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**FUTURE RENT-SEEKING AND CURRENT  
PUBLIC SAVINGS**

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Working Paper 08-20  
October 9, 2008

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Cambridge, MA 02142

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# FUTURE RENT-SEEKING AND CURRENT PUBLIC SAVINGS

Ricardo J. Caballero and Pierre Yared\*

This draft: October 9, 2008

## Abstract

The conventional wisdom is that politicians' rent-seeking motives increase public debt and deficits. This is because myopic politicians face political risk and prefer to extract political rents as early as possible. An implication of this argument is that governments will under-save during a boom, leaving the economy unprotected in the event of a downturn. This view motivates a number of fiscal rules which are aimed at cutting deficits and constraining borrowing so as to limit the size of this political distortion. In this paper we study the determination of government debt and deficits in a dynamic model of debt which characterizes political distortions. We find that in our model the conventional wisdom always applies in the long run, but only does so in the short run when economic volatility is low. Instead, when economic volatility is high, a rent-seeking government *over-saves* and *over-taxes* along the equilibrium path relative to a benevolent government. Paradoxically, the over-saving bias can also be solved in this case by a rule of capping deficits, although the mechanism operates through its effect on expectations of future rent extraction rather than through the contemporary constraint. However, these rules are ineffective in solving the high taxation problem caused by the political friction, which in the short run is more acute in the high income volatility scenario.

**JEL Codes:** E6, H2, H6

**Keywords:** Public debt, politicians, economic and political risk, rent-seeking, precautionary savings, starve-the-beast, fiscal rules

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\*MIT and NBER, and Columbia University, respectively. We are grateful to Stefania Albanesi, Marco Battaglini, Patrick Bolton, V.V. Chari, Tito Cordella, Michael Golosov, Simon Johnson, Narayana Kocherlakota, Jose Tessada, and Aleh Tsyvinski for comments. Caballero thanks the NSF for financial support. First draft: November 2007. This paper replaces NBER WP #13379 which circulated under the title of "Inflating the Beast: Political Incentives under Uncertainty."



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# 1 Introduction

The conventional wisdom is that the rent-seeking motives of politicians increase public debt and deficits. This is because myopic politicians face political risk and prefer to extract rents as early as possible. An implication of this argument is that governments will under-save during a boom, leaving the economy unprotected in the event of a downturn.<sup>1</sup> This view is not only of theoretical interest, but it motivates a number of fiscal rules in the world which are aimed at cutting deficits and constraining borrowing so as to limit the size of this political distortion.<sup>2</sup>

In this paper we study the determination of government debt and deficits in a dynamic model that characterizes political distortions.<sup>3</sup> In a nutshell, we find that the conventional wisdom always applies in the long run, but only does so in the short run when economic volatility is low. In contrast, the conventional wisdom does not hold in the short run when economic volatility is high since politicians choose public debt and deficits which are *too low*. Paradoxically, the over-saving bias can also be solved in this case by a rule of capping deficits, although the mechanism operates through its effect on expectations of future rent extraction rather than through the contemporary constraint. However, these rules are ineffective in solving the high taxation problem caused by the political friction, which in the short run is more acute in the high income volatility scenario.

More specifically, we study an economy managed by a sequence of politicians who face political risk and who care about household welfare and rents. In contrast to the previous work on the political economy of debt, we consider the interrelated implications of three important features: economic uncertainty, incomplete markets, and transitional dynamics. The economy begins in a boom, and this boom can come to a permanent end at any date. Throughout the length of the boom, the benevolent government gradually reduces its debt in order to prepare for the potential downturn. We compare this optimal behavior to that of a rent-seeking government managed by politicians.

Our first result is that while a rent-seeking government reduces its debt at the beginning of the boom, it stops reducing its debt if the boom is sufficiently prolonged.

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<sup>1</sup>See Battaglini and Coate (2008) and the survey article of Alesina and Perotti (1994) for a discussion of this view.

<sup>2</sup>Chile provides a recent example which has become a reference for fiscal reforms in Latin America and commodity producing economies more broadly. The fiscal rule establishes a structural (i.e., at “normal” terms of trades) surplus of 0.5 percent of GDP. Thus, when terms of trade rise as a result of a commodity boom, the state runs very large fiscal surpluses (the sum of the structural surplus target plus the excess fiscal income due to high commodity prices).

<sup>3</sup>Acemoglu, Golosov, and Tsyvinski (2007a, 2007b) also study the effect of political economy distortions on taxes, though they do not consider the effect on government debt.

This is because beyond a certain date, government resources become so abundant that rent-seeking considerations come to dominate intertemporal smoothing considerations. A rent-seeking government realizes that if it were to save more, then a future replacement government would use the additional funds for rent-seeking (which only benefits incumbent politicians) as opposed to tax-cutting (which benefits households), and the government therefore restrains its savings in order to starve the future government of funds. Therefore, in the long run, a prolonged boom always leads a benevolent government to hold more assets and to tax less than a rent-seeking government. This result is consistent with that emphasized by Battaglini and Coate (2008). Our main contribution is to show that while this characterization applies to the long run fairly generally, whether or not it applies to the transitional dynamics of the economy depends on the level of economic volatility.

Our second result is that if economic volatility is sufficiently low relative to political uncertainty, then the rent-seeking government *over-borrows* and *under-taxes* along the equilibrium path relative to a benevolent government. This insight—which is consistent with the conventional wisdom—emerges because low economic volatility implies that politicians are biased toward extracting rents today versus in the future since political risk is high and the cost of leaving the economy exposed in the downturn is low. This causes governments to over-borrow and under-taxes at later stages of the boom when debt is driven down sufficiently and the prospect for rent-seeking approaches. Politicians at early stages of the boom anticipate this behavior of politicians in the future, and for this reason, they choose to over-borrow and to under-tax themselves. Thus the prospect of future rent-seeking reinforces over-borrowing and under-taxation in the present.

Our third and most important result—which stands in contrast to the conventional wisdom—is that if economic volatility is sufficiently high relative to political uncertainty, then the rent-seeking government *over-saves* and *over-taxes* along the equilibrium path relative to a benevolent government. Whenever economic volatility is high, politicians are less likely to consume rents today and more likely to consume them tomorrow since this simultaneously protects the economy while providing them with potential rents in the event of a boom during which they are not replaced. In anticipation of these rents in the future, the rent-seeking government actually over-saves relative to a benevolent government since the marginal value of additional funds in the future boom due to rent-seeking exceeds the marginal value of additional funds for a benevolent government who would instead use the additional savings to cut taxes. This causes governments to over-save and over-tax at later stages of the boom when debt is driven down sufficiently and the prospect for rent-seeking approaches. Politicians at early stages of the boom anticipate

this behavior of politicians in the future, and for this reason, they choose to over-save and to over-tax themselves. The prospect of future rent-seeking therefore reinforces over-saving and over-taxing in the present.

Our last result is that the popular fiscal rule of capping deficits brings deficits and surpluses closer to those of the benevolent government, although the mechanism is different in the under-saving and over-saving cases. In the under-saving region, the government would like to save less in order to starve the future government of resources which it would otherwise squander on rents. However, the rule does not permit the government to do this, so that it must necessarily bind and it forces the rent-seeking government to save more and to behave more like a benevolent government. In the over-saving region, the rule works through expectations by reducing the value of *future* public funds. More specifically, unconstrained governments over-save because they look forward to squandering public funds in the future if the boom persists for sufficiently long. The fiscal rule however makes it impossible to squander these public funds in the future since it forces a future government to save more. Therefore, the rule reduces the value of future funds from today's perspective, and this induces today's government to save less. Part of this reduction in savings comes not from deep tax cuts but from earlier and higher levels of rent extraction relative to the economy in the absence of fiscal rules. More generally, on its own, the fiscal rule cannot force governments to cut taxes when resources become sufficiently abundant, and in the long run, additional increases in savings are used purely for rent-seeking.

This paper builds on the literature on optimal fiscal policy and debt management dating back to the classical work of Barro (1979) and Lucas and Stokey (1983).<sup>4</sup> We depart from this work by relaxing the assumption of a benevolent government and by assuming that the economy is managed by politicians who derive partial utility from rents and who face potential replacement. In this regard, this paper is most closely related to the literature on the political economy of debt. More specifically, our work complements that of Battaglini and Coate (2008). As in our work, they consider a setting in which current governments face economic risk and political risk. They show that the presence of political risk implies that in the long run, a rent-seeking government holds a level of debt which exceeds that of the benevolent government. We depart from their work by focusing on the implications of political economy distortions *along the equilibrium path* and away from steady state. In the process, we describe a novel over-saving mechanism. Our work is also related to that of Song, Storesletten, and Zilibotti (2007) who show that intergenerational

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<sup>4</sup>See also Aiyagari, Marcet, Sargent, and Seppala (2002), Bohn (1990), and Chari and Kehoe (1993a, 1993b).

conflict in a dynamic model can cause a government to under-save or over-save relative to the social optimum. We depart from their work by abstracting from intergenerational conflict and considering instead the impact of political and economic risk.<sup>5</sup> Finally, our over-saving result is related to the work of Yared (2008) who argues that prescribing high levels of savings in the presence of rent-seeking politicians is distortionary since it is associated with the anticipation of future rents. In contrast, in the current paper we explain these high savings as an endogenous mechanism to extract future rents when effective economic uncertainty is high.

This introduction is followed by five sections and an appendix. Section 2 describes the environment and Section 3 describes the corresponding equilibrium under a benevolent government. Section 4 describes the equilibrium under a rent-seeking government and compares it to that of a benevolent government. Section 5 describes a simulation of our economy and discusses policy implications. Section 6 concludes. The Appendix contains the proofs and additional material.

## 2 Model

### 2.1 Economic Environment

There are discrete time periods  $t = \{0, \dots, \infty\}$  and a continuum of mass 1 of identical households with the following period 0 welfare:

$$E_0 \left( \sum_{t=0}^{\infty} \beta^t u(c_t) \right), \quad \beta \in (0, 1), \quad (1)$$

for  $c_t \geq 0$  which represents consumption and for  $u(\cdot)$  which satisfies  $u'(\cdot) > 0$ ,  $-u''(\cdot) > 0$ ,  $u'(\infty) = 0$ , and  $u'(0) = \infty$ . Households hold a constant endowment  $e > 0$ , they pay lump sum taxes to the government  $\tau_t \leq e$ , and they balance their budget so that  $c_t = e - \tau_t$ . Since  $\tau_t$  can be negative, it can also be interpreted as negative of public spending.

There is a large number of potential and identical politicians who derive the flow utility  $u(c_t)$  when out of power and who derive the flow utility  $u(c_t) + \theta x_t$  when in power for  $x_t \geq 0$  which represents socially wasteful rents.<sup>6</sup>  $\theta \geq 0$  and we refer to the special case

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<sup>5</sup>For additional work on the political economy of debt, see for example Aghion and Bolton (1990), Alesina and Perotti (1994), Alesina and Tabellini (1990), Amador (2003), Lizzeri (1999), and Persson and Svensson (1989).

<sup>6</sup>While the linearity of rents in the utility function is important for the full characterization of the model, the over-saving mechanism we describe depends on the existence of a region in which rents are zero. Specifically if  $v(x)$  represents the flow utility of rents, we require  $v'(0) < \infty$ . Details available upon

of  $\theta = 0$  as a benevolent government since it corresponds to the case in which incumbent politicians have the same preferences as households. Levels of  $\theta$  which exceed 0 captures the inverse cost of rent-seeking for the politician so that higher levels of  $\theta$  are associated with less costly rent-seeking.

A politician in power in period  $t$  is permanently removed from office and replaced with an identical politician from  $t + 1$  onward with exogenous probability  $1 - q \in (0, 1)$ , so that  $q$  represents the survival rate of a politician.<sup>7</sup> Therefore, the welfare of the incumbent at  $t = 0$  can be written as

$$\mathbf{E}_0 \left( \sum_{t=0}^{\infty} \beta^t (u(c_t) + q^t \theta x_t) \right), \quad (2)$$

where we have taken into account that a politician in period zero survives to period  $t$  with probability  $q^t$ .

In every period, the government finances rents  $x_t \geq 0$  and debt  $b_t \geq 0$  by raising revenue  $\tau_t \leq e$  and borrowing  $b_{t+1} \geq 0$  from international markets at a price  $\beta \in (0, 1)$ . In addition, the government experiences an exogenous endowment shock  $y_t$ .<sup>8</sup> The government's dynamic budget constraint is

$$\beta b_{t+1} = b_t + x_t - (\tau_t + y_t) \quad (3)$$

for a given  $b_0$  subject to  $\lim_{t \rightarrow \infty} \beta^t b_{t+1} \leq 0$ .

The endowment  $y_t$  is stochastic and depends on the state  $s_t \in \{L, H\}$  with  $y(H) = -y(L) = \sigma > 0$ . The government therefore exists to smooth household's consumption.  $s_t$  follows a first order Markov process and is independent of the political replacement shock. Let  $s_0 = H$ . We simplify our discussion by assuming that  $\Pr\{s_t = L | s_{t-1} = L\} = 1$  and that  $\Pr\{s_t = H | s_{t-1} = H\} = \alpha \in (0, 1)$ . We refer to state  $H$  as the boom and state  $L$  as the downturn. We will focus on the path of the economy with  $s_0 = H$ . Therefore, the economy is experiencing a temporary boom which may permanently end at any date with probability  $1 - \alpha$ .<sup>9</sup>

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request.

<sup>7</sup>We could instead allow politicians to return to power, and none of our results would change if the probability of holding power at any  $t$  is i.i.d.

<sup>8</sup>There is no difference between letting the government or the households experience this endowment shock.

<sup>9</sup>This formulation allows for tractability. We have numerically simulated economies in which  $s_t = L$  is not absorbing and achieved similar characterization to our analytical results here. Details available upon request.

## 2.2 Political Environment

The order of events at every period  $t$  is as follows:

1. Nature determines  $y_t$  and potentially replaces the period  $t - 1$  incumbent.
2. The period  $t$  politician chooses policies  $\{\tau_t, x_t, b_{t+1}\}$ .
3. Markets open and clear.

Given that there are many potential equilibria which can emerge in this setting, we consider the symmetric Markov Perfect Equilibrium which coincides with the limit of our economy with  $T$  periods as  $T \rightarrow \infty$ .<sup>10</sup> In this equilibrium, the incumbent politician— independently of identity and of past political shocks—chooses policies as a function of the state  $s_t$  and the level of debt  $b_t$ . Note that in choosing  $\tau_t$ , the incumbent effectively chooses  $c_t$ , so that without loss of generality, we will refer to  $c(b, s)$ ,  $x(b, s)$  and  $b'(b, s)$  as the politician's choices of  $c_t$ ,  $x_t$ , and  $b_{t+1}$ , respectively, conditional on  $b_t = b$  and  $s_t = s$ . Define  $V^N(b, s)$  and  $V^P(b, s)$  as the continuation value of being out of office and in office, respectively, with debt  $b$  in state  $s$ . The set of policies  $\{c(b, s), x(b, s), b'(b, s)\}_{s=L,H}$  constitutes a Markov Perfect Equilibrium if  $\{c(b, s), x(b, s), b'(b, s)\}$  maximizes  $V^P(b, s)$  given  $b$  and  $s$  and subject to the government's dynamic budget constraint.

## 3 Benevolent Government Benchmark

We begin by considering the policies of the benevolent government which corresponds to a special case of our economy with  $\theta = 0$ . In this circumstance,  $V^P(b, s)$  equals  $V^N(b, s)$ , and to facilitate future discussion, we let the superscript  $B$  denote the continuation value and the policies of the benevolent government. The problem of the government in the *downturn* can be written as

$$V^B(b, L) = \max_{c, x, b'} u(c) + \beta V^B(b', L) \quad (4)$$

s.t.

$$\beta b' = b + x - (e - \sigma), \quad (5)$$

Since households are always better off consuming more, the solution to this problem assigns  $x^B(b, L) = 0$ . Conditional on  $b'$ , the politician is always better off taxing less versus

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<sup>10</sup>That is, subject to the constraint that  $\beta^T b_{T+1} \leq 0$ .

extracting more rents. Therefore, the problem is mathematically equivalent to a personal consumption problem in which smoothing consumption is optimal. Thus,  $c^B(b, L) = e - \sigma - b(1 - \beta)$ ,  $b'^B(b, L) = b$ , and  $V^B(b, L) = u(e - \sigma - b(1 - \beta)) / (1 - \beta)$ .

Using this characterization, we can now consider the government's problem during the preceding *boom*:

$$V^B(b, H) = \max_{c, x, b'} u(c) + \beta \mathbf{E}_s V^B(b', s) \quad (6)$$

s.t.

$$\beta b' = b + x - (e + \sigma). \quad (7)$$

As in the downturn, the solution to this problem yields  $x^B(b, H) = 0$ , and optimality requires  $c^B(b, H)$  to be defined by the following Euler equation:

$$u_c(c^B(b, H)) = \alpha u_c(c^B(b', H), H) + (1 - \alpha) u_c(c^B(b', H), L). \quad (8)$$

**Lemma 1**  $c^B(b, H)$  is strictly decreasing in  $b$ ,  $b'^B(b, H)$  is strictly increasing in  $b$ , and  $b'^B(b, H) < b$ .

The government taxes more and saves less when government debt is high since the economy is relatively poor. The government always raises its savings in the boom in preparation for the downturn and it continues to drive down its debt until the boom ends. Note that as the boom persists, the size of the government asset position approaches infinity since the government always benefits from saving more in preparation for the downturn.

## 4 Rent-Seeking Government

We now consider the behavior of a government more generally for all  $\theta > 0$ . Here we write the problem of the government recursively (Section 4.1), characterize the dynamics of consumption and debt (Section 4.2), and compare these policies to those of a benevolent government (Section 4.3).

## 4.1 Recursive Program

Conditional on entering a downturn, the incumbent politician solves the following problem:

$$\begin{aligned}
 V^P(b, L) &= \max_{c, x, b'} \{u(c) + \theta x + \beta (qV^P(b', L) + (1 - q)V^N(b', L))\} & (9) \\
 &\text{s.t. } x \geq 0 \text{ and} \\
 &\beta b' = b + x - (e - \sigma). & (10)
 \end{aligned}$$

The government clearly wishes to smooth consumption, though it is also interested in rent-seeking which provides a marginal utility of  $\theta$  and sets a lower bound for the marginal utility of consumption. This means that during the downturn, politicians choose the following policies, where the superscript  $P$  denotes the policies of a rent-seeking government:

$$\begin{aligned}
 c^P(b, L) &= \min \{e - \sigma - (1 - \beta)b, u_c^{-1}(\theta)\} \\
 x^P(b, L) &= \max \left\{ 0, \frac{e - \sigma - u_c^{-1}(\theta)}{1 - \beta} - b \right\} \\
 b'^P(b, L) &= \max \left\{ b, \frac{e - \sigma - u_c^{-1}(\theta)}{1 - \beta} \right\}
 \end{aligned}$$

The rent-seeking government follows the same smooth policies with zero rent-seeking as those of a benevolent government as long as its initial stock of debt  $b$  is above a threshold  $(e - \sigma - u_c^{-1}(\theta)) / (1 - \beta)$ . In this case, the government is relatively poor and any additional reductions in  $b$  are used for reducing taxes on households as opposed to raising rents (since the marginal benefit of cutting those taxes exceeds  $\theta$ ).

If  $b$  is below this threshold, then the government is rich. Politicians extract positive rents, they tax households more than the benevolent government, and they borrow more than the benevolent government. More specifically, consumption is held at  $u_c^{-1}(\theta)$ , so that the marginal benefit of rent-seeking equals the marginal benefit of consumption. Moreover, debt is held at  $(e - \sigma - u_c^{-1}(\theta)) / (1 - \beta)$ . Therefore, any additional reductions in  $b$  are used only for rent-seeking as opposed to tax or debt reduction. By following this strategy, the incumbent politician who may be replaced in the future chooses to frontload all rent-extraction and leaves all future politicians with zero rents. Note that the threshold which separates the zero rent region from the positive rent region rises with the rent-seeking bias  $\theta$ .

Given these policies, we can characterize  $V^P(b, L)$  and  $V^N(b, L)$ .

**Lemma 2** *The following conditions hold for  $j = P, N$ :*

1.  $V^j(b, L)$  is strictly decreasing in  $b$ , strictly concave in  $b$ , and continuously differentiable in  $b$  for  $b > (e - \sigma - u_c^{-1}(\theta)) / (1 - \beta)$ ,
2.  $V^P(b, L)$  is linear in  $b$  and continuously differentiable in  $b$  for  $b \leq (e - \sigma - u_c^{-1}(\theta)) / (1 - \beta)$  with  $V_b^P(b, L) = -\theta$ ,
3.  $V^N(b, L)$  is linear in  $b$  and continuously differentiable in  $b$  for  $b < (e - \sigma - u_c^{-1}(\theta)) / (1 - \beta)$  with

$$\lim_{b \rightarrow [(e - \sigma - u_c^{-1}(\theta)) / (1 - \beta)]^+} V_b^N(b, L) = -\theta \text{ and } \lim_{b \rightarrow [(e - \sigma - u_c^{-1}(\theta)) / (1 - \beta)]^-} V_b^N(b, L) = 0.$$

The important feature of Lemma 2 is that  $V^N(b, L)$  is not differentiable at the cutoff point  $(e - \sigma - u_c^{-1}(\theta)) / (1 - \beta)$  where rent-seeking begins. This is because additional resources are no longer used for cutting taxes and are instead used for raising rents which does not benefit society. We will see that an analogous result to Lemma 2 holds in the boom.

Given the behavior of the economy in the downturn, we characterize the policy of the benevolent government in the boom. The incumbent politician solves the following problem:

$$V^P(b, H) = \max_{c, x, b'} \{u(c) + \theta x + \beta \mathbf{E}_s \{ (qV^P(b', s) + (1 - q)V^N(b', s)) \} \} \quad (11)$$

s.t.

$$\beta b' = b + x + c - (e + \sigma) \quad (12)$$

To facilitate discussion, we define the following cut-off point:

$$\underline{b} = (e + \sigma - \max \{ u_c^{-1}(\theta), 2\sigma + u_c^{-1}(\theta(1 - \alpha q) / (1 - \alpha)) \}) / (1 - \beta). \quad (13)$$

We will show that  $\underline{b}$  represents the steady state level of debt to which the economy converges during a sustained boom. Note that the exact characterization of  $\underline{b}$  depends on the level of volatility  $\sigma$ , and this is important since there are two cases to consider. Specifically, define  $\sigma^*$  as

$$\sigma^* = \frac{1}{2} \left( u_c^{-1}(\theta) - u_c^{-1} \left( \theta \frac{1 - \alpha q}{1 - \alpha} \right) \right).$$

Note that since  $q < 1$ ,  $\sigma^* > 0$ . The cutoff value  $\sigma^*$  decreases in  $q$ , so that as political survival  $q$  goes to 1,  $\sigma^*$  goes to 0. Moreover, as the persistence of the boom  $\alpha$  increases,  $\sigma^*$  increases. Finally, it can be shown by implicit differentiation given that  $u'''(\cdot) > 0$  that  $\sigma^*$  is decreasing in the rent-seeking bias  $\theta$ .<sup>11</sup> Therefore,  $\sigma$  is more likely to exceed  $\sigma^*$  if political risk is low, the boom is temporary, and the rent-seeking bias  $\theta$  is high.

As we will show, rent-seeking begins at levels of debt below  $e + \sigma - u_c^{-1}(\theta) + \beta \underline{b}$ . Thus, an analogous result to Lemma 2 holds and we can characterize  $V^P(b, H)$  and  $V^N(b, H)$ .

**Lemma 3** *The following conditions hold for  $j = P, N$ :*

1.  $V^j(b, H)$  is strictly decreasing in  $b$ , strictly concave in  $b$ , and continuously differentiable in  $b$  for  $b > e + \sigma - u_c^{-1}(\theta) + \beta \underline{b}$ ,
2.  $V^P(b, H)$  is linear in  $b$  and continuously differentiable in  $b$  for  $b \leq e + \sigma - u_c^{-1}(\theta) + \beta \underline{b}$  with  $V_b^P(b, H) = -\theta$ ,
3.  $V^N(b, H)$  is linear in  $b$  and continuously differentiable in  $b$  for  $b < e + \sigma - u_c^{-1}(\theta) + \beta \underline{b}$  with

$$\lim_{b \rightarrow [e + \sigma - u_c^{-1}(\theta) + \beta \underline{b}]^+} V_b^N(b, H) = -\theta \text{ and } \lim_{b \rightarrow [e + \sigma - u_c^{-1}(\theta) + \beta \underline{b}]^-} V_b^N(b, H) = 0.$$

The first order conditions and the envelope condition imply that if  $b^P(b, H) > e + \sigma - u_c^{-1}(\theta) + \beta \underline{b}$ , then

$$u_c(c^P(b, H)) = \alpha u_c(c^P(b^P(b, H), H)) + (1 - \alpha) u_c(c^P(b^P(b, H), L)) , \quad (14)$$

so that the Euler equation holds with equality as under a benevolent government. Moreover, if  $b^P(b, H) < e + \sigma - u_c^{-1}(\theta) + \beta \underline{b}$ , then

$$u_c(c^P(b, H)) = \alpha q \theta + (1 - \alpha) u_c(c^P(b^P(b, H), L)) . \quad (15)$$

These two equations relate the marginal cost of public funds today to the expected marginal cost of public funds tomorrow. They show that the marginal cost of public funds tomorrow depends on whether or not rent-seeking takes place during the boom.<sup>12</sup> If no

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<sup>11</sup>Formally,  $\frac{d\sigma^*}{d\theta} < \frac{1}{2} \left( (u_{cc}(u_c^{-1}(\theta)))^{-1} - \left( u_{cc} \left( u_c^{-1} \left( \theta \frac{1 - \alpha q}{1 - \alpha} \right) \right) \right)^{-1} \right) < 0$ .

<sup>12</sup>Savings are never high enough for rent-seeking to occur both in the boom and in the downturn since this is suboptimal for today's government.

rent-seeking takes place, the marginal cost of public funds equals the marginal utility of consumption since additional resources are used to boost consumption (equation (14)). In contrast, if rent-seeking takes place, the marginal cost of public funds is  $q\theta$  since today's politician maintains power with probability  $q$  and extracts rents in the future which provide marginal benefit  $\theta$  (equation (15)).<sup>13</sup>

## 4.2 Transitional Dynamics

We begin by describing the transitional dynamics of policies under a rent-seeking government.

**Proposition 1 (dynamics)** *Policies satisfy the following properties for some  $\bar{b} > \underline{b}$ :*

1.  $b'^P(b, H) = \underline{b}$  if  $b \leq \bar{b}$ ,  $b'^P(b, H) < b$  if  $b > \bar{b}$ , and  $b'^P(b, H)$  weakly increases in  $b$ ,
2. If  $\sigma \leq \sigma^*$ , then  $c^P(b, H) < (=) u_c^{-1}(\theta)$  and  $x^P(b, H) = (>) 0$  if  $b > (<) \underline{b}$ , and
3. If  $\sigma > \sigma^*$ , then  $c^P(b, H) < (=) u_c^{-1}(\theta)$  and  $x^P(b, H) = (>) 0$  if  $b > (<) \bar{b}$ .

Figures 1 and 2 display this proposition graphically. Specifically, they depict  $b'^P(b, H)$  as a function of  $b$  for  $\sigma \leq \sigma^*$  and  $\sigma > \sigma^*$ , respectively. Much like the benevolent government, the rent-seeking government lets debt decline monotonically throughout the boom, but *unlike* the benevolent government, government assets do not rise forever. Beyond  $\bar{b}$ , a prolonged boom causes the government to stabilize tomorrow's debt at a minimum point  $\underline{b}$ . These figures also depict the rent-seeking regions for different levels of  $\sigma$ . If  $\sigma \leq \sigma^*$ , then rent-seeking begins when debt goes below  $\underline{b}$ . In contrast, if  $\sigma > \sigma^*$ , then rent-seeking begins when debt drops below  $\bar{b} > \underline{b}$ .

The implied dynamics of consumption and rents depend crucially on the degree of economic uncertainty  $\sigma$ . If  $\sigma \leq \sigma^*$ , then starting from  $b_0 > \underline{b}$ , the government saves and it never extracts rents along the path. Once debt  $b$  first reaches  $\bar{b}$ , the government chooses  $b'^P(b, H) = \underline{b}$  so that the economy reaches the steady state with zero rents. The government never saves beyond  $\underline{b}$  since politicians know that rents would be extracted by a likely replacement government, and the additional benefit of making these savings

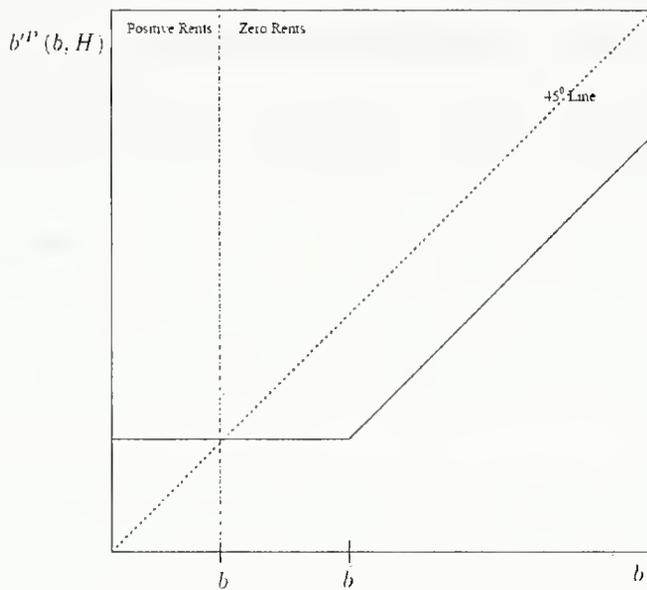
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<sup>13</sup>Note that if  $b'^P(b, H) = e + \sigma - u_c^{-1}(\theta) + \beta \underline{b}$ , then  $V^N(b, H)$  is not differentiable, though  $u_c(c^P(b, H))$  must be in the range between the right hand side of (15) and the right hand side of (14). Specifically,

$$u_c(c^P(b, H)) \in [\alpha q \theta + (1 - \alpha) u_c(c^P(b'^P(b, H), L)), \alpha \theta + (1 - \alpha) u_c(c^P(b'^P(b, H), L))].$$

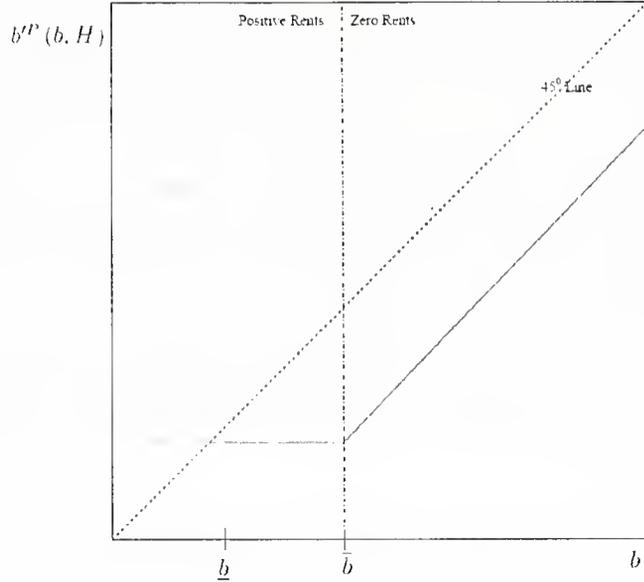
available for a downturn do not outweigh the cost of leaving additional rents for a replacement government in a boom. For the same reason, if the economy starts from  $b_0 < \underline{b}$ , the government chooses  $c^P(b_0, H) = u_c^{-1}(\theta)$ ,  $x^P(b_0, H) = \underline{b} - b_0$ , and  $b'^P(b_0, H) = \underline{b}$ , in order to starve the future government of resources. In summary, a prolonged boom in this environment leads debt to  $\underline{b}$  and to zero rent-seeking.

Figure 1:  $b'^P(b, H)$  vs.  $b$  for  $\sigma \leq \sigma^*$



These dynamics are different if  $\sigma > \sigma^*$ . Starting from  $b_0 > \bar{b}$ , the government chooses zero initial rents, and it gradually saves during the boom until debt eventually reaches  $\bar{b}$ . Once debt  $b$  drops below  $\bar{b}$ , the government chooses positive rents so that  $c^P(b, H) = u_c^{-1}(\theta)$ ,  $x^P(b, H) = \bar{b} - b$ , and  $b'^P(b, H) = \underline{b}$ . Therefore, the government reaches a steady state with positive rents, which is in contrast to the  $\sigma \leq \sigma^*$  case. Thus, even if the economy starts from zero rents, there is a possibility that rents may be positive in the future if the boom persists for sufficiently long. The current politician does not want to fully starve the future government of rents since he knows that it would expose the economy to too much volatility, and he may as well postpone rent-seeking given that he has a sufficiently high survival probability and is likely to consume these rents himself.

Figure 2:  $b'^P(b, H)$  vs.  $b$  for  $\sigma > \sigma^*$



### 4.3 Comparison to Benevolent Government

In this section, we compare the path of debt and consumption under a rent-seeking government to that under a benevolent government. We begin by considering the implications of the equilibrium if the boom is prolonged. Let  $\{c_t^B\}_{t=0}^\infty$  and  $\{b_{t+1}^B\}_{t=0}^\infty$  correspond to the equilibrium sequence of consumption and debt, respectively, conditional on a boom persisting forever under a benevolent government starting from some initial debt  $b_0$ . Define  $\{c_t^P\}_{t=0}^\infty$ , and  $\{b_{t+1}^P\}_{t=0}^\infty$  analogously for a rent-seeking government.

**Proposition 2** (*long run*)

$$\begin{aligned} \lim_{t \rightarrow \infty} b_{t+1}^B &= -\infty < \lim_{t \rightarrow \infty} b_{t+1}^P = \underline{b} \text{ and} \\ \lim_{t \rightarrow \infty} c_t^B &= \infty > \lim_{t \rightarrow \infty} c_t^P = u_c^{-1}(\theta). \end{aligned}$$

Proposition 2 implies that a prolonged boom leads a rent-seeking government to hold more debt than a benevolent government and to consume less (tax more) than a benevolent government. Though a rent-seeking government reduces its debt at the beginning of the boom, it stops reducing its debt if the boom is sufficiently prolonged. This is because beyond a certain date, government resources become so abundant that rent-seeking

considerations come to dominate intertemporal smoothing considerations. A rent-seeking government realizes that if it were to save more, then a future replacement government would use the additional funds for rent-seeking (which only benefits incumbent politicians) as opposed to tax-cutting (which benefits households), and the government therefore restrains its savings in order to starve the future government of funds. Therefore, in the long run, a prolonged boom always leads a benevolent government to hold more assets and to tax less than a rent-seeking government. This result is consistent with that emphasized by Battaglini and Coate (2008). Our main contribution is to show that while this characterization applies to the long run fairly generally, whether or not it applies to the transitional dynamics of the economy depends on the level of economic volatility.

Next we consider the dynamics of public debt and taxes along the equilibrium path. With some abuse of notation, let  $u_c(c^B(b, H; \sigma))$  represent the value of  $u_c(c^B(b, H))$  for a benevolent government facing uncertainty  $\sigma$ . Define  $\underline{\sigma}$  and  $\bar{\sigma}$  as the unique solutions to the following two equations:

$$\begin{aligned}\underline{\sigma} & : u_c \left( c^B \left( \frac{e - \underline{\sigma} - u_c^{-1}(\theta(1 - \alpha q) / (1 - \alpha))}{1 - \beta}, H; \underline{\sigma} \right) \right) = q\theta \\ \bar{\sigma} & : u_c \left( c^B \left( e + \bar{\sigma} - u_c^{-1}(\theta) + \beta \frac{e - \bar{\sigma} - u_c^{-1}(\theta(1 - \alpha q) / (1 - \alpha))}{1 - \beta}, H; \bar{\sigma} \right) \right) = q\theta\end{aligned}$$

**Lemma 4** (i)  $0 < \sigma^* < \underline{\sigma} < \bar{\sigma}$ , (ii)  $\underline{\sigma}$  and  $\bar{\sigma}$  are decreasing in  $q$  and increasing in  $\alpha$ , (iii)  $\underline{\sigma}$  and  $\bar{\sigma}$  approach 0 as  $q$  approaches 1, (iv)  $u_c(c^B(\underline{b}, H)) < q\theta$  iff  $\sigma > \underline{\sigma}$ , and (v)  $u_c(c^B(\bar{b}, H)) < q\theta$  iff  $\sigma > \bar{\sigma}$ .

The lemma states that like  $\sigma^*$ , the cutoff points  $\underline{\sigma}$  and  $\bar{\sigma}$  decrease in political risk  $q$  and increase in the persistence parameter  $\alpha$ .<sup>14</sup> Moreover, like  $\sigma^*$ , these converge to zero as  $q$  approaches 1, so that any positive value of  $\sigma$  must necessarily exceed  $\bar{\sigma}$  as  $q$  approaches 1. The parameter  $\underline{\sigma}$  is the level of volatility for which  $\sigma > \underline{\sigma}$  implies  $u_c(c^B(\underline{b}, H); \sigma) < q\theta$ .  $u_c(c^B(\underline{b}, H); \sigma)$  decreases in  $\sigma$  since as economic volatility  $\sigma$  increases, the steady state level of debt  $\underline{b}$  decreases, and it decreases by an amount large enough to cause the benevolent government's consumption at  $\underline{b}$  to rise. Eventually, the marginal utility of this consumption goes below  $q\theta$ . Analogous arguments hold for the level of debt  $\bar{b}$ , where  $\bar{\sigma}$  is the level of volatility such that  $\sigma > \bar{\sigma}$  implies  $u_c(c^B(\bar{b}, H)) < q\theta$ .

The interpretation of these cutoff points for economies with  $\sigma > \sigma^*$  is as follows: If  $\sigma < \underline{\sigma}$ , then  $u_c(c^B(b, H)) > q\theta$  for  $b \in [\underline{b}, \bar{b}]$ , which is the region in which debt exceeds steady state debt and in which rent-seeking is positive. Therefore, the marginal

<sup>14</sup>Comparative statics with respect to  $\theta$  are ambiguous.

value of public funds for a benevolent government in the boom exceeds the (expected) marginal value of public funds for a rent-seeking government in the boom who survives with probability  $q$  and who values marginal rents with weight  $\theta$ . In contrast, if  $\sigma > \bar{\sigma}$ , then  $u_c(c^B(b, H)) < q\theta$  for  $b \in [\underline{b}, \bar{b}]$ . In this case, the marginal value of public funds for a benevolent government in the boom is below the (expected) marginal value of public funds for a rent-seeking government in the boom.

As we will show, whether the marginal value of public funds for a benevolent government exceeds or is below  $q\theta$  in the rent-seeking region affects whether or not the rent-seeking government saves less or more than a benevolent government. We show that economies with  $\sigma < \underline{\sigma}$  feature over-borrowing along the equilibrium path (Section 4.3.1), and we show that economies with  $\sigma > \bar{\sigma}$  feature over-saving along the equilibrium path (Section 4.3.2). In the Appendix, we consider economies with  $\sigma \in (\underline{\sigma}, \bar{\sigma})$ , and we show that both over-borrowing or over-saving can occur along the equilibrium path, and this depends on initial condition  $b_0$ .

### 4.3.1 Low Economic Volatility

We begin by showing that the rent-seeking government over-borrows if economic volatility is low.

**Proposition 3** (*starve the beast*) *If  $\sigma < \bar{\sigma}$ , then  $b^P(b, H) > b^B(b, H) \forall b$  and  $c^P(b, H) > c^B(b, H) \forall b \geq \bar{b}$ .*

This proposition states that if economic volatility is low, then the rent-seeking government always borrows more than the benevolent government, and it consumes more than the benevolent government for levels of debt which exceed  $\bar{b}$ .<sup>15</sup> Therefore, the transition path starting from  $b_0 > \bar{b}$  features over-spending and over-borrowing, which is in line with the conventional wisdom in the political economy literature.

The intuition for this result is that low economic volatility implies that politicians are biased towards extracting rents today versus in the future, since political risk is high and the cost of leaving the economy exposed in the downturn is low. This causes governments to over-borrow and over-consume at later stages of the boom when debt is driven down sufficiently and the prospect for rent-seeking approaches. Politicians at early stages of the boom anticipate this behavior of politicians in the future, and for this reason, they choose to over-borrow and to over-consume. The prospect of future rent-seeking therefore reinforces over-borrowing and over-consumption in the present.

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<sup>15</sup>Whenever  $\sigma < \underline{\sigma}$ , there is some cutoff level of debt below which  $c^P(b, H) < c^B(b, H)$ . For the  $\sigma < \sigma^*$  case, this cutoff point is below  $\underline{b}$ .

More formally, imagine if volatility is so low that rents are never extracted under levels of debt which exceed  $\underline{b}$  (i.e.,  $\sigma < \sigma^*$ ). Since  $x^P(b, H) = 0 \forall b \geq \underline{b}$ , then  $c^P(b, H) > c^B(b, H)$  if and only if  $b^P(b, H) > b^B(b, H)$  from the dynamic budget constraint of the economy. Since  $b^P(\underline{b}, H) > b^B(\underline{b}, H)$ , the rent-seeking government must be choosing  $c^P(\underline{b}, H) > c^B(\underline{b}, H)$ . Therefore, in steady state, the government over-borrows and over-consumes, and the marginal cost of public funds at  $\underline{b}$  under a benevolent government which equals  $u_c(c^B(\underline{b}, H))$  exceeds the marginal cost of public funds under a rent-seeking government which equals  $u_c(c^P(\underline{b}, H)) = \theta$ . This affects savings decisions for all levels of debt above  $\underline{b}$ . Consider the Euler conditions of the benevolent and rent-seeking government, (8) and (14), respectively, for  $b \in [\underline{b}, \bar{b}]$ . Since  $b \geq \underline{b}$ ,  $c^P(b, L) = c^B(b, L)$  because debt is never sufficiently low in the downturn to induce rent-seeking. Therefore, satisfaction of (8) and (14) implies that  $b^B(b, H) < b^P(b, H) = \underline{b}$ , since the benevolent government perceives a higher marginal cost of public funds in the future than the rent-seeking government. Thus,  $u_c(c^P(b, H)) < u_c(c^B(b, H))$  so that the marginal cost of public funds is higher at  $b$  under a benevolent government. Forward iteration of this argument implies that all rent-seeking governments perceive a lower marginal cost of public funds in the future than the benevolent government, and they consequently save less than the benevolent government.

An analogous argument holds if instead volatility is low, though rents are extracted under levels of debt that exceed  $\underline{b}$  and are below  $\bar{b}$  (i.e.,  $\sigma^* < \sigma < \underline{\sigma}$ ). In this case,  $x^P(b, H) > 0$  for some  $b$  and it is no longer the case that  $c^P(b, H) > c^B(b, H)$  if and only if  $b^P(b, H) > b^B(b, H)$ . Nonetheless, note that the marginal cost of public funds in the boom for the rent-seeking government for  $b \in [\underline{b}, \bar{b}]$  equals  $q\theta$  since the government expects to survive with probability  $q$  and to extract rents which provide marginal utility  $\theta$ . However, given the definition of  $\underline{\sigma}$ ,  $u_c(c^P(b, H)) > q\theta$  in this region so that the benevolent government values public funds more on the margin than the rent-seeking government. Therefore, analogous arguments to the previous case comparing (8) and (15) imply that for  $b > \bar{b}$  for which  $b^P(b, H) \in [\underline{b}, \bar{b}]$ , it is the case that  $b^P(b, H) > b^B(b, H)$  and  $c^P(b, H) > c^B(b, H)$  (since  $x^P(b, H) = 0$ ) so that the rent-seeking government over-borrows and over-consumes. Since  $u_c(c^P(b, H)) < u_c(c^B(b, H))$ , the rent-seeking government under-values public funds at  $b$  and forward iteration on this argument implies that over-borrowing occurs for all  $b$ .

### 4.3.2 High Economic Volatility

The previous picture changes dramatically for high levels of economic volatility.

**Proposition 4 (feed the beast)** *If  $\sigma > \bar{\sigma}$ , then  $b'^P(b, H) < b'^B(b, H) \forall b \geq \bar{b}$  and  $c^P(b, H) < c^B(b, H) \forall b$ .*

This proposition states that if economic volatility is high, then the rent-seeking government saves *more* than the benevolent government for levels of debt which exceed  $\bar{b}$ , and it consumes less (taxes more) than the benevolent government.

The intuition for this result is as follows. Whenever economic volatility is high, the politician is less likely to consume rents today and more likely to consume them tomorrow since this simultaneously protects the economy while providing him with potential rents in the event of a boom during which he is not replaced. In anticipation of these rents in the future, the rent-seeking government may actually over-save relative to a benevolent government since the marginal value of additional funds in the future boom due to rent-seeking exceeds the marginal value of additional funds for a benevolent government who would instead use the additional savings to increase consumption. This causes governments to over-save and under-consume at later stages of the boom when debt is driven down sufficiently and the prospect for rent-seeking approaches. Politicians at early stages of the boom anticipate this behavior of politicians in the future, and for this reason, they choose to over-save and to under-consume themselves. The prospect of future rent-seeking therefore reinforces over-saving and under-consumption in the present. Future governments are not cutting taxes during the boom in response to additional savings—the natural response of a benevolent government—and this provides additional incentives for savings today.

More formally, consider the government at values of debt  $b \in [\underline{b}, \bar{b}]$ . In this region, the government chooses positive rents, and the marginal value of public funds for a rent-seeking government who may be potentially replaced prior to entering the boom is  $q\theta$ . Moreover, by the definition of  $\bar{\sigma}$ , the benevolent government is so wealthy in this region that its marginal value of public funds  $u_c(b, H)$  is below  $q\theta$ . The rent-seeking government is extracting rents and also over-taxing in order to do so. Now consider values of  $b > \bar{b}$  for which  $b'^P(b, H) \in [\underline{b}, \bar{b}]$ . In this region,  $x^P(b, H) = 0$  so that  $c^P(b, H) < c^B(b, H)$  if and only if  $b'^P(b, H) < b'^B(b, H)$ . Given (8) and (15), it must be the case that  $b'^P(b, H) < b'^B(b, H)$  and  $c^P(b, H) < c^B(b, H)$  so that the rent-seeking government over-saves and under-consumes. Since  $u_c(c^P(b, H)) > u_c(c^B(b, H))$ , the rent-seeking government over-values public funds at  $b$  and forward iteration on this argument implies that over-borrowing occurs for all  $b$ .

Note that even though the rent-seeking government over-saves along the equilibrium path, in steady state it over-borrows relative to a benevolent government who instead

drives its asset position to infinity.<sup>16</sup> In a sense then, it is the prospect of rent-seeking and over-borrowing in the future which induces politicians to over-save in the present. This induces the rent-seeking government to over-tax both when it is anticipating future rent-seeking and also in steady state when rent-seeking takes place.

## 5 Policy Implications and Discussion

A central implication of our model is that rent-extraction does *not* actually have to take place for distortions to emerge. The main mechanism in our framework operates through *expectations*. For example, when debt is sufficiently high, there are no rents independently of the regime. However, there are important distortions in both the low and high volatility scenarios.

In the low volatility scenario there is a wedge pushing the government to tax and save too little, since the government is worried that its potential replacement will squander everything. That is, the current government is too expansionary and borrows too much. In contrast, in the high volatility scenario, there is a wedge pushing the government *to tax and save too much*. Here, fiscal policy is actually too contractionary, and society would benefit from cutting taxes and saving less. In what follows, we illustrate these scenarios and conclude by analyzing the impact of standard fiscal rules.

### 5.1 The Two Scenarios

Consider an economy with  $u(c) = \log(c)$  and  $\{\beta, e, \sigma, \alpha, \theta\} = \{.95, 100, 1.5, .95, .001\}$ , where we have chosen  $\theta$  such that the long run level of debt in a boom in the  $\sigma < \sigma^*$  case is equal to 10. Consider two economies:  $q = .2$  and  $q = .99$ , so that in one economy, the current incumbent has an 80% chance of being replaced and in the other economy the incumbent has virtually no chance of being replaced. Under this parameterization, the low  $q$  case corresponds to an economy with  $\sigma < \underline{\sigma}$ , so that the government under-taxes and over-borrows, and the high  $q$  case corresponds to an economy with  $\sigma > \bar{\sigma}$  so that the government over-taxes and over-saves.

Figures 3 and 4 illustrate the path of debt and consumption in the  $q = .2$  economy during a prolonged boom starting from a level of debt  $b_0 = 30$  for a rent-seeking and a benevolent government. The rent-seeking government over-borrows relative to the benevolent government. This difference can be substantial. For example, at  $t = 40$ , the rent-

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<sup>16</sup>Formally, there exists a cutoff point in the range  $[\underline{b}, \bar{b}]$  below which the rent-seeking government over-borrows.

seeking government holds a level of debt equal to 10 whereas the benevolent government holds a level of debt equal to  $-33$ , a difference equal to over 40% of the endowment of the economy. The counterpart of the path of debt is not rent extraction (since  $\sigma < \sigma^* < \underline{\sigma}$ ) but excessive consumption (low taxes) during the transition (Figure 4), and economic fragility during the downturn (not shown).

Figure 3: Path of Debt ( $\sigma < \underline{\sigma}$ )

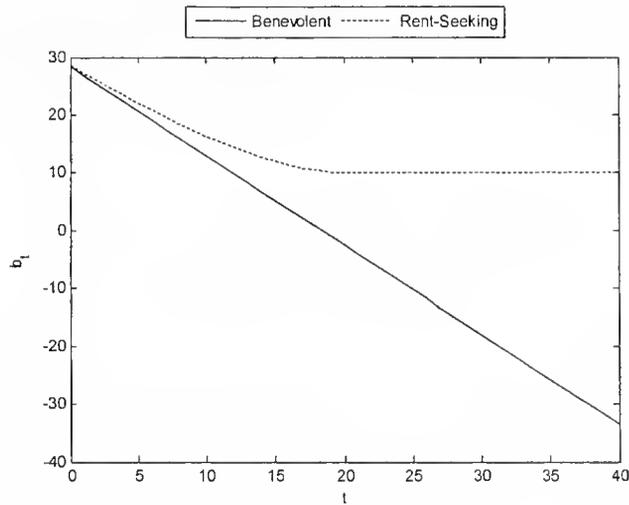
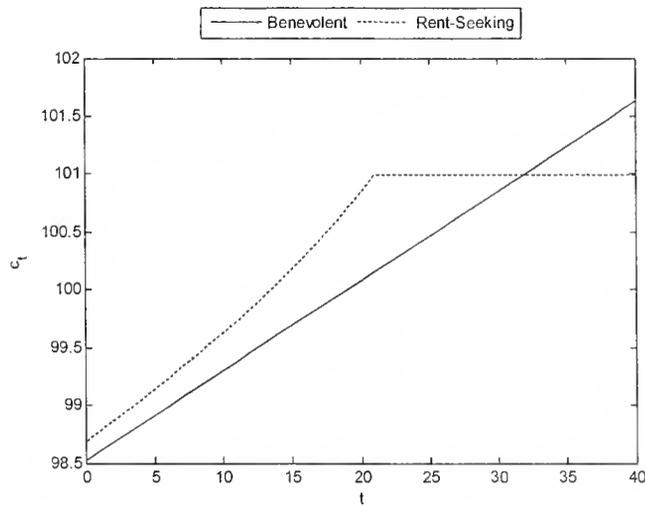


Figure 4: Path of Consumption ( $\sigma < \underline{\sigma}$ )



In contrast, Figures 5 and 6 consider the  $q = .99$  economy during a prolonged boom also starting from a level of debt  $b_0 = 30$ . In this situation, the rent-seeking government oversaves early on relative to the benevolent government (Figure 5). The difference between the two governments can be substantial. For example, at  $t = 40$  the rent-seeking government holds level of debt equal to  $-46$  whereas the benevolent government holds a level of debt equal to  $-33$ , a difference equal to over 10% of the endowment of the economy. Early on, the high taxes are used to reduce debt but later on they finance government rents. As a result, consumption is lower than under the benevolent government throughout the boom (Figure 6). Early on, when no rents are extracted, the economy gains in terms of extra protection against the contraction. Later on, consumption is lower both during the boom and the contraction.

Figure 5: Path of Debt ( $\sigma > \bar{\sigma}$ )

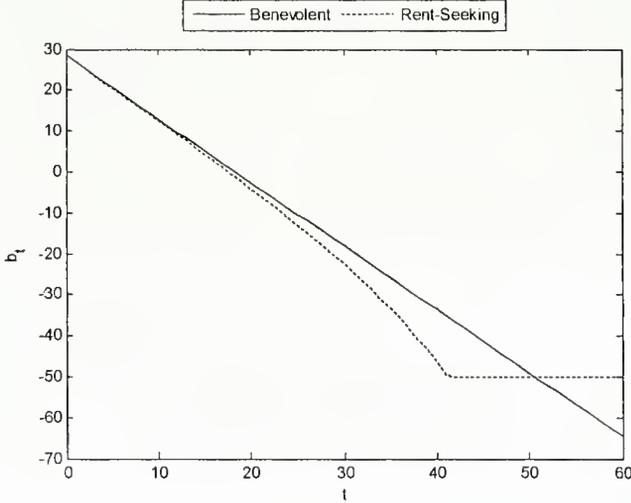
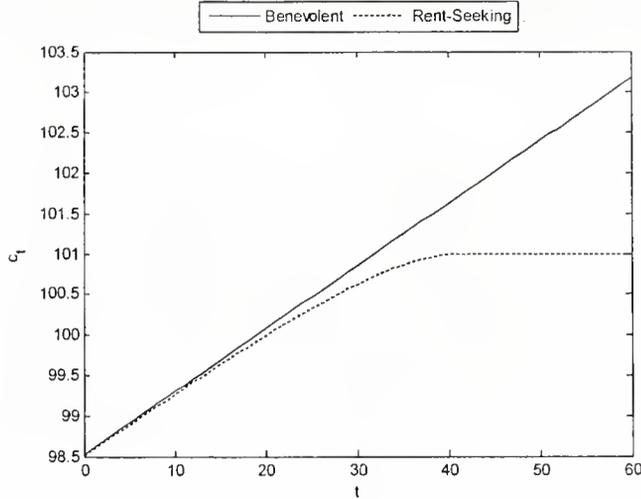


Figure 6: Path of Consumption ( $\sigma > \bar{\sigma}$ )



## 5.2 Fiscal Rules

The conventional view, captured in Figures 3 and 4, has given support to the increasingly popular policy option of adopting fiscal rules that essentially cap deficits (or require surpluses) during booms (the budget, surplus or deficit rules). A natural question concerns the degree to which such fiscal rules are useful in economies in which over-saving occurs along the equilibrium path as in Figures 5 and 6. This question is particularly relevant for commodity-economies which experience high economic volatility.

More specifically, consider an economy starting from  $b_0$  in which a benevolent government would choose a sequence of consumption  $\{c_t^B\}_{t=0}^\infty$  in the boom. Imagine a fiscal rule whereby the rent-seeking government in period  $t$  is allowed to choose any policy subject to the constraint that such a policy must satisfy

$$c_t + x_t \leq c_t^B, \quad (16)$$

so that the government effectively cannot run a primary deficit above that of the benevolent government at any given date. The political environment is as described in Section 2.2 with the exception that (16) must be satisfied by every government in every period. Since rents are zero under a benevolent government, (16) implies that the rent-seeking government must save at least as much as the benevolent government at every date. The next proposition characterizes the behavior of the economy under the fiscal rule where  $\{\tilde{c}_t^P\}_{t=0}^\infty$  and  $\{\tilde{x}_t^P\}_{t=0}^\infty$  correspond to the path of consumption and rents, respectively,

during the boom under a rent-seeking government subject to the fiscal rule.

**Proposition 5** (*fiscal rules*)  $\tilde{c}_t^P + \tilde{x}_t^P = c_t^B$  at every  $t$  in the economy under the fiscal rule and

$$\begin{aligned}\tilde{c}_t^P &= \min \{c_t^B, u_c^{-1}(\theta)\} \text{ and} \\ \tilde{x}_t^P &= \max \{0, c_t^B - u_c^{-1}(\theta)\}.\end{aligned}$$

Proposition 5 states that the fiscal rule (16) binds, and  $\tilde{c}_t^P$  and  $\tilde{x}_t^P$  are chosen as in Section 4 so that rents are only positive if the marginal value of consumption equals  $\theta$ . The rule binds in economies in which  $\sigma < \underline{\sigma}$  since the unconstrained rent-seeking government has higher equilibrium path deficits than the benevolent government. Thus the fiscal rule reduces the government deficit along the equilibrium path and increases public saving.

More surprisingly, the rule binds in economies in which  $\sigma > \bar{\sigma}$  so that the unconstrained rent-seeking government has a *lower* equilibrium path deficit than the benevolent government in the early phase of the boom. Therefore, even though the fiscal rule imposes a cap on deficits, it actually induces the rent-seeking government to borrow *more* than it would if it were unconstrained. The reason for this is that the rule works through expectations by reducing the value of *future* public funds. More specifically, in this region unconstrained governments over-save because they look forward to squandering public funds in the future if the boom persists for sufficiently long. The fiscal rule however makes it impossible to squander these public funds in the future since it forces a future government to save more. Therefore, the rule reduces the value of future funds from today's perspective, and this induces today's government to save less.

Note that the rule induces the government to consume more (tax less) and to extract more rents than it would if it were unconstrained along the equilibrium path.<sup>17</sup> This is because since the marginal value of funds in the future is lower, the current government decides to use funds for itself today, and it does so in the form of higher consumption and higher rent-seeking. This means that the government will begin to extract rents at an earlier date than it would in the absence of rules, since rent-seeking begins at higher levels of debt in comparison to an economy in the absence of rules.

Finally, note that while a fiscal deficit rule can force a rent-seeking government to save in the same fashion as the benevolent government, it cannot control the composition of public spending. Specifically, the government continues to squander resources on rents as

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<sup>17</sup>More specifically, the fiscal rules induce more consumption at high levels of debt and more rent-seeking at intermediate levels of debt.

opposed to cutting taxes if the boom is sufficiently prolonged or if initial resources are very abundant. This suggests that a deficit rule must be combined with a cap on taxes, so as to achieve the social optimum.

## 6 Final Remarks

We developed a dynamic political economy model of debt that characterizes public debt and deficits along the transitional path and in the long run. This allowed us to re-examine the conventional wisdom regarding the nature of political distortions. Our main result is that in the short run phase of a boom—when the level of public debt is still high—it matters whether the government faces high or low economic volatility. While the conventional wisdom of under-saving holds in the latter case, it does not in the former. If economic volatility is high, politicians *over-save* in the short run by keeping taxes too high.

In future work we intend to extend our analysis of fiscal policy in high economic volatility environments. The natural next steps are to study the qualitative and welfare properties of a broad class of fiscal rules found in practice,<sup>18</sup> and to pursue empirical work aimed at aligning these different rules with the characteristics of different countries and regions.

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<sup>18</sup>See for example Azzimonti, Battaglini, and Coate (2008) for an analysis of a balanced budget amendment to the US constitution.

## 7 Appendix

### 7.1 Proofs

#### 7.1.1 Proof of Lemma 1

**Step 1.**  $V^B(b, s)$  is strictly decreasing in  $b$  since a potential solution for  $b - \epsilon$  for  $\epsilon > 0$  arbitrarily small lets  $c^B(b - \epsilon, H) = c^B(b, H) + \epsilon$  and  $b'^B(b - \epsilon, H) = b'^B(b, H)$  which satisfies all constraints and strictly raises welfare. It is strictly concave in  $b$  since the objective function is strictly concave and the constraint set is convex. Differentiability follows from the standard arguments of Benveniste and Sheinkman (1979).

**Step 2.** First order conditions and the envelope condition imply that  $V_b^B(b, H) = -u_c(c^B(b, H))$ , which by step 1 implies that  $c^B(b, H)$  is decreasing in  $b$ . These also imply that  $V_b^B(b, H) = \alpha V_b^B(b', H, H) + (1 - \alpha) V_b^B(b', H, L)$  so that  $b'^B(b, H)$  is strictly increasing in  $b$ .

**Step 3.** If  $b'^B(b, H) \geq b$ , then from step 2,  $u_c(c^B(b', H)) \geq u_c(c^B(b, H))$ , which from (8) implies  $c^B(b', L) \geq c^B(b, H)$ . However, given the budget constraint (3), this contradicts  $b'^B(b, H) \geq b$ . **Q.E.D.**

#### 7.1.2 Proof of Lemma 2

**Step 1.** Given the characterization of policies in the text and the dynamic budget constraint, we can write

$$V^P(b, L) = \frac{u(\min\{e - \sigma - b(1 - \beta), u_c^{-1}(\theta)\})}{1 - \beta} + \theta \max\left\{0, \frac{e - \sigma - u_c^{-1}(\theta)}{1 - \beta} - b\right\} \text{ and}$$

$$V^N(b, L) = \frac{u(\min\{e - \sigma - b(1 - \beta), u_c^{-1}(\theta)\})}{1 - \beta}.$$

**Step 2.** All of the properties follow from this characterization. **Q.E.D.**

#### 7.1.3 Proof of Lemma 3

**Step 1.** In a  $T$  period economy, define

$$\underline{b}_t = (e + \sigma - \max\{u_c^{-1}(\theta), 2\sigma + u_c^{-1}(\theta(1 - \alpha q)/(1 - \alpha))\}) \left(\sum_{k=0}^{T-t} \beta^k\right) \quad \forall t \leq T \text{ and}$$

$$\underline{b}_{T+1} = 0.$$

Define  $\widehat{b}_t = e + \sigma - u_c^{-1}(\theta) + \beta b_{t+1}$

**Step 2.** Let the economy begin in the boom in period 0. If  $T = 0$  and  $b_T \geq \widehat{b}_T$ , then the policies of the benevolent government are chosen since those entail  $c_T^P(b_T, H) \leq u_c^{-1}(\theta)$  and  $x_T^P(b_T, H) = 0$ . If  $b_T < \widehat{b}_T$ , then first order conditions and the budget constraint yield  $c_T^P(b_T, H) = u_c^{-1}(\theta)$  and  $x_T^P(b_T, H) > 0$ . Let  $V_t^j(b_t, s)$  correspond to the value of  $V^j(\cdot)$  at date  $t$  in a finite period economy and let  $V_{tb}^j(b_t, s)$  correspond to its derivative with respect to  $b$ . We can write

$$V_{tb}^P(b_t, H) = \begin{cases} -u_c(c_t^P(b_t, H)) & \text{if } b_t \geq \widehat{b}_t \\ -\theta & \text{if } b_t \leq \widehat{b}_t \end{cases} \quad \text{and} \quad (17)$$

$$V_{tb}^N(b_t, H) = \begin{cases} -u_c(c_t^P(b_t, H)) & \text{if } b_T > \widehat{b}_T \\ 0 & \text{if } b_T < \widehat{b}_T \end{cases}, \quad (18)$$

and all of the the properties follow from this characterization at  $T$ .

**Step 3.** If  $T = 1$ , consider  $V_t^P(b_t, H)$  for  $t = 0$  given step 2 and Lemma 2.  $V_t^P(b_t, H)$  is decreasing in  $b_t$  since the constraint set is tighter and it is continuously differentiable in  $b_t$  by the arguments of Benveniste and Sheinkman (1979). If  $x_t^P(b_t, H) > 0$ , then necessarily  $b_t^{P'}(b_t, H) = \underline{b}_{t+1}$ . To see why, first order conditions imply that if  $x_t^P(b_t, H) > 0$ , then  $c_t^P(b_t, H) = u_c^{-1}(\theta)$ . Consider first if  $\sigma \leq \sigma^*$  so that  $\underline{b}_{t+1} = \widehat{b}_{t+1}$ . If  $b_t^{P'}(b_t, H) > \underline{b}_{t+1}$ , then (14) cannot hold. If  $b_t^{P'}(b_t, H) < \underline{b}_{t+1}$ , then (15) cannot hold, where we have used the fact that  $\sigma \leq \sigma^*$ . If instead  $\sigma > \sigma^*$ , then if  $b_t^{P'}(b_t, H) > \underline{b}_{t+1}$ , neither (14) nor (15) can hold and if  $b_t^{P'}(b_t, H) < \underline{b}_{t+1}$ , then (15) cannot hold. This implies that  $x_t^P(b_t, H) > 0$  only if  $b_t < \widehat{b}_t$  and  $x_t^P(b_t, H) = 0$  otherwise. This implies the properties of the lemma for  $V_t^P(b_t, H)$  and  $V_t^N(b_t, H)$  for  $b_t < \widehat{b}_t$ . If  $b_t > \widehat{b}_t$ , then  $V_t^P(b_t, H)$  is strictly concave in  $b_t$  since  $x_t^P(b_t, H) = 0$  and the objective is strictly concave. Therefore, (17) holds at  $t = 0$ . Moreover, since  $V_t^N(b_t, H)$  equals  $V_t^P(b_t, H)$  plus expected future rents, the arguments of Benveniste and Sheinkman (1979) imply that  $V_{tb}^N(b_t, H) = V_{tb}^P(b_t, H)$  in this region.

**Step 4.** Successive application of Step 3 taking  $T$  to  $\infty$  yields the result. **Q.E.D.**

#### 7.1.4 Proof of Proposition 1

**Step 1.** Define  $\bar{b}$  as

$$\bar{b} = \begin{cases} e - u_c^{-1}(\alpha u_c(\theta) + (1 - \alpha) u_c(u_c^{-1}(\theta) + 2\sigma)) + \sigma + \beta \underline{b} & \text{if } \sigma \leq \sigma^* \\ e - u_c^{-1}(\theta) + \sigma + \beta \underline{b} & \text{if } \sigma > \sigma^* \end{cases}$$

**Step 2.** The fact that  $b^P(b, H) = \underline{b}$  if  $b \leq \bar{b}$  and property (iii) for  $\sigma > \sigma^*$  follows

from step 3 of the proof of Lemma 3. The fact that  $b'^P(b, H) = \underline{b}$  if  $b \leq \bar{b}$  and property (ii) for  $\sigma \leq \sigma^*$  follows from (14) and (15) together with the envelope condition that  $V_b^P(b, H) = -u_c(c(b, H))$ .

**Step 3.** Given (14), if  $b'^P(b, H) \geq b$ , then necessarily  $u_c(c^P(b'^P(b, H), H)) \geq u_c(c^P(b, H))$  from the envelope condition since  $x^P(b, H) = 0$  for  $b > \bar{b}$ . This then implies  $c^P(b'^P(b, H), L) \geq c^P(b, H)$ , but this is a contradiction given the dynamic budget constraints. Therefore,  $b'^P(b, H) < b$  if  $b > \bar{b}$ .

**Step 4.** Substitute the envelope condition into (14) and (15) to achieve:

$$V_b^P(b, H) = \alpha V_b^P(b'^P(b, H), H) + (1 - \alpha) V_b^P(b'^P(b, H), L) \quad \text{and} \quad (19)$$

$$V_b^P(b, H) = \alpha q V_b^P(b'^P(b, H), H) + (1 - \alpha) V_b^P(b'^P(b, H), L), \quad (20)$$

respectively. Since the right hand side of (19) and (20) are strictly decreasing in  $b'^P(b, H)$ , it must be that  $b'^P(b, H)$  is single valued, continuous, and is strictly increasing in  $b$  whenever  $b'^P(b, H) \neq e + \sigma - u_c^{-1}(\theta) + \beta \underline{b}$ . If  $b'^P(b, H) = e + \sigma - u_c^{-1}(\theta) + \beta \underline{b}$ , then  $V_b^P(b'^P(b, H), H) \in [-\theta, -q\theta]$  and  $V_b^P(e + \sigma - u_c^{-1}(\theta) + \beta \underline{b}, L)$  is single-valued, which implies that

$$b \in \left[ \begin{array}{l} (e + \sigma)(1 + \beta) + u_c^{-1}(\alpha q \theta + (1 - \alpha) V_b^P(e + \sigma - u_c^{-1}(\theta) + \beta \underline{b}, L)) + \beta u_c^{-1}(\theta) + \beta \underline{b}, \\ (e + \sigma)(1 + \beta) + u_c^{-1}(\alpha \theta + (1 - \alpha) V_b^P(e + \sigma - u_c^{-1}(\theta) + \beta \underline{b}, L)) + \beta u_c^{-1}(\theta) + \beta \underline{b} \end{array} \right],$$

so that  $b'^P(b, H)$  is continuous in this region of  $b$  and is constant. **Q.E.D.**

### 7.1.5 Proof of Proposition 2

**Step 1.** Proposition 1 implies that  $b_{t+1}^P \in [\underline{b}, b_t^P)$  if  $b_0 \geq \underline{b}$ . Since  $b_{t+1}^P$  is monotonic and bounded it must converge. It cannot converge to any point other than  $\underline{b}$  since  $b'^P(b, H) < b$  for  $b > \underline{b}$ . If  $b_0 < \underline{b}$ , then Proposition 1 implies that  $b_{t+1}^P = \underline{b} \forall t$ . Finally, Proposition 1 implies that  $c^P(\underline{b}, H) = u_c^{-1}(\theta)$ .

**Step 2.** Lemma 1 implies that  $b_{t+1}^B \in (-\infty, b_t^P)$ . It cannot be that  $\lim_{t \rightarrow \infty} b_{t+1}^B = b_\infty^B > -\infty$  since  $b'^P(b, H) < b$  for all  $b$ . Given (3), this implies that  $\lim_{t \rightarrow \infty} c_t^B = \infty$ .

### 7.1.6 Proof of Lemma 4

**Step 1.** We first show that  $\underline{\sigma}$  and  $\bar{\sigma}$  exist and are uniquely defined. Let  $c_t^j$  for  $j = H, L$  correspond to the equilibrium value of consumption at date  $t$  as a function of the shock

$j$  for an economy beginning with debt  $b_0$  and state  $s_0$ . We can manipulate (3) to write

$$-\left(b_t - \frac{b_{t-1}}{\beta}\right) = -\frac{1}{\beta}c_{t-1}^H + \frac{1}{\beta}(e + \sigma).$$

Note that  $c_0^L = e - \sigma - b_0(1 - \beta)$ , and more generally  $c_t^L = e - \sigma - b_t(1 - \beta)$ . Substitution into the above equation then yields a difference equation for consumption in the downturn

$$c_t^L = \frac{1}{\beta}c_{t-1}^L - \frac{1}{\beta}(1 - \beta)c_{t-1}^H + \frac{1}{\beta}2\sigma(1 - \beta). \quad (21)$$

Therefore  $c_t^L$  is increasing in  $c_{t-1}^L$  and decreasing in  $c_{t-1}^H$ . Substitution of this equation into the Euler equation yields

$$u'(c_t^H) = \frac{u'(c_{t-1}^H) - (1 - \alpha)u'\left(\frac{1}{\beta}c_{t-1}^L - \frac{1}{\beta}(1 - \beta)c_{t-1}^H + \frac{1}{\beta}2\sigma(1 - \beta)\right)}{\alpha}. \quad (22)$$

Therefore  $c_t^H$  is increasing in  $c_{t-1}^H$  and decreasing in  $c_{t-1}^L$ .

**Step 2.** The path of consumption follows (21) and (22) subject to  $c_0^L = e - \sigma - b_0(1 - \beta)$  and  $c_0^H$  chosen to satisfy the present value budget constraint of the government

$$\sum_{t=0}^{\infty} \beta^t c_t^H = \sum_{t=0}^{\infty} \beta^t (e + \sigma) - b_0. \quad (23)$$

Define  $\underline{b}$  and  $\bar{b}$  as under the case for  $\sigma > \sigma^*$ . Consider  $b_0 = \underline{b}$ . An increase in  $\sigma$  leaves  $c_0^L$  unchanged and raises the right hand side of (23). If  $c_0^H$  weakly declines then forward iteration on (21) and (22) implies that  $c_t^H$  decline for all  $t$ , violating (23). Therefore  $c_0^H$  strictly increases in  $\sigma$ . If  $\sigma = 0$ , then  $u_c(c_0^H) = \theta(1 - \alpha q) / (1 - \alpha) > \theta$ . If  $b_0 = \bar{b}$ , an increase in  $\sigma$  reduces  $c_0^L$ . If  $c_0^H$  weakly declines then forward iteration on (21) and (22) implies that  $c_1^L$  weakly increases so that  $c_t^H$  weakly decreases for all  $t$ , violating (23). Therefore  $c_0^H$  strictly increases in  $\sigma$ . If  $\sigma = 0$ , then  $u_c(c_0^H) > q\theta$  under either  $b_0 = \underline{b}$  or  $b_0 = \bar{b}$ . As  $\sigma$  approaches  $\infty$ ,  $\underline{b}$  and  $\bar{b}$  approach  $-\infty$  so that  $c_0^L$  approaches  $\infty$ , and  $u_c(c_0^H)$  approaches  $0 < q\theta$ . Therefore,  $\underline{\sigma}$  and  $\bar{\sigma}$  exist and are uniquely defined.

**Step 3.** Properties (iv) and (v) follow from steps 1 and 2.

**Step 4.** An increase in  $q$  reduces  $\underline{b}$  and  $\bar{b}$  with no effect on  $c_0^L$ . This raises  $c_0^H$  by the arguments of step 2, so that  $u_c(c_0^H)$  decreases whereas  $q\theta$  increases. An increase in  $\alpha$  raises  $\underline{b}$  and  $\bar{b}$  with no effect on  $c_0^L$ . This reduces  $c_0^H$  by the arguments of step 2, so that  $u_c(c_0^H)$  decreases whereas  $q\theta$  is unchanged. This establishes property (ii).

**Step 5.** As  $q$  approaches 1,  $\underline{b}$  approaches  $(e - \sigma - u_c^{-1}(\theta)) / (1 - \beta)$  and  $\bar{b}$  approaches

$2\sigma + (e - \sigma - u_c^{-1}(\theta)) / (1 - \beta)$ , establishing property (iii). For any  $\sigma$ ,  $u_c(c_0^H) < \theta$ , since otherwise  $c_1^L > c_0^H$ , yielding a contradiction.

**Step 6.** To establish property (i), imagine if  $\sigma \geq \bar{\sigma}$  so that  $u_c(c^B(\bar{b}, H)) \leq q\theta$ . By Lemma 3,  $u_c(c^B(\underline{b}, H)) < u_c(c^B(\bar{b}, H)) \leq q\theta$ , which from step 3 implies that  $\sigma > \underline{\sigma}$ . Therefore,  $\bar{\sigma} > \underline{\sigma}$ . Imagine if  $\sigma = \sigma^*$ . Then  $\underline{b} = (e + \sigma - u_c^{-1}(\theta)) / (1 - \beta)$  and  $u_c(c^B(\underline{b}, H)) > \theta > q\theta$  since  $b'^B(b, H) < \underline{b}$  by Lemma 3. Therefore,  $\sigma^* < \underline{\sigma}$ . **Q.E.D.**

### 7.1.7 Proof of Proposition 3

**Step 1.** Since  $b'^P(b, H) = \underline{b} \forall b \leq \underline{b}$  from Proposition 1, then from Lemma 1,  $b'^P(b, H) > b'^B(b, H) \forall b \leq \underline{b}$ .

**Step 2.** Imagine if  $\sigma \leq \sigma^*$ . Then  $c^P(\underline{b}, H) = u_c^{-1}(\theta) > c^B(\underline{b}, H)$  since  $x^P(\underline{b}, H) = 0$  from Proposition 1. If  $b \in [\underline{b}, \bar{b}]$ , then from Proposition 1,  $b'^P(b, H) = \underline{b}$ , and since  $x^P(\underline{b}, H) = 0$ , the Euler equation implies that

$$u_c(c^P(b, H)) \leq \alpha u_c(c^P(\underline{b}, H)) + (1 - \alpha) u_c(c^P(\underline{b}, L)). \quad (24)$$

Since  $c^P(\underline{b}, L) = c^B(\underline{b}, L)$  but  $c^P(\underline{b}, H) > c^B(\underline{b}, H)$ , then in order that (8) hold given (24), it must be that  $b'^B(b, H) < b'^P(b, H)$  and  $c^B(b, H) < c^P(b, H)$  in this region.

**Step 3.** If  $b \in [\bar{b}, b'^{P-1}(\bar{b}, H)]$ , then from Proposition 1  $b'^P(b, H) \in [\underline{b}, \bar{b}]$ , and from step 2,  $c^P(b'^P(b, H), L) = c^B(b'^P(b, H), L)$  but  $c^P(b'^P(b, H), H) > c^B(b'^P(b, H), H)$ . Therefore, in order that (8) hold, it must be that  $b'^B(b, H) < b'^P(b, H)$  and  $c^B(b, H) < c^P(b, H)$  in this region. Successive applications of this argument until the natural debt limit implies that  $b'^B(b, H) < b'^P(b, H)$  and  $c^B(b, H) < c^P(b, H) \forall b \geq \bar{b}$ .

**Step 4.** Imagine if  $\sigma \geq \sigma^*$ . For any  $b \in [\bar{b}, b'^{P-1}(\bar{b}, H)]$ ,  $b'^P(b, H) \in [\underline{b}, \bar{b}]$ , and (15) holds since  $x^P(b'^P(b, H), H) > 0$  from Proposition 1. Since  $c^P(b'^P(b, H), L) = c^B(b'^P(b, H), L)$  but  $c^B(b'^P(b, H), H) < q\theta$ , then in order that (8) hold given (15) it must be that  $b'^B(b, H) < b'^P(b, H)$  and  $c^B(b, H) < c^P(b, H)$  in this region.

**Step 5.** If  $b'^P(b, H) = \bar{b}$ , then since  $c^B(b'^P(b, H), H) < c^P(b'^P(b, H), H)$ , then given (14) and (15), in order that (8) hold it must be that  $b'^B(b, H) < b'^P(b, H)$  and  $c^B(b, H) < c^P(b, H) \forall b \geq \bar{b}$ .

**Step 6.** Successive applications of step 4 can be applied then to show that  $b'^B(b, H) < b'^P(b, H)$  and  $c^B(b, H) < c^P(b, H) \forall b$  s.t.  $b'^P(b, H) > \bar{b}$ . **Q.E.D.**

### 7.1.8 Proof of Proposition 4

**Step 1.** Given the definition of  $\bar{\sigma}$ ,  $c^P(b, H) = u_c^{-1}(\theta) < u_c^{-1}(q\theta) < c^B(b, H)$  for  $b \leq \bar{b}$ .

**Step 2.** For any  $b \in [\bar{b}, b'^{P-1}(\bar{b}, H))$ ,  $b'^P(b, H) \in [\underline{b}, \bar{b})$ , and (15) holds since  $x^P(b'^P(b, H), H) > 0$  from Proposition 1. Since  $c^P(b'^P(b, H), L) = c^B(b'^P(b, H), L)$  but  $c^B(b'^P(b, H), H) > q\theta$ , then in order that (8) hold it must be that  $b'^B(b, H) > b'^P(b, H)$  and  $c^B(b, H) > c^P(b, H)$  in this region.

**Step 3.** If  $b'^P(b, H) = \bar{b}$ , then since  $u_c^B(b'^P(b, H), H) < \theta$ , then given (14) and (15), in order that (8) hold it must be that  $b'^B(b, H) > b'^P(b, H)$  and  $c^B(b, H) > c^P(b, H) \forall b \geq \bar{b}$ .

**Step 4.** Successive applications of Step 2 then imply that  $b'^B(b, H) > b'^P(b, H)$  and  $c^B(b, H) > c^P(b, H) \forall b$  s.t.  $b'^P(b, H) > \bar{b}$ . **Q.E.D.**

### 7.1.9 Proof of Proposition 5

**Step 1.** Given (3), (16) implies that  $\tilde{b}_{t+1}^P \leq b_{t+1}^B$  along the equilibrium path, where  $\tilde{b}_{t+1}^P$  corresponds to the equilibrium level of debt under a politician constrained by the deficit rule and  $b_{t+1}^B$  corresponds to the equilibrium level of debt under a benevolent government.

**Step 2.** Consider an economy in final period  $T$  in which  $\tilde{b}_T^P \leq b_T^B$ . If (16) does not bind, then this implies that  $\tilde{b}_{T+1}^P < b_{T+1}^B = 0$ , implying that the rent-seeking government can strictly raise welfare by raising  $\tilde{c}_T^P$  or  $\tilde{x}_T^P$  and increasing  $\tilde{b}_{T+1}^P$ . Therefore, (16) binds at  $T$ .

**Step 3.** Consider an economy in period  $t < T$  in which (16) binds for all  $k > t$  if  $\tilde{b}_k^P \leq b_k^B$ . If (16) does not bind at  $t$ , then this implies that  $\tilde{b}_{t+1}^P < b_{t+1}^B$ . Given that (16) binds for all  $k > t$ , this implies that  $\tilde{b}_{T+1}^P < b_{T+1}^B = 0$ . This implies that the rent-seeking government strictly raise welfare by raising  $\tilde{c}_t^P$  or  $\tilde{x}_t^P$  and increasing  $\tilde{b}_{t+1}^P$ , leaving  $\tilde{c}_k^P$  and  $\tilde{x}_k^P$  unchanged for all  $k > t$  since this increases  $\tilde{b}_{T+1}^P$ . Therefore, (16) binds at  $t < T$ .

**Step 4.** By forward induction, (16) binds for all  $t$  and as  $T \rightarrow \infty$ . **Q.E.D.**

## 7.2 Intermediate Volatility: $\sigma \in (\underline{\sigma}, \bar{\sigma})$

In this section, we briefly describe the region of intermediate volatility which we do not consider in the text. Given the definitions of  $\underline{\sigma}$  and  $\bar{\sigma}$ , there exists a cutoff point  $\tilde{b} \in [\underline{b}, \bar{b}]$  s.t.  $u_c(c^B(\tilde{b}, H)) = q\theta$  so that  $u_c(c^B(b, H)) < (>) q\theta$  if  $b < (>) \tilde{b}$ . Consider  $b \in [\underline{b}, b'^{P-1}(\tilde{b}))$ . The application of step 2 in the proof of Proposition 4 implies that  $b'^B(b, H) > b'^P(b, H)$  and  $c^B(b, H) > c^P(b, H)$  in this region. Moreover, for  $b \in (b'^{P-1}(\tilde{b}), b'^{P-1}(\bar{b}))$ , then the application of step 4 in the proof of Proposition 3 implies that  $b'^B(b, H) < b'^P(b, H)$  and  $c^B(b, H) < c^P(b, H)$  in this region.

Now consider the region for which  $b'^P(b, H) = \bar{b}$ . Equation (15) holds with equality

at a minimum value of  $b$  in this region, which means that  $b'^B(b, H) < b'^P(b, H)$  and  $c^B(b, H) < c^P(b, H)$  at this point. Equation (14) holds with equality at the maximum point in this region, which means that  $b'^B(b, H) > b'^P(b, H)$  and  $c^B(b, H) > c^P(b, H)$  at this point. Since  $b'^P(b, H) = \bar{b}$  in this region and since  $b'^B(b, H)$  is monotonically increasing, there exists a cutoff point  $\tilde{\tilde{b}}$  which splits the region such that if  $b < \tilde{\tilde{b}}$  and  $b$  is in this region then  $b'^B(b, H) < b'^P(b, H)$  and  $c^B(b, H) < c^P(b, H)$ , and if  $b > \tilde{\tilde{b}}$  and  $b$  is in this region then  $b'^B(b, H) > b'^P(b, H)$  and  $c^B(b, H) > c^P(b, H)$ .

Therefore, we can apply step 4 in the proof of Proposition 3 to the set of  $b$ 's for which  $b'^P(b, H) \in \left(\tilde{\tilde{b}}, \tilde{\tilde{b}}\right)$  and show that  $b'^B(b, H) < b'^P(b, H)$  and  $c^B(b, H) < c^P(b, H)$ .

Analogously, we can find a cutoff  $\tilde{\tilde{b}}$  such that we can apply step 2 in the proof of Proposition 4 to the set of  $b$ 's for which  $b'^P(b, H), H \in \left(\tilde{\tilde{b}}, \tilde{\tilde{b}}\right)$  and show that  $b'^B(b, H) > b'^P(b, H)$  and  $c^B(b, H) > c^P(b, H)$ . Forward iteration on this argument implies that there is a sequence of regions between  $\bar{b}$  and the natural debt limit in which there is either over-borrowing and over-spending or over-saving and under-spending.

Thus, the path taken by the economy depends on the region in which  $b_0$  is located. If  $b_0$  is in the over-borrowing region, then over-borrowing occurs along the equilibrium path until  $\bar{b}$  is passed and if  $b_0$  is in the over-saving region, then over-saving occurs along the equilibrium path until  $\bar{b}$  is passed.

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