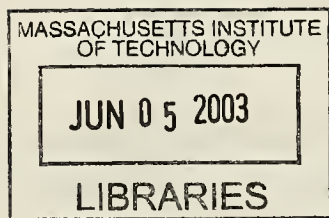Massachusetts Institute of Technology
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# MODERATION OF AN IDEOLOGICAL PARTY

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# Moderation of an Ideological Party*

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## Abstract

It is a common fear in many countries that ideological parties will
come to power through elections but will implement extreme policies
or even end the democratic regime. Many countries cope with this
problem by overriding the election results when such parties are elected.
In a two-period model, we demonstrate that the alternative approach of
containing these parties within the democratic system is more effective.
In equilibrium, if an ideological party (IP) comes to power in the first
period, depending on its type (i.e., its extremity), it either reveals its
type or chooses a moderate policy in order to be elected again. We
show that, as the probability of state's intervention in the next elections
increases, IP's policy becomes more extreme: fewer types choose to
moderate and, and when they do, they moderate less. This hurts the
median voter. It also remains true when the probability of intervention
depends on IP's policy. We further show that from the median voter's
perspective, the optimal intervention scheme can be implemented by
committing not to intervene and adjusting election times appropriately.
That is, elections are a better incentive mechanism than the threat of a
coup. Our results are extended to a model in which IP can try a coup.

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# 1    Introduction

How should a democratic regime defend itself against those political parties that would come to power through democratic means but would implement their extreme policies—policies inconsistent with the state's fundamental principles—or even end the democratic regime in order to establish their own ideological system? A major approach towards addressing this problem is that of confrontation: the state bans such parties, prosecutes individuals who form such parties, and intervenes in election results whenever such parties come to power. This approach is apparently taken by many countries, such as Algeria, Chile, and Turkey. For example, in Turkey, the national intelligence agency is controlled by the army, which has carried out several coups; the army has its own courts and is allowed to defend the system against internal enemies according to its internal code. Moreover, there are state security courts that regularly outlaw such parties (mainly Kurdish or Islamic) and prosecute their leaders. An alternative approach to addressing this problem would aim to contain ideological parties within the system by allowing them to come to power. In this paper, we analyze a formal game theoretical model that supports this latter approach. In equilibrium, if an ideological party comes to power, it implements moderate policies in order to win the next elections.[1] Moreover, when the probability of a state-intervention in election results increases, elections become less important, and thus the ideological party moderates less. This suggests that the state's interventionist organization might even be causing the polarized political spectrum in the above countries.[2] Most importantly, from the median voter's point of view, the optimal organization of the state can be reached by committing not to intervene and adjusting election times appropriately. Surprisingly, schemes that respond to an ideological party's previous policies are not optimal. Therefore, elections are better incentive mechanisms than threats of intervention.

We consider a one-dimensional policy space and an ideological party (IP),

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[1] This form of moderation is very familiar in Turkish political life. Pro-Islam Welfare Party, which was outlawed after the army's intervention in February 28th, 1998, was accused of *Takiyye*, the practice of hiding one's beliefs in order to survive (and perhaps to change the conditions in the future). On November 3rd, 2002, another pro-Islam party (AKP) has been elected, and many observers (and apparently the markets) expect this party to choose moderate policies—at least for a while.

[2] We show this by taking individuals' political preferences as given. These preferences may have deeper economic, ethnic and cultural roots. For example, both Turkey and Algeria have experienced radical transformation in their history. Turkey has transformed from multiethnic, Islam based Ottoman Empire to a secular nation state. Today, Turkey's two major conflicts are Secularism vs. religious rights and Nationalism vs. ethnic rights of Kurds.

whose preferences are its private information. (The policy that IP finds best is called its type.) The alternative to IP is a fixed point in the policy space. We have two periods. In each period, there is an election between IP and its alternative. If IP wins, it implements a policy for that period, which is observable. Otherwise, the alternative is implemented. After the second-period election, if IP wins, the state overrides the election results with some probability $q$ and implements the alternative.

We first characterize the set of sequential equilibria of this basic game that pass the Intuitive Criterion of Cho and Kreps (1987) for signaling games and in which each voter votes as if he is pivotal. Each such equilibrium can be summarized with two parameters $a$ and $b$, where $a < b$. IP wins the next election if and only if its policy in the first period is not more extreme than $a$, and IP chooses to implement such a policy if IP is not more extreme than $b$. In the first period, if its type is in between $a$ and $b$, IP chooses to moderate and implements $a$; otherwise, it implements the policy it finds best. (In the last period, if IP comes to power, it implements its best policy.) Here, $a$ is the least moderate policy that the median voter expects IP of a moderate type to implement in order to signal its type convincingly, and $b$ is the most extreme type who is willing to moderate in order to win the next election.

These parameters change with respect to the probability $q$ of a state intervention in the next election as follows. If $q$ decreases, then winning the next elections becomes more important, and hence IP's gain from moderation increases regardless of its type. Now IP can afford to moderate even if it is of more extreme types that could have not moderated before. Knowing this, voters now expect IP to implement more moderate policies if it is of a moderate type, i.e., $a$ is lower. Even though $a$ is lower now, given the high gain from moderation, IP can now afford to implement $a$ even if it is of some more extreme types that could not afford to moderate before, i.e., $b$ is now higher. A lower $a$ and a higher $b$ mean that now the set of types of IP who choose to moderate at the first period is larger, and they moderate more. Hence, IP of any given type now implements (if anything) a more moderate policy. Therefore, a lower probability of the state's intervention in the next elections causes IP to implement a less extreme policy in the first period. This remains true when $q$ depends on the first-period policy: when we decrease $q$ equally at each first-period policy, IP implements more moderate policies in the first period. We further show that the median voter gains from such moderation, provided that it does not lead IP to implement policies on the other side of the median, an event that will be referred to as overmoderation.

Now suppose that we can institute any function $q$ of the first-period policy as the probability of the state's intervention, by choosing an appropriate

organization of the state. What would be the optimal $q$—i.e., the $q$ that maximizes the median voter's payoff? We show that the optimal $q$ is a constant function: there exists some $q^*$ in $(0,1)$ with some equilibrium $e^*$ such that for every function $\tilde{q}$ and every associated equilibrium $\tilde{e}$, the median voter prefers the equilibrium $e^*$ under constant probability $q^*$ of intervention to the equilibrium $\tilde{e}$ under $\tilde{q}$. The parameters for equilibrium $e^*$ are $a^*$ and $b^*$ where $a^*$ is the median voter's ideal policy and $b^*$ is such that the median voter would be indifferent between IP and the alternative if he just knew that IP is not more extreme than $b^*$. We show that there cannot be any equilibrium (under any function $q$) in which IP moderates when it is more extreme than $b^*$. Therefore, $e^*$ both leads to the ideal policy of the median voter in the first period *and* allows moderation for any type that could possibly moderate. In order this to be an equilibrium, $q^*$ must be very small: for any $q < q^*$, IP loses the next election in any equilibrium if it implements the median voter's ideal policy. In reality, such a probability of intervention could be implemented by committing not to intervene and adjusting the election times so that the discount rate of IP is $1 - q^*$.

It is rather surprising that $q^*$ is constant—not increasing. One may naively think that optimal probability $q^*$ must be increasing as such a scheme would lead IP to moderate more. It turns out that this is not good in equilibrium. In that case, for a given $a$, when IP's type is in between $a$ and some $\tilde{a} > a$, IP will implement policies that are more moderate than $a$ in order to decrease the probability of intervention. Then, when the median voter observes that $a$ is implemented, he knows that IP is more extreme than $\tilde{a}$. In order for him to vote for IP, we must have smaller $b$. Therefore, in such equilibria, we will either have over-moderation of relatively moderate types or non-moderation of relatively extreme types—and typically both.

We also consider the case that IP can try to end the democratic system. We consider a game in which, if IP comes to power in the first period, before the next elections, it can try a coup with cost $C$ (borne by IP) and with probability $p$ of success. If IP's coup is successful, it cancels the next elections and implements the policy that it finds best; if it fails, it also loses the next elections. In equilibrium, some very extreme types of IP try a coup; these types do not moderate, either. When $C/p$ is sufficiently low, IP tries a coup whenever it chooses not to moderate. Keeping $C$ and $p$ constant, we again show that, as $q$ increases, the set of equilibria shifts in the direction of less moderation (with higher $a$ and lower $b$). In that case, the probability that IP tries a coup also weakly increases.

Another natural question is how a change in $q$ affects the ideology of IP, i.e., its preferences. To address this question, we take IP to be an organization of its

members. We assume that, prior to the basic game described above, a member is chosen (via some unspecified mechanism) to control IP. The chosen member then imposes his own preferences on IP. Assuming that the voting population cannot distinguish between the party members, we derive members' induced preferences for candidates, for a fixed $q$ and equilibrium $e$ that will be played in the basic game. We show that members' preferences on candidates (induced by $e$ and $q$) are single peaked, and each member finds the candidates of his own type best. Therefore, the members who are located at the median of members' locations (with respect to their preferences on the policy space) are Condorcet winners.[3] This median is independent of both $e$ and $q$. Therefore, assuming that a Condorcet winner is chosen, IP's ideology is invariant to the equilibrium $e$ and to the probability $q$ of the state's intervention. This is despite the fact that equilibrium parameters affect the shape of the utility functions. Each member finds the candidates located in between $a$ and $b$ similar, as they choose the same policy in the first period. Likewise, since IP's policy choice is discontinuous at $b$, any two candidates located at either sides of $b$ will be viewed significantly different, even though their political views may be almost identical.

## 2    Basic Model

We consider a single-dimensional policy space $X = \mathbb{R}$ with a generic member $x$,[4] and a time space $T = \{0,1\}$ with a generic member $t$. Our main actors are *voters* (who are distributed on a line $Y = \mathbb{R}$ with total population-measure of 1) and an *ideological party* (denoted by IP). The alternative to IP is a given policy $s \in X$, which is the ideal policy of the state. We will assume that the median voter is located at 0 and that $s < 0$. The justification for this assumption is that the median voter's ideal policy is likely to differ from the policy that is best for the representative bureaucrat (representing the state officials who can carry out a coup), which can also be observed from the voting

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[3] A candidate is a Condorcet winner if and only if, for any other candidate (located at any other point), there exists a majority of the members who (strictly) prefer him to the other. When there is a Condorcet winner, the voting mechanisms typically choose a Condorcet winner under various solution concepts utilized in the literature. Note that the single-peakedness would not yield Condorcet winners when the policy space is not one-dimensional. In fact, for a multi-dimensional policy space, presence of Condorcet winner is rather an exception (see for instance, Plott (1967) and McKelvey,1976).

[4] Here. $\mathbb{R}$ denotes the set of real numbers. Given any $f : \mathbb{R}^n \to \mathbb{R}$, $f_i$ denotes the partial derivative with respect to $i$th coordinate, and $f_{ij}$ denotes $(f_i)_j$. When $n = 1$, we will simply write $f'$ for the derivative of $f$.

data in the above countries. (When $s = 0$, the problem becomes trivial; IP loses the elections.)

The order of the events is as follows.

1. There is an election at $t = 0$; each voter located at some $y \in Y$ privately votes — either for IP or for the alternative. If a majority[5] vote for IP, then IP wins; otherwise, alternative $s$ is chosen.

2. If IP wins, then it chooses a policy $x_0 \in X$, which becomes public information; otherwise $s$ is implemented.

3. At $t = 1$, there is another election as in period 0.

4. If IP wins the election at $t = 1$, then there will be a coup with probability $q \in (0, 1)$, yielding $s$.

5. If IP wins and there is no coup, then IP chooses some $x_1 \in X$.

IP has a type $z$, which is its private information, drawn from the set $Z = (s, \infty)$ with support $\mathbb{R}_+$, cumulative distribution function (CDF) $F$, and probability distribution function (pdf) $f$.[6] Notice that we assume that $f(z) > 0$ for each $z > 0$ and $\Pr(z < 0) = 0$. Here, $F$ represents the voters' common belief at the beginning; after observing IP's choice $x_0$, they update their beliefs. We assume that, if IP loses the election at $t = 0$, the voters adhere to their initial beliefs—consistent with our assumption that no voter has any private information about IP. We write $E$ and $E[\cdot|\cdot]$ for the unconditional and conditional expectations, respectively.

Assuming that the agents care only about the policy implemented, we write $u(x, y)$ and $w(x, z)$ for the per-period benefit of any policy $x$ for the voters located at any $y \in Y$ and for IP of any type $z \in Z$, respectively. We normalize $u$ and $w$ so that $u(s, y) \equiv w(s, z) \equiv 0$. Each agent maximizes the sum of his two per-period benefits. Everything described above is common knowledge.

Throughout the paper, we will assume that $u$ and $w$ are twice continuously differentiable and satisfy the following properties.

**A1.** Both $w$ and the logarithm of $w$ are supermodular: $\partial^2 w(x, z) / \partial x \partial z > 0$ and $\partial^2 \log(w(x, z)) / \partial x \partial z > 0$ whenever they are defined.

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[5] By a majority, we mean a group of voters whose measure is at least $1/2$.

[6] We restrict $z$ to $(s, \infty)$ for simplicity. In equilibrium, if allowed, IP of any type $z < s$ would always choose $x_0 = z$, revealing her type and losing the next election.

**A2.** For each $y \in Y$ and $z \in Z$, the functions $u(\cdot, y)$ and $w(\cdot, z)$ are single-peaked with maximum at $y$ and $z$, respectively.

**A3.** At any $x > s$, $u(x, \cdot)$ and $w(x, \cdot)$ are strictly increasing, i.e., $u_2(x, \cdot) > 0$ and $w_2(x, \cdot) > 0$.

**A4.** $E\left[u\left(z, 0\right)\right] < 0$.

The part $u_2(x, \cdot) > 0$ of A3 will guarantee that IP wins an election if and only if the median voter votes for it; the assumption $w_2(x, \cdot) > 0$ and A1 will play a crucial role in our monotone comparative statics and in separating IP's types (see Lemma 4). A4 states that the median voter would not vote for IP if he had no information about IP and believed that IP would implement the policy that it finds best.

**Example** A standard special case of these utility functions can be derived from Euclidean preferences as follows. Let the utility of an individual located at some $y$ from a policy $x \in X$ be $-v(x - y)$ where $v : \mathbb{R} \to \mathbb{R}$ is an even and strictly convex function. After the normalization $u(s, y) \equiv w(s, y) \equiv 0$, we have

$$u(x, y) = w(x, y) = -v(x - y) + v(s - y) \tag{1}$$

at each $x, y \in \mathbb{R}$. One can easily check that these functions satisfy our assumptions A1-A3 whenever the mapping $\zeta \mapsto 1/v(\zeta)$ is convex on $\mathbb{R}_+$, the canonical case.

To simplify our exposition, we assume that the measure of the voters who vote for IP is not observed, and we allow only two outcomes for each election: either IP wins, or IP does not win. Under this restriction, $x_0$ is a function of $z$. If IP comes to power at $t = 0$, then $x_1$ is a function of $x_0$ and $z$, and each voter's second-period vote is conditioned on $x_0$ and his location $y$; otherwise, $x_1$ is a function of $z$, and a voter's vote only depends on his location.

A sequential equilibrium $e^*$ is a pair of a *sequentially rational* strategy profile and posterior beliefs for the voters (after observing $x_0$) that are *consistent* with the strategy profile. That is, at each history each player maximizes his expected utility given his beliefs at that history and given that that history is reached, and the voters' beliefs are derived through Bayes' rule at each $x_0$ that is implemented by IP of some type $z$.

# 3 Equilibria

In this section, we characterize the sequential equilibria that satisfy the two requirements below. We also explore certain properties of these equilibria.

## 3.1 Equilibrium Criteria

Our game contains voting processes with infinitesimal voters and a signalling game, each generating an excessive number of equilibria. Therefore, we will refine our equilibria by imposing the following two requirements. First, we require that *each voter votes as if he is pivotal*. That is, given his beliefs, at any time he will vote for IP if and only if his expected benefit from IP's victory at that election is at least as high as his expected benefit from its loss. Since we have only two alternatives, the other actions are weakly dominated given the voter's beliefs at each history. Under this assumption, IP wins if and only if the median voter votes for IP, and the game is reduced to a game between IP and the median voter.

Towards describing our second requirement, we observe that, at any sequential equilibrium $e^*$, the policy chosen by IP at $t = 1$ is

$$x_1^*(z) = z \quad \text{for each } z \in Z. \tag{2}$$

That is, in the last period, IP implements the policy that it finds best. Therefore, after the history that IP comes to power at $t = 0$, there is a signaling game $S$: IP with private information $z \in Z$ chooses some $x_0(z)$; observing $x_0$, voters vote — for or against IP. Our second requirement is that *the substrategy profile for $S$ passes the Intuitive Criterion* of Cho and Kreps (1987), defined in the appendix. We write $SE(q)$ for the set of sequential equilibria that satisfy our two conditions.

The payoffs of voters and IP are as in Table 1. The future benefits are discounted by the effective discount rate $\delta = 1 - q$, because with probability $q$, there will be a coup and everyone will get 0.[7]

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[7] This is only because we have assumed that individuals are indifferent between IP's defeat and a coup. In reality, a coup has its social cost, and voters would prefer $s$ being implemented without a coup to the one with a coup. In that case, scared by the possibility of a coup following IP's victory, voters would vote against IP, even when they believed that, given the chance, IP would choose some $x$ that is better than $s$. In such cases, those who oppose IP would threaten the voters by a coup. See Ellman and Wantchekon (2000) for a similar point.

| | If IP wins | If IP loses |
|---|---|---|
| payoff of IP of type $z$ | $w(x_0, z) + \delta w(z, z)$ | $w(x_0, z)$ |
| payoff of a voter at $y$ | $u(x_0, y) + \delta u(z, y)$ | $u(x_0, y)$ |

Table 1: Payoffs for the imbedded signaling game $S$, where $\delta = 1 - q$.

## 3.2  Characterization of Equilibria

We now characterize the sequential equilibria that satisfy our two requirements. We first derive the basic necessary conditions for an equilibrium. In any equilibrium, IP wins the elections at $t = 1$ if and only if $x_0 \leq a$ for some $a \in X$, and IP implements such a policy if and only if $z \leq b$ for some $b \in \mathbb{R}$. We present some necessary and sufficient conditions on $a$ and $b$. The proofs are in the appendix.

Notice that, by (2) and our first requirement, after observing $x_0$, a voter at a location $y$ votes for IP at $t = 1$ if and only if $E[u(z,y)|x_0] \geq 0$. By A3, $u(z,y)$ is strictly increasing in $y$, hence $E[u(z,y)|x_0]$ is strictly increasing in $y$, too. Consequently, the median voter is indicative: when the median voter votes for IP, the voters who are located to the right of the median also vote for IP; when the median voter does not vote for IP, neither do the voters located to the left of the median. Therefore, given $x_0$, IP wins the elections at $t = 1$ if and only if

$$E[u(z,0)|x_0] \geq 0. \tag{3}$$

**Theorem 1** *IP chooses policies $x_0^*$ and $x_1^*$ in an equilibrium at periods 0 and 1, respectively, if and only if*

$$x_0^*(z) = \begin{cases} a & \text{if } z \in [a, b], \\ z & \text{otherwise} \end{cases} \quad \text{and} \quad x_1^*(z) = z \quad \forall z \in Z$$

*for parameters $a$ and $b$ that satisfy the conditions*

$$\int_a^b u(z,0)f(z)dz \ \geq \ 0, \tag{4}$$

$$u(b, 0) \ \leq \ 0, \tag{5}$$

*and*

$$w(a, b) = qw(b, b). \tag{6}$$

*In any such equilibrium IP wins the election at $t = 1$ if and only if $x_0 \leq a$.*
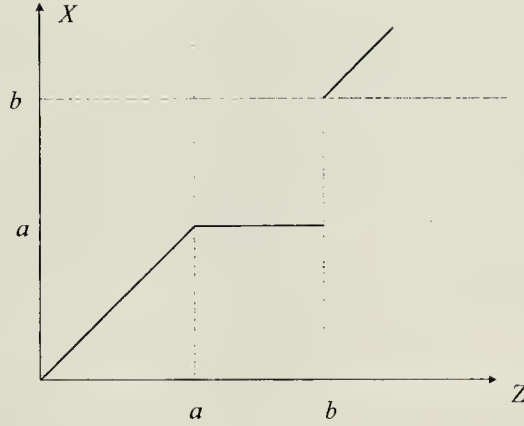
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Figure 1: The policy $x_0^*$ chosen by IP at $t = 0$, as a function of her type $z$.

Here, $a$ is the most extreme policy that the median voter tolerates; $b$ is the most extreme type who can afford to moderate. That is, if it is of some "moderate" type $z \leq b$, IP implements a moderate policy $x_0(z) \leq a$ at $t = 0$ so that it wins the elections at $t = 1$.[8] If it is of some "extreme" type $z > b$, then it implements the "extreme" policy $z$, losing the next elections. This is a partially separating equilibrium: IP reveals its type by implementing its best policy at $t = 0$, except when $z \in [a, b]$, in which case it implements $a$.

Consistent with $x_0^*$, the voters infer that IP's type $z$ is $x_0$ when they observe that IP implements some $x_0 < a$ or $x_0 > b$. If they observe that IP implements the policy $x_0 = a$, all they can infer is that $z \in [a, b]$. That is,

$$E\left[u(z,0)|x_0^*(z) = a\right] \equiv \frac{1}{F(b) - F(a)} \int_a^b u(z,0)f(z)dz. \tag{7}$$

Hence, (4) is equivalent to the requirement that the median voter votes for IP in the next elections if IP implements $x_0 = a$ in the first period. Likewise, (5) requires that, if the median voter observes that IP is of type $b$, he does not have a strict incentive to vote for IP. This is necessary in equilibrium because, by definition of $b$, the median voter is not supposed to vote for IP when IP implements $x_0 = b + \epsilon$ and reveals that its type is $b + \epsilon$ for any $\epsilon > 0$. Finally,

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[8]It will be clear that IP of type $b$ is indifferent between implementing $a$ and $b$ at $t = 0$. Since $z = b$ is a zero probability event, this yields a multiplicity of equilibria in which IP of type $b$ mixes between $a$ and $b$. We will ignore this multiplicity at this zero-probability event.

(6) is implied by (and actually equivalent to) the equilibrium condition that $x_0^*$ is a best response to the belief that IP wins the elections at $t = 1$ if and only if it implements a policy $x_0 \leq a$. To see this, take any $z \in Z$ with $z > a$. Now IP of type $z$ has two options: (i) it can either implement policy $a$ at $t = 0$, win the elections at $t = 1$, and thereby (if there is no coup) implement $z$ at $t = 1$, or (ii) it can implement $z$ at $t = 0$ and lose the next elections, yielding the policy $s$ at $t = 1$. (No other strategy can be a best response.) Its payoffs for these two strategies are $w(a, z) + (1 - q) \, w(z, z)$ and $w(z, z)$, respectively. Hence, its *net gain from moderation* is

$$R(a, z) = w(a, z) - qw(z, z). \tag{8}$$

Now IP of a given type $z > a$ may choose to moderate if and only if $R(a, z) \geq 0$. Since IP of type $b$ chooses to moderate (i.e., $x_0^*(b) = a$), we must have $R(a, b) \geq 0$. Likewise, for any $z > b$, since $x_0^*(z) = z$, we must have $R(a, z) \leq 0$. Since $R$ is continuous, we therefore have $R(a, b) = 0$, which is equivalent to (6).

By Theorem 1, we can summarize equilibria with two real numbers, namely, $a$—the most extreme policy level median voters can tolerate at $t = 0$, and $b$—the most extreme type who can afford to implement $a$ so that it can win the next election, after which it chooses its best policy. Our next result presents simpler conditions that $a$ and $b$ must satisfy; these conditions are equivalent to the conditions (4), (5), and (6).

**Lemma 1** *Given any $q$ and $b > s$, there exists a unique solution $a^{IP}(b, q)$ to (6), where $a^{IP}(b, q) \in (s, b)$. The function $a^{IP}$ is differentiable with partial derivatives*

$$\frac{\partial a^{IP}}{\partial b} = -\frac{R_2(a^{IP}(b, q), b)}{w_1(a^{IP}(b, q), b)} > 0 \tag{9}$$

*and*

$$\frac{\partial a^{IP}}{\partial q} = \frac{w(b, b)}{w_1(a^{IP}(b, q), b)} > 0. \tag{10}$$

*Moreover, (4) and (5) are satisfied by $\left(a^{IP}(b, q), b\right)$ if and only if $\underline{b} \leq b \leq \bar{b}(q)$ where $u(\underline{b}, 0) = 0$ and $\bar{b}(q)$ is the unique solution to $E\left[u(z, 0) | a^{IP}(b, q) \leq z \leq b\right] = 0$. Hence, the set of all equilibrium parameters is*

$$SEP(q) = \{(a, b) \in X \times Z \, | \, a = a^{IP}(b, q), \underline{b} \leq b \leq \bar{b}(q)\}. \tag{11}$$

*Here, $SEP(q)$ is an ordered set with maximal member $\left(\bar{a}(q), \bar{b}(q)\right)$ where $\bar{a}(q) = a^{IP}\left(\bar{b}(q), q\right)$. Finally, $\bar{b}$ is decreasing in $q$.*

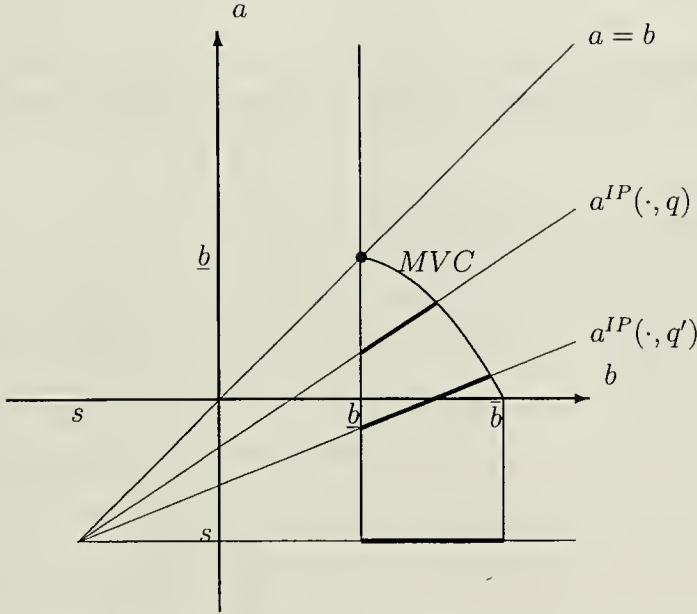The functions in our lemma take a simple form in our canonical example.

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Figure 2: The set of equilibrium parameters (with homogenous $v$ in the Example), and its change with respect to $q$.

**Example** (*continued*) Since $v$ is even, $u(-s, 0) = 0$, hence $\underline{b} = -s$. Equation (6) becomes $v(a - b) = (1 - q) v(s - b)$, hence $a^{IP}$ has a simple explicit form:

$$a^{IP}(b, q) = b - v^{-1}\left((1 - q) v(s - b)\right),$$

where $v^{-1} : \mathbb{R}_+ \to \mathbb{R}_+$ is the inverse of $v$ on $\mathbb{R}_+$. When $v$ is homogenous,

$$a^{IP}(b, q) = (1 - v^{-1}(1 - q))b + v^{-1}(1 - q) s.$$

This case is illustrated in Figure 2.

Here, $a^{IP}(z, q)$ is the most moderate policy level IP of type $z$ can afford in order to win the next election. By (9), $a^{IP}(z, q)$ is increasing in $z$. That is, IP of a more moderate type (with a lower $z$) can afford to moderate more. It also follows that, for each $a$, there exists a unique $b$ that satisfies (6). Moreover, by (10), $a^{IP}$ is also increasing in $q$. Hence, as the probability $q$ of a coup decreases, the graph of $a^{IP}$ shifts down, enticing IP of each type to moderate more. In that case, the voters will also expect IP of a moderate type to implement a more moderate policy $x_0$ to signal his type convincingly. Note that the equilibrium parameters are the part of the graph of $a^{IP}$ that is in the region bounded by $\underline{b}$ and the curve $MVC$, defined by $E[u(z, 0)|a \le z \le b] = 0$. Note also that $\underline{b}$ and $MVC$ are solely determined by the median voter's preferences

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and do not depend on $q$. Therefore, when we decrease $q$, $\bar{b}(q)$ increases and $\bar{a}(q)$ decreases. This leads to a shift in equilibria in the direction of more moderation of IP's policy.

# 4   Moderation of Policy

In this section, we will show that, if the probability $q$ of a coup decreases, or equivalently if the effective discount rate $\delta = 1 - q$ increases, IP chooses a more moderate policy.

Since IP is typically located above 0 (the policy that the median voters find best), we say that IP *chooses a more moderate policy* if and only if IP of each type chooses the same or a lower policy-level at each date. In equilibrium, in the last period, IP always implements the policy that it finds best. Hence, given any two equilibria $e^*$ and $e^{**}$ with first-period policy levels $x_0^*$ and $x_0^{**}$, respectively, we say that *IP chooses a more moderate policy in $e^*$ with respect to $e^{**}$* and write $e^* \succeq e^{**}$ if and only if $x_0^*(z) \leq x_0^{**}(z)$ at each $z \in Z$. By Theorem 1, there exist $(a, b)$ and $(a', b')$ such that

$$x_0^*(z) = \begin{cases} a & \text{if } z \in [a, b], \\ z & \text{otherwise} \end{cases} \quad \text{and} \quad x_0^{**}(z) = \begin{cases} a' & \text{if } z \in [a', b'], \\ z & \text{otherwise.} \end{cases}$$

Clearly, $x_0^*(z) \leq x_0^{**}(z)$ at each $z \in Z$ if and only if $a \leq a'$ and $b \geq b'$. Therefore,

$$[e^* \succeq e^{**}] \iff [a \leq a' \text{ and } b \geq b']. \tag{12}$$

Since we have multiple equilibria, we extend $\succeq$ to the sets of equilibria. Given any two sets $SE^*$ and $SE^{**}$ of equilibria (of games with possibly different parameters), we write $SE^* \succeq SE^{**}$ if and only if the following two properties hold:

1. For each $e^* \in SE^*$, there exists some $e^{**} \in SE^{**}$ such that $e^* \succeq e^{**}$, and

2. for each $e^{**} \in SE^{**}$, there exists some $e^* \in SE^*$ such that $e^* \succeq e^{**}$.

In that case, we say that IP chooses (if anything) more moderate policies in equilibria $SE^*$ with respect to $SE^{**}$. Note that, if $SE^* = \{e^*\}$ and $SE^{**} = \{e^{**}\}$, then $SE^* \succeq SE^{**}$ if and only if $e^* \succeq e^{**}$. Graphically, in Figure 2, IP chooses more moderate policies if and only if the set of equilibria shifts towards the south-east. In Figure 2, this happens when $q$ decreases. Our next result shows that this is true in general.

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**Theorem 2** *Given any two probabilities $q, q' \in (0, 1)$ of coup with $q \geq q'$, we have $SE(q') \succeq SE(q)$. That is, IP chooses more moderate policies in equilibria when the probability of a coup is lower.*

**Proof.** (See Figure 2.) Take any $q$ and $q'$ as in the hypothesis. To check the second condition towards $SE(q') \succeq SE(q)$, take any equilibrium $e \in SE(q)$ with parameters $(a, b) \in SEP(q)$. Since $\bar{b}(q) \leq \bar{b}(q')$, we have $(a^{IP}(b, q'), b) \in SEP(q')$. Consider the equilibrium $e' \in SE(q')$ with parameters $(a^{IP}(b, q'), b)$. By (10), $a^{IP}$ is increasing in $q$, hence $a^{IP}(b, q') \leq a^{IP}(b, q) \equiv a$. Therefore, $e' \succeq e$. To check the first condition, take any $e' \in SE(q')$ with parameters $(a', b') \in SEP(q')$. If $b' \leq \bar{b}(q)$, the equilibrium $e \in SE(q)$ with parameters $(a^{IP}(b', q), b') \in SEP(q)$ satisfies the condition $e' \succeq e$. So, assume that $b' > \bar{b}(q)$. Now, if $\bar{a}(q) \equiv a^{IP}(\bar{b}(q), q) < a^{IP}(b', q') \equiv a'$, then $(\bar{a}(q), \bar{b}(q)) < (a', b')$ contradicting the maximality of $(\bar{a}(q), \bar{b}(q))$. Hence, $\bar{a}(q) \geq a'$. Thus, the equilibrium $\bar{e} \in SE(q)$ with parameters $(\bar{a}(q), \bar{b}(q))$ satisfies our required relation $e' \succeq \bar{e}$. Therefore, $SE(q') \succeq SE(q)$. ∎

Intuitively, as the probability $q$ of a coup decreases, the discount rate $\delta$ increases. In that case, given any required level of moderation in order to win the next elections, the gain from moderation for IP of any given type increases, and hence IP of more extreme types can afford to chose moderate policies. In that case, the voters expect IP of a "moderate type" to implement more moderate policies in order to signal its type convincingly. This typically results in two changes:

1. The required level of moderation $a$ in order to win the next election becomes more moderate.

2. Given the high gain from moderation, IP of some more extreme types can now afford to implement even this more moderate required policy (i.e., $b$ is now higher).

These two changes have two important consequences. First, at each date, if IP comes to power, IP of any given type implements (if anything) some more moderate policy. Second, IP reveals less information about its type, in the sense that whatever a voter can learn could have been learned before. In a multi-period model this gives IP more room for moderation.

As $q$ approaches 1, the set of equilibrium parameters $SEP(q)$ approaches $\{(\underline{b}, \underline{b})\}$, where there is no moderation, i.e., IP always implements $z$. (See Figure 2.) In that case, since $E[u(z, 0)] < 0$, IP loses the election at $t = 0$. On the other hand, as $q$ approaches 0, $SEP(q)$ approaches the set $\{s\} \times [\underline{b}, \bar{b}(0)]$, where IP is required to imitate its alternative $s$ in order to win the next election.
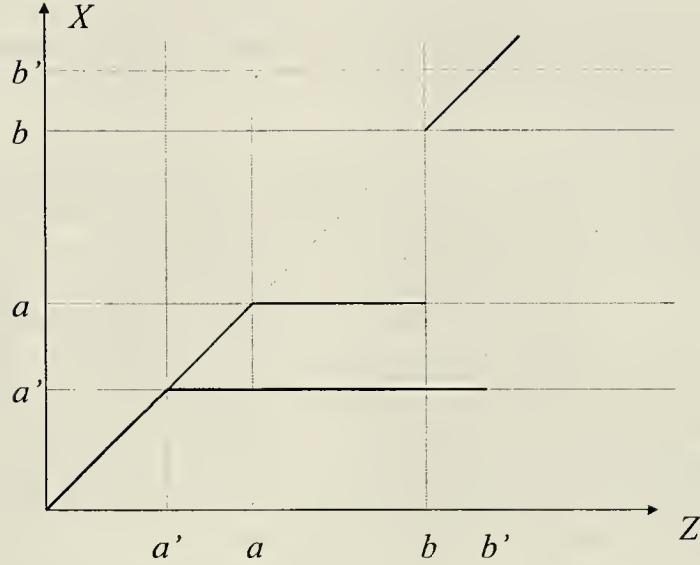
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Figure 3: The policies chosen by IP in the first period in equilibria $(a, b)$ and $(a', b')$ under $q$ and $q'$, respectively.

**Median Voter's Welfare** Since IP wins the next election when it chooses a moderate policy, the moderation is not necessarily good for everybody, even for some moderate voters. We now show that the median voter gains when the probability $q$ of a coup decreases, enticing IP to choose a more moderate policy, provided that this does not result in "overmoderation," i.e., $a$ remains non-negative.

**Theorem 3** *Given any $q, q' \in (0, 1)$ with $q > q'$, let $(a, b)$ and $(a', b')$ be any two equilibrium parameters corresponding to the probabilities $q$ and $q'$ of coup, respectively. Consistent with Theorem 2, assume that $a > a' \geq 0$ and $b < b'$. Then, the median voter prefers the equilibrium with $(a', b')$ and the lower probability $q'$ of coup to the equilibrium with $(a, b)$ and the higher probability $q$ of coup.*

**Proof.** (See Figure 3 for illustration.) Let us change $(a, b, q)$ to $(a', b', q')$ as in the theorem and compare the equilibrium outcomes from the median voter's point of view for each possible type $z$ of IP. If $z \leq a'$, then IP would chose $z$ in both periods independent of the probability of coup. Hence, the only change is now we have a lower probability of coup, when the inferior

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policy $s$ is implemented in the second period rather than $z \le a'$. (Recall that $u(z,0) > u(a',0) \ge 0 = u(s,0)$ for $z < a'$.) We refer the welfare effect of decreasing the probability of implementing $s$ after a coup as the direct effect. Consider the case $a' < z < a$. Before the change, the policies chosen in the first and the second periods were both $z$, but now IP chooses $a' < z$ in the first period, benefitting the median voter. The lower probability of coup also benefits the median voter. Now consider the case that $a \le z \le b$. Before the change, IP was choosing the policies $a$ and $z$ in the first and the second periods, respectively. Under $q'$, it chooses $a' < a$ in the first period and $z$ in the second period, benefiting the median voter once again. Now the direct effect of lowering the probability of a coup depends on $z$. This effect is positive for lower values of $z$, and negative for the higher values of $z$. But by the equilibrium condition (7), the expected value $E[u(z,0)|a \le z \le b]$ of this direct effect is non-negative. Now consider the case $b < z \le b'$. Before the change, IP would have chosen $z$ in the first period and lost the elections, after which $s$ would have been implemented, yielding the negative payoff of $u(z,0)$ for the median voter. Now IP chooses $a'$ in the first period and $z$ in the second period. The payoff of the median voter is $u(a',0) + (1 - q')u(z,0)$. Now the median voter gets the negative payoff $u(z,0)$ only with probability $1 - q'$, and gets the positive payoff $u(a',0)$ extra; his overall payoff is higher. Finally, if $z > b'$, nothing has changed—IP chooses $z$ in the first period and $s$ is implemented in the second period. Since the moderation benefits the median voter at each region, it benefits him in expectation. ∎

Theorem 3 establishes that the median voter would prefer the state to commit to low probabilities of a coup in order to entice IP to moderate, so long as it does not lead IP to overly moderate and implement policies that are close to the status quo—a contingency that is usually ignored in public discourse. Overmoderation becomes an issue because of our simplifying assumption that IP does not discount the future; if IP discounted its second period payoffs with discount rate $\delta \le 1 - q^*$, then the median voter would want the state to commit zero probability of coup, where $q^*$ is the optimal probability of coup in the present model, as defined below.

# 5   General Case and Optimal Coup Scheme

We take the probability $q$ of a coup exogenous so that we can show the causality —that increasing $q$ causes more extreme policies. In previous sections, we further assumed that this probability did not depend on $x_0$, but one might expect $q(x_0)$ to be increasing. In this section, we allow probability of coup to

be any function $q\left(\cdot\right)$ of $x_0$ and illustrate how our result can be extended to the general case. We then show that, among all functions of $x_0$, the optimal probability of coup (from the median voter's point of view) is indeed a constant.

Notice that the probability of a coup may be independent of $x_0$, and yet may vary as the organization of the state varies. For example, when a coup is motivated by the coup leaders' desire for power and these leaders do not need the approval of the voters, $q$ will not depend on $x_0$. But $q$ will still depend on the organization of the state, e.g., it will depend on the organization of the agencies required for a successful coup, such as the army, the intelligence agencies, and the police. When the players discount the future, we can again vary $q$ independently of $x_0$ by varying the frequency of elections.

Our next result characterizes the equilibria when $q$ is an increasing and differentiable function of $x_0$.

**Theorem 4** *Given any twice differentiable, increasing function $\tilde{q}$ as the probability of a coup, assume that the maximand $\hat{x}\left(z\right)$ of*

$$w\left(x_0, z\right) + \left(1 - \tilde{q}\left(x_0\right)\right) w\left(z, z\right) \tag{13}$$

*is given by the first-order condition $w_1\left(x_0, z\right) = \tilde{q}'\left(x_0\right) w\left(z, z\right)$ and the second-order condition $w_{11}\left(x_0, z\right) < \tilde{q}''\left(x_0\right) w\left(z, z\right)$. Then, IP chooses policy $x_0^*$ in an equilibrium at $t = 0$ if and only if*

$$x_0^*\left(z\right) = \begin{cases} \hat{x}_0\left(z\right) & \text{if } z \leq \tilde{a}, \\ a & \text{if } \tilde{a} < z \leq b, \\ z & \text{otherwise} \end{cases} \quad \forall z \tag{14}$$

*for parameters $a$ and $b$ that satisfy the conditions*

$$w(a, b) = \tilde{q}\left(a\right) w(b, b), \tag{15}$$

$$u\left(b, 0\right) \leq 0, \tag{16}$$

*and*

$$\int_{\tilde{a}}^b u(z, 0) f(z) dz \geq 0, \tag{17}$$

*where $\tilde{a}$ is defined by $w_1\left(a, \tilde{a}\right) = \tilde{q}'\left(a\right) w\left(\tilde{a}, \tilde{a}\right)$. In any such equilibrium, $x_0^*\left(z\right)$ is increasing in $z$, and IP wins the election at $t = 1$ if and only if $x_0 \leq a$.*

**Sketch of the proof and explanation.** Similarly to the proof of Theorem 1, one can show that for any equilibrium there exists some $a$ such that IP wins the election if and only if $x_0 \leq a$. (Such monotonic voting behavior can

also be taken as a regularity condition.) Then, IP's payoff for any policy level $x_0 \leq a$ is given by (13). In considering policy levels $x_0 < a$, IP now also takes into account that, by varying $x_0$, it affects the probability $\tilde{q}(x_0)$ of a coup. Hence, when $\tilde{q}$ is strictly increasing, IP will choose a policy $\hat{x}_0(z)$ that is more moderate then its type $z$ and the required level $a$ for reelection. Since $w$ is log-supermodular, $\hat{x}_0(z)$ is increasing in $z$, and hence there will be some $\tilde{a} \geq a$ such that IP will implement such policies whenever $z \leq \tilde{a}$. Here, $\tilde{a}$ is computed through $\hat{x}_0(\tilde{a}) = a$. In considering policy levels $x_0 \geq a$, IP will make its policy decision on the basis of whether it will win the next election (at the fixed probability $\tilde{q}(a)$ of a coup). This is because, whenever IP implements a policy $x_0 > a$, it will lose the next elections anyway. Hence, as before, there will be a unique cutoff value $b \geq a$ defined by (15), the same condition as (6) for the probability $\tilde{q}(a)$ of coup fixed at $x_0 = a$. This cutoff value is such that the types that are more moderate than $b$ prefer to implement $x_0 = a$ and win the next election rather than implementing their best policy and losing the next election, while the types more extreme than $b$ prefer to implement their best policy and reveal their extreme type. Therefore, at $t = 0$, in equilibrium IP implements $x_0^*$, as defined in (14). Moreover, we will have (16), so that the median voter does not vote for IP when he observes $x_0 > b$, and (17), so that the median voter votes for IP when he observes $x_0 = a$. Notice that the integral in (17) is taken from $\tilde{a}$ to $b$ (rather than $a$ to $b$) because now the median voter knows that the types in $[a, \tilde{a})$ implement policies more moderate than $a$. ∎

Now the equilibrium behavior depends both on the level of $\tilde{q}$ and its slope. Our next result states that if we increase the probability of coup for each $x_0$ by a constant amount so that the slope does not change, then the equilibria will shift in the direction of less moderation.

**Theorem 5** *For functions $\tilde{q}$ and $\bar{q} = \tilde{q} + \Delta$ as the probabilities of a coup for some constant $\Delta \geq 0$, under the assumptions of Theorem 4, we have $SE(\tilde{q}) \succeq SE(\bar{q})$.*

**Sketch of the proof and explanation.** (See Figure 4 for the illustration.) Firstly, as before, for each $b$ there exist $a^{IP}(b, \tilde{q})$ and $a^{IP}(b, \bar{q})$ as solutions to (15) for $\tilde{q}$ and $\bar{q}$, respectively. Using the implicit function theorem, one can again show that $a^{IP}(b, \tilde{q})$ and $a^{IP}(b, \bar{q})$ are increasing in $b$ and $a^{IP}(b, \tilde{q}) \geq a^{IP}(b, \bar{q})$. For increasing $\tilde{q}$, the equilibrium condition (17) is different from its counterpart for constant $q$. But once again, for any $a$, there exists $b^{MV}(a, \tilde{q}')$ such that $(a, b)$ satisfies (17) if and only if $b \leq b^{MV}(a, \tilde{q}')$. Notice that $b^{MV}$ is decreasing in $a$ and depends on the derivative $\tilde{q}'$ and not the level $\tilde{q}$. Hence,
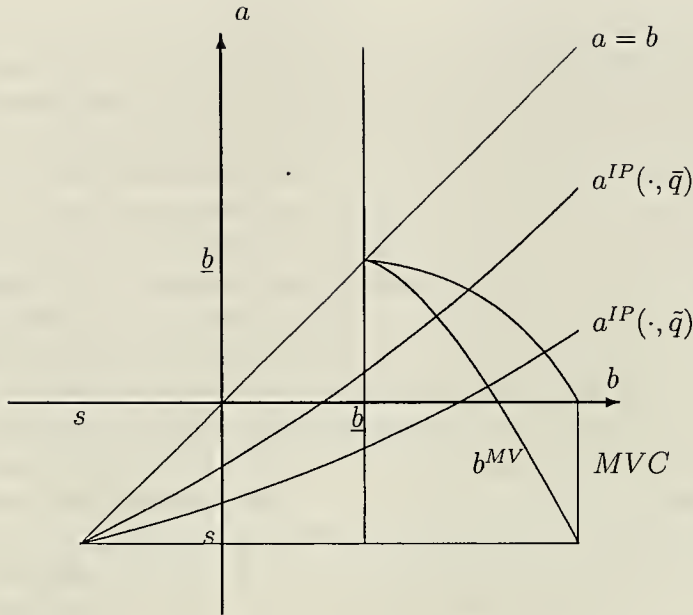
Figure 4: The set of equilibrium parameters for general case, which are given by the graph of $a^{IP}$ in the region bounded by $\underline{b}$ and $b^{MV}$.

we have $b^{MV}(a, \tilde{q}') = b^{MV}(a, \bar{q}')$. (Since $\tilde{a} \geq a$, the curve $b = b^{MV}(a, \tilde{q}')$ is below $MVC$.) Therefore, the equilibrium parameters are given by the graph of $a^{IP}$ in the region bounded by $\underline{b}$ and $b^{MV}$, as in Figure 4. Using the same arguments in the proof of Theorem 2, we can then complete the proof. ∎

**Optimal Probability of Coup**  Imagine the founders of the state, designing the state organization. If they want to make sure that the ideal state policy, namely $s$, is implemented, they either choose a very high probability $q \cong 1$ of coup, i.e., essentially a non-democratic state, or choose a very low probability $q \cong 0$ of coup, a state that is committed to democracy. In either case, IP will lose the elections at $t = 0$, and $s$ will be implemented throughout, because IP is assumed to be worse than the status quo for the median voter in expectation. What will they choose if they want to maximize the median voter's payoff? This question is of interest because the median voter is taken to be the representative agent in political science. Moreover, if the state's organization is determined through a democratic process, we would expect the selected organization to maximize the median voter's payoff at the time of organization. [The future median voter's payoffs will not be known at the time the state is organized, and in the future the median voter's ideal policy will likely to be different from $s$, the policy that is best for the representative

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bureaucrat (representing the state officials who can carry out a coup).] We will now show that the optimal coup scheme for the median voter is a constant $q^*$, which can be implemented by banning coups and setting election frequencies appropriately.

Towards stating our result, given any coup probability $\tilde{q} : X \to R$ (as a function of $x_0$), we write $SE_0(\tilde{q})$ for the set of all sequential equilibria in which each voter votes as if he is pivotal under $\tilde{q}$, and for any $e \in SE_0(\tilde{q})$, we write $U_0(e, \tilde{q})$ for the median voter's expected payoff at the node "IP comes to the power at $t = 0$" when equilibrium $e$ is played and the probability of coup is $\tilde{q}$. Notice that the median ex ante voter's payoff is $\max\{U_0(e, \tilde{q}), 0\}$. Notice also that by dropping the intuitive criterion in definition of $SE_0(\tilde{q})$, we obtain a stronger optimality result.

**Theorem 6** *The optimal coup scheme for the median voter is a constant. That is, there exists $q^* \in (0, 1)$ with equilibrium $e^* \in SE(q^*)$ such that for every integrable $\tilde{q} : X \to R$ and every $\tilde{e} \in SE_0(\tilde{q})$,*

$$U_0(e^*, q^*) \geq U_0(\tilde{e}, \tilde{q}).$$

**Proof.** In the appendix. ∎

The inequality states that the median voter prefers equilibrium $e^*$ and constant probability $q^*$ of a coup to every coup scheme $\tilde{q}$ and every associated equilibrium $\tilde{e}$. Recall that when the agents discount the future, we can implement such constant probability by making sure that coup does not occur and adjusting election times appropriately. Moreover, as will be clear in a moment, increasing coup schemes are typically inefficient. Therefore, elections are better incentive schemes than threats of a coup.

The equilibrium $e^*$ is defined by the parameters $(a^*, b^*) \in SEP(q^*)$ where $a^* = 0$ and $b^*$ is the unique intersection of the curve $MVC$ with the horizontal axis (see Figure 4); $b^*$ is defined by $E[u(z, 0) | 0 \leq z \leq b^*] = 0$. The optimal probability of coup is given by IP's optimization condition (6): $q^* = w(a^*, b^*)/w(b^*, b^*)$. Notice that, among all equilibria in $SE(q^*)$, $e^*$ is the equilibrium in which the median voter is most lenient towards IP, i.e., $a \leq a^*$ for each $(a, b) \in SEP(q^*)$. But $q^*$ is so small that even in this lenient equilibrium the median voter has very high expectations and will not vote for IP in the next elections if IP implements $x_0 > 0$; for any $q < q^*$, IP loses the next elections even when it implements the ideal policy $x_0 = 0$ of the median voter.

In our proof we show that for any $(\tilde{e}, \tilde{q})$ as in the theorem, there exists $\tilde{b} \leq b^*$ such that IP wins the elections at $t = 1$ if $z < \tilde{b}$ and loses the elections

at $t = 1$ if $z > \tilde{b}$.[9] This already shows that $e^*$ both induces the best possible first period policy *and* allows moderation for all types that can moderate in any equilibrium. Hence, as will be clearer in a moment, excluding the direct effect of the coup (defined in the proof of Theorem 3), the equilibrium $e^*$ is better than $\tilde{e}$ for each $z$, establishing an even stronger optimality result.

To elaborate further, let us compare equilibrium outcomes for each $z$. When $z \geq b^*$, the equilibrium outcome is the same in both equilibria. If $\tilde{b} < z < b^*$, in $(e^*, q^*)$, IP chooses $x_0^*(z) = 0$ at $t = 0$, yielding the highest possible payoff $u(0,0) > 0$ for the median voter, and $x_1^*(z) = z$ at $t = 1$, yielding a payoff of $(1 - q^*) u(z,0) < 0$, as there will be a coup with probability $q^*$. On the other hand, in $(\tilde{e}, \tilde{q})$, IP chooses $\tilde{x}_0(z) = z$ and loses the next elections yielding the very low payoff of $u(z,0) < 0$ for the median voter, and leaving him clearly worse off. When $z \leq \tilde{b}$, in the first period IP implements again the best policy for the median voter in $e^*$, and in both equilibria $e^*$ and $\tilde{e}$, IP wins the next elections and implements $x_1^*(z) = \tilde{x}_1(z) = z$. Now, although the median voter gets the best possible payoff at $t = 0$ in $(e^*, q^*)$, there is a potential advantage of $\tilde{q}$, as $\tilde{q}(\tilde{x}_0(z))$ may be increasing in $z$, making more extreme policies less likely to be implemented in the second period. It turns out that this advantage is small compared to all of these inefficiencies introduced—as our result establishes.

# 6   Hitler Syndrome

It is a common fear that an ideological party may come to power and end the democratic regime in order to establish its own ideological system. Kalaycioglu and Sertel (1995) call this Hitler syndrome. In this section, in the case that IP comes to power at $t = 0$, before the election at $t = 1$, we allow IP to try a coup that will succeed with some small probability $p$ and will cost $C$ to IP. If IP's coup is successful, IP cancels the election at $t = 1$ and implements a policy $x_1$, which will be $z$ in equilibrium. If its coup is unsuccessful, it loses the election.

As before, there exists a policy level $a \in X$ such that IP wins the election at $t = 1$ if and only if $x_0 \leq a$ and it has not attempted a coup. Therefore, IP of a type $z > a$ has the following options: it can either (i) choose $x_0 = a$ and

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[9] We use the assumption that $\Pr(z < 0) = 0$ for this result. Otherwise, the optimal coup scheme will be *decreasing* to prevent over-moderation!

In Figure 4, $\tilde{b}$ is given by the intersection of the graphs of $a^{IP}(\cdot, \tilde{q})$ and $b^{MV}$. As shown in the figure, when $\tilde{q}$ is increasing, the inequality is typically strict, preventing the types $z \in (\tilde{b}, b^*)$ from moderation.

not try a coup, which yields $w(a, z) + \delta w(z, z)$, or (ii) choose $x_0 = z$ without attempting a coup, which yields $w(z, z)$, or (iii) choose $x_0 = z$ and try a coup, which yields $(1 + p)w(z, z) - C$. No other strategy can be a best response.

To determine which option is best for which type, we first write

$$R(a, z; \pi) = w(a, z) - \pi w(z, z)$$

for the gain from moderation when the total probability of a coup by anybody is $\pi$, and the effective discount rate is $1 - \pi$. We already know that the option (i) is at least as good as (ii) if and only if $z \leq b$ where $b$ is defined by

$$R(a, b; q) = 0.$$

The option (ii) is at least as good as the option (iii) if and only if $z \leq d$ where $d$ is defined by

$$w(d, d) = \frac{C}{p}.$$

Finally, (i) is at least as good as (iii) if and only if $R(a, z; p + q) \geq -C$. By Lemma 4 in the appendix, this inequality holds if and only if $z \leq c$ where $c$ is defined by

$$R(a, c; p + q) = -C.$$

In summary, (i) is a best response when $z \leq \min\{b, c\}$; (ii) is a best response when $b \leq z \leq d$, and (iii) is a best response when $z \geq \max\{c, d\}$.

Define $a^*$ by

$$R(a^*, d; q) = 0.$$

Notice that $R(a^*, d; p + q) = -C$. Hence, when $a = a^*$, we have $b = c = d$. Moreover, when $a < a^*$, we have $b < c < d$, and when $a > a^*$, we have $b > c > d$. Therefore, we have two types of equilibria.

**Type 1 equilibria $(a \leq a^*)$.** As in our basic model, we have

$$x_0^*(z) = \begin{cases} a & \text{if } z \in [a, b], \\ z & \text{otherwise.} \end{cases}$$

IP tries a coup if and only if it is of some more extreme type $z > d$. Note that, if it is of some type $z \in (b, d]$, it does not try a coup even though it reveals its extreme type by implementing $z$.

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**Type 2 equilibria** $(a \geq a^*)$. We have

$$x_0^*(z) = \begin{cases} a & \text{if } z \in [a, c], \\ z & \text{otherwise,} \end{cases}$$

and IP tries a coup whenever it reveals its extreme type.

Which types of equilibria we have depends on the size of $d$, determined by $\frac{C}{p}$. If $d \leq \underline{b}$ (i.e., if the cost of a coup relative to the probability of success is very low), then we must have $d \leq b$, and hence we have only equilibria of second type, where IP tries a coup whenever it reveals its extreme type. When $d \geq \bar{b}$ (i.e., when the cost of a coup relative to the probability of success is very high), we have only equilibria of the first type. When $d \in (\underline{b}, \bar{b})$, both types exist.

In equilibrium, IP's policy remains essentially unchanged — except for a new cutoff value for large values of $a$. The new cutoff value, $c$, exhibits the same properties as $b$: For any total probability $p+q$ of a coup, define $a^H(\cdot; p+q)$ by $R(a^H(c; p+q), c; p+q) = -C$. By the implicit function theorem,

$$\frac{\partial a^H}{\partial c} = -\frac{R_2}{R_1} > 0 \tag{18}$$

and

$$\frac{\partial a^H}{\partial q} = -\frac{R_3}{R_1} > 0. \tag{19}$$

Thus, $a^H$ has all the properties of $a^{IP}$ that have been used in order to prove Theorem 2. Moreover, the other conditions on $(a, c)$ are determined by the median voter's incentives as before. Since $d$ is determined by $C/p$, assuming that these parameters do not vary when we change $q$, we get an equivalent of Theorem 2 in our extended model: *When the probability $q$ of a coup (by the state) decreases, IP implements more moderate policies in equilibrium. In that case, the probability that IP tries a coup, namely $\Pr(z > \max\{c, d\})$, also weakly decreases.*

# 7    Moderation of Ideology

Can the ideology of IP (i.e., its type) be changed by a change in the probability $q$ of a coup or by a change in the equilibrium $e$ that will be played? To answer this question, we now recognize that IP is an organization of its members, who determine IP's ideology. We assume that a member becomes the *leader* of the party, and plays the basic game above using his own preferences. We

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show that the member located at the median of the members with respect to their preferences on policy space $X$ is the Condorcet winner with respect to the members' induced preferences on the leader types, provided that the voters' beliefs about IP do not depend on who the leader is. In that case, if IP is organized in a way that the Condorcet winner is selected whenever it exists, IP's ideology will be independent of the probability $q$ of a coup and the equilibrium $e$ that will be played.

We assume that IP is an organization of its finitely many members $m \in M$, each of which is located in $Z$. We take $w(x, z)$ as the per-period benefit of a policy $x$ for a member located at some $z \in Z$. We assume that there exists a unique member $m^0$ located at the median $z^0$ of the members' locations. A member is selected as the *leader* and plays the basic game for IP. Since the leader is chosen once and for all, in equilibrium, he maximizes his own payoffs, effectively imposing his own type on IP. Finally, we assume that each member's location is common knowledge among themselves, but voters cannot distinguish the members from each other.

Take any equilibrium $e$ of the basic game with parameters $(a, b)$. If a member located at some $z \in Z$ becomes the leader and plays equilibrium $e$ for IP (using his own preferences), and if IP comes to power at $t = 0$, then the net benefit for a member located at some $y \in Z$ will be

$$
W^0(z, y; a, b, \delta) = \begin{cases} (1+\delta)w(z, y) & \text{if } z < a, \\ w(a, y) + \delta w(z, y) & \text{if } a \le z \le b, \\ w(z, y) & \text{if } b < z. \end{cases} \tag{20}
$$

If IP wins the election at $t = 0$ according to $e$, then the expected benefit for a member at $y$ from a leader located at $z$ is $W^0(z, y; a, b, \delta)$; otherwise, it is identically 0. We only consider the case that IP wins the election at $t = 0$; the other case is trivial. Two properties of $W^0$ are noteworthy. Firstly, if IP comes to power, all the members located in $[a, b]$ choose the same policy $a$ at $t = 0$, hence they are viewed similarly. (For instance, the slope of $W^0(\cdot, y; a, b, \delta)$ is $\delta w_1(z, y)$ at any $z \in (a, b)$, while it is $(1 + \delta) w_1(z, y)$ at any $z < a$.) Second, $W^0$ is typically discontinuous at $z = b$. Hence, the members located on the opposite sides of $b$ are viewed very differently even if their preferences on the policy space are very similar. The following property of $W^0$ is central to this section.

**Lemma 2** *Given any $y \in Z$, the function $W^0(\cdot, y; a, b, \delta)$ is a single-peaked function, and takes its maximum at $y$, i.e., it is strictly increasing on $(-\infty, y)$ and strictly decreasing on $(y, \infty)$.*

Lemma 2 is due to A1, and the assumption that the voters' beliefs about IP are independent of the leader. It states that each member prefers the members who are located more closely to themselves to be the leader. In particular, a member's best candidate is himself, because he can always choose the strategy that any other candidate plays. Since we have single-peaked preferences on a single-dimensional space of candidates (i.e., the members), the median candidate $m^0$ is the Condorcet winner. That is, given any other member $m$, a majority of the members finds $m^0$ strictly better than $m$. For $m^0$ and the members located on the other side of $m^0$ —a majority— prefer $m^0$ to $m$. But when there is a unique Condorcet winner (and if the preferences of members are common knowledge), voting mechanisms typically select the Condorcet winner. Examples of such mechanisms are binary-agenda voting with unlimited amendment rights (under subgame perfection, see Miller, 1980 and McKelvey, 1986) and the majoritarian voting rule with simultaneous voting (under strong Nash equilibrium, see for instance Sanver and Sertel, 1997). Therefore, under this very general prediction of social choice theory, $m^0$ will be selected as a leader, rendering $z^0$ as IP's ideology, independent of the probability $q$ of a coup and the equilibrium $e$ that will be played.

**Theorem 7** *Assume that IP is organized in such a way that, if there is a unique Condorcet winner as a candidate for leadership, then the Condorcet winner becomes the leader. Then, the median member $m^0$ becomes the leader and imposes his type $z^0$ as IP's type — independent of the probability $q$ of a coup and of the equilibrium $e$ that will be played in the basic game.*

In conclusion, if the internal politics are not observable by the voters, independent of the environment, the same member $m^0$ will be selected, and will impose the same ideology $z^0$. Nevertheless, this same leader will implement more moderate policies when the probability $q$ of a coup is lower.

# A    Appendix–Omitted Proofs

## A.1    Proof of Theorem 1

We first show that the conditions (4), (5), and (6) are sufficient for an equilibrium as described in the statement. We then complete the description of the equilibrium. Finally, we show that all equilibria that satisfy our criteria are in this form.

**Sufficiency of conditions (4), (5), and (6).**    The characterizing conditions for equilibrium as in our theorem are (i) IP wins the elections at $t = 1$ if and only if

$x_0 \leq a$, and (ii) $x_0^*$ is a best response to (i). (Below, inequalities (21) and (22) are (4) and (5), respectively.)

**Lemma 3** *The condition that IP wins the elections at $t = 1$ if and only if $x_0 \leq a$ is equivalent to*

$$E\left[u(z,0)|x_0^*(z) = a\right] \equiv E\left[u(z,0)|a \leq z \leq b\right] \equiv \frac{1}{F(b) - F(a)} \int_a^b u(z,0)f(z)dz \geq 0 \tag{21}$$

*and*

$$E\left[u(z,0)|x_0^*(z) = b\right] \equiv u(b,0) \leq 0. \tag{22}$$

**Proof.** By (3), (i) is equivalent to $E[u(z,0)|x_0] \geq 0 \iff x_0 \leq a$, which implies (21) and (22) as special cases. But (21) and (22) are also sufficient: We have $u(z,0) > u(b,0)$ at each $z \in [a,b)$, because $s \leq a < b$, $b > 0$, and $u(\cdot,0)$ is single-peaked with a maximum at 0 (by A2). Thus, if both (21) and (22) hold, then $u(a,0) > 0$, hence by A2 again, we have $u(z,0) \geq \min\{u(a,0), u(s,0)\} \geq 0$ at each $z \leq a$, showing that $x_0 \leq a \Rightarrow E[u(z,0)|x_0] \geq 0$. Since $b \geq 0$ and $u(\cdot,0)$ is decreasing on $\mathbb{R}_+$, (22) also implies that $u(z,0) < 0$ at each $z > b$, showing that $x_0 > a \Rightarrow E[u(z,0)|x_0] < 0$. ∎

Towards showing that (6) is sufficient for $x_0^*$ being a best response, we first obtain the following lemma.

**Lemma 4** *Take any $\bar{z} \in Z$ with $R(a, \bar{z}) \leq 0$. Then, $R(a, \cdot)$ is decreasing on $[\bar{z}, \infty)$, and therefore $R(a, z) < 0$ at each $z > \bar{z}$.*

**Proof.** Take any $z \in (a, \infty)$ with $R(a, z) \leq 0$. We **claim** that $R_2(a, z) < 0$. Since this will be true for arbitrary $z' \in (a, \infty)$ with $R(a, z') \leq 0$, this will imply that $R(a, z') < 0$ (and hence $R_2(a, z') < 0$) thereafter, which will prove our Lemma. In order to prove our **claim**, we note that, since $R(a, z) = w(a, z) - qw(z, z) \leq 0$, we have $qw(z, z)/w(a, z) \geq 1$. Since $w$ is log-super-modular (i.e., $\partial^2 \log\left(w\left(x, z\right)\right)/\partial x \partial z > 0$), $\frac{\partial \log(w(x,z))}{\partial z} = \frac{w_2(x,z)}{w(x,z)}$ is increasing in $x$, and hence we have

$$\frac{w_2(a, z)}{w(a, z)} < \frac{w_2(z, z)}{w(z, z)} \leq \frac{w_2(z, z)}{w(z, z)} \frac{qw(z, z)}{w(a, z)} = \frac{qw_2(z, z)}{w(a, z)}.$$

Thus $w_2(a, z) < qw_2(z, z)$, and therefore we have $R_2(a, z) = w_2(a, z) - qw_2(z, z) < 0$, proving our Lemma. ∎

Since $R(a, a) = \delta w(a, a) > 0$, Lemma 4 implies that Equation (6) is sufficient for $x_0^*$ to be a best response. (Recall that $\delta = 1 - q$.) To see this, note that, if we had $R(a, z) \leq 0$ at any $z \in [a, b)$, then by Lemma 4 we would also have $R(a, b) < 0$, which is false by definition. Hence, we need to have $R(a, z) > 0$ at each $z \in [a, b)$, which in turn implies that $x_0^*(z) = a$ is a best response at each $z \in [a, b)$. On the other hand, by the second part of our Lemma 4, since $R(a, b) = 0$, we have $R(a, z) < 0$ at each $z > b$, which implies that now $x_0^*(z) = z$ is a best response, showing that $x_0^*$ is a best response.

**Completing the description of the equilibrium.** We now specify the voters' behavior at $t = 0$. If IP of type $z$ comes to power at $t = 0$, the benefit for a voter at $y$ will be

$$U^0(z,y) = \begin{cases} (1+\delta)u(z,y) & \text{if } z < a, \\ u(a,y) + \delta u(z,y) & \text{if } a \leq z \leq b, \\ u(z,y) & \text{if } b < z. \end{cases} \tag{23}$$

The expected benefit will be

$$E[U^0(\cdot,y)] = [F(b)-F(a)]u(a,y) + \int_s^a u(z,y)f(z)dz + \int_b^\infty u(z,y)f(z)dz + \delta \int_s^b u(z,y)f(z)dz. \tag{24}$$

We assume that $E[u(z,0)] < 0$. Thus, if IP does not come to power at $t = 0$, it loses the election at $t = 1$. Therefore, in equilibrium a voter at $y$ will vote for IP if and only if $E[U^0(\cdot,y)] \geq 0$. IP wins the election at $t = 0$ if and only if voters at median vote for IP, i.e., $E[U^0(\cdot,0)] \geq 0$.

**Converse** We now show that all SPE are as described above. Fix any equilibrium $e \in SE(q)$ with first-period policy $\tilde{x}_0$. We will show that there exists $a \in Z$ such that IP wins the next election if $\tilde{x}_0(z) \leq a$, and loses if $\tilde{x}_0(z) > a$; therefore $\tilde{x}_0 = x_0^*$ for some $(a,b) \in SEP(q)$. Write $A$ for the set of policy levels $x$ such that IP wins the next election if it implements $x$ at time 0. The expected benefit of IP is

$$W(x,z) = \begin{cases} w(x,z) + \delta w(z,z) & \text{if } x \in A, \\ w(x,z) & \text{otherwise.} \end{cases}$$

We first observe some basic properties of $e$:

**Lemma 5** (1) $(A \cap Z) \cup \{s\}$ is closed. (2) $A \cap Z \neq \emptyset$. (3) $\tilde{x}_0(z) = z$ for each $z \in A \cap Z$. (4) $Z \backslash A \neq \emptyset$.

**Proof.** (1) Otherwise, we would have $z_n \rightarrow z$ for some sequence with $z_n \in A \cap Z$ and $s < z \notin A$. But then the best-response correspondence would be empty at $z$. (2) Otherwise we would clearly have $\tilde{x}_0(z) \equiv z$, which would imply that $0 \in A$. (4) Otherwise, we would have $\tilde{x}_0(z) \equiv z$ by part 3, and thus $z \notin A$ for each $z > \underline{b}$. ∎

The **Intuitive Criterion** for our game is defined as follows. For every $x \in X$ and $z \in Z$, we define

$$I(x,z) = w(x,z) + \delta w(z,z) - W(\tilde{x}_0(z),z).$$

Note that $I(x,z)$ is the best increment IP of type $z$ can get by implementing $x$ at $t = 0$. Take any $x \in Z \backslash \tilde{x}_0(Z)$, which is not implemented in equilibrium, and therefore by Lemma 5.3, $x \notin A$. We write $\tilde{Z}(x) = \{z \in Z | I(x,z) < 0\}$ for the set

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of types who would never want to deviate to $x$. Equilibrium $e$ *fails* the Intuitive Criterion if $u(z,0) > 0$ at each $z \in Z \backslash \tilde{Z}(x)$, i.e., IP wins the election when it implements $x$ no matter how voters interpret this as long as they are convinced that IP is not of some type $z$ in $Z \backslash \tilde{Z}(x)$.

**Lemma 6** *If $z^* \notin A$ for some $z^* \in Z$, then $z \notin A$ at every $z \geq z^*$.*

**Proof.** Take any $z^* \in Z \backslash A$, and write $\bar{z} = \min\{x \in A | x \geq z^*\}$ and $\underline{z} = \max\{x \in A | x \leq z^*\}$, where we use the convention that $\min \emptyset = \infty$ and $\max \emptyset = -\infty$. By Lemma 5.1, $\bar{z}$ exists and is greater than $z^*$. We will show that $\bar{z} = \infty$. Suppose that $\bar{z} < \infty$, i.e., our lemma is false. Then, $\bar{z} \in A \cap Z$. By supermodularity of $w$, there exists $z_0 \in (\underline{z}, \bar{z})$ such that

$$W(\underline{z}, z) \gtreqless W(\bar{z}, z) \iff z \lesseqgtr z_0. \tag{25}$$

Hence, $\tilde{x}_0^{-1}(\bar{z}) \subseteq [z_0, \infty)$. Likewise, there exists $z_1 \in (z_0, \bar{z})$, such that $\tilde{x}_0^{-1}(\bar{z}) \supseteq (z_1, \bar{z})$. But, since $\bar{z} \in A$, $E[u(z,0)|x_0(z) = \bar{z}] \geq 0$, and thus $u(z_0, 0) > 0$. Therefore, by continuity, there exists $z_2 > z_0$ such that

$$u(z,0) > 0 \qquad \forall z < z_2. \tag{26}$$

Moreover, by (25) and continuity, there exists $x \in (\underline{z}, z_0)$ such that $0 > w(x,z) + \delta w(z,z) - W(\bar{z}, z) \geq I(x,z)$ for each $z \geq z_2$, i.e., $\tilde{Z}(x) \supseteq [z_2, \infty)$. In summary, we have $x \in Z \backslash \tilde{x}_0(Z)$ (by definition) with $u(z,0) > 0$ for each $z \in Z \backslash \tilde{Z}(x)$ (by (26)), showing that $e$ fails the Intuitive Criterion, a contradiction. ∎

  **To complete the proof of the theorem,** define $a \equiv \sup A$. By Lemmas 5.4 and 6, $a \in Z$, and in fact, $A \cap Z = (s, a]$. Then, by Lemma 5.3, $x_0(z) = z$ whenever $z \leq a$. Since $a > s$, there exists (a unique) $b > a$ such that $R(a,b) = 0$. [If there were no such $b$, since $R(a,a) > 0$, by Lemma 4 we would have $R(a,z) > 0$ at each $z > a$, and hence we would have $x_0(z) = a$ at each $z > a$, therefore $E[u(z,0)|x_0(z) = a] = E[u(z,0)|z \geq a] \leq E[u(z,0)] < 0$, which contradicts that $a \in A$. By Lemma 4, $b$ must be unique.] Now by Lemma 4, we have $R(a,z) > 0$ at each $z \in [a,b)$ and $R(a,z) < 0$ at each $z > b$. Hence $x_0^*$ (defined in the statement of the theorem) is the only best response, and therefore $\tilde{x}_0 = x_0^*$ (see Footnote 8). ∎

## A.2   Proof of Lemma 1

Existence of $a^{IP}(b,q)$ follows from the continuity of $R$ in $a$ and the fact that $R(s,b) = -qw(b,b) < 0$. The uniqueness is by Lemma 4. The differentiability and the expressions for partial derivatives come form the implicit function theorem. We now derive the inequalities in (9) and (10). Since $a^{IP}(b,q) < b$, by A2, $w_1(a^{IP}(b,q),b) > 0$. By Lemma 4 (in the Appendix), $R_2(a^{IP}(b,q).b) < 0$. Hence (9) holds. Also, by A2, $w(b,b) > 0$, yielding (10).

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Consider (5): $u(b, 0) \leq 0$. We must have $b > 0$, and $u(\cdot, 0)$ is decreasing on this region. Hence we have (5) if and only if $b \geq \underline{b}$ where $\underline{b}$ is defined by $u(\underline{b}, 0) = 0$. Now, consider (4): $E[u(z, 0)|a \leq z \leq b] \geq 0$. Given any $(a, b)$ and $(a', b')$ with $(a, b) \geq (a', b') \geq (s, \underline{b})$, if $(a, b)$ satisfies (4), so does $(a', b')$.[10] Therefore, given the fact that $(a, b) \geq (s, \underline{b})$, the set of parameters that satisfy (4) is the region under the curve $MVC$. Since $a^{IP}$ is strictly increasing in $b$, the graph of $a^{IP}$ intersects the curve $MVC$ at a unique point $(\overline{a}(q), \overline{b}(q))$, which is the upper bound of the equilibrium parameters.

To see that $\overline{b}$ is decreasing in $q$, take any $q, q' \in (0, 1)$ with $q > q'$. Since $a^{IP}$ is increasing in $q$, $a^{IP}(\overline{b}(q), q) > a^{IP}(\overline{b}(q), q')$, hence $(a^{IP}(\overline{b}(q), q'), \overline{b}(q))$ is under the curve $MVC$, showing that $\overline{b}(q') \geq \overline{b}(q)$. ∎

## A.3   Proof of Theorem 6

The proof consists of three steps.

*Step 1* (Construction of $(e^*, q^*)$): For each $q \in (0, 1)$, we have maximal equilibrium $(\overline{a}(q), \overline{b}(q))$ — at the intersection of $MVC$ and the graph of $a^{IP}(\cdot, q)$. Since $MVC$ is connected, $\lim_{q \to 0} \overline{a}(q) = \underline{b} > 0$, and $\lim_{q \to 1} \overline{a}(q) = s < 0$, there exists $q^* \in (0, 1)$ such that $\overline{a}(q^*) = a^* \equiv 0$. We have $b^* = \overline{b}(q^*)$, where

$$E[u(z, 0)|0 \leq z \leq b^*] = 0. \tag{27}$$

*Step 2* (Existence of $\tilde{b} \leq b^*$): For any $z$ and $z' < z$, in equilibrium $\tilde{e}$, if IP wins the next elections when its type is $z$, it wins the next elections if its type is $z'$, too. This is because if IP wins at $z$, we must have $w(\tilde{x}_0(z), z) - \tilde{q}(\tilde{x}_0(z)) w(z, z) \geq 0$, and by Lemma 4, we have $w(\tilde{x}_0(z), z') - \tilde{q}(\tilde{x}_0(z)) w(z', z') \geq 0$, showing that IP prefers implementing $\tilde{x}_0(z)$ to losing the next election at $z'$. Define $\tilde{b}$ as the supremum of $z$'s at which IP wins the next elections in equilibrium $\tilde{e}$. We have just established that IP wins the next election if $z < \tilde{b}$, and it loses when $z > \tilde{b}$. Now for any $z < \tilde{b}$, the median voter votes for IP when he observes $\tilde{x}_0(z)$; hence $E[u(z, 0) : x_0 = \tilde{x}_0(z)] \geq 0$. Integrating both sides over $x_0 < \tilde{x}_0(\tilde{b})$, and observing that $\Pr(z < 0) = 0$, we obtain that $E\left[u(z, 0) : 0 \leq z < \tilde{b}\right] \geq 0$. By (27), this yields $\tilde{b} \leq b^*$.

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[10] We have

$$\int_{a'}^{b'} u(z, 0) f(z) dz = \int_{a'}^{a} u(z, 0) f(z) dz + \int_{a}^{b} u(z, 0) f(z) dz - \int_{b'}^{b} u(z, 0) f(z) dz \geq \int_{a}^{b} u(z, 0) f(z) dz.$$

Since $a' \geq s$, $u(z, 0) \geq 0$ at each $z \in [a', a]$, and since $b' \geq \underline{b}$, $u(z, 0) \leq 0$ at each $z \in [b', b]$, yielding the inequality above.

*Step 3* (Main): Compute that

$$U_0\left(e^*, q^*\right) = \int_0^{b^*} u\left(0,0\right) dF\left(z\right) + \left(1 - q^*\right) \int_0^{b^*} u\left(z,0\right) dF\left(z\right) + \int_{b^*}^{\infty} u\left(z,0\right) dF\left(z\right)$$

$$= \int_0^{b^*} u\left(0,0\right) dF\left(z\right) + \int_{b^*}^{\infty} u\left(z,0\right) dF\left(z\right),$$

where the last inequality is by (27). Using (27) one more time, we compute that

$$U_0\left(\tilde{e}, \tilde{q}\right) = \int_0^{\tilde{b}} u\left(\tilde{x}_0\left(z\right),0\right) dF\left(z\right) + \int_0^{\tilde{b}} \left(1 - \tilde{q}\left(\tilde{x}_0\left(z\right)\right)\right) u\left(z,0\right) dF\left(z\right) + \int_{\tilde{b}}^{\infty} u\left(z,0\right) dF\left(z\right)$$

$$= \int_0^{\tilde{b}} u\left(\tilde{x}_0\left(z\right),0\right) dF\left(z\right) - \int_0^{\tilde{b}} \tilde{q}\left(\tilde{x}_0\left(z\right)\right) u\left(z,0\right) dF\left(z\right) + \int_{b^*}^{\infty} u\left(z,0\right) dF\left(z\right).$$

Hence,

$$U_0\left(e^*, q^*\right) - U_0\left(\tilde{e}, \tilde{q}\right) = \int_0^{\tilde{b}} \left[u\left(0,0\right) - u\left(\tilde{x}_0\left(z\right),0\right)\right] dF\left(z\right) + \int_{\tilde{b}}^{b^*} u\left(0,0\right) dF\left(z\right)$$

$$+ \int_0^{\tilde{b}} \tilde{q}\left(\tilde{x}_0\left(z\right)\right) u\left(z,0\right) dF\left(z\right).$$

Since $\tilde{b} \leq b^*$ and $u\left(0,0\right) \geq u\left(\cdot,0\right)$, the first two terms are clearly non-negative. It thus suffices to show that $I \equiv \int_0^{\tilde{b}} \tilde{q}\left(\tilde{x}_0\left(z\right)\right) u\left(z,0\right) dF\left(z\right) \geq 0$. But at each $x_0 \in \tilde{x}_0([0,\tilde{b}))$, the median voter votes for IP when he observes that $\tilde{x}_0\left(z\right) = x_0$, and hence

$$\int_{\tilde{x}_0(z)=x_0} \tilde{q}\left(\tilde{x}_0\left(z\right)\right) u\left(z,0\right) dF\left(z\right) \geq 0.$$

By integrating both sides over $x_0$, we obtain $I \geq 0$,[11] completing the proof. ∎

## A.4   Proof of Lemma 2

The function $W^0(\cdot, y; a, b, \delta)$ has three pieces — on $(-\infty, a)$, on $[a, b]$, and on $(b, \infty)$. Clearly, each piece has the same shape as $w\left(\cdot, y\right)$; a single-peaked function with maximum at $y$. Moreover, at $z = a$, $W^0(\cdot; a, b, \delta)$ is continuous. Therefore we only need to check that $W^0(b, y; a, b, \delta) \geq \lim_{z \downarrow b} W^0(z, y; a, b, \delta) \equiv w(b, y)$ if and only if $y \leq b$. [To see that this is sufficient consider the case $z' > b > z'' > y$. Then, $W^0(z', y) < \lim_{z \downarrow b} W^0(z, y) \leq W^0(b, y) < W^0(z'', y)$. The other cases are checked similarly.] To this end, define the function $r : Z \to \mathbb{R}$ by setting

$$r(y) = W^0(b, y; a, b, \delta) - \lim_{z \downarrow b} W^0(z, y; a, b, \delta) = w(a, y) - qw(b, y) \qquad (28)$$

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[11] More formally, we first assume that $\tilde{x}_0$ is simple in this region, i.e., $\tilde{x}_0([0, \tilde{b})) = \{x^1, \ldots, x^n\}$. Writing $Z_k = \tilde{x}_0^{-1}\left(\{x^k\}\right)$, we obatin $I = \sum_{k=1}^n \int_{Z_k} \tilde{q}\left(\tilde{x}_0\left(z\right)\right) u\left(z,0\right) dF\left(z\right) = \sum_{k=1}^n \tilde{q}\left(x^k\right) \int_{Z_k} u\left(z,0\right) dF\left(z\right) \geq 0$. We then apply the usual machinery.

at each $y \in Z$. We will show that, if $r(y) \leq 0$ at any $y \in Z$, then $r'(y) < 0$. This will imply that $r(y') < 0$ at each $y' > y$. Since $r(b) = R(a,b) = 0$, this will complete the proof. Assume that $r(y) \leq 0$ at some $y \in Z$. Then, $\frac{qw(b,y)}{w(a,y)} \geq 1$. Moreover, since $\log(w(x,y))$ is super-modular (by A1), $\partial \log(w(x,y))/\partial y = \frac{w_2(x,y)}{w(x,y)}$ is increasing in $x$. Hence, we have

$$\frac{w_2(a,y)}{w(a,y)} < \frac{w_2(b,y)}{w(b,y)} \leq \frac{w_2(b,y)}{w(b,y)} \frac{qw(b,y)}{w(a,y)} = \frac{qw_2(b,y)}{w(a,y)}.$$

Thus $w_2(a,y) < qw_2(b,y)$. Therefore, $r'(y) = w_2(a,y) - qw_2(b,y) < 0$. ∎

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