14.02 Principles of Macroeconomics

Problem Set # 7, Questions

Posted during Week # 14, due in the middle of Week # 15.
You must staple a copy of this frontpage on your problem set.
Remember to write down your name.

Name: _____

(Table is for corrector use only)

	1	2	3	4	5	6	7	Total
I								
II								
III								
Total								

1 Economic Development and the Wage Rate (5 points each, 25 total, 5 extra points)

Assume that the real wage rate (w) is determined on the labor market to equal the marginal product of labor. Assume that the interest rate (r) is determined in the capital market to equal the marginal product of capital. Assume a constant depreciation rate δ and a constant savings rate s. Also assume that the production function is

$$y = AK^{\alpha}L^{1-\alpha}$$

(a) Calculate the real wage as a function of capital per capita. Make a graph of it against capital per capita (put capital per capita in the x axis).

(b) Calculate the interest rate as a function of capital per capita. Make a graph of it against capital per capita (put capital per capita in the x axis).

(c) Consider an economy with zero population growth and no technological progress. What happens during the convergence process to the real wage rate? How does the real wage behave in the steady state? What happens to the interest rate?

(d) Assume that this economy has an exogenous rate of population growth n every period. How does the real wage behave in the steady state? What happens to the interest rate?

(e) Assume that this economy has an exogenous rate of technological progress such that A grows at a rate a every period. Assume there is no population growth. How does the real wage behave in the steady state? What happens to the interest rate?

(f) EXTRA: Suppose that 1% of the population of this country owns all of its capital and the remaining 99% are workers. Assume that there is technological growth and no population growth. How will income distribution change during the convergence process?

2 Savings, Aid and Development (5 points each, 35 total, 10 extra points)

Suppose that the gross savings rate, s can change as an economy develops.

(a) In what way will this affect the capital accumulation equation of the neoclassical growth model.

(b) Suppose that s declines as an economy develops (because the saving rate is low when the marginal peoduct of capital is low). How does this property affect the tendency for the growth rate of capital to decline as the levels of K and Y increase? What does this behavior mean for the convergence property? Show graphically.
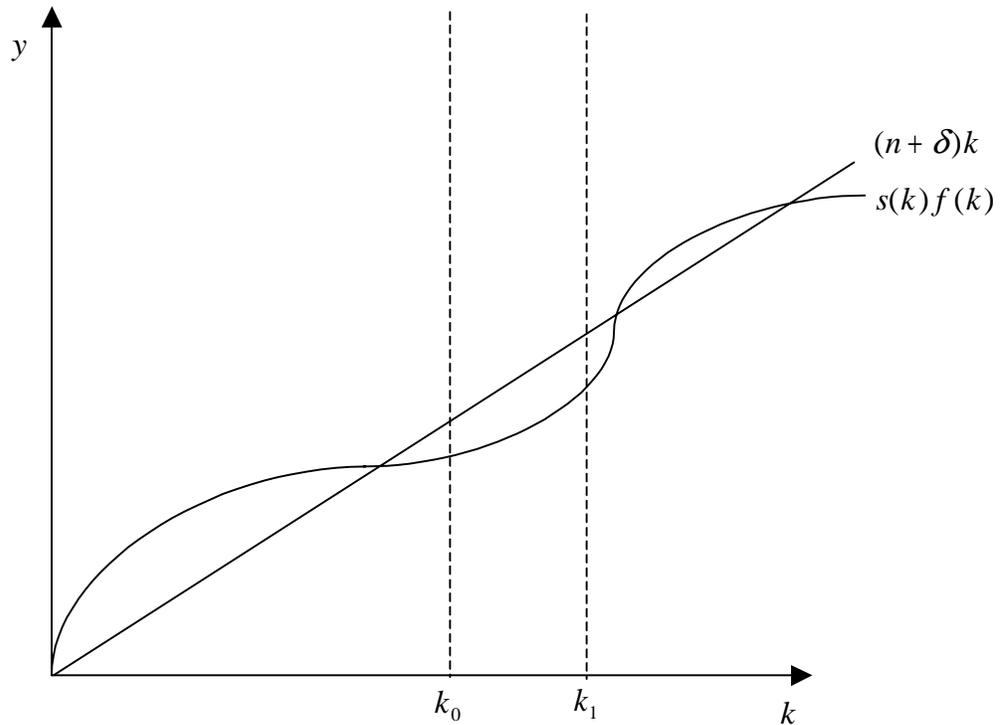
Figure 1:

(c) Suppose that s rises as the economy develops (because the saving rate is high when the level of income is closer to its steady-state value) What does this behavior mean for the convergence property? Show graphically.

In the real world both (b) and (c) can be true at the same time. Suppose that the gross savings rate is a function of capital per capita in the way described by the graph.

(d) How will the existence of this savings function affect convergence? Show how many potential steady states this economy has. Discuss the qualitative differences between the different potential steady states.

Now assume that poor economies (lower capital per capita than k_1) are recipients of foreign aid for investment from international institutions such as the World Bank.

(e) Suppose that the poorest (lower than k_0) economies receive close to no aid (because the World Bank does not trust their governments) but middle

income (between k_0 and k_1) economies recieve some aid? How will this affect the convergence property? Show graphically. How many potential steady states are there now? Discuss the qualitative differences between the different potential steady states.

(f) What should be done so that middle income countries converge to the steady state of the industrialized countries? What can we do about the poorest countries?

(g) Show graphically the effect of a civil war on a middle income country. (Hint: capital is destroyed and international aid is suspended)

(h) EXTRA: Suppose that the *probability* of a civil war increases when the country is poorer. Suppose that the occurrence of a civil war also involves lower savings rates (due to military expenditure). How would you incorporate this into the model.

(i) EXTRA: Explain the emphasis of the World Bank on government reform for the poorest countries in the context of this model.

3 Private and Social Optima (5 points each, 40 total)

Assume a country with M firms. Firm j has the following production function.

$$Y_j = K_j^\alpha (AN_j)^{1-\alpha}, \ 0 < \alpha < 1$$

Assume that productivity of labor depends on the total capital of the country according to the following equation.

$$A = \left(\sum_{j=1}^{M} K_j\right)^\beta, \ 0 < \beta < 1$$

(a) Calculate the steady state output per capita, capital per capita and consumption per capita for each firm. Assume that they are each so small that they consider A as a parameter of the economy that they cannot affect.

(b) Calculate the golden rule for each firm. Show your mathematical steps and explain them.

(c) Explain graphically how to find the golden rule for each firm.

(d) Assume that all firms are identical. In particular that $N_j = N$ and $K_j = K \ \forall j = 1...M$. Find an expression for output per capita as a function of capital per capital in this economy (that is not a function of A).

(e) Now imagine that this economy (with the production function you have just derived in part (d)) is one big firm. Calculate the steady state output per capita, capital per capita and consumption per capita of this economy.

(f) Calculate the golden rule for the economy as a whole?

(g) Is it different from the one derived in (c)? How so? Why?

(h) What would you have to impose on the parameters of this economy to induce endogenous growth? Why does endogenous growth appear when you make this assumption?