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Educational Policy and the Economics of the Family

Abhijit V. Banerjee^{*}

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Abstract

This paper analyzes the implications of alternative ways to model decisionmaking by families for educational policy. We show that many of the policy implications associated with credit constraints cannot be distinguished from the implications of models of the family that differ from the conventional Barro-Becker model. We then argue that it is the combination of credit constraints and non-conventional preferences that provides a robust basis for government intervention to promote educational investment.

Keywords: Family; Intergenerational Transactions; Education. JEL Codes: O15; O16; D13.

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1 Introduction

The family plays a crucial role in decisions about investment in education, at least in developing countries. The reasons are simple: Education is expensive and there is only a very limited scope for borrowing in order to invest in education. Education is expensive because free and functioning public schools do not exist in many places, and even when they do they are often free only in name (The Probe Report, 1999, Kochar, 2000). One cannot borrow to invest in education because human capital provides very poor collateral, and, in any case, credit markets function very poorly in developing countries. Most of the financial investment in education therefore has to be funded by the family.

As a result, most theoretical and empirical studies of investment in education have to take a stand on how they think the family makes its decisions. This is an issue especially with respect to decisions about education that are taken when the children are young and parents are, at least nominally, in charge of financial decisions for the family. There is, however, very little agreement about how families make these decisions. Even a cursory glance at the literature on investment in education reveals a wide variety of views ranging from the Barro-Becker model (Becker, 1981; Loury, 1981; Mookherjee and Ray, 2000), where parents and children share a single unified utility function, to the pure overlapping generation model, where the family is purely a nexus for transactions—the old "lend" to their children who repay them with old age care (Kotlikoff and Spivak, 1981; Cox, 1987; Cox, 1990; Cremer, Kessler and Pestieau, 1992; Cox and Jakubson, 1995, Barham et al, 1995). There is also a range of formulations that are in between these two—formulations where parents put some weight on the income, consumption or human capital of their children,¹ for example because they like the "warm-glow" they get from what they have given the child.²

However, to the best of our knowledge, there have been no systematic attempts to

¹See Behrman. Pollak and Taubman (1982), and Ermisch and Francesconi (2000) for examples of formulations of this class.

²See Glomm and Ravikumar (1992), Galor and Zeira (1993). Banerjee and Newman (1994), and Galor and Moav (1999).

understand whether and in what way these things matter—in other words, what are the positive and normative implications of making each of these alternative assumptions. The goal of this paper is to fill a part of this gap in the literature. We present a simple one-good, two-sector growth model where goods are produced using skilled and unskilled labor. Everyone is endowed with a unit of unskilled labor, but skill has to be accumulated by investing in human capital. We consider both the case where the credit markets are perfect and the case where there are no credit markets, and investment in human capital is either financed by the state³ or by the parents of the child who is getting the human capital. In addition, we make the plausible and standard⁴ assumption that the production of skills is more intensive in human capital than the production of goods. Finally, we assume that the production function for skills exhibits decreasing returns and that there are no human capital externalities. We recognize that each of these last assumptions is potentially controversial. They are made mainly for strategic reasons. The assumption of non-convexities in the educational production function has been shown by Galor and Zeira (1993) to have a number of strong implications when combined with the assumption of imperfect credit markets—in particular, there are poverty traps and public investment in education that can increase steady state output. Moreover, they argue that their results are independent of the way we model parental preferences as long as the non-convexity is large enough. Since we are interested in the differences between the implications of different types of parental preferences, avoiding non-convexities clearly helps. We do, however, take comfort in the fact that the evidence for non-convexities in education is rather mixed. Psacharapoulus (1994) argues on the basis of evidence from a number of countries that the returns are highly concave. On the other hand, a couple of detailed studies from the U.S. find evidence of small non-convexities (Angrist and Krueger, 1999; Card and Krueger, 1992). Likewise, the decision not to allow for human capital externalities was driven in part by the fact that the consequences of such

³Where the state could be the local government.

⁴This assumption goes back at least to Uzawa (1965). Rebelo (1991), in his well-known study of education policy in growth models also makes this assumption.

externalities have been widely studied and in part by the fact that there seems to be very little micro-evidence for the presence of large externalities of this type.⁵

We study this model under different assumptions about family decision-making. We distinguish between the models of the family along three dimensions: First, whether altruism is perfect in the sense that each generation maximizes a discounted sum of the private utilities of future generations—the formulation associated with the work of Barro (1974). Second, the nature of the inter-generational contract within the family: Is it complete in the sense of binding all present and future generations; if incomplete, how many generations does it span? Third, is there symbolic consumption in the sense of people caring directly about consumption or investment outcomes, and not only because they are a source of utility to someone — the archetypal example is someone caring about their son going to Harvard because it is Harvard, and not because of he will be richer because of having been to Harvard.

We take as benchmark the case where there is perfect altruism but no contracts and no symbolic consumption, i.e., the so-called Barro-Becker model, combined with perfect credit markets. In this model (see section 3.1) the investment in human capital at any point of time is independent of current family wealth and parental preferences. This contrasts with what we find when we shut down the credit markets in section 4.1—in this case both parental preferences and family wealth matter for educational investment. However, when we allow for more general preferences (section 4.2) this simple contrast between the perfect and imperfect credit market cases no longer holds. It is still true that when there are credit constraints, there is a family wealth effect and a parental preference effect. However, both of these effects can arise even with perfect credit markets if (and only if) there is symbolic consumption of educational investment. This raises questions about using these effects as a way of identifying credit constrained households.

The steady state implications of the benchmark model, given in section 3.2, are perhaps

⁵See Acemoglu and Angrist (1999) and Duflo (2000). Bils and Klenow (1998) make a case against educational externalities using cross-country data.

even more striking.⁶ First, there is no inequality in the steady state: Every dynasty, irrespective of where they started, will end up with the same level of education.⁷ Second, the steady state is unique: Economies that start with very few highly skilled people (and therefore very few teachers) will end up with the same level of skills as an economy which starts with an abundance of them.⁸. Third, lump-sum taxes and subsidies have no long-run effects, while a proportional tax on human capital reduces human capital investment even if it is then redistributed as a lump-sum educational subsidy. Finally, policies that increase the rate of return on human capital necessarily raise investment.

Section 5.1 shows that these results also hold when we shut down the credit markets, as long as we have either perfect altruism, no symbolic consumption and no contracting, or no altruism and perfect contracting.⁹ To put things in perspective, it is worth underscoring that some of these results are very strong indeed. In particular the idea that an increase in the rate of return on education should always raise the level of investment is not at all obvious, since raising the rate of return also raises the price of teachers, which discourages investment.

Section 5.2 turns to the case of symbolic consumption of educational expenditures. In this case none of the five properties listed above necessarily hold with or without perfect credit markets and irrespective of whether there is no altruism or perfect altruism: Longrun inequality and multiplicity of steady states are now real possibilities and lump-sum taxes and subsidies have long-run effects. Proportional taxes on human capital may

 $^{^{6}}$ The steady state, as is well-known, is an inadequate proxy for what happens after the short run, but it is the only one we have.

⁷Remember there is no randomness in our model.

⁸This says that even if there is a shock which wipes out a huge part of a country's human capital (as AIDS seems to be in the process of doing in sub-Saharan Africa), the country will recover (actually it says less than that because even a unique steady state need not be a global attractor).

⁹The fact that the absence of credit markets does not make any difference in the long run, absent non-convexities, is in Loury (1981). The similarity of the two polar cases of no altruisms and complete contracts and perfect altruism and no contracts, arises from the fact that both lead to a perfect alignment of the incentives of parents and children. In this sense, these results are related to the intuitions behind Becker's famous "Rotten Kid Theorem" (Becker, 1981).

sometimes raise average investment and increases in the returns to human capital may *reduce* investment in education by making teachers expensive.

Finally section 5.3 looks at the case of imperfect altruism: While this case is harder to solve because we have to deal with time consistency issues, the results appear to be similar to those in section 5.2.

Taken together these results suggest that the educational policy debate is substantially handicapped by the fact that we have no settled model of the family. Even relatively elementary questions like whether it would be good to raise the rate of return on human capital (as recommended, for example, by Foster and Rosenzweig, 2000), seem to depend on what we believe about preferences. On the other hand, it suggests that the importance of credit constraints by themselves may be slightly exaggerated, at least if we mainly care about the long run.

2 The Basic Model

Production Technology: Consider a world where there is only one final good but two types of human inputs—skill and unskilled labor. The final good is produced by combining the two types of inputs using a technology:

$$y = f(H, L, \alpha)$$

where H and L are, respectively, the amount of skill and the amount of unskilled labor used in production, and α is a parameter describing the nature of the technology. We assume that the production function exhibits constant returns with respect to the two inputs together but diminishing returns with respect to each individual input. This formulation also imposes the restriction that all levels of human capital are perfect substitutes—one of the insights of the paper by Mookherjee and Ray, cited above, is that relaxing this assumption introduces an additional source of persistent inequality.

Labor Supply: Each person in the economy is assumed to own one unit of unskilled labor. In addition, they own a certain number of units of skills, which correspond to the amounts they invested in human capital.

The Life Cycle: Agents in this economy live for three periods. In the first period of their lives, there is no consumption or work—all they do is acquire skills. The second period is when people work and earn an income. We assume that labor has no disutility associated with it. This income can be spent on current consumption or saved and invested for consumption in the next period, when they no longer have wage income. In the second period of their lives they also give birth to exactly one child. The population therefore stays unchanged over time and each cohort is assumed to be size 1.

Human Capital: Human capital is produced using a combination of human capital and unskilled labor. The cost of acquiring h units of human capital is $\gamma s(h, h^-)$ units of human capital and $(1 - \gamma)s(h, h^-)$ units of unskilled labor, where $0 \le \gamma \le 1$ and $h^$ is the level of human capital of the parent of the person who is acquiring the human capital. We assume that $\frac{\partial s}{\partial h} > 0$, $\frac{\partial^2 s}{\partial h^2} > 0$, $\frac{\partial s}{\partial h^-} < 0$, $\frac{\partial^2 s}{\partial h^{-2}} > 0$, and $\frac{dS(h)}{dh} > 0$, where S(h) = s(h, h). The second and third assumptions tell us that the cost function is increasing and convex, therefore ruling out non-convexities. The next two assumptions capture the idea that the family atmosphere matters—children of skilled parents find it easier to acquire skills—though at a diminishing rate. The last assumption imposes the condition that the increased cost from increasing h is not outweighed by the reduction in cost coming from the fact that the next generation will now have more education and therefore find it easier to educate their children.

Markets: Throughout this paper we will make the assumption of perfectly competitive markets for labor and skills, with the price of labor at time t being w_t^L and that of skill w_t^H . Except in the next section we will make the extreme assumption that there are no capital markets and no assets other than human capital.

Policy Instruments: We assume that there is an educational subsidy of $e_0 + e_1 E$, where $E = (\gamma w_t^H + (1 - \gamma) w_t^L) s(h_{t+1}, h_t)$ is the amount the family spends on education. This is financed partly or entirely by a tax on the earning members of society that is partly lump-sum and partly a function of the taxpayer's human capital: $T = \tau_0 + \tau_1 h_t w_t^H$. Lumpsum taxes are of course standard while the tax on human capital earnings will turn out to be a simple way to introduce redistributive taxation. In order to get sharper results and limit the number of cases, we will, for the most part, focus on the case where we start from a situation where the family was already spending some amount on education and then look at the effect of very small changes in the taxes and subsidies. This allows us to avoid the issue of corner solutions.¹⁰

Preferences: We allow people to get utility both from private and collective (familylevel) outcomes. Private outcomes includes both material consumption and symbolic consumption. Material consumption is the consumption of the one final good when one is middle-aged and when one is old: U_t^M , the utility from material consumption accruing to a person of generation t, is $U_t^M(c_t, p_{t+1})$, where c_t is his consumption in middle-age and p_{t+1} his consumption in the last period of his life. Symbolic consumption covers things like the "warm glow" of giving to one's children (Andreoni, 1989), pride in having children who are well-educated or rich and the pleasure of being able to say that one's children go to an expensive school. In other words, the utility from private outcomes may take the form:

$$U_t^S = U_t^S(h_{t, h_{t+1}}, c_{t+1}, (\gamma w^H + (1 - \gamma) w^L) S(h_{t+1})),$$

where h_t is the human capital level of current generation, h_{t+1} is that of the next ("my son has a Ph.D."), c_{t+1} is the consumption level of the next generation ("my daughter drives a Porsche" and $(\gamma w^H + (1 - \gamma) w^L) S(h_{t+1})$ is the amount spent on the education ("my son goes to Exeter"). Total private utility is therefore $U_t^P = mU_t^M + sU_t^S$.

The utility from collective outcomes, is given by

$$U_t^C = \sum_{s=1} \delta_s U_{t+s}^P + \sum_{s=1} \widetilde{\delta}_s U_{t+s}^C.$$

Finally let U represent an individual's total utility, i.e. $U_t = U_t^P + cU_t^C$.

While this formulation is relatively general, it still imposes a number of stringent restrictions. In particular, we have assumed additive separability at a number of different

¹⁰This formulation does, however, come with the cost that there is no way to distinguish between educational subsidies and general income subsidies since even if the subsidy were earmarked for education, the family could always cut back on what it was already spending on education.

levels and we have ruled out children caring about the welfare of parents.

A special case of this is the standard Barro-Becker formulation of altruism: This is the case where c = 1, $\delta_1 = \tilde{\delta}_1 = \delta$ and $\delta_s = \tilde{\delta}_s = 0$ for s > 1. We will call this case "perfect altruism" to distinguish it from the case where either $\delta_s \neq \tilde{\delta}_s$, or at least one of δ_s and $\tilde{\delta}_s \neq 0$, for s > 1, which we call "imperfect altruism". An even more special case is one where symbolic consumption is ruled out, i.e., s = 0 and m = 1. It is easily checked that this yields the standard Barro-Becker preferences, which is a utility function of the form:

$$U_t = \sum_{s=0} \delta^s U^M(c_{t+s}, p_{t+s+1}).$$
 (1)

Inter-generational Contracting: Since each generation may have to rely on educational investment decisions taken by previous generations, inter-generational contracts can potentially play an important role in this analysis. Suppose, for example, generation t has to rely on generation t - 1 for its human capital investment. But if generation t - 1does not care about the well-being of the next generation, why would it invest? One possible solution is that generation t contracts with the previous generation to provide it with old-age care (p_t) in return for the human capital investment (h_t) . This would clearly provide some incentives to invest. However, the amount of old-age care generation t is happy to provide will depend on how much he gets to consume, c_t , which, given that he has a budget constraint, clearly depends on how much he will have to invest in the human capital of the next generation (h_{t+1}) . In other words, it will depend on the contract between generation t and generation t + 1, which, in turn, will depend on the contract

The same issues arise even when there is some altruism. The basic point is that the preferences of the current generation over future outcomes can be very different from that of the generation who has control over that decision. Of course, there could be contracts (implicit or otherwise) between generations which would minimize these issues but there are obvious difficulties with contracts that span many generations.

The fact that there are externalities across contracts suggests that the ideal contract may be one that covers all the generations. Under such a contract, $\{p_t, c_t, h_{t+1}\}_{t=0}^{t=\infty}$ will be simultaneously chosen to maximize $\sum_{t=0}^{\infty} \lambda^t U_t$. Obviously, one should not think of this as a real contract: It is perhaps best thought of as a norm that binds all generations of particular dynasty. This is what we will call a complete contract.

This is obviously an extreme assumption. One alternative would be to go to the other extreme: Simply assume that inter-generational contracts are impossible, i.e., no contracting. Then each generation would have to be assigned control rights over a set of decisions. We impose the sensible restriction that this set should not include decisions about actions taken when they are not living. We define the potential control set of generation $t D_t = \{p_{t-1}, p_t, p_{t+1}, c_{t-1}, c_t, c_{t-1}, h_t, h_{t+1}, h_{t+2}\}$. The actual control rights would be an allocation of a subset of D_t to generation t in such a way that each generation gets to take the exact corresponding set of decisions.¹¹

A third alternative, which allows some scope for contracting, is to assume bi-generational contracting—we assume that generation t and t + 1 are jointly allocated control over a subset of $D_t \cup D_{t+1}$ in such a way that each pair of generations get to take the exact corresponding set of decisions. They then choose the contract to maximize a weighted average of their utilities, $U_t + \lambda U_{t+1}$, taking as given the contract between generations t and t-1. Note that this makes no contracting a special case of bi-generational contracting where $\lambda = 0$.

3 The Benchmark: The Beckerian Model

3.1 Properties of the Short Run Equilibrium

Here we consider the case of our model defined by standard preferences, i.e., no symbolic consumption and perfect altruism combined with perfect credit markets and no contracting. The first two assumptions imply that the preferences of each generation are given by 1. Perfect credit markets amount to assuming that anyone can borrow and lend as much

¹¹In other words, if generation t gets to decide on a vector $X_t = \{p_t, c_t, h_t\}$ then generation t + s will get to decide on a vector X_{t+s} .

as they want at the market interest rate r. Under this assumption, each dynasty faces the inter-temporal budget constraint:

$$\omega_{t+1} = r_t (\omega_t - c_t - p_t + w_t^L + h_t (1 - \tau_1) w_t^H - (\gamma w_t^H + (1 - \gamma) w_t^L) s(h_{t+1}, h_t) (1 - e_1) + e_0 - \tau_0), \forall t.$$
(2)

where ω_t is the starting wealth of the tth generation. The term $(\gamma w_t^H + (1-\gamma)w_t^L)s(h_{t+1}, h_t)(1-e_1)$ represents the investment in the next generation's human capital measured in units of the good.

Since there is no contracting, each generation is assigned control rights over its own consumption when middle-aged (c_t) and old (p_{t+1}) , and the education of its child (h_{t+1}) . Maximizing the utility function given in Equation 1 under Equation 2 gives us:

$$\frac{\partial U_m(c_t, p_{t+1})}{\partial c_t} = \delta r_t \frac{\partial U_m(c_{t+1}, p_{t+2})}{\partial c_{t+1}}$$
(3)
$$\frac{\partial U_m(c_t, p_{t+1})}{\partial p_{t+1}} = \delta r_t \frac{\partial U_m(c_{t+1}, p_{t+2})}{\partial p_{t+2}}$$

$$(\gamma w_t^H + (1 - \gamma) w_t^L) s_1(h_{t+1}, h_t) (1 - e_1)$$

$$= \frac{1}{r_t} [w_{t+1}^H (1 - \tau_1) - (\gamma w_{t+1}^H + (1 - \gamma) w_{t+1}^L) s_2(h_{t+2}, h_{t+1}) (1 - e_1)].$$
(4)

Note that h_{t+1} is completely determined by the last equation, taking as given the current endowment h_t and its future expected level, h_{t+2} . Remarkably no utility term enters this equation. This has two implications: First, there are no income effects, since the choice of h_{t+1} is unaffected by whether the marginal utility of consumption is high or low. Second, parental preferences do not affect the decision to invest in human capital. Both these results follow from the fact that under perfect capital markets, the decision to invest in human capital can be analyzed entirely in terms of its net present value of income—the person who invests in his son's education today can immediately borrow back the implied increase in his son's future income and consume it.¹²

Observation 1: In a model with no symbolic consumption, perfect altruism, no contracting and perfect credit markets, investment in human capital neither depends on how much money the parents have nor on their preferences.

3.2 Properties of the steady state

As I say in the introduction, the main focus here is on what happens in the steady state. A steady state is defined to be a state of the economy where $r_t = r_{t+1} = r$, $c_t = c_{t+1}$, $w_t^H = w_{t+1}^H = w^H, w_t^L = w_{t+1}^L = w^L$ and $h_t = h_{t+1} = h_{t+2} = h$. Assuming that such an outcome exists, the above conditions reduce to one key condition:

$$s_1(h,h) + \delta s_2(h,h) = \delta \frac{(1-\tau_1)/(1-e_1)}{\gamma + (1-\gamma)w^L/w^H}.$$
(5)

We are now in a position to ask a number of questions about the nature of the steady state.

Dependence on Parental Preferences: Preferences enter equation 5 only because they help determine the steady state interest rate, which turns out to be $1/\delta$. Preferences of individual parents do not enter anywhere into this equation.

Inequality: Is the steady state perfectly egalitarian or is it possible inequality persists in the sense that different dynasties within the same economy converge to different steady state levels of human capital? Or in other words, are there multiple steady states for a fixed value of w^L/w^H ?¹³

¹²However, while there are no income effects in this economy, there can be a parental human capital effect, which in the data may look like a wealth effect: For example, if $s_{12} < 0$, an increase in h_t , keeping h_{t+2} fixed, lowers the marginal cost of investing in education (without affecting the benefits) and therefore raises h_{t+1} . In fact, the amount of human capital in all generations will go up. The increase in h_{t+1} causes h_{t+2} to go up, which causes $s_2(h_{t+2}, h_{t+1})$ to go down, which encourages further increases in h_{t+1} , etc.

¹³We also have to check that the value of w^L/w^H so generated is consistent with the amount of human capital that we would have in the steady state, but we can always choose the production function to

The argument, made above, suggesting that the children of educated parents get more education, tells us that such multiplicity may be possible and indeed this is the case if δ is small enough and the ratio s_{12}/s_{11} is positive and large enough. However, in the rest of this paper we focus on the case where such multiplicity cannot arise, namely the case where $s_{12} = 0$. There are two reasons why we make this choice: First Galor and Tsiddon (1997) have already developed the possibility of multiple steady states when $s_{12} < 0$ and we have nothing to add to their results. Second, it can be shown that the multiplicity only exists when the dynasties are sufficiently impatient—for δ close to 1, we have the equivalent of a Turnpike Theorem, which tells us that the steady state is independent of initial conditions. However, making this separability assumption has the unfortunate consequence that $s_2(0, h)$ can be strictly negative, implying that a more educated parent would have a lower educational expenditure than a less educated parent even when both are providing zero human capital.¹⁴

Under the assumption that $s_{12} = 0$, all dynasties converge to the same level of h. To see this, observe first that under this condition, neither h_t nor h_{t+2} enters Equations 3 and 4, which tells us that every dynasty chooses the same level of h_{t+1} . Therefore, everyone must have the same level of human capital in every period other than the very first.

Uniqueness: Do otherwise identical economies that start with very different average levels of human capital eventually end up with same average level? To answer this question we need to show that the steady state equation:

$$s_1(h,h) + \delta s_2(h,h) - \delta \frac{(1-\tau_1)/(1-e_1)}{\gamma + (1-\gamma)g(h,\alpha)} = 0$$
(6)

has a unique solution where the function $g(h, \alpha) \equiv \frac{w^L}{w^H}$ denotes the steady state relative to wages. To derive the function $g(h, \alpha)$ note that in a steady state where everyone has h units of human capital, the net supply of unskilled labor and human capital to the production sector are given by $1 - S(h)(1 - \gamma)$ and $h - S(h)\gamma$, respectively. Therefore

$$w_H = f_H(h - S(h)\gamma, 1 - S(h)(1 - \gamma), \alpha) \text{ and } w_L = f_L(h - S(h)\gamma, 1 - S(h)(1 - \gamma), \alpha).$$

¹⁴In some sense, this is just an interpretation of what zero human capital means.

What happens to $g(h, \alpha)$ when h goes up? Since $f(\cdot)$ is homogeneous of degree one, $g(\cdot)$ depends only on the ratio $\frac{h-S(h)\gamma}{1-S(h)(1-\gamma)}$. Since, by assumption S(h) is increasing in h, the denominator of this expression clearly goes down when h goes up. The effect on the numerator is, however, potentially ambiguous. However, if h is in the neighborhood of a steady state,

$$\frac{d(h-S(h)\gamma)}{dh} = 1 - s_1\gamma - s_2\gamma > 1 - s_1\gamma - \delta s_2\gamma = 1 - \frac{\delta(1-\tau_1)/(1-e_1)}{1 + \frac{(1-\gamma)}{\gamma}g(h,\alpha)}.$$

Therefore, in the neighborhood of a steady state, g(h) is increasing in h as long as the expression on the right is positive. This is true as long as $(1-\tau_1)/(1-e_1)$ is not too much larger than 1; in other words the subsidy cannot be too large. Basically, if the subsidy is too large there may be over-investment in education to the point where an extra unit of human capital costs more than one unit of human capital to produce. When this happens, adding an extra unit of human capital to the economy will lead to a *reduction* in the number of units of human capital available to the production sector. As a result, the relative price of skills will go up when the level of human capital in the economy goes up. For the rest of the paper we will assume that subsidy is never so large that we get into this case and therefore we will assume that $g(h, \alpha)$ is always increasing as a function of h, the per capita endowment of human capital in the economy.

To complete the argument, we differentiate the steady state map (Equation 6) with respect to h and use the fact that $s_{12} = 0$, to get the expression

$$s_{11} + \delta s_{22} + \delta \frac{1 - \gamma}{\gamma} \frac{(1 - \tau_1)/(1 - e_1)}{(\gamma + (1 - \gamma)g(h, \alpha))^2} g_1(h, a).$$

In the neighborhood of a steady state this expression is always positive since s_{11} , s_{22} and $g_1(h)$ are all positive. Therefore, the steady state has to be unique.

The Effects of Taxes and Subsidies: We are now in a position to look at some policy experiments. Consider first a small increase in the lump-sum educational subsidy e_0 or a fall in the lump-sum tax τ_0 . Since neither enters Equation 6, they clearly have no effect on the long-run level of investment in education, as long as the non-negativity constraint on investment in education is not binding.¹⁵

Changes in e_1 and τ_1 clearly do have an impact on educational investment. It is easily checked that the first-best level of investment will be achieved when $\tau_1 = e_1$. Since tax collection is costly, the optimum is presumably $\tau_1 = e_1 = 0$. Consequently, the only reason to subsidize education is to counteract the effects of pre-existing taxes on human capital. Moreover, an increase in e_0 financed by an increase in τ_1 clearly reduces educational investment. We interpret this as saying that if education subsidies are harder to target than taxes on human capital, then the net effect of the subsidy may be to discourage education.¹⁶

Raising Returns to Education: Finally, we can look at what happens when the rate of return to education goes up. We capture this by assuming that raising α lowers $g(h, \alpha)$ for all values of h: This amounts to an increase in the relative price of skills, for each level of h, and can be interpreted as the result, for example, of a government policy promoting the adoption of a new skill-intensive technology.¹⁷ It is clear from Equation 6 that if $g(\cdot)$ goes down, h will have to up. Therefore, an increase in the rate of return on education does increase investment in education in this model.

Observation 2: In the benchmark model the steady state is unique. Every dynasty has the same human capital level in the steady state irrespective of where they started and their preferences. Lump sum taxes and subsidies have no effect, while proportional taxes and subsidies tend to distort investment in human capital. Higher returns to human capital are associated with higher investment.

¹⁵In the short run, however, changes in $e_0 - \tau_0$, keeping all other policies fixed, do affect the interest rate and through it investment in education.

¹⁶Once again, this is only true if the non-negativity constraint does not bind.

¹⁷Foster and Rosenzweig (2000) suggest that the green revolution in India was an example of such a policy.

4 Beyond the Beckerian Model: Short Run Properties

4.1 The Role of Credit Constraints

Let us now consider a world where there are no credit markets, or at least the existing credits markets are too inefficient to be of relevance to most people. At this point we impose no restrictions on the class of preferences beyond what has already been imposed in section 2. In this world, people invest in their children's schooling using their family resources. Moreover, investing in education is the only investment opportunity available to the family. There are, however, also taxes and subsidies, just as in the previous section. Therefore, its budget constraint in any period t can be written as:

$$w_t^L + h_t w_t^H (1 - \tau_1) - (\gamma w_t^H + (1 - \gamma) w_t^L) s(h_{t+1}, h_t) (1 - c_1) - p_t + e_0 - \tau_0 = c_t.$$
(7)

This constraint holds for every t. Moreover, consumption is assumed to be always non-negative in order to make the credit constraint meaningful.

This assumption rules out the possibility that the decision to invest in human capital could be evaluated in terms of its effect on the present value of income—changing the level of investment must necessarily alter the distribution of consumption over time and/or across people. Marginal utilities of consumption must therefore enter the calculation that determines investment in education, and since changes in income have an effect on the marginal utility of consumption, they will affect investment in human capital. More specifically, from Equation 7 it is clear that any increase in current income (say e_0 or w_t^L has gone up) must either go into a higher h_{t+1} or a higher c_t or p_t . Suppose h_{t+1} did not go up. Then at least one of c_t and p_t would have to go up and the corresponding marginal utilities would have to go down. Yet, because of our separability assumptions, the marginal gain from additional investment in human capital is unchanged and therefore the new outcome with h_{t+1} unchanged and c_t or p_t having gone up, cannot be an optimum. Therefore h_{t+1} must go up. An increase in income must be associated with an increase in investment in human capital. Moreover the extent to which it would go up will clearly

depend on the relative weight given to these various outcomes in the collective preferences and therefore will typically depend on parental preferences.

This logic is quite easy to formalize, but the wide range of preferences permitted here makes it notationally cumbersome. We therefore limit ourselves to stating the implied result informally.

Observation 3: As long as credit markets are imperfect, two dynasties with different income levels and/or different preferences will typically invest different amounts in the human capital. This remains true even if there is perfect altruism and no symbolic consumption.

4.2 The Role of Non-standard Preferences

In this section we go back to the assumption of perfect credit markets, but now expand the set of preferences to allow for symbolic consumption and imperfect altruism. This is not entirely straightforward, in particular because even small departures from the Beckerian formulation can lead to unexpected complexities. To see this, combine the Barro-Becker assumption about preferences (i.e., the utility function of generation t is $\sum_{s=0} \delta^s U^M(c_{t+s}, p_{t+s+1}))$ with the assumption of bi-generational contracting (instead of no contracting). Let generations t and t + 1 have control over c_t, p_{t+1} and h_{t+1} . Then generations t and t + 1 will want to maximize $\sum_{s=0} \delta^s U^M(c_{t+s}, p_{t+s+1}) +$ $\lambda \sum_{s=1} \delta^{s-1} U^M(c_{t+s}, p_{t+s+1})$, which gives them an effective utility function:

$$U^{M}(c_{t}, p_{t+1}) + (\lambda + \delta) \sum_{s=0} \delta^{s} U^{M}(c_{t+1+s}, p_{t+s+2}).$$

This utility function builds in a form of hyperbolic discounting. As is well-known from the literature (Harris-Laibson, 2001) this generates decision rules that are not time-consistent and therefore the current generations, in taking their decisions, have to take account of the reaction function of the next set of decision-makers. The Beckerian model avoids this issue by making the very specific assumption that $\lambda = 0$.

We now consider a model where there is symbolic consumption of educational spending

and potentially imperfect altruism:

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$$U_{t}^{P} = U^{M}(c_{t}, p_{t+1}) + U^{S}((\gamma w_{t}^{H} + (1 - \gamma)w_{t}^{L})s(h_{t+1}, h_{t})).$$
$$U_{t}^{C} = \sum_{s=1}^{S} \delta_{s}U_{t+s}^{P} + \sum_{s=1}^{S} \widetilde{\delta}_{s}U_{t+s_{s}}^{C'}$$
$$U_{t} = U_{t}^{P} + cU_{t}^{C}$$

It is implicitly assumed here that there are no taxes and subsidies. To complete the description of the model, assume bi-generational contracting with generation t and t + 1 having control over c_t, p_{t+1} and h_{t+1} . As noted above, this subsumes the case of no contracting. Maximizing $U_t + \lambda U_{t+1}$ subject to the budget constraint (Equation 7), tells us that at an interior maximum it must be the case that:

$$\frac{\partial U^{M}(c_{t}, p_{t+1})}{\partial c_{t}} \times$$

$$[w_{t+1}^{H}(1-\tau_{1}) - (\gamma w_{t+1}^{H} + (1-\gamma)w_{t+1}^{L})s_{2}(h_{t+2}, h_{t+1})(1-e_{1}) - r_{t}(\gamma w_{t}^{H} + (1-\gamma)w_{t}^{L})s_{1}(h_{t+1}, h_{t})(1-e_{1})] - U^{\prime S}((\gamma w_{t}^{H} + (1-\gamma)w_{t}^{L})s(h_{t+1}, h_{t}))(\gamma w_{t}^{H} + (1-\gamma)w_{t}^{L})s_{1}(h_{t+1}, h_{t})(1-e_{1})]$$
(8)

Note that in writing down this condition we have not had to deal with the time consistency issues. This is because we assume both perfect credit markets and $s_{12} = 0$. If s_{12} were not equal to zero, the marginal cost of educational investment in the future would depend on how much the current generation invested and therefore the current generation's investment decision would have to take account of the responsiveness of future investment to current investment

It follows from equation 8 that as long as U^M and U^S are subject to diminishing marginal utility, any increase in family income (say h_t goes up) necessarily must increase h_{t+1} . Moreover, parental preferences, given by the nature of the U^M and U^S functions, clearly play a role. Moreover, it is evident that the result here is uninfluenced by the nature and form of altruism that we have assumed, since these terms do not enter the above condition. The key condition is that $U_h^s \neq 0$: When $U_h^s = 0$, the above condition reduces to equation 4, i.e., we collapse back to the benchmark case. Once again, this remains true whether we have perfect or imperfect altruism and whether we have no contracting ($\lambda = 0$) or bi-generational contracting. Moreover since symbolic consumption of other people's material consumption is effectively just a form of altruism, adding this type of symbolic consumption does not alter the result. Finally, while not shown here, it is easy to show that the same result (that we get same results as the benchmark case unless there is symbolic consumption of investment in human capital) holds in the case of multigenerational contracting the case of complete contracts.

This leads us to the following.

Observation 4: Given perfect credit markets, there can be income effects and parental preference effects on investment in human capital if and only if there is symbolic consumption of investment in human capital.

It follows from this observation and the previous one that the evidence of income effects on educational investment (see Jacoby, 1994; Glewwe and Jacoby, 2000; and Carvalho, 2000) can only be interpreted as evidence for credit constraints if we are prepared to assume that there is no symbolic consumption of educational investment in itself.

5 Beyond the Beckerian Model: Steady State Properties

5.1 Credit Constraints in the Beckerian Model

5.1.1 Case 1: Altruism Without Contracting

We are interested in the long run properties of the Beckerian Model if borrowing and ending are ruled out. We start with the case where altruism is perfect but there is no contracting with generation t having control over c_t , p_{t+1} and h_{t+1} . We continue to assume that there is no symbolic consumption. Using the budget constraint we can rewrite $\sum_{t=0}^{\infty} \delta^t u(c_t, p_{t+1})$ as $\sum_{t=0}^{\infty} \delta^t u(w_t^L + h_t(1 - \tau_1)w_t^H - (\gamma w_t^H + (1 - \gamma)w_t^L)s(h_{t+1}, h_t)(1 - e_1) + e_0 - \tau_0, p_{t+1})$. Maximizing this with respect to the sequence $\{h_t\}$ yields the first order conditions:

$$(\gamma w_{t-1}^{H} + (1-\gamma)w_{t-1}^{L})s_{1}(h_{t}, h_{t-1})(1-e_{1}) = \delta \frac{\partial U(c_{t}, p_{t+1})/\partial c_{t}}{\partial U(c_{t-1}, p_{t})/\partial c_{t-1}}[w_{t}^{H}(1-\tau_{1}) - (\gamma w_{t}^{H} + (1-\gamma)w_{t}^{L})s_{2}(h_{t+1}, h_{t})(1-e_{1})].$$

Using the fact that in the steady state $c_{t-1} = c_t$, $p_t = p_{t+1}$, $w_t^H = w^H$ and $w_t^L = w^L \forall t$, this reduces to the condition

$$s_1(h,h) + \delta s_2(h,h) - \delta \frac{(1-\tau_1)/(1-e_1)}{\gamma + (1-\gamma)g(h,\alpha)} = 0,$$

which will be recognized as Equation 6, the steady state condition under altruism and perfect credit markets. It follows that under the assumption that $s_{12} = 0$, there is complete equalization in the long-run, even though the rich invest more than the poor in the human capital of their children. Moreover, the steady state is unique and the level of investment and consumption in the limit are what they would be in a first-best world.¹⁸

The long-run effects of policy in this model, not surprisingly, are the same as in the benchmark case—lump-sum taxes and subsidies have no effect, and while proportional taxes and subsidies do have an effect, no taxes and no subsidies remain optimal. The general policy bias that comes from this model is that, at least in the long run, what matters is removing barriers that prevent the returns to human capital from being as high as possible.¹⁹

5.1.2 Case 2: Complete Contracts with No Altruism

In this subsection we consider the exact opposite model—a model where people are completely selfish in the sense that parents do not care at all about what happens to their

¹⁸This is essentially a deterministic version of the result in Loury (1981) showing that in the long run all dynasties have the same distribution of consumption even in the presence of credit constraints.

¹⁹The short-run effects of policy are, however, potentially different from the first-best case. In particular, because human capital is not instantly equalized, a subsidy financed by a tax on human capital does redistribute and the resulting increase in the income of the poor will typically increase their investment in human capital and thus hasten convergence.

children, but where there is a complete contract covering all generations. We stil rule out all forms of symbolic consumption. In this world, parents will invest in the education of their children, because the family here acts as an imperfect substitute for the missing credit market. In other words, parents invest in the education of their children in return for old age care. There are several plausible reasons for why such a transaction may be possible in an economy where there are no other credit transactions: First, parents typically know a lot more about their children, which may limit the amount of adverse selection; second, there may be social norms that punish children who do not take care of their parents, but do not apply to those who fail to honor their debts; finally, there may be emotional ties between parents and children which enable parents to "punish" children who do not take care of them.

Given these assumptions, we can now write down the exact maximization problem facing a particular dynasty at time 0, given that it started with h_0 amount of human capital and an obligation of p_0 towards its parent. It has to choose $\{h_t, p_t\}_{t=1}^{t=\infty}$ to maximize $\sum_{t=0}^{\infty} \lambda^t U(c_t, p_{t+1})$ given the initial values h_0 and p_0 and the budget constraint

$$w_t^L + h_t (1 - \tau_1) w_t^H - (\gamma w_t^H + (1 - \gamma) w_t^L) s(h_{t+1}, h_t) (1 - e_1) + e_0 - \tau_0 - p_t = c_t, \forall t.$$
(9)

To see what happens in the long-run, observe that the first order conditions for the above maximization problem (assuming an interior solution exists) are:

$$\lambda \partial U(c_t, p_{t+1}) / \partial c_t = \partial U(c_{t-1}, p_t) / \partial p_t$$

$$(\gamma w_t^H + (1 - \gamma) w_t^L) s_1(h_{t+1}, h_t)(1 - e_1) \partial U(c_t, p_{t+1}) / \partial c_t$$

= $\lambda [(1 - \tau_1) w_{t+1}^H - (\gamma w_{t+1}^H + (1 - \gamma) w_{t+1}^L) s_2(h_{t+2}, h_{t+1})(1 - e_1)] \partial U(c_{t+1}, p_{t+2}) / \partial c_t.$

In a steady state the second equation reduces to the much more compact condition:

$$s_1(h,h) + \lambda s_2(h,h) = \lambda \frac{(1-\tau_1)/(1-e_1)}{\gamma + (1-\gamma)g(h,\alpha)}$$

The expression defining the steady state level of human capital is exactly the same condition as in the case of true altruism, with λ replacing δ . A fall in λ , much like a fall in δ , discourages investment in human capital—greater power in the hands of the older generation leads to less education.

The other properties of the steady state are relatively unsurprising: Under the maintained assumption that $s_{12} = 0$, there is no inequality in the long-run, and the steady state is unique. Moreover, the changes in policy parameters have effects in the same direction as they would have in the case of true altruism.

However, a number of new policy issues arise: First, if the government initiates a payas-you-go social security system, investment in education is likely to collapse since parents will no longer feel the need for transfers from their children. Second, it raises the issue of whether the government should try to alter the balance of power within the family: There is no a priori reason why λ , which measures the bargaining power of the young, should also be the weight children get in the social welfare function. For example, the government may want to maximize steady state welfare rather than the maximand that actually gets maximized. In this case, unlike in the case of true altruism, a proportional subsidy to education or a mandatory schooling law may be optimal. In other words, the government may push people to invest more in education in order to safeguard the interests of the young.

Observation 5: The introduction of credit constraints alone do not alter the properties of the steady state as long as there is no symbolic consumption, and we have either perfect altruism and no contracting or complete contracting and no altruism.

One might wonder why the results described here do not extend to the case where we have both complete contracting and perfect altruism. The problem is that it is not clear that there will be a steady state in this case, because once we combine altruism and complete contracting the weight on generation t is no longer a constant multiple of weight on generation t + 1.

5.2 Beyond the Beckerian Model: Symbolic Consumption of Human Capital

5.2.1 An example

To see what changes with symbolic consumption of human capital, let us start with a case where the analytics are particularly straightforward: Consider a model where parents symbolically consume the amount spent on the child's human education. To avoid the issue of whether we should consider pre-tax expenditure or post-tax, we start by assuming that there are no taxes and subsidies. In addition, we assume no credit markets, no altruism and no contracting. Finally, we assume that there is no consumption in old age and that the cost of education is unaffected by parental education, i.e., that we can drop h_t from the $s(h_{t+1}, h_t)$ function. With these assumption, each generation maximizes a utility function of the form $U^M(c_t) + U^S((\gamma w_t^H + (1 - \gamma) w_t^L)s(h_{t+1}))$ subject to the budget constraint (Equation 7). It is easy to see the first order conditions for this problem is

$$\frac{U'^{S}((\gamma w_{t}^{H} + (1 - \gamma)w_{t}^{L})s(h_{t+1}))}{U'^{M}(w_{t}^{L} + hw_{t}^{H} - (\gamma w_{t}^{H} + (1 - \gamma)w^{L})s(h_{t+1}))} = 1,$$

which yields the steady state condition

$$\frac{U^{\prime S}((\gamma w^{H} + (1 - \gamma)w^{L})s(h))}{U^{\prime M}(w^{L} + hw^{H} - (\gamma w^{H} + (1 - \gamma)w^{L})s(h))} = 1.$$
(10)

Parental preference effect: It is evident from the form of equation 10 that parental preferences, represented in the intensity of desire for symbolic consumption relative to material consumption will affect the accumulation of human capital even in the long run.

Inequality: Both the numerator and the denominator of the expression on the left of equation 10 are typically going to be decreasing as a function of h^{20} . Whether the curve given by this expression slopes up or down depends on which of the two decreases faster. In particular, it is not hard to find cases where it goes up and down, generating

²⁰This is not always true because for high enough values of h, it is conceivable that the numerator increases with h. This requires, however, that the family is heavily over-investing in education. In other words, it must be the case that an increase in h actually *reduces* output net of educational expenditure.

the possibility of several solutions, which imply that there can be inequality in the steady state.

To see more precisely what is going on here, consider first the case where there is constant relative risk aversion, i.e., $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, and moreover $v(\cdot) = \delta u(\cdot)$. In this case, condition 10 can be rewritten in the form:

$$\frac{w^L + hw^H}{(\gamma w^H + (1 - \gamma)w^L)S(h)} = 1 + (\frac{1}{\delta})^{\frac{1}{\sigma}}.$$
(11)

It is easily checked, using the fact that the function $S(h) \equiv s(h, h)$ is increasing and convex, and the fact that S(0) = 0, that the left-hand side is decreasing as a function of h, and therefore the value of h that solves this equation is unique.

Consider next the other standard formulation for preferences, namely the constant absolute risk aversion family: i.e., $u(c) = 1 - e^{-\sigma c}$ and $v(\cdot) = \delta u(\cdot)$. In this case, Equation 10 can be written in the form:

$$-(w^{L} + hw^{H}) + 2((\gamma w^{H} + (1 - \gamma)w^{L})S(h)) = \frac{1}{\sigma}\log_{e}\delta.$$
 (12)

The left-hand side of this equation defines a convex function of h. If S'(0) = 0, it always starts by being downward-sloping but eventually slopes up (because S(h) grows faster than h). Assuming $-w^L > \frac{1}{\sigma} \log_e \delta$ and $S(h)/h \to \infty$ as $h \to \infty$, we have the case that is in Figure 1. There are three potential stationary values of h in this case— $h = 0, h = h_1$ and $h = h_2$. Equation 10 does not actually hold at h = 0: What happens is that the marginal utility of bequests is strictly less than the marginal utility of consumption, but since bequests cannot be negative, h = 0 is the optimal choice. Of the other two steady states, the one at h_2 is stable and therefore perhaps more worthy of interest.

In both the CARA and CRRA cases the basic forces are the same—there is an income effect of being more educated, which makes educated parents more willing to invest. This, however, is counteracted by the fact that an educated parent has to invest more just to make sure that his child is not less educated than he is, and, moreover, the marginal cost of investment is rising. Which of these effects dominates and therefore whether there is one or more stationary level of h, depends on the exact curvature of the utility functions.²¹

²¹This source of long run inequality is closely related to that in Galor and Moav (1999), though their

Uniqueness: The argument in the previous paragraphs shows that there can be inequality in the long run for CARA preferences. Dynasties that start with more human capital will also tend to end up with more human capital. Given that there is inequality, it is likely that uniqueness will also fail: An economy that starts with a large number of high human capital dynasties will tend to have a high long-run level of human capital.

There is actually a second, independent reason why uniqueness might fail: To highlight this reason we choose CRRA preferences so that in the long run there is perfect equality. Moreover assume (purely for convenience) that $\gamma = 1$. Under these conditions we can rewrite Equation 11 in the form:

$$\frac{g(h,a)+h}{S(h)} = 1 + (\frac{1}{\delta})^{\frac{1}{\alpha}}.$$
(13)

Both the denominator and the numerator of the ratio $\frac{g(h,a)+h}{S(h)}$ are increasing in h. We know that the ratio h/S(h) declines as h goes up since S is convex. However, it is entirely possible that $\frac{g(h,a)}{S(h)}$ increases in h at least over a range—this will be the case when the degree of substitutability between skilled and unskilled labor is relatively small. It is therefore quite straightforward to construct examples where the function $\frac{g(h,a)+h}{S(h)}$ has the form given in Figure 2. There are clearly three steady states in this case, of which the two extreme ones are stable. The multiplicity of steady states reflects the following simple intuition: Teachers are cheap in an economy where there is a lot of human capital and therefore the same level of bequests buys more human capital, and as a result the initial high level of human capital is reproduced.²²

Taxes and Subsidies: We now need to address the question of how taxes and subsidies enter the preferences. One possibility is that $v = v((\gamma w_t^H + (1 - \gamma) w_t^L)s(h_{t+1}, h_t)(1 - e_1) - e_0)$. This assumption says that people count as their bequest the entire amount they have spent on education, but not the taxes they have paid which are then spent on subsidizing education. An alternative possibility would be that people count the taxes they

model is based on Stone-Geary type preferences for bequest and is therefore analytically more demanding. ²²This type of multiplicity of steady states, sustained by price effects, is similar to the multiplicity in Banerjee and Newman (1993).

have paid as part of their bequest. In this case $v = v((\gamma w_t^H + (1 - \gamma) w_t^L)s(h_{t+1}, h_t)(1 - e_1) - e_0 + \tau_0 + \tau_1 h_t w_t^H)$. While the second assumption is more conventional, we prefer the first of these assumptions on the grounds that it is very difficult to imagine that people can identify the part of their tax payments that is going to pay for the educational subsidies, but we allow for both.

Under the first assumption, the steady state condition turns out to be:

$$\frac{u'(w^L + h(1 - \tau_1)w^H - (\gamma w^H + (1 - \gamma)w^L)S(h)(1 - e_1) + e_0 - \tau_0)}{v'((\gamma w^H + (1 - \gamma)w^L)S(h)(1 - e_1) - e_0)} = 1$$

It is clear from this expression that changes in lump-sum taxes and subsidies have long-run effects in this model: An increase in e_0 and a cut in τ_0 promote investment in education. Moreover, an increase in e_0 and τ_0 that leaves the government deficit unchanged ($\Delta e_0 = \Delta \tau_0$) increases investment in human capital. Indeed, any increase in subsidies and taxes that keep everyone's net income unchanged (at the original steady state value of h) will increase investment. This is because people in this model do not make the connection between the increases in subsidy and the increase in taxes.

Under the second assumption, the steady state condition is going to be:

$$\frac{u'(w^L + h(1-\tau_1)w^H - (\gamma w^H + (1-\gamma)w^L)S(h)(1-e_1) + e_0 - \tau_0)}{v'((\gamma w^H + (1-\gamma)w^L)S(h)(1-e_1) - e_0 + \tau_0 + \tau_1h_tw_t^H)} = 1.$$

In this case, changes in taxes and subsidies that do not change the public deficit have no effect on investment as long as there is no inequality in the steady state. On the other hand, an increase in e_0 without any other changes in taxes or subsidies does increase investment in human capital (at the expense of increasing the public deficit). If, however, there is inequality in the steady state, changes in proportional taxes and subsidies have distributional effects—and as a result there may be an effect on average investment even if there is no change in the public deficit. Consider the case in Figure 1, where we have CARA preferences and multiple steady states. To simplify matters, assume that subsidies are lump-sum but taxes are proportional, i.e., $e_1 = 0$ and $\tau_0 = 0$, and that there is a balanced budget, i.e., $e_0 = \tau_1 \bar{h} w^H$, where \bar{h} is the mean level of human capital. With these assumptions we can rewrite Equation 12 as

$$-w^L - hw^H + 2(\gamma w^H + (1-\gamma)w^L)S(h) + 2(h-\overline{h})\tau_1 w^H = \frac{1}{\sigma}\log_e \delta.$$

First consider what happens for fixed values of w^L and w^H . The effect of an increase in τ_1 is shown in Figure 1. An increase in τ_1 shifts the left-hand side of the above equation down for low values of h and up for high values of h. For small changes in τ_1 starting at $\tau_1 = 0$, the stable steady state at h = 0 remains and the stable "good" state as h_2 moves left. If we think of the population as being initially distributed across these two steady states, it is clear that the effect of increasing τ_1 is to reduce the mean level of human capital. However, for larger increases in τ_1 , the steady states at h = 0 and h_1 vanish, and the entire population ends up at h_2 . This tells us that higher taxes can bring about more investment. However, note that we can minimize the changes in w^L and w^H by making the elasticity of substitution in production high enough. Therefore, the same result, namely that a lump-sum subsidy financed by a proportional tax on human capital, can increase investment in human capital holds even when w^L and w^H are endogenous. Essentially, redistribution here helps start a virtuous cycle and this leads to a large increase in human capital.

The fact that the steady state is not unique even when there is no inequality in the steady state suggests a different possibility for a virtuous cycle. Suppose starting at a "bad" steady state, the government borrows money abroad and invests it in a proportional subsidy to human capital. If the subsidy is large enough, this can set off a virtuous cycle where there is more and more human capital and therefore investment in human capital becomes cheaper and cheaper. As the economy gets richer, the government can raise taxes and repay the initial debt.

Raising returns to education: The effect of changes in returns to education are best seen by looking at the steady state condition with CRRA preferences (with $\gamma = 1$):

$$\frac{g(h,a)+h}{S(h)} = 1 + (\frac{1}{\delta})^{\frac{1}{\alpha}}.$$

It is clear that a shift in α that shifts $g(h, \alpha)$ down (a skill-biased change), has to reduce investment in education, since at any stable steady state the function $\frac{g(h,a)+h}{S(h)}$ has to be a declining function of h. An increase in the rewards for human capital always reduces investment in human capital in this case, because it makes teachers more expensive and education is produced using teachers. Clearly the strength of this result derives from the fact that given the specification of preferences, the size of the rewards from education do not matter to the parents. While this is clearly an extreme assumption, it is not implausible that parents will care less about the returns to education than about the costs, given that they pay the costs and their children get the returns.

5.2.2 Symbolic Consumption of Human Capital: The General Case

The problem with this example is that it builds in a number of assumptions (in particular, no credit markets, no altruism) in addition to the assumption of symbolic consumption. Let us now consider the case where we introduce perfect altruism and perfect credit markets into this scenario. Each generation now maximizes $\sum_{s=0} \beta^s [U^M(c_{t+s}) + U^S((\gamma w_{t+s}^H + (1-\gamma)w_{t+s}^L)s(h_{t+s+1}))]$ subject to the budget constraint 2.

As already noted, one of the first-order conditions for this problem is Equation 8 which, in steady state reduces to:

$$\frac{U^{\prime S}((\gamma w^{H} + (1 - \gamma) w^{L})s(h))}{U^{\prime M}(w^{L} + hw^{H} - (\gamma w^{H} + (1 - \gamma) w^{L})s(h))} = r - \frac{w^{H}}{(\gamma w^{H} + (1 - \gamma) w^{L})s^{\prime}(h)}.$$
 (14)

It is easy to show that another first-order condition is that $U'^{M}(c_{t}) = r_{t}\beta U'^{M}(c_{t+1})$, which implies that in a steady state $r = 1/\delta$.

Note that the left-hand side of equation 14 is exactly what we had in equation 10 and for the same reasons may be non-monotonic. The expression on the right of equation 14 is however clearly increasing as a function of h (because s'' > 0). This probably makes it less likely that the curves representing the two sides will intersect more than once (compared to equation 10, where the right-hand side is a constant), but is by no means impossible: In particular, since we are essentially free to choose the shape of the U'^S/U'^M function we can always generate cases where there are multiple steady sates and therefore inequality in the steady state.

In terms of the question of the uniqueness of the average level of human capital, the fact that w^L/w^H is an increasing function of h does make the right-hand side of Equation 14 steeper, which probably goes against multiplicity; but on the other hand, at least in the CRRA case, it also makes the left-hand side go up more steeply, which tends to favor multiplicity.

The fact that there are these multiplicities is clearly related to the presence of strong income effects, even in the steady state, and given that these income effects are important, the distribution of income will matter for the steady state level of human capital investment. Hence, policies that redistribute can improve the efficiency of investment, exactly as in the example above.

Observation 6: In the presence of symbolic consumption of human capital, the steady state may not be unique. The history of each dynasty may matter for their longrun levels of investment in human capital as will their preferences. Lump sum taxes and subsidies will affect long run investment in human capital, and proportional taxes and subsidies may increase the efficiency of investment in human capital. Higher returns to human capital are not necessarily associated with higher investment. These results are consistent with there being perfect credit markets and perfect altruism.

5.3 Beyond the Beckerian Model: Imperfect Altruism, Incomplete Contracts

The steady state analysis has so far avoided the problems that arise from the fact that the current generation's preference over future outcomes may not be the same as the preferences of the generations who have control over those outcomes. The assumption of perfect altruism avoids this problem because all generations, in effect, have the same preferences. The problem obviously also does not arise when a complete contract is possible. Finally, in the example of symbolic consumption in the previous subsection, parents only care about the amount spent on the education of their children, which is entirely under their control.

However, this issue becomes unavoidable once we depart from the assumptions of perfect altruism and complete contracting and allow each generation to have preferences over outcomes controlled by future generations. In effect, this sets up a game between the different generations. To see what this might imply, consider a particularly simple example which combines symbolic consumption and imperfect altruism: We assume that generation t cares only about $U(\mu c_t) + \delta U(1-\mu)c_{t+1}$. In the case where $\mu = 1/2$, this reduces to the case where there is imperfect altruism but no symbolic consumption, since generation t values the next generation's consumption exactly as they themselves do. By contrast if $\mu \neq 1/2$ there is necessarily an element of symbolic consumption since generation t cares about the next generation's consumption but differently from how generation t cares about their own consumption.²³ Note, however, that the symbolic consumption here involves the material consumption of others, rather than how much human capital they have, as in the previous example. As we saw above, this distinction is very important in understanding short run behavior in a model without credit constraints—the model without symbolic consumption of human capital was much better behaved. We are now interested in whether this is still true when there are credit constraints.

It is evident that the preferences assumed here build in a conflict of interest between the generations—the current generation cares only about the next, while the next generation cares about the one after.²⁴ This raises the issue of strategic interactions between the generations—ex ante each generation would like to promise that it will consume every dollar that it gets from its parent's investment (this maximizes the parent's willingness to invest). Ex post, it will want to share some of that money with their own children. Since contracts are ruled out, each generation's maximization problem should take account of

²³A potential interpretation of these preferences is that each generation is purely selfish but depends on the next generation for old age support. The next generation is also selfish and does not want to give anything to its parent, but consumption is a family public good and therefore the parent's consumption in old age is proportional to his son's consumption in middle age.

²⁴See Bernheim and Ray (1987) for a different model of this class, which, once again illustrates the difficulties of working with such models.

the next generation's equilibrium decision rule for allocating their earnings between consumption and human capital investment. Unfortunately, this gives rise to the possibility of multiple equilibria, which differ in the decision rule adopted by each generation. To limit the number of equilibria, we confine ourselves to looking only at the set of Markovian equilibria. These are equilibria where each generation uses a decision rule of the form $h_{t+1}(h_t)$. In other words, the human capital investment by each generation depends only on the amount of human capital that it got from its parent and the index of time.²⁵ It is easily checked that such an equilibrium exists.

However, this by itself does not tell us very much about the steady states of this economy since everything depends on the function $h_{t+1}(h_t)$, which itself is determined in equilibrium. In order to understand the possibilities that arise in this case, we need to look at a parametric example: We show that for a specific choice of parameters, $h_{t+1}(h_t)$ is linear and this allows us to characterize the solution in more detail.

5.3.1 A Linear-quadratic Example

Assume that $u(x) = x - \frac{1}{2}\phi x^2$. Assume also that $s(h_{t+1}, h_t) = s \cdot h_{t+1}$ and that $\gamma = 1$. Using the budget constraint, we rewrite the family's maximand as:

$$u(\mu(w_t^L + h_t(1 - \tau_1)w_t^H - (\gamma w_t^H + (1 - \gamma)w_t^L)s(h_{t+1}, h_t)(1 - e_1) + e_0 - \tau_0)) + \delta u((1 - \mu)(w_{t+1}^L + h_{t+1}(1 - \tau_1)w_{t+1}^H - (\gamma w_{t+1}^H + (1 - \gamma)w_{t+1}^L)s(h_{t+2}, h_{t+1})(1 - e_1) + e_0 - \tau_0))$$

The first order condition for utility maximization with respect to h_{t+1} turns out to be:

$$u'(\mu c_t)\mu(\gamma w_t^H + (1-\gamma)w_t^L)s(1-e_1)$$

$$= \delta u'((1-\mu)c_{t+1})(1-\mu)[w_{t+1}^H(1-\tau_1) - s(\gamma w_{t+1}^H + (1-\gamma)w_{t+1}^L)(1-e_1)\frac{dh_{t+2}}{dh_{t+1}}].$$
(15)

We now claim, that given our assumptions about preferences and the production technology for human capital, there is a linear solution to this maximization problem. In

²⁵The index of time is there to allow for the possibility that w^H and w^L change along the equilibrium path.

other words, there is a solution that takes the form $\frac{dh_{t+1}}{dh_t} = \eta_t, \forall t$. To show this, assume that $\frac{dh_{t+2}}{dh_{t+1}} = \eta_{t+1}$ and differentiate Equation 15 with respect to h_t to get (using also the fact that $u''(c_t) = u''(c_{t+1}) = \phi$):

$$\mu^{2} [w_{t}^{H}(\gamma w_{t}^{H} + (1 - \gamma)w_{t}^{L})s(1 - e_{1})(1 - \tau_{1}) - (\gamma w_{t}^{H} + (1 - \gamma)w_{t}^{L})^{2}s^{2}(1 - e_{1})^{2}\frac{dh_{t+1}}{dh_{t}}]$$

= $(1 - \mu)^{2}\delta \frac{dh_{t+1}}{dh_{t}} [w_{t+1}^{H}(1 - \tau_{1}) - s(1 - e_{1})(\gamma w_{t}^{H} + (1 - \gamma)w_{t}^{L})\eta_{t+1}]^{2}.$ (16)

This equation defines $\frac{dh_{t+1}}{dh_t}$ in terms of a set of constants and η_{t+1} . As long as η_{t+1} is a constant, $\frac{dh_{t+1}}{dh_t}$ will also be a constant, which confirms that there is a solution with the property that $\frac{dh_{t+1}}{dh_t} = \eta_t$, $\forall t$.

Using the steady state conditions (including the fact that $\eta_t = \eta_{t+1} = \eta$), Equation 16 reduces to:

$$s\frac{1-e_1}{1-\tau_1}\left[\frac{1}{(\gamma+(1-\gamma)g(h))}-s(\frac{1-e_1}{1-\tau_1})\eta\right] = \delta\eta(\frac{1-\mu}{\mu})^2\left[\frac{1}{(\gamma+(1-\gamma)g(h))}-s(\frac{1-e_1}{1-\tau_1})\eta\right]^2$$

or

$$s = \frac{\delta\eta}{(\frac{\mu}{1-\mu})^2 + \delta\eta^2} \frac{1-\tau_1}{1-e_1} \left[\frac{1}{(\gamma + (1-\gamma)g(h))}\right]$$
(17)

where h is the average amount of human capital in the economy in the steady state.

The two sides of this equation are represented in Figure 3. The right-hand side, represented as $H(\eta)$, defines a non-monotonic function of η for any fixed value of h. It is close to zero and increasing for η in the neighborhood of zero but decreases towards zero for high values of η . Therefore, if there is at least one value of η that solves this equation there must be a second. In Figure 3, these two steady states are identified as $\underline{\eta}$ and $\overline{\eta}$. Note that both $\underline{\eta}$ and $\overline{\eta}$ are positive: As in the previous models, parents with more education will have children with more education.

The multiplicity of possible values for η is not particularly surprising—a high value of η means that each generation responds to an increase in its own human capital by substantially increasing investment in its children's human capital. This amounts to saying that the current generation of the middle-aged get a low rate of return on their investment in children which may lead them to invest less, but may also make their investment more responsive to their own human capital. This is what gives us the multiplicity.

We do not take a stand on which of $\underline{\eta}$ and $\overline{\eta}$ is the right solution. The usual method of using the stability properties to eliminate solutions is problematic here given the complex, forward-looking dynamics associated with Equation 16.

However we can say something more about the properties of η . Rewrite steady state condition 15 in the form

$$s = \frac{\delta}{\frac{\mu u'(\mu c)}{(1-\mu)u'((1-\mu)c)} + \delta\eta} \frac{1-\tau_1}{1-e_1} [\frac{1}{(\gamma + (1-\gamma)g(h))}],$$
(18)

and combine it with equation 17 to get:

$$\eta \frac{u'(\mu c)}{u'((1-\mu)c)} = \frac{\mu}{1-\mu}.$$
(19)

This immediately implies that $\eta \geq 1$ if and only if $\mu \geq 1/2$. In the case where $\eta > 1$ dynastic trajectories will tend to diverge: A positive shock to h_t , starting at the steady state, will generate an even larger increase in h_{t+1} , and so on. The case where $\mu < 1/2$ which corresponds to the case, not implausible in the context of developing countries where the balance favors the elderly, is better behaved since it implies $\eta < 1$. In this case, dynasties will return to their steady state values following a shock. Henceforth, we will focus on the case where $\mu < 1/2$ and $\eta < 1$. Note that this rules out the case where $\mu = 1/2$, which is the case where there is only imperfect altruism and no symbolic consumption. In this particular case it is evident that $\eta = 1$ and there is no symbolic consumption.

Finally, observe that the steady state condition 18 is not unlike the corresponding condition for the first-best case, the one difference being that δ has been replaced by $\frac{\delta}{(1-\mu)u'((1-\mu)c)}+\delta\eta}$. It is not clear how these numbers compare since there is no reason why the $\delta's$ should be the same, but if they were the same, then, at least for μ not too different from 1/2, it can be shown that $\frac{\delta}{(1-\mu)u'((1-\mu)c)}+\delta\eta} > \delta$. In other words, a result of the strategic interaction between the generations is that there is underinvestment in education.

Parental Preference Effect: Parental preferences clearly play a role in the determination of η through the term $\frac{u'(\mu c)}{u'((1-\mu)c)}$, and therefore have an effect on investment in human capital even in steady state.

Inequality: These different values of η represent different dynastic strategies. They will typically lead to long-run inequality in the sense that those dynasties will have different steady state levels of consumption and human capital. To see this, observe that as long as $\mu < 1/2$, $\frac{u'(\mu c)}{u'((1-\mu)c)} = \frac{1-\phi\mu c}{1-\phi(1-\mu)c}$ is increasing in c. It follows from Equation 19 that in this case, high values of η will be associated with low values of steady state consumption.²⁶

Uniqueness: As suggested in the previous sub-section, once there is inequality in the steady state, the presumption is that there will also be multiple steady states: Intuitively, an economy which starts with only high η dynasties will be different from an economy which starts with only low η dynasties, even in the long-run. However, as before, it is also worth checking whether there can be multiple steady states without any inequality. In equation 18, there is yet another possible source of multiplicity. However actually demonstrating multiplicity will require a more involved discussion than we feel is appropriate here. We therefore confine ourselves to explaining the logic of the construction: Suppose h were to go down for some reason. This pushes the curve up in Figure 3 because g(h) goes down. As we know, the effect this has on η depends on the initial value of η . Assuming all dynasties started with $\eta = \underline{\eta}$, η goes down, which means that in the steady state, h has to go up (assuming $\mu < 1/2$), contradicting our premise that h had gone up. This tells us that there cannot be multiple steady states in this case. Now consider the case where initially everyone was at $\overline{\eta}$. The reasoning is exactly the same but now η goes up, pushing h down. There is "positive feed-back", creating the possibility of multiplicity.

The basic intuition for the multiplicity is closely related to the explanation for why there are two equilibrium values of η . An increase in h lowers the rate of return to human

²⁶Since c is typically increasing as a function of h, this implies that high values of the marginal propensity to invest in education will go with less overall investment. Moreover, since $h_{t+1}(h_t) = h_0 + \eta h_t$, this implies that high values of η will go with low values of h_0 .

capital, which has both an income effect and a substitution effect. Investment goes up when the income effect dominates.

Taxes, subsidies and returns to human capital: Lump-sum taxes and subsidies do not enter Equations 17 and 19 and therefore have no direct effect on the steady state values of η and c. However since in steady state $c = w^L + hw^H(1 - \tau_1) - sh(\gamma w^H + (1 - \gamma)w^L)(1 - e_1) + e_0 - \tau_0$, an increase in the net (lump-sum) subsidy to the household sector will lead to a decrease in h for any fixed c, which raises the curve in Figure 3. This will lead to a fall in $\underline{\eta}$ and an increase in $\overline{\eta}$. For those who were initially at $\underline{\eta}$, the effect will be to increase c and h (again assuming $\mu < 1/2$). For those who were initially at $\overline{\eta}$, there will be a fall in both c and h.

An increase in τ_1 has a direct negative effect on h, which can be seen from equation 18, under the assumption that c and η are fixed. This is the standard disincentive effect. However it also pushes the curve down in Figure 3, which raises $\underline{\eta}$ and reduces $\overline{\eta}$. For those who started at $\underline{\eta}$, this will lead to a further fall in h (assuming $\mu < 1/2$). For those who started at $\overline{\eta}$, this would counteract the initial fall in h though we do not yet know whether the net effect can be positive. The same logic also applies to the case of an increase in the return to human capital. Starting at $\underline{\eta}$, the effect is always positive, but we cannot rule the possibility that it is negative when $\eta = \overline{\eta}$.

Observation 7: None of the long run properties of the Beckerian model (equality, uniqueness, the effects of taxes and changes in the return on human capital) necessarily hold once we introduce imperfect altruism, incomplete contracts and symbolic consumption of other people's consumption.

This result is, however, relatively tentative, since we have yet to understand the dynamics of this model—the steady state with the perverse properties may be globally unstable. We also looked at a very special example and we do not know how far these results generalize.

6 Conclusion

The main point of this paper was to underscore the important differences between the implications of alternative ways of modeling the family in a world where educational investment is largely financed by the family.

In part, this suggests that we need more theoretical research aimed toward putting some structure on the nature of family decision-making. Rangel (1999) is one example of this type of work, arguing that the requirement that the implicit contract within the family be enforceable imposes important restrictions on the family's choices. In part, it suggests an empirical agenda aimed toward identifying and testing the peculiar implications of these and other models of the family. As pointed out already, because many of these models display income effects, their short run implications tend to be quite similar. However, there may be other implications which are quite different, say with regard to the reaction to an increase in the child's income (see Ermisch, 1996, for example).²⁷ Both this kind of empirical work and related experimental work have an enormous contribution to make to the way we think about education policy.

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²⁷His conclusion on the basis of his empirical analysis favors altruism.

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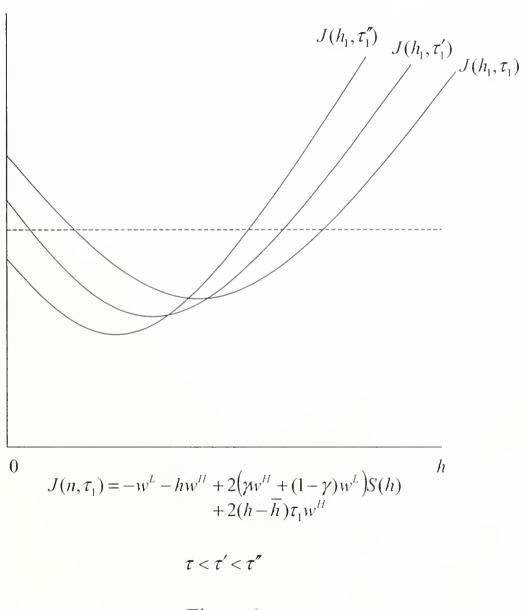
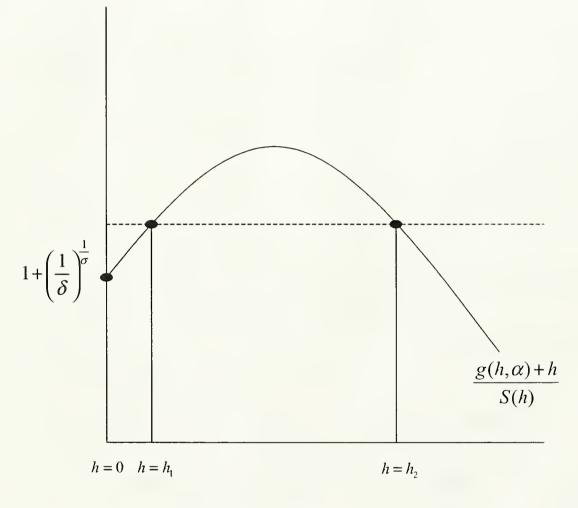


Figure 1





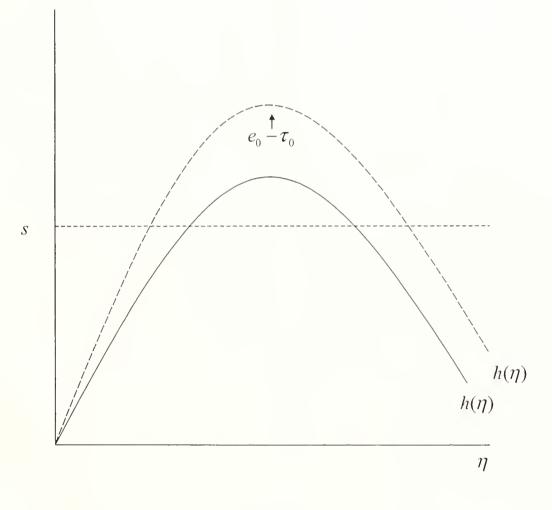


Figure 3

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