working paper
department
of economics

An Equilibrium Analysis of Search and
Breach of Contract, I

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Number 221 July 1978

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The literature on markets where agents have imperfect information about their trading possibilities has been growing considerably.[1/] Many models of this literature depend fundamentally on asymmetries: either buyers or sellers set prices but not both. We, however, shall consider a symmetric model where individuals meet pairwise and negotiate contracts. Individuals find potential contracting partners in a costly, stochastic search process. The purpose of a contract is to carry out a single project.[2/] The worth of a project depends on the quality of the match between the two individuals.[3/] Our model, in fact, assumes for simplicity precisely two qualities: good (project with large output) and poor (project with small output). An individual, therefore, can be in any of three positions: without a partner, in a poor partnership, or in a good partnership.

Individuals can continue to search after joining a partnership. Therefore, one of the parties to a contract may later come upon a better match and desire to break his contract. In practice, the possibility that individuals may wish to breach their current contracts to form better ones is recognized by provisions for payment of damages. The common law, for instance, stipulates damage payments for breach of contract. Such

*Financial assistance from the NSF is gratefully acknowledged.

1/ For examples, see the October, 1977 issue of the Review of Economic Studies and the discussion and reprints in Diamond and Rothschild [1978].

2/ Seasonal opportunities generate many examples of markets with this single-project feature; e.g., summer house rentals. Other examples include hiring architects for home renovations or painters for portraits. In later work we plan to analyze the case of continuous production.

3/ Here we follow Satterthwaite [1977].

damages are frequently compensatory in the sense that they exactly compensate for breach; i.e., they leave the breached-against partner in the same financial position as before the breach.[1/] As an alternative to externally determined damages, parties to a contract may write damage rules into the contract itself.[2/] Such provisions are called liquidated damage rules. We examine both compensatory and liquidated damages in this paper. We also consider efficient rules: the search and breach behavior that would be set by a central planner who maximized net social output.

We are concerned with the equilibrium steady states[3/] of a model where individuals are perfectly informed about the distribution of possible partners they might meet. We consider two distinct, simple meeting technologies. In one, the probability of an individual's meeting any given potential partner is independent of the number of other potential partners. In this case, the individual's probability of meeting someone at all rises linearly with the number of potential partners. The aggregate number of meetings (which we assume, by appeal to large numbers, to equal the expected number) increases with the square of the number of searchers.

[1/] The law is not uniform in fixing the point just before breach from which compensation is measured. A partner may be restored to the position he held either before or after he signed the contract which was breached. We shall confine our discussion of compensatory damages to post-signature compensation.

[2/] For a discussion of damages for individually optimal contracts, see Mortenson [1978]. We build on Mortenson's analysis by considering equilibrium with many partnerships.

[3/] In our companion piece (Diamond and Maskin [1978]) we substitute an evolving economy for a steady state.

We shall refer to this technology as the quadratic case. It is reasonable when there is a low density of potential partners.[1]

Notice that an additional searcher raises the meeting probabilities of all his potential partners. Since meetings are of value, a searcher thus creates a positive externality for other searchers. The presence of this externality suggests that efficiency would call for more search than occurs in equilibrium with compensatory damages. In our two papers, we examine several circumstances where such a divergence occurs.

Our other meeting technology corresponds to a higher density of potential partners. In this case, we postulate that an individual's probability of meeting someone at all is independent of the number of potential partners, when that number is positive. We shall refer to this technology as the linear case, since the aggregate number of matches increases linearly with the number of searchers. Adding potential partners to the search process does not alter the probability of an individual's meeting someone but does affect the chances of his meeting someone in any given position. When individuals who already have partners continue to search, they/may impose a negative externality on potential partners; for, as we shall see below, an individual would/generally prefer, for given quality of match, to be paired with someone who does not already have a partner. An influx of searchers with partners reduces the proportion of partnerless searchers and therefore makes all searchers worse off. Again we expect a discrepancy between the efficient and compensatory damage rules. We shall examine this discrepancy in Section 14.

The presence of search externalities means that not only will the decision to search generally be inefficient under compensatory damages but that so will be the decision whether to breach. To understand this second inefficiency, consider th

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1/ For discussions of allocations where individuals are assumed never to meet more than one potential partner, see Diamond and Mirrlees [1975] and Landes and Posner [1977].

2/ We emphasize that we consider only pairwise matches in this model. One-to -many matches create the possibility of competition among searcher, and may negate the positive externality.

meeting of two searchers, i and j, each of whom already has a partner. (To underscore the distinction between inefficient search and breach decisions, assume that it is, in fact, efficient for these individuals to be searching.) Let the values of i's and j's current positions be V^i and V^j, respectively, and the same for their partners. Let V_1 be the value (the same for everyone) of being without a partner. Suppose that if i and j breach their current contracts to form a new partnership, it would never be worthwhile thereafter for either to search, and suppose that the product of the new partnership is 2X. Individuals i and j will elect to breach and form a new contract if the surplus they derive from the new contract is positive; i.e., if $S = 2X - V^i - V^j$ - damage payments > 0. Under compensatory damages $S = 2X + 2V_1 - 2V^i - 2V^j$. Now, to breach if and only if $2X + 2V_1 - 2V^i - 2V^j$ is positive may appear to be efficient: before the breach the total positional value of the four individuals involved is $2V^i + 2V^j$ and afterwards $2X + 2V_1$. So breach according to compensatory damages occurs if and only if it increases total positional value of these four individuals. These calculations, however, fail to account for the changes in the external effects exerted by the four agents on the rest of the economy. When individuals change positions, they may alter the positional values of others as described above. That is, others may prefer to search in a steady state where this breach does not occur. Since these changes are not incorporated by the compensatory breach rules breach will, in general, be inefficient (see Section 14).

Allowing individuals to stipulate their own damages (i.e., liquidated damages) complicates matters still further. In our model, individuals who breach to form a new contract divide the product of that contract according to a fixed rule (for symmetry, we use the 50-50 rule) for splitting the surplus (the excess of the sum of new positional values over old values and damage payments).[1/] Because the surplus depends on damage payments, an individual who breaches can, in effect, get his new partner to share the burden of the damage payments to his old partner. In this way, a pair of individuals in a contract exerts some monopoly power over potential partners, and we expect liquidated damages, ceteris paribus, to be higher than compensatory damages. These higher damages tend to encourage search and induce countervailing effects on the incentives for breach. On the one hand, breach is discouraged because higher damages makes profitable breach more difficult. On the other hand, search and breach are promoted because higher damages raise the values of new contracts too. That is, an individual may sign a new contract at least in part because of the damage payments he anticipates receiving if his new partner later breaches. Either effect on breach can, in principle, outweigh the other, and once again, the analysis is rendered more elaborate by the general equilibrium effects that one individual's search behavior and positional value have on the rest of the economy.

After setting up our model (sections 1-3), we begin analysis with the case of a quadratic meeting technology and compensatory damages (sections 3 and 4). We then take up liquidated damages (sections 6-8). In both cases we compare the results to those with efficient search and breach rules. We then repeat the entire analysis (sections 10-13) for a linear technology (constant probability of finding a match). In the models used through Section 12, either no one searches with a poor contract or no poor project is carried out. This structure limits the range of inefficiencies which can occur. When search is undertaken by those with poor contracts who might carry out their projects, breaching behavior with compensatory damages can be inefficient. To illustrate this point we change the model to (stochastically) require some existing contracts to be carried out. The same issue arises in the companion paper where the absence of a steady state may alter the desirability of continued search even though search has been worthwhile. In the appendix we consider an alternative to the use of liquidated damages, where individuals may not be truthful about the value of a match they are in.

[1/] For a more detailed discussion of liquidated damages, see section 5.

1. Outputs

We consider a model of two types of individuals. Individuals are distinguished by type only in that each partnership (contract) requires exactly one partner of each type. Otherwise types are symmetric. Individuals search for a partner (of their opposite type) with whom to undertake a single project. If partners are well-matched, the project is worth 2X. If they are not well-matched, output is 2X'. We assume $X > X' > 0$. After partners have stopped searching -- and only then -- the project corresponding to their partnership is completed. Individuals are risk neutral and are able to make side payments with no bankruptcy constraint. Each individual can engage in at most one project and belong to at most one partnership.

2. Search: Quadratic Meeting Technology

Individuals can meet new potential partners only if they search, and the cost of search is a flow, c, per unit time. For any two searchers (of opposite types), the probability of their meeting, per unit time, is a. a is sufficiently small so that we can ignore the possibility that two partners who are both searching will simultaneously find new potential partners.[1/] When two individuals meet, the probability of their being a poor match is p, and 1-p is the probability they are a good match. There is an inflow per unit time of ab new individuals (of each type); individuals initially have no partners.

To avoid trivial equilibria, we consider, throughout this paper, only those parameter values for which a steady state equilibrium exists where

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1/ We have implicitly modeled contracting as instantaneous. Without instantaneous contracting, the assumption of no simultaneous meeting is less plausible.

partnerless individuals find it worthwhile to search.[1/] Partnerless individuals will wish to form contracts with any willing potential partners they meet. Clearly, individuals with good matches have no reason to search. An individual with a poor match may or may not find search profitable (we assume that either both partners search or neither does). If search is profitable for him, then it is surely worthwhile for him to breach his contract if he meets a partnerless individual with whom he makes a good match. (If search is worthwhile, then there must exist some potential partner for whom he would breach his contract. A currently partnerless individual who makes a good match is the most advantageous potential partner: highest output, lowest damage payments. Hence breach is worthwhile when meeting such a potential partner.) If damages are at least compensatory, an individual in a poor match will never wish breach to form a new poor match, since the surplus to be gained from such a match is zero or negative. Finally, an individual in a poor partnership may or may not find it worthwhile to breach to form a new contract with a presently poorly-matched potential partner with whom he makes a good match. If he does, we shall say that a double breach has occurred, since two contracts are broken.

Summing up, there are three possible search/breach configurations, detailed below. We can describe the dynamics for each of these configurations in terms of the number of partnerless searchers (of a given type), h_1, and number of searchers (of a given type) in poor partnerships, h_2.

Let M denote an individual who is partnerless and let N denote one in a poor partnership. Configuration A obtains when N's search and breach

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[1/] By this assumption and our focus on steady states, we rule out consideration of cyclical processes in which first no search occurs while the stock of partnerless individuals builds up, followed by a period of search and matching until too few agents are left to make further search worthwhile, at which point the process begins again.

their contracts whenever they find a good match. Under Configuration A, dynamics are given by

(1) $$\dot{h}_1 = -ah_1^2 + a(1-p)h_2^2 + ab$$

$$\dot{h}_2 = aph_1^2 - 2a(1-p)h_1h_2 - 2a(1-p)h_2^2 \quad \textbf{implying that}$$

$$\dot{h}_1+\dot{h}_2 = -a(1-p)\ (h_1+h_2)^2 + ab.$$

The number of M's grows through new entrants and double breaches by N's. It declines through matches between M's. The number of N's grows because of new, poor matches between M's and declines because of good matches from single or double breaches. In a steady state, $\dot{h}_1 = \dot{h}_2 = 0$, so that, for a steady state under Configuration A, we have (from (1))

(2) $$\frac{h_1}{h_2} = \frac{1-p}{p}\left[1 + \left(\frac{1+p}{1-p}\right)^{\frac{1}{2}}\right]$$

$$h_1^2 = \frac{b(h_1/h_2)^2}{(h_1/h_2)^2 - 1 + p}$$

$$h_1+h_2 = \left(\frac{b}{1-p}\right)^{\frac{1}{2}}$$

In Configuration B, N's search but breach only when they make good matches with M's. Configuration B differs from A in that there are no double breaches. The dynamics are described by

(3) $$\dot{h}_1 = -ah_1^2 + ab$$

$$\dot{h}_2 = aph_1^2 - 2a(1-p)h_1h_2$$

For a steady state under Configuration B,

(4) $$h_1 = b^{\frac{1}{2}}$$

$$h_2 = \frac{pb^{\frac{1}{2}}}{2(1-p)}$$

In Configuration C, only M's search. The dynamics are given by

(5) $$\dot{h}_1 = -ah_1^2 + ab$$

In a steady state under Configuration C,

(6) $$h_1 = b^{\frac{1}{2}}$$

3. Contracting with Compensatory Damages

If two M's meet and make a good match, they form a partnership and divide the value of the project, 2X, equally. If two M's meet and make a poor match, they form a partnership which calls for equal division if the project is completed and for damage payments if the contract is breached by one of the partners. Compensatory damages are those which exactly compensate the partner who is breached against. Thus if V_1 is the (expected) value of being an M and V_2 is the value of being an N, then the compensatory damages to be paid to the partner of a breaching N are $V_2 - V_1$.

Consider next two N's who meet and make a good match. They will breach their old contracts and form a new partnership if and only if the aggregate value of their position increases by more than the damages that they have to pay. That is, iff

(7) $$2X - 2V_2 \geq 2D$$

where D represents damages. Compensatory damages to each breached-against partner are $V_2 - V_1$. Therefore, (7) becomes

(8) $$S = 2X - 4V_2 + 2V_1 \geq 0,$$

where S represents the "surplus" from the new match.[1/] We postulate that division of output in the new contract is made so as to split the surplus evenly. (Since, in this case, the partners enter the contract from equal

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1/ In a competitive equilibrium without search costs, there is no surplus in this sense, since a contract of the same quality can be costless arranged with someone else.

positions, halving the surplus is equivalent to halving the product.) We note that with compensatory damages, the surplus is positive -- i.e., breach by the N's is worthwhile -- if and only if breach leads to an increase in the sum of the positional values of the four parties to the original two contracts. As we argued in the introduction and as we shall formally demonstrate below, however, the rule, "breach iff the surplus is positive," need not maximize the sum of positional values over <u>all</u> individuals.

The final meeting possibility of interest is an encounter between an M and N which makes a good match. Once again, breach is worthwhile if the contracting surplus, in this case $2X - V_2 - V_1 - D$, is positive. With compensating damages, the surplus is $2X - 2V_2$. We again assume that the new contract (if signed) provides that the surplus is evenly split between the parties. This rule gives the M partner a position value of $V_1 + \frac{1}{2}(2X - 2V_2) = X - V_2 + V_1$ and the N partner, $V_2 + \frac{1}{2}(2X - 2V_2) = X$.[1/]

4. Steady States: Compensatory Damages and Efficiency (Quadratic Technology)

To see whether a steady state can occur for a particular configuration, we need to check that the breach and search rules defined by that configuration are indeed individually optimal, given the numbers of searchers implied in the equations $\dot{h}_1 = 0$ and $\dot{h}_2 = 0$. We first examine steady states for compensatory damages.

Configuration C

In a steady state under Configuration C, only M's find search worthwhile. The M's sign contracts with whomever they first meet and cease

[1/] Note that the M partner bears the full brunt of the damage payments to N's old partner.

search. For someone following this behavior, the expected payoff when entering the process (i.e., the positional value of being an M) is half his expected project output $pX' + (1-p)X$ less the per unit search cost c times the mean expected time for a meeting $(ah_1)^{-1}$.

$$(9) \qquad V_1^C = pX' + (1-p)X - c(ah_1)^{-1}$$

$$= pX' + (1-p)X - \frac{c}{ab^{\frac{1}{2}}}$$

The first condition for a Configuration C steady state is that V_1^C be non-negative. However, because earlier we postulated that search is always worthwhile for M's, we shall assume this requirement is automatically met. The second condition requires that those who have made a poor match do not find continued search worthwhile. If they do not search (i.e., if they follow the behavior dictated by Configuration C) their positional value is just $V_2^C = X'$. If some partnership of N's does continue to search for time Δt, each partner incurs the cost $c\Delta t$, has a probability $a(1-p)h_1\Delta t$ of making a good match and increasing his positional value by $X - X'$.

Thus an N's expected net gain from continued search is

$$(10) \qquad a(1-p)h_1\Delta t(X-X') - c\Delta t$$

(Note that because damages are compensatory, breach by one's partner does not affect one's positional value.) The condition that continued search be unprofitable becomes

$$(11) \qquad c \geq ab^{\frac{1}{2}} (1-p)(X-X')$$

Given V_1^C non-negative, a Configuration C steady state with compensatory damages can occur for any combination of parameter values satisfying (11). Let the set of such parameters be called the compensatory Region C. Then, for given values of a, c, X, and X', this Region is depicted in b-p space in Figure 1.

Configurations A and B

In a steady state with compensatory damages under Configurations A or B, individuals continue to search until they find a good match. This behavior implies that the positional value of an N is no greater than that of an M. The compensatory damages corresponding to a poor match are, therefore, zero, and, so, an N will find it advantageous to breach for any good match. A Configuration B steady state with compensatory damages is, consequently, impossible. We have a steady state under A if N's wish to continue searching. An individual (either M or N) who searches until finding a good match has an expected payoff of $X - \frac{c}{a(h_1+h_2)(1-p)}$ where the second term represents expected search costs. Therefore N's wish to continue searching iff $X - \frac{c}{a(h_1+h_2)(1-p)} \geq X'$, and so the compensatory Region A, depicted in figure 2, is defined by

$$(12) \qquad a(X-X')b^{\frac{1}{2}} (1-p)^{\frac{1}{2}} \geq c$$

Since $(1-p)^{\frac{1}{2}} \geq (1-p)$, with equality at 0 and 1, Regions A and C overlap, as illustrated in figure 1.

Efficiency

For any given combination of parameters, the <u>efficient</u> steady state is the Configuration A, B, or C steady state which maximizes the aggregate net output flow.[1/] Once again, we can rule out Configuration B right away; since the projects of poor contracts are never completed under Configuration B, there is clearly no reason, from the standpoint of efficiency, for poor contracts to be made. For Configuration A, the steady-state outflow is

$$Q^A = abX - c(h_1 + h_2)$$

$$= abX - cb^{\frac{1}{2}} (1-p)^{-\frac{1}{2}}$$

where, because of the steady state, the project completion rate equals the entrance rate. For Configuration C, we have a net outflow of

$$(13) \qquad Q^C = ab(pX' + (1-p)X) - cb^{\frac{1}{2}}$$

Thus, the Configuration A steady-state is more efficient than that of Configuration C if and only if

$$(14) \qquad ab^{\frac{1}{2}} (X-X')p((1-p)^{-\frac{1}{2}} - 1)^{-1} \geq c$$

This locus is illustrated in figure 2. The important feature is that the efficiency locus lies below both the lower border of compensatory Region A and the upper border of compensatory Region C. Notice that this feature implies that if there are multiple compensatory steady states for some combination of parameters, the one with more search is efficient. That is, the result demonstrates that there is a bias towards too little

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[1/] Because of increasing returns to numbers of searchers, a social planner could, in general, increase the flow of net product by calling for non-steady state behavior. Rather than having individuals search continually, he could halt search to allow the stock of potential searchers to grow. During this time, of course, no search costs would be incurred. After the population grew to a sufficient size search could resume with rapid -- hence low cost -- meetings. For convenience, we rule out such policies.

search (relative to efficiency) in compensatory equilibria with quadratic meeting. The bias derives from the fact that decision by an individual to search always makes potential partners better off and never harms individuals of the same type. This positive externality is simply not captured by compensatory damages, which concern only the immediate parties to a breach. Consequently individuals do not receive sufficient incentive to search. We should emphasize that this unambiguous bias towards too little search depends crucially on the unambiguous positive externality of search under the quadratic technology. Indeed we shall show below (see section 13) that with a linear technology and a slightly more elaborate model, the results of this section can be reversed and that, for some parameter values, there can be too much search in a compensatory equilibrium.

5. No Damage Payments

In some circumstances individuals do not use formal contracts to reserve their partners while searching for better deals. Rather, they maintain their contacts, which may or may not be available at later times.[1/] In terms of the model as described above, damage payments are equal to zero. Surprisingly the Regions of different equilibria are the same with compensatory damages as with no damages. This equivalence does not generally carry over once poor contracts may be completed, as is shown in the companion paper.

With compensatory damages, a poor match which will not be carried out has no value. Thus, apart from Region C, we have V_2 equal to V_1. This implies zero compensatory damages and so the same damages with and without formal contracts. Thus in both settings, there is no Region B, and Region A has the borders given by equation (12). In the interior of Region C, V_2 exceeds V_1 and compensatory damages are positive. Thus there is less incentive to search with contracts than without. However, on the border of Region C, V_2 again equals V_1 since individuals are indifferent to continued search. Thus, the equation for the border is unchanged. We note that this argument has not made use of the details of the technology and remains true with the linear technology. We also note that the possibility of completion of such contracts would raise V_2 over V_1 if the project were of positive value, ending the equivalence.

6. Liquidated Damages

The common law bases damage payments on the need to compensate for a breach of contract. In theory, therefore, courts allow the substitution of privately set damage levels only when these approximate a suitable level for compensation which itself is difficult to measure. In practice, there is some opportunity for divergence between privately contracted damages (liquidated damages) and perfect compensation. The legal doctrine against liquidated damages in excess of the level needed for compensation is essentially paternalistic. One argues that individuals must be prevented from mistakenly promising large compensation, because they do not fully anticipate events which might make them unable or unwilling to carry out the contract.

There are at least two additional arguments in favor of compensatory damages. One is the assertion that they are efficient. We saw above that with compensatory damages breach will occur if and only if the sum of the positional values of the principal parties to a breach -- the breachers and those they breach against -- increases. In this sense compensatory damages are efficient. What this analysis leaves out, as we have already noted, is the external effect these parties have on the rest of the market. Thus, equilibrium with compensatory damages is not necessarily efficient.

The other argument in support of compensatory damages is the claim that they are identical to the damages that rational parties to a contract would themselves choose. It is useful to review this argument. Suppose that i and j are negotiating a contract G which yields them positional values V^i and V^j, respectively, and suppose that, if either of them breaches , the payoff that he receives in his new contract is independent of the damages set in the old. In such a case, individual i, say, will be willing to breach in order to sign a new contract of positional value $\bar{V}^i$ iff $\bar{V}^i - D^i > V^i$, where D^i is the damage payment that i makes to j. If damages are compensatory, $D^i = V^j - V_1$, where V_1 is value of being partnerless. Thus i will breach iff $\bar{V}^i + V_1 > V^i + V^j$. Analogously for j. We see that with compensatory damages, i and j will breach precisely in those cases where they can increase the sum of their positional values. Since advance sidepayments are possible, it is clearly in i's and j's joint interest to set damages at the compensatory level.

The preceding argument is correct given the assumption that deals are independent of the damage payments currently set. In many instances, however, this independence is implausible. If 1 has to pay very high damages to form a new partnership with 2, while 2 is initially partnerless, 1 can forcefully argue that he should receive a larger share of the product of their partnership. Indeed, this is the result if the new partners divide the surplus (as defined) between them in some fixed proportion. One may try to rebut this argument by suggesting that 1's share be tied to his previous positional value rather than to the damage payments he makes. Damage payments, however, are probably far more readily observable than the values of previous positions.[1/] In any event, once shares in new deals become tied to previous damage payments, a pair of individuals in a contract has monopoly power over potential partners in a sense described in the introduction. Raising damage payments by one dollar increases the payment to the previous partner by one dollar. But the burden of payment is shared by the new partner. Damages cannot be raised without limit, because higher damages mean that breach is less likely and only when breach occurs can monopoly power be exerted. Still, they will be higher than compensatory damages.

As explained in the introduction, we assume in this paper that parties to a contract split the surplus equally. An individual's share, then, <u>does</u> depend on the damages set in his previous contract, and so monopoly extraction becomes possible. Such extraction alters positional values (except of the value of a good contract) and thus alters search strategies. We illustrate this point rather starkly in the model of the next section, where individuals make poor contracts but never complete them. The rationale for these contracts is solely to "milk" future partners for damage payments. While such contracts may seem artificial in our very simple setting -- after all, everyone should be aware that no poor contract will be carried out -- the artificiality disappears in somewhat more elaborate models where poor contracts <u>are</u> sometimes fulfilled. One such model is presented

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[1/] See the appendix for a model which explains how the observability of damage payment and the difficulty of observing project product may generate the behavior under liquidated damages that we discuss in this paper.

in section 14. We saw in section 4 that, because of externalities, compensatory damages provide too little incentive for search with quadratic meeting. We shall now see that the possibility of exercising monopoly power over potential partners has a mitigating effect on these externalities and that, consequently, a liquidated damage rule may be more efficient than compensatory damages.

7. Contracting with Liquidated Damages

We shall first calculate the damages which individuals would choose to stipulate in their poor contracts when search by N's is worthwhile. Notice that the level of damages cannot be optimal if an increase in the level does not diminish the possibility of a new contract because an increase raises the profit accruing to the contracting pair from any new contract.[1/] Thus, there are two possibilities. One is that damages are set precisely equal to the surplus that derives from a good match with an M. In this case, equilibrium is under Configuration B; there are single breaches (breaches where one of new partners is an M) but no double breaches (breaches where both new partners are N's).

(15) $$D^B = 2X - V_2 - V_1$$

The second possibility is that the partners forego some of the profit from single breaches for the opportunity to make matches which result from double breaches. In this case, the steady state is under Configuration A and damages are set at

(16) $$D^A = X - V_2$$

1/ We assume that the level of damages called for by the contract cannot depend on the quality of the new match of the breaching party.

We note that compensatory damages, $V_2 - V_1$, are always less than optimal liquidated damages. For D^B, this is clear since $X > V_2$ when search is costly. Note that $D^A = V_2 - V_1$ when the surplus from a double breach is zero. Raising damages above this level prevents double breaches but does not reduce profit because of the zero surplus. On the other hand, this increase in liquidated damages increases the return from single breaches. Thus D^A must exceed $V_2 - V_1$.

8. Equilibrium with Liquidated Damages (Quadratic Technology)

As before, we proceed by checking the conditions for equilibrium under each configuration.

Configuration C

We have an equilibrium under C if search is worthwhile for an M but not for an N. We assume, again, that the former condition holds. This latter condition differs from its counterpart for compensatory damages in that the gain to a pair of N's from a breach by one of them equals the (liquidated) damage payment minus compensatory damages (since liquidated damages are at a level which makes the surplus zero). This gain, of course, exceeds the gain occurring with compensatory damages. Therefore, the upper boundary of the liquidated Region C lies strictly below that of the compensatory C. Equating per unit search cost with expected return, we obtain the equation for the boundary

$$c = (1-p)ah_1(2X - V_2 - V_1 - (V_2-V_1)) \tag{17}$$

$$= 2(1-p)ab^{\frac{1}{2}} (X-X')$$

Configuration B

We have an equilibrium under Configuration B if two conditions are satisfied. One is that search is worthwhile for an N. The second is that individuals prefer to set damages at D^B rather than D^A. Now, when all other poor contract pairs set damages at D^B, there is no possibility of a profitable double breach $(2X - 2V_2 - (2X - V_2 - V_1) < 0)$. Therefore, the second condition is automatically satisfied. Because N's can make new contracts only with M's, the lower border of Region B is defined by the same expression, in terms of h_1, as the upper border of C. Moreover, the number of M's is the same in the two regions, since the $\dot{h}_1 = 0$ equations are the same for Configurations B and C. Therefore, liquidated Regions B and C partition b-p space, as figure 3 illustrates.

For later use, we derive the formula for V_2^B. V_2^B equals an N's expected positional value after a brief time less search costs. If an N finds a good match, he and his old partner extract all the surplus and, between them, share a positional value of 2X. Thus,

$$(18) \qquad V_2^B = -c\Delta t + ah_1^B\Delta t(1-p)(2X) + (1 - 2ah_1^B\Delta t(1-p))V_2^B$$

or

$$(19) \qquad V_2^B = X - \frac{c}{2ah_1^B(1-p)}.$$

Efficiency between B and C

Above we determined the locus of parameters separating the regions where output is greater under Configuration A than under C. We now compare output in B with C. Under Configuration B, only good contracts are carried out and, in the steady state, the flow of completions equals the flow of new entrants. Therefore

$$(20) \qquad Q^B = abX - c(h_1^B + h_2^B)$$

$$= abX - cb^{\frac{1}{2}} \left(\frac{2-b}{2-2p}\right)$$

and using (13))

$$(21) \qquad Q^B - Q^C = abp(X-X') - cb^{\frac{1}{2}} \left(\frac{p}{2-2p}\right)$$

Setting $Q^B - Q^C$, as given by (21), equal to zero, we note that we obtain the same equation as that defining the liquidated B-C border (18). Thus in the choice between Configurations B and C, liquidated damages result in the efficient option, whereas compensatory damages tend to promote too little search.

We note in passing that a steady state under Configuration A is more efficient than under B. Both configurations give rise to the same gross output flow. Under Configuration A, search costs are lower. Since search continues until a good match is made, there is no social value in passing up a good match between N's, since poor contracts will not be carried out. With more elaborate models, the comparison of efficiency between A and B becomes more interesting.

Configuration A

The boundaries for steady states in Configuration A, where all good matches are made, are set by two conditions -- the willingness of N's to continue searching and a preference by a pair of N's for damages at D^A rather than D^B. The latter condition requires the expected profit from a good match with damages set at D^A to exceed that of a good match with damages D^B, given that everyone else uses D^A. With probability h_1/h_1+h_2, any good match will be with an M. The gain to the pair is one-half the surplus -- $\frac{1}{2}(2X - V_2 - V_1 - D)$ -- plus the excess of the surplus over compensatory damages, $D - V_2 + V_1$ or $\frac{1}{2}(2X - 3V_2 + V_1 + D)$. With damages set at D^A, this is $\frac{1}{2}(3X - 4V_2 + V_1)$. With damages at D^B, this is $\frac{1}{2}(4X - 4V_2)$. With probability h_2/h_1+h_2, any good match is with an N. In this case, the gain to the pair is $X - 2V_2 + V_1$ for damages D^A. There is no surplus, and so no new match, if the damages are D^B. Thus, the condition for preferring D^A to D^B is

$$\frac{1}{2} h_1(X-V_1) \leq h_2(X - 2V_2 + V_1) \tag{22}$$

That is, the condition states that the expected gain from a match with an M accruing from higher damages be less than the foregone profits from possible double breaches. To evaluate (22) we must determine the values, V_i.

To calculate positional values, let us consider the possible positions an M could attain in a brief time Δt. He pays search cost $c\Delta t$ for this time and could meet another M to form a good or poor partnership, or could meet an N to form a good contract, or could form no new partnership at all. Thus

$$\begin{aligned} V_1^A = {} & -c\Delta t + ah_1\Delta t(1-p)X + ah_1\Delta tpV_2 + \tfrac{1}{2}\, ah_2\Delta t(1-p)(X+V_1) + \\ & (1 - ah_1\Delta t - ah_2(1 - p)\Delta t)\, V_1^A \end{aligned} \tag{23}$$

Solving for V_1^A, we obtain

$$V_1^A = (-ca^{-1} + X(h_1 + \tfrac{1}{2} h_2)(1-p) + V_2(ph_1))(h_1 + \tfrac{1}{2} h_2(1-p))^{-1} \tag{24}$$

Similarly, for an N, the expected cost of further search equals the expected gain.

$$\frac{c}{a} = \tfrac{1}{2}h_1(1-p)(X-V_1) + (h_1+h_2)(1-p)(X - 2V_2 + V_1) \tag{25}$$

or

(26) $$V_2^A = X - \left(\frac{c}{a(1-p)}\right)\left[\frac{h_1(\frac{3}{2} - \frac{1}{2}p) + h_2(\frac{3}{2} - \frac{3}{2}p)}{h_1^2(2 - \frac{1}{2}p) + h_1h_2(3-2p) + h_2^2(1-p)}\right]$$

Let us reconsider (22), the condition under which D^A is preferable to D^B, using (24) and (26). We obtain

(27) $$X - \frac{ca^{-1}}{2h_1(1-p)} \leq X - \frac{ca^{-1}}{(1-p)}\left[\frac{h_1(3-p) + 3h_2(1-p)}{h_1^2(4-p) + h_1h_2(6-4p) + 2(1-p)h_2^2}\right]$$

Rearranging terms and making use of (2) to eliminate h_2^2, we have

(28) $$(1-p)h_1 + (1-2p)h_2 < 0$$

Substituting from (2), (28) becomes

(29) $$p > p^*,$$

where p^* is the (positive) solution to

$$(1-p)^2\left[1 + \left(\frac{1+p}{1-p}\right)^{\frac{1}{2}}\right] = p(1 - 2p)$$

One can verify that $\frac{2}{3} < p^* < \frac{3}{4}$. Note that (29) states the liquidated equilibrium under A becomes possible only for "high" values of p. This may seem strange because the higher p, the less likely a good match. The explanation is that, as p rises, so does h_2 relative to h_1 in a steady state (See (2)). (22) shows that a larger h_2 makes double breaches more valuable.

The second condition for a Configuration A equilibrium is that V_2^A exceed X', so that search is worthwhile. This condition can be written

(30) $$(X-X')\,\frac{a}{c} \geq \frac{(3-p)h_1 + 3(1-p)h_2}{h_1(1-p)(4h_1 + (4-2p)h_2)}$$

or

$$(X-X')\ \frac{a}{c}\ b^{\frac{1}{2}} \geq \frac{(3-p)R + 3(1-p)}{(1-p)(4R + 4 - 2p)} \left[\frac{R^2 - (1-p)}{R^2}\right],$$

where $R = \frac{1-p}{p}\left[1 + \left(\frac{1+p}{1-p}\right)^{\frac{1}{2}}\right] = \frac{h_1}{h_2}.$

9. Choice of Damages

The above analysis of compensatory and liquidated damages shows that, with a quadratic technology, neither leads necessarily to efficient behavior. One may ask, therefore, if there exists a rule which does. There are two ways of asking this question: whether an efficient equilibrium is always attainable with some damage rule and whether an inefficient equilibrium can always be prevented. For the second question, the second answer is no, as we shall now show. The absence of efficient damages should not be terribly surprising. As we have noted, damages affect both search and breach decisions. Only by happy coincidence could a single instrument prevent the wrong decisions in both categories.

To see that inefficiency cannot necessarily be prevented with a quadratic technology, recall from (14), that a Configuration A steady state is more efficient than one of Configuration C if and only if $ab^{\frac{1}{2}}(X-X')p(1-p)^{\frac{1}{2}}-1)^{-1}>c$. Therefore if damages are to prevent inefficiency, we should have $\hat{V}_2^C>V_2^C$ when the previous inequality holds, where $\hat{V}_2^C$ is the positional value of an N who continues searching while everyone behaves under Configuration C. Otherwise, an inefficient equilibrium under C can occur. For $\hat{V}_2^C$ to exceed V_2^C, the surplus from a single breach must be positive or else there is no reason for search. If damage payments are D and single breaches are worthwhile, $\hat{V}_2^C = X- \frac{c}{ab^{\frac{1}{2}}(1-p)} + \frac{1}{3-p}D$. Thus $\hat{V}_2^C$ is increasing in D up to the point where the surplus from a single breach is zero. But liquidated damages are precisely those which make this surplus zero. Thus an equilibrium in C cannot be prevented for those parameter values for which there exists a liquidated damage equilibrium under Configuration C. The border for Region C coincides with the efficiency border between Regions B and C. But Configuration A is always more efficient from Configuration B. Thus from figure 3, we see that there are parameters such that no matter how damages are chosen, the economy has a Configuration C equilibrium when A would be more efficient.

A requirement on damages less demanding than to rule out all inefficient equilibria is simply to ensure that an efficient equilibrium exists. If the efficient Configuration is C, there is no problem, since compensatory damages will guarantee that the equilibrium is under Configuration C. The question is whether Configuration A behavior can be induced when it is efficient. Zero (or compensatory) damages lead to Configuration A equilibria in the compensatory Region A (see figure 1). There is a gap, however, (see figure 2) between the lower border of compensatory Region A and the efficient border. We ask how much of this gap can be covered by raising D above zero. Increasing D makes search by an N more worthwhile. D, however, is bounded from above by the requirement that double breaches yield a non-negative surplus. D is at the maximal level permitting such a surplus when set as in a Configuration A liquidated damages equilibrium, because then damages make the surplus zero. 1/ Therefore, the relevant comparison is between the liquidated A-C border ($V_2^A = X'$ where V_2^A is given by (26)) and the efficient border, given by (14). If the former lies entirely on or below the latter, efficient Configuration A equilibria can always be induced. Otherwise, not. As D approaches either zero or one, efficient equilibria can be supported in this way. We have not yet compared the two conditions for intermediate values of p.

1/ Since damages are centrally set, we need not be concerned about the individual advantage which might come from choosing D^B rather than D^A, as in the liquidated damages.

10. Linear Meeting Technology

In markets where potential traders are hard to find, the quadratic technology we have examined may be a plausible approximation of the process of traders' meeting. When there are many traders, however, an individual's problem is less one of finding a potential partner than of finding a partner who makes a good match. In such markets, we may represent the meeting technology by assuming that the rate of finding potential traders is independent of the number of potential traders searching.[1/] Then additional searchers do not raise the probability of others' finding trading partners. What is crucial with this "linear" technology is not the number of searchers but their distribution between M's and N's. Additional searchers are influential through their effect on this distribution. Additional M's make trade more valuable for potential partners, while additional N's have the opposite effect when their positional values differ.

To study the linear technology, we follow the same procedure as before. We first consider equilibrium with compensatory damages. Since in equilibrium poor matches are never made, the issue of M-N distribution does not arise; all searchers are M's. Therefore, searchers exert no externality on others, and equilibrium is efficient. We then examine liquidated damages, where we demonstrate that the incentives for search and contract formation may actually be too great.

[1/] One could, of course, consider more general technologies.

11. Dynamics of the Linear Technology

We define the linear technology so that a searcher has a probability $\frac{ah_1}{h_1+h_2}$ of finding a potential partner who is an M and a probability $\frac{ah_2}{h_1+h_2}$ of finding an N (assuming $h_1 + h_2 \cdot 0$), where h_1 and h_2, as before, denote the number of M's and N's, respectively, among searchers. In Configuration A, which is defined as above, the decline per unit time in the number of M's equals the number of new contracts between M's less those who become partnerless due to double breaches less new entrants (observe that single breaches do not affect the number of M's). Thus,

$$(31) \qquad \dot{h}_1 = -ah_1^2(h_1+h_2)^{-1} + a(1-p)h_2^2(h_1+h_2)^{-1} + ab$$

The number of N's increases by the number of poor matches made by M's and decreases because of single and double breaches.

$$(32) \qquad \dot{h}_2 = aph_1^2(h_1+h_2)^{-1} - 2a(1-p)h_1h_2(h_1+h_2)^{-1} - 2a(1-p)h_2^2(h_1+h_2)^{-1}.$$

In a steady state, we have, of course, $\dot{h}_1 = \dot{h}_2 = 0$, so that

$$(33) \qquad -h_1^2 + (1-p)h_2^2 + b(h_1+h_2) = 0$$

$$ph_1^2 - 2(1-p)h_1h_2 - 2(1-p)h_2^2 = 0$$

For some calculations, we are interested in only the total number of searchers h. Only good matches decrease the number of searchers. Thus, under Configuration A,

$$(34) \qquad \dot{h} = -a(1-p)h + ab$$

In the steady state,

$$(35) \qquad h^A = \frac{b}{1-p}.$$

Under Configuration B, there are no double breaches. Therefore, the equations of motion differ from those above by the elimination of terms corresponding to double breaches. Thus,

$$(36) \qquad \dot{h}_1 = -ah_1^2(h_1+h_2)^{-1} + ab$$

$$\dot{h}_2 = aph_1^2(h_1+h_2)^{-1} + 2a(1-p)h_1h_2(h_1+h_2)^{-1}$$

In a steady state we have

$$(37) \qquad h_1^B = b\,\frac{2-p}{2-2p}$$

$$h_2^B = b\,\frac{p(2-p)}{4(1-p)^2}$$

Under Configuration C, only M's search. Therefore,

$$(38) \qquad \dot{h}_1 = -ah_1 + ab$$

In the steady state

$$(39) \qquad h_1^C = b.$$

12. Compensatory Damages with a Linear Technology

As with the quadratic meeting process, the linear technology produces no steady state equilibria with compensatory damages under Configuration B. Under Configuration C, the value of search is the expected output $pX' + (1-p)X$ less expected search costs $\frac{c}{a}$ (we assume that this number is positive). An N would gain $X-X'$ from further search and would expect to incur search costs $\frac{c}{a(1-p)}$. Thus the condition for equilibrium under Configuration C is

$$(40) \qquad X - X' \leq \frac{c}{a(1-p)}.$$

Observe that this condition is independent of b. This independence derives from a search technology where the number of potential partners does not affect the probability of meetings. We note that net output under Configuration C is

$$(41)\qquad Q^C = abV_1^c$$

$$= ab(pX' + (1-p)X) - bc.$$

Under Configuration A, everyone searches until finding a good match. Thus a poor match has the same positional value as no match at all. An N finds further search worthwhile if the expected gain, X-X', exceeds expected search costs $\frac{c}{a(1-p)}$. This condition is the complement of that for equilibrium under C. Thus, as figure 5 illustrates, compensatory Regions A and C form a partition of b-p space.

Calculating net output under Configuration A, we obtain

$$Q^A = abX - ch_1^A$$

$$= abX - \frac{cb}{1-p} = abV_1^A.$$

Comparing the equations for Q^A and Q^C with those defining Regions A and C, we find that the efficiency border coincides with the compensatory A-C border.

For later reference, we note that aggregate net output under Configuration B is given by

$$(42)\qquad Q^B = abX - c\left(h_1^B + h_2^B\right)$$

$$= abX - c\left(\frac{2-p}{2(1-p)}\right)^2$$

Thus the efficient B-C border is defined by

$$(43)\qquad X - X' = \frac{c}{a}\left(\frac{4-3p}{4(1-p)^2}\right)$$

13. Liquidated Damages with a Linear Meeting Technology

As before, the use of liquidated damage rules may, in our model, lead to the signing of a class of contracts which are never carried out. That they are never carried out and yet are valuable underscores the fact that, with liquidated damages, at least part of the value of contract is the profit extracted from a new partner if the contract is breached.

Configuration C

For an equilibrium under Configuration C, a pair must not find continued search profitable. If they do continue to search, they will set damages equal to $2X - X' - V_1$ to extract all surplus from a new match. The pair incurs costs $2c\Delta t$ to search for time Δt. Their gain from a good match is the excess of liquidated damages over compensatory damages (which are $X'-V_1$). Thus the condition for an equilibrium is

$$c \geq 2a(1-p)(X-X') \tag{44}$$

The border defining the liquidated Region C is, again, independent of b and lies to the right (see figure 6) of the compensatory A-C border. Since for N's, search with liquidated damages is more valuable than search with compensatory damages, the liquidated Region C is smaller than its compensatory counterpart.

Configuration B

For an equilibrium under Configuration B, search must be worthwhile to an N whose contract sets damages at $2X - X' - V_1$. In contrast with the quadratic technology, liquidated Regions B and C are not contiguous for the linear technology. The gap between the regions derives from the fact that when other N's are searching (as in Configuration B) search (with a linear technology) is less worthwhile than when they are not (as in Configuration C). Under Configuration B, search by a pair of N's costs 2c per period and yields profit X-X' with probability $2a(1-p)h_1^B(h_1^B + h_2^B)^{-1}$. Thus, for an equilibrium under Configuration B, we have

$$(44) \qquad \frac{c}{a(X-X')} \leq \frac{4(1-p)^2}{2-p}$$

We note that the efficient A-C border always lies to the left of the liquidated C border and may lie to the left or right of the liquidated B border, depending on the values of parameters a, c, and X-X'. The case where it lies to the left is of particular interest because then, for some values of p, efficient behavior falls under Configuration C, while liquidated damage rules lead to equilibrium in B. That is, the liquidated damage rule in fact encourages too much search and, hence, too much breach of contract because of monopoly profit. Whether the liquidated B border lies to the right or left of the efficient A-C border, liquidated damages give rise to equilibria under Configuration B for parameter values for which efficiency requires Configuration A behavior. This means that, although individuals are efficiently searching, they may tend to form too few new contracts.

Configuration A

For a steady state under Configuration A two conditions must hold: damages $D^A = X-V_2$ must be preferable to $D^B = 2X - V_2 - V_1$ and continued search by N's must be worthwhile. The first condition will determine the left border of liquidated Region A and the second, the right border. If the two borders are in the wrong relative positions, there is no liquidated Region A in the b-p plane.

The preference for D^A over D^B is equivalent to

$$\tfrac{1}{2} h_1(X-V_1) \leq h_2(X - 2V_2 + V_1)$$

or

$$(45) \qquad (\tfrac{1}{2} h_1 + h_2)(X-V_1) \leq 2h_2(X-V_2)$$

This condition has the same form as (22), which determines one border of the liquidated A region with a quadratic technology. Indeed, we shall now show that the two conditions are identical. Choose b^Q and b^L so that for a quadratic meeting technology with entry rate ab^Q, the meeting rates ah_1^{AQ} and ah_2^{AQ} are the same, respectively, as the meeting rates $\dfrac{ah_1^{AL}}{h_1^{AL} + h_2^{AL}}$ and $\dfrac{ah_2^{AL}}{h_1^{AL} + h_2^{AL}}$ for a linear technology with entry rate ab^L. Since h_1^A/h_2^A is the same for two economies, independent of b, such a choice is possible. One can verify that equations (23) and (26), defining V_1^{AQ} and V_2^{AQ}, also define V_1^{AL} and V_2^{AL} when the quadratic meeting rates are replaced by the linear. Thus V_1^A and V_2^A are the same for the two economies, completing the argument. Thus, as with the quadratic technology, the equation $p = p*$, where $p*$ is as in (29), defines one of the borders of the liquidated Region A. Since the other liquidated borders all depend on $c/a(X-X')$, $p = p*$ may have any position relative to these borders.

The other border of Region A is determined by the desirability of search. From the argument just given, we have the same expression for V_2^A in terms of h_1 and h_2 for the linear technology with meeting rate a as for the quadratic technology with meeting rate $a/(h_1+h_2)$. Thus, from (30), the condition for indifference to search is

$$(46)\qquad \frac{c}{a(X-X')} = \frac{h_1(1-p)(4h_1 + (4-2p)h_2)}{((3-p)h_1 + 3(1-p)h_2)(h_1+h_2)} = \frac{R(1-p)(4R + (4-2p))}{((3-p)R + 3(1-p)(R+1))},$$

where

$$R = \frac{1-p}{p}\left[1 + \left(\frac{1+p}{1-p}\right)^{\frac{1}{2}}\right].$$

14. Completion of Poor Contracts

In the case of a linear technology and compensatory damages, equilibrium in the model is efficient. We suggested above that this result is not robust to elaborations in the model because it depends on a poor contract's being of no greater positional value than no contract at all. Once poor contracts have incremental value, N's would tend to search too much, relative to efficiency, since their presence reduces the value of search to potential partners. M's are more valuable than N's as partners because for them the trading surplus is larger. This point is illustrated below by an equilibrium under Configuration B when Configuration C behavior is more efficient. Similarly, private incentives for double breach will be too small. Adding M's and removing N's from the search process--as a double breach does--enhances the value of search for potential partners, but such an effect is not taken into account by individuals. We shall illustrate this point by providing an example of a compensatory equilibrium under Configuration B where, however, efficiency requires Configuration A behavior.

There are several ways to alter our model to introduce search by individuals with contracts of incremental value. One way is to make production continuous. We hope to study such a feature in a future paper. Another method is to postulate rising search costs for each individual. Instead, we will make a smaller, but more artificial, change in the model. We assume that, with probability K per unit time, any given individual must leave the search market. If he is an M, he exits with zero payoff. If he is an N, both he and his partner[1/] leave and carry out the project with value X'.

15. Steady States with a Probability of Leaving

We shall provide an example of a compensatory equilibrium under Configuration B where efficient behavior is given by Configuration C. We first calculate the equilibrium numbers of searchers. Under Configuration C, we have the single equation

$$\dot{h}_1 = -ah_1 + ab - aKh_1 \tag{47}$$

In steady state equilibrium

$$h_1^C = \frac{b}{1+K}.$$

Net output under C satisfies

$$\begin{aligned} Q^C &= ah_1^C(pX' + (1-p)X) - ch_1^C \\ &= \frac{ab}{1+K}(pX' + (1-p)X) - \frac{bc}{1+K} \\ &= h_1^C V_1^C \end{aligned} \tag{48}$$

Under Configuration B, we have two equations of motion

$$\dot{h}_1 = -\frac{ah_1^2}{h_1+h_2} + ab - aKh_1$$

$$\dot{h}_2 = \frac{aph_1^2}{h_1+h_2} - \frac{2ah_2h_1(1-p)}{h_1+h_2} - 2aKh_2$$

[1/] We could, alternatively, have specified that an individual could choose whether to carry out the project with a partner who is forced to leave. He would opt to carry it out if $X' - V_1^B$ were positive. This expression is, in fact, positive in the example which follows.

These equations yield the steady states

$$(49)\qquad \frac{h_1^B}{h_2^B} = \frac{1 - p + K + ((1-p+K)^2 + 2pK)^{\frac{1}{2}}}{p} \equiv R$$

$$(50)\qquad h_1^B = \frac{b(1+R)}{R + KR + K}$$

Aggregate net output is given by

$$(51)\qquad Q^B = a(1-p)X\left(\frac{2+R}{1+R}\right)\left(\frac{b(1+R)}{R + KR + K}\right) + 2aKX'\left(\frac{b(1+R)}{R(R+KR+K)}\right)$$

$$- c\left(\frac{1+R}{R}\right)\left(\frac{b(1+R)}{R + K + RK}\right) = h_1^B V_1^B$$

We will have a compensatory equilibrium under Configuration B if search by N's is worthwhile and double breaches are not worthwhile; that is, if $V_2^B \geq X'$ and $2V_2^B \geq X + V_1^B$. To check these two conditions, we need an expression for V_2^B. We have

$$V_2^B = -c + \frac{h_1^B}{h_1^B + h_2^B}\, a(1-p)X + 2aKX + \left[1 - \frac{h_1^B a}{h_1^B + h_2^B}(1-p) - 2aK\right] V_2^B$$

or

$$(52)\qquad V_2^B = \frac{\frac{R}{1+R}(1-p)X + 2KX' - c/a}{\frac{R}{1+R}(1-p) + 2K}$$

Consider the choice of parameters $a = 1$, $b = 1$, $c = 1$, $K = \frac{1}{2}$, $p = \frac{1}{2}$, $X' = 20$, $X = 23$. These numbers give rise to $R = 4.45$, $V_1^B = 16.9$, $V_2^B = 20.2$, $Q^B = 12.8$, $Q^C = 13.7$. Thus, we have an example of a compensatory equilibrium under Configuration B, where, in fact, behavior under Configuration C provides greater efficiency. Moreover, for these parameters, there is no compensatory equilibrium under C (i.e., $c \leq a(1-p)(X-X')$).

We also note that with these parameters, Q^A is 12.83. To calculate this we note that the equations of motion satisfy

$$\dot{h}_1 = -\frac{ah_1^2}{h_1 + h_2} + \frac{a(1-p)h_2^2}{h_1 + h_2} + ab - aKh_1$$

$$\dot{h}_2 = \frac{aph_1^2}{h_1 + h_2} - \frac{2a(1-p)h_1h_2}{h_1 + h_2} - \frac{2a(1-p)h_2^2}{h_1 + h_2} - 2aKh_2$$

This gives the steady state values

$$\frac{h_1^A}{h_2^A} = \frac{1 - p + K + ((1-p+K)(1+p+K))^{\frac{1}{2}}}{p}$$

$$h_2^A = \frac{b(1 + \frac{h_1}{h_2})}{(1+K)(\frac{h_1}{h_2})^2 - 1 + p + K(\frac{h_1}{h_2})}$$

Steady state output is

$$Q^A = a(1-p)X(h_1 + h_2) + 2aKh_2X' - c(h_1 + h_2) = h_1^A V_1^A$$

We do not have an equilibrium in A since $X + V_1 < 2V_2$ where

$$V_2^A = (-c + a(1-p)X + \frac{h_2}{h_1 + h_2} a(1-p)V_1 + 2aKX')/$$

$$(a(1-p)\frac{h_1 + 2h_2}{h_1 + h_2} + 2aK) = 20.14$$

$$V_1^A = 16.82$$

Appendix: A Model of Exaggeration

In this appendix we present a simple model of damage setting which explains, when damage payments but not positional values can be readily observed by everyone, how the equilibrium search and breach behavior under liquidated damages that we have described in the text may arise even with compensatory damages. To determine whether to a sign a contract, potential partners must evaluate the surplus of their match. Implicitly, we have until now assumed observability of the three components of surplus: the aggregate positional value of the new match, the levels of any damages to be paid to former partners if the contract is signed, and the pre-contract positional values of the potential partners. While retaining observability of the first two items, we now suppose that potential partners cannot assess each other's current position. Instead we assume that they simultaneously announce the value of their own current match, if any. These (possibly) strategically misrepresented announcements[1/] are then used to calculate the surplus. Not having a current match, an M is not permitted to misrepresent.

In the examination of liquidated damages above, we assume implicitly that the search decision is made jointly by the two partners to a contract and that either both or neither search. Such an assumption is required because under Configuration B, for example, all the gains from search accrue to the partner breached-against, so that it would be in an individual's interest to remain at home while his partner searched (Note that this complication does not arise with compensatory damanges because neither partner cares whether the other searches). If in our exaggeration model we assume that search and exaggeration are both joint decisions and that partners are free to set any damages, then search and breach behavior coincides with that of our previous liquidated damages model. In fact, the same conclusion holds even when damages are fixed exogenously (as long as they are not set above the liquidated level) because the partners can choose an exaggeration term so that the apparent surplus (from single breaches for a Configuration B equilibrium and from

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1/ The assumption that individuals can misrepresent their positional values in our simple model may not seem especially plausible, since one can infer that any searcher with a contract has value V_1.. In a world of many qualities, however, it makes sense.

breaches for an A equilibrium) is zero. That is, optimal search and breach behavior is not affected as long as the sum of exaggeration and damages remains constant. Changing the magnitudes of these addends alters only the distribution of gains between the two partners.

Interestingly, when we drop the assumption that exaggeration is a joint decision and leave it to the discretion of the exaggerator himself (but retain search as a joint decision), the Region borders under liquidated damages still coincide with those of the non-exaggeration liquidated model. Moreover, one particular optimal choice of damages is to set damages at the compensatory level (and, therefore, for these damages, the decision to search need not be joint). We shall demonstrate these results for the quadratic technology; they are also true for the linear.

That compensatory damages are also optimal liquidated damages in the exaggeration model suggests that observability of potential partners' current positions is essential to generate the compensatory damage behavior of Section 4. A central planner who naively imposes compensatory damages when observability does not hold will induce behavior corresponding to liquidated rather than compensatory damanges.

To establish our assertions, suppose that an N who meets a new potential partner claims to have a positional value $V_2 + E$, where E is the amount of exaggeration. On the border of Region C, optimal exaggeration makes the surplus from a single breach zero. Thus $E = 2X - V_2 - V_1 - D$. A pair of N's is just willing to search if $\frac{c}{a}$ equals $(1-p)h_1^C(E + V_1 + D - V_2)$. But $\frac{c}{a} = (1-p)h_1^C\ (E + V_1 + D - V_2) = 2(1-p)h_1^C\ (V_2 - 2V_1)$ is the same as (17), the equation for the upper boundary of liquidated Region C. Under Configuration B, if all other N's set E to make the apparent surplus from a single breach zero, the apparent surplus from a double breach will be negative. Therefore, double breaches will never occur, and taking $E = 2X - V_2 - V_1 - D$ is again optimal. The lower border of Region B, is, again, given by (17). Notice that for both Regions B and C, the choice of D is immaterial as long as $D < 2X - V_2 - V_1$. In particular D could be compensatory.

Consider, finally, an equilibrium under Configuration A, since double breaches are not profitable, a pair of partners will maximize their joint return if exaggeration is set at $E = X - V_2^A - D$. An N who has met a potential partner will clearly wish to set either the double breach or single breach apparent surplus to zero. That is, he will choose either $E^A = X - V_2^A - D$ or $E^B = 2X - V_2^A - V_1^A - D$. Notice that if damages are compensatory, E^A is preferable because then the N's expected gain is

$$\tfrac{1}{2}(1-p) \frac{h_1^A}{h_1^A - h_2^A} (3X - 4V_2^A + V_1^A) + (1-p) \frac{h_2^A}{h_1^A + h_2^A} (X - 2V_2^A + V_1^A),$$

which is larger than

$$\frac{h_1^A}{h_1^A + h_2^A} (2X - 2V_2^A),$$

the expected gain from $\bar{E}^B$. Therefore compensatory damages are optimal, and the lower Region A border is defined by

$$\frac{c}{a} = (1-p)h_1^A (E + \tfrac{1}{2}(2X - V_1^A - V_2^A - E - D)) + (1-p)h_2^A E$$

$$= \tfrac{1}{2}(1-p)h_1^A (3X - 4V_2^A + V_1^A) + (1-p)h_2^A (X - 2V_2 + V_1^A),$$

which is the same as for liquidated damages (see equation (25)).

Thus, in all cases the Regions are the same. This argument works only for damages less than or equal to the compensatory level, since otherwise an individual gains from his partner's breach, which must enter the calculations.

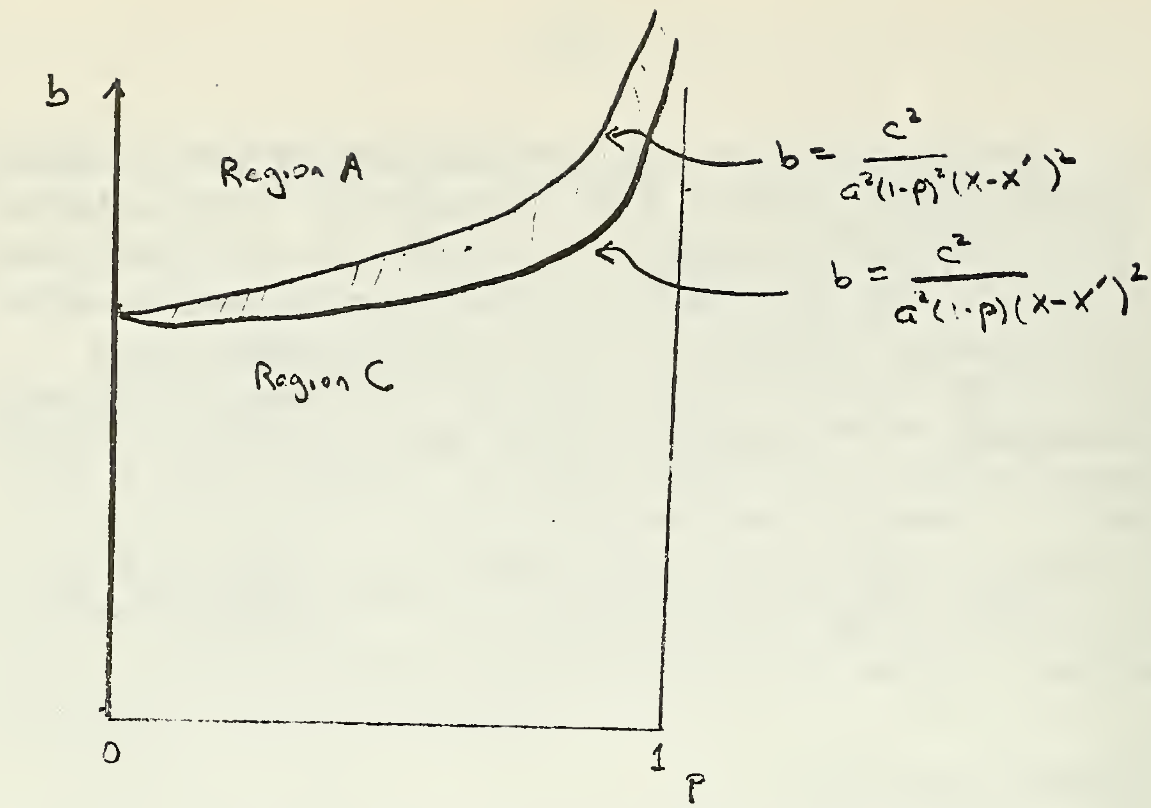

Figure 1 Equilibrium with Compensatory Damages and Quadratic Technology

The shaded region represents the intersection of compensat Regions A and C

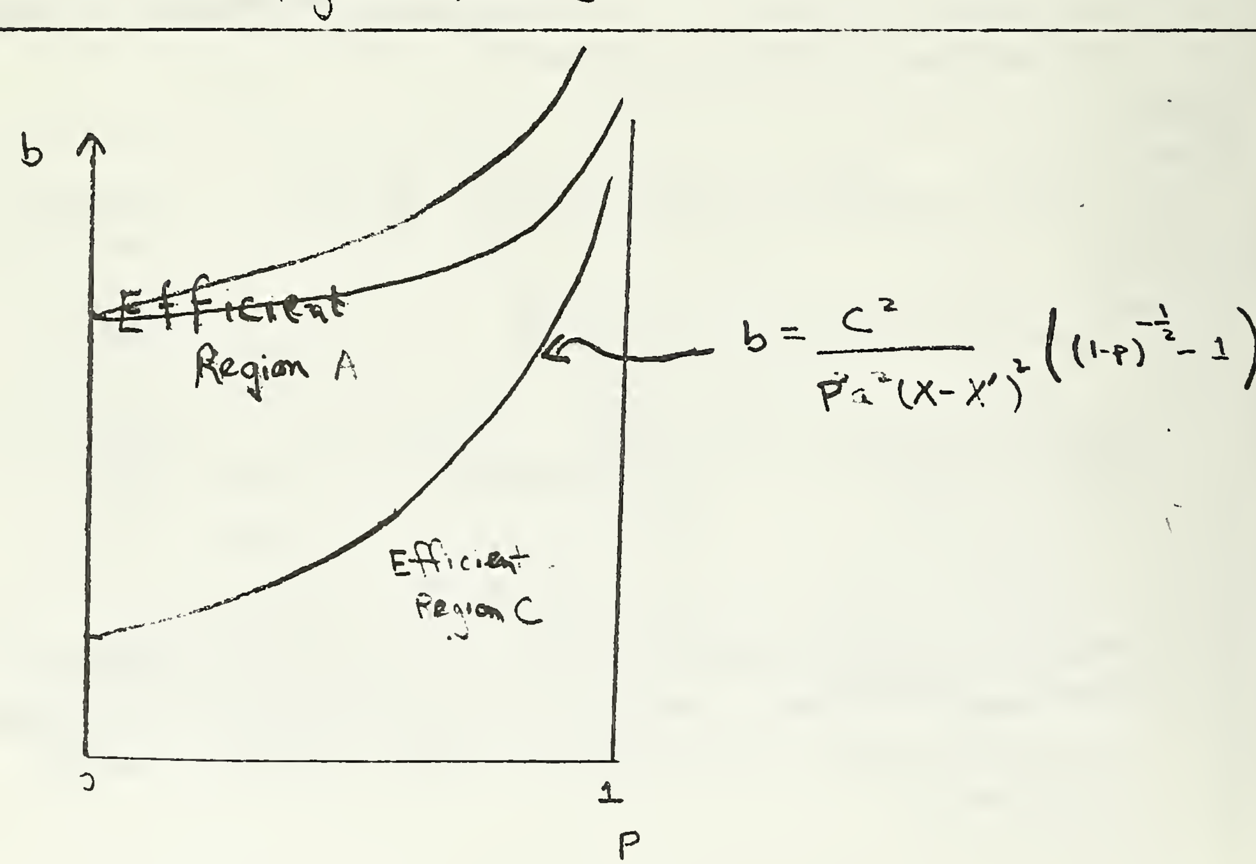

Figure 2 Compensatory Damages and Efficiency

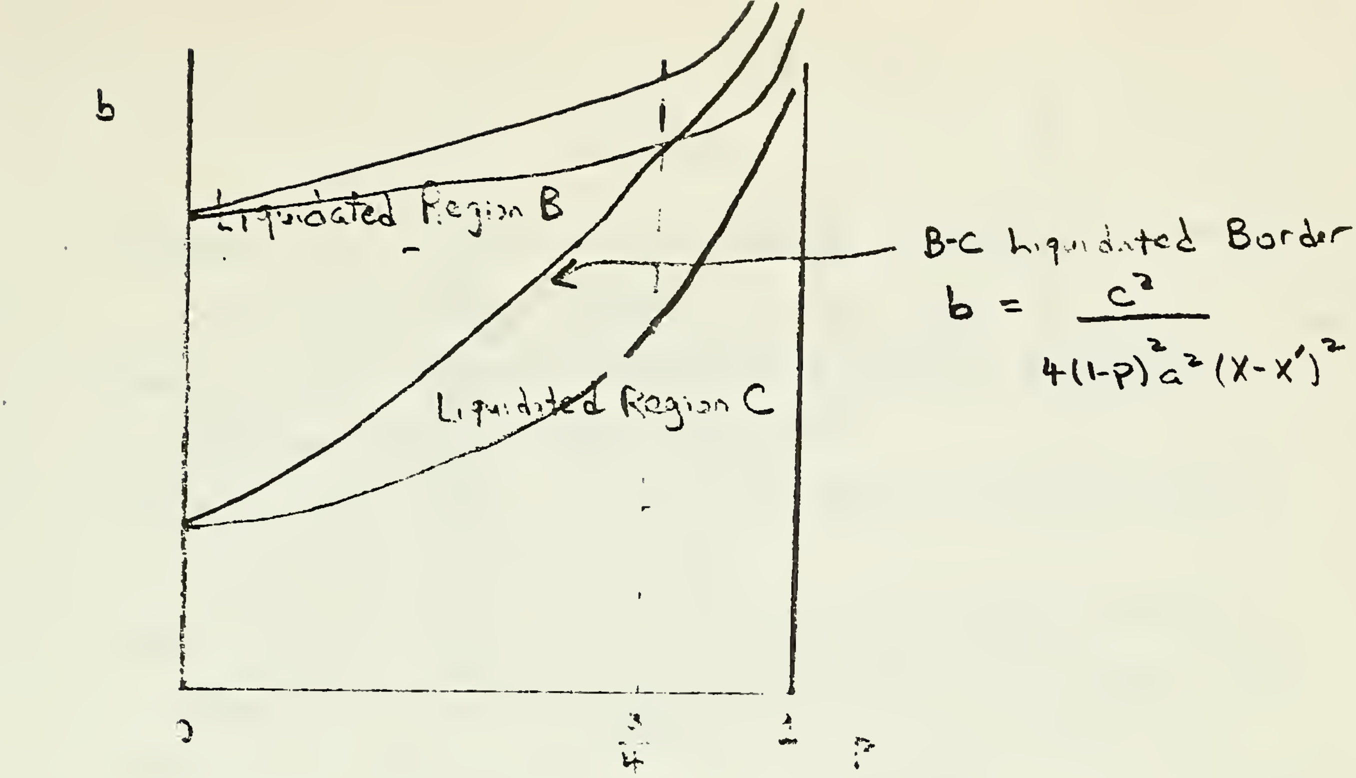

Figure 3 Regions B and C for Liquidated Damages with Quadratic Technology

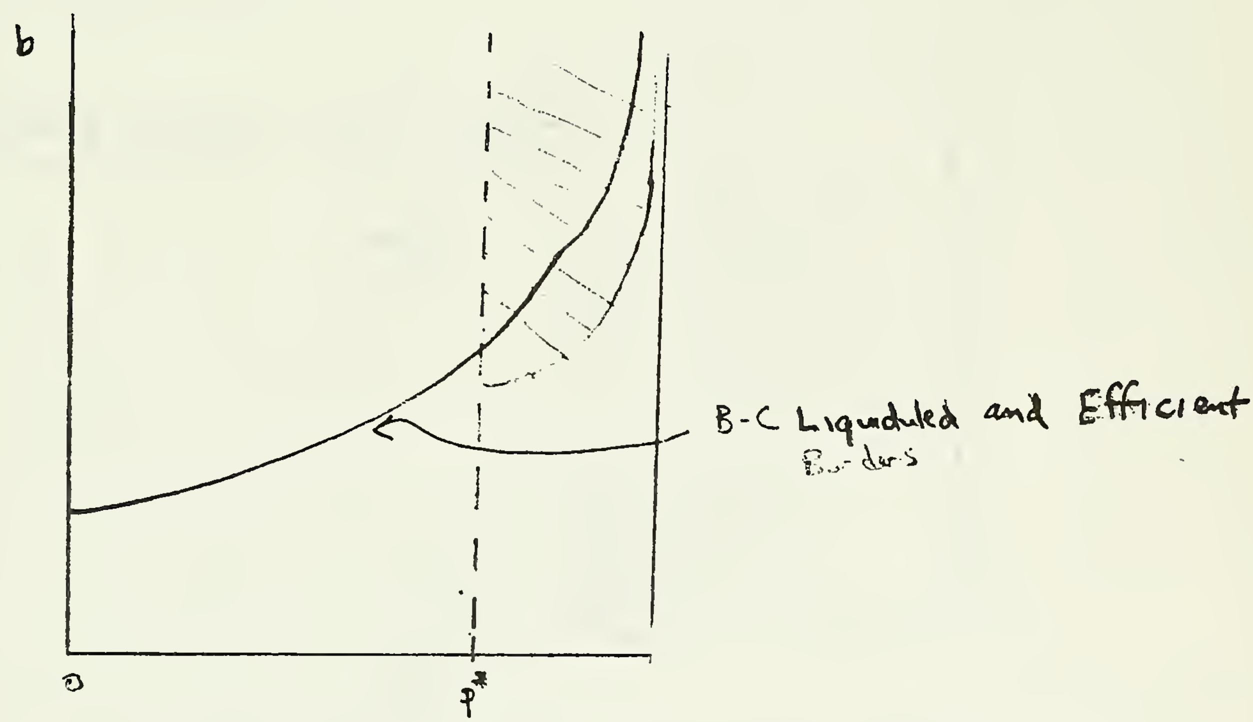

Figure 4 Region A for Liquidated Damages with Quadratic Technology

Shaded area represents Region A

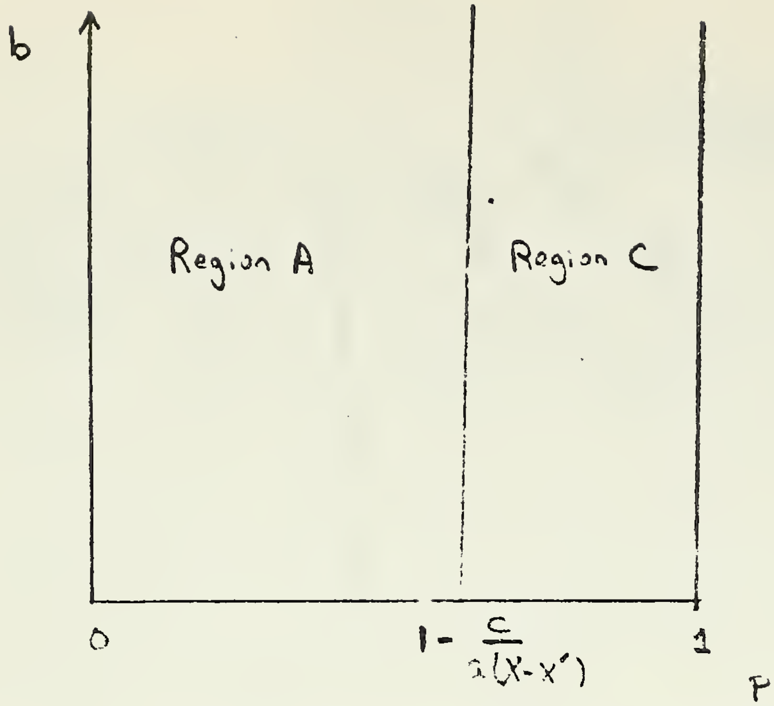

Figure 5 Equilibrium with Compensatory Damages and Linear Technology

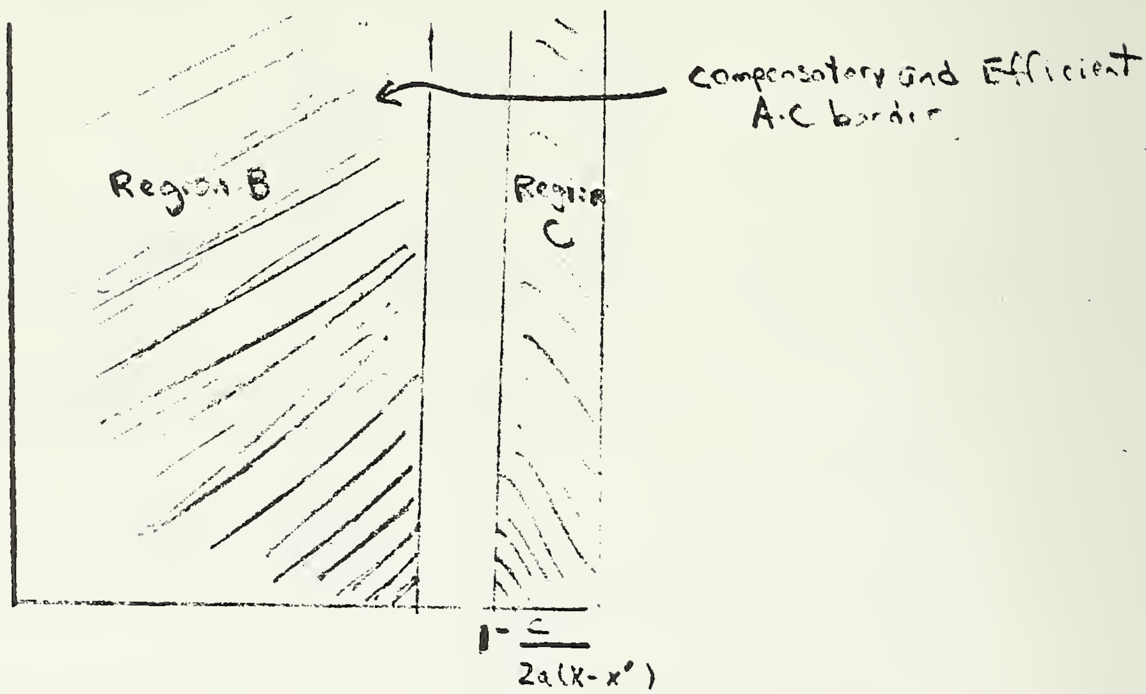

Figure 6 Regions B and C for Liquidated Damages and Linear Technology

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