F}_j^I = -\frac{1}{2} \rho k \zeta_j \iint_{S_B} n_j \omega \phi_j^* ds \quad (289)$$

Equation (289) can also be expressed using the complex amplitude of excitation force due to incident waves, in other words using the Froude-Krylov part of the excitation force. If the Froude-Krylov part of the excitation force is written as the following,

$$F_j^I = -i \rho \omega \iint_{S_B} n_j \phi_I ds \quad (290)$$

\mathcal{F}_j^I can be written as

$$\mathcal{F}_j^I = \frac{i}{2} k \zeta_j (F_j^I)^* \quad (291)$$

Here, $(F_j^I)^*$ represents the complex conjugate of Froude-Krylov excitation force.

The second contribution to the steady state force comes from (283). The normal derivative of ϕ_j^* in (283) can be expressed as the following

$$\frac{\partial \phi_j^*}{\partial n} = k(n_3 + i n_2 \sin \beta) \phi_j^* \quad (292)$$

including the slender body assumption given in (184). Substituting (292) into (283) and replacing ϕ_j with speed independent oscillatory potential ϕ_j^0 for $j=2, 3, 4$ as given in (159) result in

$$\mathcal{F}_j^D = -\frac{i}{2}\rho k^2 \zeta_j \iint_{S_B} \phi_j^0 (n_3 + in_2 \sin \beta) \phi_l^* ds \quad (293)$$

For $j=5$ and 6 , \mathcal{F}_j^D can be expressed as the following using the equations given in (160) and (161).

$$\mathcal{F}_5^D = -\frac{i}{2}\rho k^2 \zeta_5 \iint_{S_B} \left(\phi_5^0 + \frac{U}{i\omega_e} \phi_3^0 \right) (n_3 - in_2 \sin \beta) \phi_l^* ds \quad (294)$$

$$\mathcal{F}_6^D = -\frac{i}{2}\rho k^2 \zeta_6 \iint_{S_B} \left(\phi_6^0 - \frac{U}{i\omega_e} \phi_2^0 \right) (n_3 - in_2 \sin \beta) \phi_l^* ds \quad (295)$$

Using the strip theory assumptions (184)-(187), if the three dimensional generalized normal n_j and three dimensional speed independent oscillatory potential ϕ_j^0 are replaced by two dimensional generalized normal N_j and potential ψ_j respectively, (293), (294) and (295) can be written as the following:

$$\hat{h}_j(x) = -\rho k \int_{C_x} \psi_j (N_3 + iN_2 \sin \beta) \phi_l^* dl \quad (296)$$

$$\mathcal{F}_j^D = \frac{i}{2} k \zeta_j \int_L \hat{h}_j(x) dx \text{ for } j = 2, 3, 4 \quad (297)$$

$$\mathcal{F}_5^D = \frac{i}{2} k \zeta_5 \int_L \left(-x + \frac{U}{i\omega_e} \right) \hat{h}_3(x) dx \quad (298)$$

$$\mathcal{F}_6^D = \frac{i}{2} k \zeta_6 \int_L \left(x - \frac{U}{i\omega_e} \right) \hat{h}_2(x) dx \quad (299)$$

The integration in (296) is along the boundary of any section. This expression is similar to sectional diffraction force defined in chapter 3, but the conjugate of incident wave potential is used in this equation.

Finally, the force component due to the diffraction potential can be expressed by substituting (292) into (285) as the following:

$$\mathcal{F}_D = -\frac{i}{2}\rho k^2 \iint_{S_B} \phi_D(n_3 - in_2 \sin \beta) \phi_I^* ds \quad (300)$$

This integral equation can be expressed as an integral along the length of the ship if a sectional force similar to (296) is written.

$$\hat{h}_D(x) = -i\rho k^2 \int_{C_x} \phi_D(n_3 + in_2 \sin \beta) \phi_I^* dl \quad (301)$$

$$\mathcal{F}_D = \frac{1}{2} \int_L \hat{h}_D(x) dx \quad (302)$$

Substitution of ϕ_I^* into (301) results in the following:

$$\hat{h}_D(x) = -\rho\omega A k e^{ikx \cos \beta} \int_{C_x} \phi_D(n_3 - in_2 \sin \beta) e^{-iky \sin \beta} e^{kz} dl \quad (303)$$

“It is assumed that in the integral in (303), e^{kz} and $e^{-iky \sin \beta}$ can be replaced by e^{-kds} and $e^{-ik(\frac{1}{2}b)s \sin \beta}$ respectively. Here, s represents sectional area coefficient, sectional area divided by sectional beam times draft, ($s = A_x/(bd)$); b the sectional beam and d the sectional draft. The first assumption is often used in strip theory calculations and has been shown to give accurate results for conventional ship hull forms.”[3] The second assumption is considered reasonable if the wave length is considerably larger than the half beam of the ship ($\lambda \gg \frac{1}{2}B$). Substituting these assumptions into (303) results in

$$\hat{h}_D(x) = -\rho\omega A k e^{ikx \cos \beta} e^{-ik(\frac{1}{2}b)s \sin \beta} e^{-kds} \int_{C_x} \phi_D(n_3 + in_2 \sin \beta) dl \quad (304)$$

Denoting the integral in (304) by I_D and substituting the first body boundary condition given in (157), this integral equation can be written as the following:

$$I_D = -\frac{i}{\omega_e} \int_{C_x} \phi_D \left(\frac{\partial \phi_3^0}{\partial n} + i \sin \beta \frac{\partial \phi_2^0}{\partial n} \right) dl \quad (305)$$

Applying Green's second identity in two dimensions, which was previously given in (228), this integral equation now turns into the following form:

$$I_D = -\frac{i}{\omega_e} \int_{C_x} (\phi_3^0 + i \sin \beta \phi_2^0) \frac{\partial \phi_D}{\partial n} dl \quad (306)$$

The following form of $\hat{h}_D(x)$ is obtained after the hull boundary condition (151) is applied to (306).

$$\hat{h}_D(x) = -\rho \omega A k^2 e^{ikx \cos \beta} e^{-ik\left(\mp \frac{1}{2}b\right)s \sin \beta} e^{-kds} \left\{ -\frac{i}{\omega_e} \int_{C_x} (\phi_3^0 + i \sin \beta \phi_2^0) [-\phi_1(n_3 - in_2 \sin \beta)] dl \right\} \quad (307)$$

Substituting incident wave potential into this equation results in

$$\hat{h}_D(x) = \rho A^2 k \frac{\omega^2}{\omega_e} e^{-2kds} \int_{C_x} (\phi_3^0 n_3 - in_2 \sin \beta \phi_3^0 + in_3 \phi_2^0 \sin \beta + n_2 (\sin \beta)^2 \phi_2^0) dl \quad (308)$$

The following relationship is assumed valid for symmetric sections by Salvasen [3].

$$\int_{C_x} \phi_3^0 n_2 dl = \int_{C_x} \phi_2^0 n_3 dl = 0 \quad (309)$$

Finally, (308) takes the following form:

$$\hat{h}_D(x) = \rho A^2 k \frac{\omega^2}{\omega_e} e^{-2kds} \int_{C_x} (\phi_3^0 n_3 + n_2 (\sin \beta)^2 \phi_2^0) dl \quad (310)$$

This equation can also be expressed using sectional added mass and damping coefficients.

Recalling the strip theory assumptions (184)-(187) and introducing the following equation

$$a_{jk} - \frac{i}{\omega_e} b_{jk} = -\rho \frac{i}{\omega_e} \int_{C_x} N_j \psi_k dl \quad (311)$$

into (310) results in

$$\hat{h}_D(x) = A^2 k \frac{\omega^2}{\omega_e} e^{-2kds} [b_{33} + (\sin \beta)^2 b_{22} + i\omega_e (a_{33} + (\sin \beta)^2 a_{22})] \quad (312)$$

Since only the real part of \hat{h}_D is needed in the calculation of steady state force, (302) can be written in its final form as the following:

$$h_D(x) = A^2 k \frac{\omega^2}{\omega_e} e^{-2kds} [b_{33} + (\sin \beta)^2 b_{22}] \quad (313)$$

$$\mathcal{F}_D = \frac{1}{2} \int_L h_D(x) dx \quad (314)$$

The total second order steady state force can now be calculated by summing (289), (297), (298), (299) and (314).

$$\mathcal{F} = Re \left\{ \sum_{j=2}^6 [\mathcal{F}_j^I + \mathcal{F}_j^D] \right\} + \mathcal{F}_D \quad (315)$$

4.2 The Steady State Moment

The hydrodynamic moment on a body in the free surface can be expressed in a similar form used to evaluate the force given in (264).

$$M = -\rho \frac{d}{dt} \iint_{S_B + S_F} \Phi (\vec{n}' \times \vec{n}) ds - \rho \iint_{S_B + S_F} \vec{r}' \left[\frac{\partial \Phi}{\partial n} \nabla \Phi - \frac{1}{2} |\nabla \Phi|^2 \vec{n} \right] ds \quad (316)$$

Here, \vec{r}' is the position vector in a coordinate system fixed in space and \vec{r} the position vector with respect to the coordinate system fixed to the mean hull position. \vec{r}' can be related to \vec{r} as

$$\vec{r}' \times \vec{n} = \vec{r} \times \vec{n} - Un_3 t \vec{j} + Un_2 t \vec{k} \quad (317)$$

If the first integral in (316) is denoted by I and only the moment about the z axis is considered, substitution of (317) into (316) results in

$$I = -\rho \frac{d}{dt} \iint_{S_B+S_F} \Phi (\vec{n} \times \vec{n})_z ds - \rho \frac{d}{dt} \iint_{S_B+S_F} \Phi U n_2 t ds \quad (318)$$

In this equation, the first term does not give any steady state contribution.[3] The contribution from the second term can be written as

$$y = \frac{i U}{2 \omega} \int_L (\zeta_3 - x \zeta_5) [k f_2^*(x) + g(x)] dx \quad (319)$$

where

$$f_j(x) = A \rho g e^{ikx \cos \beta} \int_{C_x} N_j e^{iky \sin \beta} e^{kz} dl \quad (320)$$

$$g(x) = -A \rho g e^{ikx \cos \beta} N_2 \cos\left(\frac{1}{2} kb \sin \beta\right) \quad (321)$$

Following the same steps in deriving the steady state force, the steady state moment can be expressed as

$$\mathcal{M} = y - \frac{1}{2} \iint_{S_B} \left(\phi_B \frac{\partial}{\partial n} - \frac{\partial \phi_B}{\partial n} \right) (\vec{n} \times \nabla)_z \phi_i^* ds \quad (322)$$

Only the moment about the z axis is considered, so that

$$(\vec{n} \times \nabla)_z = x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \approx x \frac{\partial}{\partial y} \quad (323)$$

since it is assumed that $x \frac{\partial}{\partial y} \gg y \frac{\partial}{\partial x}$ for slender ships.

Substitution of (323) into (322) gives the following expression.

$$\mathcal{M} = y - \frac{i}{2} \rho k \sin \beta \iint_{S_B} \left(\phi_B \frac{\partial}{\partial n} - \frac{\partial \phi_B}{\partial n} \right) x \phi_i^* ds \quad (324)$$

Equation (324) is similar to (276) except for the presence of moment arm x . If the same steps which were taken in the calculation of steady state force are taken and the total moment is written as

$$m = y + \sum_{j=2}^7 m_j \quad (325)$$

and the moment components can be expressed as the following:

$$\mathcal{M}_j = \frac{1}{2} k \cos \beta \zeta_j \int_L x (f_j + \hat{h}_j) dx \text{ for } j = 2,3,4 \quad (326)$$

$$\mathcal{M}_5 = -\frac{1}{2} k \cos \beta \zeta_5 \int_L x \left(x f_3 + \left(x + \frac{iU}{\omega_e} \right) \hat{h}_3 \right) dx \quad (327)$$

$$\mathcal{M}_6 = \frac{1}{2} k \cos \beta \zeta_6 \int_L x \left(x f_2 + \left(x + \frac{iU}{\omega_e} \right) \hat{h}_2 \right) dx \quad (328)$$

$$\mathcal{M}_7 = \frac{1}{2} \cos \beta \int_L x h_D(x) dx \quad (329)$$

Here $f_j(x)$, $\hat{h}_j(x)$ and $h_D(x)$ are defined by (320), (296) and (313) respectively.

4.3 Results and Discussions

The added resistance of Mariner type ship in head waves for various speeds was calculated using SMA. Figure 15 shows the results both obtained by SMA and given by Salvassen [3]. It can be seen that the results are close to each other; however, the greatest differences occurred near the maximum values of added resistance. Similar differences can also be seen in Figure 16 in which the results were calculated by using MIT-5D seakeeping program. In addition, added resistance of Mariner type ship at 15 knots for various headings was also calculated by SMA. It can be seen in Figure 17 that the results are very close to the ones given by Salvassen[3], even though SMA does not include viscous roll damping in the calculations which affects the results obtained for sway, roll and yaw motions. Since the steady state force is calculated using the results obtained from close-fit source distribution method and the linear strip theory, its accuracy

depends on the accuracy of the responses, sectional hydrodynamic coefficients and the excitation forces and moments.

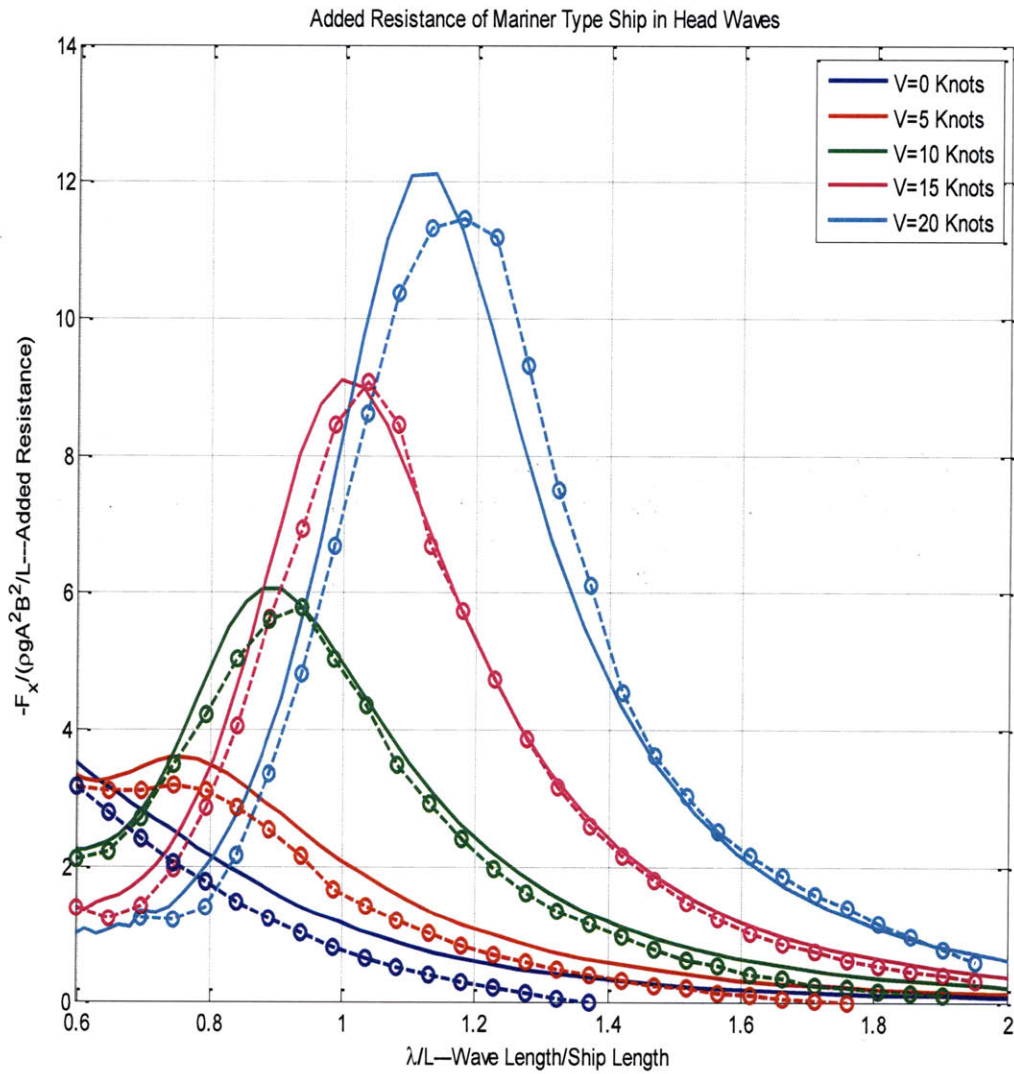


Figure 15: Added resistance of Mariner type ship in head waves at various speeds. Solid lines represent the results obtained by SMA and dashed lines are adapted from Salvasen [3].

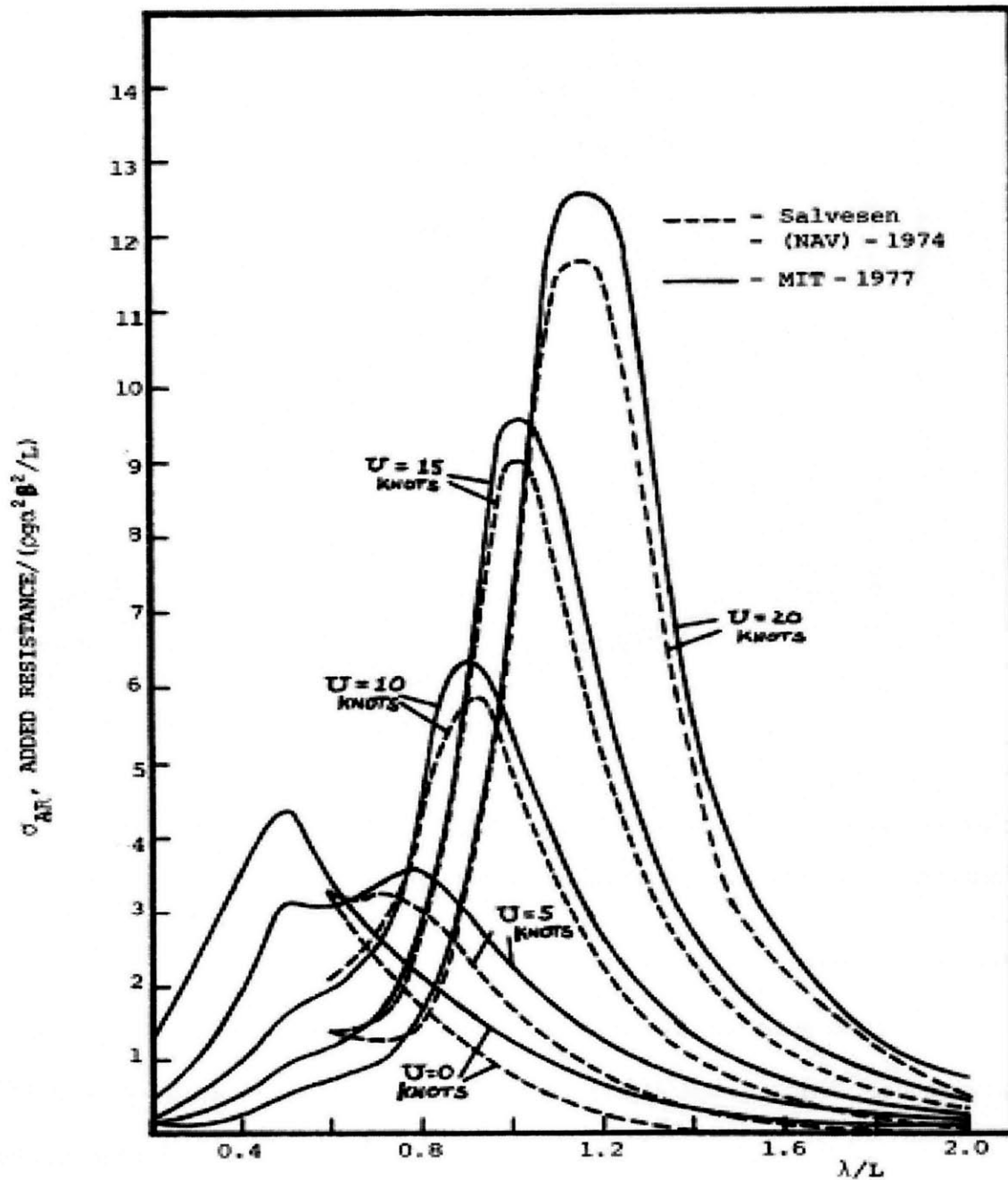


Figure 16: Added Resistance of Mariner type ship in head waves at various speeds (Adapted from Erb [21].)

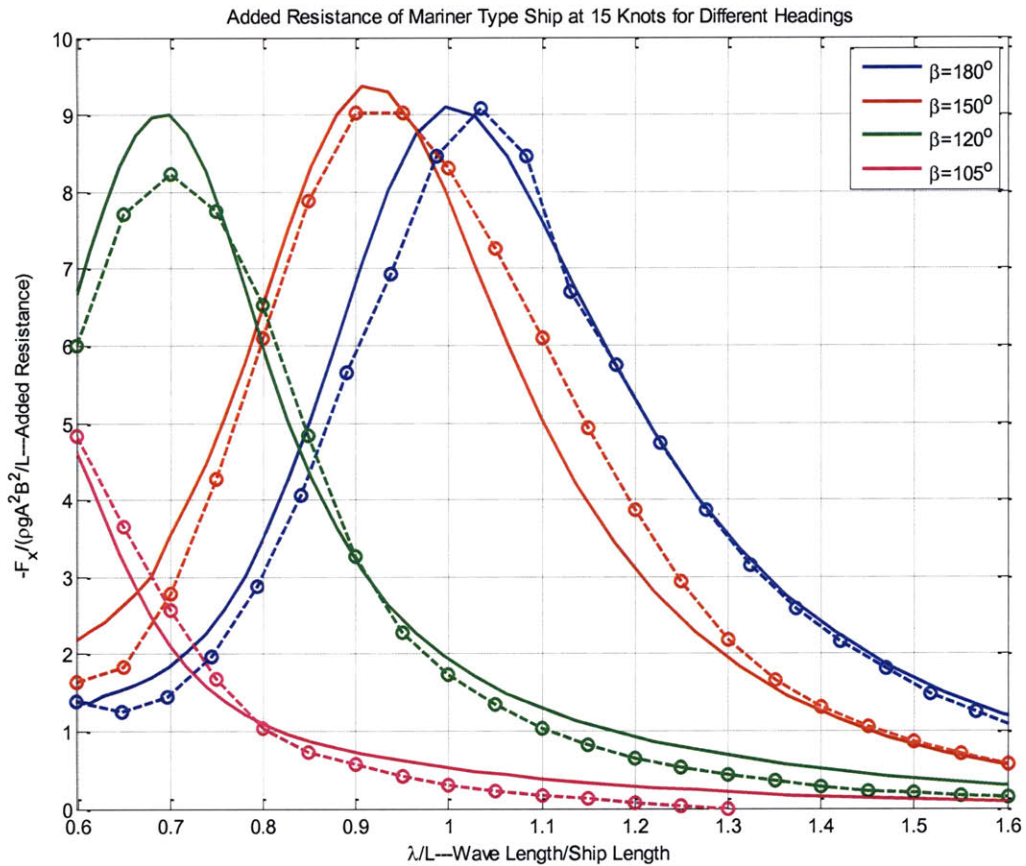


Figure 17: Added Resistance of Mariner type ship at 15 Knots for different headings (Solid lines represent results obtained by SMA and dashed lines are adapted from Salvasen[3])

4.4 Conclusions

The results presented here are only for one type of ship. Even though very close results were obtained during the calculations, more calculations should be carried out using different hull forms and the results should be compared to experimental results. As mentioned before, SMA does not take viscous roll damping into account at the moment, so this factor must be included first to obtain better results for sway, roll and yaw motions.

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