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Daron Acemoglu[†] Robert Shimer[‡]

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Abstract

In market economies, identical workers appear to receive very different wages, violating the 'law of one price' of Walrasian markets. We argue in this paper that in the absence of a Walrasian auctioneer to coordinate trade: (i) wage dispersion among identical workers is very often an equilibrium phenomenon; (ii) such dispersion is necessary for a market economy to function.

We analyze an environment in which firms post wages and workers may at a small cost observe one or more of the posted wages, i.e. search, before deciding where to apply. Both with homogeneous and heterogeneous firms, equilibrium wage dispersion is *necessary* for the economy to approximate efficiency. Without wage dispersion, workers do not search, and wages are depressed. As a result: *(a)* there is excessive entry of firms; *(b)* because in the absence of search, high productivity firms cannot attract workers faster than low productivity firms, their relative profitability is reduced, and technology choices are distorted.

Keywords: Efficiency, Search, Search Intensity, Sorting, Wage Dispersion, Wage Posting.
JEL Classification: D83, J41, J31.

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1 Introduction

In market economies, identical workers appear to receive widely different wages (Krueger and Summers, 1988). This dispersion is remarkably persistent over time and similar across countries (Krueger and Summers, 1987), and is mostly due to different firms paying different wages for the same jobs in the same line of business (Abowd and Kramartz, 1994 and Groshen, 1991). Although inequality of wages among observationally identical workers may be explained by unobserved worker or job heterogeneity (Murphy and Topel, 1986), this does not seem to be the whole story. First, workers moving from a low wage to a high wage industry receive a wage increase in line with the wage differential between these two sectors (Gibbons and Katz, 1992). Second, vacancies in high wage jobs attract significantly more applicants than low wage sectors (Holzer, Katz and Krueger, 1991). Existing explanations view this inequality as a failure of market economies (Bulow and Summers, 1986, using efficiency wage theories, Acemoglu, 1996b, using search and bargaining). Essentially, the same commodity, a homogeneous worker, trades at two different prices, distorting the allocation across different firms or sectors. In other words, an important cornerstone of Walrasian markets, *the law of one price*, fails, and this distorts the allocation of resources.

In this paper we argue that when the Walrasian auctioneer is not around to *coordinate* trade, wage dispersion among identical workers will arise naturally. Furthermore, this departure from the law of one price is necessary for the decentralized equilibrium to achieve some degree of efficiency. Without the Walrasian auctioneer, it is important for workers to search, and they will only do so if there is wage dispersion. Therefore, wage dispersion is often efficient from a 'third-best' perspective.

We model the coordination problem of a decentralized economy as a search problem with fully optimizing agents on both sides of the market. Firms decide to open jobs (of possibly varying qualities), and as is the case in most real world labor markets, they post wages. Workers, as in the partial equilibrium search literature pioneered by Stigler (1961), sample a number of firms and learn their wage offers. Then, using sampled information, they make their application decisions. As appears relevant to most labor markets, we assume that firms cannot hire an unlimited number of applicants; thus workers anticipate that they may be turned down by the jobs to which they apply. Therefore, in the absence of the Walrasian invisible hand, there is a fundamental coordination problem: some workers will be unemployed, because too many of them have applied to the same firm. Recognizing this possibility, in their application decisions, workers will trade-off higher wage earnings for lower probabilities of obtaining a high wage job. This trade-off will encourage firms to 'locate' in different parts of the wage distribution and attract workers at different rates.

Search intensity (effort) is a crucial decision in our economy. Previous analyses have incorporated this margin by introducing a search effort decision that increases the probability of a 'match' (see also Pissarides, 1990). Yet, this reduced form modelling assumption ignores important strategic considerations: search intensity is likely to be higher when there is more wage dispersion. But which distribution of wages will prevail in equilibrium will also depend on the search intensity of workers. This reasoning underlies many of our results: a disperse distribution of wages is only possible if workers are *informationally heterogeneous* — some workers locate multiple jobs while others take the first one that comes along. Workers, on the other hand, only choose to become informationally heterogeneous if there is sufficient but not excessive wage dispersion. Therefore, in our equilibrium, some workers will always choose not to search intensively, and the presence of such informational heterogeneity will induce firms to offer a disperse wage distribution.

Wage dispersion is necessary for the labor market to work efficiently for two reasons. The first is simplest to see with homogenous firms. In this case, search intensity is a pure rent-seeking mechanism. As more workers search intensively, firms face more severe competition for the workers they are trying to attract, and thus are forced to offer higher wages. This increases social surplus, because in the absence of this effect firms do not internalize that entry crowds the market for other firms and enter in excessive numbers. The second reason is perhaps even more important, and arises when there is potential heterogeneity among firms, e.g., firms choosing from a menu of heterogeneous technologies. Ideally, society prefers that good jobs are more likely to start up than bad jobs, and that firms opening good jobs are able to select workers who are more suited to these tasks. When there is wage dispersion, workers search intensively. Firms that offer high wages attract more applicants, and thus have a higher probability of being productive. In contrast, in the absence of search intensity, firms are unable to fill their vacancies faster by offering higher wages. Since capital-intensive (high productivity) firms would like to attract workers with higher probability, the absence of search intensity reduces their relative profitability, and as a result, firms are discouraged from choosing these technologies. Therefore, in our economy, a more efficient allocation with higher output, capital-intensive technologies, and higher labor productivity requires search and wage dispersion.

Our paper is related to the search literature. Other papers including Albrecht and Axell (1984), Butters (1977), Burdett and Judd (1983), Lang (1991), Salop and Stiglitz (1982), and Sattinger (1991) analyze search models in which firms post wages or prices, and obtain a non-degenerate equilibrium distribution. Mortensen and Pissarides (1994) obtain different wages for identical workers because firms have different productivity levels, and bargain with their employees. Burdett and Mortensen (1989) generate a distribution of wages because low wages increase the probability of quits; some firms choose to pay higher wages, attract more workers and face lower quits. Lang (1991) and Butters (1977) generate equilibrium distributions due to informational heterogeneity, and Montgomery (1991) obtains a non-degenerate distribution due to productivity differences across firms. None of these models endogenize search intensity, and therefore, none reaches our crucial conclusion regarding the link between wage dispersion and efficiency.

Our model is most closely related to the important work by Burdett and Judd (1983). They also show the presence of an equilibrium price distribution with homogeneous and optimizing agents on both sides of the market. However, there is an important difference between the two approaches. In Burdett and Judd (1983) there are no search frictions other than the inability of consumers to observe prices; in particular, a consumer knows that when he walks into a store, he will purchase the good at the posted price. In contrast, search frictions in the form of coordination problems are present in our model even when workers observe the whole distribution of wages. As in Peters (1991) and Montgomery (1991), firms face 'capacity constraints'. A worker has to anticipate how many other workers will apply to a particular wage, and then decide whether it is worth applying for a high wage job. This difference underlies the fact that in our model, there are always unemployed workers, and the limit point of our equilibrium as the cost of sampling wages disappears is a well-behaved search economy. In contrast, in Burdett and Judd, there is never any unsatisfied consumer demand, and the limit point is the Walrasian allocation. More significantly, this difference enables us to endogenize entry and analyze meaningfully the efficiency of the decentralized allocation.[1] This leads to our main conclusion, not shared by Burdett and Judd nor by the other papers mentioned above, that in the absence of wage dispersion, there will be a very inefficient allocation.

Perhaps most importantly, and differently from all other contributions mentioned so far, we study an environment in which firms endogenously choose from a menu of different technologies. This enables us to analyze the joint determination of search intensity, the wage distribution and the allocation of workers to heterogeneous firms. This is an important innovation, since we argue that wage dispersion and search are required to ensure 'sorting', and through this channel have an important impact on firms' technology choices.

The paper proceeds as follows. In section 2 we describe the basic static environment with homogeneous firms, and we characterize the equilibrium of this economy in some detail. We establish that there will always exist a no search equilibrium with no wage dispersion, and for moderate values of search costs, equilibria with search and wage dispersion. We provide closed form solutions for the distribution of wages and the number of firms in the economy. The analysis of the dynamic environment and of the economy with heterogeneity will have many parallels to this section, and this justifies our detailed analysis of this simplest case. Furthermore, some of the arguments we develop in this section can be of independent interest.

[1] There would be two problems if one endogenized entry and analyzed efficiency in Burdett and Judd (1983). First, since firms are making positive profits, there should be a cost of entry to close the model (otherwise there would be an infinite number of infinitesimal firms). But when search costs disappear, price converges to the Walrasian price level, and firms make zero gross profit, thus no firm would want to enter. Second, since costs are linear, a firm could split into two and increase its profits, unless the costs of posting prices were significant. We show that our model collapses into the Burdett and Judd's when costs of entry disappear.

Section 3 contains one of the key results of our analysis. It demonstrates equilibria with wage dispersion and search dominate the equilibrium without wage dispersion. Section 4 extends the basic model to an infinite horizon economy. Section 5 contains another important part of our analysis. It introduces a menu of technologies from which firms choose. It establishes that wage dispersion and search enable high productivity firms to fill their vacancies faster (what we refer to as *sorting*), and thus encourages the choice of better technologies. Section 6 briefly discusses variable match quality and two sided heterogeneity. Section 7 concludes.

2 The Static Model With Homogeneous Firms

2.1 The Environment

We will first present a one-period economy with homogeneous firms. There is a continuum 1 of workers and a larger continuum of firms. Each firm has access to a simple Leontieff technology; it produces 1 unit of output if it employs a worker and cannot employ more than 1 worker. All agents have linear utility. The disutility of work and unemployment benefit for workers are normalized to 0.

The sequence of events is as follows.

1. Firms decide whether to post a vacancy or not. Posting a vacancy costs γ. We denote the set of firms that post a vacancy by $\hat{V}$ and the measure of the set $\hat{V}$ by V.

2. Each firm $i \in \hat{V}$ posts a wage $w_i \in \mathbb{R}$.

3. Each worker decides how many vacancies to sample (or to *locate*). A worker who samples $n \geq 1$ firms learns the wage offered by n randomly chosen firms.

4. Each worker applies to at most one of the firms she has located. Workers rationally anticipate that a high wage will also attract other applicants; since each vacancy corresponds to 1 job, the worker will then have a lower probability of actually obtaining a high wage job. The exact form of this trade-off is determined in equilibrium.

5. Finally, each vacancy chooses one of the applicants, and pays the posted (promised) wage. Since applicants are homogeneous, the decision of whom to hire is arbitrary. Naturally there may also be some firms without applicants.

The need for sampling comes from the fact that, although workers rationally anticipate the equilibrium *distribution* of wages, they do not know which wage a *particular* firm is offering (see Stigler, 1961). Because all firms appear to be identical before individual wages are observed, a worker randomly samples n of them, and learns the wage offered by the firms that it locates.[2] For now, we assume that all workers locate at least one wage,[3] and the cost of locating the n^{th} job is $c_n > 0$. We assume that c_n is weakly increasing, which is equivalent to the cost of search being weakly convex.

Point 4 expresses the fundamental coordination problem in our economy. Even if there are N jobs and N workers, not all workers will be employed nor will all jobs be filled. This is because some workers will apply to the same jobs, and some firms will not receive any applicants (see also Peters, 1991). In contrast, a Walrasian auctioneer would ensure that each job receives one worker, and this coordination problem would not have arisen.

Let λ_n be the proportion of workers locating n jobs (or equivalently, the probability that a representative worker locates n jobs). We naturally have $\sum_{n=1}^{\infty} \lambda_n = 1$, and we let $q \equiv 1/V$ be the inverse of the tightness of the labor market. We also denote the distribution function of wages by $\hat{G}(w)$, and let $\mathcal{W}$ be the closed support of $\hat{G}$.[4] Finally, G is a transformation of $\hat{G}$ which will play an important role in our analysis. $G(w)$ is the probability that a worker who has located two jobs, one at wage w and the other with a wage drawn randomly from the distribution $\hat{G}$, applies to w instead of to the latter. If workers always apply to higher wages, and the wage distribution is atomless, then $G = \hat{G}$, but in general the two functions may differ. Observe that the wages that a worker locates are independent random variables. Also, whether a worker prefers to apply to a job offering w_1 or to one offering w_2 is independent of irrelevant alternatives; in particular, it is independent of the wages offered by the other jobs she locates. Then it follows that a worker who locates n jobs, including one offering w, applies to the job offering w if she prefers that job to each of the other possibilities, an event with probability $G(w)^{n-1}$.

2.2 Basic Results

Let us define $\Sigma(w) = \sum_{n=1}^{\infty} n\lambda_n G(w)^{n-1}$. Since each worker randomly locates a subset of the jobs, $\Sigma(w)d\hat{G}(w)$ is the density of job applications by a 'representative' worker. This is essentially the probability that an average worker applies to a firm offering wage w, given her probabilistic search intensity decision $\{\lambda_n\}$ and the preferences captured by G. Thus $\Sigma(w)$ is simply the probability that a representative worker applies to a firm offering w, conditional on locating such a firm. Dividing this

[2] This is an *urn-ball* search technology; each of qN workers throws n balls with equal probability into the N urns representing active firms. We take the limit as N goes to infinity. We work with a continuum of agents, rather than a countable infinity, for notational convenience.

[3] We discuss how the results change when this assumption is relaxed in sections 2.7 and 2.9. Also note that we assume a worker must decide how many firms to locate *before* sampling any wages. In the partial equilibrium/decision theoretic search literature, this corresponds to 'simultaneous', rather than 'sequential', search. Sequential as well as simultaneous search is present in our dynamic extension of section 4.

[4] There is only a trivial loss of generality in thinking about $\mathcal{W}$ as closed. Whether a countable number of wage levels, each offered by a zero measure of firms, is contained in the support of the wage distribution, can have no quantitative or qualitative effect on the equilibrium.

by the vacancy-unemployment ratio, or equivalently multiplying this by q, we obtain the expected number of applications that a firm posting w receives. $e^{-q\Sigma(w)}$ is then the probability that the firm receives no applications. Therefore, the probability that a firm posting the wage w gets at least one applicant is:[5]

$$P(w) \equiv 1 - e^{-q\Sigma(w)} \tag{1}$$

The gross expected profit of a firm offering wage w can be written as:

$$\pi(w) = \left(1 - e^{-q\Sigma(w)}\right)(1-w) \tag{2}$$

Similarly, the expected return of a worker applying to a job offering w is:

$$\rho(w) = \frac{1 - e^{-q\Sigma(w)}}{q\Sigma(w)} w \tag{3}$$

where $\Sigma(w)$ is defined as above to be the conditional probability that a worker locating a job offering w applies to that firm. In words, the return from applying to a vacancy posted at w is the probability of obtaining this job times the wage. The fraction in (3) is the probability that a worker applying to w is hired, which is equal to the probability that a firm posting w hires a worker, $P(w)$, divided by expected number of workers applying to a firm posting w.

We also denote the overall expected return of a worker as a function of the number of firms n that she decides to locate as R_n. Then the return to sampling n jobs is:

$$R_n = n \int_{\mathcal{W}} \rho(w) G(w)^{n-1}\, d\hat{G}(w) - \sum_{i=1}^{n} c_i \tag{4}$$

Next, we define an equilibrium for the one-period game outlined in this section:

Definition 1 *An equilibrium consists of a measure of active firms V, a distribution of posted wages $\hat{G}(w)$ with support $\mathcal{W}$, an expected profit function for firms $\pi(w)$, an expected return function for workers $\rho(w)$, and search intensity decisions for workers $\{\lambda_n\}$ such that:*

1. *$\forall w \in \mathcal{W}$ and $\forall w'$, $\pi(w) \geq \pi(w')$*
2. *$\forall w \in \mathcal{W}$, $\pi(w) = \gamma$.*
3. *A worker who learns about wages $w_1, w_2, ..., w_n$ applies to wage w_i only if $\rho(w_i) \geq \max_j \langle \rho(w_j) \rangle$ and $\rho(w_i) \geq 0$.*
4. *$\lambda_{n^*} > 0$ only if $n^* \in \arg\max_{n \in \mathbb{N}} R_n$.*

[5]Since all workers are homogeneous, the firm is indifferent between any positive number of applicants. Also, note that workers do not observe how many other workers have located a particular firm, and hence cannot correlate their actions in this way.

2.3 Distribution of Wages and Application Decisions

We now characterize the support of the equilibrium wage distribution $\mathcal{W}$, and prove that each worker applies to the highest wage that she locates.

First, combining equation (2) with the second condition in the definition of equilibrium, that all active firms make gross profit γ, we obtain that $\forall w \in \mathcal{W}$:

$$\gamma = \left(1 - e^{-q\Sigma(w)}\right)(1 - w) \tag{5}$$

The right hand side is the gross profit level of a firm offering wage w. For all wages offered in equilibrium, this must equal γ.

Next, we define $\tilde{\rho}(w|\gamma)$, the return of a worker applying to a wage w, under the (possibly hypothetical) assumption that the firm offering this wage makes gross profits equal to γ. Mathematically, this function is obtained by substituting for $q\Sigma(w)$ from (5) into (3). Intuitively, it is the return to a worker applying to a wage w once the probability that other workers apply for wage offer w is adjusted such that (5) holds. The expression for $\tilde{\rho}(w|\gamma)$ is given as:

$$\tilde{\rho}(w|\gamma) = \frac{\gamma w}{(1 - w)(\log(1 - w) - \log(1 - w - \gamma))} \tag{6}$$

It can be verified easily that $\tilde{\rho}$ is a strictly quasiconcave function of w. Also $\tilde{\rho}$ is equal to 0 when $w = 0$ or when $w = 1 - \gamma$, and is maximized at an intermediate point w^* satisfying

$$(1 - w^* - \gamma)\left(\log(1 - w^*) - \log(1 - w^* - \gamma)\right) = \gamma w^* \tag{7}$$

Using this, we can prove a key result for the result of our analysis:

Lemma 1 *In equilibrium, ρ is strictly increasing on $\mathcal{W}$.*

Proof. First, we claim that ρ is nondecreasing on $\mathbb{R}$. Assume to the contrary that there is a $w_1 < w_2$, with $\rho(w_1) > \rho(w_2)$. From the third condition in the definition of an equilibrium, a worker is more likely to apply to w_1 than to w_2, and so $G(w_1) \geq G(w_2)$. Then by definition, $\Sigma(w_1) \geq \Sigma(w_2)$. Then equation (3) implies $\rho(w_1) < \rho(w_2)$, a contradiction.

Next, $\tilde{\rho}(w|\gamma) \leq \rho(w)$ for all w, with equality if $w \in \mathcal{W}$. If the inequality were reversed, a firm offering wage w would earn gross profits in excess of γ, which is inconsistent with the second part of the definition of an equilibrium.

Finally, we prove that ρ is strictly increasing on $\mathcal{W}$. Since $\rho(w) = \tilde{\rho}(w|\gamma)$ if $w \in \mathcal{W}$, this is equivalent to proving that $\tilde{\rho}$ is strictly increasing on $\mathcal{W}$. Since $\tilde{\rho}$ is strictly quasiconcave, this is true if $\mathcal{W} \subseteq (-\infty, w^*]$. Suppose not. Take $w > w^*$, with $w \in \mathcal{W}$. Then since $w \in \mathcal{W}$ and $\{w^*\} = \arg\max \tilde{\rho}(w|\gamma)$, it follows that $\rho(w) = \tilde{\rho}(w|\gamma) < \tilde{\rho}(w^*|\gamma) \leq \rho(w^*)$, violating weak monotonicity. ■

This lemma establishes that *in equilibrium*, the fact that there is more competition for high wage jobs does not deter applicants. Workers expect a higher return from applying to a higher wage, and always apply to the highest wage they locate. That workers weakly prefer higher wages — the first step in the proof — is not surprising. However, our result is stronger: workers are never indifferent between two wages that are offered in equilibrium. In general, competition for a higher wage may be sufficiently severe so that the decreased chance of being accepted just offsets the increased reward from being hired. In fact, this is a very common phenomenon for wages not in $\mathcal{W}$. Nevertheless, Lemma 1 precludes this possibility for wages in equilibrium, and therefore simplifies our analysis considerably.

Using Lemma 1, we can fully characterize the support of the wage distribution.[6]

Lemma 2 *If $\lambda_1 < 1$, the support of the equilibrium wage distribution $\mathcal{W}$ consists of a convex non-empty interval $[0, \bar{w}]$ and possibly the point $w^* \geq \bar{w}$. The wage distribution is atomless on $[0, \bar{w}]$ but may have an atom at w^*.*

Proof. From Lemma 1, we know that $\mathcal{W} \subseteq (-\infty, w^*]$. In fact, since the return to applying for a negative wage is negative (equation (3)), we know from the third part of the definition of an equilibrium that no worker makes such an application. Thus $\Sigma(w) = 0$ if $w < 0$, and so $\pi(w) = 0 < \gamma$. Hence the support of the wage distribution is a subset of $[0, w^*]$.

We now prove that if there is an atom in the wage distribution, it must occur at w^*. If a wage $w \in [0, w^*)$ is offered by a positive mass of firms, then $\rho(w') \geq \tilde{\rho}(w'|\gamma) > \tilde{\rho}(w|\gamma) = \rho(w)$ for all $w' \in (w, w^*]$ implies that any higher wage offer would attract discretely more applicants. A sufficiently small increase in the wage above w would be nearly costless, and hence any w' slightly larger than w would be strictly more profitable.

Next observe that a positive measure of firms must offer some wage besides w^*. Suppose not. Then, $R_1 > R_n$ for all n since locating multiple firms would offer no benefit, and yet would be costly. Therefore, $\lambda_1 = 1$ by part four of the definition of an equilibrium, contradicting our starting assumption.

Now we prove that $\mathcal{W}$ consists of a convex interval $[0, \bar{w}]$ and possibly the point $w^* \geq \bar{w}$. Suppose this were not the case. Then since $\mathcal{W}$ contains points besides w^*, there will be points $w_1, \bar{w} \in (0, w^*)$, $w_1 \notin \mathcal{W}$, $\bar{w} \in \mathcal{W}$. Since $\mathcal{W}$ is closed, there is a smallest point $w_2 \in [w_1, \bar{w}]$, $w_2 \in \mathcal{W}$. Now suppose a firm offering w_2 cut its wage to w_1. It would lose no applicants, but it would save $w_2 - w_1$ on labor costs if it hires a worker. Then $\pi(w_1) > \pi(w_2)$, contradicting the first condition in the definition of an equilibrium. ∎

This lemma proves that as long as some workers sample more than one job, only two types of wage distributions are admissible in equilibrium: a continuous distribu-

[6] If λ_1 is small, then the equilibrium distribution of wages would be degenerate at w^*, but the proof establishes that this is not possible in equilibrium. See Lemma 3 and Proposition 2 for details.

tion on a convex support without any mass; and a distribution that is continuous on a convex support and then has a positive mass at w^*. Because the wage distribution is continuous everywhere except at w^*, $\hat{G}(w) = G(w)$ for all $w \neq w^*$. Defining μ to be the proportion of firms that post wage w^*, we also have $G(w^*) = 1 - \mu/2$ and $\hat{G}(w^*) = 1$.

At this point it is useful to recall Rothschild's (1974) criticism of search models: if all firms offer the same wage, why should anyone search? Lemma 2 captures this criticism. If *some* workers search for multiple jobs, then the equilibrium wage distribution must not be degenerate. This intuition also relates to the informational externality identified in Grossman and Stiglitz (1980) in the context of a stock market. They show that when prices transmit all the relevant information, no trader will exert effort to find out additional information. Thus for traders to invest in information, there must be a sufficient degree of noise in the system. Similarly, in our economy, for workers to invest in higher search intensity, there must be a sufficient wage dispersion.

2.4 Search Intensity

In this subsection, we establish that in equilibrium no worker ever locates more than two wages, and some workers always locate only one wage. The first step in the proof is:

Lemma 3 *In any equilibrium, $\lambda_1 > 0$.*

Proof. By the way of contradiction, suppose $\lambda_1 = 0$. Lemma 2 applies, and thus $0 \in \mathcal{W}$ and $G(0) = 0$. But Lemma 1 implies that workers will always apply to the higher wages, thus $\pi(0) = 0 < \gamma$: a contradiction. ■

Next, we show that there are decreasing returns to search intensity:

Lemma 4 *In equilibrium, $R_2 - R_1 \geq R_{n+1} - R_n$ $\forall n \geq 2$, and strictly if $\lambda_1 < 1$.*

Proof. Integration by parts gives:

$$\int_0^{\bar{w}} (1 - G(w))G(w)^n \rho'(w)dw = \rho(w)G(w)^n(1 - G(w))\Big|_0^{\bar{w}} + (n+1)\int_0^{\bar{w}} \rho(w)G(w)^n\,dG(w) - n\int_0^{\bar{w}} \rho(w)G(w)^{n-1}dG(w) \quad (8)$$

The first term on the right hand side of (8) is equal to $\rho(\bar{w})\mu^n(1-\mu)$. The second term is $R_{n+1} - \rho(w^*)\left(1 - \mu^{n+1}\right) + \sum_{i=1}^{n+1} c_i$; and the third term is $R_n - \rho(w^*)\left(1 - \mu^n\right) + \sum_{i=1}^{n} c_i$. Then equivalently,

$$R_{n+1} = R_n + \int_0^{\bar{w}} (1 - G(w))G(w)^n\,\rho'(w)dw + \left(\rho(w^*) - \rho(\bar{w})\right)\mu^n(1-\mu) - c_{n+1} \quad (9)$$

Since $G(w) \in [0,1]$ for $w \in [0, \bar{w}]$, $\mu \in [0,1]$ and $c_{n+1} \geq c_n$, these two equations imply that $R_{n+1} - R_n \leq R_n - R_{n-1}$ for $n \geq 2$. If $\lambda_1 < 1$, the inequality is strict, since the wage distribution is not degenerate (Lemma 2). The rest of the result follows by transitivity. ■

Looking for an additional job is only worthwhile if the job turns out to be superior to the other jobs that a worker has located. This becomes less and less likely as the worker locates more jobs. Equivalently, there are decreasing returns to search, because the chance that the worker decides to apply for the last job she locates is decreasing in the total number of jobs she locates, and since marginal search costs are increasing.

Lemmata 3 and 4 imply an important result for our analysis.

Lemma 5 *In equilibrium, $\lambda_1 + \lambda_2 = 1$. That is, no worker locates more than two jobs.*

Proof. If $\lambda_1 = 1$, the result is immediate. Otherwise, Lemma 3 implies $\lambda_1 > 0$. Then according to the fourth condition for an equilibrium, it must be the case that $R_1 \geq R_n$ for all n, and in particular that $R_1 \geq R_2$. Therefore, Lemma 4 immediately implies that for all $n > 2$, $R_2 > R_n$ and so again by the fourth condition for an equilibrium, $\lambda_n = 0$ for all $n > 2$. ■

Search intensity is costly. For there to be sufficient benefits to high search intensity, there needs to be a sufficiently disperse distribution of wages. This is only possible when some workers must take the first job that comes along. Therefore, irrespective of how low the cost of sampling wages may be, as long as it is strictly positive, a number of workers will not undertake any search effort. This will support a distribution of wages which will make it worthwhile for others to search. In other words, in equilibrium it must be the case that some workers *free-ride* on the search intensity of others.

Since in any equilibrium, $\lambda_n = 0$ for all $n > 2$, for the remainder of this section we adopt the notation $z \equiv \lambda_2$ and $\lambda_1 = 1 - z$. A 'search intensive' equilibrium is one in which z is strictly positive.

2.5 Equilibrium With No Search

We now establish that there always exists an equilibrium in which all workers locate only one job.

Proposition 1 *There always exists an equilibrium with $z = 0$. In this equilibrium the wage distribution is degenerate at $w = 0$, and $q = -\log(1 - \gamma)$.*

Proof. If $z = 0$, all firms offer $w = 0$ in equilibrium, since all workers will apply for any nonnegative wage. Now consider a deviation by a worker locating two jobs.

This will cost c_2, but because all firms offer the same wage, it will bring no benefit. Since firms have to make zero-profit, inverting (5) with $w = 0$ and $\Sigma(0) = 1$ gives the equilibrium tightness of the labor market. ■

The existence of an equilibrium without search is not surprising. A worker will find it profitable to search only when there is a disperse distribution of wages. However, firms are only willing to offer a non-degenerate distribution when there are workers who search; otherwise, they can exercise maximal monopsony power and pay $w = 0$. Therefore, when firms expect no worker to search, there is no dispersion in wages, and thus no worker finds it beneficial to search.

2.6 Equilibria With Search and Wage Dispersion

In this subsection, we characterize the wage distribution in equilibria with $z > 0$. We first solve explicitly for $G(w)$ from the zero-profit condition of firms, and then find the tightness of the labor market consistent with equilibrium. Finally, we prove the existence of this equilibrium.

Using z to denote the fraction of workers who search intensively, substitute for $\Sigma(w)$ in equation (2). This gives:

$$\pi(w) = \left(1 - e^{-q((1-z)+2G(w)z)}\right)(1-w) \tag{10}$$

Inverting the zero-profit condition $\pi(w) = \gamma$ gives an expression for the probability that a worker who locates two wages applies for $w \in \mathcal{W}$ instead of a randomly drawn alternative:

$$G(w) = \frac{\log(1-w) - \log(1-w-\gamma) - q(1-z)}{2qz} \tag{11}$$

It is easily verified that $G(w)$ is an increasing function of w. Also, recall that because in equilibrium workers always apply to the highest wage, this function is identical to the wage distribution $\hat{G}$ for all $w \in [0, \bar{w}]$, i.e. except at the point of the atom, w^*.

Now by Lemma 2, $0 \in \mathcal{W}$ and by Lemma 1, $G(0) = 0$. Using equation (11), we can immediately solve for the equilibrium tightness of the labor market only as a function of the search behavior of workers, z:

$$q = \frac{-\log(1-\gamma)}{1-z} \tag{12}$$

From (12), q is positive and increasing in z (recall $\log(1-\gamma) < 0$). Note that at $z = 0$, this expression corresponds to the equilibrium tightness with no search. It can be seen from (12) that the higher is the proportion of workers using high intensity z, the less tight is the labor market (i.e. the smaller is the number of vacancies per worker). The intuition for this result is simple but instructive: when workers search more intensively, firms are induced to pay higher wages in order to attract workers who now have more options. In other words, the probability that the firm offering a zero wage will obtain a worker goes down. As a result, rents are shifted from firms to workers, and this discourages entry. A slightly different way of expressing this intuition is as follows. There are rents in this economy that need to be dissipated. The dissipation of rents either happens by firms entering in larger numbers until the fixed costs of entry (γ) exhaust the rents, or it takes the form of workers searching intensively for high wage jobs, which induces firms to offer higher wages.

We can now use equation (12) to eliminate q from (11). This gives an expression for G in terms of the parameter γ and the endogenous search intensity z:

$$G(w) = \frac{1-z}{2z}\left(\frac{\log(1-w)+\log(1-\gamma)-\log(1-w-\gamma)}{-\log(1-\gamma)}\right) \tag{13}$$

Observe that in response to changes in z, G shifts in the sense of first order stochastic dominance. When there is higher search intensity (higher z), the entire wage distribution function shifts to the right. This again captures the notion of *free-riding* in this model. When a worker decides to search more, he improves the distribution of wages for all other workers.

For a certain distribution of wages to be an equilibrium, we require that $\tilde{\rho}(w|\gamma)$ is strictly increasing on the support of the wage distribution $\mathcal{W}$ (Lemma 1). For an atomless wage distribution, this condition obtains if and only if $w^* \geq \bar{w}$, where w^* is defined in equation (7) to maximize $\tilde{\rho}$, $\bar{w}$ satisfies $G(\bar{w}) = 1$, and G is defined by equation (13). Equivalently, a necessary condition for there to be an atomless wage distribution with a particular value of z is $G(w^*) \geq 1$, that is:

$$z \leq \frac{\log(1-w^*)+\log(1-\gamma)-\log(1-w^*-\gamma)}{\log(1-w^*)-\log(1-\gamma)-\log(1-w^*-\gamma)} \equiv \underline{z} \tag{14}$$

It can be easily confirmed that $\underline{z} \in (0,1)$. More generally, for an equilibrium with an atomless wage distribution to exist, we require the proportion of workers who search intensively to be small; otherwise, firms are forced to offer relatively high wages, and this implies a mass of firms at w^*.

In equilibria with $z > \underline{z}$, in which a positive mass μ of firms offer wage w^*, these firms must still earn gross profits γ. Recall $G(w^*) = 1 - \mu/2$. That is, a firm offering w^* receives applications from all workers who contact it, except for half of those who receive an identical offer. Then we can re-express equation (10), the zero-profit condition of firms offering w^*, as:

$$\pi(w^*) = \left(1 - e^{-q(1+z(1-\mu))}\right)(1-w^*) = \gamma \tag{15}$$

From Lemma 2, we know that, even when a positive mass of firms offer w^*, there must be a firm offering $w = 0$ and earning gross profits γ. Thus equation (12) still obtains. Substituting this into equation (15) yields:

$$\mu = \max\left\langle \frac{1+z}{z} - \frac{1-z}{z}\left(\frac{\log(1-w^*-\gamma)-\log(1-w^*)}{\log(1-\gamma)}\right); 0\right\rangle \tag{16}$$

The 'max' operator takes care of the case where there is no positive mass of firms at w^*.

The upper support of the convex portion of the wage distribution, $\bar{w}$, satisfies $G(\bar{w}) = 1 - \mu$; any worker who does not locate a firm offering w^* will apply for a wage in $[0, \bar{w}]$. Using the definition of G in (13), we can solve for $\bar{w}$ as

$$\bar{w} = 1 - \frac{\gamma}{1 - (1-\gamma)^{(1-z(2\mu-1))/(1-z)}} \tag{17}$$

One can confirm that when $z = 0$, $\mu = 0$ and $\bar{w} = 0$. When no worker locates more than one job, as characterized above, the wage distribution is degenerate at 0. On the other hand, if $\mu = 1$, again $\bar{w} = 0$. The 'convex' portion of the wage distribution disappears if all firms offer w^*.

Finally, in order for there to be an equilibrium with positive search intensity ($z > 0$), workers must be indifferent between locating one or two jobs, $R_1 = R_2$. Then from equation (9), we have:

$$\int_0^{\bar{w}} G(w)(1-G(w))\tilde{\rho}'(w|\gamma)dw + (\tilde{\rho}(w^*|\gamma) - \tilde{\rho}(\bar{w}|\gamma))\,\mu(1-\mu) = c_2 \tag{18}$$

We can now state the main result of our analysis in this section. Essentially, as long as the cost of sampling a second firm, c_2, is not too large, there always exists a 'stable' equilibrium with positive search intensity and a non-degenerate distribution of wages. There also exists another 'unstable' equilibrium with positive search intensity. This equilibrium is unstable, in the sense that if an arbitrarily small fraction of the agents changed their search behavior, the desired behavior of all other agents would also change in the same direction (e.g. if additional agents decided to search for two jobs, the remaining agents would also want to do so).

Proposition 2 *$\exists \bar{c}$ such that $\forall c_2 > \bar{c}$, there exist no equilibrium with search, and $\forall c_2 \in (0, \bar{c})$, there exist at least two equilibria with positive search intensity. One of these equilibria is unstable. In each of these equilibria:*

1. *the support of the wage distribution $[0, \bar{w}] \cup w^*$ is defined by (17) and (7);*
2. *the wage distribution is characterized by a distribution G (equation (13)) on $[0, \bar{w}]$ and by a mass μ (equation (16)) at w^*;*
3. *z satisfies equation (18); and*
4. *z is always less than a cutoff level $\bar{z} \in (\underline{z}, 1)$.*

Proof. We have proven most of this result by construction. In an equilibrium with search, the continuous portion of the wage distribution must be given by G in equation (13), and the mass μ at w^* by equation (16), in order for active firms to earn zero net profits. w^* is necessarily the unique mass point by Lemma 2; and $\bar{w}$ is determined to ensure that the wage distribution is in fact a distribution function.

Also, in equilibrium z must satisfy (18), so that workers are willing to locate either one or two jobs. We now prove that $z < \bar{z}$ in any equilibrium; and that if $c_2 \in (0, \bar{c})$, there are at least two equilibria with search.

To see that z can never exceed some threshold $\bar{z}$, impose the restriction that $\mu < 1$ in equation (16). If this restriction failed, then the wage distribution would be degenerate at w^*, violating Lemma 2. The restriction is equivalent to:

$$z < \frac{\log(1 - w^*) + \log(1 - \gamma) - \log(1 - w^* - \gamma)}{\log(1 - w^*) - \log(1 - w^* - \gamma)} \equiv \bar{z} \tag{19}$$

One can easily confirm that $\bar{z} \in (\underline{z}, 1)$.

Next, we prove existence. First note that the left hand side of equation (18) is directly and indirectly a continuous function of the endogenous variable z. Moreover, the left hand side of equation (18) is nonnegative, and evaluates to 0 at $z = 0$ and $z = \bar{z}$ (see Figure 1). Therefore, this expression must have an interior maximum, whose value we denote by $\bar{c}$. For all $c_2 > \bar{c}$, there can be no solutions to (18), and hence no equilibria with search. For all $c_2 \in (0, \bar{c})$, continuity implies that there must be at least two solutions to (18), and thus at least two equilibria with search. At one intersection, the left hand side of equation (18) is decreasing in z. A small increase in the number of workers searching for two jobs z reduces the return to searching for two jobs by a small amount (since the function is decreasing). Hence, equilibria where the left hand side of equation (18) is decreasing in z are 'stable'; conversely, equilibria where this expression is increasing in z are unstable. ■

The proposition establishes that for any value of the search cost c_2 that is not excessively high, there exists a distribution of wages such that some workers find it profitable to locate two jobs while others do not search (i.e. only sample one job and accept this). The diversity in workers' search behavior induces firms to offer a distribution of wages. Not all firms offer w^*, because there always exist some workers who will take the first job they see, and therefore it is profitable to try to attract these workers at low wages. Low wage firms are therefore trading off lower probability of hiring (since some of the workers will have also sampled a higher wage) versus higher net revenues conditional on hiring.

The proposition characterized *all* the equilibria, and showed that there must be at least two equilibria with search (plus the no search equilibrium characterized above). We also conjecture that there are only two equilibria with search in total; in other words, (18) has at most two solutions. This could be formally established if the left hand side of (18) were strictly quasiconcave in $z \in [0, \bar{z}]$, as appears to be the case in our simulations; see Figure 1. We have been unable to prove this result analytically.

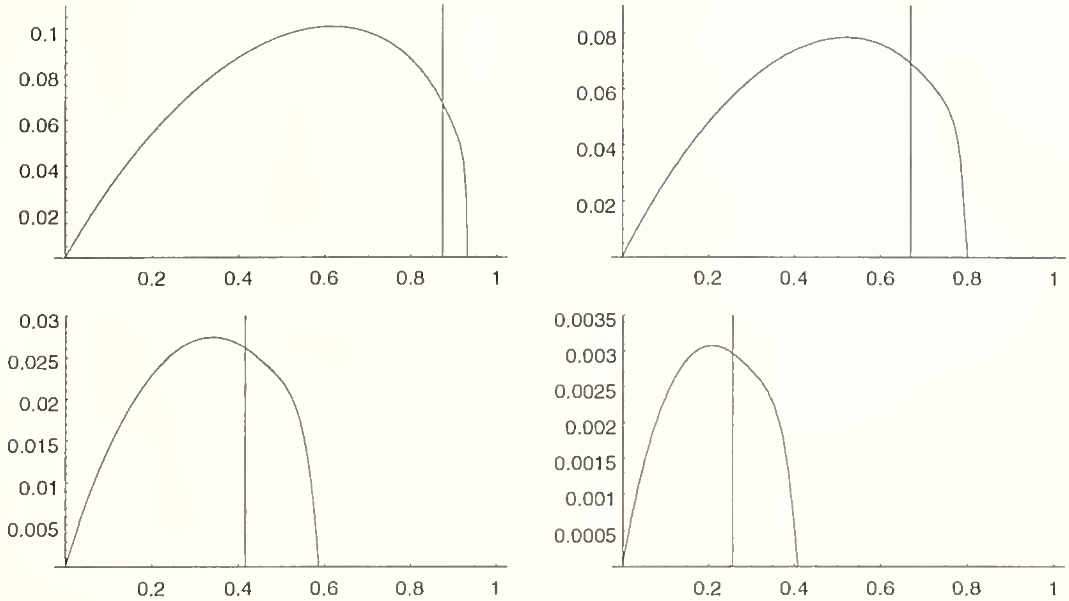

Figure 1: These panels plot the gross return to searching for a second job as a function of the equilibrium value of $z \leq \bar{z}$, for four different values of γ. The vertical lines show $\underline{z}$. In equilibrium, this gross return must equal the cost of searching for a second job, c_2; see equation (18). The top row shows the limit as γ converges to 0, and $\gamma = 0.1$ while the bottom row shows $\gamma = 0.5$ and $\gamma = 0.9$. The general shape of this figure is remarkably independent of γ.

2.7 Discussion

Our main result in this section is that in an equilibrium with search, there must be wage dispersion. If a single wage were offered in equilibrium, search efforts would go unrewarded. On the other hand, equilibrium wage dispersion is necessarily limited, because if dispersion were so great that all workers searched intensively, a firm at the bottom of the wage distribution would never hire anyone.

As well as the equilibrium with wage dispersion, there also exists an equilibrium with no search and a degenerate wage distribution (at $w = 0$). But this latter equilibrium suffers from the Diamond's (1971) paradox. Suppose workers have an option not to participate in the market; exercising this option yields a deterministic payoff of 0. If $c_1 > 0$, so even locating one wage is costly, then, the 'no-search' equilibrium disappears and is replaced by a 'no-activity' equilibrium, $\lambda_0 = 1$. In contrast, the search intensive equilibrium with wage dispersion survives as long as c_1 is not too large, because wages are strictly positive, and as a result R_1 and R_2, which are the ex ante equilibrium payoffs excluding c_1, are strictly positive too.

The intuition for multiplicity of equilibria in our economy is also worth discussing. Workers want to search when the distribution of wages is disperse. How disperse this distribution is depends on how intensively workers are searching. Essentially, it is this general equilibrium interaction that ensures the multiplicity of equilibria. However, there is more to the economics of this result.

The dispersion of wages is non-monotonic in the search intensity of workers. When no one searches intensively, the distribution of wages is degenerate at 0 (with $c_1 = 0$). When at least a fraction $\bar{z}$ of workers locate two or more jobs, there is a single wage at w^*. In contrast in all intermediate cases, the distribution is diffuse. Thus, in some sense, the incentive to search is strongest when an intermediate number of other agents are searching intensively. Put differently, when $z = 0$ — when no one is locating more than one job — search intensity decisions are '*strategic complements*': a worker only searches when others do so, because high search intensity by others induces a distribution of wages. In contrast when z is positive, search intensity decisions are '*strategic substitutes*'; when others search intensively, this limits the dispersion of the distribution of wages, and a worker does not have much incentive to search.

Finally, it is informative to consider the limit point as γ, the cost of opening an additional vacancy, disappears. In this limit, there will be an infinite number of open vacancies per worker, and all vacancies will expect zero profit. Because there is an infinite number of vacancies, the probability that a vacancy gets two applications is zero. Therefore, workers do not have to worry about the fundamental coordination problem of our economy. The return to applying to a wage at w is now $\rho(w) = w$. As a result this limit point corresponds to Burdett and Judd's (1983) model where all workers would obtain jobs. The equilibrium wage distribution is also well-behaved in this limit, and is given by $G(w) = \frac{(1-z)w}{2z(1-w)}$. Thus, our model nests the Burdett-Judd economy. However, as we show in the next section, our efficiency conclusions, even for γ arbitrarily close to 0, differ radically Burdett and Judd's.

2.8 Equilibria as Search Costs Fall

What happens when the cost of search, c_2, falls? First, as is usual in models of multiple equilibria, in order to answer this question we concentrate on stable equilibria. As c_2 falls, the equilibrium with no search (or no activity) continues to exist; however small the cost of search may be, the strategic complementarity at the point of no search is sufficiently strong to preserve this equilibrium. However, at the limit when $c_2 = 0$ — and only at this limit point — this equilibrium involves workers using *weakly dominated* strategies. By searching for two jobs, they have nothing to lose, but a lot to gain if some firms were to post positive wages.

Next consider the stable equilibrium with search, where z is decreasing in c_2. As one would expect, lower search costs lead to more search. Since the wage distribution depends on z as remarked above, changes in search costs also impact the distribution of wages. In particular, as z increases, G shifts to a new distribution that first-order stochastically dominates the old one. The wage distribution becomes increasingly concentrated at w^*, but the lower support remains at $w = 0$. Finally, in the limit point of $c_2 = 0$, the distribution is degenerate at w^*.

There are a number interesting points to note about the limit as $c_2 \to 0$. To start with, note that w^* is the same wage that Moen (1995) and Shimer (1996) obtain as the unique equilibrium wage rate when firms post wages, and workers observe all the posted wage offers (see also Mortensen and Wright, 1995). This upper hemicontinuity of the set of equilibria is reassuring.[7] Yet, there are important differences between this paper and Moen's and Shimer's earlier work. First, we have demonstrated that it is not necessary for workers to observe all the posted wages in order to ensure that all firms post w^*. It is sufficient for at least a proportion $\bar{z} < 1$ of workers to observe *two* of the wage offers, a much less stringent condition than the one required by these papers. Second, this is not the only equilibrium that is a limit point of our model as search costs fall. As noted above, the allocation in which no worker searches and the wage distribution is degenerate at 0, is also an equilibrium. The reason that this equilibrium does not exist in Moen's and Shimer's papers, is that they do not model the choice of how many wages to observe. Finally, explicitly or implicitly, these papers impose the condition first proposed by Peters (1991) that search strategies should be *anonymous*; one worker cannot decide to apply only to 'blue' firms, while an identical worker applies only to 'green' firms. If each worker could search for a job that bore her name, for example, the coordination problem would be solved. However, this solution is only possible when workers observe all the wage offers, and hence know which job bears her name. In our model, even in the limit with $c_2 = 0$, workers would observe a finite number of wage offers from a continuum (especially if for some $n > 2$, $c_n > 0$). Coordination using names or other non-wage characteristics of jobs would remain impossible. Therefore, we do not require this additional anonymity restriction.

2.9 Employment and Search Intensity

Note an interesting implication of our model: as search costs fall, more workers search and average wages increase in the stable search-intensive equilibrium. This is because a reduction in c_2 always leads to more workers searching intensively, thus to a higher value of z. From (13), this leads to a downward shift of $G(w)$, thus to higher average wages. Because the associated reduction in profits attracts entry by fewer firms, this reduces the tightness of the labor market, and employment. This result is stark in our model because higher search intensity does not directly increase the number of applications — each worker still applies to one job. Instead, as we have emphasized before, search intensity is a mechanism for sharing rents. Because high search intensity transfers rents from firms to workers, it reduces vacancy creation, and thus employment.

Some simple extensions to our model allow higher search intensity also to have a positive impact on employment, without qualitatively changing our other results.

[7]The set of equilibria is however not *lower* hemicontinuous. When $c_2 = 0$, any search behavior with $z \geq \bar{z}$ is an equilibrium. In all of these equilibria, wages are degenerate at w^*.

However, note that even with these modifications, the impact of search intensity on employment through the rent redistribution channel remains. This effect, ignored in the existing literature, will always be present when search frictions are explicitly modelled.

We turn now to the extensions:

1. There is a distribution of costs of entering the market for workers (e.g. costs of locating the first wage), given by $\Phi(c)$. In this case, only workers with $c \leq R^*$ enter, where R^* is the gross return to participating in the labor market, $R^* \equiv R_1 + c_1 \geq R_2 + c_1 + c_2$. Let N denote the measure of active workers in a proposed equilibrium, and define $q \equiv N/V$, the ratio of active workers to active firms. We can then compute the equilibrium as before, and impose the additional condition that $N = \Phi(R^*)$ to solve for the equilibrium number of active workers. Since total employment is equal to the number of firms that manage to fill their vacancy, employment can be expressed as $Ve \equiv Ne/q$, where $e \equiv \int_0^{w^*} \left(1 - e^{-q((1-z)+2zG(w))}\right) dG(w)$ is the probability that an active firm fills its vacancy. A decline in c_2 or a move to a search intensive equilibrium reduces the probability that an individual worker is hired, e/q; however, by raising R^*, it raises N. Therefore, the overall impact on total employment is ambiguous.

2. The second alternative is to assume that there is probability χ that the match between a worker and a firm is not successful, and that workers learn about the success of a match after locating the firm, but before deciding where to apply. Our analysis so far can be considered as the special case with $\chi = 0$. With more search intensity, the number of successful matches, and therefore employment, may increase. Again, this effect needs to be strong enough to outweigh the decline in vacancies associated with more intense search. We conjecture that our main results are unchanged, although this variation introduces new complexities. For example, it is no longer true that all workers locate either one or two jobs.

3 Welfare with Homogeneous Agents

In this section, we investigate whether the decentralized equilibria we characterized in the previous section have some desirable efficiency properties. We show that no equilibrium is socially optimal, even in a constrained sense that accounts for the immutable coordination problem. However, a search intensive equilibrium with wage dispersion Pareto dominates the no-search equilibrium with a degenerate wage distribution. Thus wage dispersion is *required* for efficiency in this market economy.

We will use the notion of efficiency that is the usual one in the literature (e.g. Hosios, 1990); we compare equilibria to the choice of a 'social planner' who maximizes the total social surplus subject to the same search frictions that the decentralized economy faces. To set the scene for our results, we remind the reader that the standard Diamond-Mortensen-Pissarides search model, in which workers and firms randomly run into each other and then determine wages via Nash Bargaining, is efficient for one particular value of the bargaining parameter. Moreover, Moen (1995) and Shimer (1996) show that the equilibrium is always efficient if firms post wages and workers costlessly observe all the wage offers before deciding which job to apply to. We will contrast our results with these findings.

3.1 Social Planner's Choice

Consider a social planner who chooses the number of firms V and the search intensity of workers $\{\lambda_n\}$. There is no reason for the social planner to incur search costs, and therefore he would choose $\lambda_1 = 1$ and $\lambda_n = 0$ for all $n > 1$. Compared to this allocation, the decentralized equilibrium is inefficient as long as $\lambda_1 < 1$, because the society is incurring additional costs of search. This result relates to our earlier observation that the role of more intensive search is not to increase the number of applications but to change the distribution of rents.

Can the equilibrium with no search, i.e. with $z = 0$, be efficient? The social planner would set V to maximize total surplus:

$$\left(1 - e^{-1/V}\right) V - \gamma V$$

Since all workers locate and apply to one randomly chosen job, the first term is the number of jobs created times the productivity of a job (= 1). The second term is the cost of the vacancies created. One can easily confirm that the social planner's objective function is concave, and achieves its maximum at V^* satisfying:

$$1 - e^{-1/V^*} - e^{-1/V^*}/V^* = \gamma \tag{20}$$

This gives the optimal 'tightness' of the labor market when the planner can regulate both the entry of firms and the search behavior of workers.

In the decentralized equilibrium with $z = 0$, we know from Proposition 1 that the wage distribution is degenerate at zero and the number of firms is given by V solving:

$$1 - e^{-1/V} = \gamma \tag{21}$$

V is always larger than V^*, because compared to (21), (20) has the additional term $-e^{-1/V^*}/V^*$, which captures the *congestion effect*. When the social planner creates an additional vacancy, she takes into account that this reduces the probability that other firms will be able to hire a worker. In contrast, in the decentralized equilibrium, if firms receive all the output from their match (which is the case with $z = 0$), this effect is ignored. We summarize this result in the next proposition (proof in the text):

Proposition 3 *The decentralized equilibria of section 2 are always inefficient. The equilibrium with $z = 0$ has too many vacancies, and in any equilibrium with z positive, workers spend excessive resources on search.*

This result is in stark contrast to the efficiency of the decentralized equilibrium without costs of locating wages. In fact, as noted in the previous section, one of the two limit points of our model with $c_2 \to 0$ is an equilibrium in which $\bar{z}$ workers search for two wage offers; at this limit, the equilibrium is socially optimal. Comparing our results to this limiting case makes it clear that all the inefficiencies arise because of the search intensity problem. Intuitively, by locating more firms, workers create a number of externalities. In particular, the change in their search behavior induces a change in the distribution of wages. As these effects are not internalized, the equilibrium is always inefficient. Either workers incur additional costs of search that the planner avoids, or due to low search intensity, firms get too large a share of the quasi-rents they create, and there is excessive entry. Moreover, a small increase in search costs starting from $c_2 = 0$ will have a first-order impact on output.[8] Thus, search costs not only change the nature of equilibrium, but also have a significant impact on total output.

It is interesting to note that even though decentralized equilibria are inefficient, there is a government intervention that decentralizes the constrained efficient allocation — a minimum wage at w^*. Then, no firm can offer less than w^*, and no firm would want to offer more (given the form of $\rho(w)$ characterized in section 2). Workers would save on search costs, and the efficient number of firms would enter.

3.2 Efficiency of Wage Dispersion

We have proven that all decentralized equilibria are inefficient. Now we prove that any equilibrium with positive search intensity and wage dispersion Pareto dominates the equilibrium without search, in the sense that ex ante all agents would prefer to be in an equilibrium with wage dispersion, and for some agents this preference is strict.

Proposition 4 $\forall c_2 \in (0, \bar{c})$, *an equilibrium with positive search intensity and wage dispersion always Pareto dominates the no search equilibrium.*

Proof. Consider the no search equilibrium. Workers get no surplus since $\hat{G}(w)$ is degenerate at 0; all firms also make zero-profit from the free-entry condition. Now consider an equilibrium with search. All workers get a level of surplus equal to $R_1 > 0$, and firms again make zero-profit. ■

Even though the social planner would never choose $z > 0$, such an equilibrium always has more social surplus than an equilibrium with $z = 0$. This is because the social planner controls V directly (e.g. by taxation or regulation), while in the decentralized equilibrium, V is a decreasing function of z. In the equilibrium with

[8] Informally, the impact of an increase in c_2 is first-order, because the derivative of total output net of search costs with respect to c_2 has an upper bound of $-\bar{z}$ at $c_2 = 0$. A small increase in c_2 leads to approximately $\bar{z}$ agents incurring the search costs. Externalities add to this inefficiency.

$z = 0$, all the rents go to the firms, whose entry behavior is highly elastic (the free-entry condition forces all firms to make zero-profits). This implies that all the rents are dissipated by further entry and that society suffers excessive costs of vacancy creation. This result is closely related to the efficiency results of search models with bargaining. As shown by Hosios (1990), when firms have all the bargaining power, there will be too much entry and excessively low unemployment. In our model, with $z = 0$, all the rents go to firms and the same result obtains. In contrast, when z is positive, some of the rents get transferred to workers; and in our model, the worker side is highly inelastic. Thus the additional rent dissipation effect on the worker side is limited. In fact, the asymmetry between the firms' and the workers' decisions is worth emphasizing because it is at least partly endogenous in our model. On the firm side, when rents are high, all firms want to enter. In contrast, on the worker side, as noted earlier, search intensities are *strategic substitutes*: when other workers are searching a lot, wages are pushed up, and the distribution of wages is concentrated. Therefore, each worker has only weak incentives to exert high search effort. This effect limits the search costs that society has to incur. Thus wage dispersion is the market's means of controlling entry.

Proposition 4 holds for any $\gamma > 0$. As a result, the welfare implications of our model differ significantly from Burdett and Judd's in the limit as γ converges to 0. Recall that at this limit, the coordination problem disappears from our model, since there is infinitely much entry; $\rho(w) = w$. Hence one might think that the crowding externality disappears as well, and so it is not necessary to have wage dispersion to approximate efficiency. Instead, in the limit each firm has an extremely small external effect on every other firm, but since there is an unbounded mass of active firms as γ converges to 0, the total size of the externality does not disappear. As a result, the search intensive equilibrium continues to Pareto dominate the no-search equilibrium. An equivalent way to say this, is that in the limit, the total cost of entry does not disappear. An 'infinite measure' of firms each spends 0 on entry, and total entry costs converge to a finite number. Therefore, even in this limit, wage dispersion and search are socially beneficial.

To conclude, we have established that with homogeneous agents, search intensity is purely a 'rent-sharing' mechanism; it transfers rents from firms to workers. This transfer of rents is nonetheless very useful from an efficiency viewpoint, because dissipation of rents by further entry of firms is socially more expensive than the search process of workers. Therefore, with homogeneous firms and no government intervention in labor markets, wage dispersion is necessary for workers to search, average wages to increase and the society to reach a more efficient allocation.

4 The Dynamic Environment

The previous two sections analyzed a one period economy. Because search decisions are dynamic, extending our analysis to a multi-period setting is important. In this section, we prove that all of our results completely generalize to this environment.

Given the parallel to the results of section 2, the analysis will be brief.

Consider the following economy. Time is discrete and infinite. There is a continuum 1 of infinitely lived workers. In each period, unemployed workers (endogenous measure U) decide how many firms to locate among the available vacancies. The matching technology is unchanged. In every period, the number of active vacancies is determined by the entry decision of firms; having an open vacancy costs γ. A worker who gets a job offer leaves unemployment and becomes employed the following period. However, each employed worker faces an exogenous probability δ that her match breaks apart in any period. In this event, the firm disappears and the worker becomes unemployed. As before, all agents are risk-neutral, but now they maximize the present discounted value of their income stream, net of search costs, using a common discount factor $\beta < 1$.

Next we specify the space of contracts that firms might post. In the previous sections, these were simply one-period wages. Now, however, they can be more complicated objects. For instance, a firm could post a vacancy that promises a constant wage for all periods in which the worker is employed by the firm. Yet other possibilities, for instance an upward-sloping wage profile, are also possible. Offers by different firms may differ in multiple dimensions, but these differences are more apparent than real. Since both firms and workers are risk-neutral and have a common discount factor, all that they care about is the expected present discounted value of the offer the firm makes. Thus we can reduce the problem to a one-dimensional one. In fact, Shimer (1996) demonstrates that any equilibrium allocation can be represented as a situation in which all firms offer 'simple contracts': promise a 'signing bonus' s to a worker in the period she accepts a job and then, in any future period that the worker is still with this firm, give her a wage that keeps her indifferent between working for this job and becoming unemployed, i.e. her 'reservation wage'. Competition is restricted to the single-dimensional signing bonus. For expositional simplicity, we restrict attention to these simple contracts, although it is straightforward to map back from these contracts to other equivalent contracts that could be offered in equilibrium. By this procedure, we are *not* ignoring any equilibrium allocations nor sacrificing any generality

We now introduce some new notation. We denote the (expected present discounted) value of an unemployed worker by J^U. Under the simple contracts, this is also equal to the value of an employed worker who has already consumed her signing bonus. The value of a worker who decides to locate n jobs is again R_n; and the value of a worker who has applied to a job posted at signing bonus s is $\rho(s)$, analogous with our earlier notation. Also, the value of profits for a vacancy that posts a job with signing bonus s, net of vacancy costs γ, is denoted by $J^V(s)$. And finally, the present value of a firm employing a worker at her reservation wage $(1-\beta)J^U$ is denoted by J^F. This is naturally equal to $\frac{1-(1-\beta)J^U}{1-\beta(1-\delta)}$.

We now define a steady state equilibrium for this economy.

Definition 2 *A steady state equilibrium consists of a measure V of firms that create vacancies each period; an unemployment rate U; a distribution of signing bonuses* *offered by each firm $F(s)$ with closed support $\mathcal{S}$; a value function for firms $J^V(s)$; a return function for unemployed workers $\rho(s)$; and search intensity decisions for unemployed workers $\{\lambda_n\}$ such that;*

1. *$\forall s \in \mathcal{S}$ and $\forall s'$, $J^V(s) \geq J^V(s')$.*
2. *$\forall s \in \mathcal{S}$, $J^V(s) = 0$.*
3. *A worker who learns about signing bonuses $s_1, s_2, ..., s_n$ applies to s_i only if $\rho(s_i) \geq \max_j \langle \rho(s_j) \rangle$ and $\rho(s_i) \geq 0$.*
4. *$\lambda_{n^*} > 0$ only if $n^* \in \arg\max_n R_n$.*
5. *U and V are constant in every period.*

Let us first define analogously to $\Sigma(w)$ in section 2, the conditional probability that a worker locating a firm offering signing bonus s applies for the job:

$$\Sigma(s) \equiv \sum_{n=1}^{\infty} n\lambda_n F(s)^{n-1}$$

where λ_n is the proportion of workers locating n vacancies. Then:

$$(1-\beta)J^V(s) = \left(1 - e^{-q\Sigma(s)}\right)\left(\beta\left(J^F - J^V(s)\right) - s\right) - \gamma$$

where q equals $\frac{U}{V}$. The free entry condition, the second condition in the definition of equilibrium, gives that for all $s \in \mathcal{S}$:

$$q\Sigma(s) = \log(\beta J^F - s) - \log(\beta J^F - s - \gamma) \tag{22}$$

With a similar reasoning to equation (3) above, we use (22) to obtain an expression for $\rho(s)$:

$$\rho(s) = \frac{1 - e^{-q\Sigma(s)}}{q\Sigma(s)} s + \beta J^U$$

Again, we can ask what the return to applying for signing bonus s must be, in order for a firm posting this signing bonus to earn zero net profits:

$$\tilde{\rho}(s|\gamma) = \frac{\gamma s}{(\beta J^F - s)\left(\log\left(\beta J^F - s\right) - \log(\beta J^F - s - \gamma)\right)} + \beta J^U \tag{23}$$

As in section 2, $\tilde{\rho}$ is a quasiconcave function of s, Moreover, $\forall s$, $\tilde{\rho}(s|\gamma) \leq \rho(s)$, with equality for $s \in \mathcal{S}$. Therefore, we can prove an analogue to Lemma 1; workers always apply to the firm offering the highest nonnegative signing bonus that they locate. From this it follows (as in Lemma 2) that if $\lambda_1 < 1$, the closure of the signing bonus distribution consists of a convex interval $[0, \bar{s}]$ and possibly the point $s^* \geq \bar{s}$ that is the unique maximizer of (23); and that the bonus distribution is atomless on $[0, \bar{s}]$ but has an atom at s^*.

Similarly, as in Lemma 3, some workers choose only to locate one firm; and as in Lemma 4, the marginal return to locating the $n+1^{\text{st}}$ firm is less than the marginal return of locating the n^{th} firm, where the total return is defined by:

$$R_n = n \int_S \rho(s) F(s)^{n-1}\, dF(s) - \sum_{j=1}^{n} c_j$$

Therefore, a result equivalent to Lemma 5 follows immediately: in equilibrium, $\lambda_1 + \lambda_2 = 1$ and $R_1 \geq R_2$. Then it follows that the value of unemployment satisfies:

$$J^U = R_1 = \int_S \rho(s)\, dF(s) - c_1$$

Next, since no worker will locate more than two jobs, we can again define $z \equiv \lambda_2$ as the proportion of workers who search intensively. Thus (22) implies:

$$F(s) = \frac{\log(\beta J^F - s) - \log(\beta J^F - s - \gamma) - q(1-z)}{2qz} \tag{24}$$

Since $F(0) = 0$, (24) yields an expression for the equilibrium tightness of the labor market, $q = \left(\log(\beta J^F) - \log(\beta J^F - \gamma)\right)/(1-z)$. Now substituting for q in (24), we obtain:

$$F(s) = \frac{1-z}{2z}\left(\frac{\log(\beta J^F - s) - \log(\beta J^F - s - \gamma)}{\log(\beta J^F) - \log(\beta J^F - \gamma)} - 1\right) \tag{25}$$

The condition that ensures an atomless wage distribution, $F(s^*) \geq 1$, again takes the form $z \leq \underline{z}$; in this case, $\bar{s}$ satisfies $F(\bar{s}) = 1$. On the other hand, if $z > \underline{z}$, then there is an atom at s^* of measure μ:

$$\mu = \frac{1+z}{z} - \frac{1-z}{z}\left(\frac{\log(\beta J^F - s^* - \gamma) - \log(\beta J^F - s^*)}{\log(\beta J^F - \gamma) - \log(\beta J^F)}\right)$$

and now $\bar{s}$ satisfies $F(\bar{s}) = 1 - \mu$.

In an equilibrium with search, workers must be indifferent between locating one or two firms, and so the following condition must hold:

$$\int_0^{s^*} F(s)(1 - F(s))\, d\tilde{\rho}(s|\gamma) = c_2$$

where c_2 is again the marginal cost of locating the second firm.

Finally, since we are looking for a steady state of this dynamic economy, the number of workers leaving and entering unemployment must be equal in any period. Thus steady state imposes $\delta(1-U) = Ue(z)/q$. The number of matches destroyed is the probability that a match breaks down in any period, δ, times the number of existing matches, $1-U$. The number of matches created is equal to the proportion of unemployed workers, times the probability that an unmatched *firm* hires a worker in a given period, $e(z) \equiv \int_0^{s^*}\left(1 - e^{-q((1-z)+2zF(s))}\right) dF(s)$, divided by the unemployment-vacancy ratio. Note that as in the standard search models (e.g. Pissarides, 1990), the steady state unemployment *rate* can be determined after all the other parameters because it is only affected by the unemployment-vacancy ratio, q, and does not impact on the other endogenous variables.

Therefore, the steady state equilibria of the dynamic economy are very similar to the equilibria of the static economy. In particular, for c_2 small enough, two equilibria with search will exist with non-degenerate distributions. Also it is straightforward to see that there will always exist an equilibrium with $z = 0$. If no one locates more than one job, $F(s)$ is Dirac at 0. Then it follows that $J^U = 0$ in this equilibrium.

To summarize:

Proposition 5 *There always exists a steady state equilibrium with $z = 0$ in which no firm offers a positive signing bonus. $\exists \hat{c} > 0$ such that $\forall c_2 \in (0, \hat{c})$, there exist (at least) two steady state equilibria in which some unemployed workers locate two vacancies and the distribution of signing bonuses is non-degenerate. One of these equilibria is stable and the other is unstable.*

Also it is straightforward to see that we can repeat the welfare analysis in this case, and the results would again parallel those of the one period economy. In particular, in this infinite horizon economy too, the equilibrium with search and wage dispersion Pareto dominates the no search equilibrium. And once again, wage dispersion is necessary for the market economy without the Walrasian auctioneer to function.

5 Heterogeneous Firms and Choice of Technology

5.1 The Environment

Our analysis so far has established that firms may want to place themselves in different parts of the wage distribution in order to attract workers. In practice, such behavior appears more common when firms have different productivities. In order to analyze this possibility, we now return to the one-period environment of sections 2 and 3, but allow firms to choose from a menu of technologies. Workers are still identical.

We assume that before contacting a worker, firms must simultaneously decide what type of job to create. For instance, firms decide what kind of equipment to buy, or choose the level of capital investments (e.g. Acemoglu, 1996a). The rest of the sequence of events is exactly the same as in section 2. In particular, each worker simply cares about the wage that a firm offers. As before, she must locate a firm in order to find out its wage. In keeping with the spirit of this paper, we assume that workers cannot condition their search on observations of firms' types.[9]

[9]This is consistent with interpreting θ's as varying within an industry; as noted above, a significant fraction of wage dispersion is within an industry. Most of our results would generalize to the

Job type is denoted by $\theta \in \mathbb{R}^+$. A type θ job will produce θ if the firm hires a worker and nothing otherwise. Let $\Gamma : \mathbb{R}^+ \mapsto \mathbb{R}^+$ denote the cost of creating a type θ job, e.g. the cost of equipment. We assume that Γ is positive, increasing, convex, and continuously differentiable. Also, we define $\underline{\theta}$ to be the unique solution to $\underline{\theta}\Gamma'(\underline{\theta}) \equiv \Gamma(\underline{\theta})$, the minimum average fixed cost technology. To allow a possibility of production, we assume that $\underline{\theta} > \Gamma(\underline{\theta})$; a firm that is guaranteed to hire a worker at zero wage can make a profit using the minimum average cost technology. Finally, we assume that

$$\frac{\theta\Gamma'(\theta) - \Gamma(\theta)}{-\log(1 - \Gamma'(\theta))}$$

is strictly quasiconcave.[10] We explain the importance of this regularity condition in the text.

The gross profit of a firm of type $\theta \in \mathbb{R}^+$ that posts wage w is:

$$\pi(\theta, w) = \left(1 - e^{-q\Sigma(w)}\right)(\theta - w) \tag{26}$$

where $q = 1/V$ and $\Sigma(w)$ is defined as in section 2 to be the probability that a representative worker applies to a firm offering w, conditional on contacting such a firm. The rest of the model is unchanged. In particular, workers are ex ante identical.

5.2 Analysis

Because the results are parallel to those of section 2, we will state many of these without proof.

Definition 3 *An equilibrium consists of a measure of active firms V, wage distributions $\hat{G}(w|\theta)$ over the support $\mathcal{W}(\theta)$ for each type of firm θ, a distribution of active firms $\hat{H}(\theta)$ over the support Θ, a profit function for firms π, an expected return function for workers ρ, and search intensity decisions for workers $\{\lambda_n\}$ such that:*

1. $\forall\theta \in \Theta$, $\forall w \in \mathcal{W}(\theta)$, $\forall\theta'$, *and* $\forall w'$, $\pi(\theta, w) - \Gamma(\theta) \geq \pi(\theta', w') - \Gamma(\theta')$.

2. $\forall\theta \in \Theta$, $\forall w \in \mathcal{W}(\theta)$, $\pi(\theta, w) = \Gamma(\theta)$.

3. *A worker who learns about wages $w_1, w_2, ..., w_n$ applies to wage w_i only if $\rho(w_i) \geq \max_j\langle\rho(w_j)\rangle$ and $\rho(w_i) \geq 0$.*

4. $\lambda_{n^*} > 0$ *only if* $n^* \in \arg\max_n R_n$.

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case where the distribution of θ varies across industries, and workers can decide to sample wages in different industries.

[10] A sufficient, but by no means necessary, condition for this to be true is that $\Gamma'''(\theta)$ is nonnegative. A less restrictive, but still sufficient condition, is that $\theta(1 - \Gamma'(\theta))$ is strictly quasi-concave. If this condition fails, the general qualitative properties of the equilibrium are unchanged; however, there may be two or more wages offered by positive measures of firms.

The important point to note about this definition is that firms *simultaneously* choose their type and the wage that they will offer. This implies that $\forall \theta \in \Theta$ and $\forall w \in \mathcal{W}(\theta)$, w and θ must maximize net profits:

$$\max_{w,\theta} \left(1 - e^{-q\Sigma(w)}\right)(\theta - w) - \Gamma(\theta)$$

This objective is a strictly concave function of θ. *If* wage w is offered in equilibrium, it must be offered by a type θ firm satisfying:

$$1 - e^{-q\Sigma(w)} = \Gamma'(\theta) \tag{27}$$

Since Γ is strictly convex, (27) defines at most one type $\theta^*(w)$ that may offer wage w in equilibrium. Equivalently, $\forall w$, and $\forall \theta \neq \theta'$, $w \in \mathcal{W}(\theta) \Rightarrow w \notin \mathcal{W}(\theta')$.

Next, the second requirement of the definition of an equilibrium, the zero profit condition, implies $\left(1 - e^{-q\Sigma(w)}\right)(\theta^*(w) - w) = \Gamma(\theta^*(w))$. Combining this with the optimal type condition (27) implies that $\forall \theta \in \Theta$, $w \in \mathcal{W}(\theta)$ if and only if

$$w = \theta - \frac{\Gamma(\theta)}{\Gamma'(\theta)} \tag{28}$$

This uniquely defines the wage offered by a type $\theta \in \Theta$ firm, $w^*(\theta)$; $\mathcal{W}(\theta)$ is a *singleton* for all $\theta \in \Theta$. Since Γ is convex, the wage is an increasing function of the firm's type. Also, given the definition of $\underline{\theta}$, $w^*(\underline{\theta}) = 0$, and is strictly positive for higher values of θ. Thus (proof in the text):

Lemma 6 *The wage distribution $\hat{G}(w|\theta)$ for all $\theta \in \Theta$ is degenerate at the point $w^*(\theta) = \theta - \Gamma(\theta)/\Gamma'(\theta)$.*

It is important to stress that $w^*(\theta)$ is independent of all other variables, including worker's search intensity and the distribution of firms in the economy. This implies that instead of focusing on wage distributions, as in the earlier sections, we can equivalently analyze the endogenous type distribution $\hat{H}$. Once this distribution is determined, (28) immediately gives the distribution of wages. We proceed using this notational simplification, which will also highlight the parallel between the results of this section and those of section 2.

The return to a worker who applies for a type θ job is

$$\rho(\theta) = \frac{1 - e^{-q\Sigma^*(\theta)}}{q\Sigma^*(\theta)} w^*(\theta) \tag{29}$$

with $w^*(\theta)$ given by (28) and $\Sigma^*(\theta) \equiv \Sigma(w^*(\theta))$. Substituting in equation (29) from equation (28) and the implicit definition of $q\Sigma(w)$ in equation (27), we obtain:

$$\tilde{\rho}(\theta|\Gamma) = \frac{\theta\Gamma'(\theta) - \Gamma(\theta)}{-\log(1 - \Gamma'(\theta))} \tag{30}$$

Again $\tilde{\rho}$ is the return to the worker from applying to a type θ job if $\pi(\theta) = \Gamma(\theta)$. Thus as in section 2, we have that $\tilde{\rho}(\theta|\Gamma) \leq \rho(\theta)$, with equality if $\theta \in \Theta$.

By assumption, $\tilde{\rho}$ is strictly quasiconcave. This allows us to prove that, analogous to Lemma 1, in equilibrium a worker always applies to the highest type of firm that she locates. Also, if $\lambda_1 < 1$, then as in Lemma 2, the support of the type distribution Θ consists of a convex interval $[\underline{\theta}, \bar{\theta}]$ and possibly a point $\theta^* \geq \bar{\theta}$ that is the unique maximizer of $\tilde{\rho}(\theta|\Gamma)$. The type distribution is atomless on $[\underline{\theta}, \bar{\theta}]$ but may have an atom at θ^*.

Because workers always apply to the highest wage, and thus to the highest type of firm that they locate, it follows that in equilibrium some workers must search for one wage. Otherwise, if all workers search intensively, there would be no equilibrium wage dispersion, and it would not pay anyone to search intensively. Then decreasing returns to search implies that no worker searches for more than two jobs.

Once again, we let z denote the fraction of workers who locate two wages, and $1-z$ denote the fraction that locate one. Thus $\Sigma^*(\theta) = (1-z) + 2zH(\theta)$, where H is the probability that a worker who locates another firm applies to a type θ firm. As was the case in section 2, the functions $\hat{H}$ and H coincide except possibly at θ^*.

It is now possible to invert equation (27) to solve for $H(\theta)$ in terms of z:

$$H(\theta) = \frac{-\log(1-\Gamma'(\theta)) - q(1-z)}{2qz} \tag{31}$$

As long as the type distribution is not degenerate at θ^*, some firms will choose to be of type $\underline{\theta}$, which parallels our result in section 2, that some firms offered 0 wage. Since we must have $H(\underline{\theta}) = 0$, the equilibrium tightness can be determined as:

$$q = \frac{-\log(1-\Gamma'(\underline{\theta}))}{1-z} \tag{32}$$

Since by assumption $\Gamma'(\underline{\theta}) = \Gamma(\underline{\theta})/\underline{\theta} < 1$, q is a positive number. Also (32) defines q increasing in z; this parallels our results of section 2. Now, substituting q back into equation (31) we obtain:

$$H(\theta) = \frac{1-z}{2z}\left(\frac{\log(1-\Gamma'(\underline{\theta})) - \log(1-\Gamma'(\theta))}{-\log(1-\Gamma'(\underline{\theta}))}\right) \tag{33}$$

which gives the equilibrium distribution of types (and therefore of wages) only in terms of the cost function, $\Gamma(\theta)$, and the proportion of workers sampling two jobs. It can easily be verified that this is a strictly increasing function with $H(\underline{\theta}) = 0$.[11]

Recall that in the homogeneous firm model, an increase in search intensity z caused a first order stochastic dominating increase in the wage distribution. Now an increase in search intensity does not affect the wage offered by any particular

[11] It can be verified that as the cost of creating jobs converges to a 'L-shape', i.e. $\Gamma(\theta) \approx \gamma$ for $\theta < 1$ and infinite otherwise, the equilibrium aggregate wage distribution converges to $\hat{G}(w)$ characterized in section 2 and the type distribution degenerates at 1 in the limit. The limiting technology is not differentiable, and hence our analysis does not apply.

type of firm (equation (28)), but instead leads to a first order stochastic dominating shift in the *type* distribution, hence to a first-order stochastic shift of the resulting 'aggregate wage distribution' faced by the workers. Therefore, in contrast to section 2, when workers search more, the distribution of jobs (and labor productivities) improves.

Next, suppose there is an equilibrium in which a mass μ of firms choose type θ^*. Then $H(\theta^*) = 1 - \mu/2$ as before. Since the cost of searching for the second job c_2 is positive, the type distribution cannot be Dirac at θ^*; therefore some firms choose to be of type $\underline{\theta}$, and (33) still applies. Hence:

$$\mu = \max\left\{\frac{1+z}{z} - \frac{1-z}{z} \cdot \frac{\log(1-\Gamma'(\theta^*))}{\log(1-\Gamma'(\underline{\theta}))}; 0\right\} \tag{34}$$

This fraction lies between 0 and 1 if $z \in (\underline{z}, \bar{z})$, where[12]

$$\underline{z} = \frac{\log(1-\Gamma'(\underline{\theta})) - \log(1-\Gamma'(\theta^*))}{-\left(\log(1-\Gamma'(\underline{\theta})) + \log(1-\Gamma'(\theta^*))\right)}$$
$$\bar{z} = \frac{\log(1-\Gamma'(\underline{\theta})) - \log(1-\Gamma'(\theta^*))}{-\log(1-\Gamma'(\theta^*))}$$

If $z \leq \underline{z}$ in equilibrium, there is an atomless distribution of firms. The supremum of the type distribution satisfies $H(\bar{\theta}) = 1$, where H is defined in equation (31). If $z \in (\underline{z}, \bar{z})$ in equilibrium, the support of the type distribution consists of an atom with mass μ at θ^*, and an atomless interval $[\underline{\theta}, \bar{\theta}]$, where $H(\bar{\theta}) = 1-\mu$. Finally, since $c_2 > 0$, there is no equilibrium with $z \geq \bar{z}$. To see this note that if there were, the type and wage distributions would be degenerate, and no one would want to search intensively.

Finally, as in section 2, for small enough search costs c_2, equilibria with positive search intensity z will exist and are characterized by workers indifference condition:

$$R_2 - R_1 \equiv \int_{\underline{\theta}}^{\theta^*} H(\theta)(1-H(\theta))\, d\tilde{\rho}(\theta|\Gamma) - c_2 = 0. \tag{35}$$

Clearly there is always equilibrium with no search intensity, $z = 0$ too. In this case, no firm would offer a positive wage, and so from equation (28), only type $\underline{\theta}$ are active in equilibrium. The type distribution is degenerate at $\underline{\theta}$, and the wage distribution is degenerate at $w = 0$. Essentially, when higher types cannot offer higher wages to fill their vacancies faster, type $\underline{\theta}$ will enter in sufficient numbers so as to make entry unprofitable for any other type. Hence the market collapses, and only the minimum average fixed cost technology is used.

Then, we can characterize the equilibria of this section (proof omitted):

Proposition 6 *1. There always exists an equilibrium with no search, thus $z = 0$. In this equilibrium, the wage and type distributions are degenerate at $\underline{\theta}$.*

[12]It can again be confirmed that $0 < \underline{z} < \bar{z} < 1$.

2. $\exists \tilde{c}$ *such that* $\forall c_2 > \tilde{c}$, *there is no equilibrium with search and* $\forall c_2 \in (0, \tilde{c})$, *there exist at least two equilibria with positive search intensity. One of these equilibria is unstable. In equilibria with positive search intensity:*

   (a) *wages are given by equation (28);*

   (b) *the support of the type distribution* $[\underline{\theta}, \bar{\theta}] \cup \theta^*$ *is defined by* $\underline{\theta}\Gamma'(\underline{\theta}) \equiv \Gamma(\underline{\theta})$, $H(\bar{\theta}) = 1 - \mu$, *and* $\theta^* \in \arg\max_\theta \tilde{\rho}(\theta|\Gamma)$;

   (c) *the type distribution is characterized by a distribution* H *(equation (33)) on* $[\underline{\theta}, \bar{\theta}]$ *and by a mass* μ *(equation (34)) at* θ^*;

   (d) z *satisfies equation (35); and*

   (e) z *is less than the cutoff level* $\bar{z} \in (\underline{z}, 1)$.

5.3 Discussion

The first point to note is the parallel with the results of section 2. In particular, there is again a multiplicity of equilibria, and the intuition is the same: search intensity decisions are strategic complements when search intensity is low, and strategic substitutes when search intensity is high. Furthermore, the equilibrium is characterized as a distribution (of types rather than wages), and firms once again care about their relative position in this distribution. Yet, changes in search intensity of workers do not affect wages posted by particular types of firms, but the technology choices of firms and the distribution of types.

More significantly, in contrast to the model with homogeneous firms, search intensity has additional effects now. Higher search intensity enables socially desirable *sorting*, that is higher productivity firms facing a higher probability of filling their vacancies. This is because workers who search intensively are more likely to locate and apply for high wage jobs. Since it is the high productivity firms that are more willing to pay high wages (recall equation (28)), wage dispersion and search intensity increase the relative profitability of high productivity firms. Through this channel, higher search intensity encourages investment in better technologies and improves the distribution of available jobs. In contrast, in the absence of intensive search, all firms pay the same wage and get filled at the same rate. Free entry leads to the creation of only the 'lowest common denominator', type $\underline{\theta}$ firms. This introduces a new reason for search and wage dispersion to be socially desirable.

Also observe that as the cost of locating firms, c_2, falls, z increases as in section 2. Now, the increase in z not only transfers rents from firms to workers, but also affects firms' investment choices. From (33), a higher z implies a better productivity distribution ($H(\theta)$ shifts to the right). Interestingly, as $c_2 \to 0$, the type distribution converges to being degenerate at θ^*, and the wage distribution converges to $w^*(\theta^*)$. Recall that in the homogeneous firm case this limit was the constrained efficient allocation. More surprisingly, the same is true here. At the limit of $c_2 = 0$, both the equilibrium number of vacancies and the investments levels are exactly the same as what the planner would choose.[13]

To see why this result is striking, recall that in models with ex ante investment and ex post bargaining (e.g. Grout, 1984, Acemoglu, 1996a), investment incentives are always distorted, because firms are not full residual claimants. This is also true in our model since $w(\theta)$ is increasing in θ. However, the probability of obtaining a worker is a function of θ as well. In the limit of $c_2 = 0$, the competition for workers is severe enough that in order to attract workers, firms are forced to choose the 'right' technology.

Having noted the efficiency of the search intensive equilibrium at $c_2 = 0$, it is also straightforward to repeat the analysis of section 3. First, for all $c_2 > 0$, equilibrium is always inefficient. More importantly, for all $c_2 > 0$, an equilibrium with search and wage dispersion Pareto dominates the no search equilibrium. That is, once again, with positive search costs, wage dispersion and search intensity are necessary to avoid highly inefficient allocations: in the no-search equilibrium, all firms open type $\underline{\theta}$ jobs, thus labor productivity and output are very low. This result is however more surprising than the one we obtained in section 3. A simple reasoning could have suggested that investment incentives would be least distorted in the no-search equilibrium, because firms are the full residual claimant of the returns they create. However, counteracting this, when $z = 0$, there is an excessive entry of firms as in sections 2 and 3. This reduces the probability that a given firm will contact a worker, making it unprofitable to invest in high productivity technologies.

To summarize, as in the model with homogeneous workers, higher search intensity transfers rents from firms to workers and improves social welfare by avoiding excessive entry. However, there is also a more interesting benefit of higher search intensity: *it leads to better sorting*, in the sense that high productivity firms have higher probability of hiring a worker. And this possibility of better sorting increases the relative profitability of high productivity jobs and encourages firms to invest in better technologies.

6 Heterogeneity and Match Quality

Our analysis so far has established that in a number of environments, wage dispersion is an equilibrium phenomenon because firms care about their relative ranking in the wage distribution. And more importantly, wage dispersion is necessary for search, therefore it is necessary for a third-best efficient allocation of resources. In order to concentrate on wage dispersion among identical workers, we have assumed that there is no ex ante nor ex post heterogeneity in the productivities of workers.

[13]Given constant returns to scale, the planner would want to create only one type of job, and would therefore like workers to locate one job only. Then the planner chooses q and θ to maximize: $\frac{1}{q}\left(1-e^{-q}\right)\theta - \frac{1}{q}\Gamma(\theta)$. The first order conditions are satisfied at θ^* and $q^* = -\log\left(1-\Gamma'(\theta^*)\right)$, that is at the search intensive decentralized equilibrium when $c_2 = 0$. Although the decentralized equilibrium 'wastes' resources on search, this is costless in the limit.

Such heterogeneity is likely to be an important component of wage distributions we observe in practice, and our key results would not be affected by these considerations. In this section, we briefly outline how these aspects could be incorporated into our model.

First, it is generally accepted that some workers are more suited to some jobs, but that whether a match between a worker and firm is good cannot be observed until it is realized. This feature can be captured by introducing a match quality α with some distribution Λ over $\mathbb{R}^+$, such that the productivity of the worker in a given job of quality θ is $\alpha\theta$. α's in different jobs are independent draws from Λ, and are only observed after the worker applies to a job. This complication changes nothing from the point of the worker.[14] His probability of acceptance in a job (before α's are observed) is still given by $\frac{1-e^{-q\Sigma(w)}}{q\Sigma(w)}$; the firm will hire the worker with the highest α and all workers draw from the same distribution Λ. The difference happens on the firm side: now firms care about how many applicants they attract, not only if they attract at least one applicant. Attracting more applicants enables a firm to select better the one most suited to the job. The profit function of a firm is more complicated now, but the implication of this extension is clear: high productivity firms will care about match quality more than do low productivity firms; therefore, they will have an added incentive to locate on the upper part of the wage distribution. As a consequence, wage dispersion and search intensity will not only enable high productivity firms to fill their vacancies faster, but also to achieve a higher match quality (i.e. select better suited candidates). Although qualitatively this effect is identical to ours, the economic interpretation is different, and may better deserve the term *sorting*. Nevertheless our main results are unaffected. There will be wage dispersion is some equilibria, and wage dispersion is necessary for search and for efficiency.

The second extension, which is more difficult to analyze formally, is to introduce ex ante heterogeneity on both sides of the market (as in Sattinger, 1995 and Shimer and Smith, 1996). For instance, we can assume that there are two types of workers. High skill workers produce θ in a job of type θ and low skill workers produce $\kappa\theta$ where $\kappa < 1$. A social planner or a Walrasian auctioneer would allocate high skill workers to high productivity firms. However, in a search economy this is only possible if they locate these jobs. As a result, high productivity firms will have yet another reason to offer high wages: to attract high skill rather than low skill workers. In this case, the worker side is no longer as simple as before, because workers may want to locate more than two jobs. Nevertheless, it is clear that in this case a firm's relative position in the wage distribution will not only affect the probability of filling its job, but also which types of workers will apply for the job. It will not necessarily be the case, for example, that low skilled workers will want to apply to a high productivity firm, where competition may be more severe.

[14] Clearly our analysis so far is the special case with Λ degenerate.

7 Conclusion

This paper has offered a model which resembles the actual practice of search for jobs in an economy without the Walrasian auctioneer to coordinate trade. Workers engage in search activity in order to find the best wage offer. They sample a number of firms, and then, anticipating the probability of obtaining these jobs, decide for which to apply. Firms, anticipating the search and application decisions of workers, offer different wage distributions. We have also analyzed the equilibrium composition of jobs when firms can choose between different technologies.

The main implication of our analysis is that in an economy without the invisible hand, wage dispersion among identical workers should be ubiquitous, because wage dispersion is necessary for intensive worker search. Symmetrically, intensive search induces firms to locate in different parts of the economy's wage distribution. This enables firms to attract workers at different rates, and also to ensure better match quality. Therefore, wage dispersion, and the search behavior that it induces among workers, enables better sorting, encourages the creation of better jobs, and improves the allocation of resources. Moreover, search intensity, which is also a rent-sharing device, pushes average wages up and prevents excessive entry by firms. Hence the title of the paper: *efficient wage dispersion.*

The analysis of this environment, where workers observe some but not all of the wage offers, can open the middle ground for the study of the decentralized allocation of heterogeneous workers to heterogeneous firms: if our model is extended to heterogeneous workers (as outlined in section 6), it would avoid the criticism of most search models that meetings are completely random; but it would also not coincide with the Walrasian allocation where a worker is immediately assigned to the firm at which her marginal product is highest. This middle ground is likely to generate a number of insights about decentralized allocations in the presence of heterogeneity.

Our line of research also invites a number of observations of potential empirical importance. Most importantly, small variations in the costs of search that individuals face may lead to important changes in average wages, wage inequality and unemployment. In the search-intensive equilibrium, a lower search cost will lead to higher search intensity, and to a shift in the wage distribution. This shift may lead to more or less wage dispersion, depending on our starting point, but it will certainly lead to higher wages, less entry, and therefore higher unemployment. Also, in the case with heterogeneous firms, it will lead to higher labor productivity. As previously commented, the impact on unemployment is extreme because there is no countervailing improvement in the number of matches due to higher search intensity. Nevertheless it is instructive of the general equilibrium connections between search intensity, average level of wages and job creation. As a result, if Gary Burtless is correct in observing that (1987, p. 149): "*compared with government employment services in Europe, the U.S. is relatively ineffective in aiding and monitoring the search for jobs*," then European workers who face lower search costs will sample more firms; this will lead to higher wages, higher productivity, lower job creation, higher unemployment and possibly lower wage inequality. It would be naive to believe that the large differentials between Europe and the U.S. are explained only by this mechanism; and yet, it also appears plausible that European workers are more 'choosy', and this will naturally have important effects on wage distribution and the composition of jobs (see also Acemoglu, 1996b). Overall, the investigation of the links between search behavior, composition of jobs, and the distribution of wages appears a fruitful and under-researched area for future work.

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