Impact of Carbon Emission Regulatory Policies on the Electricity Market: A Simulation Study

by

Sandeep Tiwari

M.Tech., Indian Institute of Technology Madras (2009)

Submitted to the School of Engineering
in partial fulfillment of the requirements for the degree of

Master of Science in Computation for Design and Optimization

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 2010

Author..
Sandeep Tiwari
School of Engineering
July 30, 2010

Certified by..
David Simchi-Levi
Professor of Engineering Systems Division
Thesis Supervisor

Accepted by...
Karen Willcox
Associate Professor of Aeronautics and Astronautics
Codirector, Computation for Design and Optimization Program

Impact of Carbon Emission Regulatory Policies on the Electricity Market: A Simulation Study

by

Sandeep Tiwari

Submitted to the School of Engineering
On July 30, 2010 in partial fulfillment of the
requirements for the degree of
Master of Science in Computation for Design and Optimization

Abstract

With ever rising concerns regarding global warming and other dangerous effects of CO_2, there had been efforts to reduce CO_2 emissions all around the world by adopting more efficient technologies and alternate green or carbon neutral fuels. However, these technologies require large investments and hence to make them economically viable there should be suitable incentives from the government in form of emission regulatory policies such as carbon taxation and carbon cap-and-trade policy.

In this research, a simulation study was carried out to analyze the impact of different carbon emission regulatory policies including cap-and-trade policy and carbon taxation policy on the utilities of various stakeholders of the electricity market. An agent based simulation approach was used to model the market where each market stakeholder was represented as an autonomous agent. We use the simulation model to compare the effectiveness of cap-and-trade policy and taxation policy in achieving emission reduction targets. We observe significant windfall profit for electricity producers under the cap-and-trade policy. Therefore for the same emission level the cost to consumers is higher under cap-and-trade policy as compared to taxation policy. Our results suggest that cap-and-trade policy might be ineffective in emission reduction when the market is not fully efficient. Moreover the simplicity of Taxation model gives government a better control on emissions.

Based on our study we recommend that the present model be extended to more efficient cap and trade mechanisms by incorporating multistage periods, auctioning of carbon emission permits and carbon emission permits banking.

Thesis Supervisor: David Simchi-Levi
Title: Professor of Engineering Systems Division

Acknowledgements

With the feeling of profound gratitude, I acknowledge the invaluable guidance and encouragement provided to me by my thesis supervisor Professor David Simchi-Levi throughout the course of the project work. He filled me with confidence and courage whenever I was let down by various problems during the course of work. Without his arduous task of reviewing the entire work, it would not have been possible to bring out the report in its present form. I consider myself to be fortunate to have a chance to work with him. His suggestions and advices, both technical and personal, have helped me in various ways. I would also like to thank Cristian Figueroa, a PhD candidate in O.R.C. for his advice and suggestions during initial stages of modeling.

I am grateful to C.D.O. co-directors, Professor Karen Willcox and Professor Tony Patera for their advice on academic affairs. I am also thankful to Singapore-MIT Alliance for providing me with financial support throughout my study at M.I.T.

I would like to thank C.D.O. administration for taking care of all formal administrative procedures and providing me with an excellent workspace in C.D.O. office. I am especially thankful to Mrs. Laura Koller for her help and support with administrative and academic affairs right from the time I have been granted admission to M.I.T. to the completion of my thesis.

I also thank all my friends from C.D.O. program, Tang Hall and my flatmates for the excellent company they gave during my stay at M.I.T. They made my stay at M.I.T. fun and memorable.

And finally I would like to thank my family for their constant support and encouragement throughout the course of project work.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

This chapter presents the detailed background of the research area. It also discusses the previous research efforts and published information in this area. And finally it presents the objective of the project and outline for the rest of the thesis.

1.1 Background

1.1.1 Menace of Rising Levels of CO_2 in Atmosphere

Carbon dioxide in earth's atmosphere is considered a trace gas currently occurring at an average concentration of about 390 parts per million by volume or 591 parts per million by mass [1]. Carbon dioxide is essential for human existence due to its key role in photosynthesis in plants (the key player in food chain) and absorbing IR radiations from the sun which helps in maintaining earth's temperature at desired level. However, human activities such as the combustion of fossil fuels and deforestation have caused the atmospheric concentration of carbon dioxide to increase by about 35% since the beginning of the age of industrialization [2].

Such rise in CO_2 concentration creates several dangerous effects on environment and health risks for humans. Prolonged exposure higher CO_2 concentrations can affect respiratory function and cause excitation followed by depression of the central nervous system. High concentrations of CO_2 can displace oxygen in the air, resulting in lower oxygen concentrations for breathing. Therefore, effects of oxygen deficiency may be combined with effects of CO_2 toxicity [3]. The harmful environmental effect of rising CO_2 concentration is mainly due to its contribution to Global Warming.

1.1.2 Global Warming

Global warming is one of the most serious challenges facing us today. Global warming is the phenomenon which involves the rise in average temperature of the earth. Climate model projections summarized in the latest IPCC report indicate that the global surface temperature is likely to rise a further 1.1 to 6.4 °C (2.0 to 11.5 °F) during the 21st century [4]. The perilous effects of Global Warming can never be over-emphasized. An increase in global temperature might cause sea levels to rise and might change the amount and pattern of precipitation, probably including expansion of subtropical deserts [5]. Warming is expected to be strongest in the Arctic and would be associated with continuing retreat of glaciers, permafrost and sea ice. Other likely effects include changes in the frequency and intensity of extreme weather events, species extinctions, and changes in agricultural yields.

Global Warming happens when greenhouse gases (carbon dioxide, water vapor, nitrous oxide, and methane) trap heat and light from the sun in the earth's atmosphere, which increases the temperature on earth. Although the Global Warming Potential (GWP) of CO_2 is less than many of the other greenhouse gases, the very long life span of its molecules (CO_2 molecules at average has lifespan of 100 yrs) and very fast rate of increase in its concentration in atmosphere makes it the most dangerous greenhouse gas and rise in global warming is suspected to be the result of rise in the CO_2 concentration in atmosphere.

1.1.3 Power Sector- The Main Contributor to CO_2 Emissions

The power industry is economically the most important sector. Almost every aspect of industrial productivity and daily life is dependent on electricity. The global demand for electricity is rising with a significant growth due to advancements in technologies and automation and the rise in demand is tremendous in the developing nations like India and China due to their enormous G.D.P growth rate. The energy demand in both countries is increasing at the rate of approximately 5% per year. World electricity demand is projected to grow at an annual rate of 2.5% to 2030 [6]. Interruption in energy supply can be highly detrimental as observed in the case of California Energy Crisis of 2001-2002 and the August 2003 Blackouts in eastern Canada and in the U.S. Northeast and Midwest [7].

The Power Sector is the major contributor of global CO_2 emissions. Power sector emissions increased by 31% between 1990 and 2002, making it the fastest growing sector over the period. Developing countries experienced the greatest growth in emissions over this period [8].

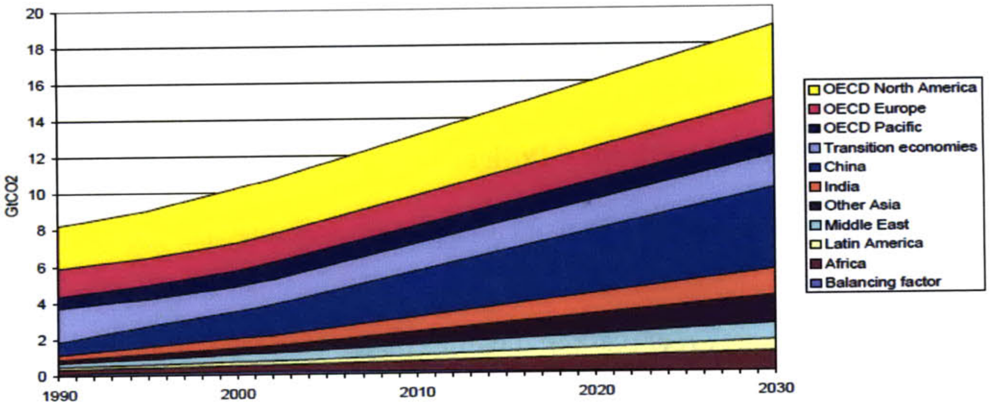

Figure 1-1 Power sector CO2 emissions by country 1990-2030 [8]

The Figure 1-1 shows the historical trend and the future projections of the GHG emissions due to power sector from year 1990-2030. Nearly 37% of GHG emissions in Unites States originate from power sector [9].

The rise in demand for electricity is inevitable and the balance between the growing electricity demand and the affordable and reliable supply while restricting CO_2 emissions is absolutely essential. One of the key solutions to this problem is adopting clean technologies and carbon capture and storage measures. The CO_2 emission of the power plant is a strong function of the fuel used and the overall efficiency of the plants. Therefore considerable reduction in CO_2 emissions can be achieved by adopting cleaner options for fuel and by advancement in generation technology in Power Plants. The profile of the power industry has changed in recent years, particularly in European Union and Japan where the carbon emission regulatory policies have been implemented successfully. The average emissions factor per utility in the EU is 353 kgCO2/MWh, whereas in the United States it is 720 kgCO2/MWh which is more than twice of European average [10]. The differences between these countries' carbon intensities are mainly due to the differences in production efficiencies and fuel mix used. And the reason why most of the power generating companies in EU opted for clean technology is the economic incentive offered by E.T.S. which offsets the extra cost incurred in shifting to cleaner but more expensive production technology and fuels and building Carbon Capture and Storage (C.C.S.) system.

Therefore, carbon emission regulation on power sector is the most promising way to control CO_2 emissions from power sector which in turn has a potential to control global warming at large.

1.1.4 Carbon Emission Regulatory Policies

Over the last decade the awareness about the global warming has increased globally. Tremendous research has been directed towards development of highly efficient energy systems and alternate green or carbon neutral fuels. However, implementation and refinement of these technologies require large investments and hence to make them economically viable there should be suitable incentives from the government. Most of the developed countries ratified the Kyoto Protocol which sets legally binding emission limitations on each country. Each country can adopt its own mechanism to control their GHG emission levels below cap value. Carbon Taxation and Emission Trading Scheme are the most common policies discussed and implemented in this direction. The two are commonly perceived as the competing policy instruments for the abatement of GHG emissions.

Carbon Taxation

Carbon Tax is analogous to any other tax where the tax is imposed at a certain rate per unit emission exceeding the pre-determined maximum non taxable value. Hence the taxation rate (or tax bracket) and Non-Taxable Emissions are the key parameter that controls the emission.

Cap-and-Trade Policy

Cap-and-Trade policy (or Emission Trading Scheme) has been designed to reduce pollution by utilizing market mechanisms. It involves initial allocation of carbon permits to all the firms and the permits are transferable in nature such that the firms can trade permits among each other. The two most prominent examples of existing cap and trade systems are the EU-ETS (European Union Emission Trading Scheme) and the US Sulfur Dioxide Trading System. In such systems, a central authority (typically national government) sets a limit (cap) on the total amount of pollutant that can be emitted by any firm within a pre-determined period. To ensure that this target is complied with, a certain number of permits or credits are allocated to the firm based on their emission profiles, and a penalty is applied as a charge per unit of pollutant emitted outside the limits of a given period. Firms may reduce their own pollution or purchase emission permits from other firms, in order to avoid accruing potential penalties. The transfer of allowances by trading helps in minimizing the costs caused by regulation as the companies that can easily reduce emissions will do so, while those for which it is harder buy permits. In a cap-and-trade system, the total initial allocation is indeed the crucial parameter that the regulator uses as a throttle to control the emission level. But while the value of the total initial allocation is driven by the emissions target, the initial distribution of these allowances among the various producers and market participant should be carefully chosen in order to create incentives to design and build cleaner and more efficient production units [11].

However, the effectiveness of these policies in achieving the goal of minimizing emission level and end consumers' cost is highly debatable.

The benefits and limitations of Emission Trading Scheme are conspicuous from the European Union Emission trading scheme which is in practice since Kyoto Protocol [12]. The key benefits observed were:

- 2-5% decline in carbon emissions over the trial period 2005-2007 is attributable to the E.T.S.
- As a result of the ETS, European power companies have begun to fully integrate the cost of carbon into their investment decisions and include more low-carbon

technologies, such as combined cycle gas turbines, high-efficiency coal and renewable energy (e.g., wind) in their future plant mix.

- Europe's ETS has promoted the development of low-carbon projects worldwide by creating a framework that allows the utilization of assets generated through the Clean Development Mechanism (CDM) and Joint Implementation (JI) for compliance purposes within the ETS.

However, several limitations of E.T.S. have also been observed:

- High price volatility of carbon assets will discourage investment in low-carbon/emission reduction projects.
- Windfall profits for the producers have been observed as naturally the companies pass the burden of penalty to the end consumers.
- It involves significant transaction cost due to the fees paid to brokers or exchange institutions.

Likewise Carbon Taxation has its own merits and demerits. Some of the key advantages of Carbon Taxation over E.T.S are as follows:

- A carbon tax has a potential to offer a broader scope for emissions reduction as it can be extended to individual consumers. Whereas trading systems can only be implemented among private firms or countries - not individual consumers.
- Taxes are non susceptible to strategic behavior by firms or non-governmental organizations that hampers the achievement of emissions reduction target.

The advantages of E.T.S. over Carbon Taxation are as follows:

- Permits adjust automatically for inflation and external price shocks, while taxes do not.
- E.T.S. has the advantage of fixing a certain emission target as the aggregate emissions levels are fixed.

1.1.5 Performance Metrics of Carbon Emission Regulatory policies

The previous section described the technical and qualitative advantages of each policy over another in a very subjective fashion. However, there are certain important parameters that measure the performance of each policy in a rather objective way:

- **Environmental Utility:** It is a measure of total CO_2 emission under the given carbon emission regulatory policy. Higher emission implies lower environmental utility.
- **Consumers' Cost:** It is a measure of total cost borne by end consumers under the carbon emission regulatory policy.
- **Producers' Profit:** It is a measure of profit realized by producers under the carbon emission regulatory policy.
- **Government Revenue:** The amount of revenue generated by government due to the carbon emission regulatory policy.

These parameters can be controlled precisely by adjusting the allocation of initial carbon permits in case of E.T.S. and by adjusting the Tax Brackets in case of Carbon Taxation.

The ideal policy should be the one that maximizes Environmental Utility, minimizes Consumers' Cost while preventing generation of windfall profits for the producers.

1.2 Literature Review

1.2.1 Studies on Oligopolistic Electricity Market Equilibrium

One of the earliest researches on oligopolistic electricity market equilibrium was carried out by von der Fehr and Harbord (1992) [13] with a particular focus on electricity market of U.K. and Wales. They conducted a research on price competition in a deregulated wholesale electricity market in U.K. and Wales by modeling it as a sealed-bid, multiple-unit auction with a random number of units. They adopted analytical game theoretic approach to solve simplified duopoly model under the production capacity constraints. And through the analysis results they raised serious doubts on the effectiveness of the competition in the new electricity supply industry for England and Wales in achieving the purposes for which it was originally designed, i.e. the efficient generation of electricity, sold at competitive prices to consumers. Richter and Sheble (1998) [14] used agent based simulation approach to model electricity market where each agent uses a Genetic Algorithm coupled with various price forecasting techniques to select appropriate bidding strategies for the current market conditions. Weber and Overby (1999) [15] used multi agent optimization approach in modeling the electricity market and showed that iteratively use of the objective of maximizing personal welfare can be an effective way of simulating electricity markets and studying the equilibrium behavior of the market. Wen and David (2001) [16] used two different methods viz. Monte Carlo Based Method and Optimization Based Method to arrive at optimal strategies for each player in oligopolistic electricity market. Through their simulation results they showed that market clearing price can be higher than competitive levels if the suppliers bid strategically, and the market power of the suppliers will be reduced if the load is elastic to the price of electricity. Gan and Bourcier (2002) [17] studied single period oligopolistic electricity market by developing and applying game theoretic models. They proved the non-existence of equilibrium under a wide range of market conditions, and characterized the equilibria of the game under strong capacity constraints, and introduced the concept of quasi-equilibrium for the study of market performance under weak capacity constraints.

1.2.2 Studies on Market Modeling using Agent Based Simulations.

The complex interactions and interdependencies among participants in today's deregulated, decentralized electricity markets can be modeled well using game theoretic approach. However, the strategies used by many power market participants are often too complex to be conveniently modeled by standard analytical game theoretic approach. In particular, the ability of market participants to repeatedly probe markets and rapidly adapt their strategies adds additional complexity. Computational science offers appealing extensions to traditional game theory with the introduction of Agent Based Modeling and Simulation (also known as Multi-Agent Modeling) [18]. Several electricity market ABMS tools have been constructed, including those created by Bower and Bunn (2000) [19], Petrov and Sheble (2000) [20], Veselka et al. (2002) [21], and North et al. (2002) [22]. Agent based Modeling approach has also been used by researchers in several other fields like logistics, manufacturing and portfolio optimization. Li and Sun (2007) [23] used parallel multi-agent simulation approach to solve complex logistics planning problem with genetic optimization. Fleury et al. (1999) [24] used multi agent approach for the stochastic estimation of consequences of random events in manufacturing systems. Plikynas [25] used neural network based multiagent system of investing agents to solve Portfolio design and optimization problem.

1.2.3 Studies on Impact of Carbon Emission Regulatory Policies on Electricity Market

Nakata and Lamont (2001) [26] conducted a research to analyze impact of carbon taxation on energy systems in Japan using multi-period market equilibrium models. They concluded on the basis of their studies that carbon tax can be effective in reducing Japan's carbon emissions and more or less it tends to eliminate coal as an energy resource for Japan. Cramton and Kerr (2002) [27] presented comparative advantages of auctioning the carbon permits rather than grandfathering (granting permits based on historical emission levels). Carmona et al. (2006) [11] presented a mathematical framework for competitive equilibrium, in which emissions trading schemes can be analyzed. They confirmed the presence of windfall profits to the producers in the standard cap and trade mechanism. Several other researchers analyzed impact of EU E.T.S scheme on electricity price. Sijm et al. (2006) [28] studied the implications of free allocation of CO_2 emissions allowances on the price of electricity in Germany and Netherlands. They recommended lesser grandfathering of permits and auctioning part of the permit requirements to ensure lesser windfall profits for the producers. Lise et al. (2010) [29] studied the impact of EU Emission Trading Scheme on the prices, utilities and emissions in the power sector across 20 countries in European Union. For their analysis they used COMPETES (COmprehensive Market Power in Electricity Transmission and Energy Simulator) model where the electricity network is aggregated into one node per country with a few exceptions. Their results showed that a significant part of the costs of (freely allocated) CO_2 emission allowances is passed through to power prices, resulting in higher electricity prices for consumers and windfall profits for power producers, even in cases of full auctioning. They also showed that the ETS-induced increases in power prices depend not only on the level of CO_2 prices but also on the structure of the power market, i.e., the incidence of market power, and the price responsiveness of power demand.

1.3 Objective of the Thesis

The ultimate objective of the thesis is to evaluate the impact of different carbon emission regulatory policies imposed on power sector on the utility of its stakeholders and environment. Going to finer details, the project is aimed to achieve the following objectives:

- To compare the performance of carbon cap-and-trade policy with carbon taxation policy
- To perform sensitivity analysis on the performance of each policy with respect to key controlling parameters like the distribution of initial carbon permits in case of cap-and-trade policy and tax rate in case of carbon taxation.
- To evaluate the efficiency of the cap-and-trade policy under different scenarios.
- Finally, to present a concluding remarks and recommendations based on the results.

1.4 Thesis Organization

Rest of the thesis is organized as follows:

- *Chapter 2* presents the computational approach in simulating the electricity market. It describes physical models, assumptions, mathematical models and the algorithms to compute equilibrium for each model.
- *Chapter 3* presents simulations results, sensitivity analysis and discussions on the results.
- *Chapter 4* presents concluding insightful remarks from the simulation results and the recommendations for the policy implementation and the future scope of work in this area of research.

Chapter 2

Methodology

This chapter introduces various market models and presents mathematical and computational formulation for each model under different carbon emission regulatory policies. Prior to introducing the actual market models, the basic computational approach for simulation has been discussed. Agent Based Modeling and Simulation (A.B.M.S.) approach has been used to arrive at market equilibrium. The stochasticity in demand has been incorporated using special class of Monte Carlo techniques viz. Stratified Sampling.

2.1 Agent Based Modeling and Simulation Approach

The best approach to simulate the complex electricity market is the use of Agent Based Modeling and Simulation (A.B.M.S.). Each stakeholder may act as an individual agent or a group of stakeholders may be represented by a single body like controller or exchange which acts as a single agent. MATLAB was used as a platform to execute the model.

2.1.1 Key Agents in the Model:

Electricity Producer: Each producer of electricity constitutes one player or agent and he makes his decisions with the sole motive of profit maximization.

Controller: Controller is the body that buys electricity from producers and distributes it among end consumers. Its role is similar to that of I.S.O. in U.S.A. The controller represents a single buying agent in the market. It makes its decisions with the sole motive of minimizing the total cost incurred in buying the electricity.

Government: The role of government in the model is to fix the tax rate, non-taxable emission value and penalty value and distribute the initial permits. Since decisions of government as reflected by policy making are the key parameters in the present analysis, this agent has not been simulated as an autonomous player but its decision parameters are varied manually to observe their impact on overall system. Therefore the decisions of government as an agent are not dynamic in the game and have to be fixed apriori.

2.2 Stochastic Modeling of Electricity Demand

Electricity demand is uncertain and producers fix their unit selling prices based on the probabilistic distribution of electricity demand. To incorporate randomness and stochastic nature of demand in the model, the Monte Carlo approach has been used. The electricity demand is assumed to follow a specific distribution which is known to all players. All players base their decisions on expected values of their utilities (obtained from Monte Carlo simulations) and also all performance parameters have been reported as their corresponding expected values.

2.3 Cap-and-Trade Game Model

All producers of electricity have the same target consumer base represented by single electricity controller. All producers have been granted certain carbon emission permits at the beginning of production phase, which fixes a cap on their emission level. Emissions not offsetted by permits are penalized at certain fixed rate. These permits are transferable and have a pre-defined expiry period. Therefore the players will tend to trade permits among each other. So basically there is a simultaneous occurrence of two processes in the game: 1) Price competition among the producers to sell electricity to controller. 2) Trade of permits among producers.

Following are the key parameters used by government to control the market:

Initial Carbon Emission Permits (Carbon Credits): The number of carbon emission permits or allowances initially allocated to each player.

Penalty: The rate of fine imposed per unit emission not offsetted by permits.

2.4 Carbon Taxation Game Model

It is similar to previous one except for the fact that in place of granting initial carbon emission permits to the producers, Government will fix certain *Non-Taxable Emission (N.T.E.)* value for each producer and will impose tax at a certain tax rate per unit emission exceeding that value. Hence the players only compete on selling electricity by adjusting their selling prices.

Following are the key parameters used by government to control the market:

Non-Taxable Emission (N.T.E.): The maximum tax free amount of emissions.

Tax Rate (T.R.): The rate at which tax is imposed on emissions exceeding N.T.E.

2.5 Key Assumptions

- Demand has to be met at any cost.
- Modeling involves single stage period. And permits expire at the end of period.
- No uncertainty in production cost or quantity.
- Emission rate of each producer is known and deterministic in nature.
- All players/agents are assumed to act rationally.
- Each player is assumed to have complete information about the structure of the game and the payoff function of the other players.

2.6 Electricity Market Models

I Isolated System

It is assumed that there is no influence from external agent and the equilibrium is achieved by exhaustive mutual interaction of only the agents described in previous section.

Three different models have been analyzed starting from simplest and theoretical model and slowly adding complexity and finally reaching very complex and most practical model:

1. **Model 1:** It is the most simplified system model consisting of just 2 producers competing for a common buyer/controller. There is no production capacity constraint on any producer. This model can help in getting key preliminary insights into nature of equilibrium and it will also help in designing strategies of each agent for the more complex models/games.

2. **Model 2:** In reality there is always a production capacity constraint on each producer. Therefore Model 2 takes the production capacity constraints into account. It consists of 2 producers competing with each other to sell electricity when each one of them has its own production capacity constraint.

3. **Model 3:** It is a complex and most practical system model consisting of 3 producers and a single buyer/controller. There is a production capacity constraint on each producer. Most of the analysis is carried out on this model as it is the most realistic case.

II Open System

The system includes the effect of external agents like central planner, carbon permit exchange and highly competitive electricity market. Following are the two models of such system:

1. **Model 4:** It consists of 3 producers who are exposed to highly competitive electricity market. Therefore, electricity price is not controlled by these producers. All producers coordinate to maximize their total combined utility. This task is performed by central

planner who reallocates permits among players and distributes production quantities among each player with the sole motive of total utility maximization.

2. Model 5: This model is similar to Model 3 except for the fact that now players have an access to central carbon permit exchange. Therefore the permit prices are no more controlled by the players in the system but it is controlled by the bigger and more competitive permits market.

Note: The above two models are specific to cap-and-trade policy.

2.6.1 Model 1

Description

Two producers of electricity compete with each other to supply electricity to controller. Demand is random with a known distribution. The producers are assumed to have infinite production capacity. The producers have different cost and emission rate functions which is a common knowledge. In rest of the thesis the term 'Producer' has been used interchangeably with the term 'Player'.

The structure of the game varies significantly with the emission regulatory policy. Therefore separate models have been developed for different policies. The following sections describe the game and present a mathematical model under each policy.

2.6.1.1 Cap-And-Trade Policy

Since there are only 2 producers in the system, at any point of time no more than 1 producer will be able to sell the permits. And only one player will be producing electricity as the production capacity of each player is assumed to be infinite and the tie is improbable without collusion. Since there is always one player selling permits to another he can set the value of permit value as high as he can as long as it is below the penalty value. If the player prices his permits above the penalty value then producing player will pay penalty rather than buying more expensive permits. Hence ideally the player should price his permits slightly below the permit price which for all practical purposes can be considered as equal to permit price.

The players can play one of the following two roles on the basis of their pricing decision:

1. Active Player: The producer who quotes lowest electricity price and hence supplies all the demanded electricity to controller.

2. Passive Player: The producer with higher quoted price and hence he doesn't get production control and rather sells off his permits to other player at price equal to penalty.

Structure of the Game

Players: Firm 1 and Firm 2 (2 players).

Strategy: Each player has only one action/strategy i.e. to fix an electricity price. The other strategy of fixing permit price is redundant as it is always equal to the penalty value.

Payoff: Each player's utility function is something like this:

Utility = Revenue from electricity sell + Revenue from Carbon permits sell – Cost of producing electricity – Cost incurred in buying permits from other players – Cost in penalty.

$$\text{Utility of active player} = \int \left((P_0 - c_a) \times D - \max(0, (e_a \times D - E_a) \times \pi) \right) f(D)dD \tag{2.1}$$

$$\text{Utility of passive player} = \int \left(\pi \times \min\left(E_p, e_a \times D - E_a\right) \right) f(D)dD \tag{2.2}$$

Where, π = Penalty value, P_0 = Equilibrium price, c_a = Production cost of active player,

E_a = Initial permits allotted to active player, E_p = Initial permits allotted to passive player, e_a = Emission rate of active player and D = Demand for electricity.

Equilibrium Price Determination – Analytical Approach

Since players are assumed to be rational they will adopt the following strategy to set their prices:

Strategy

To produce only at a price greater than or equal to the price that renders profit equal to the value of permits that he can sell to another player if he doesn't produce (the other player produces).

Let us call this price as "critical price". And the profit at the critical price as the Bottom-line Profit of the player.

So, logically the player with minimum critical price will be producing at second lowest critical price and the player with higher critical price should act as a passive player.

Bottom-line profit for Player1, $BL_1 = \min(E_1, e_2 \times D - E_2) \times \pi$ (2.3)

Bottom-line profit for player 2, $BL_2 = \min(E_2, e_1 \times D - E_1) \times \pi$ (2.4)

Where E_i is the total number of permits initially granted to i^{th} player, e_i is the emission rate per unit production for i^{th} player, D is demand for electricity and π is penalty value. Therefore, the critical prices of the players are:

Critical price for player 1, $$P_{cr1} = \frac{\int [BL_1 + (c_1 \times D + \pi \times \max(0, e_1 \times D - E_1))] f(D) dD}{\int D \times f(D) dD} \qquad (2.5)$$

Critical price for player 2, $$P_{cr2} = \frac{\int [BL_2 + (c_2 \times D + \pi \times \max(0, e_2 \times D - E_2))] f(D) dD}{\int D \times f(D) dD} \qquad (2.6)$$

Equilibrium Price or market price, $P_0 = \max(P_{cr1}, P_{cr2})$ (2.7)

Equilibrium Determination – Computational Approach

The model has been implemented using MATLAB. The action or strategy space of each player i.e. the electricity price they quote is discretized into several nodes. The price range considered ranges from the minimum production cost to the maximum price allowed by regulatory bodies. The best response of each player is the price that maximizes his expected payoff value for the given selling price of another player as obtained from utility function discussed in previous section.

There are two main computational approaches to obtain Nash Equilibrium:

Graphical Approach: The best response of each player is plotted as a function of price set by another player on a single graph. The point where the two curves intersects, represent the Nash Equilibrium.

Iterative Approach: This approach involves modeling of equivalent extensive form game where each player set their prices in turn just on the basis of current and previous prices set by the another player without any knowledge of payoff function of other player. The initial prices are set at highest possible value and then each player sets their price as the best response value to the price set by another player. After several iterations the prices set by each player converge to a constant value and that confirms the existence of Nash Equilibrium at that strategy as none of the players has any incentive to deviate from that strategy profile.

Modeling Demand Uncertainty

Monte Carlo method has been used to incorporate demand uncertainty in the model.

Monte Carlo Method: It is the most common approach for numerical integration. Large sample of random inputs is generated (over which integration has to be performed) from underlying probability distribution. Then the function is evaluated at each input value and the average value of the function over the sample distribution gives the expected value of the function for the given stochastic input. The special class of Monte Carlo Method viz. Stratified Sampling has been used for the modeling.

Stratified Sampling: Stratified sampling is a special class of Advanced Monte Carlo Methods which gives very less sampling error as compared to crude Monte Carlo Method for the same sample size. In this approach range interval of distribution is divided into several number of equiprobable sub-intervals (typically equal to sample size) and then one sample is randomly selected from each sub-interval. Therefore samples generated from stratified sampling comply well with the underlying probability distribution. The considerable reduction in computational efforts is realized by the use of stratified sampling as it allows relatively lower sample size with the same level of accuracy.

2.6.1.2 Carbon Taxation

Under the carbon taxation policy each player will be penalized with the same rate per unit emission exceeding the predefined N.T.E. The structure of the game under carbon taxation is presented below:

Structure of the Game

Players: Firm 1 and Firm 2 (2 players)

Strategy: Each player has only one action/strategy i.e. to fix an electricity price.

Payoff: Each player's utility function is something like this:

Utility = Revenue of electricity sell – Cost of producing electricity – Tax paid.

$$\text{Utility of active player} = \int\left(\left(P_0 - c_a\right)\times D - \max\left(0,\left(e_a \times D - E_a\right)\right)\times \alpha\right) f(D)d(D) \qquad (2.8)$$

$$\text{Utility of passive player} = 0 \qquad (2.9)$$

Here, E_a is the N.T.E. value for the active producer and α is the tax rate.

Equilibrium Price Determination: Analytical Approach

Since players are assumed to be rational they will adopt the following strategy to set their prices.

Strategy

The profit of player is zero if he doesn't produce. Therefore the player will be willing to produce at any price higher than or equal to the price that gives him overall zero utility.

Therefore critical prices of the players are:

$$\text{Critical price for player 1}, P_{cr1} = \frac{\int [c_1 \times D + \alpha \times \max(0,(e_1 \times D - E_1))] f(D)dD}{\int Df(D)dD} \qquad (2.10)$$

$$\text{Critical price for player 2}, \; P_{cr2} = \frac{\int [c_2 \times D + \alpha \times \max(0,(e_2 \times D - E_2))] f(D)dD}{\int Df(D)dD} \qquad (2.11)$$

The equilibrium price of electricity should be equal to second lowest critical price and the player with lowest critical price will take production control.

Equilibrium Price Determination – Computational approach

The equilibrium for model with carbon taxation can be found by the same methods that were used for model with emission trading scheme but with the modified utility functions.

Model 1 assumes infinite production capacity for each producer which makes it too unrealistic. Therefore we need to consider model with capacity constraint.

2.6.2 Model 2

This model is identical to Model 1 except for the fact that here each player has certain production capacity limitation. Now due to capacity constraint, there might be a situation where both of the players will be producing. Therefore on the basis of pricing decision each player can fall into one of these 2 categories:

1. **Primary Producer:** The player who quotes the lowest price and hence he takes the primary production control. He supplies electricity up to his production capacity or demand whichever is lesser.

2. **Secondary Producer:** The player who quotes the second lowest (or highest in this case) and hence he takes the secondary production control. He supplies electricity for the excess demand that could not be met by primary producer.

2.6.2.1 Cap-And-Trade Policy

The players can trade permits among each other. Typically the primary producer runs out of permits and buys permits from secondary producer to offset the penalty cost. Now since at any point of time there is only one seller of permits, he has monopolistic power to set the permit prices up to penalty rate. Let us consider key scenarios which differ on the basis of demand

There are two key scenarios based on demand value:

Scenario 1 Demand is less than individual capacity of all the players

Under this scenario the production capacity constraint becomes redundant and the model becomes identical to model 1.

Scenario 2 Demand is higher than the production capacity of atleast one of the players

Under this scenario electricity market becomes monopolistic and equilibrium price reaches the maximum level allowed by regulatory bodies. The player with the lower critical price will take the primary production control and the other player will take secondary production control and sell the permits demanded by primary producer at penalty value. Therefore explicit calculation of equilibrium price and payoffs is possible analytically for this model:

$$\text{Equilibrium Price, } P_0 = \text{Ceiling Price} \tag{2.12}$$

Where, Ceiling Price is the maximum price allowed by regulatory bodies

Utility of primary player,

$$U_p = \int \left(\left(P_0 - c_p \right) \times \min\left(k_p, D \right) - \max\left(0, \left(e_p \times \min\left(k_p, D \right) - E_p \right) \right) \times \pi \right) f(D) dD \tag{2.13}$$

Utility of secondary player,

$$\begin{aligned} U_s = & \int \left(P_0 - c_s \right) \times \min\left(k_s, \max\left(0, D - k_p \right) \right) f(D) dD + \ldots. \\ & \int \min\left[E_s - \max\left(0, e_s \times \min\left(k_s, (D - k_p) \right) \right), \max\left(0, e_p \times \min\left(k_p, D \right) - E_p \right) \right] f(D) dD \end{aligned} \tag{2.14}$$

Where c_p and c_s are the production costs for primary and secondary producers respectively, k_p and k_s are the production capacities of primary and secondary players respectively, and e_p and e_s are the emission rate per unit production for primary and secondary players respectively.

2.6.2.2 Carbon Taxation

Under the Carbon Taxation policy each player will be penalized with the same rate per unit emission exceeding the predefined N.T.E. When demand is less than the individual production capacities of the both players, this model becomes identical to model 1 and when the demand is more than the minimum production capacity of any player then the electricity market becomes monopolistic and market price reaches highest level.

Model 2 doesn't produce any interesting insights as it gives only trivial equilibria. Therefore to get the more interesting (non-trivial) equilibrium one needs to analyze model with 3 players.

2.6.3 Model 3

Description

There are three producers of electricity with same target consumer base represented by single controller. These players involve in price competition to sell electricity to the controller. Each of these players has his own production capacity constraint. The following sections describe the game and present the mathematical formulation and computational approach to find the market equilibrium under different policies.

2.6.3.1 Cap-And-Trade Policy

All three players have been granted certain carbon emission permits at the beginning of production phase to offset the penalty imposed on carbon emissions. These permits are transferable and have a pre-defined expiry period. Therefore the players will tend to trade permits among each other. The player who is emitting more than his allotted permits will buy permits from the player with extra permits to offset the penalty cost. The initial allocation of permit should never exceed the total emission value at production capacity for any player otherwise the emission reduction constraint will become redundant.

Basically there are two simultaneous processes occurring in the model:

1) Electricity Price Competition: The three players compete among each other to sell electricity to controller. Since each player has a production capacity constraint, there can be a situation where more than one player are producing. The market price of electricity will be equal to the highest price among the producing players. Each active producer will be able to sell electricity at the market price. The players can have one of the following three roles on the basis of their pricing decision:

Primary Producer: The player with the lowest quoted price. Hence it is the primary supplier of electricity and it supplies up to his production capacity level or demand whichever is lesser.

Secondary Producer: The player with second lowest quoted price. It is the secondary supplier of electricity who supplies the excess demand that could not be fulfilled by primary producer.

Contingency Producer: The player with the highest quoted price who supplies only if all other players have run out of their capacity. If this player is producing, the market turns into monopoly. Therefore in most of the practical cases demand shouldn't exceed the capacity of other 2 players and this player solely indulge in selling permits to other 2 players and hence makes a profit.

2) Permits Trading: The electricity producers will indulge in the trade of emission permits where the player emitting more and consequentially running out of permits can buy permits from the player emitting lesser than his allotted permits and therefore in the process the player who buys permits avoid the higher penalty cost and the player who sells permit gets windfall profit. When the demand of permits by primary producer is so high that none of the other 2 players are capable of meeting demand individually, then both of the non-primary players will be supplying permits to primary producer through individual direct contracts and hence their selling prices may differ. And logically primary producer will give higher preference to the seller (of permits) with lower quoted price.

Approaches to find Nash Equilibrium in a Multi-Agent Game:

1. Infinite Regress: Each player knows the payoffs of all other players. And hence he sets his optimum strategy assuming that the other players act rationally, and they will set their optimum strategy based on their belief about rest of the players being a rational. And by performing this infinite regress one can reach a unique set of optimum strategy (optimum strategy profile) or in some cases more than one from which none of the players has any incentive to deviate. That strategy profile corresponds to Nash Equilibrium. The computational implementation of this approach is very difficult in a game with three players and two strategies and therefore the alternate approach should be considered as discussed below.

2. Equivalent Extensive Form Game: The alternate method to find Nash Equilibrium is by formulating an equivalent extensive form game. Here we assume that player doesn't know about the payoffs or equilibrium strategy of other players but they respond strategically to the past observations of the game. So in our problem player takes turn to set up their price and the process continues until steady state is reached which corresponds to Nash equilibrium. This approach is convenient from computational implementation point of view.

In the present analysis, the second method has been used to simulate a game and finding equilibrium.

Structure of the Game:

Players: Firm 1, Firm 2 and Firm 3 (3 players)

Strategy: Each player has 2 strategies: 1) Fixing electricity price and 2) Fixing permit's price.

Payoff: Each player's utility function is something like this:

Utility = Revenue of electricity sell + Revenue of Carbon permits sell – cost of producing electricity – Cost incurring in buying permits from other players – Cost in penalty.

The analytical expression for the utility of the players has been presented in the subsequent section.

Since this game involves multiple strategies and multiple players with random input, finding a closed form analytical solution for equilibrium is very difficult. Hence only the computational approach has been used and mathematical models are used just to get insight into simulation results.

Equilibrium Determination – Computational Approach

Each player has two strategies and the decision on any one strategy is influenced by the other strategy. Therefore the game should be modeled as two-level game where players converge to equilibrium profile for one strategy at each level

Two-Level Game

All players fix their permit prices and then enter the game of electricity price competition with the fixed set of permit prices and arrive at electricity price equilibrium. Therefore, there is an equilibrium electricity price corresponding to every set of permit prices and using that the utility of each player can be evaluated corresponding to each set of permit prices. Then at the second level players can adjust their permit prices to maximize their utility and eventually moving towards steady state which might be a Nash Equilibrium. Ideally at equilibrium all players should have local maximum of utility i.e. none of the players have any incentive to deviate from the equilibrium

Finding an Electricity Price Equilibrium

The equilibrium electricity price is obtained by simulating a hypothetical extensive form game as discussed earlier. The detailed description of game is presented in stepwise manner below:

1. Prices by all three players are initialized at highest possible value allowed by regulatory board.

2. Player 1 begins the game by setting his price slightly lower than that of Player 2 and Player 3 to take the primary production control. To define "slightly" here I have used a certain stepsize for each players. These stepsizes have been chosen to be a little different for each player to avoid possible ties.

3. Player 2 responds by either setting his price slightly lesser than Player 1 or he sets it very high (such that he doesn't produce) depending on his utility function.

4. Player 3 has now 3 strategies: Setting his price either lower than both of the players, or intermediate or the highest. He will go for the one which will give maximum utility.

5. *General Decision Making:* The game will continue till steady state. Now let us look at the general process. The first 3 steps are close to upper boundary and hence don't give general picture. At any general point where Player 1 has to set his price P_1 observing prices of other two players as P_2 and P_3, he has basically 4 alternative strategies: 1) P_1 =

min $(P_2, P_3) - ss_1$, 2) $P_1 = \max(P_2, P_3) - ss_1$, 3) $P_1 = \max(P_2, P_3) + ss_1$ and 4) $P_1 =$ ceiling price. Where ss_1 is a step size for player 1. Strategy 1 will give player 1 a primary production control where he will be supplying electricity at the price quoted by the player with highest price supplying the controller. Strategy 2 will give Player 1 a secondary control which implies he will produce only if the primary producer has run out of his capacity. And strategy 3 and 4 gives player a contingency producer role where he will produce only if other two players run out of their capacities. Strategy 3 and 4 are similar from utility point of view when demand is not high enough to cause the other two players run out of their capacities. However when demand is that high, strategy 4 clearly gives higher utility. It can be easily concluded that strategy 4 always dominates strategy 3 but we have included strategy 3 because Player1 shouldn't jump to highest price possible unless he is producing i.e. he has incentive to set that price. Because if he does so then other players get tremendous room to set their prices higher and the whole process will get involved in a loop as we are using iterative approach.

6). Stopping Criterion: After several iterations, one of the players decides to choose strategy 3 or 4. That implies they want to withdraw from price competition at the current price set by other two players. Let say Player 3 sets his strategy as strategy 3 (fixing price higher than other 2 players). Now Player 3's decision is not just an explicit function of other two players' prices. It also depends on the permit prices set by other two players. Hence his response is not just dependent on the current price of electricity but also on the permits which in turn depends on which player has the permits left to sell (in general primary producer run out of his permits granted as he generates most of the time at the capacity). Hence the price of the permit is the price set by the other two players and production decision is highly influenced by the lowest permit price. Now although we have fixed permit prices of each player before we started analyzing electricity price equilibrium, the shift in production control changes the minimum permit price. And hence Player 3's response to give up on price competition is sensitive to production control. Therefore we need to interchange price of electricity of the other two players (which will shift the production control) and observe Player 3's response. If Player 3 again chooses strategy 3 or 4 as his optimum strategy, then it can be concluded that irrespective of who is producing Player 3 will never have an incentive to take production control (as a primary or secondary producer) at price lower than or equal to the current market price. Hence we can freeze Player 3's price at the response price corresponding to this iteration. This price is the critical price of Player 3. And Player 3 exits the competition.

Now Player 1 and Player 2 may still compete with each other to get a primary production control till the point where one of the players chooses not to lower price further and maximize his utility by setting price at the maximum level keeping the role of secondary producer. Let us assume Player 2 chooses to be a secondary producer. Hence, $P_2 = P_3 - ss_2$ Now we can freeze the price of Player 2 to this value.

Finally Player 1 is given a chance to optimize his price based on the frozen prices of Player 2 and Player 3 which should be $P_2 - ss_1$. And hence we find the equilibrium with price approximately equal to P_3 and the Player 1 is acting as a primary producer, Player 2 as a secondary producer and Player 3 as a contingency producer.

Finding a Permit Price Equilibrium

Since the equilibrium electricity price is a function of permit prices set by each player, we can't use the same discretization approach as was used for electricity price equilibrium and the action or strategy space corresponding to permit price should be discretized extensively such that it spans all possible permit prices below the penalty value. Following is the stepwise description of process:

1. Initial permit price of each player is set to the highest value (equal to the penalty value).

2. Player 1 sets his permit price from the available set of discretized prices (which spans his whole decision space) that maximizes his utility for the current permit prices set by Player 2 and Player 3.

3. Now Player 2 sets his permit price to maximize his utility for the given prices of Player 1 and Player 3.

4. Player 3 sets his permit price in the similar manner.

5. The steps 1-2-3 will repeat until the permit prices converge to the unique set (typically close to zero) or set of permit prices get in a loop.

6. In most of the cases especially when the demand of permit can't be met by a single player the permit prices set by the players repeats in a cyclic manner. Therefore, in order to avoid the ambiguity of permit price decision the players coordinate among each other to decide on the set of strategy in a loop that maximizes total utility of all players combined together.

Sample iterations to find the equilibrium permit price are presented in the next section.

Sample Result:

Following is the sample result for the game with the parameters given as:

Capacity, k = (1.4, 1.2, 1.0), Cost, **c** = (1, 1.5, 2), Emission per unit, e = (2, 1.5, 1)

Initial Permits, E = 1.3*Capacity and Penalty, π = 1/unit of emission.

Note: First entry in each set represents parameter for player 1, second entry for player 2 and third one for player 3. All values are normalized for the ease of calculations.

Table 2-1 Sample iterations for finding permit equilibrium price in Model 3

Iteration No.	Permit Price			Equilibrium Electricity Price			Total Utility Eut
	Player 1 Ep1	Player 2 Ep2	Player 3 Ep3	Player 1 P1	Player 2 P2	Player 3 P3	
1	0	0.98	1.02	2.092	2.094	2.093	1.586
2	0	0.98	1.02	2.092	2.094	2.093	1.586
3	0	0.98	0.273	2.025	2.026	2.027	1.752
4	0.273	0.98	0.273	2.068	2.069	2.070	1.838
5	0.273	0.268	0.273	2.068	2.069	2.070	1.838
6	**0.273**	**0.268**	**1.02**	**2.090**	**2.091**	**2.092**	**1.881**
7	0	0.268	1.02	2.090	2.091	2.092	1.881
8	0	0.98	1.02	2.092	2.094	2.093	1.586

The highlighted row in bold letters is the equilibrium condition of the game. It can be observed that permit strategy runs in a loop and after iteration 8 we observe the same state as after iteration 1. Therefore, we look for the strategy that maximizes the total utility within a loop and that corresponds to iteration 6 with a maximum utility of 1.8812. The electricity price corresponding to equilibrium permit price is the equilibrium market electricity price.

Grid Convergence

Results are highly sensitive to step size used in electricity and permit price strategy space. If the grid is not sufficiently fine, the model might stuck in false equilibrium. Therefore we tried with different grid sizes and found out an optimal step size that gives same equilibrium irrespective of starting point and the sequence.

Scenario analysis and characteristics of equilibria:

Various scenarios were analyzed which differ on the electricity demand

Scenario 1 <u>Demand exceeds capacities of any of the 2 players put together.</u>

It results into monopolistic market and electricity price reaches highest level. Therefore this scenario doesn't give any interesting insight into equilibrium.

Scenario 2 <u>Demand is less than the individual capacity of all the players.</u>

Under this scenario, capacity constraint becomes redundant and the model behaves like infinite capacity model. Primary producer remains the only active producer in the game. Since, the demand is low the amount of emissions exceeding the cap should be low and hence emission cost becomes insignificant in production decision. Therefore, in this scenario, typically player with least production cost takes the primary production control.

Scenario 3 <u>Demand is intermediate.</u>

It is interesting scenario. Therefore one needs to explore sub-scenarios on the basis of number of initial permits granted to each player.

a) *Only one player supplying permits:*

When initial permit allocation is such that each of the non-primary producing players are capable of supplying all the permits required by the primary producer. Then the permit suppliers (non-primary producers) compete vigorously to sell their permits to primary producers leading to equilibrium permit price of zero. Therefore the permit price decision is practically decoupled from electricity price decision. The equilibrium electricity price will be the highest critical price as in this situation two players are producing. The equilibrium electricity price can be found analytically as follows:

Equilibrium Price = Highest Critical Price (2.18)

Utility of primary producer, $U_p = \left(P_0 - c_p\right) \times k_p$ (2.19)

Utility of secondary producers, $U_s = \left(P_0 - c_s\right) \times \min(k_s, D - k_p)$ (2.20)

Utility of contingency producer is zero.

b) *Both players supplying permits:*

None of the non-primary producing players can supply all the permits required by the primary producer. This is clearly the most interesting and complex scenario because the strategies of fixing permit price and that of fixing electricity price are coupled together. Each of the non primary players has two strategies of exploiting his limited permits: 1) Take a role of secondary producer and offset penalty by using these permits. 2) Adopt a role of contingency producer and sell off all the permits to the primary producer.

The decision of every player can be framed as a simple optimization problem where the key decision variables are the amount of permits used to offset their emissions and the amount of permits used for trade with other players.

Utility function of each player: U = U_{prod} + U_{sell} (2.21)

Where, U_{prod} = Profit in producing electricity and U_{sell} = Profit in selling permits

Let us assume the total permits granted to player = E

The amount of permits used for production = E_{prod}

The amount of permits used for trade = E_{trade} (positive implies selling, negative means buying)

Total emission that has been penalized on the player = $E_{penalty}$

Hence objective function is:

$$Max\left((P-c)\frac{E_{prod}}{e}+E_{trade}.E_p+\pi.E_{penalty}\right) \tag{2.22}$$

Where, P : Price of electricity

c: Cost of production

e: Emission per unit production

Ep : Market Permit price

π: Penalty

Subject to: $E_{prod}+E_{trade}=E+E_{penalty}$ (2.23)

$$E_{trade} <= E \tag{2.24}$$

$$\delta_2 \times \min\left(D-k_p,k\right)+\delta_1 \times \min\left(D,k\right) < \frac{E_{prod}}{e} < k \tag{2.25}$$

Where,

E = Amount of initial permits allotted.

k_p = capacity of primary producer.

k = capacity of the player in consideration.

δ_2 = 1 if player is a secondary producer, 0 otherwise.

δ_1 = 1 if player is a primary producer, 0 otherwise.

The optimum solution to this MLP is not straightforward as it is a multi-agent optimization problem and Ep (Permit price) and P (Electricity price) are the function of strategy of other players. Hence it is impossible to solve it as optimization problem in isolation unless all these players act towards common goal of total utility optimization.

Hence simulation is the only way to find equilibrium.

Insights from the simulation analysis

Each player faces same set of options to pick any of the roles (primary, secondary or contingency producer) by adjusting his electricity selling price accordingly. Now as we are considering case where none of the players is able to meet total permit demand. Therefore whenever player is producing electricity, he is producing at the cost of permits which otherwise could have been sold at the current market price.

Player chooses to play a role of primary producer when the electricity price is high, and gradually the market electricity price goes down due to competition to the level where:

$$P_{crit} < c + e \times Ep \tag{2.26}$$

$$\Rightarrow U_{sell} > U_{prod} \tag{2.27}$$

At this point, the production of electricity is less desirable as just selling permits can fetch more money than utilizing them in production. And since the player can sell all the permits (as demand of permits exceeds his supply), he will try to set price to the highest and be a contingency producer such that he can use all his permits in trade. Similarly every player will have such critical price. The player with the lowest critical price will produce as a primary producer followed by the one with the second lowest critical price who will produce as a secondary producer. And both of them will produce at the critical price of the third player.

Non-Existence of Equilibrium:

When the price of permits set by each player is such that the critical prices of two players turn out to be approximately equal. In this situation both players would either want to be a primary producer or contingency producer and since no one would want to take a role of secondary producer, equilibrium is non-existent.

Mathematical Model for utilities and performance measures

Let us redefine players as follows (just for utility calculations):

Player 1: Player with least selling price for electricity (Primary Producer).

Player 2: Player with intermediate electricity selling price (Secondary Producer).

Player 3: Player with highest electricity selling price (Contingency Producer).

Utility of each player is a function of electricity prices and permit prices set by all the players in a game. Hence there is a unique set of payoffs/utilities associated with every strategy profile consisting of fixed set of electricity prices and permit prices. Players are sorted in ascending order of their electricity prices for the computing convenience.

Amount of electricity supplied by player 1, $q_1 = \min(D, k_1)$ (2.28)

Amount of electricity supplied by player 2, $q_2 = \min(\max(0, D - k_1), k_2)$ (2.29)

Amount of electricity supplied by player 3, $q_3 = \min(\max(0, D - (k_1 + k_2)), k_3)$ (2.30)

Market Electricity Price, $P = \delta_1 \times P_1 + \delta_2 \times P_2 + \delta_3 \times P_3$ (2.31)

Where, D = Electricity demand and k_i is the production capacity of i^{th} player.

$\delta_1 = 1$ if $D <= k_1$ and $\delta_1 = 0$ otherwise (2.32)

$\delta_2 = 1$, if $k_1 < D <= k_1 + k_2$ and $\delta_2 = 0$ otherwise (2.33)

$\delta_3 = 1$, if $k_1 + k_2 < D$ and $\delta_3 = 0$ otherwise

(2.34)

P_i is the electricity price set by Player i

Demand of permits by player 1, $D_{perm1} = \max(0, e_1 \times q_1 - E_1)$ (2.35)

Demand of permits by player 2, $D_{perm2} = \max\left(0, e_2 \times q_2 - E_2\right)$ (2.36)

Demand of permits by player 3, $D_{perm3} = \max\left(0, e_3 \times q_3 - E_3\right)$ (2.37)

Where e_i = emission rate of the i^{th} player and E_i = initially allotted permits to i^{th} player.

Let E_{ij} be the number of permits that player i is willing to sell to player j

Since the initial allotment of permits can't exceed the emission at the production capacity level, the flow of permits can only be from player 3 (contingency player) to player 1 (primary) or player 2 (secondary) and from player 2 to player 1.

Therefore, $E_{12} = 0,\ E_{13} = 0 \text{ and } E_{23} = 0$ (2.38)

$E_{21} = \max\left(0, E_2 - e_2 \times q_2\right)$ (2.39)

For player 3 the decision is tricky as there can be a situation where both players 1 and 2 are seeking permits. Since the permit price is same and fixed by the Player 3, the only way to decide on allocation of permits is on basis of demand i.e. the Player 3 will give higher preference to player with higher demand for permits.

Case I $D_{perm1} \geq D_{perm2}$

$E_{31} = \max\left(0, E_3 - e_3 \times q_3\right)$ (2.40)

$E_{32} = \max\left(0, E_{31} - D_{perm1}\right)$ (2.41)

Case II $D_{perm2} > D_{perm1}$

$E_{32} = \max\left(0, E_3 - e_3 \times q_3\right)$ (2.42)

$E_{31} = \max\left(0, E_{32} - D_{perm2}\right)$

(2.43)

Each player will tend to meet majority of his demand from the supplier selling permits at lower price. Since player 1 will never be able to sell his permits his permit selling price doesn't affect utility functions in any way. However the amount of electricity traded between different pairs of players is the strong function of the relative values of permit prices set by Player 2 and Player 3 i.e. E_{p2} and E_{p3}.

Let q_{ij} be the amount of permits that player i sells to player j

Case 1 $E_{p2} \leq E_{p3}$

$$q_{21} = \min\left(D_{perm1}, E_{21}\right) \tag{2.44}$$

$$q_{31} = \min\left(\max\left(D_{perm1} - E_{21}, 0\right), E_{31}\right) \tag{2.45}$$

$$q_{32} = \min\left(D_{perm2}, E_{32}\right) \tag{2.46}$$

Case 2 $E_{p2} > E_{p3}$

$$q_{21} = \min\left(\max\left(D_{perm1} - E_{31}, 0\right), E_{21}\right) \tag{2.47}$$

$$q_{31} = \min\left(D_{perm1}, E_{31}\right) \tag{2.48}$$

$$q_{32} = \min\left(D_{perm2}, E_{32}\right) \tag{2.49}$$

Let say U_{prod-i} be the net utility of i^{th} player by producing electricity.

And U_{perm-i} be the utility of i^{th} player by selling permits.

$$U_{prod-1} = \left(P - c_1\right) \times q_1 - q_{21} \times E_{p2} - q_{31} \times E_{p3} - \max\left(0, D_{perm1} - E_{21} - E_{31}\right) \times \pi \tag{2.50}$$

$$U_{prod-2} = \left(P - c_2\right) \times q_2 - q_{32} \times E_{p3} - \max\left(0, D_{perm2} - E_{32}\right) \times \pi \tag{2.51}$$

$$U_{prod-3} = \left(P - c_3\right) \times q_3 - D_{perm3} \times \pi \tag{2.52}$$

$$U_{perm-1} = 0 \tag{2.53}$$

$$U_{perm-2} = q_{21} \times E_{p2} \tag{2.54}$$

$$U_{perm-3} = \left(q_{31} + q_{32}\right) \times E_{p3} \tag{2.55}$$

Total utility of the i^{th} player, $U_i = U_{prod-i} + U_{perm-i}$ (2.56)

Total Emissions, $E_{tot} = e_1 \times q_1 + e_2 \times q_2 + e_3 \times q_3$ (2.57)

Total Cost to Consumers, $C_{cons} = P \times (q_1 + q_2 + q_3)$ (2.58)

Total Government Revenue (Earned from penalties),

$R_{gov} = \pi \times \max(0, E_{tot} - E_1 - E_2 - E_3)$ (2.59)

2.6.3.2 Carbon Taxation

Tax is imposed per unit emission for each unit exceeding the N.T.E. for each player. Each player has only one strategy in this case i.e. to fix the electricity price. Based on this decision each player takes a role of primary, secondary or contingency producer

Structure of the Game:

Players: Firm 1, Firm 2 and Firm 3 (3 players)

Strategy: Each player has only one strategy i.t. to fix electricity price.

Payoff: Each player's utility function is something like this:

Utility = Revenue of electricity sell – Cost of producing electricity – Tax paid.

Utility of Primary Producer,

$$U_p = \int \left((P - c_p) \times \min(D, k_p) + \alpha \times \max\left(0, \left(e_p \times \min(D, k_p) - E_p\right)\right) \right) f(D)dD \tag{2.60}$$

Utility of Secondary Producer,

$$U_s = \int \left((P - c_s) \times \min(D - k_p, k_s) + \alpha \times \left(\max\left(0, e_s \times \min(D - k_p, k_s) - E_s\right) \right) \right) f(D)dD \tag{2.61}$$

Utility of Contingency Producer,

$$U_c = \int \left((P - c_c) \times \min(D - k_p - k_s, k_c) + \alpha \times \left(\max\left(0, e_c \times \min(D - k_p - k_s, k_c) - E_c\right) \right) \right) f(D)dD \tag{2.62}$$

Approach to find Equilibrium

The same approach has been used as in previous model to find the price equilibrium.

However this model is much simplified as the production control and the price decision of each player is not a function of any other strategy.

2.6.4 Model 4

Description:

This model consists of three electricity producer, central planner and a highly competitive electricity market. The electricity price is decided by the market and hence it is external to the system and is taken as a fixed input. However players still can trade permits among themselves. The role of central planner is to maximize the total combined utility of all the producers. Since permit trade among producers do not affect the total combined utility the planner can reallocate the permits to minimize the penalty imposed by the government. Secondly, the planner maximizes total utility by optimal division of demand among producers. This model is specific to cap-and-trade policy.

Optimization Approach:

This problem can be solved using linear optimization as it involves single linear objective and linear constraints. The LP formulation is as follows:

$$Max\sum_i\left((P-c_i)\times q_i - y_i\right) \tag{2.63}$$

Subject to:

$$y_i \geq e_i \times x_i - E_i \qquad \forall i \tag{2.64}$$

$$y_i \geq 0 \qquad \forall i \tag{2.65}$$

$$\sum_i q_i = D \qquad \forall i \tag{2.66}$$

$$\sum_i E_i = \sum_i E_{initial-i} \qquad \forall i \tag{2.67}$$

$$0 \leq q_i \leq K_i \qquad \forall i \tag{2.68}$$

$$E_i \geq 0 \quad \forall i \tag{2.69}$$

$i = \{1, 2, 3\}$

The key decision variables are:

q_i = Quantity of production by i^{th} player

E_i = The amount of emission permits with i^{th} player after reallocation,

The other constant input patrameters are :

P = Electricity price, c_i = Cost of Production for i^{th} player, e_i = emission rate of i^{th} player, $E_{initial-i}$ = Amount of emission permits allotted to i^{th} player initially,

K_i = Production capacity of the i^{th} player and D = Electricity Demand

2.6.5 Model 5

This model consists of three producers competing to sell their electricity to common controller. All the producers have an access to centralized carbon permit exchange. Therefore the price of permits is fixed by exchange and is input to the system. Hence this model involves competition among players only for electricity sell and hence its formulation is very similar to Model 3 under Carbon Taxation. Each player tries to maximize his utility and in the process equilibrium is achieved. The utility of each producer is as follows:

$$U_i = (P - c_i) \times q_i + (E_{sell} - E_{buy}) \times E_{pm} \tag{2.70}$$

Where, U_i = Utility of i^{th} producer, P = Electricity price, q_i = Amount of electricity produced by i^{th} producer, E_{sell} = Amount of permits sold by i^{th} producer to the carbon permits exchange, E_{buy} = Amount of permits bought by i^{th} producer from the carbon permits exchange and E_{pm} = Market price of Permit.

Chapter 3

Results and Discussions

In this section we discuss the results obtained by implementing the models discussed in Chapter 2 on various hypothetical electricity markets. We present the sensitivity analysis on the equilibrium conditions for each situation with respect to government policies and uncertainty in electricity demand. All the parameters pertaining to market have been scaled down for the ease of calculations. In the subsequent sections we describe the market corresponding to each model as discussed in Chapter 2 and present the results and analysis obtained from market simulation.

3.1 Analysis of Market Based on Model 1

The market includes two producers of electricity who use different generation technologies and hence have different production costs and emission rates. One of them uses Gas Based Technology while the other producer uses Coal Based Technology for electricity production. Gas based technology is cleaner and more expensive than coal based technology. The demand for electricity by controller is assumed to follow a normal distribution with a known mean and standard deviation. The Table 3-1 describes the parameter values for each producer:

Table 3-1 Production related parameters of each producer (Model 1)

Parameter	Player 1	Player 2
Technology	Coal Technology	Gas Technology
Unit Production Cost, c	1	1.5
Emission Rate, e	2	1

Note: All parameter values are scaled down.

Each player is assumed to have infinite production capacity and demand possesses normal distribution with mean = 1 unit and standard deviation about mean = 0.1 unit.

The Model 1 as discussed in Chapter 2 has been used to model this situation. The graphical approach has been adopted for equilibrium analysis as it gives clear insight into the process. The following sections present the equilibrium analysis of the electricity market under different emission regulatory policies.

3.1.1 Cap-and-Trade Policy

It is assumed that government will distribute equal amount of initial permits among all players. Since there are only two players in the market, at any point of time not more than one player will be selling his permits. Therefore, this monopoly in permit market enables the player to set his permit price very close to penalty value. For our analysis we can consider it equal to penalty value. The government can control electricity market by adjusting the amount of initial permits and the penalty (π). Following sections present a detailed result for one base case with particular penalty value and initial permits and sensitivity analysis of the equilibrium condition with respect to policy parameters as they deviate from the base case value.

Detailed Result – Base Case

Initial Permits, E = 1.0 unit (for both players) and Penalty Value, π = 1/unit above cap value.

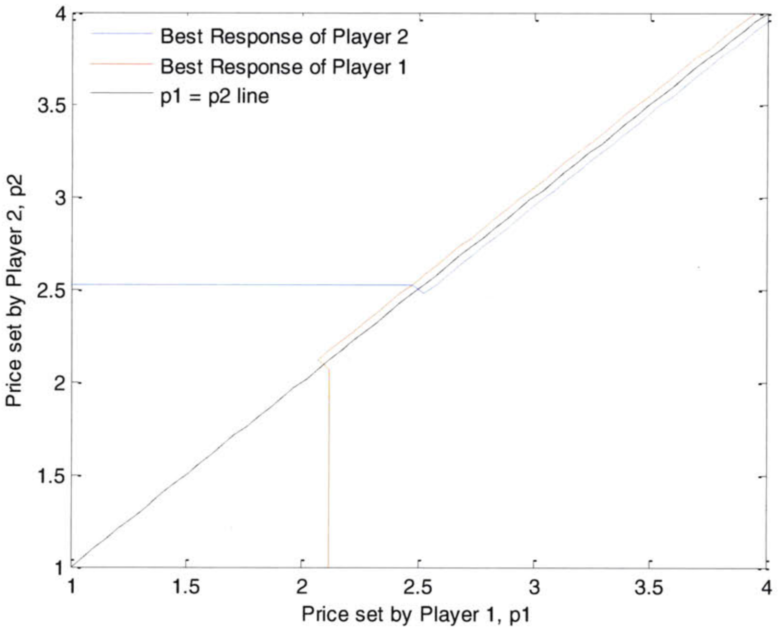

Figure 3-1 Graphical representation of strategy profiles of each player

In Figure 3-1. the blue curve represents the best response of Player 2 (Y-axis) for a given strategy (electricity price) of Player 1 (X-axis). The red curve represents the best response of Player 1 (X-axis) for every strategy of Player 2 (Y-axis). The black dotted line passes through origin making an angle of 45^o with the positive X-axis. Therefore this line consists of reference points at which p1 = p2. It can be observed from the above figure that the optimum strategy for Player 2 is to fix his price slightly lower than the price set by player 1 (p1) but not lesser than 2.515. Therefore if the Player 1 sets his price lower than 2.515, Player 2 should forego production control and should take a role of passive producer and just sell his permits to other player. Hence, the critical price for Player 2 is 2.515. Similarly it can be concluded that the critical price of Player 1 is 2 (as Player 1 will never set his price less than that.)

The Nash Equilibrium in the Two-player game is the point of intersection of the best response curves of both players. Therefore the Nash Equilibrium is obtained at critical price of Player 2. It has also been logically derived in Chapter 2 that the market electricity price should be the second lowest critical price and the production control is taken by the player with lowest critical price. Hence, observation supports the logical reasoning. In the current situation under study the Player 1 will produce electricity and will sell it at the market price of 2.515. And Player 2 will adopt a passive role and will supply excess permits required by Player1.

The following section presents the characteristics of market equilibrium.

Equilibrium Characteristics

Following are the key characteristics of the electricity market at equilibrium under the given scenario:

Production Control:

Producer	Role/Control
Player 1	Active
Player 2	Passive

Market Electricity Price = 2.515 units

Expected utility of the Player 1 = 0.515 units

Expected utility of the Player 2 =0.920 units

Total combined expected utility of all Producers =1.435 units

Expect amount of permits sold by player 2 to player 1 = 0.5004 units

Expected total cost to consumers = 2.515 units

Expected total Government's Revenue = 0.08 units

Expected Total Emission = 2.0 units.

The above results indicate that Player 2 is making huge profits without actually producing anything. This windfall profit is attributed to free allocation of permit whose cost is eventually borne by the end consumers.

In the following section the above numerical results have been validated against analytical solution.

Validation with Analytical Solution

The analytical expressions for equilibrium price and utility of each player have been derived in Chapter 2. The simulation model was validated by comparing simulation results with the analytical solution for the market under Base Case.

Table 3-2 Comparison of simulation result with analytical solution

	Analytical Solution	Simulation Result
Equilibrium Price	2.5	2.515
Production Control	Player 1 – Active Player 2 - Passive	Player 1 – Active Player 2 - Passive
Utility of Player 1	0.5	0.515
Utility of Player 2	1.0	0.920

It is observed from Table 3-2 that numerical results deviate slightly from the analytical solution due to limited discretization of strategy space for each player and the limited sampling for the stochastic modeling of demand. However taking these limitations into account, the numerical model approximates the analytical solution quite well.

In the next section sensitivity analysis of the market model has been discussed.

Sensitivity Analysis

The market equilibrium conditions are highly sensitive to policy parameters.

Sensitivity to the amount of Initial Permits

The effect of initial permits allocation on the equilibrium condition has been analyzed by varying the amount of initial permits (E) keeping it same for both the players and for this analysis penalty value has been kept constant at $\pi = 1$

1. Production Control Vs Initial Permits

Table 3-3 Production as a function of amount of initial permits (Model 1)

	Initial Permits, $0<E<0.75$	Initial Permits, $E>=0.75$
Active	Player 2	Player 1
Passive	Player 1	Player 2

When the amount of initial permits (E) is low, Player 2 has a competitive edge over Player 1 due to his lower emission rates and hence he takes production control. The critical price of both players are equal at E = 0.75 after which the critical price of Player 1 is lower than that of Player 2 as the benefit of lower production cost overcomes the loss due to emission penalty. Therefore for E>=0.75, Player 1 takes an active production control.

2. Electricity Price Vs Initial Permits

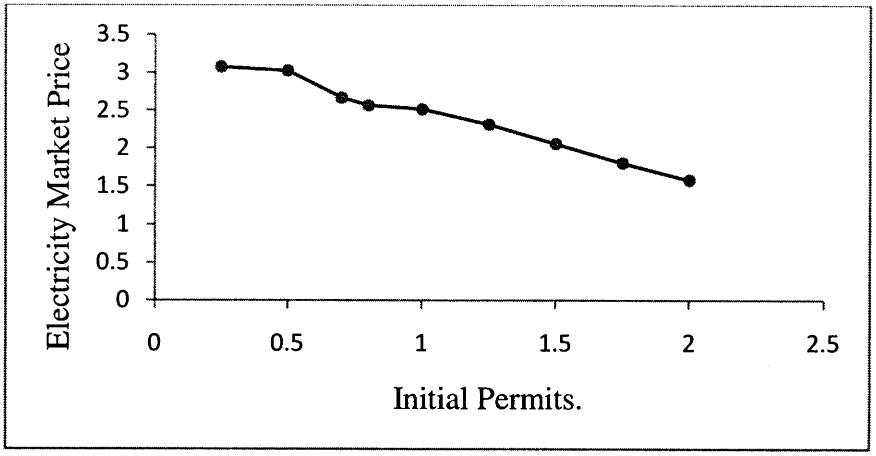

Figure 3-2a Variation of electricity price with the amount of initial permits (Model 1)

The larger number of initial permits results into lesser cost of buying the permits from other players and the lesser cost incurred in penalty. And the market price is directly proportional to cost of each producer. Therefore, we observe the decrease in the price of electricity with the increase in the amount of initial permits allotted. However it can also be observed from the figure that the electricity price remains constant for certain range of the number of initial permits. It can be explained by the following reasoning. The market electricity price is the critical price of the passive player. When the passive player is able to sell all his permits to the active player his bottom-line profit becomes independent of the initial permits allotted. Moreover, the critical price is the sum of cost and bottom-line profit, therefore the critical price becomes independent of the number of initial permits allotted. When the number of initial permits lies between 0 and 0.5 or between 0.75 and 1, the passive player is able to sell all his permits to the active player and hence we observe constant electricity price.

3. Net Consumers' Cost Vs Initial Permits

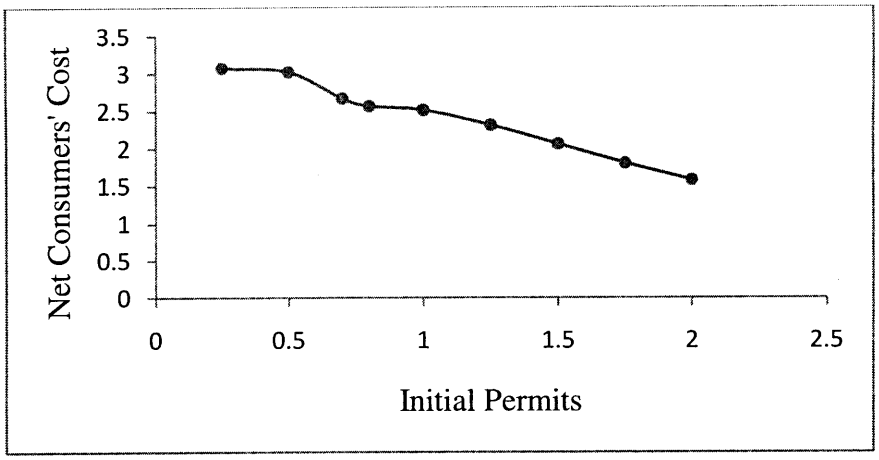

Figure 3-2b Variation of net consumers' cost with the amount of initial permits. (Model 1)

Net consumers' cost is directly proportional to the price of electricity. Since the demand distribution is constant, the variation of net consumers' cost with respect to amount of initial permits is exactly similar to that of electricity market price.

4. Total Producers' Utility Vs Initial Permits

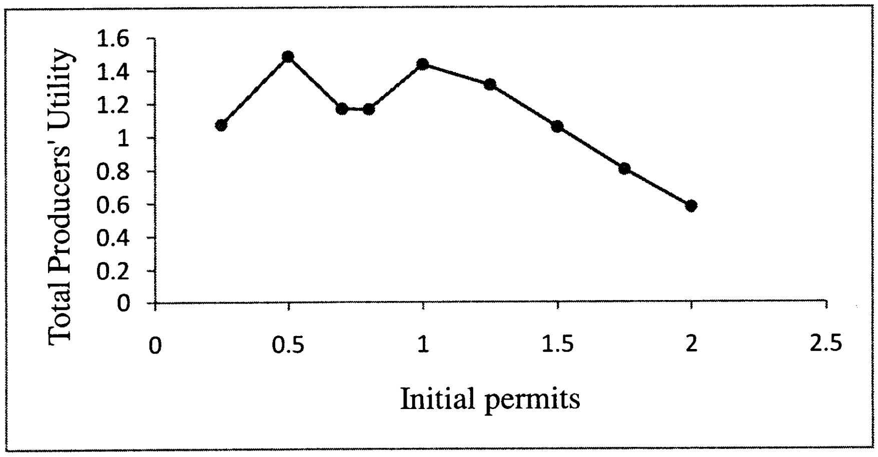

Figure 3-2c Variation of total producers' utility with the amount of initial permits (Model 1)

The total profit increases linearly with the increase in initial permits if and only if the passive player is able to sell all his permits to the active player. Otherwise the total profit decreases linearly with the increase in number of initial permits due to linear decrease in electricity price. The points of local maxima of total profit correspond to the points of local maxima of the amount of permits trade where the windfall profit is maximum. The point of local minimum of total utility corresponds to the value of the initial allotted permits where the critical prices of both players become almost equal and the shift of production control takes place. The local minimum occurs due to the fact that the active player looses the advantage of fixing price above his critical price by utilizing the gap between his critical price and that of the other player.

5. Government's Revenue Vs Initial Permits

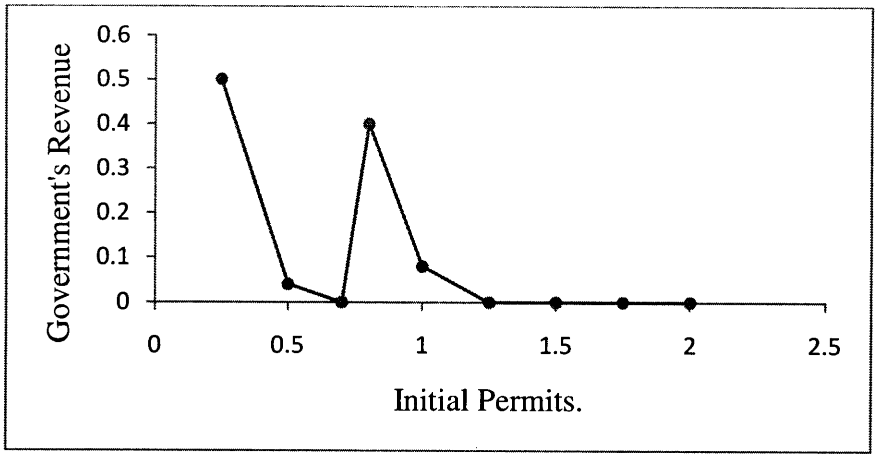

Figure 3-2d Variation of government's revenue with the amount of initial permits (Model 1)

Government's revenue should decrease with the increase in number of initial permits due to decrease in penalty. However the discontinuity or the sudden jump in government's revenue as observed in Figure 3-2d is due to shift in production control resulting into sudden rise in total emission.

6. Total Emissions Vs Initial Permits

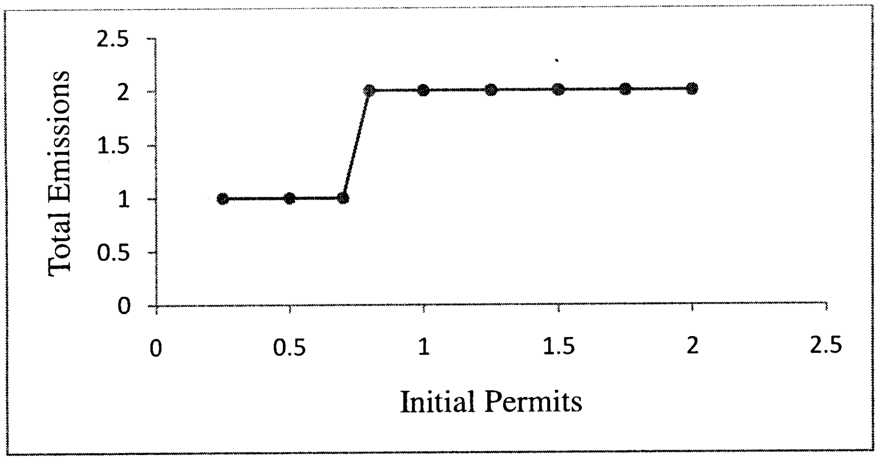

Figure 3-2e Variation of total emissions with the amount of initial permits (Model 1)

A total emission is just the function of the production control because the demand distribution is same.

Sensitivity to Penalty Value

The effect of penalty value on the equilibrium condition has been analyzed by varying penalty value keeping all other parameters same as in base case.

The number of initial permit allotted to each player, E = 1

Penalty, π = Variable

1. Production Control Vs Penalty

The Production Control is independent of the penalty value. Because any change in penalty value leads to exactly similar changes in utility functions and critical prices of both the players. Following production control was observed:

Active: Player 1

Passive: Player 2

2. Electricity Price Vs Penalty

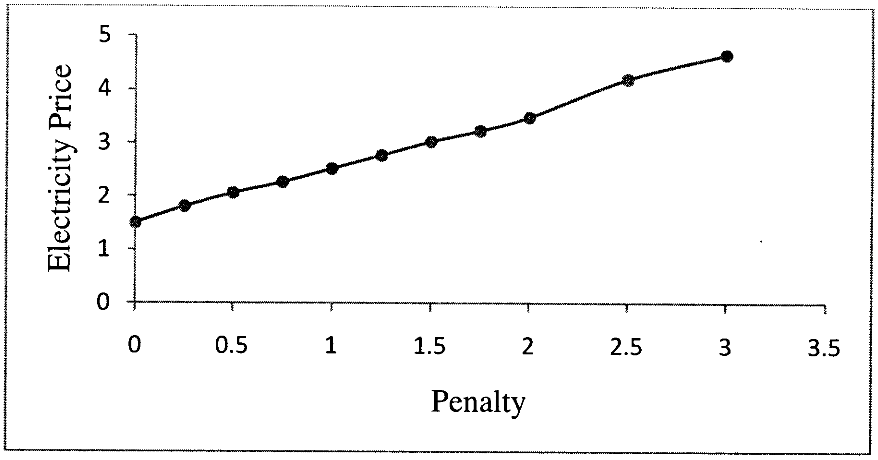

Figure 3-3a Variation of electricity price with the penalty (Model 1)

The critical price of each player is a linear function of penalty value. Therefore the market electricity price increases linearly with the increase in penalty value.

3. Net Consumers' Cost Vs Penalty

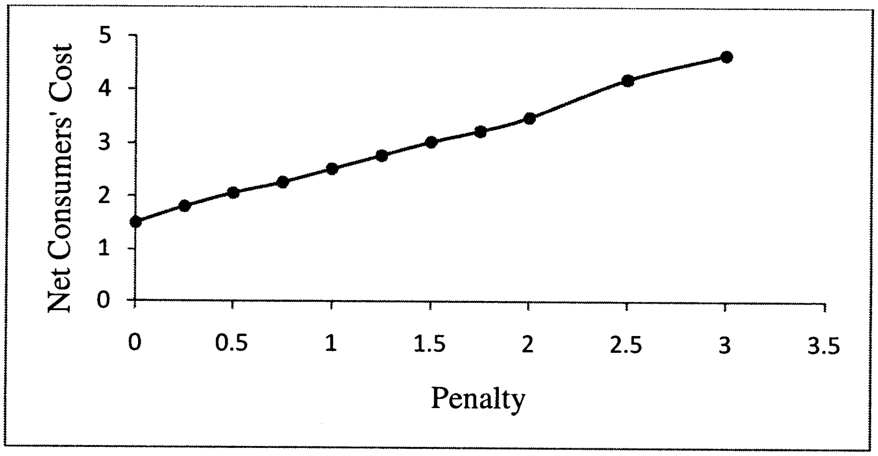

Figure 3-3b Variation of net consumers' cost with the penalty (Model 1)

Net consumers' cost is a direct function of electricity price and therefore we observe a similar variation with penalty value.

4. Total Producers' Utility Vs Penalty

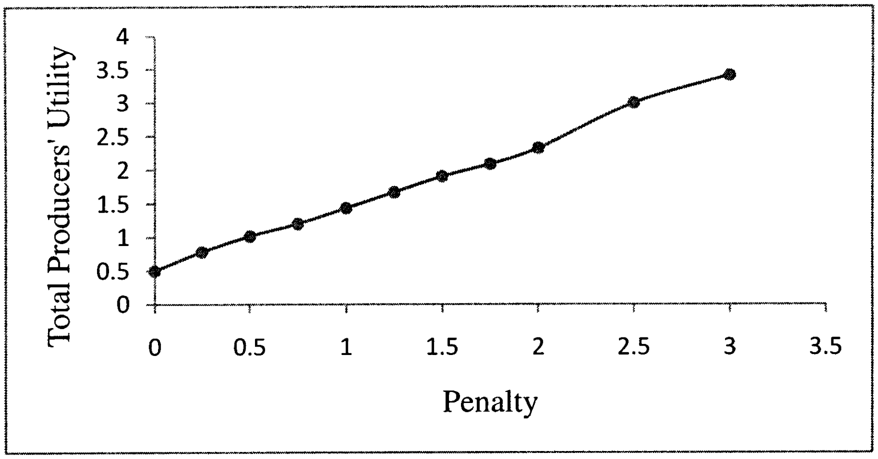

Figure 3-3c Variation of total producers' utility with the penalty (Model 1)

In the Two-player model the price of permits is always equal to penalty value. Therefore the rise in penalty value results into similar increase in windfall profit of the passive player due to increase in permit prices. Hence the overall utility increases with the increase in penalty.

5. Government's Revenue Vs Penalty

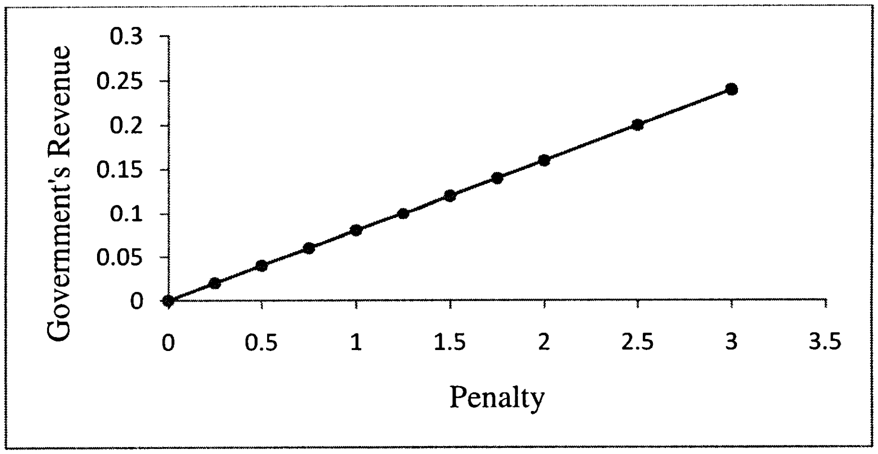

Figure 3-3d Variation of net government's revenue with the penalty (Model 1)

Because demand distribution and production control are independent of the penalty value and it is always the same, the total amount of emission that has been penalized is also constant. Therefore government's revenue is a linear function of penalty.

6. Total Emission Vs Penalty

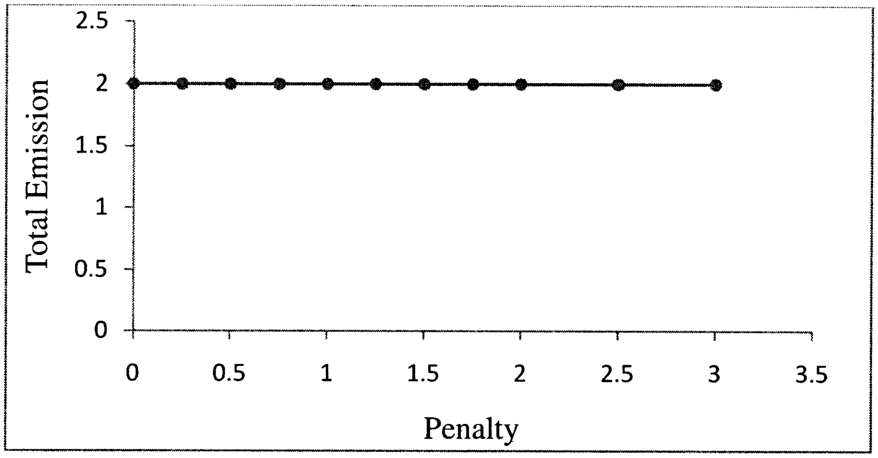

Figure 3-3e Variation of total emissions with the penalty (Model 1)

Since production control is independent of penalty, the total emission is also constant with respect to variation in penalty.

3.1.2 Carbon Taxation

Under the Carbon taxation policy each player is levied upon a carbon tax per unit emission exceeding pre-determined Non-Taxable Emissions (N.T.E.). The Government can control market equilibrium by adjusting carbon tax rate and N.T.E. Let us consider a base case with the N.T.E., $E = 1$ unit for each player and the tax rate, $\alpha = 3$/unit.

Detailed Result – Base Case

Non-Taxable Emission, $E = 1$ and Tax Rate, $\alpha = 1$/unit

Again the graphical approach has been employed to arrive at market equilibrium.

Following are the key characteristics of the electricity market at equilibrium under the given scenario:

Production Control:

<u>Producer</u>	<u>Role/Control</u>
Player 1	Active
Player 2	Passive

Market Electricity Price = 2.0 units.

Net Consumers' Cost = 2.0

Expected utility of Player 1 = 0 units.

Expected utility of Player 2 = 0.453 units.

Total expected utility of players = 0.453units.

Expected total Government's Revenue = 0.04 unit.

Expected Total Emission = 1.0 units.

In the subsequent sections sensitivity of the model has been analyzed with respect to policy parameters.

Sensitivity Analysis

In the Carbon taxation policy, government can control equilibrium condition by adjusting tax rate and the N.T.E. Sensitivity of the equilibrium conditions with respect to these two parameters has been presented in the next two sections.

Sensitivity to the Carbon Tax Rate

N.T.E., E = 1 and Tax Rate, α = variable

1. Production Control Vs Tax Rate

Table 3-4 Production control as a function of tax rate (Model 1)

	Tax Rate, $0 < \alpha \leq 0.5$	**Tax Rate, $\alpha > 0.5$**
Active	Player 1	Player 2
Passive	Player 2	Player 1

Since Player 1 has higher emission rates he will be producing only when the tax rate is lower than 0.5. For tax rate higher than 0.5, Player 2 gets competitive edge over Player 1 due to his lower emission rates.

2. Electricity Price Vs Tax Rate

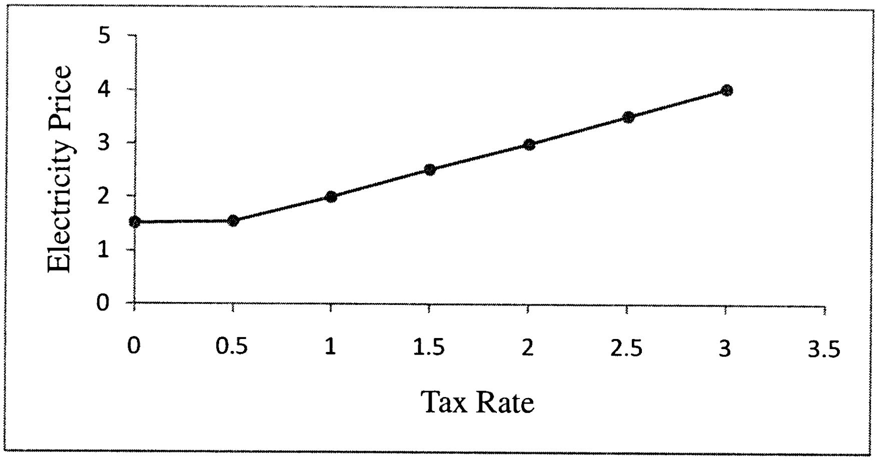

Figure 3-4a Variation of electricity price with the tax rate (Model 1)

The electricity market price is the critical price of the passive player. When Tax Rate is less than or equal to 0.5, Player 1 takes a production control as the advantage of lower cost of production overcomes the additional cost of tax (due to higher emissions) with the lower tax rate. Hence the critical price of Player 2 decides the equilibrium price of electricity. For the current demand distribution, only the small fraction of emission by Player 2 falls above the N.T.E. value (if player 2 produces). Therefore critical price of Player 2 is not very sensitive to the tax rate. And therefore we observe almost constant electricity price for tax rate below 0.5. However approximately at tax rate = 0.5, the shift of production control occurs which gives Player 1 a role of passive producer. The emission of Player 1 exceeds the N.T.E. value considerably for the current demand. Therefore his critical price is highly sensitive to tax rate. Hence we observe a linear increase in price with the increase in tax rate when it is greater than 0.5.

3. Net Consumers' Cost Vs Tax Rate

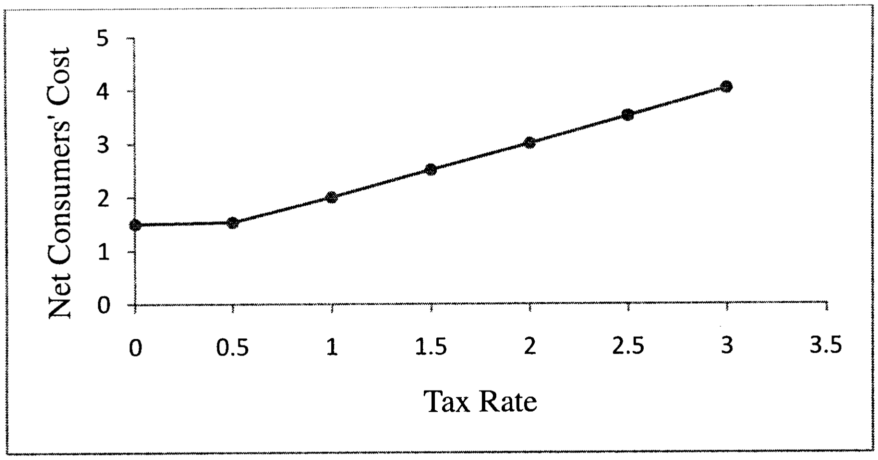

Figure 3-4b Variation of net consumers' cost with the tax rate (Model 1)

4. Total Producers' Utility Vs Tax Rate

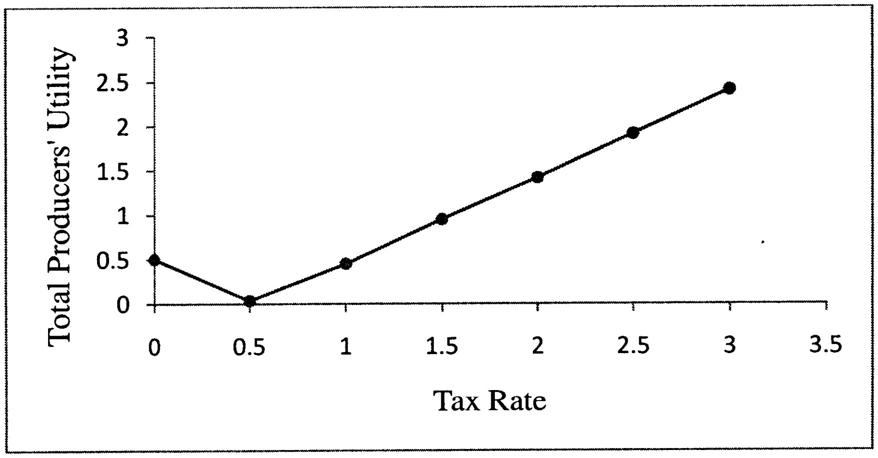

Figure 3-4c Variation of total producers' utility with the tax rate (Model 1)

The total utility of producers decreases with the increase in tax rate if the price is constant. However when the price of electricity starts increasing with the tax rate, the total utility of producers also increases.

5. Government's Revenue Vs Tax Rate

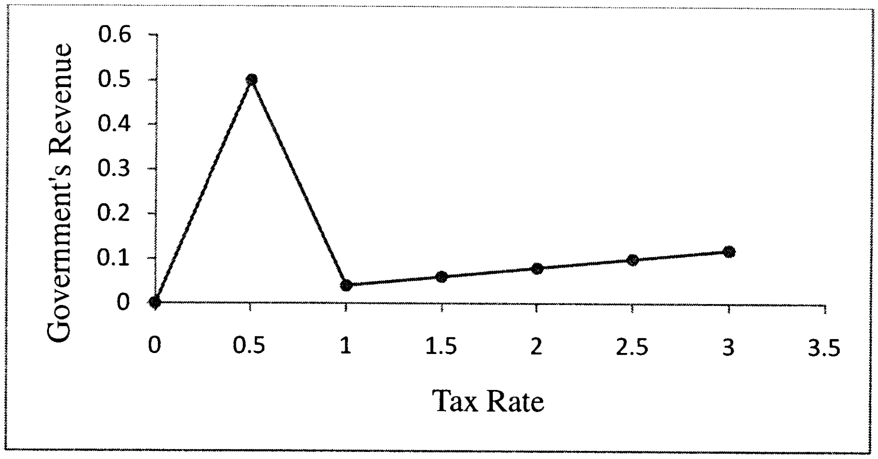

Figure 3-4d Variation of government's revenue with the tax rate (Model 1)

Government's revenue should increase with the increase in tax rate. The point of discontinuity and the sudden drop in the government's revenue at tax rate = 0.5 is due to the sudden drop in emission because of shift of production control from Player 1 to Player 2. The larger slope is observed when Player 1 is an active producer because slope is the measure of taxable emissions which is definitely much higher in the production control of Player 1.

6. Total Emissions Vs Tax Rate

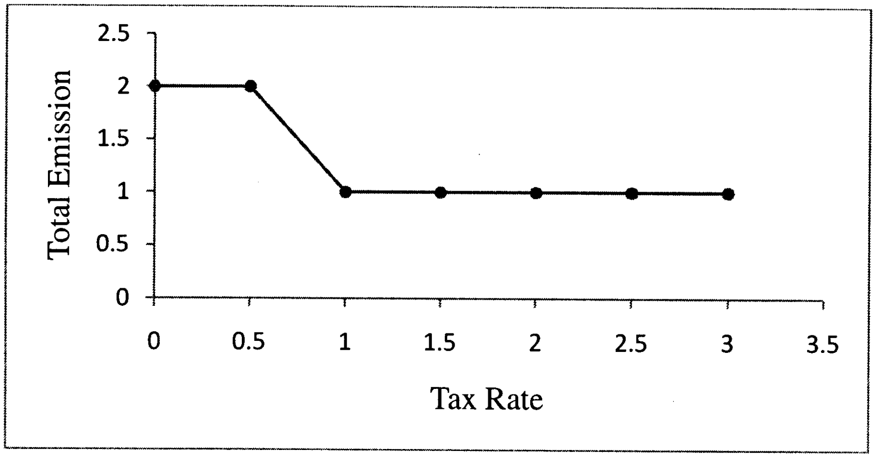

Figure 3-4e Variation of total emissions with the tax rate (Model 1)

Total emission drops at tax rate = 0.5 because of shift in production control from Player 1 to Player 2.

Sensitivity to Non-Taxable Emissions (N.T.E.)

Tax Rate, alpha = 1 and N.T.E., E = variable

Increasing N.T.E. has exactly the reverse effect of increasing Tax Rate as also observed in the figures below

1. Production Control Vs N.T.E.

Table 3-5 Production control as a function of N.T.E. (Model 1)

	N.T.E., $0<E\leq1.5$	**N.T.E., $E>1.5$**
Active	Player 2	Player 1
Passive	Player 1	Player 2

2. Electricity Price Vs N.T.E.

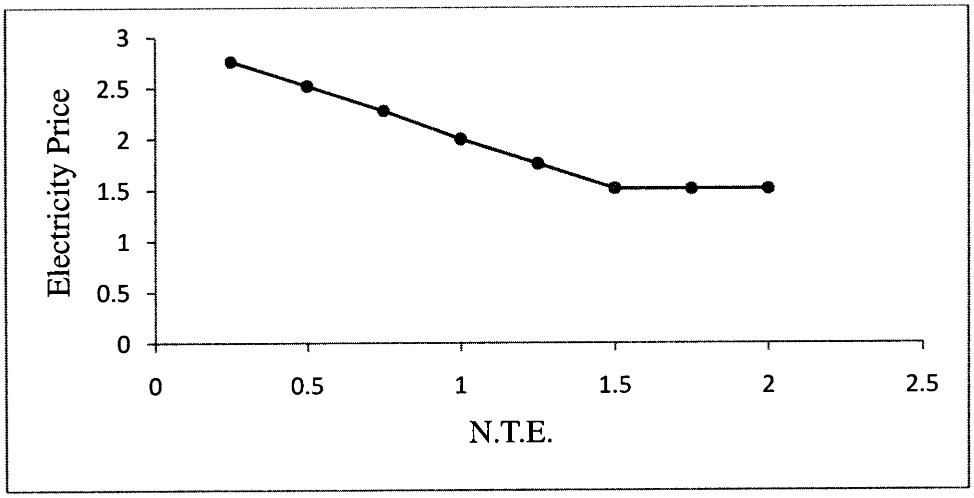

Figure 3-5a Variation of electricity price with N.T.E. (Model 1)

The market price is equal to the passive player's critical price. When the N.T.E. is low (less than or equal to 1.5), Player 1 acts as a passive player and his critical price is a strong function of N.T.E. for the given range of N.T.E. in which he acts as a passive player. Therefore electricity price decreases linearly with the increase in N.T.E. However for N.T.E. > 1.5, Player 1 takes a production control and Player 2 becomes a passive producer. The critical price of Player 2 is not very sensitive to the N.T.E. when N.T.E. is greater than 1.5 as in this case majority of his emission always lie within the N.T.E. Therefore the electricity price is almost constant for the N.T.E. >= 1.5.

3. Net Consumers' Cost Vs N.T.E.

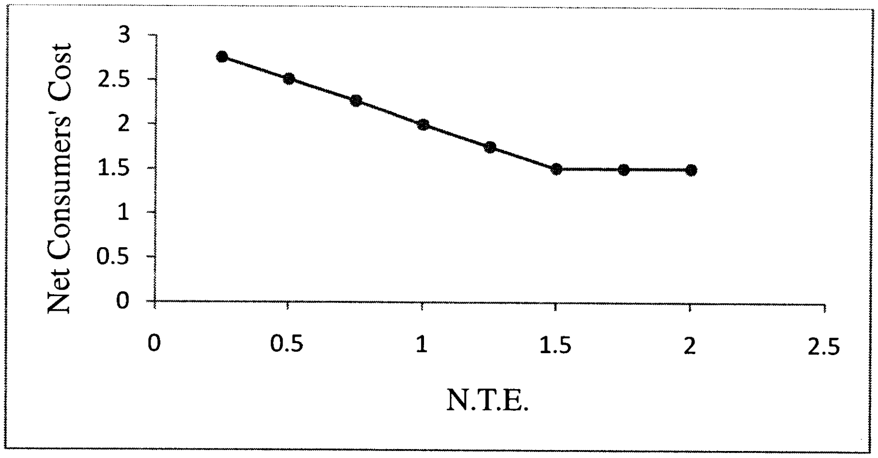

Figure 3-5b Variation of net consumers' cost with N.T.E. (Model 1)

4. Total Producers' Utility Vs N.T.E.

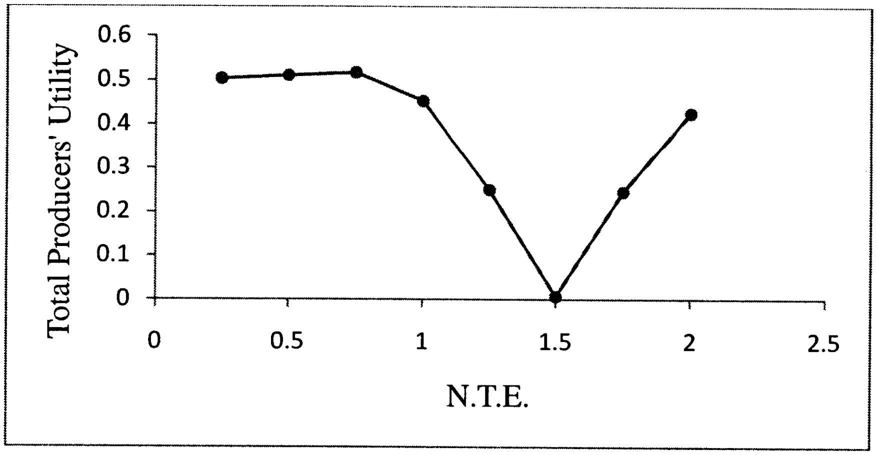

Figure 3-5c Variation of total producers' utility with N.T.E. (Model 1)

For the low values of N.T.E., the electricity price linearly decreases with increase in N.T.E. Therefore the benefit of increase in N.T.E. to the producers is neutralized by the loss due to reduction in price. And hence we observe almost constant total producers utility for N.T.E. less than or equal to 0.75. However for the N.T.E. > 0.75, Player 2 doesn't get much advantage from increase in N.T.E. However electricity price still decreases at the same rate. Therefore the increase in N.T.E. over 0.75 leads to linear decrease in the total producer's utility. Total utility suddenly increases at N.T.E. = 1.75 dues to shift in production control which makes the electricity price constant.

5. Government's Revenue Vs N.T.E.

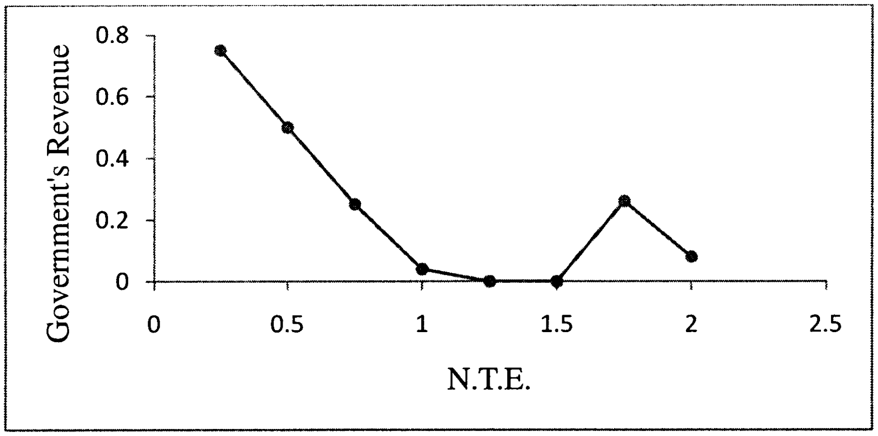

Figure 3-5d Variation of government's revenue with N.T.E. (Model 1)

It is intuitive that government's revenue drops with the rise in N.T.E. The sudden rise in revenue at N.T.E = 1.75 is due to production control shift from Player 2 to Player1.

6. Total Emissions Vs N.T.E.

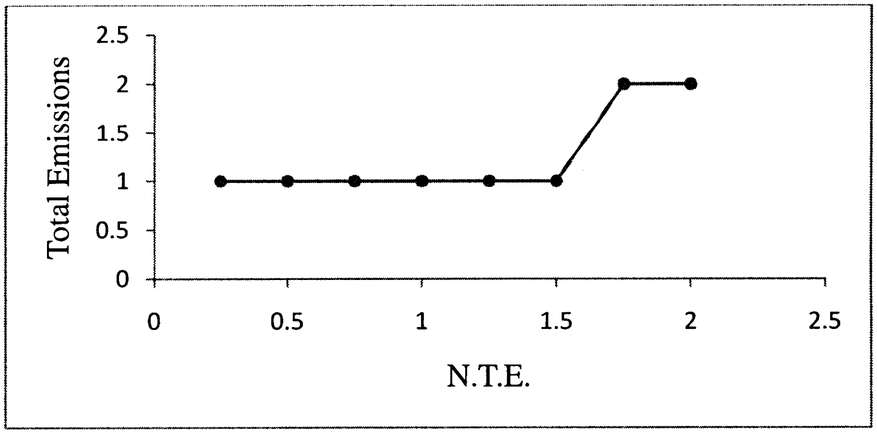

Figure 3-5e Variation of total emissions with N.T.E. (Model 1)

Total emission is a function of the production control. Therefore we observe the above trend.

Following section presents comparison of market equilibrium condition under Taxation and Cap and Trade Policy.

3.1.3 Comparison of Policies

For the same level of emission, carbon taxation policy ensures lesser cost to consumers as compared to cap-and-trade policy. Total producers' utility is higher in case of cap-and-trade policy due to significant amount of windfall profit earned by passive player. For the given N.T.E. (E) = 1 units for each player, the optimum tax rate that ensures least price with least emission is $\alpha = 1$ and corresponding electricity price and total emission are P = 2 and Etot = 1 unit respectively. For the same total emission of 1 unit under cap-and-trade policy the minimum electricity price is P = 2.67 at Initial Permits, E = 0.5.

To sum up, cap-and-trade policy generates higher total producers' utility and higher net consumers' cost for the same emission level as compared to carbon taxation policy. This is due to the fact that one of the producers makes profit without even producing any quantity as he sells the permits allotted to him free of cost. The cost of these permits is then transferred to end consumer by the active producer by incorporating this cost in his selling price. Therefore the brunt of windfall profit by producers is borne by the end consumer.

3.2 Analysis of Market Based on Model 3

The Market includes three competing producers with different generation technologies and hence they posses different cost and emission parameters. Each of them competes to supply electricity to a single controller whose demand is random and is assumed to have normal distribution with known mean and standard deviation. The three producers namely, Player 1, Player 2 and Player 3 use the following three technologies respectively: 1) Supercritical Pressure Coal Fired Power Plants, 2) Integrated Gasification Combined Cycle (I.G.C.C.) and 3) I.G.C.C with carbon capture and storage system. It is obvious that the production cost increases and the emission rate decreases as we move from Technology 1 to Technology 3. Each producer has certain production capacity constraint. Table 3-6 illustrates the parameter values associated with each producer:

Table 3-6 Production parameters of players (Model 3)

Parameter	Player1	Player 2	Player 3
Production Technology	Coal Fired	I.G.C.C.	I.G.C.C. with C.C.S.
Unit Production Cost, c	1	1.5	2
Emission Rate, e	2	1.5	1

This market situation is modeled using Model 3 as discussed in Chapter 2

Let us consider two key scenarios which differ only in terms of individual production capacities of the producers and the electricity demand of the controller. Scenario 1 involves equal capacity for all the three producers. However in reality different producers have different capacities especially when they use different technologies. Therefore Scenario 2 is introduced to model the market where each player has different production capacity.

3.2.1 Scenario 1: Players with Equal Production Capacities

All producers have same production capacity = 1 unit. The demand is normally distributed with mean = 1.5 and standard deviation about mean = 0.1. Following sections presents results and analysis on this scenario under different regulatory policies:

3.2.1.1 Cap-and-Trade Policy

Since all producers have equal production capacity, it is logical and fair to allot equal initial emission permits to each player. The Government can adjust the amount of initial permits to be allotted to each player to control the equilibrium conditions. The other critical policy parameter that influences equilibrium condition greatly is the penalty value. The following sections present a detailed result for one base case with particular penalty value (π) and initially allotted permits (E) and sensitivity analysis of the results w.r.t. policy parameters and demand uncertainty as they deviate from the base case values.

Detailed Result - Base Case

Initial Permits, $E = 1.3$ and Penalty, $\pi = 1$

Standard deviation of Demand Distribution, Std.Dev. (D) = 0.1

The equivalent extensive form game has been modeled to find the equilibrium. Each iteration corresponds to each stage of the game where player sets his permit price in turn as a best response to permit prices set by other players. And within each stage there is another extensive form game for electricity price competition. Therefore it is a two level extensive form game. The Nash equilibrium is possible for the electricity price competition. However in the upper level game (permit trade) pure Nash Equilibrium is non-existant and the strategy profile runs in a loop. Therefore players coordinate among themselves to fix a strategy profile that maximizes their combined utility within a loop.

Table 3-7 Iterations to find equilibrium in base case (Model 3 and Scenario 1)

Iteration No.	Permit Price			Equilibrium Electricity Price			Total Utility Eut
	Player 1	Player 2	Player 3	Player 1	Player 2	Player 3	
1	0	0.98	1.02	2.0409	2.0432	2.042	1.0630
2	0	0.7711	1.02	2.0741	2.0764	2.0777	1.3646
3	0	0.7711	0.3735	2.0139	2.015	2.0162	1.2725
4	0.3737	0.7711	0.3735	2.0751	2.0762	2.0774	1.3643
5	0.3737	0.3723	0.3735	2.059	2.0601	2.0613	1.3401
6	0.3737	0.3723	0.3634	2.0715	2.0726	2.0738	1.3589
7	0.3636	0.3723	0.3634	2.0715	2.0726	2.0738	1.3589
44	0.2424	0.2394	0.2422	2.0384	2.0395	2.0407	1.3092
45	0.2424	0.2394	1.02	2.0391	2.0402	2.0414	1.3103
46	0	0.2394	1.02	2.0391	2.0402	2.0414	1.3103
47	**0**	**0.7711**	**1.02**	**2.0741**	**2.0764**	**2.0777**	**1.3646**

If the Game continues, the strategies from iteration 2 to iteration 47 will repeat indefinitely in a cycle. Therefore the players will opt for strategy that maximizes their overall utility. The highlighted row in bold letters in Table 3-7 represents the equilibrium condition of the game as it gives total maximum utility within the loop.

However, the individual profits of the producers are not necessarily maximum at this point as observed in Figure 3-6.

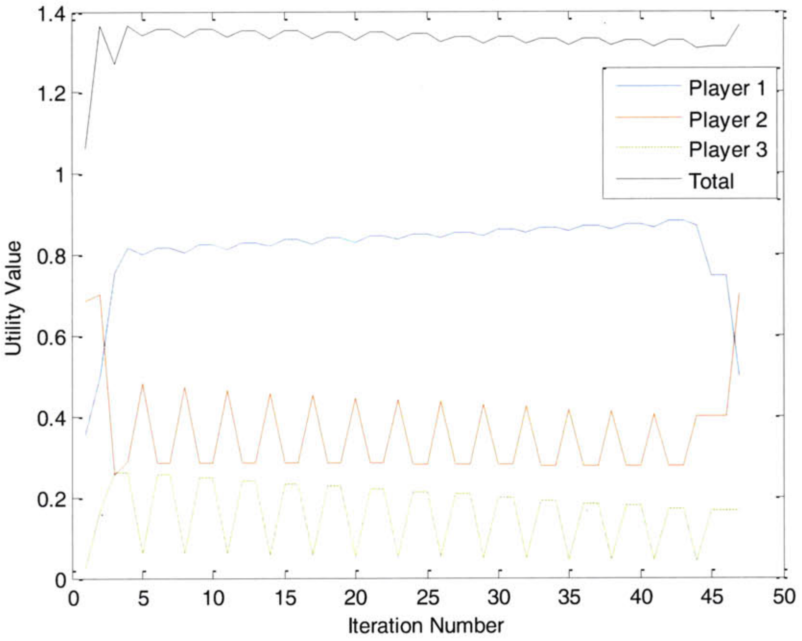

Figure 3-6 Utility of Players Vs Number of Iterations (Model 3 & Scenario 1)

Equilibrium Characteristics

Following are the key characteristics of the electricity market at equilibrium under the given scenario:

Production Control:

Producer	Role/Control
Player 1	Primary
Player 2	Secondary
Player 3	Contingency

Market Electricity Price = 2.0764 units

Expected utility of the Player 1 = 0.4962 units

Expected utility of the Player 2 = 0.7027 units

Expected utility of the Player 3 = 0.1657 units

Expected total combined utility of all producers = 1.3646 units

Average amount of permits sold by Player 2 to Player 1 = 0.5376 units

Average amount of permits sold by Player 3 to Player 1 = 0.1624 units

Expected total cost to consumers = 3.1146 units

Expected total Government's Revenue = 0

Expected total emission = 2.75 units

The following section presents the sensitivity analysis of the market equilibrium conditions with respect to policy parameters and demand uncertainty

Sensitivity Analysis

The equilibrium condition is highly sensitive to amount of initial permits allotted, penalty value and uncertainty in demand

Sensitivity to the amount of initial permits allotted:

Penalty value is kept constant at 1/unit and the amount of initial permits allotted has been varied from 1.2 to 1.5 with an interval of 0.05.

1. Production Control Vs Initial Permits (E)

Production control is always the same irrespective of amount of Initial Permits:

Primary Producer: Player 1
Secondary Producer: Player 2
Contingency Producer: Player 3

The optimal permit price strategy for equilibrium has been defined as a strategy profile that maximizes the total combined utility of all the producers. The overall utility is maximum if and only if the Player 1 produces as a primary producer and Player 2 produces as a secondary producer. When the amount of initial permits is low, the above production control ensures the maximum volume of permit trade resulting into highest total utility. Whereas when the amount of initial permits is high, then players with lesser production cost have a natural advantage to take production control. Therefore the same production control still gives the overall maximum utility. Hence the production control follows the same pattern irrespective of the amount of initial permits.

2. Electricity Price Vs. Initial Permits (E)

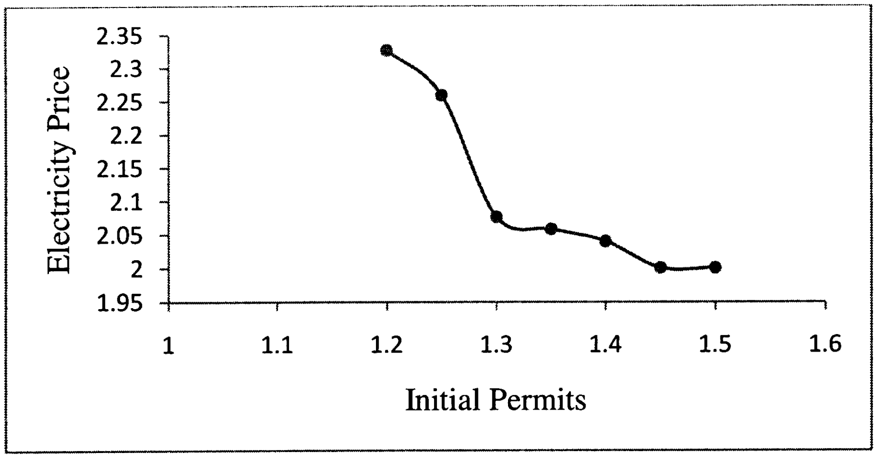

Figure 3-7a Variation of electricity price with Initial Permits (Model 3 and Scenario 1)

The decrease in electricity price with increase in initial permits is due to its direct impact on the total cost. Higher amount of initial permits results into lesser emission cost which reduces overall cost of electricity production. Therefore electricity price decreases.

3. Consumers' Cost Vs Initial Permits

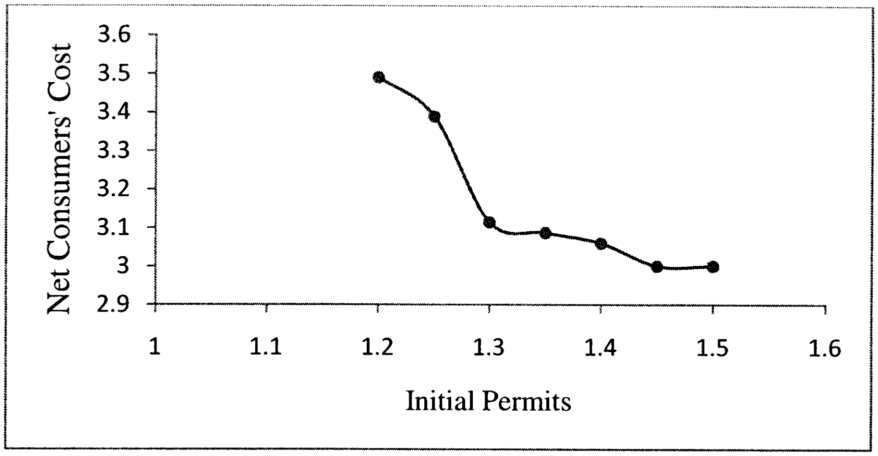

Figure 3-7b Variation of net consumers' cost with initial permits (Model 3 and Scenario 1)

4. Total Producers' Utility Vs Initial Permits

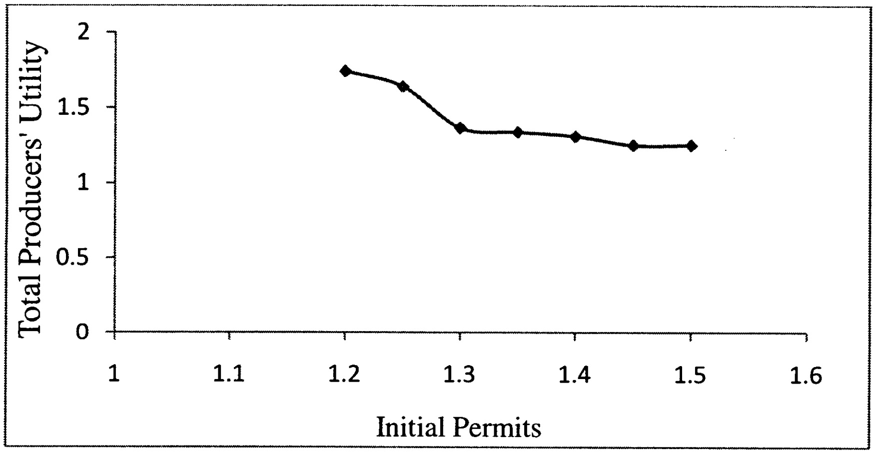

Figure 3-7c Variation of total producers' utility with initial permits (Model 3 and Scenario 1)

It can be observed from the above figure that total producers' utility drops down with the increase in the amount of initial permits. For the studied range of initial permits the net penalty paid to government is always zero. Besides the reduction in the amount of initial permits results into higher volume of permits trade among the players. And since these permits have been granted free of cost, it leads to significant windfall profits at lower amount of initial permits resulting into higher total producers' utility. Moreover the increase in cost incurred by active producers due to smaller amount of initial permits is counterbalanced by the increase in electricity price.

5. Emission Rate Vs Initial Permits

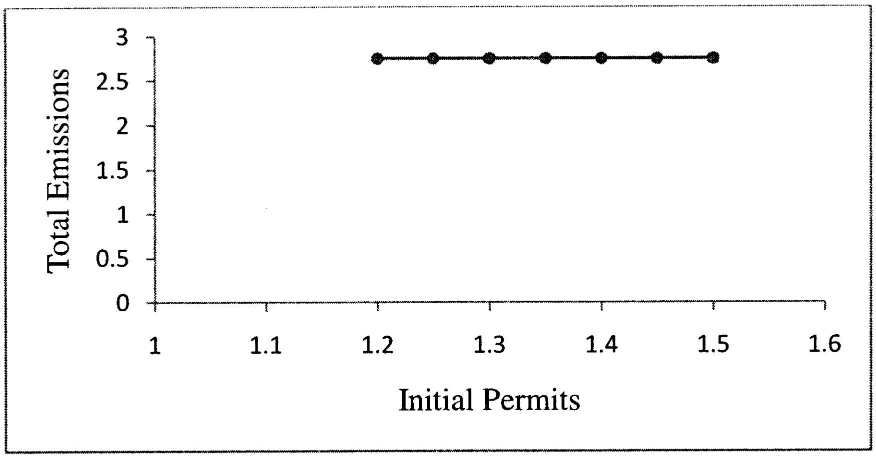

Figure 3-7d Variation of total emissions with initial permits (Model 3 and Scenario 1)

Since the production control and demand is always the same, the amount of total emissions is also constant.

Sensitivity to Penalty Value:

1. Production Control Vs Penalty

Production control is always the same irrespective of amount of penalty:

Primary Producer: Player 1

Secondary Producer: Player 2

Contingency Producer: Player 3

The penalty value affects critical price of each player similarly and hence it doesn't induce any impact on production control.

2. Electricity Price Vs Penalty

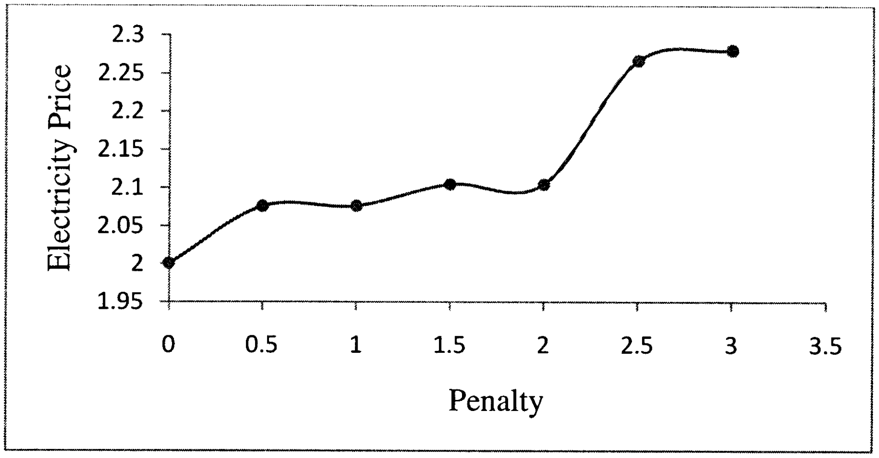

Figure 3-8a Variation of electricity price with penalty (Model 3 and Scenario 1)

Penalty value doesn't have any direct influence on the electricity price because total government's revenue is zero for the case under study. Penalty value provides the upper limit for the permit prices. Therefore the effect of increase in penalty value is transferred by the equivalent increase in the equilibrium permit price which in turn leads to increase in electricity price.

3. Net Consumers' Cost Vs Penalty

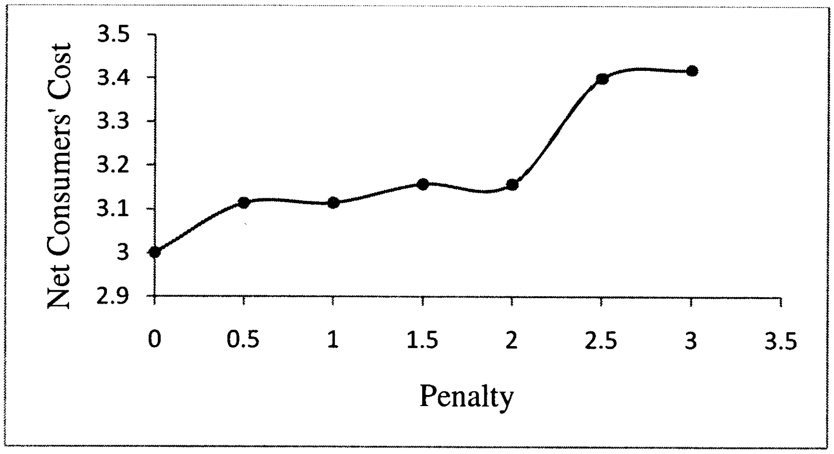

Figure 3-8b Variation of net consumers' cost with penalty (Model 3 and Scenario 1)

4. Total Producers' Utility Vs Penalty

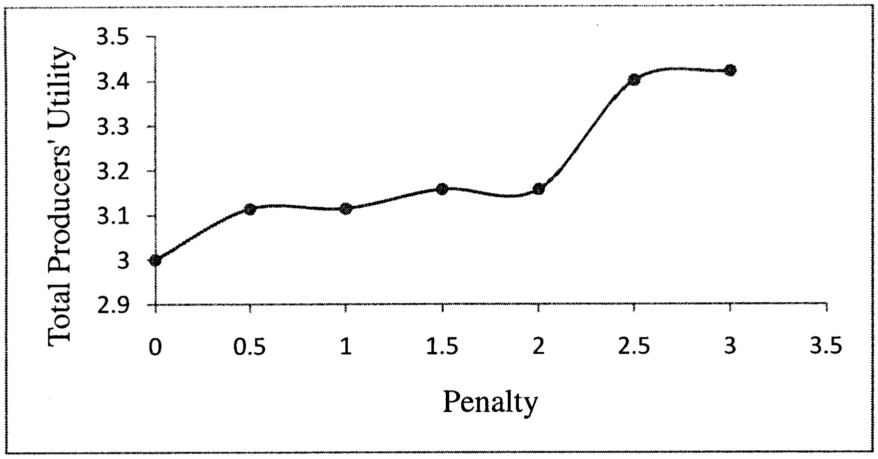

Figure 3-8c Variation of total producers' utility with penalty (Model 3 and Scenario 1)

Increase in penalty value causes increase in permit prices which in turn ensures larger windfall profits. Besides the primary producer transfers the cost of the increased permit prices to the end consumer by raising his price accordingly. Therefore the overall utility of producers increases with the increase in penalty value.

5. Total Emission Vs Penalty

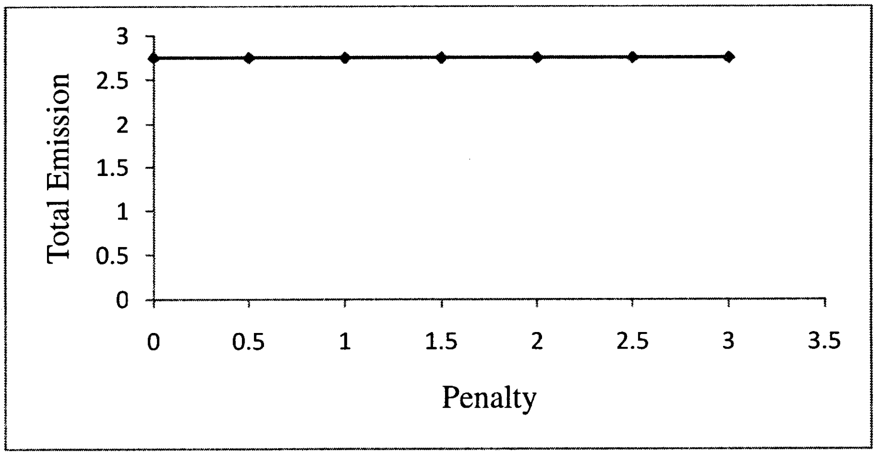

Figure 3-8d Variation of total emissions with penalty (Model 3 and Scenario 1)

Since production control and demand is same, total emission is also constant.

Sensitivity to Demand Uncertainty

1. Production Control Vs Standard deviation of demand distribution

Production control is always the same for all values of standard deviation of demand distribution:

Primary: Player 1

Secondary: Player 2

Contingency: Player 3

Demand uncertainty doesn't affect the production control.

2. Electricity Price Vs. Standard deviation of demand distributiom

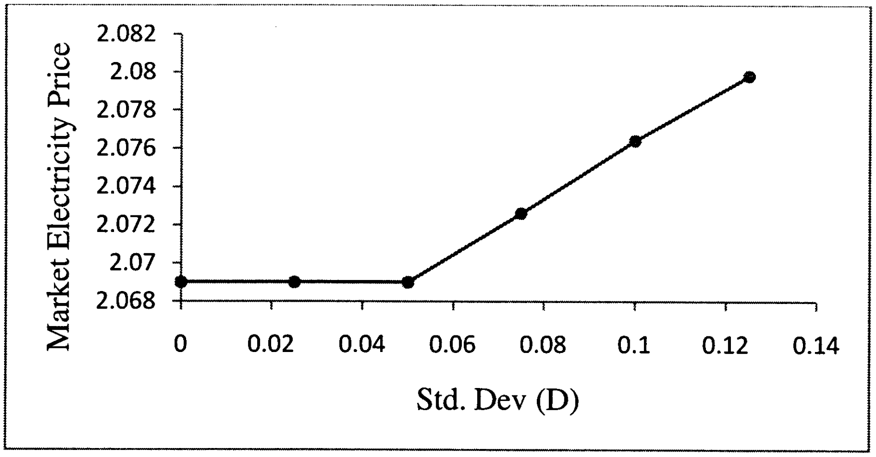

Figure 3-9a Variation of electricity price with Std. Dev (D) (Model 3 and Scenario 1)

The higher uncertainty in demand results into higher market equilibrium price for electricity due to the higher cost of risk involved.

3. Consumers' Cost vs. Standard deviation of demand distribution

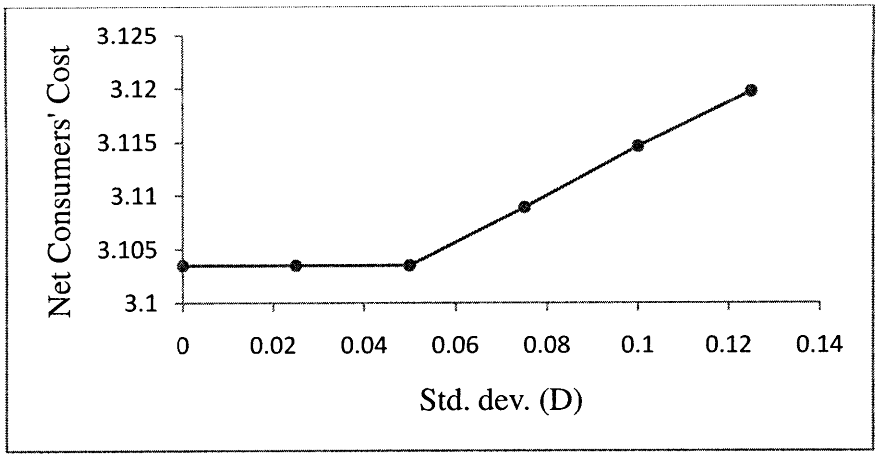

Figure 3-9b Variation of net consumers' cost with Std. Dev (D) (Model 3 and Scenario 1)

4. Producers' Utility Vs Standard deviation of Demand

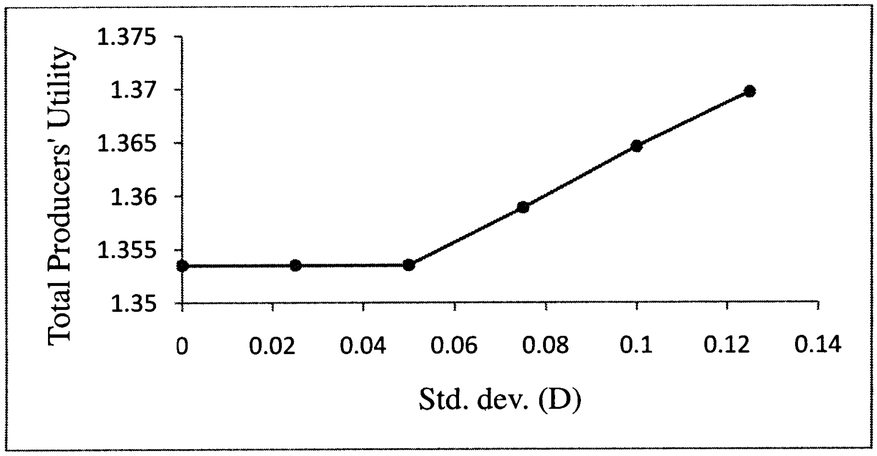

Figure 3-9c Variation of total producers' utility with Std. Dev (D) (Model 3 and Scenario 1)

Higher uncertainty in demand increases producers' total profit due to higher electricity market electricity price.

5. Emission Rate Vs Standard deviation of demand distribution

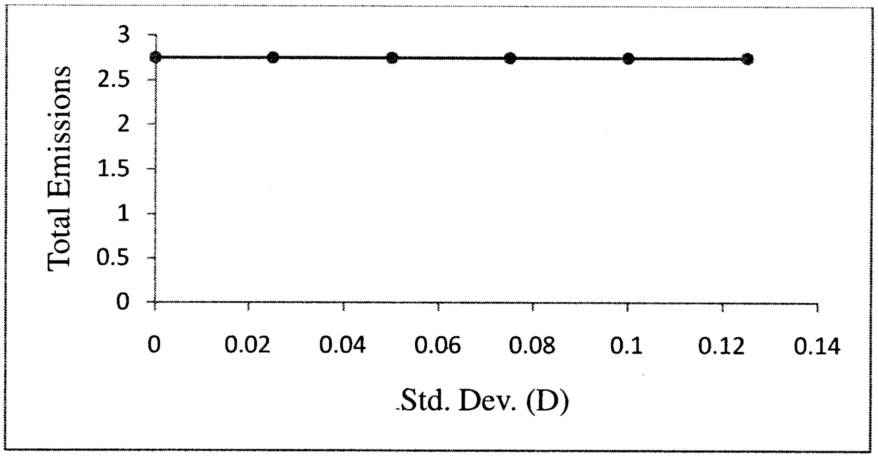

Figure 3-9d Variation of total emissions with Std. Dev (D) (Model 3 and Scenario 1)

Production Control is independent of uncertainty in demand and therefore the emission rate is also constant.

3.2.1.2 Carbon Taxation Policy

Carbon Taxation Policy has two critical parameters that influence equilibrium greatly: 1) Non-Taxable Emissions (E) and 2) Carbon Tax Rate(α). The Government can adjust N.T.E. and the Carbon Tax Rate to control the equilibrium conditions. The following sections present the detailed results and analysis on the sample case with a particular N.T.E. (E) and Carbon Tax Rate(α).

Detailed Results - Base Case

N.T.E., E = 1.3 and Tax Rate, $\alpha = 3$

The equivalent extensive form game has been used to arrive at equilibrium.

Following equilibrium characteristics were obtained on simulating the market under the above policy:

Production Control:

Producer	Role/Control
Player 1	Secondary
Player 2	Contingency
Player 3	Primary

Market Electricity Price = 2.098 units

Expected utility of the Player 1 = 0.5316 units

Expected utility of the Player 2 = 0 units

Expected utility of the Player 3 = 0.0978 units

Expected total combined utility of all producers = 0.6294 units

Expected total cost to consumers = 3.1467 units

Expected total Government's Revenue = 0.0173 units

Expected total emission = 2 units

Sensitivity Analysis

The equilibrium conditions are highly sensitive to policy parameters and demand uncertainty.

Sensitivity to Carbon Tax Rate

N.T.E. = 1.3 and Tax Rate = Variable

1. Production Control Vs Carbon Tax Rate

Table 3-8 Production control as a function of tax rate (Model 3 and Scenario 1)

	Tax Rate, $0 < \alpha \leq 0.5$	**Tax Rate,** $0.5 < \alpha \leq 2.7$	**Tax Rate ,** $2.7 < \alpha$
Primary	Player 1	Player 2	Player 3
Secondary	Player 2	Player 1	Player 1
Contingency	Player 3	Player 3	Player 2

At the lower values of tax rate, the effect of production cost is more predominant than that of the emission cost and therefore for tax rate less than 0.5 the players with lesser production cost take a production control. However, as the tax rate increases the emission cost becomes more significant and therefore the players with lesser emission rate take a production control for the higher values of tax rate.

2. Electricity Price Vs Tax Rate

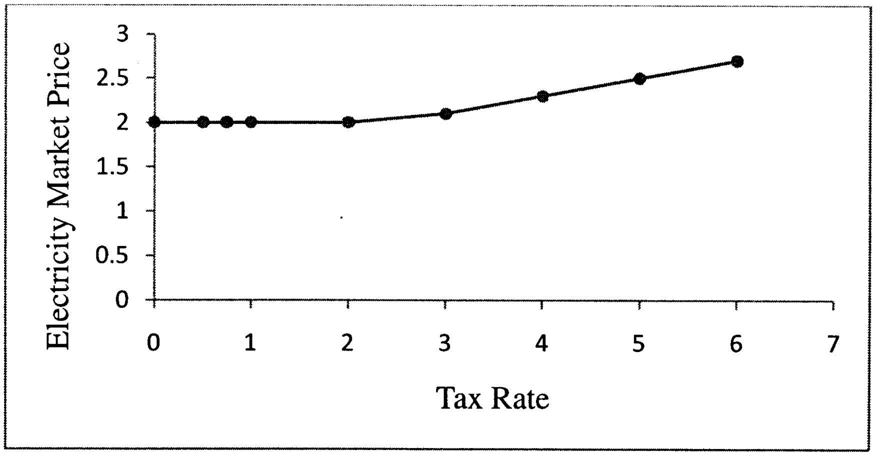

Figure 3-10a Variation of electricity price with tax rate (Model 3 and Scenario 1)

To meet the current demand with the given production capacity of each player almost always two players will be producing electricity. Therefore the equilibrium price should be the critical price of the Contingency Producer. When the tax rate is lower than 2 Player 3 acts as a contingency producer. Since Player 2 has his emission level well within the N.T.E. value, his critical price is independent of tax rate and is precisely equal to his production cost i.e. 2/units. Thatswhy we observe constant electricity price of 2/unit for tax rate less than 2. However when tax rate exceeds 2, the Player 3 takes a primary production control and Player 2 becomes contingency producer. The considerable fraction of emission by Player 2 exceeds the N.T.E. value and therefore his critical price is the strong function of the tax rate. Therefore electricity price increases linearly with tax rate when its value is greater than 2.

3. Consumers' Cost Vs Tax Rate

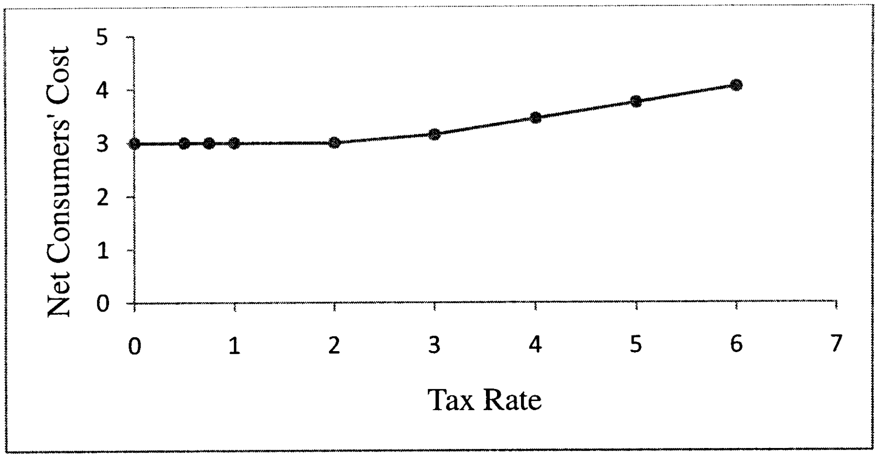

Figure 3-10b Variation of net consumers' cost with tax rate (Model 3 and Scenario 1)

4. Total Producers' Utility Vs Tax Rate

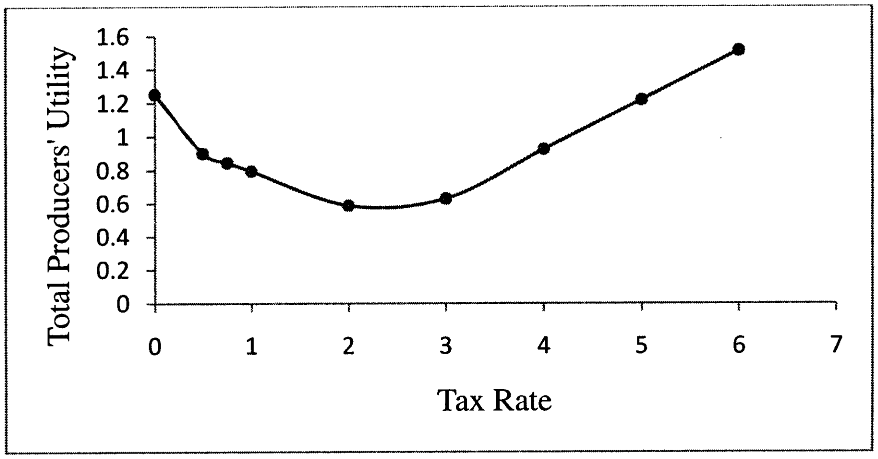

Figure 3-10c Variation of total producers' utility with tax rate (Model 3 and Scenario 1)

The total utility of producers decreases with the increase in tax rate if the price is constant. However when the price of electricity starts increasing with the tax rate, the total utility of producers also increases. This is because the primary producer's critical price is lesser sensitive to tax rate as compared to that of contingency producer which decides the equilibrium price. Therefore although the price of electricity increases with the increase in tax rate the cost borne my primary producer is almost the same resulting into linear rise in his profit with the increase in tax rate.

5. Government's Revenue Vs Tax Rate

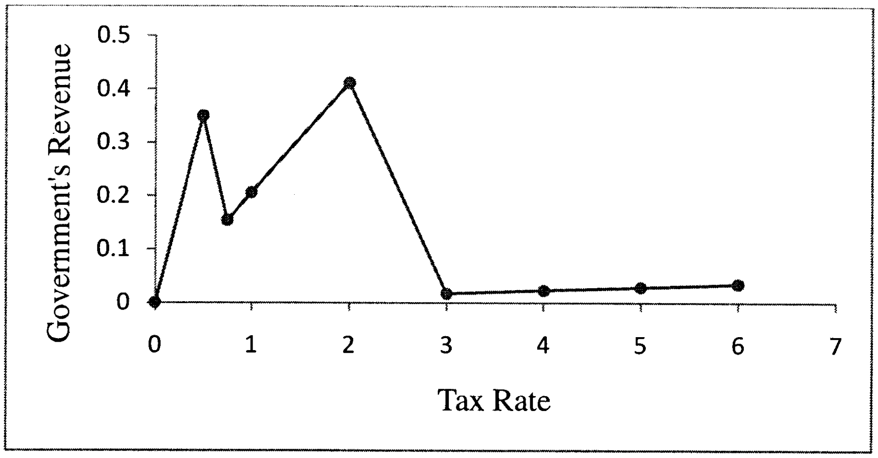

Figure 3-10d Variation of government's revenue with tax rate (Model 3 and Scenario 1)

As discussed earlier the breaks and sudden jump and decline in government's revenue attributes to the shift in production control. Higher slope indicates the higher emissions. And the slope decreases as the emission decreases.

6. Emission Rate Vs Tax Rate

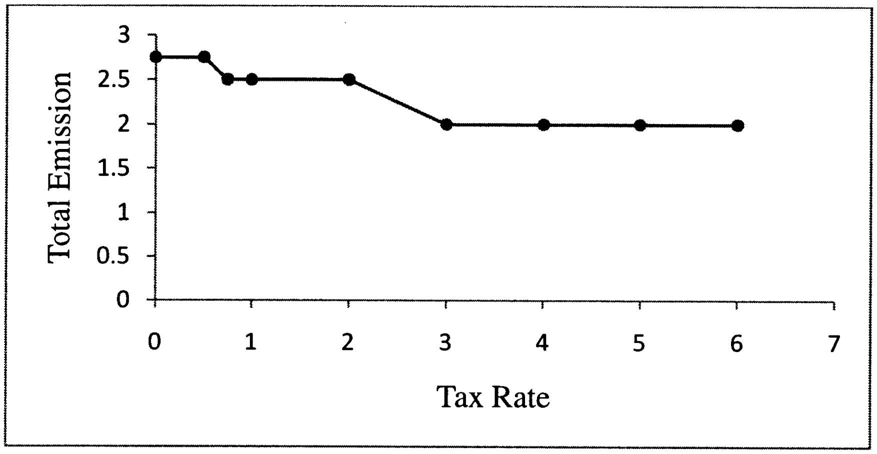

Figure 3-10e Variation of total emissions with tax rate (Model 3 and Scenario 1)

It is quite intuitive that as the tax rate increases, Players with lesser emission has a greater advantage to produce and hence it results into lesser total emission.

Sensitivity to N.T.E.

Tax rate = 3 and N.T.E. = variable

The variation of N.T.E. produces exactly reverse effect of that of varying Tax Rate as can be observed in the figures below.

1. Production Control Vs N.T.E.

Table 3-9 Production control as a function of N.T.E. (Model 3 and Scenario 1)

	N.T.E., $0 < E \leq 1.3$	**N.T.E., $1.3 < E \leq 1.9$**	**N.T.E., $1.9 < E$**
Primary	Player 3	Player 2	Player 1
Secondary	Player 1	Player 1	Player 2
Contingency	Player 2	Player 3	Player 3

2. Electricity Price Vs N.T.E.

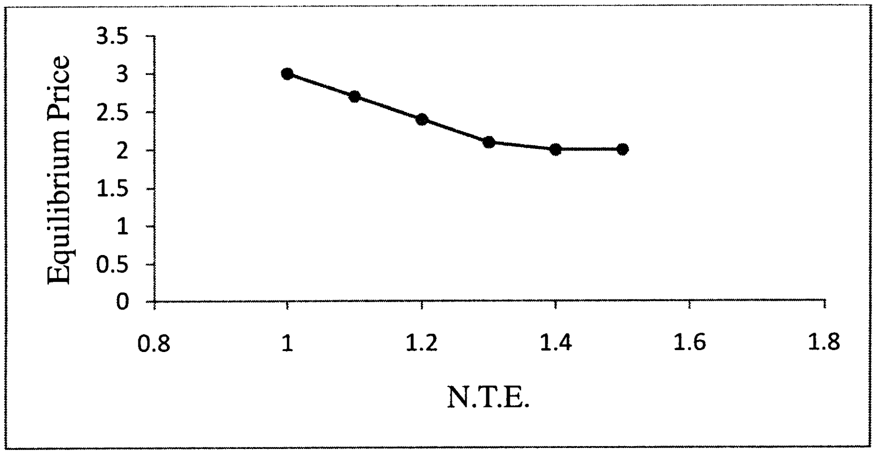

Figure 3-11a Variation of electricity price with N.T.E. (Model 3 and Scenario 1)

3. Consumers' Cost Vs N.T.E.

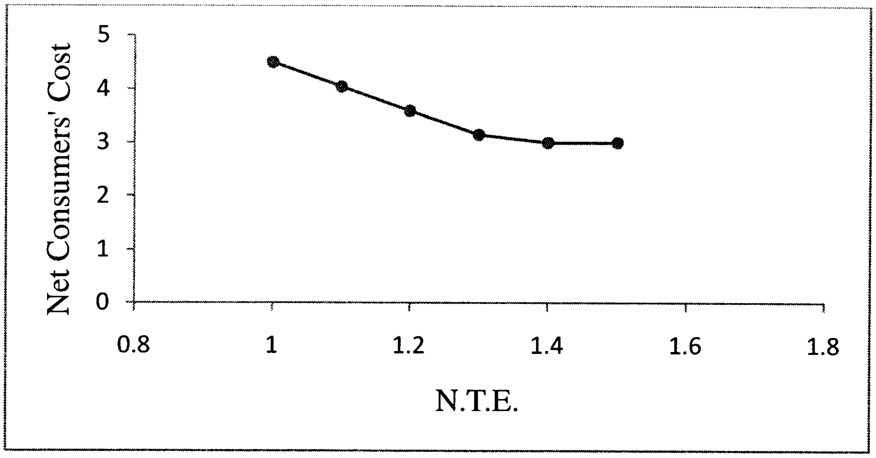

Figure 3-11b Variation of net consumers' cost with N.T.E. (Model 3 and Scenario 1)

4. Producers' Utility Vs N.T.E.

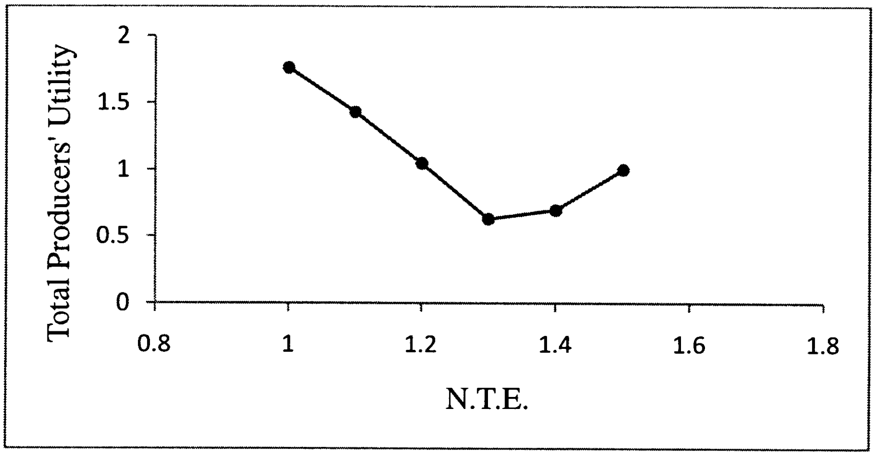

Figure 3-11c Variation of total producers' utility with N.T.E. (Model 3 and Scenario 1)

5. Government's Revenue Vs N.T.E.

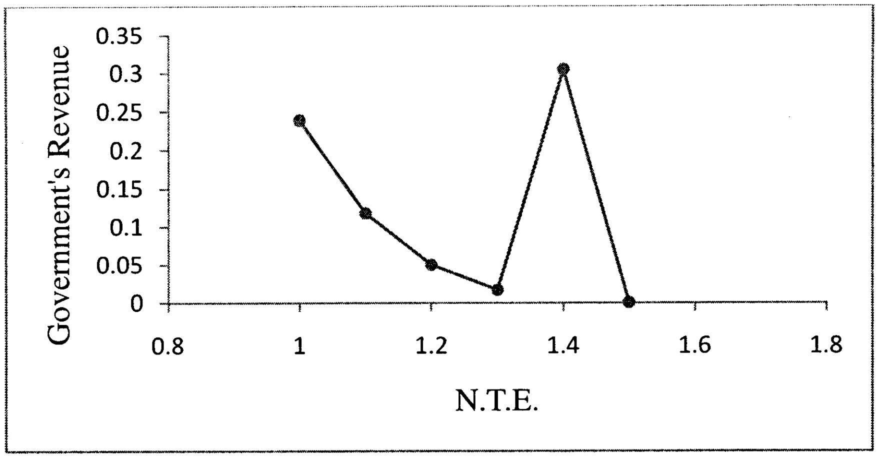

Figure 3-11d Variation of government' revenue with N.T.E. (Model 3 and Scenario 1)

6. Emission Rate Vs N.T.E.

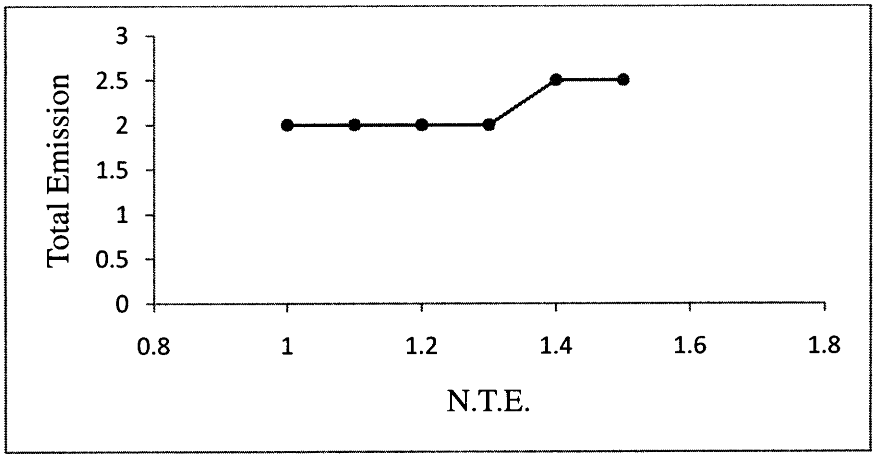

Figure 3-11e Variation of total emissions with N.T.E. (Model 3 and Scenario 1)

3.2.2 Scenario 2: Players with Different Production Capacities

In Scenario 1 all players had equal production capacity. However in most of the practical situations, different producers have different production capacities especially when they use different technologies. Therefore in Scenario 2 we release the assumption of equal capacity and present a situation where each player has different capacity from the other two players. The production capacities of the players are as follows:

Producer	**Production Capacity**
Player 1	1.4 units
Player 2	1.2 units
Player 3	1 units

Electricity Demand has a normal distribution with mean = 2 units and standard deviation about mean = 0.1. Rest all parameters are exactly the same as in Scenario 1.

3.2.2.1 Cap-and-Trade Policy

Since the production capacities of players are different, it is logical and fair to allot initial permits in proportion to the individual production capacity of the player. Let us call the proportionality constant as "Initial Permits Factor (I.P.F.)" and denote it with "p"

$$\text{Initial Permits Factor (I.P.F.) for Player i, } p_i = \frac{\text{Initial permits allotted to Player i}}{\text{Production capacity of Player i}}$$

Since I.P.F is same for all players, $p_i = p \quad \forall i$

Hence government can use I.P.F. as a throttle to control equilibrium conditions. The other critical policy parameter that influences equilibrium condition greatly is the penalty. The following sections present a detailed result for one sample case with particular penalty and I.P.F. and sensitivity analysis of the model with respect to I.P.F.

Detailed Results - Base Case

I.P.F., $p = 1.3$ and Penalty, $\pi = 1$

The same approach has been used to find equilibrium as discussed in Scenario 1. The Figure 3-12 shows the individual and total utilities of the players with respect to iteration number within the loop:

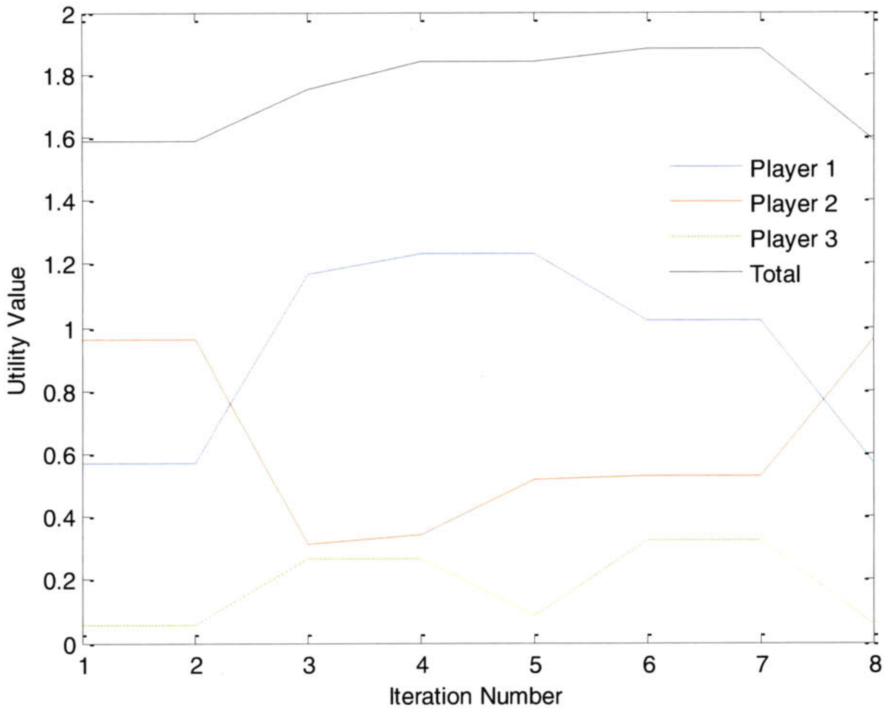

Figure 3-12 Utility of players Vs Number of iterations (Model 3 & Scenario 2)

The equilibrium is observed at 7th iteration which gives the maximum overall utility.

Following are the key characteristics of the electricity market at equilibrium under the given scenario:

Production Control:

Producer	Role/Control
Player 1	Primary
Player 2	Secondary
Player 3	Contingency

Market Electricity Price = 2.0906 units

Expected utility of the Player 1: 1.023 units

Expected utility of the Player 2: 0.5309 units

Expected utility of the Player 3: 0.3273 units

Expected total combined utility of all producers: 1.8812 units

Average amount of permits sold by Player 2 to Player 1 = 0.6592

Average amount of permits sold by Player 3 to Player 1 = 0.3208

Expected total cost to consumers = 4.1812 units

Expected Total Government's Revenue = 0 units

Expected Total Emission = 3.7 units

Significant windfall profits can be observed in the utilities of Player 2 and Player 3.

Sensitivity Analysis

The equilibrium condition is highly sensitive to amount of initial permits, penalty and uncertainty in demand. The sensitivity with respect to demand uncertainty and penalty value is similar to what observed for Scenario 1. Therefore only the sensitivity to initial permits has been presented.

Sensitivity to Initial Permits:

It is assumed that the amount of initial permits allotted (E) to any player is always proportional to his production capacity and the proportionality constant is Initial Permits Factor (p).

$$E = p \times k$$

Since production capacity is always constant for the given scenario, amount of initial permits allotted varies linearly with I.P.F. And therefore analyzing sensitivity of the results to the initial permits allotted is equivalent to analyzing its sensitivity with respect to I.P.F.

1. Production Control Vs Initial Permits Factor

Due to same reason as discussed for Scenario 1, the production control for Scenario 2 also stays the same irrespective of the I.P.F.:

Primary: Player 1

Secondary: Player 2

Contingency: Player 3

2. Electricity Price Vs I.P.F.

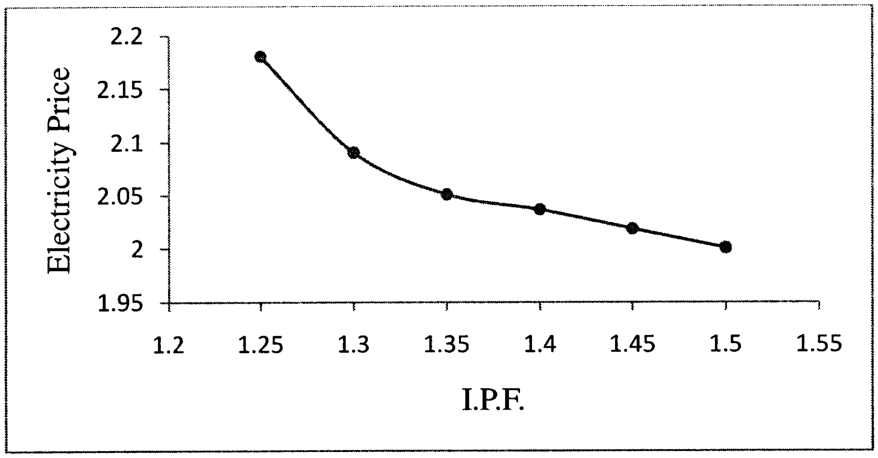

Figure 3-13a Variation of electricity price with I.P.F. (Model 3 and Scenario 2)

It is quite intuitive that the increase in I.P.F. leads into reduction in electricity market price.

3. Net Consumers' Cost Vs I.P.F.

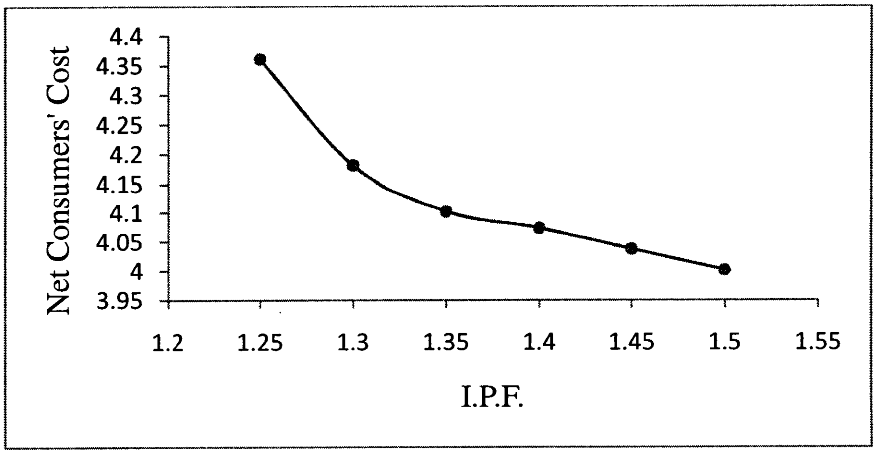

Figure 3-13b Variation of net consumers' cost with I.P.F. (Model 3 and Scenario 2)

4. Total Producers' Utility Vs I.P.F.

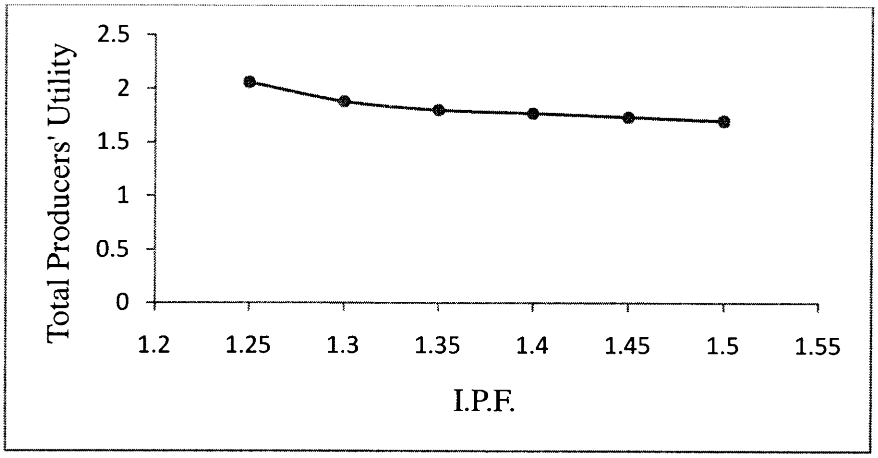

Figure 3-13c Variation of total producers' utility with I.P.F. (Model 3 and Scenario 2)

The total producers' utility decreases with the increase in I.P.F. because of reduction in trading volume of the carbon permits.

5. Total Emissions Vs I.P.F.

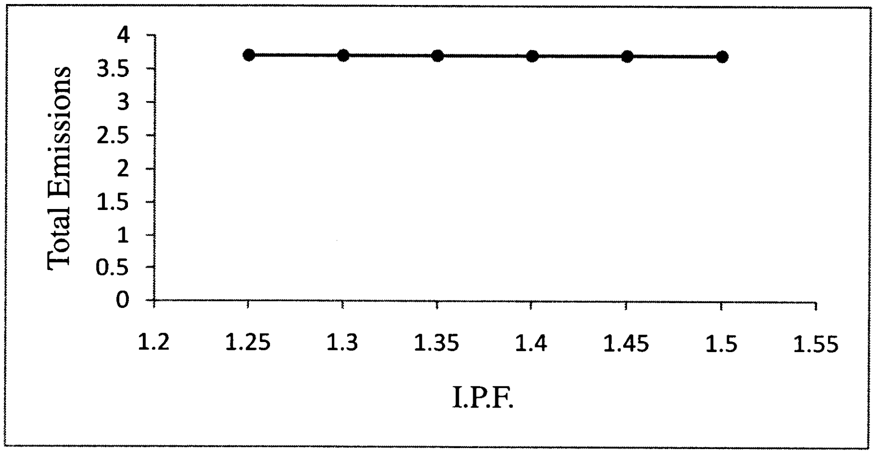

Figure 3-13d Variation of total emissions with I.P.F. (Model 3 and Scenario 2)

Total Emission is constant because the production control is precisely the same irrespective of I.P.F.

3.2.2.2 Carbon Taxation Policy

Carbon taxation policy has two critical parameters that influence equilibrium greatly: 1) Non-Taxable Emission (N.T.E.) and 2) Carbon Tax Rate(α). The N.T.E should be set in proportion to the individual capacity of the player and let us call the proportionality factor as Non-Taxable Emissions Factor (N.T.E.F.) and denote it with "r":

$$\text{Non-Taxable Emissions Factor (N.T.E.F.) for Player i, } r_i = \frac{\text{Non-Taxable Emissions for Player i}}{\text{Production capacity of Player i}}$$

Since N.T.E.F. is equal for all the players, $r_i = r \quad \forall i$

The Government can adjust N.T.E.F. (r) and the tax rate (α) to control the equilibrium conditions. The following sections present the detailed results and analysis on the sample case with a particular N.T.E.F. (r) and the tax rate (α)

Detailed Results - Base Case

N.T.E.F., $r = 1.3$ and Tax Rate, $\alpha = 3$

Following are the key characteristics of equilibrium under the given scenario:

Production Control:

Producer	Role/Control
Player 1	Secondary
Player 2	Contingency
Player 3	Primary

Market Electricity Price = 2.099 units

Expected utility of the Player 1: 0.499 units

Expected utility of the Player 2: 0 units

Expected utility of the Player 3: 0.099 units

Expected total combined utility of all producers: 0.5985 units

Expected total cost to consumers = 4.198 units

Expected total Government's Revenue = 0.6

Expected total emission = 3 units

Sensitivity Analysis

The equilibrium conditions are highly sensitive to policy parameters and demand uncertainty.

Sensitivity to Carbon Tax Rate

N.T.E.F., r = 1.3 and Tax Rate, α = Variable

1. Production Control Vs Tax Rate

Table 3-10 Production control as a function of tax rate (Model 3 and Scenario 2)

	Tax Rate, $0<\alpha\leq 0.35$	**Tax Rate,** $0.35<\alpha\leq 2.3$	**Tax Rate,** $2.3<\alpha\leq 2.5$	**Tax Rate ,** $2.5<\alpha$
Primary	Player 1	Player 2	Player 3	Player 3
Secondary	Player 2	Player 1	Player 2	Player 1
Contingency	Player 3	Player 3	Player 1	Player 2

The pattern of production control variation with tax rate in Scenario 1 is exactly identical to that of Scenario 2. Therefore the same logic can be used to explain the observation.

2. Electricity Price Vs Tax Rate

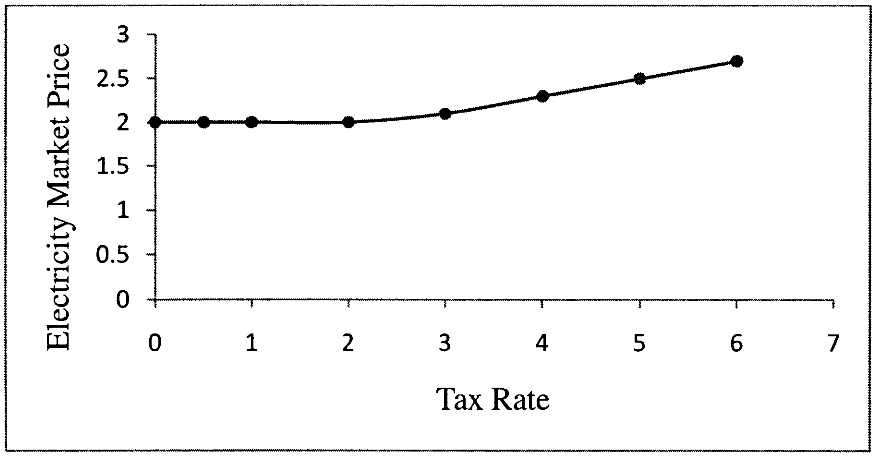

Figure 3-14a Variation of electricity price with tax rate (Model 3 and Scenario 2)

The equilibrium electricity price is equal to the third lowest critical price in the market as demand is such that almost all the time there are exactly two players actively producing electricity. When the tax rate is low, players with lower production cost are at advantage inspite of their high emissions. Therefore Player 1 and Player 2 are active producers. And Player 3 acts as a contingency producer. Therefore for the very low values of tax rate the critical price of Player 3 decides the equilibrium price. And as shown in Chapter 2 the critical price of Player 3 is constant at 2. Therefore the electricity price has a constant value of 2 as long as Player 3 is acting as a contingency producer which happens when tax rate is less than or equal to 2. However as soon as tax rate exceeds the value of 2, Player 3 takes a primary production control and Player 2 takes a role of contingency producer. Since the critical price of Player 2 is a strong function of tax rate the price increases linearly with tax rate when its value is greater than 2.

3. Net Consumers' Cost Vs Tax Rate

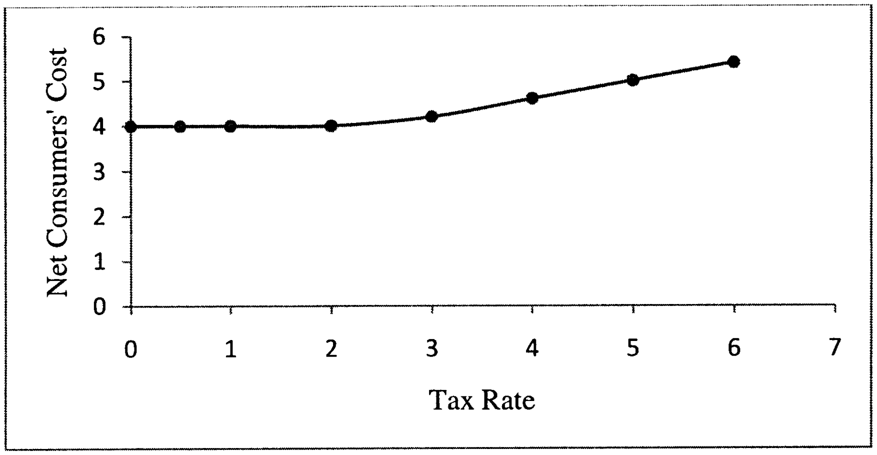

Figure 3-14b Variation of net consumers' cost with tax rate (Model 3 and Scenario 2)

4. Producers' Utility Vs Tax Rate

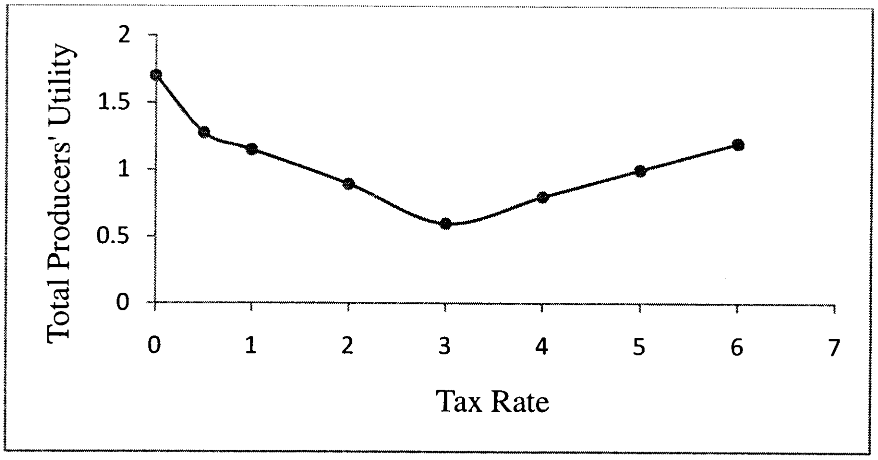

Figure 3-14c Variation of total producers' utility with tax rate (Model 3 and Scenario 2)

At lower tax rate the effect of constant price and increasing tax rate results into lower total producers' utility. However when tax rate is greater than or equal to 3 the electricity price also starts rising considerably with the increase in tax rate resulting into similar rise in total producers' utility.

5. Government's Revenue Vs Tax Rate

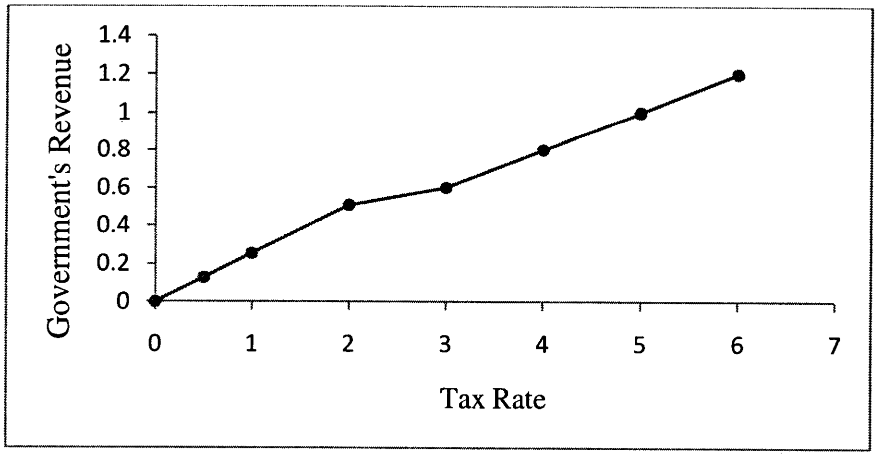

Figure 3-14d Variation of government's revenue with tax rate (Model 3 and Scenario 2)

It is obvious that government's revenue should increase with the increase in tax rate

6. Total Emissions Vs Tax Rate

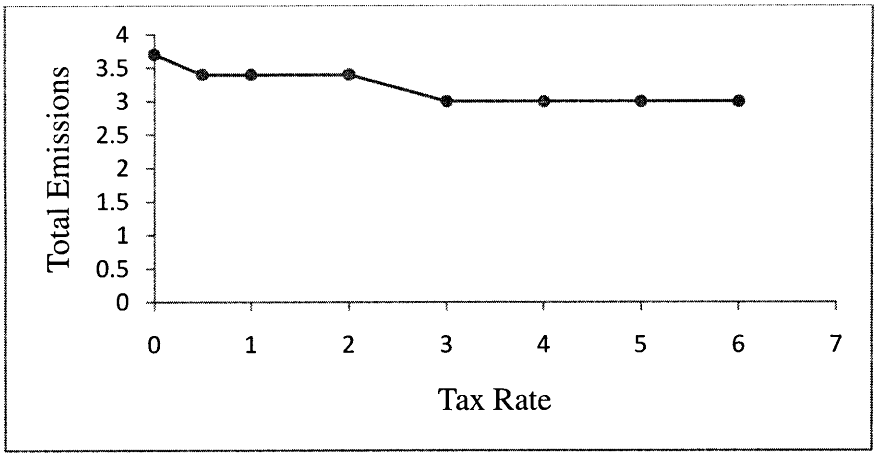

Figure 3-14e Variation of total emissions with tax rate (Model 3 and Scenario 2)

Total emissions decreases with increase in tax rate because higher tax rate gives the player with lesser emission a competitive edge in production over the players with higher emissions.

Sensitivity to N.T.E.F.

Tax rate = 3 and N.T.E.F. = variable

The effect of varying N.T.E.F. is just reverse of that of varying the Tax Rate

1. Production Control Vs N.T.E.F.

Table 3-11 Production control as a function of N.T.E.F. (Model 3 and Scenario 2)

	N.T.E.F., $0 < r \leq 1.3$	**N.T.E.F.**, $1.3 < r \leq 1.9$	**N.T.E.F.**, $1.9 < r$
Primary	Player 3	Player 2	Player 1
Secondary	Player 1	Player 1	Player 2
Contingency	Player 2	Player 3	Player 3

2. Electricity Price Vs N.T.E.F.

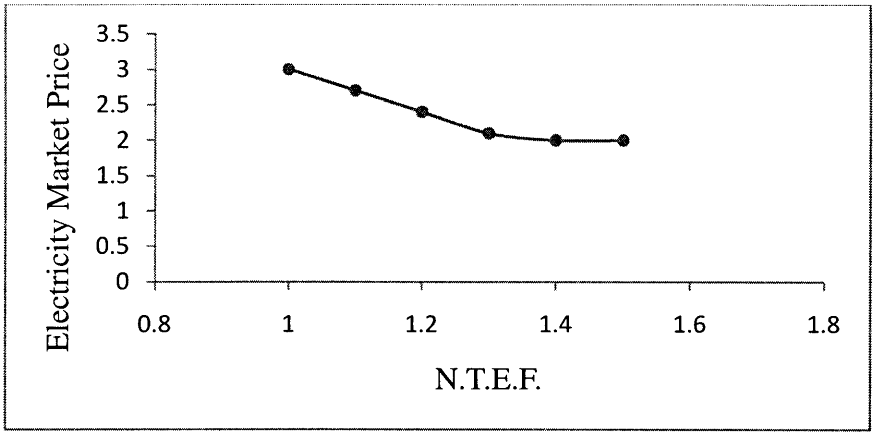

Figure 3-15a Variation of electricity price with N.T.E.F. (Model 3 and Scenario 2)

3. Net Consumers' Cost Vs N.T.E.F.

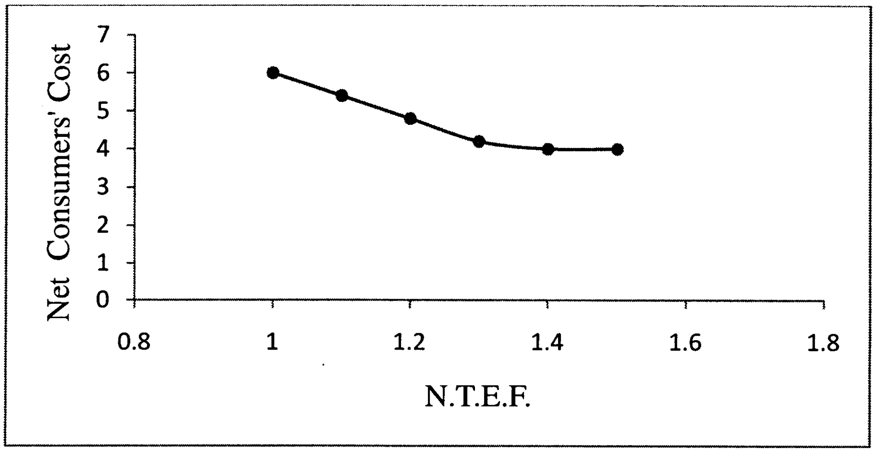

Figure 3-15b Variation of net consumers' cost with N.T.E.F. (Model 3 and Scenario 2)

4. Producers' Utility Vs N.T.E.F.

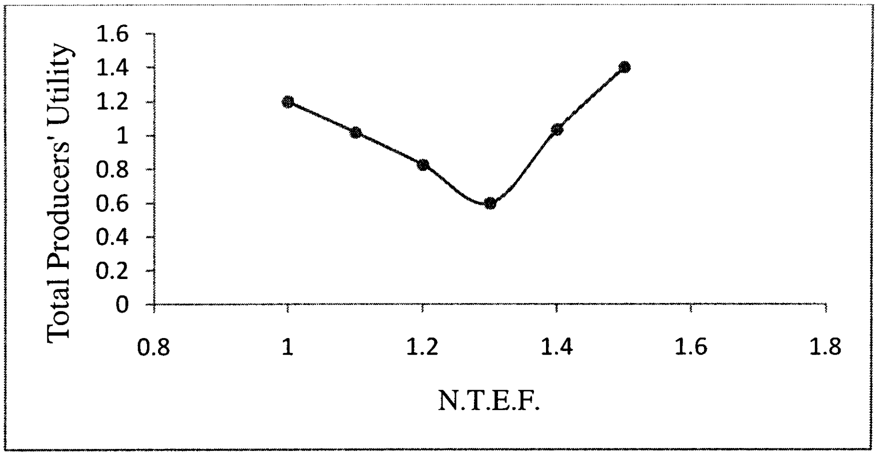

Figure 3-15c Variation of total producers' utility with N.T.E.F. (Model 3 and Scenario 2)

5. Government's Revenue Vs N.T.E.F.

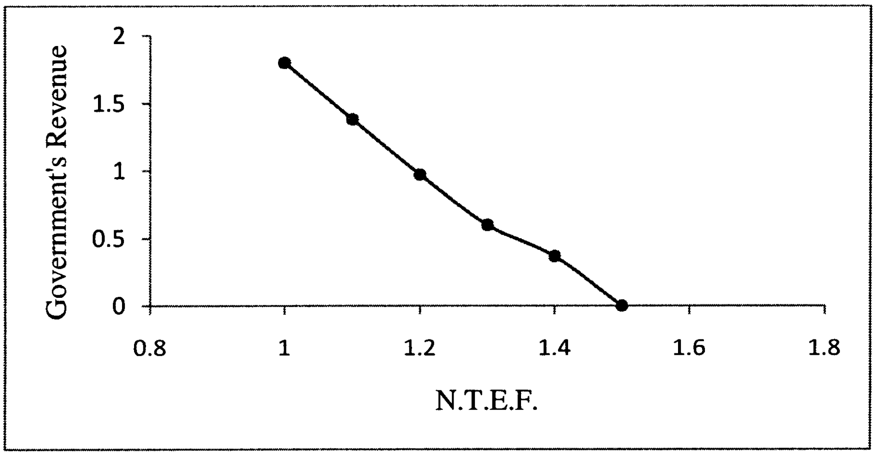

Figure 3-15d Variation of government's revenue with N.T.E.F. (Model 3 and Scenario 2)

6. Total Emissions Vs N.T.E.F.

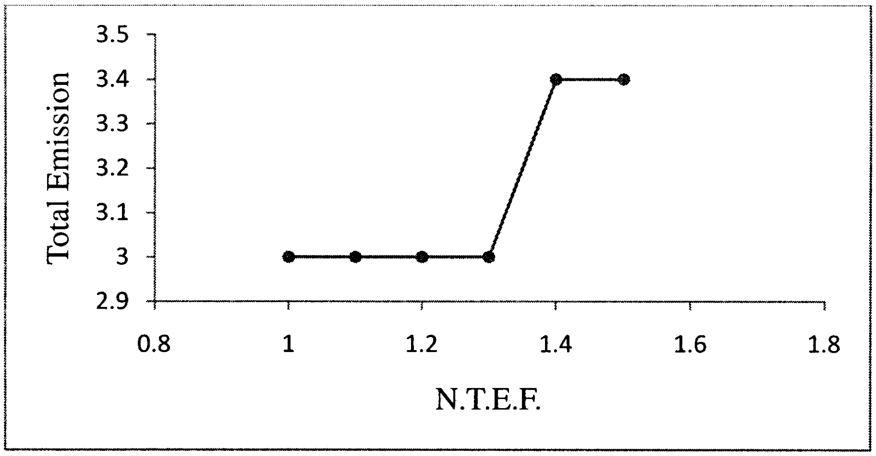

Figure 3-15e Variation of total emissions with N.T.E.F. (Model 3 and Scenario 2)

3.2.3 Comparison of Policies

The total emission is always highest in case of cap-and-trade mechanism. Under the cap-and-trade policy the optimal permit price strategy for equilibrium has been defined as a strategy profile that maximizes the total combined utility of all the producers. The overall utility is maximum if and only if the Player 1 produces as a primary producer and Player 2 produces as a secondary producer as discussed earlier. Infact this production control causes highest possible emission for the given electricity demand. However under the carbon taxation policy, lower emission target can be achieved by imposing stricter emission regulation polices like higher tax rates or lower N.T.E.

This limitation of cap-and-trade policy mainly attributes to the equilibrium selection criterion. Cap-and-trade policy might help in emission reduction if suitable equilibrium criterion is designed. The other reason for this limitation is the fact that all the players were isolated from the external world in electricity price competition and permit trade. Such isolation granted oligopolistic power to the players which reduced the functionality of cap-and-trade policy. However in reality producers are exposed to highly competitive and efficient permit exchange and electricity market. Therefore, Model 4 introduces highly competitive electricity market and Model 5 includes highly competitive and efficient permit exchange to observe the performance of cap-and-trade policy in such situations.

3.3 Analysis of Market Based on Model 4

This market involves two additional agents viz. central planner and electricity exchange. In this market the electricity producers don't have oligopolistic power as they are not the only suppliers to the electricity exchange and the price of electricity is fixed by exchange. However, the players still trade permits among themselves with complete isolation to external world in permits trade. The role of central planner is to facilitate permits trade among the players and also to divide the total demand of electricity among the players to maximize the total utility of all producers put together. Hence this problem is more or less a resource allocation problem and hence it can be solved using standard optimization methods like Linear Programming.

This market situation is modeled using Model 4 as described in Chapter 2. The MATLAB has been used to execute the stochastic model for demand coupled with the optimization model for resource allocation.

This model is specific to cap-and-trade policy. There is no competition among the players as the electricity price is fixed by the competitive market. Permit trade is facilitated by central planner to achieve maximum total utility of all the producers.This model is executed on the market with the following parameters:

Demand (D) is normally distributed with mean 1.5 units and standard deviation about mean 0.1 units. The production parameters of players are presented in table below:

Table 3-12 Production parameters of players (Model 4)

Parameter	Player 1	Player 2	Player 3
Production Cost	1	1.5	2
Emission Rate	2	1.5	1
Production Capacity	1	1	1

Note: All parameters are normalized for ease of calculations.

Since the production capacity of all players is same and all of them are facing same demand, the government should distribute equal number of permits to each player. Government can control market equilibrium by adjusting the amount of initial permits (E) and penalty (π).

In this model, the electricity price is fixed by external market and therefore it is an input to the system. The optimization model is compared with the game theoretic model with the equilibrium electricity price realized in game theoretic model as an input to the optimization model. The results are presented in Table 3-13:

Table 3-13 Comparison of results from optimization model with that of game theoretic model

I.P.F.	Game Theoretic Model		Optimization Model	
	Total Utility	Total Emission	Total Utility	Total Emission
1.2	1.7405	2.75	1.7405	2.75
1.25	1.639	2.75	1.639	2.75
1.3	1.3646	2.75	1.3646	2.75
1.35	1.3373	2.75	1.3373	2.75
1.4	1.3103	2.75	1.3103	2.75
1.45	1.2509	2.75	1.2509	2.75
1.5	1.2509	2.75	1.2509	2.75

The results from optimization model indicate that for the same electricity price and other parameters Model 4 possess exactly the same equilibrium characteristics as Model 3 as can be observed in Table 3-13. Hence the coordination strategy for permit trade in Model 3 indeed resulted into global maximum of combined utility of all the players.

It is also interesting to observe the effect of initial permit allocation on equilibrium condition when the electricity price is fixed and constant (as it is fixed by external market).

Sensitivity to Initial Permits

Electricity Price = 3/unit (fixed) and Initial Permits, E = variable

1. Total Emission Vs Initial Permits

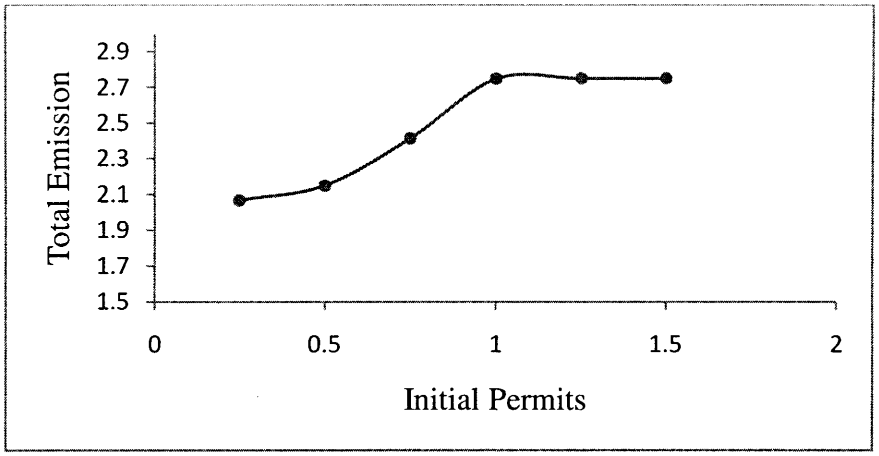

Figure 3-16a Variation of total emissions with initial permits (Model 4)

It is intuitive that lower the initial permits allotted, lower would be the total emission. Because lower amount of initial permits allocation makes it advantageous for player with least emission to meet the biggest fraction of demand.

2. Total Producers' Utility Vs Initial Permits

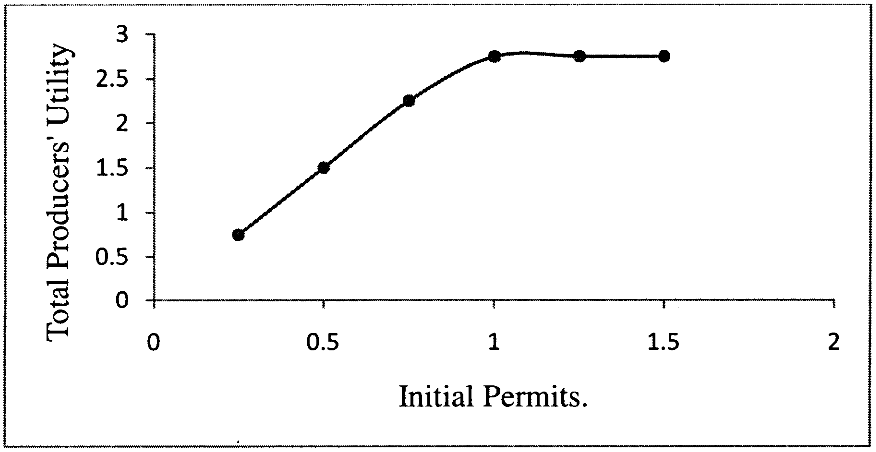

Figure 3-16b Variation of total producers' utility with initial permits (Model 4)

Total producers' utility increases with the increase in amount of initial permits to the point where the total initial permits (of all players combined) exceeds the maximum possible emission. Any permits more that that remain unused and don't add to producers' utility.

3.4 Analysis of Market Based on Model 5

The electricity market consists of three producers with single electricity buyer or controller and all producers have an open access to carbon permits exchange. This market has been introduced to analyze the impact of fixed permit price in cap-and-trade policy. In this market scenario the players will compete only in selling electricity. However permit trade will not be restricted to these three players and all players are able to buy and sell their permits to the carbon permits exchange at the price fixed by exchange market. It is a justifiable assumption that the permit price is always less than the penalty value imposed by government. Hence the government imposed penalty has no role in this model and government's revenue is always zero. This situation can be modeled using Model 5.

This model has also been implemented in MATLAB. The equilibrium condition has been analyzed for different permit price values. The price range of permit has been considered from 0 to 2 per unit. The permit price higher than 2 per unit is impractical when the ceiling on electricity price is fixed at 4/unit.

1. Production Control Vs Permit Price

Table 3-14 Production control as a function of permit price (Model 5)

	Permit Price $0 < E_{pm} \leq 0.75$	Permit Price $0.75 < E_{pm} \leq 2$
Primary	Player 1	Player 3
Secondary	Player 2	Player 2
Contingency	Player 3	Player 1

The order of production control is from the player with least production cost to the highest production cost when the market permit price is lower. However when market permit price becomes higher than 0.75 the order gets reversed and the player with the least emission takes a primary production control followed by the second lowest emitter and so on. Hence, market permit price plays a critical role in emission control.

2. Electricity Price Vs Permit Price

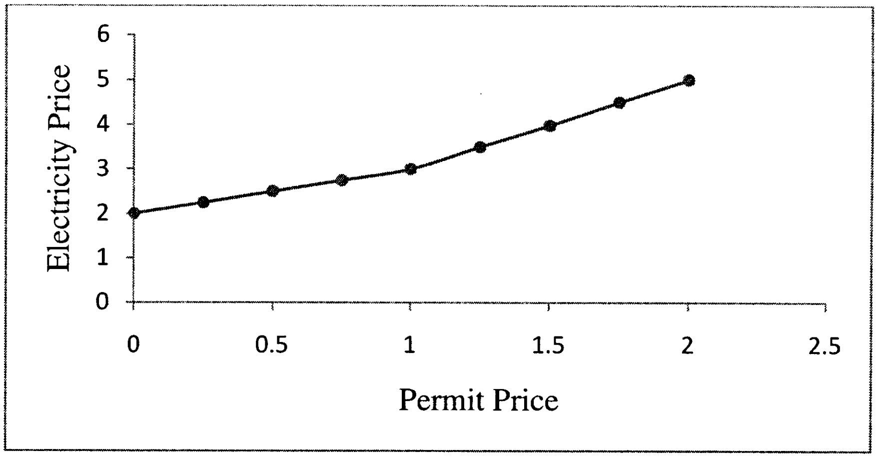

Figure 3-17a Variation of electricity price with permit price (Model 5)

The increase in permit price increases the overall cost of the producer and therefore it leads into increase in his critical price. Therefore, electricity price increases with the increase in permit price.

3. Net Consumers' Cost Vs Permit Price

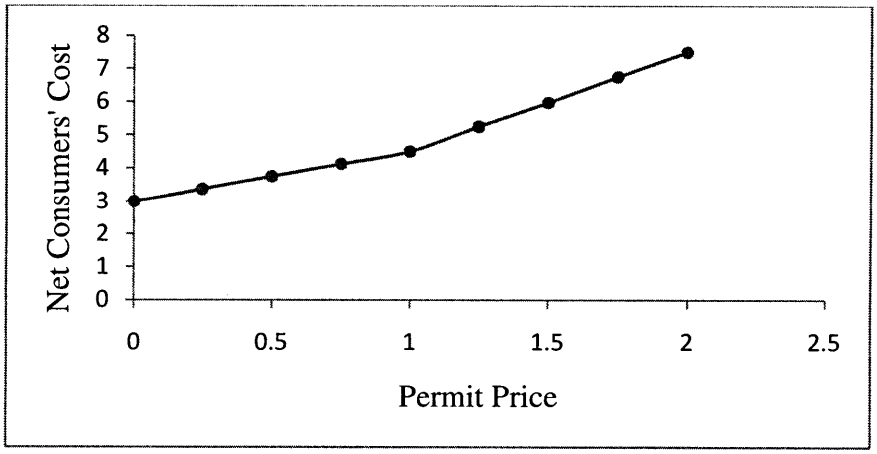

Figure 3-17b Variation of net consumers' cost with permit price (Model 5)

4. Producers' Utility Vs Permit Price

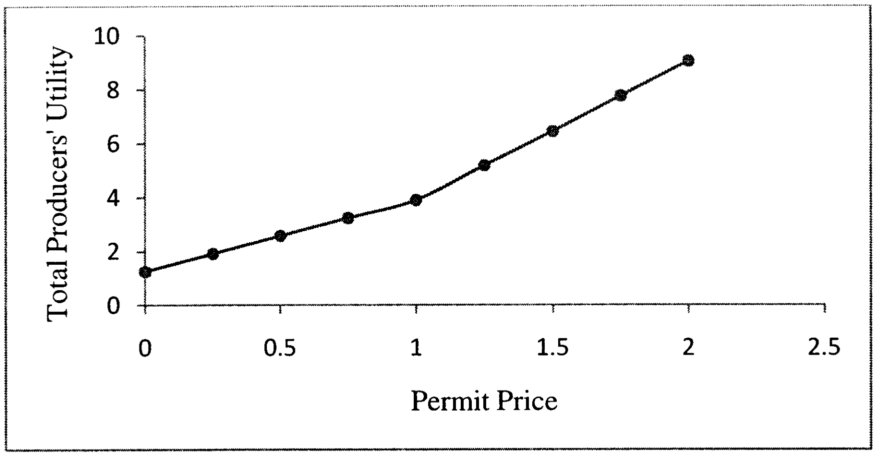

Figure 3-17c Variation of total producers' utility with permit price (Model 5)

Revenue of producers with excess permits is directly and positively linked with the permit price. And the primary producer (who generally falls short of permits) transfers the cost of permits to the electricity price. Therefore the total producers' utility always increases with the increase in permit price.

5. Total Emissions Vs Permit Price

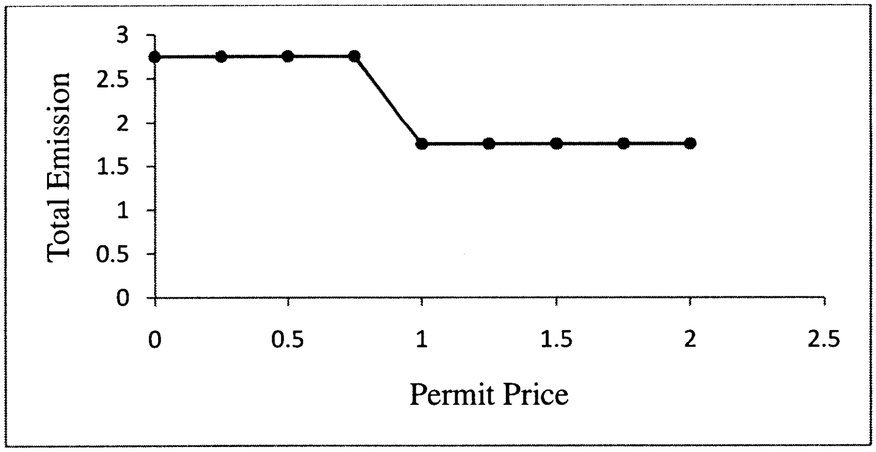

Figure 3-17d Variation of total emissions with permit price (Model 5)

Just like tax rate and penalty value, increase in permit price gives an advantage to the player with lesser emission to take a primary production control resulting into lower overall emission.

Chapter 4

Conclusion

This chapter reiterates the problem statement and presents the summary of the work done to address the problem. It also summarizes the results and the key insights developed from them. Finally it presents the recommendations for the future work.

4.1 Thesis Summary

This thesis aimed to analyze the impact of various carbon emission regulatory policies on the environment and various stakeholders of electricity market. Agent based modeling and simulation approach was adopted to model the electricity market where each stakeholder was modeled as an autonomous agent or player who aims to maximize his own utility and in the process market equilibrium was realized. As electricity market differs considerably from one region to another. Therefore to account for different possibilities, five different market scenarios were studied. Modeling of all these markets under cap-and-trade policy and taxation policy provided a more thorough insight into their influence of these policies on the market equilibrium conditions.

Market with just two producers was analyzed using conventional graphical method of plotting best response of each player. However multi-agent (more than two producers) single stage games have been modeled as an equivalent extensive form games as graphical approach is very difficult for such models. To account for multiple strategies of fixing electricity price and the permit price, two level extensive form game has been designed and modeled where producers first enters the game of electricity price competition for the given set of permit prices fixed by each player. Hence an equilibrium electricity price and the utilities associated with each set of permit price were obtained. At the second level players enter another extensive coordination based game with a strategy of fixing their permit price and equilibrium was achieved when their overall combined utility is maximum.

All these market scenarios involve uncertainty in electricity demand. This uncertainty was modeled using special class of Advanced Monte Carlo methods viz. Stratified Sampling. The utility or payoff of the player is calculated as an expected value over the demand distribution. The performance metrics have been defined to evaluate the efficiency of the policies in achieving the goal of emission reduction without exerting excessive financial burden on consumers. Expected values of these performance parameters (over the demand distribution) was presented for each market under each policy and also the sensitivity analysis was carried out with respect to policy parameters like initial permits, penalty, tax rate, N.T.E. etc and uncertainty in demand.

4.2 Key Insights from Results

The performance metrics such as consumer cost, emission level etc. are greatly influenced by the market type, type of regulatory policies and the policy parameters.

Total Emission:

In the Two-player model (Model 1) the reduction in emission can be achieved under both cap-and-trade policy and carbon taxation policy by adjusting parameters such as initial permits (in cap-and-trade model) and N.T.E. and the tax rate (in taxation model) accordingly. The lower amount of initial permits and N.T.E. ensures lesser emission as it creates incentive for the player with least emission rate to take an active production control. However the higher amount of initial permits and N.T.E. makes the cost of emission penalty insignificant in comparison to production cost and hence in such a situation, player with least production cost should take an active production control. In our market model and also in most of the cases in reality, the players with relative lower production cost tends to have higher emission rates. Therefore higher amount of initial permits and N.T.E. results into higher emissions.

In Model 3 (Three-player model), the total emission is always at the highest level under the cap-and-trade policy. Whereas emission reduction can be achieved under the carbon taxation policy by increasing tax rate and reducing N.T.E. The inefficiency of the cap-and-trade policy in achieving the target of emission reduction is due to the fact that the prices of permits are fixed based on coordination strategy among the players (as no pure Nash Equilibrium is possible for this game). When players coordinate to maximize their combined total utility there is only one production control in which it is maximum irrespective of policy parameters and demand uncertainty. Under this production control, the player with least production cost and highest emission rate takes the primary production control and the player with the intermediate production cost and intermediate emission rate takes the secondary production control. It was observed from Model 4 that if the price of electricity is fixed and regulated, the reduction in emissions is possible by adjusting the amount of initial permits even though if players completely coordinate with each other.

In all the first four models there was a key assumption that the system is isolated from the external world for permit trade. But in reality there are highly competitive permit exchanges in operation. Model 5 entails the interaction of the all three producers with the highly competitive permit exchange. The total emission is not constant in this case and it reduces with the increase in market permit price.

Net Consumers' Cost:

Net consumers' cost is a direct function of the market electricity price because demand distribution is assumed to be same for all the cases. The reduction in the amount of initial permits (in cap-and-trade policy) and N.T.E. (in taxation policy) causes increase in market electricity price and hence the net consumers' cost. Similarly the increase in penalty (in cap-and-trade policy) and tax rate (in taxation policy) results into increase in electricity price and net consumers' cost. In Model 5, net consumers' cost increases with the increase in permit price.

Total Producers' Utility:

Presence of Windfall Profits: Significant windfall profits have been realized by the players who don't actively participate in electricity production just by selling their permits to other players. Windfall profit is the main limitation of the cap-and-trade policy with grandfathering of permits. The cost of this profit is eventually borne by the end consumers.

In the Model 1 (Two-player model) the total utility of producers increases linearly with the increase in initial permits allotted as long as passive player can sell all his permits to the active player. Otherwise the total utility of producers decreases with the increase in initial permits. The local minima in total producers' utility are observed when the shift in production control occurs and the local maximum of the total producers' utility corresponds to the point where the total permit trade is locally maximum.

In the Model 3 (Three-player model), the total utility of producers decreases with the increase in the amount of initial permits (in cap-and-trade policy). This is due to the fact that increase in the amount of initial permits results into decrease in the volume of permits trade and hence the windfall profits are also reduced leading to reduction in total utility of the producers. Under the carbon taxation policy the variation of total producers' utility with the tax rate is convex in nature with the local minimum at the point where the shift in production control occurs.

It was observed that for the same electricity price Model 3 (Game Theoretic Model) gives the similar emission rate and total producers' utility as Model 4 (Optimization Model). This phenomenon indicates that coordination strategy adopted for fixing permit price indeed resulted into global maximum of their combined utility.

Government's Revenue:

Government's Revenue is typically larger in taxation policy as compared to cap-and-trade policy. The observations supports the logical reasoning that the Government's revenue should increase when the regulations become tighter (increased tax rates, reduced initial permits etc.).

Policies Comparison:

For the same level of emissions, taxation policy ensures lesser cost to consumers as compared to cap-and-trade policy. Total producers' utility is higher in case of cap-and-trade policy due to significant windfall profits involved. In Model 3 (Three-player model) cap-and-trade policy is ineffective in reducing emission as the equilibrium criterion for permit price strategy always ensures a same production control profile, which also happens to be the one that causes maximum total emission. However total emission can be controlled under the carbon taxation policy by adjusting tax rates and N.T.E.

In addition to equilibrium criterion, the isolation of the players with the external world for the permit trade and electricity price competition was also one of the main factors contributing to uniform production control irrespective of the variation in cap-and-trade policy parameters. Such an isolation provided players with oligopolistic power. The oligopolistic nature of electricity market rendered players with tremendous power to transfer the brunt of penalty to the end consumers by adjusting their electricity price accordingly. It has been shown with the help of Model 4 that the reduction in emissions is possible under cap-and-trade policy with Model 3 (Three-player model) if the electricity price is fixed and regulated by an external agent like a highly competitive electricity market or government. The effect of fixing permit price was observed by introducing highly competitive external permits exchange in Model 5. It was observed that the total emission decreases with the increase in market permit price. Therefore cap-and-trade policy becomes efficient if the oligopolistic power is removed from atleast one of the strategies of the players.

It can also be concluded that in taxation policy government has much better control over the emission due to simplicity of the model. However in cap-and-trade policy the efficacy of government's policy is highly subjective and depends on the market efficiency. The extreme volatility of permit prices in the recent past indicates that permits market is not yet very efficient.

4.3 Recommendations for Future Work

In the present study, analysis has been carried out on basic single stage period models to lay down the methodology to model such markets in general and to obtain key insights into the response of market to the regulatory policies in such basic market scenarios. However, this methodology can be extended to more complex market scenarios which involves multiple stage period in which the demand is met and permits are traded at each stage but the permits expire only after the end of several stages constituting period. Multistage period models should be modeled using dynamic programming. But conventional dynamic programming may become computationally prohibitive due to multi-dimensionality and continuity of state space. Therefore approximate dynamic programming should be used to get a reasonably good solution for such model.

As it has been discussed in the earlier section, the grandfathering of permits result into excessive windfall profits for producers at the cost of consumers' utility. Therefore inorder to deal with this situation, grandfathering of permits should be replaced by auctioning of permits. Government can auction the permits to the producers in the same way as controller takes a bid for electricity. Modeling of market with auctioning of permits is computationally much more expensive as it involves additional layer/level of competition among producers to buy the permits. The equilibrium buying price of the permits from the auction market can then serve as a floor for the selling price of permits.

More efficient equilibrium criterion should be designed for multi-agent multi-strategy (Model 3) as it doesn't have pure Nash Equilibrium. The coordination among players for fixing permit price hinders any prospective for emission reduction.

The more recent concept of permits banking should also be incorporated in the model. In the banking mechanism, players get an extra flexibility to deposit their unused permits in bank. Similarly he can also request permits from bank as a loan. It is intuitive that with the introduction of banking, permit prices will become more stable and hence it will increase viability of cap-and-trade policy.

Appendix A

Glossary

Following are the special terms that have been used repeatedly in Thesis:

1. *Active Player:* The producer who quotes lowest electricity price and hence supplies all the demanded electricity to controller.

2. *Passive Player:* The producer with higher quoted price and hence he doesn't get production control and rather sells off his permits to other player at price equal to penalty.

3. *Primary Producer:* The player with the lowest quoted price. Hence it is the primary supplier of electricity and it supplies up to his production capacity level or demand whichever is lesser.

4. *Secondary Producer:* The player with second lowest quoted price. It is the secondary supplier of electricity who supplies the excess demand that could not be fulfilled by primary producer.

5. *Contingency Producer:* The player with the highest quoted price who supplies only if all other players have run out of their capacity. If this player is producing, the market turns into monopoly. Therefore in most of the practical cases demand shouldn't exceed the capacity of other 2 players and this player solely indulge in selling permits to other 2 players and hence makes a profit.

6. *Initial Permits:* The number of carbon emission permits or allowances initially

allocated to each player. It is denoted by symbol E.

7. *Initial Permits Factor (I.P.F.):* The ratio of intial permits to the production capacity of each player. It is denoted by symbol p.

8. *Penalty:* The rate of fine imposed per unit emission not offsetted by permits. It is denoted by symbol π.

9. *Non-Taxable Emissions (N.T.E.):* The maximum tax free amount of emissions. It is also denoted by symbol E as it is equivalent to initial permits in cap-and-trade model

10. *Non-Taxable Emissions Factor (N.T.E.F.):* It is the ratio of N.T.E. to the production capacity of each player. It is denoted by symbol r.

Bibliography

[1] Trends in Carbon Dioxide. *Earth system research laboratory, NOAA* http://www.esrl.noaa.gov/gmd/ccgg/trends/#mlo. Accessed June 2010.

[2] After two large annual gains, rate of atmospheric CO_2 increase returns to average. NOAA News Online, Story 2412. http://www.noaanews.noaa.gov/stories2005/s2412.htm, March 2005.

[3] Canada's national occupational health and safety resource. http://www.ccohs.ca/ oshanswers/chemicals/chem_profiles/carbon_dioxide/health_cd.html, 1997.

[4] Summary for Policymakers. *Climate Change 2007: The Physical Science Basis. Contribution of Working Group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change.* http://www.ipcc.ch/pdf/assessment-report/ar4/wg1/ar4-wg1-spm.pdf , 2007.

[5] J. Lu, G.A.Vecchi, and T. Reichler. Expansion of the Hadley cell under global warming. *Geophysical Research Letters* 34: L06805, 2007.

[6] *World Energy Outlook,, International Energy Agency* http://www.worldenergyoutlook.org/docs/weo2009/WEO2009_es_english.pdf, 2009.

[7] S. Labatt and R.R. White, *Carbon Finance: The Financial Implications of Climate Change*, Wiley Finance 2007.

[8] N. Stern. The Stern Review: The Economics of Climate Change. Annex 7.b. Emissions from the Power Sector. www.sternreview.org.uk, 2006.

[9] Table ES-7. Inventory of U.S. Greenhouse Gas Emissions and Sinks: 1990-2007. *2010 U.S. Greenhouse Gas Inventory Report U.S. Environmental Protection Agency (EPA)*. 2009.

[10] *PricewaterhouseCoopers and Enerpresse 2003: Climate Change and the Power Industry*. 2003.

[11] R. Carmona, M. Fehr, J. Hinz, and A. Porchet, Market Design for Emission Trading

Schemes. *SIAM Review.* To appear.

[12] State and Trends of the Carbon market. *Carbon Finance at the World Bank,* 2010.

[13] N-H. M. Von der Fehr, D. Harbord. Spot market competition in the UK electricity industry. *The Economic Journal*, 103, pp. 531-546, 1993.

[14] C.W. Richter Jr. and G.B. Sheble. Genetic Algorithm Evolution of Utility Bidding Strategies for the Competitive Marketplace. *IEEE Transactions on Power Systems,* Vol. 13, No. 1, February 1998.

[15] J.D. Weber and T.J. Overbye. A Two-Level Optimization Problem for Analysis of Market Bidding Strategies. *in Proc. 1999 IEEE Power Engineering Society Summer Meeting*, pp. 682 – 687, July 1999.

[16] F. Wen and A.K. David, Optimal Bidding Strategies and Modeling of Imperfect Information Among Competitive Generators. *IEEE Transactions on Power Systems,*Vol. 16, No. 1, February 2001.

[17] D. Gan and D.V. Bourcier. A Single-Period Auction Game Model for Modeling Oligopolistic Competition in Pool-Based Electricity Markets. *in Proc. 2002 IEEE Power Eng. Soc. Winter Meeting*, 2002.

[18] Conzelmann, Guenter, North, J. Michael, Boyd, A. Gale, Cirillo, R. Richard, Koritarov, Vladimir, Macal, M. Charles, Thimmapuram, R. Prakash, & Veselka, D.Thomas. Simulating strategic market behavior using an agent-based modeling approach. *In Proc., 6th IAEE European Conference, Zurich, Switzerland*, 2004.

[19] J. Bower, and D.W. Bunn. A Model-Based Comparison of Pool and Bilateral Market Mechanisms for Electricity Trading. Energy Journal, Vol. 21, No. 3, July 2000.

[20] V. Petrov, and G.B. Sheblé. Power Auctions Bid Generation with Adaptive Agents using Genetic Programming. *Proceedings of the 2000 North American Power Symposium, Institute of Electrical and Electronic Engineers, Waterloo-Ontario, Canada.* Oct. 2000.

[21] T.D.Veselka, G. Boyd, G. Conzelmann, V.S. Koritarov, C.M. Macal, M.J. North, B.Schoepfle, and P. Thimmapuram. Simulating the Behavior of Electricity Markets with an Agent-Based Methodology: The Electricity Market Complex Adaptive Systems (EMCAS) Model. *Proceedings of the 22nd Annual USAEE/IAEE Conference, Vancouver, Canada,* Oct 2002.

[22] M.J. North, G. Conzelmann, V.S. Koritarov, C.M. Macal, P. Thimmapuram, and T.D.Veselka. E-Laboratories: Agent-Based Modeling of Electricity Markets. *Proceedings of the American Power Conference, Chicago*, April 2002.

[23] H. Li and F. Sun. A Parallel Multi-Agent Simulation Planning Approach to Complex Logistics System with Genetic Optimization. *Wireless Communications, Networking and Mobile Computing. WiCom 2007.* Pages 4843 – 4846, 2007

[24] G. Fleury, Jean-Yves Goujon, M. Gourgand and P. Lacomme, Multi-agent approach and stochastic optimization: random events in manufacturing systems. *Journal of Intelligent Manufacturing Vol 10*, Pages 81-101, 1999

[25] D. Plikynas. Portfolio Design and Optimization using Neural Network based Multi-agent System of Investing Agents. *The 20th International Conference, EURO Mini Conference "Continuous Optimization and Knowledge-Based Technologies" (EurOPT-2008)*, Pages 137-142, 2008

[26] T. Nakata and A. Lamont, Analysis of the impacts of carbon taxes on energy systems in Japan. *Energy Policy, Vol. 29, Issue 2,* Pages 159-166, 2001

[27] P. Cramton and S. Kerr. Tradeable carbon Permit Auctions. *Energy Policy Vol 30* Pages 333-345, 2002.

[28] J.P.M. Sijm, Y. Chen, M. Donkelaar, J.S. Hers, and M.J.J. Scheepers. CO2 price dynamics: A Follow-up Analysis of the Implications of E.U. Emissions Trading for the Price of Electricity. *Energy Research Centre of the Netherlands (ECN),* 2006.

[29] W. Lise, J. Sijm, and B. F. Hobbs. The Impact of the EU ETS on Prices, Profits and Emissions in the Power Sector: Simulation Results with the COMPETES EU20 Model. *Environmental and Resource Economics*, 2010.