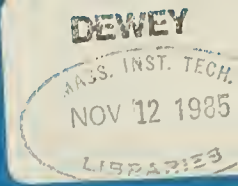


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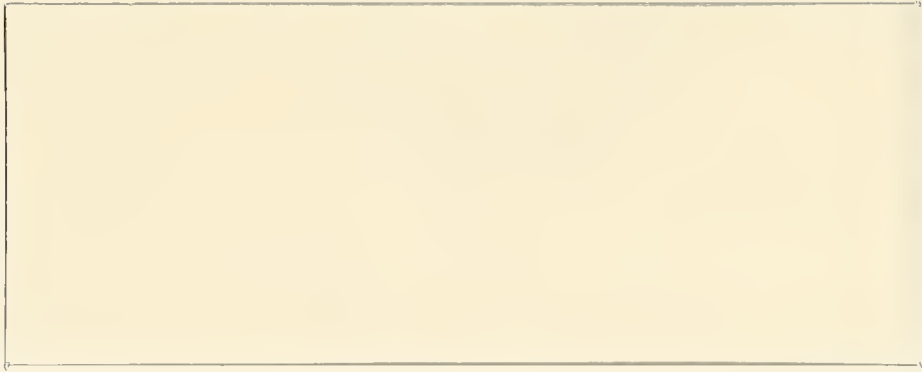
DUOPOLY WITH TIME-CONSUMING PRODUCTION

Garth Saloner*

M.I.T. Working Paper #341
July 1983
(Revised January 1984)

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1. INTRODUCTION

In static oligopoly models in which output is the strategic variable, it is typically assumed that the firms' output choices are made at an instant in time. This is equivalent to assuming either that production is instantaneous or that a firm is committed to producing its entire chosen output. Furthermore, although the order in which the firms make these decisions will usually have a large impact on the outcome (compare the Cournot (1838) and Stackleberg (1934) outcomes), the order is usually exogenously specified. This paper presents a model in which production is time-consuming and where firms can modify their production plans at various times during the production period. Although the production of all firms is assumed to take place contemporaneously, under certain circumstances identified here, one firm may achieve a de facto first-mover advantage. In addition, "degrees" of first-mover advantage emerge endogenously.

The model is essentially a Cournot model in structure with the exception that the single output choice is replaced by a sequence of output choices. Thus this is basically a one-period game with markets clearing only once. Only one aspect of the dynamics of these markets is studied here, namely the time consumed in production. The fact that markets clear only once provides the firms with a commitment to sell everything they produce since unsold goods cannot be carried to a later period. (An explicit multiperiod model with inventory is presented in Saloner (1983)). This considerably simplifies the analysis and focuses attention on the production decision.

The methodology adopted in this model corresponds closely to that developed

in Spence (1979) and Fudenberg and Tirole (1981) with two exceptions: (i) the focus here is on production rather than investment, and (ii) the model presented here employs a discrete-time formulation. In the spirit of those models we assume that the amount any firm can produce in a production "sub-period" is bounded above. (If it is unbounded then the standard Cournot model emerges.) Consequently it is the relative production rates of the firms that are important in determining the outcome.

We show that a firm with a relatively high rate of production has a degree of first-mover advantage over its opponent. Indeed if the relative rates of production become arbitrarily large, the firm with the higher production rate gains an absolute first-mover advantage as in the Stackelberg model. On the other hand, if the firms are symmetric both in costs and speed of production, the Cournot outcome emerges. The outcome varies continuously as the relative production rate varies between these extremes. Thus the degree of first-mover advantage and hence the appropriate oligopoly solution are endogenously determined and are quite simply related to the production capacity of the firm.

An important result that allows us to refer to the outcome is that here there is a unique perfect equilibrium path for production. This is in contrast to the multiplicity of equilibria that arise in the Fudenberg and Tirole model. The uniqueness derives from the finite time-horizon rather than the discrete-time formulation. The infinite-horizon setting of the Fudenberg and Tirole model admits implausible perfect equilibria in which each firm invests forever. Given that a firm's opponent is investing, to continue to invest as well is an optimal response. While equilibria of this kind are implausible and uninteresting in

themselves, since they are perfect equilibria they can be invoked as perfect equilibrium strategies off-the-equilibrium-path in other equilibria, i.e. the threat to invest forever if an opponent deviates from some specified strategy is credible. These threats can thus be used to sustain other equilibria. The finite-horizon employed here, in contrast, does not admit threats of that kind and hence eliminates equilibria sustained by them. The result is a unique equilibrium for each set of values of the firms' production rates.

The final result of the model is that unlike the standard static models, the notion of the firm's "size" (as measured by its maximum production rate) takes on some meaning. "Larger" firms in terms of production rates, ceteris paribus, also account for larger shares of total output. Thus if two firms merge, since their combined production rate is at least the sum of their individual production rates, the merged firm will be larger in a meaningful sense. Consequently it is quite simple to generate examples in which firms find it profitable to undertake mergers that result in a decrease in aggregate consumer surplus. This is in contrast to mergers in the Cournot model where a merger essentially results in the disappearance of one of the merging firms.

2. THE MODEL WITH TIME-CONSUMING PRODUCTION

This paper is concerned with providing a means of characterizing a timing of moves that neither endows one of the firms with an absolute first-mover advantage in production (as does the Stackelberg model) nor robs both firms of any first-mover advantage (as does the Cournot-Nash model). The method of doing this closely follows the methodology for studying the investment rates of firms in a

growing industry developed by Spence (1979) and refined by Fudenberg and Tirole (1981). In order to focus on the production timing issue, this paper analyzes a single-period model.

The model has the same basic structure as the standard static quantity-setting models. There are two major differences. Firstly, the output and sales decisions of the firms are made separate.¹ Secondly, production is assumed to take time. Formally, the production period consists of T sub-periods with production being allowed each sub-period. After the production period is over the firms then make their decisions about how much to sell. Thus each firm chooses a sequence $\{q_1^i, q_2^i, \dots, q_T^i, x^i\}$ where q_j^i is the output of firm i in sub-period j and x^i is the amount put up for sale by firm i . The firms make each sub-period choice simultaneously knowing the entire history of all firms' choices to that stage.

Clearly if both firms have unbounded rates of production, each firm can carry out its total desired production in the first sub-period and the situation would be no different from the standard Cournot model. Instead we will assume that there is some upper bound on the production rate, Q^i , for each firm such that $q_s^i \leq Q^i \forall i$ and s , and we assume constant returns to scale up to Q^i .² Our intention is not to impose capacity constraints but rather differential rates of production. Accordingly we will assume that TQ^i is "large" for all i . (If we define the Stackelberg outcome with Firm i as leader as $S^i = (S_1^i, S_2^i)$, as will be apparent shortly, for our purposes capacity constraints will not be binding if $TQ^i > S^i$ for $i=1$ and 2). We will ignore the complicating factors of inventory carrying costs and discounting during the production period, both in order to focus on the speed of production and for simplicity.

The standard static best-response functions serve as useful benchmarks. We refer to these as R_s^1 and R_s^2 . The equilibrium strategies will result in discrete-jump production paths moving out from the origin in a northeasterly direction. We will refer to such a path as a production growth path (PGP). We are interested in the properties of equilibrium PGP's. Let $\bar{q}_t^i = \sum_{s=1}^{t-1} q_s^i$ (for $t = 1, \dots, T$), i.e. $(\bar{q}_t^1, \bar{q}_t^2)$ is the point in output space that the PGP has reached at time t .

Several benchmark regions will prove useful in what follows. These are illustrated in Figure 1 along with a typical PGP.

--FIGURE 1--

Regions A, B, & C all lie within the outer envelope of R_s^1 and R_s^2 . The borders of the regions are defined by the Nash and Stackelberg points.

The equilibrium concept used is that of a pure strategy Perfect Nash equilibrium. Equilibrium strategies will be denoted by superscripting variables with a star. Where it is important to include the border of a region within that region, the closure will be denoted by a bar. The intuition underlying each result is given in the body of the text and proofs are provided in the appendix.

Lemma 1: If any equilibrium PGP reaches region D, both firms cease production.

Lemma 1 states that if an equilibrium PGP "jumps over" the outer envelope of

the best-response functions, all production must cease thereafter. The argument relies on the finite time-horizon and uses a simple backwards-induction argument. If the PGP has crossed both firms' best-response functions at the beginning of the final sub-period, in the final sub-period it will be a dominant strategy for both firms to produce nothing. Thus, under the assumption that it is common knowledge that there will be no production in the final sub-period, regardless of the players' actions in all the previous sub-periods, it will again be a dominant strategy to produce nothing in the second-to-last period, and so on.

Lemma 1 establishes that the first sub-period in which the PGP enters the region beyond the outer envelope of the best-response functions is also the last sub-period in which there is production. This result is used in Lemma 2 to establish that an equilibrium PGP will never terminate in the interior of that region, i.e. if an equilibrium PGP reaches \bar{D} it must terminate on the outer envelope of the best-response functions. By Lemma 1 it is common knowledge that any PGP entering region \bar{D} will terminate on entry. But given this, the last firm to produce a positive output can increase its payoff by reducing its output slightly in the last sub-period in which it produced a positive output. Provided the amount by which this production choice is decreased is sufficiently small, after that production has taken place the PGP will be in \bar{D} and thus (by Lemma 1) will be the terminal point. Since this new terminal point will be closer to the firm's best-response function, this point is preferred by the firm. This result is given as Lemma 2:

Lemma 2: No equilibrium PGP ends in the interior of region D.

The argument behind Lemma 1 is straightforward but it does display the

essential difference between the finite-horizon and infinite-horizon versions of the model. In the infinite-horizon version there are perfect equilibrium paths in which both firms produce as rapidly as possible, even beyond both of their best-response functions. Not only does this mean that there can exist equilibria with terminal points in region D, but 'early-stopping' equilibria in which the firms stop production before the outer envelope of their best-response functions is reached can also be sustained. In these early-stopping equilibria both firms threaten to continue production into region D if both firms do not stop at the target early-stopping point. Since there do exist perfect equilibria that have paths into region D, these threats constitute credible off-the-equilibrium path strategies and indeed are mutually self-enforcing. If one of the firms deviates by producing beyond the target point, it will then be rational for it to participate in its own 'punishment'. In the finite-horizon model, by contrast, there are no equilibrium paths that lead into region D and hence threats to carry production into that region are not credible. As Lemma 3 shows, a major consequence of this is that the 'early-stopping' equilibria disappear.

Lemma 3: Any equilibrium PGP ends in \bar{D} .

It is straightforward to show that an equilibrium PGP cannot terminate before the inner envelope of the best-response functions. At such a terminus, given the point reached by its opponent, each firm would prefer to have attained a larger aggregate output. However by assumption both firms have sufficient capacity to have produced more. Thus consider the last sub-period for which one of the firms failed to produce to its sub-period capacity, Q^i . That firm can improve its outcome by increasing its output in that sub-period. Notice that in

so doing it need not be concerned that it will be 'punished' for this action since along the original path its opponent was already producing to capacity from that time on.

It requires a more detailed argument to show that there cannot exist early-stopping equilibria on the inner envelope, or between the inner and outer envelopes, of the best-response functions. In the previous case both firms would have preferred larger outputs at the terminal point. In this case this is true for only one firm (say Firm 2). Further it does not follow immediately that Firm 2 should raise its output in the last sub-period for which it produced less than Q^2 since Firm 1 may be able and willing to respond with larger outputs in the subsequent sub-periods. In fact however (as is shown in the proof) a sufficiently small deviation by Firm 2 will not trigger an unfavorable response by Firm 1. The logic of the proof uses the fact that if Firm 1 had a credible response of this kind to a very small deviation, it would have wanted to adopt this course of action as its strategy along the equilibrium path. (The formal proof checks that there is a deviation of $\epsilon > 0$ sufficiently small to make Firm 2 better off without triggering an unfavorable response by Firm 1).

Lemmas 1, 2 and 3, taken together, imply that any equilibrium PGP, if one exists, terminates on the outer envelope of R_s^1 and R_s^2 . In fact an equilibrium does exist and it is just the discrete-time analog of the equilibrium in Spence's model (1979). Roughly speaking, in equilibrium both players produce as rapidly as possible until the outer envelope of the best-response functions is reached at which time both stop (or until one of the players reaches his Stackelberg outcome in which case he stops and the other continues until his

best-response function is reached). Formally, however, the point at which a best-response function is "reached" is not well-defined in a discrete-time model since it may be "jumped over". The following proposition provides the necessary formalization to take account of this.

Consider the PGP that results when both players produce as rapidly as possible, and then "connect the dots". Define the player whose best-response function is the last to be crossed to be the 'outer' player (if the connected PGP passes through N both players are 'outer' players). Without loss of generality assume that player 2 is the outer player. Let sub-period $l[j]$ be the first sub-period during which the outer [inner] player can 'reach' his best-response function if the inner [outer] player produces nothing in that sub-period.³ The following proposition describes an equilibrium PGP:

Proposition: The following strategies form a perfect pure-strategy Nash

equilibrium: Case (a) - The connected PGP passes to the southeast of S_1^1 :

Player 1 produces as rapidly as possible until he has produced S_1^1 . Player 2 produces as rapidly as possible until he has produced S_2^1 .

Case (b) - The connected PGP passes to the northwest of S_1^1 :

for $t < j$: $q_t^{1*} = Q^1$, $i=1,2$

for $t > j$: if $j < 1$, $q_j^{1*} = (R_S^2)^{-1} (\bar{q}_j^{2*} + Q^2) - \bar{q}_j^{1*}$

$q_j^{2*} = Q^2$

if $1 < j$, $q_1^{1*} = \max \{0, N_1 - \bar{q}_1^{1*}\}$

$q_1^{2*} = \min \{R_S^2 (\bar{q}_1^{1*}), N_2\} - \bar{q}_1^{2*}$.

Some typical equilibrium PGP's are illustrated in Figure 2 (with 'dots' connected).

--FIGURE 2--

Notice that in the special case in which $Q^i > N_i$ for both players, the Nash outcome results as would be expected. Further, in Case (b) as Q^1 and Q^2 approach zero with Q_2/Q_1 remaining constant, the equilibrium PGP approaches the straight line connecting the origin and R_S^2 with slope Q_2/Q_1 i.e. the continuous-time limit of this discrete-time model is the point on the outer envelope reached when both firms produce as rapidly as possible. This is the outcome used in Spence's (1979) continuous-time model and one point in the continuum of equilibria that exists in the Fudenberg and Tirole (1981) model.

As the proof of the proposition illustrates, not only are those strategies mutual best-response but either player can guarantee at least this equilibrium outcome by playing the suggested strategy. Since any equilibrium outcome must lie along the outer envelope of the best-response function and because the players' interests are strictly opposing along this outer envelope, this equilibrium is also the unique equilibrium PGP.

In this model the relative rates of production determine the outcome. If the production rate of one of the firms is sufficiently large relative to its opponent, it will be able to achieve its Stackelberg outcome and it will effectively have a first-mover advantage. As the ratio Q_2/Q_1 rises, the equilibrium outcome changes continuously from S^1 to S^2 . The Cournot-Nash outcome

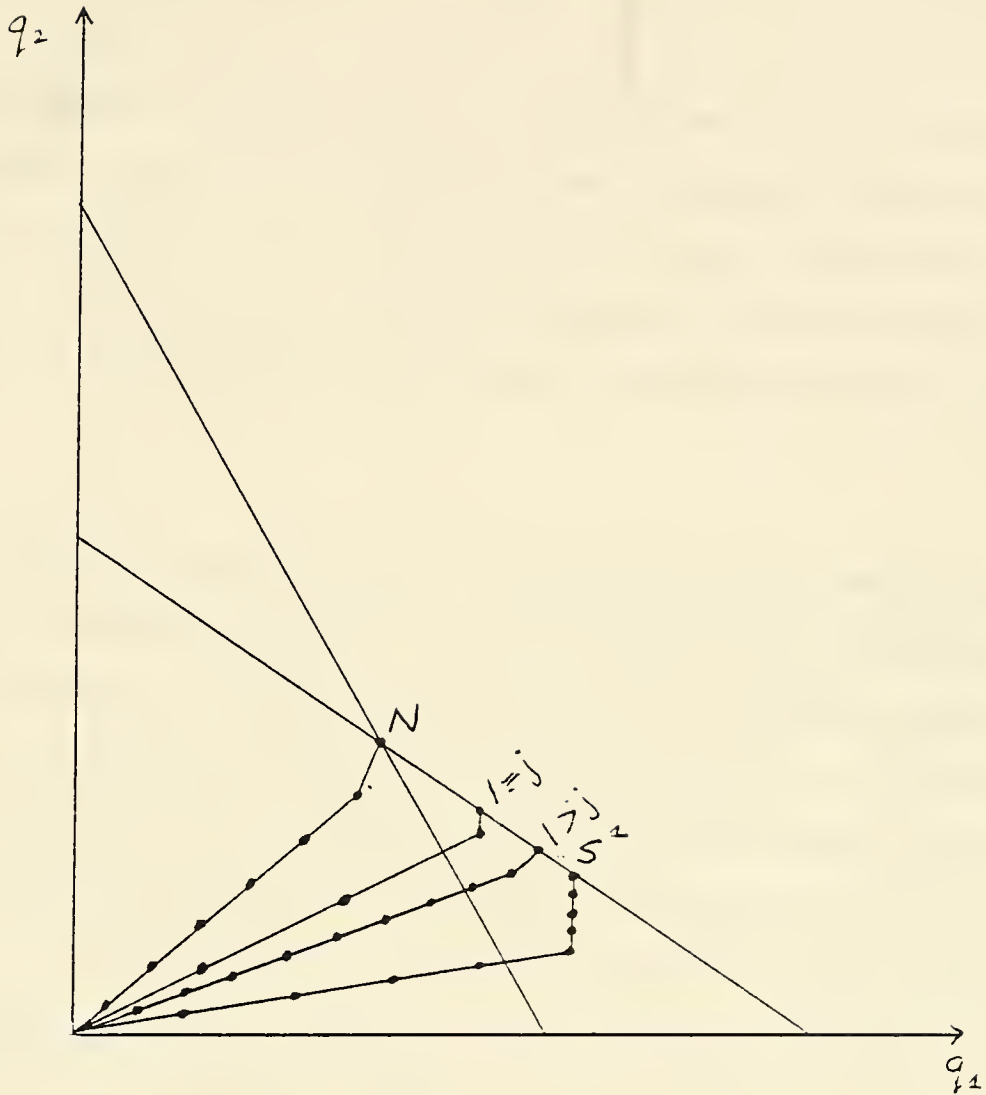


FIGURE 2

has no special significance and emerges only when the relative rate of production of Firm 1 to Firm 2 is in the ratio N_1/N_2 .

As was mentioned in the introduction, the fact that the outcome depends on the relative rates of production means that the concept of 'size' takes on some meaning here. Suppose that when two firms merge that the production rate of the merged entity is the sum of the individual rates. In this case the new firm will be 'larger' than the merging firms and may attain a more dominant position in the industry.

As an example consider an industry of three firms where the rates of production are in the ratio $Q^1:Q^2:Q^3 = 2:1:1$. Let all the firms have constant marginal costs (normalized to be zero) and suppose the industry inverse demand function is $P = 15 - q$, where $q = q_1 + q_2 + q_3$ is the total industry output at the end of the period. The outcome in this model has $q_1 = 6$, $q_2 = q_3 = 3$, $P = 3$, consumer surplus of 72 and profits of 18, 9 and 9 for Firms 1, 2 and 3 respectively. Suppose now that Firms 2 and 3 merge to form Firm 0. In this case the rates of production of Firms 0 and 1 are equal. The equilibrium outcome has $q_0 = q_1 = 5$, $P = 5$, consumer surplus of 50 and profits of 25 for each firm. Notice that Firm 0 earns more than the sum of the pre-merger profits of Firms 2 and 3. Thus an incentive to merge exists and the merger leads to a decrease in both consumer surplus and social welfare (as measured by the sum of consumer and producer surplus).

In this example the merger both destroys Firm 1's leadership and reduces the competitiveness of the industry. It is the combination of both of these effects

that leads to the decrease in social welfare. To illustrate this, suppose instead that Firms 1 and 2 merge to form a new Firm 0. In equilibrium $q_0 = 9$ and $q_3 = 3$. Thus the output of the merged firms is merely the sum of the pre-merger firms' outputs. Accordingly prices and welfare are unchanged. The reason for this is straightforward. In equilibrium Firm 3's output will be one-fourth of the total, both pre- and post-merger. Thus from Firm 3's perspective the PCP remains the same. Further, in both cases Firm 3 is the 'outer' firm. Thus the equilibrium, which is at the intersection of the PCP and the outer firm's best-response function, remains unaltered.

Thus the example is not meant to illustrate that an incentive to undertake welfare-reducing mergers always exists, but rather that a key ingredient to a theory of mergers is a notion of firm size that is preserved (at least to some degree) when mergers take place. In this model this is achieved by merging the firms' production rates. More generally, merged firms should also be expected to preserve existing customer relationships and brand loyalties.

3. CONCLUSION.

Traditional models of oligopoly usually do not embody any notion of firm size. A result of this is the fact that firms that have equivalent cost structures are placed in a symmetric position with respect to the shares of final output that they command regardless of their capacities. Thus when two firms merge the result is simply that one decision-making agent who was previously active disappears. Therefore in a Cournot model with more than two symmetric firms, a merger by any two firms results in a decrease in profits to the merging

firms. There are, however, a number of reasons why firm size may be important. The reason explored in this paper relates to the relative speeds of production of the competing firms in a model in which production is time-consuming. In such a setting large firms, i.e., those with high maximum production rates, are able to gain a degree of first-mover advantage. Other reasons for the importance of firm size include customer loyalties that are not entirely lost when firms merge and the reputations that the firms' products themselves enjoy. Any of these factors will be sufficient for profitable mergers that are socially detrimental to be possible.

It is not unusual for the number of equilibria in games with finite time horizons to be much smaller than in their infinite-horizon counterparts. This has been shown to be the case here and the factors leading to the difference have been identified. However the reduction of the multiplicity of equilibria to the unique equilibrium found here is only of interest if one is prepared to rule out the perfect equilibrium strategies that give rise to the multiplicity or if the finite time-horizon intrinsically has more appeal. In product markets where there are frequent model changes, either by the firms' choice or due to obsolescence, the appropriate model has a finite time-horizon and thus also a unique equilibrium.

Finally, the model presented here provides a means of endogenizing the degree of first-mover advantage a firm possesses in a quantity-setting oligopoly model. A firm's degree of leadership or followership is linked here to its production capacity. This raises a number of issues for entry deterrence and strategic choice of capacity that have not been addressed here.

APPENDIX

Lemma 1: If any equilibrium PGP reaches region D, both firms cease production (i.e. if $\bar{q}_s^{1*}, \bar{q}_s^{2*} \in \bar{D}$, $q_t^{i*} = 0 \forall t > s, i=1,2$).

Proof: Without loss of generality, suppose that region D is reached on sub-period T-n. Consider sub-period T and suppose that $(\bar{q}_T^{1*}, \bar{q}_T^{2*}) \in \bar{D}$. Since both firms are on or beyond their best-response functions it is a dominant strategy for each to set $q_T^{i*} = 0$. Now consider sub-period T-1 and suppose that $(\bar{q}_{T-1}^{1*}, \bar{q}_{T-1}^{2*}) \in \bar{D}$. Regardless of the move made in sub-period T-1 at sub-period T we will have $(\bar{q}^{1*}, \bar{q}^{2*}) \in \bar{D}$. Therefore neither firm will produce in sub-period T. Therefore the choices at time T-1 will not affect the choices at time T. But then again $q_{T-1}^{1*}, q_{T-1}^{2*} = 0$ are dominant strategies. Clearly this reasoning extends to any sub-period from T-n on. Q.E.D.

Lemma 2: No equilibrium PGP ends in the interior of region D (i.e. $(q_s^{1*}, q_s^{2*}) \in$ interior of D for any s).

Proof: Suppose that a PGP ends in the interior of region D. Let sub-period s be the last sub-period in which there was a strictly positive output by either firm. Then it must be the case that $(\bar{q}_s^1, \bar{q}_s^2) \in \bar{D}$ by Lemma 1. But then at best one of the following must have occurred:

a) $q_s^i < R^i(\bar{q}_s^j) - \bar{q}_{s-1}^i$ for i or j, or (b) $\bar{q}_{s-1}^i > R^i(q_s^i)$ and $q_s^i > 0$.

Statement (a) merely says that at least one firm produced an output that took it 'beyond' its best-response function. Statement (b) says that a firm

already 'beyond' its best-response function produced a positive output. In case (a) this strategy is dominated by producing $q_s^i = R^i(\bar{q}_s^j) - \bar{q}_{s-1}^i$, and in case (b) by setting $q_s^i = 0$. Q.E.D.

Lemma 3: Any equilibrium PGP ends in \bar{D} (i.e. $(\bar{q}_T^{1*} + q_T^{1*}, \bar{q}_T^{2*} + q_T^{2*}) \in \bar{D}$).

Proof: Suppose to the contrary that there is an equilibrium PGP that ends in the interior of A, B or C. Suppose firstly that it ends within the inner envelope of R_s^1 and R_s^2 . Consider the last sub-period s for which $q_s^{i*} < Q^i$ for at least one i . (One such sub-period exists since TQ^i has been assumed to be sufficiently large.) This strategy for i is dominated by replacing q_s^{i*} in its sequence of moves by $q_s^i = \min \{Q^i, q_s^{i*} + R_s^i(\bar{q}_T^{j*} + q_T^{j*}) - (\bar{q}_T^{i*} + q_T^i)\}$. (This simply says that player i can move the outcome in the direction of his best-response function.)

Suppose then that the PGP ends in the interior of B or C. This means that the PGP crosses exactly one player's best-response function. Without loss of generality let this be player 1. Consider the sub-period T . Regardless of q_T^{1*} , it was a dominant strategy for player 2 to set $q_T^{2*} = Q^2$. Given that it was a dominant strategy for player 2 to set $q_T^{2*} = Q^2$, player 1 must have set $q_T^{1*} = 0$ (for otherwise a reduction by player 1 would have resulted in an outcome nearer his reaction function along the locus $q_2 = \bar{q}_T^{2*} + Q_2$). Consider sub-period $T-1$ and suppose that $(\bar{q}_T^{1*}, \bar{q}_T^{2*}) \in B$ or C . Then it must have been the case that $q_{T-1}^{2*} = Q^2$. By the argument for period T player 1 will set $q_T^{1*} = 0$ if region B or C has been reached. But then for any choice of q_{T-1}^{1*} it is a dominant strategy for player 2 to choose $q_{T-1}^{2*} = Q^2$ since the

outcome $q_2 = (\bar{q}_{T-1}^{1*} + q_{T-1}^{1*} + 0, \bar{q}_{T-1}^{2*} + 2Q^2)$ is preferable to the outcome $q_2 = (\bar{q}_{T-1}^{1*} + q_{T-1}^{1*} + 0, \bar{q}_{T-1}^{2*} + q_{T-1}^2 + q_T^2) \forall q_{T-1}^2, q_T^2 < Q^2$. Extending this argument backwards in time yields the conclusion that the first step into region B or C and all succeeding steps must have had $q^{1*} = 0$ and $q^{2*} = Q^2$. Now consider the last sub-period s for which $q_s^{1*} \neq 0$ and/or $q_s^{2*} \neq Q^2$ (such a sub-period exists for if $q_t^{1*} = 0 \forall t$, the PGP could not cross firm 1's best-response function). If $q_s^{1*} \neq 0$ then firm 1 could do better by reducing q_s^{1*} and if $q_s^{2*} < Q^2$ firm 2 could do better by increasing q_s^{2*} .

The final possibility to consider is that the equilibrium PGP ends on the inner envelope of the best-response functions at a point other than N. Suppose that there does exist such an equilibrium PGP and call the terminal point A as in Figure 3.

--FIGURE 3--

Let sub-period s be the last sub-period for which $q_s^{2*} < Q^2$. Let $B \equiv (\bar{q}_s^{1*} + q_s^{1*}, \bar{q}_s^{2*} + q_s^{2*})$ and let K be the point B shifted vertically by ϵ . At point B change player 2's strategy for the rest of the game to $q_s^2 = q_s^{2*} + \epsilon$, $q_t^2 = \min \{Q^2, R_s^2(\bar{q}_t^{1*}) - \bar{q}_t^{2*}\} \forall t > s$. Given this strategy of player 2, player 1 can guarantee the outcome given by C in the figure by producing a total of $(L - q_s^{1*})$ in the remaining periods. It remains to show that player 1 can do no better than C for if this is the case point C will result from 2's suggested deviation, an outcome that is strictly preferred by player 2. Given player 2's suggested strategy, the outcome will lie on the locus HIJ.

All the outcomes on HI with the exception of C are dominated by C itself. Suppose then that player 1 can force an outcome on IJ that is preferred by player 1 to C i.e. that lies on IJ to the southwest of D. Note however that starting from B even by producing as rapidly as possible player 1 cannot attain an output of E_1 before player 2 attains an output of E_2 . This follows since if this was possible, adopting that strategy at B would have resulted in a superior outcome for player 1. Also, starting from K, player 2 can attain a total output of exactly C_2 . Therefore when player 2 adopts the suggested strategy and player 1 produces as rapidly as possible, the PGP lies on a dotted line originating at K and passing between $H(\epsilon)$ and $G(\epsilon)$. Call the point at which this dotted line intersects R_s^2 , F. The closest to J along R_s^2 that F can lie results when this dotted line passes through G when $\epsilon=0$ when $G(0) \equiv G'$. Note that this value of F'_1 is strictly less than E_1 by the strict concavity of player 1's profit function. Thus facing player 2's suggested strategy, the best outcome that player 1 can attain is F'_1 , strictly less than E_1 . By choosing $\epsilon > 0$ small enough, player 2 can ensure that $F_1^i < D_1$ and hence that C is preferred by player 1 to F. Thus player 2's deviation results in point C which with $\epsilon > 0$ is strictly preferred by player 2 to the original outcome A. Q.E.D.

Proposition: The following strategies form a perfect pure-strategy Nash equilibrium: Case (a) - The connected PGP passes to the southeast of S_1^1 : Player 1 produces as rapidly as possible until he has produced S_1^1 . Player 2 produces as rapidly as possible until he has produced S_2^1 .

Case (b) - The connected PGP passes to the northwest of S_1^1 :

for $t < j$: $q_t^{1*} = Q^1$, $i=1,2$

for $t > j$: if $j < 1$, $q_j^{1*} = (R_S^2)^{-1} (\bar{q}_j^{2*} + Q^2) - \bar{q}_j^{1*}$

$$q_j^{2*} = Q^2$$

if $1 < j$, $q_1^{1*} = \max \{0, N_1 - \bar{q}_1^{1*}\}$

$$q_1^{2*} = \min \{R_S^2 (\bar{q}_1^{1*}), N_2\} - \bar{q}_1^{2*}.$$

Proof of Proposition 1 (SKETCH ONLY): Consider case (b). (Case (a) is obvious.) Firstly consider sub-period 1. By the argument of Lemma 1, once R_S^2 is reached both players will stop producing. If N can be reached within sub-period 1 it is clearly a Nash equilibrium for the players to produce the outputs that will take them to N (by the very definition of N). Suppose then that $\bar{q}_1^{1*} > N_1$ and that $j=1$. If player 1 produces nothing in sub-period 1 it is clearly optimal for player 2 to produce up to his best-response function. Similarly if player 2 produces up to his best-response function, since all production will then cease (because of the requirement of perfectness), it is optimal for player 1 to produce nothing. (Any positive output would result in a terminal point on a lower isoprofit curve for player 1.) Suppose then that $j < 1$. If player 2 produces Q^2 player 1 can choose to produce an amount that will bring the players exactly to R_S^2 . Clearly player 1 would not choose to produce any more than that. Further if player 1 produces anything less, player 2 will produce a strictly positive amount in the following sub-period. Since player 2 will not stop until R_S^2 is reached, this outcome is strictly worse for player 1. Given this strategy of player 1, player 2 can do no worse by setting $q_j^{2*} = Q^2$. Now consider any sub-period before j and suppose

that player 2 produces less than Q^2 in that sub-period. Given the strategy of player 1 this means that when sub-period j is reached the outputs will be at a point due south of where it would otherwise have been. By virtue of the way sub-period j is played, the terminal point on R_S^2 that is reached can be no better for player 2. Similarly for player 1. Q.E.D.

Notes

1. This follows the methodology used in Saloner (1983).
2. This assumption follows the Fudenberg and Tirole (1981) model.
3. Formally, l is the smallest value of s such that $R^i(\bar{q}_{Os}^*) < \bar{q}_{Os}^{i*} + Q^i$ when i is the outer player.

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