A Tactical Planning Model for a Serial Flow Manufacturing System

by

Bin Huang

B.Comp., Computer Engineering
National University of Singapore, 2009

Submitted to the School of Engineering
in partial fulfillment of the requirements for the degree of

Master of Science in Computation for Design and Optimization

At the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 2010

Author..
School of Engineering
July 30, 2010

Certified by..
Stephen C. Graves
Abraham J. Siegel Professor of Management Science
Thesis Supervisor

Accepted by..
Karen Willcox
Associate Professor of Aeronautics and Astronautics
Codirector, Computation for Design and Optimization Program

A Tactical Planning Model for a Serial Flow Manufacturing System

by

Bin Huang

Submitted to the School of Engineering
on Aug 2, 2010, in partial fulfillment of the
requirements for the degree of
Master of Science in Computation for Design and Optimization

Abstract

This project aims to improve the operation and planning of a specific type of manufacturing system, a serial flow line that entails a sequence of process stages. The objective is to investigate inventory policy, raw material ordering policy, production planning and scheduling policy, in the face of demand uncertainty, raw material arrival uncertainty and in-process failure.

The tactics being explored include segmenting the serial flow line with decoupling buffers to protect against demand and raw material arrival uncertainty, and production smoothing to reduce production-related costs and the variance in upstream processes. Key policies for each segment include a work release policy from the decoupling buffer before the segment, and a production control policy to manage work-in-process inventory level within the segment and to meet inventory targets in each downstream decoupling buffer. We also explore raw material ordering policy with fixed ordering times, long lead-times and staggered deliveries in a make-to-order setting.

A tactical model has been developed to capture the key uncertainties and to determine the operating tactics through analysis and optimization. This study also includes extensive numerical tests to validate the output of the tactical model as well as to gain a better understanding of how the tactical model reacts to different parameter variations.

Thesis Supervisor: Stephen C. Graves
Abraham J. Siegel Professor of Management Science

Acknowledgement

First of all, I thank my advisor, Prof. Stephen C. Graves, for his guidance over the past year and making this thesis a great learning experience. His analytical insights and professional experience in the manufacturing system industry is inspirational. He is also one of the most caring professors I have ever worked with who pays close attention to his students and would not hesitate to help if I address my concerns. This experience already has a big impact on my personal growth and will definitely benefit my career in the future.

Secondly, I am thankful to Mr. Pallav Chhaochhria, one of Ph.D. candidates under Prof. Graves' supervision, who I have been working closely with on this thesis. Without his help and guidance along the way, I would not be able to grasp the fundamental concepts of manufacturing system optimization and dive into the research within a short time period.

I am also grateful to Ms. Laura Koller, who has been supportive and caring throughout the entire period of my studies at MIT.

I would like to show my appreciation to Singapore-MIT Alliance fellowship, which allows me to study and carry out research at one of the most prestigious universities in the world.

Last but not least, I dedicate this work to my parents back in China for their unconditional love and support.

Table of Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Project Motivation

A serial flow line in a manufacturing context consists of a set of process steps or stages in sequence, which covers machining, heat treatment, inspection and rework operations. In the case that a work piece fails inspection, it may go through several pre-specified rework processes depending on why it failed and join the main routing afterwards. At each process step, there can be a set of parallel machines that perform the same task. Some processes can be batch processes wherein the process takes in a batch of work pieces and outputs them at the same time upon finishing.

This project aims to optimize the total cost of the serial flow line mentioned above in the face of various uncertainties. Firstly, the demand of finished goods can vary from month to month. A placed order can be cancelled, or the due date can be advanced or delayed by a few months. To accommodate this variability and maintain a robust manufacturing system, we have to forecast the demand and plan our production accordingly. Secondly, there is uncertainty in the procurement times for the raw material; there can also be some yield uncertainty with the received raw material. Thirdly, in the production line there can be yield uncertainty, which is detected at inspection processes. A product might fail inspection and become scrap or go through a series of rework processes. The in-process failure uncertainty has a significant impact on the estimation of the lead time of production.

The costs involved in a typical manufacturing system include penalty cost, overtime cost and inventory cost. Penalty cost occurs when an order is not fulfilled at the due date, which is particularly significant when there is an unexpected spike in the demand. Overtime cost occurs when the regular capacity is not enough to produce to the desired production rate. In addition, there are three types of inventory, raw material inventory, work in process inventory and end item inventory. Each of these three types of inventory incurs an inventory holding cost, due to the capital invested in the inventory as well as handling and storage related costs.

To minimize these costs while taking uncertainties into account, there are a few questions to be addressed.

1. Where in the serial flow line do we place decoupling buffers to protect against demand and raw material arrival uncertainty, and reduce variance in upstream processes?
2. What level of work in process inventory do we aim at? How much inventory do we keep at each decoupling buffer?
3. To maintain the inventory level in the segment as well as in the decoupling buffer, what kind of work release policy do we apply to each decoupling buffer? What production rate control policy is good for each segment?
4. How much raw material do we order at the fixed ordering dates, considering the long lead time and staggered, yet possibly delayed, deliveries?

This project aims to address all the questions above through theoretical analysis and mathematical modeling. Before we go into details of how the optimization is carried out, we shall review several key concepts involved.

1.2 Literature Review

1.2.1 MMFE

The Martingale Model of Forecast Evolution (MMFE), developed by Graves et al. (1986) [1] and Heath and Jackson (1994) [2], provides a framework to model the evolution of a demand forecast process in a discrete time setting. For the serial flow line, we model the demand process as an MMFE process and develop the tactical model based on that assumption. Thus, it is important to review what an MMFE process is. We define the following variables.

H : Forecast horizon;

D_t : Demand at time t;

$f_t(t+i)$: Forecast at time t for demand in period $t+i$;

$\Delta f_t(t+i)$: Forecast revision at time t for the demand forecast for period $t+i$;

Here we only consider the time period from now until H units of time later. Thus at time t, $f_t(t)$ is simply the actual demand D_t at time t and the forecasts are $f_t(t+1)$, $f_t(t+2)$, ..., $f_t(t+H)$. Assuming the next revision happens at time $t+1$, we revise the nearer term forecasts and make the following amendment.

$$
\begin{gathered}
f_{t+1}(t+1) = f_t(t+1) + \Delta f_{t+1}(t+1) = D_{t+1} \\
f_{t+1}(t+2) = f_t(t+2) + \Delta f_{t+1}(t+2) \\
\vdots \\
f_{t+1}(t+H) = f_t(t+H) + \Delta f_{t+1}(t+H)
\end{gathered}
$$

Then at period $t+1$ we need to make the first forecast for period $t+H+1$, namely $f_{t+1}(t+H+1)$. This is how we model the forecast process. To arrive at the actual demand of time t, we have the previous forecast at time t-H and then make H revisions. Thus, we can express the demand as the following.

$$
f_t(t) = D_t = f_{t-H}(t) + \sum_{i=1}^{H} \Delta f_{t-H+i}(t)
$$

In each time period, we assume that the vector of H revisions, $\overline{\Delta f_t}$, is an independent, identically distributed (i.i.d.) random vector with $E[\Delta f_t(j)] = 0, \forall t, j$. Under this assumption, Graves et al. [1] and Heath and Jackson [2] have established several properties for this forecast evolution model.

Property1. $f_t(t+i)$ *is a martingale and an unbiased estimate of* D_{t+i}*;*

Property 2. The variance of the forecast error $D_{t+i} - f_t(t+i)$ *increases in i;*

Property 3. The variance of the random variable D_t is the trace of the covariance matrix for $\overline{\Delta f_t}$, which we denote by Σ.

For this project, the initial forecast $f_t(t+H)$ is assumed to be the average demand μ for all *t*. Combined with Property 1, it is evident that $E[D_t] = \mu$ and $Var[D_t]$ is equal to the trace of Σ.

1.2.2 Safety Stock

Graves [3] gives a summary of the previous work on Safety Stock and suggests that if everything is deterministic, there would be a minimum inventory level that a manufacturing system would need to satisfy the fixed demand. However, in reality we need a certain amount of excess inventories besides the minimum inventories in order to buffer the uncertainties in raw material arrival, production and demand, and also due to the inflexibility of manufacturing system. That excess inventory, namely the *Safety Stock*, is used to fulfill customer's demand at a satisfactory performance level and also to reduce production costs under those uncertainties.

This broader definition of Safety Stock includes not only the stocks that protect against various uncertainties, but also the stocks that help perform production smoothing or serve the purpose of decoupling the line. In a real scenario, factories do not label any part of their inventory explicitly as the Safety Stock; instead, they simply have in-process inventories to perform the functions intended for safety stock.

1.3 Assumptions

Before developing the tactical model, we need to make several assumptions.

First of all, we assume a discrete time model with an underlying time period, for example, one month. It is the same frequency at which we would make the release decisions and production rate decisions for each segment. Forecasts get updated at this frequency as well or maybe less frequently.

The demand process for the end items is assumed to be a MMFE process. Thus, given the covariance matrix Σ for the forecast updates, the variance of the demand process is simply the trace of Σ. At times it might be preferable to use standard deviation, i.e. the square root of the trace.

Moreover, we assume that an inventory target will be set for each decoupling buffer in terms of a safety factor, z. A typical z value of 2 corresponds to a protection level of two times the standard deviation to the right of the mean value and provides a service level (probability of not stocking out in a period) of 98% of the time, under the assumption that the forecast revisions are normally distributed. Even with a high buffer inventory target, it can still happen that the buffer does not have enough inventory to release to the downstream segment. For this project, we assume that the upstream buffer never starves the segment; in other words, the desired release rate is always realizable.

We also assume that we only have one product type. However, the tactical model can be extended to multiple product types fairly easily. These assumptions make it easier to develop an effective tactical model that well serves the purpose of this project.

Chapter 2

Tactical Model

In this chapter, the tactical model will be described in detail to address the objectives mentioned in Chapter 1.

2.1 Dynamic Programming Approach

This part describes how to determine the location of each decoupling buffer, which breaks the serial flow line into segments. Each segment has several process stages followed by a decoupling buffer. Two important parameters of a segment are σ_{in} and σ_{out}, corresponding to the standard deviation of the release and demand processes for the segment, respectively. Given two potential buffer locations at i and j, $i < j$, and a pair of values set for σ_{in} and σ_{out}, we define $C(i, j, \sigma_{in}, \sigma_{out})$ as the inventory and production cost for the segment consisting of steps $i+1, \ldots, j$ for this specific set of parameter values.

How to calculate this cost function will be explained step by step in the following sections. For now let us assume that we know how to calculate this cost function. Thus, we have the DP structure as:

$$G(i,\sigma_{in}) = \min_{j,\sigma_{out}}\{C(i, j,\sigma_{in},\sigma_{out}) + G(j,\sigma_{out})\} \tag{2.1}$$

$$\forall j,\sigma_{out}\{j,\sigma_{out} \mid j = i+1,\ldots,N; \sigma_{in} \le \sigma_{out} \le \sigma_{demand}\}$$

where σ_{demand} is the standard deviation for the end item and $G(i,\sigma_{in})$ is the cost of the optimal solution for process stages $i+1, \ldots, N$ (process N is the end of serial flow line), under the assumption that there is a decoupling buffer after process i and the standard deviation of the release process into step $i+1$ is σ_{in}.

To find the optimal solution for the entire serial flow line, we solve for

$$G(0) = \min_{\sigma}\{G(0,\sigma)\} \quad \forall\sigma\{\sigma \mid 0 < \sigma \le \sigma_{demand}\}$$

with boundary condition

$$G(N,\sigma) = \begin{cases} 0 & for\ \sigma = \sigma_{demand} \\ \infty & otherwise \end{cases}$$

We now have the big picture of how to find the optimal configuration for the entire serial flow line. The following sections will explain how to determine the operating policy of each segment and evaluate the cost function based on the policy parameters. From this point onwards, our discussion is mostly within the scope of a single segment.

2.2 Release of Work

We introduce a smoothing parameter for the release rule, α. The following release rule is applied.

$$r_t(t) = \alpha \times f_t(t) + (1-\alpha) \times r_{t-1}(t-1) \qquad (2.2)$$

where $r_t(s)$ is the planned release rate for time s determined at period t, $s>t$ and $r_t(t)$ is simply the release rate for period t; $f_t(s)$ is the demand forecast for time s determined at period t, $s > t$ and $f_t(t)$ is the demand at period t.

It is obvious that with a larger α, the new release rate reflects more of the current demand whereas smaller α means that the new release rate is closer to the previous release rate. In other words, larger α indicates more responsiveness to the demand while smaller α leads to more smoothing and flatter production rate. It can be derived from the release rule (2.2) that the release process is also an MMFE process if the demand is an MMFE process.

$$
\begin{aligned}
r_t(t) &= \alpha \times f_t(t) + (1-\alpha) \times r_{t-1}(t-1) = \sum_{i=0}^{t-1} \alpha (1-\alpha)^i f_{t-i}(t-i) + \alpha (1-\alpha)^t r_0(0) \\
&= \sum_{i=0}^{t-1} \alpha (1-\alpha)^i \left(\sum_{j=0}^{t-i-1} \Delta f_{t-i-j}(t-i) + f_0(t-i) \right) + \alpha (1-\alpha)^t r_0(0) \\
&= \sum_{i=0}^{t-1} \sum_{k=i}^{t-1} \alpha (1-\alpha)^i \Delta f_{t-k}(t-i) + \sum_{i=0}^{t-1} \alpha (1-\alpha)^i f_0(t-i) + \alpha (1-\alpha)^t r_0(0) \\
&= \sum_{k=0}^{t-1} \sum_{i=0}^{k} \alpha (1-\alpha)^i \Delta f_{t-k}(t-i) + r_0(t) \\
&= \sum_{k=0}^{t-1} \Delta r_{t-k}(t) + r_0(t)
\end{aligned}
$$

where

$$
\begin{aligned}
r_0(t) &= \sum_{i=0}^{t-1} \alpha (1-\alpha)^i f_0(t-i) + \alpha (1-\alpha)^t f_0(0) \\
\Delta r_{t-k}(t) &= \sum_{i=0}^{k} \alpha (1-\alpha)^i \Delta f_{t-k}(t-i) \text{ for } 0 \le k \le t-1
\end{aligned}
$$

The second equation can be rewritten as

$$
\Delta r_t(t+k) = \sum_{i=0}^{k} \alpha (1-\alpha)^i \Delta f_t(t+k-i) \text{ for } 0 \le k \le t-1 \tag{2.3}
$$

We observe that the revision vector $\overline{\Delta r_t}$ is an i.i.d. vector with zero mean value, similar to $\overline{\Delta f_t}$ as shown in Section 1.2.1. Thus, from Equation (2.3), $\overline{\Delta r_t}$ can be expressed as a transformation of $\overline{\Delta f_t}$.

$$
\overline{\Delta r_t} = M_1 \, \overline{\Delta f_t}
$$

where M_1 has zeros above the diagonal and α on the diagonal, followed by geometric weights $\alpha(1-\alpha)^i$ for *i* rows below the diagonal.

However, since we are only considering the revision within the horizon *H*, Equation (2.3) does not hold for *k*>*H*. We can argue that each column of M_1 has to sum up to one since the sum of every possible adjustment to $r_t(s)$ must be equal to one. Thus, the last row of M_1 can be shown to be $(1-\alpha)^{H-1}$, $(1-\alpha)^{H-2}$, …, $(1-\alpha)$, 1.

As mentioned in Section 1.2.1, the variance for f_t is simply the trace of Σ. Thus, the variance for r_t is the trace of $M_1\Sigma M_1^T$. Here we propose an approximation as a simpler way of calculating the variance.

$$
\begin{aligned}
Var(r_t(t)) &= tr(M_1\Sigma M_1^T) \cong \left(\frac{\alpha}{2-\alpha}\right)(\alpha \times tr(\Sigma) + (1-\alpha)\times sum(\Sigma)) \\
&= \left(\frac{\alpha}{2-\alpha}\right)(\alpha \times Var(f_t(t)) + (1-\alpha)\times sum(\Sigma))
\end{aligned}
\tag{2.4}
$$

where $sum(\Sigma)$ represents the sum of all elements in Σ.

Since each column of M_1 sums up to one, $sum(M_1\Sigma M_1^T)$ is equal to $sum(\Sigma)$. Thus, the sum of covariance matrix remains constant as we move upstream from the last segment. Rewriting Equation (2.4), we have

$$
\sigma_{in}^2 \cong \left(\frac{\alpha}{2-\alpha}\right)\left(\alpha \times \sigma_{out}^2 + (1-\alpha)\times sum(\Sigma)\right)
$$

If we are given the variance of demand σ_{out} and a possible smoothing parameter α, we can calculate σ_{in}. If we are given σ_{in} and σ_{out}, we can solve for α directly, which simplifies the computation of DP significantly.

2.3 Inventory Target and Safety Stock for Decoupling Buffer

We define the following notation.

$x_t(t)$: Finished goods inventory after a line segment, as of the end of period *t*; subscript *t* can be omitted;

$x_t(t+k)$: Forecast of finished goods inventory for period *t*+*k*, as of period *t*;

$r(t)$: Release into the segment for period *t*;

$r_t(t+k)$: Forecast of the planned release for period $t+k$;

$p(t)$: Production rate for the segment for period t;

$p_t(t+k)$: Forecast of the planned production rate for period $t+k$;

X : Finished goods inventory target for the decoupling buffer;

μ : Average demand rate.

The inventory variability is largely correlated with the production rule and, therefore, we need to define the production rule first. Assuming a linear control, we have the production rule as

$$p(t) = \mu + \beta \times (X - x(t)) \tag{2.5}$$

The interpretation of production rule (2.5) is that as β goes down, we have greater production smoothing and the production becomes less responsive to the variability of the demand. That in turn requires more inventory to accommodate more variability so as to assure some desired service level.

We assume that the following balance equation holds.

$$x(t) = x(t-1) + p(t) - f_t(t) \tag{2.6}$$

Substitute Equation (2.5) into (2.6), we have

$$\begin{aligned}
x(t) &= x(t-1) + \left(\mu + \beta \times \left(X - x(t-1)\right)\right) - f_t(t) \\
&= (1-\beta) \times x(t-1) + \left(\mu - f_t(t)\right) + \beta X \\
&= (1-\beta) \times \left((1-\beta) \times x(t-2) + \left(\mu - f_{t-1}(t-1)\right) + \beta X\right) + \left(\mu - f_t(t)\right) + \beta X \\
&= \sum_{k=0}^{t-1} (1-\beta)^k \times \left(\mu - f_{t-k}(t-k)\right) + (1-\beta)^t \times x(0) + \sum_{k=0}^{t-1} (1-\beta)^k \times \beta X \\
&= \sum_{k=0}^{t-1} (1-\beta)^k \times \left(\mu - f_{t-k}(t-k)\right) + X
\end{aligned}$$

where we assume $x(0) = X$. Similar to the release rule deduction, we can show that the inventory process is also an MMFE process.

$$
\begin{aligned}
x(t) &= \sum_{k=0}^{t-1}(1-\beta)^k \times \left(\mu - f_{t-k}(t-k)\right) + X \\
&= X + \sum_{k=0}^{t-1}(1-\beta)^k \times \left(\mu - \sum_{j=0}^{t-k-1} \Delta f_{t-k-j}(t-k) - f_0(t-k)\right) \\
&= X - \sum_{k=0}^{t-1}(1-\beta)^k \times \left(\sum_{j=0}^{t-k-1} \Delta f_{t-k-j}(t-k)\right) \\
&= X - \sum_{j=0}^{t-1}\sum_{k=0}^{t-1-j}(1-\beta)^k \times \Delta f_{t-k-j}(t-k) \\
&= X - \sum_{i=0}^{t-1}\sum_{k=0}^{i}(1-\beta)^k \times \Delta f_{t-i}(t-k) \\
&= x_0(t) + \sum_{i=0}^{t-1} \Delta x_{t-i}(t)
\end{aligned}
$$

where we assume

$$f_0(t-k) = \mu, \forall k$$

$$x_0(t) = X$$

$$\Delta x_{t-i}(t) = -\sum_{k=0}^{i}(1-\beta)^k \times \Delta f_{t-i}(t-k)$$

The last equation shows that $\overline{\Delta x_t}$ can be expressed as $-M_2\overline{\Delta f_t}$, where M_2 has zeros above the diagonal and 1 on the diagonal, followed by geometric weights $(1-\beta)^i$ for i rows below the diagonal. Similar to the release process, we have an approximation for the variance of inventory.

$$Var(x_t(t)) = tr(M_2 \Sigma M_2^T) \cong \left(\frac{1}{2\beta - \beta^2}\right)\left(\beta \times tr(\Sigma) + (1-\beta) \times sum(\Sigma)\right) \qquad (2.7)$$

where Σ is the covariance matrix of f_t.

The implication of this approximation is that given the demand standard deviation of a segment, σ_{out}, i.e. $\sqrt{tr(\Sigma)}$, the variance of finished goods inventory becomes a function of β. Moreover, given the variance of finished goods inventory, finished goods safety stock should be set to $z\sqrt{Var(x_t(t))}$ where z is the safety factor as mentioned in Section 1.2.2. From this we can calculate the FGI cost.

Here we choose 0.1 for the lower limit of β because the decoupling buffer at the end of the segment experience extremely high variance as β goes lower than 0.1. This can be observed from Equation (2.7). Intuitively, it is obvious that the production rate is close to average demand μ with a small β, which can result in a very low FGI inventory level when there is a spike in the demand. It will take a long time to recover from this situation, which also increases the penalty cost. Thus, β cannot be lower than 0.1.

2.4 Production Planning and Smoothing

Using the result for *x(t)* in Section 2.3, we have

$$\begin{aligned}
p(t) &= \mu + \beta \times (X - x(t)) \\
&= \mu + \beta \times \left(X - \left(X - \sum_{i=0}^{t-1} \sum_{k=0}^{i} (1-\beta)^k \times \Delta f_{t-i}(t-k) \right) \right) \\
&= \mu + \beta \times \sum_{i=0}^{t-1} \sum_{k=0}^{i} (1-\beta)^k \times \Delta f_{t-i}(t-k) \\
&= p_0(t) + \sum_{i=0}^{t-1} \Delta p_{t-i}(t)
\end{aligned}$$

where

$$\begin{aligned}
&p_0(t) = \mu \\
&\Delta p_{t-i}(t) = \beta \times \sum_{k=0}^{i} (1-\beta)^k \times \Delta f_{t-i}(t-k) \\
&or\ \Delta p_t(t+i) = \beta \times \sum_{k=0}^{i} (1-\beta)^k \times \Delta f_t(t+i-k)
\end{aligned}$$

This implies that production process is an MMFE process as well as other process proven to be MMFE in previous sections. $\overline{\Delta p_t}$ can be expressed as $\beta M_3 \overline{\Delta f_t}$, where M_3 has zeros above the diagonal and 1 on the diagonal, followed by geometric weights $(1-\beta)^i$

for i rows below the diagonal. Similar to the approximation of release process and finished goods inventory, we have

$$
\begin{aligned}
Var(p(t)) &= \beta^2 Var(x(t)) \cong \beta^2\left(\frac{1}{2\beta-\beta^2}\right)\left(\beta\times tr(\Sigma)+(1-\beta)\times sum(\Sigma)\right) \\
&= \frac{\beta}{2-\beta}\left(\beta\times tr(\Sigma)+(1-\beta)\times sum(\Sigma)\right)
\end{aligned}
\tag{2.8}
$$

It is worthwhile to take note that given the demand standard deviation of a segment, σ_{out}, i.e. $\sqrt{tr(\Sigma)}$, the variance of production rate becomes a function of the smoothing parameter β.

2.5 Work in Process Inventory in a Segment

Work in process inventory in a segment is defined as follows.

$$
W = \mu \times \text{segment lead time} + z\times\sigma_{out}\sqrt{\frac{(1-\beta)^2}{1-(1-\beta)^2}-\frac{2(1-\beta)(1-\alpha)}{1-(1-\beta)(1-\alpha)}+\frac{(1-\alpha)^2}{1-(1-\alpha)^2}} \tag{2.9}
$$

where μ is the average demand rate.

The segment lead time is equal to the sum of the lead times at each process in the segment. The following notations are defined.

ϕ : Number of times a work piece visits the process;

t: Process time for a work piece;

C: Capacity of the process in time units per period;

c_a: Coefficient of variation for arrivals to the process;

ρ : Utilization for the process;

To estimate the waiting time at each process, we use a G/D/1 queuing approximation.

$$\text{Waiting time} = \frac{c_a^2}{2} \cdot \frac{t\rho}{1-\rho}$$

$$\text{where } c_a = \frac{\sigma_{in}}{\mu} \text{ and } \rho = \frac{\phi\mu\, t}{C}$$

The second term in Equation (2.9) is a function of α and β, which represents additional WIP needed to accommodate variability due to the release and production rules. An intuitive way of understanding this is that as the difference between α and β gets larger, more WIP is needed in the segment. However, we note that when one of α and β approaches 0, the term goes to infinity.

2.6 Raw Material Ordering Policy

We are working with a specific raw material order generation process as follows. Raw material orders are placed N times a year, with the time between orders being the same. L denotes the lead time of the first raw material arrival after order placement. Each raw material order has staggered deliveries, i.e., an order placed at month t will be delivered in month $t+L$, $t+L+1$, ..., $t+L+(12/N)-1$. If N is 2 and L is 6 months, an order placed in month 3 will have six installments from month 9 until month 14, or month 2 of next year.

We define the following terms.

$v_t(t)$: Quantity of raw materials on-hand at the end of month t; the subscript can be dropped;

$v_t(t+k)$: Forecast of raw material inventory at the end of month $t + k$ as of month t;

$r_t(t)$: Quantity of raw materials to be released into production during month t; the subscript can be dropped;

$r_t(t+k)$: Forecast for the quantity of raw materials to be released during month $t + k$ as of month t;

$q(t+k)$: Quantity of raw material ordered for delivery at month $t + k$. We assume delivery occurs in the beginning of the month.

Thus we can model the inventory dynamics as follows.

$$\begin{aligned}
&v(t) = v(t-1) + q(t) - r(t) \\
&\Rightarrow v_t(t+L) = v(t) + q(t+L) + \sum_{i=1}^{L-1} q(t+i) - \sum_{i=1}^{L} r_t(t+i) \\
&\Rightarrow v_t(t+k) = v(t) + q(t+k) + \sum_{i=1}^{k-1} q(t+i) - \sum_{i=1}^{k} r_t(t+i) \qquad k \geq L
\end{aligned} \tag{2.10}$$

We take note that the forecasts $v_t(t+k), r_t(t+i)$ in Equation (2.10) are random variables. If N is 2 and L is 6, at time t the future orders $q(t),...q(t+5)$ have been previously determined and are scheduled for receipt in the next 6 months. However, we need to determine the orders $q(t+6),...q(t+11)$ at time t. More generally, we need to set the quantities for $q(t+L),...,q(t+12/N-1)$.

It is obvious that we need to know the safety stock target prior to determining the quantity of raw material orders. From Equation (2.10), if we assume that there is no uncertainty in raw material delivery time, the mean and variance for the inventory random variable can be characterized as follows.

$$E\left[v_t(t+k)\right] = v(t) + q(t+k) + \sum_{i=1}^{k-1} q(t+i) - E\left[\sum_{i=1}^{k} r_t(t+i)\right] \tag{2.11}$$

$$Var\left[v_t(t+k)\right] = Var\left[\sum_{i=1}^{k} r_t(t+i)\right] \tag{2.12}$$

We propose that the order quantities should be set in the following way.

$$E[v_t(t+k)] = z_\alpha \sqrt{Var[v_t(t+k)]} = z_\alpha \sqrt{Var\left[\sum_{i=1}^{k} r_t(t+i)\right]} = z_\alpha \sigma\left(\sum_{i=1}^{k} r_t(t+i)\right) \tag{2.13}$$

where z_α is the safety factor to satisfy a protection level of α percentile. In this project we use a z_α of value 2.4. This can be interpreted as the safety stock target for month $t+k$, which we denote by $SS(t+k)$.

$$SS(t+k) = z_{\alpha}\sqrt{Var\left[\sum_{i=1}^{k} r_t(t+i)\right]} \qquad for \qquad k = L,...,L+\frac{12}{N}-1$$

Thus, the safety stock target level is dependent on the cumulative variance term, which in turn requires us to determine the covariance matrix of the release into the first segment. The covariance matrix of release process is

$$\Sigma_r = M_3 \Sigma_f M_3^T$$

where Σ_f is the covariance matrix of demand process at the end of the first segment.

However, there are numerous ways that the processes from process 2 to the end of the line can be segmented and, therefore, the first segment can end at any process. Meanwhile, we do not track segmentation, which means that Σ_f and σ_{out} of segment 1 cannot be easily determined. As a result, when the DP proceeds to the first process, we have to handle things a little differently. In Section 2.2, we had

$$\sigma_{in}^2 \cong \left(\frac{\alpha}{2-\alpha}\right)\left(\alpha \times \sigma_{out}^2 + (1-\alpha) \times sum(\Sigma)\right)$$

Given the fact that σ_{out} cannot be predetermined for segment 1, we assume that σ_{out} is simply σ_{demand}, the standard deviation of demand process at the end of the serial flow line. Hence for each possible segment that starts at process step 1 and σ_{in}, we can calculate the estimated α_{est}.

$$\sigma_{in}^2 \cong \left(\frac{\alpha_{est}}{2-\alpha_{est}}\right)\left(\alpha_{est} \times \sigma_{demand}^2 + (1-\alpha_{est}) \times sum(\Sigma)\right)$$

By doing this, we reduce the search space of DP enormously. Now we can set M_3 to be a weight matrix with zeros above the diagonal, α_{est} on the diagonal and $\alpha_{est}(1-\alpha_{est})^i$ for i rows below the diagonal. After we obtain Σ_r, we can calculate the cumulative variance term.

$$
Var\left[\sum_{i=1}^{k} r_t(t+i)\right] = Var\left[\sum_{i=1}^{k}\left(E\left[r_t(t+i)\right] + \Delta r_{t+1}(t+i) + \Delta r_{t+2}(t+i) + \ldots + \Delta r_{t+i}(t+i)\right)\right]
$$

$$
= Var\left[\sum_{i=1}^{k}\Delta r_{t+1}(t+i) + \sum_{i=2}^{k}\Delta r_{t+2}(t+i) + \ldots + \sum_{i=k-1}^{k}\Delta r_{t+k-1}(t+i) + \sum_{i=k}^{k}\Delta r_{t+k}(t+i)\right]
$$

$$
= \sum_{j=1}^{k} Var\left[\sum_{i=j}^{k}\Delta r_{t+j}(t+i)\right]
$$

We observe that

$$
Var\left[\sum_{i=j}^{k}\Delta r_{t+j}(t+i)\right] = Var\left[\sum_{i=0}^{k-j}\Delta r_t(t+i)\right]
$$

which is exactly the sum of all elements in the $(k\text{-}j+1)$ principal minor of the covariance matrix Σ_r.

After the cumulative variance term and the safety stock target for $SS(t+k)$ for $k=L, \ldots, L+12/N-1$ are obtained, we can determine $q(t+L),\ldots,q(t+12/N-1)$.

If we set $v_t(t+k)$ in Equation (2.10) to be the safety stock target, we have

$$
q(t+k) = SS(t+k) - v(t) - \sum_{i=1}^{k-1} q(t+i) + \sum_{i=1}^{k} r_t(t+i) \tag{2.14}
$$

As we can determine $q(t+L),\ldots,q(t+k-1)$ before we determine $q(t+k)$, we see that $q(t+k)$ is the only unknown variable in Equation (2.14). Hence, we can use (2.14) to iteratively solve for each $q(t+L), \ldots, q(t+L+12/N-1)$.

At this point, the raw material ordering policy is well defined. We determine the smoothing parameter α, the covariance matrix of release into the first segment and the cumulative variance term, which we eventually use to determine the safety stock target level and the ordering quantity for each ordering month, to be received L months later in several installments.

2.7 Evaluation of Cost Function

Having analyzed each component of the tactical model, we are now able to evaluate the entire cost structure.

Given two potential buffer locations at *i* and *j*, $i < j$, and a pair of values set for σ_{in} and σ_{out}, $C(i, j, \sigma_{in}, \sigma_{out})$, as defined in Section 2.1, is the inventory and production cost for the segment from process $i+1$ to process j. To evaluate this cost, we go through the following steps.

1. Given σ_{in} and σ_{out}, we determine α by solving this quadratic equation.

   $$\sigma_{in}^2 \cong \left(\frac{\alpha}{2-\alpha}\right)\left(\alpha \times \sigma_{out}^2 + (1-\alpha) \times sum(\Sigma)\right)$$

   There are two solutions for the value of α, and at times some extra work needs to be done to find out which value is better.
2. Determine the value of z. If the segment contains the last process of the serial flow line, we need to determine z by solving an optimization problem over minimizing FGI holding cost and penalty cost, for which the solution is attached in Appendix A. We will have the penalty cost determined after this step. If the segment is not the last segment, we simply use a predetermined fixed safety factor, for instance $z = 1.6$.
3. Determine the value of β by minimizing the inventory cost and overtime cost. To do this, we do a line search over $0.1 \le \beta \le 1$. How to calculate overtime cost is attached in Appendix B. We will have determined the overtime cost after this step.
4. Part of inventory holding cost is due to decoupling buffer safety stock, which is determined by

   $$\begin{aligned} X &= z \times \sqrt{Var\left(x_t(t)\right)} \\ &= z \times \sqrt{\frac{1}{2\beta - \beta^2} \times \left(\beta \times \sigma_{OUT}^2 + (1-\beta) \times sum(\Sigma)\right)} \end{aligned}$$

5. Inventory holding cost also include work in process inventory, which is described in Section 2.5. After step 4 and 5, we multiply the total inventory level by inventory cost per day per work piece to get the total inventory cost.

After we determine all the parameters using a given pair of values for σ_{in} and σ_{out} and two potential decoupling buffer locations *i* and *j*, we have a minimum cost for $C(i, j, \sigma_{in}, \sigma_{out})$. For the special case when the DP goes upstream to the first process, we need to determine the raw material ordering policy as specified in Section 2.6 and hence calculate the raw material inventory cost. Now we can construct the DP cost table and determine the optimal configuration with minimum total cost for the entire serial flow line.

Chapter 3

Model Output Analysis

In the following section we report on the test results of our tactical model and investigate how the operation policies vary depending on different statistics of the serial flow line. Our test cases are based on a serial flow line of 30 processes, each of which has its own inventory cost, overtime hourly cost, process time, regular capacity and overtime capacity.

For the simplicity of this project, we assume that the batch size is 1 for all processes and work pieces go through each process exactly once. Another assumption is that raw material orders are delivered with no possibility of being delayed. Based on these assumptions, an initial base case is set up, which will be described in the next section. We build other test cases upon the base case by varying the utilization level, inventory cost, overtime cost and penalty cost.

3.1 Case 1 - Base Case

The base case is described as follows.

1) The horizon H is 6 months;
2) The number of times raw material orders are placed per year N is 4 and delivery lead time L is 3 months;
3) The average monthly demand μ is 300 with a standard deviation σ of 76.4;
4) The production line consists of 30 processes. Assuming that there is only one shift of 7.6 hours per day for 22 days per month, the maximum regular capacity of each process is around 10,000 minutes/month. We want 60% of the regular capacity to be able to cover average demand μ; thus for all process steps the process time for one work piece should be around 60%$\times$10,000 minutes /300, which is 20 minutes;
5) Since the value of the work pieces increases as the production goes on, the conventional way is to have uniformly increasing inventory holding cost from process 1 to process 30. Here we set the holding cost of process 1 as 200 Japanese

yen/(day · piece) and it reaches 1000 yen/(day · piece) at process 30 with an increment of 27.59 per process step. Following this trend, finished goods inventory holding cost is set to 1020 yen/(day · piece);

6) OT cost is set to 100,000 yen/hr;
7) Penalty cost is set to 100,000 yen/(day · piece);
8) For all the cases, we assume there is no delay for raw material;
9) Possible value of ratio σ_{in}/σ_{out} in each segment can be 0.1, 0.2, …, 1; this is a setting for determining how we decide the state space for the DP.

Table 3-1 summarizes the most important information above, and this format is used in all test cases to describe the selected values for different parameters.

Table 3-1: Case 1 description

Utilization to produce μ	Inventory cost	FGI inventory cost	Penalty cost	OT cost
60%	200-1,000 yen/(day·piece)	1,020 yen/(day·piece)	100,000 yen/(day·piece)	100,000 yen/hour

After running tactical model, the optimal serial flow line configuration and cost structure breakdown is shown as follows.

Table 3-2: Case 1 cost structure breakdown

Raw Material Inv Cost	WIP Cost	Buffer Cost	OT Cost	Penalty Cost	Total Cost
36,229	14,612	385,210	1,076	59,228	496,354

Table 3-3: Case 1 overview of segmentation and parameters

Segmentation	No segmentation
σ_{in}	22.93
α	0.26
β	0.25
z	1.7

Here we give an explanation of the summary of results and the results of following test cases will be reported in the same manner. The detailed data is attached in Appendix C for reference.

Table 3-2 shows the optimal overall cost structure breakdown, which allows us to see the different cost entities and analyze the tradeoff between different costs. The three types of costs are overtime cost, penalty cost and inventory cost. We report the values of three sub categories under inventory cost, namely, raw material inventory cost, WIP inventory cost and decoupling buffer cost, denoted as "Buffer cost" for conciseness. The total sum of all the costs is given as well.

Table 3-3 contains information on how the flow line is segmented and what values the production parameters are set to in each segment. The standard deviation of release into the segment, σ_{in}, is given. This table also shows the value for work release smoothing parameter, α, production rate smoothing parameter, β, and FGI safety factor, z, in each segment. If the segment is not the last segment in the line, z is omitted because we use a fixed value of 1.6 for it.

These data elaborate the work release policy, production planning policy and how the demand variance is smoothed out as we move from the end of the line towards upstream processes.

Now let's analyze the results for Case 1. We can see that there is no decoupling buffer added in the line, and therefore, there is only one segment. In addition, α and β are fairly low, indicating a high smoothing effect. As a result, σ_{in} is as low as 22.93 while σ_{out} for the segment is the standard deviation of customer's demand, 76.4.

3.2 Variation of the Utilization Level

In the base case all processes have utilization level of 60% to fulfill mean demand μ. In this section we are going to vary the utilization level across the serial flow line. To increase the utilization level to 100%, we set each process time to be 10,000 minutes /300, which is around 33 minutes.

3.2.1 Case 2 - Utilization Level Variation

Table 3-4: Case 2 description

Utilization to produce μ	Inventory cost	FGI inventory cost	Penalty cost	OT cost
100%	200-1,000 yen/(day·piece)	1,020 yen/(day·piece)	100,000 yen/(day·piece)	100,000 yen/hour

If we set all the processes to 100% utilization level, the result is as follows.

Table 3-5: Case 2 cost structure breakdown

Raw Material Inv Cost	WIP Cost	Buffer Cost	OT Cost	Penalty Cost	Total Cost
33,801	34,766	715,358	745,984	67,086	1,596,995

Table 3-6: Case 2 overview of segmentation and parameters

Segmentation	Before process 30
Segment 1	
σ_{in}	15.28
α	0.19
β	0.15
Segment 2	
σ_{in}	15.28
α	0.14
β	0.15
z	1.7

If we increase the utilization level of all processes to 100%, there is a breaking point before process 30, as shown in Table 3-6. Thus there are now 2 segments in this case. Segment 2 smooth out the production significantly and has σ_{in} as 15.28 while σ_{out} is 76.4. Therefore, σ_{out} of segment 1 is also 15.28 and process steps 1 to 29 experience low variance in the demand. Putting a short segment at the end of the line is an effort to reduce overtime cost in as many processes as possible.

We plot the cost structure of Case 2 against that of base case.

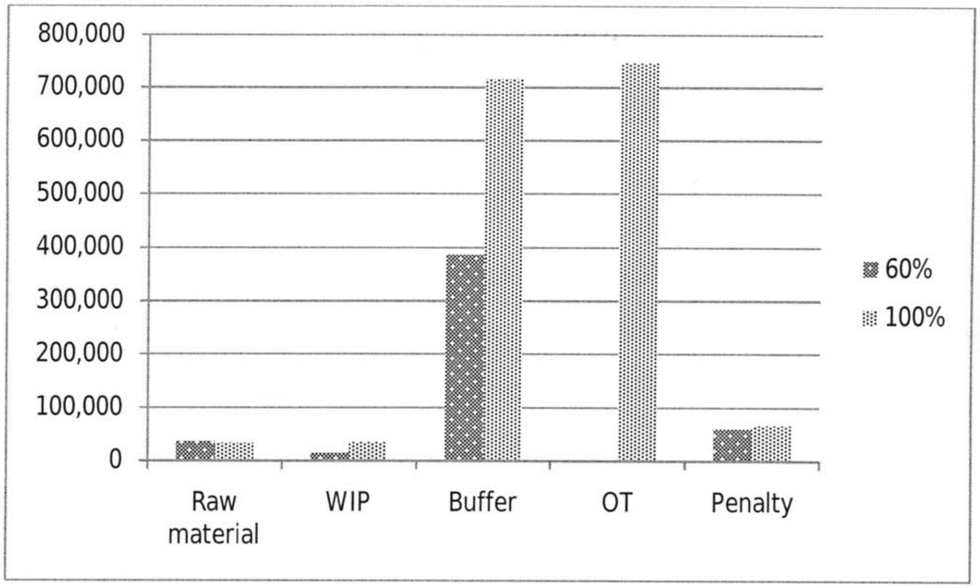

Figure 3-1: Individual cost comparison of Case 1 vs. Case 2

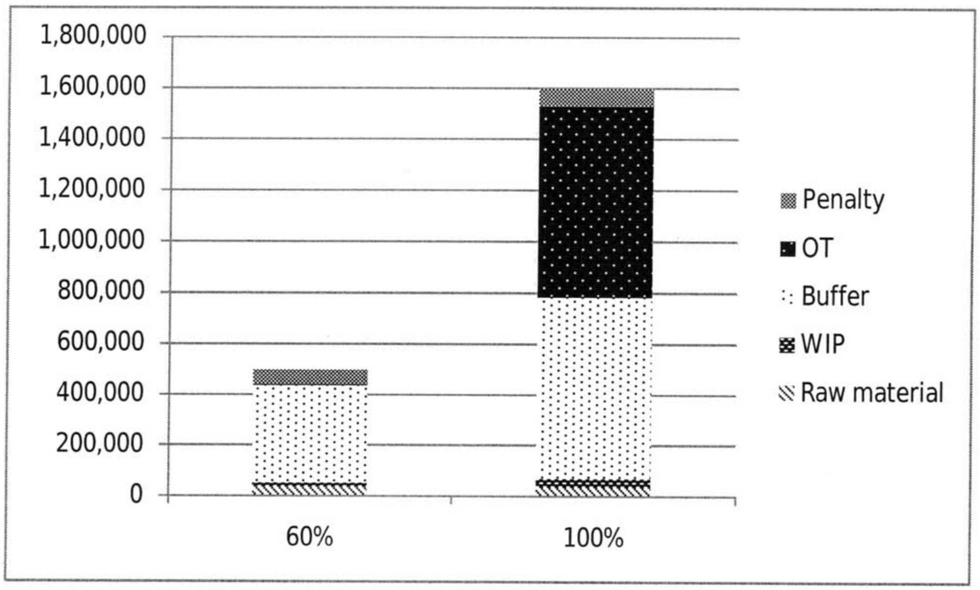

Figure 3-2: Overall cost comparison of Case 1 vs. Case 2

Since the processes are at 100% utilization level in order to meet the average demand μ, the production line runs into overtime easily. Thus, compared with Case 1, the production line in Case 2 has much higher overtime cost. Although adding a decoupling buffer doubles the buffer inventory cost compared to Case 1, it is necessary to do so in order to reduce overtime cost, due to the high utilization level here. Overall speaking, the result is consistent with our expectation that the production should be smoothed out more.

3.2.2 Case 3 - Utilization Level Variation

Table 3-7: Case 3 description

Utilization to produce μ	Inventory cost	FGI inventory cost	Penalty cost	OT cost
P1-P15 100% P16-P30 60%	200-1,000 yen/(day·piece)	1,020 yen/(day·piece)	100,000 yen/(day·piece)	100,000 yen/hour

From the base case, if we only modify the process time of process 1 to process 15 to be 33 minutes to increase the utilization level to 100%, the result is as follows.

Table 3-8: Case 3 cost structure breakdown

Raw Material Inv Cost	WIP Cost	Buffer Cost	OT Cost	Penalty Cost	Total Cost
33,801	22,660	571,567	382,274	58,930	1,069,231

Table 3-9: Case 3 overview of segmentation and parameters

Segmentation	Before process 16
Segment 1	
σ_{in}	15.28
α	0.19
β	0.15
Segment 2	
σ_{in}	22.93
α	0.26
β	0.25
z	1.7

The difference in the utilization level breaks the line into 2 segments. For segment 1, 100% utilization means there has to be greater smoothing to reduce the overtime cost. To achieve this, segment 2 with 60% utilization has high smoothing, which brings down σ_{in} of segment 2, i.e. σ_{out} of segment 1, to 22.93. Thus, segment 1 experiences low demand variance and in addition, it also has low values for α and β, thus the production is made rather flat to reduce overtime cost within the segment.

3.2.3 Case 4 - Utilization Level Variation

Table 3-10: Case 4 description

Utilization to produce μ	Inventory cost	FGI inventory cost	Penalty cost	OT cost
P1-P15 60% P16-P30 100%	200-1000 yen/(day·piece)	1,020 yen/(day·piece)	100,000 yen/(day·piece)	100,000 yen/hour

From the base case, if we only modify the process time of process 16 to process 30 to be 33 minutes, the second half of the line will have 100% utilization level. The result is as follows.

Table 3-11: Case 4 cost structure breakdown

Raw Material Inv Cost	WIP Cost	Buffer Cost	OT Cost	Penalty Cost	Total Cost
33,801	22,080	418,536	624,664	67,728	1,166,809

Table 3-12: Case 4 overview of segmentation and parameters

Segmentation	No segmentation
σ_{in}	15.28
α	0.14
β	0.15
z	1.7

There is no segmentation in this case. Since processes from P16 to P30 have 100% utilization, the production line will easily run into overtime. Thus, we have low α and β value to smooth out the production line and control the overtime cost.

It is interesting to observe the difference in the cost structure of Case 3 versus that of Case 4.

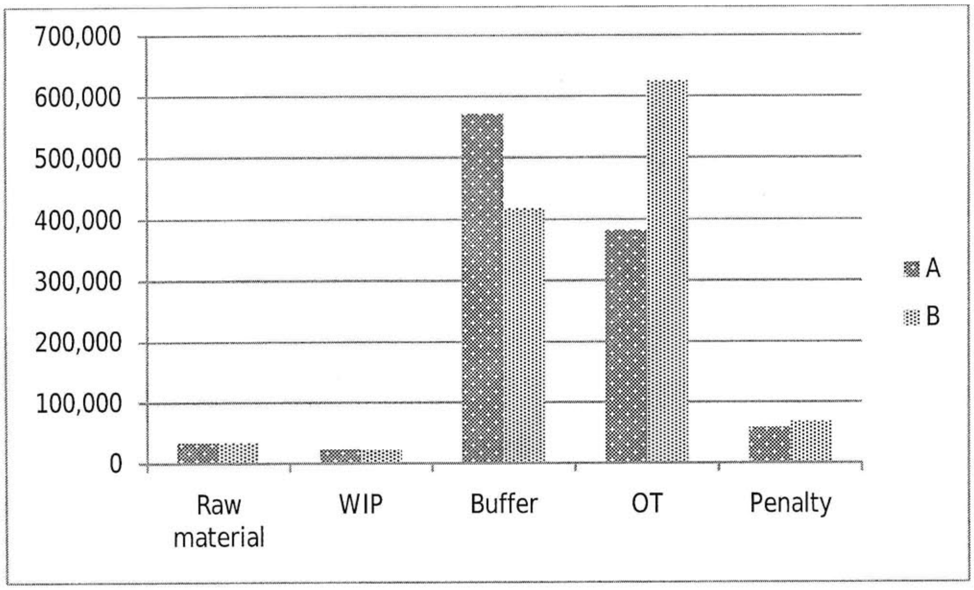

Figure 3-3: Individual cost comparison of Case 3 vs. Case 4

A: P1-P15 100% P16-P30 60%

B: P1-P15 60% P16-P30 100%

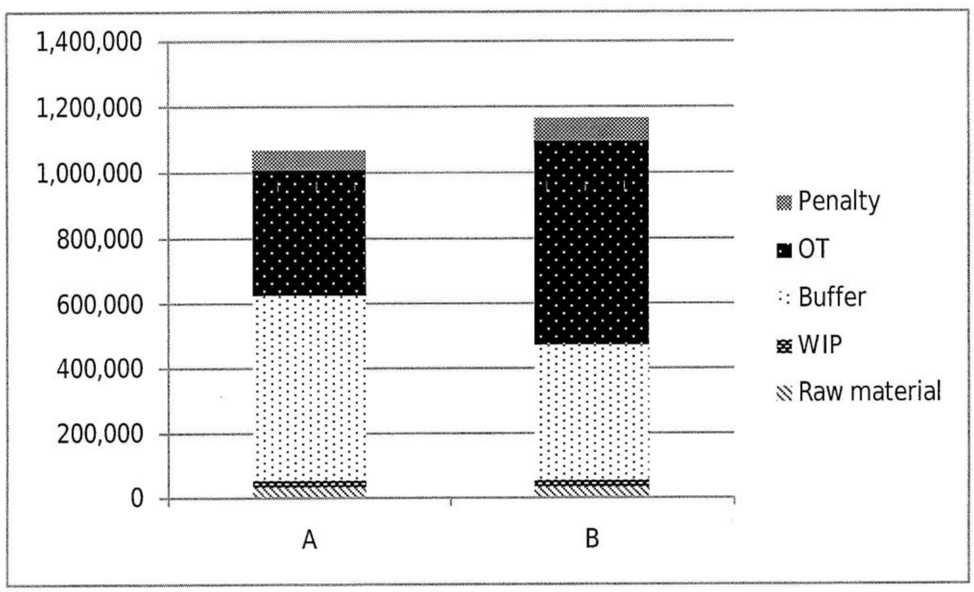

Figure 3-4: Overall cost comparison of Case 3 vs. Case 4

A: P1-P15 100% P16-P30 60%

B: P1-P15 60% P16-P30 100%

Compared with Case 3, which has a decoupling buffer before process 16, Case 4 has no decoupling buffer and, therefore, much less inventory but more overtime cost. The overall cost for Case 4 is higher because processes with 100% utilization level are nearer to FGI, where the inventory holding is more expensive and it is more difficult to do production smoothing.

3.2.4 Case 5 - Utilization Level Variation

Table 3-13: Case 5 description

Utilization to produce μ	Inventory cost	FGI inventory cost	Penalty cost	OT cost
P1-P10 100% P11-P20 80% P21-P30 60%	200-1000 yen/(day·piece)	1,020 yen/(day·piece)	100,000 yen/(day·piece)	100,000 yen/hour

Building upon the base case, if we modify the process time of process 1 to process 10 to be 33 minutes and that of process 11 to 20 to be 26.7 minutes, the utilization level is modified as shown in the shaded box. The result is as follows.

Table 3-14: Case 5 cost structure breakdown

Raw Material Inv Cost	WIP Cost	Buffer Cost	OT Cost	Penalty Cost	Total Cost
33,801	24,030	555,524	260,325	66,353	940,032

Table 3-15: Case 5 overview of segmentation and parameters

Segmentation	Before process 11
Segment 1	
σ_{in}	15.28
α	0.19
β	0.15
Segment 2	
σ_{in}	15.28
α	0.14
β	0.15
z	1.7

Process 1 to 10 are grouped as a segment and differentiated from other processes because the utilization level of 100% is very high. Due to the smoothing effect of segment 2, segment 1 has flat production rate and although it runs into overtime easily, the overtime cost will be scaled down.

3.2.5 Case 6 - Utilization Level Variation

Table 3-16: Case 6 description

Utilization to produce μ	Inventory cost	FGI inventory cost	Penalty cost	OT cost
P1-P10 60% P11-P20 80% P21-P30 100%	200-1000 yen/(day·piece)	1,020 yen/(day·piece)	100,000 yen/(day·piece)	100,000 yen/hour

Building upon the base case, if we modify the process time of process 11 to process 20 to be 26.7 minutes and that of process 21 to 30 to be 33 minutes, the result is as follows.

Table 3-17: Case 6 cost structure breakdown

Raw Material Inv Cost	WIP Cost	Buffer Cost	OT Cost	Penalty Cost	Total Cost
33,801	22,092	418,572	435,959	67,737	978,162

Table 3-18: Case 6 overview of segmentation and parameters

Segmentation	No Segmentation
Segment 1	
σ_{in}	15.28
α	0.14
β	0.15
z	1.7

Similar to Case 4, there is no segmentation and the entire production line is smoothed out at a fairly high level because of the utilization level bottleneck at the end of the line.

3.3 Variation of Overtime Cost and Penalty Cost

3.3.1 Case 7 – Overtime Cost Variation

Table 3-19: Case 7 description

Utilization to produce μ	Inventory cost	FGI inventory cost	Penalty cost	OT cost
P1-P10 100% P11-P20 80% P21-P30 60%	200-1000 yen/(day·piece)	1,020 yen/(day·piece)	100,000 yen/(day·piece)	500,000 yen/hour

Building upon Case 5, if we increase the overtime cost to 500,000 yen/hour, the result is as follows.

Table 3-20: Case 7 cost structure breakdown

Raw Material Inv Cost	WIP Cost	Buffer Cost	OT Cost	Penalty Cost	Total Cost
31,247	58,705	590,598	1,230,625	66,329	1,977,504

Table 3-21: Case 7 overview of segmentation and parameters

Segmentation	Before process 11, 12
Segment 1	
σ_{in}	7.64
α	1
β	1
Segment 2	
σ_{in}	7.64
α	0.05
β	1
Segment 3	
σ_{in}	15.28
α	0.14
β	0.15
z	1.7

If we increase the overtime cost to 500,000 yen/hour, the production line needs to be smoothed out even more compared to Case 5 and the demand variance needs to go lower. In order to achieve this, there is an additional decoupling buffer before process 12, besides the decoupling buffer before process 11 which is already in Case 5. Having this short segment, consisting of only process 11, makes it possible to lower σ from 15.28 to 7.64 and hence, processes 1 to 10 can have a flatter production rate.

It might sound reasonable to have process 11 to 30 grouped as one segment and this segment can reduce σ from 76.4 to 7.64. However, in order for this to happen, β has to go lower than 0.15, which leads to enormous growth in the end decoupling buffer inventory level as explained in Section 2.3. Thus, the last segment can only lower σ from 76.4 to 15.28 and we need to have a short segment to double smooth the production.

3.3.2 Case 8 – Overtime Cost Variation

Table 3-22: Case 8 description

Utilization to produce μ	Inventory cost	FGI inventory cost	Penalty cost	OT cost
P1-P10 100% P11-P20 80% P21-P30 60%	200-1000 yen/(day·piece)	1,020 yen/(day·piece)	100,000 yen/(day·piece)	1,000,000 yen/hour

Building upon Case 5, if we increase the overtime cost to 1,000,000 yen/hour, the result is as follows.

Table 3-23: Case 8 cost structure breakdown

Raw Material Inv Cost	WIP Cost	Buffer Cost	OT Cost	Penalty Cost	Total Cost
31,247	64,789	694,085	2,282,856	58,831	3,131,808

Table 3-24: Case 8 overview of segmentation and parameters

Segmentation	Before process 11, 21
Segment 1	
σ_{in}	7.64
α	1
β	1
Segment 2	
σ_{in}	7.64
α	0.04
β	0.15
Segment 3	
σ_{in}	22.93
α	0.26
β	0.25
z	1.7

If we increase the overtime cost to 1,000,000 yen/hour, the difference in utilization level has more impact and the line breaks into three segments at the exact places where the utilization level changes. Segment 2 and 3 largely smooth out the demand signal and hence segment 1 has very low variance in the demand even though α and β are both 1. Thus the entire production is flat and the tactics try to scale overtime cost down as much as possible.

3.3.3 Case 9 – Penalty Cost Variation

Table 3-25: Case 9 description

Utilization to produce μ	Inventory cost	FGI inventory cost	Penalty cost	OT cost
P1-P10 60% P11-P20 80% P21-P30 100%	200-1000 yen/(day·piece)	1,020 yen/(day·piece)	1,000,000 yen/(day·piece)	100,000 yen/hour

Building upon Case 6, if we only increase the penalty cost to 1,000,000 yen/(day·piece), the result is as follows.

Table 3-26: Case 9 cost structure breakdown

Raw Material Inv Cost	WIP Cost	Buffer Cost	OT Cost	Penalty Cost	Total Cost
33,801	22,092	570,466	435,959	39,266	1,101,584

Table 3-27: Case 9 overview of segmentation and parameters

Segmentation	No Segmentation
Segment 1	
σ_{in}	15.28
α	0.14
β	0.15
z	2.7

When penalty cost increases we will keep more FGI and hence z is increased to 2.7 compared to 1.7 in Case 6.

3.4 Variation of the Inventory Cost

3.4.1 Case 10 – Inventory Cost Variation

Table 3-28: Case 10 description

Utilization to produce μ	Inventory cost	FGI inventory cost	Penalty cost	OT cost
P1-P15 100% P16-P30 60%	P1-P11 200-300 P12-P30 820-1000 yen/(day·piece)	1,020 yen/(day·piece)	100,000 yen/(day·piece)	100,000 yen/hour

From Case 3, if we modify the inventory cost so that there is a leap from process 11 to process 12, the result is as follows.

Table 3-29: Case 10 cost structure breakdown

Raw Material Inv Cost	WIP Cost	Buffer Cost	OT Cost	Penalty Cost	Total Cost
33,801	25,851	507,231	450,086	67,456	1,084,425

Table 3-30: Case 10 overview of segmentation and parameters

Segmentation	Before process 11
Segment 1	
σ_{in}	15.28
α	0.19
β	0.15
Segment 2	
σ_{in}	15.28
α	0.14
β	0.15
z	1.7

The breaking point for Case 3 is before process 15. The leap in inventory cost shifts the breaking point to before process 11. In this case, the leap in inventory cost is more significant than the difference in utilization level. We note that the decoupling buffer should not be placed at the process where the inventory cost increases much, because the inventory cost at this process is high. We select the process upstream instead to place a decoupling buffer. In this case, we place the decoupling buffer before process 11 instead of before process 12.

3.4.2 Case 11 - Inventory Cost Variation

Table 3-31: Case 11 description

Utilization to produce μ	Inventory cost	FGI inventory cost	Penalty cost	OT cost
P1-P10 100% P11-P20 80% P21-P30 60%	P1-P20 200-390 P21- P30 910-1000 yen/(day·piece)	1,020 yen/(day·piece)	100,000 yen/(day·piece)	100,000 yen/hour

Building upon Case 5, we modify the inventory cost so that there is a leap from process 20 to process 21 and the result is as follows.

Table 3-32: Case 11 cost structure breakdown

Raw Material Inv Cost	WIP Cost	Buffer Cost	OT Cost	Penalty Cost	Total Cost
33,801	19,683	510,581	265,466	60,811	890,341

Table 3-33: Case 11 overview of segmentation and parameters

Segmentation	Before process 20
Segment 1	
σ_{in}	15.28
α	0.19
β	0.15
Segment 2	
σ_{in}	22.93
α	0.26
β	0.25
z	1.7

Case 5 initially has the breaking point before process 11. If we vary the inventory cost such that it increases significantly at process 21, the breaking point shifts to before process 20.

3.5 Variation of Standard Deviation

3.5.1 Case 12 – Standard Deviation Variation

Table 3-34: Case 12 description

Utilization to produce μ	Inventory cost	FGI inventory cost	Penalty cost	OT cost	σ_{out}
60%	200-1,000 yen/(day·piece)	1,020 yen/(day·piece)	100,000 yen/(day·piece)	100,000 yen/hour	150

Table 3-35: Case 12 cost structure breakdown

Raw Material Inv Cost	WIP Cost	Buffer Cost	OT Cost	Penalty Cost	Total Cost
36,526	24,403	631,424	8,764	122,340	823,458

Table 3-36: Case 12 overview of segmentation and parameters

Segmentation	No segmentation
σ_{in}	30.17
α	0.16
β	0.15
z	1.7

Compared to Case 1, α goes down from 0.26 to 0.16 and β goes down from 0.25 to 0.15, which indicates more smoothing. In order to reduce overtime cost, the production line is largely smoothed out and less responsive to demand variance. We note that FGI safety stock increases from 225 to 464 to accommodate larger demand variance.

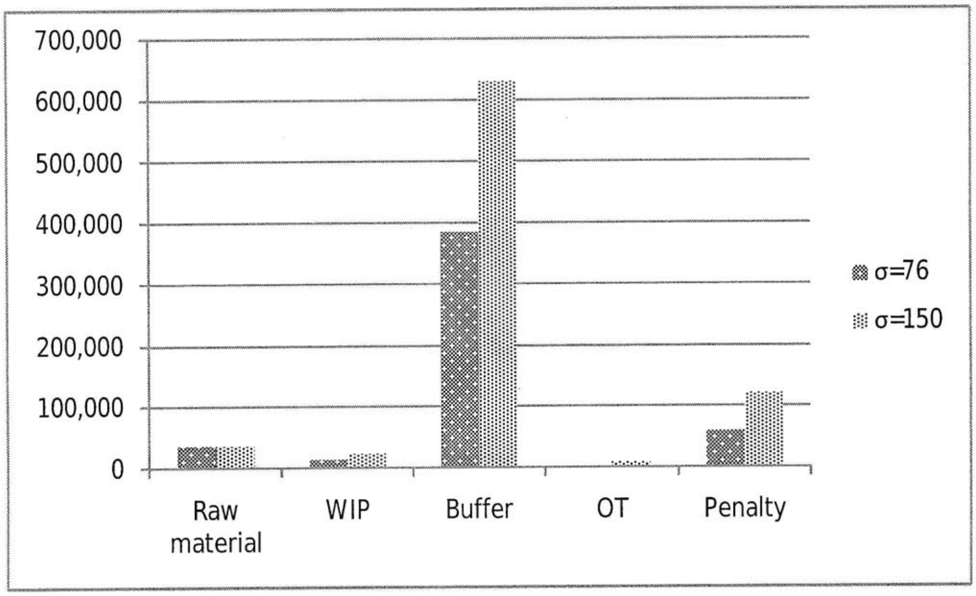

Figure 3-5: Individual cost comparison of Case 1 vs. Case 12

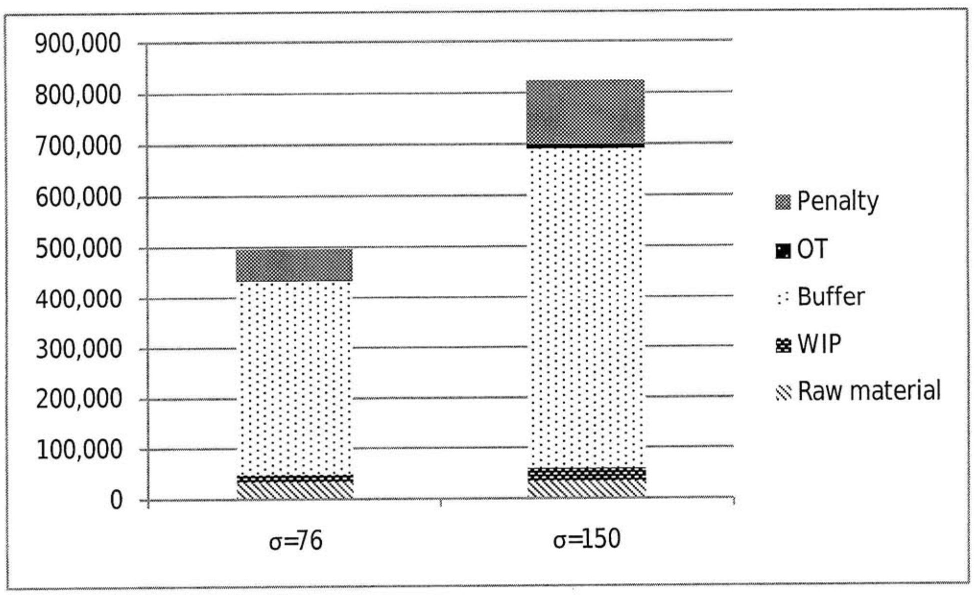

Figure 3-6: Overall cost comparison of Case 1 vs. Case 12

The cost breakdown shows some increase in the penalty cost, and the increment is primarily in the buffer cost, which is due to higher FGI safety stock target.

3.5.2 Case 13 – Standard Deviation Variation

Table 3-37: Case 13 description

Utilization to produce μ	Inventory cost	FGI inventory cost	Penalty cost	OT cost	σ_{out}
P1-P15 100% P16-P30 60%	200-1,000 yen/(day·piece)	1,020 yen/(day·piece)	100,000 yen/(day·piece)	100,000 yen/hour	150

Table 3-38: Case 13 cost structure breakdown

Raw Material Inv Cost	WIP Cost	Buffer Cost	OT Cost	Penalty Cost	Total Cost
32,148	61,144	882,239	674,895	121,774	1,772,200

Table 3-39: Case 13 overview of segmentation and parameters

Segmentation	Before process 16
Segment 1	
σ_{in}	15.08
α	0.16
β	0.15
Segment 2	
σ_{in}	30.17
α	0.16
β	0.15
z	1.7

Compared to Case 3, segment 1 decoupling buffer safety stock and segment 2 decoupling buffer safety stock, i.e. FGI safety stock, experience significant increment, which can be verified by the detailed results provided in Table C-13 of Appendix C. α in segment 1 goes down from 0.19 to 0.16 while both α and β of segment 2 go down. The smoothing effect is a lot stronger in this case compared to Case 3. This is similar to the comparison between Case 1 and Case 12.

3.5.3 Case 14 – Standard Deviation Variation

Table 3-40: Case 14 description

Utilization to produce μ	Inventory cost	FGI inventory cost	Penalty cost	OT cost	σ_{out}
P1-P10 100% P11-P20 80% P21-P30 60%	200-1000 yen/(day·piece)	1,020 yen/(day·piece)	100,000 yen/(day·piece)	100,000 yen/hour	150

Table 3-41: Case 14 cost structure breakdown

Raw Material Inv Cost	WIP Cost	Buffer Cost	OT Cost	Penalty Cost	Total Cost
32,148	66,830	938,392	477,492	121,585	1,636,448

Table 3-42: Case 14 overview of segmentation and parameters

Segmentation	Before process 21
Segment 1	
σ_{in}	15.08
α	0.06
β	0.15
Segment 2	
σ_{in}	30.17
α	0.16
β	0.15
z	1.7

In Case 5, because processes of utilization level 80% and 60% do not experience high variance in the demand, they still have sufficient capacity and are grouped as one segment. In this case, as we increase the demand variance, processes of 80% utilization level are not able to handle the high variance and hence are grouped with the first 10 processes of 100% utilization level. This shifts the breaking point from before process 11 to before process 21.

To summarize the above, the test case results are mostly consistent with our expectations and the tactics work well in terms of determining the best locations of decoupling buffer and the optimal policies.

Chapter 4

Conclusion

In this project we have explored the tactics of a serial flow line that entails a sequence of process stages. We aim to improve the operation and planning by looking into inventory policy, raw material ordering policy, production planning and scheduling policy. Our goal is to develop a robust model that determines the optimal policies to minimize inventory cost, penalty cost and overtime cost, in the face of demand uncertainty, raw material arrival uncertainty and in-process failure.

To achieve this, we model the demand process, release process, inventory process and production process as MMFE processes and investigate the dynamics and correlation of them. The concept of decoupling buffer and safety stock is crucial because it helps smooth out the production and prepare for unexpected spike in demand.

At the top level, we use dynamic programming to locate breaking points to place decoupling buffer. In between every possible pair of decoupling buffer locations, we formulate sub optimization problems and equations to search for the ideal parameters that minimize the overall cost within the segment. By constructing the cost table, we eventually obtain the optimal configuration for the entire serial flow line.

We certainly acknowledge the limitations of this theoretic model, one of which being that the production rate is set based on the mean demand and the difference between decoupling buffer inventory level and target safety stock level. This production rate is not always realizable in real life due to constraints on workers, WIP and other resources. Another limitation is that we assume the prescribed production in the previous period will enter the end decoupling buffer in the current period. This assumption is based on the ideal case that there is enough WIP and production capacity within the segment so that the same number of work pieces as the released amount in the previous period will arrive at the end of the segment in current period. Again this ideal case is not always true in real life situations. Besides looking into these assumptions, we may also work on finding a better approach to estimate WIP and gaining a better understanding of the relationship between WIP and two parameters, α and β.

Overall speaking, this project is meaningful as it links several key concepts successfully and forms a systematic way of improving the operation and planning of a serial flow line.

References

[1] Stephen C. Graves, Harlan C. Meal, Sriram Dasu, Yuping Qiu. *Two-Stage Production Planning in a Dynamic Environment*. Lecture Notes in Economics and Mathematical Systems, *Multi-Stage Production Planning and Inventory Control*, edited by S. Axsater, Ch. Schneeweiss, and E. Silver, Springer-Verlag, Berlin, Vol. 266, 9-43, 1986.

[2] Heath, D. C., and P. L. Jackson. *Modeling the Evolution of Demand Forecasts with Applications to Safety Stock Analysis in Production/Distribution Systems*. IIE Trans. 26(3):17–30, 1994.

[3] Stephen C. Graves. *Safety Stocks in Manufacturing Systems*. Journal of Manufacturing and Operations Management, Vol. 1, No. 1, pp. 67-101, 1988.

[4] Stephen C. Graves, David B. Kletter, William B. Hetzel. *A Dynamic Model for Requirements Planning with Application to Supply Chain Optimization*. Operations Research, Vol. 46, No. 3, Supplement: Focus Issue on Manufacturing, 1998.

[5] Stephen C. Graves. *A Tactical Planning Model for a Job Shop*. Operations Research, Vol. 34, No. 4, pp. 522-533, 1986.

[6] Lawrence M. Wein. *Scheduling Semiconductor Wafer Fabrication*. IEEE Transactions on Semiconductor Manufacturing, VOL. I , NO. 3, 1988.

[7] Xiangwen Lu, Jing-Sheng Song, Amelia Regan. *Inventory Planning with Forecast Updates: Approximate Solutions and Cost Error Bounds*. Operations Research, Vol. 54, No. 6, pp. 1079–1097, 2006.

Appendix A

Optimization Problem to Determine z

The variance of the end-item demand is known to be $tr(\Sigma)$, where Σ is the covariance matrix of the demand process revision vector $\overline{\Delta f_t}$. In 2.3, we have shown that

$$E[x_t(t)] = X$$

$$Var(x_t(t)) = tr(M_2 \Sigma M_2^T) \cong \left(\frac{1}{2\beta - \beta^2}\right)(\beta \times tr(\Sigma) + (1-\beta) \times sum(\Sigma))$$

We also suggested that $X = z\sqrt{Var(x_t(t))} = z\sigma_x$. Now we have to find out the optimal value for z so that it minimizes the FGI cost and penalty cost.

We define the following terms.

h: FGI holding cost per work piece per time period;

Q: Number of work pieces per order;

τ: Expected time for the segment to produce an order of size Q;

p_j: Probability that there is a j-th order delayed, in other words, the probability that the number of orders delayed is j or more;

π: Penalty cost per order per time period;

The optimization problem to be solved is

$$Min\left(hX + \pi Q\tau \sum_j p_j \times \frac{2j-1}{2}\right)$$

where $X = z\sigma_x$, $p_j = \Pr[x < -(j-1)\times Q] = 1 - \Phi(z_j) = 1 - \Phi\left(\frac{X + (j-1)\times Q}{\sigma_x}\right)$.

Thus z is in both X and p_j of the objective function. There is no analytical solution for this problem and we need to do a line search over a range of values for z.

Appendix B

Calculation of Overtime Cost

To determine the optimal value of β for each choice of $(i, j, \sigma_{in}, \sigma_{out})$, we need to do an optimization over possible values of β.

$E[p(t)]$ denotes the average number of work pieces to be produced per time unit, which can be obtained from the average demand. If ω is the average processing time per work piece at a particular process stage, $\omega E[p(t)]$ is the expected production in time units for this process step. The variance of production rate $Var(p(t))$ can be obtained easily since it is a function of β as shown in 2.4.

$$Var(p(t)) \cong \frac{\beta}{2-\beta}\left(\beta \times tr(\Sigma) + (1-\beta) \times sum(\Sigma)\right)$$

Thus we can model the actual desired production (in time units) as a random variable g with mean value as $\omega E[p(t)]$ and variance as $\omega^2 Var[p(t)]$.

We can easily calculate the nominal capacity χ at each process step based on the number of parallel machines at the process step and the number of shifts. If the machines are shared by other processes, the capacity has to be divided proportionally among the sharing process steps.

Thus, the overtime per period is the difference between the desired production rate g and the nominal capacity χ, denoted by $E[(g-\chi)^+]$.

Appendix C

Table C- 1: Case 1 detailed results

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
1	1	30	0.26	0.25	22.93	76.42

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	1,076	496,354	24	378	225	14,612	385,210	1.7	59,228

Raw Material Type	Raw Material Inv	Raw Material SS	Raw Material Inv Cost	Safety Factor
1	181	31	36,229	2.4

Table C- 2: Case 2 detailed results

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
1	1	29	0.19	0.15	15.28	15.28

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	704,769	1,060,738	39	299	149	22,831	299,338	1.6	0

Raw Material Type	Raw Material Inv	Raw Material SS	Raw Material Inv Cost	Safety Factor
1	169	19	33,801	2.4

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
2	30	30	0.14	0.15	15.28	76.42

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	41,216	536,257	12	408	255	11,935	416,020	1.7	67,086

Table C- 3: Case 3 detailed results

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
1	1	15	0.19	0.15	15.28	22.93

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	381,767	613,393	26	306	156	10,303	187,523	1.6	0

Raw Material Type	Raw Material Inv	Raw Material SS	Raw Material Inv Cost	Safety Factor
1	169	19	33,801	2.4

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
2	16	30	0.26	0.25	22.93	76.42

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	507	455,837	15	377	224	12,357	384,043	1.7	58,930

Table C- 4: Case 4 detailed results

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
1	1	30	0.14	0.15	15.28	76.42

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	624,664	1,166,809	34	410	258	22,080	418536	1.7	67,728

Raw Material Type	Raw Material Inv	Raw Material SS	Raw Material Inv Cost	Safety Factor
1	169	19	33,801	2.4

Table C- 5: Case 5 detailed results

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
1	1	10	0.19	0.15	15.28	15.28

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	242,772	424,994	19	299	149	6,043	142,379	1.6	0

Raw Material Type	Raw Material Inv	Raw Material SS	Raw Material Inv Cost	Safety Factor
1	169	19	33,801	2.4

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
2	11	30	0.14	0.15	15.28	76.42

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	17,554	515,038	25	405	252	17,987	413,144	1.7	66,353

Table C- 6: Case 6 detailed results

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
1	1	30	0.14	0.15	15.28	76.42

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	435,959	978,162	35	410	258	22,092	418,572	1.7	67,737

Raw Material Type	Raw Material Inv	Raw Material SS	Raw Material Inv Cost	Safety Factor
1	169	19	33,801	2.4

Table C- 7: Case 7 detailed results

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
1	1	10	1	1	7.64	7.64

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	1,135,057	1,250,832	10	171	21	3,104	81,424	1.6	0

Raw Material Type	Raw Material Inv	Raw Material SS	Raw Material Inv Cost	Safety Factor
1	156	6	31,247	2.4

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
2	11	11	0.05	1	7.64	15.28

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	16,718	150,676	80	191	41	37,836	96,122	1.6	0

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
3	12	30	0.14	0.15	15.28	76.42

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	78,850	575,996	24	405	252	17,765	413,052	1.7	66,329

Table C- 8: Case 8 detailed results

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
1	1	10	1	1	7.64	7.64

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	2,270,115	2,385,889	10	171	21	3,104	81,424	1.6	0

Raw Material Type	Raw Material Inv	Raw Material SS	Raw Material Inv Cost	Safety Factor
1	156	6	31,247	2.4

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
2	11	20	0.04	0.15	7.64	22.93

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	9,430	289,349	85	305	155	50,911	229,008	1.6	0

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
3	21	30	0.26	0.25	22.93	76.42

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	3,312	456,570	12	376	224	10,774	383,653	1.7	58,831

Table C- 9: Case 9 detailed results

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
1	1	30	0.14	0.15	15.28	76.42

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	435,959	1,101,584	35	559	409	22,092	570,466	2.7	39,266

Raw Material Type	Raw Material Inv	Raw Material SS	Raw Material Inv Cost	Safety Factor
1	169	19	33,801	2.4

Table C- 10: Case 10 detailed results

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
1	1	10	0.19	0.15	15.28	15.28

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	242,772	370,901	19	299	149	4,568	89,761	1.6	0

Raw Material Type	Raw Material Inv	Raw Material SS	Raw Material Inv Cost	Safety Factor
1	169	19	33,801	2.4

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
2	11	30	0.14	0.15	15.28	76.42

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	207,314	713,524	25	409	257	21,283	417,470	1.7	67,456

Table C- 11: Case 11 detailed results

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
1	1	19	0.19	0.15	15.28	22.93

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	255,462	416,464	28	306	156	8,039	119,163	1.6	0

Raw Material Type	Raw Material Inv	Raw Material SS	Raw Material Inv Cost	Safety Factor
1	169	19	33,801	2.4

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
2	20	30	0.26	0.25	22.93	76.42

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	10,005	473,877	13	384	231	11,643	391,418	1.7	60,811

Table C- 12: Case 12 detailed results

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
1	1	30	0.16	0.15	30.17	150.85

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	8,764	823,458	41	619	464	24,403	631,424	1.7	122,340

Raw Material Type	Raw Material Inv	Raw Material SS	Raw Material Inv Cost	Safety Factor
1	183	33	36,526	2.4

Table C- 13: Case 13 detailed results

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
1	1	15	0.06	0.15	15.08	30.17

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	670,682	991,485	91	412	263	35,628	253,026	1.6	0

Raw Material Type	Raw Material Inv	Raw Material SS	Raw Material Inv Cost	Safety Factor
1	161	11	32,148	2.4

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
2	16	30	0.16	0.15	30.17	150.85

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	4,213	780,715	32	617	462	25,515	629,213	1.7	121,774

Table C- 14: Case 14 detailed results

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
1	1	20	0.06	0.15	15.08	30.17

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	474,721	858,564	94	412	263	41,776	309,919	1.6	0

Raw Material Type	Raw Material Inv	Raw Material SS	Raw Material Inv Cost	Safety Factor
1	161	11	32,148	2.4

Segment	StartProc	EndProc	Alpha	Beta	Sigma_in	Sigma_out
2	21	30	0.16	0.15	30.17	150.85

Product Type	OT Cost	Total Cost	Target WIP	Buffer Inv	Buffer SS	WIP Cost	Buffer Cost	Safety Factor	Penalty Cost
1	2,772	777,884	29	616	461	25,054	628,473	1.7	121,585