The Effect of Taxation on Labor Supply:
Evaluating the Gary Negative Income Tax Experiment

by

Gary Burtless and Jerry Hausman

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The economic theory of consumer choice derives consumer demands under the assumption of a constant price which is independent of quantity demanded by the consumer. Empirical estimation of consumer demand functions then depends on a functional relationship between the price of a commodity and the amount of the quantity which is consumed. Many actual situations do not conform to these classical assumptions. Totally within the private sector, nonlinear prices may arise in industries with substantial fixed costs. Here average price and marginal price are not equal so the choice of the "correct" price to put in the demand function is not straightforward. Electricity prices are the most important example of this situation. The existence of government income tax and income transfer programs, however, are the primary source of nonlinear consumer prices. Net, after tax, wage rates almost always depend on the number of hours of work supplied. For instance, workers facing a progressive income tax have net wage rates that decline as gross earnings rise. Other important examples are AFDC programs and social security payments for individuals between 65-72 years of age. Here income transfers are accompanied by high marginal tax rates. These programs not only cause prices to be nonlinear, but also cause budget sets to be nonconvex, which further complicates the theory and estimation of labor supply.

Most econometric studies of labor supply assume that the most preferred level of work effort for an individual depends on a single wage rate that is independent of the chosen level of hours of work. Thus the endogeneity of the net wage rate is ignored. When it has been considered, only reduced form estimates have been computed. These estimates have the disadvantage that they depend on the particular sample information used and cannot be used to evaluate the expected effect of policy changes. In this paper we

propose an alternative method of estimating labor supply functions in the
presence of nonlinear net wages. The technique follows from a structural
model of individual labor supply choice when the net wage depends on hours
of work supplied. Thus individual choice depends on all net wages which
comprise the budget set so that policy changes can be evaluated using the
parameter estimates. The model we estimate has one other important dif-
ference from usual models of labor supply. We allow for a distribution of
preferences in the population for the labor-leisure choice. This broad-
ening of the traditional model seems called for by the observed data in
which otherwise identical individuals have widely differing labor supply
choices. Our findings confirm this observation since a very skewed dis-
tribution of preferences is observed.

The model developed here is used to evaluate the effects of a Negative
Income Tax (NIT) experiment in Gary, Indiana. In this experiment, income
transfer payments were made to families on the basis of an income support
level which depended on family size and a tax rate of either 40% or 60%.
Beyond a given number of hours worked, an individual's earnings were taxed
at the usual federal and state tax rates. A complicated budget constraint
resulted which consisted of linear segments connected by kink points. We
estimate the unknown parameters of the uncompensated labor supply function
together with the associated indirect utility function to evaluate the
income and substitution effects of the NIT. An important question con-
sidered is how large are the appropriate income and wage elasticities.
We further consider the likely pattern of response to an NIT in the popula-
tion. The estimates found here can be compared to labor supply response
estimates from other NIT experiments in New Jersey and in Seattle and
Denver. Our findings imply that a NIT has only a very small effect on a

substantial proportion of the population; but that a significant number of individuals' labor supply decisions may be affected quite substantially.

The first section of the paper discusses the problem of nonlinear net wages. It shows how a progressive income tax leads to a convex budget set while government transfer programs like a NIT lead to nonconvex budget sets in which certain choices of hours worked can never be optimum. Thus, the importance of knowing the form of both the labor supply function and the associated indifference curve map is emphasized. In Section 2 we use the modern theory of duality to derive the indifference curve map through the indirect utility function. This utility function is derived from an ordinary specification of labor supply. Restrictions from the theory of consumer demand are derived so that the parameter estimates will not lead to violation of the theory of individual choice. Next we consider utility maximization and consumer choice in the presence of nonlinear net wages. We calculate the preferred point along a budget line at which individual choice jointly determines both hours worked and net marginal wage. To complete the specification of the model, we propose a stochastic specification which allows for both individual variation in preferences and the more traditional deviation between preferred hours of work and actual observed hours of work. Section 4 discusses the operation of the Gary NIT experiment and the actual calculation of individual budget sets. Since some individuals acted as controls, we also consider budget sets of individuals facing only a progressive income tax. Potential data problems are also discussed. In Section 5 we present the results of maximum likelihood estimation of our structural model. While an important income elasticity response is found, no associated uncompensated wage elasticity response seems to be present. Lastly, we discuss the policy implications of the results and indicate possible future research.

## 1. Labor Supply With Nonlinear Net Wages

The economic theory of labor supply is a straightforward application

of utility maximization. Individuals face a given market determined

wage along with prices of other consumption goods. Workers are assumed to

choose the desired amount of hours of work which corresponds to the

most preferred point on their budget sets.[1] In the familiar two-good

diagram of hours supplied and expenditure on other goods, the slope of the

budget set is the normalized wage $w = \tilde{w}/p$ and the intercept $y = \tilde{y}/p$ is

normalized nonlabor income where $\tilde{w}$ and $\tilde{y}$ are the market wage and nonlabor

income, respectively, and we use the price of the consumption good as the

numeraire. In Figure 1.1, $-H^*$ is the point which corresponds to the most

preferred point created by the tangency of the indifference curve to the
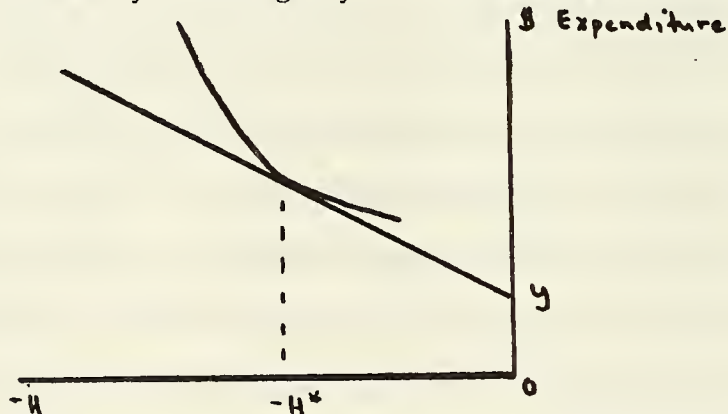
budget set:



Fig 1.1

However, an important shortcoming of this analysis is the failure to

incorporate the fact that the individual faces a nonconsistent net wage,

$w(1-t)$, where t is the marginal tax rate. If t were a constant independent

of labor supplied and y were also independent of $-H$, then the budget line

would simply be rotated counterclockwise. The previous analysis would be

correct using the correctly measured net wage.

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[1] The theory of labor supply is sometimes stated as a theory of leisure
demand given full income, see Becker [1965]. However, we will treat
hours supplied as the variable of interest using a minus sign for hours
worked in the utility function to maintain the usual monotonicity con-
ventions. Also, in the econometric estimation, we will account for the
fact that individuals may not be able to work their desired amount.

However, proportional tax systems are used only at the state and
local tax level so that the analysis must account for nonlinear budget
sets created by progressivity in tax formulae. Thus, the budget set
is piecewise linear, with kinks at points where income rises sufficiently
to put the individual into the next higher tax bracket. The effect of
a progressive tax system is to create a quasiconvex budget set like the
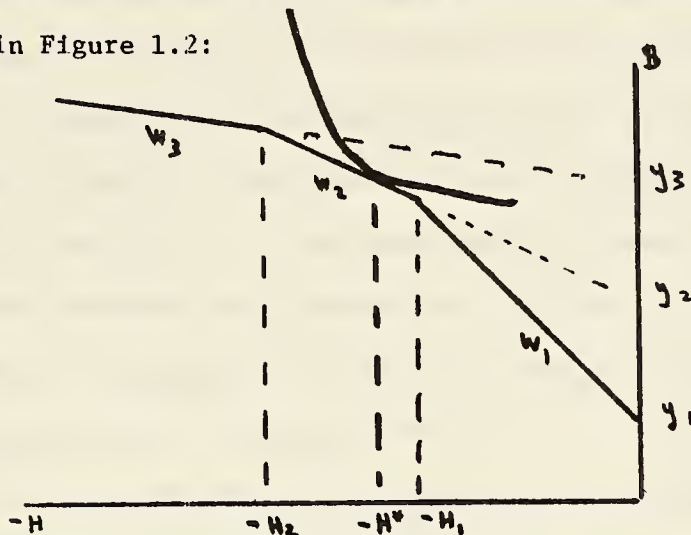one shown in Figure 1.2:



Fig. 1.2

In this highly simplified version of a progressive tax system,
$-H_1$ and $-H_2$ correspond to the kink points induced by the tax system
and $-H*$ is the preferred amount of labor supply. This convex nonlinearity
creates problems for the theory and estimation of labor supply. Theoreti-
cally, the usual comparative statics results must take account of the kink
points and how their location depends on the gross wage. For estimation
the problems are especially severe. Typical labor supply specifications
have the form

(1.1) $$H = g(w,y,z,\epsilon) \ ,$$

where z is a vector of individual characteristics, and $\epsilon$ is a stochastic
term. Within the context of this type of labor supply function it is
not obvious which net wage should be included as the variable explaining
labor supply. Nor is it straightforward to decide which level of nonlabor

income to specify. For instance if the net wage that corresponds to the second segment of the budget set of Figure 1.2, $w_2$, were chosen we might want to use "virtual income", $y_2$, that corresponds to the intercept which equals nonlabor income of the budget set that the individual faces at the margin.[1.]

Hall [1973] noted that a worker can be considered to be facing a linear budget constraint that is tangent to his actual budget set at the observed level of hours of work. For example, the individual facing the budget set drawn in Figure 1.2 and observed to be working H* hours can be considered to be facing a single wage rate, $w_2$, and a single level of virtual income, $y_2$. While this procedure is an important advance over using the gross wage, it cannot yield unbiased single-equation estimates because of the presence of the stochastic term $\varepsilon$ in equation (1.1). Since both the net wage and virtual income are functions of hours worked H, they will be correlated with $\varepsilon$ inducing a simultaneous equation or errors in variables problem into the estimation procedure. In a study of tax response among married women Rosen [1976] attempted to avoid the simultaneity problem by using the slope and intercept of the budget line at 1500 hours of work per year. Considered as an instrumental variables procedure, it is not clear how highly correlated Rosen's measure is with the actual net wage. Hausman-Wise [1976] using time series-cross section data used a more specific instrumental variable, basing their instrument on a prediction of past hours of work. However, this approach is not fully satis-

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[1.] Certainly the gross wage should not be used if substantial divergence exists between the gross wage and the net wage since an upward biased errors in variables problem will result. Hurd [1976] has included all wages in equation (1.1) but this specification can be considered, at best, a reduced form estimate where the estimated values of the parameters depend on the specific budget sets faced by individuals in the sample.

factory since the stochastic term ε is found to be correlated over time.
Another approach is to estimate the probability that an individual's
preferred point is on a particular linear segment, and then to estimate
hours worked as a random variable that is censored at the kink points and
that is a function of the net wage and virtual income for that segment.
Again, this approach is only a reduced form approach since the probability
model parameter estimates depend on the particular budget sets faced by
individuals in the sample and would change with different budget sets.

This problem of a nonlinear budget set would be solved if we had
sufficient knowledge about the form of the utility function. Suppose we
choose a particular form of the utility function and then adopt an additive stochastic specification of the labor supply function. The labor
supply function can be written

$$(1.2) \qquad H = h(w_1, w_2, w_3, y_1, y_2, y_3, z) + \varepsilon$$

where $h(\cdot)$ is the consumer's supply function for labor, derived from
maximizing the individual utility function.[1] Discrepancies from utility
maximization would be represented by ε; but since all the right hand
side variables can be treated as exogenous, the unknown parameters of
the utility function could be estimated. No problems of "which" net
wage or "which" level of nonlabor income to use would arise since they are
fully taken into account in the utility maximization.

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[1] Note that for nearly all utility functions $h(\cdot)$ will be a complicated
function of net wages and nonlabor income. In fact, it will probably
not exist in closed form for nonlinear budget constraints. However, it
can be calculated easily by numerical techniques on a computer. The
actual procedure used will be discussed in Section 3. Heckman [1974]
proposed an alternative procedure in which net wages are estimated as
a function of hours worked. If the form of the budget set is known,
this relationship would be exact so that a random variable, hours worked,
and an exact nonlinear transformation of it appear on both sides of the
equation.

When government programs beyond the progressive income tax are considered, the situation becomes even more complex since the resulting budget set may be nonconvex. Consider the operation of the NIT program that we study empirically in this paper. An income transfer, T, is calculated on the basis of a family's income guarantee, the NIT marginal tax rate, and family income. At the breakeven point, $\tilde{H}$, in Figure 1.3, the income transfer is completely taxed away due to high family earnings, and the earner returns to the federal and state income tax schedules. At this point the net wage rate rises, creating a nonlinearity in the budget set. Not only are four wage rates encountered, but an additional problem
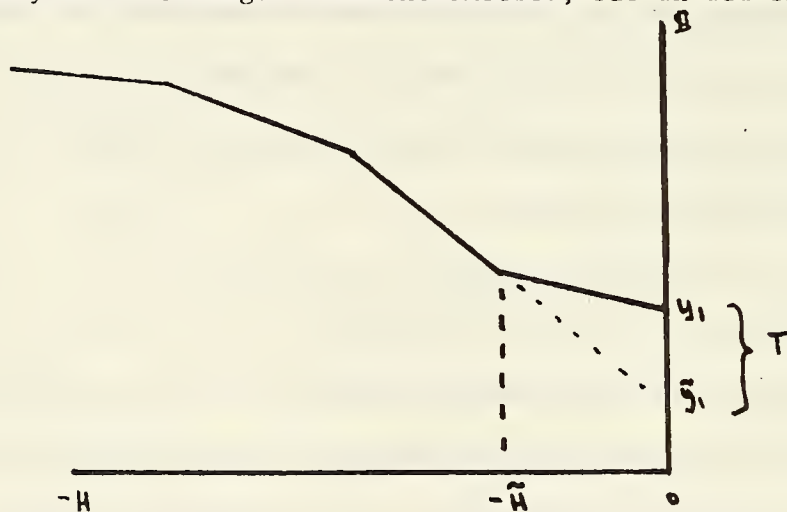


Fig 1.3

arises. Since the breakeven point represents a nonconvex kink point, there exists an interval along the budget line in the vicinity of break-even that may never contain a global maximum if, as is generally assumed, indifference curves are convex. Furthermore, the exact size and location of this interval depends on the specification and unknown parameters of the underlying utility function. Observed hours of work may sometimes fall in this interval if there are errors in optimization or institutional rigidities, but the implied restrictions on globally optimal hours must be taken into account in the estimation of labor supply.

The case of a nonconvex budget set emphasizes the importance of knowledge about the underlying utility function. This type of nonconvexity is encountered not only in the negative income tax, but also in other earnings related taxes or subsidies — for example, child care payments, social security payments for individuals from 62-72 years old, AFDC, and food stamp subsidies. Knowledge about the form of the utility function would permit estimation within the context of equation (1.2) although determination of the utility maximizing point would be more complicated than in the previous case due to the nonconvexity of the budget set.

In this section we have specified the complications in the theory of labor supply which arise when individuals face nonlinear budget sets due to government tax and subsidy programs where the marginal tax rate is determined by earnings. The main problem for econometric purpose is the multiplicity of net wage rates which the individual faces in deciding on his labor supply. We have emphasized how knowledge of the utility function, up to its unknown parameters, would help to solve the problem. Yet all our empirical knowledge arises from observing the uncompensated labor supply function of equation (1.1), since utility is never observed. In the next section we show how knowledge of the uncompensated labor supply function can be used to derive knowledge of the utility function for purposes of econometric estimation.

## 2. Derivation of the Indirect Utility Function

Nonconvexity of the budget set requires specification of a parametric form for the indifference curves to determine the set of points which cannot correspond to utility maximizing behavior. In the case of both convex and nonconvex budget sets, a complete model of consumer behavior requires knowledge of the indifference curve to determine the appropriate prices that the consumer faces. Two possible approaches to the problem are apparent. The direct approach is to specify a form of the direct or indirect utility function and then to derive the consumer demand equations. For example, a Cobb-Douglas utility function could be specified leading to a leisure demand (labor supply) equation which can be estimated. This approach, taken by Burtless [1976], places strong restrictions on the labor supply elasticities.[1] Less restrictive utility specifications such as the second order flexible form utility functions outlined by Diewart [1974] might also be used. However, the flexible form specifications lead to complicated labor supply equations which would be extremely difficult to estimate given a nonlinear budget set.

A second approach, which we will use here, arises from the theory of duality. In the context of consumer demand theory, Roy [1947] did pioneering research including the derivation of the identity relating consumer demand to the indirect utility function.[2] Define the consumers utility maximization problem as maximizing a utility function u(x) where

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[1] Since the first draft of this paper was completed, we have found Wales and Woodland [1977] using a CES utility function. However, they do not permit variation in preferences in the population as Burtless does.

[2] Hotelling, Wold, Samuelson, and Houthakker all made important contributions to the use of duality in consumer demand theory. Other contributions and a review of the theory are found in Diewert [1974].

$x = (x_1, \ldots, x_N)$ an N-dimensional utility function, subject to the budget constraint $p \cdot x \leq y$ where $p = (p_1, \ldots, p_N)$ is the vector of prices and $y$ is the individual's income. Then the indirect utility function relates the maximum utility the consumer can attain, as a function of the exogenous variables $p$ and $y$.[1] The function is determined by the solution to the utility maximization problem

$$(2.1) \qquad\qquad v(p,y) \equiv \max_{x} \, [u(x): \quad p \cdot x \leq y]$$

Because utility is an unobserved variable, estimation can take place only by observing consumer demand. Here Roy's Identity simplifies matters since it relates consumer demand to the indirect utility function by the formula

$$(2.2) \qquad\qquad x_i \equiv - \frac{\partial v(p,y)}{\partial p_i} \Big/ \frac{\partial v(p,y)}{\partial y} \qquad i = 1, \ldots, N \ .$$

We propose to use Roy's Identity "in reverse". That is, considerable empirical knowledge has been built up about labor supply together with the functional forms useful in estimating it. In fact, all our knowledge about the specific form of utility functions, both direct and indirect, must arise from observations of consumer demand. Thus, our approach is to integrate Roy's Identity to derive the form of the indirect utility function rather to specify the utility function a priori. The resulting indirect utility function will be consistent with consumer theory since it is derived using only the assumptions of utility maximization, and consistent with the data to the extent that the specified labor supply function is supported by observed behavior.[2]

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[1] For the present section we are assuming linear prices and a straight line budget set.

[2] Rosen [1974] emphasizes the importance of using the observed consumer behavior to achieve a proper specification of the utility function.

Integration of equation (2.2) over all N goods raises the integrability problem that the function obtained must satisfy restrictions on the Slutsky matrix: rank N-1, symmetry, and negative semi-definiteness. This integration would be difficult and also fruitless since we do not have observations over consumer demand for all goods. Instead, we simplify to a two good case where the goods are labor supply and expenditure on all other goods. This two good approach has been (implicitly) taken in almost all studies of labor supply. In order to aggregate N-1 goods, justifications may be offered. A possible justification arises from assuming homogeneous weak separability of preferences between labor supply and the other N-1 goods. This approach, due to Leontief, seems a reasonable assumption since labor supply differs so much from other consumption goods. It also permits the two good approach to be applied to cross section data where individuals do not face approximately the same prices.

Given a two good model, the integrability problem dissolves. One diagonal element of the two by two Slutsky matrix determines the other 3 elements, since all expenditure not made on labor supply (leisure demand) must go to the remaining good. Thus, the only requirements imposed by the theory of consumer demand are that the compensated labor supply derivative with respect to the wage be greater than or equal to zero and that the indirect utility function be monotone nondecreasing in the wage and in nonlabor income.[1] Once we have estimated the unknown parameters in the indirect utility function we have obtained all the observable information possible about the consumer's indifference map. This information will, however, be sufficient to estimate the labor supply effects of other government tax and subsidy programs.

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[1] The compensated derivative is positive, not negative, since work is supplied while leisure is demanded. These requirements correspond to the quasiconvexity and monotonicity properties of indirect utility functions. For further details see Diewert [1974, pp. 120-133].

To obtain the indirect utility function, we need to specify a model of labor supply. We use the constant elasticity specification because it has been successfully used in other labor supply investigations (Hausman and Wise [1976], Lillard [1977]) and leads to a convenient indirect utility function. The labor supply function is

(2.3)
$$h = kw^{\alpha}y^{\beta}$$

where h is hours worked over the appropriate period, k is a constant determined by individual characteristics, w is the net wage, and y is non-labor income.[1] Using Roy's identity of equation (2.3) with the insertion of a negative sign since labor supply is a "bad"

(2.4)
$$-kw^{\alpha}y^{\beta} = - \frac{\partial v(w,y)}{\partial w} \bigg/ \frac{\partial v(w,y)}{\partial y} .$$

To derive the indirect utility function we use the implicit function theorem

(2.5)
$$kw^{\alpha}dw = -y^{-\beta}dy .$$

Then integrating both sides using the separability of the differential equation where c is the constant of integration[2]

(2.6)
$$k \frac{w^{1+\alpha}}{1+\alpha} = - \frac{y^{1-\beta}}{1-\beta} + c .$$

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[1] Wage and income are both divided by the consumer goods price deflator so we take the composite price of other goods as numeraire.

[2] Separability of the labor supply function in the wage and nonlabor income is the crucial simplification which permits this approach. A more general specification is $h = kr(w)s(y)$. Note that the integration is only done locally over the range of the observed data so that boundary conditions can be ignored.

We choose the constant of integration c as our cardinal measure of utility and rearranging terms leads to the indirect utility function[1].

$$(2.7) \qquad c = v(w,y) = k \frac{w^{1+\alpha}}{1+\alpha} + \frac{y^{1-\beta}}{1-\beta}$$

Equation (2.8) is thus the indirect utility function that corresponds to the constant elasticity labor supply function. The monotonicity properties are satisfied if utility is nondecreasing in w and y.[2] To derive the Slutsky matrix restriction on the indirect utility function, we use the Slutsky equation

$$(2.8) \qquad \frac{\partial h}{\partial w} = s_{ww} + h \frac{\partial h}{\partial y}$$

where $s_{ww}$ is the compensated wage derivative. Upon taking derivatives and simplifying

$$(2.9) \qquad s_{ww} = \frac{1}{h} \left( \frac{\alpha}{w} - \frac{\beta h}{y} \right) .$$

Then the Slutsky restriction $s_{ww} > 0$ implies $\alpha > \beta h w / y$. Taking the expected case of $\beta < 0$,

$$(2.10) \qquad \frac{k w^{1+\alpha}}{y^{1-\beta}} > \frac{\alpha}{\beta} .$$

This quasi-convexity condition is automatically satisfied if $\alpha \geq 0$ as we would expect for a sample of low income males since $k > 0$. If $\alpha < 0$, then we would need to check the observations in the sample to make certain that inequality (2.12) holds.

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[1] In principle the direct utility function may be derived from the indirect utility function by a constrained minimization problem. For the constant elasticity specification a closed form does not exist. However, nowhere is the direct utility function needed since labor supply and the effects of alternative tax and subsidy programs can be calculated solely from the indirect utility function and the expenditure function.

[2] These properties are satisfied globally with the limiting case of $\alpha = -1.0$ and $\beta = +1.0$ corresponding to the Cobb-Douglas specification.

In this section we have derived the indirect utility function corresponding to an ordinary model of labor supply using Roy's Identity. The sign restrictions to insure monotonicity and quasi-convexity of the indirect utility function have also been derived. However, these derivations were based on the classical assumptions of linear prices and a straight line budget constraint. Yet almost all government income tax and income transfer programs create nonlinear prices, leading to convex or nonconvex budget sets. In the next section we demonstrate how to calculate labor supply is determined in the more complicated case using the derived indirect utility function.

## 3.    Nonlinear Budget Sets and Stochastic Specification

A consumer is assumed to select his most preferred level of hours, $H^*$, on the basis of a set of preference parameters $\theta = (k,\alpha,\beta)$. Given values of these parameters, a worker facing a convex budget set of the type pictured in Figure 1.2 or a nonconvex budget set of the type in Figure 1.3 will choose a certain level of labor supply, and this choice may be calculated by using the indirect utility function. Note that direct use of the uncompensated labor supply function is impossible due to the multiplicity of net wages faced by the consumer. In the case of a nonconvex budget set we must also ascertain the range of the interval around the breakeven point that can never contain a utility maximizing choice. Once the utility maximization problem is solved for a single individual, we proceed to a statistical specification that permits differences among individuals to be reflected in differences in the parameters of their indirect utility functions. Given the significant variation observed in hours worked for observationally equivalent individuals, it seems inappropriate to simply add an additive stochastic disturbance to the estimated labor supply function. Allowing for a distribution of preferences for work in the population is therefore an important component of our model of labor supply.
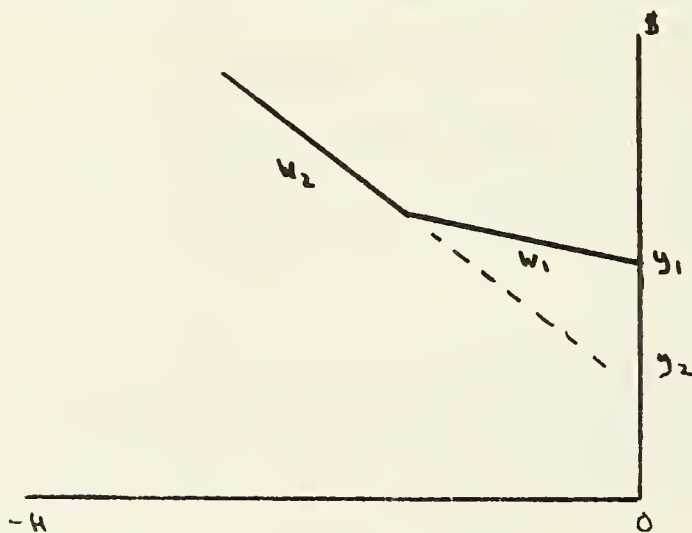


Fig. 3.1

To demonstrate how preferred hours of work H* are derived given
an indirect utility function with parameters $\theta$ when an individual is
faced with a nonlinear budget set, consider Figure 3.1. The first budget
segment is described by the slope, representing the net wage $w_1$, and
the intercept $y_1$, which represents nonlabor income at zero hours work.
Similarly, the second budget segment is described by net wage $w_2$ and by
virtual nonwage income $y_2$. For a given set of preference parameters the
maximum indirect utility on segment one may be calculated as $v_1(w_1,y_1)$
along with associated hours of work $h_1(w_1,y_1)$. This indirect utility may
then be compared to the corresponding maximum indirect utility on segment
two, $v_2(w_2,y_2)$ which has preferred hours of work $h_2(w_2,y_1)$. The maximum
maximorum, $v*(w_1,w_2,y_1,y_2)$, equals the greater of $v_1$ or $v_2$, which in turn
determine the global maximum of hours of work $H*(w_1,w_2,y_1,y_2)$. The case of
the convex budget set of Figure 1.2 is treated in the same manner, and the
extension to an indefinite number of budget segments is immediate.[1] All
that is required is the comparison of maximized utility on each budget
segment, a comparison that is easily made using the indirect utility func-
tion.

Since the goal of empirical work is to estimate the unknown parameters
of the uncompensated labor supply function, we now specify a stochastic
theory of labor supply variation in a cross-section of individuals. In-
dexing individuals by i, we expect random differences to occur between ob-
served hours supplied, $H_i$, and preferred hours of work, $H_i^*$. This random
variation is caused by institutional factors and by measurement error.

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[1] The convex budget set case does differ to the extent that an optimum
at the kink point does not in general correspond to a unique indifference
curve. This problem is solved in equations (3.4) and (3.5).

A worker may not be observed to be working at exactly H* because of the inflexibility of hours of work in most jobs or because hours are not ac- curately measured on the survey questionnaire. These sources of random variation are not affected by the location of the budget sets or by the values of the preference parameters, $\theta$.

Another form of randomness in the data occurs because of individual variation in tastes. Two individuals with the same personal character- istics who face the same budget sets may prefer to work substantially different amounts. From a policy standpoint these individual differences are very important in determining the response to alterations in the budget set induced by government programs. In estimating the unknown parameters k, $\alpha$, and $\beta$, all may be specified to be functions of measurable and unmeasurable individual differences. However, this very general spec- ification leads to an intractable estimation problem. In our empirical work a number of specifications were attempted using an instrumental vari- able estimator for the uncompensated labor supply function. This ex- perimentation suggested that $k_i$ may best be treated as a function of meas- urable individual differences, while $\alpha_i$ and $\beta_i$ are apparently independent of differences in measured personal characteristics. The actual specifica- tion used in this paper is presented, however, with the caveat that sig- nificant additional research is needed.

The constant term in the labor supply function of equation (2.4) is there- fore specified to be a function of both measured and nonmeasured individual differences. Since we also assume that there will be random variation in uncompensated labor supply due to errors in measuring preferred hours, it will be convenient to subsume this random disturbance in our specifica- tion of the constant term. Thus,

$$(3.1) \qquad\qquad k_i = \exp(Z_i \delta + \varepsilon_{2i}) ,$$

where $z_i$ is a vector of individual characteristics and $\varepsilon_{2i}$ is assumed
to be distributed $N(0,\sigma_2^2)$.[1] The two other individual parameters $\alpha_i$
and $\beta_i$ may both be expected to vary in the population. Technically
both distributions are identified so that given ideal data the parameters
of both distributions can be estimated. As a practical matter, estimation
currently seems limited to allowing one of the two parameters to vary.
Thus, we decided to permit either $\alpha_i$ or $\beta_i$, but not both, to vary in
the population. Empirical results led us to specify $\alpha_i = \bar{\alpha}$ a constant
and to specify $\beta_i$ as a random variable in the population.

The integrability conditions discussed in Section 2 impose restric-
tions arising on the distribution of the $\beta_i$. The wage elasticity $\bar{\alpha}$
is expected to be nonnegative in a sample of low income workers. In
addition, there is a strong expectation that the income elasticity is
nonpositive which leads to the integrability inequality of equation (2.12)
to be satisfied globally. A convenient distribution which imposes the
negativity restriction on $\beta_i$ is the truncated normal with the truncation
point at zero. As Figure 3.2 show a wide variety of shapes of probability
densities can be accommodated with this specification. The individual
parameter $\beta_i$ can then be written as $\beta_i = \bar{\beta} + \varepsilon_{1i}$ where $\varepsilon_{1i} \sim TN(0,\sigma_1^2)$



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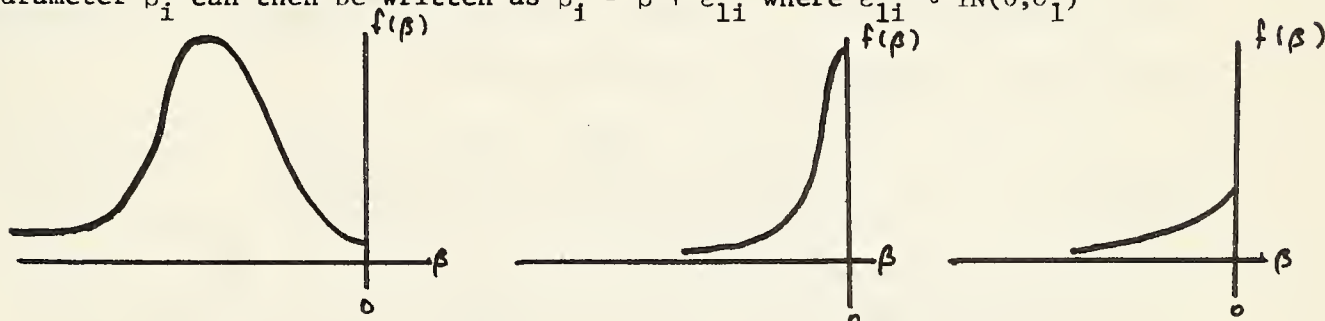
[1] The effect of unmeasured characteristics in determining $k_i$ will be ob-
servationally equivalent to the first source of random variation $\log H_j - \log H_i^*$
which arises from differences of hours supplied from the utility maximizing
point.

with a truncation point from above of $-\bar{\beta}$. We assume that $\varepsilon_{1i}$ and $\varepsilon_{2i}$ are independent sources of random variation. Given this stochastic specification, the unknown parameters of the model are $\theta = (\delta, \bar{\alpha}, \bar{\beta}, \sigma_1^2, \sigma_2^2)$. We now use this stochastic specification to derive the likelihood function for a sample of observations.

The analysis is confined to budget constraints with only two linear segments, although generalization to more segments is straightforward. A control observation faces a convex budget set while an experimental faces a nonconvex budget set of the type drawn in Figure 3.1. The probability of the point actually observed, $H_i$, depends on the unknown parameters $\delta$ and $\alpha$ and the densities for $\beta_i$ and $\varepsilon_{2i}$. Neglecting $\varepsilon_{2i}$ momentarily, let us calculate the probability that a particular point, $H$, is the global maximum. For large negative values of $\beta_i$ the individual will have a global maximum on the first segment; $H*$ the global maximum will be less than the breakeven point, $\tilde{H}$, and the net wage on the margin will be $w_1$ with associated virtual income $y_1$. As $\beta_i$ increases toward zero the global maximum point moves along the first segment toward $\tilde{H}$, until a critical $\beta_i$ is reached at which the individual is indifferent between a solution on the first segment and a solution on the second segment. This critical $\beta_i$, say $\beta_i^*$, depends on the underlying utility function and on the unknown parameters in that function. Using equation (2.8), $\beta_i^*$ is calculated quite easily by solving the following equation:

$$(3.2) \qquad \frac{e^{z_i \delta} w_{1i}^{1+\bar{\alpha}}}{1+\bar{\alpha}} + \frac{y_{1i}^{1-\beta_i^*}}{1-\beta_i^*} = \frac{e^{z_i \delta} w_{1i}^{1+\bar{\alpha}}}{1+\bar{\alpha}} + \frac{y_{2i}^{1-\beta_i^*}}{1-\beta_i^*} .$$

For every experimental observation, this equation must be solved for $\beta_i^*$ each time the parameters change; however, the solution may be cheaply obtained on a computer. As $\beta_i$ rises from $\beta_i^*$ to its limiting value,

the global maximum moves upward along the second segment. Each value of $\beta_i$ between $-\infty$ and zero therefore has an associated global maximum level of hours; the probability that a particular level of hours is a global maximum is the same as the probability of the associated income elasticity, $\beta_i$. We may now extend the analysis to observed hours of work by noting the relationship between observed hours, $H_i$, and desired hours of work effort, $H_i^*$,

$$(3.2a) \qquad \log H_i = \log H_i^* + \varepsilon_{2i} \ .$$

For any particular $H_i$, there are an infinite number of combinations of $H_i^*$ and $\varepsilon_{2i}$ that satisfy (3.2a). By successively determining, for every possible $H_i^*$, the probability that the global maximum is $H_i^*$ and the stochastic term $\varepsilon_{2i}$ equals the difference between the logs of $H_i^*$ and $H_i$, we can ascertain the probability that actual hours, $H_i$, will be observed. Letting $f(\beta)$ be the truncated normal density with associated distribution $F(\beta)$, and $\phi(\cdot)$ and $\Phi(\cdot)$ be the standard normal density and distribution, respectively, the probability of observing $H_i$ is

$$(3.3) \qquad PNC_i = \int_{-\infty}^{\beta_i^*} \frac{1}{\sigma_2} \phi \left( \frac{\log H_i - \log H_{1i}^*}{\sigma_2} \right) f(\beta) d\beta$$

$$+ \int_{\beta_i^*}^{0} \frac{1}{\sigma_2} \phi \left( \frac{\log H_i - \log H_{2i}^*}{\sigma_2} \right) f(\beta) d\beta$$

where $\log H_{ij}^* = Z_i \delta + \bar{\alpha} \log w_{ij} + \beta \log y_{ij}$ where $j$ is an index of the budget segment. Evaluation of these integrals is equivalent to evaluation of a normal distribution $\Phi(z)$ and is thus inexpensive to perform on a computer.[1] The truncated density for $\beta$ poses no problem since it is a

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[1] See the appendix for derivation of the evaluation procedure for the integrals.

normal density divided by a standard normal distribution which remains constant across all observations.

The case of a convex budget set is slightly different because there exists a range for $\beta$, say $BL_i$ to $BU_i$, for which the utility maximizing point is at the kink point $\tilde{\tilde{H}}$ in Figure 3.3. If the individual's $\beta_i$ is con-
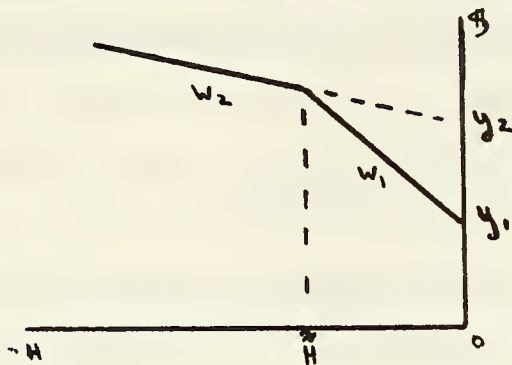


Fig 3.3

siderably less than zero, the global maximum $H_i^*$ will be on the first segment with associated net wage $w_1$ and nonlabor income $y_1$. If his income elasticity is quite near zero, the utility maximum will lie along the second segment corresponding to net wage $w_2$ and virtual and nonlabor income $y_2$. The range of $\beta_i$ that places the utility maximum at the kink point is easily computed from the uncompensated labor supply function. The lower point of the range, $BL_i$, is the greatest $\beta_i$ on the first budget segment that leads to a utility maximum at the kink point

$$(3.4) \qquad BL_i = \frac{\log \tilde{\tilde{H}}_i - Z_i \delta - \bar{\alpha} \log w_{1i}}{\log y_{1i}} .$$

Correspondingly, the upper point of the range, $BU_i$, is the smallest $\beta_i$ consistent with a global maximum on the second segment, and therefore

$$(3.5) \qquad BU_i = \frac{\log \tilde{\tilde{H}}_i - Z_i \delta - \bar{\alpha} \log w_{2i}}{\log y_{2i}} .$$

All $\beta_i$'s that lie between $BL_i$ and $BU_i$ thus lead to a utility maximum at the kink point, $\tilde{\tilde{H}}_i$. The probability that the observed level of hours

$H_i$ corresponds to a utility maximum at the kink point is

$$(3.5) \qquad \text{pr}(\log H_i^* + \log \varepsilon_{2i} = \log H_i \,|\, \log H_i^* = \log \tilde{\tilde{H}}_i) \cdot \text{pr}(\log H_i^* = \log \tilde{\tilde{H}}_i) =$$

$$\frac{1}{\sigma_2} \, \phi\left(\frac{\log H_i - \log \tilde{\tilde{H}}_i}{\sigma_2}\right) \int_{BL_i}^{BU_i} f(\beta) d\beta \ .$$

For observed hours of work $H_i$ corresponding to $\beta_i$'s outside the range $BL_i$ to $BU_i$, the probabilities are similar to those calculated in equation (3.3). Thus, for the case of a convex budget set the probability of observing actual hours worked $H_i$ is

$$(3.6) \qquad PC_i = \int_{-\infty}^{BL_i} \frac{1}{\sigma_2} \, \phi\left(\frac{\log H_i - \log H_{1i}^*}{\sigma_2}\right) f(\beta) d\beta$$

$$+ \frac{1}{\sigma_2} \, \phi\left(\frac{\log H_i - \log \tilde{\tilde{H}}_i}{\sigma_2}\right) \left[F(BU_i) - F(BL_i)\right] + \int_{BU_i}^{0} \frac{1}{\sigma_2} \, \phi\left(\frac{\log H_i - \log H_{2i}^*}{\sigma_2}\right) f(\beta) d\beta$$

Given our stochastic specification of the model, we are able to specify the probability of observing actual hours worked as a function of the unknown parameter values. The natural method of estimation is then maximum likelihood estimation, in which the unknown parameter values are chosen so as to maximize the probability of observing the sample. Our method can be extended, in principle, to cover the case of an arbitrarily large number of budget segments per individual although this extension will not be undertaken here. In the next section we apply our methodology to evaluate the results of the Gary Income Maintenance Experiment.

4.    Calculation of Budget Sets, Data, and Sample Considerations

We use the labor supply model to estimate a structural model of labor

supply for adult married males who participated in the Gary Income Main-

tenance experiment.  The experiment, which took place from 1971 to 1974, had

as participants residents of low income neighborhoods in Gary, Indiana.

All participants were black.  The families were not chosen for the experiment

at random, a problem which we will discuss later in this section.  Participants

in the Gary experiment were randomly assigned to one of four NIT plans or to

control status.  (Control families received no benefits except a small pay-

ment for their continued participation.)  Each of the NIT plans can be

described in terms of two parameters:  the constant marginal tax rate and

the basic support level.  In two of the plans, wage and nonwage income was

subject to a 40 percent tax rate; in the remaining two, income was taxed

at a 60 percent rate.  Two of the Gary NIT plans offered basic income supports,

scaled according to family size, that were equal to slightly more than the

poverty level.  The other two plans offered basic supports, also scaled to

family size, that were one-quarter less.  All federal, state, and F.I.C.A.

income tax liabilities were fully reimbursed for income up to the breakeven

point $\tilde{H}$.  Earned income above the breakeven point was taxed according to

the federal, state, and F.I.C.A. tax tables.

Thus for individuals eligible for NIT payments, the intercept $y_1$ in

Figure 3.1 equals the NIT income guarantee plus net (after tax) nonwage

income.  The slope of the first budget segment, $w_1$, is determined by the

worker's gross wage rate times one minus the experimental NIT tax rate.

For the second segment of the NIT budget line, the virtual income intercept,

$y_2$, and net wage, $w_2$, are calculated in the same manner as the second

segment of a control individual's budget set, the calculation of which we

now describe.

Control families are assumed to face a budget line with only two linear segments. This assumption results in a substantial simplification in budget lines, since the federal income tax schedule has a large number of kinks at lower levels of taxable income. However, low income families face only one important kink in this schedule, one which occurs at the point where family exemptions and deductions are equal to countable family income. At that point the federal tax rate rises from 0% to 14%. Thereafter, the tax rate changes in relatively small steps. In calculating the budget lines for control individuals, we assume that the marginal tax rate along the first segment equals 5.85% for F.I.C.A. plus the 2% Indiana state income tax rate. The second budget segment is calculated on the assumption that workers face an additional 18% marginal tax rate because of the federal income tax. The kink point $\tilde{H}$ is calculated by assuming that workers took standard income exemptions and used the low-income tax deduction available in 1973. Nonwage income is assumed to be nontaxable.

Data on workers' hours, wages, nonwage income, and personal characteristics were taken from the first, fourth, and seventh of the periodic interviews administered to participants in the experiment during the period of NIT payments. To be included in the sample, workers must have responded to at least two of the three interviews.[1] Since we are interested in long-run labor supply response, the measure of labor supply is an average of working hours in the three representative weeks during the experiment.[2]

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[1] Since approximately 35% of the individuals dropped out of the experiment with attrition of controls 10% higher than attrition for experimentals, a problem of attrition bias might occur. However, Hausman and Wise [1977a] in a study of possible attrition bias on this sample concluded that while it is serious for analysis of variance models, it does not pose a problem for structural models which control for individual characteristics and experimental design parameters.

[2] A more long-run measure of labor supply is desirable, but such data are not available at the present time.

Individuals not observed to be working are omitted from the current sample. This feature of the labor supply model may be modified by specifying and estimating a separate behavioral equation for an individual's participation in the labor force as Heckman [1974] and Hanoch [1976] have done for women. This extension is not undertaken here since the sample consists of prime-age males who are heads of household. The proportion of the sample that does not participate in the labor force in this sample is quite small; and presumably for a sample of average yearly hours worked, the number of nonparticipants would decline even further. Nevertheless, we intend to add the behavioral equation for labor force participation in future work on average yearly labor supply when such data become available.

A problem referred to in passing is that initial sample selection was not random but was based on current earnings. Thus the possibility of substantial bias exists since hours worked is one component of earnings.[1] Sample truncation did not occur in Gary, but families whose earnings are above 2.4 times the poverty limit were undersampled by a factor of three. Note, that the cutoff line was substantially higher in Gary than in New Jersey so that even if total truncation had occurred, the effect on the conditional mean would be less.[2] In estimating the effects of the Gary NIT we used the consistent weighted estimator proposed by Hausman and Wise [1977b]. Since the results differed only slightly from the nonweighted estimates, in discussing our results we will present only the nonweighted estimates. Apparently, the combination of the high cutoff line of 2.4

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[1] In fact, in the New Jersey Negative Income Tax Experiment where sample truncation occurred at 1.5 times the poverty limit Hausman and Wise [1976, 1977] found that the ratio of estimated coefficients rose by a factor as high as 200% when sample truncation was accounted for.

[2] Hausman and Wise [1977a, 1977b] propose and estimate a maximum likelihood estimator which accounts for the nonrandom sample selection. However, they find little indication of bias in an earnings equation.

times the poverty limit and the presence of an undersampled group above the cutoff led to little or no truncation bias.

In this section we have discussed computation of the nonlinear budget set for each individual pointing out how marginal tax rates are computed. We then discussed the sample used from the Gary Income Maintenance Experiment as well as possible biases resulting from attrition bias, nonlabor force participation, and truncation bias. In the next section we present the specification of the individual intercept $k_i$ in the uncompensated labor supply function as a function of individual characteristics. We then present and discuss our results, concluding with policy implications and ideas for future research.

## 5.   Results

Both the uncompensated labor supply function, equation (2.4), and the associated indirect utility function, equation (2.8), include the unknown parameters k, $\alpha$, and $\beta$.   It will be recalled that $\alpha$ and $\beta$ are assumed to be independent of individual characteristics while k is specified to depend on these characteristics through $k_i = \exp(Z_i\delta + \varepsilon_{2i})$.   A wide variety of personal characteristics may affect tastes for work; the following list was chosen by reference to earlier research on the NIT experiments:

Constant

Education:   A dummy variable is used for individuals whose educational attainment is less than nine years.

Number of Adults:   Number of persons aged 16 or more residing in the household.

Poor Health:   A dummy variable is used if the individual reported his health to be "poor in relation to others" and zero otherwise.

Age:   A variable equal to the age of the respondent minus 45 years was used if this age exceeded 45 years.   Otherwise the age variable was set to zero.   The other unknown structural parameters of the model are the wage elasticity $\bar{\alpha}$ and parameters in the distribution of the income elasticity $f(\beta)$.

The sample consists of 380 individuals assumed to be independent observations.   Once the budget lines have been determined for each individual, the unknown parameters can be estimated by the method of maximum likelihood.   The log likelihood equals the sum of the logs of the probabilities of actual hours worked by the NIT-eligible individuals, log $PNC_i$, from equation (3.3), and the logs of the probabilities of actual hours

worked by the control individuals, $\log PC_i$, from equation (3.6). Thus the log likelihood function has the form

(5.1)
$$L = \sum_{i=1}^{N_1} \log PNC_i + \sum_{i=1}^{N_2} \log PC_i$$

where the number of experimentals $N_1$ and controls $N_2$ equals 247 and 133, respectively. Approximately 65 percent of the sample was eligible for the NIT. Because of technical reasons discussed in the appendix, the likelihood function was maximized using the gradient method of Berndt, Hall, Hall, and Hausman [1974] as well as the no derivative conjugant gradient method of Powell [1964]. Both techniques converged to the same maximum of the likelihood function. A variety of starting points converged to the same optimum leading us to conclude we have found the global maximum.

Results are presented in Table 1. All the elements of $k_i$ believed to affect tastes for work have the expected effects. Poor health reduced expected labor supply by 2.25% while a 60 year old is expected to work 12% less due to his age, other things equal. Increased family size, on the other hand, is related to higher levels of expected work effort which leads to the conclusion that endogeneity of nonwage income is probably not a serious problem. Moreover, the effect of relatively low levels of educational attainment is in the expected direction under the assumption that more educated workers have a wider variety of activities to pursue in their nonwork time. The estimates of these parameters are relatively precise, except for the effect of increased age; all except the coefficient of age are significantly different from zero at the 5% level.

The parameter estimates most important to the design of a negative income tax are the ones that measure work response to the level of the income guarantee and to the marginal tax rate. Our first finding is the

Table 1

| Variable | Parameter Estimates (Asymptotic Standard Errors) |
| --- | --- |
| Constant | 3.75043 (.02555) |
| Primary Education | .01078 (.00558) |
| Number of Adults | .03300 (.01272) |
| Poor Health | -.02224 (.00438) |
| Age | -.00869 (.01347) |
| Wage Elasticity, $\bar{\alpha}$ | .00003 (.01632) |
| Mean Income Elasticity, $\bar{\beta}$ | -.04768 (.00465) |
| Variance of $\beta$ distribution, $\sigma_1^2$ | .06751 (.00399) |
| Variance of $\varepsilon_{2i}$, $\sigma_2^2$ | .00135 (.00022) |

Number of Observations = 380
Log of the Likelihood Function = -196.27

lack of a perceptible effect on labor supply of variations in the NIT

tax rate. The estimated wage elasticity is .00003 and even at a range of

two standard errors is less than .04 in magnitude.[1] This estimate

is well below the Hausman-Wise [1976] estimate for white males in the

New Jersey NIT. This finding is consistent, however, with the Hausman-Wise

[1977a] findings for the same Gary sample. Using a reduced form earnings

specification, they found the labor response to the level of the income

guarantee to be much more important than the response to the marginal tax

rate.[2] While no direct effect of different marginal tax rates is found

here, an indirect effect is present through the effect of taxes on a

family's nonwage income. Consider two individuals with identical gross

wage rates and nonwage income who are offered identical NIT income guaran-

tees but have different NIT tax rates. They will have substantially dif-

ferent budget sets because the tax rate affects the locus of the breakeven

point $\tilde{H}$ in Figure 3.1. The individual with the lower tax rate is more

likely to work less than the breakeven level of hours and is therefore

likely to respond to a higher level of nonwage income than the individual

with the higher NIT tax rate. Nonetheless, the finding of essentially

zero wage elasticity leads to the conclusion that the wide variation in

after tax wage rates had little effect on labor supply among the black

males in the Gary NIT experiment.

On the other hand, the estimate of the income elasticity was found

to be quite significantly different from zero. The average income elasticity

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[1] Note that the integrability condition of equation (2.10) is satisfied
in the sample. When the distribution $f(\beta)$ was not truncated at zero,
the estimate of $\bar{\alpha}$ was .0305 although the estimate was not very precise.

[2] In fact, for low levels of income guarantee Hausman-Wise found the re-
sponse to a higher marginal tax rate among experimental individuals to be
in the wrong direction, although the response was only estimated very
imprecisely.

in the sample is estimated at -.04768. To assess the effect of the income guarantee under the NIT, consider a family of four with one other adult present with the worker in good health, under 45 years of age, and with a ninth grade education (all near the mean of the sample). Using our estimates, the worker's expected preferred hours of work at a pretax wage of $3.50 per hour with the (convex) income tax budget set are 39.943 hours per week.[1] For comparison to a NIT plan, we assign an income guarantee of $3500 and a marginal tax rate of 60 percent. Preferred hours of work fall to 36.985, a change of 2.958 hours per week or 7.69 percent.[2] Weekly earnings under the NIT rise to $119.07 from after tax earnings of $115.54 without the NIT. Thus, a significant work response to introduction of a NIT is found to exist, although its magnitude is not especially large. The effects of other NIT plans may be estimated in a similar manner, averaging over different family characteristics to find the average population response.

Since a distribution of income elasticities was estimated for the population, it is interesting to consider the estimated density $f(\beta)$. As Figure 5.1 shows the truncated normal distribution consists of the extreme left tail of a regular normal distribution.
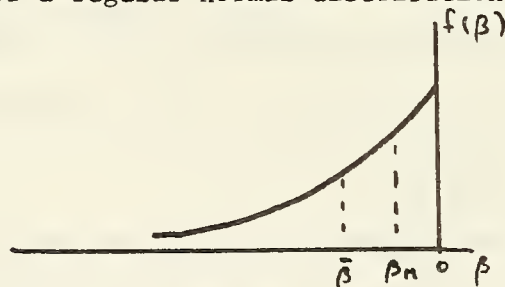


Fig 5.1

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[1] It is important to take the expectation with respect to $f(\beta)$ rather than at the mean $\bar{\beta}$ due to the skewness of the $f(\beta)$ distribution. Thus, we calculate $EH^* = \int H^*(\beta)f(\beta)d\beta$.

[2] This estimate is approximately equal to earlier estimates for the total response in the Gary sample by Kehrer et al. [1976] and by Hausman and Wise [1977c] who found an average response of 7.97 percent among NIT individuals.

Thus while the mean $\bar{\beta} = -.04768$ the median $\beta_M = -.03331$ which means

a substantial proportion of the population had a very low income elasticity.

In fact, about 20% of the population is estimated to have an income

elasticity of between zero and 1%. Since the variance of $\varepsilon_{2i}$ is very small

compared to the variance of the $\beta$ distribution, we can conclude that most

of the observed variation in response to the NIT experiment results from

differences in individual preferences rather than from random difference

between the utility maximum and observed hours of work. Given this con-

clusion, we can interpret the estimate of the $\beta$ distribution as suggesting

that a small proportion of the Gary sample is substantially more responsive

to the presence of an income guarantee in making their labor supply deci-

sion than is the rest of the population. This pattern of response indicates

that most individuals will vary their labor supply very little in response

to the introduction of NIT plans similar to those plans used in the Gary

experiment. A few individuals, however, will react with large reductions

in labor supply. From estimates in the Gary sample, this responsiveness

seems to be the result of increases in nonwage income rather than increases

in the result of the marginal tax rate on earned income. A possible ex-

planation of this result is that some individuals take an increased amount

of time in between jobs if they have an income guarantee. They do not

search and find jobs with higher wages since the wage distribution re-

mains virtually identical for control individuals and NIT individuals.

Thus the income effect is much more important than the uncompensated

wage effect in determining the response to introduction of a NIT.

## 6. Conclusion

Given estimates of the unknown parameters in the uncompensated labor supply function of equation (2.4) and the associated indirect utility function of equation (2.8), we could, in principle, do an applied welfare economics analysis in designing a Negative Income Tax to maximize various welfare measures subject to a budget constraint. However, since our estimate of $\alpha$ is zero, the derivative of the indirect utility function with respect to changes in the marginal tax rate is simply a constant proportional to the tax change since no labor supply response is expected. Nor is the estimated income response very high for most individuals. Thus, we might more simply conclude that within the range of guarantees and marginal tax rates considered in the Gary NIT experiment, the combination of a high guarantee and a high tax rate would lead to the fulfillment of one goal of a Negative Income Tax, which is to provide a basic level of income support at the poverty line, without at the same time causing payments to be made to families with relatively high levels of earnings or causing a substantial reduction in population labor supply.

Considerable future research is desirable in estimating the effect on labor supply of government programs which create nonconvexities in the budget set. These programs may induce large distortions on individual economic activity, and the size of this effect is an important consideration in evaluating such programs. The type of model developed here can be extended to cover a wide variety of such situations.

Appendix

Evaluation of the log likelihood function requires evaluation of the integrals in equation (3.3) for nonconvex budget sets and equation (3.6) for convex budget sets. Two types of integrals are present. The more complicated integral has the form

(A.1)
$$\int_{-\infty}^{\beta_i^*} \frac{1}{\sigma_2} \, \phi\left(\frac{\log H_i - Z_i\delta - \alpha \log w_{1i} - \beta \log y_{1i}}{\sigma_2}\right) f(\beta)\,d\beta \ .$$

The truncated normal density has the form

(A.2)
$$f(\beta) = \frac{\phi\left(\frac{\beta - \mu_\beta}{\sigma_\beta}\right)}{\sigma_\beta\left[1 - \phi\left(\frac{\mu_\beta}{\sigma_\beta}\right)\right]}$$

where $\mu_\beta$ and $\sigma_\beta$ are the parameters of the corresponding untruncated distribution. The standard normal density in the numerator can be combined with the other normal density in the integral. To combine the two densities note that without truncation $\log H_i$ is distributed normally with mean $x_i\delta + \alpha \log w_{1i} + \mu_\beta \log y_{1i}$ and variance $\sigma_\beta^2(\log y_{1i})^2 + \sigma_2^2$. Now considering the joint distribution of $\beta$ and $\log H_i$, we write it as

(A.3)
$$f(\beta, \log H_i) = f(\beta|\log H_i)f(\log H_i)$$

where $f(\cdot)$ stands for the appropriate density. The conditional density $f(\beta|\log H_i)$ (without truncation) is districuted normally with conditional mean $\mu_\beta + [\sigma_\beta^2 \log y_{1i}]/[\sigma_\beta^2(\log y_{1i})^2 + \sigma_2^2](\log H_i - Z_i\delta - \alpha \log w_{1i} - \mu_\beta \log y_{1i})$ and conditional variance $\sigma_\beta^2\sigma_2^2/[\sigma_\beta^2(\log y_{1i})^2 + \sigma_2^2]$. Using equation (A.3) and equation (A.2) to simplify the integral of equation (A.1) and evaluating it yields

$$(A.4) \qquad \frac{1}{\left[1 - \Phi\left(\frac{\mu_\beta}{\sigma_\beta}\right)\right]} \int_{-\infty}^{\beta_1^*} f(\varepsilon_2|\beta) f(\beta) d\beta =$$

$$\frac{1}{\left[1 - \Phi\left(\frac{\mu_\beta}{\sigma_\beta}\right)\right]\sqrt{\sigma_\beta^2\tilde{y}^2 + \sigma_2^2}} \phi\left[\frac{\log H_i - Z_i\delta - \bar{\alpha}\tilde{w} - \mu_\beta\tilde{y}}{\sqrt{\sigma_\beta^2\tilde{y}^2 + \sigma_2^2}}\right] \Phi\left[\frac{(\beta_1^* - \mu_\beta)\sqrt{\sigma_\beta^2\tilde{y}^2 + \sigma_2^2}}{\sigma_\beta\sigma_2}\right.$$

$$\left. - \frac{\sigma_\beta\tilde{y}(\log H_i - Z_i\delta - \alpha\tilde{w} - \mu_\beta\tilde{y})}{\sigma_2\sqrt{\sigma_\beta^2\tilde{y}^2 + \sigma_2^2}}\right].$$

where $\tilde{w} = \log w_{1i}$ and $\tilde{y} = \log y_{1i}$. The somewhat formidable expression on the right hand side of equation (A.4) is quite simple to evaluate, requiring evaluation of one normal density and two normal distributions where the distribution in the denominator remains constant across observations. The only other type of integral appears as the middle term in the convex budget set probability of equation (3.6). It is easily evaluated as

$$(A.5) \qquad \frac{1}{\sigma_2} \phi\left[\frac{\log H_i - \log \tilde{\tilde{H}}_i}{\sigma_2}\right]\left[F(BU_i) - F(BL_i)\right] =$$

$$\frac{1}{\sigma_2\left[1 - \Phi\left(\frac{\mu_\beta}{\sigma_\beta}\right)\right]} \phi\left(\frac{\log H_i - \log \tilde{\tilde{H}}_i}{\sigma_2}\right)\left[\Phi\left(\frac{BU_i - \mu_\beta}{\sigma_\beta}\right) - \Phi\left(\frac{BL_i - \mu_\beta}{\sigma_\beta}\right)\right].$$

Two techniques were used to maximize the likelihood function. Convergence was obtained using the modified scoring method proposed by Berndt, Hall, Hall, and Hausman [1974]. Only first derivatives are required for this algorithm. However, as a check to make certain that the global maximum was achieved, the no derivative conjugate gradient

routine of Powell [1964] was also used to verify the parameter estimates.
The reason for this caution is that while the log likelihood function of
equation (5.1) is everywhere differentiable in the parameters, the deriva-
tives are not everywhere continuous because of the kink point.  While
proofs of the usual large sample properties of maximum likelihood were not
attempted in this nonregular case, consistency of the estimates would
follow from the usual type of proof.  However, proof of asymptotic nor-
mality of the estimates is complicated by the lack of continuous deriv-
atives, and the reported asymptotic standard errors should be interpreted
with this problem in mind.  Starting values for the maximum likelihood
programs were estimated using an instrumental variable technique to predict
the net wage and nonlabor income at the sample mean of 35 hours of work.

One last econometric note concerns the question of whether the gross
wage should be treated as endogenous.  Many studies in the past have
treated it as endogenous, but the reasons advanced for the usual spec-
ification of a triangular system of a wage equation excluding hours in
addition to the hours equation are not present here since we use the
appropriate net wages.  Previous studies used only one net wage and since
the level of utility maximizing labor supply is observed with error the
single net wage rate is also observed with an error that is correlated with
the error between actual and preferred hours.  Thus a simultaneous equation
problem existed.  Here since all appropriate net wages are observed and
used in the labor supply specification the main cause of the simultaneous
equation problem will not occur.  Nevertheless, in our preliminary in-
vestigations with the Gary data we did specify and estimate such a system
making the gross (market) wage a function of personal characteristics.  The
results of the joint estimation were similar to those obtained when the

gross wage was taken to be exogenous, and a specification error test of

Hausman [1976] failed to reject the null hypothesis that the market wage

could be treated as exogenous and measured without significant error.

Thus, simultaneous equation estimation results are not presented in the

paper.

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