Can We Use Cap Rates To Better Allocate Investments in
Commercial Real Estate in a Dynamic Portfolio?

by

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Submitted to the Program in Real Estate Development in Conjunction with the Center for Real Estate in Partial Fulfillment of the Requirements for the Degree of Master of Science in Real Estate Development

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ABSTRACT

This thesis has a two-fold objective, namely to explore the role of cap rates in predicting the returns to commercial real estate, and to identify how cap rates can be used to improve the allocation of real estate in a dynamic investment portfolio.

Seeking an answer to the first question, we run predictive regressions using data for real estate "All Properties" and for all four major property types, examining the predictability power of cap rates for a forecasting horizon from one to four quarters in the future. Moreover, we examine whether or not stock dividend-price ratio can predict real estate returns, and examine the predictability of stock returns by cap rates and dividend-price ratio.

The analysis confirms that both cap rates and the dividend-price ratio can predict real estate "All Properties" returns for up to one year in the future. Concerning the analysis per property type, the results vary from property type to property type, and for different forecast horizons. Moreover, the analysis shows that stock returns can be predicted by the dividend-price ratio at all forecast horizons, whereas the cap rates seem to have somewhat limited predictive power regarding the stock returns.

We approach the second question by following the dynamic portfolio allocation methodology proposed by Brandt and Santa-Clara (2006). We expand the existing set of "basis" assets comprised of stocks and real estate to include "conditional" portfolios, and then compute the portfolio weights of this expanded set of assets by applying the Markowitz solution to the optimization problem. We apply this methodology to three different portfolio rebalancing horizons. Moreover, we work with three cases for each portfolio, i.e. with the unconditional case, with the case where the dividend-price ratio is the only conditioning variable, and with the case where the cap rate is the second conditioning variable.

In almost all instances the results confirm that, by adding the cap rate as an additional state variable, the performance of the portfolios increases significantly. The same conclusion stands when we impose a "no shorting" restriction to real estate, although now the role of cap rates seems somewhat less significant.

Thesis Supervisor: Walter N. Torous
Title: Visiting Professor, Center for Real Estate

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1. Introduction and Background

1.1. Introduction

This thesis has a two-fold objective, namely to explore the role of cap rates in predicting the returns to commercial real estate, and to identify how cap rates can be used to improve the allocation of real estate in a dynamic investment portfolio.

The first question of how, and to what degree, do cap rates predict the returns to commercial real estate, is a question of major importance for investors. A similar question of how the dividend-price ratio can predict returns in the stock market has been extensively explored, and the results are supportive of the opinion that this kind of relation should be examined in the world of commercial real estate, too.

In order to deal with that issue we follow a simplified version of the predictive regressions methodology implemented by Plazzi, Torous and Valkanov (2010), and run a series of predictive regressions to examine the predictability power of cap rates for periods ranging from one to four quarters in the future. This forecasting horizon was chosen because it covers the most commonly used timeframe in portfolio analysis and portfolio rebalancing, which is explored in the second part of this thesis.

Additionally, since the results from this part of the thesis will be used to confirm our findings in the second part, it was considered necessary to expand our analysis in identifying whether or not the stock market's dividend-price ratio can predict real estate returns. Moreover, and for the same reason as above, we further explore the predictability of stock returns by the cap rates and dividend-price ratio.

The second question which will be examined in this thesis is how investors can use the results from the aforementioned analysis so as to better allocate commercial real estate in a dynamic, risky asset portfolio, so as to achieve better portfolio performance.

In recent years, a growing number of institutional investors, acknowledging the diversification benefits of real estate as a major asset class, have started including commercial real estate in their portfolios. This fact has major implications for portfolio managers, since it influences the strategic decision making of how to allocate the wealth of these large institutional “players” among the various investment classes, so as to gain the maximum possible returns while being exposed to the minimum possible risk. Moreover, nowadays there are new approaches proposed by the academic world that can help portfolio managers apply a dynamic portfolio selection, which is considered a much better approach to portfolio analysis, without facing the severe computational difficulties of past methodologies.

One of the novel methodologies proposed is that of Brandt and Santa-Clara (2006), and is the one we will follow in the second part of the current thesis. This approach, although quite data intensive for long horizons, is very intuitive and simple to apply. It is based on the rationale that we can expand an existing set of “basis” assets so as to include mechanically managed, “conditional” portfolios, and then compute the static portfolio weights of this expanded set of assets by applying the traditional Markowitz solution to the optimization problem.

The organization of this thesis is as follows: Chapter 1 includes a brief introduction to the concepts the current thesis will be dealing with. Chapter 2 describes the real estate and stock databases and discusses the data used. Chapter 3 describes the predictive regression methodology followed and includes the analysis results from the series of predictive regressions we have run. Chapter 4 outlines the dynamic portfolio methodology of Brandt and Santa-Clara (2006) and includes the results of its application to the data of this thesis. Chapter 5 concludes and expresses some thoughts on topics that might be of interest for future research. Chapter 6 shows the references used.

1.2. Addition to Current Body of Knowledge

Of course, there has been significant work done on this important topic by, among others, Plazzi, Torous and Valkanov, (2010); this thesis is intended to add to the current body of knowledge by using techniques, database recourses, and a historical examination period which differ from those utilized in previous papers. For a more detailed description of the data and methodology employed in each case, please refer to the corresponding chapters.

2. Research Data

2.1. Data Description

The data requirements for this thesis reflect the quantitative nature of the topic, and require access to both real estate and stock databases. More specifically, in the case of real estate we are using the historical data on cap rates from the NPI index, and data on returns from the TBI index. The data collected concern the main "All Properties" index and the four major real estate property types (Apartment, Industrial, Office, and Retail buildings) for the 1984Q2-2010Q1 and 1994Q2-2010Q1 periods, respectively.

As far as the data for stocks is concerned, we use the historical S&P500 total returns and the corresponding dividend-price ratios, which are made publicly available by Standard & Poor's for the aforementioned historical periods. Moreover, we use the US Treasury T-Bills with a 3-month maturity period as an approximation of the risk-free rate; data on the T-Bills were collected from the Federal Reserve Bank website.[1]

Furthermore, please note that the "raw" data, as provided by the sources above, are reported in monthly and quarterly time intervals, which differ from those we utilize in our analysis. Therefore, the data were adjusted accordingly to match the time intervals used in this thesis (our analysis is on a quarterly, semi-annual and annual basis).

For definitions and more detailed descriptions on the aforementioned databases and on the specific data used for the purposes of this thesis, the reader can refer to Sections 2.2. and 2.3. to follow.

2.2. Commercial Real Estate Data

At that point we start by briefly introducing the reader to the NCREIF database and, more specifically, to the characteristics of the NPI index, which provides us with some of the data that will be used in the current thesis.

[1] http://www.federalreserve.gov/

The National Council of Real Estate Investment Fiduciaries (NCREIF) is an institutional real estate investment association which dates its origins to the mid 1970s. NCREIF collects data from its contributing members that concern both individual commercial properties and investment funds, and utilizes these for creating the quarterly NCREIF Property Index (NPI), among other indices.[2]

As of the first quarter of 2010, there are 6,067 individual properties in the NCREIF database from which the NPI is constructed. Moreover, and in addition to the main NPI "All Properties" index, NCREIF publishes sub-indices for the four main property types (Apartment, Industrial, Office, and Retail buildings).

From this database we will be using the quarterly historical cap rates for each of the real estate categories above, and more specifically the "equal-weighted cap rate four-quarter moving average". Based on the methodology applied by NPI, the quarterly cap rates are calculated by dividing the actual accounting NOI income from properties that are revalued during each quarter. Then, the "four-quarter moving average cap rate" is set up to be equal to the average of the cap rate of each quarter and the quarterly cap rates from three periods back in the past.

The NPI is the most widely used indicator measuring the performance of institutional quality commercial real estate in the US. However, the NPI is, by definition, an "appraisal-based index with staggered appraisals", a fact which means that it is based on the cross-sectional aggregate appraisal values at the property level, however without all properties included in the index being reappraised as of the same point in time [Geltner, Miller, Clayton and Eichholtz (2006), pp. 676-677]. As a consequence, the NPI index is based on "stale appraisals", thus suffering from "lagging" and "smoothing", which makes the index ineffective in some applications, such as portfolio optimization.[3]

In order to address the issues that arise from the appraisal-based nature of NCREIF, the MIT Center for Real Estate launched in 2006 a new index, the "Transactions-Based Index (TBI) of Institutional

[2] For more details on the organization, and on the process it follows to produce the aforementioned index, we recommend that the reader refer to the NCREIF website: www.ncreif.org

[3] Fisher, Jeff D., Geltner, David M., and Henry O. Pollakowski, 2007, A Quarterly Transactions-Based Index (TBI) of Institutional Real Estate Investment Performance and Movements in Supply and Demand, Journal of Real Estate Finance and Economics 34, 5-33

Commercial Property Investment Performance". The TBI is a transactions-based index complementary to the NPI, which uses econometric techniques to estimate the market movements and returns on investment based on transaction prices of properties sold from the NPI database each quarter [Fisher, Geltner, and Pollakowski (2007)].

Since a large part of the current thesis involves portfolio optimization, the TBI database was considered more precise and appropriate than the NPI. Thus, from the former database we will be using the quarterly historical returns for the "All Properties" and for each of the four main property types mentioned above.[4] Please note that the TBI returns are comparable to the so-called NPI's "equal-weighted cash-flow based returns, with appreciation including capital expenditures" [Fisher, Geltner, and Pollakowski (2007)].

2.3. Data on Stocks

In this thesis we will be using the historical S&P500 database for the data on stocks that we need for our analysis. The S&P 500 is one of the most commonly used indices in analyzing the performance of the US-based, large cap common stocks, and it includes data since 1957. It is a capitalization-weighted index that considers the 500 large-cap American stocks, and its database is publicly available by Standard & Poor's.[5]

From this database we will be using the historical total returns of that index for the aforementioned period, namely from 1984Q2 until 2010Q1. The data are reported on a monthly basis, so they were adjusted accordingly in the time intervals we are considering in this paper. Moreover, for the same period, we use the "four-quarter moving average" dividend-price ratio reported by S&P, which has a similar structure to that of the NPI's "four-quarter moving average" cap rates.

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[4] The TBI database can be found at MIT/CRE's website: http://web.mit.edu/cre

[5] Data available in the S&P's website: http://www.standardandpoors.com

3. Prediction of Future Returns in Commercial Real Estate and Stocks

3.1. Predictive Regression Methodology

In this chapter we examine the ability that the cap rate and dividend-price ratio have in predicting real estate and stock returns, following a simplified version of the predictive regressions methodology implemented by Plazzi, Torous and Valkanov (2010). More specifically, we will be focusing on the relation between the asset returns and the lagged predictive variables, however without creating a system of "structural" equations, which is normally required to apply additional restrictions so as to express the dynamics among the various predictive variables used in each case.[6]

The form of the predictive regression model we will be using involves a linear relation between the value of an asset and a lagged variable, which variable's value should already be known at the beginning of each period, so as to play the role of a predictor. The form of this OLS regression model is given by the expression [Stambaugh (1999), p.376]:

$$y_{t+1} = \alpha + \beta x_t + \varepsilon_{t+1} \qquad (1)$$

where:

y_{t+1} reflects a change in the asset's value during the period from time *t* to *t+1*

x_t is the lagged regressor-variable, related to the asset's value, and known at time *t*

a, β are coefficients to be estimated

ε_{t+1} are the regression "residuals"

The equation above, although it refers to a single time period, it can be set up in an even more general way, which will enable us to examine the predictability power of the lagged variables at longer time horizons. That being the case, we will use the following regression formula to predict an asset's value over a longer horizon of *k>1* periods [Plazzi, Torous and Valkanov (2010), p.7]:

[6] The Plazzi, Torous and Valkanov (2010) paper includes a detailed methodology on how to construct a system of "structural" equations for predictive purposes, and is focused on the case of real estate. For an application of a similar methodology to the case of stocks, the reader may refer to:
Campbell, John Y., and Robert J. Shiller, 1988b, The Dividend-Price Ratio and Expectation of Future Dividends and Discount Factors, Review of Financial Studies 1, 195-228.

$$y_{t+1\to t+k} = \alpha_k + \beta_k x_t + \varepsilon_{t+1\to t+k} \tag{2}$$

where:

$$y_{t+1\to t+k} = \sum_{i=0}^{k} y_{t+1+i} \tag{3}$$

3.2. Prediction of Expected Returns to Commercial Real Estate and Stocks using Historical Data on Cap Rates and the Dividend-Price Ratio

By using the predictive regression models given by equations (1) and (2), we will now test whether or not, and to what degree, cap rates and the dividend-price ratio can predict the returns to commercial real estate and stocks. Our focus will be to examine the predictability of the returns to these two asset classes using quarterly time intervals within a one year timeframe, which is the most commonly used timeframe in portfolio analysis and portfolio rebalancing.

At this point it is also worth mentioning that the predictive regressions expressed by formula (2), combined with equation (3), require using overlapping data for returns in the left-hand side of equation (2). In order to deal with this issue, we apply the methodology proposed by Newey-West (1987b)[7], setting the Newey-West estimator q in each regression equal to the number of lags that correspond to the number of periods we want to predict the returns for, going forward in the future.

So, in the case of real estate, we will be applying the following regression formulas:

$$\left\{\begin{array}{r} r_{t+1}^{RE} = \alpha_1 + \beta_1 \left(cap_t\right) + \varepsilon_{t+1} \\ r_{t+1\to t+k}^{RE} = \alpha_k + \beta_k \left(cap_t\right) + \varepsilon_{t+1\to t+k} \end{array}\right\} \tag{4}$$

and

$$\left\{\begin{array}{r} r_{t+1}^{RE} = \alpha_1 + \beta_1 \left(dy_t\right) + \varepsilon_{t+1} \\ r_{t+1\to t+k}^{RE} = \alpha_k + \beta_k \left(dy_t\right) + \varepsilon_{t+1\to t+k} \end{array}\right\} \tag{5}$$

[7] Newey, Whitney K., and Kenneth D. West, 1987b, A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix, Econometrica 55, 703-708.

As far as the case of stocks is concerned, the forecasting regressions take the form:

$$\left\{\begin{array}{c} r^S_{t+1} = \alpha_1 + \beta_1 \left(dy_t\right) + \varepsilon_{t+1} \\ r^S_{t+1\to t+k} = \alpha_k + \beta_k \left(dy_t\right) + \varepsilon_{t+1\to t+k} \end{array}\right\} \tag{6}$$

and

$$\left\{\begin{array}{c} r^S_{t+1} = \alpha_1 + \beta_1 \left(cap_t\right) + \varepsilon_{t+1} \\ r^S_{t+1\to t+k} = \alpha_k + \beta_k \left(cap_t\right) + \varepsilon_{t+1\to t+k} \end{array}\right\}. \tag{7}$$

In this section we will be using historical quarterly data for the returns, cap rates and the dividend-price ratio to study the predictability of returns for *k=1, 2, 3* and 4 quarters going forward at each point in the sample period. Accordingly, we will be regressing the log excess returns (gross returns minus the risk-free rate) of each asset with each of the prediction variables, also expressed in logarithm form. The results of the forecasting regressions are shown in Exhibit 1 to follow.

3.3. Predictive Regression Results

Exhibit 1: Predictive Regression Summary Output

Table 1.1: Commercial Real Estate Excess Returns regressed against corresponding Cap Rates

Real Estate "All Properties" Returns against "All Properties" Cap Rates										
k	Coeff. α	Std. Error α	t Stat α	Coeff. β	Std. Error β	t Stat β	R^2	Adj. R^2	S.E. of Regr.	F Stat
1	-0.073343	0.029424	-2.492604	4.216335	1.550554	2.719244	0.068217	0.058991	0.042962	7.394289
2	-0.154187	0.084598	-1.822593	8.860487	4.115906	2.152743	0.136867	0.128236	0.061466	15.857062
3	-0.240755	0.128668	-1.871131	13.816903	6.214027	2.223502	0.191049	0.182878	0.078547	23.380766
4	-0.343539	0.167673	-2.048871	19.567742	8.031953	2.436237	0.252913	0.245290	0.092562	33.176188

Real Estate Apartment Returns against Apartment Cap Rates										
k	Coeff. α	Std. Error α	t Stat α	Coeff. β	Std. Error β	t Stat β	R^2	Adj. R^2	S.E. of Regr.	F Stat
1	-0.031590	0.027541	-1.147015	2.717027	1.573870	1.726335	0.046581	0.030951	0.043399	2.980232
2	-0.067767	0.090985	-0.744809	5.655661	4.642791	1.218160	0.083835	0.068566	0.066248	5.490390
3	-0.098103	0.138535	-0.708142	8.299472	7.045621	1.177962	0.099264	0.083997	0.088783	6.501984
4	-0.138343	0.184372	-0.750348	11.593448	9.323532	1.243461	0.124255	0.109156	0.109141	8.229303

Real Estate Industrial Returns against Industrial Cap Rates										
k	Coeff. α	Std. Error α	t Stat α	Coeff. β	Std. Error β	t Stat β	R^2	Adj. R^2	S.E. of Regr.	F Stat
1	-0.077065	0.045291	-1.701555	4.522622	2.257248	2.003600	0.061747	0.046365	0.055014	4.014413
2	-0.160956	0.136531	-1.178899	9.442792	6.244187	1.512253	0.126978	0.112428	0.077502	8.726815
3	-0.250420	0.214819	-1.165727	14.638700	9.773935	1.497728	0.186417	0.172628	0.095773	13.518735
4	-0.349182	0.285446	-1.223285	20.261914	12.931948	1.566811	0.224084	0.210706	0.117471	16.750368

Real Estate Office Returns against Office Cap Rates										
k	Coeff. α	Std. Error α	t Stat α	Coeff. β	Std. Error β	t Stat β	R^2	Adj. R^2	S.E. of Regr.	F Stat
1	-0.062702	0.032178	-1.948597	3.937124	1.633635	2.410039	0.086940	0.071971	0.042726	5.808289
2	-0.140087	0.124349	-1.126569	8.568009	5.797620	1.477849	0.149817	0.135647	0.068685	10.573047
3	-0.226884	0.186085	-1.219248	13.799716	8.628788	1.599265	0.191623	0.177922	0.095496	13.985779
4	-0.316520	0.241569	-1.310271	19.166161	11.146828	1.719427	0.229043	0.215751	0.117845	17.231190

Real Estate Retail Returns against Retail Cap Rates										
k	Coeff. α	Std. Error α	t Stat α	Coeff. β	Std. Error β	t Stat β	R^2	Adj. R^2	S.E. of Regr.	F Stat
1	-0.039722	0.037399	-1.062095	2.722393	1.874764	1.452126	0.033413	0.017568	0.044762	2.108671
2	-0.092331	0.092947	-0.993366	6.076085	4.330518	1.403085	0.068128	0.052597	0.068866	4.386545
3	-0.152183	0.142045	-1.071368	9.833005	6.596278	1.490690	0.099409	0.084145	0.090766	6.512565
4	-0.216236	0.188786	-1.145405	13.838795	8.750302	1.581522	0.124986	0.109900	0.111894	8.284662

Table 1.2: Commercial Real Estate Excess Returns regressed against the Stock Dividend-Price Ratio

Real Estate "All Properties" Returns against Stock Dividend-Price Ratio										
k	Coeff. α	Std. Error α	t Stat α	Coeff. β	Std. Error β	t Stat β	R^2	Adj. R^2	S.E. of Regr.	F Stat
1	0.038301	0.012213	3.136159	-5.352705	1.889520	-2.832839	0.073607	0.064435	0.042838	8.024975
2	0.073492	0.025572	2.873977	-10.056804	3.825154	-2.629124	0.118505	0.109690	0.062116	13.443607
3	0.102557	0.040843	2.510975	-13.681025	5.739261	-2.383761	0.127089	0.118272	0.081593	14.413604
4	0.133131	0.056342	2.362909	-17.715683	7.610046	-2.327934	0.142957	0.134212	0.099140	16.346698

Real Estate Apartment Returns against Stock Dividend-Price Ratio										
k	Coeff. α	Std. Error α	t Stat α	Coeff. β	Std. Error β	t Stat β	R^2	Adj. R^2	S.E. of Regr.	F Stat
1	0.056414	0.020310	2.777680	-8.991614	4.251444	-2.114955	0.068319	0.053045	0.042901	4.473035
2	0.104688	0.042166	2.482797	-16.357559	10.008137	-1.634426	0.094638	0.079549	0.065856	6.271844
3	0.150487	0.061957	2.428897	-23.025431	14.925958	-1.542643	0.103263	0.088064	0.088586	6.794093
4	0.169113	0.081730	2.069155	-23.517863	20.629286	-1.140023	0.066553	0.050459	0.112679	4.135297

Real Estate Industrial Returns against Stock Dividend-Price Ratio										
k	Coeff. α	Std. Error α	t Stat α	Coeff. β	Std. Error β	t Stat β	R^2	Adj. R^2	S.E. of Regr.	F Stat
1	0.065666	0.025947	2.530777	-11.521615	5.431524	-2.121249	0.068698	0.053431	0.054809	4.499698
2	0.111188	0.046496	2.391334	-18.384387	11.511819	-1.597001	0.083233	0.067954	0.079420	5.447413
3	0.144309	0.062330	2.315260	-22.519304	16.138478	-1.395380	0.076668	0.061019	0.102029	4.899030
4	0.175028	0.082311	2.126437	-26.271439	21.779142	-1.206266	0.063517	0.047370	0.129055	3.933834

Real Estate Office Returns against Stock Dividend-Price Ratio										
k	Coeff. α	Std. Error α	t Stat α	Coeff. β	Std. Error β	t Stat β	R^2	Adj. R^2	S.E. of Regr.	F Stat
1	0.061076	0.020213	3.021642	-10.276307	4.231204	-2.428696	0.088172	0.073224	0.042697	5.898562
2	0.117494	0.063304	1.856035	-19.768289	14.564321	-1.357309	0.119324	0.104646	0.069906	8.129515
3	0.151616	0.094276	1.608214	-23.867290	22.246494	-1.072856	0.086068	0.070577	0.101540	5.556202
4	0.162139	0.118412	1.369271	-22.781599	28.792842	-0.791224	0.047157	0.030729	0.131010	2.870490

Real Estate Retail Returns against Stock Dividend-Price Ratio										
k	Coeff. α	Std. Error α	t Stat α	Coeff. β	Std. Error β	t Stat β	R^2	Adj. R^2	S.E. of Regr.	F Stat
1	0.060658	0.020642	2.938591	-10.139883	4.321019	-2.346642	0.082800	0.067763	0.043603	5.506728
2	0.134338	0.053889	2.492874	-23.181406	10.372238	-2.234947	0.178907	0.165222	0.064643	13.073319
3	0.197964	0.079145	2.501278	-33.912509	14.886267	-2.278107	0.214287	0.200970	0.084779	16.091051
4	0.252170	0.102639	2.456874	-42.587954	19.322254	-2.204088	0.207465	0.193801	0.106490	15.182916

Table 1.3: Stock Excess Returns regressed against the Dividend-Price Ratio

Stock Returns against Dividend-Price Ratio										
k	Coeff. α	Std. Error α	t Stat α	Coeff. β	Std. Error β	t Stat β	R^2	Adj. R^2	S.E. of Regr.	F Stat
1	-0.028930	0.023636	-1.223984	7.252082	3.656930	1.983106	0.037478	0.027948	0.082907	3.932711
2	-0.061311	0.046998	-1.304557	14.896803	5.894222	2.527357	0.073559	0.064294	0.119725	7.939931
3	-0.093673	0.071812	-1.304419	22.492736	8.748794	2.570953	0.108302	0.099295	0.146871	12.024098
4	-0.120939	0.095543	-1.265812	29.000101	11.427090	2.537838	0.134539	0.125708	0.168109	15.234455

Table 1.4: Stock Excess Returns regressed against Commercial Real Estate Cap Rates

Stock Returns against R.E. "All Properties" Cap Rates

k	Coeff. α	Std. Error α	t Stat α	Coeff. β	Std. Error β	t Stat β	R^2	Adj. R^2	S.E. of Regr.	F Stat
1	-0.055457	0.057441	-0.965455	3.754737	3.026941	1.240439	0.015006	0.005254	0.083870	1.538690
2	-0.112757	0.139460	-0.808527	7.550466	7.252586	1.041072	0.028117	0.018398	0.122626	2.893018
3	-0.167885	0.194810	-0.861788	11.205192	10.248192	1.093382	0.039614	0.029913	0.152423	4.083499
4	-0.243320	0.235491	-1.033248	15.821476	12.596839	1.255988	0.058069	0.048457	0.175379	6.041567

Stock Returns against R.E. Apartment Cap Rates

k	Coeff. α	Std. Error α	t Stat α	Coeff. β	Std. Error β	t Stat β	R^2	Adj. R^2	S.E. of Regr.	F Stat
1	-0.056919	0.055463	-1.026256	3.991180	3.169465	1.259260	0.025337	0.009359	0.087397	1.585736
2	-0.112562	0.128363	-0.876904	7.825852	7.072960	1.106447	0.041714	0.025742	0.132909	2.611783
3	-0.175321	0.181300	-0.967022	12.024128	10.174006	1.181848	0.058060	0.042095	0.171990	3.636707
4	-0.251686	0.222712	-1.130096	16.771231	12.796994	1.310560	0.079240	0.063365	0.202726	4.991430

Stock Returns against R.E. Industrial Cap Rates

k	Coeff. α	Std. Error α	t Stat α	Coeff. β	Std. Error β	t Stat β	R^2	Adj. R^2	S.E. of Regr.	F Stat
1	-0.092273	0.071628	-1.288228	5.235265	3.569878	1.466511	0.034056	0.018221	0.087005	2.150655
2	-0.185500	0.173679	-1.068060	10.443422	8.436366	1.237905	0.057970	0.042269	0.131776	3.692236
3	-0.283808	0.240795	-1.178631	15.856373	11.818173	1.341694	0.078522	0.062904	0.170111	5.027598
4	-0.410806	0.282799	-1.452646	22.487847	14.129493	1.591554	0.109983	0.094638	0.199313	7.167267

Stock Returns against R.E. Office Cap Rates

k	Coeff. α	Std. Error α	t Stat α	Coeff. β	Std. Error β	t Stat β	R^2	Adj. R^2	S.E. of Regr.	F Stat
1	-0.094923	0.065222	-1.455381	5.481838	3.311224	1.655532	0.042999	0.027310	0.086601	2.740786
2	-0.194252	0.162665	-1.194186	11.115536	8.034730	1.383436	0.075903	0.060501	0.130516	4.928253
3	-0.299425	0.220699	-1.356714	16.992507	10.959485	1.550484	0.104376	0.089196	0.167708	6.875846
4	-0.435894	0.249608	-1.746315	24.237208	12.560427	1.929648	0.147819	0.133126	0.195030	10.060655

Stock Returns against R.E. Retail Cap Rates

k	Coeff. α	Std. Error α	t Stat α	Coeff. β	Std. Error β	t Stat β	R^2	Adj. R^2	S.E. of Regr.	F Stat
1	-0.058617	0.073404	-0.798548	3.557159	3.679638	0.966714	0.015089	-0.001057	0.087855	0.934536
2	-0.120209	0.190945	-0.629548	7.195606	9.492769	0.758009	0.026379	0.010152	0.133968	1.625610
3	-0.190712	0.268642	-0.709910	11.243125	13.512334	0.832064	0.037858	0.021551	0.173824	2.321541
4	-0.283006	0.326456	-0.866905	16.183756	16.744495	0.966512	0.054796	0.038500	0.205399	3.362429

In order to make it easier to follow the author's comments on the findings of the regression analysis above, the results will be presented separately for each table.

Comments on Table 1.1

As we can see in Table 1.1, where the results of the Commercial Real Estate returns regressed against cap rates are shown, the "All Properties" returns can be predicted by the corresponding cap rates at all time horizons (k=1 to 4), showing statistically significant t-stats (larger than 2) for the cap rate coefficient β. Moreover, we can see that the longer the forecasting horizon, the larger the R^2 gets, which starts with a value of 6.8% for k=1, reaching 25.2% for k=4.

As far as the Industrial and Office returns are concerned, we see that at the short horizon of a quarter (k=1), cap rates can predict returns, explaining 6.17% and 8.69% of the variability in returns, respectively. Furthermore, for the longer forecasting horizons we see that the predictability power decreases (lower t-stats), however it remains somewhat significant (t-stats around 1.5 to 1.7), with the R^2 increasing throughout, being for both cases more than 22%, for k=4.

Finally, for the cases of Apartment and Retail, we see that the cap rates can somewhat predict returns, since we have the t-stat of the β coefficient moving within a wide range of values, i.e. between 1.1 and 1.7. In this case we observe an increase in the R^2 as the forecasting horizon increases, but with lower R^2 values compared to the previous case of Industrial and Office buildings.

Comments on Table 1.2

Checking the results from Table 1.2, where Real Estate returns are regressed against the Stock dividend-price ratio, we can see that the "All Properties" and Retail returns can be predicted by the dp ratio at all forecasting horizons (k=1 to 4), having statistically significant t-stat values for the dp ratio β coefficient. Furthermore, for the longer forecasting horizons we see that the R^2 is increasing, and even reaching 20% in the case of Retail with k=4.

For the case of Apartment, Industrial and Office buildings, we see they follow a prediction "pattern", where the dividend-price ratio can predict the returns in the short-run (k=1), and as the forecasting horizon increases, the t-stats are getting less significant. Please note that the R^2 in all of these cases appears to be much less volatile compared to the case of "All Properties" and Retail discussed above.

Comments on Table 1.3

Moving to stocks, we can see from Table 1.3 that the dividend-price ratio can predict stock returns at all forecasting horizons (k=1 to 4), having statistically significant t-stat values for the dp ratio β coefficient. As far as the R^2 is concerned, its value increases significantly as the forecasting horizon increases, starting with a value of just 3.74% for k=1, and reaching up almost 13.5% for k=4.

Comments on Table 1.4

Turning our attention to the ability of the cap rates to predict stock returns, we can see in Table 1.4 that the results are less significant, compared to the cases of the previous tables. More specifically, the t-stats show a limited ability of the cap rates to predict stock returns for all cases, since the t-stats that correspond to the cap rates range almost exclusively between 1.0 and 1.5.

We can see two exceptions to the aforementioned "pattern". One has to do with the fact that the t-stats in Retail properties have all insignificant values, less than 1.0. The other exception is the t-stat value we got for Office in the case of k=4, where we have a significant t-stat value almost equal to 2, explaining about 15% of the variability in returns.

4. Dynamic Allocation of Commercial Real Estate in a Risky Asset Portfolio

4.1. Dynamic Portfolio Allocation Methodology

We now introduce the reader to the dynamic portfolio allocation methodology followed in this chapter, and more specifically, to the Brandt and Santa-Clara (2006) approach outlined in their paper the "Dynamic Portfolio Selection by Augmenting the Asset Space" (reference [1]).

The intuition behind this methodology is to expand the existing set of "basis" assets used in the dynamic portfolio analysis so as to include mechanically managed, "conditional" portfolios, and then compute the static portfolio weights of this expanded set of assets by applying the traditional Markowitz solution to the optimization problem. Hence, the time-varying weights invested in each "basis" asset in the dynamic portfolio are expressed as a linear function of the parameterized weights of the "basis" asset and its corresponding "conditional" assets in the aforementioned static portfolio.

To make things easier, we start by defining a few concepts which will be used throughout this chapter. Firstly, "conditional" portfolios are defined as mechanically constructed portfolios that invest in each "basis" asset (for instance stocks) an amount proportional to the level of the conditioning variable in each time period. Secondly, the conditioning variables are those factors, for which there is evidence that they can predict the returns of the "basis" assets (such is the case of dividend-price ratio, which can predict the stock market returns, as we saw in the previous chapter).

Moreover, please note that throughout this chapter we will be working with asset excess returns, which are calculated by the equation:

$$r_{t+1}^{i} = R_{t+1}^{i} - R_{t}^{f} \tag{8}$$

where:

r_{t+1}^{i} is the excess return of the "basis" asset i at time $t+1$

R_{t+1}^{i} is the total (gross) return of asset i at time $t+1$

R_{t}^{f} is the risk-free rate at time t, known prior to time $t+1$.

The rationale for using excess returns is that by doing so, we implicitly presume that the remainder of the portfolio's value is invested in the risk-free asset (3-month US T-Bills), with a return equal to R_t^f [Brandt and Santa-Clara (2006), p.2191].

Now, assuming that we have N "basis", risky assets and K "conditional", state variables, we define the time-varying vectors of excess returns r_{t+1} $(N\times1)$ and state variables z_t $(K\times1)$ as in (9) below. Please note that the first value of the state variables is taken to be a constant equal to 1, which reflects the static mean-variance case (no conditioning variables applied).

$$r_{t+1}=\begin{bmatrix} r_{t+1}^1 \\ r_{t+1}^2 \\ \cdots \\ r_{t+1}^N \end{bmatrix} \quad , \quad z_t=\begin{bmatrix} 1 \\ z_t^1 \\ \cdots \\ z_t^{K-1} \end{bmatrix} \tag{9}$$

where:

r_{t+1}^i is the excess return of the "basis" asset *i* at time *t+1* (*i*=1, 2…N denotes a different "basis" asset)

z_t^j is the value of the conditioning variable *j* at time *t* (*j*=1, 2…K-1 denotes a different state variable).

Furthermore, we construct a $(N\times1)$ vector of the parameterized, time-varying portfolio weights x_t, and a $(N\times K)$ coefficient matrix θ. These matrices are related to each other in such a way so as to satisfy equation (10) given below, thus defining a system of linear equalities, and relating the state variables vector z_t to the portfolio weights vector x_t (ibid, p.2190):

$$x_t=\theta z_t \Rightarrow \begin{bmatrix} x_t^1 \\ x_t^2 \\ \cdots \\ x_t^N \end{bmatrix}=\begin{bmatrix} \theta_{11} & \theta_{12} & \cdots & \theta_{1\mathrm{K}} \\ \theta_{21} & \theta_{22} & \cdots & \theta_{2\mathrm{K}} \\ \cdots & \cdots & \cdots & \cdots \\ \theta_{\mathrm{N}1} & \theta_{\mathrm{N}2} & \cdots & \theta_{\mathrm{NK}} \end{bmatrix}\times\begin{bmatrix} 1 \\ z_t^1 \\ \cdots \\ z_t^{K-1} \end{bmatrix}\Rightarrow\left\{\begin{matrix} x_t^1=\theta_{11}+\theta_{12}z_t^1+\cdots+\theta_{1\mathrm{K}}z_t^{K-1} \\ x_t^2=\theta_{21}+\theta_{22}z_t^1+\cdots+\theta_{2\mathrm{K}}z_t^{K-1} \\ \cdots \\ x_t^N=\theta_{N1}+\theta_{N2}z_t^1+\cdots+\theta_{N\mathrm{K}}z_t^{K-1} \end{matrix}\right\} \tag{10}$$

where:

x_t^i is the weight of the "basis" asset *i* at time *t* (*i*=1, 2…N denotes a different "basis" asset)

θ_{ij} is a coefficient of the matrix θ.

Next we define the expanded static portfolio, which is equivalent to the dynamic portfolio with *N* "basis" assets. This static portfolio is a portfolio which includes (*NK*) assets, and is comprised of both "basis" and "conditional" assets. For this expanded portfolio, the returns of the "basis" assets at time *t+1* are taken from the database used in each case, whereas the returns of the "conditional" assets are created by multiplying the returns of the "basis" assets at time *t+1* with the value of the state variables at time *t* (ibid, p.2198). Therefore, we now have an expanded static portfolio, which has a time series of *t+1* observations of the form below (ibid, p.2199):

$$\begin{array}{c}
\begin{array}{ccccc}
\overbrace{\hspace{6em}}^{N\text{ "basis" Assets}} & \overbrace{\hspace{8em}}^{N\text{ Assets "conditional" on } z^{1}} & \hspace{2em} & \overbrace{\hspace{10em}}^{N\text{ Assets "conditional" on } z^{k-1}}
\end{array}\\
\begin{bmatrix}
r_1^1 & r_1^2 & \cdots & r_1^N & z_0^1 r_1^1 & z_0^1 r_1^2 & \cdots & z_0^1 r_1^N & \cdots & z_0^{K-1} r_1^1 & z_0^{K-1} r_1^2 & \cdots & z_0^{K-1} r_1^N \\
r_2^1 & r_2^2 & \cdots & r_2^N & z_1^1 r_2^1 & z_1^1 r_2^2 & \cdots & z_1^1 r_2^N & \cdots & z_1^{K-1} r_2^1 & z_1^{K-1} r_2^2 & \cdots & z_1^{K-1} r_2^N \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
r_{t+1}^1 & r_{t+1}^2 & \cdots & r_{t+1}^N & z_t^1 r_{t+1}^1 & z_t^1 r_{t+1}^2 & \cdots & z_t^1 r_{t+1}^N & \cdots & z_t^{K-1} r_{t+1}^1 & z_t^{K-1} r_{t+1}^2 & \cdots & z_t^{K-1} r_{t+1}^N
\end{bmatrix}
\end{array} \tag{11}$$

where:

r_{t+1}^i is the excess return of the "basis" asset *i* at time *t+1* (*i*=1, 2…N denotes a different "basis" asset)

z_t^j is the value of the conditional variable *j* at time *t* (*j*=1, 2… K-1 denotes a different state variable).

Please note that the difference in dating between the returns and the state variables reflects the fact that the state variables should be known at the beginning of each return period, as they play the role of predictors (ibid, p.2198).

Considering now the assumptions of the static portfolio theory [Markowitz (1952)] that the investor's goal is to maximize the expected returns over a single period going forward in the future, and at the same time minimize the risk, the optimization problem of the dynamic portfolio can be expressed as an optimization problem of the newly created static portfolio. Therefore, the optimization problem can be expressed as [Brandt and Santa-Clara (2006), p.2191]:

$$\max_{x_t} E_t\left[x_t^{\mathbf{T}} r_{t+1} - \frac{\gamma}{2} x_t^{\mathbf{T}} r_{t+1} r_{t+1}^{\mathbf{T}} x_t \right] \tag{12}$$

where:

x_t is the time-varying portfolio weights vector, as defined above ($x_t = \theta z_t$)

r_{t+1} is the excess returns vector of the "basis" assets, as defined above

γ is a constant reflecting the representative investor's risk tolerance.

In order to solve expression (12) analytically we need to make some additional transformations. For this reason, we define two additional vectors, namely the $(NK \times 1)$ vectors $\tilde{x}$ and $\tilde{r}_{t+1}$ as follows:

$$\tilde{x} = \begin{bmatrix} \tilde{x}_1 & \tilde{x}_2 & \cdots & \tilde{x}_{NK} \end{bmatrix}^{\mathbf{T}} = vec(\theta) = \begin{bmatrix} \theta_{11} & \cdots & \theta_{N1} & \theta_{12} & \cdots & \theta_{N2} & \cdots & \theta_{1K} & \cdots & \theta_{NK} \end{bmatrix}^{\mathbf{T}} \quad (13)$$

$$\tilde{r}_{t+1} = z_t \otimes r_{t+1} \Rightarrow \tilde{r}_{t+1} = \begin{bmatrix} r_{t+1}^1 & \cdots & r_{t+1}^N & z_t^1 r_{t+1}^1 & \cdots & z_t^1 r_{t+1}^N & \cdots & z_t^{K-1} r_{t+1}^1 & \cdots & z_t^{K-1} r_{t+1}^N \end{bmatrix}^{\mathbf{T}}. \quad (14)$$

By using linear algebra, expression (12) can now be re-written in the form of (15) below (ibid, p.2192), providing an analytical solution, as expressed in equation (16):

$$\max_{\tilde{x}} E\left[\tilde{x}^{\mathbf{T}} \tilde{r}_{t+1} - \frac{\gamma}{2} \tilde{x}^{\mathbf{T}} \tilde{r}_{t+1} \tilde{r}_{t+1}^{\mathbf{T}} \tilde{x} \right] \quad (15)$$

$$\tilde{x} = \frac{1}{\gamma} \left[\sum_{t=0}^{T} \left(z_t z_t^{\mathbf{T}} \right) \otimes \left(r_{t+1} r_{t+1}^{\mathbf{T}} \right) \right]^{-1} \left[\sum_{t=0}^{T} z_t \otimes r_{t+1} \right]. \quad (16)$$

In essence, the values of vector $\tilde{x}$, calculated by using equation (16), represent the optimal portfolio weights of each asset in the augmented asset space of the static portfolio.

Moreover, $\tilde{x}$ is a "vectorized" version of the coefficients matrix θ, as defined in formula (13). Therefore its values can be used in expression (10) to calculate the time-varying portfolio weights x_t of the "basis" assets, by simply placing the values of the state variables at each point in time:

$$\begin{Bmatrix} x_t^1 = \theta_{11} + \theta_{12} z_t^1 + \cdots + \theta_{1\mathrm{K}} z_t^{K-1} \\ x_t^2 = \theta_{21} + \theta_{22} z_t^1 + \cdots + \theta_{2\mathrm{K}} z_t^{K-1} \\ \cdots \\ x_t^N = \theta_{N1} + \theta_{N2} z_t^1 + \cdots + \theta_{N\mathrm{K}} z_t^{K-1} \end{Bmatrix}. \tag{17}$$

Finally, in order to estimate the expected performance of the dynamic portfolio, we can use, once again, the equivalent expanded static portfolio returns and the corresponding "basis" and "conditional" asset weights from vector $\tilde{x}$, in order to compute all the first and second moment statistics.

4.2. Application of the Dynamic Asset Allocation Methodology

In this section we will describe how we have applied the general methodology above, seeking an answer to the main question that this thesis examines, namely whether or not we can use cap rates to better allocate real estate in a dynamic portfolio.

Our analysis concerns portfolios of two risky assets (stocks and real estate) and a risk-free asset (US T-bills with a 3-month maturity period), considering three different portfolio rebalancing periods. The most commonly used rebalancing horizons for a portfolio are the quarterly, semi-annual and annual rebalancing, and for that reason our work will be focused on these horizons. Furthermore, in the case of real estate, and for each of the aforementioned rebalancing periods, we will be working with the "All Properties" data and with the four major property types, forming five different portfolios in total, that include stocks, and one of the real estate categories.

Additionally, and for comparison reasons, in each analysis presented in this chapter we will be examining three different cases for each portfolio: the first case refers to the unconditional-static case, the second involves considering the dividend-price ratio as the single conditioning variable in the dynamic portfolio, whereas the third case takes into account both the dividend-price ratio and cap rate as conditioning variables.

Finally, note that in our analysis we use the notation "*S*" and "*RE*" to refer to the "basis" assets, namely stocks and real estate. Additionally, we name the corresponding "conditional" assets on dividend-price ratio and on cap rates for each of the aforementioned "basis" assets as "*S-dp*", "*S-cap*" and "*RE-dp*", "*RE-cap*", respectively.

We now briefly describe how the general methodology of Section 4.1 is adjusted to each of the aforementioned unconditional, conditional on *dp* and conditional on *dp* and *cap* cases. After this, we discuss the restrictions and additional assumptions that are applicable to the dynamic allocation of commercial real estate in a portfolio.

Starting with the unconditional-static portfolio case, we can see that the optimization problem [expression (12)] with a state variables vector of the form $z_t = [1]$, and with no time-varying weights $x_t = x$, corresponds to the traditional, Markowitz portfolio optimization problem. So, in this case, expression (12) is simplified, providing the following analytical solution for *x* [Brandt and Santa-Clara (2006), p.2191]:

$$x = \frac{1}{\gamma}\left[\sum_{t=1}^{T-1} r_{t+1} r_{t+1}^{\mathbf{T}}\right]^{-1}\left[\sum_{t=1}^{T-1} r_{t+1}\right]. \tag{18}$$

In the second case with the two "basis" assets *S* and *RE* being conditional on *dp*, we have both the number of assets and state variables equal to 2. (*N*=2, *K*=2). The extended static portfolio will be a portfolio with four assets, two "basis" and two "conditional" ones, with a time series of observations:

$$\begin{bmatrix} r_1^S & r_1^{RE} & dp_0 r_1^S & dp_0 r_1^{RE} \\ r_2^S & r_2^{RE} & dp_1 r_2^S & dp_1 r_2^{RE} \\ \cdots & \cdots & \cdots & \cdots \\ r_{t+1}^S & r_{t+1}^{RE} & dp_t r_{t+1}^S & dp_t r_{t+1}^{RE} \end{bmatrix}. \tag{19}$$

For this portfolio, if we apply the analytical solution given by equation (16), we can calculate the optimal weights $\tilde{x}$ for these four assets, and hence the values of the coefficients vector θ from the equality:

$$\tilde{x} = \left[\tilde{x}_S \quad \tilde{x}_{RE} \quad \tilde{x}_{S-dp} \quad \tilde{x}_{RE-dp} \right]^{\mathbf{T}} = vec(\theta) = \left[\theta_{11} \quad \theta_{21} \quad \theta_{12} \quad \theta_{22} \right]^{\mathbf{T}} . \tag{20}$$

Therefore, from equation (10) we can compute the time-varying weights of the “basis” assets:

$$x_t = \theta z_t \Rightarrow \begin{bmatrix} x_t^S \\ x_t^{RE} \end{bmatrix} = \begin{bmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{bmatrix} \times \begin{bmatrix} 1 \\ dp \end{bmatrix} \Rightarrow \begin{Bmatrix} x_t^S = \theta_{11} + \theta_{12} dp_t \\ x_t^{RE} = \theta_{21} + \theta_{22} dp_t \end{Bmatrix} \tag{21}$$

Similarly, when working on the case where the “basis” assets are conditional on both *dp* and *cap* (N=2, K=3), we solve the optimization problem for the following expanded set of assets:

$$\begin{bmatrix} r_1^S & r_1^{RE} & dp_0 r_1^S & dp_0 r_1^{RE} & cap_0 r_1^S & cap_0 r_1^{RE} \\ r_2^S & r_2^{RE} & dp_1 r_2^S & dp_1 r_2^{RE} & cap_1 r_2^S & cap_1 r_2^{RE} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{t+1}^S & r_{t+1}^{RE} & dp_t r_{t+1}^S & dp_t r_{t+1}^{RE} & cap_t r_{t+1}^S & cap_t r_{t+1}^{RE} \end{bmatrix} . \tag{22}$$

By using equation (16) as before, we calculate the optimal weights $\tilde{x}$ for the six assets, and hence the values of the coefficients vector θ from the equality:

$$\tilde{x} = \left[\tilde{x}_S \quad \tilde{x}_{RE} \quad \tilde{x}_{S-dp} \quad \tilde{x}_{RE-dp} \quad \tilde{x}_{S-cap} \quad \tilde{x}_{RE-cap} \right]^{\mathbf{T}} = vec(\theta) = \left[\theta_{11} \quad \theta_{21} \quad \theta_{12} \quad \theta_{22} \quad \theta_{13} \quad \theta_{23} \right]^{\mathbf{T}} . \tag{23}$$

We can then calculate the time-varying weights of the “basis” assets:

$$x_t = \theta z_t \Rightarrow \begin{bmatrix} x_t^S \\ x_t^{RE} \end{bmatrix} = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & \theta_{23} \end{bmatrix} \times \begin{bmatrix} 1 \\ dp_t \\ cap_t \end{bmatrix} \Rightarrow \begin{Bmatrix} x_t^S = \theta_{11} + \theta_{12} dp_t + \theta_{13} cap_t \\ x_t^{RE} = \theta_{21} + \theta_{22} dp_t + \theta_{23} cap_t \end{Bmatrix} . \tag{24}$$

The final part of this section deals with some assumptions and realistic constraints imposed to the analysis mainly due to the particularities of real estate as an asset class.

As far as the analysis assumptions are concerned, following Brandt and Santa-Clara (2006) and Plazzi, Torous and Valkanov (2010), we set the constant γ reflecting the representative investor's risk tolerance equal to 5.

Moreover, we follow the approach of Plazzi, Torous and Valkanov (2010), p.34 in order to account for the transaction costs in commercial real estate, which based on Geltner, Miller, Clayton and Eichholtz (2006), p.275, can be at a minimum, around 5% for a typical "roundtrip" (i.e. buying and selling) transaction. Therefore, for the unconditional-static case we apply a 5% per annum reduction to the commercial real estate returns. On the other hand, when we are dealing with the conditioning cases, we follow a more general approach, as proposed by Brandt, Santa-Clara and Valkanov (2009), so as to estimate the portfolio returns net of trading costs with equation (25), and then solve the optimization problem (ibid, pp. 3423-3424).

$$r_{t+1}^{p} = \sum_{i=1}^{N} \tilde{x}_i r_{t+1}^{i} - c_t^{i} \left| x_t^{i} - x_{t-1}^{i} \right| \tag{25}$$

where:

r_{t+1}^{p} is the portfolio's excess return at time *t+1*

$\tilde{x}_i$ is the optimal weight of asset *i* included in the expanded portfolio

r_{t+1}^{i} is the excess return of asset *i* at time *t+1* included in the expanded portfolio

$\left| x_t^{i} - x_{t-1}^{i} \right|$ is the absolute difference in weights of asset *i* included in the expanded portfolio, from period *t-1* to period *t*

c_t^{i} reflects the transaction costs for asset *i* at time *t*, taken equal to 0 for stocks and 2.5% per annum ("single-trip") for real estate (ibid, p.3424).

Furthermore, we will also be examining the performance of the dynamic portfolios under the realistic restriction of "no shorting" of the real estate assets [Plazzi, Torous and Valkanov (2010), p.36]. This is a reasonable restriction, given the illiquidity issues that investors face in the case of investing in real estate. In order to deal with that issue, we prohibit short positions in real estate whenever they occur. Thus, whenever the time varying weight x_t^{RE} takes negative values we set the weights of the real estate-related assets equal to zero, and assume that all of these funds are invested in T-bills, with return equal to the risk-free rate.

Finally, note that in this thesis we are following the Plazzi, Torous and Valkanov (2010) approach and use standardized values for the state variables, a fact which enables us to interpret the values of $\tilde{x}_S$ and $\tilde{x}_{RE}$ of each "basis" asset as "a time-series average of weights" (ibid, p.34). In addition, we will be following this paper's approach, and use the Sharpe ratio and the certainty equivalent return as the performance and comparison criteria.

In the sections to follow we provide the reader with the results from the analysis, applying the dynamic allocation methodology described in this Section. More specifically, for each portfolio we have included two charts that show the time series of the portfolio weights of the two "basis" assets [expressions used: (18), (21), (24)], a table that shows the basic first and second moment statistics for the expanded portfolio, a table with the optimal portfolio weights of the augmented asset space [expressions used: (16), (18), (20), (23)] and a summarizing table of the portfolio performance.

4.3. Results of the Dynamic Portfolio Analysis with Quarterly Rebalancing

Exhibit 2: Stocks and R.E. "All Properties" Dynamic Portfolio Optimization

(Unconditional and Conditional Portfolios with Quarterly Rebalancing)

Figure 2.1: Time Series of Portfolio Weights on Stocks

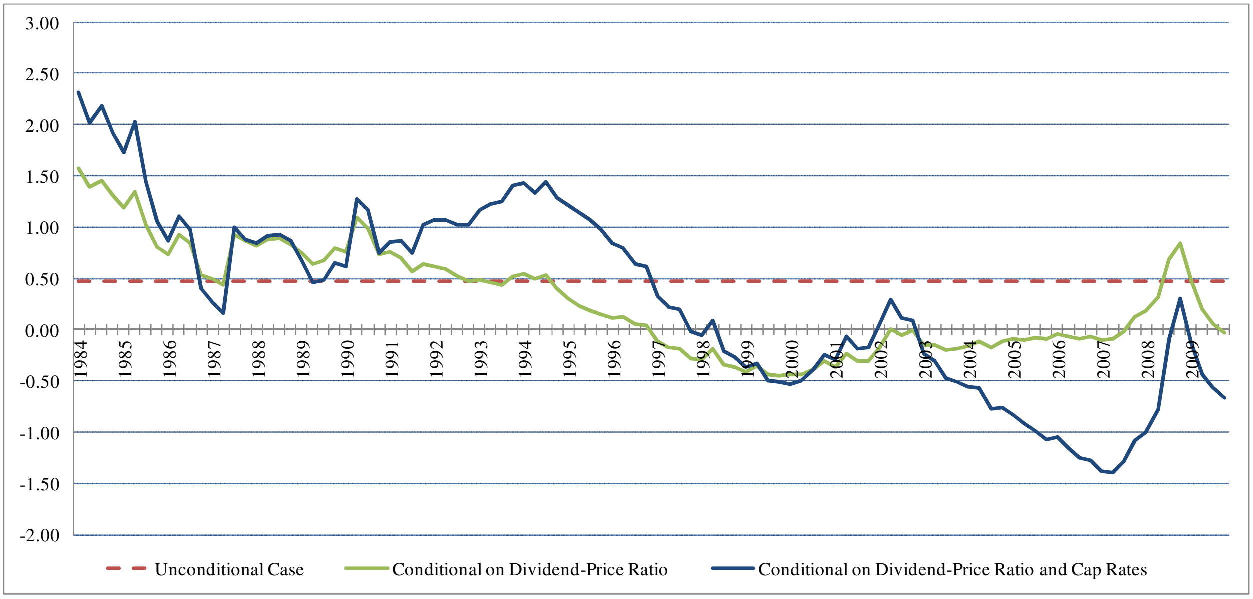

Figure 2.2: Time Series of Portfolio Weights on R.E. "All Properties"

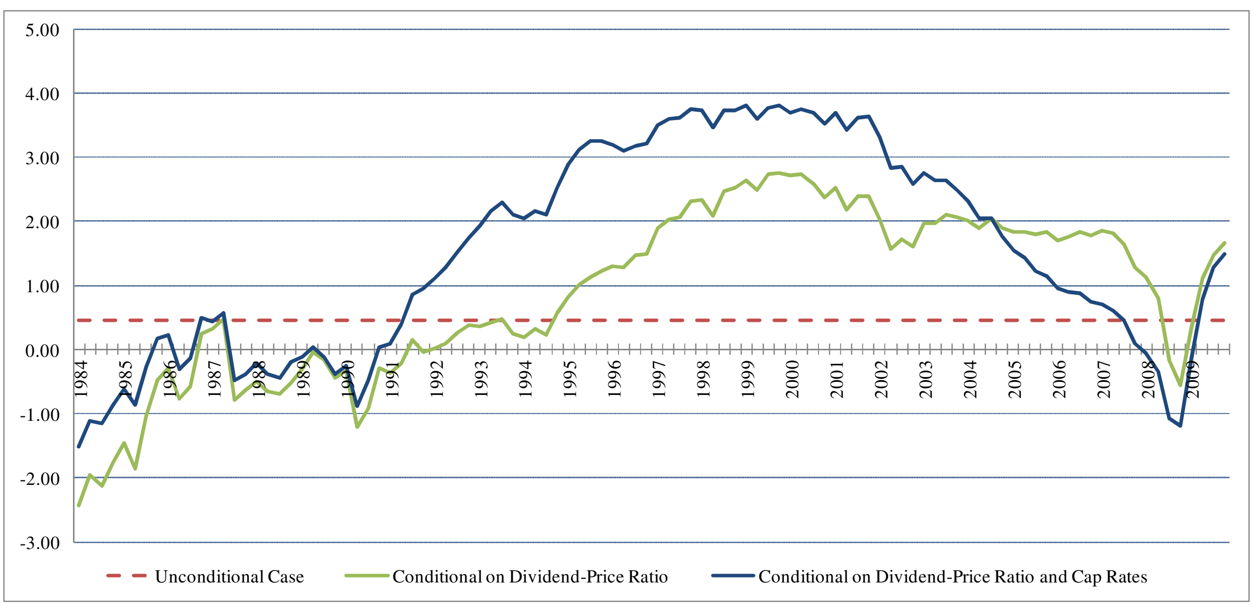

Table 2.1: Major First and Second Moment Statistics

Return, Volatility and Sharpe Ratio of "Basis" and "Conditional" Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
$E[r_i]$	0.017992	0.006655	-0.004154	-0.013054	0.010135	0.009290
σ_i	0.082510	0.043877	0.085830	0.035473	0.084570	0.045650
Sharpe Ratio	0.218053	0.151664	-0.048394	-0.367989	0.119839	0.203500
Correlation Matrix of Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
S	1.000000	0.185825	0.045493	-0.006098	-0.122552	-0.077165
R.E.		1.000000	-0.060296	0.050983	-0.051200	-0.553929
S-dp			1.000000	0.290012	-0.621519	-0.148331
R.E.-dp				1.000000	-0.162190	-0.137489
S-cap					1.000000	0.352786
R.E.-cap						1.000000

Table 2.2: Optimal Portfolio Weights

		Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
Stocks	$\tilde{x}_S$	0.473687	0.286200	0.296939
	$\tilde{x}_{S-dp}$	—	0.508612	0.871828
	$\tilde{x}_{S-cap}$	—	—	0.586544
R.E.	$\tilde{x}_{R.E.}$	0.463154	0.859254	1.499485
	$\tilde{x}_{R.E.-dp}$	—	-1.301297	-1.074407
	$\tilde{x}_{R.E.-cap}$	—	—	0.933634

Table 2.3: Dynamic Portfolio Performance

	Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
	Allowing Shorting of Commercial Real Estate		
$E[r_p]$	0.011394	0.024353	0.038685
σ_p	0.047243	0.070103	0.087359
Sharpe Ratio	0.241175	0.347392	0.442831
Certainty Equivalent Return	0.010947	0.023370	0.037159
	Restricting Shorting of Commercial Real Estate		
$E[r_p]$	0.011394	0.035583	0.046182
σ_p	0.047243	0.072075	0.087541
Sharpe Ratio	0.241175	0.493692	0.527550
Certainty Equivalent Return	0.010947	0.034544	0.044650

Exhibit 3: Stocks and R.E. Apartment Dynamic Portfolio Optimization

(Unconditional and Conditional Portfolios with Quarterly Rebalancing)

Figure 3.1: Time Series of Portfolio Weights on Stocks

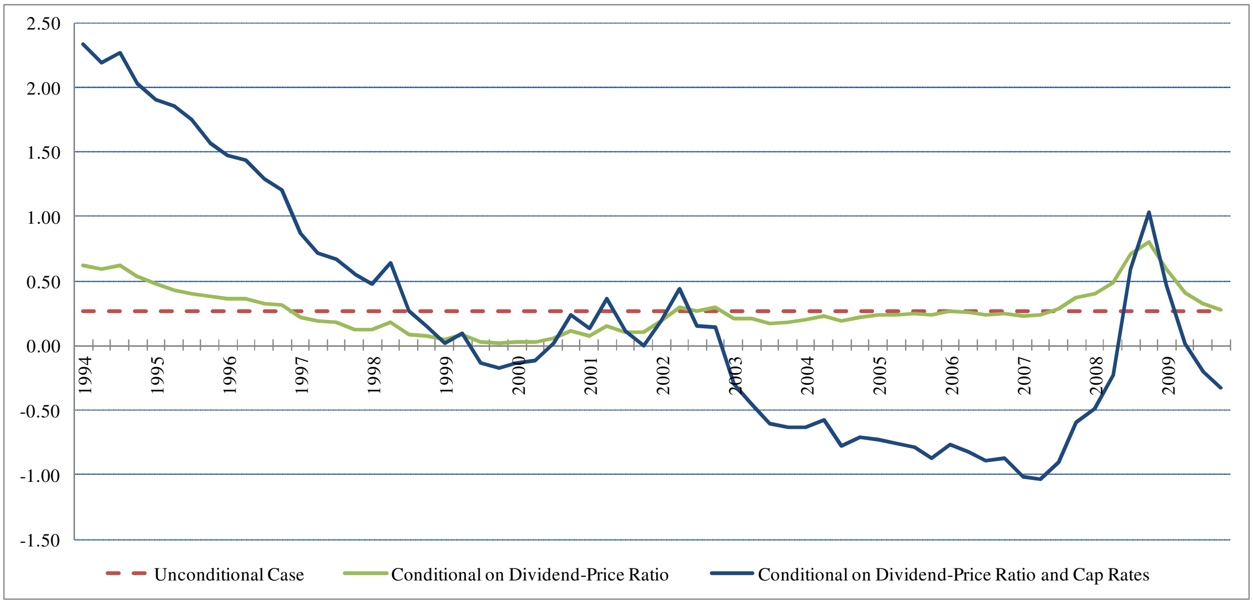

Figure 3.2: Time Series of Portfolio Weights on R.E. Apartment

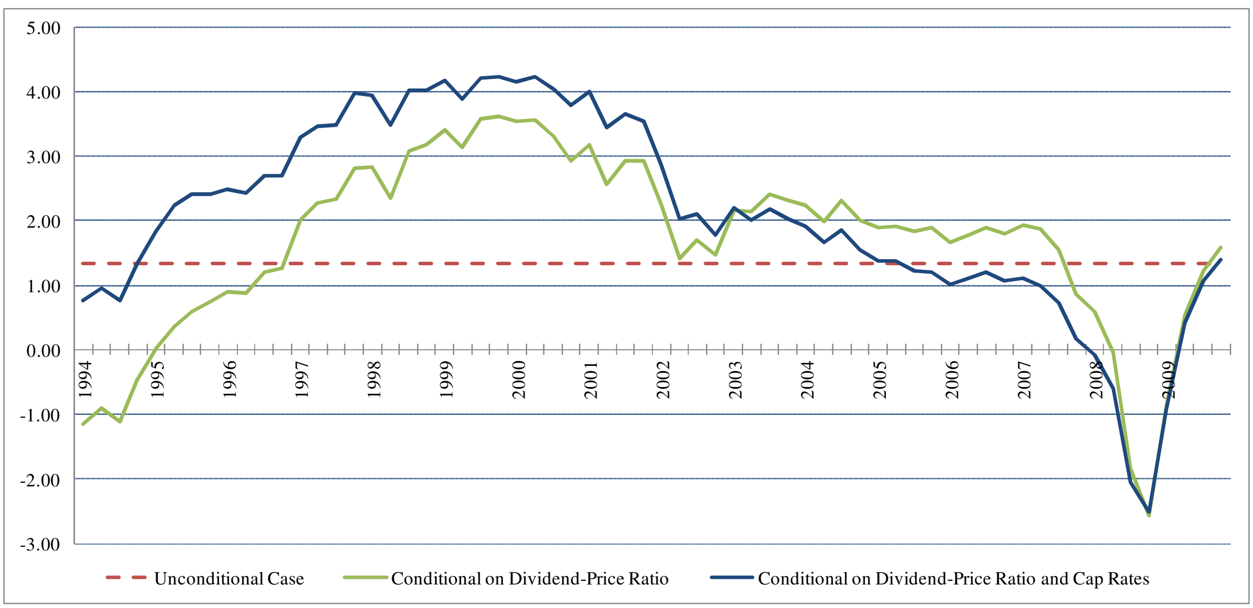

Table 3.1: Major First and Second Moment Statistics

Return, Volatility and Sharpe Ratio of "Basis" and "Conditional" Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
$E[r_i]$	0.015030	0.017185	-0.011410	-0.010269	0.013995	0.009011
σ_i	0.086030	0.045118	0.110394	0.058516	0.081503	0.050998
Sharpe Ratio	0.174712	0.380882	-0.103355	-0.175496	0.171715	0.176697
Correlation Matrix of Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
S	1.000000	0.133788	0.167570	0.252729	-0.037577	-0.187480
R.E.		1.000000	0.263950	0.426161	-0.252709	-0.359911
S-dp			1.000000	0.436184	-0.691200	-0.262944
R.E.-dp				1.000000	-0.301678	-0.147555
S-cap					1.000000	0.318281
R.E.-cap						1.000000

Table 3.2: Optimal Portfolio Weights

		Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
Stocks	$\tilde{x}_S$	0.266058	0.268787	0.204847
	$\tilde{x}_{S-dp}$	—	0.173866	0.604269
	$\tilde{x}_{S-cap}$	—	—	0.799177
R.E.	$\tilde{x}_{R.E.}$	1.332134	1.651428	2.083799
	$\tilde{x}_{R.E.-dp}$	—	-1.377659	-1.128597
	$\tilde{x}_{R.E.-cap}$	—	—	0.883890

Table 3.3: Dynamic Portfolio Performance

	Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
	Allowing Shorting of Commercial Real Estate		
$E[r_p]$	0.026221	0.042425	0.060577
σ_p	0.066875	0.083322	0.099744
Sharpe Ratio	0.392084	0.509174	0.607328
Certainty Equivalent Return	0.025326	0.041037	0.058588
	Restricting Shorting of Commercial Real Estate		
$E[r_p]$	0.026221	0.042358	0.054784
σ_p	0.066875	0.084063	0.103159
Sharpe Ratio	0.392084	0.503881	0.531063
Certainty Equivalent Return	0.025326	0.040944	0.052656

Exhibit 4: Stocks and R.E. Industrial Dynamic Portfolio Optimization (Unconditional and Conditional Portfolios with Quarterly Rebalancing)

Figure 4.1: Time Series of Portfolio Weights on Stocks

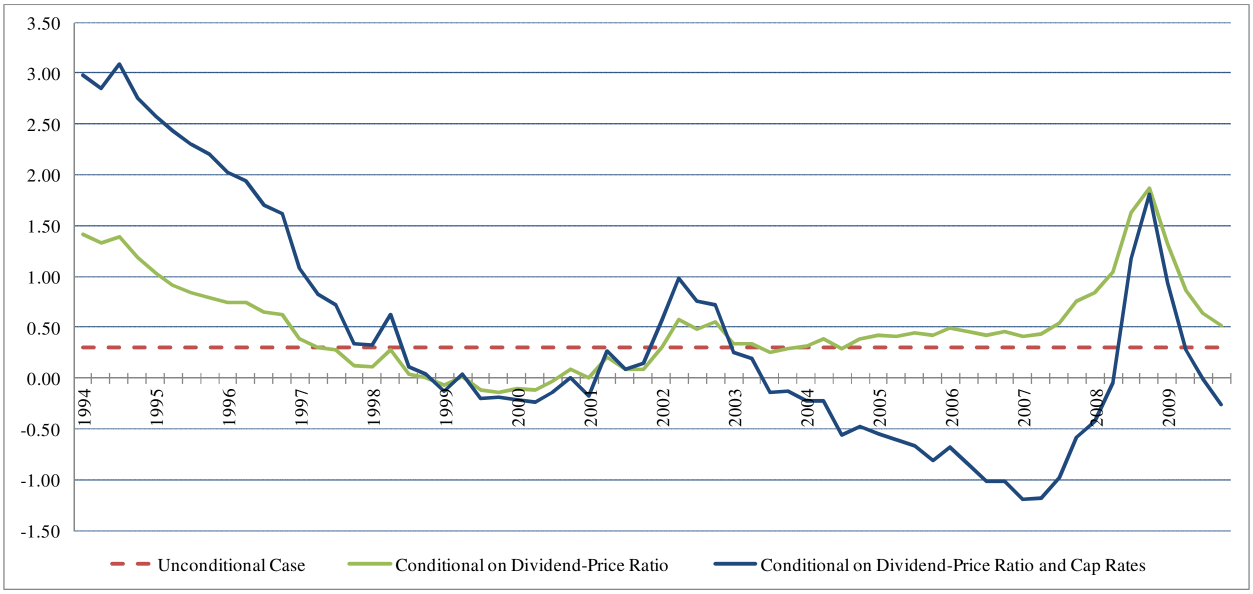

Figure 4.2: Time Series of Portfolio Weights on R.E. Industrial

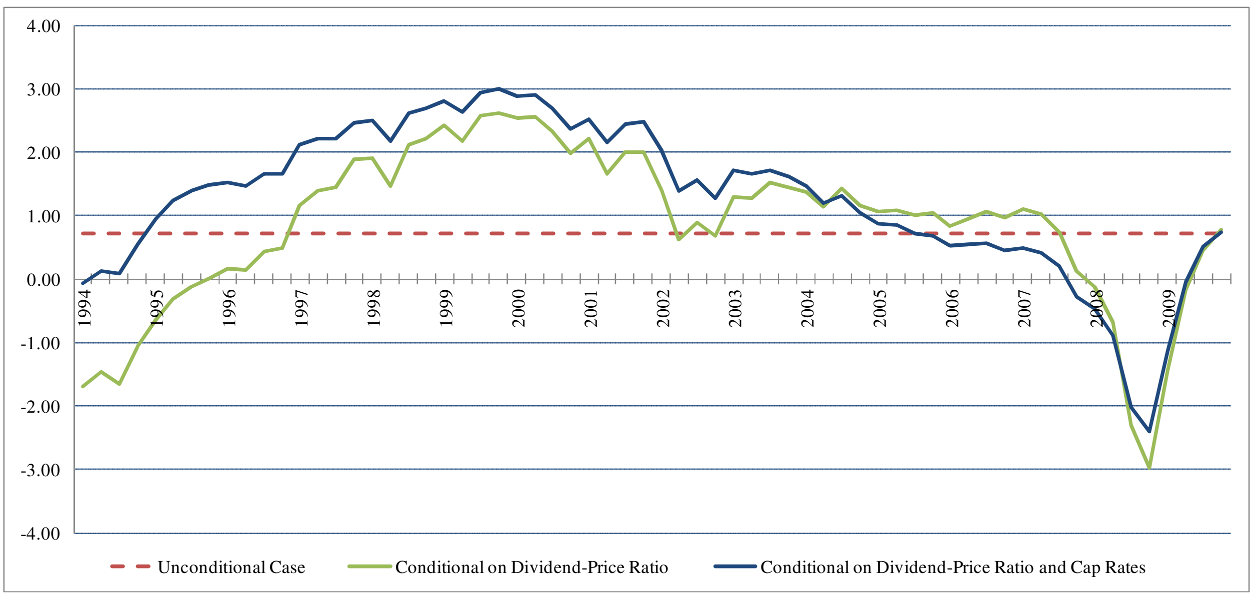

Table 4.1: Major First and Second Moment Statistics

Return, Volatility and Sharpe Ratio of "Basis" and "Conditional" Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
$E[r_i]$	0.015030	0.014058	-0.011410	-0.018013	0.016335	0.010409
σ_i	0.086030	0.056489	0.110394	0.082989	0.084045	0.064111
Sharpe Ratio	0.174712	0.248871	-0.103355	-0.217057	0.194358	0.162354
Correlation Matrix of Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
S	1.000000	0.186425	0.167570	0.345068	-0.079308	-0.265670
R.E.		1.000000	0.377090	0.392704	-0.324303	-0.682864
S-dp			1.000000	0.563814	-0.732877	-0.362355
R.E.-dp				1.000000	-0.342130	-0.593581
S-cap					1.000000	0.422885
R.E.-cap						1.000000

Table 4.2: Optimal Portfolio Weights

		Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
Stocks	$\tilde{x}_S$	0.297519	0.499072	0.419238
	$\tilde{x}_{S-dp}$	—	0.445469	0.892207
	$\tilde{x}_{S-cap}$	—	—	0.906579
R.E.	$\tilde{x}_{R.E.}$	0.719029	0.839149	1.268901
	$\tilde{x}_{R.E.-dp}$	—	-1.242635	-0.895767
	$\tilde{x}_{R.E.-cap}$	—	—	0.616064

Table 4.3: Dynamic Portfolio Performance

	Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
	Allowing Shorting of Commercial Real Estate		
$E[r_p]$	0.014193	0.034654	0.049611
σ_p	0.051797	0.087395	0.098215
Sharpe Ratio	0.274008	0.396518	0.505124
Certainty Equivalent Return	0.013656	0.033126	0.047682
	Restricting Shorting of Commercial Real Estate		
$E[r_p]$	0.014193	0.028089	0.040444
σ_p	0.051797	0.093974	0.105035
Sharpe Ratio	0.274008	0.298905	0.385052
Certainty Equivalent Return	0.013656	0.026323	0.038237

Exhibit 5: Stocks and R.E. Office Dynamic Portfolio Optimization

(Unconditional and Conditional Portfolios with Quarterly Rebalancing)

Figure 5.1: Time Series of Portfolio Weights on Stocks

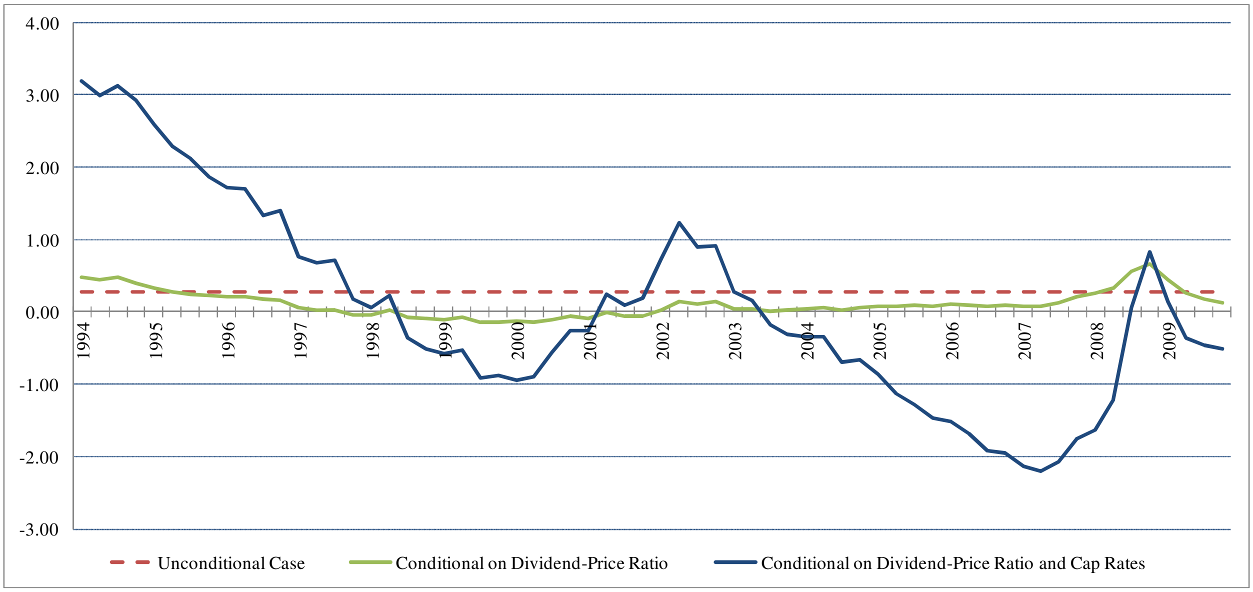

Figure 5.2: Time Series of Portfolio Weights on R.E. Office

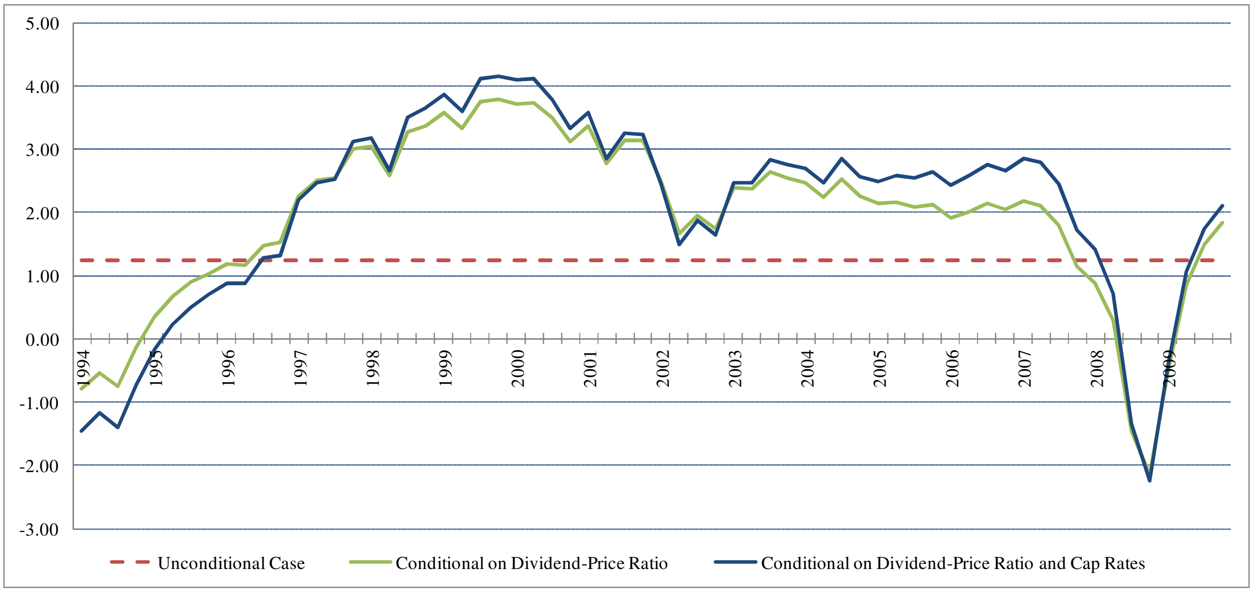

Table 5.1: Major First and Second Moment Statistics

Return, Volatility and Sharpe Ratio of "Basis" and "Conditional" Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
$E[r_i]$	0.015030	0.014324	-0.011410	-0.013963	0.016676	0.009305
σ_i	0.086030	0.044327	0.110394	0.044334	0.088459	0.049061
Sharpe Ratio	0.174712	0.323144	-0.103355	-0.314946	0.188515	0.189664
Correlation Matrix of Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
S	1.000000	0.157020	0.167570	0.127588	-0.085438	-0.065658
R.E.		1.000000	0.089806	0.461992	-0.096325	-0.304750
S-dp			1.000000	0.324724	-0.714967	-0.120002
R.E.-dp				1.000000	-0.132915	-0.302438
S-cap					1.000000	0.356509
R.E.-cap						1.000000

Table 5.2: Optimal Portfolio Weights

		Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
Stocks	$\tilde{x}_S$	0.272811	0.111047	0.034330
	$\tilde{x}_{S-dp}$	—	0.179860	0.898094
	$\tilde{x}_{S-cap}$	—	—	1.222042
R.E.	$\tilde{x}_{R.E.}$	1.234449	1.903514	2.032296
	$\tilde{x}_{R.E.-dp}$	—	-1.322216	-1.552409
	$\tilde{x}_{R.E.-cap}$	—	—	-0.304212

Table 5.3: Dynamic Portfolio Performance

	Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
	Allowing Shorting of Commercial Real Estate		
$E[r_p]$	0.021146	0.043274	0.055974
σ_p	0.062641	0.079141	0.102295
Sharpe Ratio	0.337583	0.546792	0.547177
Certainty Equivalent Return	0.020362	0.042021	0.053881
	Restricting Shorting of Commercial Real Estate		
$E[r_p]$	0.021146	0.042796	0.057359
σ_p	0.062641	0.081323	0.109255
Sharpe Ratio	0.337583	0.526252	0.525002
Certainty Equivalent Return	0.020362	0.041474	0.054972

Exhibit 6: Stocks and R.E. Retail Dynamic Portfolio Optimization

(Unconditional and Conditional Portfolios with Quarterly Rebalancing)

Figure 6.1: Time Series of Portfolio Weights on Stocks

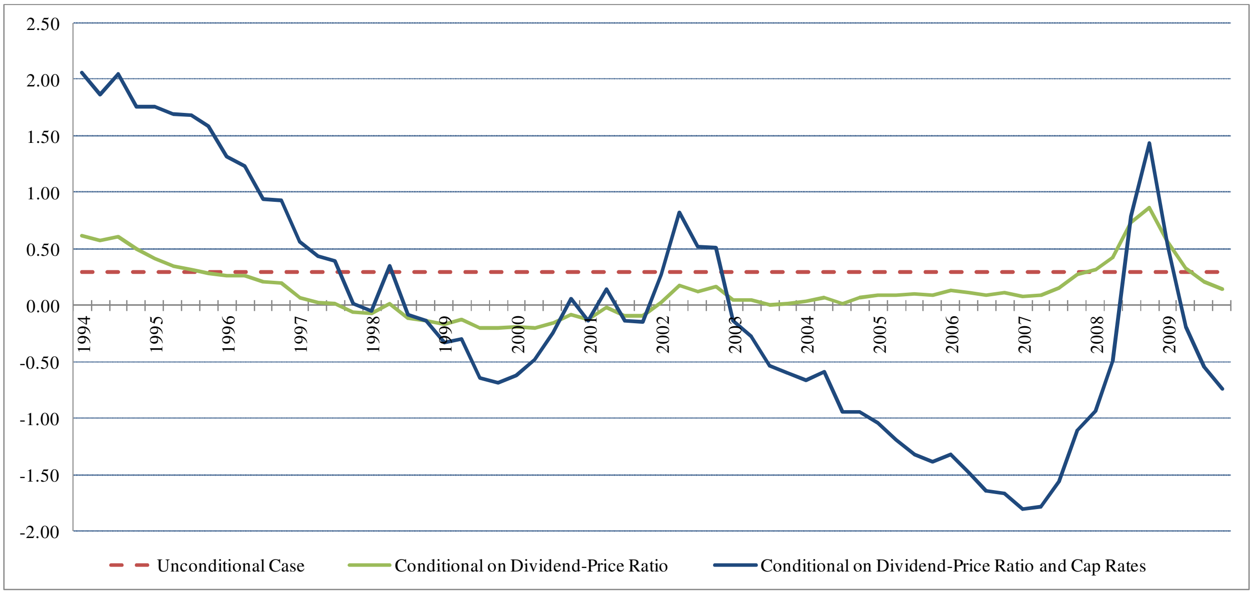

Figure 6.2: Time Series of Portfolio Weights on R.E. Retail

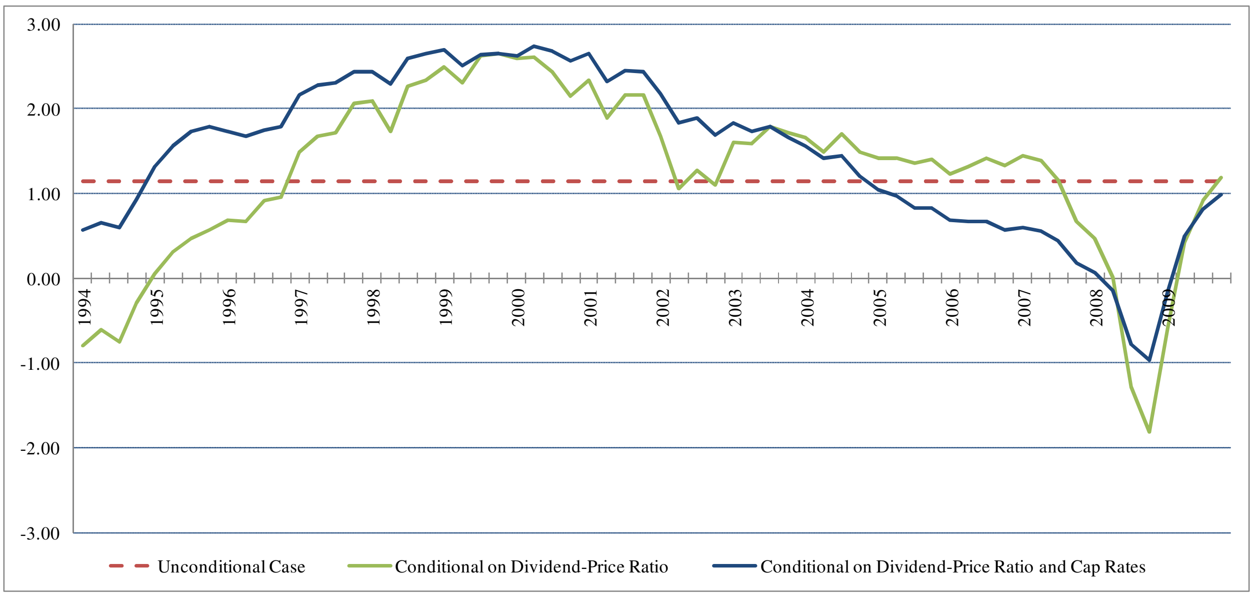

Table 6.1: Major First and Second Moment Statistics

Return, Volatility and Sharpe Ratio of "Basis" and "Conditional" Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
$E[r_i]$	0.015030	0.015274	-0.011410	-0.011782	0.011291	0.005326
σ_i	0.086030	0.046930	0.110394	0.035066	0.088045	0.040871
Sharpe Ratio	0.174712	0.325459	-0.103355	-0.335991	0.128241	0.130314
Correlation Matrix of Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
S	1.000000	0.126646	0.167570	-0.201803	0.051735	0.064683
R.E.		1.000000	-0.118062	-0.061494	0.032324	-0.375799
S-dp			1.000000	0.226485	-0.769867	-0.140185
R.E.-dp				1.000000	-0.181089	-0.332163
S-cap					1.000000	0.169478
R.E.-cap						1.000000

Table 6.2: Optimal Portfolio Weights

		Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
Stocks	$\tilde{x}_S$	0.293507	0.131116	-0.050390
	$\tilde{x}_{S-dp}$	—	0.237271	0.916848
	$\tilde{x}_{S-cap}$	—	—	0.948056
R.E.	$\tilde{x}_{R.E.}$	1.139415	1.232106	1.479135
	$\tilde{x}_{R.E.-dp}$	—	-0.994804	-0.522064
	$\tilde{x}_{R.E.-cap}$	—	—	0.599651

Table 6.3: Dynamic Portfolio Performance

	Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
	Allowing Shorting of Commercial Real Estate		
$E[r_p]$	0.021290	0.028245	0.030264
σ_p	0.061563	0.072739	0.085450
Sharpe Ratio	0.345815	0.388313	0.354177
Certainty Equivalent Return	0.020532	0.027187	0.028804
	Restricting Shorting of Commercial Real Estate		
$E[r_p]$	0.021290	0.029397	0.029578
σ_p	0.061563	0.073384	0.086651
Sharpe Ratio	0.345815	0.400592	0.341350
Certainty Equivalent Return	0.020532	0.028320	0.028077

General Comments

In general, we can say that the results from the analysis of all the portfolios with quarterly rebalancing reveal some very interesting issues, but also some weaknesses related to the data, methodology and time period taken into consideration for the purposes of this thesis.

We start by making two comments of general nature that also apply to Sections 4.4 and 4.5 to follow. More specifically, as we can see from the analysis results, in almost all cases the portfolios that include stocks with one of the four major property types have higher allocation to real estate, compared to the allocation proposed by the more diversified portfolio with stocks and real estate "All Properties", for both unconditional and conditional cases.

There are two possible explanations for that. The first one has to do with the limited diversification at the portfolio level, since we are using only two risky assets in our analysis, namely stocks and real estate. However, the results of this analysis also confirm one of the remarks made by Plazzi, Torous and Valkanov (2010), that has to do with the significance of diversification of commercial real estate in both across property types and across locations, and the consequences of not doing so. From this paper we can see that "to the extent that commercial real estate investments tend to be specialized by either property type or by location, the resultant investments are subject to idiosyncratic risk…" [Plazzi, Torous and Valkanov (2010), p.36], which is probably the case in our analysis.

The second explanation has to do with the different time period taken into consideration for the analysis of the Stocks-"All Properties" portfolio and the portfolios of stocks with the specialized property types. In order to give the reader a sense of the role the different time period has played in these results, we can say that the former Stocks-"All Properties" portfolio analyzed for the 1984Q2-2010Q1 period considers two additional "Big Bear" historical events [Geltner (2010)], which had a large impact on the returns of Stocks (-22.9% in 1987Q4) and real estate "All Properties" (-22.0% in period 1987-1992), and which the specialized portfolios do not account for.

The next issue we need to address has to do with the fact that our analysis, in general, provides us with optimal portfolio weights for real estate that are undoubtedly high compared to those for stocks. This issue has to do with the limited time periods taken in our analysis (26 years of observations for the Stocks-"All Properties" portfolio, and just 16 years for the specialized per property type portfolios). None of these periods consist a full market cycle, and both of these consider the deep, 2008-2010 recession, a fact which makes the data used somewhat non-representative of a normal market cycle.

In order to give the reader a sense of how the latter issue influences our results, we can mention the fact that, based on Geltner, (2010), within the ten-year period between 2000-2009 the stock market faced two "Big Bear" events, showing -46.8% losses in 2000-2002, and -46.9% losses in 2007-2009, whereas for the same time period, real estate faced one "Big Bear" event, from 2007 until 2009, losing 36.3% of its value. Therefore, we can conclude that, by no means can this kind of performance be considered representative of the long-term performance of stocks and real estate, especially in the case one wants to base an analysis on a normal market cycle.

With these in mind, we now focus on the analysis results, by checking the tables that include the optimal portfolio weights of the augmented asset space for the unconditional and conditional cases, in conjunction with the tables that show the portfolio performance. Moreover, we will separate the analysis comments in two parts, dealing with the Stocks and "All Properties" portfolio first, and then check the specialized per property type portfolios.

Comments on the Stocks and R.E. "All Properties" Dynamic Portfolio Optimization

In the unconditional case, the Stocks-"All Properties" portfolio appears to be balanced, allocating the portfolio's wealth in almost equal shares between the two asset types (0.47 in stocks, 0.46 in real estate), whereby the proposed portfolio has a Sharpe ratio of 0.24 (expressed quarterly), and a certainty equivalent return of around 1.09% (expressed quarterly).

By adding *dp* as a conditioning variable, we can now see that the weights allocation has changed significantly, as the real estate weight experienced a 53% increase, as opposed to a 60% decrease observed in the weight of stocks. At the same time, the Sharpe ratio increased from 0.24 to 0.34, yielding a certainty equivalent return of 2.33%, as opposed to the static case where it was 1.09% (expressed quarterly). This means that the portfolio performance improved significantly by adding *dp* as a conditioning variable.

By adding *cap* as the second conditioning variable we observe an even better performing portfolio, with an allocation to real estate and stocks equal to almost 1.5 and 0.3, respectively. The new Sharpe ratio increased from 0.34 to 0.44, yielding a certainty equivalent return of 3.71%. What makes things even more interesting is that the same conclusions apply in the case of "no shorting" for real estate, although the role of the cap rate seems less significant than without the restriction, since adding the cap increases the Sharpe ratio by "only" 0.0338 (from 0.49 to 0.52), however yielding a higher certainty equivalent return of 4.46% (expressed quarterly).

In general, adding conditioning variables to this portfolio greatly increases the allocation to real estate, providing the investor with a significant increase in the performance of the portfolio, in terms of increases in both the Sharpe ratio and the certainty equivalent return. Note that these results are supported by the fact that both *dp* and *cap* can largely predict the returns for stocks and real estate "All Properties" in a quarterly horizon, as we saw in Chapter 3.

Comments on the Stocks and R.E. Specialized, Per Property Type Dynamic Portfolio Optimization

Continuing our analysis results for the specialized, per property type portfolios, we can say as a general impression that all specialized portfolios analyses recommend that it is optimal to allocate a significant percentage of the portfolio's wealth to real estate, at the expense of stocks.

More specifically, for the unconditional case, the proposed real estate weights for all property types range between 0.71 and 1.33, whereas, the allocation to stocks is between 0.266 and 0.297. Moreover, we see a portfolio Sharpe ratio range between 0.27 and 0.39, and a certainty equivalent return within a 1.3% and 2.5% range (expressed quarterly).

Adding *dp* as a conditioning variable in these portfolios, we observe that the aforementioned ranges of weights change to a different extent for each property type portfolio. As a general tendency, we see the weights in real estate now being between 0.83 and 1.90, showing an increase in all portfolios. Concerning the allocation to stocks, though, we observe that in the case of the portfolios that include Apartment and Industrial buildings, the allocation to stocks increases, whereas the opposite happens to the portfolios that include Office and Retail. However, in all cases we have a significant improvement in Sharpe ratios and certainty equivalent returns, which now range between 0.39 and 0.55, and 2.71% and 4.2%, respectively.

The most interesting finding is when we add *cap* as the second state variable. Once again, we see that all portfolios show an increase in allocation to real estate and a decrease in allocation to stocks, compared to the corresponding cases with one conditioning variable. Moreover, we see that, with the exception of the Stocks-Retail portfolio, both the Sharpe ratio and certainty equivalent return are increasing, and moving now within the range of 0.50-0.60 and 4.76%-5.85%, respectively (expressed quarterly).

However, there's only one exception, that of the Stocks-Retail portfolio, where we actually see the Sharpe ratio decreasing from 0.388 with one conditioning variable, to 0.354 in this case. The explanation to this exception probably has to do with the fact that the retail cap rates do not show significant evidence that they can predict returns in stocks and retail properties, as we have already seen in Chapter 3. Moreover, please note that all the other results are supported by the fact that both *dp* and *cap* can, to a significant degree, predict the returns for stocks and real estate in the short, horizon k=1.

Finally, we can see that, in general, the same conclusions as above apply to the case of "no shorting" of real estate, although the role of the cap rate seems less significant than without the restriction.

4.4. Results of the Dynamic Portfolio Analysis with Semi-Annual Rebalancing

Exhibit 7: Stocks and R.E. "All Properties" Dynamic Portfolio Optimization

(Unconditional and Conditional Portfolios with Semi-Annual Rebalancing)

Figure 7.1: Time Series of Portfolio Weights on Stocks

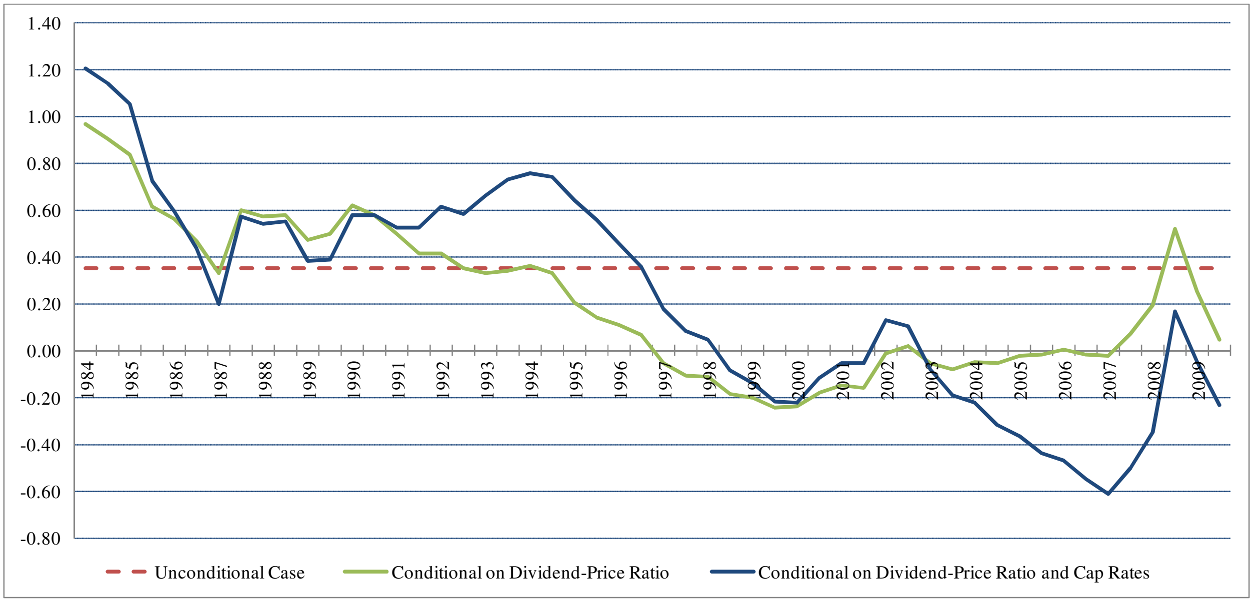

Figure 7.2: Time Series of Portfolio Weights on R.E. "All Properties"

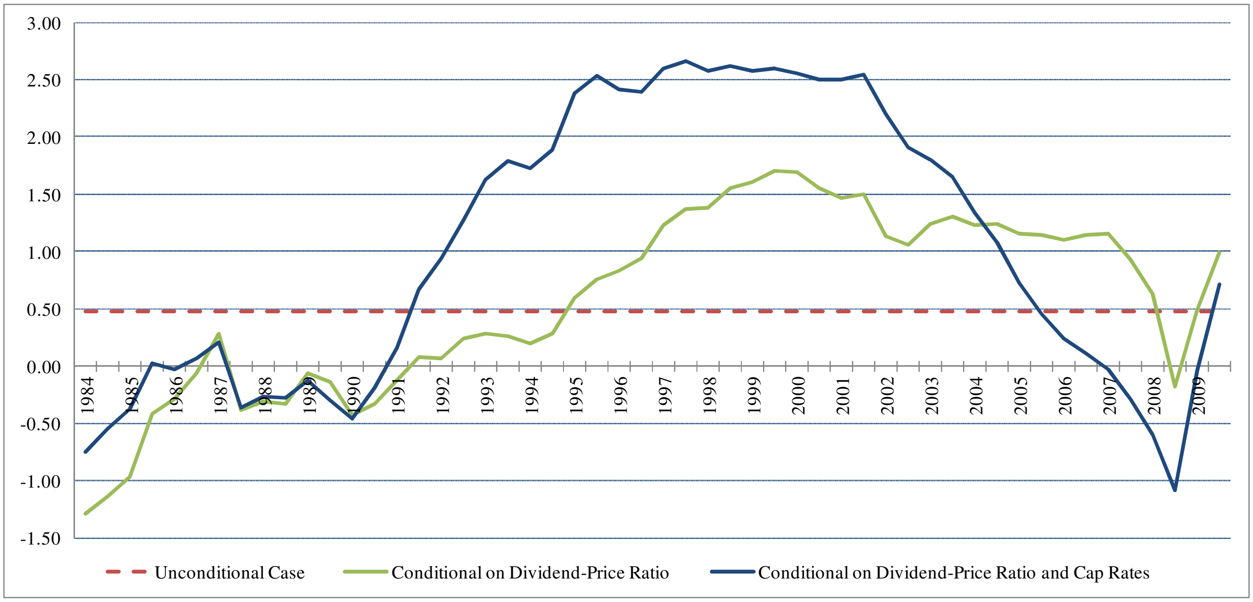

Table 7.1: Major First and Second Moment Statistics

Return, Volatility and Sharpe Ratio of "Basis" and "Conditional" Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
$E[r_i]$	0.038466	0.013233	-0.002146	-0.025684	0.018930	0.016954
σ_i	0.136810	0.067054	0.133563	0.054557	0.146243	0.070160
Sharpe Ratio	0.281162	0.197347	-0.016068	-0.470772	0.129441	0.241642
Correlation Matrix of Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
S	1.000000	0.097243	0.021393	-0.011258	-0.173212	-0.080116
R.E.		1.000000	-0.118624	-0.102890	-0.036659	-0.524105
S-dp			1.000000	0.191509	-0.630664	-0.167309
R.E.-dp				1.000000	-0.139020	-0.005128
S-cap					1.000000	0.422503
R.E.-cap						1.000000

Table 7.2: Optimal Portfolio Weights

		Unconditional Case	Conditional on dp	Conditional on dp and cap
Stocks	$\tilde{x}_S$	0.350624	0.216613	0.221355
	$\tilde{x}_{S-dp}$	—	0.318579	0.453418
	$\tilde{x}_{S-cap}$	—	—	0.280141
R.E.	$\tilde{x}_{R.E.}$	0.479483	0.564561	1.005730
	$\tilde{x}_{R.E.-dp}$	—	-0.789727	-0.635749
	$\tilde{x}_{R.E.-cap}$	—	—	0.864040

Table 7.3: Dynamic Portfolio Performance

	Unconditional Case	Conditional on dp	Conditional on dp and cap
	Allowing Shorting of Commercial Real Estate		
$E[r_p]$	0.019108	0.033353	0.053813
σ_p	0.060111	0.074890	0.089184
Sharpe Ratio	0.317886	0.445365	0.603389
Certainty Equivalent Return	0.018386	0.032231	0.052222
	Restricting Shorting of Commercial Real Estate		
$E[r_p]$	0.019108	0.047628	0.059961
σ_p	0.060111	0.077487	0.089248
Sharpe Ratio	0.317886	0.614662	0.671850
Certainty Equivalent Return	0.018386	0.046428	0.058368

Exhibit 8: Stocks and R.E. Apartment Dynamic Portfolio Optimization

(Unconditional and Conditional Portfolios with Semi-Annual Rebalancing)

Figure 8.1: Time Series of Portfolio Weights on Stocks

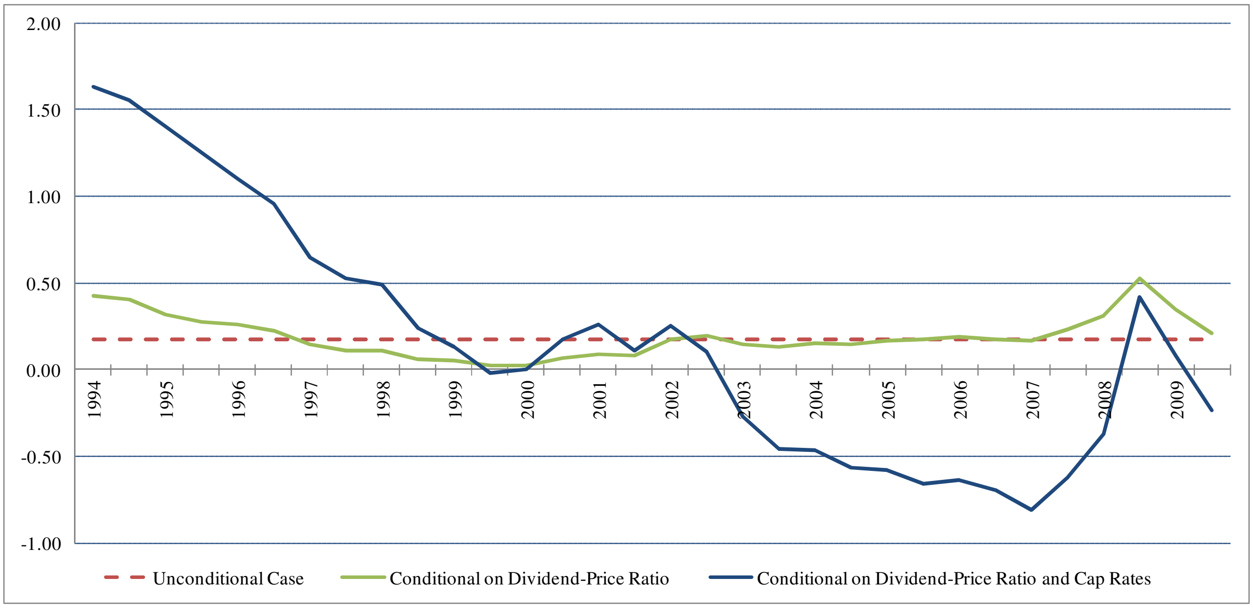

Figure 8.2: Time Series of Portfolio Weights on R.E. Apartment

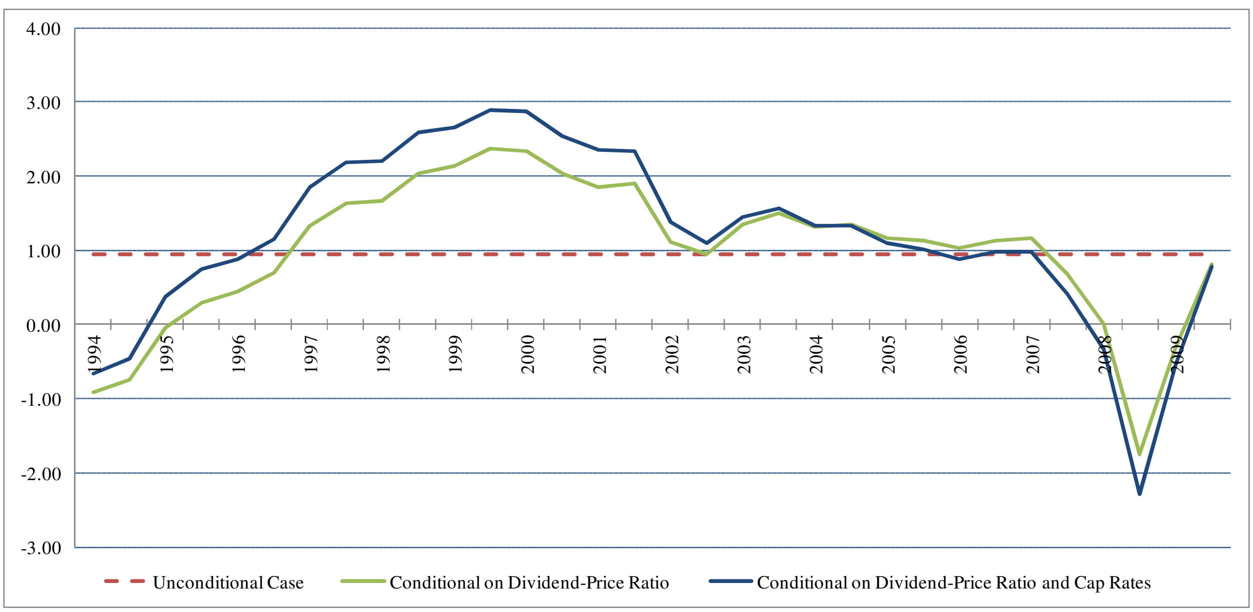

Table 8.1: Major First and Second Moment Statistics

Return, Volatility and Sharpe Ratio of "Basis" and "Conditional" Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
$E[r_i]$	0.033611	0.035330	-0.015088	-0.020540	0.024562	0.018134
σ_i	0.147640	0.073859	0.202603	0.102335	0.142045	0.086087
Sharpe Ratio	0.227656	0.478345	-0.074470	-0.200716	0.172919	0.210644
Correlation Matrix of Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
S	1.000000	0.170062	0.282477	0.393762	-0.133248	-0.180217
R.E.		1.000000	0.386718	0.453336	-0.243741	-0.340335
S-dp			1.000000	0.568215	-0.705957	-0.213409
R.E.-dp				1.000000	-0.239664	-0.133498
S-cap					1.000000	0.411798
R.E.-cap						1.000000

Table 8.2: Optimal Portfolio Weights

		Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
Stocks	$\tilde{x}_S$	0.174798	0.190395	0.154335
	$\tilde{x}_{S-dp}$	—	0.118887	0.398552
	$\tilde{x}_{S-cap}$	—	—	0.637239
R.E.	$\tilde{x}_{R.E.}$	0.937781	0.989846	1.176270
	$\tilde{x}_{R.E.-dp}$	—	-0.973086	-1.092644
	$\tilde{x}_{R.E.-cap}$	—	—	0.279688

Table 8.3: Dynamic Portfolio Performance

	Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
	Allowing Shorting of Commercial Real Estate		
$E[r_p]$	0.037283	0.055508	0.078941
σ_p	0.077194	0.086391	0.110404
Sharpe Ratio	0.482982	0.642517	0.715024
Certainty Equivalent Return	0.036091	0.054015	0.076504
	Restricting Shorting of Commercial Real Estate		
$E[r_p]$	0.037283	0.056069	0.075279
σ_p	0.077194	0.089822	0.112764
Sharpe Ratio	0.482982	0.624222	0.667582
Certainty Equivalent Return	0.036091	0.054455	0.072736

Exhibit 9: Stocks and R.E. Industrial Dynamic Portfolio Optimization

(Unconditional and Conditional Portfolios with Semi-Annual Rebalancing)

Figure 9.1: Time Series of Portfolio Weights on Stocks

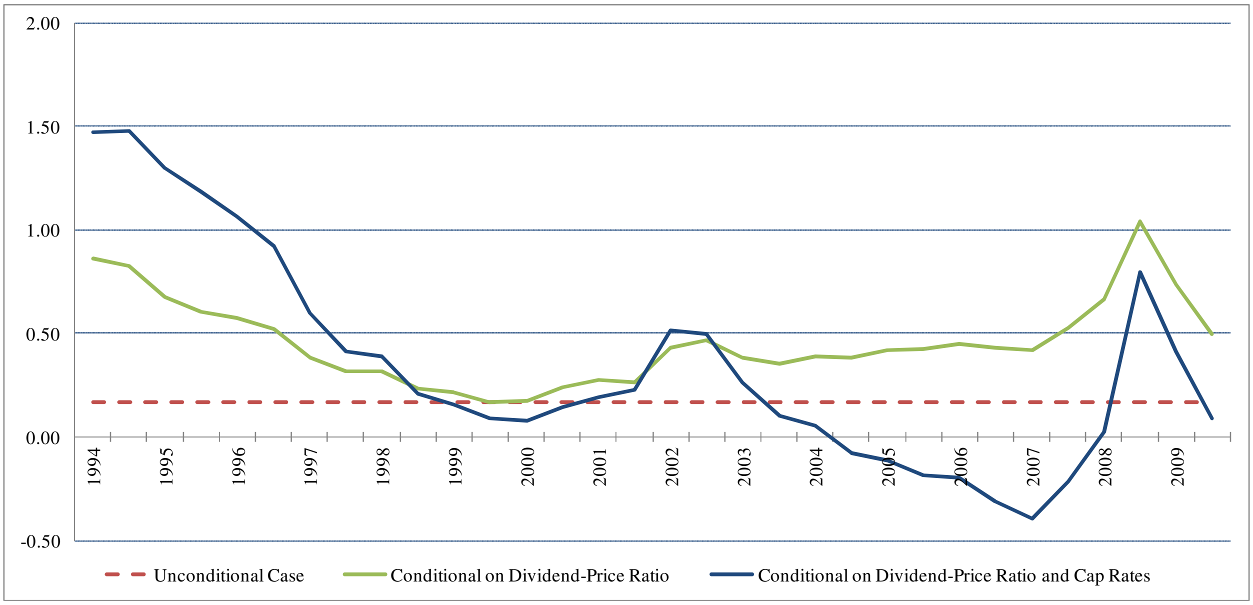

Figure 9.2: Time Series of Portfolio Weights on R.E. Industrial

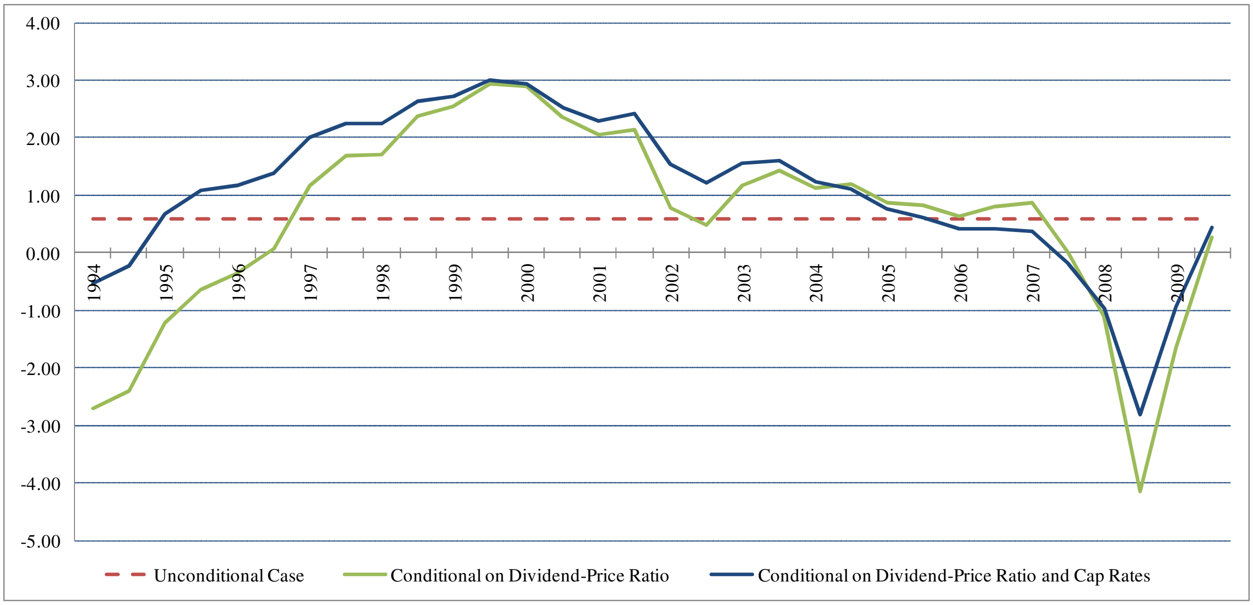

Table 9.1: Major First and Second Moment Statistics

Return, Volatility and Sharpe Ratio of "Basis" and "Conditional" Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
$E[r_i]$	0.033611	0.028920	-0.015088	-0.035088	0.030573	0.019957
σ_i	0.147640	0.086486	0.202603	0.135393	0.149362	0.104157
Sharpe Ratio	0.227656	0.334390	-0.074470	-0.259154	0.204693	0.191604
Correlation Matrix of Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
S	1.000000	0.306914	0.282477	0.494429	-0.169686	-0.383981
R.E.		1.000000	0.520275	0.553817	-0.474160	-0.784563
S-dp			1.000000	0.780572	-0.749055	-0.542559
R.E.-dp				1.000000	-0.526777	-0.676861
S-cap					1.000000	0.629742
R.E.-cap						1.000000

Table 9.2: Optimal Portfolio Weights

		Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
Stocks	$\tilde{x}_S$	0.168338	0.457804	0.348439
	$\tilde{x}_{S-dp}$	—	0.207059	0.393676
	$\tilde{x}_{S-cap}$	—	—	0.423833
R.E.	$\tilde{x}_{R.E.}$	0.579652	0.566972	1.094295
	$\tilde{x}_{R.E.-dp}$	—	-1.673542	-1.067920
	$\tilde{x}_{R.E.-cap}$	—	—	0.574531

Table 9.3: Dynamic Portfolio Performance

	Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
	Allowing Shorting of Commercial Real Estate		
$E[r_p]$	0.021317	0.080404	0.093857
σ_p	0.062354	0.151524	0.119812
Sharpe Ratio	0.341864	0.530639	0.783367
Certainty Equivalent Return	0.020539	0.075812	0.090986
	Restricting Shorting of Commercial Real Estate		
$E[r_p]$	0.021317	0.062614	0.075401
σ_p	0.062354	0.113218	0.114174
Sharpe Ratio	0.341864	0.553041	0.660406
Certainty Equivalent Return	0.020539	0.060051	0.072794

Exhibit 10: Stocks and R.E. Office Dynamic Portfolio Optimization

(Unconditional and Conditional Portfolios with Semi-Annual Rebalancing)

Figure 10.1: Time Series of Portfolio Weights on Stocks

Figure 10.2: Time Series of Portfolio Weights on R.E. Office

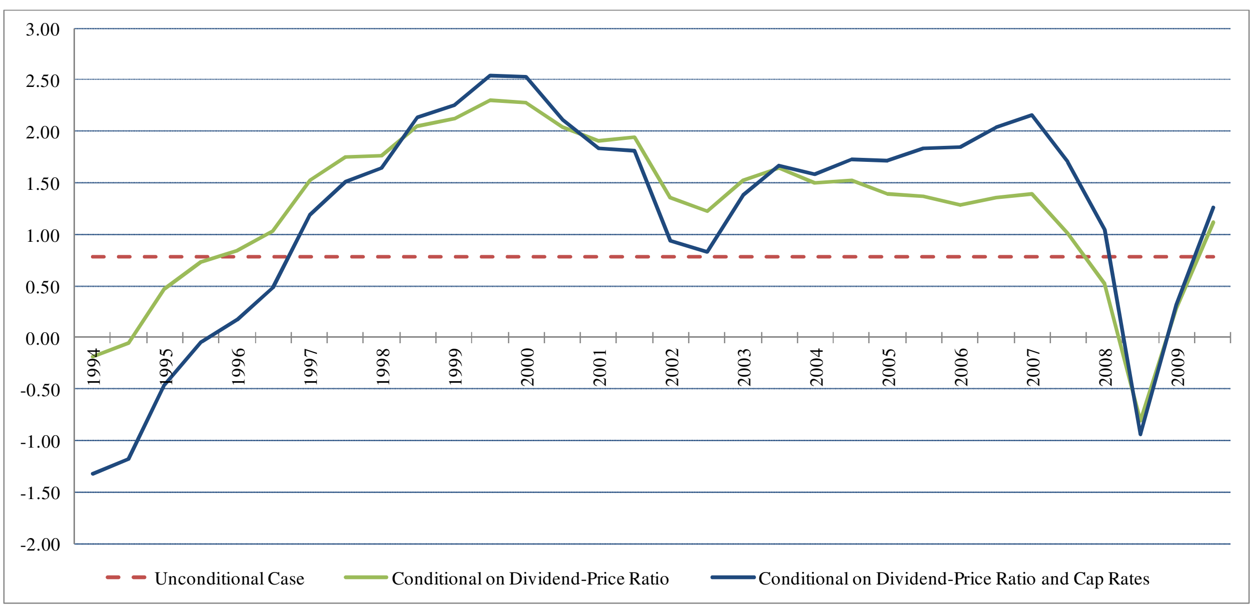

Table 10.1: Major First and Second Moment Statistics

Return, Volatility and Sharpe Ratio of "Basis" and "Conditional" Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
$E[r_i]$	0.033611	0.029613	-0.015088	-0.026650	0.031003	0.016973
σ_i	0.147640	0.075232	0.202603	0.070196	0.156199	0.084120
Sharpe Ratio	0.227656	0.393628	-0.074470	-0.379649	0.198485	0.201776
Correlation Matrix of Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
S	1.000000	0.222201	0.282477	0.125582	-0.180779	-0.030334
R.E.		1.000000	0.054985	0.428804	-0.062595	-0.423668
S-dp			1.000000	0.470710	-0.736098	-0.198813
R.E.-dp				1.000000	-0.255357	-0.429068
S-cap					1.000000	0.424609
R.E.-cap						1.000000

Table 10.2: Optimal Portfolio Weights

		Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
Stocks	$\tilde{x}_S$	0.177452	-0.039096	0.009299
	$\tilde{x}_{S-dp}$	—	0.156242	0.656761
	$\tilde{x}_{S-cap}$	—	—	0.901730
R.E.	$\tilde{x}_{R.E.}$	0.787770	1.255926	1.197660
	$\tilde{x}_{R.E.-dp}$	—	-0.734865	-1.038830
	$\tilde{x}_{R.E.-cap}$	—	—	-0.462835

Table 10.3: Dynamic Portfolio Performance

	Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
	Allowing Shorting of Commercial Real Estate		
$E[r_p]$	0.027851	0.050041	0.068363
σ_p	0.069339	0.084512	0.116470
Sharpe Ratio	0.401663	0.592120	0.586952
Certainty Equivalent Return	0.026889	0.048613	0.065649
	Restricting Shorting of Commercial Real Estate		
$E[r_p]$	0.027851	0.051771	0.077131
σ_p	0.069339	0.087362	0.130517
Sharpe Ratio	0.401663	0.592600	0.590965
Certainty Equivalent Return	0.026889	0.050244	0.073724

Exhibit 11: Stocks and R.E. Retail Dynamic Portfolio Optimization

(Unconditional and Conditional Portfolios with Semi-Annual Rebalancing)

Figure 11.1: Time Series of Portfolio Weights on Stocks

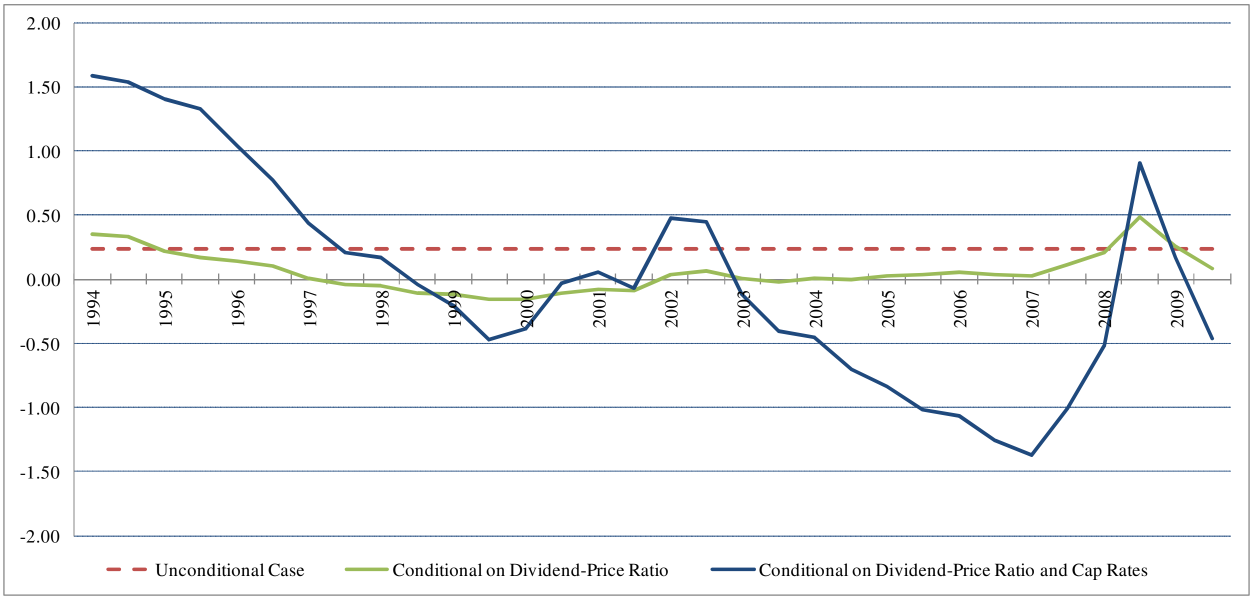

Figure 11.2: Time Series of Portfolio Weights on R.E. Retail

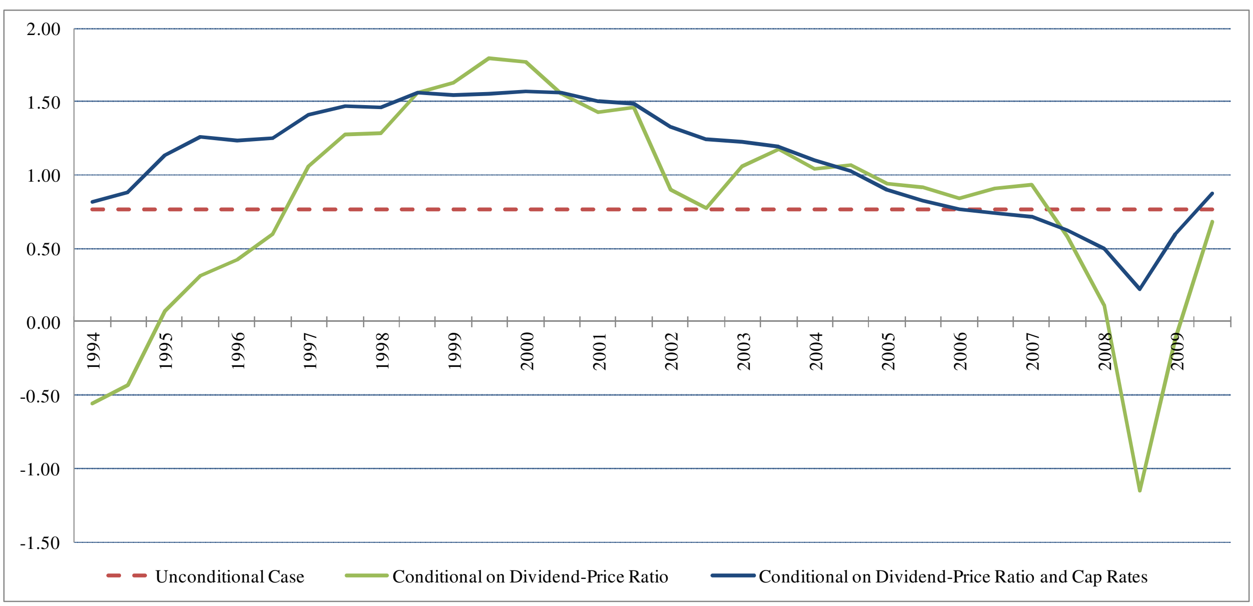

Table 11.1: Major First and Second Moment Statistics

Return, Volatility and Sharpe Ratio of "Basis" and "Conditional" Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
$E[r_i]$	0.033611	0.031515	-0.015088	-0.022138	0.019765	0.008895
σ_i	0.147640	0.079654	0.202603	0.058126	0.155000	0.074338
Sharpe Ratio	0.227656	0.395653	-0.074470	-0.380864	0.127514	0.119660
Correlation Matrix of Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
S	1.000000	0.062567	0.282477	-0.139143	-0.049097	0.037247
R.E.		1.000000	-0.091171	0.020017	0.006028	-0.448261
S-dp			1.000000	0.326475	-0.778193	-0.144003
R.E.-dp				1.000000	-0.205973	-0.245349
S-cap					1.000000	0.187720
R.E.-cap						1.000000

Table 11.2: Optimal Portfolio Weights

		Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
Stocks	$\tilde{x}_S$	0.237641	0.056611	0.003337
	$\tilde{x}_{S-dp}$	—	0.151924	0.714356
	$\tilde{x}_{S-cap}$	—	—	0.750838
R.E.	$\tilde{x}_{R.E.}$	0.766017	0.809312	1.111400
	$\tilde{x}_{R.E.-dp}$	—	-0.696394	-0.181592
	$\tilde{x}_{R.E.-cap}$	—	—	0.259343

Table 11.3: Dynamic Portfolio Performance

	Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
	Allowing Shorting of Commercial Real Estate		
$E[r_p]$	0.030830	0.037630	0.044237
σ_p	0.071354	0.076584	0.112040
Sharpe Ratio	0.432064	0.491359	0.394837
Certainty Equivalent Return	0.029811	0.036457	0.041727
	Restricting Shorting of Commercial Real Estate		
$E[r_p]$	0.030830	0.040340	0.044237
σ_p	0.071354	0.079173	0.112040
Sharpe Ratio	0.432064	0.509517	0.394837
Certainty Equivalent Return	0.029811	0.039086	0.041727

At this point note that, in order to keep the analysis as simple as possible, we will be presenting only the key differences in results between this case and the case with quarterly rebalanced portfolios of Section 4.3. In general, a "smoothening" in the analysis results is now anticipated due to the longer portfolio rebalancing periods. Moreover, based on what we saw in Chapter 3, we expect to start seeing the consequences of the gradual weakening in the ability of the conditioning factors *dp* and *cap* to predict the stock and real estate returns, again due to the longer rebalancing periods.

Indeed, by comparing the "big" picture from the current section with that of Section 4.3 we can see that all portfolios behave in an almost similar way to that they behaved when they were quarterly rebalanced. More specifically, we observe very few differences between the same quarterly and semi-annually rebalancing portfolios in terms of how the weights allocation between stocks and real estate change when adding the conditioning variables. The general impression that we have confirms our line of thought that, adding conditional variables can increase the portfolio performance.

Furthermore, we see again the inability of cap rates to improve the performance of the Stocks-Retail portfolio, only this time we see this magnified, since the portfolio with two conditioning variables now performs even poorer than the unconditional one.

Additionally, what is new here is that now cap rates show an inability to improve the performance of the Stocks-Office portfolio, too; adding the cap rate as the second conditioning variable slightly decreases the portfolio's Sharpe ratio compared to that with one conditioning variable. Nevertheless, the Sharpe ratio of the former case is still 46% larger than that of the unconditional case, greatly improving the portfolio performance.

The most logical explanation for these cases can be found in what we have mentioned earlier, namely that for k=2 (predicting two quarters in the future) we see a somewhat more limited ability of the cap rates to predict retail, office and stock returns.

4.5. Results of the Dynamic Portfolio Analysis with Annual Rebalancing

Exhibit 12: Stocks and R.E. "All Properties" Dynamic Portfolio Optimization

(Unconditional and Conditional Portfolios with Annual Rebalancing)

Figure 12.1: Time Series of Portfolio Weights on Stocks

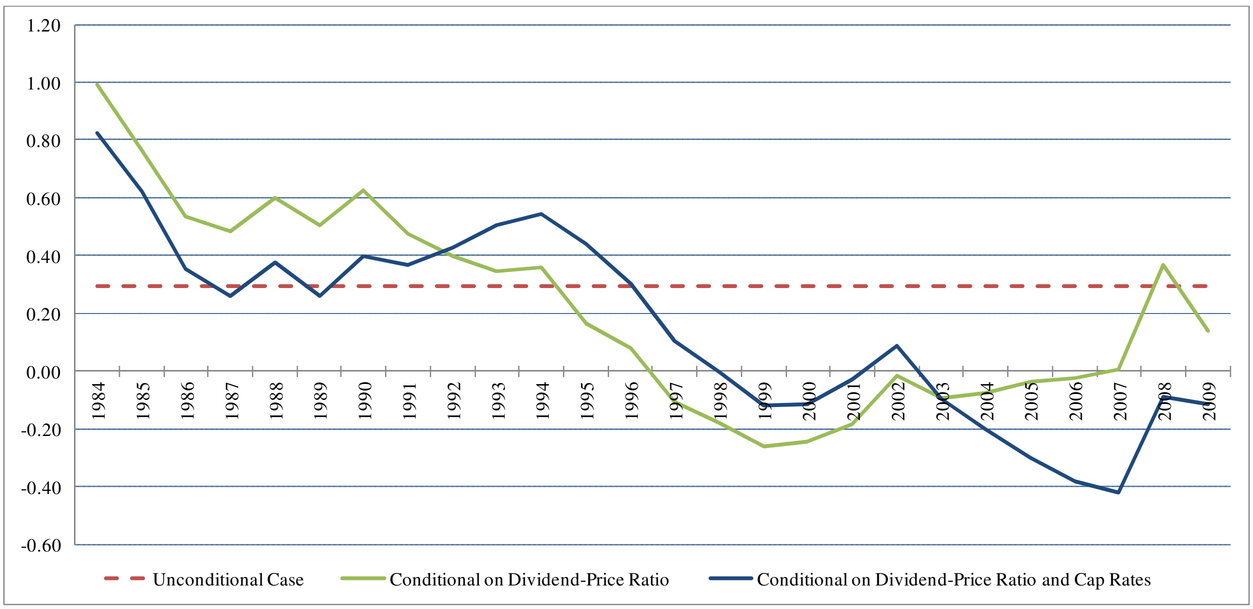

Figure 12.2: Time Series of Portfolio Weights on R.E. "All Properties"

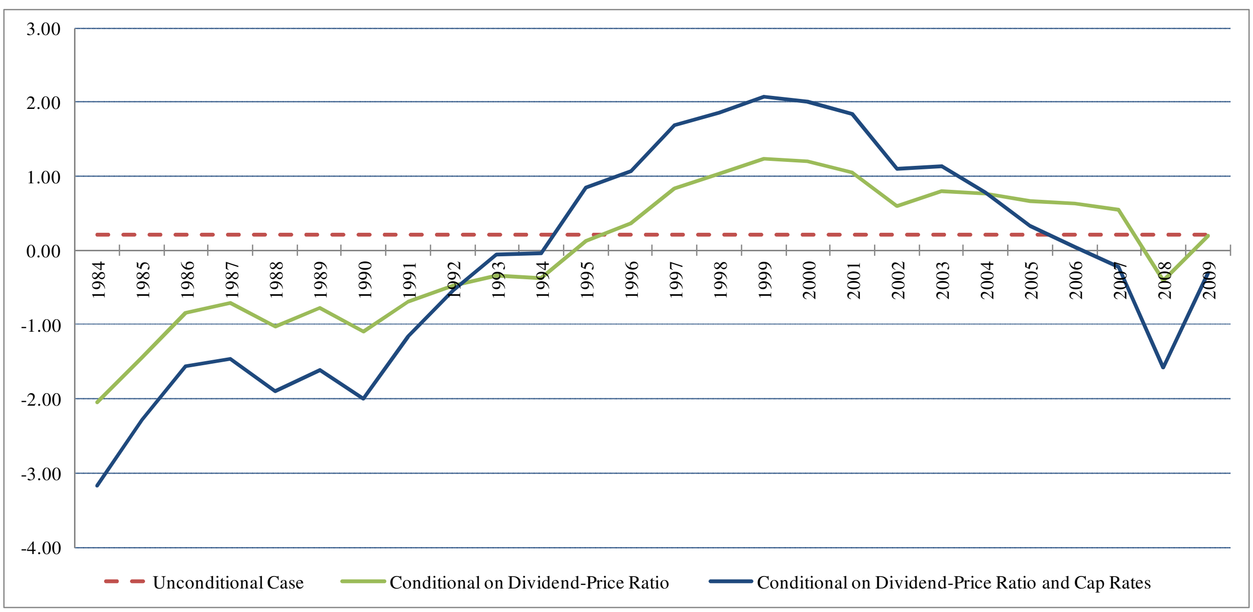

Table 12.1: Major First and Second Moment Statistics

Return, Volatility and Sharpe Ratio of "Basis" and "Conditional" Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
$E[r_i]$	0.079373	0.026823	0.009556	-0.051696	0.042075	0.034329
σ_i	0.206639	0.118278	0.188985	0.071104	0.240534	0.133687
Sharpe Ratio	0.384114	0.226782	0.050567	-0.727042	0.174923	0.256786
Correlation Matrix of Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
S	1.000000	0.178436	-0.262466	0.001648	-0.146893	-0.100452
R.E.		1.000000	-0.201753	-0.647361	-0.039189	-0.574886
S-dp			1.000000	0.329142	-0.484984	-0.217294
R.E.-dp				1.000000	-0.168780	0.359273
S-cap					1.000000	0.525487
R.E.-cap						1.000000

Table 12.2: Optimal Portfolio Weights

		Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
Stocks	$\tilde{x}_S$	0.295568	0.216002	0.153645
	$\tilde{x}_{S-dp}$	—	0.342167	0.319517
	$\tilde{x}_{S-cap}$	—	—	0.212217
R.E.	$\tilde{x}_{R.E.}$	0.208426	-0.007204	-0.119845
	$\tilde{x}_{R.E.-dp}$	—	-0.898521	-1.285572
	$\tilde{x}_{R.E.-cap}$	—	—	0.500179

Table 12.3: Dynamic Portfolio Performance

	Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
	Allowing Shorting of Commercial Real Estate		
$E[r_p]$	0.027984	0.060273	0.092228
σ_p	0.069474	0.078618	0.104818
Sharpe Ratio	0.402800	0.766657	0.879890
Certainty Equivalent Return	0.027019	0.059037	0.090031
	Restricting Shorting of Commercial Real Estate		
$E[r_p]$	0.027984	0.076114	0.098641
σ_p	0.069474	0.101366	0.109900
Sharpe Ratio	0.402800	0.750884	0.897547
Certainty Equivalent Return	0.027019	0.074059	0.096225

Exhibit 13: Stocks and R.E. Apartment Dynamic Portfolio Optimization

(Unconditional and Conditional Portfolios with Annual Rebalancing)

Figure 13.1: Time Series of Portfolio Weights on Stocks

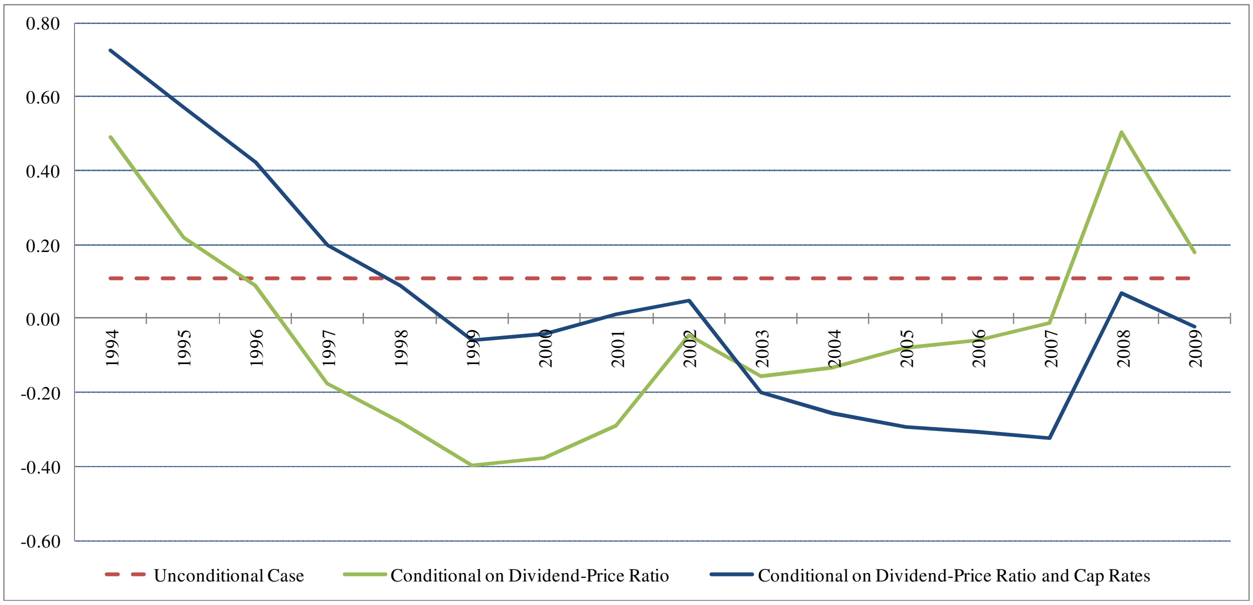

Figure 13.2: Time Series of Portfolio Weights on R.E. Apartment

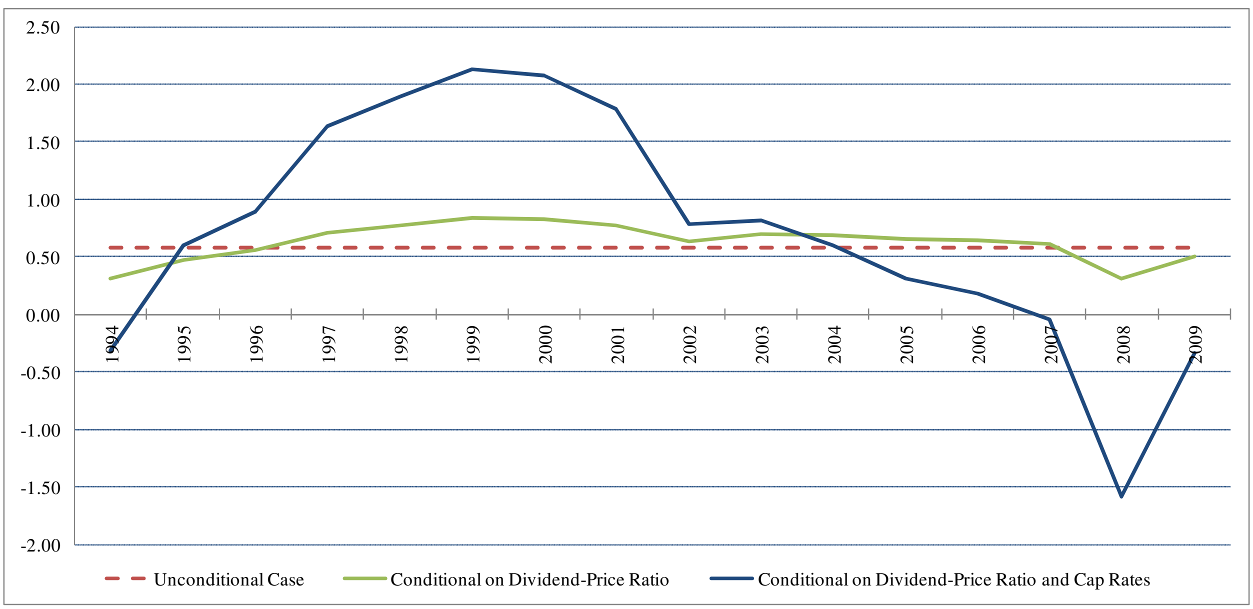

Table 13.1: Major First and Second Moment Statistics

Return, Volatility and Sharpe Ratio of "Basis" and "Conditional" Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
$E[r_i]$	0.074628	0.074184	-0.014059	-0.036954	0.057427	0.035258
σ_i	0.251203	0.130758	0.280153	0.129991	0.246141	0.156804
Sharpe Ratio	0.297084	0.567335	-0.050182	-0.284283	0.233309	0.224851
Correlation Matrix of Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
S	1.000000	0.221623	0.346059	0.467199	-0.218267	-0.033526
R.E.		1.000000	0.366526	0.442453	-0.095016	-0.545845
S-dp			1.000000	0.655270	-0.617838	-0.166374
R.E.-dp				1.000000	-0.174201	-0.099160
S-cap					1.000000	0.503542
R.E.-cap						1.000000

Table 13.2: Optimal Portfolio Weights

		Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
Stocks	$\tilde{x}_S$	0.109248	-0.033068	0.039983
	$\tilde{x}_{S-dp}$	—	0.272801	0.204913
	$\tilde{x}_{S-cap}$	—	—	0.271293
R.E.	$\tilde{x}_{R.E.}$	0.575207	0.625649	0.712888
	$\tilde{x}_{R.E.-dp}$	—	-0.161705	-0.815713
	$\tilde{x}_{R.E.-cap}$	—	—	0.502209

Table 13.3: Dynamic Portfolio Performance

	Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
	Allowing Shorting of Commercial Real Estate		
$E[r_p]$	0.046616	0.044218	0.104751
σ_p	0.084037	0.115940	0.122801
Sharpe Ratio	0.554712	0.381386	0.853018
Certainty Equivalent Return	0.045204	0.041530	0.101735
	Restricting Shorting of Commercial Real Estate		
$E[r_p]$	0.046616	0.044218	0.096286
σ_p	0.084037	0.115940	0.118341
Sharpe Ratio	0.554712	0.381386	0.813639
Certainty Equivalent Return	0.045204	0.041530	0.093486

Exhibit 14: Stocks and R.E. Industrial Dynamic Portfolio Optimization

(Unconditional and Conditional Portfolios with Annual Rebalancing)

Figure 14.1: Time Series of Portfolio Weights on Stocks

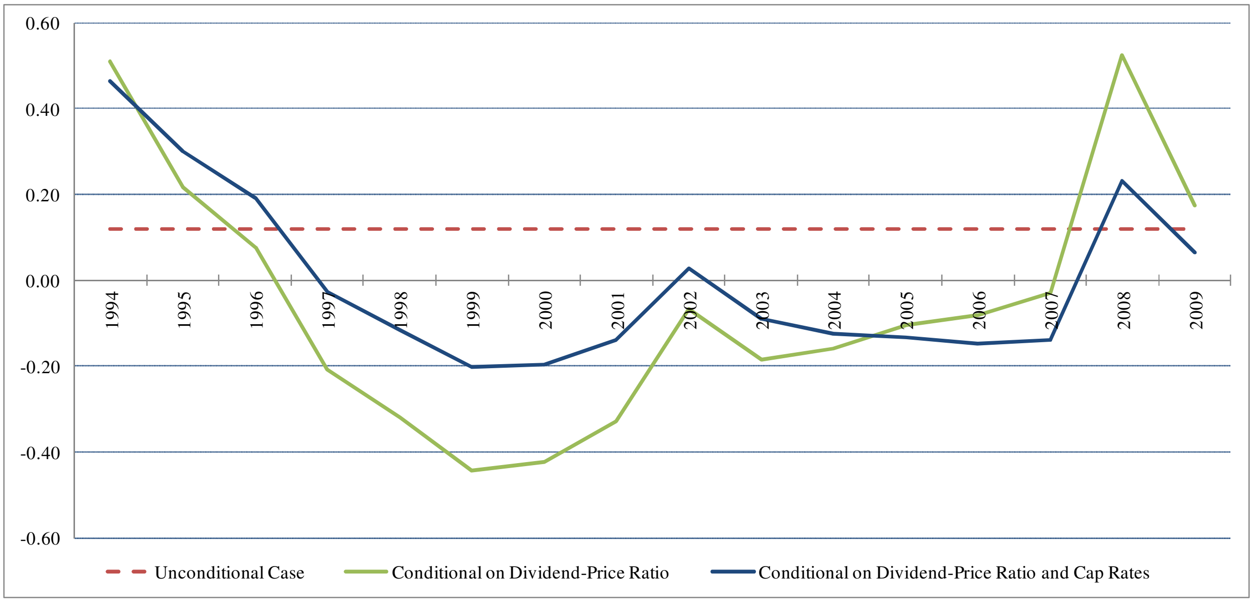

Figure 14.2: Time Series of Portfolio Weights on R.E. Industrial

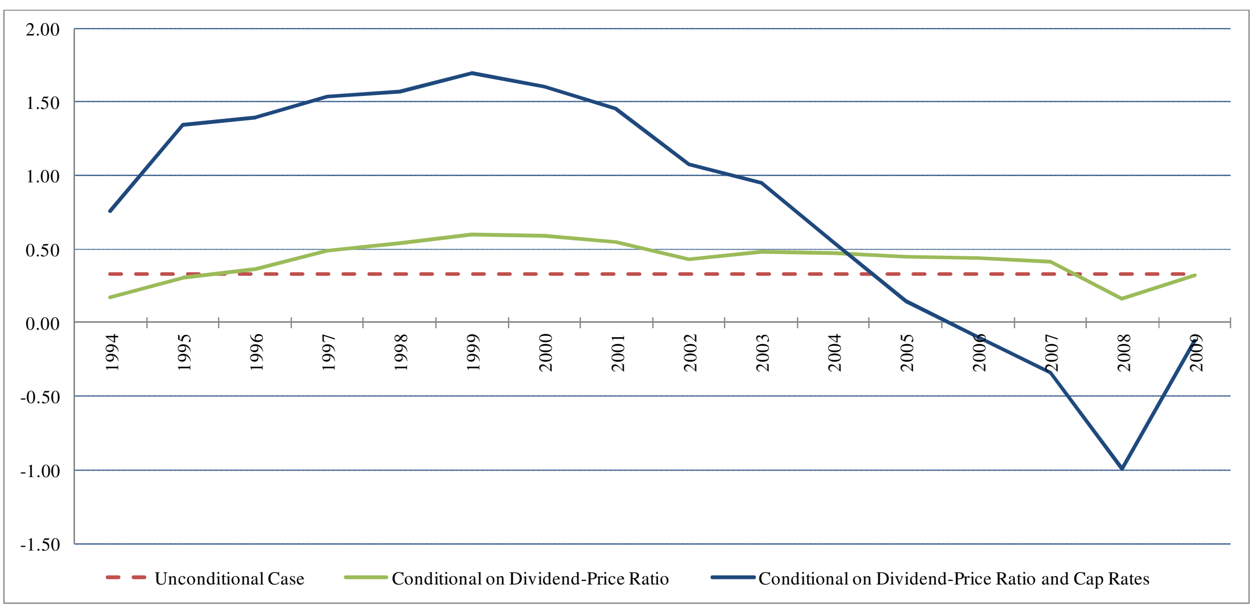

Table 14.1: Major First and Second Moment Statistics

Return, Volatility and Sharpe Ratio of "Basis" and "Conditional" Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
$E[r_i]$	0.074628	0.062030	-0.014059	-0.064123	0.063444	0.036279
σ_i	0.251203	0.156818	0.280153	0.177352	0.254942	0.193328
Sharpe Ratio	0.297084	0.395553	-0.050182	-0.361557	0.248858	0.187657
Correlation Matrix of Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
S	1.000000	0.377026	0.346059	0.491985	-0.233317	-0.311506
R.E.		1.000000	0.403897	0.566953	-0.411161	-0.857436
S-dp			1.000000	0.771749	-0.693847	-0.485137
R.E.-dp				1.000000	-0.497191	-0.631070
S-cap					1.000000	0.640466
R.E.-cap						1.000000

Table 14.2: Optimal Portfolio Weights

		Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
Stocks	$\tilde{x}_S$	0.118029	-0.052788	-0.002019
	$\tilde{x}_{S-dp}$	—	0.292349	0.196671
	$\tilde{x}_{S-cap}$	—	—	0.095397
R.E.	$\tilde{x}_{R.E.}$	0.328293	0.423341	0.783354
	$\tilde{x}_{R.E.-dp}$	—	-0.130913	-0.338735
	$\tilde{x}_{R.E.-cap}$	—	—	0.690557

Table 14.3: Dynamic Portfolio Performance

	Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
	Allowing Shorting of Commercial Real Estate		
$E[r_p]$	0.026801	0.025093	0.091109
σ_p	0.068054	0.102000	0.091686
Sharpe Ratio	0.393819	0.246007	0.993708
Certainty Equivalent Return	0.025875	0.023012	0.089428
	Restricting Shorting of Commercial Real Estate		
$E[r_p]$	0.026801	0.025093	0.070549
σ_p	0.068054	0.102000	0.092231
Sharpe Ratio	0.393819	0.246007	0.764916
Certainty Equivalent Return	0.025875	0.023012	0.068848

Exhibit 15: Stocks and R.E. Office Dynamic Portfolio Optimization

(Unconditional and Conditional Portfolios with Annual Rebalancing)

Figure 15.1: Time Series of Portfolio Weights on Stocks

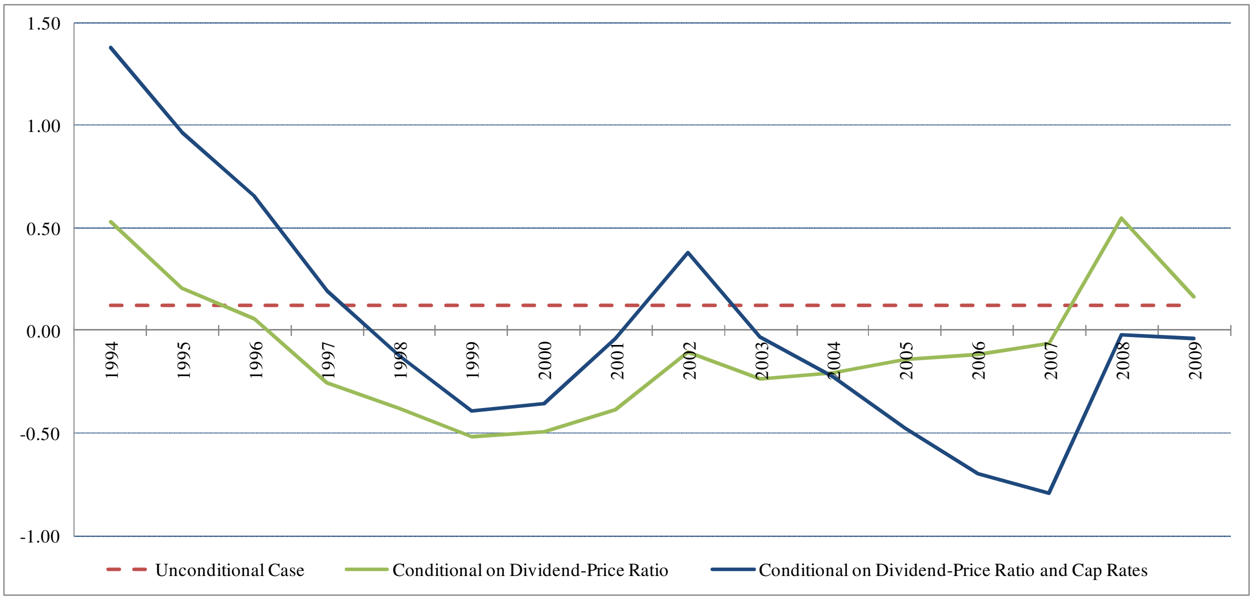

Figure 15.2: Time Series of Portfolio Weights on R.E. Office

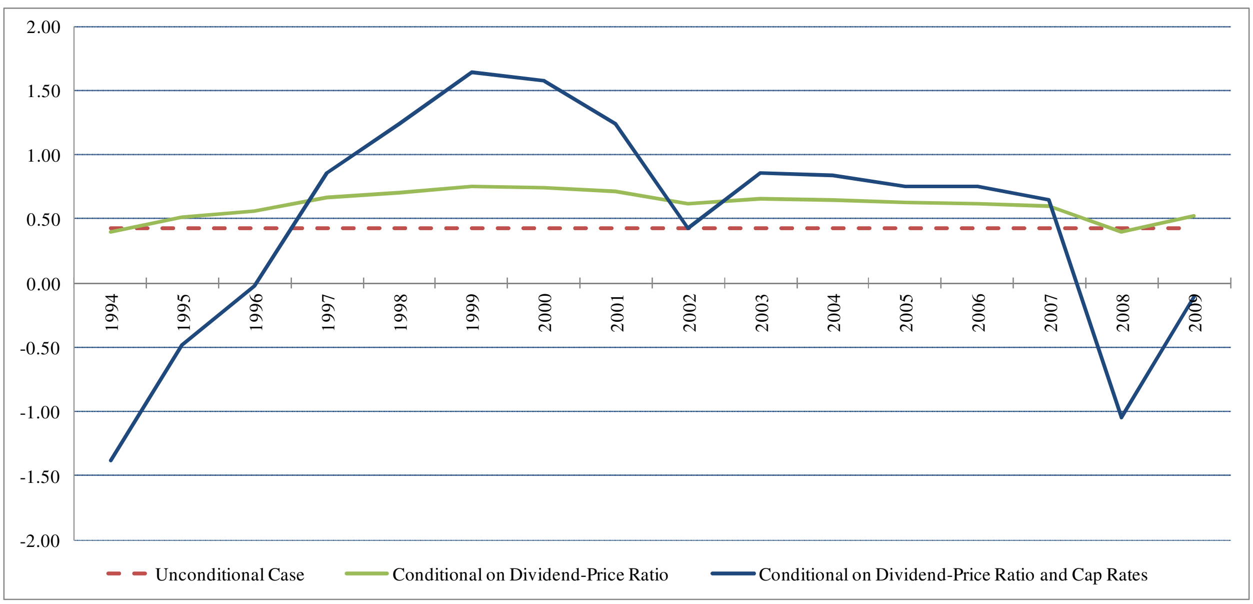

Table 15.1: Major First and Second Moment Statistics

Return, Volatility and Sharpe Ratio of "Basis" and "Conditional" Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
$E[r_i]$	0.074628	0.064106	-0.014059	-0.048369	0.068765	0.030409
σ_i	0.251203	0.148944	0.280153	0.116827	0.259161	0.161780
Sharpe Ratio	0.297084	0.430403	-0.050182	-0.414021	0.265338	0.187967
Correlation Matrix of Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
S	1.000000	0.256262	0.346059	0.278260	-0.229166	-0.033331
R.E.		1.000000	0.126468	0.359408	-0.094196	-0.491156
S-dp			1.000000	0.646986	-0.709160	-0.365380
R.E.-dp				1.000000	-0.444839	-0.519141
S-cap					1.000000	0.572283
R.E.-cap						1.000000

Table 15.2: Optimal Portfolio Weights

		Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
Stocks	$\tilde{x}_S$	0.124170	-0.087990	0.023761
	$\tilde{x}_{S-dp}$	—	0.320947	0.426593
	$\tilde{x}_{S-cap}$	—	—	0.492202
R.E.	$\tilde{x}_{R.E.}$	0.423917	0.608609	0.487835
	$\tilde{x}_{R.E.-dp}$	—	-0.108912	-0.896834
	$\tilde{x}_{R.E.-cap}$	—	—	-0.129462

Table 15.3: Dynamic Portfolio Performance

	Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
	Allowing Shorting of Commercial Real Estate		
$E[r_p]$	0.033389	0.031947	0.089205
σ_p	0.075857	0.120452	0.124063
Sharpe Ratio	0.440156	0.265225	0.719026
Certainty Equivalent Return	0.032238	0.029045	0.086126
	Restricting Shorting of Commercial Real Estate		
$E[r_p]$	0.033389	0.031947	0.098156
σ_p	0.075857	0.120452	0.138936
Sharpe Ratio	0.440156	0.265225	0.706480
Certainty Equivalent Return	0.032238	0.029045	0.094295

Exhibit 16: Stocks and R.E. Retail Dynamic Portfolio Optimization

(Unconditional and Conditional Portfolios with Annual Rebalancing)

Figure 16.1: Time Series of Portfolio Weights on Stocks

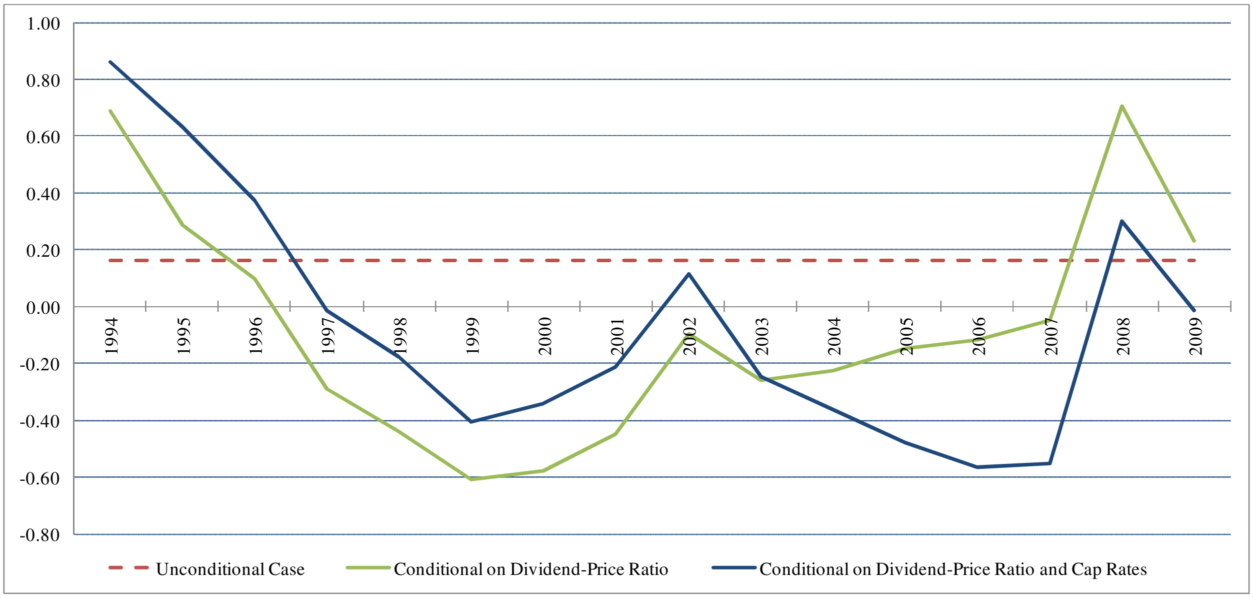

Figure 16.2: Time Series of Portfolio Weights on R.E. Retail

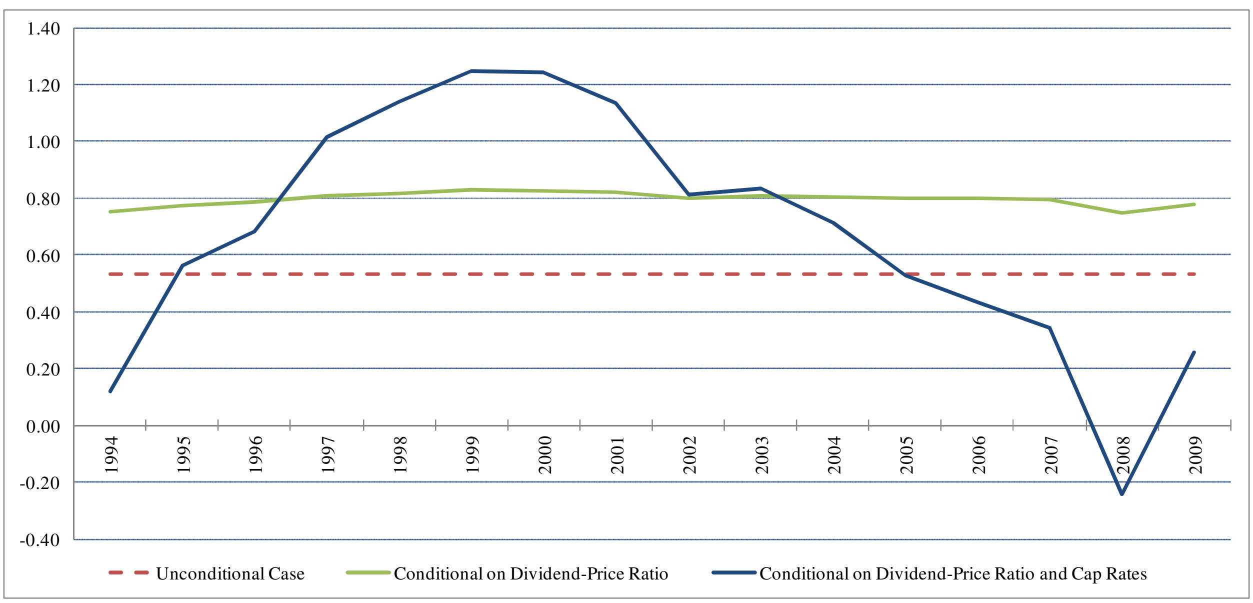

Table 16.1: Major First and Second Moment Statistics

Return, Volatility and Sharpe Ratio of "Basis" and "Conditional" Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
$E[r_i]$	0.074628	0.065683	-0.014059	-0.039565	0.043926	0.014996
σ_i	0.251203	0.132697	0.280153	0.075966	0.254978	0.136542
Sharpe Ratio	0.297084	0.494984	-0.050182	-0.520824	0.172275	0.109824
Correlation Matrix of Portfolio Assets						
Asset	S	R.E.	S-dp	R.E.-dp	S-cap	R.E.-cap
S	1.000000	0.071266	0.346059	-0.187766	-0.170917	-0.000876
R.E.		1.000000	-0.154610	-0.021200	-0.056565	-0.483284
S-dp			1.000000	0.447489	-0.755792	-0.160693
R.E.-dp				1.000000	-0.233248	-0.202692
S-cap					1.000000	0.241673
R.E.-cap						1.000000

Table 16.2: Optimal Portfolio Weights

		Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
Stocks	$\tilde{x}_S$	0.161808	-0.077858	-0.068091
	$\tilde{x}_{S-dp}$	—	0.397236	0.435400
	$\tilde{x}_{S-cap}$	—	—	0.315749
R.E.	$\tilde{x}_{R.E.}$	0.533420	0.797214	0.676886
	$\tilde{x}_{R.E.-dp}$	—	-0.023982	-0.318097
	$\tilde{x}_{R.E.-cap}$	—	—	0.189638

Table 16.3: Dynamic Portfolio Performance

	Unconditional Case	**Conditional on dp**	**Conditional on dp and cap**
	Allowing Shorting of Commercial Real Estate		
$E[r_p]$	0.043525	0.041640	0.057761
σ_p	0.081985	0.135435	0.084519
Sharpe Ratio	0.530888	0.307455	0.683408
Certainty Equivalent Return	0.042181	0.037972	0.056332
	Restricting Shorting of Commercial Real Estate		
$E[r_p]$	0.043525	0.041640	0.057491
σ_p	0.081985	0.135435	0.085096
Sharpe Ratio	0.530888	0.307455	0.675604
Certainty Equivalent Return	0.042181	0.037972	0.056043

The results when we annually rebalance portfolios show some differences in how the portfolios behave, revealing some interesting issues, which are somewhat difficult to interpret.

Interestingly, concerning the Stocks-"All Properties" portfolio, we can see that, when adding the state variables in the conditional cases, the model proposes as an optimal solution the shorting of real estate, having negative $\tilde{x}_{RE}$ values. Regardless of that though, the Stocks-"All Properties" portfolio performs in a similar way to that we saw it performing in the previous two Sections, with the Sharpe ratio and certainty equivalent return increasing as we increase the number of conditioning variables, confirming once again the significance of adding conditioning variables in a dynamic portfolio. Also note that in this case we have significant evidence that both state variables can predict stock and real estate returns.

On the other hand, the results from the analysis of the portfolios that include stocks and one of the major real estate property types seem somewhat counter-intuitive. Up to now we have seen few cases where adding the cap rate as a second conditioning variable did not improve the performance of the portfolio. However, now we see the opposite happening, namely in all of these portfolios we observe a decline in portfolio performance as soon as we add *dp* as the only conditioning variable (both Sharpe ratio and certainty equivalent return are lower that the corresponding values of the unconditional case). Moreover, as soon as we add *cap* as the second state variable, all portfolios greatly improve their performance, outperforming the unconditional case by at least 28.7% in terms of Sharpe ratio.

Although these results contradict what we have seen in the previous parts of our analysis, we try to identify the potential reasons for these. One explanation has to do with the longer period we are considering in this case, which "smoothens" the results, and also makes the idiosyncratic risk even more apparent. Moreover, maybe this has to do with the fact that the cap rates seem to have more significant ability to predict stock and real estate returns as the dividend-price ratio does; we can observe that by comparing the t-stats in Exhibit 1 for k=4. One last explanation has to do with the limited number of historical observations we have for the annually rebalancing portfolios (26 observations for the "Stocks-"All Properties" portfolio, and just 16 observations for the other four portfolios), a fact which might be distorting our findings or call into question the significance of our results.

5. Summary and Conclusions

This thesis has a two-fold objective, namely to explore the role of cap rates in predicting the returns to commercial real estate, and to identify how cap rates can be used to improve the allocation of real estate in a dynamic investment portfolio.

5.1. The Use of Cap Rates and Dividend-Price Ratio to Predict Future Returns to Commercial Real Estate and Stocks

Seeking an answer to the first question of how, and to what degree, can cap rates predict the returns in commercial real estate we have run a series of predictive regressions to examine the predictability power of cap rates for a forecasting horizon from one to four quarters in the future. Moreover, we continued this part of our research by examining whether or not stock dividend-price ratio can predict real estate returns. Furthermore, we examined the predictability of stock returns by cap rates and dividend-price ratio.

The results show that the ability of cap rates and dividend-price ratio to predict real estate returns depend on two factors, namely the type of real estate we are considering ("All Properties" versus specialized per property type real estate) and the forecasting horizon (how far into the future we try to predict the returns).

The analysis results confirm the fact that both cap rates and the dividend-price ratio can predict the real estate "All Properties" returns for up to one year in the future. Concerning the analysis per property type, we saw that the results vary from property type to property type, and for different forecast horizons. Generalizing our findings, we can say that both of the aforementioned ratios can predict returns in the short-run, quarterly horizon, whereas for the longer horizons have a somewhat less ability to do so.

As far as stock returns are concerned, the analysis has shown us that they can be predicted by the dividend-price ratio in all forecast horizons, whereas the cap rates seem to have somewhat limited predictive power regarding the stock returns.

5.2. The Use of Cap Rates in Allocating Investments in Commercial Real Estate in a Dynamic Portfolio

The second question we tried to answer had to do with the ability of cap rates to improve the allocation of real estate in a dynamic investment portfolio, so as to achieve better portfolio performance.

Following the dynamic portfolio allocation methodology proposed by Brandt and Santa-Clara (2006), we examined three different portfolio rebalancing horizons (quarterly, semi-annual and annual) for each of the Stocks-"All Properties" and Stocks-specialized per property type portfolios. Moreover, in order to make the role of the cap rate in improving a portfolio's performance more distinct, we worked with three cases for each portfolio, i.e. with the unconditional case, a conditional case with one conditioning variable (the dividend-price ratio), and a case where we added the cap rate as the second conditioning variable.

In almost all cases we can see that the analysis results fully confirmed our initial line of thought. We saw that by adding the cap rate as an additional state variable the performance of the portfolios, as measured by the Sharpe ratio and certainty equivalent return, has shown significant increase. We also came up with the same conclusion even when we checked the portfolio performance after imposing the realistic "no shorting" restriction to real estate, although in this case the role of the cap rate seemed somewhat less significant than without the restriction.

The few exceptions we faced can be largely explained by two elements, namely the lack of predictability of returns by the cap rates, and the gradual 'smoothing" in analysis results due to the longer portfolio rebalancing period employed in some cases.

5.3. Study Limitations and Scope for Further Research

Throughout this thesis we have had the chance to discuss a few issues that arose during the analysis, and which are related to the data we are using, the historical period we are considering, the predictive regression model we followed, and the way we set up the portfolios.

The issues concerning the data and the historical period used in this thesis are interrelated, and have to do with the limited time span of historical observations included in the databases. More specifically, the TBI index is a new index launched in 2007, a fact that inevitably led us using only small parts of the other databases as well. The latter issue also had as a consequence the use, on our behalf, of a historical period which, not only it is not a full market cycle, but it also includes the recent 2008-2010 deep recession.

Related to the previous comment, we can also mention the fact that the NPI database, which comprises the basis for the TBI data, and provides us with values on cap rates, has experienced significant changes throughout time. These changes concern both the number of properties considered in the index (6,067 properties considered in 2010 versus 1,159 properties in 1985), and their representativeness in the database (equal-weighted versus value-weighted methodology used). Nevertheless, it would be very interesting to see how all the methodologies of this thesis apply to a different time period, to different data, or even in the case of data from a different country.

The second issue has to do with the simplification that we did in the predictive regression chapter. There we set up the predictive regression model implicitly presuming that no additional relations and restrictions apply among the cap rates and other predictive variables of real estate returns, such as rent growth, for example.[8] Of course, the same thing applies to the use of the dividend-price ratio as a stock returns predictor, since there's significant evidence that its predictive ability is influenced and restricted by the dividend growth rate.[9] One might want to consider these factors, too, and build a more complex and accurate predictive regression model, and a system of "structural" equations that take more variables into consideration.

Finally, it would be of interest to examine the use of cap rates as a conditioning variable in more complicated portfolios that include additional representative "basis" assets such as bonds, and more conditioning variables such as the bond's yield. This kind of a more complex portfolio can be of great interest, not only because it can diversify away the idiosyncratic risk of the "basis" assets, as it might be the case in some portfolios considered in this thesis, but also because it implements the dynamic portfolio methodology described in this thesis in a more "real world" portfolio.

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[8] For more details on that issue, please refer to:
Plazzi, Alberto, Torous, Walter N., and Rossen Valkanov, 2010, Expected Returns and Expected Growth in Rents of Commercial Real Estate, UCLA Working paper.

[9] For more information on that issue, please refer to:
Campbell, John Y., and Robert J. Shiller, 1988b, The Dividend-Price Ratio and Expectation of Future Dividends and Discount Factors, Review of Financial Studies 1, 195-228.

6. References

[1] Brandt, Michael W., and Pedro Santa-Clara, 2006, Dynamic portfolio selection by augmenting the asset space, Journal of Finance 61, 5.

[2] Brandt, Michael W., Santa-Clara, Pedro and Rossen Valkanov, 2009, Parametric portfolio policies: Exploiting characteristics in the cross-section of equity returns, Review of Financial Studies 22, 3411-3447.

[3] Federal Reserve Bank, website: <http://www.federalreserve.gov/>, accessed June 2010.

[4] Fisher, Jeff D., Geltner, David M., and Henry O. Pollakowski, 2007, A Quarterly Transactions-Based Index (TBI) of Institutional Real Estate Investment Performance and Movements in Supply and Demand, Journal of Real Estate Finance and Economics 34, 5-33

[5] Geltner, David M., 2010, Class Lectures, Real Estate Capital Markets, MIT Center for Real Estate, Cambridge, MA

[6] Geltner, David M., Miller, Norman G., Clayton, Jim, and Piet Eichholtz, 2006, Commercial Real Estate Analysis and Investments 2nd edition, Southwestern Educational Publishing, Cincinnati, OH.

[7] Markowitz, Harry, 1952, Portfolio Selection, Journal of Finance 7, 77-91.

[8] MIT Center for Real Estate, website: <http://web.mit.edu/cre/>, accessed June 2010.

[9] National Council of Real Estate Investment Fiduciaries, website: <https://www.ncreif.org>, accessed June 2010.

[10] Plazzi, Alberto, Torous, Walter N., and Rossen Valkanov, 2010, Expected Returns and Expected Growth in Rents of Commercial Real Estate, UCLA Working paper.

[11] Stambaugh, Robert F., 1999, Predictive regressions, Journal of Financial Economics 54, 375-421.

[12] Standard & Poor's, website: <http://www.standardandpoors.com>, accessed June 2010.