

**The Real Estate Risk Management Process:  
Integrating Tools from Other Disciplines**

by

**Paul E. Adornato**

**Bachelor of Arts  
Dartmouth College  
(1986)**

Submitted to the Department of Urban Studies and Planning  
in Partial Fulfillment of the Requirements for the Degree of

**Master of Science in Real Estate Development**

at the  
**Massachusetts Institute of Technology**

**July 1992**

© Paul E. Adornato 1992  
All Rights Reserved

The author hereby grants to MIT permission to reproduce and to  
distribute publicly copies of this thesis document in whole or in  
part.

Signature of Author \_\_\_\_\_  
Department of Urban Studies and Planning  
July 31, 1992

Certified by \_\_\_\_\_  
Thomas A. Steele III  
Chairman, Center for Real Estate

Accepted by \_\_\_\_\_  
Lawrence S. Bacow  
Chairman, Interdepartmental Degree Program in  
Real Estate Development

Rotch

MASSACHUSETTS INSTITUTE  
OF TECHNOLOGY

SEP 02 1992

LIBRARIES

**The Real Estate Risk Management Process:  
Integrating Tools from Other Disciplines**

by  
Paul E. Adornato

Submitted to the Department of Urban Studies and Planning in  
Partial Fulfillment of the Requirements for the Degree of  
Master of Science in Real Estate Development.

**ABSTRACT**

Risk, defined as the volatility of investment returns, is an important consideration in the real estate decision-making process. Current real estate risk management models are largely based upon securities investment models of Modern Portfolio Theory (MPT). However, these models have two principal drawbacks: they ignore some of real estate's unique characteristics, and they require data which are unavailable or difficult to estimate for real estate. This thesis introduces tools from other disciplines to model the early stages of the risk management process: risk identification and categorization.

Four distinct models are explored. Chapter 2 applies the theory of comparative advantage to real estate risk and suggests an analytical tool for risk management. Chapter 3 considers the options characteristics of real estate and explores how options pricing models may apply to real estate valuation. Chapter 4 notes empirical findings which question the applicability of expected utility theory and highlights the advantages of explicitly considering downside risk. Chapter 5 reports on a generalization of a widely-accepted investment model, the Capital Asset Pricing Model (CAPM), which may be more suitable to real estate than its more narrow form. Chapter 6 concludes.

Thesis Supervisor: Thomas A. Steele III  
Chairman, Center for Real Estate

## Table of Contents

Abstract.....	2
Chapter 1: Real Estate Risk.....	4
Chapter 2: Comparative Advantage in Real Estate.....	14
Chapter 3: The Options Characteristics of Real Estate.....	25
Chapter 4: Modified Expected Utility Theory.....	33
Chapter 5: A Consumption-Based CAPM.....	44
Chapter 6: Conclusions.....	51
Bibliography.....	54

## Exhibits

1.1 Systematic and Non-Systematic Risk.....	5
1.2 Sources of Return Variation.....	7
1.3 The Risk Management Process.....	9
2.1 Plot of Real Estate Participants.....	17
2.2 Plot of Components of Real Estate Risk.....	19
4.1 Classic Expected Utility Theory.....	34
4.2 Probability Distribution of Expected Returns.....	38
4.3 Portfolio Returns.....	39
4.4 Volatility Around the Mean.....	41
4.5 Comparison of Risk Measures.....	42
5.1 Capital Asset Pricing Model.....	46

## Chapter 1

### **Real Estate Risk**

**Chapter Summary:** The concept of risk in real estate investment and development is defined. The risk mitigation process is outlined. The shortcomings of current real estate risk mitigation strategies are noted, and a framework for different risk identification and categorization techniques is set.

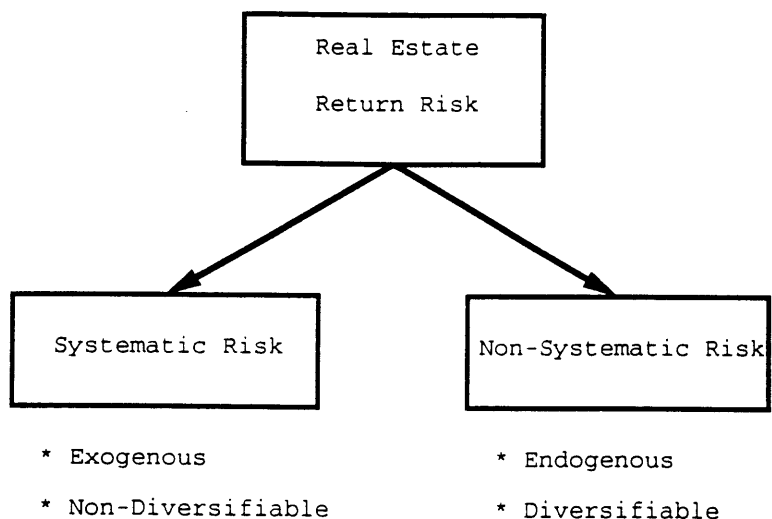
**1.1 Definition of Risk** Risk is common term in everyday language which has a variety of connotations. Risk can be defined in a number of ways, such as: the probability of loss, the probability of not receiving what is expected, or the probability that an investor will not achieve his required rate of return on an investment.

Real estate investment and development is a complex activity which usually spans many months or years. Consequently, real estate investment returns tend to be volatile; that is, returns often vary from period to period. This variation is often difficult to predict. For the purposes of many real estate professionals (and for the purpose of this thesis), risk is defined as the possibility that actual returns are different from expected returns.

This difference may be positive or negative. Real estate returns vary from period to period due to a variety of causes, and it is the goal of this thesis is to introduce tools to better define and evaluate real estate risk.

Just as there are many ways to define risk, there are numerous ways to categorize elements of risk. Many real estate professionals first categorize elements of risk into systematic risks and specific risks. Systematic risks are those risks which affect the entire system or universe of which the investment is a part. Since these risks are exogenous to the system and affect all members of a particular universe, it is not possible to avoid these risks. An example of a systematic risk might be inflation, since inflation would affect all real estate investments, and furthermore, inflation is considered beyond the control of individual investors.

**Exhibit 1.1: Systematic and Non-Systematic Risk**



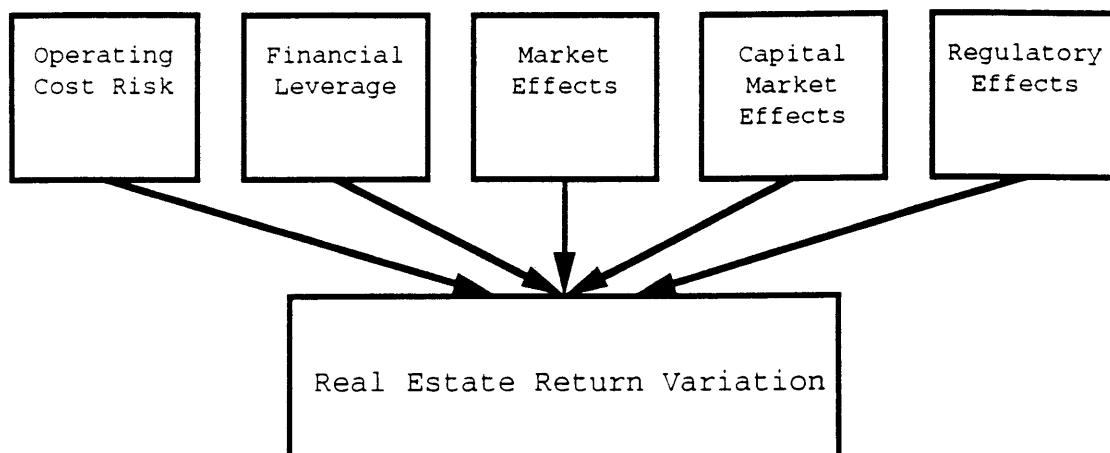
Alternatively, leasing risk is an example of a specific or endogenous risk. Leasing a project is generally within the control of the developer of the project who can use his skill and knowledge of the market and of potential tenants to secure the highest rental rates, the longest lease terms, or the most creditworthy tenants possible.

The division between exogenous and endogenous risks (or, non-diversifiable and diversifiable risks) is often blurred and depends on the boundaries of the universe of the case. For example, a person who invests only in real estate may consider real estate market values an exogenous risk, since variability in the level of real estate values is beyond his control. Alternatively, another investor who owns a portfolio of investments including stocks, bonds, and real estate may not be as impacted by volatile investment returns because she owns assets other than real estate. Real estate market variability is endogenous to her because her portfolio returns may not vary due to changing real estate market values; other assets' returns may offset real estate market variability. Thus, the same element of risk, real estate market variability, may be exogenous to one investor and endogenous to another.

**1.2 Types and Sources of Risk** There are three components of real estate investment returns: the capital investment, the

operating cash flows, and the residual asset value. Each component can be volatile. Capital investment variations may affect the construction or renovation expenses and alter investment return. Cash flow variations are changes in the ongoing operating return stream. Residual value variations arise from appreciation or depreciation of the market value of the property.

**Exhibit 1.2: Sources of Return Variation**



These variations of real estate investment returns have many sources. These are some major categories of sources of variation:

**Operating cost risks** include those sources of variation in the cost of providing the day-to-day services to operate a property, such as janitorial services and security services.

**Financial leverage** exposes investment returns to movements in interest rates or changes in financing terms. The amount of debt and financing terms determine the direction and magnitude of return variations.

**Market effects** such as the local supply and demand for space affect real estate returns.

**Capital market effects** such as the terms and availability of investment monies from banks, insurance companies, pension funds, and the public securities market are likely to affect real estate investment returns.

**Regulatory effects** such as investment tax credits for low income housing can also greatly influence investment returns.

**1.3 The Risk Management Process** Given the complexity of real estate investment, it is important to approach risk management in a consistent and logical manner. Just as there are different ways to define risk and to describe the sources of risk, there are various methods to manage risk. It is important to note that risk management is an ongoing process which should be reviewed frequently, as the sources of risk continually change and interact. The steps in the risk management process are as follows:

**Identify** the broad concepts of real estate investment risk, such as systematic risk and specific risk.

**Categorize** the risks by their source.

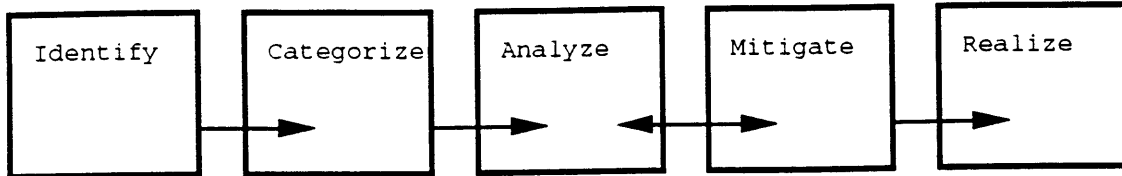
**Analyze** the risks in each category to determine their characteristics, magnitude, and sensitivity under different scenarios.

**Mitigate** the risks by carrying out specific strategies, such as diversification under modern portfolio theory or structuring the deal to hedge or to shift risk.



**Realize** investment returns. The process should be continually reviewed so that risk mitigation strategies can be adjusted to reflect current conditions.

**Exhibit 1.3: The Risk Management Process**



**1.4 Risk Mitigation and Modern Portfolio Theory** Modern Portfolio Theory (MPT) is a collection of investment principles which has gained wide acceptance by institutional investors over the last 20 years. MPT is considered the standard investment philosophy among institutional stock and bond investors. The first principle of MPT was described by Harry Markowitz<sup>1</sup> in 1959; it is the concept of the "efficient frontier." Markowitz held that in order to minimize variability of investment returns between periods, investors should own a portfolio of investments whose returns are not correlated. Further, the efficient frontier represents a trade-off between risk (variability of returns)

---

<sup>1</sup> \_\_\_\_\_, Portfolio Selection - Efficient Diversification of Investments, 1959, Yale University Press, New Haven.

and expected rate of return which maximizes the expected return for each level of risk. Theorists have extended MPT with other investment models.

Applying MPT to stocks and bonds was relatively straightforward, given the liquidity of the securities, the frequency of transactions, the substantial historical trading data, and the overall efficiency of the securities markets. Soon institutional investors extended the diversification theory to argue in favor of real estate investment. Real estate was included in the investment portfolios of institutional investors in order to further diversify their portfolio returns beyond stocks and bonds, due to preliminary data showing low or negative real estate return correlation to stocks and bonds.

Once large-scale institutional real estate investment was accepted, academicians and practitioners sought to apply MPT within their real estate portfolios. Thus, the focus of applying MPT to real estate was narrowed; that is, the diversification principles of MPT were applied within the real estate portfolio rather than viewing real estate as one component of a larger portfolio consisting of stocks, bonds, and other asset classes.

The nature of real estate as an investment vehicle has presented a number of problems in applying MPT. The data required to rigorously implement MPT is lacking for real estate. Standardized operating data and reliable capital appreciation estimates are missing, while the unique characteristics of real estate investments and the illiquidity of real estate further complicate investment analysis for real estate.

A recent article by Eugene Fama and Kenneth French<sup>2</sup> casts serious doubt on the usefulness of the capital asset pricing model (CAPM) as an investment tool. The CAPM is widely used by securities analysts to assess the risk of equity portfolios, and its extension to real estate has been a major goal of real estate investment professionals. (See Chapter 5 for a complete discussion.) The researchers present empirical evidence for stocks which shows that size (measured by the dollar value of traded stock) and book-to-market equity are better predictors of stock variation than beta (historical variation from market returns.) By questioning the usefulness of beta within the CAPM for equity investment management, the paper may also undermine the foundation of one of the major ideas in real estate investment management. Real estate academics and professionals

---

<sup>2</sup> \_\_\_\_\_, "The Cross Section of Expected Stock Returns," J. of Finance, 1992, v.47, n.2, pp.427-465.

have devoted considerable attention to devising a suitable beta, or measure of variability from the market, for real estate.

### **1.5 Risk Identification and Categorization Revisited**

Despite attempts by real estate academics and practitioners, there does not exist a widespread, practical model to manage real estate risk. This paper attempts to survey the literature from fields other than real estate investment in order to introduce notions of risk identification and categorization from those disciplines to real estate investment. Structuring less developed country investments, venture capital investing, option theory, and modified expected utility theory are some fields which may provide a helpful framework in which to consider the identification and categorization of real estate investment risk.

Rather than propose a single model for risk management, this thesis will survey several separate avenues which may suggest direction for further research. Chapters 2 through 5 each adapt a concept of risk from another discipline to real estate. These models attempt to stimulate thought in the early phases of risk management: the risk identification and categorization stages. Chapter 2 applies a fundamental economic principle, the theory of comparative advantage, to real estate risk management. Chapter 3 considers financial

options theory in relation to real estate. Modifications to expected utility theory and downside variance are explored in Chapter 4, and a more generalized version of the CAPM is described in Chapter 5. Chapter 6 provides concluding observations.

## Chapter 2

### **Comparative Advantage in Bearing Risk**

**Chapter Summary:** Comparative advantage is a fundamental economic principle which provides the basis for specialization and trade between parties. Real estate development and investment is a complex activity in which numerous entities interact. Applied to real estate, the concept of comparative advantage in bearing risk provides a useful framework in which to identify and categorize elements of risk in a multi-party context. Furthermore, the framework can provide a basis for devising strategies to mitigate those risks.

**2.1 Introduction** The concept of comparative advantage was applied to structuring alternative financing arrangements for investors in less developed countries by Donald R. Lessard<sup>3</sup>. The idea is extended to real estate in this chapter by providing a map of how real estate fits the concept, providing some examples, discussing how the concept can be applied within the context of modern portfolio theory, and showing what the framework implies for deal structure.

**2.2 The Theory of Comparative Advantage** Eighteenth century economic theorist David Ricardo first described the theory

---

<sup>3</sup> \_\_\_\_\_, "Alternative Finance for Less Developed Countries: A Primer," 1991, unpublished working paper, MIT.

of comparative advantage, a fundamental economic principle which has important implications for specialization and trade. The theory is easily understood by way of a simple example. Suppose that two countries, Japan and China, produce only two goods, televisions and rice. Japan can produce televisions more efficiently than China, and China can produce rice more efficiently than Japan. Each country has an **absolute advantage** in the production of one good. It is to Japan and China's mutual advantage to specialize in producing the good which they can produce most efficiently and to trade for the good they lack. In this way, each country is better off producing what it produces best, and trading some of the surplus for their neighbor's product.

Suppose, however, that Japan is more efficient in making televisions and growing rice; that is, Japan has an absolute advantage in producing both goods. China can produce both goods, too, but it is extremely inefficient in producing TVs but only slightly less efficient than Japan in growing rice. Both countries can be better off (that is, the sum of all goods produced in both countries is maximized) when each specializes in what it has a **comparative advantage**. Japan should produce only televisions and sell some to China. China is less inefficient in producing rice than it is in producing televisions. Japan's gain in producing televisions

more than offsets China's loss of producing rice, and with trade, both countries are better off.

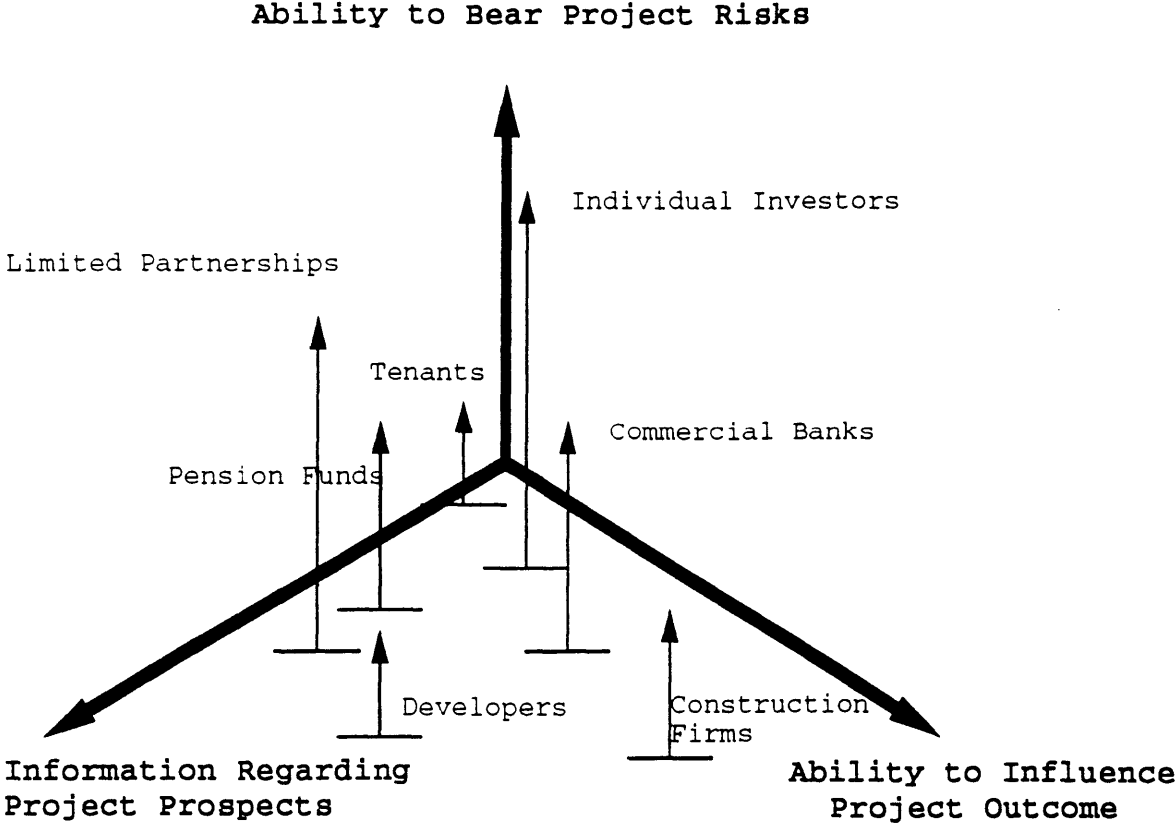
**2.3 Comparative Advantage in Real Estate** Real estate investment and development is rarely a single-party enterprise and often involves many important participants. Various entities supply one or more inputs in the development, operation, and investment process. Such inputs include investment capital, design expertise, construction knowledge, market research, brokerage services, management of the development process, management of the operational phase, and other factors of production. Each participant brings a set of skills and resources which may or may not be deployed in any particular project. There is significant diversity in the goals, skills, resources, and experiences of the participants.

To understand the array of potential resources represented by participants in a development or investment project requires explicit differentiation. One way to compare the participants is to plot them in a three dimensional chart, as shown in Exhibit 2.1. The axes used to place the participants are: the ability to diversify, the ability to influence project outcome, and the level of information regarding the project. These axes are now more fully defined.



For an investor, the **ability to diversify** can be thought of as the ability to hold assets of different types or geographic locations, or the ability to hold a diverse portfolio of stocks, bonds, and other investment vehicles. For a construction firm, ability to diversify might mean the ability to engage in projects spanning different time periods or the ability to seek projects of varying construction types (industrial site-cast buildings and multi-family masonry structures.)

**Exhibit 2.1: Plot of Real Estate Participants**



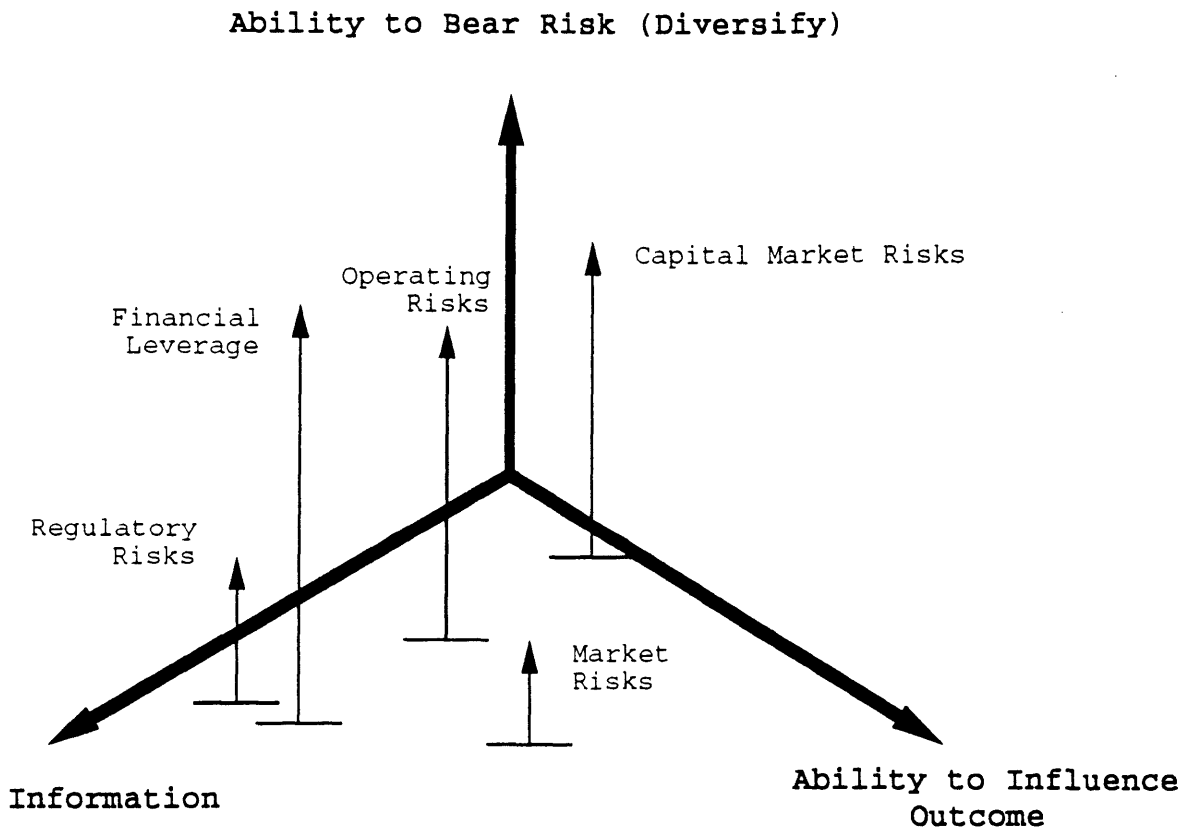
The **ability to influence project outcome**, the second axis, is fairly self-explanatory. It is the ability to influence any of the three characteristics of investment return from Chapter 1: the capital investment, the ongoing cash flow, or the residual value. A property manager may have the ability to keep operating costs low, while a large insurance company may be able to provide a favorable financing package.

The third axis, the **ability to gain information regarding the project prospects**, may be thought of as the ability to predict outcomes which might affect the project. A large brokerage firm may have information regarding future market demand, rental rates, and tenant needs. An architectural firm may know how a project design can accommodate future telecommunication or computer expansion lines.

As illustrated in the diagram, participants of a specific type tend to occupy certain sections of the diversification-influence-information space; however, among different players of the same general type, there may be considerable variation in position. The size of the firm, the capital resources and the human talent all affect firms' delineation.

The components of risk, such as operating risk, financial leverage, local market effects, capital market effects, and regulatory effects, can be similarly graphed in the three dimensional space. As illustrated in Exhibit 2.2, the position of the sources of risk may vary substantially for different participants.

**Exhibit 2.2: Plot of Sources of Real Estate Risk**



For example, one development company may be a large nationwide firm with considerable experience and expertise building large projects throughout the country. It may have relationships which allow it to gain an informational advantage or which allow it to exert influence over other participants. Alternatively, another developer may be a smaller regional player which seeks a project outside of its primary market area where its information and ability to influence are relatively low. This would suggest that the same project would have a different configuration of the components of risk in the three dimensional space, based on the individual firm's characteristics and expertise. In the matrix, it is better to be far from the origin (the point of greatest risk.)

In a similar fashion, the traditional components of real estate risk may be evaluated from the perspective of the individual parties. Factors such as inflation, tax effects, financing, leasing, and market risks may be either endogenous (specific) or exogenous (systematic) to different players depending upon their size, experience or expertise. While the large national developer may consider regional economic trends diversifiable, a small regional developer operating entirely within the region may not be able to diversify away from the effects regional economy. For the same potential project, each developer would have a

different matrix, and should seek other participants -- financial partners, contractors, etc. -- who have the best complementary "fit" with their project risk matrix.

In current real estate investment literature, the analysis often begins after the categorization into endogenous or exogenous risks has been made. This approach avoids explicit acknowledgement of the richness of the possibilities, thereby neglecting the relative nature of risk categorization. This, in turn, limits the range possible strategies for mitigation of risk.

**2.4 Real Estate Applications** Applying the concept to real estate development and investment confirms why deal structuring plays such an important role in risk management. Development deals or real estate investments should be "engineered" so that ideally the party best able to mitigate a particular risk bears that risk.

For example, in a joint venture between a developer and a financial partner, the developer should bear the risk of (and receive a reward for) the maintaining the construction budget, since the developer has the power to control subcontractor construction costs and oversee the day-to-day construction process. Likewise, the financial partner presumably has substantial capital resources and cash

management expertise, which implies that it should be able to diversify and hedge interest rate fluctuation. Even inflation, which is typically considered an exogenous risk to most real estate market participants, may be better tolerated by international investors who may diversify globally.

The crisis unwinding in the real estate and savings and loan industries provides an example of poor allocation of risk. Developers eager to build and bankers eager to lend found a mutually beneficial partnership in which profits from development and lending exceeded the perceived risks. Depositors did not need to scrutinize the lending activity of their bank, since deposit insurance eliminated their risk. The Federal government was an absentee partner in the deals, and in the form of low cost deposit insurance, assumed much of the risk of real estate development from all of the other participants with little ability to control or mitigate that risk.

**2.5 Implications for Deal Structure and Evaluation** The framework suggests that potential parties in a development or investment deal not only analyze the deal in the context of their optimal portfolio and preferred risk profile but also consider the risk mitigation ability of their co-participant. Those risks which cannot be borne efficiently

by one participant should be shifted to another party. This "risk engineering" is the key risk mitigation strategy of the comparative advantage framework. In this way, the sum of the risk of all parties is minimized. Under comparative advantage, the sum of the risk of both parties is minimized when each party assumes the risk it is best able to manage.

The risk engineering aspect of comparative advantage, in which participants actively seek to construct a portfolio of investments or business relationships, implies that the nature of prospective real estate deal evaluation may change. In addition to finding the team of participants which can best bear the project risks, each participant must incorporate the project within its portfolio of projects. The matrix analysis which determines the optimal fit between parties can also be helpful in evaluating how the project fits the rest of the portfolio.

**2.6 Integration within Modern Portfolio Theory** The comparative advantage framework is consistent with risk mitigation according to the principles of modern portfolio theory. As currently practiced by institutional real estate investors, MPT aims to reduce the volatility of the portfolio return by including investments in the portfolio whose returns are not correlated. Risk engineering and the comparative advantage framework may enhance the portfolio

returns by allowing investment managers to structure a deal to fit the goals of entire portfolio. Thus, the concept not only enhances for portfolio diversification to mitigate risk, but also suggests risk engineering strategies to select an individual's optimal risks.

**2.7 Conclusion** Comparative advantage in real estate provides an additional framework through which to consider risk. The concept has appeal because it explicitly evaluates risk from the risk bearer's perspective. The categorization provides a starting point to perform risk engineering to mitigate risk, the next step in the risk management process. With each party bearing the risk it is best able to mitigate, the technique implies that the total risk of the deal is minimized. Additionally, the comparative advantage concept is not inconsistent with MPT and may enhance attempts to apply MPT by engineering a portfolio which achieves the diversification goals of MPT.



## Chapter 3

### **The Options Characteristics of Real Estate**

**Chapter Summary:** Real estate investment and development analysis uses securities investment models which ignore some unique characteristics of real estate, namely the ability to delay or modify decisions. Option pricing models may be extended to real estate and can be useful to real estate decision makers in identifying and categorizing risk.

**3.1 Introduction** Traditional Net Present Value (NPV) analysis holds that a project should be undertaken if the probability-adjusted present value of all expected cash flows, discounted at an appropriate rate, is greater than the project cost. However, there are other characteristics of real estate investment that make this decision rule less than complete. Two of these characteristics are: expenditures that are largely irreversible, and decisions that can often be delayed.

Real estate decision makers often use securities investment models or capital budgeting models, which do not emphasize the unique characteristics of real estate. By considering models which incorporate these unique characteristics, real estate decision makers may better understand the elements of risk in the identification and

categorization phases of risk management. Robert S. Pindyck<sup>4</sup> applies option pricing theory to investment decisions and extends the model to consider a multi-period, multi-option case. The extensions capture some unique characteristics of real estate which traditional securities investment models ignore.

**3.2 The Irreversible Nature of Real Estate** Although most types of institutional-quality real estate, namely office, industrial, retail, and multi-family residential, are long lived assets, the notion of irreversibility has some relevance to real estate. Office and retail space often require significant capital expenditure to refit them if there is a change in tenancy. Industrial space is frequently built to the specifications of a particular industry or production process, and would lose value if it had to be adapted to another user. Repositioning multi-family residential units by reconfiguring unit layouts or making other structural changes similarly can incur significant costs, which are sometimes borne by the investor.

Although some value remains in repositioned real estate projects, the expected profit margin may shrink

---

<sup>4</sup> \_\_\_\_\_, "Irreversibility, Uncertainty, and Investment," J. of Economic Literature, 1991, v.29, pp.1110-1148.

significantly or disappear. Even if improvements to the land can be reused or adapted for a different use, there may often be significant erosion in value to the initial investor. The case can be made that real estate may lose much of its value if the original user or intended function changes. In this way, real estate development resembles a sunk cost.

**3.3 Options in Real Estate** Many real estate development decisions clearly have the characteristics of financial options. Site control is often obtained by paying the owner for an option to purchase the land within a specified time period for a specified price. This is similar to a call option in securities markets, in which the holder of the option retains a right to purchase a specified security at a specified price within some predetermined time period.

There are other aspects of real estate development and investment which have characteristics of options. Consider that projects can be built in phases in order to obtain more information about market demand before committing more funds to the project. Developers of residential subdivisions can gain insights into the home buyers' preferences before building later stages. Suburban mall developers often own adjacent land and may choose to expand the mall if conditions are favorable.

Thus, many development decisions may be delayed to gain more information and to reduce uncertainty about the project. In so doing, developers undertake a project when the risk profile of the project meets their desired risk profile. Market uncertainty, a systematic risk, becomes more of a specific or diversifiable risk when more information is obtained. Thus, although the risk is still present, there may be more effective alternatives to mitigate that risk later in the risk management process.

Similarly, investors may choose to purchase a fully leased and operating project rather than invest during the development stage in order to achieve a lower risk profile. In a real estate portfolio context, the volatility of returns and the correlation of returns to other assets of a new project may be unknown, so waiting to observe actual performance provides insight into the project's pattern of volatility. The decision to delay an investment decision may be considered a risk mitigation strategy.

**3.4 A Numerical Example** The concepts introduced thus far can be made clear through a simple numerical example. Consider a retail development in which the cost of construction is \$1,000,000, construction is instantaneous (that is, construction is completed this year, Year 0) and the rental income realized from the development will be

\$100,000 in the first year. Next year, the rental income will either rise to \$150,000 or fall to \$50,000 per year, and remain at that level forever. The probability of a rise is 50%, and the probability of a fall is 50%. Further assume a 10% discount rate. Calculating the NPV yields a result of:

$$-1,000 + \sum_{t=0}^{\infty} \{100/(1.1)^t\} = \$100$$

The NPV is positive, so the project should be undertaken.

This conclusion is misleading because it ignores an option, the option of waiting one year to see if the market rental rates will increase or decrease. The cost of this option is the opportunity cost of not investing now (that is, not receiving the first year's rental income) and possibly missing the "window of opportunity" due to the entry of other competitors. To see this, consider the NPV of waiting one year and investing only if the market rental rates increase. Note that in Year 0 there is no expenditure and no income, because we have decided to wait. In Year 1, there is only expenditure if the rental income increases. This has a probability of 50%.

$$.5 [-1,000/1.1 + \sum_{t=1}^{\infty} \{150/(1.1)^t\}] = \$295$$

The NPV today is higher (\$295 instead of \$100) if we wait one year before making the investment decision. Waiting is clearly better than investing now.

Note that if the only choice was between investing today or not, we would still invest today. If we had the option to wait until next year to make the decision, we would prefer to wait. What is the value of having the flexibility to invest later? The option must have some value because there is a higher NPV when waiting than not waiting. The value of the flexibility is simply the difference in the NPVs, namely,  $\$295 - \$100 = \$195$ . The cost of the option is the opportunity cost of not completing the project in the earlier period: foregone income, possible loss of favorable financing terms, or discouraging the entry of competitors.

Pindyck also proves that traditional securities option pricing models give the same result as the NPV analysis. Although these option pricing models hold appeal as a framework in which to evaluate real estate risk, there are some conceptual omissions in applying them to real estate. Option pricing real estate assumes that a portfolio consisting of the hard asset (the retail center) and the income stream (from the rentable square feet) can be rebalanced from period to period. In real life it is difficult to imagine easily changing a project's rentable square feet.

Majd and Pindyck<sup>5</sup> introduce three characteristics to the options pricing model which makes it much more appealing for real estate applications. First, in the modified model, both investment decisions and cash outlays occur sequentially over many time periods. The decision to continue investment takes place in stages as more information becomes available. This gives the project the properties of a complex option. Second, there is a maximum rate at which cash outlays and construction can occur; that is, the project takes time to build. Third, the project does not yield any cash return until it is completed.

The investment decision model has two input variables: the total amount of investment remaining for completion, and the current market value of the completed project. The control variable is the rate of investment. The problem is to solve for the rate of investment which maximizes the value of the completed project. Majd and Pindyck solve the equation and demonstrate how the total value of the development program can be determined for various remaining amounts of investment and various market values of the completed project. The work is important because it represents a realistic model to quantify real estate investment risk.

---

<sup>5</sup> \_\_\_\_\_, "Time to Build, Option Value, and Investment Decisions," J. of Financial Economics, 1987, v.18, pp.7-27.

**3.6 Conclusion** The modified options decision model holds appeal as a tool to evaluate real estate risk identification and categorization. Investors can consider elements of risk, such as future market rental rates, construction costs, or interest rates, in the context of delaying investment decisions.

The model holds practical appeal as well, because the estimates required to produce a sensitivity analysis are typically made in traditional NPV analysis. The model also holds promise because it explicitly considers elements of real estate development and investment, namely the multi-period construction phase and the multi-option decision array. The model departs from the current focus of real estate investment theory by shifting estimates away from real estate market estimates (estimating a real estate beta) to property-based estimates (estimating a range of potential project costs and returns.)



## Chapter 4

### **Modified Expected Utility Theory**

**Chapter Summary:** There is empirical evidence which suggests that classic expected utility theory does not accurately model investment behavior in some extreme situations. Modifications of classic expected utility theory are proposed and described, and their implications for real estate risk identification and categorization are discussed.

**4.1 Introduction** Classic expected utility theory has long been a part of investment decision making, including real estate investment decisions. 18th century theoretician Bernoulli described the way people made decisions under risk or uncertainty by quantifying possible investment returns and assigning probabilities to those outcomes. The product of the possible outcome and the probability of occurrence yields the expected utility, and the decision maker would choose the project which maximizes the expected utility. Exhibit 4.1 provides a simple illustration.

### Exhibit 4.1: Classic Expected Utility Theory

<b>Project A</b> -----		<b>Project B</b> -----	
50% chance to earn \$1,000		60% chance to earn \$900	
50% chance to earn \$0		40% chance to earn \$0	
<b>Expected Utility</b> -----			
Project A	$(.5) \times (1,000)$	+	$(.5) \times (0) = \$500$
Project B	$(.6) \times (900)$	+	$(.4) \times (0) = \$540$

Project B has a higher expected utility, therefore, all other things equal, it is preferred over Project A.

**4.2 Modifications** Recent empirical research by Kahneman and Tversky<sup>6</sup> suggests that expected utility theory does not adequately model some special cases of decision-making. They have described several effects which suggest that classic expected utility theory may not hold in boundary situations.

The first anomaly is called the "**certainty effect.**" Subjects were asked to choose between (a) an 80% chance of winning \$4,000 or (b) a 100% chance of winning \$3,000. More than 80% of the experimental subjects chose option (b) even though the expected utility of (b),  $(1.0) \times (3,000) = \$3,000$ , is less than the expected utility of option (a),

---

<sup>6</sup> \_\_\_\_\_, "Prospect Theory: An Analysis of Decision Under Risk," Econometrica, 1979, v.47, pp.263-291.

$(0.8) \times (4,000) = \$3,200$ . Thus, given the choice between a greater but uncertain gain and a lesser but certain gain, most subjects in the sample tended to be biased toward risk-aversion, weighting the certainty of gain more than the magnitude of potential gain.

The certainty effect suggests that there may be systematic mispricing of assets by market participants. This means that "risky" investments (those with volatile returns) may be priced cheaply; that is, an investor who purchases such an asset may receive returns which more than compensate for the higher level of risk or volatility because other investors avoid them and drive the price down. For example, if institutional real estate investors shun hotel investments because of the high volatility of returns, those investors who do purchase hotels will receive returns which overcompensates them for the amount of risk they have assumed.

A second interesting anomaly is called the "**reflection effect**," which deals with risk preference in an environment of likely loss. Faced with the choice between (a) a loss of \$4,000 with 80% probability or (b) a loss of \$3,000 with 100% probability, most test subjects chose (a) even though its expected loss (\$3,200) is greater than option (b)'s

\$3,000 loss. Apparently, most subjects became more risk seeking when the prospect of loss was high.

Again, this finding may have relevance for real estate investors. Faced with the prospect of realizing returns slightly below expectations or making an additional capital investment (with potentially larger loss) to possibly achieve expected returns, the theory suggests that developers would choose the latter, all else equal. These two modifications suggest that there may be a bias to behave differently from what the decision rules prescribe.

In terms of assembling a portfolio of investments, Ruhnka and Young<sup>7</sup> hypothesize that an investor undertakes a two-step process. First, the investments are screened according to the potential magnitude of loss. Those possessing an acceptable level of risk in terms of the probability and magnitude of potential loss are then evaluated in terms of highest expected gain or maximum expected utility. Thus, downside risk becomes an important determinant of investment behavior and an important subject for investment behavior research.

---

<sup>7</sup> \_\_\_\_\_, "Some Hypotheses about Risk in Venture Capital Investing," J. of Business Venturing, 1991, v.6, pp.115-133.

**4.3 The Importance of Downside Risk** The empirical studies discussed above provide evidence that investors behavior does not fit traditional expected utility theory. The traditional CAPM definition of risk as variability of expected returns has been challenged by some researchers. Sortino and Van der Meer<sup>8</sup> observe that investors are not disappointed about variability of returns on the upside. They argue that downside variability is more important in decision-making than variability in general.

Other academicians have also explored the way downside risk affects investment decision-making. Arnott and Bernstein<sup>9</sup> believe that elimination of risk does not refer to elimination of variability; rather, risk refers to the risk of having insufficient assets to meet obligations or to achieve desired minimum returns.

Hagigi and Kluger<sup>10</sup> fault the traditional risk definition and offer a "safety first" rule which avoids downside risk. Such models hold intuitive appeal, especially for real

---

<sup>8</sup> \_\_\_\_\_, "Downside Risk," J. of Portfolio Management, 1991, v.17, n.4, pp.17-21.

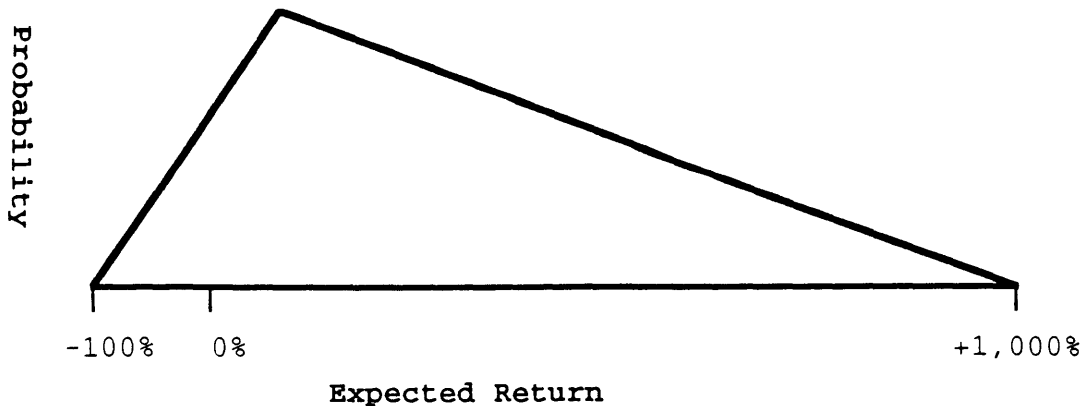
<sup>9</sup> \_\_\_\_\_, "The Right Way to Manage Your Pension Fund," Harvard Business Review, 1988, n.1, pp.95-102.

<sup>10</sup> \_\_\_\_\_, "Safety First: An Alternative Performance Measure," J. of Portfolio Management, 1987, v.13, n.4, pp.34-40.

estate portfolio managers who strive to beat a real estate market proxy or invest to meet some minimum return.

A typical simplification of the traditional investment models assumes that security returns are normally distributed. Most practitioners would agree that the probability of investment returns is more often skewed, with more variability on the upside as shown in Exhibit 4.2. If risk is defined as variability of return, then the best possible outcome, +1,000%, would be deemed the most risky outcome.

**Exhibit 4.2: Probability Distribution of Expected Returns**

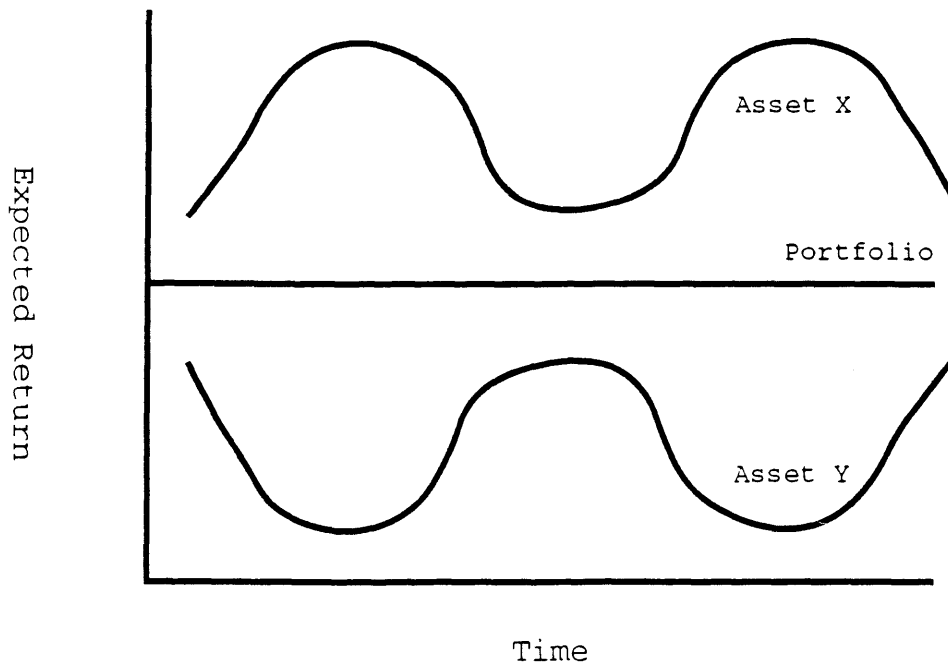


Sortino and Van der Meer introduce the concept of Minimal Acceptable Return, which is described as the minimum return

required to accomplish some investment goals. Only potential returns which fall below the MAR are defined as risky, and the farther below they fall, the greater the risk. Variation above the MAR and variation below the MAR can be explicitly considered.

Other scenarios in a portfolio context underscore the shortcomings of the traditional definition of risk. In a two asset portfolio, traditional definitions of risk mitigation in an MPT context would strive to minimize variability of returns. Exhibit 4.3 shows how returns on two assets are expected to vary over time.

**Exhibit 4.3: Portfolio Returns**

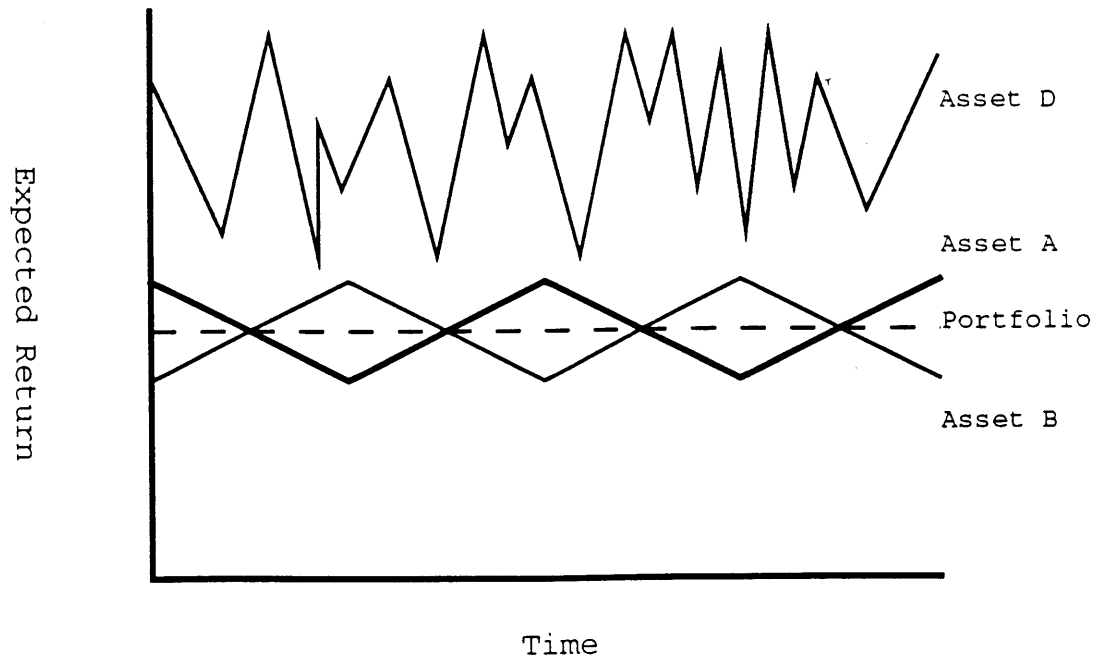


In order to minimize variability of returns, an investor should invest equal amounts in asset X and asset Y. According to Markowitz' mean-variance efficiency (a cornerstone of MPT), the combined portfolio is preferable to holding only asset X, because its expected volatility is zero. This is clearly misleading, because all investors should choose to invest only in Asset X regardless of risk tolerance or MAR.

Another illustration points out the problem of identifying riskless assets in terms of variability. As Exhibit 4.4 shows, combining assets A and B in a portfolio would provide a portfolio with zero variance and low volatility about the mean; however, the better investment is Asset D. Clearly, traditional measures of volatility do not capture the distinct effect of downside volatility. Corporate pension fund advisors, subject to the Employee Retirement Income Security Act (ERISA) of 1974, became particularly concerned with meeting minimum investment goals by avoiding downside risk. Although ERISA's "prudent man" rule was initially misinterpreted among pension fund trustees to imply avoidance of downside volatility on an asset-by-asset basis, subsequent clarifications by the Department of Labor relaxed this strict interpretation. Nevertheless, pension fund sponsors remain concerned with meeting minimum return objectives.



#### Exhibit 4.4: Volatility Around the Mean

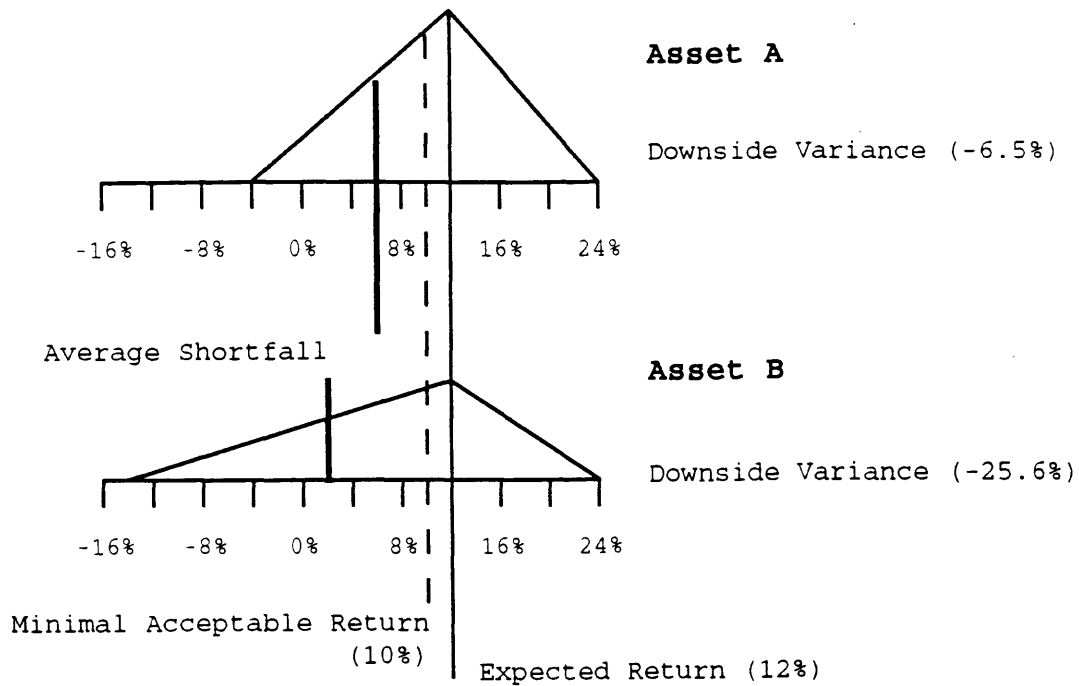


A measure of the risk of falling below the MAR is provided by Fishburn<sup>11</sup>. His calculation of downside variance is based on a probability-weighted function of deviations below a specified level, the MAR. The model addresses the weaknesses of the traditional mean-variance risk model while remaining compatible with Modern Portfolio Theory (MPT). Exhibit 4.5 illustrates the difference in downside risk measures. Although both assets have the same expected return of 10% and the same standard deviation of 6%, the two assets clearly are not equivalent. Asset B has much more downside variance.

---

<sup>11</sup> \_\_\_\_\_, "Mean-Risk Analysis with Risk Associated with Below Target Returns," Am. Economic Review, 1977, v.67, n.2, pp.116-125.

**Exhibit 4.5: Comparison of Risk Measures**



Research by Harlow and Rao<sup>12</sup> demonstrate that the downside variance measure can be incorporated into a CAPM framework. Whereas the traditional CAPM models variability around the mean, the downside variance method models downside variability below any arbitrary level. Conceptually, the downside variance model is simply a more general version of the traditional CAPM. It allows practitioners to measure risk as variability below an arbitrary level, rather than measuring risk below the market index.

---

<sup>12</sup> \_\_\_\_\_, "Asset Pricing in a Generalized Mean-Lower Partial Moment Framework: Theory and Evidence," J. of Financial and Quantitative Analysis, 1989, v.24, n.3, pp.285-310.

**4.4 Application to Real Estate** The discussion of downside risk, or volatility of returns below some minimal acceptable return, has relevance to real estate decision-makers. The assumption of normally distributed returns, and the smoothing of classic expected utility analysis is avoided by explicit consideration of downside risk. The initial stages of the risk management process, risk identification and categorization process can benefit from explicit consideration of downside risk by increasing the precision of sensitivity analysis. Additionally, the downside variance model is similar in form to the traditional CAPM, so practitioners can integrate the model into the familiar Modern Portfolio Theory milieu.

## Chapter 5

### **A Consumption-Based Capital Asset Pricing Model**

**Chapter Summary:** The consumption-based CAPM, a generalized form of the CAPM which relates an asset's price to its covariance of returns to the level of national consumption, is described. Its practical and theoretical advantages over the traditional CAPM are noted.

**5.1 Introduction** The previous chapters of this thesis explored some of the problems in attempting to apply securities investment models to real estate. The dissimilarity of the asset classes and the lack of comparable returns data for real estate remain major stumbling blocks of this effort. Consequently, real estate risk identification and categorization techniques within these frameworks are subject to the same conceptual and practical limitations.

Recent research by David M. Geltner<sup>13</sup> attempted to bridge both the informational and conceptual gaps by modifying the traditional investment models to incorporate observable data

---

<sup>13</sup> \_\_\_\_\_, "Estimating Real Estate's Systematic Risk from Aggregate Level Appraisal-Based Returns," AREUEA Journal, 1989, v.17, n.4, pp.463-481.

and aid investment decision-making for unsecuritized real estate. This suggests that the risk identification and categorization process can use observable cash flow, lease term, and market data, along with consistent indices released by the U.S. government, to quantify risk.

As discussed in Chapter 1, the Capital Asset Pricing Model (CAPM) has long been the favored investment model for securities analysts. Attempts to apply the CAPM to real estate has necessitated the estimation of a "market return" index for real estate. Although the Russell-NCREIF Index attempts to give a normalized view of investment returns, there are serious data limitations to the practical implementation of the CAPM to real estate. Geltner suggests that a more generalized version of the CAPM, the Consumption CAPM (CCAPM) developed by Breeden<sup>14</sup>, provides a more pragmatic way to evaluate the risk characteristics of unsecuritized real estate.

**5.2 The Capital Asset Pricing Model** The CAPM is an investment theory which describes the way prices for assets are determined in a two-period time frame. Investors seek to minimize risk while maximizing return. Assuming that markets are efficient, then the "market" (a portfolio

---

<sup>14</sup> \_\_\_\_\_, "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," J. of Financial Economics, 1979, v.7, pp.265-296.

consisting of all risky assets within the market) will represent an optimal risk/reward tradeoff. Each asset's measure of risk is captured by the statistic beta ( $\beta$ ), which estimates the asset's expected variability from the market return.  $\beta$  is estimated by observing historical covariance to the market. The following simplified equation summarizes how this relationship is applied:

**Exhibit 5.1: The Capital Asset Pricing Model**

$$R_e = R_f + \beta(R_m - R_f)$$

Where:  $R_e$  = expected return  
 $R_f$  = risk-free investment return  
 $R_m$  = the market return  
 $\beta$  = beta = the level of risk

For example, suppose the stock market's expected rate of return is 10%, and the risk-free investment rate (the yield on Treasury bills) is 6%. If the stock of General Motors has a  $\beta$  of 1.5 based on its historic performance relative to the market, then the CAPM estimates General Motor's stock return to be:  $6\% + 1.5(10\% - 6\%) = 12\%$ .

**5.3 The Consumption-Based CAPM** The CCAPM is a more general form of the traditional CAPM. The CAPM uses an index of stock market returns to derive an estimate of an individual stock's risk. Similarly, applying the CAPM to real estate necessitates the estimation of a real estate market index of returns.

The CAPM uses stock market returns as an index of wealth, and the attempted application of CAPM to real estate similarly has used real estate market returns as an index of wealth. This implies that total wealth is contained within the asset class, whether it is stocks or real estate. Although many investment management firms do separate investment decisions by asset class, this fragmented view misses the point of the CAPM. Wealth is spread among many asset classes, and the CAPM, too should be applied at the portfolio level consisting of many asset classes.

The CCAPM, in contrast, generalizes that wealth beyond a portfolio of stocks or a portfolio of real estate. National consumption serves as a proxy for total wealth. The CCAPM uses a readily available and objective data stream, national consumption as reported quarterly by the U.S. Bureau of Economic Analysis, to estimate individual wealth.

The concept behind the CCAPM is simple. Utility is defined as consumption, and less volatility of consumption results in greater utility. When national consumption is greater than expected (as reported by the government consumption index), individuals' consumption is greater than expected, and individuals are better off. Similarly, when the level of national consumption is below expectations, individuals are worse off. Any asset that achieves higher

returns when consumption is high, or achieves lower returns when consumption is low increases individuals' consumption volatility and results in lower individual utility. Conversely, an asset that achieves higher returns when consumption is below expectations and that achieves lower returns when consumption is higher than expected will decrease the volatility of individual consumption and increase individual utility. Thus, an asset that can smooth an individual's expected consumption pattern will be worth more than an asset which does not.

As mentioned in Chapter 1, recent research by Fama and French has questioned the robustness of the CAPM as it applies to the stocks. Their arguments against beta, the estimate of a stock's volatility relative the stock market volatility, do not refute the concept of the CCAPM. The researchers argued that estimation of volatility of an asset within its asset class (one stock's volatility within the stock market) is not a strong indication of future volatility. The CCAPM avoids this problem by relating an individual asset's volatility (a real estate investment) to an aggregate measure of volatility of wealth (the national consumption.)

**5.4 Application to Real Estate Investment** The CCAPM provides an opportunity to apply traditional investment



techniques to unsecuritized real estate. The earlier problem of finding an adequate real estate market index (as a proxy for wealth) is circumvented by generalizing the CAPM to use national consumption as a proxy for wealth. The historical property return stream, often known or reasonably estimated, is also used. Thus, the data adequacy problem may be alleviated using the CCAPM.

Ideally, application of the CCAPM to real estate would resemble a simple equation similar to the CAPM equation of Exhibit 5.1.:  $R_e = R_f + \beta(R_m - R_f)$ .  $R_m$ , the return on the market, would instead be the estimated change in national consumption (using seasonally-adjusted quarterly personal consumption expenditures from the Bureau of Economic Analysis.)  $\beta$  would be an estimate of the asset's return variability relative to  $R_m$ , estimated by measuring the asset's ex post covariation with the changes in national consumption. Based on these estimates, one could estimate the ex ante value of the real estate asset.

Of course, there remain a number of obstacles in the practical application of the CCAPM. First, although the preliminary empirical study by Geltner showed that the CCAPM may hold for valuing unsecuritized real estate, the size and scope of the study was narrow. Second, the model must be specified more precisely; Geltner does not specify the

optimal number of time period lags between real estate returns and market consumption. Third, the real estate return data, though more "knowable" than other data streams required in other models, are not consistent throughout the industry and require substantial preparation before use in the model. Thus, although much needs to be done, the CCAPM appears a model worthy of continued research.

**Chapter 6**  
**Conclusions**

This thesis attempted to describe tools outside of the real estate mainstream which may be useful to a wide variety of real estate decision-makers. Emphasis was placed on applying these tools to the early stages of the risk management process: the risk identification and categorization stages. By focusing on the front-end, this thesis attempted to introduce broad concepts to a field in which both academics and practitioners express dissatisfaction with the status quo techniques of risk management.

The models described in Chapters 2 to 5 represent a wide variety of conceptual bases, as different from each other as from current analytical techniques. Although each represents a departure from the status quo, none is wholly inconsistent with MPT. This is important to a field which is rooted in the securities investment models.

The comparative advantage model of Chapter 2 provides a broad framework which stresses the multi-party aspect of

real estate investment and development decisions. The methodology is qualitative and does not necessarily have the new or better data requirements which have slowed implementation of other models to real estate.

The extension of options pricing models to real estate recognizes the option-like characteristics of real estate and attempts to model them for decision-makers. To apply the technique, its quantitative approach demands a precision in its input data which is difficult to achieve in real estate. However, this tool may provide helpful approximations and estimates.

The modifications to expected utility theory and explicit consideration of downside risk explored in Chapter 4 suffer from the same limitations as its predecessor. Expected utility theory is a simple and well-understood investment concept, but its practical application is limited by its simplicity.

The consumption CAPM of Chapter 5 requires more rigorous testing to determine its robustness. The generalization of the CAPM has intuitive appeal for its application to unsecuritized real estate valuation.

Overall, the continuing search by practitioners and academics for models of risk identification and categorization underscores the difficulty of such an undertaking. The most significant aspect of this thesis, then, may be in its approach: an attempt to borrow respected concepts from other investment decision fields and apply them in a meaningful way to real estate.

## Bibliography

- Apgar, M., "A Strategic View of Real Estate," Real Estate Issues, 1986, v.11, n.2, pp.6-11.
- Arnott, R.D., and Bernstein, P., "The Right Way to Manage Your Pension Fund," Harvard Business Review, 1988, v.88, n.1, pp.95-102.
- Bawa, V.S., and Lindenberg, E.B., "Capital Market Equilibrium in a Mean-Lower Partial Moment Framework," J. of Financial Economics, 1977, v.5, pp.189-200.
- Blitzer, C.R., Lessard, D., and Paddock, J.L., "Risk Bearing and the Choice of Contract Forms for Oil Exploration and Development," working paper, 1982, MIT International Energy Studies Program.
- Breeden, D.T., "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," J. of Financial Economics, 1979, v.7, pp.265-296.
- Cole, R., Guilkey, D. and Miles, M., "Pension Fund Investment Managers' Unit Values Deserve Confidence," Real Estate Review, 1987, v.17, n.1, pp.84-89.
- \_\_\_\_\_ et. al. and Webb, B., "More Scientific Diversification Strategies for Commercial Real Estate," Real Estate Review, 1989, v.19, n.1, pp.59-66.
- Del Casino, J., "On Assembling Real Estate Portfolios," Real Estate Issues, 1985, Fall/Winter, pp.47-49.
- Fama, E., and French, K., "The Cross-Section of Expected Stock Returns," J. of Finance, 1992, v.42, n.2, pp.427-465.
- Firstenberg, P., Ross, S., and Zisler, R., "Real Estate: The Whole Story," J. of Portfolio Management, 1988, v.14, n.3, pp.22-34.
- \_\_\_\_\_, and Wurtzbaach, C., "Managing Portfolio Risk and Reward," Real Estate Review, 1989, v.19, n.2, pp.61-65.
- Fishburn, P., "Mean-Risk Analysis with Risk Associated with Below-Target Returns," Am. Economic Review, 1977, v.67, n.2, pp.116-125.
- Geltner, D., "Estimating Real Estate's Systematic Risk from Aggregate Level Appraisal-Based Returns," AREUEA Journal, 1989, v.17, n.4, pp.463-481.

\_\_\_\_\_, "Risk and Returns in Commercial Real Estate: An Exploration of Some Fundamental Relationships," 1989, Ph.D. thesis, MIT.

Hagigi, M., and Kluger, B., "Safety First: An Alternative Performance Measure," J. of Portfolio Management, 1987, v.13, n.4, pp.34-40.

Harlow, W., and Rao, R., "Asset Pricing in a Generalized Mean-Lower Partial Moment Framework: Theory and Evidence," J. of Financial and Quantitative Analysis, 1989, v.24, n.3, pp.285-310.

Kahneman, D., and Tversky, A., "Prospect Theory: An Analysis of Decision Under Risk," Econometrica, 1979, v.47, pp.263-291.

Kloppenburger, W., "Real Estate Portfolio Management: One Approach for Diversification of Specific Risk," 1990, Master's thesis, MIT.

Lessard, D., "The Global Financial Revolution and the New Development Finance," 1992, unpublished working paper, MIT.

\_\_\_\_\_, "Alternative Finance for Less Developed Countries: A Primer," unpublished working paper, 1991, MIT.

\_\_\_\_\_, "Evaluating Foreign Projects: An Adjusted Present Value Approach," 1979, unpublished working paper, MIT.

Louargand, M., "Characteristics of Real Estate Investment Risk," 1988, unpublished paper, MIT.

Lusht, K., "The Real Estate Pricing Puzzle," AREUEA Journal, 1988, v.16, n.2, pp.95-104.

Majd, S., and Pindyck, R., "Time to Build, Option Value, and Investment Decisions," J. of Financial Economics, 1987, v.18, pp.7-27.

Markowitz, H., Portfolio Selection - Efficient Diversification of Investments, 1959, Yale University Press, New Haven.

O'Connor, J., "Real Estate Development: Investment Risks and Rewards," Real Estate Issues, 1986, Spring/Summer, pp.6-11.

Roulac, S., and Cirese, R., "A Risk Analysis Matrix to Improve Investment Decisions," Real Estate Review, v.16, pp.36-40.

Ruhnka, J., and Young, J., "Some Hypotheses about Risk in Venture Capital Investing," J. of Business Venturing, 1991, v.6, pp.115-133.

Pindyck, R., "Irreversibility, Uncertainty, and Investment," J. of Economic Literature, 1991, v.29, pp.1110-1148.

\_\_\_\_\_, "Irreversible Investment, Capacity Choice, and the Value of the Firm," Am. Economic Review, 1988, v.78, n.5, pp.969-985.

Pyhrr, S., and Cooper, J., Real Estate Investment: Strategy, Analysis, Decisions, 1982, John Wiley & Sons, New York.

Shukla, R., and Trzcinka, C., "Research on Risk and Return: Can Measures of Risk Explain Anything?" J. of Portfolio Management, 1991, Spring, pp.15-21.

Sortino, F., and Van der Meer, R., "Downside Risk," J. of Portfolio Management, 1991, v.17, n.4, pp.17-21.

Trigeorgis, L., "A Log-Transformed Binomial Numerical Analysis Method for Valuing Complex Multi-Option Investments," J. of Financial and Quantitative Analysis, 1991, v.26, n.3, pp.309-326.

Vernor, J., An Introduction to Risk Management in Property Development, 1981, Urban Land Institute, Washington, D.C..

Webb, J., and Rubens, J., "Portfolio Considerations in the Valuation of Real Estate," AREUEA Journal, 1986, v.14, n.3, pp.465-495.

Zerbst, R., and Cambon, B., "Real Estate: Historical Returns and Risks," J. of Portfolio Management, 1984 Spring, pp.5-20.