Global Terrestrial Uranium Supply and Its Policy Implications: A Probabilistic Projection of Future Uranium Costs

by

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ABSTRACT

An accurate outlook on long-term uranium resources is critical in forecasting uranium cost-resource relationships, and for energy policy planning as regards the development and deployment of nuclear fuel cycle alternatives. In this study, which was part of the MIT Study on the Future of the Nuclear Fuel Cycle, uranium production cost projections over the next half-century are enabled through the development of a comprehensive model for resource cost ($/kg of U_3O_8) versus cumulative energy generation (GWe-yr). The probabilistic cost model incorporates three sub-models including Deffeyes’ crustal abundance model, learning/experience effects, and economies/diseconomies of scale. Using Monte Carlo techniques to develop a cdf of the resource cost correlation factor (θ), in the expression $(\$ / kg) \propto (GWe \cdot yr)_{\text{cumulative}}^\theta$, the resulting model encompasses three probabilistic industry growth scenarios, pessimistic, prudent, and optimistic, representative of confidence levels of ≤ 85%, ≤ 50%, and ≤ 15%, respectively.

The impacts of current domestic and international nuclear policies on industry growth (and subsequently uranium market pricing) are also evaluated, considering waste management, uranium stockpiling, and proliferation. Moreover, various options to optimize natural uranium usage including the reduction of tails during the enrichment phase, recycling re-enriched uranium from SNF, recycling TRU in LWRs, and optimizing fuel burn-up are presented. Further insight is provided to examine the energy balance and environmental impacts of once-through fuel cycles as compared to recycling/reprocessing options and other nuclear and non-nuclear fuel-cycle alternatives. The economic viability of SNF recycling and reprocessing, deployment of breeder reactors, and use of unconventional resources including thorium and seawater uranium are discussed in the conclusions of this study. The results of the study confirm that once-through LWR fuel cycles can sustain aggressive expansion of nuclear power and can remain competitive well beyond the mid-century mark; however, volatility of uranium spot prices is expected until uranium resource production/consumption equilibrium is reached.

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"Consider it pure joy, my brothers, whenever you face trials of many kinds, because you know that the testing of your faith develops perseverance. Perseverance must finish its work so that you may be mature and complete, not lacking anything."

-James 1:2-4

As my recent endeavors in the research enterprise culminate with the completion of this thesis, I would like to take the opportunity to appreciate the support from many individuals and organizations, without whom my progress would not be possible.

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I would like to also thank my parents and family, my “Joy”, and friends whose love and encouragement over the years has kept me grounded, and whose belief in my potential has challenged me to reach for distances well beyond conceivable limits.

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I present this thesis as my contribution to nuclear engineering and technical policy, with the hope that this work adds to the edification of the reader, and is of benefit to those following related research paths.

"We need to encourage the future we want, rather than try to prevent the future we fear."

- Bill Joy (Sun Microsystems)
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CHAPTER 1: INTRODUCTION

1.1 Motivation

In a climate of increased consciousness concerning global climate change coupled with a growing domestic public approval rating for nuclear energy generation, the nuclear industry appears poised for a renaissance. Globally, 180 new reactors are on order, planned, or already under construction [1]. The International Atomic Energy Agency (IAEA) projects an average annual growth rate of 1-3% within the nuclear energy sector. World nuclear energy generating capacity is expected to reach levels between 509 and 663 GWe by 2030, corresponding to upwards of an 80% increase from a current level of 372 GWe. Such growth would increase annual fuel requirements from 70,000 MT U_{nat} to between 94,000 and 122,000 MT U_{nat}. The resurgence of the civilian nuclear program in the U.S. is thus hinged upon a critical point, whether or not aggressive nuclear expansion is sustainable considering next generation reactor technology relying on terrestrial uranium resources. Current literature assessing uranium supplies reflects dichotomous views on the sustainability of long-term uranium resources; on one hand, “uranium resources are seen as plentiful and not in themselves to constitute a constraint on nuclear power development [2].” (2007 Survey of Energy Resources) The “Red Book”, a leading resource detailing global uranium production developed by the Organization for Economic Cooperation and Development’s (OECD) Nuclear Energy Agency (NEA) and the IAEA recently affirmed notions of resource security projecting over a century’s worth of uranium for the existing fleet of reactors at current consumption rates [3].

On the other hand, another belief is that, “nuclear energy is unlikely to be a substantial part of the solution to our climate and energy problems. Worldwide supplies of cheap uranium will last no more than a few decades [4].” J.W. van Leeuwen, an energy and technology analyst at Ceedata Consultancy in the Netherlands sides with the sentiments found in NewScientist and suggests that, “high [ore] grades will be depleted within a decade”. Van Leeuwen supports these claims, arguing as one exploits lower quality ore-grades (≤0.01% U_{3}O_{8}), the environmental advantage provided by nuclear energy is diminished and there is simply no strong evidence that new uranium-dense deposits can be found. Ambivalent speculations have given rise to two schools of thought; optimistic of the industry’s longevity, nuclear enthusiasts and economists support the idea that high uranium prices will drive the discovery and production of new uranium-rich deposits; there is an implied reliance on secondary supplies, the prospect of seawater uranium, and the introduction of advanced reactor technology. Those skeptical of nuclear’s clean and safe energy generation express incredulity, and support the notion that economics simply cannot “circumvent” physical and geological limitations to the supply.

Although both economic and physical arguments are viable, neither side has been able to determine the exploitable resource limit or, even more difficult, predict spot-market prices of uranium. Unfortunately, the mining industry hasn’t been able to provide definitive support either; market analysts point to myopia within the mining industry. Relatively low (and volatile) uranium prices, slow nuclear power growth, and HEU down-blending provide no incentive for miners to look far into the future for reserves.
Additionally, imbalance between uranium production and consumption adds to the complexity; over the past decade, global production of uranium met only 60% of the industry’s annual consumption. In effect, resource price and supply predictions have been met with great uncertainty. The ability to provide objective insights to the abundance of global terrestrial uranium, incorporating both physical and economic arguments, will be critical in forecasting the price of the resource with a level of confidence. Such perspicacity is key in determining industry directives at the mid-century mark and beyond.

1.2 Background

*Market Activity [5]*

Market pricing of uranium serves an important gauge of the balance between supply and demand of the resource. Essentially, periods of increased prices signal perceived supply shortage relative to industry demands; price reductions conversely indicate a perceived surplus. Factors governing the price of the resource include the *supply*, determined by production rates, secondary supplies and various supply disruptions (i.e. mine closings and accidents); *production cost factors*, including deposit concentrations (average ore-grade) and mining operations; and *demand*, determined by cumulative generating capacity and advancements in reactor technology. Spanning as far back at 1945, as illustrated in Figure 1.1, early market prices for uranium were driven primarily by military interest in the buildup of atomic arms until the end of the Cold War in 1989. The market peak in the mid-1970s was the result of a supply strain due to limited production capacity in the face of a burgeoning civilian nuclear industry combined with military demands; the price sensitivity during this period also reflected the growing concern over the long-term supply of exhaustible resources like oil and gold (discussed further in Chapter 2).

Towards the end of the Cold War Era in the late 1980s, price behavior shifted to be more like that of a commodity. Prices during this period (leading into the new millennium) show rapidly declining prices due to the emergence of secondary uranium supplies from uranium stockpiles and HEU from the dismantling of nuclear weapons. However, the Three Mile Island (1979) and Chernobyl (1986) accidents provided the biggest shock to the market, leading to new reactor order cancellations and ultimately freezing industry expansion for nearly two decades.
An outlook of historical data reveals the characteristically volatile international uranium spot-market. Observed in Figure 1.2, following the Cigar Lake flooding of late-2006 (the largest mine in Canada), the market responded with a price spike on June 18, 2007 of 136 $/lb U₃O₈; however, as a result of increased long-term contract volumes and a 17% increase in the identified resource base (‘Reasonably Assured’ and ‘Inferred’), the peak was followed by a depression of prices to nearly 40 $/lb U₃O₈ on April 13, 2009.

Spot-market analysis is extremely useful for broader industry analysis as spot trends signal exploration expenditures and drilling (generally the largest share of the exploration
Expenditure). Exploration expenditures, comprised of land acquisition and maintenance, ground and air geophysical and geochemical sampling, drilling (cumulative hole depth), and other personnel costs, tend to lag market price trends by one to two years; drilling trends, on the other hand, closely match that of the uranium spot-price. From an economics standpoint, the selling price of U3O8 drives mine activity; so long as the uranium market price is set above the potential exploration, production, and expansion costs, mine operation is profitable, thus providing an incentive for increased mining activity. Because uranium is exhaustible and not fungible, the market price for uranium strictly follows industry demand. The interplay between this demand and supply has been complicated in the past few decades by the introduction of secondary supplies. As a means to moderate this market volatility, utilities have recently increased volumes of long-term (one to three year) contracts. Unfortunately, companies and countries have not chosen to disclose any long-term contract pricing information. As a result, using only spot-market information to form conclusions about resource availability is not totally accurate; increased maturation and disclosure (with resource reporting and pricing) will enable development of more deterministic price-forecasting models in the future.

Terrestrial Uranium Deposits [7]

Uranium is a relatively abundant metal resource, approximately as plentiful as zinc or tin, with an average crustal concentration of 2.76 parts-per-million (ppm). Occurring in various sedimentary, hydrothermal and igneous environments, the major ore types are uraninite (UO2) and pitchblende (U3O8), with great variations in the mineral ore over differing deposits. Parts per million (ppm) concentrations of uranium in various deposits are classified to serve as a proxy for the economic viability of production of the deposits. Table 1.1 summarizes typical uranium concentration classifications.

<table>
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<th>Type of Deposit</th>
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<td>High-grade ore (&gt; 2% U)</td>
<td>20,000 ppm U</td>
</tr>
<tr>
<td>Low-grade ore (&gt; 0.1% U)</td>
<td>1,000 ppm U</td>
</tr>
<tr>
<td>Granite</td>
<td>4 ppm U</td>
</tr>
<tr>
<td>Sedimentary rock</td>
<td>2 ppm U</td>
</tr>
<tr>
<td>Average crustal concentration</td>
<td>2.8 ppm U</td>
</tr>
<tr>
<td>Seawater</td>
<td>0.003 ppm U</td>
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Unconformity-Related Deposits

Mineralisation of unconformity-type deposits generally occurs due to pressure and temperature changes at the geological boundary with a meta-sedimentary layer. This type of uranium deposit encompasses uraninite and pitchblende ores. Accounting for some of the richest and largest deposits, unconformity deposits account for nearly 33% of the uranium in the western world, with the deposit holding zones as rich as 20% to 50% U3O8. Some particularly well-known deposits of this type include Canadian mines in the
Athabasca Basin, Saskatchewan and the Northwest Territories; in Australia, the Alligator Rivers region is another major deposit.

Hematite Breccia Deposits

Although there is no solid geological understanding of how these deposits form, some geologists propose hydraulic fracturing, corrosion, faulting, and gravitational collapse to be principal mechanisms. Breccia-complex deposits contain gold, silver, copper, iron, uranium and other rare earth elements; though containing only 0.04 to 0.08% U$_3$O$_8$ (400-800 ppm), co-product extraction enables economic production of uranium, accounting for approximately 17% of the world’s uranium resource. The most significant deposit of the hematite breccia type is Olympic Dam in Australia, known to be the largest reserve of uranium worldwide.

Sandstone Deposits

Constituting approximately 27% of the world’s uranium resource, sandstone deposits predominantly develop in medium to coarse grain sandstone; primary uranium minerals in sandstone include uraninite and coffinite, ranging from low to medium grades of 0.05 to 0.4% U$_3$O$_8$ in the deposit. Although often occurring at low grades, advanced mining technologies such as in-situ leaching (ISL) has permitted economic extraction of the uranium resource deposited in sandstone. Large sandstone deposits are available in the United States (the Powder River Basin), Niger, South Africa, and Kazakhstan.

Quartz-Pebble Conglomerate Deposits

Constituting almost 7% of the world’s uranium resources, uranium production from conglomerate deposits are economically feasible as a by-product of gold mining; uranium concentrations are very low in these deposits from 0.01 to 0.15% U$_3$O$_8$. Witwatersrand in South Africa and Elliot Lake in Canada are sites where quartz-pebble conglomerate deposits occur.

Vein Deposits

Despite hosting among the most uranium-dense ore, near 10% U$_3$O$_8$, vein deposits comprise only 10% of the total world resource. Vein deposits are those where uranium fills cavities and fissures such as veins, pores and breccias. Predominantly deposits of pitchblende, vein deposits have been found in the Democratic Republic of the Congo, Canada, and the Jachymov deposit in the Czech Republic. The remaining worldwide resource is found in surficial, volcanic, intrusive, metasomatite, collapse breccia pipe, phosphorite, metamorphic, lignite, and black shale deposits. Figure 1.3 is a depiction of the uranium resource breakdown by deposit category as a function of ore grade.
Figure 1.3: Uranium distribution as a function of ore-grade, for various deposit types [8]

Mining and Production Techniques [9]

Familiarity with current mining techniques and their association with various deposit types will aid in the comprehension of the uranium production-cost breakdown. Considering the viability of various mining and production techniques, not only operational costs, but also environmental costs are important in assessing the production-cost fraction affecting uranium market pricing. Currently, there are two types of mining approaches that are predominantly employed; conventional, comprised of open-pit (surface) and deep (underground) mining, and solution mining, such as in situ leaching, are used to extract uranium to produce "yellow cake". Figure 1.4 illustrates an open-ended cycle characterizing the fuel fabrication process for traditional LWR fuel.
Underground and Open-Pit Mining

Uranium production has been accomplished principally through underground and open-pit mining. Deep and surface mining accounted for approximately two-thirds of the world’s production of predominantly 0.1% to 0.2% uranium ore in 2005 [5]; these conventional mining processes accounted for an average of 75% of all produced uranium from 1993 to 2003 [10]. In surface or open-pit mining, first a layer of overburden, the geological layer(s) above the formation of economic [or geological] interest, is removed through drilling, blasting, and/or other excavation techniques. In deep (underground) mining, the analogous overburden is comprised of hollowed tunnels of rock containing little to no ore. Next, the remaining ore is excavated using the same array of techniques; this stream of waste, denoted ‘waste rock’ as it typically contains higher concentrations of radioisotopes such as radium (and daughter radon) as compared to normal rock (overburden), is traditionally returned to the pit and covered with overburden. Additional waste is accumulated consisting of ore of grades too low to be economically processed. A stripping ratio, overburden tonnage to ore tonnage, is an important metric in assessing the economic feasibility of a potential mine or open-pit.
After stockpiling the uranium-bearing ore, the rock is transported to (usually onsite) mills for conversion to yellow cake. In the milling process, ore is pulverized into powder and roasted to remove impurities and other organic materials. Chemical leaching is also employed to acquire pure uranium as well as other elements such as iron, lead, molybdenum, and vanadium by dissolving these elements from the rock. Sulfuric acid is generally used as a leaching acid, but also may include alkaline solutions as a leaching agent. Using mainly ion exchange or solvent extraction processes, uranium is precipitated from the acidic solution, washed, centrifuged, and dried into purified yellow cake, ready for conversion. The remaining acidic solution or ‘sludge’ is disposed of in a tailings pond (a dam-like structure).

Some benefits afforded by underground and surface mining include a high rate of recovery of the ore, as well as minimal surface intrusion (for deep mining). However, because mining occurs at low-grade ore deposits, large throughputs of ore must be mined and milled to economically acquire pure uranium. As a result, a large tailing stream of waste ore is expected; a grade of 0.1% U₃O₈ corresponds to 99.9% of leftover material, which is released to a tailings pond in the form of a radioactive sludge. The great volume of the hazardous solution is observed in the 60 million metric ton Olympic Dam tailings pond, which covers nearly 2 square miles. These sludge ponds contain virtually all of the radium and decay products in the original ore, heavy metals, arsenic and other contaminants, as well as reagents used in the milling products such as the acids (leaching liquids) used in the ion exchange or solvent extraction processes. Some additional concerns with deep mining include personnel and environmental exposure to the harmful tailings, potential groundwater contamination, mine cave-ins, as well as radon gas emissions during mining.

In-Situ Leaching

First employed in the early 1960s, in situ leaching (or in situ recovery) has been increasingly utilized to produce uranium; currently accounting for over 20% of global and nearly 85% of U.S. uranium production [11], in situ leaching output increased over 15% from 1990 to 2003 with the introduction of the mining activities in Kazakhstan and Uzbekistan [5]. ISL, also referred to as solution mining, involves reiterative chemical leaching to dissolve uranium (and other elements) from an ore body, pumped through an arrangement of injection and recovery wells. Generally, borehole grid networks with 30 meter spacing characterize a typical recovery site; in the lattice, four orbiting wells are used as injection wells, and a central well is used for extraction. Solution mining is used to extract uranium below water tables, from permeable sandstone deposits surrounded above and below by impermeable geology. There are two leaching reagents used during the extraction, depending on the type of sandstone deposit. For ores containing considerable levels of calcium such as limestone or gypsum, alkaline agents, including NaHCO₃ (sodium bicarbonate) and CO₂, are used; otherwise, sulfuric acid (H₂SO₄) and O₂ are used. Similar to conventional mining, ion exchange and solvent extraction is used to precipitate uranium from the leaching solution. Finally, the uranium is further processed to produce yellow cake in preparation for conversion. Figure 1.5 illustrates a typical ISL setup.
Figure 1.5: Typical in situ leaching operation [12]

The benefits afforded by ISL include minimized surface disturbance (with reduced radiation exposure at surface) as well as an eliminated overburden; additionally, solution-mining practices reduce risks to personnel and miners through increased automation and elimination of large-scale excavation. In situ recovery lowers the carbon footprint as compared to conventional mining activities since ISL removes the need for excavation, stockpiling and transport of uranium-rich ore using large trucks and energy-intensive excavating equipment. This leads to reduced production costs as well as lower secondary waste volumes. Unfortunately, employment of ISL is not without the introduction of other risks; in situ recovery is associated with increased risk of groundwater contamination and leaching liquid excursion. Additionally, it is nearly impossible to restore the affected strata once leaching operations are complete. Finally, optimum uranium recovery reaches only up to 50% as compared to upwards of 90% using conventional mining.

Other Methods

Although typically the production of uranium is reported for uranium as a main product, dictating the economic viability of a particular mine, co-/by-production of uranium comprises a significant contribution to the resource base. Co-production involves simultaneous production of two or more metals or commodities in a mine to economically justify mining activity; by-product prices, on the other hand, have no influence on a mine’s overall output [12]. Co-/by-product mining of copper and gold
comprised over a tenth of global uranium production in 2005, almost entirely attributable to Olympic Dam's uranium co-production with gold. In-place leaching, heap leaching, stope/block leaching, phosphate recovery and mine water recovery make up the remaining production techniques currently employed in the production of uranium. Table 1.2 summarizes the global uranium production contribution in 2005 by method.

<table>
<thead>
<tr>
<th>Production Method</th>
<th>$10^3$ tU @ &lt;130 $/kgU</th>
<th>% Contribution (2006)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Underground</td>
<td>1992.5</td>
<td>39.9</td>
</tr>
<tr>
<td>Open-pit</td>
<td>1049</td>
<td>24.2</td>
</tr>
<tr>
<td>In-situ Leach</td>
<td>809.4</td>
<td>24.9</td>
</tr>
<tr>
<td>Co-/by-product</td>
<td>1099.7</td>
<td>8.9</td>
</tr>
<tr>
<td>Heap Leach</td>
<td>77.5</td>
<td>2.2</td>
</tr>
<tr>
<td>Other*</td>
<td>407.4</td>
<td>0.2</td>
</tr>
<tr>
<td>In place Leach</td>
<td>33.4</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td><strong>Total:</strong></td>
<td><strong>5469</strong></td>
<td></td>
</tr>
</tbody>
</table>

*Includes phosphate recovery, stope/block leaching, and mine water recovery
**Disparities may be attributed to reporting issues and unspecified methods

Resource/Reserve Cost Categories [14]

Uranium reserves and [conventional] resources are published in terms of various cost categories. Implicitly a supply/demand dynamic, the evaluation of resource availability incorporates the economic feasibility of resource recovery. In addition to geological criteria, the cost categories imply various levels of confidence in potential production of the commodity. Framing the discussion of the cost basis for uranium, an important distinction must first be made between reserves and resources. As defined by the Canadian Institute of Mining, Metallurgy and Petroleum, a mineral reserve is the quantity of economically mineable mineral determined by a feasibility study; reserves are reported as a distribution of confidence in proven and probable reserves. Mineral resources on the other hand, under less economic scrutiny (determined to have reasonable economic viability), are the quantities of a mineral (based on grade, location, geology, etc.) indicated, inferred or measured with various levels of confidence. Because of the correlations between mineral geology and production cost, mineral reserves and resources are often used interchangeably. With greater specificity, the IAEA established five resource categories to evaluate the available mineral supply since 1965.

Reasonably Assured Resource (RAR)

With the highest confidence of existence, RAR characterizes the uranium supply in known geological deposits of recognized size and grade; reasonably assured resources are reported within various cost brackets corresponding to probable production costs using
current mining and processing technology. RAR evaluations are closest to the definition of mineral reserve.

_Estimated Additional Resource Category I (EAR-I)_

In addition to the RAR, EAR-I refers to inferred resources on the basis of direct geological evidence. Less assurance is placed on this cost category, as the supply estimates are found for the best-known deposit zones or similar deposits, and are inadequate for RAR denotation. EAR-I is known as the ‘inferred’ resource and combines with the reasonably assured resource to comprise the identified resource base.

_Estimated Additional Resource Category II (EAR-II)_

EAR-II is in addition to EAR-I and RAR categories and refers to the supply estimates based on more indirect geological findings, and mineralogical trends. This cost category is also termed ‘prognosticated’ supply. As a side note, the U.S. combines and reports EAR-I and EAR-II resources as solely EAR-II supply.

_Speculative Resource_

Speculated supply is based on geological trending and extrapolations in addition to the EAR-II category. The resource approximation is reached by estimating potential supply using existing exploration techniques. Speculative and EAR-II combine to comprise the undiscovered resource base.

_Unconventional Resource_

The remaining resource category refers to the potential supply, not presently considered economically viable, often due to the low grades of the resource; the resource category also encompasses the resource recoverable as an insignificant by-product. Some deposits considered in this category include lignite, black shale, coal, and phosphates. Table 1.3 summarizes the estimated resource for the cost categories of <$40/kgU, <$80/kgU, and <$130/kgU (<$15.38/lb \text{U}_3\text{O}_8, <$30.77/lb \text{U}_3\text{O}_8, and <$50/lb \text{U}_3\text{O}_8, respectively), adjusted for expected mining and milling losses.
Table 1.3: Uranium Resource Categorization (10^6 tU) [10][15]

<table>
<thead>
<tr>
<th>Recovery Cost ($/kgU)</th>
<th>Identified</th>
<th>Unconventional**</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RAR</td>
<td>EAR-I</td>
</tr>
<tr>
<td>&lt;40</td>
<td>&gt;1.766</td>
<td>1.204</td>
</tr>
<tr>
<td>&lt;80</td>
<td>2.598</td>
<td>&gt;1.858</td>
</tr>
<tr>
<td>&lt;130</td>
<td>&gt;3.339</td>
<td>&gt;2.130</td>
</tr>
<tr>
<td>&gt;130/ Cost Unassigned</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Subtotal</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>5.469*</td>
<td>10.54</td>
</tr>
</tbody>
</table>

* As a result of steady price rises, increased explorative efforts have led to an expected 17% increase in the Identified Resource Category; thus, this category is projected to yield 5.5 million metric tons. Assuming a constant global consumption of the resource at 70,000 tU/yr, conventional supplies of uranium are expected to suffice beyond two centuries.

** The unconventional resource category is dominated by uranium recovery from phosphates; seawater uranium prospects are not reported as an unconventional resource base, but are hypothesized to yield as much as 4 billion tU.

Resource Bracket Creep

An observation regarding the resource cost categorization over the Red Book’s forty-year publishing history involves the concept of bracket creep. Between 1986 and 1989, the Red Book published expected resource projections with a cost category up to $260/kgU as a result of the price boom over the period. However, the OECD decided to drop the upper cost category in response to lower market prices in the 1990s. Question regarding whether or not the OECD should reinstate the upper cost category has arisen largely due to the record setting price increases in the 2000s. At present, the <130 $/kgU cost level is the upper limit of longest standing. Additionally, the cost category benchmarks have been set in nominal, or current market dollars rather than real, or constant dollars. This implies the negation of inflation and effects of increased constant dollar production costs. Thus, due solely to inflation, resources will move to above benchmark levels over time and hence “disappear” from lower cost brackets. Nevertheless, categorized amounts have either increased or remained the same since the 1980’s. Over the Red Book’s forty-year run, the US GDP price deflator increased by a factor of five and the nominal price of electricity rose by a factor of 4.8. Consistency with this trend suggests an upper cost category of <$650/kgU. For an upper cost limit of $650/kgU ($250/lb U₃O₈), economists suggest the identified resource base will reflect great enhancements in expected yield, putting to bed the argument of resource scarcity, at least from an economics standpoint.

One circumstance where constant cost brackets would be appropriate is when the price of electricity remains constant. As long as the price of electricity is constant, and the share

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1 Confirmed in email by Fritz Barthel (co-author of Redbook 2006) to M.J. Driscoll on 2/3/09
of nuclear generation also remains the same, the constant cost cutoffs are suitable. In presenting the most appropriate cost categories over time, one must consider the consumer price index (CPI), electricity cost history, and/or the producer price index (PPI). Table 1.4 reports the identified uranium resource available under a cost category of 130 $/kg U, as a function of deposit type.

<table>
<thead>
<tr>
<th>Table 1.4: Identified Resources by Deposit [5] (&lt;130 $/kg U)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reasonably Assured Resources (10³ tU)</td>
</tr>
<tr>
<td>----------------------------------------</td>
</tr>
<tr>
<td>Unconformity-related</td>
</tr>
<tr>
<td>Sandstone</td>
</tr>
<tr>
<td>Hematite breccia complex</td>
</tr>
<tr>
<td>Quartz-pebble conglomerate</td>
</tr>
<tr>
<td>Vein</td>
</tr>
<tr>
<td>Intrusive</td>
</tr>
<tr>
<td>Volcanic and caldera-related</td>
</tr>
<tr>
<td>Metasomatite</td>
</tr>
<tr>
<td>Other²</td>
</tr>
<tr>
<td>Unspecified</td>
</tr>
<tr>
<td>Total³</td>
</tr>
</tbody>
</table>

Additionally, total and identified worldwide resources by nation are presented in Table 1.5 and Figure 1.6, respectively.

² Includes surficial, metamorphic, limestone, collapse breccia pipe, and uranium coal deposits; this does not include pegmatite, black shale, granites.
³ An update to the Redbook suggests a 17% increase in identified resources at present (2009)
Table 1.5: Worldwide Uranium Resources (106 tU) @ <$130/kg U [10]

<table>
<thead>
<tr>
<th>Country</th>
<th>Identified</th>
<th>Undiscovered</th>
<th>Total Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>1.243</td>
<td>Not Reported</td>
<td>1.243</td>
</tr>
<tr>
<td>Kazakhstan</td>
<td>0.817</td>
<td>0.8</td>
<td>1.617</td>
</tr>
<tr>
<td>Canada</td>
<td>0.423</td>
<td>0.85</td>
<td>1.273</td>
</tr>
<tr>
<td>Russia</td>
<td>0.546</td>
<td>0.991</td>
<td>1.537</td>
</tr>
<tr>
<td>South Africa</td>
<td>0.435</td>
<td>0.11</td>
<td>0.545</td>
</tr>
<tr>
<td>USA</td>
<td>0.339</td>
<td>2.131</td>
<td>2.47</td>
</tr>
<tr>
<td>Remaining</td>
<td>1.666*</td>
<td>2.685**</td>
<td>4.351</td>
</tr>
<tr>
<td>Total</td>
<td>5.469</td>
<td>7.567</td>
<td>13.036</td>
</tr>
<tr>
<td>GWe Reactor Years***</td>
<td>27,000</td>
<td>37,800</td>
<td>65,000</td>
</tr>
</tbody>
</table>

* Reported from 38 nations
** Reported from 28 nations
*** 1 GWe @ 200MT/GWe-yr

Figure 1.6: Global distribution of Reasonably Assured Resources (<130 $/kg U) by Country [6]
Demand Scenarios

In evaluating the adequacy of worldwide uranium supplies meeting [nuclear] industry demands, detailed projections of long-term reactor requirements must be assessed making an allowance for uncertainties in future energy demands. Over the past decade, concerns have mounted over the imbalance between growing reactor requirements for uranium and the downscaled production of the same mineral. From 1995 to 2008, global uranium production met, on average, less than 60% of total reactor requirements (see Figure 1.7). Although a temporary match is seen in 1990, the demand for uranium has continued to outpace mine production to the present, with the remaining uranium balance satisfied by secondary supplies, comprised of commercial and government inventories and down-blended highly enriched uranium (HEU) from decommissioned military arms; in effect, these additional supplies have, “displaced comparable amounts of newly produced uranium [17].” Assuming limited to no changes in reactor technology in the near future, energy security concerns will not be assuaged unless the production-demand gap at present is closed with an increased supply of fresh uranium.

![Annual uranium demand and production, 1985-2009](image)

* World Nuclear Association values used in absence of Red Book data
** WNA projection (July 2009)

**Figure 1.7:** Annual uranium demand and production, 1985-2009 [5]

The IAEA, in its *Analysis of Uranium Supply to 2050*, deductively forecasts reactor requirements to 2050 beginning with mid-century energy demand estimates, followed by a nuclear density outlook to meet those energy demands; of considerable difficulty,
according to the study, is fuel cycle modeling, resolving the number and type of deployed reactors, burn-up and load factors, as well as recycling and reprocessing strategies.

![Figure 1.8: Published projections for worldwide annual uranium needs to 2020 [17]](image)

There is no shortage of projections, as featured in Figure 1.8, which displays a “diversity” of demand projections from 5 organizations including the IAEA, the USDOE’s Energy Information Administration (USEIA), the OECD's NEA (‘99 Red Book), the Uranium Institute (UI) and the International Institution of Applied Systems Analysis (IIASA). Dating back to projections made between 1995 and 1999, the considerable agreement between projections to 2005 and 2006 (ignoring the USEIA low case) is due to delayed generating capacity responses to policy changes in reactor license renewals and construction permits, influencing the international energy portfolio. However, by 2020, these projections greatly diverge, with high forecasts topping 100,000 tU (IAEA hi case) and low forecasts dipping to nearly 25,000 tU (USEIA low case). The majority of research agencies provide forecasts that end in 2020; thus, the IAEA published an extended projection, relying on IIASA’s projections of uranium demand to 2050. Figure 1.9 displays three (out of a total of six) demand scenarios developed by the IIASA, forecasting nuclear generating capacity to 2050 (converted for equivalent uranium consumption).
In Figure 1.9, the conservative, low demand case is based on expected moderate economic growth, energy policies driven by environmental consciousness aimed at the reduction and minimization of greenhouse gas emissions, and a complete phase-out of nuclear power by the end of the twenty-first century. The middle demand case is characterized by increased clean energy technology development with energy policies again aimed at carbon emission reduction, moderate economic growth, and moderate nuclear expansion; the middle demand scenario assumes next generation reactors will be increasingly modular, with vast improvements in passive and active safety, and with an overall heightened sense of public acceptace. Additionally, the IIASA analysts suggest on the supply side that secondary supplies will have limited long-term impact on overall supply, with uranium production in the Commonwealth of Independent States (CIS) also limited in contribution, cooperatively having insignificant impact on market price trends. Lastly, the high demand case is characterized by aggressive development and deployment of biomass and nuclear, high economic growth (2 to 4% annual growth), low energy prices, and weak energy policy emphasis on environmental sustainability (coupled with substantial supplies of important fossil fuels).

The IAEA analysis set out to evaluate the credence of analysts’ estimates of a 1 to 3% nominal growth in generating capacity to 2050; assuming a 60,000-tU requirement in 2000, the projected nominal growth corresponds to a cumulative uranium consumption of
between 3.9 and 6.9 million tonnes of uranium by 2050. Although the identified (5.5 million tU) resource (Table 1.3) may satisfy modest industry growth, presently undiscovered resources may be necessary to fuel a potential resurgence in nuclear energy generation. The attractiveness of additional fuel sustaining options, including deployment of breeder and burner technology, is largely dependent on expected future costs of the uranium resource. In MIT’s *The Future of Nuclear Power* (2003), “the amount of known resources depends on the intensity of the exploration effort, mining costs, and the price of uranium. Thus, any predictions of the future availability of uranium that are based on current mining costs, prices and geological knowledge are likely to be extremely conservative”, in effect suggesting great uncertainty in projections of future uranium supplies.

1.3 Thesis Objectives

The ability of the nuclear industry to meet its demand for uranium at acceptable costs is limited by the global availability of this resource. Thus, an updated uranium assessment is necessary to reduce uncertainties in the study. The focus of the following report is on global uranium supply as opposed to solely domestic U.S. production of the mineral resource. However, special interest in the global scarcity or abundance of affordable uranium will give insights to the energy security provided by the U.S.’s nuclear industry; undeniably, over the past decade, the U.S. has imported nearly 80% of its uranium without controversy. Although numerous front-end stages lead to production of reactor fuel, conversion, enrichment and fabrication steps are not addressed in this study, as these steps are not resource-limited, but rather services.

Development of a comprehensive resource cost vs. cumulative energy generation model will be the basis by which uranium pricings over the next half-century will be forecasted. The function of the cost model is to assuage concerns of resource scarcity, enhanced by spot price volatility and consumption/production market imbalance, by providing an objective and insightful approach for price projection of terrestrial uranium. Uranium resource cost modeling will be developed through the integration of 3 sub-models including Uranium ore grade vs. resource availability [Deffeyes], learning effects [Schneider and Sailor], and economies/ diseconomies of scale. Deffeyes’ model is still by far the best-known analytic model for prediction of Uranium supply as a function of ore grade. According to Deffeyes, distributions of Uranium (varied by ore grade) can be modeled using a single lognormal curve [18]; proposed by this model is a 300-fold increase in Uranium abundance per 10-fold decrease in ore grade. These claims by Deffeyes will be reviewed by revisiting the concepts involved in the development of his uranium resource model. The model forecasting probabilistic distributions of uranium spot prices vs. cumulative energy generation will include uranium ore pricing ($/kg) relative to a base year (2005) as well as the consumption of uranium in GWe-yr beyond base year (2005). The model will bracket the probabilistic cost distributions over a range of projected demand scenarios, pessimistic, prudent, and optimistic, determined by statistical assessments of uranium markets using Monte Carlo techniques.
Various options to optimize current uranium resources are important considerations in the study. Reprocessing SNF and recycle into LWRs will reduce ore demand by 30%, but is a considerably expensive option; breakeven analysis with once-through fuel cycles must be completed to speculate on the viability of this option. Reducing enrichment tails composition potentially saves 30% of uranium ore. Furthermore, uranium from seawater should not be discarded; potentially economically feasible extraction methods are being developed in Japan. These options will be further explored in the study. The culminating analysis is hence focused on addressing industry concern regarding the competitiveness of the once-through LWR fuel-cycle beyond mid-century, in response to aggressive nuclear expansion.

1.4 Thesis Organization

Chapter 2 discusses the underlying concepts behind, and provides the development of, the cost versus cumulative consumption resource model. In this chapter, Deffeyes’ model, Schneider’s learning effects, and economies/diseconomies in production scale are further explored. Sample calculations using various growth scenarios are completed and the implications of the findings are summarized.

Chapter 3 discusses the environmental externalities and energy balance issues relating to various uranium production techniques. In situ leaching versus conventional mining approaches, as well as environmental viability of various enrichment methods are addressed in this section. The implications of the findings are concluded.

Chapter 4 reports the breakeven economic evaluation for various nuclear fuel options such as LWR recycle and seawater uranium usage compared to traditional terrestrial ore use in the once-through fuel cycle.

Chapter 5 presents various policy issues and arguments surrounding uranium production and fuel cycle issues. Proliferation, ground water contamination, toxic tailings exposures, and production safety concerns are addressed and the connection to the uranium supply is discussed in chapter 5.

Chapter 6 discusses various resource conservation alternatives including stockpiling strategies, optimum tails assays, increased fuel burn-up, and use of unconventional fuels. Safety, cost, and potential uranium ore savings are evaluated for each option.

Chapter 7 summarizes the findings of the report, discusses the implications of the findings, and presents recommendations for future work.

Appendix A presents the development and characterization of the lognormal relation utilized in Deffeyes’ uranium grade distribution model; various techniques involved in the characterization of the lognormal parameters are included. Finally, a relation for the cumulative resource distribution is resolved, revealing the resource elasticity.
Appendix B reviews the concepts and presents the methods for integration of the three sub-models (ore grade elasticity, learning, and economy of scale).

Appendix C summarizes the Monte Carlo methods used to develop the composite resource cost correlation factor ($\theta$).
2.1 Chapter Introduction

In this chapter, the conceptual and analytical development of the uranium resource model is provided. The development of a geo-statistical model, based on Deffeyes’ work, was motivated by the tenuous nature of resource cost forecasting, accurately projecting price trends beyond several decades. Cohen, in a piece titled *Earth Audit*, published in the NewScientist in 2007, shows that the resource-exhaustion time horizon for nineteen widely-used metals varies extensively, with a median of roughly six decades; uranium coincidentally falls at the median, found by dividing the reserve base by the annual global consumption rate (assuming global consumption is met only by global production, no recycle or secondary sources). Unfortunately, Cohen’s calculation is based on a snapshot of the industry and does not take into account changes in market supply or demand [19].

Sustainable development (first coined in 1987) by the Brundtland Commission (formally the World Commission on Environment and Development WCED) is the, “development that meets the needs of the present without compromising the ability of future generations to meet their own needs.” Therefore, there is a need to evaluate today’s fuel cycle options not only within the scope of immediate political, economic and environmental concerns, but from a vantage point considering prospects for the future. The ability to effectively discern the sustainability of nuclear growth with regard to the long-term uranium supply prospects is enabled through use of accurate long-term supply and cost models. Considering Cohen’s findings, the time horizon necessary to develop effective strategies for sustainable nuclear development must extend well beyond 60 years.

As discussed in the previous chapter, great uncertainties must be accommodated in the development of cost and resource forecasts. Considerable uncertainties are introduced in the discussion of resource availability and projections of future uranium demand. Traditionally, market price escalations are linked to mineral scarcity. Although a viable practice, the common failure of resource models to credit ingenuity and efficiency gains through experience, contributes greatly to the uncertainty and inaccuracy of current models. Shropshire and others have pointed out falling metal prices over the course of the past century; an anecdote underlines this phenomenon: In 1980, two prominent figures emerged in the debate over the future of exhaustible resources. Ecologist Paul Erlich argued that the world was reaching its carrying capacity and thus prices (in real time) would rise as supplies of food and commodities were depleted. Economist Julian Simon argued the converse, insisting prices would drop overtime, as populations became more effective at improving the availability of very abundant, although exhaustible, resources (through improved exploration, extraction of resources, etc.). Simon proposed a bet to Erlich and called for him to pick specific materials as case studies. Erlich settled on copper, nickel, tin, tungsten, and chrome. In 1990, Erlich paid up - all of the prices had fallen. Figure 2.1 shows the 20th century price history of various metals including aluminum and zinc [9].
2.2 Review of Model Concepts

There is a continuum of ideologies concerning strategic development assuring resource sustainability; at one end of the spectrum is what Deutch and Lester term prohibition, or cessation of nonrenewable resource exploitation [20]. At the other end of the spectrum is the notion that mineral resources are inexhaustible, whereby based on ingenuity and economic expansion, extraction of increasingly lower grade and more expensive resources, as well as the creation of alternative resources, is economically viable in the face of increasing resource scarcity. The incorporation of economic assessments in the debate over resource abundance dates back to the 1930’s with Harold Hotelling’s “The Economics of Exhaustible Resources”; however, the interest in resource availability predates even this. Princeton professor Frank Notestein has been quoted [by J. Simon] in the 1970’s asserting, “we’ve been running out of oil ever since I’ve been a boy”; Notestein was in his 80s when he gave such remarks [21].

Two alternative views on exhaustible resources, termed ‘fixed stock’ theory and ‘expansion’ theory, commonly arise in the discussion. In fixed stock theory, earth’s finite resource endowment is depleted along a defined cost and production trajectory, often associated with peak production theories within the oil industry. According to Simon, if resource scarcity progressively increased, one would expect escalating prices (although physical theorists would assert this to be inconclusive), a declining ratio of remaining stock to production, declining production, and a diminishing reserve. On the other hand, theorists claim the recoverable resource limit is inconceivable, relying heavily on technological improvements, which are highly unpredictable. Here, an emphasis is placed
on resource price, a proxy for society’s valuation of a mineral. This study will further explore the technology improvements, efficiency gains, and production cost reductions facilitating resource expansion.

Lognormal Frequency Distribution of Uranium

Geologist M. King Hubbert developed a prominent model attempting to predict oil’s peak production in 1956. Working with Shell Oil, he predicted a peak in U.S. oil production in the 1970’s. Better known as the Hubbert Curve, Hubbert’s analysis was based on a bell-shaped logistics curve (contemporaries use Gaussian curves), displaying a distribution for annual resource production. Figure 2.2 displays Hubbert’s 1956 graph.

![Figure 2.2: Hubbert Curve of 1956](image)

The peak production of the resource was estimated using Hubbert linearization. Hubbert linearization is founded on the idea that oil production is at maximum when the cumulative production of the resource equals the resource base remaining. One must however, have an accurate knowledge of the potential resource availability (the geological limit). Linearization works by plotting the annual production as a percentage of the running cumulative production on the vertical axis, against total production on the horizontal axis. The result is a graph of the change in growth rate of total production as the resource is exploited (analogously, the percentage change in uranium resource as lower concentrations of ore are exploited). By drawing out the line to the axis intercept, one may deduce the exhaustible limit of the resource. This model improves upon former economics-driven models (which considered foreign competition and resource substitutions) and is, “fundamentally geological, reflecting increasing difficulty of
increasing production while exploiting resources of progressively poorer quality [23].” Many scientists and economists recognized the success of Hubbert’s model and have since adopted some form of Hubbert’s linearization; D. Rutledge of Caltech carved out a study to forecast coal production using Hubbert linearization.

![Graph](image)

**Figure 2.3:** Hubbert Linearization for the U.S. lower 48 crude oil production [24]

Although Hubbert’s logistics curve representation of annual resource production would not be applicable to uranium, geologist Kenneth Deffeyes (along with Ian McGregor) postulated the applicability of Louis H. Ahrens’ 1954 geological model in representing uranium resource distribution as a function of crustal concentration. Ahrens’ research findings uncovered a lognormal correlation between the abundance of trace elements in granites as a function of their concentrations; Ahrens suggested such a Gaussian distribution is a fundamental geochemical law. Combining the groundbreaking work of Hubbert and Ahrens, Deffeyes sought out to determine if in fact the distribution of uranium can be reasonably approximated by a log-normal curve; additionally, Deffeyes questioned if current exploited mines are at the high-grade tail of the distribution. Deffeyes concluded that the uranium crustal distribution could be represented by a single lognormal curve, showing a 300-fold increase in recoverable resource per tenfold decrease in ore grade, in the range of current interest.

Deffeyes disagreed with Ahrens’ assertion that the lognormal relationship was representative of a geochemical law; instead, Deffeyes reasoned that the correlation was the result of the central–limit theorem in statistics. The deposition of high concentrations of uranium is the result of compounded, unlikely events; from a source element, water must then transport this element through subsurface conduits, and into zones where the mineral is precipitated in high concentration. All of these improbable steps are required
for successful uranium deposition. Multiplying the probabilities of the required steps, the central-limit theorem suggests the subsequent Gaussian (bell-shaped) distribution, also evident on a logarithmic scale. For this study, a unimodal distribution is assumed for uranium crustal distribution; Yale geologist Brian J. Skinner postulated a bi-modal distribution for trace elements, where one peak represents distribution in ore deposits and the other in common rock [25]. Figure 2.4 illustrates the Gaussian distribution for crustal uranium abundance, as presented by Deffeyes. Integration of the frequency distribution reveals the cumulative resource distribution. Aligned with Hubbert’s linearization principles, the [ore-grade based] resource elasticity (a ratio of change in resource abundance to change in ore grade) can be determined from the cumulative lognormal uranium distribution. In return, production limits based on extracted ore grades may be resolved from this elasticity.

Figure 2.4: Deffeyes Lognormal Frequency Model for the Global Crustal Uranium Distribution [25]
Economy of Scale

Economies of scale are known in economics as the cost advantage gained by the increase (or decrease for diseconomy) in the scale of production, or simply a firm’s expansion of operation. Economies of scale have even been observed in purchasing with the buying of long-term contracts and bulk ordering. In conventional mining operations, economies of scale have driven the upscaling of excavation tools and machinery; 77 cubic yard shovels and trucks with a 360-ton capacity currently dominate the landscape of open-pit mines. Moreover, as published in The Economist’s article, Some Miner Concerns, exploiting potential economies of scale is an explanation offered for BHP Billiton’s attempt to acquire Rio Tinto, respectively the industry’s top and 3rd based on mining revenues; in the upscaling of companies, larger companies benefit from economies of scale effectively reducing administrative costs as well as other operational costs [26].
Figure 2.5: Evolution of the Excavator, Exemplifying Economy of Scale [27][28]

_Diseconomy of Scale_

Although conventional logic suggests great reductions in production costs as one optimizes operation scale, increasingly stringent standards for the handling of mine and mill tailings have been observed to actually increase production costs. Where many deposits contain less than 1% uranium (<10,000 ppm), the amount of tailings is nearly
equivalent to the total ore mined. In fact, the amount of ore handled increases as \(1/x\), where \(x\) is the ore grade (ppm). Thus, the overall economy of scale enjoyed by up-scaled operations competes with the radionuclide burden in the waste rock and mine and mill tailings. These environmental standards have also affected the cost and time to bring new mines and mills online; 10 to 15 years is the current estimate.

**Learning/ Experience Effects**

Schneider points out a lack of experience and learning effects in current uranium cost models. Experience and learning effects were first introduced in the 19th century, by German psychologist Hermann Ebbinghaus (although he used a ‘forgetting curve’). The learning curve effect explains the relationship between experience and efficiency gains in a particular activity. As one gains experience, the rate of reduced cost is proportional to the instantaneous cost (initial cost). Fundamentally, as one increases the iterations by which a task is performed, there will be a reduction in time required for each subsequent iteration of the task; this timesaving translates directly to cost, especially with production. Learning effects are proposed to incorporate market competition, labor force, social, policy and tax structures. Total production costs are comprised of development (research and development, prototyping and testing), construction (labor, materials, manufacturing, installation, licensing), engineering (design, management, sequencing and layout), operating (equipment, supplies, staff, maintenance and fuel), and financing (borrowing, interest, risk premium, profit and investor return) costs [29].

Quantified in the 1930's, the experience curve was used to explain a phenomenon observed in the production of airplanes; for every doubling in units of product, an improvement of 10-15% in the time to produce the same number of units was afforded. On a broader scope, learning curve theory asserts that as the number of units produced doubles, costs are reduced at a predictable rate. By the 1960’s, learning theory caught the interest of Boston Consultant Group (BCG) consultant Bruce Henderson, where he utilized the curve for strategic planning. The power law relation commonly used today is termed Henderson’s Law, due to his efforts. NASA has published a list of learning rates for various industries; learning rates typically range from 70% to 90%, corresponding to 30% and 10% cost reductions for a doubling of produced units [30]. Among some of the benefits through increased experience in uranium production are improved labor efficiencies, excavation tool design, equipment utilization, and exploration efforts. Also critical in plant construction calculations, concepts of first-of-a kind (FOAK) and nth-of-a-kind (NOAK) are discussed for generation IV reactor deployment, explaining deployment and standardization costs, differentiated from research and development costs [9].

2.3 Modeling Methods and Results

In this section, a summary of the methodology employed in the development of the cost vs. cumulative generation model is provided; a more rigorous and thorough assessment is provided in Appendix sections A through C. The approach adopted involved development of a price elasticity of cumulative uranium consumption based on Deffeyes’ model of reserves as a function of ore grade [31]. His work extended the lognormal
model previously applied to individual mined deposits (e.g., by Krige for gold) [32] to a global ensemble of uranium deposits (Figure 2.4). The region of interest in Figure 2.4 is the left-hand side of the plot, denoted ‘current mines’; this region, bounded above by the richest ore deposits geologically feasible, has a lower cutoff just above 100 ppm uranium, below which grade the energy expended to extract the uranium will approach a significant fraction of that recoverable by irradiation of fuel in LWRs. Numerical integration of Deffeyes’ log-normal frequency distribution gives the cumulative reserves as a function of ore grade (ppm). This result can then be manipulated to yield the ore-grade elasticity, \( s \). Numerical analysis validated the following semi-analytic approximation in the range of interest (between \( 10^2 \) and \( 10^4 \) ppm, typical concentrations of ore within currently exploited mines).

\[
\begin{align*}
\frac{\text{% increase in cumulative reserves}}{\text{% decrease in ore grade}} &= s \\
&= \frac{\sqrt{\pi}}{2\sigma^2} \left[ \ln x - \bar{v} + \frac{\sigma}{\sqrt{2}} \right]
\end{align*}
\]

(2.1)

where
\( x = \text{ore grade, ppm U} \)
\( \bar{v} = \text{mean value of } \ln(x) : 2.48 \)
\( \sigma^2 = \text{variance of } \ln(x) : (1.51)^2 \)

note \( \ln(x) = 2.303 \times \log(x) \), since base 10 plotting is most common.

Figure 2.6 displays the results; at about 1000 ppm, supply is predicted to increase approximately 2\% for every 1\% decrease in average ore-grade mined. Note, \( s \) is positive (as opposed to conventional elasticity, which is its negative), and a linear function of \( \log(x) \).
An important factor overlooked here in the prediction of uranium resources is the recovery of uranium as a co-product or by-product of other mining operations; the most important category in this regard involves phosphate deposits. A recent CEA assessment projects 22 million MT from this source - enough to power 1000 one-GWe reactors for 100 years, so long as co-production is fully pursued [33]. Additionally, several authors have noted that Deffeyes' assessment was completed before the discovery of rich ore deposits in Canada, at grades in excess of 3% (30,000 ppm). This could imply that the projected cost escalation based on his results would, in effect, be postponed for a period.

The predicted ore-grade elasticity in Figure 2.6 was then combined with the classical economy of scale and learning curve models of engineering economics (see Appendix B) to obtain a relation between cost C, $/kgU and cumulative consumption of nuclear electricity, G, GWe-yr (directly related uranium demand):

\[
\left( \frac{C}{C_r} \right) = \left( \frac{G}{G_r} \right)^\theta
\]

(2.2)

where

\[
\theta = \left( \frac{n}{s} - \alpha \right)
\]

in which
n - economy of scale exponent (typically 0.7)
\(\alpha\) - learning exponent \(= -\ln(f)/\ln(2)\) (hence 0.23 for \(f = 0.85\), where \(f\) is taken as a decimal)

In Equation 2.2, \(C_r\) and \(G_r\) are reference (start of interval) values, $/kg U_{\text{NAT}}$ and cumulative GWe-years of electricity generated, respectively. Note that \(G\) can also be expressed as cumulative uranium consumption, since we assume a constant proportionality of 200 MT/GWe-yr at full power. It should be evident that extrapolation into an ill-defined future is not properly a deterministic undertaking. Hence, following the lead in a similar effort in 1980 by Starr and Braun of EPRI, a probabilistic approach was adopted [34].
Figure 2.7: Relative Uranium Cost vs. Normalized Cumulative Nuclear Energy Generation

Key to Figure:

\[
\text{LINES PLOT } \left[ \frac{C, \$/kgU}{100} \right] = \left[ \frac{G, \text{GWe-yr}}{10^4} \right]^{\theta}
\]

\(\theta = -0.10\) for 15% CPDF Percentile, Optimistic Choice
\(\theta = 0.11\) for 50% CPDF Percentile, Median Case
\(\theta = 0.29\) for 85% CPDF Percentile, Conservative Choice (Appendix C)

where \(G1 = 100\) years at today’s rate of uranium consumption and electricity generation
\(G5 = 100\) years at \(5 \times \) today’s rate (equivalent to 2.7%/yr exponential growth)
\(G10 = 100\) years at \(10 \times \) today’s rate (equivalent to 3.6%/yr exponential growth)

also: \(\text{RBI} = 2007\) Red Book, Identified, < 130 \$/kg
\(\text{RBU} = \text{RBI} +\) Undiscovered, < 130 \$/kg

BASE YEAR point @ [1,1] is 2005: 100 \$/kg \& \(10^4\) GWe-yr

Figure 2.7 plots Equation 2.2, where a straight line may be observed on the log-log plot. Values of \(C_r = 100\) \$/kg and \(G_r = 10^4\) GWe-yr are assigned based on 2005 as the reference year. Trend lines for three values of \(\theta\) are shown, based on the probabilistic assessment described in Appendix C. The plot is to be interpreted as the probability (e.g., 85%) that the cost (e.g., 200 \$/kg) will be less than the value on the trace plotted (in this example supporting \(\sim 10 \times 10^4\) GWe yr). Note that the 100% probability line (not shown)
is given by $\theta = 0.5$, which matches the $0.40 \leftrightarrow 0.52$ values in four of the (non-probabilistic) models surveyed by Schneider [35]. Our value of 0.29 matches his “optimistic” value of 0.30. Points are plotted on Figure 2.7 corresponding to 2007 Red Book values for identified and identified-plus-undiscovered resources under 130 $/\text{kg} - 5.5$ and 13.0 million metric tons, respectively. Also shown are cumulative consumption indicators for 100 years at one, five and ten times today’s rate. These benchmarks support the expectation that uranium production costs should be tolerable for the remainder of the 21st century, long enough to develop and smoothly transition to a more sustainable nuclear energy economy.

**Model Application**

To employ this figure in scenario analysis, one merely integrates under a postulated GWe vs. time history (starting at 2005), divides by $10^4$, adds 1 (to include pre-2005 consumption), and reads off the projected cost of natural uranium in 2005 dollars as $(C/C_r) \times 100$ $/\text{kg}$. Values for different values of $\theta$ are readily plotted. In the following, the “conservative” 85th percentile value (i.e., median plus approximately one-sigma) of $\theta = 0.29$ is used.

For example: A scenario gives 50,000 GWe yr between 2005 and 2050, Hence $(G/G_r) = 5 + 1 = 6$. For $\theta = 0.29$, Fig. 3.4 gives $(C/C_r) = 1.7$, thus $C = 170$ $/\text{kg}$ in 2005 dollars as of 2050.

Scenarios are often based on simple exponential growth:

$$E(t) = E_r e^{\gamma t}, \text{ GWe}$$

(2.3)

Thus cumulative energy generation over a period of $T$ years is:

$$\left( \frac{G}{G_r} \right) = 1 + \left( \frac{E_r}{\gamma G_r} \right) [e^{\gamma T} - 1]$$

(2.4)

For example:

Let $E_r = 400 \text{ GWe},$
$G_r = 10^4 \text{ GWe-yr},$
$\gamma = 0.04 \text{ yr}^{-1}$, the annual demand growth rate
$T = 80$ years

Then

$$\frac{G}{G_r} = 24.53$$
Again, assuming that $\theta = 0.29$, the plot gives $C = 250 \ $/kg (using Equation 2.2, $C = 252.94 \ $/kgU), which would warrant serious consideration of timely development of alternatives to once-through LWRs. Again note that our estimates are in constant dollar values: using nominal dollars would give much higher values decades from now. By introducing the further approximation that the reference condition is just the integral of the exponential scenario from $-\infty$ to 0, the following analytic relations can be derived (which obviates the need for the graphic method):

Cost at time $T$:

$$\frac{C(T)}{C_r} = e^{\theta y T} \quad (2.6)$$

Average cost, 0 to $T$:

$$\overline{C} = \frac{e^{\theta y T} - 1}{\theta T} \quad (2.7)$$

Thus for $\theta = 0.29, \gamma = 0.04$/yr, $T = 80$ yrs as in our earlier example:

$$\frac{C(80)}{C_r} = 2.53; \overline{C} = 1.65$$

$C(80)/C_r$ agrees within readable precision with the value given by the plot in Figure 2.7.

2.4 Implications

*How to explain falling metal costs*

Our simple model gives a cost relation as

$$C = C_r e^{\theta n} \quad (2.8)$$

where

$$\theta = \left(\frac{n}{s} - \alpha\right)$$

Let's assume $\gamma = 0.03$/yr, equivalent to the GDP nominal growth rate (this 3% growth corresponds to a 6 fold increase in demand over the span of a century). From 1900 to 2000, $\theta \gamma = -0.012$, the mean for all metals studied by Shropshire [9]. Thus, $\theta = -0.4$, quite a large value, given the model optimistically predicts a value of $\theta = -0.10$ at a 15% confidence level. If solely learning effects are observed in the resource price elasticity exponent for $\theta = \alpha = -0.4$, one would find:
Although a price elasticity of -0.4 is extremely (almost unreasonably) optimistic, the resulting value for learning is typical of many industries, assuming the n/s term is negligibly small. If \( n = 0.5 \), Simon’s industrial scaling factor (low-end limit), and \( s = 3 \), the high end of Deffeyes’ reserve correlation, \( n/s = 0.17 \); in this case, \( \alpha = -0.57 \) to give \( \theta = -0.4 \) revealing \( f = 67\% \), which again is not inconceivable. The annual consumption growth rate, \( \gamma \), is a key variable in determining the maximum parameter values; table 2.1 presents the maximum allowable learning rates for various annual demand growth scenarios, according to Shropshire’s average price trend for metals (\( \theta \gamma = -0.012 \), from 1900 to 2000).

Table 2.1: Learning Rate as a function of annual Growth Rate

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( \theta )</th>
<th>max ( f ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>-0.60</td>
<td>66.0%</td>
</tr>
<tr>
<td>0.03</td>
<td>-0.40</td>
<td>75.8%</td>
</tr>
<tr>
<td>0.04</td>
<td>-0.30</td>
<td>81.2%</td>
</tr>
<tr>
<td>0.05</td>
<td>-0.24</td>
<td>84.7%</td>
</tr>
</tbody>
</table>

Thus, high growth rates in consumption imply increasingly less aggressive learning, assuming the n/s term is small. However, there is a plausible reason for expecting strong learning. As demand grows, there is incentive to develop better methods for locating and delineating potential mining regions. This is in addition to the conventional concept of learning as it applies to production (e.g. assembly line production of cars, refrigerators, etc...). Hence, we benefit from “how to improve exploration” in addition to “how to improve production”. Although not completely analogous, the example of oil discovery and production exemplify a similar concept of duality with uranium production.

Another mechanism which will act to reduce \( \theta \) is the recovery of uranium from mine and mill tailings; here, low grade production is mitigated by the fact that the ore is already mined and crushed into slurry. This effect is related to the production of uranium through by-production, where sharing of mining/milling resources with a principal or co-product reduces production cost. In theory, the recovery of uranium from tails as well as uranium by-production of lower grade ores effectively narrows the production cost-gap between high and low grades. This can be expressed as an increase in the ore grade elasticity, \( s \).
Resource Elasticity Exponent Values from other Studies

Schneider and Sailor published a review of various supply-cost curves. The models included in their study can be used as a benchmark for the price elasticity values obtained in this study. Figure 2.8 illustrates an updated ensemble of uranium supply curves presented in Schneider’s study. It is worth noting that the plots are linear on log-log scale. Table 2.3 summarizes the various resource elasticity values obtained from various sources used by Schneider and Sailor.

<table>
<thead>
<tr>
<th>Study</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starr (1980)</td>
<td>0.5*</td>
</tr>
<tr>
<td>Schneider- Conservative</td>
<td>0.4</td>
</tr>
<tr>
<td>Schneider- Optimistic</td>
<td>0.3</td>
</tr>
<tr>
<td>Kim &amp; Edmonds</td>
<td>0.52</td>
</tr>
<tr>
<td>Red Book (2007): Conventional</td>
<td>0.803</td>
</tr>
<tr>
<td>Red Book (2007): Conventional + Unconventional + Seawater</td>
<td>0.278</td>
</tr>
<tr>
<td>FCCCG 1</td>
<td>0.43</td>
</tr>
<tr>
<td>MIT (1978) Ghahramani &amp; Driscoll</td>
<td>0.5, 0.63</td>
</tr>
<tr>
<td>MIT 2009</td>
<td>0.29*, 0.5**</td>
</tr>
</tbody>
</table>

* Estimates at a 50% confidence level
** Estimates at a 100% confidence level

Our results are at the lower end of other similar studies in the “optimistic” range, except when we observe the most pessimistic conditions in our model, where \( n=1, s=2, \) and \( \alpha=0. \)
2.5 Chapter Summary

Distance vision is blurred for all resource predictions because of the lack of commercial incentives to search for and prove out reserves for more than several decades into the future. Cohen predicts current uranium reserves will last for just six decades; this motivated the development of a geostatistical model, based on Deffeyes’ work, extending beyond this time horizon. An even greater contributor to uncertainty is the usual failure in resource modeling to credit learning (ingenuity) in process evolution. Shropshire et al have pointed out industrial metal prices have actually decreased (in constant dollars) over the course of the 20th century; Schneider and Sailor credit learning to the production cost reductions.

Although four metals presented in the introduction were used to suggest similar price reducing agents in uranium markets, the market history of other metals may not be a valid predictor for uranium for several reasons:

(a) Uranium is not fungible and has a very inelastic demand profile. Additionally, nuclear reactors have a long lead-time, long life, and are base-load units. Hence, once built, use rate is extremely predictable.
(b) Resource substitution and conservation are not presently relevant. Current LWR physics and fuel management practices are at near-optimum with respect to uranium utilization [36].
(c) There is no comparable market for inexpensive “scrap metal” recycle. Spent fuel reprocessing and recycle is expensive, currently employed in fewer than 10% of all reactors, and of limited efficacy. Reducing enrichment plant tails composition is equally effective and more easily implemented on a wide scale.
(d) Uranium has a unique mill tailings remediation problem due to radium (hence radon) content, which adds costs.
(e) About half of uranium resources are government owned, distorting free market (perfect competition) models useful in mapping price trends for other metals.

Diminishing constant dollar exponential curve fits for metal prices is a purely empirical construct. Resource economists have to date only provided qualitative theory-based guidance in this regard. Krautkraemer starts with Hotelling’s Rule (advanced in 1931) that under perfect competition, the profit, or 'rent', defined as the difference between the price and the production cost of a depletable resource, whose transportation and extraction costs are negligible, must rise at the rate of interest [37]. Krautkraemer goes on to show that significant extraction costs imply price paths that rise at less than the rate of interest and should instead follow a U-shape (i.e., fall, then rise). In the future, more work along the lines of Hubbert’s approach to oil production can be anticipated.

Understandably, past performance is no guarantee of future success and few have the temerity to extrapolate declining production cost trends into the future with assurance. However, an “optimistic” low-probability (15%) decreasing cost trajectory (for \( \theta = -0.1 \)) is shown in Figure 2.7 - the constant cost line \( (\theta = 0) \) corresponds to a 30% probability. One motivation is that over the past decade or so there has been increased deployment of a significant innovation in uranium mining as seen with in situ leaching (ISL). Not all deposits are suited to this approach however (some estimates are \(~20\%) , but significant operations are underway in the U.S. (five such operations in 2008) and Kazakhstan (20 ISL sites). Another promising development is the GE-Hitachi Silex laser isotopic separation process, which is set for performance tests in 2008-2009; advanced enrichment methods effectively lower enrichment costs, making lower grades of U-235 economically accessible.
CHAPTER 3: ENERGY AND ENVIRONMENTAL EXTERNALITIES ASSOCIATED WITH URANIUM PRODUCTION

3.1 Chapter Introduction

Strategic planning for sustainable provision of “clean” energy well into the distant future demands credible estimates of the energy and environmental externalities involved. In the debate concerning the energy and environmental advantage of nuclear over its baseload counterparts, coal, natural gas, and oil, a great deal of misleading information is prevalent regarding the environmental costs of supplying nuclear fuel to LWRs and subsequently the life-cycle emissions involved in generating electricity, the mainstay of current and anticipated future nuclear power systems. On one hand, nuclear proponents claim nuclear electricity generation does not produce criteria pollutants or greenhouse gas emissions responsible for widespread global climate change [38]; environmentalist reject this assertion and point to the carbon-intensive front and back-ends of the nuclear fuel cycle, where plant construction and decommissioning and uranium mining, milling, and enriching require great amounts of hydrocarbon-based energy. For example in a September 2001 issue of the Boston Globe, Dr. Helen Caldicott insisted, “the cleanliness of nuclear power is nonsense. Not only does it contaminate the planet with long-lived radioactive waste, it significantly contributes to global warming” [39]. In this chapter, a summary of current literature dealing with energy and environmental externalities is provided. An objective discussion of the construction, enrichment, mining and milling, and decommissioning (including waste transport) footprints from the once-through LWR lifecycle is developed to assess the environmental competitiveness of nuclear generation. Finally, as pointed out in the expressed concerns of Dr. Caldicott, the increased radionuclide exposure and radiation release to the environment will be evaluated, and associated health risks to the public will be assessed.

3.2 Nuclear Fuel Cycle Lifecycle Emission Methods

Evaluating the lifecycle emissions for nuclear generation, implying the potential abatement of CO₂ emissions compared to other energy systems, a standard approach to the analysis must be employed; such a necessity is evident from Benjamin Sovacool’s survey of 103 nuclear lifecycle emissions studies with estimates ranging from 1.4 g of carbon dioxide equivalent per kWh of generated electricity (gCO₂e/kWh) to nearly 300 gCO₂e/kWh (mean of 66 gCO₂e/kWh). Carbon dioxide equivalence is a metric used to compare the global warming potentials (GWP) of various greenhouse gases (CO₂, methane, etc.), accounting for their radiative forcing potential; radiative forcing is a measure of the rate at which the earth retains heat and energy (signaled by changes in radiation levels) between the troposphere and stratosphere from the emission of particular greenhouse gases including nitrous-oxide, perfluorocarbons, and methane. The composite CO₂e is thus a measure of the gas concentration within the earth’s atmosphere over a defined period of time, calculated as the integration of the product between the mass of the constituent gases and their respective GWP scores. Since gases remain in the

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4 Criteria pollutants are used by the EPA to suggest air quality, comprised of the six most prevalent air contaminants in the U.S.: carbon monoxide, lead, nitrogen dioxide, ozone, particulate matter, and sulfur dioxide.
atmosphere for varying periods of time (thus the CO$_2$e value is time-variant), GWPs are taken for three common time horizons - twenty, one hundred, and five hundred years; the most common, as assumed in this lifecycle study, is 100 years. The following are the equivalent GWPs for various gasses, as defined by the Kyoto Protocol over a span of 100 years [40]:

- CO$_2$ has a GWP of 1 for all time horizons
- CH$_4$ has a GWP of 21, meaning it is 21 times more hazardous in the atmosphere than CO$_2$
- N$_2$O has a GWP of 310
- Halocarbons (HFC) has a GWP between 140 to 11,700
- SF$_6$ has a GWP of 23,900

Sovacool observed three general approaches to lifecycle greenhouse gas emission estimation – economic input/output (EIO), process-based lifecycle analysis (LCA), and some combination of the two. EIO, developed in the 1930s by Wassily Leontief, involves linear modeling of an industry sector’s monetized consumption of various goods and services over its lifecycle. Presented in matrix form, EIO-LCA models enable the decomposition and delineation of industry growth scenarios into its sector-specific inputs, accounting for changes in demand within each defined sector; for example, increases in demand for nuclear generation can be linked to relative demand increases throughout the supply chain, from uranium exploration and mining, to enrichment and HLW disposal. Accounting for direct (first-tier), and indirect (second-tier and higher) influences of industry changes, the total effects of cooperative industrial transactions are integrated. Unfortunately, although EIO-LCA models provide comprehensive, high-level economic maps for an industry, these models are often extremely conservative regarding environmental impacts of power generation, which may lead to either overlooked or overestimated economic, environmental, and human health effects. The conservative nature of EIO models is seen in uranium mining; principal production is typically recognized, and thus the shared environmental impact from co-production or by-production is often overlooked in such analysis.

The process-based LCA approach involves the itemization of input and output of goods or services within a sector. The issue that often arises with process-based LCAs is the level of inclusiveness that is appropriate. For example, in evaluating the environmental impact of nuclear generation, it is disputable whether or not it is appropriate to include the environmental impact of particular material consumption, electricity and labor in the construction of drills and excavators used for the production of uranium. The assumed limits in scope of the model are critical, as the cyclical nature of extensive models may overly complicate a process-based evaluation. Although process-based models are very detailed, model development is time intensive, costly, and may be difficult to duplicate, often relying on proprietary data. As a result, EIO-LCA models have been employed as an alternative to process-based models, and are much simpler. Also commonplace is the cooperative use of both models.

To develop a robust LCA, one must specify the scope and assumptions made within the study. In this chapter a review of LCA models for the greenhouse gas emissions associated with the nuclear lifecycle will be limited to a once-through LWR fuel cycle as
presented in Figure 3.1. After reviewing Sovacool’s survey, five models were selected for closer examination. Although some models chosen contained more than enough data, model evaluation was limited to uranium mining, milling, conversion, and enrichment, fuel fabrication, and plant construction, operation, and decommissioning. Variations found in the studies were attributed to plant and mine location (local geology, energy portfolio, and valuation of goods and services), the type of LCA model used, uranium origin (deposit type), employed enrichment technology, and plant specifications (lifetime, rating, average discharge burn-up, enrichment level, and thermal efficiency). Furthermore, this study is limited to carbon dioxide equivalency measurements; specific criteria pollution contribution is outside the scope of this review.
Figure 3.1: LWR Once-through Fuel Cycle (outlined processes are used in the LCA comparative study) [41]
3.3 GHG Emissions over the Nuclear Power Generating Lifecycle

Mining and Milling

During mining, large amounts of electricity (equivalent emissions deduced from the local generation mix), and fossil fuels (liquefied petroleum gas, diesel fuel, coke, etc.) are consumed to haul and transport the ore and raw material, drill, blast, and excavate overburden and ore, and to indirectly facilitate mining operations. Once the uranium ore is mined from the deposit, it is ground, crushed and processed into slurry where the solution is then leached, washed, and filtered to chemically dissociate uranium from the host rock; as a result, a concentrated solution of U₃O₈, “yellow cake”, is produced, typically between 70 and 90 percent uranium oxide. The energy expenditure during mining closely correlates with ore grade; to optimize profits from mining activities, mining companies, “pick the eyes out of the mountain”, or exploit rich ores containing uranium grades between 0.1 and 10% U₃O₈ with minimal ore throughput and energy exertion. Many highly productive deposits contain ores as low as 500 ppm (such as Olympic Dam), with production hinged on whether or not the mineral is profitably extracted through principal production, co-production or by-production. The emission rates for mining and milling were assumed for conventional open-pit and underground mines; in situ leaching methods were not considered in any models though it is assumed to leave a lighter carbon footprint than traditional mining techniques.

A distinction between rich and low grade deposits is made in the literature to develop energy cost models; the ore qualifiers, hard and soft, proxy for production effort, resolved by geology and uranium concentration (shaping the ore tonnage necessary to obtain a unit of uranium product). In energy assessments noting such distinction, low energy costs are connected to soft ore, generally found in shale and sandstone between 0.01 and 2% U₃O₈; conversely, higher energy expenditures are associated with hard ore, characteristically found in granites and quartz-pebble conglomerates (some vein deposits as well) at concentrations typically 0.2% and less. Storm van Leeuwen introduces the concept of a mineralogical barrier to mining, where mining operations would become progressively less economical over a decreasing ore grade. His model predicted an exponential decline in mining effectiveness below an ore grade of 0.02% (Leeuwen assumed a “mineralogical barrier” of 100 ppm, at which the resource ore matrix changes). This model was based on data gathered on mining production yields and milling dilution (as a function of ore grade) from reports dating back to the late 1960s through 1980. Although van Leeuwen’s formulations were extremely cogent, his model fails to account for learning effects and effects of co-production and by-production (enabling exploitation of poor grades). Additionally, there is a need for current data resolving the present capability of mining technologies (such as in situ leaching) and processes, which achieve higher yields with reduced energy needs, as noted earlier.

Using our resource model, we can project the nuclear growth by which exploitation of 100-ppm resources will be expected. Manipulating the model slightly to account for cumulative generation as a function of the ore distribution:
\[
\frac{G^*}{G_0} = \left( \frac{x_o}{x^*} \right)^{\bar{s}}
\]

where

\( \bar{s} \) = the mean grade elasticity in the range of interest (for this example 100-2000 ppm)
\( G \) = cumulative energy generation
\( G_0 = 10^4 \) GWe-yr as of 2005 (worldwide), characteristic of 400 reactors powered for 25 years

Setting

\( x_o = 2000 \) ppm, the average strip mine ore grade
\( x^* = 100 \) ppm, the point at which the nuclear CO\(_2\) footprint surpasses coal

Then

\[
\frac{G^*}{G_0} = (20)^2 = 400
\]

Hence, \( G^* = 4 \times 10^6 \) GWe-yr. This value is representative of 4000 one GWe reactors powered for 1,000 years, or 400 one GWe reactors powered for 10,000 years! Clearly, from the model, nuclear will remain cleaner than coal. Completing the same calculation for the estimated CO\(_2\) breakeven point with natural gas (400 ppm), one obtains \( G^* = 2.5 \times 10^5 \) GWe-yr, representative of 2500 reactors for 100 years! For such demand scenarios, our model projects nearly ten times today’s generation demand for coal, and six times current generation for natural gas.

**Conversion, Enrichment and Fuel Fabrication**

During the conversion stage, U\(_3\)O\(_8\) is further purged of impurities, and converted to uranium hexafluoride, UF\(_6\), commonly used as feed material in enrichment facilities. The UF\(_6\) is then heated to form vapor and subsequently cooled and condensed, to loading cylinders in preparation for enrichment. During the enrichment phase, the U\(_{235}\) component of the UF\(_6\) is increased from 0.711% to between 3 and 5 weight percent. Two enrichment methods are most commonly used in this process; dominating the U.S. enrichment landscape, gaseous diffusion currently comprises approximately 40% of worldwide enrichment capacity. Developed during the Second World War, the process involves the channeling of “hex” (UF\(_6\)) through a cascade of porous membranes. The lighter U\(_{235}\) molecules are propelled to higher speeds than the heavier U\(_{238}\), enhancing the likeliness of U\(_{235}\) passing through the membrane (by increasing the rate of U\(_{235}\) collisions with the semi porous membrane wall), thus “enriching” the U\(_{235}\) quality of the UF\(_6\); as an approximation, the rate of effusion is found proportional to the inverse square of the molecular mass. What makes gaseous diffusion extremely energy intensive is the gas recompression, required as a result of pressure losses suffered through each stage. The second system conventionally employed is gas centrifuge, accountable for over 40% of the global enrichment capacity [42]. Also developed during the 1940s, this method
relies on centripetal forces (accelerations of nearly 100 times that of gravity) to separate the U238 molecules from the lighter 235 molecules.

Unanimously, enrichment costs (in energy) were the heaviest contributors at the front end of the fuel cycle; according to Hondo, enrichment contributed to nearly 90% of the front-end emissions from energy consumption. The majority of surveyed studies assumed some blend of diffusion and centrifuge for enrichment. Forward looking in the studies however forecasted preference for centrifuge (between 40 and 100 kWh/SWU [43]) over the more costly gaseous diffusion (2400-3000 kWh/SWU). In fact, Hondo and Fthenakis projected large emission reductions, in excess of 10 gCO₂e/kWh, by switching from diffusion to entirely gas centrifuge within the next decade. Additionally, expansion in plant capacity is more economically tolerable using gas centrifuge enrichment [44]. Other innovative methods are being developed (although not discussed in the surveys) including SIlex, Separation of Isotopes by Laser excitation originally developed in Australia. SIlex is anticipated to be over an order of magnitude more efficient than the conventional methods of enrichment, and is currently in a pre-deployment phase in a GE-Hitachi collaboration [45]. Additional methods include atomic vapor laser isotope separation, molecular laser isotope separation, aerodynamic processes, electromagnetic isotope separation, plasma separation and other chemical methods.

Fuel fabrication completes the front-end of the fuel cycle, where the enriched UF₆ is converted to UO₂, and pressed into various reactor fuel element geometries. Front-end emission rate estimates, including mining, milling, conversion, enrichment, and fuel fabrication ranged from 9.5 to 31.5 gCO₂e/kWh (a mean of 19.4 gCO₂e/kWh); the high-end of the estimates typically included land reclamation (sometimes accounted for during the decommissioning phase), transport, and exploration costs into the calculations. EIO-LCA studies accounting for soft and hard ore production, and a large share of diffusion in the enrichment mix revealed elevated emission rates. To account for large variations in similar fuel cycles, front-end emissions projections must account for domestic energy portfolios used to generate upstream electricity demanded by fuel cycle facilities. The literature found Switzerland and Sweden (Vattenfall) to operate the cleanest of nuclear fuel cycles, due to their nearly fossil-fuel-free domestic energy economies.

Reactor Construction and Nuclear Power Plant (NPP) Operation

The construction stage involves the creation of the reactor and power plant housing, as well as the transport, fabrication, and use of various construction materials, with large contributions from concrete, steel, copper, and other carbon-intense materials.⁵ During operation, energy is consumed in the cooling, maintenance, quality control and monitoring of the plant during performance periods as well as reactor inactivity due to emergency, repair, refueling, or shutdown. Emissions attributed to power plant construction ranged from 1.9 to 20 gCO₂e/kWh, with an average slightly over 8 gCO₂e/kWh according to Sovacool. Most studies assessing the emissions impacts of plant construction were EIO-based, integrating carbon-intensities associated with various

---

⁵ Calculation conducted in a study by White (2007) for a 1GWe reactor—170,000 tons of concrete, 32,000 tons of steel, 1,363 tons of copper, and 205,464 tons of other materials.
material and fossil fuels directly used during the construction of the nuclear power plant. Operations estimates for carbon dioxide emission ranged from 2.2 to 24.4 gCO₂e/kWh with an average value of 10.76 gCO₂e/kWh (among the five studies examined). Important factors in the determination of the emissions from construction and operation include operating lifetime, plant capacity factor (van Leeuwen found that a 3% increase in capacity factor resulted in a 28% drop in expected emission per kWh for construction and operation [46]), labor intensity (Chinese reactor studies show labor intensive construction techniques result in augmented emission rates), as well as the LCA model used, as EIO-LCA models give rise to higher projections.

The Fuel Cycle Backend and Decommissioning

Sovacool distinguishes plant decommissioning from the rest of the fuel cycle backend, although many LCA studies offer no such distinction; other studies combine construction and decommissioning impacts. The backend of the cycle incorporates spent fuel handling, interim storage and long-term waste management. Decommissioning involves the closing and dismantling of the reactor facility, decontamination of plant components and waste packaging; some studies include land reclamation and environmental restoration at this stage of the fuel cycle. Because of the lack of standardization in calculating lifecycle emissions, coupled with widespread inexperience with operating a waste repository (some nations have not defined policies for handling spent fuel), emissions projections for plant dismantling and other backend stages exhibit large variance. Additionally, the inclusiveness of such evaluations are open-ended, as Storm van Leeuwen argues inclusion of repository construction in the emissions estimates as depicted in Figure 3.1. Decommissioning emissions ranged from 0.01 to 44.3 gCO₂e/kWh (average of 9.3 gCO₂e/kWh) while other backend emission rates were estimated in the range of 0.8 to 28.13 gCO₂e/kWh (9 gCO₂e/kWh sample average). Figures 3.2 and 3.3 illustrate the total lifecycle emissions per kWh of generated electricity. Note that unlike dollar expenditures, far future emissions are not discounted.
Figure 3.2: Lifecycle emission estimates from various LCA studies

Figure 3.3: Range in individual LCA emission estimates
3.4 Waste Impact and Lifecycle Exposure Risk

Aside from the potential nuclear lifecycle emissions, perhaps even greater environmental concern is aimed at the environmental impacts and risks associated with the “invisible bullet”, radiation exposure from various waste streams throughout the nuclear fuel cycle. Historically, attention has been placed on the fuel cycle front end, where mining and milling tails exposed to the atmosphere have contaminated ground water and caused cancer and other impairments to nuclear industry employees and the public. During mining operations, radioactive dust and radon gas (produced by the decay of Ra226) in waste rock piles containing relatively harmless uranium traces and its radioactive progeny is released to the atmosphere. Dating back to early mining operations, occupational health hazards to uranium miners were exceptionally high (45% chance of contracting lung cancer over a 10 year employment period) largely due to poor mine ventilation and weak site remediation standards (Figure 3.4).

![Figure 3.4: Excess Relative Risk vs. Year of Risk, Jacobi Study [47]](image)

Improved operating standards significantly lowered occupational hazards; today, the global average excess risk associated with a 20-year employment ranges between 0.16 (1.56 mSv/yr exposure) and 0.44% (4.45 mSv/yr) for open-pit and underground mining operations, respectively. Background radiation, although varying with location, is commonly accepted as 2.4 mSv/yr. After undergoing the milling process, the waste stream in the form of slurry, referred to as mill tailings, is released to an open embankment called a tailings dam. These tailing piles, nearly equal to the original ore throughput, contain all of the radium and nearly all of the other highly radioactive nuclides (85% of original radioactivity) [48]. At the surface of the tailing pond, gamma and beta radiation is of immediate concern, but in limited proximity; however, toxic
arsenic and radon gas from Ra226 decay is of serious concern, as it is capable of being transported over large distances as radioactive dust in the waste pile.

To mitigate the spread of such harmful toxins to the environment, tailings dams are commonly covered with water (intermediately), which over the lifetime operation of a mine serves an effective shield to radon emission, also further reducing surface radioactivity. However, such arrangement is susceptible to water leakage, reducing shielding effectiveness over time, and does not provide an effective shield from potential physical contact. A more effective tailings cover has proven to be soil, particularly clay. In general, the stability of tailings dams is susceptible to failure from environmental interference (earthquake, erosion, heavy rain, etc.). Moreover, if abandoned without cover after mill shutdown, mill tailings would be responsible for 91% of the global collective dose, projected to cause approximately 23 deaths from cancer (long-term) per year of reactor operation, assuming a linear no-threshold dose effect (measured through the fuel cycle front end, reactor operation and spent fuel management) for a 1300 MWe reactor and a mine producing 1000 tU/yr; long term collective dose per GWe-yr is expected to reach near 470 mSv/GWe-yr, with almost 430 mSv/GWe-yr coming from mill tailings alone. For a covered dam (at an EPA standard of 0.74 Bq/m²s), a near nine-fold reduction in the global collective dose is observed. In this case, 72% of the dose would be from reactor operation, projecting 3 cancer-related deaths in the long-term, per year of reactor operation [47].

Figure 3.6 illustrates the normalized collective effective dose commitment in man sievert per GWe-yr. A lethal dose of radiation to an individual is approximately 5 sievert; a level of acute radiation poisoning, between 60 and 90% of sufferers will succumb to injuries within 30 days (chances are improved with immediate and intensive medical care).
Figure 3.5: Radiation exposures to nuclear workers and the public, expressed in terms of collective dose per GWe-yr

Figure 3.6 displays the results of a comparative study on the impacts of various electricity generating options on wildlife. As expected, the front-end of the nuclear fuel cycle presents the greatest risk of harm to wildlife; however, the study conducted by the New York State Energy Research and Development Authority concludes nuclear generation has the least impact on wildlife.
3.5 In-Situ Leaching Generic Environmental Impact

In-situ recovery, outlined in chapter 1, has gained significant attention in recent years, especially following Kazakhstan’s prompt ascension to the forefront of the uranium market, with production of their resource driven by in-situ leach mining. Tighter remediation standards and prospects for lower capital and operating costs for uranium mines have resulted in the increased ISL mining efforts. Although intended to reduce surface radionuclide exposure and disturbance to the environment, solution mining has been purported to introduce new risks to the environment, including groundwater contamination from solution excursions. The NRC has recently issued a report on the generic environmental impacts (GEIS) of ISL (in-situ leach) mining as a result of concerns surrounding ground water contamination, site reclamation, and other imminent risks to the environmental posed by the construction, operation, and decommissioning of ISL mines. In the GEIS, potential environmental impacts at each stage of the ISL lifecycle have been identified using NUREG-1748 significance levels [51]:

**Small Potential Impact** - The environmental effects are not detectable or are so minor that they will neither destabilize nor noticeably alter any important attribute of the resource considered.

**Moderate Potential Impact** - The environmental effects are sufficient to alter noticeably, but not destabilize, important attributes of the resource considered.

**Large Potential Impact** - The environmental effects are clearly noticeable and are sufficient to destabilize important attributes of the resource considered.
Table 3.1 summarizes the potential environmental impacts of ISL discussed in NUREG-1910.

Table 3.1: NRC GEIS: Outlook of ISL Lifecycle Potential Impacts to Environment [51]

<table>
<thead>
<tr>
<th>Category</th>
<th>Construction</th>
<th>Operation</th>
<th>Aquifer Restoration</th>
<th>Decommission</th>
</tr>
</thead>
<tbody>
<tr>
<td>Waste Management</td>
<td>Small Potential</td>
<td>Small Potential</td>
<td>Small Potential</td>
<td>Small Potential</td>
</tr>
<tr>
<td>Socioeconomic</td>
<td>Small-Moderate Potential</td>
<td>Small-Moderate Potential</td>
<td>Small-Moderate Potential</td>
<td>Small-Moderate Potential</td>
</tr>
<tr>
<td>Visual &amp; Scenic</td>
<td>Small Potential</td>
<td>Small Potential</td>
<td>Small Potential</td>
<td>Small Potential</td>
</tr>
<tr>
<td>Historical &amp; Cultural Resources</td>
<td>Small-Large Potential</td>
<td>Small Potential</td>
<td>Small Potential</td>
<td>Small-Large Potential</td>
</tr>
<tr>
<td>Noise</td>
<td>Small-Moderate Potential</td>
<td>Small-Moderate Potential</td>
<td>Small-Moderate Potential</td>
<td>Small-Moderate Potential</td>
</tr>
<tr>
<td>Air Quality</td>
<td>Small Potential</td>
<td>Small Potential</td>
<td>Small Potential</td>
<td>Small Potential</td>
</tr>
<tr>
<td>Aquatic Ecology</td>
<td>Small Potential</td>
<td>Small Potential</td>
<td>Small Potential</td>
<td>Small Potential</td>
</tr>
<tr>
<td>Threatened &amp; Endangered Species</td>
<td>Small-Large Potential</td>
<td>Small-Large Potential</td>
<td>Small Potential</td>
<td>Small-Large Potential</td>
</tr>
<tr>
<td>Groundwater</td>
<td>Small Potential</td>
<td>Small-Large Potential</td>
<td>Small-Moderate Potential</td>
<td>Small Potential</td>
</tr>
<tr>
<td>Surface Water</td>
<td>Small-Moderate Potential</td>
<td>Small-Moderate Potential</td>
<td>Small-Moderate Potential</td>
<td>Small-Moderate Potential</td>
</tr>
<tr>
<td>Geology &amp; Soils</td>
<td>Small Potential</td>
<td>Small Potential</td>
<td>Small Potential</td>
<td>Small Potential</td>
</tr>
<tr>
<td>Transportation</td>
<td>Small-Moderate Potential</td>
<td>Small-Moderate Potential</td>
<td>Small-Moderate Potential</td>
<td>Small-Moderate Potential</td>
</tr>
<tr>
<td>Land Use</td>
<td>Small-Large Potential</td>
<td>Small Potential</td>
<td>Small Potential</td>
<td>Small-Moderate Potential</td>
</tr>
</tbody>
</table>

The ISL lifecycle involves drilling of potential fields to gain access to aquifers containing exploitable ore. Currently, solution mining is only operable in semi-porous deposits like sandstone, confined above and below by impermeable strata. A leaching solution, containing oxidizing agents to dissolve and mobilize the uranium, is injected into injection wells; this solution is called a lixiviant. The lixiviant is composed of a groundwater base (taken from a local water table) mixed with hydrogen peroxide, bicarbonate, sulfuric acid, gaseous oxygen, and ammonia (a variety of solutions can be used, common solutions are alkaline and acidic solutions). Once the solution has become impregnated with uranium (containing the dissolved mineral), it is extracted and processed into yellowcake (Figure 3.7).
The nature of the lixiviant is subject to public and occupational health and safety concern, effecting soil toxicity, groundwater, and even air quality. Operational leaks and off-normal excursions of mobilized uranium and heavy metal slurries could potentially contaminate groundwater and associated soils. Causes of operational failures associated with lixiviant excursions are known to involve the following:

- Thinning or discontinuous confinement
- Improperly abandoned wells that may provide vertical flow pathways
- Casing failure or other well leaks
- Natural fluctuations in groundwater quality
- Improper balance of well field hydrologic gradients

Such spills have unfortunately been quite common; in the U.S., Smith-Ranch Highland in Wyoming, the largest NRC licensed ISL facility, reported more than 80 spills from 1988 to 2007; the magnitude of the accidents ranged from 50 to almost 200,000 (June, 2007) gallons of solution, and although generally concluded with quick cleanups (on the order of weeks to months), some messy excursions have taken upwards of 8 or more years. While Australia has experienced similar excursion events, the lack of an abundant clean water supply has made ISL mining accidents a critical environmental policy concern, especially for hydrologists. Typically, horizontal excursions are quickly cleaned, where lixiviant spills migrate laterally, remaining in the intended ore-zones. Vertical excursions
on the other hand, are characterized by lixiviant leakage into other aquifers, migrating outside of intended production zones. The largest recorded spill occurred in the Czech Republic, where upwards of 200 billion liters of groundwater were contaminated in a vertical excursion, expected to take decades, perhaps centuries to remediate. As a preventative measure, monitoring wells are designed to detect lixiviant excursions so that expedient cleanups mitigate environmental impacts of such accidents.

Other environmental impacts from ISL mining include fugitive dust, uranium particulate, and radon gas (and other radium progeny) emissions from well-fields (although a fraction of conventional mining exposure). Moreover, slow and incomplete site reclamation has heightened public concerns over the security of local clean water supplies. Further improvements to the mechanical integrity of well assemblies during operation are necessary before ISL gains both an economical and environmental advantage over conventional mining methods. Improved well maintenance and excursion avoidance as well as speedy and full site reclamation are critical steps to assuage environmental concerns at the front end of the ISL fuel cycle.

3.6 Chapter Summary

The environmental benefits of nuclear energy generation are significant. The U.S. fleet of nuclear power plants avoids 681 million tons of CO₂ emissions annually [53]. However, many nuclear critics have claimed that with growing uranium demand, construction of new nuclear plants will call for production of lower grade ore resources. Since production of poor uranium grades is increasingly more energy intensive, new plant construction would imply higher emissions for the nuclear fuel cycle. Van Leeuwen has predicted the energy inputs necessary to produce an equivalent uranium product based on ore grade; he concluded a tenfold increase in necessary ore throughput per tenfold decrease in ore grade. Leeuwen then argues, with such an increase in ore throughput, a tenfold increase in required mining and milling efforts (energy) results. Various concepts discussed in this chapter challenge Leeuwen’s theory, including learning (advancing new technologies), and co-/by-production, which curb mining energy needs. For example at Olympic Dam, grades of 0.06% are exploited economically through co-production and by-production of copper, gold, and silver. In response to Leeuwen, British Energy conducted a follow-up study that showed even with a very low uranium ore grade, CO₂ emissions would remain very small [31].

The economics of electricity generation are critical in comparing power conversion systems for economic competitiveness; however, because these systems function to provide electricity, a viable metric is to compute the energy balance between inputs and outputs. The task of providing an energy balance is extremely difficult because of the complexity of the inputs, whether direct or indirect inputs of energy to the system, and generally it is difficult to decide how inclusive the analyses should be (e.g.- should calculations include emissions from exploration, construction of transport vehicles and excavation tools, construction of conversion and enrichment plants as well as waste management facilities). Above energy costs, there are external costs to be considered, those environmental and health consequences of energy production, which do not appear
in the financial accounts. The energy and environmental externalities involved in the generation process include costs borne by the general public and are in the form of criteria pollutant and greenhouse gas emissions, as well as increased radiation exposure. The ExternE project is a leading study resolving the health and environmental costs of energy generation across industries (nuclear, coal, hydropower, etc), commonly unnoticed in policy making processes; part of the European Commission’s ‘Joule Programme’, the study assesses and monetizes the impacts of various fuel cycles within the European Union. Parallel analysis for the United States and North America will enable energy policy makers in promulgating appropriate legislation.

The following are key findings in this chapter:

* The amount of GHG emissions from the nuclear chain associated with LWRs is controlled by several parameters: the nuclear cycle considered, the average enrichment and burn-up at discharge of the fuel, the lifetime of the plant, the mined ore grade of the resource, and most importantly, the enrichment process used.
* Although a large portion of the enrichment mix, diffusion enrichment is expected to be phased out by 2017, whereas centrifuge methods now widely used are up to 50 times more energy efficient (less than 50 instead of 2400 kWh/SWU operationally) and are expected to make up 93% of the enrichment share by 2017.
* The future use of new reactor designs, including fast reactors, is dismissed on the grounds that some research programs in Europe have been closed down. However, Russia has been operating a 600 MW commercial fast reactor at Beloyarsk in the Urals for decades and on the basis of its operating success is now building a new larger version on the same site. The main reason there are not more fast reactors is that they are uneconomic in an era of low uranium prices. Should uranium ever look like becoming scarce, there is over 200 reactor-years of operating experience, including some in breeder reactor mode, on which to base a new generation of fast breeder reactors.
* Over the shorter term, no allowance is made for plant life extension of nuclear reactors, although this is now commonplace and extends operating life significantly, typically to 60 years.
* In uranium mining, although energy costs are now very well quantified, no consideration is given to relatively new technologies such as in-situ leaching which is more efficient than traditional mining methods in terms of both cost and energy use.
CHAPTER 4: URANIUM RESOURCE EXTERNALITIES: POLICY ISSUES AFFECTING URANIUM PRODUCTION

4.1 Chapter Introduction

Throughout the period covered by this study, the nuclear industry has experienced many watershed moments, as major nuclear policy decisions have been made. Anticipated technical advancements, new insights, and new political direction, critical questions related to uranium supply, non-proliferation, and waste management have been part of an increasingly open conversation between nuclear utilities, politicians, and the public. In the previous chapters, the focus of the discussion has been the supply, economics, environmental, and health and safety impacts of the once-through LWR fuel cycle. In this chapter, the public safety and security of the fuel cycle is further underscored in the discussion of public policy as it presents an additional ‘pressure point’ for the expansion of nuclear power generation. Key topics covered here are non-proliferation, waste management, and mining and milling safety policies; additionally, public support, correlated with political support, provides critical insights qualifying the relationship between policy and projected nuclear growth.

4.2 Non-Proliferation

The longest standing policy concern connected to nuclear generation is the threat of weapons proliferation, its inception dating back to the creation of the atom bomb during World War II. The modern-day “Prometheans”, as articulated by Joseph Angelo, nuclear physicists associated with the splitting of the atom in the Manhattan Project are responsible for the opening of “Pandora’s Box” and the global threat of a nuclear apocalypse upon our modern civilization [54]. The perceived threat of a nuclear war capable of destroying nearly a fifth of the world’s population is sobering enough to warrant immediate action and international cooperation to prevent such an event. In a 1982 article entitled “Twilight at Noon: The Atmosphere After a Nuclear War”, two Nobel laureates, John W. Birks and Paul J. Crutzen spelled out an additional catastrophe connected to nuclear fallout; in the article, the scientists exclaimed, “...it is likely that the agricultural production of the Northern Hemisphere would be almost entirely eliminated, so that those who might have survived the immediate effects of the war would have nothing to eat”. Table 4.1 compares the collateral damage (death toll) associated with various major catastrophes. Garwin in his book “Megawatts + Megatons” predicts one billion will perish from a nuclear war, more than twenty times the death toll of WWII.
Table 4.1: Impacts of Various Major Catastrophes [55]

<table>
<thead>
<tr>
<th>Cause</th>
<th>Location</th>
<th>Date</th>
<th>Number of Deaths</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black Plague pandemic</td>
<td>Europe</td>
<td>1347-51</td>
<td>25 million</td>
</tr>
<tr>
<td>Earthquake</td>
<td>Shaanxi, China</td>
<td>1556</td>
<td>830,000</td>
</tr>
<tr>
<td>Volcanic eruption</td>
<td>Mount Tambora, Indonesia</td>
<td>1815</td>
<td>160,000</td>
</tr>
<tr>
<td>Famine</td>
<td>Northern China</td>
<td>1876-79</td>
<td>10 million</td>
</tr>
<tr>
<td>World War I</td>
<td>World- Mainly Europe</td>
<td>1914-18</td>
<td>20 million</td>
</tr>
<tr>
<td>Accidental chemical explosion</td>
<td>Halifax Harbor, Canada</td>
<td>1917</td>
<td>1,654</td>
</tr>
<tr>
<td>Flood</td>
<td>Huang He Basin, China</td>
<td>1931</td>
<td>3.7 million</td>
</tr>
<tr>
<td>World War II</td>
<td>World</td>
<td>1939-45</td>
<td>40 million</td>
</tr>
<tr>
<td>Nuclear weapon explosion</td>
<td>Hiroshima, Japan</td>
<td>1945</td>
<td>200,000</td>
</tr>
<tr>
<td>Cyclone</td>
<td>Bangladesh</td>
<td>1970</td>
<td>300,000</td>
</tr>
<tr>
<td>AIDS</td>
<td>World</td>
<td>1980-</td>
<td>&gt;3 million/yr</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>seropositive*</td>
</tr>
<tr>
<td>Chemical discharge</td>
<td>Bhopal, India</td>
<td>1984</td>
<td>5,000</td>
</tr>
<tr>
<td>Nuclear power plant accident</td>
<td>Chernobyl, Soviet Union</td>
<td>1986</td>
<td>56-4,000**</td>
</tr>
<tr>
<td>Nuclear war</td>
<td>World</td>
<td>-</td>
<td>Estimated 1 billion</td>
</tr>
</tbody>
</table>

* Refers to the presence of particular antibodies associated with HIV
** Chernobyl resulted in 56 direct deaths, but the IAEA predicted 4,000 cancer-related deaths (in the Chernobyl Forum)

In response to the U.S.‘s campaign in Japan, measures were taken by the international community to acquire nuclear technology, for purposes of peaceful energy generation as well as weapons proliferation. In response, although the U.S. concluded the desire of nations to develop civilian nuclear programs was legitimate, the necessary reduction of incentives for nations to acquire nuclear arms was paramount, to prevent nuclear cataclysm. The first official set of legislation was “Atoms for Peace”, announced in 1953 by President Eisenhower. In Atoms for Peace, the cornerstone of U.S. nuclear policy, the U.S. agreed to trade nuclear technology and materials with nations under the condition that nuclear programs would be for peaceful generation with abandonment of nuclear armament. *Inter alia*, the U.S. wanted to control the spread of plutonium, as well as enrichment and reactor technologies capable of producing weapons grade materials. Unfortunately, perhaps the only notable success surrounding Atoms for Peace was the establishment of the IAEA (International Atomic Energy Agency) in 1957, a Vienna-based international body authorized to oversee and regulate nuclear technology and material trades, inspect nuclear facilities, and prevent nuclear material diversion. The failures of Atoms for Peace may be attributed to:

1) Perceived U.S. duplicity – nations were prohibited from building nuclear arms, but the U.S. did not agree to disarm or even cease their weapons development.
2) Perceived U.S. policy restriction – nations felt the U.S. held too much authority in controlling re-transfer operations of nuclear material and technology.

3) Lack of effective oversight/foresight
   a) IAEA officials would inspect only nuclear facilities declared official by the host nation. This left the door wide open for clandestine and covert efforts in weapons proliferation.
   b) PNE's, peaceful nuclear explosives were acceptable for domestic activities as mine blasting; again, this policy left the door open for weapons testing.

In 1970, a newer policy was drafted, the Treaty on the Nonproliferation of Nuclear Weapons (NPT). A more successful attempt to gain international cooperation, the NPT of 1970 sought to establish and maintain peaceful nuclear technology use internationally, gain a committed renunciation of nuclear weapon development among non-nuclear weapon states, eliminate existing nuclear weapon arsenals throughout the world (among the 5 official nuclear weapons states), prevent the trade of nuclear weapon technologies between states, and establish and strengthen IAEA safeguards and inspection mandates. Improvements from Atoms for Peace were observed by IAEA’s increased authority to inspect all facilities, as well as the expressed commitment from nuclear weapons powers to abandon weapon developments. The goals of the NPT of 1970 would be accomplished through increased secrecy and restrictive regulation of nuclear technologies and materials, the reduction of incentives for nuclear weapon acquisition, promotion of peaceful civilian nuclear programs, and the establishment of international cooperation against the spread of nuclear weapons. Continued success from the 1953 Atoms for Peace Treaty was achieved by the emphatic Russian- U.S. support of IAEA jurisdiction and NPT policy.

Gaining solidarity with 43 nations in agreement, special trade arrangements between experienced nuclear states France, Germany and the U.S. and nuclear “newbie” nations South Korea, Pakistan, and Brazil proved that the NPT was working. However, discovery of Indian missile tests in 1974 once again shook the NPT at its very foundations. What was becoming apparent was a constant and irreversible threat of proliferation - Pandora’s Box had been opened. Confidence in the NPT eroded as evidence of proliferation plans surfaced; Taiwan, South Korea, Argentina, Brazil, Israel, and Pakistan were recognized nations planning or already capable of deploying nuclear weapons. This matched with the realization that under the auspices of a civilian nuclear program, nuclear proliferation would be feasible, even if not covert. The U.S. moved fast to counter prospects of new nuclear weapons programs - nuclear related technology and material exports were cancelled⁶, intensive negotiations between the U.S. and weapon developing states (Brazil, Taiwan, South Korea, and Pakistan) for the abandonment of nuclear weapon initiatives began, and an initiative to modify research reactors globally was led by the U.S.

Moreover, although largely attributed to the Carter administration, the stance against reprocessing technologies began at the end of the Ford administration. Subsequent policies not only sought to discourage reprocessing, but also restrict development of new enrichment technologies and curb development of fast breeder reactors (using plutonium

⁶ Nuclear Suppliers Group subsequently formed to monitor transactions
fuel). The international community, however, was unmoved; foreign nuclear enthusiasts, in Japan and Europe, saw the policy move as a power play, convinced the U.S. was hoarding its nuclear technology, already with a robust energy portfolio, as compared to the dearth of fossil fuel resources associated with other states. Initially, the policy change was seen as a breach of the U.S.'s agreement under the NPT to provide peaceful nuclear technology, as opposed to ensuring its proscription. Global consent was gained only through a diligent campaign of credible expert explanation of the complex relationship between the nuclear fuel cycle and inherent security implications. Restrictions under the NPT were bolstered by conjunctive application with the Comprehensive Nuclear Test Ban Treaty, SPNFZT (South Pacific Nuclear Free Zone Treaty), and the Convention on the Physical Protection of Nuclear Material [56]. Coincidentally, qualms were further assuaged regarding the perceived capricious nature of U.S. policy when NRC studies such as GESMO (generic environmental statement on mixed oxide fuel) found that reprocessing, advanced enrichment, and fast breeder technologies were far from cost competitive with conventional once-through fuel cycles (nor were these technologies expected to be economical decades into the future). As a result of these efforts, the NPT has experienced international support, signed by 189 nations; progress is still needed as India, Israel, Pakistan and North Korea have yet to comply.

Although far removed from the Cold War, a reemergence of proliferation concern has materialized with the growth of the global nuclear community. September 11th has heightened concern over proliferation by rogue states within the U.S. and Britain; conversely, nations (such as North Korea) purport nuclear armament "preempts preemption" of the U.S. (nuclear weapons are still perceived as the sine qua non in the deterrence of aggression, or in international negotiations). Additionally, advanced enrichment technologies (SILEX), reprocessing and recycling (France and Japan) as well as fast breeder technologies are either in operation or being further developed on the world stage. The threat of proliferation from the implementation of these technologies will intensify unless major policy moves are made to ensure proliferation risks are at a minimum. Also notably, the collapse of the Soviet Union left Russia's stockpile of nuclear weapons and plutonium at increased risk of diversion from under-supervised and understaffed facilities. Mitigating this proliferation threat, the U.S. agreed to purchase 500 tons of HEU from Russia through the "Megawatts-to-megatons" initiative. Additional support for Russia's weapons control endeavors have been administered through the "cooperative threat reduction program", established by Senators Lugar, Nunn, and Domenici.

As a result of long-standing proliferation threats associated with nuclear generation, the growth of nuclear generation through global technology transfer has been minimized. In effect, supplies of fuel for the once-through LWR fuel cycle have been extended through use of secondary stockpiles accumulated in anticipation of accelerated nuclear power growth, and through the down blend of HEU from Russian (and American) warheads.

Two opposing views on plutonium usage have recently surfaced in political debate: plutonium recycling and deep geological waste management. The LWR plutonium recycle option was likely born out of the global conscience effort to reduce nuclear arms combined with the growing inventory of plutonium, delay of fast reactor deployment, and incredulity surrounding the notion of safe deep geological waste storage. Perhaps the
leading defense for recycling is that not only does recycling decrease the radioactivity of long-lived wastes, but it also increases the nuclear fuel resource base [57]. Although deep geological waste management has been an option for decades, the recent move by the U.S. government to abandon its Yucca Mountain project may suggest plutonium recycling and MOX fuel production is currently the politically favored option.

4.3 Waste Management

Resolving a strategy to close the nuclear fuel cycle has been a major challenge for the nuclear industry over the past few decades; long-term radioactive waste monitoring and storage is seen as an Achilles heel of sorts for the nuclear industry. In the U.S. a ‘not in my backyard’ sentiment has permeated the general public, as assertions of proposed health and environmental risks regarding a radioactive waste repository have often been tied to past nuclear plant accidents such as Chernobyl and Three Mile Island. This public opposition has spilled over into the political arena, where candidates have used anti-Yucca Mountain rhetoric to gain political momentum. Most recently, on March 5, 2009, Energy Secretary Steven Chu announced at a Senate hearing, “the Yucca Mountain site no longer was viewed as an option for storing reactor waste.” The direction of the Obama Administration materialized with the defunding of the Yucca Mountain Program, decided by overwhelming margins in House and Senate votes in July of this year; a quick turn of events, since the Yucca Mountain License Application was docketed just last year on September 8, 2008.

There are three contributing sources of waste - the irradiation of nuclear fuel in civilian nuclear fuel cycles, fuel cycle operations (from uranium mining and milling to fuel fabrication), and defense activities (such as naval vessel power generation, and, historically weapons production). Discussion of sensitive waste handling generally refers to the waste stream from civilian nuclear power generation, classified as high-level waste (HLW). The spent nuclear fuel (SNF) is composed of long-lived, highly radioactive fission products, minor actinides such as Neptunium and Americium, transuranics (TRU), and Plutonium, among other constituents. Typically, three waste management strategies are employed to handle spent fuel – biosphere isolation, dilution, and various destruction or treatment techniques such as vitrification; primarily, waste isolation has been proposed for long-term waste management. The Yucca Mountain Repository, the USDOE’s deep geological waste storage facility, is a prime example of such long-term waste storage solution. Originally to be a 77,000-ton waste capacity facility located ninety miles from Las Vegas, Nevada, the Yucca Mountain facility has been outfitted for various disposal options, technically capable of recoverable and non-recoverable emplacement, as well as enlarged capacity (115,000 ton). Identified by the 1987 Nuclear Waste Policy Act (NWPA) as a viable prospect for geological waste storage, the Yucca Mountain Project was first approved for further evaluation during the Bush Administration (2002).

Although considered by many as the likely long-term nuclear waste storage solution, Obama Administration assertions in February of 2009 foreshadowed the demise of the Yucca Mountain Project. Under the direction of the Obama Administration, DOE officials announced that, "the Yucca Mountain program will be scaled back to those costs necessary to answer inquiries from the Nuclear Regulatory Commission (NRC), while the
administration devises a new strategy toward nuclear waste disposal." Just five months later in July, the House of Representatives voted the Yucca Mountain Project out of existence, 388 to 30; the Senate placed the nail in the coffin when it passed a $34.3-billion energy-spending bill, designed to remove funding for the Yucca Mountain Project, by an 85-9 vote. President Obama will have an opportunity to close on his pre-inaugural promise to close the facility once the measures are reconciled in Congress.

With the Yucca Mountain option abandoned, a plan to resolve the once-through (or recycle) fuel cycle has become increasingly elusive. The lack of strategy (besides the prolongation of interim waste storage at the U.S.’s 104 power plants) has severe national security and proliferation implications as outlined in the previous section. Since no other nations have successfully operated a deep geological long-term HLW storage facility, the feasibility of geological waste storage is not yet convincingly demonstrated. However, the U.S. has successfully operated a defense-related waste facility, the Waste Isolation Pilot Plant (WIPP), thus the challenge becomes focused on the feasibility of deep geological storage for civilian nuclear waste. Table 4.2 outlines the status of international repository programs. Although France, Finland, and Sweden have identified viable storage sites, none of them are designated HLW repositories.

Table 4.2: Organization of National Repository Programs [58]

<table>
<thead>
<tr>
<th>Country</th>
<th>Manage Funds</th>
<th>Implement Disposal</th>
<th>Transfer of Liability</th>
<th>Siting Success</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>State</td>
<td>State</td>
<td>SNF leaving reactor</td>
<td>No</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>State</td>
<td>Special State Authority</td>
<td>After fee payment</td>
<td>No</td>
</tr>
<tr>
<td>France</td>
<td>Utility</td>
<td>Special State Authority</td>
<td>Repository acceptance</td>
<td>Yes</td>
</tr>
<tr>
<td>Japan</td>
<td>State</td>
<td>Utility</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Finland</td>
<td>Utility and State</td>
<td>Utility</td>
<td>Utility until repository validation</td>
<td>Yes</td>
</tr>
<tr>
<td>Sweden</td>
<td>State</td>
<td>Utility</td>
<td>-</td>
<td>Yes</td>
</tr>
<tr>
<td>Germany</td>
<td>Operator</td>
<td>Utility and State</td>
<td>After repository closure decision</td>
<td>-</td>
</tr>
<tr>
<td>Switzerland</td>
<td>State</td>
<td>Utility and State</td>
<td>Utility for 50 years after acceptance</td>
<td>-</td>
</tr>
<tr>
<td>Belgium</td>
<td>Operator</td>
<td>State</td>
<td></td>
<td>-</td>
</tr>
</tbody>
</table>

Undoubtedly, nuclear power growth prospects are related to the waste storage issue. As nuclear power is ramped up, the back-end waste accumulation issue will progressively become more urgent. In fact, even with recycling incorporated into the nuclear fuel cycle, long-term [geological] storage of the terminal waste stream will be necessary. At some point, the technical requirements must be addressed by nuclear policy. Moreover, until the backend waste issue is resolved, nuclear growth will be confined by fuel cycle “loose ends”. Looking back to the proliferation question, two solutions have been proposed to effectively reduce the proliferation risk posed by available quantities of weapons grade Plutonium – deep geological waste storage, and plutonium recycling. Because Yucca Mountain, and thus deep geological storage options in the U.S. have been indefinitely postponed to the future, the remaining [default] proliferation-minimizing action is
plutonium recycle, effectively burning the fissile isotope in mixed-oxide fuel (MOX). This requires spent fuel reprocessing, which unfortunately heightens the risk of proliferation; however, a full-scale example is available in France, as both the French and Japanese utilities recycle fuel to cut down on the overall waste volume and radio-toxicity.

A 2009 MIT fuel cycle study suggests a three-tier solution to resolve waste management uncertainties [59]:

1) “Development of a risk-based “Integrated Waste Management Strategy” should be adopted with (a) a waste classification system based on the radionuclide and physical characteristics of each waste stream and (b) corresponding disposal facilities for the disposal of all wastes from each category of wastes.

2) Creation of an independent authority for the management of all long-lived commercial radioactive wastes and defense HLW SNF. This includes long-term storage of SNL and HLW.

3) Incorporation into fuel cycle studies, waste management must become an integral part of the development of any fuel cycle, including an open fuel cycle. The impact of waste management must be included in cost and risk evaluations of alternative fuel cycles.”

4.4 Mining/Milling Safety

Whereas the back end of the fuel cycle has involved sensitive handling of material to ensure public health and safety, the front-end of the nuclear fuel cycle is of similar concern. Increased mining and milling site remediation standards have been enacted to reduce the environmental and public health impacts from mining and milling operations, known to emit large amounts of gamma radiation and radon fumes; radionuclides in milling tailings contain anywhere from 20 to 100 times as much gamma-radiation as typical background levels at the surface of undisturbed deposits [60]. Regulation of uranium mining and milling operations were introduced to correct three deplorable mining practices - abandonment of mine and mill facilities after the cessation of mining operations, abandonment of exhausted deposits without site reclamation or cleanup, and the dumping of milling tails into lakes and streams. The acknowledged threat to the public and environment led to the promulgation of an EPA regulation (EPA: 40 CFR 192) and NRC’s (NRC: 10 CFR 40) 1978 Uranium Mill Tailings Radiation Controls Act (UMTRCA). In this legislation, maximum soil concentrations were set at 5 and 15 pCi/g of Radium 226, for surface and subsurface soils, respectively [61]; the stated reclamation standards must be satisfied for between 200 and 1000 years, depending on the facility.

The UMTRCA is broken into two parts, Title I and Title II. In Title I, under NRC licensing (and DOE rules for cleanup and site remediation), special remediation guidelines are provided for reclamation of abandoned milling sites (formerly used for military purposes); UMTRCA Title II details EPA authority in setting radiological and non-radiological hazard standards, NRC authority to control those hazards, and the eventual ownership of the disposal sites. Additional reclamation requirements for the

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7 The determination of whether SNF is a liability or an energy resource has not been decided at present; thus, recoverable SNF management may be necessary to ensure waste management decisions are optimal
decontamination of neighboring residences are included in the legislation. This applies to the U.S. Neighboring Canada does not enforce similar reclamation requirements; their Atomic Energy Control Board (AECB) has only promulgated lax guidelines resulting in a lower level of protection for the public and the environment.

Controversy surrounding mining and milling risks to the public has effectively reduced enthusiasm for nuclear expansion. In North America, the health and safety at the fuel cycle front-end has been at odds with public standards, especially for those populations residing in close proximity to mines and mills. In a Boston Globe article in March of 2008, a Navajo Nation ban on uranium mining and milling on its land was detailed. The Ambrosia Lake, New Mexico mining region, responsible for the production of nearly 400 million pounds of uranium over a thirty-year span was also the site responsible for high rates of birth defects, lung cancer, and kidney disease due to tailings mismanagement. Uranium Resource Inc. CEO Richard Van Horn noted the ban would effectively limit the company’s production, although not sufficient to stop mining in the region; nonetheless, resistance against milling and mining operations directly led to reductions in uranium product.

The Sundance Film Festival award-winning documentary The Return of the Navajo Boy, by Jeff Spitz depicts the imminent risks to the public during mining and milling operations. A revisit from an earlier documentary in the 1950s entitle The Navajo Boy, the film released in 2000 follows a family reunion, underscoring the prevalent threat of Uranium mining in the area, responsible for the lung cancer which claimed the life of subject Elsie Mae Cly’s mother. Many other documented cases exist, such as in the documentary Uranium, by Magnus Isacson, berating current mining and milling practices leading to the demise of uninformed and underprivileged residents. Ostensibly, improving public opinion about mining, milling, and other geological operations may also improve public perspective on long-term geological waste storage.

4.5 Public Support

In this section, the time history of public support for nuclear generation will be briefly summarized; these findings may enable projections of support in the future, thereby also strengthening projections of nuclear growth. All of the topics mentioned in this chapter, proliferation, waste management, and mining and milling, are major weaknesses in the current fuel cycle; significant improvements to policy and operations are needed to gain support in the endeavor to expand peaceful nuclear power generation. Unquestionably, public attitudes towards nuclear power, influenced by the aforementioned issues, have fashioned local and national nuclear policies. From the inception of nuclear technology encapsulated in the shell of a warhead, the public stance of cynicism and pessimism towards the nuclear industry has proliferated through each stage of nuclear development. A significant reduction in public support followed the Three Mile Island and the Chernobyl accidents in the 1970s and early 1980s. Following these accidents, nuclear growth has failed to bounce back to growth rates observed in the 60s and early 70s.

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8 Findings from the MIT 2009, Future of the Nuclear Fuel Cycle Study were summarized in this section
Figure 4.1 displays the historical trend in public opposition and support of the construction of additional nuclear power plants. The culmination of a variety of polls and studies conducted from the mid 1970s through 2002, nuclear support was expressed in response to the question “do you support building nuclear power plants?”9 From the figure, inflection points for both curves are found over the period of TMI and Chernobyl; this inflection marks a significant loss of momentum for the nuclear industry, with disapproval peaking in the mid to late 80s (the likely response to Chernobyl). From 1990 to 2000, few surveys were either administered or available concerning the public’s position on nuclear energy growth, as public attention to the industry was minimal. However, at the dawn of the new millennium, discussions of potential nuclear growth rapidly reemerged. Following the terrorist attacks of September 11th, rising international proliferation concerns paired with a growing consciousness of global climate change led to an even divide in the public attitude towards industry growth. Survey results during this period reflect the public’s ambivalence; however, more stratified survey results were necessary.

Figure 4.1: Public Attitudes Toward Building New Nuclear Plants [59]

9The results of earlier polls and trends are reviewed by Eugene A. Rosa and Riley E. Dunlap, "Nuclear Power: Three Decades of Public Opinion Trends," Public Opinion Quarterly 58 (1994): 295. Figure represents findings of Cambridge Energy Research Associates (CERA) and Gallup. NEI’s surveys ask somewhat different questions, especially whether the respondent supports nuclear power, period, or expansion of capacity at existing facilities.
From 2002 to 2007, the survey question was altered – now, instead of only having a yes/no choice, individuals surveyed could now express interest in nuclear growth, reduction, stagnation, or elimination.10

Do you think the US should reduce or increase its use of nuclear power?

<table>
<thead>
<tr>
<th></th>
<th>2002</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increase</td>
<td>28%</td>
<td>34%</td>
</tr>
<tr>
<td>Not Change</td>
<td>25%</td>
<td>25%</td>
</tr>
<tr>
<td>Reduce</td>
<td>38%</td>
<td>29%</td>
</tr>
<tr>
<td>Not Use</td>
<td>9%</td>
<td>12%</td>
</tr>
</tbody>
</table>

The six point swing in opinions from 2002 to 2007 showed an increasing support of nuclear in the recent past; this change can be attributed largely to the growing sensitivity to carbon footprinting coupled with a strong government backing for nuclear, expressed through various inducements to spark new nuclear construction. However, despite showing increased support for nuclear, the results of the study still show nuclear critics outnumber nuclear supporters (“increase” compared to “reduce” + “not use”). The most recent study conducted in the 2009 Energy Survey show an improved approval margin; 61% of the respondents were enthusiastic about the prospects of nuclear growth. The jump in support has been attributed to the coverage given to the topic throughout the presidential campaigns and the election. Another reasonable idea considers the increased understanding of TMI and Chernobyl accidents, greater global acceptance of NPT and non-proliferation stances, as well as communicated advances in the technology and its societal benefits.

4.6 Chapter Summary

The theme of this chapter concerned waste as the critical lynch pin to nuclear growth. Not only is proliferation-risk directly influenced by waste management strategies, mining/milling environmental impacts are additionally associated with tailings (perceived waste) management strategies. Skip Bowman (immediate past head of NEI), in his address to the Commonwealth club offered a different insight on a waste repository; although supportive of all waste management options including plutonium recycling, Bowman emphasizes the necessity to view deep geological repository as storing “monitored and retrievable” waste. However, a large majority of Americans remain skeptical about waste storage, and that skepticism, as well as concerns about cost and local environmental impacts, dampens support for expanding the use of nuclear power in the United States.

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10 A second part of the question was appended to allow respondents to give perspective on other major power sources
CHAPTER 5: BREAKEVEN ECONOMICS: NATURAL URANIUM WITH LWR RECYCLE

5.1 Chapter Introduction

Recent abandonment of the Yucca Mountain Project, pointed out in the previous chapter, has led to growing debate regarding the cost competitiveness and feasibility of incorporating plutonium recycling and fuel reprocessing into the LWR fuel cycle. On one hand, advocates insist reprocessing contributes to the sensible stewardship of uranium resources and the fuel cycle by stretching the fixed resource base, lowering the radio-toxicity and heat-load of the terminal waste stream, and effectively progressing towards the closing of the fuel cycle; in fact, France, Japan, and India currently rely on fuel reprocessing to bring closure to their fuel cycles. Opponents point out the high costs of reprocessing as well as the heightened proliferation risk. Additionally, in the U.S., the combined low long term- market price for uranium and [low] waste disposal charge to utilities (1 mill/kWh) currently keep the cost competitiveness of reprocessing and recycling options distant from the once-through cycle. In this chapter, various breakeven scenarios are outlined to quantify breakeven points for mixed oxide fuels (MOX) with natural uranium. Additionally, breakeven points for nuclear with coal-powered generation options are briefly presented.

5.2 Terrestrial Ore vs. Reprocessing

By balancing the undiscounted cost of natural uranium (fresh reload fuel) with MOX reload fuel, one can make a rough estimate of the cost threshold at which fresh reload fuel is no longer economically purchased for once-through LWR fuel cycles. The breakeven cost for natural uranium vs. MOX reload fuel is represented by the following in $/kg:

$$C_{ab} = \frac{C_R R + (C_{RF} - C_F) - C_S (S/P) - (C_D - C_{DR})}{(F/P)}$$

(5.1)

where

- $C_R$ = cost of reprocessing, $/kg
- $R$ = kg reprocessed to obtain enough TRU for 1kg of equivalent reload fuel. (Note that 100/R is the percent ore savings.)
- $C_{RF}, C_F$ = fabrication cost of MOX and fresh fuel, respectively, $/kg.$
- $C_S$ = cost of separative work, $/kg$
- $C_D, C_{DR}$ = cost of HLW disposal, intact spent fuel and reprocessed waste form, respectively, $/kg$
- $F/P$ = natural uranium required per kg of fresh reload fuel
- $S/P$ = separative work required per kg of fresh reload fuel

Conversely, the breakeven cost for reprocessing, given a natural uranium price is represented by:
\[ C_{RB} = \frac{C_U \left( \frac{F}{P} + 1 \right) - (C_{RF} - C_p) + C_p (S/P) + (C_D - C_{DR})}{R} \]  
\hspace{1cm} (5.2)

where

\[ C_U = \text{cost of natural uranium, } \$/\text{kgU} \]

Furthermore, one may obtain the natural uranium per kg of fresh reload fuel cost by completing a mass balance over the enrichment facility as follows:

\[ \left( \frac{F}{P} \right) = \frac{x_p - x_w}{x_F - x_w} \]  
\hspace{1cm} (5.3)

where \( x_p, x_w, \) and \( x_F \) are the enrichment of reload fuel, separation plant tails and natural uranium (0.711 w/o) weight fractions of U-235, respectively. In fuel cycle economics, the separative work ratio, measured in separative work units (SWU) is calculated as a proxy for enrichment effort, directly related to the overall cost of the enrichment operation. The following relation calculates the separative work:

\[ S/P = (2x_p - 1) \ln \left( \frac{x_p}{1-x_p} \right) + \left( \frac{x_F - x_p}{x_w - x_F} \right) \left( 2x_w - 1 \right) \ln \left( \frac{x_w}{1-x_w} \right) - \left( \frac{x_F - x_w}{x_F - x_w} \right) \left( 2x_F - 1 \right) \ln \left( \frac{x_F}{1-x_F} \right) \]  
\hspace{1cm} (5.4)

Additionally, the waste fee disposal fee charged by the government to nuclear utilities is given by:

\[ C_D = 24\eta B_df \]  
\hspace{1cm} (5.5)

in which

\[ \eta = \text{reactor thermodynamic efficiency (~0.32 for a PWR)} \]
\[ B_d = \text{fuel discharge burnup, MWd/kg} \]
\[ f = \text{waste disposal fee, mills/kWh (currently 1.0 in the U.S.)} \]

In calculating the breakeven point, the following values are used, which are representative values for PWRs

\[ C_r = 1000 \ \$/\text{kg} \]
\[ R = 6 \]
\[ C_{RF} = 800 \ \$/\text{kg} \]
\[ C_F = 250 \ \$/\text{kg} \]
\[ C_S = 80 \ \$/\text{kg} \]
\[ C_D = 384 \ \$/\text{kg} \] (where \( \eta=0.32, B_d=50 \ \text{MWd/kg}, f=1 \ \text{mill/kWh} \))
\[ C_{DR} = 100 \ \$/\text{kg} \]
\[ x_w = 0.25\% \]
Assuming a U235 weight percent (w/o) of 0.711 in natural uranium, x_F, and 4.8% enrichment, x_p, one finds the undiscounted breakeven cost of uranium:

\[ C_{\text{uBe}} = 579 \, \$/\text{kg} \]

Hence, so long as natural uranium costs less than this value, reprocessing provides no cost advantage. Moreover, according to our model in chapter 2, with aggressive growth (at an 85% confidence level) recall the following relation:

\[ \left( \frac{C}{C_r} \right)^{\theta} = \left( \frac{G}{G_r} \right) \]

At a price of 579 \$/kgU and a confidence level of 85% (\( \theta = 0.29 \)), the expected cumulative global generation is 4.3x10^6 GWe-yr. This is representative of 4300 one GWe reactors powered for 1000 years, or 430 one GWe reactors powered for 10,000 years: i.e. a virtually unlimited amount. It is also reasonable to speculate that a fuel substitute such as uranium from seawater would enter the market long before this price threshold. Furthermore, at these time scales, advancements in reactor technology are expected to be online. Note, however that the breakeven calculation takes no credit for recycle of the uranium in spent fuel. If reprocessed uranium is roughly equivalent to natural uranium, then one merely divides by (F/P+1) in place of (F/P), in which case:

\[ C_{\text{uBe}} = 526 \, \$/\text{kg} \, U_{\text{nat}} \]

Which is still a very large value. For a cost of natural uranium at 100 \$/kg \, U_{\text{nat}}, the cost by which reprocessing substitutes for natural uranium is:

\[ C_{\text{RBe}} = 229 \, \$/\text{kg} \]

Essentially linear tradeoff graphs suggested by expressions of the above type are commonly seen in the literature. Figure 5.1 summarizes the breakeven cost relationship as a function of the reprocessing cost.
The calculations completed in this section are ballpark approximations, based on representative industry parameters. The largest uncertainty in such analyses is the projected cost of reprocessing; D. Shropshire has observed that a wide variation in facility costs can result from the nature of the ownership (government vs. private), with expected reprocessing costs ranging from 500 to 2600 $/kgHM (Figure 5.2).
A wide variation in facility costs can result from the type of facility ownership

![A wide variation in facility costs can result from the type of facility ownership](image)

Figure 5.2: Ore vs. MOX Recycle in LWRs [62]

5.3 Ore vs. MOX Recycle in LWRs

Based only on undiscounted direct costs, one can obtain a rough estimate of UOX vs. MOX fuel costs. Itemized cost estimates are from D. Shropshire et al’s 2008 INL, advanced fuel cycle cost study [9].

### Table 5.1: Uranium Oxide Fuel Cost Estimate [6] ($ USD), August 31, 2009

<table>
<thead>
<tr>
<th>Stage</th>
<th>Mass (kg)</th>
<th>Price ($/kg), ($/SWU)</th>
<th>Total ($ USD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uranium purchase*</td>
<td>9.2</td>
<td>46</td>
<td>423</td>
</tr>
<tr>
<td>Conversion to UF₆</td>
<td>9.2</td>
<td>6.25</td>
<td>58</td>
</tr>
<tr>
<td>Enrichment</td>
<td>6.9</td>
<td>160</td>
<td>1,104</td>
</tr>
<tr>
<td>Fabrication</td>
<td>1</td>
<td>240</td>
<td>240</td>
</tr>
<tr>
<td>Waste Fee**</td>
<td>1</td>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td><strong>2,225</strong></td>
</tr>
</tbody>
</table>

* For 4.5 w/o enriched fuel and 0.25 w/o enrichment plant tails, one has F/P= 9.22, S/P= 6.87
**Based on 1 mill/kWh, C=24Bₙθ, where η=(1/3), Bₙ discharge burnup (50 MWd/kg)
Based on reprocessing six assemblies to obtain sufficient Pu for one MOX assembly (minor actinides are not recycled in this case). A fundamental assumption here is that spent UOX is provided free of cost from the government.

**The reduction of six fuel assemblies produce two for waste disposal; therefore, (6-4)+1(MOX)=3**

Thus, based on these estimates (albeit highly uncertain, as pointed out in the previous section), MOX fuel will cost nearly 2.25 times that of UOX fuel. The cost for MOX is expected, under these circumstances, to break even with natural uranium when the cost of fresh natural uranium rises to 386 $/kg.

If instead no waste credit were offered to utilities:

<table>
<thead>
<tr>
<th>Stage</th>
<th>Mass (kg)</th>
<th>Price ($/kg)</th>
<th>Total ($ USD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reprocess UOX Spent Fuel*</td>
<td>6</td>
<td>502</td>
<td>3,012</td>
</tr>
<tr>
<td>Fabrication</td>
<td>1</td>
<td>3,200</td>
<td>3,200</td>
</tr>
<tr>
<td>Net Credit for Waste**</td>
<td>-3</td>
<td>400</td>
<td>-1,200</td>
</tr>
<tr>
<td>Total:</td>
<td></td>
<td></td>
<td>5,012</td>
</tr>
</tbody>
</table>

*Based on reprocessing six assemblies to obtain sufficient Pu for one MOX assembly (minor actinides are not recycled in this case). A fundamental assumption here is that spent UOX is provided free of cost from the government.

**The reduction of six fuel assemblies produce two for waste disposal; therefore, (6-4)+1(MOX)=3**

With these conditions and imposing no waste fee, utilities can tolerate natural uranium prices up to 802 $/kg before reprocessing becomes economical. Another market scenario to consider is where the UOX operator is charged 1,000 $/kg as an added waste conditioning fee; and additionally, gives the plutonium (TRU) free of charge to the MOX operator.

<table>
<thead>
<tr>
<th>Table 5.3: UOX vs. MOX Recycle into LWR ($ USD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UOX</td>
</tr>
<tr>
<td>U_{nat} (9.2 kg/kg fuel)</td>
</tr>
<tr>
<td>423</td>
</tr>
<tr>
<td>SWU (@160 $/kg + conversion)</td>
</tr>
<tr>
<td>1,162</td>
</tr>
<tr>
<td>Fabrication</td>
</tr>
<tr>
<td>240</td>
</tr>
<tr>
<td>Total: 1,825</td>
</tr>
</tbody>
</table>
Table 5.4: UOX vs. MOX Recycle into LWR with Waste Fee ($ USD)

<table>
<thead>
<tr>
<th></th>
<th>UOX</th>
<th>MOX</th>
</tr>
</thead>
<tbody>
<tr>
<td>U_{nat} (9.2 kg/kg fuel)</td>
<td>423</td>
<td>Reprocess (6 kg @ 1000 $/kg)</td>
</tr>
<tr>
<td>SWU (@160 $/kg + conversion)</td>
<td>1,162</td>
<td>Fabrication</td>
</tr>
<tr>
<td>Fabrication</td>
<td>240</td>
<td>Waste Credit</td>
</tr>
<tr>
<td>Waste Conditioning Fee</td>
<td>1,600</td>
<td>-</td>
</tr>
<tr>
<td>Total:</td>
<td>3,425</td>
<td>Total: 4,400</td>
</tr>
</tbody>
</table>

Thus, the cost of the MOX recycle is only 30% higher than that of the UOX.

5.4 Uranium Breakeven Price as Determined by Dispatch Order

Plants are put on line for base load operation based on operating cost (approximately O&M + fuel). Hence there is a limit on the U_{nat} price beyond which coal is preferred. Using results from MIT’s Future of Nuclear Power (2003) and Coal (2007) studies among others, the following are operating costs (mills/kWh) for nuclear, coal and coal with CO₂ capture:

Table 5.5: Operating Costs Comparison: Nuclear vs. Coal vs. Coal+CO₂ Capture (mills/kWh)

<table>
<thead>
<tr>
<th></th>
<th>Nuclear</th>
<th>Coal</th>
<th>Coal+CO₂ Capture</th>
</tr>
</thead>
<tbody>
<tr>
<td>O&amp;M</td>
<td>8</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>Fuel*</td>
<td>6</td>
<td>13</td>
<td>18</td>
</tr>
<tr>
<td>Total:</td>
<td>14</td>
<td>21</td>
<td>34</td>
</tr>
<tr>
<td>Difference:</td>
<td>-</td>
<td>7</td>
<td>20</td>
</tr>
</tbody>
</table>

* U_{nat} at 50 $/kg per 2 mills/kWh

Thus, the breakeven cost of natural uranium, C_u ($/kg) is:
- Nuclear vs. Coal = 225 $/kg
- Nuclear vs. CO₂ Capture = 550 $/kg

Thus, nuclear will not lose its dispatch order ranking to coal until natural uranium becomes very expensive. Further, if a carbon tax of 25 $/tCO₂e were imposed on coal plants, which emit roughly 0.85 kg CO₂e/kWh (making a crude assumption of no emissions for nuclear), one finds that 2.13 $/kWh more can be spent on U_{nat}. Also assuming that at 100 $/kg U_{nat} has a busbar contribution of 0.25 $/kWh, nuclear becomes even more affordable, breaking even with coal at a price of 940 $/kg U_{nat}. 
5.6 Chapter Summary

At present, reprocessing and MOX recycling do not appear to be economical alternatives to closing the fuel cycle unless substantial waste fees are imposed to serve as a credit for the recycle. Compared to coal and coal with carbon capture, the LWR fuel cycle can sustain significant increases in $U_{nat}$ prices before its operating costs become level with coal generation. These rough calculations suggest uranium prices will not become intolerable in the near future.
CHAPTER 6: URANIUM CONSERVATION

6.1 Chapter Introduction

The long-term prospects of supplying natural uranium to meet a growing global demand for the resource is challenging without a definitive long-range plan to conserve the resource. In previous chapters, spent fuel reprocessing and recycling have been discussed as options; additional studies have revealed several other propositions to extend the resource base, secure a limited supply within a particular market, or introduce conventional fuel alternatives. In this chapter, several strategies proposed in the literature as feasible strategies will be evaluated.

6.2 Stockpiling and Fuel Banking Strategies

As it is conventionally believed that costs will rise significantly in the future, stockpiling (of natural or enriched uranium) would make sense as a business venture. Additionally, it would introduce some energy security analogous to the U.S. Strategic Petroleum Reserve, but with a far longer period of coverage. However, in order for stockpiling to be worthwhile, the price would have to rise faster than inflation, by which marginal rates of return are attractive to an investor. Since SWU prices in constant dollars are expected to decrease because enrichment is a manufacturing process (adhering to experience and learning effects), and as technology continues to improve, it makes more sense to stockpile natural uranium, especially in avoidance of the volatile spot market. Mining companies and utilities currently do some stockpiling as indemnity against supply interruptions – certainly these stockpiles are for temporary resolution of supply interruptions, and not of long-term interest in this case. However, stockpiling of this sort should set a limit on the real dollar annual price escalation at approximately 10% per year if carried out at a large scale [63].

Regarding the U.S. Strategic Petroleum Reserve (SPR), at 50 $/bbl, the 727 million barrel stockpile is worth an estimated 36.4 billion dollars (USD); currently 97% full, this oil SPR is only enough to replace approximately 70 days of oil imports [64]. Conversely, investing equally in a uranium stockpile would result in a 364,000 ton stockpile of natural uranium, valued at 100 $/kg. Noting an average reactor requirement of 200 tons of \( U_{nat} \) per year, such a stockpile would last over 1,800 reactor years, capable of supporting the entire U.S. fleet of (104) reactors for over 17 years! A stockpile scaled down by a factor of ten would still provide a significant buffer against supply interruption or shortfall.

If supply interruptions should become of great concern (or the resource price increases faster than interest rates), stockpiling of either natural uranium or fuel-ready LEU would surely be a prudent option to ensure immediate relief within an isolated market. Two additional stockpiling sources (U.S. based) are currently considered, but are not fully exploited at present:

1) Enrichment plant tails, amounting to nearly 2 million metric tons (containing 5,000 tons of \( U_{235} \)), could significantly support 50 one GWe reactors for 100
years. Cheaper SWU should eventually permit cost-effective access to this material.

2) Domestic in situ ore reserves amounting to nearly $2 \times 10^6$ MT $U_{nat}$, if eventually recovered, could support 100 reactors for 100 years; cheaper Canadian and other international fuel supplies lowers incentive for U.S. miners to exploit the domestic in situ base.

Recently, the Department of Energy released a report detailing a proposed strategy for the management of its excess uranium inventory. Table 6.1 is a summary of the U.S.'s excess uranium inventories – in total about 300 reactor years [65].

<table>
<thead>
<tr>
<th>Inventory</th>
<th>MTU</th>
<th>Enrichment Level</th>
<th>Natural Uranium Equivalent (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unallocated U.S. HEU</td>
<td>67.6</td>
<td>HEU</td>
<td>12,485</td>
</tr>
<tr>
<td>U.S.-Based $U_{nat}$ as $UF_6$</td>
<td>5,156</td>
<td>$U_{nat}$</td>
<td>5,156</td>
</tr>
<tr>
<td>Russian-Based $U_{nat}$ as $UF_6$</td>
<td>12,440</td>
<td>$U_{nat}$</td>
<td>12,440</td>
</tr>
<tr>
<td>Off-Spec Non-$UF_6^*$</td>
<td>4,461</td>
<td>DU/$U_{nat}$/LEU</td>
<td>2,900</td>
</tr>
<tr>
<td>Depleted Uranium (DU) as $UF_6^{**}$</td>
<td>75,300</td>
<td>DU</td>
<td>25,950</td>
</tr>
</tbody>
</table>

* $U_{nat}$ equivalent corresponds to $U_{nat}$ and LEU material only
** $DU$ as $UF_6$ having an assay $\geq 0.35\%$ and $< 0.711\% U_{235}$; $U_{nat}$ equivalent based on 0.2\% tails assay

The above considerations bolster the assertion that uranium supply will not be a major constraint for the remainder of this century.

**Effect of Weapon Stockpile Reduction [66]**

As pointed out in chapter 4, the U.S. and Russia reached and agreement in the 1993 Megatons to Megawatts program, to blend down 500 tons of 90\% enriched uranium (from dismantled weapons) for consumption in the U.S.’s LWR reactor fleet. One metric ton of HEU can sustain a 1 GWe LWR for approximately 1 year. Hence, until complete in 2013, this source can support 50 reactors for 10 years – this unfortunately is only a minor satisfaction of the demand, considering a global reactor fleet of 500 within a few decades. Russia and the U.S. have retained a stockpile of between 600 and 1200 MT of HEU, IPFM estimating more than 1700 tons total; again, the world uranium market could easily absorb this reserve. Further, the current global stockpile of separated plutonium is approximately 500 tons, about half of which is civilian. As MOX LWR fuel, one metric ton would support a 1 GWe reactor for a year.

In total, these stockpiles could support about thirty reactors for 100 years. As a whole, this supply is significant if available over a small time frame (a decade or so). In the longer term, this inventory is progressively insignificant, especially under a robust growth scenario.
6.3 Optimum Tails and Tails Recovery

Recovery of enrichment plant tails, as aforementioned, will potentially contribute a considerable measure of natural uranium, hinged upon SWU pricing. The minimum cost of reactor reload fuel is achieved at an enrichment plant tails composition, which is a function of the ratio of natural uranium to separative work unit costs. Optimum tails value is given by the following relation [67]:

$$\frac{C_F}{C_s} = \frac{(x_F - x_o)(1 - 2x_o)}{x_o(1 - x_o)} - (1 - 2x_F)\ln\left(\frac{x_F(1 - x_o)}{x_o(1 - x_F)}\right)$$

(1)

where

$C_F$ = cost of natural uranium, ($/kg$)
$C_s$ = cost of separative work, ($/kg$)
$x_F$ = enrichment of natural uranium (0.711%)
$x_o$ = optimum tails enrichment (fraction)

Since both $x_F$ and $x_o$ are very small ratios of U235 (at least 2 orders of magnitude smaller than 1), one can simplify Equation 1. For enrichment in weight percent:

$$\left(\frac{0.711}{x_o}\right) \approx 1 + \left(\frac{C_F}{C_s}\right) + \ln\left(\frac{0.711}{x_o}\right)$$

(2)

Further, if the range of values is restricted, the optimum tails takes the following closed form, accurate within a percent:

$$x_o \approx \frac{1}{2.94 + 1.69\left(\frac{C_F}{C_s}\right)}$$

(3)

$$x_o \approx \frac{1}{2.22 + 2.19\left(\frac{C_F}{C_s}\right)}$$

(4)

Natural uranium savings by lowering uranium tails assay (increasing enrichment effort) can further be found through calculations of the separative work requirements. The amount of natural uranium needed per kg of reactor reload fuel at $x_p$ w/o enrichment is:

$$\frac{F}{P} = \frac{x_p - x_o}{x_F - x_o}$$

(5)

For advanced technology, and hence cheaper separative work, one can recover more U235 from old tails. From a batch of tails, the mass of which is $w$, original assay is $x_w$, 

87
and natural uranium enrichment, $x_F$, going to a lower tails enrichment $x_w'$ will enable the recovery of additional “ore”. Then the additional uranium recovered relative to the original ore consumption is given by:

$$\frac{F'}{F} = \frac{x_w - x_w'}{x_F - x_w'} \frac{x_p - x_F}{x_p - x_w}$$

(6)

and where

$x_w = 0.3\%$

$x_w' = 0.1\%$

$x_F = 0.711\%$

$x_p = 5\%$

the result is

$$\frac{F'}{F} = 0.299$$

Thus, for $x_p = 5\%$ w/o, decreasing tails from 0.3 to 0.1% will reduce natural uranium requirements by 30%. Further improvements in enrichment cost reduction can have a significant impact. One can even go back and strip old tails to recover additional fuel-grade material. It should be noted that the cost prescription for separative work was originally developed for multi-stage enrichment cascades, whether diffusion or centrifuge. This method of cost assignment will not necessarily be appropriate for laser isotope separation. Another option is to use the old tails as feed, and enrich up to the reactor grade $x_p$, using the new, lower tails enrichment, $x_w'$. In this case one creates new fuel relative to the old fuel in which created the original tails, in the ratio

$$\frac{P'}{P} = \frac{x_w - x_w'}{x_F - x_w'} \frac{x_p - x_F}{x_p - x_w}$$

(7)

for the same conditions as previously mentioned, resulting in

$$\frac{P'}{P} = 0.426$$

Thus, remarkably, 43% more fuel can be produced, albeit at the expenditure of considerable separative work. Using highly-advanced separations technology, such that all residual $^{235}$U can be recovered ($x_w' \rightarrow 0$) one finds:

$$\frac{P_{\text{max}}'}{P} = 0.626$$
Hence, upwards of 63% more reload fuel can be created. Note however, such technology would greatly intensify proliferation concerns.

6.4 Increased Fuel Burnup

Often overlooked, today's PWRs operate very close to optimum with regard to natural uranium utilization and fuel burnup, defined as the following:

\[ U_u = \frac{B_d}{(F/P)} \text{, MWd}_{\text{th}}/\text{kgU} \]  

where

\[ B_d = \text{discharge burnup (MWd/kgU)} = nB_c = \left(\frac{2n}{n+1}\right)B_i \]

\[ B_c \text{ is the cycle burnup, and 1/n^th of the core is refueled each shutdown. Starting with the feed-to-product mass ratio:} \]

\[ \frac{F}{P} = \frac{x_p - x_w}{x_f - x_w} \]

an additional curve-fit to PWR burnup calculations using CASMO/SIMULATE gives [68]:

\[ B_i \approx 28\sqrt{x_p - 0.88} - 19.9, \text{ MWd/kgHM} \]  

Thus, we may compare a current PWR to one for which burnup is doubled, at the same cycle length (keeping the capacity factor constant). For 0.25% tails:

<table>
<thead>
<tr>
<th>Table 6.2: Double Burnup Impact on Uranium Utilization</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ B_d \text{ (MWd/kgHM)} ]</td>
</tr>
<tr>
<td>---------------------------------</td>
</tr>
<tr>
<td>53.76</td>
</tr>
</tbody>
</table>

Hence within model accuracy, the uranium utilization is about the same (~0.6% relative difference). Another check on the burnup effect, a relation for the product enrichment as a function of discharge burnup and number of batches is [69]:
\[ x_p = 0.41201 + 0.11508 \cdot \left( \frac{n+1}{2n} \cdot B_d \right) + 0.00023937 \cdot \left( \frac{n+1}{2n} \cdot B_d \right)^2 \]  \hspace{1cm} (10)

Comparing the same burnup cases found in Table 6.2

<table>
<thead>
<tr>
<th>Table 6.3: Check on Double Burnup Impact on Uranium Utilization</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>B_d (MWd/kgHM)</strong></td>
</tr>
<tr>
<td>--------------------------</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>x_p (% U235)</strong></td>
</tr>
<tr>
<td><strong>F/P (@ x_p=0.25%)</strong></td>
</tr>
<tr>
<td><strong>U_d (MWd/kgU)</strong></td>
</tr>
</tbody>
</table>

Using Equation 10 results in an improvement of roughly 12.6%, a moderately larger improvement than reported in Table 6.2. The uncertainty in actual ore usage (from discharge burnup ratings) has been reported in the IAEA’s Water Reactor Fuel Extended Study. Figure 6.1 illustrates the wide uncertainty band in actual usage.
Figure 6.1: Enrichment versus burnup, revealing large ranges in burnup for a given enrichment [70]

Furthermore, extended studies have shown that we are nearly optimized at approximately 5.7 MWd/kg\textsubscript{U\text{nat}} (Figure 6.2). For a 1 GWe reactor ($\eta_{\text{th}}=1/3$), operating for a year at a 90% capacity factor (329 effective days), the annual uranium requirement is approximately 175 tons of U\text{nat} per GWe-yr. This number is very close to IAEA’s figure of roughly 180 metric tons per GWe-yr. Both values support our use of 200 tons/GWe-yr at 100% CF.
Thus, no large improvements in uranium utilization are likely in the future. Additionally, higher burnups might be seen as a safety risk. Irradiating fuel for longer periods of time increases the radiotoxicity of the waste. According to a UK study (Nirex), 55 GWd/tU in a PWR would increase the radiotoxicity of spent fuel by 50% as compared to a burnup of
33 GWd/tU (throughout the duration of the storage). Additionally, in the event of LOCA (loss of coolant accident), high fuel burnups might exceed NRC safety standards; such unforeseen safety problems are said to come into play above a burnup threshold of 45 GWd/tU [71]. In summary, increasing the burnup of the fuel does not significantly improve uranium utilization and should not be rigorously pursued solely for that purpose.

6.5 Alternative Fuel Options

The following section presents the latest fact-find in literature regarding various resource substitutes considered feasible for natural terrestrial uranium.

Uranium from Seawater

Uranium extraction from seawater is a compelling topic arising in the discussion of long-term uranium resource production. What makes the seawater uranium an interesting topic is that although only present at a concentration of 3.3 ppb (by weight) total oceanic reserves of uranium are about 4.5 billion tons. Practically inexhaustible, extraction of uranium from seawater virtually represents the uranium market-price ceiling. Work on uranium extraction from seawater in the 1980s confirmed the technical feasibility of extraction; however, long-range recovery-cost predictions are highly speculative, since they are the result of extrapolations from small-scale research demonstration. Over the past 18 years, persistent Japanese researchers, whose findings have brought seawater extraction closer to the goal of full-scale deployment, have made the most technological advancements in seawater extraction.

A Japanese study, by Masao Tamada, employing a fibrous acrylic amidoxime ion exchange resin, presents the results of sea trials on a braided absorbent, using an approach similar to one evaluated at MIT in 1984 [72]. In the study, the researchers achieved module loadings of 2000 to 3000 ppm of uranium over the course of around 60 days submerged at sea. Figure 6.1 presents the results of the demonstration.
From this work, adsorbent durability drives production costs. For a loading rate of 100 ppm/day, we see very competitive cost-of-production estimates flooring around $150/kgU. Table 6.3 provides the cost of yellowcake production based on the analysis performed above. The grayed portion of the table is the desirable range of prices that will make Uranium extraction from seawater an affordable option.

| Table 6.4: Production Cost as a Function of Loading rate and Sorber Capacity ($/kgU) |
|-------------------------------------|---------------|---------------|---------------|---------------|---------------|
| Sorber Cap, Q (ppm)                | Loading Rate, q (ppm/day) | 2              | 5              | 20             | 100           | Infinity      |
| 100                                 | 4,290.10       | 2,877.28      | 2,170.87      | 1,982.49       | 1,935.4       |
| 200                                 | 3,322.40       | 1,909.58      | 1,203.17      | 1,014.79       | 967.70        |
| 500                                 | 2,741.78       | 1,328.96      | 622.55        | 434.17         | 387.08        |
| 1000                                | 2,548.24       | 1,135.42      | 429.01        | 240.63         | 193.54        |
| 1500                                | 2,483.73       | 1,070.91      | 364.50        | 176.12         | 129.03        |
| 2000                                | 2,451.47       | 1,038.65      | 332.24        | 143.86         | 96.77         |
| 5000                                | 2,393.41       | 980.59        | 274.18        | 85.80          | 38.71         |
| Infinity                            | 2,354.70       | 941.88        | 235.47        | 47.09          | 0             |

Although extensive research has been conducted on Uranium extraction from seawater, considerable efforts are still needed to confirm the viability of uranium from seawater as
a primary source for nuclear fuel. From literature and the conclusions of this study, some key options and considerations requiring further resolution include use of synergistic energy systems, additional organic resin/adsorber options, as well as byproduct profitability. The synergistic systems considered include offshore wind farms, which would potentially serve as a power source powering a pump, in effect increasing the throughput of seawater. As for organic options for uranium adsorption, black sea algae, which consume uranium in seawater in the Black Sea, may be collected from the seabottom at high concentrations with moderate effort. Lastly, Vanadium and Molybdenum are picked up as trace elements in the extraction process. Vanadium and Molybdenum currently sell at about $12/kg and $5/kg respectively. These trace elements may further provide cost credits to the seawater extraction system. However, because these trace elements are essential for various oceanic organisms and vegetation, more research is needed to forecast environmental effects of large-scale removal. Assuming continued efforts over the next ten to twenty years, a sufficiently accurate cost for uranium from seawater should be at hand, to permit choosing between this approach and for example, deploying fast breeder reactors.

Thorium

The option of thorium (Th232) as a nuclear fuel has been proposed based on its neutronic properties and similarity with uranium concerning maximum energy content. Additionally, thorium substitution for natural uranium has been considered because of its greater crustal abundance, roughly three times as abundant as uranium. Moreover, although formerly considered detrimental, the buildup of U232 in irradiated fuel can lower proliferation risk, and thorium neutron captures reduce production of TRU and plutonium. Moreover, the reduced production of minor actinides enables improved long-term HLW confinement. However, thorium use as a primary fuel in the once-through fuel cycle does not appear to be a productive measure in offsetting uranium cost escalation for several reasons:

1) Because U233 does not occur in nature, thorium cycles must be started up using enriched uranium or recycled TRU, posing a considerable impediment.

2) Processing thorium-based fuels is expensive; further, a significant benefit is achieved solely through the introduction of reprocessing and recycling.

3) Since U233 must be denatured to less than 13% by admixture with U238 (to comply with safeguards), complications to reprocessing and recycling are introduced by varying degrees of thorium-uranium miscegenation.

During the past decade, there was a period of renewed interest in thorium. Literature reporting positive implications for thorium-use include papers citing increased natural uranium utilization (by 10 to 25%) from thorium admixture, depending on burnup (U_n decreases with increases in burnup). However, this savings is extinguished if the bred U233 is sufficiently denatured [73]. Therefore, it is concluded that the contention for thorium as a substitute for natural uranium is weakened, especially if restricted to a once-through fuel cycle.
6.6 Chapter Summary

Presented in this chapter were various options, which if employed, would effectively extend the supply of natural uranium. Anticipated as the most effective resource-enhancing technology, breeder technology is expected to increase natural uranium savings by up to approximately 95%; unfortunately this technology is not at a stage of development expected to be online for several decades and thus was outside the scope of this study. Recovery and utilization of enrichment tails is promising, but although very effective at conserving the natural uranium supply, this option relies heavily on the expenditure of large amounts of separative work; further, improvements in separation technology (necessary for the reduction of tails assays to become economical) inevitably warrant concerns over proliferation hazard. In summary, the conservation of uranium must be efficiently optimized through the use of a supply portfolio rather than any one measure.
CHAPTER 7: SUMMARY, CONCLUSIONS & FUTURE WORK

7.1 Thesis Summary and Conclusions

On the cusp of significant developments and growth, the nuclear industry has been the subject of heavy political debate and speculation. The prospects of rapid growth in global nuclear generating capacity, to meet a growing clean and sustainable energy demand, have been constrained by concerns about waste management, proliferation, and uranium supply. According to the International Energy Agency, global energy demand is expected to increase 50% by 2030. However, the uranium market is in serious (production/supply) imbalance and vulnerable to price volatility until current efforts to expand production come to fruition in ten or fifteen years.

The principal accomplishment of this thesis project was the development of a probabilistic cost model to project production price trends for natural uranium as a function of cumulative consumption of the resource. Incorporating economic models of experience (learning effects) and scale effects with Deffeyes’ geophysical resource distribution model, Monte Carlo techniques were used to develop a cost correlation factor correlating uranium production cost with cumulative nuclear power generation; this relationship was determined with varying levels of confidence. Moreover, this model enables breakeven cost calculations to predict cost competitiveness of the once through LWR fuel cycle with various fuel cycle alternatives.

As a result of literature review, and the present cost/resource modeling carried out in the present work, there is a high degree of confidence that natural uranium will be available at tolerable costs well into the future. Compared to coal generation, reprocessing and recycle, and other fuel cycle alternatives, the LWR fuel cycle can sustain significant increases in $U_{nat}$ prices before its operating costs are matched by such alternatives. However, the calculations using the model developed in this project suggest uranium prices will not become intolerable in the near future. These findings generally agree with the conclusions of E. Schneider, a leading U.S. uranium resource analyst [35]. Our findings support a conclusion that concerns over resource depletion should not motivate premature large scale deployment of alternatives: there are sufficient reserves of natural uranium, allowing for further R&D and well-paced introduction of fuel cycle alternatives.

Motivations such as closure of the fuel cycle through the facilitation of HLW disposal have not yet been resolved; the recent defunding of the Yucca Mountain project has delayed those prospects. Further, at present, reprocessing and MOX recycling do not appear to be economical alternatives to closing the fuel cycle unless substantial waste fees are imposed to credit reprocessing and recycle. Comprehensive breakeven assessments must be established to identify future research, development, demonstration, and deployment of fuel cycle alternatives, compatible with uranium consumption scenarios. Finally, as a measure to increase uranium utilization, expenditures of additional separative work are found to significantly free up more natural uranium; therefore expansion of enrichment facilities and R&D should be encouraged, respecting proliferation concerns.
7.2 Future Work

Looking ahead, from this study, we find that mining is a core issue at the heart of the discussion, impacting the environment and public health, nuclear power growth, and subsequently employed strategies to secure low-cost reserves of uranium fuel; hence, mining will likely play an increasing role in regulation - impacting costs, and scheduled development of nuclear fuel cycle options. Moreover, the use of resource cost modeling will play a principal role in projecting the feasibility of various fuel cycle programs. Although inclusion of learning effects and production scale effects on uranium production costs are important models in resource cost models, the three-tier model developed in this study only provides crude estimates of probabilistic cost distributions for uranium given a particular nuclear power growth scheme. In conclusion, this study recommends three critical issues be resolved in developing more robust and deterministic cost/resource models:

1. An obvious recommendation is that the practice of making running updates of the uranium resource situation continues. This should be preceded by an intensive analysis of the reserves available in the U.S. and worldwide - analogous to the U.S. National Uranium Resource Evaluation (NURE) program carried out in the 1970s and 80s. Since Deffeyes’ work dates back to the 1980s, an update is dearly in order. This is essential for projecting resource recovery costs beyond the several decade time-horizon provided by commercial exploration activities. The first, and last, major coordinated assessments of existing and projected uranium ore resource were made over several years bracketing 1980. They consisted of the U.S. National Uranium Resource Evaluation (NURE), the International Uranium Resource Evaluation Program (IUREP), and the more wide-ranging U.S. Non proliferation Alternative Systems Assessment Program (NASAP) and the International Nuclear Fuel Cycle Evaluation (INFCE). Since then, the less-well-funded biennial IAEA Red Book has compiled voluntarily submitted information from participating countries: a valuable but necessarily constrained contribution. These running updates effectively provide an up to date resource baseline, by which deterministic, systems dynamic approaches to cost/resource estimation are enabled.

2. There is a clear need for geotechnical and enhanced geostatistical modeling – for example, development of a relationship for drilling depth and number of holes, related to production cost and the cost of uranium. Additionally, cost models for in situ recovery are critical as ISL progresses as a preferred mining technique. New discoveries and technical advancements such as the increased use of in situ leaching, as well as the discovery of Saskatchewan-based unconformity-related deposits, with grades as high as 70%, necessitate major modifications to cost-resource models.

3. Finally, there is a need to sharpen parameter ranges and distribution functions. A particular limitation is reflected by the maintenance of the same upper cost cutoff of 130 $/kg since (at least) 1978 – roughly 300 $/kg in today’s dollars. This has presumably resulted in progressive compression of the range of compiled resources. A new set of upper cost categories, such as 200 and 300 $/kg, has the added benefit of providing an upper benchmark, which is close to a breakeven value versus alternatives such as reprocessing and recycle.
REFERENCES


Displaying the derivation of the normal probability distribution function and its transformation into the lognormal pdf is extremely helpful in understanding the inner-workings of the equation and its application. Further, because this work uses K. Deffeyes' lognormal curve for resource tonnage as a function of ore grade, the derivation is central to the process of developing the cumulative resource elasticity model.

To begin, the fundamental assumptions based on first principles must be developed. In the reference provided, a framework is developed for characterizing the accuracy of a dart throw; this renders the dart’s probability of hitting its target region, the bull’s eye, or graphically the origin of a 2-D Cartesian space. The following three assumptions will be used to frame the derivation:

1) Independence: In 2-D Cartesian space, the probability of x (abscissa) is independent of y (ordinate); change in the ordinate dimension, y, has no affect on the probability of x.

2) Arbitrary Orientation: Probabilities of x and y are independent of the orientation of the coordinate system.

3) Small errors are more probable than large errors in accuracy.

In Figure A1.1 (assuming all of the regions are of equal size), the event of landing in region A is more probable than region B, and region B is more probable than region C. Additionally, if the regions are of equal displacement from the origin, landing within the larger region V is more probable than the smaller regions U and T. Figure 2 illustrates a

---

11 Analogously, the presented framework aims to characterize resource abundance over a range of ore grades.
region of space within a two dimensional Cartesian plane from which the normal pdf will be derived. The region is defined with dimensions from $x$ to $x+\Delta x$ and from $y$ to $y+\Delta y$ along the abscissa and ordinate, respectively.

\[ \Delta y / t_0 \]

\[ \Delta x / t_0 \]

**Figure A1.2:** Representation of spatial occupation within a Cartesian coordinate system

Since the orientation of the coordinate system has no effect on probability (from the principle of independence), it is convenient to decompose the Cartesian space into polar coordinates where

\[ x = r \cos(\theta) \]  
\[ y = r \sin(\theta) \]

The probability of occupying the above Cartesian space is represented by $p(x)\Delta x$ for the abscissa and $p(y)\Delta y$ for the ordinate. Therefore, the probability of landing within the represented region, $\Delta x\Delta y$ is $p(x)\Delta x \ast p(y)\Delta y$. Additionally, from the principle of arbitrary orientation, any region $r$ units from the origin with dimensions $\Delta x$ and $\Delta y$ will express the same probability:

\[ p(x)\Delta x \ast p(y)\Delta y = g(r)\Delta x\Delta y \]  

where:

\[ g(r) = p(x)p(y) \]

Note, $g(r)$ has no angular dependence. Differentiating equation 2b with respect to coordinate orientation, $\theta$:

\[ 0 = p(x)\frac{dp(y)}{d\theta} + p(y)\frac{dp(x)}{d\theta} \]

Completing the chain rule and substituting equations 1a and 1b into 3a, one obtains:

\[ 0 = p(x)p'(y)(r\cos(\theta)) + p(y)p'(x)(-r\sin(\theta)) \]
Rewriting equation 3b:

\[ 0 = p(x)p'(y)x - p(y)p'(x)y \]  
(A1.3c)

Finally:

\[ \frac{p'(x)}{xp(x)} = \frac{p'(y)}{yp(y)} \]  
(A1.3d)

The above differential equation is true for all combinations of \( x \) and \( y \), implying the ratio expressed is constant; therefore:

\[ \frac{p'(x)}{xp(x)} = \frac{p'(y)}{yp(y)} = C \]  
(A1.4a)

Focusing on only the left-hand equation with respect to \( x \), solving the differential equation:

\[ \ln(p(x)) = \frac{Cx^2}{2} + c \]  
(A1.4b)

then:

\[ p(x) = Ae^{\frac{Cx^2}{2}} \]  
(A1.4c)

Noting assumption 3, events further from the origin are less probable, thus the sign of constant \( C \) must be negative. Replacing \( C \) with \( k \) reveals the basic form of the normal distribution:

\[ p(x) = Ae^{\frac{-kx^2}{2}} \]  
(A1.5)

Now, the constants \( A \) and \( k \) may be solved for. As a probability distribution, the total area under the curve must equal 1; integrating the function over an infinite domain:

\[ \int_{-\infty}^{\infty} Ae^{-\frac{kx^2}{2}} dx = 1 \]  
(A1.6a)

Through symmetry, the integral can be divided in half:

\[ \int_{0}^{\infty} e^{-\frac{kx^2}{2}} dx = \frac{1}{2A} \]  
(A1.6b)
Equation 6b is also true for \( y \), thus the total probability of landing within the defined region is represented by:

\[
\left( \int_{0}^{\infty} e^{-\frac{ky^2}{2}} dy \right) \left( \int_{0}^{\infty} e^{-\frac{kx^2}{2}} dx \right) = \frac{1}{4A^2} \quad (A1.6c)
\]

Through independence, equation 6c may be rewritten as:

\[
\int_{0}^{\infty} \int_{0}^{\infty} e^{-\frac{k(x^2+y^2)}{2}} dy dx = \frac{1}{4A^2} \quad (A1.6d)
\]

Switching now to polar coordinates using equations 1a and 1b:

\[
\int_{0}^{\infty} \int_{0}^{\infty} e^{-\frac{k(x^2+y^2)}{2}} dy dx = \int_{0}^{\infty} \int_{0}^{\pi/2} e^{-\frac{k}{2}r^2} r dr d\theta \quad (A1.7a)
\]

Employing u-substitution to solve equation 7a:

\[
\int_{0}^{\pi/2} \int_{0}^{\infty} e^{-\frac{k}{2}r^2} r dr d\theta = \int_{0}^{\pi/2} \left[ \int_{0}^{\infty} e^{-u} du \right] d\theta = \int_{0}^{\pi/2} \frac{d\theta}{k} = \frac{\pi}{2k} \quad (A1.7b)
\]

where:

\[
u = -\frac{k}{2} r^2
\]

\[-\frac{1}{k} du = r dr
\]

Therefore:

\[
\frac{1}{4A^2} = \frac{\pi}{2k} \quad (A1.8a)
\]

Solving for A:

\[
A = \sqrt{\frac{k}{2\pi}} \quad (A1.8b)
\]

Plugging the result in 8b into equation 5:

\[
p(x) = \sqrt{\frac{k}{2\pi}} e^{-\frac{kx^2}{2}} \quad (A1.9)
\]
Now, the constant \( k \) may be evaluated to obtain the standard form of the normal pdf, first defining the mean \((\mu)\) and variance \((\sigma^2)\). Because \( p(x) \) is an even function, the mean is defined by an odd function, and equals zero.

\[
\mu = \int_{-\infty}^{\infty} xp(x)dx = \sqrt{\frac{k}{2\pi}} \int_{-\infty}^{\infty} xe^{-\frac{k x^2}{2}} dx = 0 \quad (A1.10a)
\]

The variance is defined as follows:

\[
\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 p(x)dx = \int_{-\infty}^{\infty} x^2 p(x)dx = \sqrt{\frac{k}{2\pi}} \int_{-\infty}^{\infty} x^2 e^{-\frac{k x^2}{2}} dx \quad (A1.10b)
\]

Further, through symmetry:

\[
\sqrt{\frac{k}{2\pi}} \int_{-\infty}^{\infty} x^2 e^{-\frac{k x^2}{2}} dx = \sqrt{\frac{2k}{\pi}} \int_{0}^{\infty} x^2 e^{-\frac{k x^2}{2}} dx = \sigma^2 \quad (A1.10c)
\]

'Integrating by parts' to solve equation 10c:

\[
\sqrt{\frac{2k}{\pi}} \left[ \frac{-x e^{-\frac{k x^2}{2}}}{k} \bigg|_{0}^{\infty} + \frac{1}{k} \int_{0}^{\infty} e^{-\frac{k x^2}{2}} dx \right] = \sqrt{\frac{2k}{\pi}} \left[ 0 + \frac{1}{2k} \sqrt{\frac{2\pi}{k}} \right] = \frac{1}{2k} \sqrt{\frac{2\pi \cdot 2k}{k \cdot \pi}} = \frac{1}{k} \quad (A1.11a)
\]

where:

\[
u = x \\
dv = xe^{-\frac{k x^2}{2}}
\]

Equating the result of 11a with the definition of 10b:

\[
\therefore k = \frac{1}{\sigma^2} \quad (A1.11b)
\]

Finally, plugging in the value for \( k \):

\[
p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2} \quad (A1.12a)
\]

Because the mean is just a horizontal shift, insert the mean into equation 12a to obtain the general form of the normal pdf.

\[
p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad (A1.12b)
\]
Now, the general form of the normal pdf may be transformed to obtain the standard form of the lognormal pdf. To avoid later confusion, replace x with t in Equation 12b.

\[ p(t) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2}(\frac{t-\mu}{\sigma})^2} \]  

(A1.13a)

To transpose the normal probability in equation 13a into lognormal form, define the probability over a specific range of values and transform the bounds of integration.

\[ p(t < a) = \int_{-\infty}^{a} \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2}(\frac{t-\mu}{\sigma})^2} \, dt \]  

(A1.13b)

From equation 13b, if t is a [natural] logarithm of x, \( x = e^t \), \( \ln(x) = t \), and \( dt = \frac{dx}{x} \).

Hence the following equalities apply:

\[ p(x < a) = p(e^t < a) = p(t < \ln(a)) \]  

(A1.14a)

Adjusting the upper bound of integration:

\[ p(t < \ln(a)) = \int_{-\infty}^{\ln(a)} \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2}(\frac{t-\mu}{\sigma})^2} \, dt \]  

(A1.14b)

Now, transform 14b by substituting x for t, using the equality in 14a

\[ p(x < a) = \int_{-\infty}^{a} \frac{1}{x\sigma \sqrt{2\pi}} e^{-\frac{1}{2}(\frac{\ln x-\mu}{\sigma})^2} \, dx \]  

(A1.14c)

\[ \therefore p(x) = \frac{1}{x\sigma \sqrt{2\pi}} e^{-\frac{1}{2}(\frac{\ln x-\mu}{\sigma})^2} \]  

(A1.14d)

Finally, the following form of the lognormal probability distribution function will be used to model the uranium resource distribution as proposed by Deffeyes [31]

\[ u(x) = \frac{A}{x\sigma \sqrt{2\pi}} e^{-\frac{1}{2}(\frac{\ln x-\mu}{\sigma})^2} \]  

(A1.15)

where A is introduced as a normalization factor.
A2 Defeyes Curve Characterization

In this section, Defeyes' Uranium resource model will be reverse-engineered to obtain the lognormal parameters used in the ore-grade vs. resource tonnage relation, since numerical values for the parameters are not given in the available reference. An explanation of the methods employed to reconstruct the ore-grade based Uranium resource curve is provided as background to the price and resource elasticity models developed. Starting with the lognormal relation for Uranium resource as a function of ore grade,

\[ u(x) = \frac{A}{x\sigma\sqrt{2\pi}} e^{-\frac{\ln(x) - \mu^2}{2\sigma^2}} \]  \hspace{1cm} (A2.1a)

taking the natural log of both sides of equation 1a will result in a quadratic relation.

\[ \ln(u(x)) = \ln\left(\frac{A}{x\sigma\sqrt{2\pi}}\right) + \left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right) \] \hspace{1cm} (A2.1b)

Equation 1b may be generalized using the standard quadratic equation as follows:

\[ \ln(u(x)) = a + b\ln(x) + c(\ln(x))^2 \] \hspace{1cm} (A2.1c)

Expanding the terms in equation 1b:

\[ \ln(u(x)) = \ln\left(\frac{A}{\sigma\sqrt{2\pi}}\right) - \frac{\mu^2}{2\sigma^2} - \ln x + \frac{\mu\ln x}{2\sigma^2} - \frac{(\ln x)^2}{2\sigma^2} \] \hspace{1cm} (A2.2a)

\[ \ln(u(x)) = \left[ \ln\left(\frac{A}{\sigma\sqrt{2\pi}}\right) - \frac{\mu^2}{2\sigma^2}\right] + \left(\frac{\mu}{\sigma^2} - 1\right)\ln x + \left(-\frac{1}{2\sigma^2}\right)(\ln x)^2 \] \hspace{1cm} (A2.2b)

therefore:

\[ a = \ln\left(\frac{A}{\sigma\sqrt{2\pi}}\right) - \frac{\mu^2}{2\sigma^2} \]
\[ b = \frac{\mu}{\sigma^2} - 1 \]
\[ c = -\frac{1}{2\sigma^2} \]

Differentiating equation 1c with respect to \( \ln(x) \) reveals the slope of the curve:

\[ s_f = \frac{d\ln(u)}{d\ln(x)} = \frac{da}{d\ln(x)} + \frac{db}{d\ln(x)}(\ln(x)) + b + \frac{dc}{d\ln(x)}(\ln(x))^2 + 2c\ln(x) \] \hspace{1cm} (A2.3a)
This simplifies to

\[
\frac{d \ln(u)}{d \ln(x)} = b + 2c \ln(x) \quad \text{(A2.3b)}
\]

Since the maximum value will occur at a zero slope, \( s=0 \), the ore grade corresponding to the maximum tonnage will be:

\[
\text{Max: } \ln(x) = \mu - \sigma^2
\quad \text{(A2.4)}
\]

Because the quadratic fit of Deffeyes’ curve will be obtained on a log-log scale, converting the above methods to equivalent log base-10 will lead to the lognormal parameters \( A, \mu, \) and \( \sigma^2 \) directly from the curve fit equation. Noting the base-10 logarithmic conversion from \( \ln(x) \), equations 1b through 3c can be converted to log-form using the following substitution

\[
\ln(x) = \ln(10) \cdot \log_{10}(x)
\quad \text{(A2.5)}
\]

where \( \ln(10) \) is approximated here as 2.303. Substituting equation 5 into 1c:

\[
\log(u) = \frac{a}{2.303} + b \log(x) + 2.303c(\log(x))^2
\quad \text{(A2.6a)}
\]

To prevent confusion with later steps, constants used for the logarithmic (base-10) quadratic fit will differ, and will be represented by the following:

\[
\log(u) = d + f \log(x) + g(\log(x))^2
\quad \text{(A2.6b)}
\]

where:

\[
d = \left[ \ln\left( \frac{A}{\sigma \sqrt{2\pi}} \right) - \frac{\mu^2}{2\sigma^2} \right] \cdot \frac{1}{\ln(10)}
\]

\[
f = \frac{\mu}{\sigma^2} - 1
\]

\[
g = \frac{-1}{2\sigma^2} \cdot \ln(10)
\]

Differentiating equation 6a:

\[
\frac{d \log(u)}{d \log(x)} = \frac{dd}{d \log(x)} + \left[ \frac{df}{d \log(x)} (\log(x)) + f \right] + \left[ \frac{dg}{d \log(x)} (\log(x))^2 + 2g \log(x) \right]
\quad \text{(A2.7a)}
\]

Again, solving for the ore grade corresponding with the maximum tonnage value of the lognormal plot:
\[ s_f = \frac{d \log(u)}{d \log(x)} = f + 2g \log(x) \]  
(A2.7b)
\[ s_f = 0 = f + 2g \log(x) \]  
(A2.7c)
\[ \Rightarrow \log(\hat{x}) = \frac{-f}{2g} \]  
(A2.7d)

Additionally, the compliance of the numerical methods may be verified by comparing the value obtained in equation 8a with that of equation 4:

\[ \hat{x} = 10^{\frac{-f}{2g}} \]  
(A2.8a)
\[ \ln(\hat{x}) = \ln \left( 10^{\frac{-f}{2g}} \right) \]  
(A2.8b)
\[ \ln(\hat{x}) = \ln 10 \frac{\mu - \sigma^2}{\ln(10)} \]  
(A2.8c)
\[ \ln(\hat{x}) = \frac{\mu - \sigma^2}{\ln(10)} \ln(10) \]  
(A2.8d)
\[ \Rightarrow \ln(\hat{x}) = \left(\mu - \sigma^2\right) \]  
(A2.8e)

Finally, after applying a quadratic fit to Deffeyes' curve, the variance \((\sigma^2)\) and mean \((\mu)\) parameters can be determined using the following relations

\[ \ln(\hat{x}) = \ln \left( 10^{\frac{-f}{2g}} \right) \]
\[ \therefore \sigma^2 = \frac{\ln(\hat{x})}{f} = \frac{\ln(\hat{x})}{b} \]  
(A2.9a)
\[ \therefore \mu = \ln(\hat{x}) + \sigma^2 \]  
(A2.9b)

Thus, it is now possible to extract the essential lognormal parameters from the quadratic fit of Deffeyes' lognormal Uranium distribution model. The normalization factor, \(A\), will eventually be obtained by substituting the values for the mean and variance parameters, evaluated as:

\[ A = \sqrt{2\pi \sigma e^{d^2 \ln(10^2) + \frac{\mu^2}{2\sigma^2}}} = \sqrt{2\pi \sigma e^{\frac{a + \mu^2}{2\sigma^2}}} \]  
(A2.10a)

A3 Deffeyes Curve Characterization: Parabola Fit

In the process of developing a model incorporating Deffeyes' ore grade elasticity, methods to reverse engineer the published uranium frequency distribution were
necessary. Although many assumptions may be employed to approximate the lognormal distribution of uranium as a function of ore-grade, a straightforward method served to be most effective; directly scaling Deffeyes’ plot by interpolating the data points from a published figure eventually led to the distribution parameters of interest. The methods employed will be fully disclosed in this section leading to the lognormal uranium frequency distribution presumed to be representative of Deffeyes’ 1978 findings.

**Figure A3.1:** Deffeyes’ Lognormal Frequency Model for the Distribution of Global Terrestrial Uranium [31]

From the above figure, the data points in Table A3.1 were read off.
Table A3.1: Interpolated Data Points from Uranium Distribution Curve

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<th>Log Tonnage</th>
<th>Tonnage</th>
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</table>

The next step involved inputting the (logarithmically scaled) data points into Matlab to obtain a quadratic fit. The following plot resulted, including the quadratic fit significant to 5 digits:

![Quadratic Fit of Deffeyes Curve Data Points in Matlab](image-url)
The quadratic curve fit returned by Matlab, as presented in the above figure, takes the following form:

\[ y = -0.50153x^2 + 0.091261x + 12.537 \quad \text{(A3.2a)} \]

Equation 2a also corresponds with the quadratic form of the lognormal equation found in the previous section

\[ \log(u) = d + f \log(x) + g(\log(x))^2 \quad \text{(A3.2b)} \]

where,

\[ d = \frac{\ln \left( \frac{A}{\sigma \sqrt{2\pi}} \right) - \frac{\mu^2}{2\sigma^2}} {\ln(10)} \cdot \left[ -1 \ln(10) \right] = 12.537 \quad \text{(A3.3a)} \]

\[ f = \frac{\mu}{\sigma^2} - 1 = 0.091261 \quad \text{(A3.3b)} \]

\[ g = \frac{1 - 1}{2\sigma^2} \cdot \ln(10) = -0.50153 \quad \text{(A3.3c)} \]

Recognizing a determinate system of linear equations formed by equations 3a through 3c, we are able to solve for a unique set of lognormal parameter values. Solving first for the variance and standard deviation of the distribution:

\[ \sigma^2 = \begin{bmatrix} -1 \\ 2g \end{bmatrix} \cdot \ln(10) = 2.2956 \quad \text{(A3.4)} \]

\[ \therefore \sigma = 1.5151 \]

Additionally, solving for the mean from equation 3b:

\[ \mu = 2.5051 \]

As noted in the previous section, a simple check of the methods requires the compliance of the standard deviation and mean with values obtained using equations A2.8b through A2.9b below

\[ \ln(\hat{x}) = \ln \left( \frac{-f}{10^{2b}} \right) = \ln(1.2331) = 0.20950 \]

\[ \sigma^2 = \frac{\ln(\hat{x})}{f} = \frac{\ln(\hat{x})}{b} = \frac{0.20950}{0.091261} = 2.2956 \]

\[ \mu = \ln(\hat{x}) + \sigma^2 = 2.5051 \]

Equation A2.10a will finally be employed to solve for the normalization factor, A.
\[ A = \sqrt{2\pi \sigma e} \left( d^{\ln(10)+\frac{\mu^2}{2\sigma^2}} \right) = 5.1303 \cdot 10^{13} \]  

(A3.5)

The following table summarizes the parameters of interest obtained by characterizing Deffeyes’ uranium distribution curve.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu \hat{x} )</td>
<td>1.2331</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>1.5151</td>
</tr>
<tr>
<td>( \sigma^2 )</td>
<td>2.2956</td>
</tr>
<tr>
<td>( \mu )</td>
<td>2.5051</td>
</tr>
<tr>
<td>( A )</td>
<td>5.1303E+13</td>
</tr>
</tbody>
</table>

Table A3.2: Lognormal Parameters

As we have set out to accomplish, the global terrestrial uranium distribution as function of ore-grade is represented by

\[ u(x) = \frac{1.3509 \cdot 10^{13}}{x} e^{-\frac{(\ln x - 2.3051)^2}{4.5911}} \]  

(A3.6)

Alternative Deffeyes Curve Approximation Using Geometry

On first principles, a convenient approach to verify our findings is to exploit parabolic symmetries, recognizing that the log-log frequency distribution of uranium as a function of ore-grade plots as a parabola. Measuring the width and height of the curve in figure A3.3 below, parameters w and h are graphically defined as follows:
Figure A3.3: Graphical Representation of the Parabola Measurements

The general equation for a parabola centered about the y-axis, and passing through the origin takes the following form:

\[ y = Cx^2 \quad \text{(A3.7a)} \]

where \( C \) is a constant. The following also gives the relationship between \( w \) and \( h \)

\[ \frac{w}{\sqrt{h}} = \beta \quad \text{(A3.7b)} \]

where

\[ C = \beta^2 \quad \text{(A3.7c)} \]

The relationship expressed in equation A3.7b may be confirmed for several measurements of \( w \) and \( h \). The bisecting line also denoted in the above figure intersects the x-axis at 1.2331 ppm given by \( x_{\text{hat}} \) from table A3.2. The scale assumed for each generation (tenfold unit increase) corresponded with 12 mm; however, measurements were taken using a digital ruler, which gave readings to the hundredth of an inch. The following table is a tabulation of 3 measurements taken, converted to cm:
Table A3.1: Parabola Measurements

<table>
<thead>
<tr>
<th>w (cm)</th>
<th>h (cm)</th>
<th>β (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.74</td>
<td>3.07</td>
<td>1.56</td>
</tr>
<tr>
<td>3.53</td>
<td>5.11</td>
<td>1.56</td>
</tr>
<tr>
<td>4.19</td>
<td>7.19</td>
<td>1.56</td>
</tr>
</tbody>
</table>

As observed by the agreement of β for the different readings, the measurements conform to equation A3.7b. From the graphical representation of h and w presented in figure A3.3, the slope of the unimodal frequency distribution may be approximated where

\[ w = \log x - \log \hat{x} = \log \left( \frac{x}{\hat{x}} \right) \]  
(A3.8a)

\[ h = \log \hat{u} - \log u = \log \left( \frac{\hat{u}}{u} \right) \]  
(A3.8b)

Substituting A3.8a and A3.8b into equation A3.7b

\[ \left[ \log \left( \frac{x}{\hat{x}} \right) \right]^2 = C \cdot \log \left( \frac{\hat{u}}{u} \right) \]  
(A3.9a)

\[ \log u = \log \hat{u} - \frac{1}{C} \left[ \log x - \log \hat{x} \right]^2 \]  
(A3.9b)

Differentiation equation A3.9b for the slope of the log-log plot

\[ s_c = \frac{d \log u}{d \log x} = -\frac{2}{C} \left[ \log x - \log \hat{x} \right] \]  
(A3.10a)

As a reference point, Deffeyes approximates the slope of the distribution over a range of uranium concentrations that are currently being mined to be between -1 and -2.5, with -2.5 corresponding to a 300-fold increase in recoverable resource per tenfold drop in ore grade [31]. Assuming an ore grade of 1000 ppm:

\[ s_c = \frac{d \log u}{d \log x} = -\frac{2}{2.43} \left[ 3 - 0.091 \right] = -2.39 \]  
(A3.10b)

Hence, the slope obtained through parabolic symmetry is reasonably close to Deffeyes’ claim of -2.5, confirming that a parabola is an exact representation of a log-log plot of a lognormal distribution. Further, the slope of -3.5 noted on figure A3.3 is closely approximated at an ore grade of 20,000 ppm. As a side-note, only metric units will work with this approximation method; although β and C are constants, they are not unit-less.
A4 Cumulative Resource, Ore-grade Elasticity, and the Complementary Error Function

In this section, the lognormal parameters will be used as inputs to develop the ore-grade elasticity. To develop the ore-grade elasticity of the resource, the complementary error function will be employed to approximate the slope of the cumulative distribution. Additionally, a rational approximation method using Chebyshev Polynomials, an asymptotic expansion of the error function, and a semi-analytical approach will also be applied to validate the findings of the slope of the cumulative lognormal distribution. The slope of the cumulative distribution is of interest because it represents the resource that becomes increasingly available in response to a drop in ore grade, revealing the ore-grade elasticity. The cumulative resource is represented as the following for a lognormal (Gaussian) distribution:

\[ U(x) = \int_x^\infty u(x) \, dx \]  

(A4.1)

where, as previously determined

\[ u(x) = \frac{A}{x\sigma\sqrt{2\pi}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}} \]

\[ \therefore U(x) = \int_x^\infty \frac{A}{x\sigma\sqrt{2\pi}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}} \, dx \]  

(A4.2)

Note, the lower-case \( u(x) \) represents uranium resource tonnage for a particular ore grade, characterized by a Gaussian distribution. For simplification, the following substitution will be used:

\[ z = \frac{(\ln x - \mu)}{\sqrt{2}\sigma} \]  

(A4.3a)

\[ dz = \frac{1}{\sqrt{2}\sigma} \frac{dx}{x} \]  

(A4.3b)

Substituting A4.3a and A4.3b into A4.2

\[ U(z) = \frac{A}{\sqrt{\pi}} \int_z^\infty e^{-z^2} \, dz \]  

(A4.4)

The slope of the cumulative distribution represents the percentage change in cumulative resource over the percentage change in ore grade, denoted by \( s_c \); remember, however, the ore-grade elasticity is defined as the percentage increase in cumulative resource over the percentage drop in ore-grade, denoted by \( s \), therefore:
Substituting equations A4.3a and A4.3b into A4.5,

\[ s_c = -s = \frac{d\ln U}{d\ln x} = \frac{dU/U}{dx/x} \quad (A4.5) \]

Applying the fundamental theorem of calculus, equation A4.4 may be reconstructed

\[ U(z) = \frac{A}{\sqrt{\pi}} \int_z^\infty e^{-z^2} \, dz = \frac{A}{\sqrt{\pi}} \left[ 1 - \int_0^z e^{-z^2} \, dz \right] \quad (A4.7a) \]

\[ U(z) = -\frac{A}{\sqrt{\pi}} \int_0^z e^{-z^2} \, dz \]

\[ \therefore \quad \frac{dU}{dz} = -\frac{A}{\sqrt{\pi}} e^{-z^2} \quad (A4.7b) \]

Combining equations A4.6 and A4.7b

\[ s_c = \frac{dU(z)/dz}{\sqrt{2\sigma(U(z))}} = \frac{-e^{-z^2}}{\sqrt{2\sigma} \int_z^\infty e^{-z^2} \, dz} \quad (A4.8) \]

As aforementioned, equation A4.8 will be evaluated using a rational approximation technique, an asymptotic series expansion, as well as a semi-analytical approach to the complementary error function.

**Complementary Error Function**

The complementary error function is defined as:

\[ \text{erfc}(z) = 1 - \text{erf}(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-z^2} \, dz \quad (A4.9a) \]

where:

\[ \text{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-z^2} \, dz \quad (A4.9b) \]

For a normalized Gaussian distribution, the complementary error function is introduced to represent the upper tail of the cumulative distribution; this function is useful
considering current uranium mining activities where high ore grades are first exploited as the “low-hanging fruit”. Further, as production (mining or milling) of lower ore grades is increased, there is an increase in total exploitable resource. Note that the Deffeyes’ plot shows a reverse ore grade domain, where the ore grade decreases from left to right.

\[
U(z) = \frac{A}{2} (erfc(z))
\]  

(A4.10)
The rational approximation method is frequently used to approximate the error function, as developed for use within the digital computer industry. The formula applied to approximate the complimentary error function takes the following form and has a maximum absolute error of 2.5\times 10^{-5}

\[
erf(z) = 1 - \left[t_1 + t^2a_2 + t^3a_3\right]e^{-z^2} + \varepsilon(z) \quad \text{for} \quad z \geq 0 \quad (A4.11a)
\]

\[
\therefore \quad \text{erfc}(z) = \left[t_1 + t^2a_2 + t^3a_3\right]e^{-z^2} - \varepsilon(z) \quad (A4.11b)
\]

where

\[
t = \frac{1}{1 + pz}
\]

\[
p = 0.47047
\]

\[
a_1 = 0.3480242
\]

\[
a_2 = -0.0958798
\]

\[
a_3 = 0.7478556
\]

\[|\varepsilon(z)| < 2.5 \times 10^{-5}\]

The following table compares the values obtained using the rational approximation to tabulated values of the complimentary error function [77]:

<table>
<thead>
<tr>
<th>z</th>
<th>t</th>
<th>erf table</th>
<th>rational approximation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.913998249</td>
<td>0.7772974</td>
<td>0.77729742</td>
</tr>
<tr>
<td>0.2</td>
<td>0.841617656</td>
<td>0.5716076</td>
<td>0.5716287</td>
</tr>
<tr>
<td>0.4</td>
<td>0.779859656</td>
<td>0.3961439</td>
<td>0.396142425</td>
</tr>
<tr>
<td>0.6</td>
<td>0.726545653</td>
<td>0.257899</td>
<td>0.257878468</td>
</tr>
<tr>
<td>0.8</td>
<td>0.680054676</td>
<td>0.1572992</td>
<td>0.157283174</td>
</tr>
<tr>
<td>1</td>
<td>0.639155701</td>
<td>0.089686</td>
<td>0.08968754</td>
</tr>
<tr>
<td>1.2</td>
<td>0.602897041</td>
<td>0.0477149</td>
<td>0.047731274</td>
</tr>
<tr>
<td>1.4</td>
<td>0.57053137</td>
<td>0.0236516</td>
<td>0.023673385</td>
</tr>
<tr>
<td>1.6</td>
<td>0.541463663</td>
<td>0.0109095</td>
<td>0.010928788</td>
</tr>
<tr>
<td>1.8</td>
<td>0.515214278</td>
<td>0.0046777</td>
<td>0.004691257</td>
</tr>
<tr>
<td>2</td>
<td>0.491392281</td>
<td>0.0018628</td>
<td>0.001870819</td>
</tr>
<tr>
<td>2.2</td>
<td>0.469675849</td>
<td>0.0006885</td>
<td>0.000692589</td>
</tr>
<tr>
<td>2.4</td>
<td>0.449797636</td>
<td>0.000236</td>
<td>0.000237872</td>
</tr>
<tr>
<td>2.6</td>
<td>0.431533723</td>
<td>0.000075</td>
<td>7.57528E-05</td>
</tr>
<tr>
<td>2.8</td>
<td>0.414695137</td>
<td>0.0000221</td>
<td>2.23581E-05</td>
</tr>
<tr>
<td>3</td>
<td>0.399121295</td>
<td>0.000006</td>
<td>6.11327E-06</td>
</tr>
<tr>
<td>3.2</td>
<td>0.384674861</td>
<td>0.000015</td>
<td>1.54797E-06</td>
</tr>
</tbody>
</table>

As expected, the propagation of the absolute error is less than 2.5\times 10^{-5}:
Figure A4.4: Accuracy using Rational Approximation, Compared with ERFC Table Values

Solving equation A4.8 for the slope of the cumulative distribution in terms of $\text{erfc}(z)$

$$s_c = \frac{-e^{-z^2}}{\sqrt{2\sigma} \int_0^z e^{-z'^2} dz} = -\frac{\sqrt{\frac{\pi}{2}}}{\sqrt{2\sigma} \text{erfc}(z)} \frac{e^{-z^2}}{\text{erfc}(z)}$$  \hspace{1cm} (A4.12)

Combining equations A4.11b and A4.12

$$s_c = -\frac{\sqrt{\frac{\pi}{2}}}{\sqrt{\sqrt{\pi\sigma} \text{erfc}(z)}} e^{-z^2} = -\frac{\sqrt{\frac{\pi}{2}}}{\sqrt{\sqrt{\pi\sigma} \text{erfc}(z)}} \frac{e^{-z^2}}{\text{erfc}(z)} = -\frac{\sqrt{\frac{\pi}{2}}}{\sqrt{\sqrt{\pi\sigma} \text{erfc}(z)}} \frac{1}{\text{erfc}(z)}$$ \hspace{1cm} (A4.13)

The final form of the elasticity is found by cutting off the error term

$$\Rightarrow -s = s_c = -\frac{\sqrt{\frac{\pi}{2}}}{\sqrt{\pi\sigma} \text{erfc}(z)} \frac{1}{\text{erfc}(z)}$$ \hspace{1cm} (A4.14)
Figure A4.5: Ore-grade Elasticity of Uranium Resources Using Rational Approximation

Plotting the slope of the cumulative distribution for ore-grades greater than 100 ppm, a logarithmic regression returned the equation of the slope in terms of ore grade:

\[ s_c = -s = -0.3927 \cdot \ln(x) + 0.6043 \]  \hspace{1cm} (A4.15)

A more accurate rational approximation formula may be employed with a maximum absolute error of \(1.5 \times 10^{-7}\). Although this polynomial fit was not utilized, the fifth-order approximation takes the following form

\[
erfc(z) = \left[ t_1 + t_2a_2 + t_3a_3 + t_4a_4 + t_5a_5 \right] e^{-z^2} - \varepsilon(z)
\]

where

\[
p = 0.3275911 \\
a_1 = 0.254829592 \\
a_2 = -0.284496736 \\
a_3 = 1.421413741 \\
a_4 = -1.453152027 \\
a_5 = 1.061405429 \\
|\varepsilon(z)| < 1.5 \times 10^{-7}
\]
**Complementary Error Function: Asymptotic Expansion**

Asymptotic series expansion was employed to approximate the complementary error function; because high-grade resources are of interest \((10^2 - 10^4 \text{ ppm})\), the subsequent truncation of higher ordered terms greatly simplifies the function. Beginning with equation A4.9a and ‘multiplying by one’ [78]

\[
erfc(z) = \frac{2}{\sqrt{\pi}} \int_{z}^{\infty} \left(\frac{1}{z}\right) e^{-z^2} \, dz
\]  

(A4.16)

Integrating by parts, using the values

\[
\begin{align*}
u &= z^{-1} \\
du &= -z^{-2} \, dz \\
v &= -\frac{1}{2} e^{-z^2} \\
dv &= z e^{-z^2} \, dz
\end{align*}
\]

Equation A4.15 simplifies to

\[
erfc(z) = \frac{2}{\sqrt{\pi}} \left[ \frac{1}{2z} e^{-z^2} - \int_{z}^{\infty} \frac{1}{2z} z^{-2} e^{-z^2} \, dz \right]
\]  

(A4.17)

Completing \(n\) number of iterations of integration, the complimentary error function becomes

\[
erfc(z) = \frac{2}{\sqrt{\pi}} \left[ \frac{1}{2z} e^{-z^2} - \frac{1}{4z^2} e^{-z^2} + \frac{3}{8z^3} e^{-z^2} - \cdots + (-1)^n \frac{1 \cdot 3 \cdots (2n-3)}{2^{n+1} z^{2n+1}} \right] + \frac{1}{2} \cdot 3 \cdots (2n-1) \int_{z}^{\infty} \frac{1}{2n} e^{-z^2} \, dz
\]  

(A4.18a)

\[
erfc(z) = \frac{e^{-z^2}}{\sqrt{\pi z}} \left[ \frac{1}{2z} - \frac{3}{4z^2} + \cdots + (-1)^n \frac{1 \cdot 3 \cdots (2n-3)}{2^{n+1} z^{2n+1}} \right] + (-1)^n \frac{1 \cdot 3 \cdots (2n-1)}{2^n} \int_{z}^{\infty} \frac{1}{2n} e^{-z^2} \, dz
\]  

(A4.18b)

\[
erfc(z) = \frac{e^{-z^2}}{\sqrt{\pi z}} \sum_{n=0}^{\infty} (-1)^n \frac{(2n)!}{n!(2z)^{2n}}
\]  

(A4.19)

For an increasing ore-grade, resulting in increased values for \(z\), the terms within the brackets of equation A4.18b approach one, and thus this equation may be approximated by

\[
erfc(z) = \frac{e^{-z^2}}{\sqrt{\pi z}}
\]  

(A4.20)

Again, comparing the asymptotic expansion with the table values for the complimentary error function, the accuracy of the method may be assessed
### Table A4.2: Asymptotic Expansion vs. Erfc Table Values

<table>
<thead>
<tr>
<th>z</th>
<th>Erfc Table</th>
<th>Asymptotic Expansion</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.7773</td>
<td>2.7103</td>
<td>248.69%</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5716</td>
<td>1.2019</td>
<td>110.27%</td>
</tr>
<tr>
<td>0.4</td>
<td>0.3961</td>
<td>0.6560</td>
<td>65.61%</td>
</tr>
<tr>
<td>0.6</td>
<td>0.2579</td>
<td>0.3719</td>
<td>44.19%</td>
</tr>
<tr>
<td>0.8</td>
<td>0.1573</td>
<td>0.2076</td>
<td>31.95%</td>
</tr>
<tr>
<td>1</td>
<td>0.0897</td>
<td>0.1114</td>
<td>24.20%</td>
</tr>
<tr>
<td>1.2</td>
<td>0.0477</td>
<td>0.0568</td>
<td>18.97%</td>
</tr>
<tr>
<td>1.4</td>
<td>0.0237</td>
<td>0.0273</td>
<td>15.25%</td>
</tr>
<tr>
<td>1.6</td>
<td>0.0109</td>
<td>0.0123</td>
<td>12.52%</td>
</tr>
<tr>
<td>1.8</td>
<td>4.678E-03</td>
<td>5.167E-03</td>
<td>10.45%</td>
</tr>
<tr>
<td>2</td>
<td>1.863E-03</td>
<td>2.028E-03</td>
<td>8.86%</td>
</tr>
<tr>
<td>2.2</td>
<td>6.885E-04</td>
<td>7.408E-04</td>
<td>7.59%</td>
</tr>
<tr>
<td>2.4</td>
<td>2.360E-04</td>
<td>2.515E-04</td>
<td>6.59%</td>
</tr>
<tr>
<td>2.6</td>
<td>7.500E-05</td>
<td>7.932E-05</td>
<td>5.76%</td>
</tr>
<tr>
<td>2.8</td>
<td>2.210E-05</td>
<td>2.321E-05</td>
<td>5.02%</td>
</tr>
<tr>
<td>3</td>
<td>6.000E-06</td>
<td>6.297E-06</td>
<td>4.94%</td>
</tr>
<tr>
<td>3.2</td>
<td>1.500E-06</td>
<td>1.583E-06</td>
<td>5.54%</td>
</tr>
</tbody>
</table>

As a result of the series expansion of the complementary error function, the cumulative lognormal distribution \( U(z) \) is given by

\[
U(z) = \frac{A}{2\sqrt{\pi z}} e^{-z^2/2} \tag{A4.21}
\]

Refring back to equation A4.6

\[
\therefore \frac{dU}{dz} = -\frac{1}{\sqrt{2\sigma(U)}} \left[ \frac{A e^{-z^2/2}}{\sqrt{2\pi z}} \right] = -\frac{2z}{\sqrt{2\sigma}} = -\frac{\sqrt{2}}{\sigma} \left[ \ln(x) - \mu \right] \tag{A4.22}
\]

\[
\Rightarrow s_c = -\frac{\ln(x) - \mu}{\sigma^2} \tag{A4.23}
\]

As evident from Table A4.2, the truncation of terms in the asymptotic series using the “large z” approximation results in an overestimate of the complimentary error function; therefore, it is expected that employing this method will result in an underestimate of the slope of the cdf. An empirical correction can be appended to Equation A4.23 to better approximate the slope of the cumulative distribution and thus the ore-grade elasticity of the Uranium resource over the range of interest.
Complementary Error Function: Semi-Analytical Approximation, Expansion Correction

As previously noted, the "large z" approximation resulted in an underestimated cumulative distribution slope. Therefore, a correction term will be provided through semi-analytical methods in this section. The values for the slope of the cumulative distribution were first linearly regressed as presented in Figure A4.6.

![Elasticity of Uranium Resources using Erfc Table](image)

Figure A4.6: Slope of Cumulative Distribution using Erfc Table Values

The slope of the cumulative distribution as a function of the z variable may be expressed as

\[
s_c = -0.8336 \cdot z - 0.4162 \tag{A4.24}
\]

\[
\Rightarrow s_c = -0.8336(z + 0.4993) \tag{A4.25}
\]

By approximating this function, a rationalized form of equation A4.24 is found. The accuracy of the method will be confirmed.

\[
s_c = -\frac{\sqrt{\pi}}{\sqrt{2\sigma}} \cdot \left( z + \frac{1}{2} \right) \tag{A4.26a}
\]

\[
s_c = -\frac{\sqrt{\pi}}{2\sigma^2} \cdot \left( \ln(x) - \mu + \frac{\sigma}{\sqrt{2}} \right) \tag{A4.26b}
\]
Therefore, applying the semi-analytical correction, the slope of the cumulative distribution is approximated by

\[
  s_c = -0.39 \cdot \ln(x) + 0.55 \\
  \therefore s = 0.39 \cdot \ln(x) - 0.55
\]  \\
(A4.26c) \\
(A4.26d)

Table A4.3 compares the values for the slope of the cdf for the linear fit and rational estimate.

<table>
<thead>
<tr>
<th>z</th>
<th>Erfc Table</th>
<th>Semi-Analytical Approximation</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.5266</td>
<td>-0.4136</td>
<td>21.46%</td>
</tr>
<tr>
<td>0.2</td>
<td>-0.6509</td>
<td>-0.5791</td>
<td>11.04%</td>
</tr>
<tr>
<td>0.4</td>
<td>-0.7851</td>
<td>-0.7445</td>
<td>5.17%</td>
</tr>
<tr>
<td>0.6</td>
<td>-0.9275</td>
<td>-0.9099</td>
<td>1.89%</td>
</tr>
<tr>
<td>0.8</td>
<td>-1.0767</td>
<td>-1.0754</td>
<td>0.12%</td>
</tr>
<tr>
<td>1</td>
<td>-1.2316</td>
<td>-1.2408</td>
<td>0.75%</td>
</tr>
<tr>
<td>1.2</td>
<td>-1.3912</td>
<td>-1.4063</td>
<td>1.08%</td>
</tr>
<tr>
<td>1.4</td>
<td>-1.5546</td>
<td>-1.5717</td>
<td>1.10%</td>
</tr>
<tr>
<td>1.6</td>
<td>-1.7213</td>
<td>-1.7372</td>
<td>0.92%</td>
</tr>
<tr>
<td>1.8</td>
<td>-1.8905</td>
<td>-1.9026</td>
<td>0.64%</td>
</tr>
<tr>
<td>2</td>
<td>-2.0620</td>
<td>-2.0680</td>
<td>0.29%</td>
</tr>
<tr>
<td>2.2</td>
<td>-2.2354</td>
<td>-2.2335</td>
<td>0.08%</td>
</tr>
<tr>
<td>2.4</td>
<td>-2.4102</td>
<td>-2.3989</td>
<td>0.47%</td>
</tr>
<tr>
<td>2.6</td>
<td>-2.5868</td>
<td>-2.5644</td>
<td>0.87%</td>
</tr>
<tr>
<td>2.8</td>
<td>-2.7642</td>
<td>-2.7298</td>
<td>1.24%</td>
</tr>
<tr>
<td>3</td>
<td>-2.9407</td>
<td>-2.8953</td>
<td>1.55%</td>
</tr>
<tr>
<td>3.2</td>
<td>-3.1345</td>
<td>-3.0607</td>
<td>2.36%</td>
</tr>
<tr>
<td>3.4</td>
<td>-3.3494</td>
<td>-3.2261</td>
<td>3.68%</td>
</tr>
</tbody>
</table>

Over the range of interest (equivalent z values of 0.98 to 3.13), the semi-analytical approximation for the slope of the cumulative distribution provides an acceptable fit to the data set and thus will be representative of the ore-grade elasticity.
B Formulation of Cumulative Resource Elasticity

The development of a comprehensive cost vs. cumulative (energy) generation model will be the basis by which Uranium market pricing will be forecasted over the next half-century. This work presents an approach to incorporate the following sub-models: ore grade elasticity (% increase in cumulative resource/ % drop in the cutoff grade of \( \text{U}_{\text{nat}} \)), learning effects, and economies of scale (scale of production). In the model, the resource demand (required \( \text{U}_{\text{nat}} \)) is expressed in terms of GWe-years of nuclear energy using a once-through fuel-cycle, assuming no contribution from secondary fuel supplies. Any superimposition of demand growth scenarios using the model will thus imply market equilibrium (\( \text{U}_{\text{nat}} \) production rates meet industry generating capacity requirements). The cost vs. cumulative consumption (energy generation) model will include Uranium ore pricing ($/kg) relative to a base year (2005) as well as the consumption of Uranium in GWe-yr beyond base year (2005).

Ore Grade Elasticity

In this study, the ore-grade elasticity of the cumulative uranium resource will be extracted from Deffeyes' analysis on uranium concentration in the earth's crust; Deffeyes' observed a lognormal distribution of resource tonnage as a function of ore-grade for the richest geological categories currently found economically exploitable. The grades currently being mined, considered in this model are those with concentrations of 0.01% to 1.0% \( \text{U}_3\text{O}_8 \) (10^2 to 10^4 parts per million). The elasticity as a function of ore-grade, in parts per million (ppm), relates the cumulative resource and ore-grade through the following power law relation:

\[
\left( \frac{U}{U_r} \right) = \left( \frac{X}{X_r} \right)^s
\]

where

\[
s_c = \frac{d\ln U}{d\ln X}
\]

In the above equations, \( U_r \) and \( X_r \) denote reference values for uranium tonnage and cutoff ore-grade, respectively. Additionally \( s_c \) represents elasticity, the slope of the cumulative distribution; \( s \) denotes the negative of the conventional elasticity, an input into the model. Detailed in Appendix A3, recognize that \( s \) will always be a positive value. Because \( U \) is a complementary cumulative distribution function (integral from \( x \) to \( \infty \)), the derivative of the cumulative distribution is a negative value. Hence, this value implies an increase in ore-grade results in a diminishing resource.

Economies of Scale

The effects of the [increased] scale of production on uranium mining activities are observed to lead to a reduction in the cost of additional units of ore excavated and
processed per kilogram of natural uranium. Thus, the unit cost of additional units of natural uranium at a grade of \(X\) ppm is:

\[
\left( \frac{C}{C_r} \right) = \left( \frac{X}{X_r} \right)^n, \quad \text{[$/kg U]} \tag{B.3}
\]

where \(n\) is the scale exponent, with an average value of i.e. 0.7 for many industrial chemical engineering processes [79][80]. If \(n\) is less than 1, economies are realized through increasing scale of production; for values of \(n\) greater than 1, economies are realized through diminishing scales of production (diseconomies of scale). When \(n\) equals 1, there are constant economies of scale, thus no advantage will be gained through increased or decreased scales of production. \(C_r\) and \(X_r\) refer to a time-reference cost and ore grade respectively. Combining the economies of scale with the ore-grade elasticity of the cumulative resource:

\[
\left( \frac{C}{C_r} \right) = \left( \frac{U}{U_r} \right)^n \tag{B.4}
\]

The above expression characterizes cost scaling due to unit operation size (scale of production). In this study, additional savings will be considered due to learning effects during long-term mining operation. Although the literature points out the difficulty in discriminating cost reductions based on economies of scale or learning effects, literature has differentiated the two model concepts by considering economies of scale to be static over a particular production period (production-scale histories are not exploited). Conversely, learning effects are considered dynamic; thus, generational production histories are used to characterize the effects of learned efficiencies on production costs. These learning effects have been observed to improve and advance prospecting for, delineating, and assaying potential deposits.

Learning Effects

Schneider and Sailor note the lack of learning effects in current uranium cost models. Considered "dynamic" economies of scale, learning effects are realized as the decline in production cost of units produced in addition to the cumulative [production history]. From first principles, a linear plot of the log of production cost [$/kg] versus the log of the cumulative units produced is expected.

\[\text{12 The assumption here is for a resource of diminishing ore grade, where the reference grade is higher than present.}\]
Figure B.1: Learning-Cost Archetype

The production cost relation observed in Figure B.1 can be characterized as

\[ \log C = a - b \log P = \log \left( \frac{P}{a^*} \right)^{-b} \]

where

\[ a^* = 10^\frac{a}{b} \]

thus,

\[ \frac{C}{C_r} = \left( \frac{P}{P_r} \right)^{-b} \]

From experience theory, the cost of production declines by a factor \( f < 1 \) for each doubling of produced units, thus:

\[ C(2q) = fC(q) \quad (B.5) \]

where \( q \) is the number of "units" of produced natural uranium, equal to the cumulative resource produced, \( U \) (in metric tons), divided by the learning unit batch size, \( m \) metric tons of \( U_{nat} \) product (not including host media), hence:

\[ q_r = \frac{U_r}{m} \quad (B.6a) \]

\[ q = 2^n q_r \quad (B.6b) \]

where the "learning" generation (compounding cost reductions) is denoted by \( n \). For \( n \) generations of learning
\[ C(2^n q_r) = f C(2^{n-1} q_r) = f^n C(q_r) \]  
(B.7a)

\[ \frac{C(q)}{C(q_r)} = (f^n) \]  
(B.7b)

For typical experience factors, \( f \) ranging from 0.7 to 1.0, \( f \) may be approximated by

\[ f \approx \exp(-4(f-1)) \]  
(B.8)

Additionally, for the total units produced, where learning effects are realized

\[ q = (2)^n q_r \]  
(B.9a)

\[ \therefore n = \ln \left( \frac{q}{q_r} \right) - 1 \ln(2) \]  
(B.9b)

Combining equations B.7b, B.8 and B.9b

\[ \frac{C(q)}{C(q_r)} \approx \left[ \exp(-4(f-1)) \right]^{\ln \frac{q}{q_r} \ln(2)} \]  
(B.10a)

\[ C(q) = C(q_r) \frac{q}{q_r} e^{\frac{(1-f)}{\ln(2)}} \]  
(B.10b)

Thus, a power-law relation is observed

\[ \frac{C(q)}{C(q_r)} = \left( \frac{q}{q_r} \right)^{-\alpha} = (2^n)^{-\alpha} \]  
(B.11)

Combining equation B.11 and B.7b

\[ f^n = (2^n)^{-\alpha} \]  
(B.12a)

\[ \alpha = \frac{-\ln(f)}{\ln(2)} \]  
(B.12b)

The power-law relationship may be confirmed graphically
Figure B.2: Power-law Relationship of Cost and Cumulative Production

Although often reported as a percentage, as an input into the model, \( f \) is to be in decimal form, e.g.- for a representative learning rate [81] of 85%, \( f=0.85 \), corresponding with an \( \alpha \) of 0.2345. Because the unit batch size, \( m \), is constant, the price ratio of the cumulative resource with progressive learning is characterized as

\[
\left( \frac{C}{C_r} \right) = \left( \frac{U}{U_r} \right)^{-\alpha} \tag{B.13}
\]

This model characterizes learning for a batch unit (mass) of processed natural uranium, implying a constant ore grade. If instead the learning unit were the mass of ore mined, scaling would differ.

\[
o_r = \frac{U_r}{X} \tag{B.14}
\]

where \( o \) is the mass of ore mined, and \( X \) is the resource ore grade [mass of \( U_{nat} / \) mass of ore], then

\[
\left( \frac{C}{C_r} \right) = \left( \frac{o}{o_r} \right)^{-\beta} = \left( \frac{U_r}{U_{nat}} \right)^{-\beta} \left( \frac{X}{X_r} \right)^{-\beta} \tag{B.15}
\]

Now assuming an ore grade relation

\[
\frac{dU}{dx} \sim \frac{1}{X^n} \tag{B.16a}
\]
\[ U \sim \frac{1}{X^{n+1}} \Rightarrow X \sim U^{-\frac{1}{n+1}} \]  
(B.16b)

Combining equations B.15 and B.16b

\[
\left( \frac{C}{C_r} \right) = \left( \frac{U}{U_r} \right)^{\frac{1}{(m-1)}} \]  
(B.17a)

\[
\therefore \left( \frac{C}{C_r} \right) = \left( \frac{U}{U_r} \right)^{\frac{\beta m}{(m-1)}} = \left( \frac{U}{U_r} \right)^{\theta} \]  
(B.17b)

Hence, a similar power-law relation is found for learning based on unit ore mass, however of course with different scaling. Although this derivation was provided, the resource elasticity model will utilize the conventional cost of production relation for a unit mass of natural uranium; this measure ensures price predictions evolve with technological improvements and production scale seen in the mining industry. For example, the institution of in situ leaching has resulted in lowered throughputs of ore to obtain an equivalent natural uranium product. As a result, the cost reductions afforded by increased experience with uranium mining must be a function of the unit mass of uranium product to effectively incorporate all types of mining technologies in price predictions.

Integration of Cost Models

Combining the learning effects (equation B.13), economies of scale (equation B.3), and ore grade resource elasticity (equation B.1)

\[
\left( \frac{C}{C_r} \right) = \left( \frac{U}{U_r} \right)^{\theta} \]  
(B.18)

where

\[
\theta = \left( \frac{n}{s} \right) - \alpha \]  
(B.19)

Because of the power law nature of Equation B.9, the model will be plotted on a log-log scale, so as to produce a linear plot. The final adjustment to the resource elasticity model is based on the requirement that LWR's consume a fixed amount of natural uranium per GWe-year of generated electricity (200 MT U\text{nat} at full power), thus:

\[
\left( \frac{C}{C_r} \right) = \left( \frac{G}{G_r} \right)^{\theta} \]  
(B.20)
where \( G \), the cumulative generating capacity is defined as

\[
G = \frac{\eta_{th} \cdot B_d \cdot U}{CF \cdot 365} \tag{B.21}
\]

In equation B.21, \( CF \) represents the nuclear plant capacity factor, \( \eta_{th} \) is the thermal efficiency, and \( B_d \) is the discharge burn-up (in MWd\(_{th}\)/tU), assumed to be constant.
C Probabilistic Estimation of Cost Correlation Parameter, \( \theta \)

Appendix A developed the lognormal equation used to model ore-grade resource elasticity, as developed by K. Deffeyes; Appendix B detailed the derivation of the cost correlation parameter (to characterize the elasticity of the Uranium resource giving the power law relation between the price of Uranium and cumulative nuclear generation) accounting for ore-grade elasticity, experience effects and economies of scale [driving down Uranium production costs]. In this section, probabilistic methods to characterize the following three input variables will guide the development of the simple power-law relation of cost versus cumulative consumption used as a price-forecasting model

- \( n \), the economy of scale exponent
- \( s \), the negative of the resource vs. ore grade elasticity
- \( f \), the percent learning (leading to a)

Detailed in Appendix B, these coefficients combine to yield the cost correlation parameter, \( \theta \)

\[
\theta = \frac{n}{s} - \alpha = \frac{n}{s} + \frac{\ln(f)}{\ln(2)}
\]

(C.1)

Because of the variability in the values for \( n \), \( s \) and \( f \), a significant degree of uncertainty is introduced into the calculation of \( \theta \). As a result, a probabilistic approach adopting Monte Carlo techniques to randomly sample from each of the three probability distributions will be employed to predict Uranium price trends with a level of confidence.

Literature Review of Parameter Values

Range and average values for the parameters \( n \), \( s \), and \( f \) were first produced following review of the applicable literature.

a) Ore-grade Elasticity, \( s \):

The determination of the mean and range of values for the grade elasticity is based on Deffeyes’ observation that uranium resource tonnage is distributed [log] normally as a function of ore-grade for the range of grades currently mined, 1,000 to 10,000 ppm. The range assumed in the price forecast model is extended to grades as low as 200 ppm to allow for technological evolution, and to consider the variability of ore grades within a given mine. Additionally, it is evident that exploitation is not necessarily in descending rank of grade. Accordingly, the grade elasticity ranges from 1.5 to 3 for the range of uranium concentrations considered.

b) Economies of Scale, \( n \):

M.S. Peters [82] reports the production scale parameter values for 28 chemical plants and processes. The mean for the data set is 0.624 and ranges from 0.38 to 0.90. Further, J.L.
Simon [83] suggests an exponent of 0.5 representing aggregate industry scale (scale of industry) as opposed to the scale of an individual production unit. Additionally, cruder models for uranium costs as surveyed by Schneider [81] do not account for scale economies, thus assuming n=1. As a result, the model assumes a uniform distribution with range of 0.5 to 1.0 for the scale economies, n.

c) Learning Effects, f:

Duffey [84] reviews the learning curve and reports a range of values consistent with other findings from 70 to 100% for f. A value of 100% represents a case of no learning, representative of many earlier uranium production cost studies.

Below is the assumed range of values for n, s, and f, suggested by the literature:

\[
\begin{align*}
0.5 & \leq n \leq 1.0 \\
1.5 & \leq s \leq 3.0 \\
0.7 & \leq f \leq 1.0
\end{align*}
\]

Analytical Approach: Uncertainty Propagation

From parameter values obtained in literature, the uncertainty of the cost correlation parameter is characterized and may be calculated at various standard deviations from the statistical mean. The cumulative variance for the dependent parameter, θ is defined as the sum of the variances for the uniformly distributed, independent variables, x_i as follows

\[
\sigma_\theta^2 = \sum_i \left( \frac{\partial \theta}{\partial x_i} \right)^2 \sigma_{x_i}^2 \tag{C.2a}
\]

\[
\sigma_\theta^2 = \left( \frac{d\theta}{dn} \right)^2 \left( \sigma_n^2 \right)_{ave} \left( \sigma_n^2 \right)_{range} + \left( \frac{d\theta}{ds} \right)^2 \left( \sigma_s^2 \right)_{ave} \left( \sigma_s^2 \right)_{range} + \left( \frac{d\theta}{df} \right)^2 \left( \sigma_f^2 \right)_{ave} \left( \sigma_f^2 \right)_{range} \tag{C.2b}
\]

Evaluating equation C.2b using the identity for θ defined in equation C.1

\[
\sigma_\theta^2 = \left( \frac{1}{s} \right)^2 \left( \sigma_s^2 \right)_{ave} \left( \sigma_s^2 \right)_{range} + \left( \frac{-n}{s^2} \right)^2 \left( \sigma_s^2 \right)_{ave} \left( \sigma_s^2 \right)_{range} + \left( \frac{1}{f \ln(2)} \right)^2 \left( \sigma_f^2 \right)_{ave} \left( \sigma_f^2 \right)_{range} \tag{C.2c}
\]

Finally, plugging in the range and mean values for each parameter, the standard deviation of the distribution is found.

\[
\sigma_\theta = 0.173
\]

Additionally, the average value of the distribution is found by evaluating equation C.1 for the average parameter values; hence, σ.
Using the standard normal distribution table, confidence levels can be assessed for various values of the cost correlation parameter. Note, when reading the normal distribution table, values should be shifted so that at 0, the confidence is 50%; additionally, 1.0 reads one deviation from the mean, 2.0 for two standard deviations, and so on. Table C.1 reports the confidence levels (%) for corresponding values of the cost correlation parameter, \( \theta \).

**Table C.1: Cost Correlation Value and Corresponding Confidence Level**

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>Confidence Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.247</td>
<td>2.28%</td>
</tr>
<tr>
<td>-0.074</td>
<td>15.9%</td>
</tr>
<tr>
<td>0.099</td>
<td>50.0%</td>
</tr>
<tr>
<td>0.272</td>
<td>84.1%</td>
</tr>
<tr>
<td>0.445</td>
<td>97.7%</td>
</tr>
</tbody>
</table>

**Monte Carlo Methods**

A common approach to measure uncertainty, useful for assessment of future options, involves Monte Carlo analysis; in such analysis, input variables are randomly sampled from their probability distributions, resulting in the characterization of uncertainty for the output variable. Random sampling of \( n, s \) and \( f \) to calculate probabilistic values of \( \theta \), the following code was written to obtain the cumulative distribution of \( \theta \).

```plaintext
% Isaac Matthews
% Monte Carlo Simulation of theta

% Plan:
% 1) random generation of n, s, and f
% 2) calculate theta (sample size = 10,000)
% 3) create a bin frequency and plot frequency and cumulative dist

iter=10000; %sample size
size=20;   %# of bins
bin=zeros(size,1);
t=zeros(iter,1); %stores all values of theta
% c=zeros(size,1);

for i=1:iter
    n = 0.5 + 0.5*rand;
    s = 1.5 + 1.5*rand;
    f = 0.7 + 0.3*rand;
    theta = n/s + (log(f)/log(2));
    t(i)=theta;
end
```
for k=-0.25:0.05:0.70
    v=round(20*k+6);
    for i=1:iter
        if t(i)<=k;
            bin(v)=bin(v)+1;
        end
    end
end

c=bin/iter; %normalizes scale

for i=1:20
    plot((i-6)/20,c(i),'--rs');
    hold on
    grid on
    axis([-0.2,0.5,0,1]);
end

x=linspace(-0.25,0.7,20);
plot(x,c)
grid on
axis([-0.2,0.6,0,1])

x=-0.25:0.05:0.70;
din(20)=10000-bin(20);
for i=19:-1:1 %discrete bin values
    din(i)=bin(i+1)-bin(i);
end

plot with data points, for purposes of curve fitting
for i=1:20
    plot((i-6)/20,din(i),'--rs');
    hold on
    grid on
    axis([-0.25,0.6,0,1200])
end

bar(x,din)

Figure C.1 plots the resulting cdf and smoothed polynomial curve fit, generated using a sample size of $10^4$, and sorted into twenty $\Delta \theta$ bins.
A ninth-order polynomial fit provided a continuous function to approximate the cdf of $\theta$; the fit was used to determine $\theta$ values corresponding to confidence levels of $\leq 15\%$, $\leq 50\%$, and $\leq 85\%$, where $\theta = -0.10$, 0.11, and 0.29, respectively. Additionally, as denoted in Figure 2.7, these theta values correspond to “optimistic”, “pragmatic” and “pessimistic” cost brackets, respectively. Note in Table C.1 as well as Figure C.1 that some values of $\theta$ are negative, implying that given appropriate values of $n$, $s$, and $f$, forecasted diminishing metal costs would be consistent with historical records over the course of the 20th century. Table C.2 compares the uncertainty propagation and Monte Carlo cdf results. In general, the leftward skew of the Monte Carlo results cause a certain degree of disagreement with the uncertainty propagation findings.
Table C.2: Monte Carlo vs. Uncertainty Propagation CDF Methods

<table>
<thead>
<tr>
<th>Confidence</th>
<th>Uncertainty Propagation</th>
<th>Monte Carlo</th>
<th>Relative Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_2$</td>
<td>2.28%</td>
<td>-0.247</td>
<td>-0.23</td>
</tr>
<tr>
<td>$\theta_\alpha$</td>
<td>15.90%</td>
<td>-0.074</td>
<td>-0.091</td>
</tr>
<tr>
<td>$\theta$</td>
<td>50.00%</td>
<td>0.099</td>
<td>0.11</td>
</tr>
<tr>
<td>$\theta_{\alpha}$</td>
<td>84.10%</td>
<td>0.272</td>
<td>0.286</td>
</tr>
<tr>
<td>$\theta_{2\alpha}$</td>
<td>97.70%</td>
<td>0.445</td>
<td>0.445</td>
</tr>
</tbody>
</table>

The probabilistic data can also be plotted as a frequency distribution (pdf), as shown in Figure C.2. As expected by the central limit theorem, the distribution of a significantly large sample size of independent random variables (with finite mean and variance) may be approximated as a Gaussian.

Figure C.2: Frequency Distribution for $\theta$

Although crude, the developed model provides a framework by which future improvements can be made. Superior understanding of uranium geochemistry and mining experience may later translate into improvements to the price forecasting model by specifying the frequency distribution functions of $n$, $s$, and $f$, beyond the assumption of uniformity.