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### *“One Second Per Second”*

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# “One Second Per Second”\*

Bradford Skow

## 1 Against Passage

Is there an objective “flow” or passage of time? It is hard to know how to answer because it is mysterious just what talk of the “flow” of time is supposed to mean in the first place. Opponents of objective passage are stuck trying to attack a theory they do not even understand.

Some opponents of passage think they have found a way around this problem. *Whatever* it means to say that time flows, they say, it will be true to say that time flows at a rate of one second per second. But *nothing* can happen at a rate of one second per second. The argument for *this* conclusion makes no assumptions about what the flow of time consists in. Here is the argument: one second per second is one second divided by one second, and one second divided by one second is the number one. But the number one is not a rate, so is certainly not the rate at which anything (the flow of time included) happens.<sup>1</sup>

This argument rests on confusions about the nature of measurable quantities and the meaning of phrases of the form “n Xs per Y.” They are honest confusions; philosophers have had little to say about these topics. So it is worth explaining what measurable quantities are and what phrases like “10 meters per second” mean—both for its own sake and as a means to showing why this argument against passage does not work.

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<sup>1</sup>Peter Van Inwagen (2009, p. 75) and Eric Olson (2009) defend this argument.

## 2 Quantities and Scales for Measuring Them

A **quantity** is a determinable property whose family of determinates has a certain kind of structure. What kind of structure? It is easiest to explain with an example. Length is a determinable property; the various maximally specific lengths are its determinates. I will call these determinate properties **values** of length. The length values have a ratio structure. Suppose, for example, that I have two sticks and the first one is twice as long as the second one. This relation between the sticks is reflected in the ratio of the length values the sticks instantiate: the ratio of the length value had by the first stick to the length value had by the second is 2. What is true here of length is true of any quantity. So a determinable property is a quantity if and only if for any two values of that determinable there is a number that is the ratio of the first value to the second, and these ratios structure the values so that their structure is relevantly like the structure of the positive real numbers.<sup>2</sup>

To say anything useful about the values of a quantity we need an informative way to refer to them. (I can always refer to my current mass as “the mass value I currently instantiate,” but this is of no practical use.) It would be especially useful if we assigned names systematically so that relations among names of values reflected the relations among the values themselves. The best way to do this is to use numbers as names for quantity values. A **scale** for measuring a quantity is an assignment of numbers to values of that quantity. A **faithful scale** (I will only be interested in

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<sup>2</sup>“Relevantly like” here means that the set of values is isomorphic to the additive semi-group of the positive real numbers. (Among the real numbers the number 1 is special, but its specialness comes from its role in the multiplicative structure of the reals. Since the structure of a set of quantity values is like the additive structure of the (positive) reals, there is no quantity value that plays the special role in the set of values that the number 1 plays in the reals.)

The general definition of quantity I have given really applies only to positive scalar quantities. I will not say anything about vector quantities or quantities with non-positive values in this paper.

A broader notion of (scalar) quantity counts determinables without ratio structures as quantities. Perhaps the values just need to be ordered, the way hardness properties are. (Stevens (1946) describes several structures one might want to allow quantities to have.) But the narrow notion of quantity is all we need for the purposes of this paper.

faithful scales) is one with the following feature: if  $n$  names value  $x$  and  $m$  names value  $y$  then  $n/m$  is equal to the ratio of  $x$  to  $y$ . That is, dividing the number that names the first value by the number that names the second gives the ratio of the first value to the second. For convenience we can use “ $x/y$ ” to name the number that is the ratio of  $x$  to  $y$ ; then the numbers  $n, m$  assigned to values  $x, y$  on a faithful scale satisfy  $n/m = x/y$ . But we should be aware that “/” has different (though clearly related) meanings on the two sides of this equation. On the left side it stands for an operation on numbers while on the right it stands for an operation on quantity values.

To set up a faithful scale for measuring some quantity we only need to select a **unit**. A unit is just a particular value for that quantity, the value that shall be assigned the number 1 on the scale. The ratios between other values and the unit then determine the numbers assigned to all other values. To set up a scale for length, for example, we might choose a particular bar made of a platinum-iridium alloy. This bar has some value  $L$  for length. We say that the number assigned to any length value  $v$  on our scale is equal to  $v/L$ . To make clear what scale we are using when we attribute numerical lengths to things we name this unit “the meter” and report lengths as “ $n$  meters.”

In many contexts (scientific research, physics homework) we need numerical values for lots of different quantities.<sup>3</sup> So we need scales for measuring all of those quantities. We could go through the quantities one by one, setting up unrelated scales for measuring each one. But that would be a lot of work and would make for a lot of extra computation when solving problems. Suppose, for example, that we measure lengths in meters and durations in seconds. We also need a scale for measuring speed. Suppose we choose some arbitrary unit for measuring speed—say, the speed of sound through air under standard conditions. We name this unit “the bleg” and report speeds on this scale as “ $n$  blegs.” Then if we want to measure something’s speed it is not enough to measure how many meters it travels in one second. We would also need to figure out how many meters something moving at 1 bleg travels in a second, and then do some calculations. What a pain.

It is easier to set up a **system of scales** (also called a **system of units**). In

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<sup>3</sup>Much of the rest of this section is adapted from (Barenblatt 1996, chapter 1).

a system of scales the units for the scales are systematically related. To set up a system of scales we choose a select few quantities and set up scales for measuring those quantities. The units we choose for these scales are the **fundamental units** of the system. These fundamental units then determine **derived units** for the scales of other quantities. The derived unit for a quantity is a value of that quantity that is singled out by the values used as fundamental units. It is easiest to explain this by looking at an example. Suppose that our system uses the meter as the fundamental unit for length and the second as the fundamental unit of duration. The derived unit for speed is then the speed value had by something that moves a distance of 1 meter during 1 second (and does so without changing its speed during that second). The scale we end up with is the meters per second scale for speed, and we report speeds as “ $n$  meters per second” (often abbreviated “ $n$  m/s”). This scale for speed is obviously superior to the bleg scale: when we use the meters per second scale, once we have measured how many meters something travels in one second we automatically know its speed in meters per second. Clearly a system of scales saves an enormous amount of time and effort. If we choose fundamental units for length, duration, and mass we get derived units for lots of quantities (speed, acceleration, frequency, force...), and can measure something’s value for one of these quantities just by measuring lengths, durations, and masses.

Why, when we use this system of scales, do we report speeds as “ $n$  meters per second”? Because reporting them this way indicates the **dimension** of speed. But what is meant by talk of the dimension of a quantity? Answering this question requires some more definitions. Suppose we have decided to adopt a system of scales that measures lengths, durations, and masses in fundamental units—we think that, for our purposes, these are the quantities it is most convenient to have fundamental units for. (In other contexts it may be more convenient to choose different quantities to measure in fundamental units.) There are lots of systems of scales we could adopt that all measure length, duration, and mass in fundamental units. Let a **class of systems of scales** be a collection of systems each of which chooses the same quantities to measure in fundamental units. A system that uses the meter, second, and kilogram as fundamental units is in the same class as a system that uses the foot, the hour, and the gram as fundamental units. Systems like these that choose

length, duration, and mass as the quantities that shall be measured in fundamental units are in the “length-duration-mass” class.

(Not all systems of units are in the length-duration-mass class. One reason to be interested in other classes is that the systems in the length-duration-mass class are not adequate to measure quantities like electric charge or electric current. But even if we ignore quantities that cannot be measured by systems in the length-duration-mass class there are other classes of systems that might be of interest. The systems of units in these classes choose different quantities to measure in fundamental units but still measure all of the quantities that can be measured by systems in the length-duration-mass class. An example is the length-duration-force class. A system of units in this class chooses a fundamental unit to measure force. Its unit for mass is a derived unit: it is the value of mass with the property that when unit force is applied to a body with that mass it produces a (constant) unit acceleration.

One might think that some select set of quantities is “metaphysically basic,” and that all other quantities are somehow “constructed from” the metaphysically basic quantities. If one thought this then one might also think that a class of systems the members of which measure only basic quantities with fundamental units better reflects the structure of reality than other classes. I am sympathetic to these claims but will not here assume that they are true.)

We might, on some occasion, decide to switch from one system in the length-duration-mass class to another. Maybe we decide to switch to a system that measuring lengths in centimeters instead of meters. Every length value will be assigned a number in the new system that is different from the one it was assigned in the old. But it is easy to calculate what these new numbers are: there are 100 centimeters in a meter, so any length that is assigned a number  $n$  on the meter scale is assigned the number  $100n$  on the centimeter scale. But it is not just length values that are assigned different numbers in the new system; lots of other quantities (quantities measured in derived units) are too. The numbers assigned to areas, speeds, forces, and so on are all different. But the way that these numbers change is completely determined by the way the numbers assigned to the values of the quantities measured in fundamental units change. The **dimension function** of a quantity contains complete information about these changes. (Strictly speaking, a quantity does not have

a dimension function *simpliciter*; it has a distinct dimension function for each class of systems of units. When I speak of “ $Q$ ’s dimension function” without qualification I shall just mean  $Q$ ’s dimension function relative to whichever class of systems is contextually salient.)

Here is how the dimension function works. I will follow common practice and use “[ $Q$ ]” to denote the dimension function of quantity  $Q$ . Suppose we change from one system of units in the length-duration-mass class to another. Then there is a number that is the ratio of the unit for length in the old system to the unit for length in the new system. (When we switch from meters to centimeters this ratio is 100.) Let  $L$  be this ratio. Similarly, let  $T$  the ratio of the old unit of duration to the new and  $M$  the ratio of the old unit of mass to the new. Then the dimension function [ $Q$ ] takes these three numbers as inputs and gives the factor by which the numbers assigned to values of  $Q$  change when we move from the old system of scales to the new. So the dimension function satisfies this equation (“# $v$ ” abbreviates “the number assigned to value  $v$  of quantity  $Q$ ”):

$$(\#v \text{ in the new system}) = [Q](L, T, M) \times (\#v \text{ in the old system}). \quad (1)$$

For example,  $[\text{speed}](L, T, M) = L/T$ . (We see here how reporting velocities as “ $n$  m/s” indicates what the dimension function of speed is.) For an example of the dimension function in action, suppose that we change from the SI system to the cgs system:<sup>4</sup> we change from using the meter, the second, and the kilogram as fundamental units to using the centimeter, the second, and the gram as fundamental units. The ratio of the meter to the centimeter is 100; the ratio of the second to itself is 1; and the ratio of the kilogram to the gram is 1000. Then the speed value assigned 5 in the SI system is assigned

$$[\text{speed}](100, 1, 1000) \times 5 = 100/1 \times 5 = 500$$

in the cgs system. That is, 5 meters per second is identical to 500 centimeters per second.

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<sup>4</sup>The SI system is not in the length-duration-mass class; it has other fundamental units, to measure (for example) electric current. But let us pretend it is.

Relative to any given class of systems of units some quantities are **dimensionless**.  $Q$  is dimensionless if and only if  $[Q] = 1$ , that is, if and only if the numbers assigned to values of  $Q$  by each scale in the system are the same. There are lots of examples of quantities that are dimensionless in the length-duration-mass class. When a weight is hung from the lower end of a bar that is oriented vertically and fixed at the upper end the bar lengthens. Strain is a dimensionless quantity had by the bar. The bar's strain is determined by how much it lengthens. If a bar's strain is .01 (if its value for strain is assigned this number by any system of units), for example, and its value for length when no weight is attached is assigned the number  $r$  on some scale, then its length when the weight is attached is assigned the number  $1.01r$ . The Reynolds number of a fluid flowing over a surface is another example of a dimensionless quantity. The Reynolds number characterizes the relative importance of viscosity in the flow, and is determined by the fluid's velocity, density, viscosity, and the size of the surface. (Roughly speaking, when a fluid's value for the Reynolds number is assigned a small number then viscosity is important.)

Though I sometimes omit the relativisation, it is worth emphasizing again that quantities are only dimensionless or dimensionful relative to a class of systems of units. Strain is dimensionless in the length-duration-mass class, but we could adopt a system from the class of systems that measure length, duration, mass, *and strain* in fundamental units. Strain is not dimensionless in this system. It would, of course, be annoying to use a system like this.<sup>5</sup>

One can make sense of the idea that some quantities are dimensionless *simpliciter* if one thinks that certain quantities are "metaphysically basic." A quantity

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<sup>5</sup>However, given the choice between a system of units in which a given quantity is dimensionless and a system in which that quantity has dimensions the scientific community does not always choose the first system. The main example of this is entropy. The SI system contains a fundamental unit for measuring temperature. As a consequence, in the class to which the SI system belongs the dimension function of entropy is  $[E]/\Theta$  (where  $[E]$  is the dimension function of energy and  $\Theta$  is the ratio of units for temperature). But entropy is dimensionless in the class of systems that is just like the class to which the SI system belongs except that it does not measure temperature in fundamental units. (The dimension function for temperature in this class is the same as that of energy.)



is then dimensionless *simpliciter* iff it is dimensionless relative to a class that measures only basic quantities with fundamental units. But, again, I am remaining neutral on whether it makes sense to speak of some quantities being metaphysically basic.

Strain and the Reynolds number are just two of the many dimensionless quantities scientists are interested in. The existence of multiple dimensionless quantities shows that distinct quantities can have the same dimension. There are also distinct dimensionful quantities with the same dimension. Thermal diffusivity, for example, is a quantity that influences how heat flows through a substance. It is measured in square meters per second. The kinematic viscosity of a fluid is also measured in square meters per second. Among other things, the kinematic viscosity controls how momentum diffuses through the fluid. Kinematic viscosity is a property only fluids can have, while both solids and fluids have thermal diffusivities; and the kinematic viscosity of a fluid need not be assigned the same number as its thermal diffusivity by any system of scales. Clearly thermal diffusivity and kinematic viscosity are different quantities.

Since distinct quantities can have the same dimension, expressions like “4 square meters per second” are not always used to denote the same thing. In one context this expression may name a value of kinematic viscosity, in another it may name a value of thermal diffusivity.

### **3 The Argument Evaluated**

Now we are in a position to see why the argument against objective becoming fails. Here again are the premises:

(P1) If time passes, it passes at a rate of 1 second per second.

(P2) 1 second per second is 1 second divided by 1 second.

(P3) 1 second divided by 1 second is 1.

(P4) The number 1 is not a rate.

To decide whether these premises are true we need to figure out what (if anything) “1 second per second” means as it is used in this argument. There are a couple

of things it might mean. One might look to the meaning that “1000 meters per kilometer” has when high school teachers, explaining the metric system to their students, say “There are 1000 meters per kilometer.” Such a sentence reports that the ratio of one value for length (the one assigned 1 on the kilometer scale) to another value (the one assigned 1 on the meter scale) is equal to 1000. So one might interpret “1 second per second” as also being about the ratio of two values of the same quantity. The first interpretation, then, says that “1 second per second” names the ratio of the second to the second. Since the ratio of any value of any quantity to itself is 1, it names the number 1.

How does the argument look if we use this interpretation? To finish evaluating the argument we need to know what “1 second divided by 1 second” means. Nowhere in the previous section did I give any meaning to expressions with this form. The closest I came was a definition of “1 second / 1 second” (which names the ratio of 1 second to itself). I said that there is a close relationship between the meaning of “/” when it occurs between two names for numbers and when it occurs between two names for values of a single quantity. So I think that the best interpretation of “1 second divided by 1 second” says that it means what I mean by “1 second / 1 second.”

Now we have interpretations for all of the tricky expressions in the argument. These interpretations clearly make both (P2) and (P3) true. And (P4) is also true: rates are values of quantities, not numbers. So on these interpretations the argument is sound.

But there is an interpretation of “1 second per second” that better fits the way believers in objective becoming use this expression. Its meaning is not similar to the meaning “1000 meters per kilometer” has in “there are 1000 meters per kilometer,” but instead is similar to the meaning of “10 meters per second.” This expression does not name a number. It names a value of speed. So a second interpretation of “1 second per second” says that this expression names the value of a speed-like quantity. Of course speed is speed-through-space. And we are discussing the rate at which time passes. This rate is not a value of speed, but is a value of some other quantity. Let us call this quantity “speed-through-time.” Now whether there is any such quantity as speed-through-time in the first place is an important question. But

let us for now grant that there is.

On this second interpretation (P2) says that a value of the quantity speed-through-time is identical to 1 second divided by 1 second. So it says that a value for a certain quantity is identical to a number. But while numbers are assigned to values of quantities by scales of measurement, no number is identical to a value of a quantity. So this interpretation makes (P2) false.

I expect the opponents of passage to deny that (P2) has two interpretations: they will say that “per” has the same meaning when it occurs in “1000 meters per kilometer” and when it occurs in “10 meters per second.” In all contexts it stands for a single function (which they call “division”) defined on pairs of values of quantities. When the values are values of different quantities, operating on them with the function yields a value for some other quantity. When the values are values of the same quantity, then operating on them with the function yields a number. Divide one meter by one second and you get a speed. Divide one second by one second and you get a number.

These claims are false. For if they were true then there could not be distinct quantities with the same dimension.<sup>6</sup> These claims entail, for example, that dividing the square meter by the second yields a value of some (one) quantity. But there are *many* quantities the values of which can be named by “*n* square meters per second.” (Thermal diffusivity and kinematic viscosity are the two examples I gave earlier.) And, in general, for any dimension you choose, there are plenty of distinct quantities that have that dimension.

The correct way to understand “*n* meters per second” is the way I described in the last section. “Meters per second” indicates what system of units is being used to measure speed, and indicates what the dimension function of speed is.

(Like me, Ian Phillips (2009) wants to reject (P2). He writes that “A rate is a ratio of two quantities, a relation one quantity bears to another....one second per second is not one second divided by one second, and it is not equal to one. One second per second is a ratio of time to unit time[...].” (By “quantity” Phillips means what I mean by “value of a quantity.”) Phillips is on the right track. It is true that one second per second is not one second divided by one second. But I do not understand

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<sup>6</sup>Barenblatt (1996, p. 33) and Palacios (1964, p. xiii) make similar points.

what he thinks one second per second *is*. He says it is a ratio of “time to unit time.” If this means that one second per second is the ratio of a value of duration to a value of duration, then he has just said that one second per second is a number and has fallen into the enemy camp.)

Computations of the following sort might be thought to support the view I oppose:

$$1 \frac{\text{meter}}{\text{sec}} \times 60 \frac{\text{sec}}{\text{min}} = 60 \frac{\text{meters}}{\text{min}}. \quad (2)$$

It looks here like we are first dividing meters by seconds and then dividing seconds by minutes, and that the product of the result is some number of meters divided by a minute. But there is no problem understanding these computations on my view. We have seen them before: the dimension function of speed is here being used to find the number assigned to the speed value 1 meter per second in the system of units that uses the minute as its unit for time. So (2) is just an instance of (1), where labels have been attached to indicate which number goes with which system of units.

Sometimes people “cancel units” when performing this computation:

$$1 \frac{\text{meter}}{\text{-sec-}} \times 60 \frac{\text{-sec-}}{\text{min}} = 60 \frac{\text{meters}}{\text{min}}. \quad (3)$$

Here there is a stronger suggestion that the first expression is the result of dividing the meter by the second. Otherwise, how can you cancel the two occurrences of “sec”? Isn’t (3) just like

$$\frac{1}{\cancel{4}} \frac{\cancel{4}}{3} = \frac{1}{3}, \quad (4)$$

with the second taking the place of four, the meter taking the place of one, and so on? The answer is “no”: the analogy between (3) and (4) is merely formal. We should not take computations like the one in (3) seriously when doing metaphysics. Compare, for example, the following computation involving derivatives:

$$\frac{dy}{dt} = \frac{dy}{dx} \frac{dx}{dt}. \quad (5)$$

(5) also looks like (4) (without the explicit cancelations). It looks like the left-hand side of (4), the derivative of  $y$  with respect to  $t$ , was obtained from the right-hand side by canceling  $dx$ . And this suggests that  $dy/dx$  is the result of dividing

something named “ $dy$ ” by something named “ $dx$ .” But while early mathematicians may have believed that this is what is going on, in modern developments of calculus “ $dx$ ” and “ $dy$ ” do not name anything, and so  $dy/dx$  is not the result of dividing  $dy$  by  $dx$ . Some math books even refuse to use this notation so that students do not become confused. Still, this notation is computationally convenient: in certain contexts if we pretend that  $dy/dx$  is a division problem and manipulate it formally like a quotient then we will get the right answer.<sup>7</sup> This is also true, I claim, about “meters/sec.”

#### 4 A Better Argument Against Passage?

Speed’s dimension function is  $L/T$  (in the class of systems of units we normally use). But the fact that speeds-through-time are measured in seconds per second indicates that this quantity’s dimension function is  $T/T = 1$ —speed-through-time is *dimensionless*. We can report something’s speed through time by saying that its speed through time is 1 (or 17, or whatever). We need not mention the units that indicate its dimension.

The fact that speed-through-time, if there is such a quantity, is a dimensionless quantity, is clearly lurking in the background of the argument we have been discussing. One can read (P2) and (P3) as an attempt to establish that speed-through-time is dimensionless. But reading them this way does not make the argument any better. (P2) still mistakenly identifies the value of a (dimensionless) quantity with a number. It is better to start over and construct an argument against objective becoming that works directly with the notion of a dimensionless quantity. It proceeds like this:

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<sup>7</sup>In his book *Calculus on Manifolds* Michael Spivak complains about the confusions that this notation can cause (1971, pp. 44-45). On the other side, one mathematician writes, “if one is concerned with computations and applications rather than the abstract theory, then one is quickly led to the conclusion that the Leibniz notation is incomparably more efficient” (Edwards 1969, p. 459). Of course, there are mathematical theories in which “ $dx$ ” does have an independent meaning: the theory of differential forms and some theories of infinitesimals. This does not affect my point, which is that in “standard calculus” (5) is correct but does not involve canceling the  $dx$ ’s.

(P5) 1 second per second is the value of a dimensionless quantity, but

(P6) no dimensionless quantity is a rate.

And if 1 second per second is not a rate, it cannot be the rate of time's passage.<sup>8</sup>

This argument is better because it does not rest on any confusions about the meaning of expressions of the form “n Xs per Y.” So what should we think of it? I am going to argue that we should reject (P6), but first let us look at (P5). (P5) is true—in *the class of systems of units we normally use*. Supposing that there is such a quantity as speed-through-time (this appears to be a presupposition of the argument), we could choose a system of units that measures this quantity with a fundamental unit. In that system it is not dimensionless and (P5) is false. So (P5) is like “Boston is on the left”—incomplete without some implicit or explicit relativization. One way to deal with this is to suppose that the premises are all implicitly relativized to some class of systems of units. But then it is hard to know what to make of (P6). We are now reading (P6) so it says “No quantity that is dimensionless is such-and-such a class is a rate.” Why think that being a rate correlates with being dimensionless in some particular class of systems of units? Perhaps those who defend this argument think that some quantities are metaphysically basic, and so think that some quantities are dimensionless *simpliciter*; then the premises do not need to be relativized, and this problem with (P6) does not arise. But instead of pursuing this issue further I am going to set it aside and present grounds for rejecting (P6) that are good whether or not some quantities are dimensionless *simpliciter*.

But first: what has been said in favor of (P6)? Van Inwagen writes in a way that makes (P6) look tautologous. He writes the premise as “No dimensionless number is a rate.” And of course numbers are not rates. But this is confused. The term “dimensionless number” is a category mistake. Only *quantities* can be dimensionless or dimensionful. Using the term “dimensionless number” encourages us to confuse values of dimensionless quantities with numbers. But values of dimensionless quantities are, again, not numbers. (If they were, then there could not be

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<sup>8</sup>This is the argument in (Price 1996, p. 13). In a footnote Van Inwagen (2009, p. 90) also endorses this argument, and says that it is just another way of formulating the earlier argument.

distinct dimensionless quantities.<sup>9</sup>) When we keep this distinction clear (P6) does not look obvious.

To evaluate (P6) we need a definition of “rate.” This should not be controversial. For any quantity  $Q$  there is another quantity that is the rate of change of  $Q$ .<sup>10</sup> If we have a unit for measuring  $Q$  and a unit for measuring duration then the unit for measuring the rate of change of  $Q$  is measured in units of  $Q$  per unit of duration. If we write  $\dot{Q}$  for the rate of change of  $Q$ , then the dimension function for  $\dot{Q}$  (in the length-duration-mass class) satisfies

$$[\dot{Q}](L, T, M) = \frac{[Q](L, T, M)}{T}.$$

It may tempting to think that a function of this form can never be the constant function always equal to 1; but this cannot be tempting for long. For if  $Q$  is any quantity that is measured in units of duration then its rate of change will be dimensionless. And there are plenty of uncontroversial examples of quantities like this. One example is the period of a pendulum. A pendulum’s period is some number of seconds. Over time the length of the pendulum may change (due to, say, changes in the temperature), causing a change in the period. If the pendulum is part of a clock then the owner may be very interested in the rate at which its period changes; one can easily imagine a homework question in a physics class asking you to calculate this rate. The rate can be reported as  $n$  seconds per second (and here  $n$  need not be 1). It can also be reported just as  $n$ —provided we remember that  $n$  here names a value of a quantity (the same value no matter what unit we use to measure duration), not a number.

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<sup>9</sup>In a discussion of this argument Tim Maudlin (2007, p. 117) also warns against this confusion by emphasizing that distinct quantities can have the same dimension.

<sup>10</sup>Not all quantities that are rates are rates of change of some other quantity. A particle’s speed is the rate at which its position is changing, but position is not a quantity as I have defined it. (*Distances* between positions are values of the quantity length (they are lengths of the lines between those positions), and it is these that figure in the definition of speed.)

## 5 Conclusion

I have looked at two arguments that aim to show that one second per second is not a rate. The first is based on confusions about the meaning of “one second per second.” The second falsely claims that dimensionless quantities cannot be rates.

Throughout my discussion I have assumed that there is such a quantity as speed-through-time. Believers in objective passage certainly accept this assumption, but I have not said anything to defend it. An argument that there is no such quantity as speed-through-time would do what the opponents of passage want: establish that time cannot pass at one second per second. But nothing they say about “one second per second” constitutes such an argument.<sup>11</sup>

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